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HYBRID MATHEMATICAL/ISOGEOMETRIC ANALYSIS OF PIEZOELECTRIC ENERGY HARVESTER COMPOSITE UNDER DYNAMIC BENDING

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ABSTRACT

In this paper, an innovative hybrid mathematical/computational scheme is implemented to evaluate the energy harvested via structural vibration. The governing voltage differential equations of the piezoelectric composite beam are coupled with the NURBS-based isogeometric analysis (IGA). Numerical experiment has been done on piezoelectric composite beams with various thickness. High order IGA element has been proven less prone to the shear locking phenomena in the literatures. The results presented in this paper show greater accuracy on dynamic responses and energy estimation are obtained on a very thin beam by means of IGA compare to standard FEM.

Keywords: piezoelectric, energy harvester, dynamic bending, NURBS, isogeometric analysis.

INTRODUCTION

In the past decade, the interest on the multifunctional structure for energy harvesting application has grown in a significant manner. The piezoelectric energy harvesting via structural vibration is the focus of the work presented in this paper. One of the earliest methodology to design the piezoelectric energy harvester was based on the mathematical model of a cantilevered piezoelectric under base excitation (Erturk & Inman, 2008). Numerous mathematical/computational models to investigate the energy harvested from piezoelectric structural vibration have been developed since.

In the most recent review articles (Abdelkefi, 2016; Li et al., 2016), it can be seen that most of the proposed mathematical models were developed for flutter-based energy harvesters. To the authors' knowledge, there are only a few articles proposed models for more operational loading conditions, i.e. gust loads on wing (Xiang et al., 2015; Tsushima & Wu, 2016; Bruni et al., 2017). In order to address more practical issue, a novel hybrid scheme for dynamic response-based energy harvester has been developed by Akbar and Curiel-Sosa (2016). These scheme conveniently coupled the piezoelectric beam voltage equation with the results of FEM. The solver used for the simulation was based on the explicit FEM (Curiel-Sosa & Gil, 2009; Curiel-Sosa et al., 2013). The scheme has been well validated and implemented for complicated configuration, i.e. aircraft wingbox, under cruise load excitation.

In this paper, further enhancement of Akbar and Curiel-Sosa's model by means of coupling with isogeometric analysis (IGA) is presented. NURBS-based IGA is proven to produce more accuracy and more efficient computational process compared to standard FEM (Hughes et al., 2005). Furthermore, NURBS-based IGA possesses the capability to construct high order elements, which is hardly suffer from shear locking phenomena (Thai et al., 2012). This is an advantage in the case of piezoelectric structures or smart structures modelling. The smart structures which are usually thin-walled structures, can be modelled more conveniently. In the recent years, IGA shell elements itself has been well developed and applied for various structural mechanic problems, i.e. free vibration and buckling for laminated composites (Thai et al., 2012; Yu et al., 2016).

In order to study the implementation of IGA elements for piezoelectric-based energy harvesting simulation, a novel investigation by means of high order IGA shell elements was carried out on piezoelectric composite beam with various thickness. Coupled with Akbar and Curiel-Sosa's energy harvester model, the results show great accuracy both for the dynamic responses and energy estimations compared to classical FEM as presented in the following section of this paper.

PIEZOELECTRIC ENERGY HARVESTER COMPOSITE BEAM MODEL

Figure 1 shows the piezoelectric energy harvester model used in the present work. The model consisted of a composite beam with substrate layer and active layer, i.e. piezoelectric material (Akbar & Curiel-Sosa, 2016). The electrical field assumed to be developed only in thickness direction due to an applied bending load.

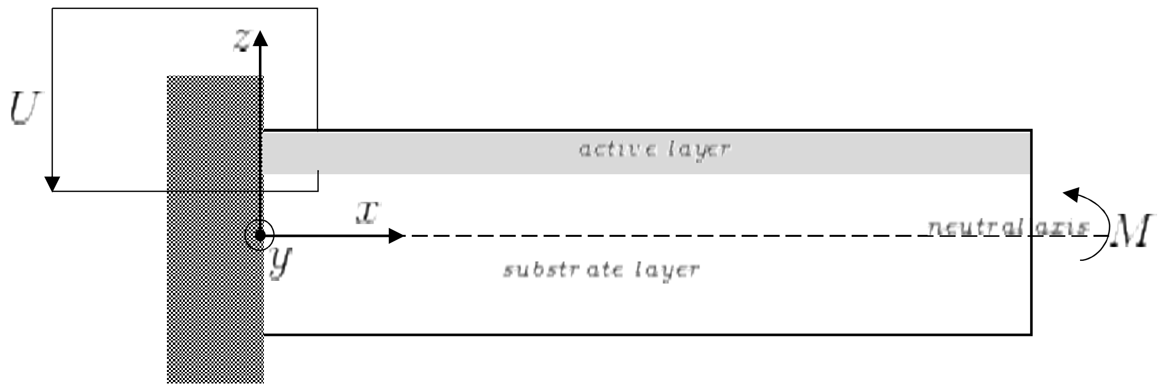

Fig. 1 - Piezoelectric Beam Bending Energy harvester

Equation 1 shows the governing voltage differential equation of the piezoelectric energy harvester composite beam (Akbar & Curiel-Sosa, 2016). The piezoelectric beam excited by a dynamic bending load with frequency ω (rad/s). The load resulted in a mechanical displacement, i.e. bending slope $\frac{\partial Z_{mech}}{\partial x}$ (rad), and a reverse piezoelectric effect, which represented by admittance function $H_{\alpha m}$ (rad/N-m). Connecting the beam with an external resistance load R (Ohm), harvested an amount of energy, i.e. voltage amplitude $\overline{U}$(volt). The parameters Γ_1 and Γ_2 are consisted of the mechanical and electrical properties of the materials and the beam geometry, with i is the imaginary value, i.e. $\sqrt{-1}$. The interested reader is referred to Akbar and Curiel-Sosa (2016) for details of these parameters and the derivation.

$$\overline{U} = \frac{i\omega\Gamma_2(x)\frac{\partial Z_{mech}(x)}{\partial x}}{-\frac{1}{R}+i\omega\Gamma_1(x)-i\omega\Gamma_2(x)^2 H_{\alpha m}(x)} \tag{1}$$

NURBS-BASED IGA/ VOLTAGE EQUATION COMPUTATIONAL ALGORITHM

In the present work, a computational code of NURBS-based IGA for shell elements is built via MATLAB®. The element is a composite shell based on first shear deformation theory (FSDT). The computational code follows the governing equations described by Thai et al. (2012).

Following the algorithm scheme of Akbar and Curiel-Sosa (2016), IGA simulations were performed for both actual mechanical load and a dummy unit of bending moment at the tip of the beam. The responses due to the dummy bending moment load is used to calculate the admittance function of reverse piezoelectric effect. In order to obtain the energy harvested, input of the material properties, structural dynamic responses and admittance function were applied to the piezoelectric voltage equation along with the resistance load value. Figure 2 shows the schematic algorithm of the procedure.

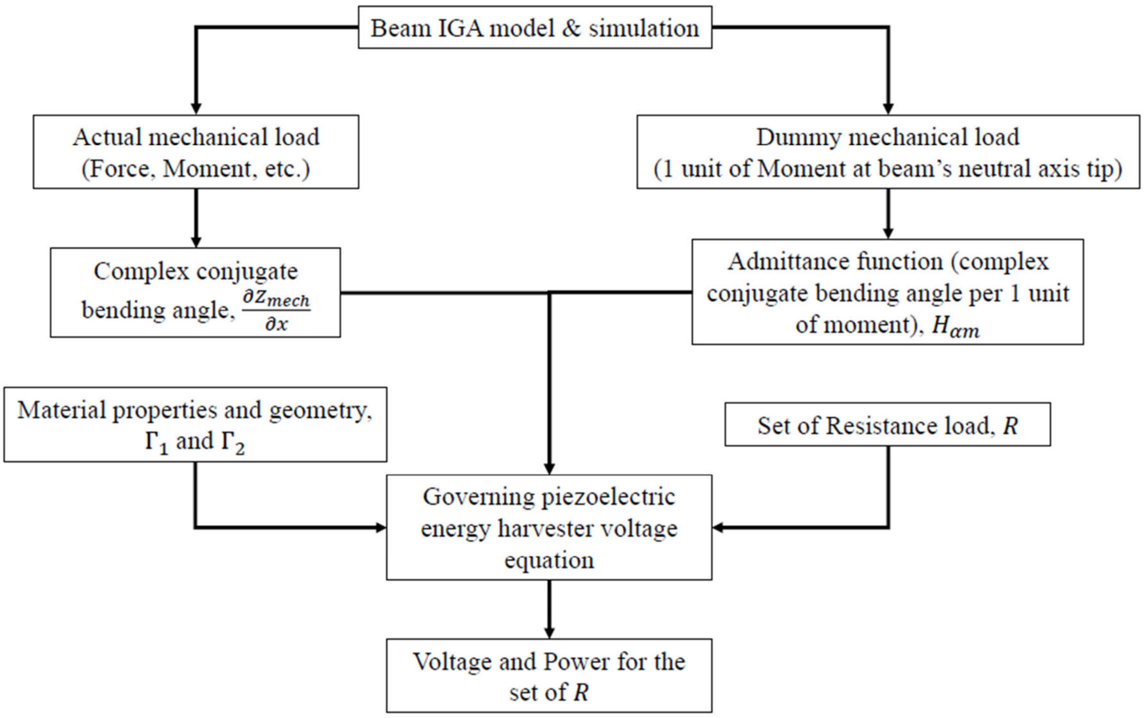

Fig. 2 - IGA and energy harvester simulation algorithm

ALGORITHM AND SIMULATION

Table 1 shows the material properties used in the present work. An 8 nodes biquadratic (Q8) FEM shell element, 9 nodes biquadratic (Q9) and 25 nodes biquartic (Q25) IGA shell elements are used for the numerical investigations. The computational model used 2 elements at y-direction (chordwise) and 12 elements at x-direction (spanwise) of the beam. Thus, the aspect ratio of the element kept to unity and the element length to thickness ratio is 7.15 for the original thickness shown in Table 1.

In the present work, the thickness of the beam is varied to a very small number to investigate the shear locking effect. A thickness ratio parameter, h_0/h, is used in the following sections. This ratio shows the original thickness ratio to the actual model thickness. For instance, $h_0/h = 10^4$ means the actual model thickness is 10^4 thinner than the original thickness shown in Table 1, and at this level the element length to thickness ratio will be more than 70000.

Table 1 - Bimorph piezoelectric energy harvester properties (Erturk & Inman, 2008)

Properties	Piezoceramics	Substructure
Length, L (mm)	30	30
Width, b (mm)	5	5
Thickness, h (mm)	0.15 (each)	0.05
Density, ρ (kg/m3)	7750	2700
Elastic Modulus, $1/S_{11}$ (GPa)	61	70
Piezoelectric constant, d_{31} (pm/V)	-171	-
Permittivity, ε_{33} (nF/m)	15.045	-

RESULTS AND DISCUSSION

As mentioned in the previous section, the dynamic response of the structure is one of the main input to calculate the energy harvested. The biquartic IGA results of the first two mode shapes of the beam with original thickness are shown in Fig. 3 and Fig. 4. The 1st bending mode is shown in Fig. 3 and 2nd bending mode is shown in Fig. 4. Table 2 shows the comparison of the natural frequencies from IGA and FEM simulations with analytical results obtained from Erturk and Inman (2008). It is obviously seen from Table 2 that the results are all in a good agreement.

Table 2 - Natural frequency (Hz) comparison

Mode Shape	Analytical	FEM Q8	IGA Q9	IGA Q25
1st bending	185.1	187.3	187.2	187.3
2nd bending	1159.8	1172.5	1172.0	1172.5

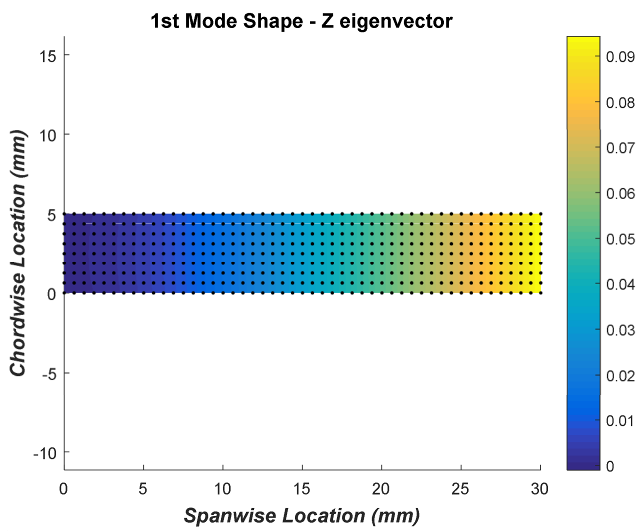

Fig. 3 - 1st mode shape eigenvector plot for beam with original thickness (h_0/h =1)

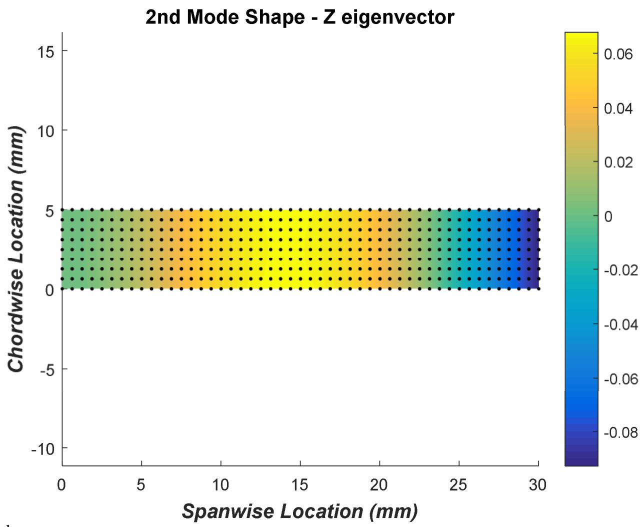

Fig. 4 - 2nd mode shape eigenvector plot for beam with original thickness ($h_0/h = 1$)

Meanwhile, Table 3 shows the comparison of the relative tip displacement of the beam with original thickness due to 1µm base excitation amplitude at the 1st resonance frequency. It can be seen that the results are all also in a good agreement. The biquartic IGA results for the beam with original thickness due to the base excitation load are depicted in Fig. 5 and Fig. 6. Fig. 5 shows the relative tip displacement amplitude at z-direction. As the excitation frequency equal with 1st bending mode, the displacement distribution follows the 1st bending mode shape such as shown in Fig. 3. The displacement angle at xz-plane (bending angle) is shown in Fig. 6, which also follows the 1st bending mode shape.

Table 3 - Relative tip displacement (µm) comparison

Excitation Frequency	Analytical	FEM Q8	IGA Q9	IGA Q25
1st bending natural frequency	78.0	78.3	78.3	78.3

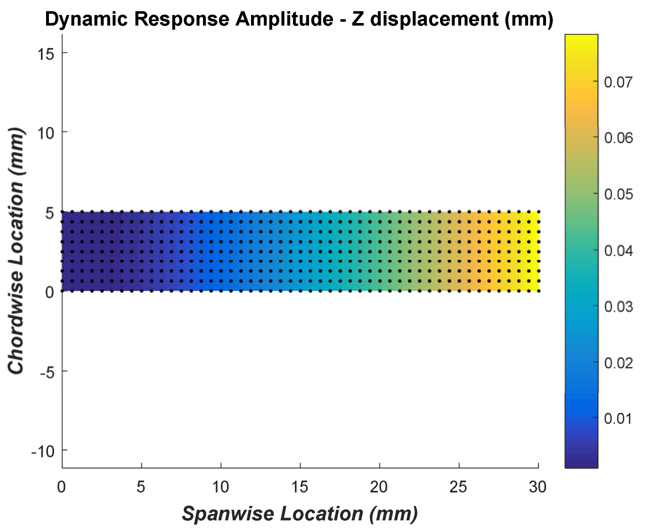

Fig. 5 - Z displacement (bending) amplitude due to base excitation at 1st resonance frequency for beam with original thickness ($h_0/h = 1$)

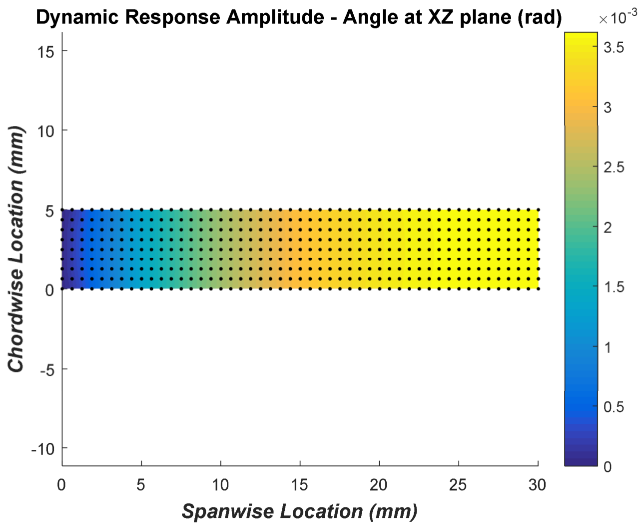

Fig. 6 - XZ plane angle (bending angle) amplitude due to base excitation at 1st resonance frequency for beam with original thickness (h_0/h =1)

From the comparison in Table 2 and Table 3, both FEM and IGA simulation obtained close results compared to the analytical ones for the beam with original thickness. However, as the thickness of the beam becomes very thin, both FEM and IGA results are shifted away from the analytical results. It can be seen in Fig. 7 and Fig. 8 until the thickness of the beam is 10^3 thinner than the original thickness, the results for both tip displacement and tip angle still show good agreement with the analytical results. At this level of thickness, shear locking phenomena has not yet occurred.

Further decreasing the level of thickness to 10^4 thinner than the original thickness, the shear part of the element becomes more dominant to the bending part of the element, thus shear locking happened. It can be seen that the FEM results dropped significantly as the shear locking occurred. Even if the thickness become thinner, the shell elements tend to behave more like plane stress elements, thus exerted by a base excitation in z-direction will not resulted in a bending displacement. Meanwhile, IGA results for both biquadratic and biquartic elements still show good comparison with the analytical results at thickness ratio 10^4.

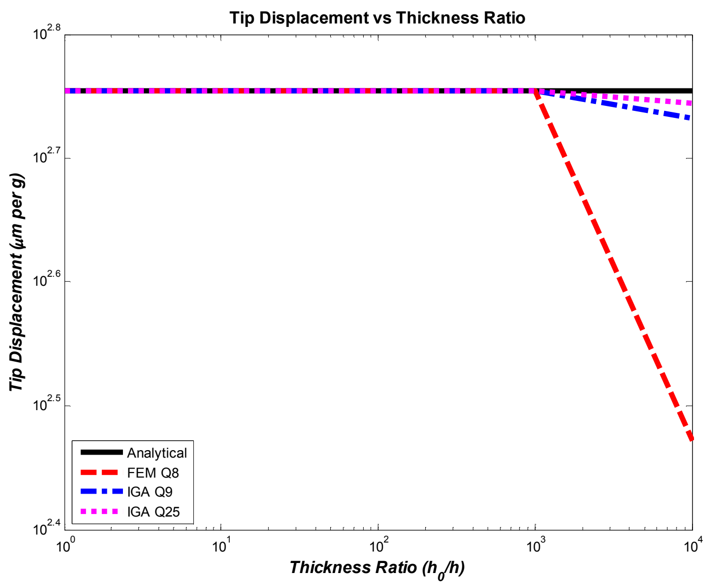

Fig. 7 - Z displacement (bending) amplitude due to base excitation at 1st resonance frequency for beam with various beam thickness

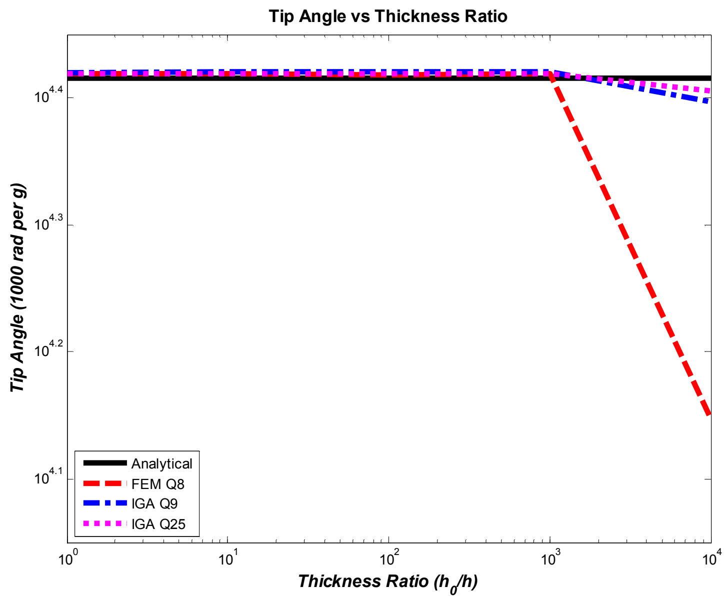

Fig. 8 - XZ plane angle (bending angle) amplitude due to base excitation at 1st resonance frequency for various beam thickness

Although the results start to get distorted, the IGA elements, are less prone to the shear locking phenomena. This results aligned with the results of Thai et al. (2012), where high order IGA elements had more resistance from shear locking.

As the structural dynamic responses of for both FEM and IGA results are in good comparison with the analytical results, thus the voltage and power amplitude are all also in good agreement as shown in Fig. 9 and Fig. 10. The figures show biquadratic FEM and biquartic IGA results are just slightly overestimate the analytical results. This is aligned with the results of Akbar and Curiel-Sosa (2016), where the numerical results also slightly overestimate the analytical results. To be noted that the figures are in logarithmic scale, where the voltage and the power are normalized per unit of g (9.81 m/s^2) and g^2. Detailed explanation of this unit normalization can be found in Akbar and Curiel-Sosa (2016).

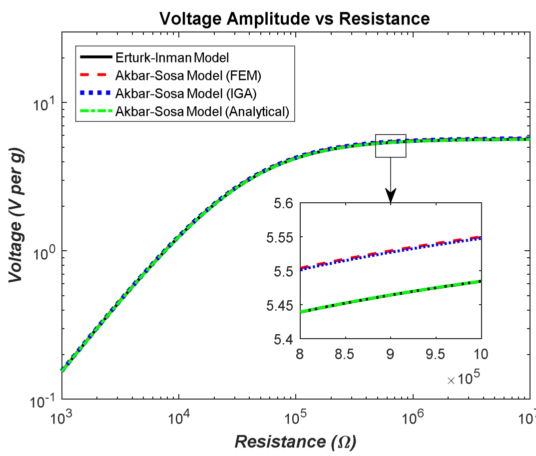

Fig. 9 - Voltage amplitude vs resistance load for beam with $h_0/h = 1$

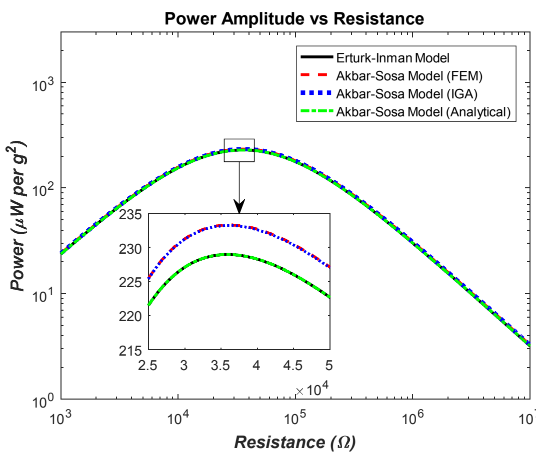

Fig.10 Power amplitude vs resistance load for beam with $h_0/h = 1$

As the beam with thickness ratio 10^4 resulted in significant variance for structural dynamic response via FEM, thus the voltage and power responses are also giving huge variance and become unreliable. The variances not also coming from the dynamic response due to base excitation, but also from the forced bending moment response which required to calculate the admittance function of reverse piezoelectric effect. Fig. 11 and Fig. 12 show both for the voltage and power response, the results via FEM completely shifted from the analytical results curve. At resistance load less than 80 kΩ the FEM results underestimated the analytical results, while at more than 80 kΩ it overestimated the analytical results. Meanwhile, the IGA results just slightly underestimate the analytical results (less than 5% variance), aligned with the trend of the structural dynamic responses.

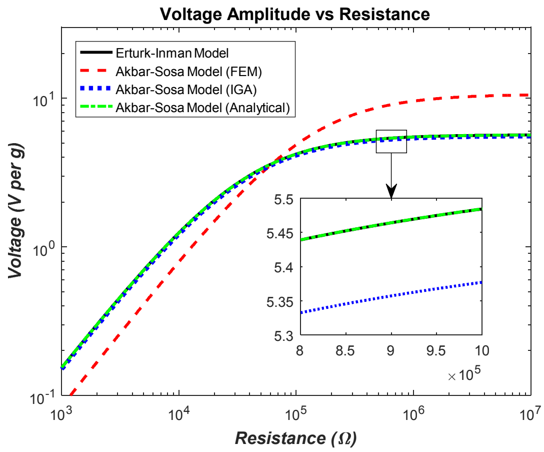

Fig. 11 - Voltage amplitude vs resistance load for beam with $h_0/h = 10^4$

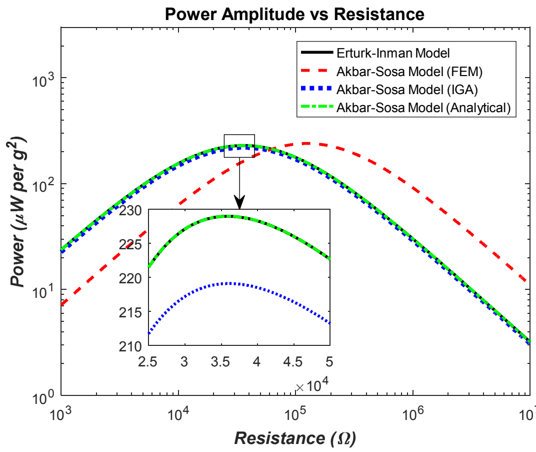

Fig. 12 - Power amplitude vs resistance load for beam with $h_0/h = 10^4$

For a standard linear (non-logarithmic) scale, Fig. 13 and Fig. 14 show clearer differences where the FEM results are shifted from the analytical results for both the voltage and power responses. Table 4 shows detailed comparison of the voltage and power response for all of the methods. It can be seen that FEM results overestimated the maximum voltage by almost twice of the analytical results. Although the maximum power seems just slightly overestimate the analytical results, however the resistance load that gives the maximum power is more than thrice the analytical results.

Table 4 - Electrical parameter comparison at $h_0/h = 10^4$

Electrical parameters	Erturk-Inman model (Analytical)	Akbar-Sosa model (Analytical)	Akbar-Sosa model (IGA Q25)	Akbar-Sosa model (FEM Q8)
Max Voltage (V)	7.80e-09	7.80e-09	7.64e-09 Δ=2.05%	1.45e-08 Δ=85.9%
Max Power (W)	4.35e-22	4.35e-22	4.17e-22 Δ=4.14%	4.55e-22 Δ=4.60%
R at max Power (kΩ)	35.90	35.90	36.09 Δ=0.53%	125.07 Δ=248%

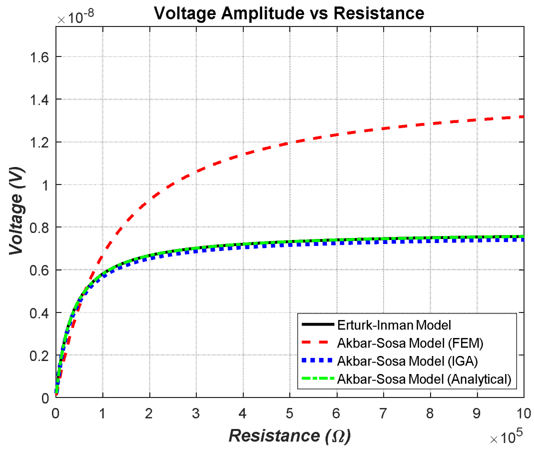

Fig. 13 - Non-logarithmic scale voltage amplitude vs resistance load for beam with $h_0/h = 10^4$

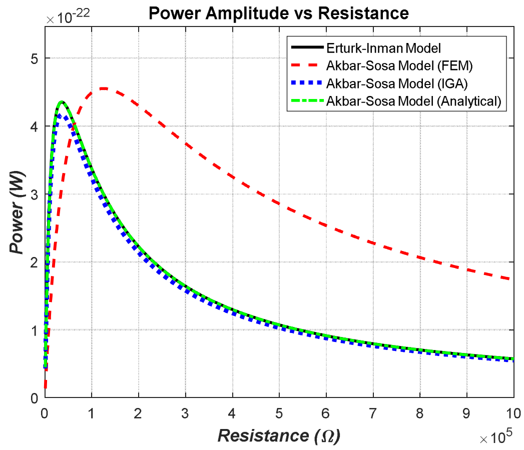

Fig. 14 - Non-logarithmic scale power amplitude vs resistance load for beam with $h_0/h = 10^4$

CONCLUSION

A computational code has been developed to investigate structural vibration by means of high order NURBS-based IGA element. Following the scheme of Akbar and Curiel-Sosa (2016), the dynamic responses of the piezoelectric structure by means of IGA applied as the input to estimate the energy harvested via the piezoelectric beam voltage equation. The numerical investigations show superiority of IGA in terms of shear locking resistance compared to classical FEM. As the beam become very thin, the IGA still accurately estimated the structural dynamic response, while FEM results altered significantly. Hence, the energy responses for IGA also show good comparisons with the analytical results, while FEM results become unreliable.

Based on these results, IGA coupled with Akbar and Curiel-Sosa's piezoelectric energy harvesting model could be beneficial in terms of computational efficiency. Akbar and Curiel-Sosa's model gives fast estimation of the energy harvested at a preliminary design stage and the thin-walled structures could be conveniently modeled by means of IGA without the needs of very dense meshes or any additional correction feature, i.e. reduced integration, to avoid shear locking.

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