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Mathematics education and the living world: a dialogic response to a global crisis

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Abstract:

Life on Planet Earth is changing in response to the actions of humans to a degree that can be considered a crisis. Mathematics educators have not yet responded adequately to the challenges of this crisis. We propose a relational, dialogic perspective on mathematics education through which mathematics educators can understand and enact their responsibilities. A dialogical stance leads to an ethics calling on our answerability and responsibility in relation to the living world. We use this perspective to reflect on previous research in mathematics education that seeks to respond to environmental issues. We look back to consider work that has addressed the relationship of mathematics and of mathematics education to the living world and draw out links to a dialogical stance. We conclude with questions for research and practice for mathematics education to respond to the ecological crisis.

Keywords:

Mathematics education, climate crisis, ecosystem, living world, dialogism, ethics

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1.0 Introduction

Life on Planet Earth is changing in response to the actions of humans to a degree that can be considered a crisis, at least for humans and many other species. Hardly a day goes by now without the publication of further evidence of this crisis, to add to the overwhelming number of studies and reports, that another aspect of the planet's ecosystem is changing in some unprecedented, problematic, disturbing and upsetting way. Human activity is changing Earth on a planetary scale. One recent study has found that the stratosphere is shrinking because of carbon emissions (Pisoft *et al*., 2021). Another has found that the Earth's axis has shifted because of melting ice and the emptying of underground aquifers (Deng *et al*., 2021). The latest reports from the Intergovernmental Panel on Climate Change highlight impacts on global climate systems, such as more frequent and more severe extreme weather events affecting all regions of the world, rapidly declining ice-fields, and global and accelerating increases in sea-level and ocean acidity (IPCC, 2021).

When it comes to the myriad forms of life on Earth, the reports of the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services have concluded that "biodiversity–the diversity within species, between species and of ecosystems–is declining faster than at any time in human history" (IPBES, 2019, p. 10). One quarter of all species groups, comprising around 1 million distinct species are estimated to be in danger of extinction in the coming decades (IPBES, 2019). The global biomass of wild mammals has declined by 82% since the rise of humans (IPBES, 2019, p. 25) and similar declines are apparent in other animal and plant groups. According to the report, most of the planet's ice-free land (72%) cannot be considered wild, in the sense that “natural processes are impaired by human activities to a significant degree” (IPBES, 2018, p. 23).

The IPBES reports paint a depressing portrait of the widespread and accelerating degradation of the planetary ecosystem. They are also quite clear that human activity is driving this degradation, through over-exploitation of animals and plants (especially harvesting, logging, hunting and fishing), agriculture, climate change, pollution, the spread of invasive species and population growth (IPBES, 2019, pp. 13–14). The reports also point out how bad all of this is for humans. That this constitutes a global ecological crisis is now widely recognised, including by the United Nations[1].

What does this global ecological crisis have to do with mathematics education? Mathematics clearly has a significant role in describing, predicting and communicating what is happening (Barwell, 2018). Moreover, mathematics is crucial to the functioning of the technological consumer capitalism that has contributed to this degradation. Supply chains, industrial production, transportation systems, communications and agriculture–every system that makes possible the way of life of most people on Earth–involve significant, often invisible, mathematics and often reinforcing or creating structural biases, discrimination and forms of oppression (see, for example, Criado Perez, 2019; Noble, 2018; O’Neil, 2017).

Given the scale of the crisis, and related crises such as climate change, and the role of mathematics in these crises, it continues to surprise us how little attention environmental,

[1] See https://www.un.org/en/un75/climate-crisis-race-we-can-win?

ecological or sustainability issues have received within the field of mathematics education[2]. There is, nevertheless, a small but growing body of work going back around a decade. If there is a theme running through this work, it is that mathematics education (teaching, learning, curriculum, assessment, etc.) needs to be completely rethought in response to the growing crisis, not simply in terms of 'content', based on new epistemologies, ontologies, ethical frameworks and pedagogies (see, in particular, Boylan, 2017b; D'Ambrosio, 2010; Gutiérrez, 2017). In this sense, then, the ecosystem perturbations currently under way on our planet herald not just an end to the material world as we know it, but, we argue, an end to mathematics education as we know it.

We are aware that the language we use to talk about crisis can be problematic. We do not wish to evoke a sense of a temporary hiatus, nor to erase the multiple ecological crises which have beset marginalised communities throughout history. A crisis discourse can be critiqued as disempowering and susceptible to behaviours that align with a gloomy future foretold, rather than being about learning to act in the present (e.g., Haraway, 2016; Hulme, 2009). What does seem new, however, is the global nature of the ecological changes taking place. We believe there is a need to find a language that is more evocative than 'the environment' and less human-centric than 'sustainability' (Boylan & Coles, 2017) to discuss our relationship as mathematics educators to these global crises. Thus, in this paper, we refer to *mathematics education and the living world*, which reflects Lovelock's (e.g., 2009) idea of our planet's ecosystem as a living organism. It must be kept in mind that the living world, as a system, involves interactions between flora, fauna and non-living matter, or, to say in another way, we see the rivers, oceans, mountains and skies as living parts of the living

[2] We observe that our field can moblise quickly around crises such as the rapid proliferation of writing about the COVID-19 pandemic that have appeared in just the past year. Hence the relative lack of attention to ecological crises is all the more puzzling.

world. For us, ecological justice is intrinsically connected to issues of social justice and, consequently, to wider, ecologically informed social change. This broadly accords with a socioecological justice standpoint (Furman & Gruenewald, 2004), although our understanding of justice differs from that of liberal humanist conceptions, given our commitment to exploring standpoints that include non-human as well as human and humanist concerns (Snaza *et al*., 2014).

In this theoretical paper, then, we propose a dialogical approach to rethinking mathematics education and the living world. This approach attends to socio-ecological justice and is rooted in Bakhtin's (1981) dialogic epistemology and the dialogical ethics it entails. Bakhtin's work is based fundamentally on the idea of relationality. A particular aspect of Bakhtin's work is the significance of alterity–the relation with the other that is encountered in every interaction. A dialogical perspective foregrounds relationships–ontologically, epistemologically and ethically. By extending these dialogic principles to consider interactions between all species, dialogicality offers a way to disrupt the ordered discourses of domination and exploitation that prevail in our society and are reflected in mathematics education.

In what follows, we expand our discussion of dialogicality and then develop the argument for a dialogical ethical stance in relation to the living world and link this to an ecological paradigm. Our dialogical stance then provides a standpoint from which to consider how mathematics education research and scholarship has–and has not–so far responded to the ecological crisis and so to the living world, including identifying contributions less or more aligned to a dialogical ethics. After providing a brief overview of relevant research and scholarship and the rationale for our selection of examples, we organise our discussion around two central themes and questions, both focused on relationality in research and scholarship in mathematics attending to ecological issues and crisis:

- What is the relationship of mathematics to the living world?
- What is the relationship of mathematics education to the living world?

We then consider implications of our proposal for future research and scholarship in mathematics education.

2.0 Dialogicality as theory, methodology and ethics

In mathematics education–and in education more generally–'dialogic' is used in at least three, albeit related, ways. One use of dialogic is focused on pedagogical relationships (for example, Bakker, Smit, & Wegerif, 2015) and is not a focus in this paper but we note it here for completeness. The two uses of dialogic we discuss are:

- A theoretical and methodological perspective on the nature of relationships;
- A normative stance with respect to ethical relationships.

In theoretical and methodological perspectives, dialogicality is fundamentally about relations. In Bakhtin's dialogic theory of language, for example, meaning does not reside in words or sounds, but in the relations between the words and sounds. These relations are multiple and located in time and space. The meaning of an utterance derives from relations between languages, accents, discourses and other features, as well as the relations between the speaker and the addressee. Meaning emerges from the relations arising through a sequence of utterances over time and in particular places. Indeed, Bakthin's thought is even stronger: existence is understood purely as systems of relations (Holquist, 2002).

In this paper, we focus attention on how relationships between mathematics, mathematics education and the living world are conceptualised. By adopting a dialogic position, we aim to bring into view things that are not visible otherwise. Common ways of thinking about human relationships with the rest of the ecosystem tend to be utilitarian and exploitative–ours to use for our welfare and pleasure. The second way in which our view is informed by dialogism is through the adoption of a dialogical ethics. We argue (normatively) that we need to look at human relationships with the ecosystem from a dialogic perspective because this means that (1) it is not understood as a relationship that is inherently and naturally one of dominance and (2) it is a mutually shaping relationship in which the ecosystem shapes who we are as much as the other way around.

Relationships imply an ethical dimension (Abtahi *et al.*, 2017; Boylan, 2016): how can these relationships be healthy? An insight we draw from Bakhtin is the importance of an ethics of answerability–the world must be answered: "An answerable act or deed is precisely that act which is performed on the basis of an acknowledgment of my obligative (ought-to-be) uniqueness" (Bakhtin, 1993, p. 42). Murray (2000) relates answerability, as a complementary notion, to Levinas's "call to responsibility" (p. 134). For Levinas (2011), it is through recognition by an Other, through responsibility, that we come to recognise ourselves as a subject. We both answer the world, in acknowledging the uniqueness of our potential, and recognise others for the uniqueness of theirs. And it is through *being* recognised in such a way by others that we gain the capacity for answering ourselves.

For both Levinas and Bakhtin, relationality comes before identity. Levinas's ethics focuses on our response to what we see in the Other and the responsibility that this entails; Bakhtin's ethics is about listening to the voice of the other and answering (Boylan, 2017a). A dialogic ethics is, therefore, a dialogue between the ethical actor and the Other and, within ethical action, between answerability and responsibility. For Levinas, our sense of ethical responsibility for the world as a whole begins in the recognition of the human Other. Similarly, for Bakhtin, it is engaging with other voices that can bring us to a recognition of our own once occurrent uniqueness of being (Bakhtin, 1993).

3.0 A dialogical relationship to the living world

In this section, we advocate for a dialogical basis for mathematics education and the living world. In the next section, we use this framing to reflect on previous contributions in mathematics education with this focus. One benefit for mathematics education in coming relatively late to ecological and environmental concerns is that much valuable work has already been undertaken in the social sciences. More than forty years ago Catton and Dunlop (1980) identified a dominant Western worldview of realist, technology-based exploitation of the ecosystem. They also proposed a more nuanced socio-culturally oriented “human exemptionalist paradigm”. Neither approach is up to the challenge, according to Catton and Dunlop, and so they proposed a “new ecological paradigm”. The main points of the three perspectives are shown in Table 1. The new ecological paradigm foreshadows subsequent important ideas in ecological thought, including the decentering of humans and the relationality and interdependence of all parts of the planetary ecosystem.

Table 1: Three views of the living world (Catton & Dunlop, 1980)

	Dominant Western Worldview	**Human Exemptionalist Paradigm**	**New Ecological Paradigm**
Assumptions about the nature of human beings	People are fundamentally different from all other creatures on Earth, over which they have dominion	Humans have a cultural heritage in addition to (and distinct from) their genetic inheritance, and thus are quite	While humans have exceptional characteristics (culture, technology, etc.), they remain one among many species that are interdependently

		unlike all other animal species.	involved in the global ecosystem.
Assumptions about social causation	People are masters of their destiny; they can choose their goals and learn to do whatever is necessary to achieve them.	Social and cultural factors (including technology) are the major determinants of human affairs.	Human affairs are influenced not only by social and cultural factors, but also by intricate linkages of cause, effect, and feedback in the web of nature; thus purposive human actions have many unintended consequences.
Assumptions about the context of human society	The world is vast, and thus provides unlimited opportunities for humans.	Social and cultural environments are the crucial context for human affairs, and the biophysical environment is largely irrelevant.	Humans live in and are dependent upon a finite biophysical environment which imposes potent physical and biological restraints on human affairs.
Assumptions about constraints on human society	The history of humanity is one of progress; for every problem there is a solution, and thus progress need never cease.	Culture is cumulative; thus technological and social progress can continue indefinitely, making all social problems ultimately soluble.	Although the inventiveness of humans and the powers derived therefrom may seem for a while to extend carrying capacity limits, ecological laws cannot be repealed.

Just how embedded the dominant western worldview is, can be seen from an examination of the widely cited UN Sustainable Development Goals. The founding declaration states:

> We envisage a world free of poverty, hunger, disease and want, where all life can thrive. We envisage a world free of fear and violence. A world with universal literacy. A world with equitable and universal access to quality education at all levels, to health care and social protection, where physical, mental and social well-being are assured. A world where we reaffirm our commitments regarding the human right to safe drinking water and sanitation and where there is improved hygiene; and where food is sufficient, safe, affordable and nutritious. A world where human habitats are safe, resilient and sustainable and where there is universal access to affordable, reliable and sustainable energy. (United Nations, 2015, Article 7)

This extract illustrates the human-centric focus of the SDGs, most strikingly in the emphasis on *human* habitats. Other worldwide initiatives, perhaps for reasons of realpolitik, adopt similar positions; the Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services, which is doing excellent work to highlight the degradation of ecosystem, frames their work, in part, in terms of 'services' that the ecosystem provides to humans. The practice of mathematics perhaps aligns most easily with this perspective; certainly, mathematics plays a determining role in work on many of the SDGs, the work of the IPBES, the IPCC and so on.

Our dialogical stance seeks to develop the new ecological paradigm. From a dialogic perspective, humans are part of the system of relations between species and no one species has a privileged position. Human action is not fully controllable, due to the "intricate linkages" in the system of ongoing dialogic responses among the entities that make up the ecosystem. Humanity is in inextricable dialogic relation with the ecosystem and human action is constrained by this relationality. The role of the Other is crucial. Without an Other, there can be no relation. Consciousness, for example, is understood as a "relation between a center and all that is not that center", where 'center' is itself a relational notion (Holquist, 2002, p. 18). It is in this sense that we can start to think about how mathematics is created by the living world.

This perspective is relevant to the ecosystem crisis that we face, as Last (2013), points out:

> Bakhtin's writing on 'cosmic terror' sensitizes us to the consequences of different imaginations of the world which place us in positions of varying vulnerability to outside threats and diminishing political agency. Climate change is perhaps the most radical and literal example of how imagined relations between oneself and the planet– or cosmos–impact on one's capacity to counter narratives which seek to close down avenues of political and ideological change […] Bakhtin appears to propose a strategy whose primary aim is to 'de-paralyse' populations, if only to create a temporary window of possibility. (p. 75)

Climate change can be understood as a response by the ecosystem to human actions; or more precisely, as emerging through a vast system of responses between humans and the rest of the ecosystem. Paralysis, here, is one possible collective human response. Our hope, in putting forward the conceptual arguments of this paper, is that mathematics education might play a role in de-paralysis and in creating temporary windows of possibilities for researchers, teachers, students. We now turn to the mathematics education literature itself in developing this line of thought.

4.0 Research and scholarship relating to mathematics education and the living world

In this section, we consider previous research and scholarship in mathematics education that has addressed issues of environment, ecology and climate change. We firstly give a brief historical overview of this work. We then consider research (from a mathematics education perspective) under two broad and (we hope) inclusive themes. The first is research into the relationship of mathematics to the living world; the second is research into the relationship of mathematics education to the living world. In these latter sections, we critically examine the approaches taken in relation to our dialogic perspective.

4.1 An historical overview of research

In this section we argue: (1) that a focus on issues related to the living world has been generally at the margins of the field of mathematics education research; and (2) that such marginalisation is also the case in the subfield of sociopolitical and critical mathematics education research and scholarship. To get a general sense of the prevalence of research on this topic, we made searches using the terms, ‘environmental’, ‘sustainable’, and ecology’, in the following journals (up until 2021): *Educational Studies in Mathematics, For the Learning* *of Mathematics*, the *Journal for Research in Mathematics Education*, *ZDM–The International Journal on Mathematics Education*, and the *Journal of Mathematical Behaviour*. This search led us to identify just seven articles which addressed the relationship of mathematics education to environmental and ecological crisis. We recognise that we have searched using English language terms and there is much important work in mathematics education outside those five journals; nonetheless, we view this limited number of contributions as support for our argument that the living world has occupied a marginal position in current research in mathematics education.

Perhaps the journals mentioned above have been less accessible to critical kinds of research. There is, after all, an important body of work that has challenged the idea that mathematics and mathematics education are neutral endeavours, and has critiqued how oppression, inequity and injustice influence or are part of the experience of learning mathematics, and how mathematics education in turn contributes to maintaining current socio-economic and cultural systems of oppression. Yet even within this area of research, so far, the take-up of ecological issues or engagement with sustainability has been more limited (Barwell, 2010; Boylan & Coles, 2017; Renert, 2011). For example, a search of proceedings of the Mathematics Education and Society conferences before 2021 highlights approximately six papers that address some aspect of mathematics education and the living world as a major focus, although several other papers mention environmental issues in passing[3]. We have been puzzled that within the large corpus of the work and research that takes place under the banner of ethnomathematics (D'Ambrosio, 1985), for example, little has addressed the living world, particularly given calls by D'Ambrosio (2010) to consider issues of human survival.

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[3] 'Approximately' because some articles mix a focus on environmental issues with other topics.

Renert (2011) and a response by Gellert (2011), in *For the Learning of Mathematics*, appears to be the first insertion of the issue into a high-profile mathematics education journal. Barwell (2013) took up the issue of climate change and connected this more transparently to the critical mathematics education tradition, also engaging with the theorising of post-normal science. Concurrently, a UK-based collection focused on classroom practice for secondary school teachers was published (Coles *et al*. 2013). In 2017, a collection of articles was published as a special issue of the *Philosophy of Mathematics Education Journal.* In addition to these articles, a number of other contributions have been made that are considered in this paper. Overall, the body of work, thus far, is relatively small and somewhat fragmented. The situation does appear to be changing, however, and in the next two sections we draw on more recent studies where we have found them. We have chosen examples of research that we view as illustrative of common approaches and where we are able to discern conceptual perspectives within the work.

4.2 Research into the nature and role of mathematics in relation to the living world

In this section, we continue our proposal of a dialogical ethical perspective by considering the nature and role of mathematics in relation to the living world as it has been conceptualised in past research. Learning mathematics is about learning to participate in certain kinds of meaning making processes. These processes involve some particular semiotic features, such as symbols, graphs, diagrams and mathematical discourses. Thinking about the living world in relation to mathematics involves making meaning with and about both. Mathematics has a significant role in describing, predicting and communicating what is happening and is central to the functioning of the consumer capitalism that has contributed to this degradation (Barwell, 2018; Skovsmose, 2021). Mathematical models have real-world impacts, for instance, when models of the impact of climate change leads to commitments to reduce carbon emissions (Barwell, 2018). And mathematics is involved in the denial of climate change, such as through the misuse of statistics. Skewed modelling of populations has contributed in the past to over-exploitation of natural resources (Coles, 2015). Mathematics produces particular versions of the living world and the living world can produce particular versions of mathematics. We consider two common views of this relationship–Platonist and social constructivist–before considering a dialogical interpretation and then interrogating current scholarship aligned with a dialogic stance.

In a Platonist view, mathematics is a timeless reality independent of human thought or perception and independent of the living world. From this perspective, discoveries about mathematics can be applied to understand different aspects of the living world, but they are not closely connected. In this sense, in a Platonist view, mathematics has no responsibility for the living world (see Ernest, 2021), and corresponds to a curriculum of accommodation in which the living world is inserted into mathematical content (to provide a real-world context for a problem, for example) or is avoided altogether (see Renert, 2011).

In a social constructivist or humanist view, mathematics is a cultural activity made up of recognisable practices that may vary from one society to another. Much research in ethnomathematics, for example, has described the variety of mathematical practices that arise around the world, leading to a view of mathematics as embedded in broader human activities. This work aligns with research in the social studies of science that examines the social organisation of mathematical activity, including the production of mathematical meaning (e.g., Roth, 2003). From this perspective, the mathematics of biodiversity or climate change is integral to the sense humans make of them and the actions humans take in response, since the investigation of these issues is a socially organised human activity. By the same token, mathematical activity is intimately related to technological innovation, with new technologies driving new mathematics and vice versa (Skovsmose, 1994). Technological innovation is also implicated in human systems of economic organisation. This view of mathematics aligns closely with the human exemptionalist paradigm. Recognition that mathematics is a socially organised human activity makes possible the inclusion of values, ethics and aesthetics in thinking about mathematics and a curriculum of reformation, in which links between mathematics, human activity and the living world are given a central role (Renert, 2011), but retains human needs as the primary focus.

Mathematics can also be thought of as a complex system that intersects, and is interdependent, with humans and technology among other things. This view of mathematics aligns with a complexity-based understanding of the living world. As such, it is understood that mathematics cannot provide definitive descriptions or solve all problems relating to the living world. Uncertainty and relationality are therefore key ideas that must be taken into account in what amounts to a "post-normal" science (Hauge & Barwell, 2017), pointing in turn to a more central role for citizens in the mathematics and science of the living world. From this perspective, school mathematics may adopt a curriculum of transformation, in which the living world is integral to every aspect of educational processes and learning mathematics coincides with healthy relationships within the living world (Renert, 2011).

We now consider what a dialogical perspective has to offer in terms of thinking about the relationship of mathematics to the living world. Bakhtin (1981) developed his dialogic theory through his analyses of texts, such as novels, epics and poems and, as such, examines the relations between author and text, and between text and world:

> But, even in the last instance [the author] can represent the temporal-spatial world and its events only as if he had seen and observed them himself, only as if he were an

> omnipresent witness to them. Even had he created an autobiography or a confession of the most astonishing truthfulness, all the same, he, as its creator, remains outside the world he has represented in his work. (p. 256)

Bakhtin's ideas suggest how we can dialogically disrupt the role of mathematics in this process of 'standing outside'; abstraction does not have to take human existence and mathematics as separate from a dialogical relationship with the world (Boylan, 2016):

> Mathematics is. It is right in front of us, at our fingertips, caught in the whorl patterns of the skin, in the symmetries of the hands, and in the rhythms of blood and breath [...] a mathematics of kinship. (Jardine, 1994, p. 112)

The flowering of 'post-human' strands of thought point to the blurring of the boundaries of the human (Harraway, 2015). A post-humanist standpoint is not in opposition to a human one, nor does it negate the vital importance of human concerns. Rather, we agree with Ulmer (2017) that, "[p]ut simply, more-than-critical methodologies are needed for a more-than-human world" (p. 3).

Gutiérrez (2017) draws on Indigenous epistemology, ontology and values, connecting these to queer and related theories that disrupt normalising discourses. She argues for an epistemological pluralism that can reconceptualise mathematics to support a sense of interdependence ('In Lak'ech' in Mayan), indeterminacy ('Nepantla' in Nahuatl) and reciprocity. Gutiérrez proposes a "new vision for practicing mathematics" (p. 2), which she labels *mathematx* (pronounced mathematesh). Mathematx begins with "principles of recognising self and/in others, responsibility towards others, and valuing tensions" (p. 15). Mathematx brings focus to joy and the aesthetics of doing mathematics; mathematx is "performance and, therefore, a verb" (p. 17), an intervention in the world, not a representation of the world. Gutiérrez calls for "living mathematx", i.e., "both that we live a version of mathematx as well as we are a living version of mathematx" (p. 18). This

Indigenous-informed, post-colonial perspective, we suggest, accords with a dialogical stance. A dialogical stance recognises, however, that any such position is situated in time and space. Guttiérez’s mathematx is in relation with the context (the people, relationships, the land) in which she works; in other contexts, different versions may arise.

4.3 Research into the relationship of mathematics education to the living world

The second dimension of our inquiry was to ask about the nature and role of education, and specifically mathematics education, in relation to the living world. For example, how are learning and knowing understood? From a dialogic perspective, systems of relations are always open-ended; there is no predetermined goal or endpoint. In the living world, species are in constant evolution and ecosystems are constant states of flux. In mathematics, too, there is a constant process of emergence of new concepts, techniques and frameworks. In education, learning can be thought of in the same way, although it is often viewed in goal-oriented terms.

As in section 4.2, we consider some common perspectives (cognitive, social, socio-political, critical) before moving on to what a dialogical stance might imply and considering work that aligns with this stance. If learning and knowing are seen in cognitive terms, then education is construed in terms of informing learners about the living world. Information and concepts about biodiversity, climate change, ecology, and so on, are provided for learners to study, having been previously established by experts through the practice of what Kuhn called normal science. In this view of learning, mathematics is part of the content needed for learners to take in information.

For many mathematics educators, learning and knowing are seen as social processes. From this perspective, education is understood as creating the conditions for learners to explore the living world, with suitable guidance, and to be socialised into appropriate forms of scientific observation and analysis or artistic appreciation. Ecosystem crises are then framed not as information to be acquired, but as situations to be explored and understood, and in which action can be taken, with an emphasis on participation. Social, participatory approaches have been extended into a broader vision of education as a democratic process, in which learners are seen as active citizens. Learning is framed within this broader democratic framework, so that learning about the living world is part of learning to be an active citizen (see, for example, Herheim, Werler & Hauge, 2022). Since the problems now arising in the living world are no longer (if they ever were) amenable to normal science, since they entail urgency, uncertainty and a role for values, approaches based on post-normal science are appropriate (Barwell, 2018; Hauge & Barwell, 2017).

Finally, socio-political perspectives on education are increasingly prominent, highlighting the dual role of schooling and education systems in both challenging and perpetuating systems of oppression such as racism, classism and sexism, among others. These approaches highlight links between education and consumer capitalism and emphasise how education for the most part socialises learners into the ways of thinking that underlie current ecosystem crises. These ways of thinking are embedded in dominant discourses and root metaphors that limit humans' collective ability to think differently (Bowers, 2001a; Martusewicz, Edmundson, & Lupinacci, 2014).

Some work in mathematics education envisages radically different eco-related pedagogies and involves an awareness that ecological and climate justice requires profound social transformation. One such response brings together perspectives from critical mathematics education and mathematics education for social justice, with theories of ecojustice generated outside mathematics education (Radakovic, Weiland & Bazzul, 2018; Wolfmeyer & Lupinacci, 2017; Wolfmeyer, Lupinacci & Chesky, 2017). Theories of ecojustice, influenced by feminist concerns, link patterns of domination and oppression in society to domination of the natural world (Wolfmeyer & Lupinacci, 2017; Wolfmeyer, Lupinacci & Chesky, 2017). For example, Radakovic, Weiland and Bazzul's (2018) examined the politics of food production and waste (see also Sriraman & Knott, 2009, and Winter, 2013, on similar themes), to propose a transdisciplinary curriculum that focuses on supporting students to transform their current situation (e.g., unsustainable food practices in school). Within this current, possibilities present themselves that are aligned with ethnomathematics or, for example, concerns with how the logic of incarceration plays out in both human society and in the relationship between humans and other living beings (Wolfmeyer, Lupinacci & Chesky 2017).

Wolfmeyer, Lupinacci and Chesky's (2017) work, building on Bower's (2001b) earlier contributions, is an explicit attempt to foster different habits of mind (which we link to Catton and Dunlop's paradigms) and, in particular, to develop a habit of mind which is supportive of both social justice and sustainability. They propose and follow an ecocritical framework for developing a curriculum, as one in which there is examination of: how "root assumptions in Western industrial culture […] lead to habits that have detrimental impacts on social and environmental systems" (p. 4); how the logic of domination leads to "inequities such as racism, classism, sexism, ableism, anthropocentrism, and so on" (p. 4); and, of "how to teach or share skills and habits of mind that support socially just and environmentally sustainable" (p. 4).

Others have drawn on ideas of post-normal science (Barwell, 2013; Hauge & Barwell, 2017; Steffensen, 2017). Funtowicz & Ravetz (2003, 2008) describe post-normal science as entailing acknowledgment that “facts are uncertain, values in dispute, stakes high and decisions urgent” (2003, p. 1). They suggest a post-normal science requires an extended peer community in order to tackle issues that are as complex as the ones now facing the world. Drawing on these ideas, Hauge and Barwell (2017) argue that mathematics education must go beyond skills and knowledge to prepare students to be engaged citizens, able to engage with uncertainty, values-based decision-making and democratic debate. Hauge and Barwell (2022) propose three principles in support of such an approach, which they illustrate with an example of a school-based discussion of the pros and cons of opening up a new area of Norway’s coastline to oil exploration. The three principles are: exploring meaningful situations of risk and uncertainty; exploring both scientific/mathematical concepts and societal perspectives; and exploring and learning through dialogue. This approach links a dialogic perspective with democratic participation. The attention to risk and uncertainty in this context is valuable. Clearly, the multiple ecosystem crises we face include high levels of risk and uncertainty, both with respect to future scenarios and political choices (including the choice of ‘business as usual’). For some social theorists, these risks, along with the uneven ways in which they are experienced, are an inherent aspect of late modern consumer capitalism (e.g., Beck, 1992). Uncertainty is a particularly challenging feature of these crises and is often, at least partially, mathematised, and yet few curricula include attention to its different faces, such as uncertainty in data, uncertainty in predictions and uncertainty about what we do not yet know (Hauge and Barwell, 2017). Risk and uncertainty are often exploited in political discourse (examples include climate change denial and COVID anti-vaccination movements), indicating the space they create for values to be deployed.

In the meeting of critical mathematics education with post-normal science and with ecojustice we see a concern for transdisciplinarity. Radakovic, Weiland, and Bazzul (2018) reflect on this in relation to theorisation of transdisciplinarity, echoing concerns in many of the scholars cited in this paper that the current challenges facing humans, in changing our relationship to the living world, cannot be addressed within disciplinary boundaries.

In a critical orientation, then, mathematics is not a neutral tool, but an ideologically laden worldview which should be questioned and critiqued with respect to human activity in the living world. Education is proposed as a process of engaging students in processes of reflection, critique and debate, in which mathematics has an important role alongside ecosystemic and societal concerns.

What can we say about the relationship of mathematics education to the living world from a dialogic perspective? Beyond our relationship with other humans, a dialogical ethics has implications for possible relations between mathematics education and the living world as a whole. The answerability of mathematics education to the living world points to the importance of recognising, as mathematics educators, the unique contribution we can make to building reparative relations with(in) the living world. The question that arises is: how can mathematics education help us to listen to the living world? The call to responsibility of mathematics education by the living world is about recognising the "face" of the living world, recognising how an ethical mathematics education does not begin with the concerns of mathematics education but rather from an awareness of how mathematics education comes into existence through its relationship to the living world. As mathematics educators, responsibility for action is not an internal event, linked to our sense of self; rather, responsibility comes *from* the living world itself; it is in the living world that ethics resides. And in the dialogue between these senses of answerability and responsibility, we reach towards a dialogic perspective on relations between mathematics education and the living world. In the context of ecological crisis, there are new criteria to consider in long-standing debates about pedagogy and curricula. Regardless of individual or societal outcomes, mathematics educators can consider if, and how, pedagogical and curricular choices support: a dialogical orientation to the living world (Boylan, 2017b); developing humans' capacity to engage with each other as citizens (Hauge & Barwell, 2017, 2022); and to live within the Anthropocene (Coles, 2017).

The discourses of mathematics education predominantly suggest that mathematics is a tool for managing the living world, making it difficult to imagine how to act differently at a time of ecosystemic transformation. Gutiérrez (2017), whose work is consistent with dialogic ethics, invites us to imagine alternative kinds of open-ended relations between us and the planet, which decentre the human as a unique ethical actor and focus of concern, and interrogates what mathematical knowledge is or could be. We also note work that supports disruptive and creative actions that embrace and engage with a world in crisis (Boylan & Coles, 2017).

Epistemological pluralism and moving away from an anthropocentric model for mathematics education, is echoed in Boylan's (2017b) argument for a mathematics education for ecological selves that embraces mathematics as an agentic material-discursive practice (Barad, 2007). Mikulan and Sinclair (2017) pose the question as to what mathematics education should be if we embrace the possibility of a future without humans. Coles (2017) argues that awareness of the Anthropocene is proof that there is not a pre-human 'natural' world that can be restored. Living in crisis is and will be our ongoing condition. Coles argues that it is fruitless to seek either a mathematics education in the form of an ideal "end", or an ideal for mathematics in the classroom in general. One thing common across the contributions we have identified as post-critical is the embrace of paradox and uncertainty as a source of energy and vitality.

4.4 Discussion

To recap: Despite increasing attention to socio-political dimensions of mathematics education, there has been surprisingly little attention to mathematics education and the living world in the mathematics education research literature. The work that has been reported is somewhat disparate. One commonality that we noticed is a sense that the mathematics curriculum needs to change, albeit with quite different views on how.

A limitation and arguably a weakness of the contributions we have reviewed, is that their origins mainly lie in wealthier minority countries (as does this paper), notwithstanding the significance of contributions, such as that of Gutiérrez (2017), that promote Indigenous perspectives. Issues of ecological crises in countries that are already economically, socially and politically fragile are 'double-headed' in that they create a high risk of violent conflict (Smith & Vivekananda, 2007, p. 3). Although a mathematics education in relation with the living world is needed globally, its local form and meanings will vary. It is important that as mathematics education addresses its ecological responsibility, the origin and location for scholarship and research is extended beyond the current base.

We have proposed a dialogic perspective as a basis for this work. From this perspective, all parts of the ecosystem, including humans, are in relation with each other. Mathematics, rather than being a tool to control an ecosystem for human goals, is part of human relationships within the ecosystem. The meaning of mathematics comes from its part in our response to the ecosystem and, in the current context, our response to the multiple ecological crises we face. Of course, this response is intertwined with human relationships with each other and all the socio-political challenges our field has highlighted. The effects of ecosystem crises disproportionately affect the poor and marginalized. Models suggest that 1 billion people may be displaced by climate change, for example[4]. In so far as mathematics education addresses the potential effects of crises, the scale of current and future suffering is not addressed. This perspective implies a mathematics education that emphasises an ethical responsibility towards the rest of the ecosystem.

5.0 Conclusion

One of the remarkable aspects of the human response to the ecological crisis is the continued collective inaction. We have become used to the annual ritual of governments and other organisation sending delegates to the Conference of Parties (COP) meetings on climate and on biodiversity, pledges and commitments being made and then nothing much happening. Relative to the current and future glocal consequences and need for action (Gupta, Van Der Leeuw & De Moel, 2007), our relationship to the living world is at the margins of political concerns. We have described how research on mathematics education and the living world is also at the margins and continues to be so. Henderson and Karger (2017) noted that:

> environmental education— [is] still primarily a cottage industry out in the hinterlands of the formal and informal educational systems. (p. 285)

[4] https://www.climate-refugees.org/spotlight/2020/5/6-4

To extend the metaphor to mathematics education for the living world, then, thus far the contributions have been from individual craft workers fashioning diverse pieces.

We consider the different approaches to research that we have identified as pointing to different important dimensions of our relationship and responsibility in relation to the living world. We have proposed a dialogic perspective as a necessary way to orient mathematics education to the living world in line with the principles of the Catton and Dunlop's new ecological paradigm. This perspective means understanding mathematics as part of a relationship between humans and the Other within our shared ecosystem and understanding that humans do not always come first.

The ethics of action in this area, as in other ethical dimensions of mathematics education, is ambiguous (Boylan 2016). The ethics of action is ambiguous because we cannot know what the effects of our actions will be and the need to navigate ethical dimensions with conflicting priorities. We have proposed a dialogical ethics as a starting point for navigating this ambiguity. Regardless of whether a dialogical ethics is adopted, as opposed to any other ethical framework, the ethical imperative to act is increasingly urgent.

With respect to this urgency, it may be that as the reality of mass extinction and risk of societal collapse grow, the demand for schools, curricula and subject disciplines to address the living world will similarly grow. However, as mathematics educators, we need not wait for these demands; we can begin now to address the increasingly urgent and important questions that the crisis prompts.

The questions implied by the current state of the world, and the ideas we have presented and reviewed include: what does a global ecological crisis mean for mathematics education? Should mathematics teachers and curricula continue as before? Should mathematics curricula include more material on environmental issues? Do we need a different kind of mathematics curriculum? Does mathematics education contribute to the crisis and can it contribute to a response? What could the contribution of mathematics education research be? What could and should mathematics education be like in a world of increasing ecological crisis? Regardless, of whether you find our arguments for a dialogical approach compelling, our central argument is that all mathematics educators have a responsibility to consider their responses to these and similar questions.

Morton (2010) writes about Nature as a reified, unnatural notion of the living world and argues that environmentalism is ill-served by its romanticised idea of nature as wild, pure and mysterious, and above all, other. We can make the same kind of point about mathematics. The increasingly detailed models of the Earth's climate, for example, are a representation from a particular perspective non-identical with the modeller's thought or understanding (or with the climate). But if our versions of 'nature' are not, in fact, anything like the living world, what does that mean for the current state of our planet? In similar terms to Last (2013), Morton (2010) critiques the whole discourse of crisis:

> Environmentalism is often apocalyptic. It warns of, and wards off, the end of the world. The title of Rachel Carson' Silent Spring says is all. But things aren't like that: the end of the world has already happened. We sprayed the DDT. We exploded the nuclear bombs. We changed the climate. This is what it looks like after the end of the world. Today is not the end of history. We're living at the beginning of history. (p. 98)

There is scope for hope, here. The world has changed irrevocably, but life and mathematics go on. We, individually and collectively, as mathematics educators and citizens, can choose how we go on. What will we do in our personal and professional lives, today, and every day, to begin a different history including a different mathematics?

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