Sheffield Hallam University

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CORNOCK, Claire <http://orcid.org/0000-0002-6918-5918>

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## CASE STUDY

# Student-Generated Examples and Group Work in Mathematics

Claire Cornock, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield. Email: c.cornock@shu.ac.uk

## Abstract

An assignment from Higher Education is presented within this paper as a case study of students generating their own examples whilst working in groups. The student perspective was gained through a questionnaire at the end of the assignment with each cohort over a three year period, which was completed by 123 students in total. The students provided insight on creating their own examples, as well as the group work aspect of the assignment. In particular, students indicated what they believe to be the most beneficial assessment approaches. Elements of learning, understanding and motivation are explored, and the student perspective is compared with the literature.

**Keywords:** Student-generated examples, problem posing, collaborative learning, student perspective, assessment for learning.

## 1. Introduction

Lecturers are usually the ones providing examples (exemplars of a topic that are then worked through), which can result in students taking a passive role in the learning process (Silver, 1994). Getting students to develop examples themselves, known as student-generated examples, is *'a particularly powerful tool in teaching'* (Watson and Mason, 2002b, p.237). It involves problem posing, which has a positive impact, particularly on understanding (Chang, Wu, Weng and Sung, 2012), and its practice is *'central'* to thinking processes required within Mathematics (Silver, 1994, p.22).

Generally working with examples can be very effective in the learning process (Anderson, Reder and Simon, 1996), but too much rote learning does not allow students to develop understanding (Lithner, 2012). In the case when students create different examples to the ones they already have, then *'knowledge transformation beyond generalizing a format is likely to take place'* (Watson and Mason, 2002b, p.246). An example generation task partly requires students to get to grips with the information they have been given (Watson and Mason, 2002a). However, the task requires far more than blindly following examples they have seen before because, as presented by Fried (2006, p.209), students *'probe and crystallize their mathematical knowledge more deeply'* when generating examples. Students make discoveries through a deeper engagement, as well as the relationships between examples and techniques (Bills et al., 2006). Anthony and Walshaw (2007) also highlight that being able to make links themselves, helps the students see connections between different ideas.

Tichá and Hošpesová (2009) say an advantage of students posing problems is that it allows the students to judge their level of understanding and the reasons behind any mistakes. There is evidence in the literature that students who create examples are better at understanding new ideas (e.g. as presented by Dahlberg and Housman, 1997). Student-generated examples also give the marker a good indication of how much the students have learnt (Watson and Mason, 2002a). This is because problem posing can be a good test of students' understanding (Silver, 1994) and assessors can distinguish between different levels (Tichá and Hošpesová, 2009).

The focus of this paper is an assignment for the formal languages and automata part of a final year Abstract Algebra optional module, and its evaluation that was carried out over a three year period (2015/16, 2016/17 and 2017/18). The assignment was designed based on the principles of assessment for learning, which aims to encourage learning (Wiliam, 2011) and is not purely to determine what students have learnt (Stiggins, 2007). The intention was to encourage students to take a deep learning approach, in which they concentrate on relationships (Smith and Wood, 2000). To encourage this type of approach, the students create their own examples of languages in the assessment. They create some recognisable languages and at least one that is not (more information on this topic can be found in Lawson, 2004). Nine methods are provided and students have to apply each at least once to show their languages are either recognisable or not recognisable, using each method a maximum of two times. They are encouraged to try lots of examples without presenting them all. They are warned that their examples may change as they develop the assignment because, as presented by Silver (1994), they may ask more questions after one particular problem is solved.

The assignment is done within groups, which is partially due to the usual benefits of group work, as explored by Laal and Ghodsi (2012). Students are more likely to do well through collaboration in assessment for learning (Hargreaves, 2007) and cognitive autonomy support encourages students to take responsibility for their learning (Stefanou et al., 2004). Some research has been undertaken surrounding student-generated examples and group work. For example, Ahn and Class (2011) present a case study of Teacher Education undergraduate students, in which they created exam questions in groups. Gaining both the teacher and student perspectives, Ahn and Class concluded that group work improves engagement, encourages deeper thinking and changes how students do their work.

The module presented in this paper is assessed via coursework (50%) and an examination (50%). There are three assignments, which are equally weighted. In 2015/16 and 2016/17, the group assignment was the second assignment, but in 2017/18 the group assignment was the final one. Other than the order of the assignments, there was very little difference in how the module was taught during the three year period. The students choose support groups of three to five people at the start of the academic year and sit in their groups throughout the taught classes. There are plenty of opportunities for students to work together on class exercises due to the workshop nature of the sessions, including during the 12 weeks before the group assignment is handed out. More information on the teaching methods, the support groups and the degree can be found in Cornock (2015).

Like all assignments for the module, marking is based on factors such as the demonstration of understanding and relevant skills, the selection of approaches, communication, the correctness of the work and explanations, the elegance and clarity of solutions, and whether sufficient detail is provided. In addition to these, the selection of examples to demonstrate the techniques, the links made between them and the sophistication of the examples are also assessed within the group assignment. The students have to fill in a contribution sheet at the end of the assignment to indicate whether members of the group provided an equal contribution, and marks are adjusted if appropriate.

Given the advantages of student-generated examples and group work, this paper will consider the student perspective in this area following an experience of example creation and whether students recognise the advantages as presented in the literature.

## 2. Methods

Marton and Säljö (2005) critique various methods for researching the student experience and present how self reports from students gives the required insight into their perspective. This study took a similar approach as students were asked for information about their experience and views.

The group assignment in this study was evaluated over a three year period through an anonymous questionnaire. This was completed by 34, 47 and 42 students in 2015/16, 2016/17 and 2017/18 respectively (89.5%, 92.2% and 80.8% of the students in the classes) on the completion of the assignment. The questions were:

- On the scale of 0 to 10 (with 0 being the least confident), how confident do you feel in tackling questions on the following topics on your own having done the recent assignment? *[List of 9 topics]*

- Which would have been more beneficial to you on this assignment? (Two separate questions)
  *Options: developing own examples; working with given examples*
  *Options: working within a group; working on your own*

- Did you feel that you were learning whilst doing this assignment?
  *Options: Yes, a lot; yes, a bit; no, not at all*

- If you answered yes [to the previous question], how were you learning?

- What motivated you to put effort into this assignment?

Throughout the questionnaire, the students were asked for reasons for their responses.

Taking the same approach as Marton and Säljö (2005) with regards to textual data, appropriate student comments were separated out into themes based on similarities. Quotes from students were used to indicate the type of comments that were being made within each theme.

## 3. Results

When asked what motivated them to put effort into the assignment, 75.4% of students who answered the question (95.9% of students provided a response to the question) said it was getting a good mark. There were a number of other responses including wanting to do well for other people in their group (18.6%) and seeing others work hard (5.1%). One student said they wanted to improve and a few (3.4%) said they wanted to increase their understanding. It is clear from the questionnaire results that the students' definition of *'beneficial'* varied and their interpretations can be seen through their responses to the open questions.

### 3.1 Students working with examples

When asked whether developing their own examples or working with given ones would be more beneficial for the assignment, there was a bigger proportion who selected given examples (Table 1):

| | Overall |
|---|---|
| **Developing own examples** | 38.2% |
| **Working with given examples** | 57.7% |
| **Both responses selected** | 4.1% |

Table 1: A breakdown of student opinions of what would be more beneficial regarding examples

#### 3.1.1 Students creating their own examples

Out of the 38.2% of students who thought that developing examples was more beneficial, 40.4% said it was because they developed a better understanding, particularly surrounding why examples worked. Student comments included that *'it's better for me…to understand why and how something is working'* and the *'element of understanding how different languages work would have been missed had there been given examples'*. Some of the students pointed out what they were able to spot why examples did not work (8.5%), with one student pointing out that this was more beneficial than seeing examples that were successful.

Students recognised that they needed a greater understanding of the topics in order to create examples. One student thought that *'there's no shortcut method'* as *'you have to know the topics well'* and another said that the assignment *'tests your understanding in further depth'*. One student provided a particularly interesting response. They expressed a preference for given examples, but then went on to say *'I definitely agree that it's more beneficial to develop own examples because it helps improves understanding of topics.*

A response made by several students was that they had to *'think'* or *'think more'* (25.5%). Comments included they had *'more freedom to think about our ideas'* and working with their own examples *'creates more independent thinking instead of the repetitive process of being told what to work on'.* In particular, they had to think about the methods and *'realise links between techniques'*. Comments included that *'it made me think more than just following steps and applying it to a given example'* and they were not just *'following steps from the book'*, they *'learned more than the basic 'how to do'',* and it made them *'look at languages in a different way'*.

Some students reported that they tried more examples (12.8%), which meant they worked with lots of languages (both ones that worked and ones that did not). *O*ne student pointed out *'with given examples, I would have only focussed on how to answer specific questions'.* A number of students thought that creating their own examples challenged them more and led to richer discussions.

#### 3.1.2 Students working with given examples

Amongst the 57.7% of students who thought that working with given examples would have been more beneficial, 22.5% of them said that creating examples took a long time and 18.4% said they found it difficult. A couple of students said they could not create any and did not fully understand the material. One comment was that it was *'difficult to learn examples and rules never mind find languages as well'* and there were other comments about how working with given examples is better if they do not fully understand them (5.6%). There were remarks about how given examples help

them to understand the material (16.9%) as it provides them with a *'basis to start from'* and *'it may be a good idea to have a mixture of the two'*.

There were concerns about marking, with 14.1% of the students commenting that they found it difficult to know whether they had done enough work and if their languages were complex enough, with one student saying it is *'hard to know what's needed for top marks'* and another saying that *'leaving it so open winded it's going to be harder to get a better grade'.* A number of students (7.0%) did not like the *'broader'* mark scheme when creating their own languages and one did not like the assessment of creativity. A couple of students said they like the security of seeing whether they are on the *'right track'*.

There were several other reasons why students preferred working with given examples. When generating examples, they did not like how there were no model solutions afterwards, there was more room for error and they were not able to just repeat the processes presented in class. Despite many students expressing that working with given examples would have been more beneficial, there was some still acknowledgement amongst the students that creating their own examples helped them.

### *3.2 Students working in a group and individually*

When asked whether working on their own or working in a group would have been more beneficial to them during the assignment, Table 2 shows more students said working in a group.

| | **Overall** |
|---|---|
| **Working in a group** | 68.8% |
| **Working on own** | 31.1% |

Table 2: A breakdown of student opinions of what would be more beneficial regarding group / individual work.

#### *3.2.1 Students working in a group*

There were a larger number of students who felt they benefitted more from the assignment being a group assignment than doing an individual one (68.8%). A common response was that they shared ideas and created examples together (27.4%). One comment was that *'bouncing ideas off each other is an effective method of doing work'.* Unlike many other assignments, they thought the assignment was *'quite creative'*. Some of the students acknowledged that they lack creativity, but that the group work element helped with this. Another student pointed out that it helped with decision making as *'without discussing it, it would be much harder to argue (with yourself) which language is best'.*

The students highlighted that they could help, support and learn from each other (23.8%). In particular, some students said it helped with the topics in which they were weaker (7.1%) as they could gain help and advice from each other, and could work through examples together. They liked that *'you get more insight from a different perspective'.* Some of the students thought the group work element helped as the techniques were difficult, and 13.1% of the students said it was easier to check and spot errors. One student said they could *'discover any misconceptions so can avoid it in the future'.* A number of students felt that the group work helped with understanding (15.5%) and an advantage was that people have different strengths (13.1%).

### 3.2.2 Students working on their own

A large reason why the students thought that working on their own would have been more beneficial was that they did not do examples for every topic, so they have gaps in their knowledge and understanding (36.8%). There were some reports of topics being strategically delegated due to the strengths of individuals. Despite lots of indications of students supporting each other in the group work, a couple of other concerns were that *'people who were comfortable with a process would complete it quickly and leave those without confidence unchanged in their abilities'* and *'sometimes group members can finish an example when you get stuck, so although they're explaining what they're doing you're not necessarily trying it yourself'.* Looking at the confidence scores that the students provided for the nine topics at the end of the assignment, about half (50.2%) of the students provided at least one score above five and one score below five. One student in particular scored ten for four topics and one for the other five. There were other reasons for the preference of individual working, which were mainly general concerns about group work.

Looking at the two factors together in Table 3, very few students (7.4%) said working with their own examples on their own would have been more useful. The biggest proportion (35.2%) said that it would have been more beneficial to work on given examples in a group.

| | **Own examples** | **Given examples** | **Both** |
|---|---|---|---|
| **Working on own** | 7.4% | 22.1% | 1.6% |
| **Working in a group** | 31.1% | 35.2% | 2.5% |

Table 3: A breakdown of student opinions of what would be more beneficial regarding examples as well as group/individual work.

## 3.3 Learning during the assignment

When asked whether they felt that they had learnt whilst doing the assignment, Table 4 shows that the majority said they did:

| | **Overall** |
|---|---|
| **A lot** | 52.0% |
| **A bit** | 46.3% |
| **Not at all** | 1.6% |

Table 4: Opinions on whether the students felt they were learning during the assignment.

There were a variety of answers when asked how they had learnt, with some students mentioning the creation of examples (14.9%) or group work (33.8%). Ways in which they were learning included that they re-capped material (30.6%), they went over parts they did not understand (14.0%) and they used the techniques (19.0%).

There were still a number of students who said they had learnt a lot despite not seeing the creation of their own examples in a group as the most beneficial way of doing the assignment. This was 36.6% of the students who said that working with given examples would have been the most beneficial, 39.5% of the students who said working on their own, and 25.9% of the students who said working on given examples on their own.

## 4. Discussion and conclusions

Following its completion, more students thought that doing the assignment with given examples would have been more beneficial than creating their own. The reasons were mainly negative views of creating examples, such as the amount of time it required and the difficulty. Despite this, findings from the student perspective seemed to match up with results from the literature on student-generated examples and problem posing, which included help with understanding (Chang et al., 2012), finding links between topics (Bills et al., 2006) and becoming comfortable with the given material (Watson and Mason, 2002a). Some students acknowledged that they were not just going through steps provided in class, which is the 'routine' approach presented by Smith et al. (1996) in which similar questions can be answered by repeating the steps in previously seen examples. Students recognised that the task of creating their own examples meant that they had to do more, which included that they had to think more about what they were doing.

The task of creating examples stretches stronger students, as indicated by comments made throughout the questionnaire. Results also suggested that students with a weaker understanding find the task of creating examples particularly difficult due to the level of understanding required. These students may benefit by building more understanding from given examples. It is hoped that the group work element and the opportunity to practice lots of questions before the assignment helps with this. However, this is an area which needs further consideration.

There were a lot of students who thought that it would have been more beneficial to have worked in a group than individually for the assignment. The main reasons were that they were able to share ideas and provide each other with support. The students who thought that working on their own would have been more advantageous mainly said it was because of general group work issues or that they did not cover all the topics in the assignment. The tactical delegation of topics within groups, based on the strengths of individuals, meant that some students were not able to look at topics they were least confident about. The confidence scores suggest that a large number of students did not cover every topic during the assignment. The strategic approach of splitting tasks may have resulted in the short term gain of a better assignment mark for the group, but also had the disadvantage of individual students not receiving the full benefit of the task. The questionnaire results suggest that many of the students thought that the task would have been too difficult as an individual assignment and the support of the group was needed. The group work aspect was typically perceived as more useful than creating their own examples.

Despite what the students saw as most advantageous, a large proportion of students said they learnt a bit or a lot during the assignment, which suggests that most students benefitted from creating their own examples in a group. It is noted that student views on learning can be inaccurate (Deslauriers et al., 2019), so there are limitations on what can be drawn from the student responses on this topic. However, the student comments indicated that deep learning had taken place. For example, some students said they could see links between the topics, saw how and why certain languages worked, and learnt from misconceptions.

When looking at the results together, working on given examples in a group was seen as the best way of doing the assignment. The students thought that creating examples was difficult and working in a group provided them with support. The main motivator for doing well in the assignment was to receive good marks, whereas building an understanding was only mentioned by a few students when asked about motivation, which provides a reason for the responses regarding assessment design and the way the assignment was approached.

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