# A Parametric Interpolation-Based Approach to Sideslip Angle Estimation

TRISTANO, Mariagrazia and LENZO, Basilio <http://orcid.org/0000-0002-8520-7953>

# A Parametric Interpolation-Based Approach to Sideslip Angle Estimation

Mariagrazia Tristano[1,2] and Basilio Lenzo[1(✉)]

[1] Università degli Studi di Padova, Padova, Italy
{mariagrazia.tristano,basilio.lenzo}@unipd.it
[2] Sheffield Hallam University, Sheffield, UK

**Abstract.** Vehicle sideslip angle has always been of interest for stability controls enhancing vehicle safety. As well-known, measuring sideslip is impractical and expensive, motivating techniques to estimate it using already-available vehicle sensors. This paper proposes a new methodology to estimate sideslip angle, separating kinematic and dynamic sideslip angle contributions with the idea that the former is straightforward and the latter may be obtained with a lateral-acceleration-based interpolation. The proposed approach is validated through experimental data on a passenger vehicle.

**Keywords:** vehicle dynamics · sideslip angle · estimation · dynamic sideslip angle · kinematic sideslip angle

## 1 Introduction

The rising interest in autonomous and semi-autonomous mobility has contributed to forging brand new research paths, aiming to improve vehicle handling and safety. When it comes to vehicle stability control, a key step is to identify meaningful vehicle states able to offer insight into the stability conditions of the vehicle. The existing literature on lateral stability controllers recurrently lists vehicle sideslip angle, $\beta$ [1]. $\beta$ is defined as the angle between the orientation of the centre-of-mass velocity vector and the longitudinal axis of the vehicle [2]. Despite knowledge of the sideslip angle being pivotal, it is a difficult quantity to monitor in real-time, as commercially available sensors are bulky and expensive. Alternatively, estimation techniques exist, which typically rely on model-based estimators, such as Kalman filters [3], or neural networks [4]. Such approaches have their own benefits and disadvantages and, to date, there is no "ultimate" solution.

This paper proposes a parametric estimation of the sideslip angle. The estimator features an interpolation-based formulation providing the dynamic sideslip angle component, while the kinematic component is provided as the result of kinematic steering. The entire strategy relies only on measurements coming from the vehicle Inertial Measurement Unit (IMU).


G. Mastinu et al. (Eds.): AVEC 2024, LNME, pp. 279–285, 2024.
https://doi.org/10.1007/978-3-031-70392-8_40

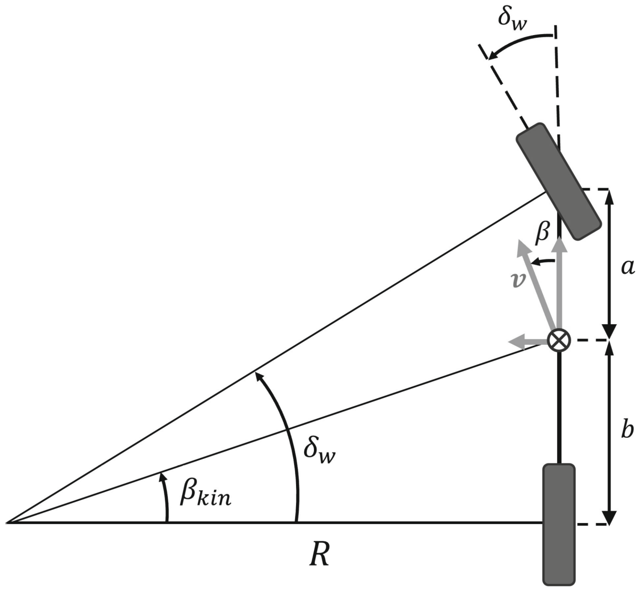



**Fig. 1.** Single-track model showing the kinematic sideslip angle.

## 2 Sideslip Angle Estimation

According to [5], $\beta$ may be split into a kinematic component, $\beta_{\text{kin}}$, an intrinsic geometrical feature, and a dynamic component, $\beta_{\text{dyn}}$, arising as a result of tire slip:

$$\beta = \beta_{\text{kin}} + \beta_{\text{dyn}} \tag{1}$$

### 2.1 Kinematic Sideslip Angle Estimate

Remarks on kinematic phenomena within this paper all refer to the single-track model [6]. Looking at Fig. 1, in kinematic conditions:

$$\tan(\delta_w) = \frac{a+b}{R} \approx \delta_w, \, \tan(\beta_{\text{kin}}) = \frac{b}{R} \approx \beta_{\text{kin}} \tag{2}$$

where $a$ and $b$ are the front and rear semi-wheelbases and $R$ the (kinematic) turning radius, $\delta_w$ the wheel steering angle. By eliminating $R$:

$$\beta_{\text{kin}} \approx \delta_w \frac{b}{a+b} \tag{3}$$

In actual driving conditions, this expression only approximates the kinematic sideslip angle [5], yet it is used in this paper as it does not require an estimation of vehicle longitudinal speed - e.g. through wheel speed sensors.

### 2.2 Dynamic Sideslip Angle Estimate

As seen above, the kinematic sideslip angle contribution at any point of the vehicle longitudinal axis depends on the distance between the rear axle and the point of interest - for the centre of mass, such distance is $b$.

Very interestingly, [5] shows that the dynamic sideslip angle contribution instead does not depend on the position along the vehicle longitudinal axis. The idea is then to find the dynamic sideslip angle contribution at the rear axle, which is easier, and that shall be the dynamic sideslip angle also at the vehicle centre of mass, to be used in Eq. 1.

The kinematic contribution at the rear axle is exactly 0. So, the overall sideslip angle at the rear axle is only made of the dynamic sideslip angle contribution - at least, for a non-rear-wheel steering vehicle. Since the rear steering angle is 0, the congruence equation [7] $\delta = \alpha + \beta$ reduces to $-\beta_R = \alpha_R$, where $\alpha$ denotes tire slip angle and the subscript R refers to the rear axle. And because at the rear axle there is no kinematic contribution, then

$$\beta_{\text{dyn}} = -\alpha_R \tag{4}$$

Below, a procedure is proposed to determine $\alpha_R$ based on lateral acceleration, $a_y$, which is readily available at the IMU of any passenger vehicle.

The key idea is that $\alpha_R$ depends on the rear axle lateral force, which in turn is a linear function of $a_y$ [7]. So any constitutive relationship between lateral force and slip, such as the well-known Magic Formula, would approximate (only at steady-state the rear axle lateral force is just proportional to $a_y$) the relationship between $a_y$ and $\alpha_R = -\beta_{\text{dyn}}$. This needs to be inverted, to provide $\alpha_R = -\beta_{\text{dyn}}$ as a function of $a_y$. Classical tire models are difficult to invert or piecewise defined. Here, we adopt the Root-Rational tire model [8], which is easy to invert, leading to:

$$\beta_{\text{dyn}} = \begin{cases} \dfrac{c_3 a_{\text{y}}}{c_1 - c_2 a_{\text{y}}} & \text{if} \quad a_{\text{y}} < 0 \\ \dfrac{c_3 a_{\text{y}}}{c_1 + c_2 a_{\text{y}}} & \text{if} \quad a_{\text{y}} \geq 0 \end{cases} \tag{5}$$

Interpolation coefficients $c_1$, $c_2$, $c_3$ are retrieved by solving a non-linear least squares problem on suitable vehicle data sets. Once both components are known, the full sideslip angle $\beta$ is reconstructed using Eq. (1).

## 3 Results

Interpolation is performed using dataset groups [9,10], herein respectively referred to as D1 and D2. Coefficient values for both datasets are reported in Table 1, with Fig. 2 showing an overlap between original and interpolated data. The validity of the estimation strategy can be assessed by comparing its outcome against the measured signal: Fig. 3a–3b display the estimation outcome on a portion of data within D1 and D2 respectively.

**Table 1.** Interpolation parameters using dataset D1 and D2.

| Dataset | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| D1 | 33901.36 | 1737.77 | 5580.54 |
| D2 | 48585.36 | 2507.95 | 12949.62 |

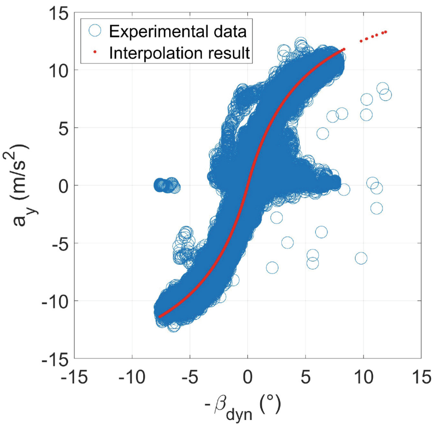
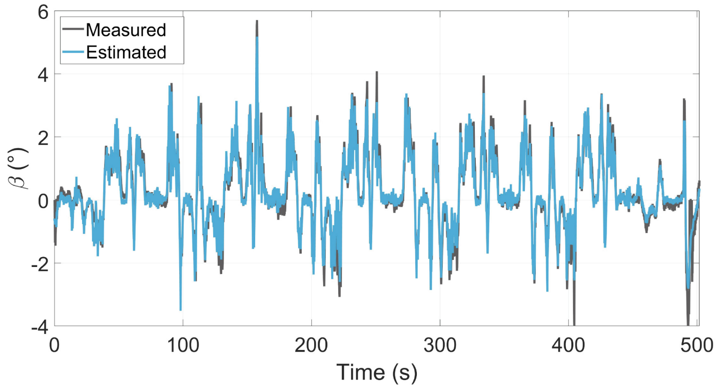
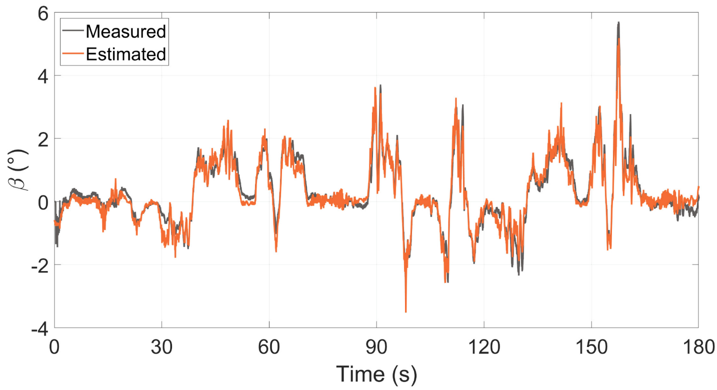



**Fig. 2.** Interpolation outcome on available datasets.

The numerical performance assessment is done through computing the Root-mean-square error (RMSE). The resulting validation RMSEs on four different runs belonging to datasets [9,10] are listed in Table 2 and Table 3, with the RMSE generally around 0.5 deg.




(a)




(b)

**Fig. 3.** Estimation outcomes from D1 (a) and D2 (b).

**Table 2.** Validation outcomes for four different datasets within D1.

| Validation dataset | RMSE (°) |
|---|---|
| 20130223_01_02_03_grandsport | 0.415 |
| 20130222_01_01_03_grandsport | 0.367 |
| 20130222_01_02_03_grandsport | 0.342 |
| 20130222_02_01_03_grandsport | 0.434 |

**Table 3.** Validation outcomes for four different datasets within D2.

| Validation dataset | RMSE (°) |
|---|---|
| `20140222_01_01_03_250lm` | 0.539 |
| `20140221_03_02_03_250lm` | 0.375 |
| `20140221_04_01_03_250lm` | 0.541 |
| `20140221_03_03_03_250lm` | 0.502 |

# 4 Conclusions

A parametric approach was proposed to tackle the challenge of estimating the vehicle sideslip angle. The estimation task was split in two sub-tasks, aiming to extract: i) the kinematic sideslip angle, as a result of kinematic steering; ii) the dynamic sideslip angle as a parametric approximation based on affinity with principles regulating tire models. Results both in terms of RMSE and normalised error distributions show there is little discrepancy with respect to the measured counterpart, hence encourage further studies.

Future work will include exploring this approach within larger sets of data and for different vehicles, and an on-vehicle implementation.

# References

1. Huang, Y., Chen, Y.: Vehicle lateral stability control based on shiftable stability regions and dynamic margins. IEEE Trans. Veh. Technol. **69**(12), 14727–14738 (2020)
2. Lenzo, B.: Torque vectoring control for enhancing vehicle safety and energy efficiency. In: Vehicle Dynamics: Fundamentals and Ultimate Trends, pp. 193–233. Springer International Publishing, Cham (2022)
3. Naets, F., Aalst, S., Boulkroune, B., El Ghouti, N., Desmet, W.: Design and experimental validation of a stable two-stage estimator for automotive sideslip angle and tire parameters. IEEE Trans. Veh. Technol. **66**, 9727–9742 (2017)
4. Novi, T., Capitani, R., Annicchiarico, C.: An integrated artificial neural network-unscented Kalman filter vehicle sideslip angle estimation based on inertial measurement unit measurements. Proc. Inst. Mech. Eng., Part D: J. Autom. Eng. **233**, 1864–1878 (2019)
5. Lenzo, B., Sorniotti, A., Gruber, P., Sannen, K.: On the experimental analysis of single input single output control of yaw rate and sideslip angle. Int. J. Automot. Technol. **18**, 799–811 (2017)
6. Genta, G.: Motor vehicle dynamics: modeling and simulation. World scientific (1997)
7. Guiggiani, M.: Correction to: The Science of Vehicle Dynamics. In: The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars, pp. C1–C1. Springer International Publishing, Cham (2023)
8. Tristano, M., Lenzo, B.: (under review) Analytical solution of vehicle phase plane equilibria through the Root-Rational tire model. Vehicle System Dynamics

9. Kegelman, J., Harbott, L., Gerdes, J.: Targa Sixty-Six. Stanford Digital Repository. (http://purl.stanford.edu/yf219gg2055, 2016) (2013)
10. Kegelman, J., Harbott, L., Gerdes, J.: Targa Sixty-Six. Stanford Digital Repository. (http://purl.stanford.edu/hd122pw0365,2016) (2014)