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ARTICLE

# Transition to clean technologies and the impact of industrial non-compliant behavior

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**Abstract**

In this paper, we adopt an evolutionary model to describe the coevolution of technological transition and pollution in a country, where the choice of technology does not only give firms access to cleaner (but more expensive) or dirtier (cheaper and illegal) forms of production, but also access to social groups and information. Firms' activity may be harmful to the environment and, due to the existence of ambient pollution charges, economic activity is affected by the level of pollution in the country. Our analysis describes how the evolution of the transition to clean technology and pollution generates a rich set of possible equilibria, which include stable pure strategies (where all firms choose the same technology) and inner equilibria (where both technologies could be adopted in the long run). We also observe more complex behavior and coexistence of different attractors as well as highlight the importance of initial conditions and uncover how the regulator may face possible pollution traps.

**Keywords:** Technological transition; non-point source pollution; strategic interaction; asymmetric information; compliance

**JEL classifications:** D43; D83; O33; Q55; Q58

## 1. Introduction

Policymakers around the world are increasingly facing critical challenges related to reducing polluting activities, controlling climate change, and incentivizing a transition to cleaner forms of production. These challenges are addressed in the 17 Sustainable Development Goals identified and adopted by all United Nations Member States in 2015. Interestingly, the 2030 Agenda for Sustainable Development, of which the 17 goals are key elements, describes the interconnectedness of environmental sustainability and economic activities, and in particular highlights how the fulfillment of the goals could trigger positive dynamics for economic growth in developing and developed countries around the world.

Of course, the way the environment and economic activities interact is complex and circular (Levin and Xepapadeas (2021)) and requires governments to appreciate the coevolution of environmental and economic variables. At the same time, the evaluation of the impact and effectiveness of environmental and economic policies must take also into account how economic agents interact strategically at the micro-level and engage with the behavior of social groups at the macro-level. At a micro-level, for example, economic agents, such as firms (their shareholders and management), will have to take strategic decisions (including the choice of investing in green and sustainable technologies), taking into account the actions of competitors and the incentives of regulatory and fiscal policies. At a more macro level, these agents belong to social groups, engage with social norms, and acquire group-specific information. It follows that economic and environmental policies will influence the conduct of each individual agent which, in turn, will impact the macro features of the economy (for example, the transition to greener technologies) and the aggregate state of environmental conditions (for example, overall levels of national pollution and quality and stock of natural resources); these conditions, of course, will recursively affect the strategic decisions of individual agents.

In this paper, we develop an evolutionary model that describes the dynamic coevolution of technological transition and pollution, where individual firms in each period engage in strategic (quantity) competition with each other based on the particular technology at their disposal. The choice of technology does not only give firms access to cleaner/more expensive or dirtier/cheaper forms of production, but also access to social networks and information sharing. In particular, we focus our attention on a scenario in which clean technology is clearly identified and mandated by the government. Producers, however, have the opportunity, engaging in illegal activities (e.g. circumventing environmental standards, falsifying documents and dishonestly reporting false information, etc.), to adopt a dirtier and cheaper form of production. In choosing to engage in illegal activities, a firm faces the risk of being audited and to pay financial (and potentially social) penalties. At the same time, however, through the interaction with illegal practices (for example, the interaction with third parties that facilitate the falsification of documents or with providers of polluting and prohibited factors of production), a dishonest firm could also acquire access to information on the composition of the group of firms who have decided to act illegally; this information may be inaccessible to firms who decide to act legally and invest in green technology.

In this setup, it follows that a national regulator has to understand the effects that environmental policy and auditing may have on the individual conduct of firms, while taking into consideration the impact created on the dynamic paths of pollution and the adoption of sustainable technology, which in our case also corresponds to the prevalence of honest behavior in the economy overall. We describe a scenario in which a regulator has a preferred technology and cannot directly assess the impact that the output choices of individual firms have on the environment; instead, what can be observed is the aggregate level of national pollution. The regulator, therefore, can impose an environmental tax based on the current level of pollution in the system, regardless of the quantity choices of the firms.[1] An environmental agency, however, can observe the technological choices of firms after an audit and, if firms are found adopting a dirty technology, they could face a penalty; the expected cost associated with an audit would reduce the desirability to cheat and adopt a dirty technology.

We show that the interplay between different factors involved in the model generates various dynamic scenarios, including stable pure strategies (where all firms choose the same technology), inner equilibria (where both technologies could be adopted in the long run), and solutions of more complex form (cycles of low or high period, closed invariant curves, etc.). We observe coexistence of different attractors, which implies the possibility of pollution traps and accentuates the importance of initial conditions. Indeed, if the initial level of adoption of green technology is sufficiently high, then in the long run the system may escape a pollution trap, converging to an asymptotic state where all firms comply with environmental standards. We also highlight certain effects of increasing the number of competing firms. In particular, our initial insights suggest that higher level of competition would allow for less stringent regulation to lead the system to a good equilibrium, where all firms adopt a clean technology.

There are multiple strands of economic literature that address the issues that we study in our work. Several contributions take a macroeconomic perspective to study the relationship between environmental quality and (exogenous[2] or endogenous[3]) economic growth.[4]

There is also a vast body of literature that studies the interplay between economic activity and environmental protection from a micro-perspective. A considerable amount of attention has been dedicated to the static strategic decision of firms to invest in emission abatement efforts in the attempt to reduce the impact of environmental taxation. These include the study of firms' behavior under international competition (see Ulph (1996) and subsequent contributions), production differentiation and firms' locations (see Bárcena-ruiz and Garzón (2003) and Espinola-Arredondo and Zhao (2012)), strategic managerial delegation (see Bárcena-Ruiz and Garzón (2002), Roelfsema (2007) and Pal (2012)), partial cross-ownership (see Bárcena-Ruiz and Campo (2012)), optimal environmental taxation and strategic regulatory pre-commitment (see Ouchida and Goto (2016), Radi *et al.*, 2025)), just to mention a few. While duopolistic competition is normally assumed, there are also works that consider oligopoly and the role that the number of firms active in the market, a proxy for the degree of competition in a market, may have on the equilibrium levels of emissions and regulatory efforts. Lambertini et al. (2017) study a static Cournot oligopoly in which firms can decide to invest in abatement efforts to reduce emissions.[5] The authors consider, in particular, the possibility that abatement efforts (akin to R&D investments) can generate spillovers in favor of competitors; they show how that, if "green" innovation generates spillovers, in equilibrium it can be observed an inverted-U relationship between innovation and competition. More recently, Buccella et al. (2024), extended a previous work (Buccella et al. (2021)) introducing oligopolist competition and the *dynamic* decision of firms to invest in abatement or not, highlighting how different approaches to environmental policy would influence the long-term decisions of firms and, consequently, social welfare and the environment.

As mentioned above and in contrast to the literature discussed so far, the model proposed in this paper considers a situation in which firms do not face emission taxes nor contemplate explicit abatement investments. Instead, we consider the case of ambient charges, imposed to address non-point source pollution, and the possibility that some firms may engage in dishonest behavior. There are several works that assume static oligopolistic strategic behavior of firms facing ambient charges. See, for example, Ishikawa et al. (2019), Matsumoto and Szidarovszky (2021), and Matsumoto et al. (2023). While these works highlight the important role that strategic interdependence may have in defining how firms respond to environmental regulation, they do not address the dynamics of technological transition and the possibility of dishonest conduct of firms. From this point of view, our model is also related to the literature that studies how auditing and transparency can influence attitudes toward corporate social and environmental responsibility. If, due to social norms and environmental awareness of consumers, markets may reward corporate engagement in social and environmental efforts, firms could exploit forms of strategic behavior such as *greenwashing*,[6] i.e. selectively engaging in observable and salient activities while neglecting unobservable investments that might be, however, more effective (and potentially more expensive). The literature highlights how the information generated by the result of auditing efforts (e.g. forms of *naming and shaming*) and the regulatory requirements on transparency around social and environmental efforts of firms can influence the intrinsic and extrinsic incentives of firms and, ultimately, social welfare.[7]

In line with what we do in this paper, the dynamic interdependence between technological transition, economic performance and environmental quality can be studied adopting an evolutionary framework, where the adoption of a technology is determined by evolutionary selection and the selection mechanism is driven by the profitability of each production process. Market structure, type of competition, available technologies (including the possibility of purchasing carbon credits) and regulatory standards affect the profitability of different production processes; in turn, assuming that different technologies may have different levels of environmental sustainability, the technology selection process influences the dynamics of environmental quality which, ultimately, also may affect the profitability of firms when consumers' preferences and the availability of production input depend on environmental quality. Among the contributions in this branch of the literature, see Zeppini (2015),[8] Zhang and Li (2018), Zhang et al. (2019), and more recently Cavalli et al. (2024). In Zhang and Li (2018) the authors study the condition for cooperation among local governments and in Zhang et al. (2019) the choice between a green technology and the purchase of carbon credits is investigated. The framework in Cavalli et al. (2024) considers the effects of an ambient tax on technology selection and it is closest to the one we adopt here. A recurring message of these contributions is that evolutionary selection may generate multiple equilibria and nontrivial dynamics characterized by endogenous oscillations.

None of the contributions mentioned above explicitly considers the possibility that firms may adopt forms of dishonest behavior and exploit sources of asymmetric information.

The rest of the paper is organized as follows. In Section 2 we state the static model for the industry that includes the coexistence of firms adopting licit technology and others employing less expensive, but illegal technology. This model is then used in the evolutionary framework in Section 3, where the fraction of firms employing each technology is a dynamic variable. Section 4 introduces into the model the component of pollution that is produced by industrial activity and results in ambient taxation to firms by the regulator. Section 5 concludes. All proofs are collected in Appendix A.

## 2. The static industry model

Consider an oligopolistic market served by $n$ identical firms that produce a homogeneous good and face a linear (inverse) demand, $P = A - Q$, where $A > 0$ represents the reservation price and willingness to pay of consumers and $Q$ is the aggregate level of output supplied by the $n$ firms in the market.

Suppose that firms face a constant marginal cost equal to $c_h$, where $A > c_h > 0$. So far, we have described a standard Cournot setup and the well-known Nash equilibrium would be described by each firm producing $q^C = \frac{A-c_h}{1+n}$ and in aggregate $Q^C = nq^C = \frac{n(A-c_h)}{1+n}$.

Suppose now that a subset $n - m$, $2 \le m \le n$, of the firms active in the market considers the possibility of engaging with forms of illegal activities, e.g. circumventing regulatory standards. Ultimately, the illegal activities we are considering here would allow these $n - m$ companies to face a lower production cost $c_d$, with $0 \le c_d < c_h$, while still earning the same uniform price as the remaining $m$ firms who continue to operate legally.

Of course, criminal activity could be detected and sanctioned. Let us assume that the additional expected cost for a firm that operates illegally is given by $F = \alpha\theta q_i^2$, where $0 \le \alpha \le 1$ represents the probability of detection of illegal activities and $\theta > 0$ represents the extent of the sanction. Notice that the expected cost of being caught engaging in criminal activities is increasing (and quadratic) in the level of production chosen by the firm; this feature reflects the fact that the production of a larger quantity of output would imply a more significant regulatory infraction or, alternatively, a more obvious case that could be the focus of monitoring and detection.[9]

Operating dishonestly, in addition, may provide a source of asymmetric information. For example, in order to circumvent regulatory requirements and standards, firms may have to interact with illegal networks (including corrupt civil servants)[10] that could provide falsified documents and fake certification. This interaction could offer the opportunity to learn, in addition to ways to circumvent regulations, also information on the other competitors engaging in dishonest behavior. Here, we consider the possibility that every firm engaging in criminal activities is made aware of the number of dishonest firms, $n - m$. Instead, firms who decided to operate honestly (and therefore have not explored the advantages and risks of criminal activities and criminal networks) do not possess this information.

Suppose that both types of firms, $m$ operating legally (h-firms) and $n - m$ operating illegally (d-firms), make output decisions targeting the maximization of profits.[11] Specifically, each of the $m$ honest firms will choose its own quantity to maximize $\pi^h = q_i\left(A - q_i - Q_i - c_h\right)$, where $Q_i$ is the aggregate quantity produced by $n - 1$ firms, assuming that the rest of the market operates legally. Consequently, honest firms behave as standard Cournot competitors, choosing in equilibrium $q_h^\star = q^C = \frac{A-c_h}{1+n}$ and producing in aggregate $Q^h = mq_h^\star = \frac{m(A-c_h)}{1+n}$.

The behavior of the honest firms is internalized by the $n - m$ firms who engage in illegal activities. Each of the $n - m$ *dishonest* firms chooses output level to maximize

$$\pi^d = q_i\left(A - q_i - \frac{m(A - c_h)}{1+n} - QI_i - c_d\right) - \alpha\theta q_i^2,$$

where $QI_i$ is the aggregate quantity of the remaining $n-m-1$ firms operating illegally. At a Nash equilibrium, the quantity chosen by each firm operating dishonestly is then

$$q_d^\star(m) = \frac{c_h m - c_d(1+n) + A(1-m+n)}{(1+n)(1-m+n+2\alpha\theta)}$$

and the aggregate quantity provided by dishonest firms is

$$Q^d(m) = (n-m)q_d^\star = \frac{(n-m)(c_h m - c_d(1+n) + A(1-m+n))}{(1+n)(1-m+n+2\alpha\theta)}.$$

It is interesting to observe, especially in relation to the results reported in Section 4, that regardless of $m$, a dishonest firm will supply a larger (lower) level of output than an honest firm when $\alpha\theta < \tilde{\beta}$ (respectively $\alpha\theta > \tilde{\beta}$), where

$$\tilde{\beta} \overset{df}{=} \frac{(c_h - c_d)(1+n)}{2(A-c_h)} > 0 \tag{1}$$

In other words, if the expected punishment is sufficiently low, then dishonest firms will produce more in equilibrium than honest ones to exploit their cost advantage (including the expected punishment). Otherwise, taking into account the quadratic punishment in quantities, in equilibrium dishonest firms will produce less than honest ones.

## 3. Industry dynamics

We employ evolutionary dynamics with heterogeneous behavior to model the choices of honest and dishonest firms along the lines in Hommes et al. (2018), Kopel and Lamantia (2018), Lamantia et al. (2018) and Radi et al. (2021). A population of firms consists of honest h-firms and dishonest d-firms. At each period, $n$ firms are randomly selected to play the one-shot oligopoly game described in Section 2. The shares of h-firms and d-firms are adjusted over time according to expected profits and the Cournot game is repeatedly played with updated fractions of firm types. Depending on the behavior employed, the *expected average profit* of a firm that knows the population shares of honest $z$ and dishonest $1-z$ firms is computed. Notice that we are not attempting to study the way individual firms change their strategies over time; instead, in each period, $n$ firms from a large population are randomly selected to play a one-shot Cournot game, having chosen whether to operate honestly or not in the market. In the next period, a new set of $n$ firms will be selected to compete and play the same one-shot game. The expected average profit is the *probability weighted sum*, over all possible market compositions, of the profits realized in each particular scenario, with $k$ honest rivals and $n-k-1$ dishonest ones. Approximating the probability of selecting an h-firm by the current fraction $z$ of h-firms in the population, an **h-firm's expected profit** is given by

$$\mathbb{E}[\pi_h(z)] = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-k-1} q_h^\star \cdot (A - c_h - Q_h^\star), \tag{2}$$

where $Q_h^\star = (n-k-1)q_d^\star(k+1) + (k+1)q_h^\star$. Similarly, a **d-firm's expected profit** is given by

$$\mathbb{E}[\pi_d(z)] = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-k-1} q_d^\star(k) \left(A - c_d - Q_d^\star - \alpha\theta q_d^\star(k)\right), \tag{3}$$

where $Q_d^\star = (n-k)q_d^\star(k) + kq_h^\star$. Firms' outputs are at the Cournot-Nash equilibrium levels, but in each period the number of firms in the population what will adopt a specific behavior will update based on average profitability based on past performance, as specified below.

The adjustment of the share of firms employing a given behavior will be governed by a *replicator-like* equation and provides the source of dynamic behavior for the model.

### 3.1 The map

We assume discrete-time adjustments for the share of honest firms. The evolutionary equation defines the population state at period $z_{t+1}$ as a function of the current population state, $z_t$ and average profits $\mathbb{E}[\pi_h(z)]$ and $\mathbb{E}[\pi_d(z)]$. Several possible specifications of the form

$$z_{t+1} = f(z_t, G_n(z_t))$$

can be considered for modeling the evolution of firms' compliance driven by expected profit differences (also referred to as the *gain* function)

$$G_n(z) = \mathbb{E}[\pi_h(z)] - \mathbb{E}[\pi_d(z)]. \tag{4}$$

In general, such dynamics follow replicator-like patterns of evolution. Since changes in firm behavior require long adjustment times, we adopt discrete-time dynamics and in particular assume a **sluggish (exponential) replicator equation**, see Cabrales and Sobel (1992) and Lamantia et al. (2018), meaning that only a share $1-\delta$, $\delta \in [0,1]$, of the firms' population updates their behavior towards the more rewarding strategy (*asynchronous updating*):

$$\begin{aligned} z_{t+1} &= f_\delta(z_t) \\ &= \delta z_t + (1-\delta)\frac{z_t \exp(\phi\mathbb{E}[\pi_h(z_t)])}{z_t \exp(\phi\mathbb{E}[\pi_h(z_t)]) + (1-z_t)\exp(\phi\mathbb{E}[\pi_d(z_t)])} \\ &= \delta z_t + (1-\delta)\frac{z_t}{z_t + (1-z_t)\exp(-\phi G_n(z_t))}, \end{aligned} \tag{5}$$

where $G_n(z_t) = \mathbb{E}[\pi_h(z_t)] - \mathbb{E}[\pi_d(z_t)]$ measures the expected extra-profits of h-firms. Parameter $\phi > 0$ models firms' propensity in adopting a different type of behavior and is often referred to as the *intensity of choice*. In the following, we propose a local equilibrium stability analysis of this map and present the main dynamic scenarios of the model also through numerical examples. These insights will be useful when we extend the study to include the evolution of behavior coupled with pollution dynamics and the impact of ambient charges.

### 3.2 Equilibrium analysis

We begin the analysis of map (5) by summarizing the structural properties of its equilibria and their stability for a generic industry size $n$. To simplify the mathematical analysis, we introduce the aggregate parameters

$$a = A - c_h, \quad b = c_h - c_d, \quad \beta = \alpha\theta. \tag{6}$$

Parameter $a$ measures the maximum margin to honest firms, $b$ is the extra marginal cost for the technology used by h-firms and $\beta$ is the expected punishment rate for each unit of quantity produced by a d-firm. In terms of the new parameters $a$, $b$ and $\beta$, which are strictly positive numbers, for a given number $k$ of honest firms, the optimal choices become

$$q_h^\star = \frac{a}{n+1} \quad \text{and} \quad q_d^\star(k) = \frac{(n+1)(a+b) - ka}{(n+1)(2\beta + n - k + 1)}, \tag{7}$$

which implies the expected profits

$$\mathbb{E}[\pi_h(z)] = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-k-1} \frac{(2\beta+1)(n-k)a^2 - (n-k-1)(n+1)ab}{(n+1)^2(2\beta+n-k)}, \tag{8}$$

$$\mathbb{E}[\pi_d(z)] = \sum_{k=0}^{n-1} \binom{n-1}{k} z^k (1-z)^{n-k-1} \frac{(\beta+1)\big((n+1)(a+b)-ka\big)^2}{(n+1)^2(2\beta+n-k+1)^2}. \tag{9}$$

The structural properties of the equilibria of map (5) are as follows.

**Proposition 1.** *For map (5) the following statements hold:*

- ***Boundary equilibria.*** *The points $\bar{z}_0 = 0$ and $\bar{z}_1 = 1$ are equilibria for any parameter constellation;*
- ***Stability of boundary equilibria.*** *The equilibria $\bar{z}_0$ and $\bar{z}_1$ can lose stability only through a bifurcation with eigenvalue +1 and $\bar{z}_0$ is stable if $\frac{a}{b} < r_1(\beta, n)$, while $\bar{z}_1$ is stable if $\frac{a}{b} > r_2(\beta, n)$ with $r_i(\beta, n)$, $i = 1, 2$ given in (29), (31) (see Appendix A);*
- ***Inner equilibrium.*** *Any point $z_* \in (0, 1)$ such that $G_n(z_*) = 0$ is an inner equilibrium;*
- ***Stability of inner equilibrium.*** *The inner equilibrium $z_*$, if exists, is locally asymptotically stable whenever*

$$-\frac{1+\delta}{1-\delta} < 1 + \phi z_*(1-z_*)G_n'(z_*) < 1. \tag{10}$$

The structural properties of boundary equilibria in Proposition 1 derive from the replicator mechanism, in which, if a behavior is not assumed in the population, it cannot spread and remains absent. The stability properties of boundary equilibria $\bar{z}_0$ and $\bar{z}_1$ change through transcritical bifurcations whenever the point $z_*$ enters/leaves the interval (0, 1) and becomes feasible/unfeasible. Such equilibrium fraction $z_*$ is an equilibrium in the model if it is feasible and if the expected profits of the different available strategies are equal. In what follows, we may refer to the boundary equilibria $\bar{z}_0$ and $\bar{z}_1$, respectively, as *good* and *bad*, since the former describes a long run scenario where all firms adopt the regulatory standards of the clean technology and the latter describes a *pollution trap*.

In the next subsection, we incorporate the new parameters in the analysis and, for analytical tractability, we focus on the case $G_2(z)$.

### *3.3 The gain function in some particular cases*

The structure of the proposed model makes it possible to consider $n$, the level of industry competitiveness, as a parameter and to show insights into the impact of the level of competitiveness on the behavior of firms. In Section 4 we shall also report of competition influences the long run level of aggregate pollution. By construction, the gain function $G_n$ is a polynomial of degree $n-1$ in $z$, so in the case with two firms $G_2$ is linear. A full analysis on the role of $n$ seems worthy of a separate investigation given also the significant complexity. Therefore, in the following, we focus on the $n = 2$ case, which corresponds to a scenario in which industrial competition is purely duopolistic, leaving room for later developments with a higher value of $n$.

**Proposition 2** (**Properties of the gain function** $G_2(z)$ **and the map** (5)). *Consider the model with $n = 2$. The following statements hold:*

- ***Linearity.*** *The gain function $G_2(z)$ is linear in $z$ of the form $G_2(z) = c_1 z + c_0$.*

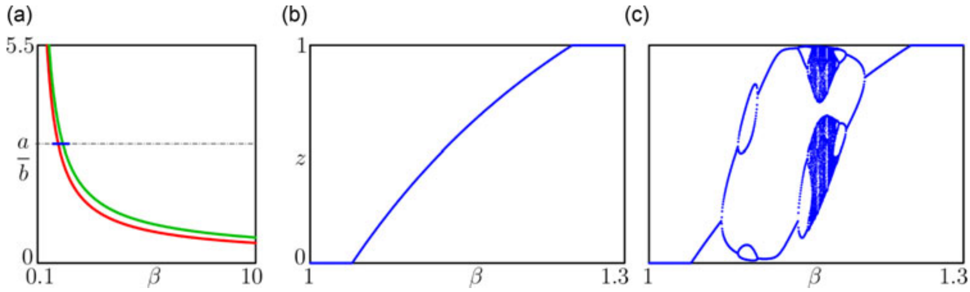


**Figure 1.** (a) The functions $r_1(\beta,2)$ (red) and $r_2(\beta,2)$ (green). (b, c) One-dimensional bifurcation diagrams versus $\beta$ along the path marked by the blue line in (a). The other parameters are $a=9$, $b=3$, and (b) $\phi=1$; (c) $\phi=13$.

- ***Inner equilibrium*** $z_*$. *For almost all parameter values, except for the case when $c_1=0$, the function $G_2(z)$ has a single zero given by*

$$z_* = 2\cdot\frac{2\beta(8\beta^2+19\beta+12)a^2-3(16\beta^2+36\beta+21)ab-18(\beta+1)^2b^2}{(2a\beta-3b)(8a\beta^2+14a\beta-12b\beta+6a-15b)}. \qquad (11)$$

- ***Existence of*** $z_*$. *If $r_1(\beta,2)<\frac{a}{b}<r_2(\beta,2)$, then $0<z_*<1$, and $z_*$ represents the unique inner equilibrium of the map $f_\delta$ in (5), where $r_1(\beta,n)$ and $r_2(\beta,n)$ are given in the Appendix.*
- ***Stability of Equilibria.***
  - *For $0<\frac{a}{b}<r_1(\beta,2)$, the point $\bar{z}_0$ is stable ($z_*<0$).*
  - *At $\frac{a}{b}=r_1(\beta,2)$, a transcritical bifurcation for $\bar{z}_0$ and $z_*$ occurs.*
  - *For $r_1(\beta,2)<\frac{a}{b}<r_2(\beta,2)$, the point $z_*\in(0,1)$ and is stable whenever $\mu(z_*)>-1$ (see (32), Appendix A.1). Otherwise, there exists another attractor located inside $(0,1)$.*
  - *At $\frac{a}{b}=r_2(\beta,2)$, a transcritical bifurcation for $\bar{z}_1$ and $z_*$ occurs.*
  - *For $\frac{a}{b}>r_2(\beta,2)$, the point $\bar{z}_1$ is stable ($z_*>1$).*

Figure 1 illustrates the dynamic scenarios of Proposition 1. In the panel a, we plot the functions $r_i(\beta,2)$, $i=1,2$. As one can see, both of them have the form of a hyperbola, and moreover, it holds that $r_1(\beta,2)<r_2(\beta,2)$ and $\lim_{\beta\to+\infty} r_i(\beta,2)=0$. Thus, with varying regulatory power $\beta$ one always observes the same bifurcation scenario. For smaller $\beta$ (when the ratio $\frac{a}{b}$ is located below the curve $r_1(\beta,2)$), the *bad* boundary equilibrium $\bar{z}_0$ is stable. An inner attractor exists for medium values of $\beta$ (the ratio is between the two curves). For $\beta$ being large enough, almost all orbits converge to the *good* boundary equilibrium $\bar{z}_1$. Whatever the ratio $\frac{a}{b}$, there always exists the level of $\beta$ that guarantees stability of $\bar{z}_1$. However, the smaller the ratio, the larger this required level of $\beta$.

Concerning the inner attractor, the following can be noted. As can be deduced from the definition of $z_*$ (11), its location depends only on $a$, $b$, and $\beta$, while its multiplier (see (32), Appendix A.1) depends in addition on $\delta$ and $\phi$. As numerical experiments show, for small $\phi$ the fixed point $z_*$ remains stable in the whole parameter range, for which $z_*$ is feasible. In Figure 1b, a 1D bifurcation diagram versus $\beta$ is plotted for $a=9$, $b=3$ and $\phi=1$. For larger $\phi$, with increasing $\beta$ above the threshold related to the first transcritical bifurcation of $\bar{z}_0$ and $z_*$, the fixed point $z_*$, being stable right after this bifurcation, later undergoes a flip bifurcation, leading to the appearance of a stable cycle of period two. When $\beta$ is increased further, there may follow a period-doubling cascade and then a period-halving cascade that ends by another (reverse) flip bifurcation of $z_*$. After this, the point $z_*$ remains stable until it becomes unfeasible (and unstable) due to the transcritical bifurcation with $\bar{z}_1$ (see Figure 1c for $\phi=13$).

Some of the bifurcation properties uncovered for $n=2$, can be generalized also for $n>2$. From Proposition 1 we know that $\bar{z}_0$ is stable if $\frac{a}{b}<r_1(\beta,n)$ and $\bar{z}_1$ is stable if $\frac{a}{b}>r_2(\beta,n)$. In Figure 2a we plot the functions $r_i(\beta,n)$ for $n=2$ (orange), $n=3$ (blue), $n=4$ (green), $n=5$ (red) with the

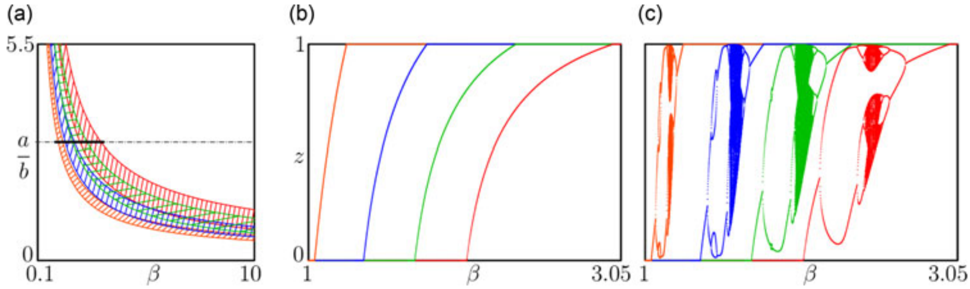


**Figure 2.** (a) The functions $r_i(\beta, n)$, $i = 1, 2$, $n = 2, 3, 4, 5$ (orange, blue, green, red). (b, c) The respective one-dimensional bifurcation diagrams versus $\beta$ along the path marked by the black line in (a). The other parameters are $a = 9$, $b = 3$, and (b) $\phi = 1$; (c) $\phi = 13$.

hatched area between these two curves. Similarly to the simplest case $n = 2$, it can be shown that $r_1(\beta, n) < r_2(\beta, n)$ for any $\beta$ and $n$ and both $r_i(\beta, n)$ tend to zero with $\beta \to \infty$ for a fixed $n$. This allows us to presume that for *any* $n$ the most probable bifurcation scenario versus $\beta$ is similar to that described for $n = 2$. In Figures 2b,c we plot 1D bifurcation diagrams versus $\beta$ for $n = 2, 3, 4, 5$ (by orange, blue, green, and red colors, respectively) and $\phi = 1, 13$ (with the same $a = 9$, $b = 3$). As one can see, the dynamics is qualitatively the same in all four cases, although the transition from *bad* equilibrium $\bar{z}_0$ to the inner attractor and then to the *good* equilibrium $\bar{z}_1$ occurs for larger $\beta$ with increasing $n$. Theoretically, for $n > 2$, when $G_n(z)$ is generically a polynomial of degree $n - 1$ in $z$ allowing for at most $n - 1$ zeros, two feasible inner fixed points $z_{*,1}$ and $z_{*,2}$ may appear due to a fold bifurcation. This may imply the coexistence of one boundary and one inner attractor or of two inner attractors (with the other inner attractor occurring after a transcritical bifurcation of $\bar{z}_0$ and $z_*$ being another zero of $G_n(z)$). However, we did not observe such situations in our numerical experiments. Apparently it happens for a rather limited parameter set.

From the dynamic point of view, given $\frac{a}{b}$, as the competitive pressure increases (higher $n$), it is necessary to increase the expected punishment $\beta$ to induce more compliance in the population. If choice intensity $\phi$ is sufficiently high, the system exhibits inner equilibrium instability and periodic/chaotic dynamics with the coexistence of the two firm behaviors for intermediate $\beta$. In any case, it remains valid that as $n$ increases, the level of punishment must increase in accordance to sustain compliance.

## 4. Pollution dynamics

Until now, we have not considered the impact of production on the environment. In this section, we will introduce pollution dynamics in the setup. Industry output levels generate a stock of pollution, which, in turn, affects profits through an environmental tax imposed on emissions.

We will reinterpret firm behavior in this new light: honest firms will follow environmental standards and adopt clean technologies, while dishonest firms will resort to dirty technologies. This approach will lead to a bi-dimensional map for industry behavior and pollution. In line with the non-point source pollution literature,[12] we assume that the government imposes an ambient charge, $\tau$ on firms based on total pollution in the market. An honest firm has a net profit

$$\tilde{\pi}^h = q_i \left(A - q_i - Q_i - c\right) - \tau p,$$

where $p$ is the current total pollution.[13] We write the stock of pollution at time $t$ as $p_t$. If caught, with probability $\alpha \in [0, 1]$, a dishonest firm pays a higher charge as a penalty for adopting a dirty technology. We have that

$$\tilde{\pi}^d = q_i \left(A - q_i - \frac{m(A-c)}{1+n} - QI_i - c_d\right) - \alpha\theta q_i^2 - ((1-\alpha)\tau + \alpha\tau_d)p$$

where $\tau_d > \tau > 0$ is the higher charge imposed to punish the adoption of a dirty technology. Pollution changes average profits through pollution taxation leading to the modified gain function:

$$\tilde{G}_n(p_t, z_t) = \mathbb{E}[\tilde{\pi}_h(z_t)] - \mathbb{E}[\tilde{\pi}_d(z_t)] = \mathbb{E}[\pi_h(z_t)] - \mathbb{E}[\pi_d(z_t)] + \alpha(\tau_d - \tau)p_t$$
$$= G_n(z_t) + \alpha(\tau_d - \tau)p_t, \qquad (12)$$

where $G_n(z_t)$ was defined in (4) of the previous section (for the model without pollution).

On average, if we assume that a unit of product by an h-firm generates a quantity of pollution normalized to 1, the total amount of pollution generated by the industry sector amounts to the quantity

$$QA_n(z) = \sum_{k=0}^{n} \binom{n}{k} z^k (1-z)^{n-k} \big[(n-k)(1+\eta)q_d^\star(k) + kq_h^\star(k)\big]. \qquad (13)$$

where $\eta \geq 0$ represents the extra contribution to the pollution by d-firms. Notice that equation (13) with $\eta = 0$ represents the average aggregate quantity provided in the market and is a polynomial of degree $n$ in $z$.

Pollution dynamics (see references) follows then the dynamic equation

$$p_{t+1} = \rho p_t + QA_n(z_t), \qquad (14)$$

where $\rho \in (0, 1)$ is the natural decay of pollution.

Equation (14), coupled with the evolutionary dynamics for the share of firms $z_t$, gives rise to a bidimensional map of the plane $F : \mathbb{R}_+^2 \ni (p_t, z_t) \to (p_{t+1}, z_{t+1}) \in \mathbb{R}_+^2$

$$p_{t+1} = \rho p_t + QA_n(z_t),$$
$$z_{t+1} = \delta z_t + (1-\delta)\frac{z_t}{z_t + (1-z_t)\mathrm{e}^{-\phi \tilde{G}_n(p_t, z_t)}}. \qquad (15)$$

In the following, we address the main dynamic properties of this bidimensional map.

To simplify analytic expressions, we again employ the aggregate parameters given in (6) and introduce a new one

$$\gamma \stackrel{df}{=} \alpha(\tau_d - \tau), \qquad (16)$$

which represents the expected extra pollution charges for adopting a dirty technology.

### *4.1 Equilibrium analysis*

Before studying the simple cases with a low value of $n$, we provide some general details concerning the fixed points. As in the one-dimensional model (5), the values $z = \bar{z}_0 = 0$ and $z = \bar{z}_1 = 1$ define fixed points with

$$\bar{p}_i = \frac{QA_n(i)}{1-\rho}, \quad i = 0, 1. \qquad (17)$$

Namely, the points $E_0(\bar{p}_0, 0)$ and $E_1(\bar{p}_1, 1)$, corresponding to the employment of only one pure strategy by firms, are always equilibria of the system. We refer to them as *boundary* equilibria as the $z$ component is equal to 0 or 1. Taking into account (13), the values $\bar{p}_i$ can be written in the general case as

$$\bar{p}_0 = n(1+\eta)q_d^\star(0) = \frac{n(1+\eta)(a+b)}{(2\beta+n+1)(1-\rho)}, \quad \bar{p}_1 = nq_h^\star = \frac{na}{(n+1)(1-\rho)}. \qquad (18)$$

Note that, with increasing $n$ and the other parameters fixed, the values $\bar{p}_i$, $i = 0, 1$, increase, but never exceed the limit values

$$\lim_{n\to\infty} \bar{p}_0 = \frac{(1+\eta)(a+b)}{1-\rho} \quad \text{and} \quad \lim_{n\to\infty} \bar{p}_1 = \frac{a}{1-\rho}.$$

Relative to their stability, the next result clarifies how these boundary equilibria can lose stability as parameters change.

**Proposition 3** (**Stability of boundary equilibria of map (15)**). *Consider map (15). Boundary equilibria* $E_0(\bar{p}_0, 0)$ *and* $E_1(\bar{p}_1, 1)$ *in (17) and (18), can lose stability only through a bifurcation with an eigenvalue* $+1$.

Let us now consider possible nontrivial fixed points of (15). From the first component of map $F$ we have

$$p_* = \frac{QA_n(z_*)}{1-\rho}. \tag{19}$$

Substituting (19) into the modified gain function with pollution (12) and recalling that inner equilibria of (15) satisfy an isoprofit condition, we obtain the following equation in $z_*$ only that characterizes the inner equilibria

$$\tilde{G}_n(p_*, z_*) = G_n(z_*) + \gamma p_* = G_n(z_*) + \gamma \frac{QA_n(z_*)}{1-\rho} \stackrel{df}{=} \hat{G}_n(z_*) = 0. \tag{20}$$

In other words, every zero $z_*$ of the function $\hat{G}_n(z)$ induces a fixed point of $F$, and this fixed point is feasible if $z_* \in (0, 1)$, since this also implies that $p_*$, defined in (19), is positive. As for the stability of these inner equilibria, it can be ascertained through the respective Jacobian given by

$$J(p_*, z_*) = \begin{pmatrix} \rho & QA'_n(z_*) \\ (1-\delta)\phi\gamma z_*(1-z_*) & 1 + (1-\delta)\phi z_*(1-z_*)G'_n(z_*) \end{pmatrix} \tag{21}$$

It is possible to show that the eigenvalues of the Jacobian can be also complex leading to a fixed point being a focus. Note that because of (20), the elements of $J(p_*, z_*)$ are all expressed in terms of $z_*$, but not of $p_*$.

### *4.2 Duopolistic industries*

Here we focus on the case of an economic system with industries characterized by a low level of competition, that is, with homogeneous sectors in which competition takes place between two firms at a time. This case is instructive as the main results can be obtained analytically. In particular, we show that the coexistence of boundary equilibria $E_0(\bar{p}_0, 0)$ and $E_1(\bar{p}_1, 1)$ both being stable can be achieved in the duopoly setting. Consequently, depending on the initial conditions, the system can be trapped in a *good* equilibrium with the predominance of clean firms or in a *bad* equilibrium with predominance of dirty firms.

**Proposition 4** (**Coexistence and stability of boundary equilibria**). *Consider map (15) with* $n = 2$. *For any given* $\beta$ *and* $b$, *an interval of the parameter* $a$ *exists such that two thresholds* $\gamma_1$ *and* $\gamma_2$ *are well defined, with* $\gamma_2 < \gamma_1$, *so that if* $\gamma \in (\gamma_2, \gamma_1)$ *then boundary equilibria* $E_0$ *and* $E_1$ *coexist and are stable.*

The previous result indicates that, under certain circumstances, the regulator's decisions on pollution control and/or taxation policies, represented in our model by parameter $\gamma$, all other economic variables in the model being equal, induce scenarios in which all firms employ only one type of technology. In other words, the economic system may be trapped in the use of dirty technologies or adopt only clean technologies depending on the ambient charges chosen by the regulator.

Note that for certain parameter constellations, it is $\bar{p}_1 > \bar{p}_0$, i.e. the level of pollution in case when all firms use clean technology is greater than the level of pollution when all firms resort to dirty production, which may appear to be rather unexpected at first sight. However, we emphasize that in the current model set up we fix the parameter $\eta = 0$ so that the pollution from production by either firm is equal. It means that we analyze the impact of pure competition between h- and d-firms without taking into account the effect of extra pollution caused by the prohibited technology. The situation with $\bar{p}_1 > \bar{p}_0$ occurs when $\beta > \tilde{\beta}$ (defined in (1)), i.e. when the expected cost of dishonest behavior is relatively high and an h-firm supplies a larger level of output than a d-firm. Then the higher level of pollution is not induced by the adoption of dirty technology, but it is the result of higher levels of production. We emphasize that the mentioned condition for $\beta$ also means that the parameter $a$ (that is the difference between $A$ and $c_h$) is large enough so that the price compensates the cost to the large extent even if the expensive technology is used. This is also the reason why h-firms tend to produce more. Since the aforementioned point requires deeper analysis, we focus on the specific ranges of parameters, which guarantee $\bar{p}_0 > \bar{p}_1$.

As for nontrivial fixed points $E^*(p_*, z_*)$, they can be at most two and are obtained from the roots of the quadratic equation

$$\hat{G}_2(z_*) = \nu d_2 {z_*}^2 + \left(c_1 + \nu d_1\right) z_* + c_0 + \nu d_0 = 0, \quad \nu = \frac{\gamma}{1-\rho}, \tag{22}$$

namely,

$$z_{*,\pm} = \frac{-c_1 - \nu d_1 \pm \sqrt{\left(c_1 + \nu d_1\right)^2 - 4\nu d_2 (c_0 + \nu d_0)}}{2\nu d_2}. \tag{23}$$

The coefficients $c_i$ and $d_i$, given respectively in (34) and in (42) (see Appendices A.2 and A.3), depend in a rather complex way on the parameters $a$, $b$, and $\beta$. Therefore, it is not easy to derive analytically the exact conditions for the points $E_{*,-}(p_{*,-}, z_{*,-})$ and $E_{*,+}(p_{*,+}, z_{*,+})$ to be feasible. We rely on numerical examples below to show the possible dynamic scenarios at hand.

#### *4.2.1 Example 1: A single attractor*

Let us fix the parameters as $a = 1.2$, $b = 2$, $\beta = 1$, $\delta = 0$, $\rho = 0.5$, $\eta = 0$. For this parameter constellation, by the results of Proposition 4, being $\gamma_1 < \gamma_2$, the boundary equilibria are not simultaneously stable. In Figure 3 we plot the asymptotic values of $p$ and $z$ versus $\gamma$ for $\phi = 1$ and $\phi = 15$. As one can see, for small $\phi$ the bifurcation scenario is analogous to the one described in Sec. 3.3 for the one-dimensional map. Namely, for small $\gamma < \gamma_1$ the fixed point $E_0$ (with all firms being dishonest) is stable, at which the pollution level is relatively high. At $\gamma = \gamma_1$, a transcritical bifurcation for the points $E_0$ and $E_{*,-}$ occurs, due to which the internal fixed point $E_{*,-}$ becomes stable and $E_0$ becomes a saddle. The point $E_{*,-}$ retains stability for $\gamma_1 < \gamma < \gamma_2$, until it undergoes another transcritical bifurcation colliding with $E_1$ at $\gamma = \gamma_2$. Finally, for $\gamma > \gamma_2$ the point $E_1$ (with all players being honest and relatively low pollution level) is stable. Note that with increasing $\gamma$ the pollution level $p$ gradually decreases with the number of honest producers. For larger $\phi$ (see panel b), the scenario is also similar to the one for the one-dimensional map $f_\delta$ (cf. Figure 1(b,c)), but now nontrivial dynamics occurs due to a Neimark-Sacker bifurcation of $E_{*,-}$ (instead of cascades of flip bifurcations as for the one-dimensional case without pollution). Notice that in this example dishonest firms will always produce more than honest firms in equilibrium, being $\beta < \tilde{\beta} = 2.5$ (see (1)). This explains why the equilibrium with all dishonest firms exhibits more pollution, despite being $\eta = 0$. The greater pollution is implied by the greater output of the dishonest firms because of the distortion on competition resulting from the use of the cheaper technology (which with $\eta = 0$ turns out to pollute as much as the other technology).

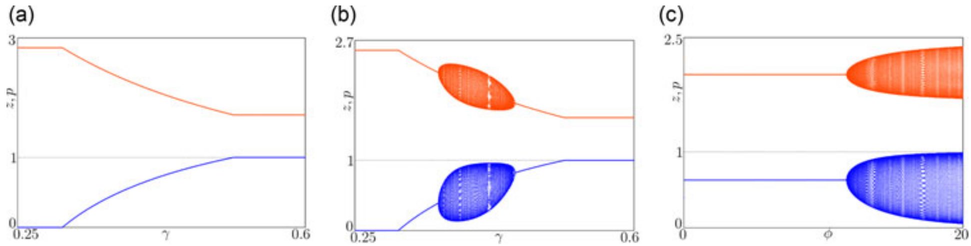


**Figure 3.** The asymptotic values of $p$ (orange) and $z$ (blue) (a) versus $\gamma$ for $\phi = 1$; (b) versus $\gamma$ for $\phi = 15$; (c) versus $\phi$ for $\gamma = 0.4$. The other parameters are $a = 1.2$, $b = 2$, $\beta = 1$, $\delta = 0$, $\rho = 0.5$, $\eta = 0$.

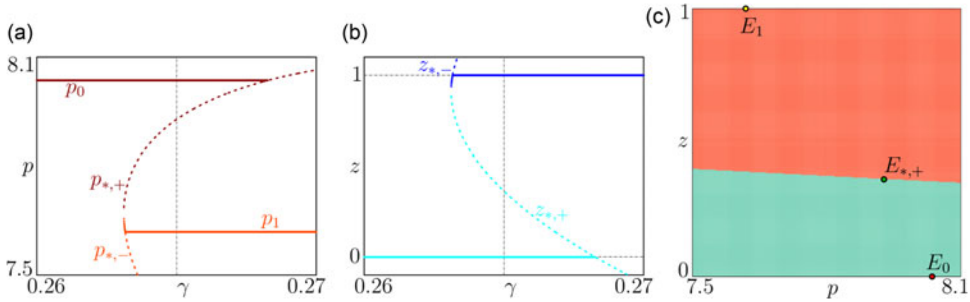


**Figure 4.** The asymptotic values of (a) $p$ (solid orange and dark red) and (b) $z$ (solid blue and cyan) versus $\gamma$. With the dashed lines of the respective colors $p_{*,\pm}$ and $z_{*,\pm}$ are shown. In (c), the basins of $E_0$ and $E_1$ for $\gamma = 0.265$ are shown in light blue and pink, respectively. The two basins are separated by the stable set of the saddle $E_{*,+}$. The other parameters are $a = 8$, $b = 1$, $\beta = 0.1$, $\phi = 1$, $\delta = 0$, $\rho = 0.3$, $\eta = 0$.

#### *4.2.2 Example 2: Coexistence of two pure strategies*

As established in Proposition 4, to observe a scenario where two boundary equilibria coexist and are simultaneously stable with $\bar{p}_1 < \bar{p}_0$, the value of $\beta$ must be sufficiently small. For that purpose, let us fix $\beta = 0.1$. Then, as explained in the proof of Proposition 4, the ratio $\frac{a}{b}$ has to be large enough and we set $a = 8$, $b = 1$. In this case, it is $\tilde{\beta} = 0.1875$ (see (1)) so, similarly to the previous example, dishonest firms will produce more than honest firms thus entailing more pollution even with $\eta = 0$. As it follows from (17), the value of $\rho$ influences the equilibrium pollution levels. Namely, the smaller $\rho$, the smaller $\bar{p}_i$, $i = 0, 1$. To keep them at moderate values, we assume $\rho = 0.3$. Under this setting, it is $\gamma_2 \approx 0.263$ and $\gamma_1 \approx 0.268$. In Figure 4 we plot the asymptotic values of $p$ (panel a) and $z$ (panel b) versus $\gamma$. As analytically established, for $\gamma < \gamma_1$ the point $E_0$ is stable, for $\gamma > \gamma_2$ the point $E_1$ is stable, and the two equilibria coexist and are stable for $\gamma_2 < \gamma < \gamma_1$. As for the nontrivial fixed points $E_{*,-}$ and $E_{*,+}$, they appear due to a fold bifurcation for some $\gamma = \bar{\gamma} < \gamma_2$. At this moment, both equilibria are feasible with $E_{*,-}$ being a stable node and $E_{*,+}$ being a saddle. Thus, there exists another very narrow interval of $\bar{\gamma} < \gamma < \gamma_2$, for which the stable $E_{*,-}$ and the stable $E_0$ coexist. Then with increasing $\gamma$, the point $E_{*,-}$ approaches the point $E_1$ and at $\gamma = \gamma_2$ the two points undergo a transcritical bifurcation, after which $E_{*,-}$ becomes a saddle and $E_1$ becomes stable. At $\gamma = \gamma_1$ the stable $E_0$ collides with the saddle $E_{*,+}$ and they switch stabilities due to a transcritical bifurcation. In Figure 4(c) we show a state space at $\gamma = 0.265$ with the two stable boundary equilibria $E_0$ and $E_1$, related to the pure strategies, basins of attraction of which (light blue and pink, respectively) are separated by the stable set of the saddle $E_{*,+}$.

From an economic standpoint, in this case, the system can have two asymptotic states, showing a *good* equilibrium such as $E_1$ with the predominant use of clean technology or it can exhibit a *pollution trap* such as $E_0$ in which firms systematically use the dirty technology. This is assuming all economic conditions of the system are fixed, but depending only on the initial conditions in terms of initial pollution and technology adoption. In particular, if the initial level of managerial

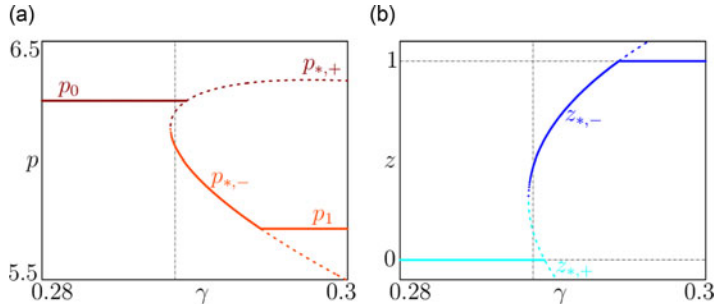


**Figure 5.** The asymptotic values of (a) $p$ (orange and dark red) and (b) $z$ (blue and cyan) versus $\gamma$. The parameters are $a = 6$, $b = 1$, $\beta = 0.1$, $\phi = 1$, $\delta = 0$, $\rho = 0.3$, $\eta = 0$.

culture sees a sufficiently high commitment to the use of clean technology, this allows the system to escape the pollution trap.

#### *4.2.3 Example 3: A different scenario of coexistence*

In this example, we let parameter $a$ decrease to $a = 6$. Now it is $\gamma_1 < \gamma_2$ and two nontrivial fixed points $E_{*,-}$ and $E_{*,+}$ occur both feasible due to a fold bifurcation for $\gamma = \bar{\gamma} < \gamma_1$, with $E_{*,-}$ being a stable node and $E_{*,+}$ being a saddle (see Figure 5). For $\bar{\gamma} < \gamma < \gamma_1$ the stable $E_{*,-}$ and $E_0$ coexist. At $\gamma = \gamma_1$ the points $E_{*,+}$ and $E_0$ undergo a transcritical bifurcation switching stabilities. After this, a single feasible attractor persists, being $E_{*,-}$ for $\gamma_1 < \gamma < \gamma_2$ and $E_1$ for $\gamma > \gamma_2$. This example shows the potential irreversibility associated with the action of the regulator. Imagine that the initial level of pollution penalty is high enough so that the dynamics converge to equilibrium $E_1$. This occurs for $\gamma_2 < \gamma$, see Figure 5. The regulator could decide to reduce $\gamma$ as it has no direct impact on the steady-state pollution level at $E_1$. This is true up to reductions in punishment at the $\gamma = \gamma_2$ level. In fact, for reductions in $\gamma$ below threshold $\gamma_2$, the level of pollution at equilibrium increases. Anyway, if the regulator reduces $\gamma$, but leaves it above the level $\gamma_1$, the reduction of the share of h-firms in the population and the increment in steady state pollution follow continuously the reduction of $\gamma$. However, if the regulator decided to reduce $\gamma$ below the $\gamma_1$ level, then there would be a jump discontinuity in the attractor of the system, which is now the equilibrium $E_0$ with more pollution and extensive use of dirty technology. A resetting of $\gamma$ to the previous level, even above $\gamma_1$, may not be sufficient now to restore the pre-existing condition of a high level of clean-tech firms and lower pollution.

In Figure 6 we plot a one-dimensional bifurcation diagram versus $\phi$ with the fixed $\gamma = 0.289$. At large values of $\phi$ the inner equilibrium $E_{*,-}$ undergoes a Neimark-Sacker bifurcation and an attracting invariant curve $\Gamma$ appears. The basins of the stable node $E_0$ and the curve $\Gamma$ are separated by the stable set of $E_{*,+}$ (see Figure 7(a)). When $\phi$ increases further, this stable set collides with the saddle fixed point $E_1$ inducing a homoclinic bifurcation, after which the basin of $\Gamma$ shrinks dramatically (see Figure 7(b)). For larger $\phi$, the invariant curve $\Gamma$ disappears due to a contact with the boundary of its basin, and a single boundary attractor $E_0$ remains.

### ***4.3 Insights with higher competition***

In the previous section, we assumed that firms typically compete duopolistically. Here we try to understand what possible effects can be induced by the presence of more competition and present some details in the case where competitions in various industries are structured in triopolies. As we shall see, the results are similar to those with two firms, but some additional insights can be obtained from this analysis.

In the case $n = 3$, the gain function $\tilde{G}_3(p, z)$ is quadratic with respect to $z$ and the aggregate output $QA_3(z)$ is cubic. The coefficients of these functions depend again on the parameters of the map

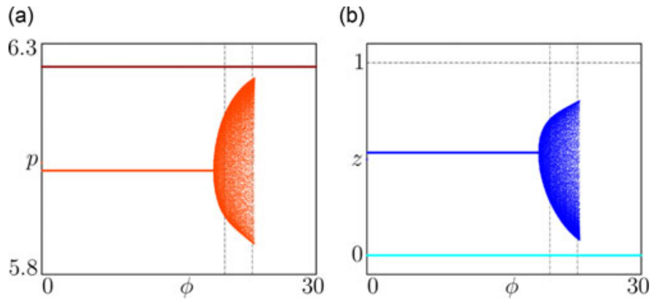


**Figure 6.** The asymptotic values of (a) $p$ (orange and dark red) and (b) $z$ (blue and cyan) versus $\phi$. The other parameters are $a = 6$, $b = 1$, $\beta = 0.1$, $\gamma = 0.289$, $\delta = 0$, $\rho = 0.3$, $\eta = 0$.

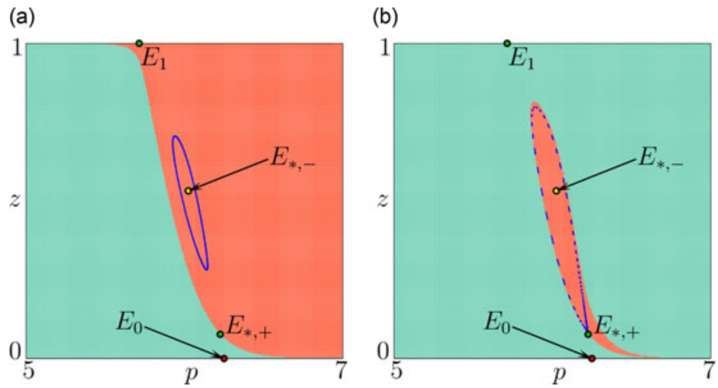


**Figure 7.** The basins of $E_0$ and $E_{*,-}$ are shown in light blue and pink, respectively, for (a) $\phi = 20$; (b) $\phi = 23$. The two basins are separated by the stable set of the saddle $E_{*,+}$. The other parameters are $a = 6$, $b = 1$, $\beta = 0.1$, $\gamma = 0.289$, $\delta = 0$, $\rho = 0.3$, $\eta = 0$.

in a complex way, and this makes it impossible to derive analytically the values $z_*$ and $p_*$ related to nontrivial fixed points. Now we are mostly restricted to the numerical analysis and we present below two examples of possible dynamic scenarios with industries having three competitors.

We can observe bifurcation scenarios that are very similar to those presented in Sec. 3.3, with either a single attractor or the coexistence of two different attractors. The former case is illustrated by Figure 8 (cf. Figure 3), where the one-dimensional bifurcation diagram is shown versus $\gamma$ for small (panel a) and large (panel b) values of $\phi$ and versus $\phi$ for a medium $\gamma$ (panel c). Similarly to Example 1 of Sec. 3.3, for smaller $\gamma < \bar{\gamma}_1$ the point $E_0$ is stable, for larger $\gamma > \bar{\gamma}_2$ the point $E_1$ is stable, while for medium $\bar{\gamma}_1 < \gamma < \bar{\gamma}_2$ the inner attractor exists (the values of $\bar{\gamma}_1$ and $\bar{\gamma}_2$ can be obtained explicitly). Note that in comparison to the case $n = 2$, the transition from the boundary equilibrium $E_0$ (pure dishonest strategy) to the inner attractor and further to the second boundary equilibrium $E_1$ (pure honest strategy) occurs for smaller values of $\gamma$. It seems that with increasing competition less stringent regulation in terms of extra pollution charges for adopting a dirty technology is needed to lead the system to a good equilibrium.[14] Likewise in the case $n = 2$, for small $\phi$ the inner attractor is a fixed point $E^*$, while for larger $\phi$ a closed invariant curve (which is born due to a Neimark-Sacker bifurcation) exists for medium values of $\gamma$. This closed invariant curve disappears due to another (reverse) Neimark-Sacker bifurcation when $\gamma$ approaches $\bar{\gamma}_2$. Note also that for $n = 3$ the critical value of $\phi$ (with a fixed $\gamma$), at which $E^*$ undergoes a Neimark-Sacker bifurcation, is slightly larger than the respective value for the case $n = 2$. This suggests that increasing $n$ may introduce an element of stability in the system, for a given $\phi$, though this point still requires additional analysis.

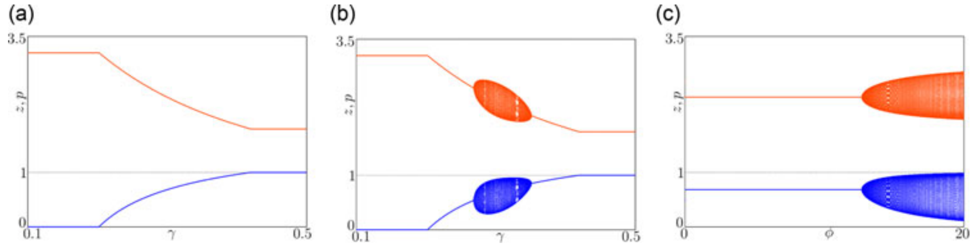


**Figure 8.** The asymptotic values of $p$ (orange) and $z$ (blue) (a) versus $\gamma$ for $\phi = 1$; (b) versus $\gamma$ for $\phi = 15$; (c) versus $\phi$ for $\gamma = 0.3$. The other parameters are $a = 1.2, b = 2, \beta = 1, \delta = 0, \rho = 0.5, \eta = 0$.

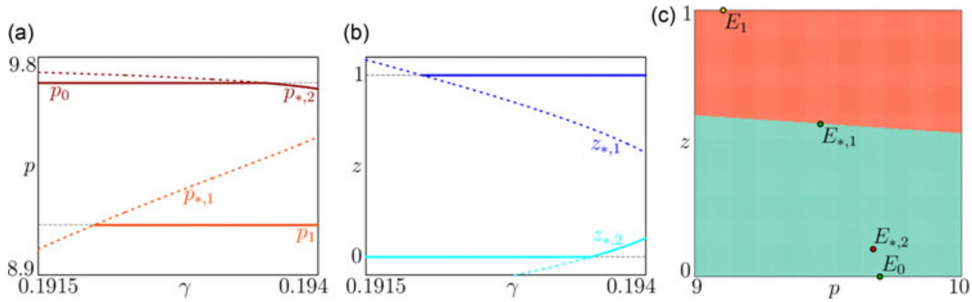


**Figure 9.** The asymptotic values of (a) $p$ (solid orange and dark red) and (b) $z$ (solid blue and cyan) versus $\gamma$. With the dashed lines of the respective colors $p_{*,i}$ and $z_{*,i}$, $i = 1, 2$, are shown. In (c), the basins of $E_{*,2}$ and $E_1$ for $\gamma = 0.194$ are shown in light blue and pink, respectively. The two basins are separated by the stable set of the saddle $E_{*,1}$. The other parameters are $a = 8.5, b = 1, \beta = 0.1, \phi = 1, \delta = 0, \rho = 0.3$.

For small values of $\beta$, one can observe a bifurcation scenario similar to the one described in Example 2 of the previous section. In Figure 9 (cf. Figure 4) we plot the asymptotic values of $p$ (panel a) and $z$ (panel b) versus $\gamma$ for $\beta = 0.1$, $b = 1$ and $a = 8.5$. For such a parameter constellation, it holds that $\bar{\gamma}_2 < \bar{\gamma}_1$ and coexistence of stable boundary equilibria occurs. Hence, for the medium values $\bar{\gamma}_2 < \gamma < \bar{\gamma}_1$ both boundary equilibria $E_0$ and $E_1$ are stable, and their basins of attraction are separated by a stable set of the inner saddle $E_{*,1}$. At $\gamma = \bar{\gamma}_1$, the point $E_{*,2}$ enters the feasible region and becomes stable due to the transcritical bifurcation. For $\bar{\gamma}_1 < \gamma < \hat{\gamma}$, with $\hat{\gamma}$ being the value of the fold bifurcation for $E_{*,1}$ and $E_{*,2}$, one observes coexistence of stable $E_1$ and $E_{*,2}$ (see Figure 9c).

This example suggests that in the presence of higher levels of competition, it might be easier to observe situations similar to the pollution trap discussed for the duopoly case, but with stable equilibria not in pure strategies. In other words, pollution traps could be compatible with a strictly positive probability of adopting clean techniques, as in the case of the equilibrium $E_{*,2}$ in the example of Figure 9c. We leave a more detailed analysis on the role of increasing competition (higher $n$) to further research.

## 5. Conclusions

In this paper, we described the insights of an evolutionary model that considered the dynamic coevolution of technological transition and pollution. For simplicity, the industrial economic sectors have been assumed to be homogeneous and the level of competition was summarized by the number of agents competing, drawn from large populations of firms. Thus, the industrial sector could be viewed as an aggregation of individual sectors. In particular, we focused our attention on a scenario that could generate possible forms of asymmetric information. In choosing to engage in illegal activities, including adopting forbidden and dirtier technologies, a firm faces the risk of being audited and paying financial (and potentially social) penalties; at the same time, however, a dishonest firm could also acquire access to information that may be inaccessible to firms who decide to act legally and invest in green technology.

We showed that the interplay between different factors involved in the model generates a rich set of possible asymptotic dynamics, which include stable pure strategies (where all firms choose the same technology), inner equilibria (where both technologies could be adopted in the long run), and more complex solutions. We observed the coexistence of different attractors and highlighted the importance of initial conditions, which may drive the regulator to face possible pollution traps. In these cases, the same economic system could have two very different outcomes in terms of long run pollution based on the initial levels of pollution and propensity of firms to adopt clean technologies. This highlights the crucial role of regulatory policy to ensure that economic activity is not locked into a pollution trap. We also pointed out the effect of an increase in the degree of competition, described by the number of firms active in the economy, which seems to indicate that less stringent environmental regulation may be needed in order to lead the system to a good equilibrium, where all firms adopt a green technology.

There are a few venues for further research that would provide additional interesting insights. Allowing illegal technologies to have a greater effect on the creation of pollution compared to cleaner ones would expand the range of feasible parameter sets and provide additional realistic scenarios to be studied. In addition, we have not considered the possibility that pollution may have a direct effect on the efficiency and production costs of firms. Such an externality would reinforce the impact that pollution has on the production choices of firms via the ambient charges that we have considered here. Finally, it may be interesting to introduce explicitly the role that reputation and social costs may have on firms' decisions and the possibility for regulators to enhance environmental compliance by coupling financial penalties with social instruments such as nudges, naming and shaming, etc. A welfare analysis of these extensions may also provide additional interesting insights.

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## Notes

**1** This environmental phenomenon is often referred to as *non-point source pollution* and *ambient charges* describe the taxation connected to the aggregate level of pollution. See Segerson (1998). For a description of the forms of regulation of point-source pollution, see Xepapadeas (2011).

**2** In these models, pollution is considered as a negative externality on households' utilities. See, for example, Antoci et al. (2011) and Antoci et al. (2021); Bosi and Desmarchelier (2018).

**3** See Fullerton and Kim (2008), Acemoglu et al. (2016) .

**4** See also John and Pecchenino (1994), Seegmuller and Verchere (2004), Xepapadeas (2005), Levin and Xepapadeas (2021), Menuet et al. (2020) and Menuet et al. (2024).

**5** See also Fujiwara (2009) for a study of optimal environmental taxation under oligopoly with product differentiation and free entry.

**6** See, for example, Lyon and Maxwell (2011) and Wu et al. (2020).

**7** See also Besley and Ghatak (2007) and Tirole and Bénabou (2006).

**8** This work, however, does not allow environmental quality to evolve in time.

**9** The quadratic nature of the expected cost of dishonest behavior is a common feature in contributions that consider the environmental damage of polluting emissions and, more broadly, works that consider penalties connected to tax evasion. In the former case, the convex nature of the cost is often connected to the increasing rate at which the environment may deteriorate due to pollution; the latter, convexity is also linked to the limited resources and the costs of auditing of tax authorities and the possible additional costs connected to the corruption of tax agents and falsification of documents. Another element that could explain the convex nature of the costs of dishonest behavior may be the existence of penalties connected to social norms. See Hashimzade et al. (2010) and Goerke and Runkel (2011).

**10** The study of the bargaining with third party agents (see Hashimzade et al. (2010)) or the strategic corruption of civil servants (see Amir and Burr (2015)) is out of the scope of our paper and will require careful study in future work.

**11** An alternative way to look at the competitive set up that we have described so far could allow the implicit introduction of emission taxes. Honest firms adopt the mandated cleaner technology, but they also produce a level of emissions which faces environmental taxation; in other words, the unit cost $c_h$ can be seen as the combination of a unit cost of production plus the emission tax. The dishonest firms declare (for example submitting fake documentation) lower emissions and, consequently, incur lower emission taxes; to some extent, the decision of engaging in illegal activities (and face the related costs) is akin to the investment in abatement technologies consider by previous literature. Since we are going to explicitly introduce pollution dynamics and ambient taxes in Section 4, to avoid confusion, for the rest of the paper will not refer to abatement nor emission taxes.

**12** See Xepapadeas (2011).

**13** The over sign has been put on profits so as not to confuse the new values with those for the one-dimensional map.

**14** See Lambertini et al. (2017) and their insights in a static oligopolistic set up with/without R&D spillovers. The authors show that when splillovers are absent, competition (measured by the number of firms active in the market) unambiguously tends to increase innovation and the adoption of clean technology. With spillovers, instead, the relationship between competition and innovation takes an inverted U-shaped form. In our dynamic set up, our expectation is that the relationship between competition and adoption of clean technologies will also increase in complexity if a detailed analysis of the case $\eta > 0$ is considered.

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## A. Technical details

### *A.1 Proof of Proposition 1.*

Let us consider an arbitrary fixed point $\bar{z}$ of the map $f_\delta$. Its multiplier is defined by the derivative of $f_\delta$, which is given as

$$\frac{\mathrm{d}f_\delta(z)}{\mathrm{d}z} = \delta + (1-\delta)\frac{\left(1+\phi z(1-z)G_n'(z)\right)\mathrm{e}^{-\phi G_n(z)}}{\left(z+(1-z)\mathrm{e}^{-\phi G_n(z)}\right)^2} = \delta + (1-\delta)\frac{\mathrm{d}f_0(z)}{\mathrm{d}z}. \tag{24}$$

Then the multiplier of $\bar{z}$ is

$$\mu(\bar{z}) = \frac{\mathrm{d}f_\delta(\bar{z})}{\mathrm{d}z} = \delta + (1-\delta)\frac{\mathrm{d}f_0(\bar{z})}{\mathrm{d}z}.$$

The point $\bar{z}$ is stable if $|\mu(\bar{z})| < 1$. For a positive multiplier the condition becomes

$$\mu(\bar{z}) < 1 \quad \Leftrightarrow \quad \delta + (1-\delta)\frac{\mathrm{d}f_0(\bar{z})}{\mathrm{d}z} < 1 \quad \Leftrightarrow \quad (1-\delta)\frac{\mathrm{d}f_0(\bar{z})}{\mathrm{d}z} < 1-\delta \quad \Leftrightarrow \quad \mu_0(\bar{z}) \overset{df}{=} \frac{\mathrm{d}f_0(\bar{z})}{\mathrm{d}z} < 1. \tag{25}$$

Therefore, for a fixed point with a positive multiplier, its stability does not depend on $\delta$.

The map $f_\delta$ has always two fixed points $\bar{z}_0 = 0$ and $\bar{z}_1 = 1$. In addition, any zero $z_*$ of the gain function $G_n$ also represents a fixed point of $f_\delta$, which is feasible in case $z_* \in [0, 1]$. The multipliers of $\bar{z}_0$ and $\bar{z}_1$ are, respectively,

$$\mu(0) = \delta + (1-\delta)\mathrm{e}^{\phi G_n(0)} \quad \text{and} \quad \mu(1) = \delta + (1-\delta)\mathrm{e}^{-\phi G_n(1)} \tag{26}$$

and they are always positive and, as shown above, their stability conditions are

$$\mu_0(0) = \mathrm{e}^{\phi G_n(0)} < 1 \quad \text{and} \quad \mu_0(1) = \mathrm{e}^{-\phi G_n(1)} < 1. \tag{27}$$

Thus, these two fixed points can undergo only a bifurcation with eigenvalue $+1$. Among those, a fold bifurcation is not possible, since $\bar{z}_0$ and $\bar{z}_1$ always exist. And it can be shown that a pitchfork bifurcation is not possible either. Hence, the boundary equilibria can change their stability properties only due to a transcritical bifurcation.

Let us obtain the respective stability conditions in terms of the map parameters. For the fixed point $\bar{z}_0$, the gain function $G_n(0)$ degenerates to a single term with $k = 0$ and the stability condition reads as

$$\mu_0(\bar{z}_0) = \frac{\mathrm{d}f_0(0)}{\mathrm{d}z} = \mathrm{e}^{\phi\left(\frac{(2\beta+1)na^2-(n^2-1)ab}{(n+1)^2(2\beta+n)} - \frac{(\beta+1)(a+b)^2}{(n+2\beta+1)^2}\right)} < 1 \quad \Leftrightarrow s_2a^2 - s_1ab - s_0b^2 < 0, \tag{28}$$

where

$$s_2 = (n^3 + (6\beta+4)n^2 + (8\beta^2 + 8\beta + 1)n - 2(\beta+1))\beta,$$
$$s_1 = (n+1)(n^3 + (6\beta+3)n^2 + (8\beta^2 + 6\beta + 1)n - 1), \quad s_0 = (\beta+1)(n+1)^2(2\beta+n).$$

This implies that the fixed point $\bar{z}_0$ is stable if

$$0 < \frac{a}{b} < \frac{s_1 \pm \sqrt{s_1^2 + 4s_2s_0}}{2s_2} \overset{df}{=} r_1(\beta, n). \tag{29}$$

Similarly, for $\bar{z}_1$ the gain function $G_n(1)$ degenerates to a single term with $k = n-1$, and hence,

$$\mu_0(\bar{z}_1) = \frac{\mathrm{d}f_0(1)}{\mathrm{d}z} = \mathrm{e}^{-\phi\frac{4\beta a^2 - 4(n+1)ab - (n+1)^2b^2}{(n+1)^2(2\beta+n)}} < 1 \quad \Leftrightarrow \quad 4\beta a^2 - 4(n+1)ab - (n+1)^2b^2 > 0, \tag{30}$$

which implies

$$\frac{a}{b} > \frac{(n+1)(1+\sqrt{\beta+1})}{2\beta} \overset{df}{=} r_2(\beta, n). \tag{31}$$

As for a zero $z_*$ of the gain function $G_n$, its multiplier simplifies to

$$\mu(z_*) = \delta + (1-\delta)\left(1 + \phi z_*(1-z_*)G_n'(z_*)\right), \tag{32}$$

which in general can be negative. Hence, the stability conditions for $z_*$ are

$$-\frac{1+\delta}{1-\delta} < \mu_0(z_*) = 1 + \phi z_*(1-z_*)G_n'(z_*) < 1. \tag{33}$$

### A.2 Proof of Proposition 2.

The linearity of $G_2(z)$ trivially follows from the general definition (4) and its coefficients can be derived explicitly:

$$G_2(z) = c_1 z + c_0, \quad c_1 = -\frac{(2a\beta - 3b)(2(4\beta^2 + 7\beta + 3)a - 3(4\beta + 5)b)}{36(2\beta + 3)^2(\beta + 1)},$$
$$c_0 = \frac{2\beta(8\beta^2 + 19\beta + 12)a^2 - 3(16\beta^2 + 36\beta + 21)ba - 18(\beta + 1)^2 b^2}{18(2\beta + 3)^2(\beta + 1)}. \quad (34)$$

The unique inner equilibrium is $z_* = -\frac{c_0}{c_1}$. Dropping the technical details, we state that $z_*$ exists for

$$r_1(\beta, 2) \overset{df}{=} 3 \cdot \frac{(16\beta^2 + 36\beta + 21) + \sqrt{(16\beta^2 + 36\beta + 21)^2 + 16\beta(\beta + 1)^2(8\beta^2 + 19\beta + 12)}}{4\beta(8\beta^2 + 19\beta + 12)}$$
$$< \frac{a}{b} < 3 \cdot \frac{1 + \sqrt{\beta + 1}}{2\beta} \overset{df}{=} r_2(\beta, 2). \quad (35)$$

with $r_i(\beta, n)$ defined in (29) and (31).

Concerning the stability of the fixed points, recall from Proposition 1 that $\bar{z}_0$ is stable if $\frac{a}{b} < r_1(\beta, n)$, while $\bar{z}_1$ is stable if $\frac{a}{b} > r_2(\beta, n)$. Finally, for $z_*$ there holds $\mu(z_*) < 1$ if $r_1 < \frac{a}{b} < r_2$. It is worth noting that while $z_*$ is feasible, it is possible to get $\mu(z_*) < -1$ for certain parameter constellations. Explicit condition for $\mu(z_*) = -1$ can be stated.

### A.3 Proof of Proposition 3.

Stability of an arbitrary equilibrium $(\bar{p}, \bar{z})$ of the map $F$ can be characterized by the eigenvalues of the Jacobian matrix

$$J(\bar{p}, \bar{z}) = \begin{pmatrix} \rho & QA_n'(\bar{z}) \\ (1-\delta)\dfrac{\phi\gamma\bar{z}(1-\bar{z})e^{-\phi\tilde{G}_n(\bar{p},\bar{z})}}{\left(\bar{z} + (1-\bar{z})e^{-\phi\tilde{G}_n(\bar{p},\bar{z})}\right)^2} & \delta + (1-\delta)\dfrac{\left(1 + \phi\bar{z}(1-\bar{z})G_n'(\bar{z})\right)e^{-\phi\tilde{G}_n(\bar{p},\bar{z})}}{\left(\bar{z} + (1-\bar{z})e^{-\phi\tilde{G}_n(\bar{p},\bar{z})}\right)^2} \end{pmatrix}, \quad (36)$$

where $G_n$, $\tilde{G}_n$, and $QA_n$ are given in (4), (12), and (13), respectively. The Jacobians of $E_0$ and $E_1$ are

$$J(\bar{p}_0, 0) = \begin{pmatrix} \rho & QA_n'(0) \\ 0 & \delta + (1-\delta)e^{\phi\tilde{G}_n(\bar{p}_0,0)} \end{pmatrix} \quad \text{and} \quad J(\bar{p}_1, 1) = \begin{pmatrix} \rho & QA_n'(1) \\ 0 & \delta + (1-\delta)e^{-\phi\tilde{G}_n(\bar{p}_1,1)} \end{pmatrix}. \quad (37)$$

Since $\rho \in (0, 1)$ is one eigenvalue, each of these two fixed points can be either a stable node or a saddle. Moreover, both eigenvalues of $E_i$ are always positive and only a bifurcation related to one eigenvalue crossing $+1$ is possible. The point $E_0$ is then stable if

$$e^{\phi\tilde{G}_n(\bar{p}_0,0)} < 1 \quad \Leftrightarrow \quad \tilde{G}_n(\bar{p}_0, 0) < 0, \quad (38)$$

while $E_1$ is stable if

$$e^{-\phi\tilde{G}_n(\bar{p}_1,1)} < 1 \quad \Leftrightarrow \quad \tilde{G}_n(\bar{p}_1, 1) > 0. \quad (39)$$

The stability conditions can be written explicitly even in general form, but we do not report them as they differ from (28) and (30) only by the term $\gamma\bar{p}_i$, $i = 0, 1$.

### A.4 Proof of Proposition 4.

Let us consider the simplest case of the gain function

$$\tilde{G}_2(p,z) = c_1 z + c_0 + \gamma p, \tag{40}$$

where $c_1, c_0$ are defined in (34), and the aggregate output function then becomes

$$QA_2(z) = d_2 z^2 + d_1 z + d_0, \tag{41}$$

with

$$d_2 = \frac{2a\beta - 3b}{3(\beta+1)(2\beta+3)}, \quad d_1 = \frac{(2\beta+3)(2a\beta-3b)}{3(\beta+1)(2\beta+3)}, \quad d_0 = \frac{2(a+b)}{2\beta+3}. \tag{42}$$

The trivial fixed points are

$$E_0 = \left(\frac{d_0}{1-\rho}, 0\right) = \left(\frac{2(a+b)}{(2\beta+3)(1-\rho)}, 0\right) \tag{43}$$

and

$$E_1 = \left(\frac{d_2+d_1+d_0}{1-\rho}, 1\right) = \left(\frac{2a}{3(1-\rho)}, 1\right). \tag{44}$$

The point $E_0$ is stable if

$\tilde{G}_2(\bar{p}_0, 0) = c_0 + \gamma \bar{p}_0 < 0 \quad \Leftrightarrow$

$$\frac{2\beta(8\beta^2+19\beta+12)a^2 - 3(16\beta^2+36\beta+21)ba - 18(\beta+1)^2 b^2}{18(2\beta+3)(\beta+1)} + \gamma \frac{2(a+b)}{1-\rho} < 0, \tag{45}$$

which is equivalent to

$$\gamma < -\frac{2\beta(8\beta^2+19\beta+12)a^2 - 3(16\beta^2+36\beta+21)ba - 18(\beta+1)^2 b^2}{36(2\beta+3)(\beta+1)(a+b)} \cdot (1-\rho) \stackrel{df}{=} \gamma_1. \tag{46}$$

The point $E_1$ is stable if

$$\tilde{G}_2(\bar{p}_1, 1) = c_1 + c_0 + \gamma \bar{p}_1 > 0 \quad \Leftrightarrow \quad \frac{4a^2\beta - 12ab - 9b^2}{12(\beta+1)} + \frac{2\gamma a}{1-\rho} > 0, \tag{47}$$

which is equivalent to

$$\gamma > -\frac{4a^2\beta - 12ab - 9b^2}{24(\beta+1)a} \cdot (1-\rho) \stackrel{df}{=} \gamma_2. \tag{48}$$

There is $\gamma_1 > \gamma_2$ provided that

$$\min\{u_1, u_2\} < \frac{a}{b} < \max\{u_1, u_2\} \tag{49}$$

$$\text{with} \quad u_1 = \frac{3\left(12\beta + 17 + \sqrt{(12\beta+17)^2 + 8(8\beta^2+13\beta+3)(2\beta+3)}\right)}{4(8\beta^2+13\beta+3)}, \quad u_2 = \frac{3}{2\beta}. \tag{50}$$

This implies the simultaneous stability of the two fixed points $E_0$ and $E_1$ for $\gamma_2 < \gamma < \gamma_1$. As follows from (49), for any fixed $\beta$ and $b$ there exists a range of $a$, for which $\gamma_2 < \gamma_1$. However, if

$a > u_2 b$, then there is $\bar{p}_1 > \bar{p}_0$, i.e. the level of pollution with all honest firms exceeds the level of pollution with all dishonest firms, which is rather odd to occur in reality. However, this result is plausible since we assumed $\eta = 0$ so that the pollution from production by either firm is equal. And provided that $a > u_2 b$, an h-firm supplies a larger level of output than a d-firm. Thus, the higher level of pollution is induced not by the dirty technology used but because the production is extensive. The opposite case $u_1 b < a < u_2 b$ implies $\bar{p}_1 < \bar{p}_0$.

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