Data-Driven Based Modelling of Pressure Dynamics in Multiphase Reservoir Model

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Abstract— **Secondary recovery involves injecting water or gas into reservoirs to maintain or boost the pressure and sustain production levels at viable rates. Accurate tracking of pressure dynamics as reservoirs produce under secondary production is one of the challenging tasks in reservoir modelling. In this paper, a data-driven based technique called Dynamic Mode Learning (DML) that aims to provide an efficient alternative approach for learning and decomposing pressure dynamics in multiphase reservoir model that produces under secondary recovery is proposed. Existing algorithms suffer from complexity and thereby resulting to expensive computational demand. The proposed DML technique is developed in the form of a learning system by first, constructing a simple, fast and efficient learning system that extracts important features from original full-state data and places them in a low-dimensional representation as extracted features. The extracted features are then used to reduce the original high-dimensional data after which dynamic modes are computed on the reduced data. The performance of the proposed DML method is illustrated on pressure field data generated from direct numerical simulations. Experimental results performed on the reference data reveal that the proposed DML method exhibits better and effective performance over standard and compressed dynamic mode decomposition (DMD) mainstream algorithms.**

I. Introduction

As the world population keeps on rising and industrial and residential activities increase, so also the demand for energy. To meet the rapidly increasing global energy demand, various forms of energy sources need to employ available and new techniques to meet this increasing demand. So far, renewable energy (solar, wind, hydro, tidal, geothermal and biomass) has accounted for up to 35% of the overall energy supply [1]. Nuclear energy is another source, however, the possible risk of contamination associated with nuclear energy is high [2]. Fossil fuels, which include oil, natural gas and coal, supply almost 65% of the total world energy. Fossil fuels in conventional and unconventional hydrocarbon reservoirs will continue to account for the large proportion of the world energy supply in the next several decades [3]. Managing hydrocarbon reservoir workflows normally involve numerous simulations for optimizing production, enhancing oil recovery and history matching [4]. Flow of fluid in porous media is governed by complex nonlinear partial differential equations (PDEs), which in practice, are spatially discretized into a high-dimensional set of nonlinear ordinary differential equations (ODEs) [5]. For consistent representation of flow dynamics and subsurface geology, grid blocks in very large numbers are required and cumbersome algorithms are employed for their spatiotemporal solutions. It requires thousands of simulations even with advanced algorithms to achieve optimal solutions when solving with nonlinear constraints [6]. The complexity in physics associated with reservoir multiphase fluid flow and the multiscale nature of the rock and fluid properties present challenges in achieving better predictive models. At early stage of production, most hydrocarbon reservoirs produce under primary recovery where a reservoir (formation) pressure forces the fluid into the well. However, since production is usually accompanied by a decrease in reservoir pressure, primary recovery via natural lift soon comes to an end. When a large portion of the oil or gas in a reservoir cannot be recovered by primary production, a method known as secondary or enhanced recovery is employed to pressurize the reservoir. Secondary recovery is achieved by injecting water (waterflooding) or gas (gas flooding) into the reservoir to displace produced fluids and hence maintain or boost the reservoir pressure [7]. The performance of a reservoir model operating under secondary recovery can be significantly influenced by different types of parameters. These parameters include static reservoir parameters (such as porosity, permeability, grid location), dynamic reservoir parameters (such as reservoir pressure, fluid composition, fluid saturation, relative permeability, and well data (such as injection rate, production rate, well radius and well patterns [1]. Conventionally, numerical simulation is used to quantify uncertainties and find the best set of parameters that give the best performing model. However, the higher the number of parameters, the more cumbersome the reservoir model becomes and consequently, the more computationally expensive the entire simulation process becomes. In comparison to numerical simulation, in which model set up is laborious and implementation is time consuming, a data-driven model that not only scales down computational complexity but also, offers accuracy without compromising results would be of great benefit [8], [9], [10]. Data-driven based machine learning techniques have been successfully applied in reservoir characterization and engineering to provide solutions to challenges that include estimation of flow rates of oil and gas in multiphase production systems [11], investigation of pressure flow in underground gas storage [12], [13], prediction of well performance [14], prediction of deliverability in underground natural gas storage [15], to mention but a few. In recent years, a large body of research called Dynamic Mode Decomposition (DMD) has emerged around modal decomposition and machine learning methods

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[16]. DMD originated as a new promising tool in the fluid dynamics community to discover spatiotemporal meaningful structures from high-dimensional fluids data. The evolving success of DMD arises from the fact that it is a data-driven and equation-free technique that is capable of discovering spatiotemporal meaningful patterns that may be used for diagnosis, control, state estimation and future-state prediction of complex dynamical systems [17]. Being a data-driven technique and despite its successful application in diverse areas, it is observed that standard DMD works best when the number of columns of the snapshot matrix, which is also the number of time steps, is smaller than the number of rows, which is also the number of observations, measurements, or samples for the purpose of retaining the spatial information wherever possible. In standard DMD, Singular Value Decomposition (SVD) is performed on the full-state snapshot data. For instance, performing SVD on a m×n snapshot matrix M results in three matrices U, S, and V, where U is an m×m unitary matrix, S is an m×n diagonal matrix, and V is an n×n unitary matrix. The entries of S are referred to as the singular values of M, the columns of U and V are referred to as the left-singular and right-singular vectors of M, respectively. After performing SVD on the full-state snapshot matrix M, one needs to truncate the components of the three matrices U, S and V according to the rank of the snapshot matrix. In a situation like this, if the number of columns of the full-state snapshot matrix M is greater than the number of rows, part of the columns of U (left-singular vector) that carries spatial information will be lost. Even though, this might not be a problem when dealing with overdetermined systems, nevertheless, this is a drawback when dealing with underdetermined systems as information regarding low level dynamics in spatiotemporal data might not be analyzed. A modified version of standard DMD called compressed DMD produces almost similar result as standard DMD but at low cost of computation. Compressed DMD integrates standard DMD with compressive sensing to achieve results by reconstructing a full-state snapshot from a random under sampling of the full-state data [18]. The basic idea behind compressed DMD method is to construct a measurement matrix C $\in R^{(p\times n)}$ of random samples first. The measurement matrix C is then used to compressed the full-state snapshot matrices after which mode decomposition is computed on the compressed representation of the original data. It is worth noting that the measurement matrix C $\in R^{(p\times n)}$ which is the cornerstone of compressed DMD is formed by drawing (p x n) independent random samples, where p stands for the number of samples randomly drawn. As such, the measurement matrix C is a kind of random sensing matrix whose number of samples (rows) randomly generated is equal to the value of p. To compress the full-state snapshot matrices, the algorithm uses the measurement matrix C to generate compressed matrices whose number of rows are randomly picked from the full-state snapshot matrices without replacement and are equal to the value of p. Even though, with compressed DMD, computational cost is reduced compared to standard DMD, nevertheless, a noticeable drawback with compressed DMD is that, the compression of the full-state snapshot data is performed by a measurement matrix whose components are randomly generated, as such, due to the randomness nature of the measurement matrix, anytime the algorithm is ran, its components change and in turn, the overall result changes.

This paper presents a new technique known as Dynamic Mode Learning (DML) that is developed based on extracting the characteristics of the full-state data and using the learned and extracted features to develop a model that accurately capture dynamics and provide behavior analysis of the full-state system. To achieve this, a simple and fast learning system that extracts important features of the full-state data is developed first. The extracted features that contain the characteristics and underlying dynamics in the full-state data are then used to reduce the full-state data, after which SVD and dynamic modes are computed on the resulting reduced data. Finally, the full-state system is reconstructed from the eigenvalues and dynamic modes of the reduced system. As such, this approach is aimed at developing a model that accurately capture the underlying dynamics in the full-state system.

The remaining parts of this paper are organized as follows: Section two presents materials and methods used to develop the proposed DML model. Section three presents the formulation and implementation of the reservoir model whose data were used to validate the proposed DML model. Section four presents experimental results and discussion. Section five ends the paper with conclusion and focus for future work.

II. Materials And Methods

This section presents the details of the proposed DML model development. This includes a description of the learning system that extracts features of the full-state data and a detailed description of the proposed DML method working principle.

A. Feature Extraction

The proposed DML method is developed based on the idea of future extraction in high-dimensional data. The main idea behind the proposed DML model is learning and extracting important features of the full-state data first, and then using the extracted features to reduce the full-state data. In contrast to the traditional compressed DMD that constructs a measurement matrix from random samples and uses it to compress the full-state data, we first developed a simple learning system that extracts and maps the underlying features of the full-state data to a low-dimensional space and saves them as a set of extracted features. This set of extracted features is then use to reduce the full-state data after which dynamic modes are computed on the reduced data. Suppose we present the full-state data which contains n samples as input data X and project it using $g(XW_i + b_i)$, to transform it into ith extracted features, F_i, where $g: R^n \rightarrow R^n$ represents the transfer function, W_i stands for the weight matrix, and b_i stands for the bias vector. The concatenation of all the first i sets of extracted features is denoted as $F^i \equiv [F_1, \dots, F_i]$. If $L \in R^{k\times n}$ is defined as the output matrix where k is the number of nodes in the extraction layer, then for any n samples of the full-state data the learning system generates k samples of extracted features which can be expressed as follows

$$F_i = g(XW_i + b_i), \quad i = 1, \dots, k \tag{1}$$

where W_i and b_i are randomly generated from the normal uniform distributions within the interval of [-1, 1]. For the transfer function, log-sigmoid is chosen to establish the samples of extracted features and to improve the generalization ability of the system, L_2-norm weight regulirizer is added to the transfer function. The architecture of the feature extraction system that describes its working process is shown in Figure 1. Once the system extracts the features and placed them in $L \in \mathbb{R}^{k\times n}$, the extracted features are then used to reduce the full-state data after which the dynamic modes are computed on the reduced data. As such, computationally expensive SVD on the high-dimensional original data is bypassed, and is rather, performed on the reduced snapshot data. Also, it is worth noting that the number of samples/rows in the reduced snapshot data will be equal to the number of samples/rows in the output layer of the feature extraction system (that is, the value of k).

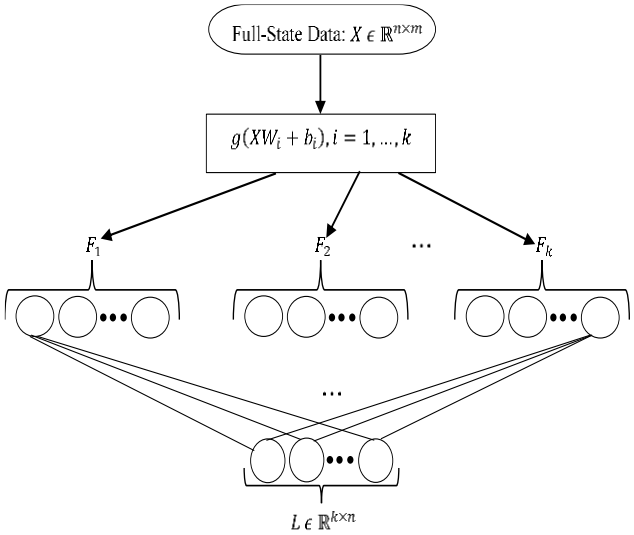

Figure 1: Illustration of the working process of the feature extraction system

B. Dynamic Mode Learning Technique

In this section, a detailed description of the proposed DML method is presented. Suppose that a high-dimensional data $X \in R^{n\times m}$ is generated and collected from numerical simulations or experiments, the features of the original data X are first extracted using (1) and placed in a low-dimensional matrix, $L \in R^{k\times n}$. Next, the original data are arranged in two snapshot matrices as follows

$$X = \begin{bmatrix} | & | & | \\ x_1 & \ldots & x_{m-1} \\ | & | & | \end{bmatrix}, \tag{2a}$$

$$X' = \begin{bmatrix} | & | & | \\ x_2 & \ldots & x_m \\ | & | & | \end{bmatrix}. \tag{2b}$$

These two matrices have large number of rows than columns, that is, $n \gg m$ and consist of the states of the system and their columns were captured in equal-spaced time, with a time step Δt. Each $X_i = X(i\Delta t)$ is a vector with components c, as such, $X, X' \in \mathbb{R}^{c\times(m-1)}$. Dynamic mode learning method attempts to construct a linear dynamical system

$$X_{t+1} \approx AX_t \tag{3}$$

and thus

$$X' \approx AX. \tag{4}$$

It is interesting to realize that the least-squares solution of (4) leads to

$$A = X'X^{\dagger} \tag{5}$$

here, $X^{\dagger}$ stands for the Moore-Penrose pseudo-inverse of X. To get an estimate of matrix A, DML method uses the matrix that contains the extracted features to reduce the snapshot matrices as follows

$$X_R = L * X \tag{6a}$$

$$X'_R = L * X', \tag{6b}$$

then Singular Value Decomposition (SVD) is computed on the reduced snapshot matrix X_R as follows

$$X_R = USV^* \tag{7}$$

where $U \in \mathbb{R}^{n\times r}, S \in \mathbb{R}^{r\times r}, V \in \mathbb{R}^{m\times r}$ and $r \leq m$ stands for the rank of truncating the snapshot matrix X_R. The columns of U are referred to as POD modes, and they satisfy $U^* \cdot U = I$. In the same manner, columns of V are orthonormal, and satisfy $V^* \cdot V = I$. The diagonal of S contains the singular values of matrix X_R. The full matrix A then can be acquired by solving the pseudo-inverse of X_R as follows

$$A = X'_R VS^{-1}U^*. \tag{8}$$

In DML method, the interest is in the leading eigenvalues r and eigenvectors of A, for this reason, A is therefore projected onto the POD modes in U as follows

$$\tilde{A} = UX'_R VS^{-1} \tag{9}$$

It is worth mentioning that the solution of (9) is the least squares fit of A. The point here is, instead of working on the full matrix A, we directly computed the reduced-order approximation $\tilde{A}$ in such a way that the full matrix A and the reduced-order matrix $\tilde{A}$ have the same nonzero eigenvalues. Thus, the spectral decomposition of $\tilde{A}$ can be computed as

$$\tilde{A}H = H\Lambda. \tag{10}$$

Here, the DML eigenvalues are the elements of the diagonal matrix Λ and the eigenvectors of $\tilde{A}$ are represented by the columns of H. The dynamic mode ϕ can then be obtained by using W of the reduced system and the snapshot matrix X' as follows

$$\phi = X'VS^{-1}H. \quad (11)$$

C. Formulation of the Multiphase Gas Injection Reservoir Model

The gas injection reservoir model used as the reference model is implemented using the benchmark data of the first SPE Comparative Solution Project [19]. The SPE 1 benchmark is a description of a depletion problem with gas injection in a $10 \times 10 \times 3$ reservoir model with injector and producer wells completed in diagonally opposite corners. The gas injection well was completed in layer 1 and is located at grid point (1, 1) while the producing well was completed in layer 3 and is located at grid point (10, 10). The reservoir has a porosity of 0.3 uniformly distributed within the grid blocks, whereas the permeability is heterogeneous with values 500, 50, and 200 mD in layers 1, 2, and 3, respectively with respective thicknesses of 20, 30, and 50 ft. Initially, the reservoir is undersaturated with a constant pressure field in each layer, a homogeneous mixture of water ($S_w = 0.12$), and oil ($S_o = 0.88$), and zero free gas $(S_g = 0.0)$ throughout the reservoir model. Detailed data that describes the petrophysical and PVT properties as well as the relative permeability of the reservoir model can be found in [45]. As mentioned above, the geological structure of the gas injection reservoir model is a three-dimensional formation that consists of 10×10 grid blocks in the $x - y$ dimension and 3 layers in the z dimension. Thus, the reservoir model consists of $10 \times 10 \times 3 = 300$ grid blocks (cells). To generate the reservoir's data at any grid cell in time, the reservoir is numerically simulated for 1200 days in 120 time-steps. The result of the numerical simulation yielded a 300×120 matrix that contains 36,000 records in the spatiotemporal database. Each record in the spatiotemporal database contains information about the reservoir's static and dynamic parameters in a single grid block in a given run and given time step. For the purpose of this study, the pressure field data of the gas injection reservoir model is retrieved and utilized as benchmark data by the proposed DML model and comparison algorithms.

III. Experimental Results And Discussion

In this section, results of experiments for the purpose of verifying the proposed DML model are presented. To validate the effectiveness of the proposed model, experiments are performed on the pressure filed data generated from simulating the multiphase gas injection reservoir model. To proof the performance of the proposed DML model, its ability to reconstruct reservoir pressure field and approximate average reservoir pressure change is compared to existing mainstream standard and compressed DMD methods. Each of the three algorithms mentioned above was evaluated using 10, 15, 20, and 25 number of modes at a time. Two classic statistical quantities are selected namely, mean square error (MSE), and root mean squared error (RMSE) to evaluate the performance of the three algorithms. The aim of performing the experiments is to measure the errors generated by each algorithm on each of the tasks performed on the reference data.

Recall that the proposed DML algorithm uses the extracted features representation matrix $L \in \mathbb{R}^{k \times n}$ to reduce the full-state snapshot matrices after which mode decomposition is computed on the reduced data. In the proposed DML algorithm, the number of rows/samples in the reduced snapshot data is equal to the number of rows/samples in $L \in \mathbb{R}^{k \times n}$ which in turn, is equal to the number of nodes in the output layer (value of k). Recall also, that in compressed DMD, a measurement matrix $C \in \mathbb{R}^{p \times n}$ is used to compress the snapshot matrices after which mode decomposition is applied on the compressed data. In compressed DMD, the number of rows/samples in the compressed snapshot data is equal to the number of rows/samples in the measurement matrix which is the number of random samples generated by the measurement matrix (value of p). For fair comparison, we set the value of p for the compressed DMD method, and k for the proposed DML model to 25 each. Thus, $C \in \mathbb{R}^{p \times n}$ and $L \in \mathbb{R}^{k \times n}$ have the same number of rows/samples so that the resulting compressed snapshot matrices that the compressed DMD algorithm utilized have the same rows/samples with the reduced snapshot matrices utilized by the proposed DML algorithm. For the standard DMD algorithm, no compression or reduction of the full-sate data is needed, thus, SVD is applied directly on the full-state snapshots.

A. Experiment on Pressure Field Data of Gas Injection Reservoir Model

First experiment is performed on the pressure field data generated from simulating the gas injection reservoir model. The original data contains 300 samples generated in 120 time steps. Thus, the original data is represented in a 300×120 matrix. To gain an insight of how the pressure evolves within the reservoir grid cells as the reservoir produces while natural gas is being injected, plot of the reservoir's average pressure change over time is shown in Figure 2 and the reservoir's pore pressure evolution for some selected days of the simulation period is shown in Figure 3.

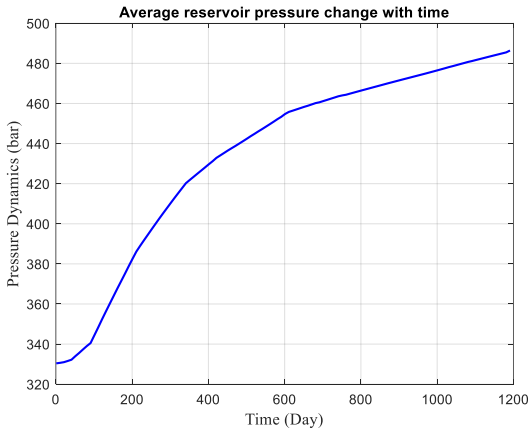

Figure 2: Plot of average reservoir pressure change with time for gas injection reservoir simulation

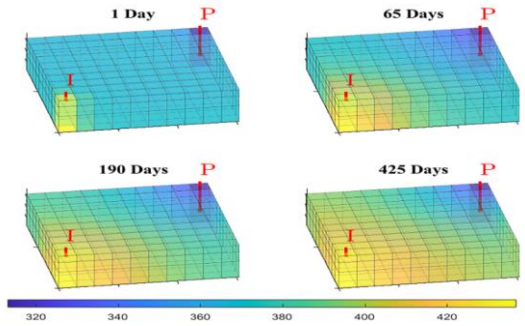

Figure 3: Reservoir pore pressure variations for some selected days of gas injection reservoir simulation

Projecting the original data on the feature extraction system described in section II(A) and setting the value of k to 25, not only reduces the dimension of the data, but also captures the characteristics of the data and stored them in 25×300 matrix. This matrix is then used to reduce the full-state 300×120 matrix after which dynamics modes are computed on the reduced data. Table I presents the experimental results performed on the pressure field data of the gas injection reservoir model by the three algorithms using 10, 15, 20, and 25 number of modes. From Table I, it can be observed that on general note, pressure field reconstruction and average reservoir pressure approximation errors for all the three methods decrease as the number of modes increases. In other words, accuracy for all the three methods improves as the number of modes increases. However, it is worth noting that the proposed DML model exhibits better performance with all the number of modes over standard and compressed DMD methods. In particular, it can be observed that with 15 modes, the proposed DML model is able to record < 1 MSE and < 1 RMSE for both pressure field reconstruction and average reservoir pressure approximation while standard and compressed DMD methods have not recorded such results even with 25 modes. Overall performance ranking for this first experiment is DML model, followed by standard DMD, then compressed DMD.

TABLE I. COMPARISON OF ALGORITHM PERFORMANCE ON PRESSURE FIELD DATA OF GAS INJECTION RESERVOIR USING DIFFERENT NUMBER OF MODES

	Reservoir's Pressure Field Reconstruction		**Average Reservoir Pressure Approximation**	
	MSE	**RMSE**	**MSE**	**RMSE**
		10 Modes		
Standard DMD	26.7476	5.1718	25.8519	5.0845
Compressed DMD	27.2875	5.2237	26.8314	5.1799
DML model	10.5455	3.2474	10.2619	3.2034
		15 Modes		
Standard DMD	9.1087	3.0181	8.9316	2.9886
Compressed DMD	12.3192	3.5099	11.8441	3.4415
DML model	0.3289	0.5735	0.2873	0.5360
		20 Modes		
Standard DMD	6.8884	2.6246	6.7435	2.5968
Compressed DMD	7.0209	2.6497	6.7113	2.5906
DML model	0.4286	0.6547	0.3841	0.6198
		25 Modes		
Standard DMD	2.2879	1.5126	2.1944	1.4813
Compressed DMD	3.5791	1.8919	3.4204	1.8494
DML model	0.3093	0.5561	0.2706	0.5202

For the sake of visualization, plots that compare the prediction of average reservoir pressure over time by all the three algorithms using 15 number of modes on the pressure field data of the gas reservoir model is presented in Figure 4.

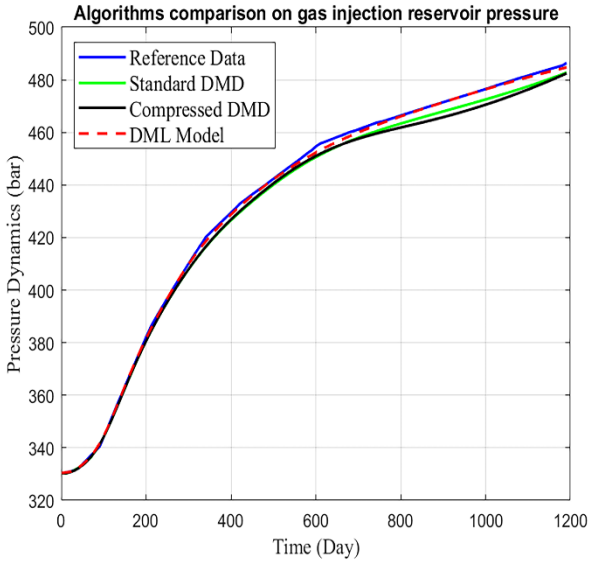

Figure 4: Comparison of algorithms performance for average reservoir pressure prediction on gas injection reservoir data

Figure 4 reports the effect of gas injection on the reservoir's pressure dynamics over the simulation period. It can be observed from Figure 4 that as gas is being injected through the injection well into the reservoir model, the formation pressure keeps raising above the initial pressure and thereby enhancing the recovery of oil through the production well. As for the ability of the three algorithms on the reservoir's pressure dynamics for the simulation period, it can be noticed that from day 1 to approximately 300 days, all algorithms perform well to capture the reservoir's pressure dynamics. However, after 300 days, standard and compressed DMD algorithms begin to lose track of the reservoir's pressure dynamics and their deviations continue to increase to the end of the simulation period. On the other hand, it can be observed that the proposed DML algorithm is able to capture the reservoir's pressure dynamics from day 1 to the end of the simulation period with a slight deviation at around day 570 to day 690.

In Figure 5, comparison is made between the reference data of the gas injection reservoir pore pressure variations for some selected days and the ones reconstructed by the proposed DML model using 15 number of modes. It can be noticed that the reservoir's pore pressure variations reconstructed by the proposed DML model is in good agreement with the reference data. Furthermore, comparisons

are made between the true eigenvalues and the ones generated by the proposed DML model using 15 modes on the pressure field data of the gas injection reservoir model and the result is presented in Figure 6. It can be observed from Figure 6 that the eigenvalues generated by the proposed DML model match the true eigenvalues.

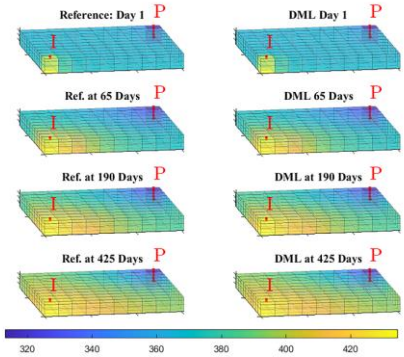

Figure 5: Comparison of gas injection reservoir pore pressure variations for some selected days between reference data and DML model

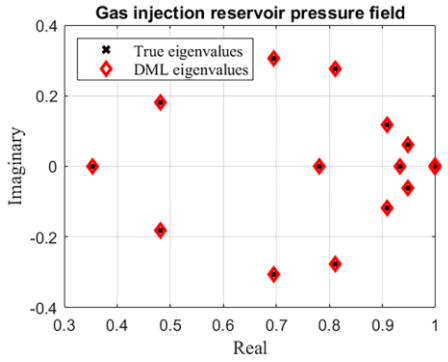

Figure 6: Comparison of reference eigenvalues and the ones generated by the proposed DML model for gas injection reservoir pressure data

IV. Conclusion

In this paper, DML method that efficiently learns and decomposes dynamic modes in high-dimensional data is proposed. Developed based on the idea of dimensionality reduction and feature extraction, the proposed model eliminates the computationally expensive SVD of standard DMD and addresses the issue of random sampling in compressed DMD. Performance of the proposed model is validated on pressure field data generated from direct numerical simulations of a benchmark multiphase reservoir model. Ability of the proposed model to keep track of reservoir pressure dynamics and reconstruct reservoir's pore pressure variations are compared to mainstream algorithms namely, standard and compressed DMD methods. Experiments performed on the pressure field data reveal that the proposed DML model exhibits better performance with the least prediction and reconstruction errors over standard and compressed DMD methods. Furthermore, eigenvalues generated by the proposed DML model are shown to match the true eigenvalues of the reference data utilized in this study. This shows that it is possible to apply the proposed model to high-dimensional systems in porous media, fluid dynamics, and to other spatiotemporal measurements. As focus for future research, consideration will be given to combining the proposed technique with other DMD innovations such as DMD with control.

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