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# **Demonstration of partially-automated assessment approach to create individualised, open-ended modelling worksheet**

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# **Demonstration of partially-automated assessment approach to create individualised, open-ended modelling worksheet**

Partially-automated assessment is implemented via the ‘Printable worksheet’ mode in the Numbas e-assessment system to create a mathematical modelling worksheet which is individualised with random parameters but completed and marked as if it were a non-automated piece of coursework, preserving validity while reducing the risk of academic misconduct via copying and collusion. A simple modelling scenario is used for this demonstration. A cylindrical tank of water is draining through a small hole in its base, with the size of the tank, size of the hole and initial volume of water randomised so that the details are different for each student. Students are guided through deriving a model of the movement of water out of their version of the tank, asked to compute the expected time for it to empty and asked to discuss some aspects of the model and their findings in a discursive narrative report.

Keywords: partially-automated assessment, assessment, e-assessment, computer-aided assessment, mathematical modelling, authentic assessment

## **1. Introduction**

Rowlett (2022) outlined a partially-automated approach to assessment and argued for its use in principle. The purpose of this article is to demonstrate a practical application of this approach by outlining the implementation of a simple mathematical modelling question written using the e-assessment system Numbas.

A partially-automated assessment uses questions set by an automated question generator which are completed by students and marked by hand as if they were a non-automated piece of coursework. Here ‘by hand’ may involve the use of a computer, for example students might submit handwritten or typeset work to a Virtual Learning Environment to be marked in an online grader tool; the crucial aspect is that answers are not marked by an automated process. This allows the advantages of automated question-setting, chiefly that questions may be individualised for different students as a deterrent

to academic misconduct via copying and collusion. At the same time, a partially-automated approach avoids several disadvantages of automated marking. The question is not limited to what can be marked by computer, so the assessor does not need to anticipate the marking engine and the task can be more open-ended. In addition, students are not required to interface with the marking system. Other methods of increasing the reliability of an assessment with respect to academic misconduct are examination conditions and e-assessment, which may impact validity by limiting what can be assessed. A partially-automated approach has the potential to maintain validity while increasing reliability by decreasing risk of academic misconduct.

In this article, a simple modelling question is designed for assessment using the partially-automated approach. The aim is to show the process for doing this using the Numbas system, though certainly other software could be used, and to demonstrate that even quite a simple question can benefit from partially-automated assessment.

## 2. Modelling scenario

Figure 1 shows a cylindrical tank containing water at depth  $h(t)$ . Consider the situation where the diameter of the tank is  $d_t$  and the initial height of the water is  $h(0) = h_0$ .

Then the volume of water at time  $t$  is given by

$$V = \pi \left( \frac{d_t}{2} \right)^2 h = \frac{\pi}{4} d_t^2 h.$$

[Figure 1 around here]

By differentiating with respect to  $t$ , we obtain the rate of change of volume of water as

$$\frac{dV}{dt} = \frac{\pi}{4} d_t^2 \frac{dh}{dt}.$$

Water drains from the tank through a circular hole of diameter  $d_h$ . Torricelli's Law says the speed of fluid outflow from a hole is  $v = \sqrt{2gh}$ , where  $g$  is the acceleration due to gravity and  $h$  is the depth of the fluid at that point. Multiplying this speed by the area of the hole, we obtain a formula for outflow from the tank as

$$\frac{\pi}{4} d_h^2 \sqrt{2gh}.$$

The rate of change of volume in the tank is given by the difference between water inflow and outflow. As there is no water flowing into the tank, we have

$$\frac{dV}{dt} = -\frac{\pi}{4} d_h^2 \sqrt{2gh}.$$

( 2 )

Equating ( 1 ) and ( 2 ), we obtain an ordinary differential equation:

$$\frac{\pi}{4} d_t^2 \frac{dh}{dt} = -\frac{\pi}{4} d_h^2 \sqrt{2gh}.$$

Separating variables, we get

$$\int \frac{1}{\sqrt{h}} dh = - \int \sqrt{2g} \left( \frac{d_h}{d_t} \right)^2 dt$$

$$\sqrt{h} = \sqrt{h_0} - \frac{\sqrt{2g}}{2} \left( \frac{d_h}{d_t} \right)^2 t.$$

( 3 )

We might observe that the time taken for the tank to empty depends on the initial height of the water and the ratio of the diameters of the tank and the hole.

One implication of equation ( 3 ) is that we can say the tank is empty ( $h = 0$ ) when

$$\sqrt{h_0} - \frac{\sqrt{2g}}{2} \left( \frac{d_h}{d_t} \right)^2 t = 0,$$

i.e. when

$$t = \sqrt{\frac{2h_0}{g}} \left( \frac{d_t}{d_h} \right)^2.$$

( 4 )

### 3. Preparing for individualisation

The amount of scaffolding required depends on the teaching context. Here we assume students have covered basic first order differential equation (ODE) methods and modelling concepts but require some guidance in attempting this specific question. First the scenario is explained and randomised parameters given. In the following text, the model parameters  $d_t$ ,  $d_h$  and  $h_0$  will be given as randomised parameters, i.e. specific numbers that are different for each worksheet; these are indicated in the question text below by enclosing them in braces.

*A cylindrical tank has diameter  $\{d_t\}$ m and contains water at a height  $h(t)$ .*

*Water drains from the tank through a circular hole of diameter  $\{d_h\}$ cm. At time  $t = 0$  the height  $h$  of the water in the tank is  $\{h_0\}$ m. Your employer is interested in what proportion of an hour the water will take to drain completely from the cylinder.*

There are three randomised parameters in this question:

- $d_t$ : a range of tank sizes might be realistic for a water tank; here we choose to vary this in steps of 0.1m from 0.5m to 2.5m. We set the randomisation to exclude two numbers in this range because these values would simplify the derivations and this would create uneven levels of difficulty between students. Thus we set  $d_t \neq 1$  and  $d_t \neq d_h$ .

- $d_h$ : this parameter is specified in cm in the question and should be small compared with the size of the tank; we vary this in steps of 0.1cm from 1.1cm to 1.9cm.
- $h_0$ : realistically, the level of water would be on comparable scale to the diameter of the tank; here we vary this in steps of 0.1m from 1.1m to 2.9m, excluding 2m because again it would simplify calculations.

The question will then proceed with a series of parts as scaffolding through the model derivation. First, we prompt the student to use the information in the question to derive equation ( 1 ).

*(a) State a formula for the volume of water in the tank,  $V(t)$ , in terms of  $h$ .*

*Differentiate this to obtain a formula for  $\frac{dV}{dt}$ .*

Next, we give information about Torricelli's Law and prompt the alternative formula for  $\frac{dV}{dt}$  given in equation ( 2 ).

*(b) Torricelli's Law says the speed of fluid outflow from a hole is  $v = \sqrt{2gh}$ ,*

*where  $g$  is the acceleration due to gravity and  $h$  is the depth of the fluid at that point.*

*Use this information and the fact that*

$$\text{volume of outflow} = \text{area of hole} \times \text{speed of outflow}$$

*to state a formula for  $\frac{dV}{dt}$ .*

In part (c), we prompt the combination of these formulas for  $\frac{dV}{dt}$  to derive equation ( 3 ).

*(c) Equate your answers from (a) and (b), obtaining an ODE in  $h$ . Solve this to get a formula for  $\sqrt{h}$  in terms of  $t$ .*

Having derived this equation, we probe for understanding by asking the student interpret their answer.

*(d) Briefly outline the parameters of the model that impact on the time taken for the tank to empty.*

Now we ask the student to derive equation ( 4 ) and use it to compute when the tank will empty. The students are given  $t_{h=0}$ , the time when  $h = 0$  in seconds. This is presented as a number automatically calculated from the randomised question information, indicated below using braces.

*(e) Show that the tank is empty after  $\{t_{h=0}\}$ s.*

Finally, following the mention of ‘Your employer’ in the question prompt, we ask for a brief report giving findings. This open-ended, narrative element could be enhanced greatly; a simple version is included here to show the possibility to ask for content an e-assessment system would not be able to mark.

*(f) Write a brief report sharing your findings, making sure to use appropriate units and accuracy.*

The partially-automated approach gives access to individualisation in contexts where automated marking is not suitable. Here, we note that part (e) in particular requires specific numbers and this is the source of individualisation. Parts (a)-(c) could operate as algebra without the randomised parameters, but it may be there is a value in the student working through the model derivation using a specific scenario. There are also parts of the question that would be difficult to mark through automation. The model derivation will produce extended mathematical work that would be hard to assess fully by inferring from an incorrect answer to part (e) alone. Parts (d) and (f) are discursive and would be extremely difficult to mark authentically using automated marking.



#### 4. Partially-automated assessment question

Numbas is a specialist mathematical e-assessment system (a description of the software is given by Graham, 2020). Numbas allows for the question content to be displayed in one of several different themes, to customise the look or functionality of an assessment. The theme used here is ‘printable worksheet’, which allows the creation of individualised worksheets.

First we create a question. In the question statement (see figure 2) we give the introductory prompt. We can include mathematics in LaTeX format which will be rendered using MathJax (Cervone, 2012), for example this is used in figure 2 in ‘height  $\backslash(h(t))$ ’. We can also include parameters to be replaced by randomised numbers in the question when it is displayed to students, which are called ‘variables’ in the software. These are included within braces, as in the reference to the variable ‘dt’ in ‘diameter {dt}m’ in figure 2 and are replaced with randomised values by Numbas (from the ranges in section 3) when the worksheets are generated.

[Figure 2 around here]

We can then add ‘parts’, which are prompts for the student to answer. We use the ‘Extension’ part type, as we don’t require the software to take input from the student. For example, in figure 3 we use this to prompt our part (a) (to derive equation (1)). Variables can be used here, including ones that were not used in the original question prompt, for example in figure 4 we use a variable ‘{empty}’ for the value in part (e) referred to as  $t_{h=0}$  above.

[Figure 3 around here]

[Figure 4 around here]

Moving to the ‘Variables’ screen, we find that Numbas has collected the variables we have used in writing the question but these are not defined. Simple random

variables can be defined by inputting values into a template such as the use of ‘Random number from a range’ in figure 5.

[Figure 5 around here]

More complicated specifications can be written using code called JME. For example, in figure 6 the variable ‘dt’ is defined using the code `dec(random(5..25 except [10,10*dh])/10)`. The function `random` defines a randomised integer and has the useful `except` keyword which allows the exclusion of some part of the range. Here we generate a number from 5 to 25 which cannot be 10 or 10 times the value of `dh`. We divide this by 10 to get a number in the desired range, and the function `dec` renders this as a decimal rather than a fraction.

[Figure 6 around here]

Variables can be defined using other variables, for example in figure 7 we create the variable ‘empty’ which depends on the variables ‘dh’, ‘dt’ and ‘h0’ and implements equation ( 4 ) (note that since ‘dh’ is given in the question in cm, there is an additional factor of  $10^{-2}$ ).

[Figure 7 around here]

Having written this template for Numbas to generate the question, we can write a template in a similar way for Numbas to generate an individualised marking sheet. This could contain answers, where these can be computed automatically, or else other information that would help the marker. This information is written in the ‘Advice’ section of the question on Numbas.

In figure 8, we begin by giving the expected answer for part (a), the derivation of equation ( 1 ). Note that in writing the LaTeX expressions a function `\var{ }` is available which can be used to refer to the Numbas variables and to do arithmetic. Here we calculate the value of  $\left(\frac{d_t}{2}\right)^2$  and use this number in the answer. Note also that

Numbas provides a live preview of LaTeX as it is typed and here the contents of `\var{ }` are rendered in braces using a different colour.

[Figure 8 around here]

In Numbas, questions are arranged into an ‘exam’, which collects them and allows control over interface settings. Thus, the final step to creating the worksheet is to place the question into an ‘exam’. In figure 9 we add the question to a newly created exam. In order to render this ‘exam’ as a printable worksheet, we must adjust the ‘Interface theme’ setting on the ‘Display’ page, setting this as ‘Printable worksheet’ (see figure 10).

[Figure 9 around here]

[Figure 10 around here]

To generate worksheets, we use ‘Test Run’ which renders the exam. This displays a simple menu (shown in figure 11) where we can ask for the number of worksheets we require. We can toggle between viewing the question and answer sheets. An ID number is used for each worksheet to assist the marker in locating the relevant answer sheet; this can also be used to regenerate a particular worksheet since the ID number defines the randomisation. Once we are happy, we can print the worksheets to PDF. Figure 12 shows an example individualised worksheet and its corresponding answer sheet.

[Figure 11 around here]

[Figure 12 around here]

An alternative to PDF worksheets is available when Numbas is integrated with a Virtual Learning Environment, either through its LTI (Learning Tools Interoperability) Tool, or a SCORM player. The questions developed for worksheets can also be presented in Numbas’ standard interface: here, questions are randomised at the start of a

student's attempt, and are therefore individualised in the same way as worksheets.

Solutions could be gathered in a number of ways, for example students could be asked to prepare and upload their solution to a Virtual Learning Environment.

This approach has the advantage that written questions could be mixed with questions that are automatically marked, allowing marking time to be efficiently focussed where it is most effective. At Newcastle University, this approach has been taken to deliver 'hybrid' assessments (some auto-marked, some hand-marked), both as coursework, for which students upload their solutions, and as exams, where students complete some questions in a standard exam booklet.

## **5. Conclusion**

The aim of this article was to demonstrate the process of creating an individualised worksheet as a partially-automated assessment using Numbas. A simple modelling scenario was encoded in an individualised question and this was rendered using Numbas 'Printable worksheet' mode. The worksheets generated could be printed and given to students (or distributed as PDFs). Work would be completed 'by hand' (could be typeset or electronic annotation) for marking by a human marker. A comparable worksheet could be created without the randomised parameters which would be less resistant to copying and collusion as a take-home piece of coursework. A pure e-assessment version of this question would not be able to ask the authentic modelling questions included here.

## **6. Disclosure statement**

Numbas is an open-source software project managed from Newcastle University by Graham and Lawson-Perfect.

## 7. References

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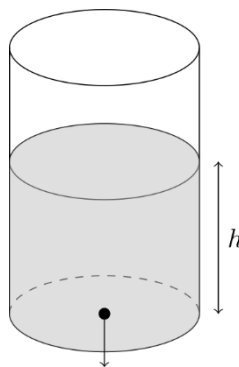


Figure 1. A cylindrical tank containing water at depth  $h(t)$  with a small hole in the bottom.

The screenshot shows the Numbas web interface. At the top is a blue header with the 'NUMBAS' logo, a search bar, and navigation links like 'Explore', '+ New', and 'Help'. Below the header, the user's workspace is shown with the title 'Peter's workspace' and a document icon next to the question title 'Water cylinder question'. A 'Status: Draft' button is on the right. On the left side, there are action buttons: 'Run', 'Make a copy', 'Delete', 'Download', and 'Add to a'. The main area is the 'Question statement' editor, which includes a rich text toolbar with options for text, bold, italic, underline, list, link, and image. The question text reads: 'A cylindrical tank has diameter  $\{d\}$ m and contains water at a height  $\{h(t)\}$ . Water drains from the tank through a circular hole of diameter  $\{d_h\}$ cm. At time  $\{t=0\}$  the height  $\{h\}$  of the water in the tank is  $\{h_0\}$ m. Your employer is interested in what proportion of an hour the water will take to drain completely from the cylinder.' To the right of the text is a small diagram of a cylinder. At the bottom, there is a note: 'Give any introductory information the student needs.'

Figure 2. Question statement input. Items in braces are randomised parameters.

Peter's workspace

## Water cylinder question

Status: Draft

**Part a) Extension**

Copy this part Replace this part with a gapfill Delete this part

Prompt

Ask the student a question, and give any hints about how they should answer this part.

Marking settings

Marking algorithm

Scripts

Statement

Parts

Part a) Extension

Alternative answers

Add an alternative answer

Steps

Add a step

Add another part

State a formula for the volume of water in the tank,  $V(t)$ , in terms of  $h$ . Differentiate this to obtain a formula for  $\frac{dV}{dt}$ .

Figure 3. Question part input for part (a).

Peter's workspace

## Water cylinder question

Status: Draft

**Part e) Extension**

Copy this part Replace this part with a gapfill Delete this part

Prompt

Ask the student a question, and give any hints about how they should answer this part.

Marking settings

Marking algorithm

Scripts

Statement

Parts

Part a) Extension

Part b) Extension

Part c) Extension

Part d) Extension

Part e) Extension

Alternative answers

Add an alternative answer

Steps

Show that the tank is empty after {empty}s.

Figure 4. Question part input for part (e).

NUMBAS
Search by name or keyword
Explore
+ New
Help

Peter's workspace

Water cylinder question

Status: Draft

Run
Make a copy
Delete
Download
Add to a queue
Add to your basket
Statement
Parts

Name

dh

Data type

Random number from a range

Value

A random number between 1.1 and 1.9 (inclusive) with step size 0.1

Description

Describe what this variable represents, and list any assumptions made about its value.

Ungrouped variables

Name	Type	Generated Value
dh	number	1.7
dt	decimal	dec("1.3")
h0	decimal	dec("1.3")
empty	number	3010.517

Add a variable

Regenerate values

Figure 5. Randomised parameter definition for ‘dh’, which corresponds to  $d_h$  in this article, using a random number template.

NUMBAS
Search by name or keyword
Explore
+ New
Help

Peter's workspace

Water cylinder question

Status: Draft

Run
Make a copy
Delete
Download
Add to a queue
Add to your basket
Statement
Parts

Name

dt

Data type

JME code

Value

dec(random(5..25 except [10,10\*dh])/10)

Description

Describe what this variable represents, and list any assumptions made about its value.

Ungrouped variables

Name	Type	Generated Value
dh	number	1.7
dt	decimal	dec("1.3")
h0	decimal	dec("1.3")
empty	number	3010.517

Add a variable

Regenerate values

Figure 6. Randomised parameter definition for ‘dt’, which corresponds to  $d_t$  in this article, using a JME code definition.

NUMBAS

Search by name or keyword

Q

Explore

+ New

Help

Peter's workspace

Water cylnder question

Status: Draft

Run

Make a copy

Delete

Download

Add to a queue

Add to your basket

Statement

Parts

Name

empty

Data type

JME code

Value

1  $\sqrt{2 \cdot h_0 / 9.81} \cdot (dt / (10^{-2} \cdot dh))^2$

JME function reference

Description

Describe what this variable represents, and list any assumptions made about its value.

Click to edit

Ungrouped variables

Name	Type	Generated Value
dh	number	1.7
dt	decimal	dec("2.4")
h0	decimal	dec("1.8")
empty	number	12073.7279896768

+ Add a variable

Regenerate values

Figure 7. Randomised parameter definition for ‘empty’, which corresponds to  $t_{h=0}$  in this article, using JME code with reference to other variables.

NUMBAS

Search by name or keyword

Q

Explore

+ New

Help

Peter's workspace

Water cylnder question

Status: Draft

Run

Make a copy

Delete

Download

Add to a queue

Add to your basket

Advice

Edit

Insert

View

Format

Table

Tools

$$\frac{dV}{dt} = \left\{ \left( \frac{dt}{2} \right)^2 \right\} \pi \frac{dh}{dt}$$

(a)  $V = \sqrt{(dt/2)^2} \pi h$

$\sqrt{\frac{\mathrm{d}V}{\mathrm{d}t}} \sqrt{\mathrm{d}t} = \sqrt{(dt/2)^2} \pi \frac{\mathrm{d}h}{\mathrm{d}t} \sqrt{\mathrm{d}t}$

Give a worked solution to the whole question.

← Variables

Move on when you're ready!

Settings →

Figure 8. Inputting the answer notes into the ‘Advice’ section.



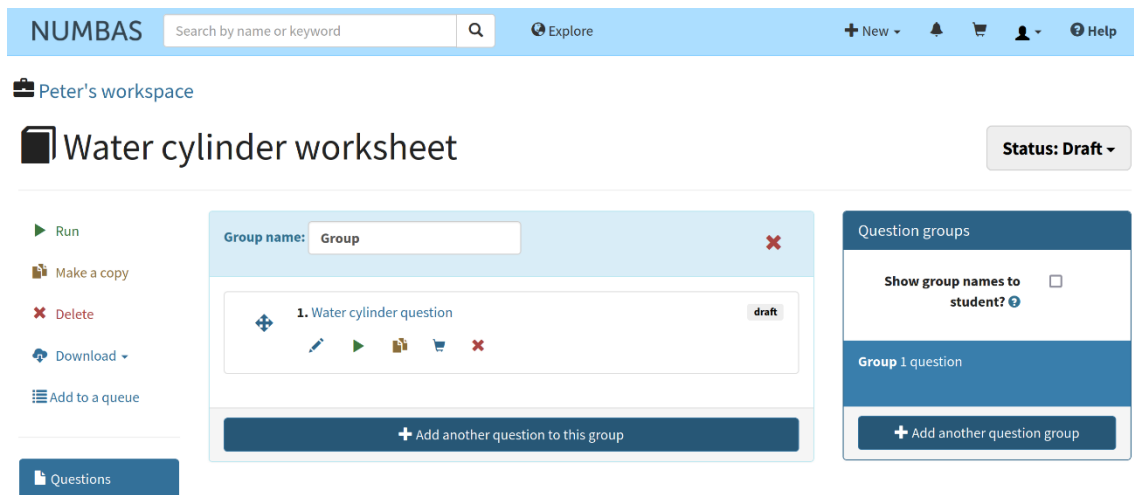


Figure 9. Adding question to an 'exam'.

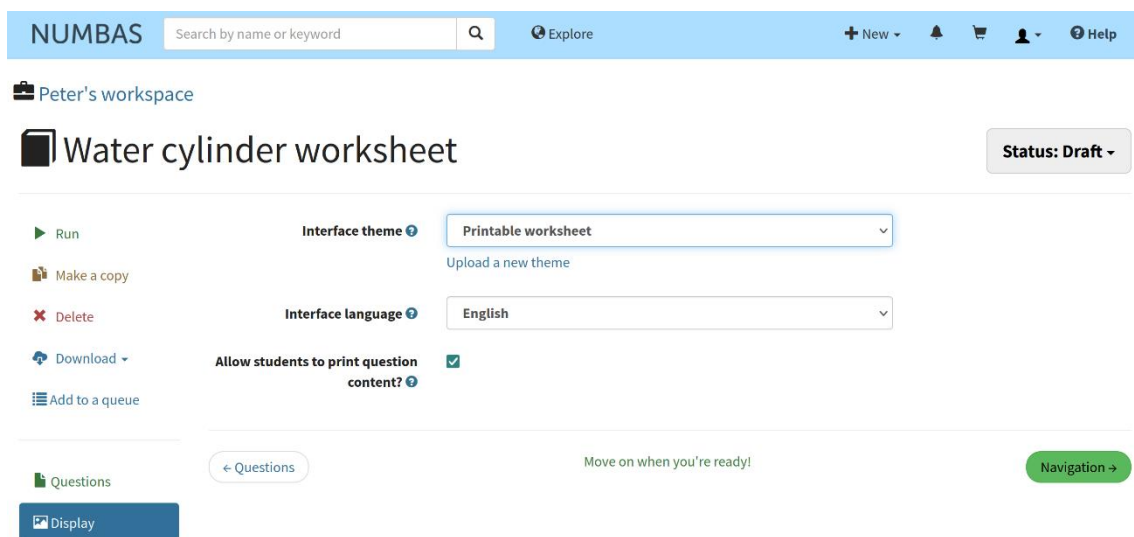


Figure 10. 'Display' settings page showing 'Interface theme' set to 'Printable worksheet'.

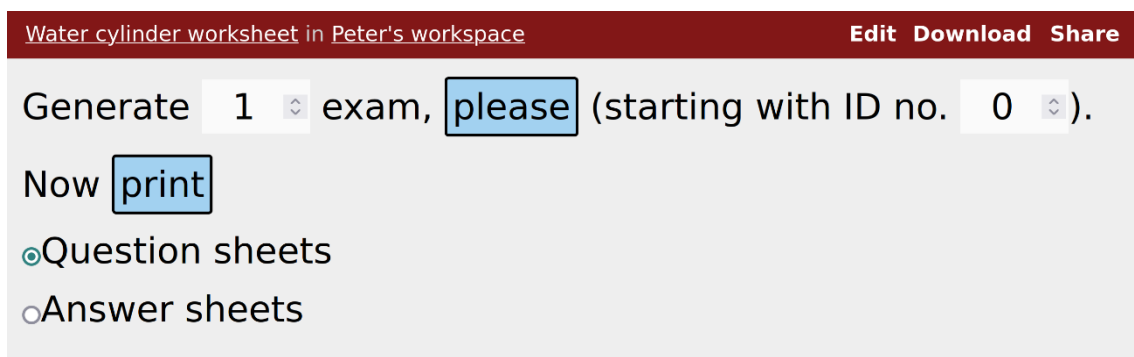
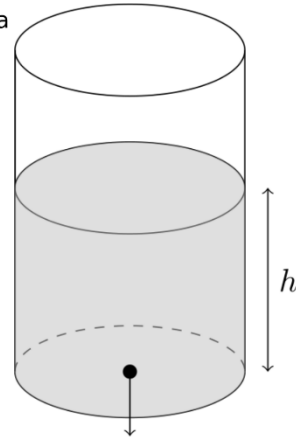


Figure 11. Worksheet generation interface.

ID: 0

## Question 1

A cylindrical tank has diameter 0.6m and contains water at a height  $h(t)$ . Water drains from the tank through a circular hole of diameter 1.1cm. At time  $t = 0$  the height  $h$  of the water in the tank is 2.2m. Your employer is interested in what proportion of an hour the water will take to drain completely from the cylinder.



a)

State a formula for the volume of water in the tank,  $V(t)$ , in terms of  $h$ . Differentiate this to obtain a formula for  $\frac{dV}{dt}$ .

b)

Torricelli's Law says the speed of fluid outflow from a hole is  $v = \sqrt{2gh}$ , where  $g$  is the acceleration due to gravity and  $h$  is the depth of the fluid at that point. Use this information and the fact that

$$\text{volume of outflow} = \text{area of hole} \times \text{speed of outflow}$$

to state a formula for  $\frac{dV}{dt}$ .

c)

Equate your answers from (a) and (b), obtaining an ODE in  $h$ . Solve this to get a formula for  $\sqrt{h}$  in terms of  $t$ .

d)

Briefly outline the parameters of the model that impact on the time taken for the tank to empty.

e)

Show that the tank is empty after 1992.549s.

f)

Write a brief report sharing your findings, making sure to use appropriate units and accuracy.

Figure 12. Worksheet generated with ID 0 showing question sheet.

ID: 0

## Question 1

(a)

$$V = 0.09\pi h$$
$$\frac{dV}{dt} = 0.09\pi \frac{dh}{dt}$$

(b)

$$\frac{dV}{dt} = -0.00003025\pi\sqrt{2gh}$$

(c)

$$\int \frac{1}{\sqrt{h}} dh = - \int \sqrt{2g} \times 0.0003361111 dt$$
$$\sqrt{h} = 1.4832396974 - \frac{\sqrt{2g}}{2} \times 0.0003361111t = 1.4832396974 - 0.0007443932t$$

(d) Should identify initial height of water and the ratio of the diameters of the tank and hole, or those parameters separately. Possibly also  $g$ .

(e)

$$1.4832396974 - 0.0007443932t = 0$$
$$t = 996.2743963139$$

(f) Gives the answer in minutes. Otherwise as marking scheme.

Figure 13. Worksheet generated with ID 0 showing answer sheet corresponding to the question sheet shown in figure 12.