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Algorithm to Determine Flow Regimes, Transition Zones and Pressure Gradient of Two-Phase Pipe Flow

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Abstract— Existing two-phase phenomenological models for predicting pressure gradient in flowlines and risers are flow regime specific, and rely on pre-knowledge of flow regime. In practical two-phase pipe flow design, several flow regimes occur in typical flowlines and risers which require different flow regime and pressure gradient phenomenological models. Several exiting flow regime prediction methods employ switching between different models with discontinuous boundaries. Therefore, a consistent method to predict flow regime transition zone and selection of appropriate pressure gradient phenomenological model is required. This work aims to provide an algorithm to predict flow regimes and transitions zones, as well as pressure gradient in a two-phase pipe flow, and to validate the developed algorithm using published experimental data. The proposed algorithm is obtained by combining existing and modified flow regimes, and pressure gradient phenomenological models. Validation of the developed algorithm shows that stratified and annular/mist flow regimes experimental data are identified as transitions flows. Results also showed that 87.87 % of slug data were correctly determined, with the remaining data identified as stratified (0.37 %), dispersed-bubble (9.51 %), and transition (2.24 %) flows. Pressure gradient predictions are within 27.36 % average absolute error. The proposed algorithm is able to determine flow regimes and transition zones for unified flowregime phenomenological prediction models, and select appropriate pressure gradient phenomenological prediction models.

Index Terms— Algorithm, Pressure gradient, Flow regime, Stratified, Slug, Annular, Dispersed bubble, Phenomenological

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1 INTRODUCTION

Accurate prediction of operational flow regime and pressure profile is required in the design of multiphase flow transportation in pipelines, flowlines, and risers. However, rigorous analytical solution of flow regime and pressure profile for multiphase system is not available [1], [2]. Multiphase flow analysis can be simplified as two-phase flow of gas and liquid ([1]–[3]). Two categories of solution methods are generally employed for practical two-phase or multiphase system design, namely: empirical, and mechanistic pressure gradient models ([1], [4]).

Empirical flow regime and pressure gradient models are derived from experiments and are only valid within the boundaries of operational parameters (flow rates of phases), geometrical variables (diameter and pipe inclination angle), and physical properties (densities, viscosities, and surface tension of phases) of the experiments; and only give reliable predictions within these boundaries ([1], [5]). This limitation led to the development of phenomenological prediction models. Phenomenological flow regime and pressure gradient models are derived based on physical behaviour of flow. These models require identification of prevalent flow regime, followed by solution of flow regime dependent momentum equations of phases ([1], [5], [6]). General classification of flow regimes includes: stratified, annular, slug, and dispersed-bubble. Stratified mechanistic modelling was pioneered by [7], with modifications made by other contributors [8]. Significant contributions have been made to annular mechanistic model by Alves et al., Oliemans et al., Xiao et al., and Petalas and Aziz. Slug phenomenological model includes the methods of Dukler and Hubbard, Taitel and Barnea [9], Sylvester, Vo and Shoham, Nicholsen et al., Fernandes et al., and Scott and Kouba [1]. For dispersed-bubble flow, homogeneous model is generally applicable [1]. In practical two-phase pipe flow design, several flow regimes occur in typical flowlines and risers which require different flow regime and pressure gradient phenomenological models. Several exiting flow regime prediction methods employ switching between different models with discontinuous boundaries. Unified phenomenological flow regime and pressure gradient models have been developed ([1], [10]–[12]). In general, liquid holdup and pressure gradient predictions depend on prior knowledge of flow regime. Although Zhang et al. [12] applied slug dynamics to predict transition between flow regimes, majority of unified phenomenological two-phase liquid-gas pipe flow prediction models exhibit discontinuous transition between flow regimes. But, the unified phenomenolical models for flow regime prediction utilize the methods Taitel & Dukler [1] and models proposed by Barnea et al., which are fundamental and widely used. Therefore, there is need to introduce transition zones between flowregimes for unified flow regime phenomenolical prediction models. In particular, there is need to capture transitions zones between annular/mist and other flow regimes. Although, transition from slug to annular is generally termed as churn flow, mechanism to determine this region is empirical [1]. Therefore, a consistent method to predict flow regimes and transition zones, and selection of appropriate pressure gradient phenomenological models is required.

This work aims to provide an algorithm to predict flow regimes and transitions zones, as well as pressure gradient in a two-phase pipe flow, and to validate the developed algorithm using published experimental data.

2 ALGORITHM DESCRIPTION

The proposed novel algorithm is obtained by combining existing and modified flow regimes, and pressure gradient phenomenological models. The flow regime models are Taitel and Dukler [13] flow regime transition mechanism and Kutateladze criterion for transition to annular flow. The pressure gradient phenomenological models employed in the proposed algorithm include: Taitel and Barnea slug model, Taitel and Barnea stratified model, and homogeneous model. Some modifications and observations are made to obtain proposed algorithm, these include:

1. In Taitel and Barnea transition mechanism, liquid holdup (H_L) is approximated using no-slip liquid holdup (λ_L) ,
2. New flow regime transition boundaries and mechanisms are incorporated to define transition zones,
3. Choice of pressure gradient phenomenological model is dependent on prevailing flow regime.

2.1 Flow regime model

Figure 1 shows flow regimes and transition boundaries for proposed algorithm. Transition zone, ADEBCF, between annular/mist and other flow regimes' region is outlined by area bounded by curves ADE and BCF. Curves ADE and BCF are also referred to, respectively, as the initiation and completion of transition to annular flow. The other flow regimes' region is further categorised into stratified, slug, and dispersed bubble based on the value of β, where β is defined for two-phase flow slug unit as L_F/L_U; β is determined using the slug phenomenological model of Taitel and Barnea. For slug flow to exist, $0 < \beta < 1$, whereas criteria for the existence of stratified and dispersed bubble flows are $\beta \geq 1$ and $\beta \leq 0$ respectively. Figure 2 illustrates changes in cross sectional geometry of flow between initiation and completion of transition to annular flow.

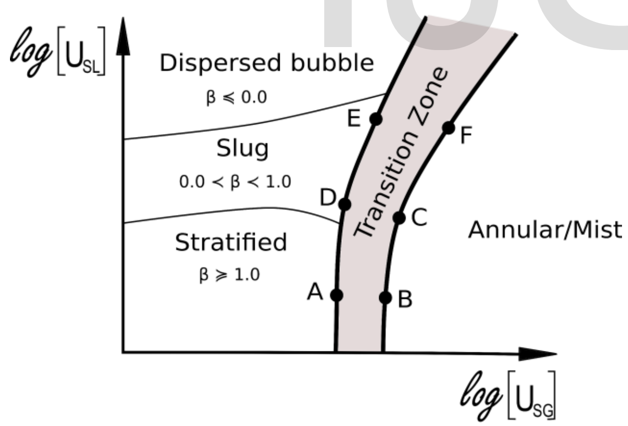

Fig. 1. Flow regimes and transition boundaries of SETM model.

For any liquid superficial velocity, U_{SL} , a unique gas superficial velocity, U_{SG}, is determined for each of curves ADE and BCF as U_{SG}^1 and U_{SG}^2 respectively. U_{SG}^1 is calculated in equation (1) as the maximum of three possible values of U_{SG} at the *initiation* of transition to annular flow. U_{SG}^2 is calculated, using equation (2), as the maximum of three possible values of U_{SG} at the *completion* of transition to annular flow. Gas superficial velocities U_{SG}^a, U_{SG}^b, U_{SG}^c, U_{SG}^d, U_{SG}^e, and U_{SG}^f are determined next.

$$U_{SG}^1 = max(U_{SG}^a, U_{SG}^d, U_{SG}^e) \quad (1)$$

$$U_{SG}^2 = max\left(U_{SG}^b, U_{SG}^c, U_{SG}^f\right) \quad (2)$$

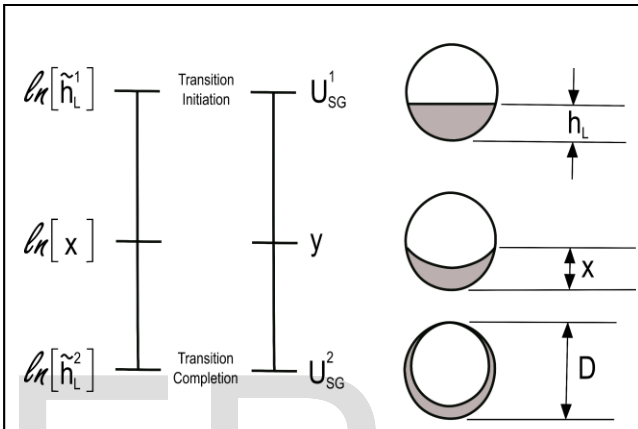

Fig. 2. Changes in flow cross sectional geometry between initiation and completion of transition to annular flow.

2.1.1 Gas velocities at the initiation of transition to annular

U_{SG}^a is determined from the criterion (equation 3) of Taitel and Dukler for transition from smooth to wavy stratified flow.

$$U_G^a = \left[\frac{4\mu_L(\rho_L - \rho_G)g\cos\theta}{s\rho_L\rho_G U_L}\right]^{0.5} \quad (3)$$

where, $U_G^a =$ gas velocity for smooth to wavy stratified $[m \cdot s^{-1}]$, $U_L =$ liquid velocity $[m \cdot s^{-1}]$, $\rho_G =$ gas density $[kg \cdot m^{-3}]$, $\rho_L =$ liquid density $[kg \cdot m^{-3}]$, $s =$ arbitrary constant taken as 0.01, $g =$ acceleration due to gravity $[m \cdot s^{-2}]$. Superficial velocities of gas and liquid are related to liquid holdup by equation (4). Substituting equation (4) into equation (3), taking $H_L \cong \lambda_L$, and simplifying gives equation (5), where λ_L is noslip liquid holdup, generally defined as $U_{SL}/(U_{SG}^a + U_{SL})$. f^a is a calibration factor, defined as $f^a = 10/(1-\lambda_L)^2$.

$$U_L = \frac{U_{SL}}{H_L}, U_G^a = \frac{U_{SG}^a}{1-H_L} \quad (4)$$

$$U_{SG}^a = (1-\lambda_L)\sqrt{\lambda_L}\left[\frac{4\mu_L(\rho_L-\rho_G)g\cos\theta}{s\rho_L\rho_G U_L}\right]^{0.5} f^a \quad (5)$$

Expression for finding U_{SG}^d is derived from the classical Kutateladze criterion for transition to annular flow in upward inclined flow [1]. In order to account for the initiation of transition to annular flow, instead of the Kutateladze number of 3.1, an arbitrary constant value lower than 3.1 is required. In this study, a value of 1.0 is used as shown in equation (6). It should be noted that an optimised constant or variable is not considered in this study.

$$U_{SG}^{d} = 1.0\left[\frac{g\sigma\sin\theta(\rho_L-\rho_G)}{\rho_G}\right]^{0.5} \quad (6)$$

U_{SG}^{e} is calculated from Taitel and Dukler stratified model, taking $H_L = 0.35$ at the initiation of transition to annular.

2.1.2 Gas velocities at the completion of transition to annular

Expression for finding U_{SG}^{b} is derived from the criterion (equation 7) of Taitel and Dukler for transition from stratified to non-stratified flow. $\tilde{h}_L$ is defined as h_L/D, where $h_L =$ liquid film height, $D =$ internal diameter of pipe.

$$U_{SG}^{b} = \left(1-\tilde{h}_L\right)\left[\frac{(\rho_L-\rho_G)gA_G\cos\theta}{\rho_G S_I}\right]^{0.5} \quad (7)$$

In this study, $\tilde{h}_L$ is replaced with liquid holdup, H_L. Using the method of Sylvester [14], H_L is expressed in equation (8) as a function of annular film thickness, δ_L. Geometric terms S_I and A_G are also expressed for annular flow (equation (9)). Then setting $H_L \cong \lambda_L$, gives final expression in equation (10). f^b is a calibration factor, defined as $f^b = 10$.

$$\delta_L = \frac{D}{2}\left[1-\sqrt{1-H_L}\right] \quad (8)$$

$$S_I = \pi(D-2\delta_L), A_G = \frac{\pi}{4}(D-2\delta_L)^2 \quad (9)$$

$$U_{SG}^{b} = (1-\lambda_L)^2\left[\frac{(\rho_L-\rho_G)g\cos\theta}{\rho_G}\cdot\frac{(D-2\delta_L)}{4}\right]^{0.5}\cdot f^b \quad (10)$$

U_{SG}^{b} is also determined from the classical Kutateladze criterion, for transition to annular flow in upward inclined flow (equation (11)).

$$U_{SG}^{c} = 3.1\left[\frac{g\sigma\sin\theta(\rho_L-\rho_G)}{\rho_G}\right]^{0.5}\cdot f^b \quad (11)$$

Based on previous studies, slug flow collapses into annular or stratified flow at $H_L \leq 0.24$. Therefore, U_{SG}^{f} is calculated from stratified model, taking $H_L = 0.24$ at the completion of transition to annular.

2.1.3 Gas liquid interface

Gas-liquid interface, S_I, of present model is determined in equations (12-15) by linear interpolation between $S_{I,f}$ and S_F (Figure (3)). It is assumed that the liquid cross-sectional area corresponding to $S_{I,f}$, S_I, and S_F are $A_{F,f}$, A_F, and 0 respectively.

First, apparent liquid height x (figure (2)) is estimated (equation (12)) from $\tilde{h}_L^1$, $\tilde{h}_L^2$, U_{SG}^1, and U_{SG}^2 by linear interpolation, and noting that $y = U_{SG}$. A_F and A_G are calculated from liquid holdup in film region, H_{LTB} (equation (13)). Once x is known, S_F is calculated using equation (14). S_I is calculated in equation (15), using the method of Zhang et al.

$$x = D\cdot exp\left\{\left(U_{SG}^2-y\right)\left[\frac{\ln(\tilde{h}_L^1)}{U_{SG}^2-U_{SG}^1}\right]\right\} \quad (12)$$

$$A_F = A\cdot H_{LTB}, A_G = A - A_F \quad (13)$$

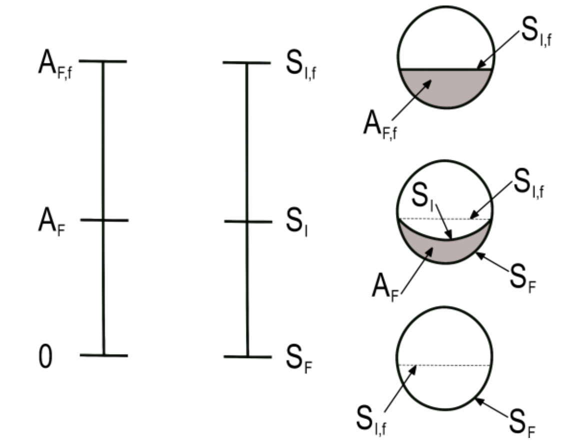

Fig. 3. Gas-liquid interface.

$$S_F = D\left[\pi-\cos^{-1}\left(2\frac{x}{D}-1\right)\right], S_G = \pi D - S_F \quad (14)$$

$$S_I = S_F - \left(S_F - S_{I,f}\right)\frac{A_F}{A_{I,f}} \quad (15)$$

2.2 Pressure gradient prediction

Based on the prevailing flow regime determined by the proposed algorithm, pressure gradient prediction adapts to slug or stratified or dispersed-bubble (for annular/mist) flow phenomenological model. The slug model of Taitel and Barnea is adopted in the algorithm for slug flow since it applies to wide pipe inclination configuration. For stratified model, pressure gradient phenomenological model of Taitel and Dukler is applied to cover wide flow and fluid properties variation. Homogeneous pressure gradient model is applied for annular/mist flow since it closely represents the dispersed nature of mist flow. Detailed description of the phenomenological models is provided by Shoham.

2.3 Proposed algorithm

Schematic illustration of the proposed algorithm is presented in figure (4). Given a set of input data, the algorithm implements the previously described equations in subsections (2.1-2.3) to implicitly obtain flow regime and pressure gradient for two-phase pipe flow. Input data include: gas and liquid superficial velocities (i.e. U_{SG}, U_{SL}), physical properties of the two fluids (i.e. $\rho_G, \rho_L, \mu_G, \mu_L, \sigma$), and geometric variables of pipe $(D, \theta, \varepsilon, L)$.

No-slip liquid holdup (λ_L) is given in equation (16)

$$\lambda_L = \frac{U_{SL}}{U_{SL}+U_{SG}} \quad (16)$$

Liquid holdup at U_{SG}^{e} (i.e. possible U_{SG} at initiation of transition from stratified to annular flow) is given as $H_L^e (\cong \lambda_L)$ (subsection (2.1.1)). Liquid holdup at U_{SG}^f (i.e. possible U_{SG} at

completion of transition from stratified to annular flow) is given as $H_L^f (\cong \lambda_L)$ (subsection (2.1.2)). It should be noted that

ALGORITHM 1
FLOW REGIME SUB-ALGORITHM.

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procedure

Enter input data:
  input ← $(U_{SG},\ U_{SG}^1,\ U_{SG}^2,\ \beta)$

Determine prevalent flow regime; based on figures (1) and (2):
  if $(U_{SG} \geq U_{SG}^2)$ **then**
    Flowregime ← *Annular/Mist*
  else if $(U_{SG}^1 < U_{SG} < U_{SG}^2)$ **then**
    Flowregime ← *TransitionZone*
  else if $(U_{SG} \leq U_{SG}^1)$ **then**
    if $\beta \leq 1$ **then**
      Flow regime ← *Dispersed bubble*
    else if $\beta \geq 1$ **then**
      Flow regime ← *Stratified*
    else if $0 < \beta < 1$ **then**
      Flow regime ← *Slug*

H_L^e and H_L^f are only required to estimate U_{SG}^e and U_{SG}^f respectively.

Flow regime is determined using criteria given in figures (1) and (2); outline for determining flow regime is given in sub-algorithm (1).

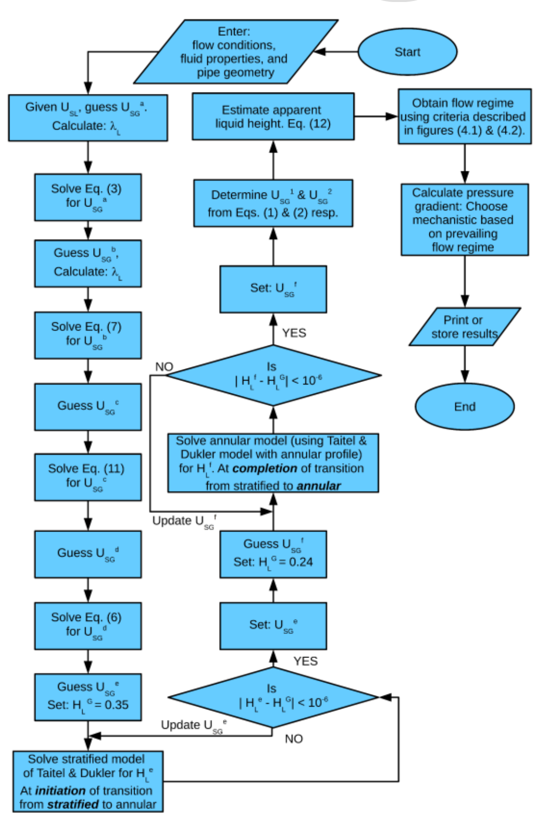

Fig. 4. Proposed algorithm to determine flow regime and pressure gradient.

3 MODEL EVALUATION

Proposed algorithm for determining flow regime and pressure gradient is validated using publicly available experimental data (table 1).

3.1 Criteria for evaluating prediction of pressure gradient

Criteria for evaluating present model is based on the following statistical parameters:

Average percentage error, ε_{ave}:

$$\varepsilon_{ave} = \left(\frac{1}{n}\sum_{i=1}^{n}\epsilon_R\right) \times 100\%$$

where,

$$\epsilon_R = \frac{\left(-\frac{dP}{dL}\right)_C - \left(-\frac{dP}{dL}\right)_M}{\left(-\frac{dP}{dL}\right)_M}$$

Absolute average percentage error, ε_{abs}:

$$\varepsilon_{abs} = \left(\frac{1}{n}\sum_{i=1}^{n}|\epsilon_R|\right) \times 100\%$$

Standard deviation of error, SD:

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(100(\epsilon_R)_i - \varepsilon_{ave}\right)^2}$$

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4 RESULTS AND DISCUSSION

This study aims to introduce transition zones between flow-regimes for unified flowregime phenomenological prediction models, as well as predict liquid holdup and pressure gradient in liquid-gas pipe flow.

Results show that all 255 stratified data are identified as transition flow (figure 5a). Ratio of dimensionless apparent liquid height (X/D) to dimensionless liquid height (h_L/D) is shown to be greater than unity for all the stratified data (figure 5b). The results also show that ratio of slug regions (i.e. $\beta = L_F/L_U$) is approximately unity for stratified data (figure 5c). This type of stratified flow is typically categorised as stratified-wavy based on the onset of wavy flow or increase in liquid height above the theoretical smooth stratified liquid height caused by increased gas flow ([1], [19]). This phenomenon is further justified with ratio of (X/D) to (h_L/D) greater than unity. Furthermore, ratio of slug regions is approximately unity since increase in liquid height has not developed into full slug flow [1].

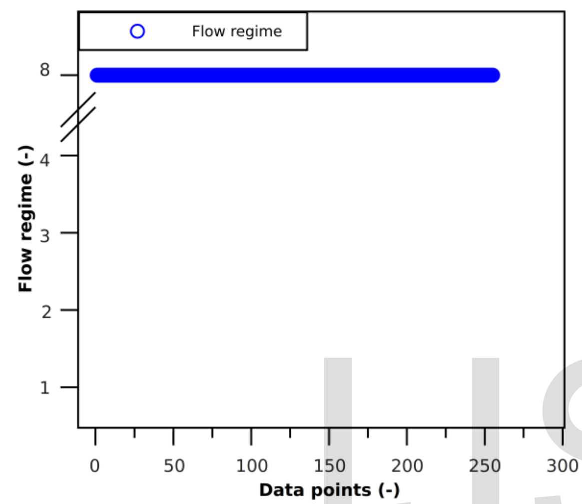


Fig. 5a. Proposed algorithm – flow regime prediction for stratified flow data.

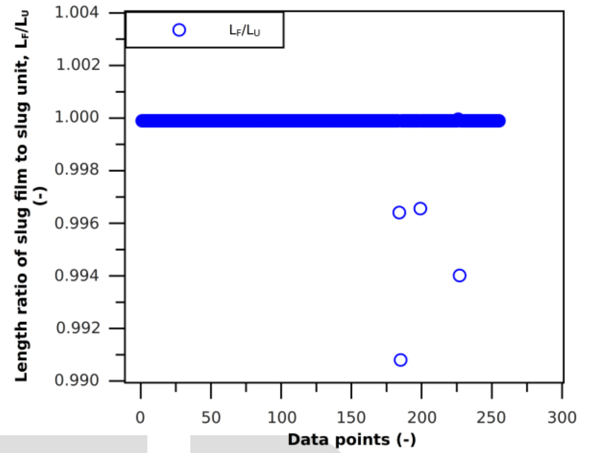


Fig. 5c. Proposed algorithm – ratio of slug regions (i.e. $\beta = L_F/L_U$) prediction for stratified flow data.

About 87.87% of the 536 slug data were correctly identified, the remaining data were identified as stratified (0.37%), dispersed-bubble (9.51%), and transition (2.24%) flows (figure 5d). Results also show that ratio of (X/D) to (h_L/D) approximately unity (figure 5e), while the ratio of slug regions exhibits a scatter distribution within the range $0 < \beta < 1$. In previous studies, slug flow has been shown to share boundaries with other flow regimes ([1], [19]), including: stratified, stratified-wavy, annular/mist and dispersed-bubble flow.

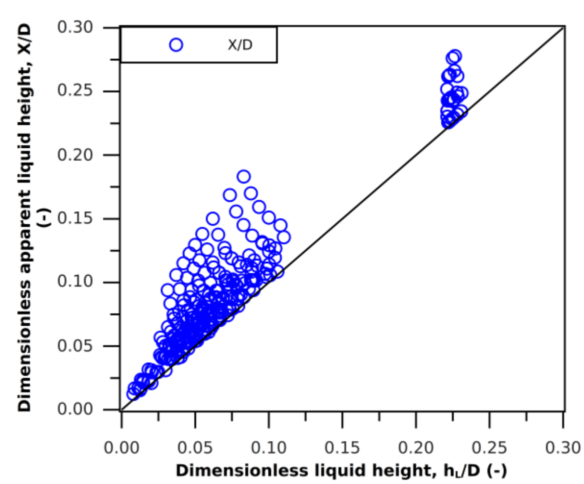
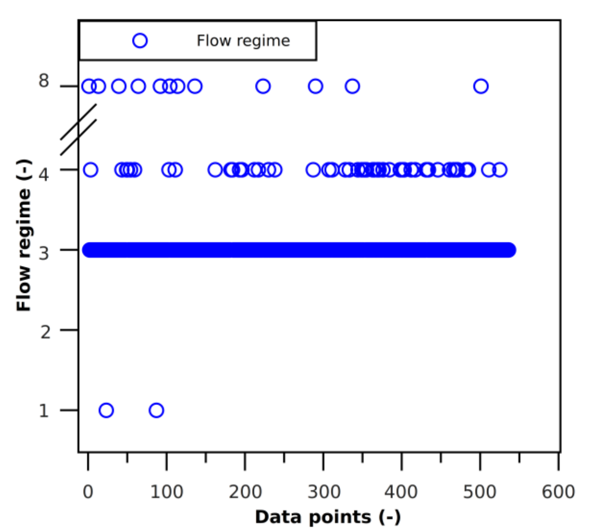


Fig. 5b. Proposed algorithm – liquid height prediction for stratified flow data.



This suggests that the slug data that were incorrectly identified share boundaries with other flow regimes that were identified instead of stratified. Ratio of (X/D) to (h_L/D) is approximately unity since apparent liquid height criterion was not applied to transition to/from stratified and dispersed-bubble flows (figure 1). Scatter distribution of ratio of slug regions within the range $0 < \beta < 1$ shows that slug represents transition zone between stratified and dispersed-bubble flow (figure 1); and as observed previously for slug data, boundary data were identified as stratified (when $\beta \geq 1$), dispersed-bubble (when $\beta \leq 0$), and transition (when criteria in subsection 2.1 are satisfied).

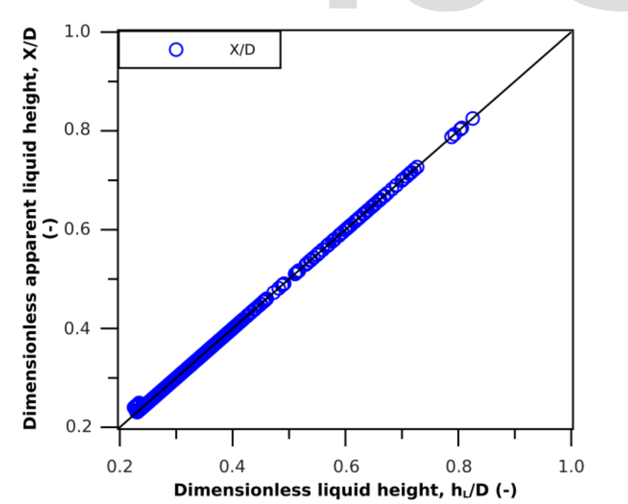


Fig. 5e. Proposed algorithm – liquid height prediction for slug flow data.

Out of 243 annular/mist data, 240 data were identified correctly, the remaining 3 data were identified as stratified flow (figure 5g). Results further show that ratio of (X/D) to (h_L/D) is greater than unity (figure 5h). Ratio of slug regions is approximately unity (figure 5i). Previous studies show that increase in gas flow results in transition from stratified to non-stratified flow. For low liquid flow, the non-stratified flow is either annular or mist flow. But transition from stratified to annular/mist occurs over a transition zone ([19], [20]), and the boundary between transition zone and annular/mist flow is however subjective [1].

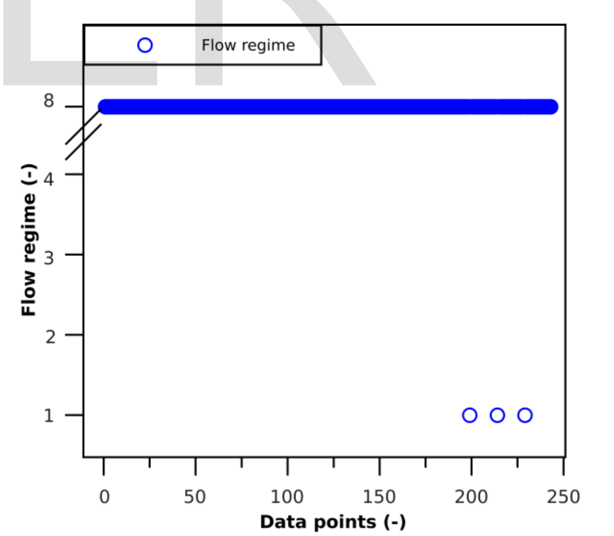
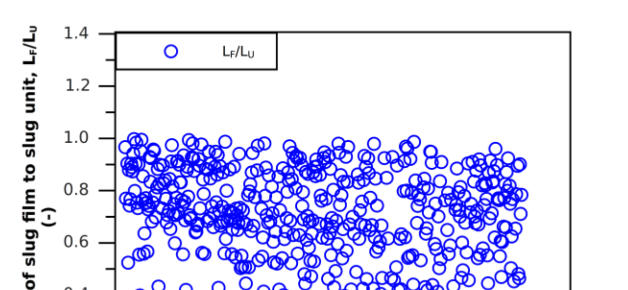
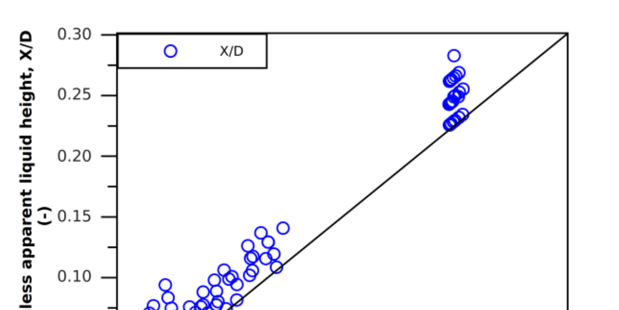


Fig. 5g. Proposed algorithm – flow regime prediction for annular/mist flow data.





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Oloruntoba and Kara showed that stratified flows with low liquid and high gas flow rates can be classified as mist flow if the criterion $\lambda_L \leq 0.0001$ is satisfied. Increase in ratio of (X/D) to (h_L/D) signifies increased shearing effect on gas-liquid interface due to high gas flow which leads to liquid entrainment to form mist flow [1]. Similar to stratified flow, ratio of slug regions is approximately unity due to low and insufficient liquid flow to form slug flow.

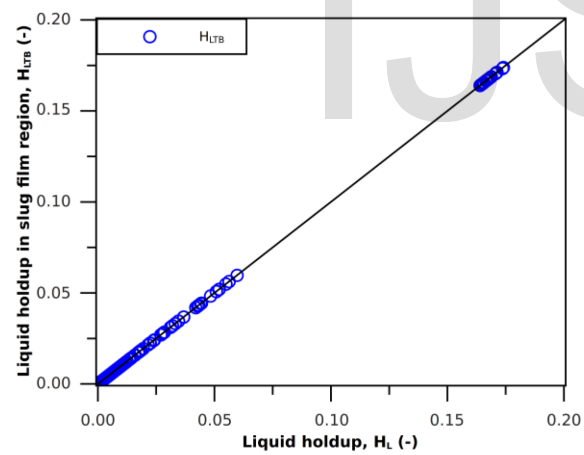

Fig. 5i. Proposed algorithm – ratio of slug regions (i.e. $\beta = L_F/L_U$) prediction for slug flow data.

Comparison between predicted pressure gradient and pressure gradient obtained from experimental data to determine prediction accuracy of the proposed algorithm is shown in figure (4). The average error of the predicted pressure gradient is −3.59%, which is close to the ideal situation of 0.00% due to the distribution of predictions about the ideal prediction line. The absolute average error is around 27.36% due to the fact 61.57% of the predictions fall within ±30% accuracy.

Therefore, the proposed algorithm is able to identify different flow regimes as well as transition zones. Furthermore, the proposed algorithm utilises relevant pressure gradient phenomenological prediction model based on prevalent flow regime. However, further validation exercise will be required to cover wider two-phase pipe flow operational envelops.

5 CONCLUSIONS

In this study, transition zones are introduced between flow-regimes for unified flowregime phenomenological prediction models, and also predict liquid holdup and pressure gradient in liquid-gas pipe flow. An algorithm to predict flow regimes and transitions zones, and pressure gradient in a two-phase pipe flow has been developed. The algorithm combines existing and modified flow regime, and pressure gradient phenomenological models. Validation of the proposed model was carried out using published experimental data. Results show that 87.87% of slug data were correctly determined, with the remaining data identified as stratified (0.37%), dispersed-bubble (9.51%), and transition (2.24%) flows. Pressure gradient predictions were shown to be within 27.36% average absolute error. Therefore, the proposed algorithm is able to determine flow regimes and transition zones, and select appropriate pressure gradient phenomenological prediction models. However, further validation will be required to cover wider two-phase pipe flow operational envelops.

NOMENCLATURE

$-\left(\frac{dP}{dL}\right)_C$	=	Computed pressure gradient, [Pa/m]
$-\left(\frac{dP}{dL}\right)_M$	=	Measured pressure gradient, [Pa/m]
β	=	length ratio of slug unit's film zone to slug body ($\beta = L_F/L_U$), [degree]
δ_L	=	liquid film thickness, [m]
μ_G	=	dynamic viscosity of gas, [Ns/m²]
μ_L	=	dynamic viscosity of liquid, [Ns/m2]
ρ_G	=	density of gas, [kg/m³]
ρ_L	=	density of liquid, [kg/m³]
σ	=	surface tension, [N/m]
i	=	counter, [−]
n	=	number of data, [−]
θ	=	inclination of pipe, [radians]
ϵ_R	=	average error, [−]
ε	=	pipe roughness, [m]
ε_{abs}	=	absolute average percentage error, [%]
ε_{ave}	=	average percentage error, [%]

$\tilde{h}_L$	=	non-dimensional liquid film height, [−]
A	=	pipe cross-sectional area (= A_P), [m²]
A_F	=	liquid film cross-sectional area, [m²]
A_G	=	cross-sectional area of gas flow, [m²]
$A_{F,f}$	=	liquid cross-sectional area corresponding to $S_{I,f}$, [m²]
D	=	internal diameter of pipe, [m]
f^a	=	calibration factor for U_{SG}^a, [−]
f^b	=	calibration factor for U_{SG}^b, [−]
g	=	acceleration due to gravity, [m/s²]
H_L	=	liquid holdup, [-]
h	=	liquid film height, [m]
H_{LTB}	=	liquid holdup in film region of slug unit, [−]
L_F	=	length of slug unit's film region, [m]
L_U	=	length of slug unit, [m]
s	=	arbitrary constant in dispersed-bubble transition criterion of Taitel and Dukler, [−]
S_F	=	liquid film perimeter, [m]
S_G	=	perimeter of gas-wall interface, [m]
S_I	=	gas-liquid interface perimeter, [m]
$S_{I,f}$	=	horizontal interface between gas and liquid, [m]
SD	=	standard deviation, [%]
U_{SG}^1	=	gas superficial velocity at *initiation* of transition to annular flow, [m/s]
U_{SG}^2	=	gas superficial velocity at *completion* of transition to annular flow, [m/s]
X, x	=	apparent liquid height, [m]
y	=	U_{SG} required to estimate X, [m/s]
λ_L	=	no-slip liquid holdup, [−]
$\tilde{h}_L^1$	=	non-dimensional height of liquid at initiation of transition to annular, [−]
$\tilde{h}_L^2$	=	non-dimensional height of liquid at completion of transition to annular, [−]
U_G, U_G^a	=	velocity of gas, [m/s]
U_L	=	velocity of liquid, [m/s]
U_G^a, U_G^d, U_G^e	=	possible values of gas superficial velocity at initiation of transition to annular flow, [m/s]
U_G^b, U_G^c, U_G^f	=	possible values of gas superficial velocity at completion of transition to annular flow, [m/s]
U_{SG}	=	superficial velocity of gas, [m/s]
U_{SL}	=	superficial velocity of liquid, [m/s]
L	=	length, [m]

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