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Abstract: *Two fundamental questions are still open about the complex relation between fluid dynamics and thermodynamics. Is it possible (and convenient) to describe fluid dynamic in terms of second law based thermodynamic equations? Is it possible to solve and manage fluid dynamics problems by mean of second law of thermodynamics? This chapter analyses the problem of the relationships between the laws of fluid dynamics and thermodynamics in both first and second law of thermodynamics in the light of constructal law. In particular, taking into account constructal law and the diffusive formulation of Bejan number, it defines a preliminary step through an extensive thermodynamic vision of fluid dynamic phenomena.*

Key Words: *fluid dynamics, aerodynamics, Bejan number, fundamental equations, first law of thermodynamics, second law of thermodynamics, constructal law*

1. INTRODUCTION

The aerodynamic science was born at the turn of XIX century, by the early work of Eiffel [1] and Joukowski, which has been considered through Jones [2]. From the very beginning fluid dynamic specialists have recognized that any aircraft must be designed to meet well defined performances related to the takeoff, landing and cruise characteristics that can be performed. Blumenthal [3] has analyzed the pressure distribution on Joukowski wings, which are designed by the three key dimensions (length, camber and radii difference). He argued that it is necessary to consider also the angle of attach. Betz [4] and Glauert [5] improved Blumenthal's model. Hence, Glauert has studied circular arc shaped wings by the well-known Glauert-Prandtl equation. Kaplan [6] determined the pressure distribution for a compressible fluid flow past a circular arc profile iterating the velocity potential in a power series of the camber coefficient. Lissaman [7] stated that the airfoil section is the quintessence of a wing or lifting surface and occupies a central position in any design discipline relating to fluid mechanics, from animal flight through marine propellers to aircraft. Carmichael [8] developed an encyclopedic work that produces an effective classification of low Re high-lift wings.

Englar [9] assessed a circulation control method for high lift and drag generation on STOL aircraft. Ellington [10] and Thomas [11] have analyzed the flight of insect and verified the effectiveness of high chamber low thickness wings at low speeds. Mateescu and Abdo [12] analyzed several airfoil profiles at very low Re. Some authors have attempted to couple lift and drag equations with the second principle of thermodynamics, effectively. Greene [13] who developed a possible solution to the aircraft minimum induced drag problem that is based on the minimum entropy production principle. Moorhouse [14, 15] stated the necessity of introducing a vision and need for energy-based design methods in aircraft design and preliminary defined a multidisciplinary analysis technique based on exergy for aircraft design. Nixon [16, 17] presents a drag formulation that considers the relation between surface pressures to entropy, which is generated predominantly by vorticity in the flow field and splits the total drag into the contributions made by different flow features. Paulus and Gaggioli [18] focused on the exergy-based systems integration, which allow characterizing and optimizing each system.

Li, Stewart and Figliola [19] have defined the preliminary guidelines in the direction of using a computational fluid dynamics (CFD) solver to assess the full field entropy generation for optimizing 2-D airfoil shapes with imposed constraints, such as maximum lift to drag ratio. It is a two-objective optimization method, which adopts an optimization scheme that is based on a genetic algorithm and couples the numerical code with a shape optimization algorithm. They also calculated the entropy generation in a 3D fully developed turbulent flow to examine the issues of accuracy that arises with turbulent modeling. Von Spakovsky et al. [18] investigated the use of exergy and decomposition techniques in the development of generic analysis, and optimization methodologies, which can be applied to the Synthesis/Design of aircraft/aerospace systems.

Drela [21] developed a thermodynamic model to assess power balance in aerodynamic flows. He has used a classical thermodynamic model considering two sub reference volumes: the aircraft and a far field surrounding air volume. He has presented the control volume around a wing (Fig. 1) and has defined two different domains:

1. outer boundary S_0 includes Trefftz Plane, which is normal to the free stream and the Side Cylinder parallel to free-stream;
2. inner body boundary S_B that includes any mechanical component including propulsion blading, to include shaft power, internal ducting to include flow losses, pump power. It reduces to one if no propulsive or internal flux is present.

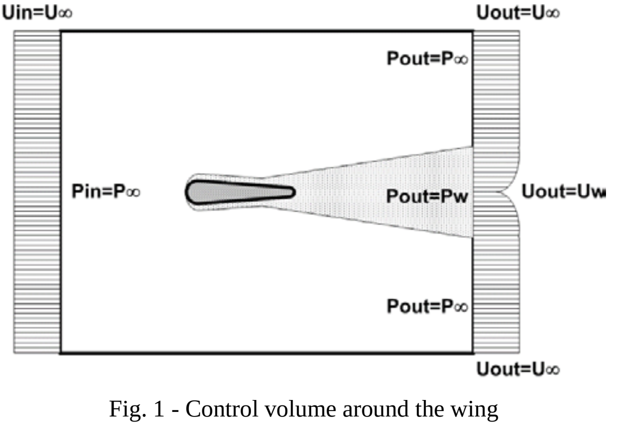

Fig. 1 - Control volume around the wing

The following governing equations can be assumed according to Guignard [22]:

1. Conservation of mass

$$\nabla \cdot (\rho \mathbf{U}) = 0 \tag{1}$$

2. Conservation of momentum

$$\rho \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla \mathbf{p} + \nabla \cdot \boldsymbol{\tau} \tag{2}$$

3. Conservation of energy

$$\rho \mathbf{U} \cdot \nabla \left(\frac{1}{2} \mathbf{U}^2 \right) = -\nabla \mathbf{p} \cdot \mathbf{U} + (\nabla \cdot \boldsymbol{\tau}) \cdot \mathbf{U} \tag{3}$$

Two different and complementary models can be considered. One is based on the first of thermodynamics and the other on the second law. Drela has developed the one according the first law. He has integrated the equation of kinetic energy to obtain the following general integral equation (4):

$$N_S + N_K + N_V = W\dot{h} + \dot{E}_a + \dot{E}_v + \dot{E}_p + \dot{E}_w + \Phi \tag{4}$$

Neglecting shaft power Ns and kinetic energy by inflow NK, equation (4) reduces to (5):

$$N_V = W\dot{h} + \dot{E}_a + \dot{E}_v + \dot{E}_p + \dot{E}_w + \Phi \tag{5}$$

The following terms can be assumed:

1. Pressure Power: $\mathrm{N_V} = \iiint (p - p_\infty) \nabla \cdot \mathbf{U} dS$
2. Potential energy related power term: $\dfrac{dPE}{dt} = W\dot{h}$
3. Axial K. E. outflow: $W_a = \dot{E}_a = \iint \frac{1}{2} \rho u^2 \cdot (U_\infty + u) dS_{\,0}$
4. Turbulent K. E. outflow: $W_v = \dot{E}_v = \iint \frac{1}{2} \rho (v^2 + w^2) \cdot (U_\infty + u) dS_{\,0}$
5. Pressure work: $W_p = \dot{E}_p = \iint \frac{1}{2} (p - p_\infty) \cdot u \; dS_{\,0}$

In case of a subsonic system (*M_a<0.3*) the wave term can be neglected. Drela has noticed that it is convenient to work with a dissipation coefficient C_D which allows obtaining equation (6)

$$\Phi_{surface} = \iint \left[\int_0^\delta \tau_{xy} \frac{\partial u}{\partial y} dy \right] dx\, dz = \iint \rho_e U_\infty^3 C_D \; dx\, dz \tag{6}$$

Φ captures all drag-producing loss mechanisms C_f and D_f still leave out the pressure-drag contribution. C_D and Φ are scalars and C_D is strictly positive.

$$\int d\Phi = \begin{cases} \int_0^x \rho_\infty U_\infty^3 C_D dx & \text{dissipation coefficient} \\ \frac{1}{2} \rho_\infty U_\infty^3 \cdot \theta^* = \int_0^{y_e} \frac{1}{2} \rho \left(U_\infty^2 - u^2 \right) u dy & \mathrm{K_E} \text{ thickness} \\ \frac{1}{2} \rho_\infty U_\infty^3 \cdot \delta_K = \int_0^{y_e} \frac{1}{2} \rho \left(U_\infty^2 - u^2 \right) u dy & \mathrm{K_E} \text{ and momentum thickness} \end{cases} \tag{7}$$

Drela has clearly demonstrated that lost of power is constant for any Trefftz plane (Fig. 2) normal to the fluid flow. It can be possible to express the losses in different formulation. The kinetic energy losses along the airfoil can be estimated by equation (18):

$$\int_0^x d\Phi = \int_0^{\vartheta_{max}} d\Phi = \frac{1}{2}\rho_\infty U_\infty^3 \cdot \theta^*(R\vartheta) \tag{8}$$

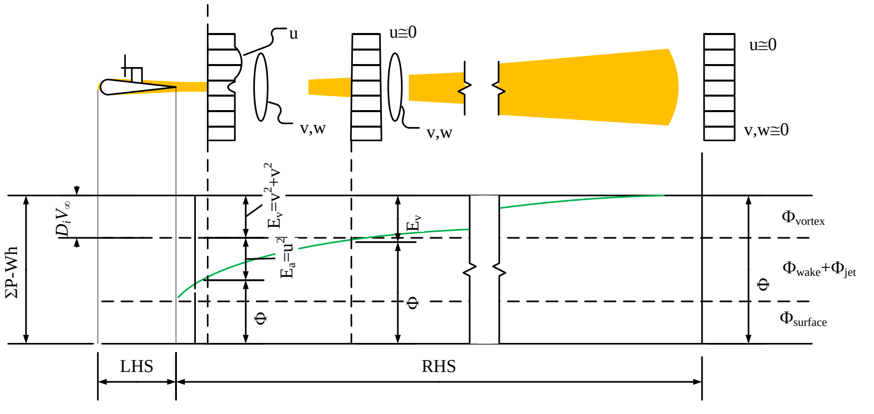

Fig. 2 - Same lost power (LHS) is obtained for any chosen RHS Trefftz Plane location (from Drela [30])

Doty et al. [23] have state the benefits produced by exergy based analysis for aerospace engineering applications with particular attention to aircraft optimization. Hayes et al. [24-25] have produced a complete review of Entropy Generation Minimization and Exergy analysis approaches for aerospace applications and a complete and exhaustive comparison of both exergy based and the traditional Breguet approaches.

Arntz and Hue [26] has accounted the models by Von Spakovsky and Drela to analyze an exergy based performance assessment. They have considered the exergy of the flow ε, which is defined as the total enthalpy h_i relative to the free stream minus the reference temperature T_∞ times the entropy relative to the free stream (Equation 9).

$$\varepsilon = (h_i - h_{i\infty}) - T_\infty (s - s_\infty) \tag{9}$$

The derivation with respect to time is

$$\nabla \cdot (\rho\varepsilon \mathbf{U}) = \nabla \cdot (\rho\delta h_i \mathbf{U}) - T_\infty \ \nabla \cdot (\rho\delta s \cdot \mathbf{U}) \tag{10}$$

and can be expressed as the sum of elementary terms.

$$\nabla \cdot (\rho\delta h_i \mathbf{U}) = \nabla \cdot (\rho\delta e \mathbf{U}) + p_\infty \nabla \cdot \mathbf{U} = \nabla \cdot (p - p_\infty) \ \mathbf{U} + \nabla \cdot \left(\frac{1}{2}\rho U^2 \mathbf{U}\right) \tag{11}$$

where $p_\infty \nabla \cdot \mathbf{U}$ has been added and subtracted.

The total enthalpy outflow has considered zero, according to the first law. It is possible to integrate Equation (11) within the control volume and apply the divergence theorem to get the following equation:

$$0 = -DU_\infty + \dot{E}_u + \dot{E}_v + \dot{E}_p + \dot{E}_{th} + \dot{E}_w, \tag{12}$$

in which

$$DU_\infty = U_\infty \int_{So} \left[\rho \cdot u \cdot (\mathbf{U} \cdot \mathbf{n}) + (p - p_\infty) n_x \right] dS \tag{13}$$

Doty et al. state that a fourth equation can be considered according to Moran and Sciubba [27] and Bejan [28].
It is the equation of the local entropy generation rate ($WK^{-1}m^{-3}$) in a three-dimensional flow field, which has been derived by a balance of the conservation equations and the second law of thermodynamics across a differential fluid volume. It assumes the following general form:

$$S_{gen}^{'''} = \frac{k}{T^2} \cdot (\nabla T)^2 + \frac{\mu}{T} \cdot \Phi \tag{14}$$

in which Φ is a dissipative term which is the viscous dissipation term.
Wing profile Drag can be related directly to entropy generation by the following equation:

$$D = \frac{1}{U_\infty} \left[\iiint_{F^*} S_{gen}^{'''} \cdot dF^* \times T_\infty \right] \tag{15}$$

The integration of the conservation of equation within the control volume and use of the divergence theorem yields the theoretical equivalence between the near-field drag (friction drag, and pressure drag) and far-field drag:

$$D_p + D_f = -\int_{F^*} \left[\rho U (U \cdot \mathbf{n}) + (\mathbf{P} - \mathbf{P}_\infty) \cdot \mathbf{n}_x \right] dF^* \tag{16}$$

where, D_p and D_f are the pressure drag and the friction drag, respectively, forming the near-field drag.
The right-hand side integral is a far-field expression of drag, which constitutes the starting point of any far-field drag formulation. The viscous forces have been neglected as they rapidly vanish after few body lengths downstream of the body. It is this latter term that will be identified in the outflow of total enthalpy. Arntz and Hue determine exergy by equation(9). It can be obtained the following expression of equation(13):

$$DU_\infty = \dot{X}_m + \dot{X}_{th} + \dot{A}_\phi + \dot{A}_{\nabla T} + \dot{A}_w \tag{17}$$

in which, the following terms are defined:
1. Mechanical exergy

$$\begin{aligned} \dot{X}_m = & \int_{F*} \frac{1}{2} \rho u^2 (\mathbf{U} \cdot \mathbf{n})\, dF* + \int_{So} \frac{1}{2} \rho \left(v^2 + w^2 \right) (\mathbf{U} \cdot \mathbf{n})\, dF* + \\ & + \int_{F*} \frac{1}{2} (p - p_\infty) \left[(U - U_\infty) \cdot \mathbf{n} \right] dF* \end{aligned} \tag{18}$$

2. Thermal exergy

$$\dot{X}_{th} = \int_{F*} \frac{1}{2} \left[\rho (\partial e - T_\infty \partial s) + p_\infty \right] (\mathbf{U} \cdot \mathbf{n}) dF* \tag{19}$$

Assuming a perfect gas, internal energy is proportional to temperature [38].

$$\partial e = c_v \cdot \partial T \tag{20}$$

The maximum amount of work that is theoretically extractable from the thermal energy is the combination of the following three terms:

1. The rate of thermal energy outflow,
2. The outflow rate (dissipation) of exergy,
3. The rate of (isobaric) surroundings work [29], which is a non-available work due to the interaction with the reference atmospheric pressure field at p∞ [30].

Hayes at al. suggest another approach, which is directly based on the determination of C_L and C_D which has the benefits of being easily coupled with CFD results. Following a similar methodology, it is possible to evaluate lift and drag coefficients:

$$C_L = \frac{2L}{\rho U_\infty^2 A_{plan}} \tag{21}$$

$$C_D = C_{Do} + \frac{C_L^2}{\pi e A} \tag{22}$$

where e is the Oswald efficiency factor.

The rate of work done (power), $\dot{N}$, on a body to move through a fluid is given as, $\dot{N} = FU$, where F is the driving force of the body at velocity U. According to Dewulf and Van Langenhove [31], the rate of exergy use can be consequently defined from (9) as:

$$\dot{X} = F\,(U - U_0) \tag{23}$$

where the reference state velocity is U_0. For a not accelerated cruise motion of a vehicle, Dewulf and Van Langenhove, and Trancossi [32, 33] derive the rates of exergy dissipation.

$$\dot{X}_D = D\,U_\infty \tag{24}$$

where the reference velocity, U_0, is equal to zero and the exergy required to keep the system in flight, also known as the exergy of lift, can be expressed as follows:

$$\dot{X}_L = L\,w \tag{25}$$

Paulus verifies that the reference velocity U_0 cannot equal zero. Given steady cruise flight, $w=0$, no exergy input seems to be required to maintain the level of flight and keep the system mass aloft, which is absurd. Otherwise, considering equation(13), it is evident that the drag force is affected by the lift coefficient.

$$\dot{X}_D = D\,U_\infty = \left[\left(C_{Do} + \frac{C_L^2}{\pi e A}\right)\rho U_\infty^2 A_{plant}\right]\cdot U_\infty \tag{26}$$

Hence, the drag related exergy can be split into two terms which depend on C_{D0} and C_L:
It is then evident that the minimum amount of exergy destroyed during flight is $\dot{X}_{Lo}$ and it can be expressed as:

$$\dot{X}_{Lo} = \frac{C_L^2}{\pi e}\rho U_\infty^3 \tag{27}$$

If the aircraft is an isolated system within the reference environment and there is no energy recovery from the aircraft wake, such as that found in formation flying patterns, it is possible to link directly power to entropy according to Oswatitsch [34], who stated that drag was simply generation of entropy, and thus the generation rate of such entropy can be assumed to be equal to the rate of exergy destruction. Thus, the drag force can be defined as an integral of entropy flow by considering the Guoy-Stodola thermodynamic theorem, which states that the decrease of useful work of a thermal machine is equal to the entropy change multiplied by the surrounding temperature (assumed as reference). In this case no useful work is done and u_∞ F corresponds to the lost energy

$$X_{des} = u_\infty F = T_\infty \iint_{F^*} \left(s - s_\infty \right) N dF^* \tag{28}$$

2. A NEW GENERAL MODEL WHICH IS BASED ON SECOND LAW

Herwig and Schmandt [35] have generalized the relation between friction phenomena and second law of thermodynamics for internal flows in terms of entropy generation or exergy disruption rate.
Fluid flows are treated according to two different categories. External flows can be expressed in terms of drag force [36, 37]:

$$F_D = C_D A_f \frac{\rho}{2} u_\infty^2 \tag{29}$$

where C_D is the drag coefficient, and A_f is the front section area.
In the case of internal fluxes, two different expressions are present in literature. One is expressed as a function of head loss [38, 39] coefficient:

$$\Delta P = K \frac{\rho}{2} {u_{av}}^2 \tag{30}$$

The other is a function of friction losses by shear stress:

$$\frac{dP}{dx} = -\frac{f}{D}\frac{\rho}{2}{u_{av}}^2 \rightarrow \Delta P = -\frac{f}{D}\frac{\rho}{2}{u_{av}}^2 \tag{31}$$

where f is the friction factor, L is flow path length and D is the hydraulic diameter.
It can be observed that the friction factor $f = \tau / (\rho u^2 / 2)$ is the friction effect of shear stress. It is not fully representative of friction phenomena.
It must be remarked that the friction factor f represents only what happens on the surface being defined by equation (31)

$$D_f = \iint \left[\tau_{xy} \right]_{y=0} dx\, dz = \iint \rho_e u_\infty^3 f \; dx\, dz \tag{32}$$

while the head loss or dissipative terms consider all the losses in the boundary layer along its development and after the detachment

$$\Phi = \iint \left[\int_0^\delta \tau_{xy} \frac{\partial u}{\partial y} dy \right] dx\, dz = \iint \rho_e u_\infty^3 C_D \; dx\, dz \tag{33}$$

It must be consequently remarked that only the dissipative model gives an exhaustive answer to the problem of the losses.
The energy analysis of fluid dynamic processes is strictly related to second law of thermodynamics [40] because it deals with energy availability [41] and usefulness. The degradation of available energy can be expressed as entropy generation, degradation of exergy,

or reduction of available work. If $\dot{S}$ it is the rate of entropy generation, the dissipated mechanical power is equal to

$$\dot{E}_L = F_D \cdot u_{ref} = \Delta P \cdot A \cdot u_{ref} = \dot{X}_{loss} = T_0 \dot{S}_{gen,f} \tag{34}$$

In the case of an external fluid dynamic problem, it results:

$$C_D = \frac{2T_0\dot{S}_{gen}}{A_f \rho u_\infty^{\ 3}} = \frac{2\dot{X}}{A_f \rho u_\infty^{\ 3}} \to F_D = \frac{T_0\dot{S}_{gen}}{u_\infty} = \frac{\dot{X}}{u_\infty} \to \Delta p_D = \frac{T_0\dot{S}_{gen}}{A_w u_\infty} = \frac{\Delta\dot{X}}{A_w u_\infty} \tag{35}$$

and in the case of internal fluid dynamic problems

$$K_0 = \frac{2T_0\dot{S}_{gen}}{A_w \rho u_{av}^{\ 3}} = \frac{2\dot{X}}{A_w \rho u_{av}^{\ 3}} \to F_K = \frac{T_0\dot{S}_{gen}}{u_{av}} = \frac{\dot{X}}{u_{av}} \to \Delta p_K = \frac{T_0\dot{S}_{gen}}{A_w u_{av}} = \frac{\Delta\dot{X}}{A_w u_{av}} \tag{36}$$

3. BEJAN NUMBER AND THE SECOND LAW OF THERMODYNAMICS

More recently Liversage and Trancossi [42] have formulated friction as a function of Bejan number by assuming the general definition by Awad and Lage [43], and obtained:

$$C_D = 2 \cdot \frac{A_w}{A_f} \cdot \frac{\mathrm{Be}_L}{\mathrm{Re}_L^{\ 2}} \tag{37}$$

where A_w is the wet area, A_f is the front area, Re_L is the Reynold Number related to fluid path length and Be_L is the diffusive Bejan number defined as follows.

$$Be = \frac{\Delta p \cdot l^2}{\mu \cdot \nu} = \frac{\Delta p \cdot l^2}{\rho \cdot \nu^2} = \frac{\rho \cdot \Delta p \cdot l^2}{\mu^2} \tag{38}$$

If u_{ref} is the reference velocity for the specific problem, is evident that the pressure losses can be generalized by equation (39):

$$\Delta p_{loss} = \frac{T}{A_w u_{ref}} \dot{S}_{gen,loss} = \frac{T}{T A_w u_{ref}} \Delta\dot{X}_{loss} \tag{39}$$

Thus, the Bejan number related to losses can be expressed in terms of entropy generation by (40),

$$\mathrm{Be}_L = \frac{l^2}{\rho \cdot \nu^2} \Delta p = \frac{1}{\rho A_w u_{ref}} \frac{l^2}{\nu^2} T \Delta\dot{S}_{gen} = \frac{l^2}{\dot{m}\nu^2} T \Delta\dot{S}_{gen} \tag{40}$$

and by equation (41) in the case of exergetic formulation:

$$\mathrm{Be}_L = \frac{l^2}{\rho \cdot \nu^2} \Delta p = \frac{1}{\rho A_w u_{ref}} \frac{l^2}{\nu^2} \Delta\dot{X}_{loss} = \frac{l^2}{\dot{m}\nu^2} \Delta\dot{X}_{loss} \tag{41}$$

Equation (40) allows demonstrating that Bejan number related to fluid-dynamic problems is a thermodynamic related magnitude that refers to entropy generation and exergy dissipation rate. Trancossi and Pascoa [44] have ensured further advancements by reaching a complete formulation of integral conservation equations in fluid dynamics in terms of Bejan number allowing a direct coupling of the fundamental equations of fluid dynamics and entropy generation or exergy disruption. If Bejan energy ξ is defined from Bejan number as follows:

$$Be_P = \frac{l^2}{\nu^2}\frac{\Delta p}{\rho_{av}} = \frac{l^2}{\nu^2}\left(P_i - P_j\right) = \frac{l^2}{\nu^2}P_i - \frac{l^2}{\nu^2}P_j = \xi_{P,i} - \xi_{P,j} \tag{42}$$

where Bejan energy is

$$\frac{l^2}{\nu^2}P_i = \xi_{P,i}\,. \tag{43}$$

Thus, by remembering the pressure equivalences in fluid dynamics

$$\Delta p = \rho g \Delta y = \frac{\rho}{2}\Delta u^2\,. \tag{44}$$

and equation (39):

$$\mathrm{Be}_L = \frac{l^2}{\rho\cdot\nu^2}\Delta p = \frac{1}{\rho A_w u_{ref}}\frac{l^2}{\nu^2}\Delta\dot{X}_{loss} = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_{loss}\,. \tag{45}$$

Hence, it is possible to express the following values of Bejan number:

1. Kinetic Bejan number:

$$Be_K = \frac{l^2}{\nu^2}\frac{\Delta p_K}{\rho} = \frac{l^2}{\rho\nu^2}\frac{1}{2}\rho\left(u_i^2 - u_e^2\right) = \frac{l^2 T}{\dot{m}\nu^2}\Delta\dot{S}_{K,gen} = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_{K,loss}\,; \tag{46}$$

2. Hydrostatic (potential) Bejan number

$$Be_H = \frac{l^2}{\nu^2}g\Delta y = \frac{l^2}{\nu^2}\frac{\Delta P_H}{\rho_{av}} = \frac{l^2}{\nu^2}g\left(y_i - y_e\right) = \frac{l^2 T}{\dot{m}\nu^2}\Delta\dot{S}_H = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_H\,; \tag{47}$$

3. Work Bejan number

$$Be_W = \frac{l^2}{\nu^2}\frac{\Delta\dot{W}}{\dot{m}} = \frac{l^2}{\dot{m}\nu^2}\left(\dot{W}_{in} - \dot{W}_{out}\right) = \frac{l^2 T}{\dot{m}\nu^2}\Delta\dot{S}_W = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_W\,; \tag{48}$$

4. Heat Bejan number

$$Be_Q = \frac{l^2}{\nu^2}\frac{\Delta\dot{Q}}{\dot{m}} = \frac{l^2}{\dot{m}\nu^2}\left(\dot{Q}_{in} - \dot{Q}_{out}\right) = \frac{l^2 T}{\dot{m}\nu^2}\Delta\dot{S}_Q = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_Q\,; \tag{49}$$

5. Loss Bejan number

$$\mathrm{Be}_L = \frac{l^2}{\rho\cdot\nu^2}\Delta p = \frac{1}{\rho A_w u_{ref}}\frac{l^2}{\nu^2}\Delta\dot{X}_{loss} = \frac{l^2}{\dot{m}\nu^2}\Delta\dot{X}_{loss}\,. \tag{50}$$

6. Reaction Bejan number

$$\xi_R = \frac{l^2}{\rho_{av}\nu^2}\frac{R}{A_w}\,. \tag{51}$$

7. Weight Bejan number

$$\xi_G = \frac{l^2}{\rho_{av}v^2}\frac{m}{A_w}g\ . \tag{52}$$

Thus conservation laws are expressed as follows:

1. Conservation of mass

$$\dot{m}_i = \dot{m}_e \rightarrow \sqrt{A_i^2\left(\frac{\rho_i u_i l}{\mu}\right)^2} = \sqrt{A_e^2\left(\frac{\rho_i u_i l}{\mu}\right)^2} \rightarrow A_i\sqrt{\rho_i}\sqrt{\xi_{K,i}} = A_e\sqrt{\rho_e}\sqrt{\xi_{K,e}}\ . \tag{53}$$

The relation between Bejan energy and Reynolds number is consequently:

$$\mathrm{Re} = \sqrt{\rho}\sqrt{\xi_K} \rightarrow \xi_K = \frac{\mathrm{Re}^2}{\rho}\ . \tag{54}$$

2. Conservation of momentum

$$\begin{cases} A_i\cos\phi_i\left(\xi_{P,i} - \xi_{K,i}\right) - A_e\cos\phi_e\left(\xi_{P,\mathrm{e}} + \xi_{K,\mathrm{e}}\right) = -A_w\xi_{R,x} \\ A_i\sin\phi_i\left(\xi_{P,i} + \xi_{K,i}\right) - A_e\sin\phi_e\left(\xi_{P,\mathrm{e}} + \xi_{K,\mathrm{e}}\right) = -A_w\xi_{R,\mathrm{y}} + A_w\xi_G \end{cases}. \tag{55}$$

$$\begin{cases} A_i\frac{\cos\phi_i l^2}{\dot{m}v^2}\left(\dot{X}_{P,i} - \dot{X}_{K,i}\right) - A_e\frac{\cos\phi_e l^2}{\dot{m}v^2}\left(\dot{X}_{P,\mathrm{e}} + \dot{X}_{K,\mathrm{e}}\right) = -A_w\xi_{R,x} \\ A_i\frac{\sin\phi_i l^2}{\dot{m}v^2}\left(\dot{X}_{P,i} + \dot{X}_{K,i}\right) - A_e\frac{\sin\phi_e l^2}{\dot{m}v^2}\left(\dot{X}_{P,\mathrm{e}} + \dot{X}_{K,\mathrm{e}}\right) = -A_w\xi_{R,\mathrm{y}} + A_w\xi_G \end{cases}. \tag{56}$$

3. Bernoulli theorem

By multiplying both terms for $l^2/\rho v^2$, Bernoulli theorem equation becomes

$$\xi_{k,i} + \xi_{z,i} + \xi_{p,i} = \xi_{k,\mathrm{e}} + \xi_{z,\mathrm{e}} + \xi_{p,\mathrm{e}} \rightarrow \xi_{K+H+P,i} = \xi_{K+H+P,e}\,. \tag{57}$$

or

$$\left(Be_K + Be_H + Be_P\right) = \left(Be_{K+H+P}\right) = 0\ . \tag{58}$$

where $\xi_{K+H+P,i} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,i} + \dot{X}_{H,i} + \dot{X}_{P,i}\right) = \xi_{K+H+P,e} = \frac{l^2}{\dot{m}v^2}\left(\dot{X}_{K,\mathrm{e}} + \dot{X}_{H,\mathrm{e}} + \dot{X}_{P,\mathrm{e}}\right)$ and

$Be_{K+H+P,i} = \frac{l^2}{\dot{m}v^2}\left[\left(\dot{X}_{K,i} - \dot{X}_{K,\mathrm{e}}\right) + \left(\dot{X}_{H,i} - \dot{X}_{H,\mathrm{e}}\right) + \left(\dot{X}_{P,i} - \dot{X}_{P,\mathrm{e}}\right)\right]$

4. Conservation of energy

Conservation of energy equation is expressed by

$$\xi_{k,i} + \xi_{z,i} + \xi_{p,i} + \xi_{W,in} + \xi_{Q,in} = \xi_{k,\mathrm{e}} + \xi_{z,\mathrm{e}} + \xi_{p,\mathrm{e}} + \xi_{W,\mathrm{out}} + \xi_{Q,\mathrm{out}} + Be_L \tag{59}$$

or

$$\left(Be_K + Be_H + Be_P\right) + Be_W + Be_Q = \left(Be_{K+H+P}\right) + Be_W + Be_Q = Be_L \tag{60}$$

$$\left(\dot{X}_{K,i} + \dot{X}_{H,i} + \dot{X}_{P,i}\right) + \dot{X}_{W,in} + \dot{X}_{Q,in} = \left(\dot{X}_{K,e} + \dot{X}_{H,e} + \dot{X}_{P,e}\right) + \dot{X}_{W,out} + \dot{X}_{Q,out} + \dot{X}_L \tag{61}$$

4. CONCLUSIONS

This paper traces a preliminary analysis of current aerodynamic models based on first and second law of thermodynamics. The activity assumes an high level of importance if coupled with the issues related to climate change. Consequently, it is a way to achieve the necessary energy efficiency improvement and to reduce emissions of aircrafts. The possibility of coupling CFD with first and second order analysis could produce an effective improvement with respect to actual state of the numerical analysis and could lead to an effective improvement of multidisciplinary optimization by introducing both first and second law of thermodynamics into the optimization process. In particular, even if it is still a pioneering area of research, the results that have been obtained by Arntz et al. (9.32) on different aircraft configurations clearly demonstrate their potential. It also demonstrates that Bejan number in its diffusive formulation belongs to both first and second law. Thus, the conservation equations, which have been expressed in a dimensionless form by mean of Bejan number, can produce results in terms of both first and second law analysis.

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