The effect of moment of inertia on the speed of swung implements

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I Abstract

The maximum swing speed of an implement is an important performance parameter in many sports. It is understood that moment of inertia (MOI) has an effect upon the swing speed of an implement and numerous studies have found a similar rate of swing speed decay (n). These studies considered different movements which suggested that skill was less important than physique to the relationship between swing speed and MOI. The aim of this project was to quantify this relationship and to determine whether the physical characteristics of a participant can be used to predict their swing speed performance. A series of eight visually identical rods with varied MOI were swung in a heavily restricted, maximal motion and trials were recorded with a motion capture system. The results found that swing speed decreased as MOI increased. It was also found that if n was assumed to be constant, the maximum work done by a participant was strongly and significantly related to their swing speed. The relationship between work done and swing speed was used to create a model to predict swing speed for an implement with a specific MOI. This model was validated for a new set of participants performing the same restricted motion and all measured data fell within the confidence intervals of the predictions. The ecological validity of the model was tested in an analysis of the swing speed of tennis groundstrokes. An impact model was used to analyse the effect of changing MOI on ball speed. It was discovered that there is an optimum MOI that produces a maximum ball speed and that this optimum MOI is dependent upon n. This makes the customisation of equipment a realistic possibility. A simple method for measuring n in a non-laboratory environment is proposed that will enable the customisation process to take place.

Key words: Moment of inertia, Swing speed, Racquets, Bats, Customisation

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VI Nomenclature

Abbreviations

ACOR	Apparent coefficient of restitution
ATP	Association of Tennis Professionals
CoM	Centre of mass
DH	Distal hand
HE	Handle end
ICOR	Instantaneous centre of rotation
ITF	International Tennis Federation
MAC	Motion Analysis Corporation
MCC	Marylebone Cricket Club
MOI	Moment of inertia (kgm^2)
MLB	Major League Baseball
NCAA	National Collegiate Athletic Association
R&A	Royal and Ancient
RFD	Rate of force development
SOM	Self-organising map

Roman letters

A_{TIP}	Acceleration at the tip of an implement (ms^{-2})
b	Distance from the centre of mass to the impact point (m)
C	Player constant
e	Coefficient of restitution
g	Acceleration due to gravity (ms^{-2})
h	Distance from the swing axis to the centre of mass (m)
I	Moment of inertia (kgm^2)
I_{CM}	Moment of inertia about the centre of mass (kgm^2)
I_{elbow}	Moment of inertia about an axis through the player's elbow (kgm^2)

I_{HE}	Moment of inertia about an the handle end of an implement (kgm^2)
k	Rate of acceleration decay with increasing moment of inertia
L	Length (m)
L_{CM}	The first moment of an implement (balance point) (m)
M	Mass of an implement (kg)
M_{IP}	Effective mass of an implement at the impact point (kg)
m	Mass of a ball (kg)
n	Rate of swing speed decay with increasing moment of inertia
P	Variable load applied to a muscle (kg)
r_c	Distance from the end of an implement to the centre of rotation (m)
T	Time period (s)
t_{swing}	Swing time (s)
V	Swing speed (ms^{-1})
V'	Implement speed post impact (ms^{-1})
V_{diff}	Difference between measured and predicted swing speeds (ms^{-1})
V_{IP}	Swing speed at the impact point (ms^{-1})
V_m	Contraction velocity of muscle fibres (ms^{-1})
V_{TIP}	Swing speed at the tip of an implement (ms^{-1})
v	Incident ball speed (ms^{-1})
v'	Ball speed post impact (ms^{-1})
WD	Work done (J)
y z	Empirical constants used in the calculation of muscle force
z_{elbow}	Elbow displacement in the sagittal plane (m)

Greek letters

ε	Coefficient of tension loss in muscle
θ_{torso}	Angle of torso rotation (degrees)
ω	Angular velocity ($rads^{-1}$)

Chapter 1: Introduction

1.1 Introduction

The following chapters detail the work of a three year research project investigating the effects of changing transverse moment of inertia (MOI) in sports that involve swinging an implement. The final study focuses on the game of tennis and how MOI can be used to maximise performance.

1.2 Motivation for the research

Modern day elite sport is often a showcase for the latest advancements in material development and manufacturing technology, such as the introduction of composite materials in tennis racquets, replacing wooden frames in the 1970s (Miller, 2006). These changes from the old to the new are instigated with athletes in mind in an attempt to assist improvements in performance.

The problem that arises for sportspeople, with continual development of sporting equipment, is that of choice. There are an ever growing number of equipment options and there is not always a method for identifying which item is best suited to an individual. This is evident in sports that involve swinging an implement. For a tennis player looking to purchase a racquet, they are presented with the question of whether they want a 'control' racquet or a 'power' racquet (Tennis Warehouse, 2015). If a player is unsure about this, they rely upon trial and error to find something that is comfortable and seems to work well. For cricket players, the decision is harder still, with the main points of consideration on offer to be budget and what type of bats the player has previously used ("Cricket bat purchasing decisions," 2015). The confusion for players is increased in sports such as cricket and baseball, where bats are sold by length and mass yet, due to the effect of moment of inertia, not all bats with the same mass will feel and perform in the same way (Russell, 2010).

One area of implement design that can influence performance in several ways is swingweight, or moment of inertia, which is the resistance of an implement to angular acceleration. There have been several studies investigating swingweight and how it affects player performance but there has been little effort to produce results that might help players choose the bat or racquet that is best suited to them. If a method could be developed that provided quantitative information to players, informing their decisions about changes in equipment, it would allow players of all abilities to achieve their maximum performance level. The information would also provide insights to equipment manufacturers about what physical properties players require in their products, which would help make optimum equipment available.

1.3 Project aim

This research aims to improve the understanding of the relationship between swing speed and moment of inertia such that players can make informed decisions when customising or replacing equipment.

1.4 Project structure

The first stage of this project will be to construct a thorough review of the literature in relevant fields of research. This summary will allow clear objectives for the project to be defined. Following the literature review, an investigation will be undertaken into the effects of swingweight on swing speed in a wide range of sports. A series of experimental studies will then be carried out to quantify the relationship between swingweight and swing speed and identify possible methods for predicting performance. Based on this work, a model will be developed to predict swing speed for specific implements and this model will be validated. The accuracy of the model will be quantified and the implications for sporting performance and equipment selection will be discussed.

Chapter 2: Literature review

2.1 Introduction

This chapter offers an analysis of research that has been carried out in the areas of moment of inertia and performance parameters in sports involving a swinging motion. When studying such sports there are three key areas of interest that could influence a player's motion and consequently, performance: the physical properties of the implement being used; the physical profile of the player; and the skill level and technique of the player. This project is primarily focussed upon the effects of implement properties on performance. Therefore state of the art research has been reviewed in this area as well as studies that link this area with physical profiling and skill investigations. The findings were then combined with the motivation for the research to provide an aim and a list of objectives for the project at the end of the chapter.

2.2 Moment of inertia

There are several physical properties of any implement which can be used to define it. The most commonly used are typically mass and length. The mass of a body can be defined as the resistance of the body to a linear acceleration. Similarly, the resistance of a body to rotational acceleration is known as the moment of inertia. A moment of inertia (MOI) describes the mass distribution of a rotating body with respect to a specific axis. A change in the value of MOI can be caused by the addition of extra mass, a change in the mass distribution or the axis about which the body rotates.

Adding or removing mass at various points along an implement can alter the mass, moment of inertia, balance point and centre of percussion of the implement (Cross, 2001). However, it is possible to modify an implement in order to change one of these variables without altering the others.

2.2.1 Regulations

Sporting governing bodies set regulations for the equipment to be used in their sports, which can limit the range of specific properties that is legal for play. Examples of the regulations set for equipment in a selection of sports is shown in Table 2.1.

In tennis, there is an upper limit on the length and head size of a racquet (ITF, 2015); in cricket and baseball, there are maximum limits on the dimensions of bats as well as regulations on the materials to be used (MCC, 2015; MLB, 2015); and in golf there are lower and upper limits on club length (R&A, 2015).

Table 2.1 - Equipment regulations for Baseball bats, Cricket bats, Golf clubs and Tennis racquets.

Sport	Baseball	Cricket	Golf	Tennis
Governing body	MLB	MCC	R&A	ITF
Length	< 0.106 m	< 0.965 m	0.457 - 1.219 m	< 0.737 m
Other	< 0.07 m diameter	< 0.108 m wide	< 460 cm^3 club head volume	< 0.317 m wide, < 0.394 x 0.292 m hitting surface

However, although there are strict limits on dimensions, there are no rules set by these governing bodies that apply to the mass or mass distribution of the implements. The only such rules found are enforced by the National Collegiate Athletic Association (NCAA) (Russell, 2010; Woodward, 2014). In addition to the MLB dimension regulations, the NCAA rule that a baseball bat's mass in ounces must be no more than three less than its length in inches. For example, a 34 inch (0.864 m) bat must have a mass of at least 31 oz (0.879 kg).

Due to the lack of ruling over mass and MOI, equipment can be designed to have very different MOI values either by a change in material or a change in material distribution. Consequently there is a large range in the inertial properties of equipment on offer to players, which could alter the speed and direction of a player's swing and how comfortable the player feels, thereby ultimately affecting the success of the shot they are executing (Brody, 1985).

2.2.2 Measuring MOI

The moment of inertia that is most commonly referred to for sports with swinging implements is the MOI defined about an axis perpendicular to the handle of the implement. This is sometimes referred to as the 'swingweight'.

The MOI of an implement can be measured by hand by setting up the implement as a pendulum (Brody, 1985; Spurr, Goodwill, Kelley, & Haake, 2014). If the implement is suspended and allowed to swing as a pendulum, the time period (T) of its oscillations can be used in Equation 2.1 to calculate moment of inertia (I), with the mass (M), gravitational acceleration (g) and distance from the swing axis to the centre of mass (h).

$$I = \frac{T^2 M g h}{4\pi^2} \tag{2.1}$$

It has been suggested that when calculating the moment of inertia for hand held swinging implements, the hand of the player should be included in the calculation as it acts as a single segment, rotating about a common axis (Cross & Bower, 2006). However, this approach is not mirrored by the majority of literature, with studies by other authors only being concerned with the MOI of the implement, usually about an axis at the butt end of the implement or the axis of rotation (Fleisig, Zheng, Stodden, & Andrews, 2000; Koenig, Mitchell, Hannigan, & Clutter, 2004; Mitchell, Jones, & King, 2000; Smith & Kensrud, 2014; Smith, Broker, & Nathan, 2003; Smith, Burbank, Kensrud, & Martin, 2012).

2.2.3 Polar MOI

As well as being able to characterise an implement by the moment of inertia along its longitudinal axis (swingweight), a polar moment of inertia can be used to describe implements with a large width, e.g. racquets. Polar moment of inertia describes the mass distribution across the face of an implement, e.g. across the width of the string bed, and is the resistance of that implement to being twisted (Brody, 1985; Spurr et al., 2014). The polar moment of inertia of a tennis racquet can be described as proportional to the width of the racquet head, with a wider racquet being less likely to twist in a player's hand. This is also referred to as 'twistweight'.

2.2.4 Player Sensitivity

While changing the properties of an implement may have performance benefits (or drawbacks), these changes may not feel the same to all players. There have been two experiments investigating the sensitivity of people to moment of inertia in sports equipment. Firstly, a study using hollow tubes with concealed masses (Kreifeldt & Chuang, 1979) and secondly a study using tennis racquets with additional lead mass (Brody, 2000). In both cases balance point and mass were kept constant across all implements and participants were asked to say how top-heavy each implement felt compared to a reference.

Both studies reported that the level of sensitivity is dependent upon the player's familiarity with the task they are performing. Non-tennis players in the first study were only able to distinguish changes in MOI of at least 25% and women were found to be 50% less sensitive than men. In contrast, competent tennis players could detect a change in MOI of as little as 2.5% and could easily determine changes of 5% or more.

For polar MOI, good tennis players were only able to distinguish between two racquets if the there was a difference of over 5%. However, because the polar MOI is an order of magnitude less than the longitudinal MOI, this means players were more sensitive to changes in polar MOI than longitudinal MOI.

The sensitivity of an average person to changing moment of inertia is also reported to be ten times the sensitivity to a lifted weight (Kreifeldt & Chuang, 1979), which makes it an important consideration in equipment design.

2.3 Effects of changing MOI

There are many potential effects of changing the MOI of a sporting implement but one of the first that a player will notice as they pick up an implement is its influence on swing speed.

2.3.1 Moment of inertia and swing speed

One of the first studies into the relationship between swingweight and swing speed tested four golfers of varying strength and skill whilst playing with six modified clubs (Daish, 1972). Each club consisted of a standard shaft with brass masses at the bottom end ranging from 0.1 to 0.35 kg. The swing speed of the clubhead was measured with light gates for ten trials with each club. Swing speed was plotted against clubhead mass on a logarithmic scale producing distinct, parallel lines for all four players, shown in Figure 2.1(a). This suggests that the differences in swing speed are down to the power of the players but there is no classification of the player's strength or skill level to test this theory. Using the principles of conservation of momentum and the coefficient of restitution, ball speed was calculated for each clubhead and the optimum clubhead mass of 0.2 kg is shown to be the same for all players. The club shaft being used was constant in this study meaning that the mass of the club and the MOI changed every time. It is therefore very difficult to know from this work how much of an effect MOI alone has on swing speed. The four players tested in this study have an extraordinarily similar rate of decreasing swing speed as the club head mass increased. However, because it is such a small sample it is difficult to conclude whether this is representative of a wider population or a fortunate coincidence.

A comparable relationship of decreasing swing speed with increasing MOI has subsequently been found in tennis (Mitchell et al., 2000), baseball/softball (Bahill, 2004; Fleisig et al., 2000; Koenig et al., 2004; Nathan, Crisco, Greenwald, Russell, & Smith, 2011; Smith & Kensrud, 2014; Smith et al., 2003, 2012) and with weighted rods (Cross & Bower, 2006; Koenig et al., 2004). The rate of swing speed decay with increasing MOI is referred to as n.

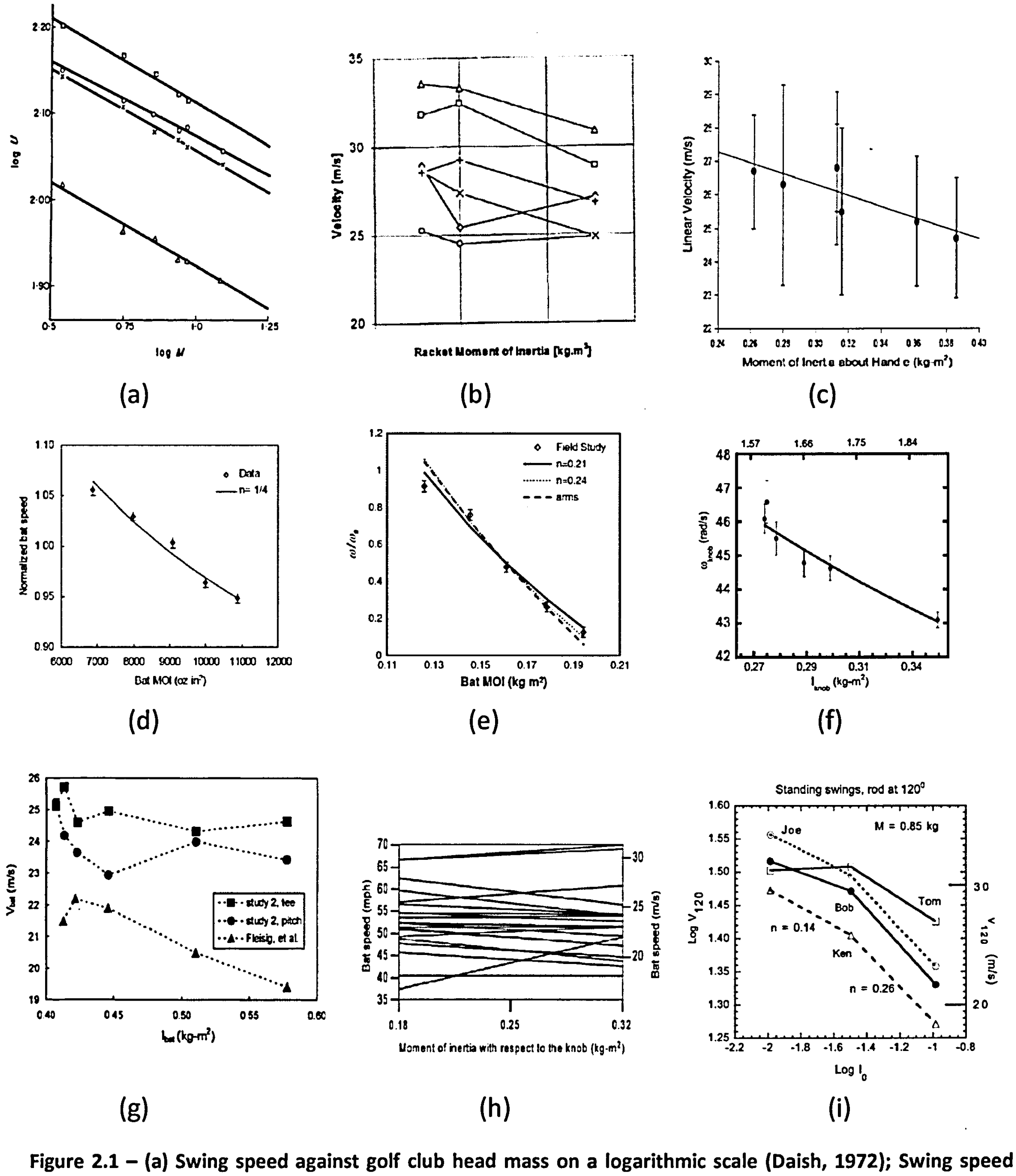

Figure 2.1 – (a) Swing speed against golf club head mass on a logarithmic scale (Daish, 1972); Swing speed against moment of inertia in: (b) Tennis (Mitchell et al., 2000); (c) Baseball/Softball (Fleisig et al., 2000); (d) Softball (Smith et al., 2003); (e) Softball (Smith & Kensrud, 2014); (f) Baseball (Nathan et al., 2011); (g) Baseball/Softball (Koenig et al., 2004); (h) Baseball (Bahill, 2004); (i) Restricted motion (Cross & Bower, 2006).

Mitchell et al. (2000) used a CODA motion capture system and a stereo high speed video system to record maximum effort serves for six county level tennis players. Four tennis racquets were tested. Three were standard racquets, weighted such that they represented the full range of racquet mass and MOI. The fourth racquet was the player's own. In Figure 2.1(b) swing speed at the impact point, normal to the racquet face, is plotted against racquet MOI, measured about an axis 10cm from the handle end. Swing speed was found to decrease with increasing MOI, across the group. However, two of the six players recorded their fastest swing speed with the racquet that was most similar to their own, suggesting familiarity could be an important factor when seeking maximal performance. Mass also varied dramatically between the racquets in this study, ranging from 0.244 – 0.360 kg and it is unknown how much this affected the outcome.

Fleisig et al. (2000) used a motion capture system to record swing speed for 17 male baseball players and 17 female softball players. Swing speed was measured at the sweet spot, which is the point along the bat that produces the highest batted ball speed. Figure 2.1(c) shows the linear swing speed at the sweet spot against MOI, about the handle end, for the baseball players. Swing speed decreases with increasing MOI and very similar results were found for the softball players. Bat mass and MOI were found to correlate strongly and significantly ($r < -0.6$) with the sweet spot swing speed for baseball and softball. In both cases MOI had the strongest correlation. However, once again, mass and MOI changed simultaneously meaning there is a possible crossover of the effects.

Smith et al. (2003) later showed that swing speed changes almost independently of mass but is greatly affected by changing moment of inertia. A high speed video system was used to measure swing speed for 14 mixed level softball players. The players swung one of two sets of bats. The first set had a constant MOI but changing mass and the second set had constant mass but changing MOI. Angular velocity was measured for each swing and normalised to the average speed for each player to allow for a comparison irrespective of skill level. This normalised swing speed is plotted against MOI in Figure 2.1(d) where a curve is fitted, with the rate of swing speed decay (n) = 0.25. The results of swing speed against bat mass are shown in Figure 2.2, which strongly suggest that mass does not influence swing speed. It therefore seems likely that the correlation found between bat mass and swing speed by Fleisig et al. (2000) was heavily linked to MOI.

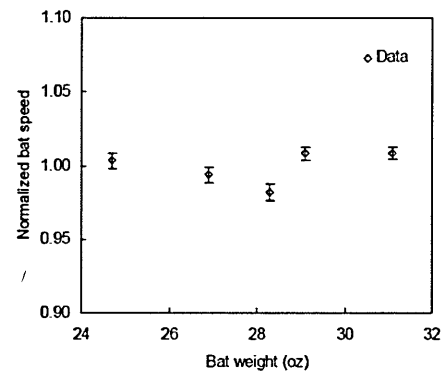

Figure 2.2 - Normalised swing speed against bat mass for 14 softball players (Smith et al., 2003).

This study has subsequently been claimed to be the most definitive evidence of swing speed being affected by moment of inertia and not mass (Cross & Nathan, 2009).

Further research by Smith and Kensrud (2014) using a stereo high speed camera system for a group of 29 softball players has found very similar results. The players ranged from expert to recreational standard and the five bats used had very similar mass but different MOI. Figure 2.1(e) shows the normalised swing speed against MOI and for this study the data fell between curves of power n = 0.21 and n = 0.24. The similarity of this result to the previous work, for a larger cohort of participants, is evidence to suggest that the rate of swing speed decay with increasing MOI might be near constant for softball players. A study of swing speed amongst 19 baseball players, of different standards, found a similar relationship (Crisco, Greenwald, Blume, & Penna, 2002; Nathan et al., 2011). Players were returning pitches from a machine using seven prepared bats with different MOI. Using the same fitting method as Smith et al. (2003), shown in Figure 2.1(f), this study found $n = 0.29 \pm 0.04$.

The results of two further studies observing the effect of MOI on swing speed in baseball and softball generally agreed with the trend already discussed. Koenig et al. (2004) presented velocity at the sweet spot against MOI measured about an axis 0.3 m towards the player from the handle end of the bat, as shown in Figure 2.1(g). The trend was less stable at the lower range of MOI, but in general swing speed decreased with MOI. Of the 20 players tested by Bahill (2004), most produced their fastest swings with the bat that had the lowest MOI. This can be seen in Figure 2.1(h) where sweet spot velocity is plotted against bat MOI measured about the handle end. However, this study found that swing speed increased with MOI for several players and there was more of a spread in the rate of swing speed decay than previously reported in literature. In this case, the positive slopes were attributed to the fact that the bats tested were all within a normal range for baseball bats and it is expected such lines would not be possible if the range extended to higher MOI.

The effect of moment of inertia on swing speed, with a restricted technique, has also been considered (Cross & Bower, 2006). This study used a set of six weighted rods, three with an approximately equal mass and three with an approximately equal moment of inertia. The rods were a mixture of wood with added weight, aluminium and brass, making them clearly distinguishable to the participants. Four participants swung all six rods in the sagittal plane using only their upper arm, forearm and wrist. This was repeated in a standing and a seated position. Swing speed was recorded by two infrared cameras. The data was extrapolated to find the theoretical impact velocities for all rods at 120° from the start of the swing and 0.6 m along the rod, representative of the impact point in baseball. The results for the standing trials with the variable MOI rods are shown in Figure 2.1(i) on a logarithmic scale. Swing speed was found to decrease with increasing MOI and the relationship was described by using Equation 2.2:

$$V = \frac{C}{I^n} \tag{2.2}$$

where V is swing speed in ms^{-1}, I is the moment of inertia in kgm^2 and C is a player constant with units of kgm^3s^{-1}. The value for n can be found by taking the gradient of the log-log plot of swing speed against swing weight. The value of n in this study was measured between each data point, of which there are three. The value of n is very different for the four participants with the lower MOI rods, but is more consistent for the higher MOI, where n = 0.26 for the standing trials and 0.36 for sitting trials.

Cross and Bower (2006) also converted the data in the study by Daish (1972) from clubhead mass to MOI and found that in that case n = 0.27. These values are highly comparable to the values of n reported by Smith et al. (2003, 2014), where n = 0.21 – 0.25. However, caution is required when analysing the results of Cross and Bower (2006) because the trend (n) has been drawn between two data points which means they cannot be considered significant without further testing with more implements.

Swing speed was found to be independent of rod mass in this study. Maximum velocity occurred with rods of different mass and there was no clear overall trend, which supports the findings of Smith et al. (2003).

It is interesting to observe that the rate of decreasing swing speed (n) is found to be so similar in studies covering a variety of sports and actions. However, many of these studies involved small numbers of participants, such as Cross and Bower (2006) and Daish (1972) where only four players were tested, meaning the results reported may not be representative of the sporting population as a whole.

A study into the changes in baseball bat acceleration with differing inertial configurations discovered that the acceleration changes more from player to player than it does between different setups (Maeda, 2010). This suggests that the effect of a change in MOI is very personal and could be different from player to player.

In the studies reviewed here, swing speed has been shown to decrease with increasing MOI. Where the rate of swing speed decay is quoted, the values are within a small range. The similar rates of swing speed decay suggest that players respond to changes of MOI very similarly, irrespective of the movement being considered. However, there has been no attempt to explain the value of n or any differences therein. The value of the player constant in Equation 2.2 is also left undefined. All of these studies used implements with visible alterations; with the exception of weighted rods used in one experiment by Koenig et al. (2004). Therefore players could be changing their approach to each trial based upon visual cues. Only the work by Smith et al. (2012) and Smith et al. (2014) tested more than 20 players and two of the studies tested only four players. There are also cases where relationships were drawn on lines of fit between as few as two data points. Consequently, the trends presented will not necessarily give an accurate representation of the relationship between MOI and swing speed and large rigorous studies in the future would be of benefit. A more thorough understanding of the variables in Equation 2.2 could be of great assistance to players and manufacturers in providing a more detailed understanding of players' requirements of their equipment.

2.3.2 Weighted warm-up

Swing speed is accepted to be one of the important factors contributing to performance in sports such as baseball and tennis. With this in mind, it is something that players and coaches attempt to train to produce performance gains. As there is an established relationship between swing speed and MOI, players have tried to use this to their advantage in warm ups. Particularly in baseball, players have turned to using differently weighted bats during warm-up in an attempt to improve their swing speed for a game (Montoya, Brown, Coburn, & Zinder, 2009). The assumed theory was that swinging a high-MOI bat during warm up would make a player's normal bat feel light and allow them to swing it faster.

There have been three studies that have investigated the effect of warming up with an altered bat on swing speed (Kim & Hinrichs, 2003; Montoya et al., 2009; Otsuji, Abe, & Kinoshita, 2002). In these studies players performed maximal swings with a normal bat before and after the warm up swings. The warm up was with a very light bat (with low MOI), a normal bat or a heavy bat (with high MOI).

Kim & Hinrichs (2003) found there to be no significant changes between pre and post warm up swing speed in the thirteen players tested. However, they found that players perceived themselves to be swinging faster after using a high MOI bat in warm up, despite there being no real performance change. Otsuji et al. (2002) reported that for the eight players they tested, swing speed decreased on the first swing after a weighted warm up but then returned to the same level as the pre warm up swings. The benefit of a weighted warm up was also found to be psychological and not biomechanical with participants reporting that the ordinary bat felt lighter and swung faster after a weighted warm up.

Montoya et al. (2009) found there to be bigger differences in swing speed after the three warm up conditions in a study of 19 recreational baseball players. Significant variation in swing speed was found between the groups that used a light bat to warm up (52.3 mph) and a normal bat to warm up (50.6 mph). The heavy bat (48.3 mph) was also found to significantly decrease swing speed post warm up, when compared to the normal bat and the light bat groups. This work strongly suggests that using a heavy (and high MOI) bat in warm up will decrease swing speed and using a light (and low MOI) bat during warm up will provide the best chance of maximising swing speed.

Furthermore, research has been undertaken to look at the effect on swing speed of training with a bat that has a dynamic MOI. An eight week study of 17 baseball players prescribed three training sessions per week of practicing swinging a bat. Eight of the group trained with a normal bat and nine players trained with the dynamic bat. The dynamic bat was constructed with a regular handle but instead of the barrel of the bat there was a large sliding mass. The effect of this bat was that at the beginning of the swing the MOI was much lower than a normal bat but the MOI increased to normal levels at the point of impact. An infrared motion capture system was used to record the swing speed at the sweet spot of the bats. The result was that the group who trained with the dynamic bat significantly increased their swing speed with a normal bat after the eight weeks, with a mean increase from 96.9 to 102.8 kmh^{-1}.

This work suggests that regular use of a certain MOI implement can have a long term effect on swing speed, as well as the immediate effects on swing speed of changing MOI. Therefore, the inertial properties of the equipment a player already uses could be an important factor to consider when calculating the implement that will produce maximum performance.

2.3.3 Moment of inertia and ball speed

As well as influencing swing speed, the MOI of an implement can have a large influence on outbound ball speed. It has been found that implements with a higher MOI can produce a higher outbound ball speed (Bahill, 2004; Cross & Nathan, 2009). However, because swing speed decreases with increasing MOI and swing speed has a direct influence on ball speed, there is a trade-off between swing speed and ball speed when changing MOI. Analysing how swing speed and ball speed change simultaneously with differing MOI would allow the balance point in the trade-off to be identified, which will be of use to players who are trying to optimise their performance.

It has been common practice in past research to record swing data for implements experimentally and then use an impact model to analyse the effect of swing changes on ball speed (Brody, 1997; Cross & Bower, 2006; Cross & Nathan, 2009; Cross, 2001; Daish, 1972; Smith & Cruz, 2008).

Daish (1972) developed a set of equations to model the impact between a golf club and ball based upon the principles of conservation of momentum (Equation 2.3) and coefficient of restitution (Equation 2.4).

The conservation of momentum principle is represented by:

$$MV + mv = MV' + mv' \qquad 2.3$$

where m is the mass of a ball, M is the mass of the implement, V is the implement velocity, v is the ball velocity and with V' and v' representing velocities after impact. The coefficient of restitution (e) is a measure of the energy dissipation of an impact, defined as:

$$e = \frac{v' - V'}{V - v} \qquad 2.4$$

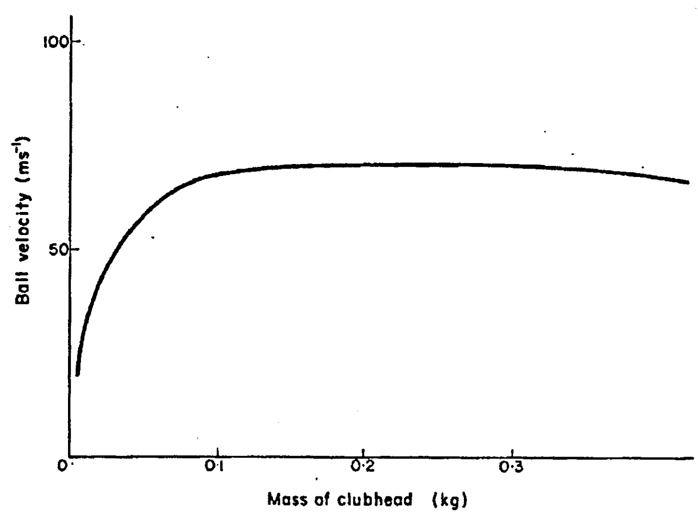

Figure 2.3 – A plot of ball speed against clubhead mass using modelled data for a golf swing (Daish, 1972).

These equations were combined to eliminate V', producing an equation for ball speed in terms of ball mass, club mass, e and club speed. The data from the swing speed experiments already discussed were placed into this equation and ball speed was plotted against clubhead mass, as shown in Figure 2.3, where an optimum value of 0.20 kg was identified. Daish concluded that this maximum ball speed was achieved with the same clubhead mass for all players. Cross & Bower (2006) repeated the same ball speed calculations as Daish and also found that the maximum in v' was very broad, making it unnecessary for a player to greatly alter the mass of their implement.

Brody (1997) developed a similar one dimensional rigid body model but also incorporated the conservation of angular momentum and the specific impact location on the implement. The equation for ball speed is as follows, where b is the distance from the centre of mass to the impact point, V_{IP} is the velocity of the impact point and I_{CM} is the MOI about the centre of mass:

$$v' = \frac{v\left(mb^2 + \frac{I_{CM}.m}{M} - e.I_{CM}\right) + V_{IP}.I_{CM}(1+e)}{mb^2 + I_{CM} + \frac{I_{CM}.m}{M}} \qquad 2.5$$

Cross & Nathan (2009) used a very similar set of equations to calculate ball speed for a set of hypothetical baseball bats when reviewing the effect of MOI on performance. Batted ball speed increased with MOI to a maximum point and then decreased with further rises in MOI, as can be seen in Figure 2.4. The point of maximum ball speed was also found to be dependent on the inbound ball velocity, with ball speed being a maximum with a higher MOI as inbound ball speed increased.

The effect of adding a point mass to a tennis racquet was studied using a laboratory based experiment to measure the ratio of outbound to inbound ball speed, known as the apparent coefficient of restitution (ACOR) (Cross, 2001). ACOR was measured for racquets with a 30g mass added at different locations and rigid body equations were used to calculate ball speed for a service shot. The results showed that the further towards the racquet tip that the mass was added, the higher the outbound ball velocity.

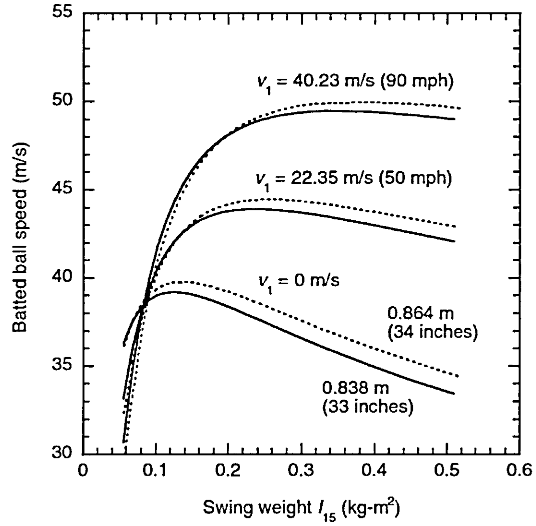

Figure 2.4 - Ball speed against MOI for a series of modelled baseball bats (Cross & Nathan, 2009).

Measurements of ACOR were also combined with rigid body modelling in a study to analyse the effect of altering softball bats (Smith & Cruz, 2008). This analysis found that if a point mass equivalent to 10% of a bat's original mass was added at the distal end, this would increase MOI by 20% and increase batted ball speed by 2.9%.

Cross (1999) proposed that modelling sporting implements as flexible beams was a more appropriate method than using rigid body mechanics. This theory was tested in a study that compared the two methods with experimental data for a freely suspended tennis racquet (Goodwill & Haake, 2001). The rigid and flexible beam models were used to calculate ball speeds for impacts at a series of locations on the stringbed. The results for both models' results were found to correlate well with the experimental data. It was also found that for impacts within 30 mm of the node point (fundamental frequency location) the velocity results for the two models were equal but beyond this point the rigid body model overestimates ball speed. Therefore, using rigid body models is an acceptable form of calculating ball speed if the modelled shots impact the implement close to the node point.

The studies considered in this section show that ball speed is affected by changing the MOI of an implement. An increase in MOI increases ball speed to a maximum point, after which further increases in MOI decrease ball speed. Finding the point at which ball speed is a maximum would be very beneficial as players could modify or change their equipment to have the required MOI to produce maximum ball speed. Rigid body modelling has been shown as an accurate method for calculating ball speed for implements, which would make determining the MOI that produces maximum ball speed simpler than if extensive experimental work was required.

2.3.4 Moment of inertia and technique

There has been very little research to investigate the effect of MOI on swing technique. However, it has been common to try to exclude technique from analysis. When analysing the effect of MOI on swing speed, Cross & Bower (2006) used a restricted motion analysis and Smith et al. (2003) normalised swing speed by a player's mean, to reduce the effects of skill on results.

A study into the effect of MOI on performance with a 5-iron in golf suggests that players with low skill levels are best to use a club head with a high polar MOI (Nesbit et al., 1996). This is because a low-skilled player will be less concerned about fine control and more concerned about producing a consistent shot with the quality of a central impact. Conversely, players with a high skill level should use a club head with a lower polar moment of inertia to allow more control of the club through the swing. An off centre impact of a tennis ball with a tennis racquet can produce a rebound ball velocity 15% lower than that of a centrally impacted ball (Cross, 2010). Tests were carried out both with a tennis racquet and a golf putter and filmed from above during off-centre impacts. In tennis, it was found that the rebound ball speed is lower near the edge of the racquet than for a central impact and increasing polar MOI increases the size of the sweet spot. In golf it was found that the off-centred impact can impart side spin onto the ball, which will affect the flight path and determine the quality of the shot.

Altering the MOI of a tennis racquet can be used to match a player's technique. Cross (2001) found that, as well as increasing ball velocity, adding mass at the tip of the racquet, and consequently increasing MOI, also moved the location on the stringbed that produced maximum ball speed. Adding mass to the racquet tip moved the sweet spot towards the tip, which would be of benefit to players who naturally strike the ball in this region of the racquet head.

2.4 Physical characteristics of players

2.4.1 Profiling athletes

There are two main methods used to profile the physical characteristics of athletes. The first is to quantify an athlete's strength, which can be done by measuring torque throughout a selected motion with an isokinetic dynamometer. For example, an isokinetic dynamometer was used to measure the torque during internal and external rotation of the shoulder of both arms in 25 professional baseball pitchers (Sirota, Malanga, Eischen, & Laskowski, 1997). Torque was considered to be a very representative indicator of shoulder strength. The players performed the movement at both 60°/s and then 120°/s. No significant differences were witnessed between the two test speeds and, surprisingly, there was no significant difference between the torque with the pitcher's dominant and non-dominant arms.

The second way in which players are profiled is to quantify the muscle activity of an athlete during a sporting motion. These methods are used to look for differences between players but also for differences within players, between dominant and non-dominant sides. As an example, shoulder function was analysed for six collegiate tennis players using electromyography (Ryu, McCormick, Jobe, Moynes, & Antonelli, 1988). This study analysed the activity of eight muscles around the shoulder for serves as well as forehand and backhand groundstrokes. The analysis allowed the researchers to break down each movement into stages based on the muscle activity, which is then of use to coaches who can train the individual segments. The results also identified that the serratus anterior muscles at the side of the chest are active in all three shot types and could therefore be susceptible to overuse injuries.

When considering the measurement of an athlete's strength, there are six categories which can be tested: maximal strength, high-load speed-strength, low-load speed-strength, rate of force development (RFD), reactive strength and skill performance (Newton & Dugan, 2002).

Maximum strength is defined as the highest force a person is capable of producing during a slow eccentric, concentric or isometric movement. Speed-strength is the highest force a person is capable of producing when performing an eccentric or concentric motion as rapidly as possible. High-load speed-strength is with weight that is greater than 30% of maximum load and low-load speed-strength is with weight less than 30% of maximum load. RFD is defined as the rate at which an athlete is able to generate force and is usually taken as the gradient of a force-time plot. Reactive strength is the ability to tolerate a high stretch load and change from a rapid eccentric movement to a rapid concentric movement. When measuring RFD, isometric tests are not relevant enough to dynamic sporting applications and it is much better to use a concentric movement test, for example a concentric only squat jump for sprinters (Wilson, Lyttle, Ostrowski, & Murphy, 1995). Skill performance refers to the ability to control the other 5 strength variables in order to best achieve the desired outcome.

2.4.2 Physical profile and performance

Strong relationships have been found between physical profile measures and performance variables in several sports with a swinging motion.

Such a relationship was found for elite tennis players without the need for laboratory based assessment (Vaverka & Cernosek, 2012). Standing height and serve speed data was collected for male and female players at the four grand slam tournaments in 2008 (78 – 84 men, 71 – 85 women). Body height was found to correlate significantly with service speed. For men, $r = 0.55$ for the relationship between body height and mean first serve speed and for women, r = 0.52. This relationship shows that taller players have a significant biomechanical advantage when it comes to generating ball speed in the serve.

A study of ten low and ten high handicap golfers profiled players using a two-handed cable wood chop that replicated the same torso and shoulder movement as a golf swing (Keogh et al., 2009). The measure of strength in this motion was the mass lifted via the cable and pulley mechanism. The 20 players also hit ten balls at a target 15m away, aiming for maximum swing velocity. A 5-iron was used by all players and swing speed was measured using a radar gun operating at 34.7 GHz. The golf specific cable wood chop strength (r = 0.706) and total arm length (r = 0.453) were both found to be significantly related to swing speed. These variables were also strongly related to player handicap.

The relationship between routinely measured sports performance variables with bat swing velocity was explored for a group of 22 division 1 collegiate baseball players (Szymanski, Beiser, Bassett, Till, & Szymanski, 2011). It was found that there were strong relationships between bat speed and a player's grip strength, peak power, body mass and 1-arm dumbbell row. It was proposed that these measures could be used as a guide to predict performance (bat velocity). However, it was pointed out that, although these measures appear significant, it is also very important to understand the swing biomechanics and bat properties to be able to fully assess performance potential.

In a study of 35 elite badminton players, who were compared to a group of recreational active people of the same age, the elite players were found to possess higher maximal and explosive muscle strength (Andersen, Larsson, Overgaard, & Aagaard, 2007). An isokinetic dynamometer was used to measure peak torque during knee extension and flexion for both groups. The measures were repeated after a 14 week programme of strength training. The reference group improved slow speed isometric muscle strength to the same level as the elite players. However the reference group did not improve during high velocity contractions (240°/s) and isometric rate of force development and peak torque at 240°/s were still significantly greater in the elite players. This shows that high level players have long-term muscle strength developments that set them apart from recreational players.

A study of 12 national level junior tennis players found that there are several lab based measures that correlate significantly with tennis ranking (Girard & Millet, 2009). Speed was quantified by measuring time for 5, 10 and 20 m sprints; vertical power was measured with squat jump, countermovement jump and drop jump heights; maximal isometric strength was measured for hand grip and plantar flexion on both sides. The results demonstrated that speed ($r = 0.74$), vertical power ($r = -0.80$) and maximal strength of the dominant side ($r = -0.73$) were all significantly correlated to player ranking but strength in the non-dominant side was not related to ranking.

Measures of strength and size are not the only characteristics of athletes that are related to performance. A study into the stability of fifteen female collegiate lacrosse players found that balance ability, measured using a Biodex balance system, and visual search ability, measured with the Trail-Making Test, are significantly related to lacrosse shot accuracy ($r = 0.76$ and $r = 0.52$ respectively) (Marsh, 2010).

Profiling players has made it possible to distinguish between elite athletes and recreational sportsmen with a wide range of laboratory measures. There are both generic and very sport specific measurements that can be made in the laboratory or in the field, which are strongly related to performance. Relationships are also found within groups of high performance athletes. Therefore, it should be possible to predict performance variables, such as swing speed, using anthropological and physiological measurements.

2.4.3 Muscle force – velocity

It is also important to understand what physical limitations there are that could affect performance. Changing the physical properties on an implement will change the load applied to the player wielding it. This changing load on muscles has been shown to be directly related to the rate of muscle shortening for isotonic contractions (Fenn & Marsh, 1935; Hill, 1938; Wilkie, 1949).

The first work in this area by Fenn & Marsh (1935), which tested frog and cat muscles, hypothesised that the relationship between load and muscle velocity would be linear but found the data was best fitted by an exponential curve. Hill (1938) best described this condition as a hyperbolic relationship explained by Equation 2.6:

$$(P + y)(V + z) = constant \qquad 2.6$$

where P is the variable load on the muscle, V is the contraction velocity of the muscle fibres and y and z are empirical constants, which are subject to temperature. This relationship was derived from experiments on the Sartorius muscle in frogs but is still widely used and accepted as a valid equation. The experiments only measured the rate of contraction for the muscle attachment at one length and without any resting load. However, subsequent experimental work has shown the relationship remains valid over a full range of muscle lengths (Matsumoto, 1967).

Wilkie (1949) tested the validity of Hill's equation for human muscle by measuring the contraction velocity during maximal elbow flexion. The movement was performed in the vertical plane with the upper arm abducted 90° from the body. Five velocity measures were recorded with a range of different loads over a two week period and the experiments showed that Hill's equation gave a good fit for the data, once the inertia of the forearm had been accounted for.

The relationship defined by Hill in Equation 2.6 has also been compared to swing speed data for baseball players at a range of skill levels (Bahill & Karnavas, 1989). A baseball swing is a much more complex motion than the simple contractions that the relationship was founded on but the results showed that a hyperbolic curve was the best fit for the swing speed – bat mass data for a large proportion of players tested. For most little league players, half of college players and a quarter of major league players tested a hyperbola gave the best fit. The data for the remaining players was best fit with a straight line. Further analysis showed that the characteristic that determined whether a player would be best fit with a straight line or hyperbola was hand to eye reaction time. Players with a short reaction time were best fit with a hyperbolic curve and players with a long reaction time found little difference between bat weights and were best fitted with a straight line.

The work in this field suggests that there are physiological limits to performance that could affect how players respond to changes in equipment. These limits may also be very individual. This suggests that the muscles of a player could pre-determine the rate at which their swing speed would decay with increasing MOI. Therefore, it may be possible to use a measure of physical profile to predict performance.

2.5 Player technique

2.5.1 Characterising skill

In the previous section, examples were shown of physical characteristics being quantified and related to performance variables. Quantifying skill or technique is a more difficult task as it is a more relative term. The approach that is often adopted is to work backwards from what is known to be a good performance. For example, taking a group of high level players and a group of low level players, observing the differences in technique and using the findings to quantify what constitutes a good technique.

One such method was to investigate timing patterns of tennis players performing forehand groundstrokes (Landlinger, Lindinger, Stöggl, Wagner, & Müller, 2010a, 2010b). An eight camera motion capture system was used to record four sets of ten forehand groundstrokes for six elite ATP professionals and seven highly ranked youth players. The elite players had higher racquet velocities than the youth players and further analysis of the movements found that this was created by differences in the timing of the swing. There were only small differences in the magnitude of the rotational velocities for the trunk and pelvis but there were large differences in the timing of these peak velocities. The elite players reached peak rotational velocity 0.03s later than the youth players, peaking 0.055s before impact.

When comparing the biomechanics of slap shots in ice hockey for different player groups, it was found that the most influential factor in determining peak puck velocity is the player's technique (Wu et al., 2003). A group of 40 players, half of whom were highly skilled and half unskilled, each performed three wrist shots and three slap shots with stick motion being recorded with high speed video, ground reaction force measured using a force plate and puck speed measured using a radar gun. The unskilled players had a mean puck speed of 16.0 ms^{-1} for wrist shots, compared to 19.7 ms^{-1} for the skilled players. The difference between the groups increased for the slap shot with a mean puck speed of 23.3 ms^{-1} for the unskilled and 30.0 ms^{-1} for the skilled players. The skilled players were able to adjust their hand positions for a slap shot to apply more vertical force through the stick, resulting in higher puck velocities. The two groups had similar strength characteristics (determined by bench press 1RM and grip strength), meaning the change in technique of the skilled players was the major difference attributed to the gap in puck speed.

When comparing elite and collegiate badminton players, it was found that elite players were able to get higher shuttle speeds in a jump smash shot by altering their technique (Tsai & Chang, 1998). Seven elite and four collegiate players were filmed with two high speed video cameras returning service shots as either a smash or jump smash. For the jump smash, the elite players lowered their centre of mass before the shot to save energy and then had faster elbow and shoulder movements during the shot to produce shuttle speeds of 67.9 ms^{-1} compared to 56.5 ms^{-1} for collegiate players.

As demonstrated by these examples, sporting movements are often very complex and defining good technique is not straightforward. A lot of background work, testing players of different standards, is required to achieve an understanding of the performance differences between good and bad technique. This means that it is possible to measure aspects of technique or skill and they could be used as performance indicators if well defined. However, it is unlikely to be worthwhile attempting to predict performance with an alternate implement based on technique measures because the technique could change with the new implement, as discussed earlier.

2.5.2 Breaking down complex movements

With so many sporting tasks being complicated movements involving multiple body segments, it can be useful to break them down into their contributing parts to gain an understanding of which smaller actions are dominating the end result.

The method of speed generation in baseball is best described as being a kinetic link, which transfers momentum from large base segments to smaller linked segments (C. M. Welch, Banks, Cook, & Draovitch, 1995). This motion is started by the acceleration of the hip segment around the trunk axis to a maximal velocity. The momentum of this is then transferred through the shoulders and arms until the bat is accelerated forward, ideally to a maximum velocity at the time of contact with the ball.

In a study of the motion of the badminton smash of thirteen male players, it was found that the wrist is the joint most responsible for the speed of the racquet head at impact (Rambely, Osman, Usman, & Abas, 2005). A six camera system was used to record smash shots in competition and the joints of the upper limbs were digitized to calculate their contributions to the racquet head velocity. It was found that the wrist action produces 26.5% of the racket head speed, compared to 9.4% and 7.4% for the elbow and shoulder respectively. Separate research analysing the smash shot for eight badminton players also found that the relationship between wrist angular velocity and shuttle velocity is highly significant ($r = 0.93$, $p<0.01$) (Hussain, 2011).

A similar analysis has been carried out for eleven high performance tennis players performing a power serve (Elliott, Marshall, & Noffal, 1995). Trials were filmed at 200 frames per second with passive markers added to the racquet and player. Body segment rotations were calculated using vector equations. The results showed that the biggest contributors to racquet head velocity were shoulder internal rotation and hand flexion, which accounted for 19.0 ms^{-1} (54.2%) and 12.6 ms^{-1} (30.6%) respectively.

These analyses could be employed when attempting to find relationships between physical profiling measures and performance variables. If the contributing elements of a complex motion are understood, strength measurements of those elements can be taken and related to the relevant performance variable. This has yet to be tested but would seem to be a logical step based upon the findings in the literature.

2.5.3 Classifying movement patterns

With studies involving complex movement patterns a method for the classification of movements is very important. This could be where a comparison needs to be made between multiple movements or to make sure that a movement is being performed as expected. There are several methods that could be used for this form of analysis.

As discussed in the previous sections, movements can be characterised by the contributions of smaller motions (Elliott et al., 1995) or by assessing timing patterns within an action (Landlinger et al., 2010b).

Alternatively there are also computational methods to classify movement patterns. The first is to use a decision tree approach, whereby the range of possible outcomes are listed and a series of yes/no questions are formed to lead each case to one of those outcomes (Wiersma, 2007). For this method a series of important sub-movements need to be identified with associated threshold values that would determine a motion as one type of movement or another. With a series of these junctions combined, with threshold values for each, an algorithm is formed that can categorise new movement patterns. This method requires human input to setup the tree and is therefore still subjective.

There are more automated methods that can train themselves on a dataset. One such example is self-organizing maps (SOMs), which are a slightly abstract method for clustering data. A SOM allows the analyst to view multidimensional data in one planar plot, which means many key variables can be considered at once to group trials as necessary (Kohonen, 1990). The map is trained on a data set and takes the form of a grid of nodes. When a vector, containing one value for each variable, is entered into the SOM, it activates one of the nodes on the map. Multiple vectors can be entered for a single movement pattern, which will activate a group of nodes on the map. The pattern of activated nodes can then be used to compare between trials or players as has previously been done for discus throwers and basketball players (Bauer & Schollhorn, 1997; Lamb, Bartlett, & Robins, 2010).

2.5.4 Speed – accuracy trade-off

Speed is not the only performance outcome that is of interest to players. In a competitive situation, accuracy can be equally important. There is a well understood principle of a speed – accuracy trade-off (Chow et al., 2003; Fitts, 1954; Freeston & Rooney, 2014; Sachlikidis & Salter, 2007).

Fitts (1954) devised a test where participants were asked to use a stylus to alternately strike two connected strips on a large metal plate. The test lasted 15 seconds and participants were asked to score as many hits as they could whilst emphasising accuracy over speed. The test was repeated for different width plates and for different tolerances on the connected target strips. The overall error across the different scenarios was very low (1.2% of taps) yet the time between taps increased dramatically between test situations (0.18s – 0.73s). This indicates that participants were able to successfully modify their speed to maintain accuracy.

This trade-off has also been observed in competitive sport. Chow et al. (2003)used two high speed cameras to film four male and four female tennis players performing serves during competition. The study found there to be large differences in first and second serves. On the second serve, ball speed was found to decrease from a mean of 46.8 ms^{-1} to 35.4 ms^{-1}. Racquet velocity only decreased by a small amount (34.7 ms^{-1} to 33.2 ms^{-1}), although the lateral and vertical components of racquet speed increased. This is because the players were placing more spin on the ball to improve their chance of landing it in the service box.

It is important to consider this relationship when making changes to equipment that could affect speed, as an increase in speed might also result in a reduction in accuracy, leaving the player in no better position than before any changes.

2.6 Measuring human motion

The tracking of swinging movements can be performed with a range of technologies, some of which have already been mentioned in this chapter. The most common methods among the literature reviewed for this study were high speed video analysis (Elliott, Fleisig, Nicholls, & Escamilia, 2003; Fradkin, Sherman, & Finch, 2004; Smith & Kensrud, 2014; Smith et al., 2003, 2012) and motion capture analysis (Crisco et al., 2002; Cross & Bower, 2006; Kwan, Andersen, Cheng, Tang, & Rasmussen, 2010; Kwan & Rasmussen, 2011; Landlinger et al., 2010a, 2010b). These are very similar camera based technologies, which track movements using one of two types of markers. Passive markers reflect light from an external source and can be tracked between frames. These can take the form of strips of tape, foam spheres (as small as 3 mm in diameter) or distinct markings. The alternative is to use active markers which transmit a light signal back to a receiver. Active markers are larger than the passive markers, have a much higher mass and are therefore more intrusive to the participants being recorded.

High speed video analysis can be executed in two dimensions where a single camera is placed normal to the plane of motion, or in three dimensions with two or more cameras. Video footage has to be digitized to retrieve location data for the points of interest. This can be done by filming with passive markers on a participant or by digitizing distinct landmarks. Research involving the filming of markers on tennis racquets during competition practice found that velocity was measured with an average error of 0.5 ms^{-1} (Choppin, Goodwill, & Haake, 2011).

Motion capture systems require a minimum of two cameras but typically have at least four. The systems are based on recording light from a set of markers in a finely calibrated capture volume. Therefore markers are required at every location for which position data is sought and these could be active or passive markers depending on the system being used. When the capture volume has been calibrated, the software linked to the cameras automatically calculates the position of every marker, which greatly reduces data processing time.

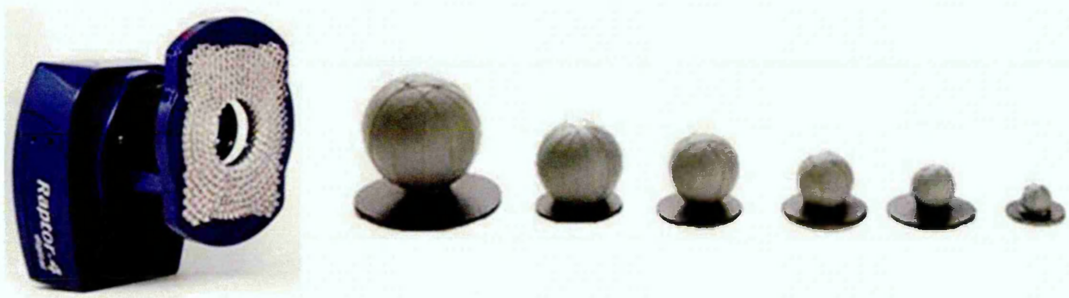

Figure 2.5 - Motion Analysis Corporation Raptor-4 camera (left) and markers (right) ("Motion Analysis," 2015).

An example of a passive marker system is that developed by the Motion Analysis Corporation (Motion Analysis, 2015). The cameras emit near red light and record it after it bounces back from the retro-reflective markers. Markers ranging from 6.4 to 25.4 mm in diameter are shown in Figure 2.5 alongside a raptor camera but they can be as small as 3 mm diameter or retro-reflective tape can be used where a marker is inappropriate or unnecessary. Active marker systems include the CODA 3D systems (Codamotion, 2015), which uses 10 mm markers that transmit light to an array of three sensors that are linked to a computer to calculate position. An advantage of this system is that it is pre-calibrated, which reduces setup time. A study analysing the kinematics of the smash shot in badminton using motion capture found that velocity measurements have a mean error of 0.15 ms^{-1} (Kwan et al., 2010).

Mitchell et al. (2000) used high speed video analysis and the CODA motion capture system for their experiments with tennis players and expressed a preference for the motion capture system. The high speed video system had the advantage that they did not need to add any mass to the racquets, whereas the CODA markers added 0.044 kg, which also altered the MOI. However, the high resolution and speed of data processing for the motion capture was a big advantage and outweighed the marker problem.

The use of light gates for measuring swing speed has also been a common technique (Bahill, 2004; Daish, 1972; Koenig et al., 2004; Montoya et al., 2009). This method is beneficial because it is unobtrusive to the participants, with no markers required. The downside is that light gates cannot be used to measure multiple locations at once and they cannot be used to track movements. They are typically only used to record the speed of a bat or club and alternate systems are needed to measure player movements. The uncertainty in swing velocity measurements when studying baseball swing speed was quoted by Koenig et al. (2004) as 4.4% (0.76 – 1.50 ms^{-1}). Radar has also been used to measure swing speed for golf clubs and is often used to measure ball speed both in research and commercially. The quoted mean error in velocity measurement for the radar gun used by Keogh et al. (2009) was 0.04 ms^{-1}. This compares very favourably with camera based systems but radar guns have the same drawback as light sensors in that they cannot track movement or give speeds for more than one point, so are of limited use when analysing sporting movements.

Alternatively, another non-cinematographic approach could be to use a magnetic tracking device. The Polhemus electromagnetic systems (Polhemus Liberty, 2015) use a hub station that transmits a magnetic field which can then be picked up by wired in sensors. These sensors detect changes in the magnetic field as they pass through it and these changes can be used to calculate position and orientation. The advantage of these systems is that multiple units can be used together without interference and line of sight is not a pre-requisite. However, a study investigating the calibrated accuracy of the Polhemus Fastrak system found that when sensors were more than 1.2 m from the hub errors rapidly increased (Day, Murdoch, & Dumas, 2000). Within that small range, the average error in position measurement was 0.012 m and 1.2° for orientation. Another restriction of these systems is that magnetic materials cannot be in range of the test equipment because they interfere with the magnetic field and skew measurements.

Inertial sensors such as accelerometers and gyroscopes can be used to track fast movements and by integrating the signal with respect to time, velocity and position data can be acquired. Accelerometers have the advantage of being able to measure at very high frequencies, with no line of sight problems and they can be wireless. The big problem with inertial sensors is that the data is prone to drifting, by which readings can become increasingly erroneous over time if a sensor has a small bias (G. Welch, 2002).

2.7 Literature summary

It has been well established that for swinging motions, there is a strong relationship between moment of inertia and swing speed, where swing speed decreases as MOI increases. The rate at which swing speed decreases (n) has been found to be very similar for studies of different sports but no attempt has been made to explain what determines n. Many of the experiments that have measured n have been with small groups of participants. Therefore, a study with a large, diverse cohort of participants may find different results.

There is also a relationship between MOI and ball speed, where increasing implement MOI produces increased ball speed. This creates a trade-off between swing speed and ball speed, with a maximum ball speed at an optimum value of MOI.

There are many physical attributes of players which can be measured in a laboratory situation and it is possible to link these attributes to performance measures. Complex movements can be broken down into contributing parts and classified to simplify analysis. This offers great potential for predicting sporting performance based upon physical profile.

Skill is a variable that is very difficult to quantify and requires measures of performance in order to rank movement patterns. Several studies assessing swing speed have normalised data to remove effects of skill from the results.

The measurement of velocities can be done simply and accurately using radar guns. However, to capture complete movements a more invasive system is required. High speed video analysis and motion capture systems both produce accurate results but motion capture has the advantage of built in data processing which reduces analysis time, compared to video techniques.

At the beginning of the chapter, implement properties, player physical profile and player technique were introduced as the main areas that would influence performance. All three have influence on performance but it is not possible to assess the effects of all three areas simultaneously. The review in this chapter has shown that quantifying physical profile and implement properties is more achievable than measuring skill. Therefore, the project will narrow to investigate these two areas.

2.8 Project aim and objectives

This research aims to improve the understanding of the relationship between moment of inertia and swing speed such that players can make informed decisions when customising or replacing equipment.

This will be achieved by delivering on the following objectives:

- Carry out rigorous experiments with a large group of participants, to quantify n and develop relationships between physical profile and MOI.
- Develop a model to predict swing speed using MOI and measures for physical profile.
- Measure the ecological validity of the predictive model.
- Propose a method to determine the optimum value of MOI for an individual.

Chapter 3: A swing speed meta-analysis

3.1 Introduction

In the previous chapter, swingweight was highlighted as one of the chief factors that can influence performance in sports with a swinging motion. The relationship between swingweight and swing speed is an important topic which requires more in depth analysis. Literature suggests that swing speed decays with increasing swingweight at a similar rate across a range of motions. This chapter aims to investigate whether a global rate of decay exists. This will be achieved by meeting the following objectives:

- Compile swing speed and swingweight data from many different sports.
- Define the most suitable definitions of swing weight and swing speed.
- Determine the optimum method for representing swing speed data.
- Calculate the global rate of swing speed decay and compare with previously reported values.

Some of the work in this chapter is presented by Schorah, Choppin, & James (2012)

3.2 Meta-analysis

Existing data from literature was used to compare the swingweight – swing speed relationships across several sports. Published data was analysed for nine different sports: badminton, baseball, cricket, field hockey, golf, ice hockey, lacrosse, tennis and tenpin bowling. All of these sports involve swinging different mass implements in different motions. Nonetheless, if swing speed is found to decrease with increasing moment of inertia at a uniform rate, this could be a very useful relationship for players, coaches and manufacturers.

In the case of tenpin bowling, the 'implement' was defined as being the bowler's arm and ball as one fixed unit which rotates around the shoulder joint. The moment of inertia data for bowling was created using mean body segment data as presented by de Leva (1996). Specific shots chosen for swing speed data were:

- Badminton smash.
- Front foot drive in cricket.
- Field hockey hit.
- Golf drive.
- Ice hockey slap shot.
- Overhead lacrosse shot.
- Forehand ground stroke in tennis.

The swings were considered up to the impact event or ball release, as appropriate.

3.2.1 Data search strategy

In order to compile data points for this analysis, an extensive database search was required. Searches were carried out using Scopus, Google Scholar and Sheffield Hallam University's Library Gateway search tool.

The aim of the search was to find values of MOI and swing speed for as many sports as possible, with the swing speed being experimentally measured. Initial searches were based around the 'Swing speed swingweight' search term. However, it soon became apparent there was a limited supply of articles with data for both MOI and swing speed so the searches were separated. Swing speed data search used 'swing speed' and 'swing velocity' search terms coupled with a sport name. The MOI search used 'swingweight' and 'moment of inertia' as search terms, coupled with the name of a sport. The sports included in the search were: badminton, baseball, cricket, fencing, field hockey, golf, hurling, ice hockey, lacrosse, racquetball, rowing, shinty, softball, squash, tennis, ten-pin bowling. However, it was not possible to find experimentally measured data for both variables in all of these sports.

The search returned data for both MOI and swing speed from nine different sports, listed in Table 3.1, spanning 31 sources. A further search was carried out, using the same databases, to find swing time data. This search used 'swing time' plus the name of the sport to find data for the nine sports that already had swing speed and MOI data. The swing time data was found in a combination of 11 sources, some of which were the same as those previously found for swing speed.

3.2.2 Defining the variables

In a comparison between such a range of sporting motions, there are many different definitions used for swingweight and swing speed. Therefore keeping the data consistent, so that any comparisons are fair, is of great importance. In order to study the effect of swingweight on swing speed, both variables need to be explicitly defined to ensure that all data describes the same property.

The immediate choice of parameter for swing weight was between the first moment (L_{cm}), or the moment of inertia (*I*) to describe the implement's rotational resistance, as defined by Brody (2000). Transverse moment of inertia (MOI) was chosen as the most appropriate measure and will be used from here on in this thesis. There were, however, four possible parallel axes about which the moment of inertia could be defined. These axes were all perpendicular to the handle and in plane with the implement's face. The locations, depicted in Figure 3.1 were:

- The centre of mass (CoM).
- The location of the distal hand (DH).
- The end of the handle (HE).
- The instantaneous centre of rotation (ICOR).

The data taken from literature was transformed to meet these four definitions using the parallel axis theorem as shown by Equation 3.1

$$I_{BB} = I_{AA} + mr^2 \quad (3.1)$$

where the moment of inertia about axis BB (I_{BB}) is equal to the moment of inertia about axis A (I_{AA}) plus the mass multiplied by the square of the distance between the two axes (r).

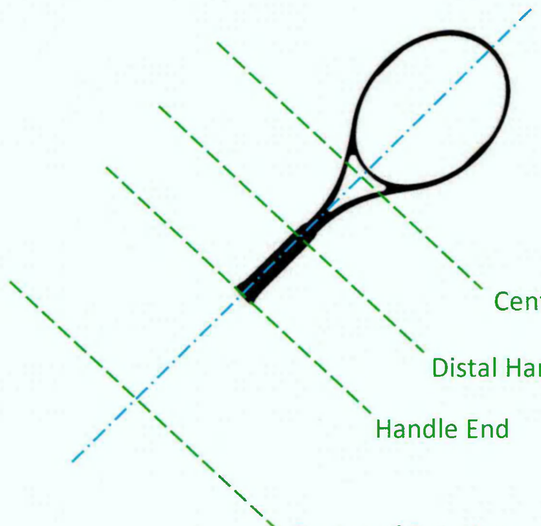

Figure 3.1 - Image to show the four proposed moment of inertia axes for a sporting implement, using a tennis racquet as an example.

For the swing speed, there were also four possible definitions for the velocity which could be used in analysis: three linear and one rotational. These options were:

- Velocity at centre of mass.
- Velocity at the impact point.
- Velocity at the implement tip.
- Angular velocity.

The linear velocities were specified as vectors normal to the implement face at the time of impact. To calculate angular velocity accurately, the centre of rotation is required and as explained previously, this data was not available for all sports making angular velocity an unreliable option.

The collated swing speed data was transformed to meet each of the four proposed definitions and the same was done for swingweight. A Pearson's correlation test was then performed between swing speed and swingweight for all possible combinations of variable definition.

The handle end moment of inertia has strong, significant relationships ($r<-0.6$, $p<0.1$) with the linear velocity at the implement's tip or impact point. The only other definition for MOI which correlated as strongly was the moment of inertia about the centre of rotation. Whilst it might be preferable to use the moment of inertia about the centre of rotation to characterise the full resistance to swinging, there was not enough data to make accurate calculations of this for all nine sports. There is insufficient data to know how each implement is held and what the centre of rotation for each swing is. Therefore, in order to transform MOI to an axis through the centre of rotation, an element of guesswork was required, which is not ideal. Consequently the MOI about the handle end (I_{HE}) was chosen as the most appropriate and reliable definition. The advantage of the handle end is that it is a definite location on every implement which will not change. Conversely the location of the distal hand is a moving position and likely to change between players, making it less reliable. MOI about the handle end and centre of mass are both simple to calculate using Equation 3.1. In all cases, the implements are being rotated at a distance from the centre of mass; therefore, I_{HE} gives a better representation of the resistance to motion.

After consideration it was decided that the most suitable velocity definition to use would be the linear velocity at the implement tip (V_{TIP}) as it is an unambiguous, unchanging location. The velocity at the impact point would be the most useful velocity to analyse and it does correlate slightly more strongly with I_{HE} than the tip velocity ($r=-0.60$, $p=0.086$ compared to $r=-0.59$, $p=0.095$). However, the location of the impact point can change between shots and even more so between players, meaning it is difficult to define a constant impact location.

The averaged data values for the most important variables across all nine sports are shown in Table 3.1, including implement length (L), mass (M), swing time (t_{swing}) and the outbound velocity of the impacted object (v').

Table 3.1 - Mean values of implement properties compiled from a literature search. Length (L), mass (M), MOI (I_{HE}), swing speed (V_{TIP}), swing time (t_{swing}) and ball speed (v') are presented for badminton, baseball/softball, cricket, field hockey, golf, ice hockey, lacrosse, tennis and tenpin bowling.

Sport	L (m)	M (kg)	I_{HE} (kgm^{-2})	V_{TIP} (ms^{-1})	t_{swing} (s)	v' (ms^{-1})
Badminton	0.67	0.090	0.013	50.4	0.048	57.4
Baseball	0.85	0.847	0.240	28.1	0.170	42.7
Cricket	0.86	1.127	0.499	16.5	0.360	20.4
Field Hockey	0.93	0.601	0.239	26.2	0.170	29.8
Golf	1.12	0.316	0.461	47.4	0.199	73.6
Ice Hockey	1.47	0.548	0.600	28.4	0.140	22.5
Lacrosse	1.10	0.370	0.400	23.7	0.199	25.7
Tennis	0.69	0.309	0.035	25.9	0.222	29.2
Tenpin Bowling	0.71	8.150	2.146	10.0	0.600	-

The data displayed in Table 3.1 is aggregated from the following sources: Badminton (Hsieh, Wong, Wang, Chung, & Wang, 2004; Kwan et al., 2010; Tang, Abe, Katoh, & Ae, 1994), Baseball (Crisco et al., 2002; Nathan et al., 2011; Nathan, 2009; Ranganathan & Carlton, 2007; Smith et al., 2003; Smith & Cruz, 2008), Cricket (Bull, 2008; Singh & Smith, 2008; Stretch, Buys, Toit, & Viljoen, 1998; Stuelcken, Portus, & Mason, 2005), Field Hockey (Mchutchon, 2006; Rai, Bhangu, Mohanty, & Goel, 2002), Golf (Daish, 1972; Miura, 2001; Quintavalla, 2006; Thompson, Cobb, & Blackwell, 2007), Ice Hockey (Anderson, 2008), Lacrosse (Crisco, Rainbow, & Wang, 2009; Hayden & Wittman, 2010; Livingston, 2006; McGinnis, 2005), Tennis (Choppin et al., 2011;

Mitchell et al., 2000; Reid, Whiteside, & Elliott, 2010; "Tennis Warehouse," 2015), Tenpin Bowling (King et al., 2010; Nathan, 2009; Wood, 2015).

3.2.3 Swing speed representation

If swing speed (V) is plotted against moment of inertia (I), an exponential decay is seen as described in the equation defined by Cross and Bower (2006):

$$V = C \times I^{-n} \qquad 3.2$$

where C is a player constant and n is the rate of decay of swing speed. As we can see in Figure 3.2, making changes to n or C can be difficult to separate from one another and it is not immediately apparent if two data sets would have a similar value for n (as this chapter seeks to explore).

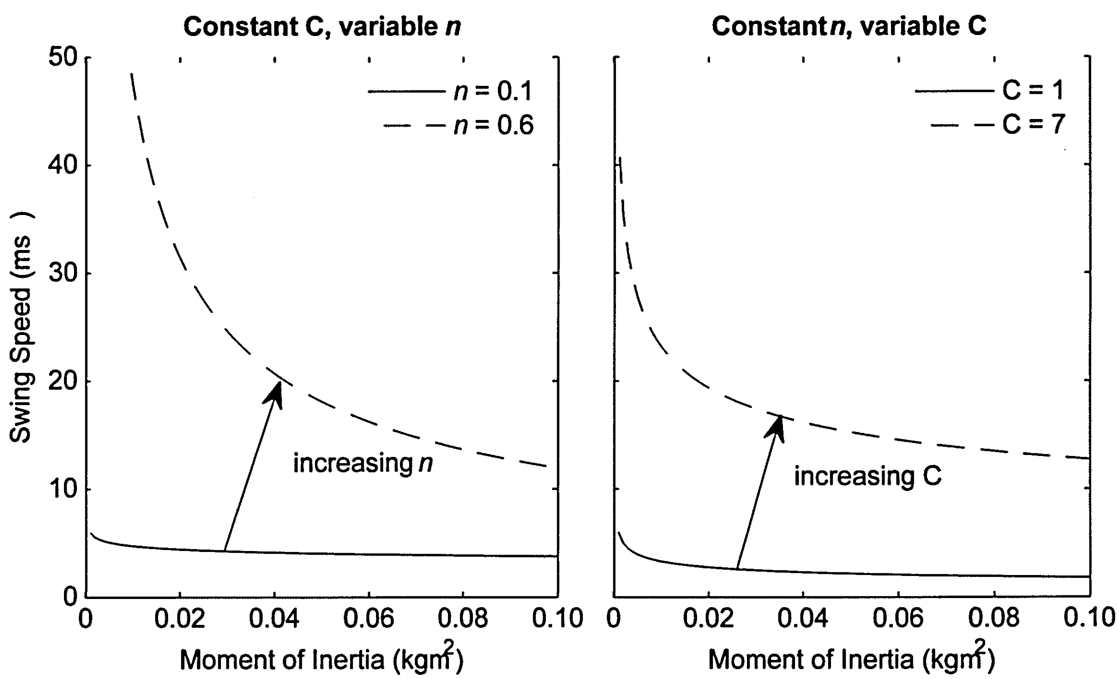

Figure 3.2. Plots of swing speed against moment of inertia on real scale axes, where C is constant (left) and *n* is constant (right).

This problem is alleviated by taking the logarithm, to base 10, of the data, producing a log-log plot, as can be seen in Figure 3.3.

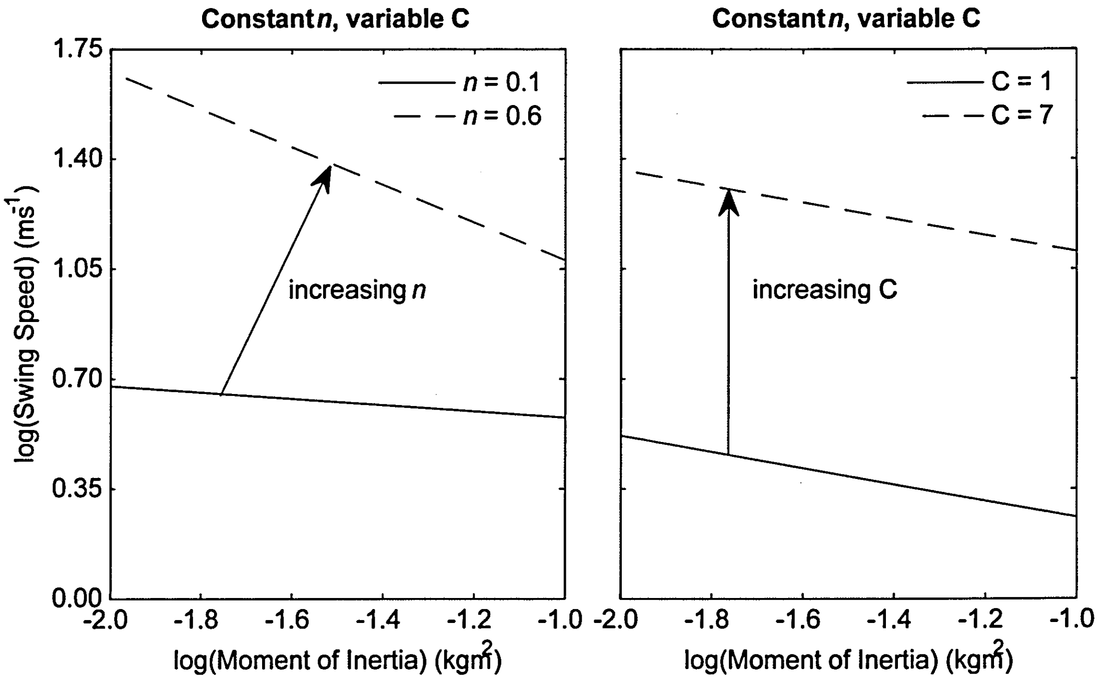

Figure 3.3. Plots of swing speed against moment of inertia on logarithmic axes, where C is constant (left) and n is constant (right)

In Figure 3.3 it is much easier to see where the lines have a common or similar *n*. Therefore these log-log plots are the best to use when we are interested in the value of *n* and will be used to display swing speed results throughout this thesis.

3.2.4 Meta-analysis results

Figure 3.4 shows the log-log plot of swing speed against moment of inertia for the nine sports considered here. The data points represent the mean of the data collected with error bars showing the range.

The value for *n* in Equation 3.2, taken from the gradient of the trend line through the data, was found to be 0.20 ($r = -0.66$, $p = 0.06$). This is comparable with the previous studies of single sports in literature that found $n = 0.20 - 0.29$ and suggests that a global value or band of values might exist across all swinging motions.

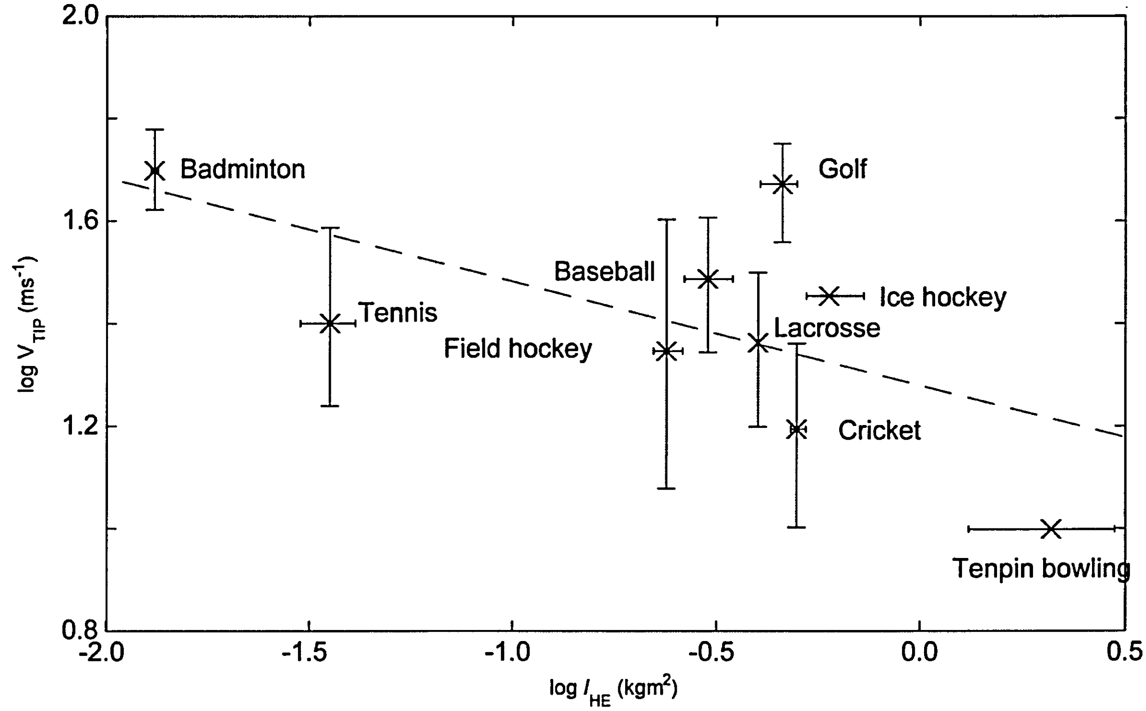

Figure 3.4. Log plot of swing speed against moment of inertia using data compiled from literature for nine sports. Here the rate of swing speed decay with increasing MOI, n = 0.20

There is obvious scatter in the data in Figure 3.4 and there are several reasons for this. It is partly due to the different way in which the games are played, with each style of action having different characteristics including type of motion and swing time, which can vary by over half a second between sports as is shown in Table 3.1.

3.2.5 Swing acceleration

It is very clear that to assume all of the sports have an equal swing time (t_{swing}) would be inappropriate as, for example, t_{swing} for tenpin bowling is 12.5 times greater than t_{swing} for badminton (King et al., 2010; Tang et al., 1994). The difference in swing time is likely to be attributable to the implement's moment of inertia but it is also partly due to the actual motion, which in this case is a gross movement compared to a fine movement. For example a badminton smash, which produces head speed almost entirely from a rotational wrist motion, as found by Kwan et al (2010), will allow a quick swing and is a simple motion to analyse. Conversely the forward drive in cricket is a much more complex movement involving the rotation of both arms and a large amount of translation with the step forwards, which is naturally a much slower movement and a more difficult motion to simplify for analysis.

The large range in swing time permitted this parameter to be used to normalise the data further. The tip velocity for each sport was divided by the swing time for that motion to produce a new variable to replace the swing speed: the apparent acceleration at the tip of the implement (A_{TIP}) or the swing acceleration.

The swing acceleration is a much more appropriate variable to use in analysis when considering multiple sports because it removes the factor of swing time. Removing the effect of swing time is important because an athlete performing a swing with a longer swing time can apply an accelerating force to the implement for a longer period of time, potentially speeding up the implement more than if they were performing a shorter swing. Taking the acceleration data also draws the data closer together which produces a more significant relationship as can be seen in Figure 3.5. Swing acceleration is more strongly correlated with the moment of inertia than the velocity data (r = -0.82, p = 0.007 compared to r = -0.66, p = 0.06). This new relationship can be defined in a similar manner to the swing speed relationship in past research as shown in Equation 3.3, where C is a player constant and k is the gradient of the log plot.

$$A_{Tip} = \frac{V}{t_{swing}} = \frac{C}{I^k} \qquad 3.3$$

The k value in this case, taken from the acceleration log plot, is 0.55. There is still scatter in the data when using the swing acceleration but the general fit is better than the previous plot. Ice hockey is one of the notable points to sit off the line, which is due to the unusually high acceleration. This could be because in the slap shot in ice hockey, the player has one hand part way down the stick which can apply an extra torque to the stick. Conversely tenpin bowling has a very low acceleration, which could be due to the fact the player has to hold a large mass with just their finger tips and a sudden acceleration could result in dropping the ball.

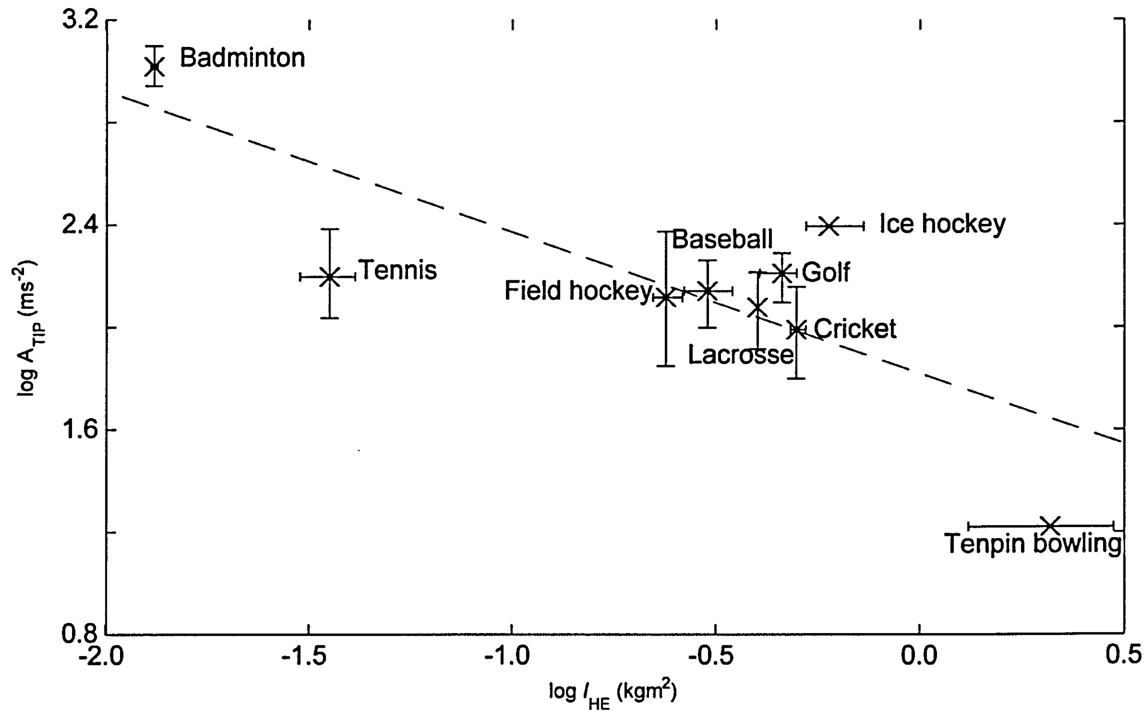

Figure 3.5. Log plot of Swing Acceleration against Moment of Inertia using data collected from literature for nine sports. Here the rate of swing acceleration decay with increasing MOI, k=0.55.

3.3 Global swing speed relationship

The results from this meta-analysis show that a relationship between swing speed and moment of inertia across a range of greatly different sports is very similar to relationships reported previously in single sport analyses. This is an interesting result and suggests that it is possible that the rate of swing speed decay (n) is either constant or sits in a narrow range, for all swinging motions.

There is a great volume of research into the relationship between force and velocity in muscle contraction and it has been found that velocity (V) decreases exponentially as load (P) increases, as shown in Equation 3.4

$$P \propto e^{\varepsilon V} \qquad 3.4$$

where ε is the coefficient of tension loss (Fenn & Marsh, 1935; Wilkie, 1949). For upper arm movements of maximum elbow flexion, ε was found to be between 0.20 – 0.48. This range of coefficients is very similar to the range of values reported for n and suggests that the muscle architecture may be at least partly responsible for the rate of change of swing speed. It has also been shown that for elbow flexion tests, maximum effort strength training can improve angular velocity, while maintaining the same velocity – load relationship. This work suggests that a player's physical profile is significant in determining the magnitude of their swing speed and could account for some or all of the player constant, C, in Equation 3.2.

3.4 Summary

A meta-analysis has been carried out assessing the relationship between swing speed and moment of inertia (swingweight) for a range of nine sports. The analysis found that the coefficient of swing speed decay, n, was equal to 0.20 across the nine sports, which coincides with the values reported in single sport studies. This suggests that a constant value for n might exist that can be applied to any swinging motion. Research into muscle force-velocity relationship has found similar rates of decay, which could explain the values for n that we are finding. Further research should be carried out to test more rigorously the value for n in a non-sport specific motion and to understand whether physical profile can explain the behaviour witnessed.

Chapter 4: Preliminary investigations into swing speed

4.1 Introduction

The previous chapter found that the relationship between swing speed and moment of inertia appears to be near constant across a range of sports. The aim for this chapter is to test the relationship between swing speed and moment of inertia experimentally, to quantify the coefficient of swing speed decay, n in Equation 3.2. This aim was achieved by delivering on the following objectives:

- Develop a rigorous method to experimentally measure swing speed in a restricted motion for implements of different MOI.
- Develop a method for creating a physical profile for participants.
- Test with more participants than some of the key studies in the literature (Cross & Bower, 2006; Daish, 1972; Mitchell et al., 2000).
- Explore the consistency of the value of n.
- Explore potential relationships between physical profile data and swing speed data.

The work presented here has partially been reported by Schorah, Choppin, & James, (2015).

4.2 Participants

Once approval was received from the faculty research ethics committee, eight participants with a range of statures and builds were recruited. The group consisted of six males and two females and all participants were healthy, active individuals with an age of 25.1 ± 5.4 years, mass of 73.3 ± 16.5 kg and height of 1.79 ± 0.27 m. The participants were all active but with no prior experience of performing the motion used in this study.

4.3 Rods

This study used eight visually identical rods made from 0.0254 m diameter, hollow aluminium tubing. In order to vary moments of inertia, a solid mass of 0.16 kg was fixed within each rod at varying locations along the length. Each rod was capped at either end, had a length of 0.506 m and a total mass of 0.32 kg (including the additional mass).

Table 4.1 shows the balance point (L_{cm}) and moments of inertia (I_{rod}, I_{elbow}) of the eight rods. The moment of inertia of the rods, I_{rod}, was defined about an axis through the butt end, perpendicular to the rod's centreline. I_{rod} values ranged from 0.0113 to 0.0495 kgm^2 (Table 1), representing the MOI of a typical badminton racket to a head heavy tennis racket. This was calculated using the time period measured for each rod swinging as a pendulum, as described by Brody (1985). A stopwatch was used to time the oscillations of the rods. When using the stopwatch to time a 10 second counter it was found that the mean difference in the timing was 0.1s. This equates to an error of less than 1% in the I_{rod} measurements.

Table 4.1 - Balance point (L_{cm}) and moments of inertia (I_{rod}, I_{elbow}) of the rods used for the restricted motion analysis in Chapter 4, all of which have length of 0.506m, mass of 0.32kg.

Rod	L_{cm} (m)	I_{rod} (kgm^2)	I_{elbow} (kgm2)
1	0.129	0.0113	0.0569
2	0.164	0.0130	0.0587
3	0.197	0.0169	0.0626
4	0.233	0.0199	0.0656
5	0.263	0.0261	0.0718
6	0.300	0.0332	0.0789
7	0.340	0.0425	0.0882
8	0.372	0.0495	0.0952

Figure 4.1 shows the bespoke attachment used to fix the rods to the back of the participant's wrist (which also restricted wrist motion during the swing). The wrist guard and rod attachment had a combined mass of 0.236 kg, positioned at the base of the rod. To account for the effect of this attachment, the rod's moment of inertia was re-calculated about the participant's elbow using the parallel axis theorem and included the mass of the attachment and wrist guard.

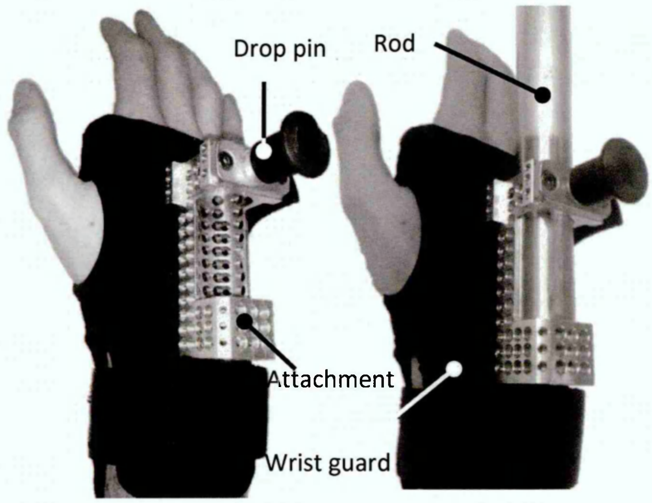

Figure 4.1 - Illustration of the mechanism used to restrain the participants' wrist joint and attach the weighted rod to their swinging arm.

The distance from a participant's elbow to wrist was calculated from motion capture data (see below). Using I_{elbow} provided a more accurate description of each rod's resistance to angular acceleration for the desired motion, but it also reduced the range in moment of inertia values. Nonetheless, the experimental range still exceeded moment of inertia values typically found in tennis, when calculated about the same elbow axis (0.055 – 0.068kgm^2)

The rods were labelled 1 to 8 in a random order and each participant swung the rods in this order. The test was carried out with a double blind protocol, where neither the participant nor observer knew the moment of inertia of the rod being swung.

4.4 Movement

Participants performed a maximal, internal rotation of the shoulder, keeping the elbow stationary, with the forearm swinging in the transverse plane. Each of the eight weighted rods was swung three times. Participants had a rest of one minute between swings to eliminate fatigue effects. To add a focal point and reduce unintentional deceleration, participants hit a ball suspended in front of them at the end of each swing. Any swings which did not visibly follow the desired motion were repeated, but these were not always easily identifiable. Participants also maintained a seated position to limit torso movement.

4.5 Motion capture setup

A motion capture system was used to track swing kinematics. Twelve Motion Analysis Corporation Eagle cameras were used, recording at 300 frames per second with a shutter speed of 1/1000s. The layout of the cameras with respect to the participant is shown in Figure 4.2. The system had a residual error of $6.24 \times 10^{-4}m$ in determining the position of markers in the 3D space.

Eight 12.7 mm diameter spherical reflective markers were used to track the movement of the participant and the rod; their locations are shown in Figure 4.3. The markers were linked in the software such that the shoulder markers were connected to the Humerus and elbow markers; the elbow marker was connected to the Humerus, wrist and rod base markers and the wrist and rod base markers were connected to the rod tip.

Before swings were recorded each participant stood in a t-pose for a static trail after which the medial epicondyle marker (5) was removed. Maximum resultant velocity of the rod tip was the key variable of interest; other markers were used to review the movement and check adherence to the protocol.

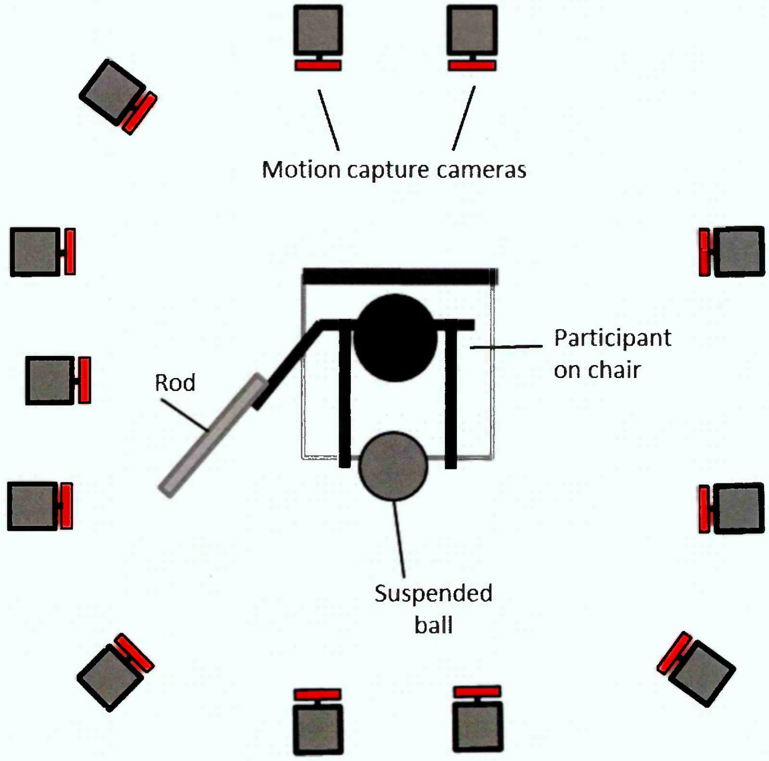

Figure 4.2. Plan view of the experimental setup for the restricted motion analysis (not to scale).

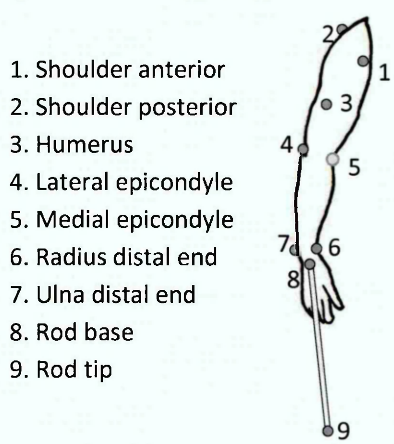

Figure 4.3. Marker arrangement used with the motion capture system in the restricted motion analysis.

The raw tracking files were initially processed using the Motion Analysis Cortex package. The cubic join function was used to fill in any short sections where the cameras had not seen a marker and the smooth function was used to reduce the noise of a trace. The smooth function consisted of a Butterworth filter with a cut-off frequency of 10Hz, which has been used effectively for upper arm motion analysis before (Hulvu & Huoshene 2007). The velocity for each point was calculated using the central difference method (Winter, 2009).

4.6 Physical profiling

Standing height, total body mass and the torque applied during maximal shoulder internal rotation were used to physically profile participants. Standing height was measured using a stadiometer and total body mass using a Weylux UK beam balance scale. A Biodex isokinetic dynamometer system, seen in Figure 4.4, was used to measure the torque with respect to both time and position applied during three maximal internal shoulder rotations (the same motion as for the swing analysis). The test on the dynamometer was an isokinetic motion with a limit of 60°/s. Participants had a short rest period between each trial. For each participant a range of movement was set that was within their comfortable limits. This made the range of motion different for each participant but reduced the risk of injury. The dynamometer was set with a restricted speed of 60 degrees per second. This has been shown to be the best method for applying resistance to maximal motion when measuring muscular strength (Davies, 1992). Applied torque was recorded at a frequency of 100 Hz. Maximum and mean torque was observed for the three trials to determine whether fatigue affected the results.

Figure 4.4. Isokinetic dynamometer (Biodex 3) used in the physical profiling test.

The raw torque data was filtered using a Butterworth filter with a cut-off frequency of 8Hz as it has been found that controlled muscle frequencies in the arm usually lie between 3 and 8Hz (Prochazka & Trend 1988).

It was decided that work done should be used for analysis instead of peak torque as it is more relevant to the sustained effort required to accelerate and move an object (i.e. a weighted rod). Work done was calculated using the trapezium rule to integrate the torque-angle data.

4.7 Kohonen self-organising maps

4.7.1 Introduction

A self-organising map (SOM) is an n-dimensional neural network which can be visualised as a 2D map of nodes. A SOM was used to ensure that only swings with good adherence to the desired movement pattern were considered in the analysis. This was necessary to ensure that a fair comparison was made between individuals when analysing swing speed. SOM analysis has been used to categorise complex sporting movements in the past (Lamb et al., 2010) and was used in a similar way here. A thorough description on the use of a SOM to investigate player technique is given by

Lees (2002). The analysis was carried out using the SOM toolbox for MATLAB (Vesanto, Himberg, Alhoniemi, & Parhankangas, 2000).

4.7.2 Map training and building

A vector, containing twelve variables, was used as the input to the SOM. These were the x, y and z positions of the shoulder, elbow and wrist joint centres during each swing and the three angles between the Humerus and the global coordinate system. Each trial was normalised to ten data points between the start of movement and peak tip velocity. This meant for each trial there were ten 12x1 input vectors for the SOM. A SOM was initialised and trained using the complete collection of input vectors, producing an 18 x 12 hexagonal map as can be seen in Figure 4.5. The U-matrix in Figure 4.5 is a version of the map showing how the Euclidean distance between nodes differs across the grid, with dark blue representing a very short distance and dark red a large distance. Two nodes that have a short Euclidean distance between them represent a similar magnitude for each variable.

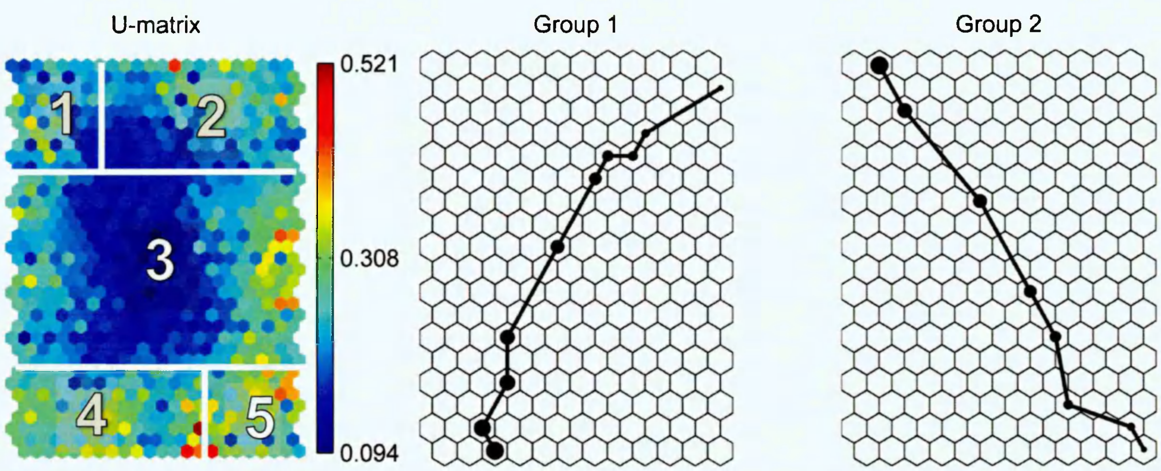

Figure 4.5. SOM trajectories showing the movement pattern of a swing at ten time points throughout the motion. (l-r) U-matrix showing Euclidean distance between nodes with overprinted section boundaries that were used to classify trajectories; example of a group one trajectory; example of a group two trajectory.

4.7.3 Exclusion criteria

Each input vector activated a node on the map, meaning for each trial a 'trajectory' could be drawn of up to ten activated nodes throughout the map (some points activate the same node). These trajectories were used to categorise the movement pattern throughout each swing. This was achieved by dividing the map into sections and classifying the trials based upon which sections the trajectories passed through. The section divisions have been overlaid in white onto the U-matrix in Figure 4.5.

This analysis produced two distinct groups, which are represented in Figure 4.5 with the trajectories starting with the largest dot and ending with the smallest. The middle map shows an example trajectory from group one, travelling from section 4 to section 2, and the right-hand map shows an example trajectory from group two travelling from section 1 to section 5. Group one accounted for 41% of data and group two accounted for 31% of data. The remaining 28% of swings did not fit into either group.

Having identified that the data fell into three groups, an understanding of what a group represents was needed. The groups can then be assessed in terms of whether or not they match the expected movement pattern for the test. This quantification can be done by comparing the trajectories for each group with the values for each of the 12 input variables.

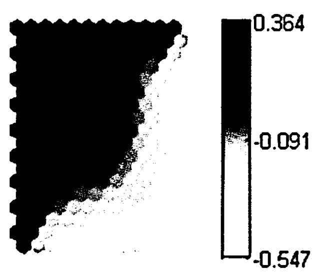

Figure 4.6. A version of the self-organising map showing the magnitude of elbow anterior-posterior position, where positive is the direction the participant was facing.

As an example, Figure 4.6 shows a version of the map which has been shaded based upon each node's value of elbow anterior-posterior position (relative to the direction the participant was facing).

The paths of typical group one and group two trajectories were analysed and compared to the values of each variable on these maps. It can be seen in Figure 4.6 that group one trajectories exhibited very little change in anterior-posterior direction elbow position. Conversely, group two trajectories went through a large change from positive to negative, meaning the elbow was being translated in the posterior direction to help produce rod velocity, rather than just using rotation of the shoulder. This was the most significant deviation from the requested motion. After analysing all variables, it was decided that group one trajectories best matched the desired motion and trials that exhibited group two or group three trajectories were eliminated.

Furthermore, participants were only included in the analysis if there was data for five or more of the eight rods from their set, which would allow for a good assessment of statistical significance. These strict criteria eliminated three participants from further analysis.

4.8 Correlation tests

The reduced data set produced by the self-organizing map method was plotted on logarithmic scale swing speed – moment of inertia graphs. In order to quantify how well correlated the data was, a 2-tailed Pearson correlation was run between the I_{elbow} values and each participant's swing speed data. The residual sum of squares, a measure of the discrepancy between data and a model, was also calculated to assess the quality of fit in the data and the square root was taken to revert the units to ms^{-1}.

The position of a participant's data in the y-axis was characterised by the swing speed value on the line of best fit, mid-way through the MOI range, labelled mid-range V. Pearson correlation tests were run between the physical profile variables, n (the rate of swing speed decay) and mid-range V.

4.9 Results

4.9.1 Physical profile

The data for the three physical profiling variables can be seen in Table 4.2. The eight participants had a range in mass of 42.8 kg, a range in height of 0.27 m and a range in work done of 43.6 J.

Table 4.2. Mass, height and work done for all eight participants.

Participant	Mass (kg)	Height (m)	Work Done (J)
1	67.8	1.78	42.46
2	51.4	1.67	19.75
3	94.2	1.94	63.34
4	60.5	1.70	22.47
5	80.7	1.75	22.62
6	70.4	1.80	33.89
7	89.3	1.87	44.93
8	72.3	1.84	29.69

4.9.2 Swing speed analysis

The swing speed results for the five participants are plotted against moment of inertia in Figure 4.7. The power relationship can be seen in the plots here but, as discussed in Chapter 3, it is best to compare the data using logarithm scale plots as shown in Figure 4.8. The n values varied from 0.19 to 0.79. The quality of fit for each participant's data is assessed in Table 4.3.

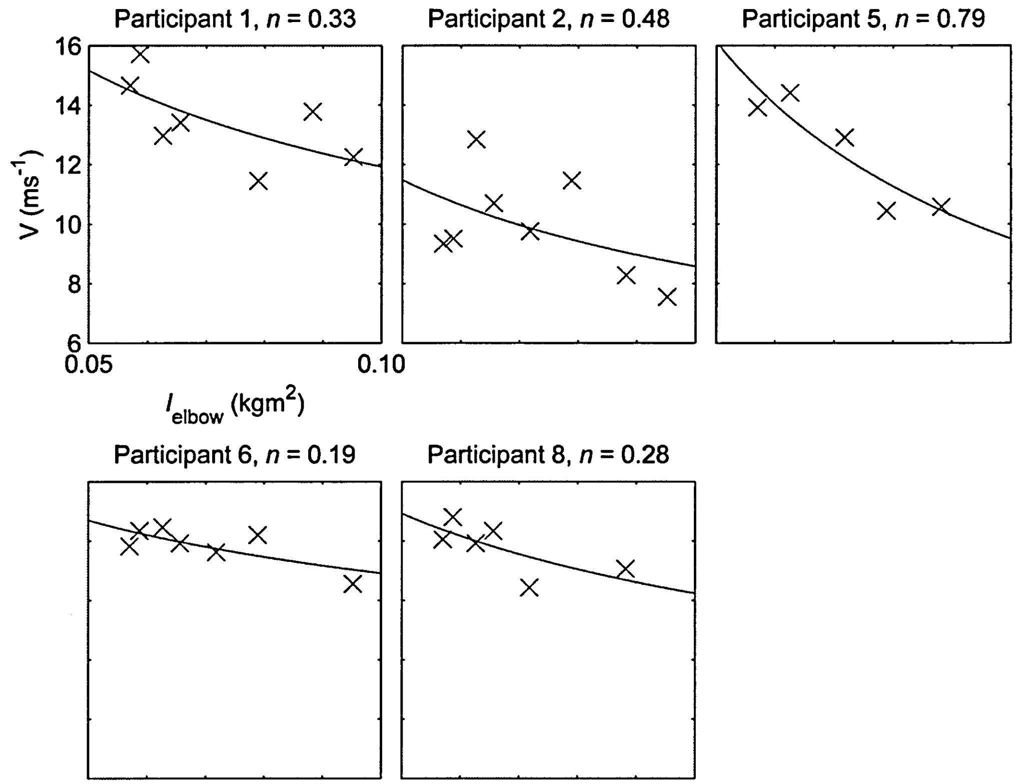

Figure 4.7. Swing speed against MOI for five participants, post data exclusion, with power relationship fit.

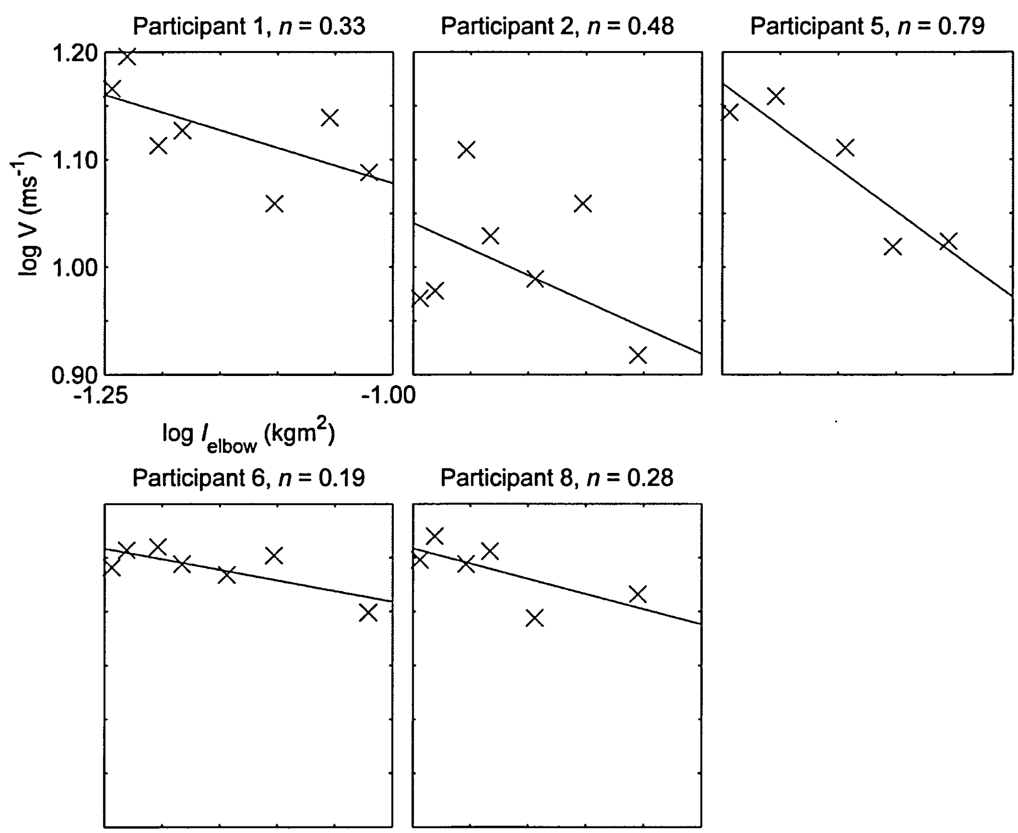

Figure 4.8. Logarithm scale plots of swing speed against MOI for five participants, post data exclusion.

4.9.3 Correlation tests

The data in Table 4.3 shows the outcome of the Pearson's correlation test run between I_{elbow} and maximum swing velocity along with the residual sum of squares for each participant. The five participants have a Pearson's correlation coefficient varying from -0.529 to -0.907 and the rooted residual sum of squares varies from 0.033 – 0.166.

The error of the motion capture system was translated to error in velocity. This velocity error was used to generate maximum and minimum potential velocity readings through which the largest and smallest gradient lines were calculated. From this analysis it was determined that the maximum potential error in the value of *n* associated with the tracking system is ±0.08.

Table 4.3. Pearson correlation coefficients and residual sum of squares for the five participant's velocity data and rod moment of inertia.

Participant's V	1	2	5	6	8
Moment of inertia	-0.629	-0.529	-0.907 *	-0.756 *	-0.605
$\sqrt{SS_R}$ (ms^{-1})	0.0878	0.166	0.0548	0.0332	0.0447

*. Correlation is significant at the 0.05 level (2-tailed).

Table 4.4 shows the Pearson's correlation coefficients between the physical profiling data and swing speed data. When tested against *n*, r ranges from -0.344 to 0.529 and when tested against the mid-range swing speed, r ranges from 0.722 to 0.914.

Table 4.4. Pearson correlation coefficients between physical profile variables and swing speed data

	n	Mid-range V
Mass	-0.344	0.722
Height	0.395	0.914*
Work done	0.529	0.758

*. Correlation is significant at the 0.05 level (2-tailed).

4.10 Discussion

A visual inspection of the results in Figure 4.7 show that the relationship between velocity and moment of inertia can be described using the power law in Equation 3.2. However, the n values are different for each participant in marked contrast with the results from previous studies (Cross & Bower, 2006; Daish, 1972; Smith et al., 2003; Smith et al., 2013). In these previous studies, the lines of best fit shown on the logarithmic plots of swing velocity against moment of inertia show near identical n values. Here, the participants have different n values and the velocity data only correlates strongly with moment of inertia for two of the five participants, as is shown in Table 4.2. The residual sum of squares values are mostly very low, with four of the five participants having $SS_R < 0.1$ showing a good fit to the trend-line. Participant 2 has a higher SS_R indicating a worse fit to the modelled trend line and this is evident in Figure 4.8.

The calibration of the motion capture system reported a residual error in the measurements of 0.62 mm. If it is assumed that the error is at a maximum across five frames, where the time period is 0.0167 s, this would produce a maximum error of 0.0749 ms^{-1} in velocity measurement. In a study where measured swing speeds were in the range of 8 – 16 ms^{-1}, this is equivalent to a 0.62% potential error in swing speed measurement due to the cameras. This level of error is acceptable as has a very small impact on the value of n and further analysis.

There are also errors associated with the isokinetic dynamometer used in the physical profiling test. It has been shown that Biodex system 3 has a maximum error in position measurement 3% (Drouin, Valovich-McLeod, Shultz, Gansneder, & Perrin, 2004). The error increases through the available range of motion to the maximum at 305°, which is well beyond the region that was tested here. The maximum error in the measurement of torque using the system was reported to be 1%. These low error values are not significant enough to greatly impact the work done calculations and are therefore acceptable for the purposes of this study.

The primary aim of this study was to observe and quantify the inverse relationship between swing speed and moment of inertia. Initial analysis found that this relationship exists but is different for different participants. As this finding contradicts the work of others, the dataset was further examined to understand whether the inter-participant differences were consistent.

For each rod, participants were ranked in order of their swing velocity. The participant with the highest swing velocity was ranked first, the participant with the second highest swing velocity was ranked second and so forth. The participant rank sets for each rod were then placed in order of their respective moment of inertia (I_{elbow}).

If the lines of best fit in Figure 4.8 had similar n values, one would not expect the participant rankings to change between rods. Conversely, if the n values were variable (as in this study) one would expect the rank sets to change. A Spearman test was implemented to determine how similar the participant rank sets were as I_{elbow} increased. The test was run between pairs of rank sets in order of increasing moment of inertia. A Spearman's rank correlation coefficient of 'one' indicates that consecutive rank sets are identical; a coefficient of 'zero' indicates that they are unrelated.

In order to confirm that the exclusion method was valid, the ranking analysis was firstly carried out for all eight participants and then repeated for the reduced dataset as specified by the self-organising map method.

Figure 4.9 shows the rank set analysis for all trials on the left and for the data set post-exclusion on the right. There are distinct differences between the Spearman's rho values for the full data set and the reduced data set.

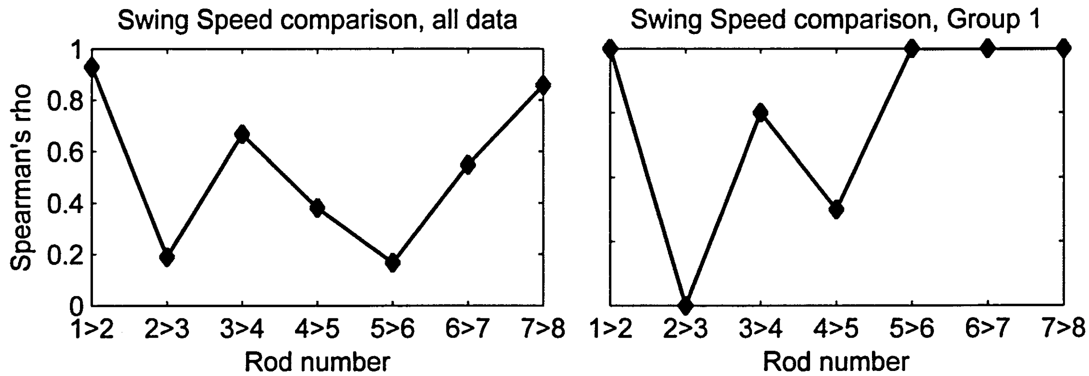

Figure 4.9. Comparison of participant rank sets for swing speed, between rods of increasing moment of inertia. For all data (left) and the data with exclusions applied (right).

There is no clear trend in the full data set; the rank sets change in a seemingly random pattern. Conversely, there is a greater consistency in the reduced data set as MOI increases, with Spearman's rho equalling 1.0 between rod 5 & 6, 6 & 7 and 7 & 8. This demonstrates the effectiveness of the exclusion criteria, justifying the decision to only analyse swings that adhered to a consistent technique.

Whilst the Spearman's rank correlation coefficient fluctuates for the low moment of inertia rods, it rests at a consistent value of 1.0 for the high moment of inertia rods. This suggests that participants swing a low moment of inertia implement in an unpredictable manner, and it is only at higher moments of inertia where a clear pattern is established. However, Figure 4.10 shows that the swing speed also became more consistent during the test. All the participants performed with the rods in the same order and the ranking consistency is 1.0 for the final three rods, suggesting there may be an effect of learning the motion or becoming more comfortable in the laboratory setting.

Figure 4.10. Spearman ranking analysis comparing swing speed ranks between rods in the order they were swung.

The second objective of this study was to test whether a measure of physical profile is strongly related to swing speed. The data in Table 4.4 shows there is no relationship between physical profile and n. There are, however, good relationships between the physical profile variables and mid-range swing speed. Both mass and work done have a strong relationship with swing speed but standing height has a very strong and significant relationship with swing speed. This finding agrees well with previous work that has found a strong relationship between body height and serve speed in elite tennis players (Vaverka & Cernosek, 2012).

The participants were also ranked by their work done values as measured by the Biodex isometric dynamometer. The participant with the highest work done was ranked first; the participant with the second highest work done was ranked second and so forth. This rank set was then compared to the swing speed rank set for each rod using a Spearman correlation test. Figure 4.11 shows a plot of the spearman's rank correlation coefficient for each rod based on the reduced data set. It can be seen that for all of the rods with higher moment of inertia (rods 5, 7 & 8), the participant rank sets for work done and swing speed were closely related with a correlation coefficient of 0.8 or higher.

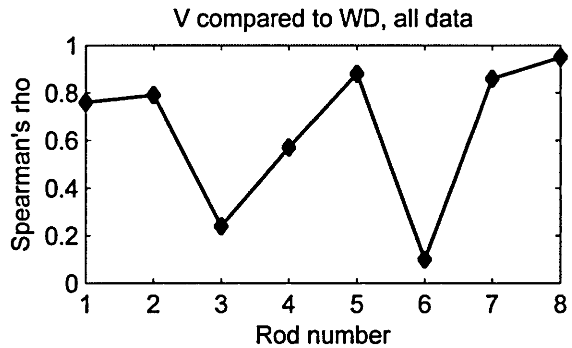

Figure 4.11. Comparison of rank sets for swing speed (V) with the rank set for work done (WD).

The results of both Pearson and Spearman correlation tests suggests that participant physique could be a key predictor of swing speed when applied at the higher moments of inertia used in this study.

It would be of value, for customisation purposes, to be able to predict swing speed for a given MOI. These findings suggest that this may be possible using physical profile measures if n is constant. The results suggest that n could be constant for a range of higher MOI implements.

Aside from swing speed, the impact characteristics in racket sports are also important performance parameters influenced by moment of inertia. It has been shown that an increase in moment of inertia can cause an increase in outbound ball velocity (Brody, 1997). This produces a trade-off in performance when changing moment of inertia and should yield an MOI value at which swing speed and impact characteristics are optimised. It is important to understand this optimum value and whether it changes for individual players of different strengths, as this could allow for customisation.

The current results support the hypothesis that for these participants and movement, it should be possible to predict a participant's swing speed for swings with implements of a relatively high MOI. This may be achievable using some measures of physical profile, for example work done. Conversely, it may not be possible for lower moment of inertia implements where there is a less consistent ranking of swing speed.

4.11 Summary

This study found that for all participants, swing speed decreased with respect to moment of inertia according to a power law. However, in marked contrast to previous studies, the rate of decrease varied from participant to participant.

Strong relationships were found to exist between physical profile and swing speed and there is a more consistent relationship for the higher MOI rods.

It was found that participants swung the high moment of inertia rods in a more consistent manner than the low moment of inertia rods. This suggests that predicting a player's swing speed may not be easily achievable for very low moment of inertia implements but could be feasible for higher moment of inertia implements.

Chapter 5: Further Investigations into swing speed and moment of inertia

5.1 Introduction

In the previous chapter, a relationship was found to exist between swing speed and moment of inertia for a restricted motion. The results also suggested physical profile is linked to swing speed. However, these relationships were based upon a small sample size and trends were only apparent at the upper range of MOI. The aim of this chapter is to examine rigorously whether n is constant and if swing speed can be predicted using strength measures. This is to be done by achieving the following objectives:

- Carry out a swing speed analysis with a larger cohort of participants.
- Repeat testing with implements that have higher moments of inertia.
- Develop a method to ensure less data is excluded from analysis.
- Investigate the relationships between swing speed and physical profile to discover whether they could be used as a performance predictor.

A subset of the data from this study has been presented previously by Schorah, Choppin, & James (2014).

5.2 Participants

Following approval from the faculty research ethics committee, twenty-five participants were recruited. The cohort had a mean height of 1.79 ± 0.08 m, a mean mass of 77.1 ± 10.9 kg and a mean age of 26.6 ± 5.4 years. The participants were all active individuals and had a large range of build and body shape.

5.3 Physical profiling

The physical profiling protocol was based upon that used in the previous chapter. The same key measures were taken: standing height, total body mass and the torque applied during maximal shoulder internal rotation. The difference was that the dynamometer test was carried out at two speeds. Standing height was measured using a stadiometer and total body mass using a Weylux UK beam balance scale, as previously.

A Biodex dynamometer system 3 was used to measure the torque applied during two sets of five maximal, isokinetic tests of shoulder internal rotation. The two sets had a limit of 60^{o}/s and 180^{o}/s respectively. This protocol was based around that previously used for baseball pitchers by Sirota et al. (1997).

The measures for standing height and mass were recorded before the participant engaged in a five minute warm up using an upper body ergometer. Range of movement was set such that it was within each participant's comfortable limits and the dynamometer test then began after 1-3 practice trials. Participants completed five maximal repetitions at 60^{o}/s with short rests between. This was followed by five maximal repetitions at 180 ^{o}/s with identical rest periods. The dynamometer was used to record torque with respect to both time and position (angle) at a frequency of 100Hz. Work done was used as the key measure of strength instead of peak torque as it better describes the effort required to accelerate an implement through a motion. Work was calculated as the area under a plot of torque against angular position, calculated using the trapezium rule.

5.4 Swing speed analysis

The swing analysis in this study was focused around the same restricted motion as the previous chapter: internal rotation of the shoulder in the transverse plane.

A series of eight weighted rods were created. The rods were visually identical and possessed a larger range in MOI than the rods used in Chapter 4. All eight rods had a mass of 0.5kg and a length of 0.7m. The rods were made from a hollow aluminium tube with a solid mass of steel secured at a different location within each rod. The rods were then capped at both ends and labelled in random order to keep the weighting hidden.

The moment of inertia of each rod was calculated from the time period of the rod swinging as a pendulum, as described by Brody (1985). The MOI was calculated about the butt end of the rod, I_{Rod}, and the values are shown in Table 5.1.

The rods were slotted into an attachment on a modified wrist guard, worn on the participant's wrist. This is the same device as was used in the previous study and is shown in Figure 5.1. The attachment and wrist guard had a combined mass of 0.236 kg and was included in the calculation of MOI about the elbow, I_{elbow}. The mean values are shown in Table 5.1 with values for I_{Rod} and the balance point, measured from the butt end of the rod. The range of MOI present here are equivalent to high MOI tennis racquets at the low end and very low MOI field hockey sticks at the high end.

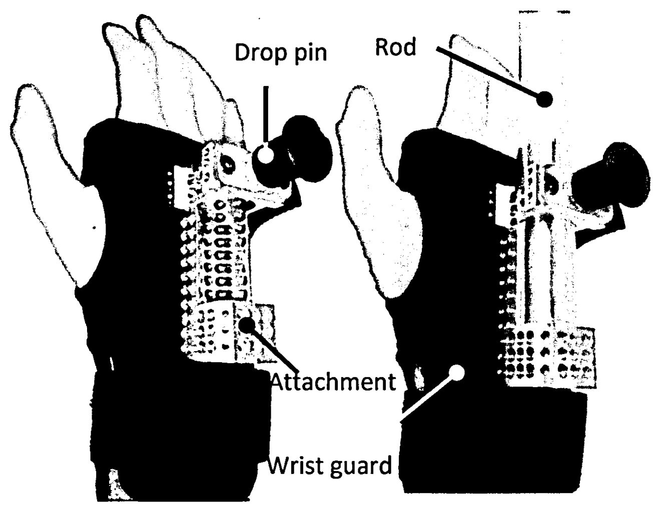

Figure 5.1 - Illustration of attachment mechanism for rods, as used in Chapter 4.

Each participant performed three maximal swings with each rod with a rest between. These were performed in a different, random order for each participant, and the real order of MOI was unknown to both the participant and test supervisor. The participants were in a seated position on a gym bench, with the back angled at 90°. The participant started each swing with their forearm as far back as was comfortable and concluded the swing by impacting their palm with a suspended ball.

Table 5.1 - Physical properties of the 8 rods, all of which have mass of 0.5kg and length of 0.7m.

Rod	L_{CM} (m)	I_{HE} (kgm^2)	I_{elbow} (kgm^2)
1	0.182	0.039	0.097
2	0.235	0.044	0.102
3	0.284	0.055	0.113
4	0.332	0.067	0.125
5	0.369	0.080	0.138
6	0.422	0.105	0.163
7	0.468	0.130	0.189
8	0.508	0.165	0.224

5.4.1 Motion capture setup

The swings were all recorded using a 12 camera Motion Analysis Corporation motion capture system. Markers with a diameter of 12.7mm and a retro-reflective coating were used to track all motion. The cameras recorded at 300 frames per second with an exposure time of 1/1000 second. For this study a capture volume of 2.5m x 2.5m with a height of 1.7m was set, with the participant located centrally in the space. A static calibration was carried out by recording an L-shaped frame with markers set at known distances and a dynamic calibration was run by moving a rod of known length, and with markers at either end, through the volume for 90 seconds. The calibration process resulted in a measure of the error in locating the centre of a marker in the capture volume (residual measurement error). The mean residual measurement error in marker position across all days of testing in this study was 0.64mm.

A set of nine markers were used at the following locations: Left shoulder anterior, Left shoulder posterior, Right shoulder anterior, Right shoulder posterior, Right lateral epicondyle, Right radius, Right ulna, Rod base, Rod tip.

The swing speed, V, was defined as the maximum velocity of the rod tip. This was calculated from the motion capture data and plotted on a log-log plot against the moment of inertia of the rod, I_{elbow}. From these plots the power of each relationship was recorded as the gradient of a linear line of best fit to provide the n value as in Equation 3.2.

5.5 Exclusion methods

In order to prevent a high percentage of data from being excluded from analysis, as occurred in Chapter 4, a new data exclusion method was employed. This method was applied during data collection, using a bespoke biofeedback system and a second check was applied using self-organising maps at the start of data analysis.

5.5.1 Biofeedback system

A biofeedback loop was set up to monitor the position of the participant and to ensure recorded trials met the requirements for the restricted motion. Acceptance thresholds were set for the angle of torso twist, θ_{torso}, and for the distance the right elbow moved in the sagittal plane, Z_{elbow}. These thresholds were determined using data from the previous study, which found that in a large proportion of the trials, participants had been moving their elbow in the posterior direction. The thresholds were measured from the position the participant started each swing with θ_{torso} set at 15° and Z_{elbow} set as the elbow joint centre passing the start location of the shoulder, in the posterior direction.

Approximately five practice swings were performed using the biofeedback system before any trials were recorded, to allow participants to understand what was expected. If a recorded swing exceeded either of these thresholds it was repeated until a valid trial was performed. A total of 23 swings were flagged as exceeding the set thresholds and were repeated, which is 3.8% of the total 600 trials captured.

5.5.2 Self-organizing maps

A self-organizing map was used to check the movement pattern of each swing and compare it to the expected motion. The SOM used was the same map, trained on the same data from Chapter 4. Using this map, trajectories were drawn for each trial and they were classified using the same method as before. All swings that fell into Group 2, an example of which is shown in Figure 5.2, were excluded from analysis. This process removed 46 trials from the data set, 7.7% of the total 600 trials. This was a great improvement on the experiment in Chapter 4, where 59% of trials had to be excluded from analysis.

To test the strength of relationships in the data, a two-tailed Pearson correlation test was run between swing speed data (*n* and mid-range V) and physical profile data (mass, height and work done at 2 speeds) for all 25 participants.

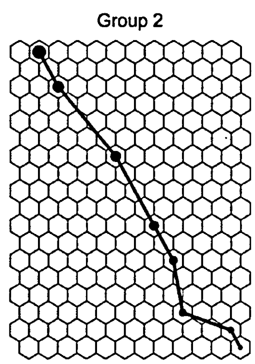

Figure 5.2 - A typical example of a Group 2 swing's movement pattern displayed on the self-organizing map. Characterised by low levels of shoulder and elbow movement.

5.6 Results

5.6.1 Physical profiling

The four measures from the physical profile test for each participant can be found in Table 5.2. Also shown here are the *n* values for the cohort, taken as the gradient of the line of best fit from the log swing speed - log moment of inertia plots. The values for work done are largely very close between the two tests for each individual, with 19 of the 25 participants doing more work in the slower test.

Table 5.2. Physical profiling results: mass, height, maximum work done at 2 speeds and coefficient of swing speed decay, *n*.

Participant	Mass (kg)	Height (m)	Work done at 60 °/s (J)	Work done at 180 °/s (J)	n
1	90.3	1.86	67.9	61.2	0.25
2	68.0	1.77	48.6	48.1	0.48
3	95.4	1.93	69.2	65.9	0.36
4	58.2	1.63	26.9	22.0	0.28
5	76.8	1.73	67.9	62.6	0.24
6	75.7	1.69	45.4	38.7	0.34
7	84.9	1.74	37.1	33.5	0.33
8	54.5	1.66	21.9	18.9	0.25
9	89.8	1.90	53.9	55.0	0.34
10	72.8	1.75	42.4	34.9	0.52
11	71.4	1.79	28.9	33.9	0.42
12	70.7	1.72	41.2	36.0	0.37
13	87.5	1.90	68.4	73.9	0.20
14	86.7	1.88	55.6	60.2	0.27
15	72.4	1.84	22.3	37.2	0.28
16	78.9	1.85	53.1	52.2	0.30
17	59.9	1.67	26.3	29.4	0.19
18	74.7	1.83	62.5	50.3	0.27
19	76.1	1.83	58.7	46.9	0.44
20	82.4	1.77	73.2	58.0	0.35
21	72.2	1.78	56.6	36.6	0.36
22	67.6	1.75	41.5	28.8	0.44
23	74.2	1.82	57.1	53.9	0.12
24	90.9	1.85	48.9	42.5	-0.19
25	95.8	1.88	77.7	73.4	0.13

5.6.2 Initial swing speed results

Figure 5.3 shows the log scale plots of swing speed against MOI for all 25 participants. For all but one of the participants (P24) swing speed decreases as MOI increases. The value of *n* ranges from 0.12 to 0.52, with the exception of participant 24 who's *n* = -0.19.

The results from Pearson correlation tests between swing speed data and physical profile data are shown in Table 5.3. The only significant relationship found was between work done at 60 °/s and mid-range swing speed. The strength of relationships seen here ranges from r = 0.065 - 0.395.

Table 5.3. Pearson correlation coefficients for tests run between swing data and physical profiling data for all 25 participants.

	Mass	Height	Work done at 60 °/s	Work done at 180 °/s
n	0.287	0.196	0.116	0.203
Mid-range V	0.193	0.065	0.395*	0.327

*. Correlation is significant at the 0.05 level (2-tailed).

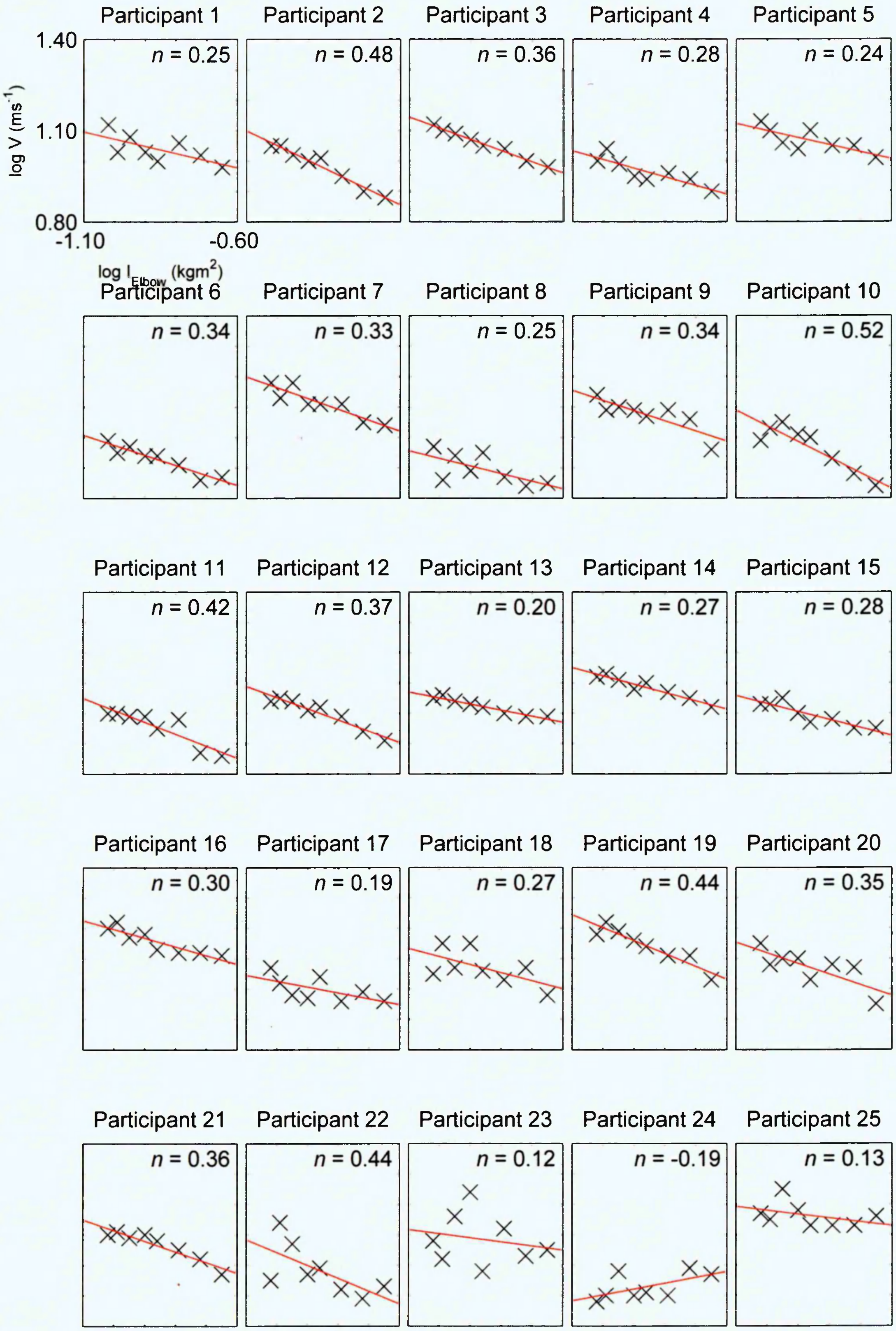

Figure 5.3. Plots of swing speed against moment of inertia for all 25 participants, on log scale axes.

5.7 Discussion

The biofeedback system used in this study had a large impact on the quality of the data collection. Participants were more thoroughly introduced to the experimental setup and the motion being asked of them and consequently a much higher proportion of data conformed to the desired movement pattern. Here, 7.7% of data was excluded using the self-organizing map method, compared to an exclusion of 59% in the study described in Chapter 4. With a much larger proportion of data to analyse, this means we should be able to draw conclusions with more confidence than in the previous study.

The physical profile results show that there is a broad range in the physical characteristics of the participants. Standing height ranges from 1.63 – 1.93 m and total body mass ranges from 54.5 – 95.8 kg. The participants also had a large range in the amount of work required to move the lever on the dynamometer. Work done ranges from 21.9 - 77.7 J for the 60°/s test and 18.9 - 73.9 J for the 180°/s test. This large range mirrors the large range in mass and height and means we should be able to find any relationships between these physical profile measures and swing speed, should they exist.

The swing speed results presented in Figure 5.3 demonstrate a strong relationship between swing speed and moment of inertia. With the exception of participant 24, swing speed decreases as moment of inertia increases and this confirms the trend found in earlier work.

It is difficult to fully explain why participant 24 was able to swing the rods faster as moment of inertia increased. They were able to produce a high level of work done during the profiling tests, when compared with the rest of the cohort (48.9 J at 60°/s and 42.5 J at 180°/s). Yet despite a seemingly good level of strength their maximum swing speed was low for all eight rods. The swing order for participant 24 was 2-5-4-3-7-6-1-8, where they swung the three highest-MOI rods in the second half of the test.

However, this is unlikely to be an explanation for their improved performance with high MOI as rod one was also towards the end of the test. It seems most likely that this participant is particularly insensitive to changes in moment of inertia and is an anomaly within the group. There have been studies in the literature that have found a similar anomalous result, where a participant has increasing swing speed as MOI increases (Bahill, 2004; Mitchell et al., 2000).

There was a large range in the values of n, from -0.19 to 0.52, with a mean and standard deviation of 0.30 ± 0.14. If participant 24 is excluded from the data set, the mean and standard deviation are 0.32 ± 0.10. This is a large range of values for n, and much broader than quoted in literature. However, the values for n in previous studies are either normalised by the mean swing speed for a player, taken between two data points, or for a very small sample of participants (Cross & Bower, 2006; Daish, 1972; Nathan et al., 2011; Smith et al., 2003). Considering the limitations of the quantification of n and the results presented in Figure 5.2, there is a strong possibility that n does not exist within a narrow range, as previously thought and is much more variable.

While several of the participants have a similar value for n, their data occupy different regions of the log-log plot. The correlation test results between mid-range swing speed, n, and the physical profile data are shown in Table 5.3. There was no correlation between the physical profile data and n. However, the mid-range swing speed does correlate significantly with work done at 60°/s but it is a weak relationship with r = 0.395. There is a similar strength relationship between mid-range V and work done at 180°/s.

There are three potential scenarios in which physical profile could be used to predict swing speed for a given implement. Scenario one is where n is a constant value and physical profile is used to predict the intercept of a player's swing speed line (C in Equation 3.2). In scenario two, C is kept constant and we accept that n is different for everyone and use physical profile to predict this value. Alternatively, in scenario three, both n and C are assumed to be variable and a combination of relationships is used to predict swing speed for a specified MOI.

Both the studies in this thesis and literature have shown C to be variable and Table 5.3 shows that there are possible relationships between work done and C but not work done and n. Therefore, scenario two is unlikely to be representative of the data and make accurate predictions. This leaves scenarios one and three as workable options for a prediction model, both of which would have C as a constant. A model with a common value for n, with variable intercept is the more logical of these two scenarios to pursue first as it would be simpler to develop and the complexity of a variable n could be added subsequently if required.

In the previous chapter there was no relationship between work done and n but the ranking analysis suggested work done was related to the mid-range velocity and here, the only significant relationship is between work done and mid-range velocity. Choosing to have a common value for n would also echo results presented in past research (Cross & Bower, 2006; Daish, 1972; Smith, Broker, & Nathan, 2003; Smith & Kensrud, 2013).

It was important to be confident that all of the data used to determine the constant n was representative of the group. Due to the data for participant 24 opposing the trend of the group, they were not included in the calculation of n. The quality of fit of a participant's data to their trend-line (Figure 5.2) was assessed, by calculating the differences between each data point and the line, to determine the residual sum of squares ($SS_{Residual}$) for each participant. As can be seen from Figure 5.4, participants 23 and 24 have distinctly worse fitting data than the rest of the group, with $SS_{Residual}$ exceeding 0.02 and they were both excluded from the n calculation.

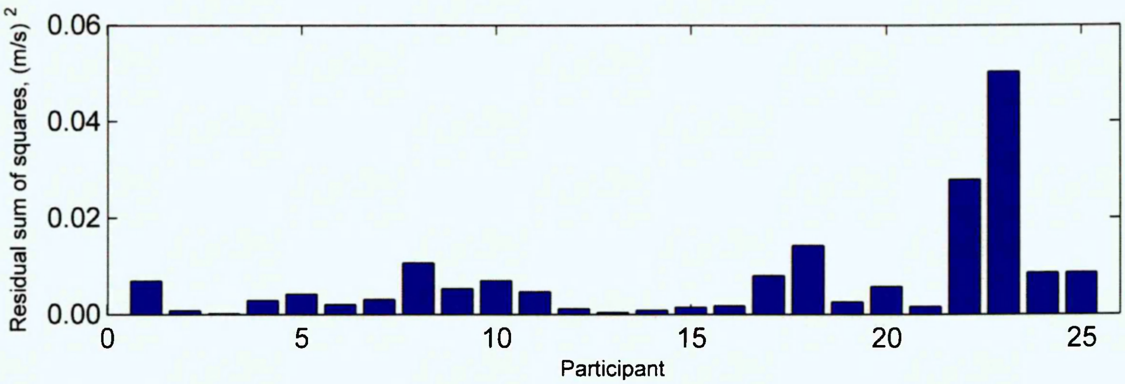

Figure 5.4. Residual sum of squares for each participant's data compared to their line of fit.

The common value of n for the 22 participants was calculated using a modified version of the least squares method. The least squares method calculates the sum of squares between all given points and a model line. The line is then optimised to the point where the least squares are a minimum and the line is the 'best fit'. For a linear scenario the equation of the line can be expressed by Equation 5.1 (Heij, Boer, Franses, Kloek, & Dijk, 2004).

$$y = \beta_2 x + \beta_1 \tag{5.1}$$

where $\beta_1$is a constant and the intercept on the y-axis and β_2 is the gradient of the line. Equation 5.1 can be expressed in matrix form and used to calculate the two unknown values β_1 and β_2. Equation 5.2, in the form XB = Y, shows the matrix form of Equation 5.1 for a line between two points: x_1, y_1 and x_2, y_2.

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{5.2}$$

The matrix of unknowns (B) is calculated as B = X^{-1}Y, which can be evaluated using the backslash operator in Matlab (B = X\Y).

When dealing with multiple lines, the matrices can be concatenated such that a constant gradient is fitted for all of the lines and the intercept is variable. An example of Equation 5.1 for three lines, each between two points, is shown in Equation 5.3

$$\begin{bmatrix} 1 & 0 & 0 & x_1 \\ 1 & 0 & 0 & x_2 \\ 0 & 1 & 0 & x_3 \\ 0 & 1 & 0 & x_4 \\ 0 & 0 & 1 & x_5 \\ 0 & 0 & 1 & x_6 \end{bmatrix} \begin{bmatrix} \beta_{1A} \\ \beta_{1B} \\ \beta_{1C} \\ \beta_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \qquad 5.3$$

where β_{1A}, β_{1B} and β_{1C} are the intercepts of the three lines and β_2 is the gradient, which is the same for all three. This method was extrapolated to take in 8 data points for each of the 22 participants and the variables were determined as explained above. The full matrices can be found in Appendix A.

The result of this calculation was that $n = 0.318$. With this new fixed value for n, the intercept of each participant's swing speed line can be used to define the player constant C, as the differentiating factor between participants. The swing speed results can be seen with the new fitted lines overlaid in Figure 5.5.

The quality of fit of the new lines was assessed in the same way as with the initial results by calculating the residual sum of squares for each participant's data. The SS_R values are shown in Figure 5.6 and while for some participants the difference between the data and the model has increased slightly, the values are all less than 0.02, which was used earlier as the cut-off point for poor fit.

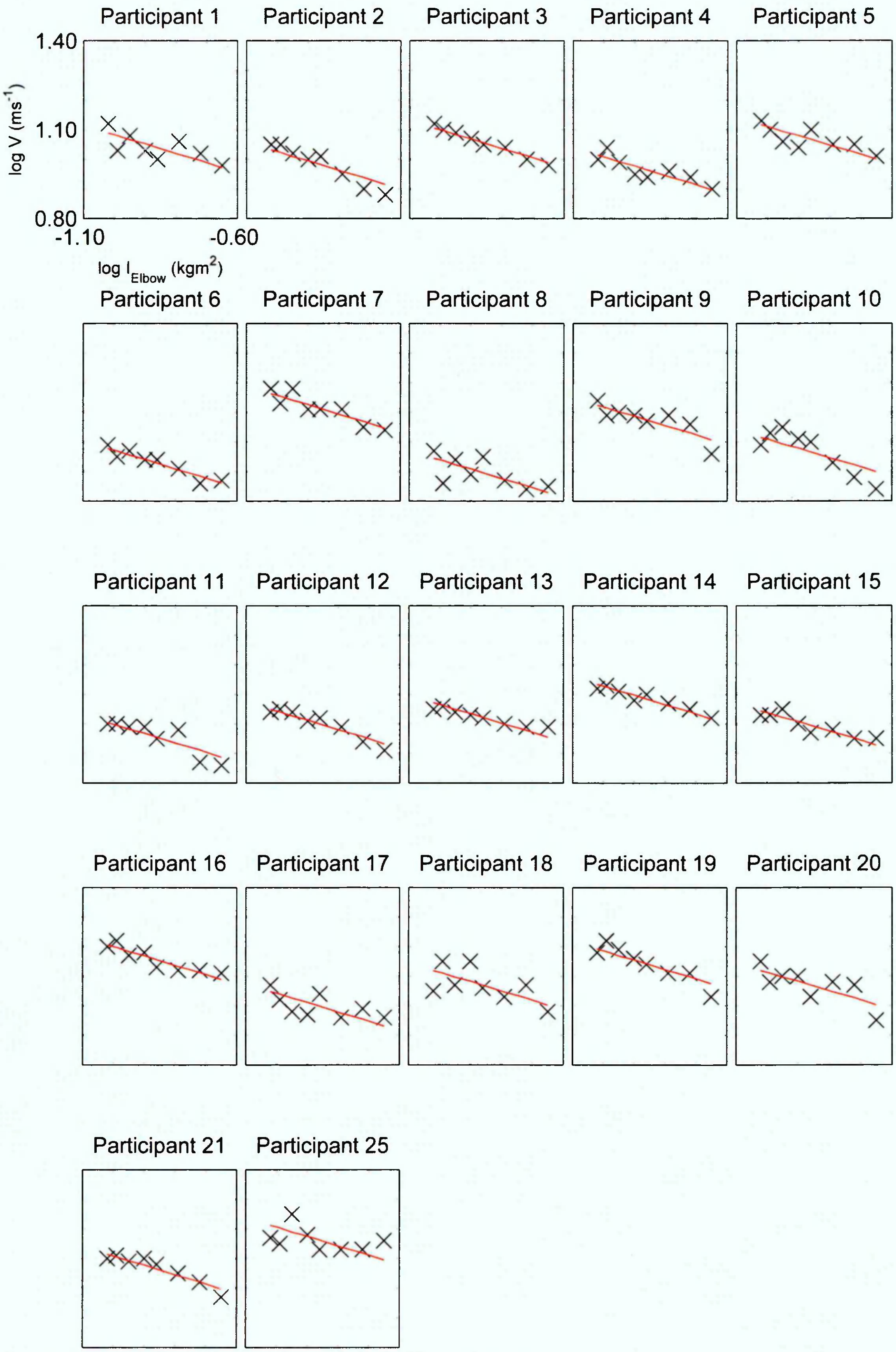

Figure 5.5. Plots of swing speed against moment of inertia on a log scale for 22 of the 25 participants, with lines of a common gradient fitted with n = 0.318.

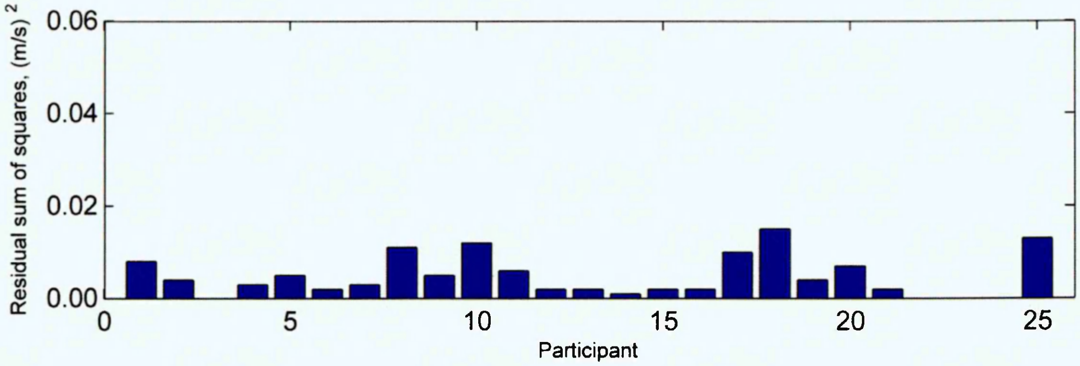

Figure 5.6. Residual sum of squares for each participant's data compared to the newly fitted line.

A second Pearson's correlation test was run between the physical profile data and the new values for the intercepts of the swing speed lines. Mass, height and work done at both speeds were all strongly and significantly related to the intercept ($0.59 \leq r \geq 0.647$, $p < 0.01$). These new results strongly suggest that if a common value for n is assumed, then player's physical characteristics could be used to predict their swing speed performance with given implements. A linear regression was run between intercept and work done at 60 °/s, as the most strongly correlated work variable, and the results can be seen in Figure 5.7. This is a strong relationship with $r = 0.635$.

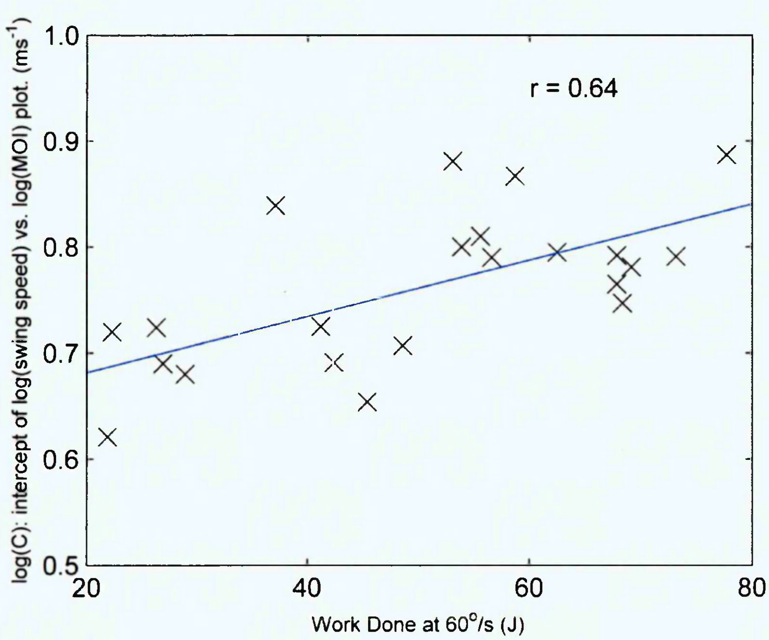

Figure 5.7. Linear regression between the intercept of log(swing speed) vs. log(MOI) plots (Figure 5.5) and work done at 60°/s

Further experimental work is required to test the strength of this relationship and if it can be used successfully to predict swing speed. If it is possible, this could have substantial implications for the racquet and bat sport industry by providing a straightforward, objective means to predict performance and customise equipment.

5.8 Summary

This study used a series of weighted rods and a restricted motion to quantify the relationship between swing speed and moment of inertia. The study also included a physical profile of the participants.

The results show that a similar trend of decreasing swing speed with increasing MOI exists for most participants but the rate of decreasing swing speed (n) is variable.

There were no strong correlations between physical profile and swing speed data. The decision was taken to assume a constant value for n in an attempt to find stronger relationships and with the new value of n = 0.32, work done was significantly related to swing speed (r = 0.64).

These findings suggest it should be possible to predict a player's swing speed for a given MOI implement using physical profile. However, more work is to be carried out to test this theory, primarily for the same restricted motion.

Chapter 6: Predicting swing speed using physical profile

6.1 Introduction

In Chapter Five a common relationship was defined between swing speed and moment of inertia for a cohort of 22 participants. The results showed that measures of work done by the participants were strongly correlated with the intercept of swing speed – moment of inertia data, when n is kept constant. Therefore, this chapter aims to make predictions for swing speed using the relationships identified in the previous chapter and to validate the predictions with new measured data. This aim will be achieved by working through the following objectives:

- Use the relationship found between work done and swing speed to create a model that will predict swing speed based upon work done and moment of inertia.
- Carry out a swing speed analysis and physical profiling for a new cohort of participants.
- Use the physical profile results to make swing speed predictions for the new cohort.
- Quantify the accuracy of the model by comparing measured and predicted data.

6.2 Model

The core of the predictive model for swing speed uses the equation for swing speed first stated by Cross & Bower (2006),

$$V = C.I^{-n} \tag{6.1}$$

where V is swing speed, C is a player constant and I is the moment of inertia. Expressing this in logarithmic form yields Equation 6.2, which describes the line of fit for a player's swing speed vs MOI data.

$$logV = logC - n.logI \quad 6.2$$

The previous chapter set a common swing speed gradient, with n = 0.318. Fixing the value for n means that the intercept for each player's line ($logC$) is the sole differentiating factor between participants' swing speed data. This allows predictions for $logC$ to be made using physical profile. In the previous chapter a regression analysis was completed between work done at 60 °/s and intercept. This was a strong, highly significant relationship with r = 0.64. The regression produced the following coefficients for predicting intercept:

$$logC = (0.00267 \times WD_{60}) + 0.627 \quad 6.3$$

where WD_{60} is the maximum work from five trials in the physical profiling test (see below for more detail). This means that, using a measurement for work done, we can now predict swing speed for a rod with a known MOI using the following equation:

$$\log(V) = ([0.00267 \times WD_{60}] + 0.627) - [0.318 \times \log(I_{elbow})] \quad 6.4$$

where I_{elbow} is the longitudinal moment of inertia calculated about an axis through the participant's elbow, perpendicular to the rod's centreline.

6.3 Validation Experiment

In order to validate this model the experiment from Chapter 5 was repeated with a new cohort of five participants. Each participant firstly underwent a physical profiling, after which swing speed predictions were made. The participant's maximum swing speed was then measured to compare against the predictions and judge the model's accuracy.

6.3.1 Test methods

Height and mass were recorded for all five participants before an upper body warm up and a strength profiling test using an isometric dynamometer. Work done was calculated as the area under the Torque - position curves for trials at 60 °/s and the maximum value was taken from the five repetitions, as was done previously.

Swing speed was measured for eight weighted rods, with the same motion capture setup as the experiment in Chapter 5. The rods were all 0.705 m in length and had a mass of 0.490 kg. The appearance of the rods was uniform meaning there were no obvious indications which might reveal the properties to the participant. MOI for the eight rods was measured about an axis through the players elbow and ranged from 0.097 - 0.224 kgm^2.

Participants wore a wrist guard to stop accelerations from the wrist and the rods were slotted into an attachment on the guard. Participants each swung the eight rods three times in a different random order. A twelve camera Motion Analysis Corporation system was used to record the kinematics. A real-time feedback system was used to monitor the participant's movement so any trials where an incorrect motion was performed could be repeated. Full details on the experimental method can be found in Chapter 5.

6.3.2 Predictions for swing speed

The maximum value for work done value was substituted into Equation 6.4 to generate a predicted swing speed for each rod used in the swing speed analysis. To quantify the precision of the predictions for swing speed a confidence interval was calculated for the regression between $log(C)$ and work done. This was done by working out the margin of error on both constants in Equation 6.3 at the 95% confidence level. This margin of error was added and subtracted from the two constants to produce lines at the upper and lower limits of the prediction confidence range. These lines are shown in Figure 6.1.

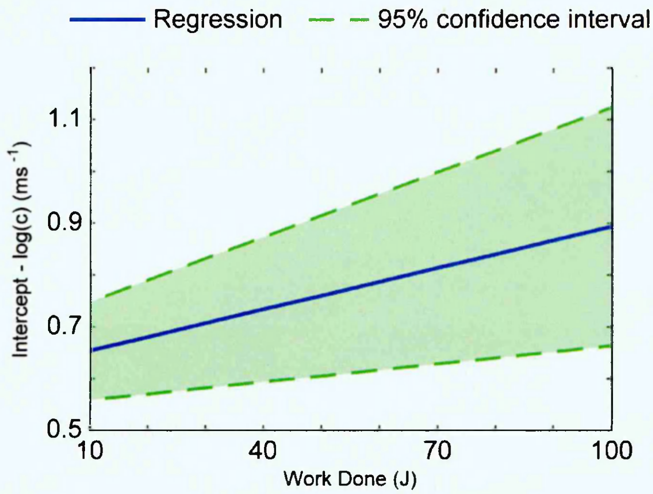

Figure 6.1 - Regression between log(C) (the intercept of swing speed vs. MOI plots) and work done, with 95% confidence interval.

The upper and lower limit values for $log(C)$ were calculated for each participant as well as the main prediction of swing speed. The confidence intervals were then plotted with the predictions for swing speed alongside the measured data.

The difference between measured and predicted data was quantified using mid-range swing speed. V_{diff} was calculated as the mid-range swing speed for the prediction line subtracted from the mid-range swing speed on the measured data line. This was expressed as a difference from the measured data.

6.4 Experimental Results

The mass, height and work done values for the five participants are displayed in Table 6.1. The range in height amongst the new cohort was 0.06 m; there was a 15.8 kg difference in mass from the lowest to highest and a range of 15.76 J in work done at 60 $^{\circ}$/s, which was enough to see distinct differences in swing speed in the previous chapter.

Table 6.1 - Mass, height and work done results for the five participants.

Participant	Mass (kg)	Height (m)	Work Done at 60°/s (J)
A	76.5	1.84	32.6
B	84.6	1.85	27.6
C	74.5	1.78	28.3
D	85.6	1.85	44.1
E	68.8	1.83	35.1

The swing speed results for all five participants can be seen on logarithmic scale plots of swing speed against MOI in Figure 6.2. All of the participants show decay in swing speed with increasing moment of inertia. This trend is the same as was witnessed in previous chapters. The range of n is 0.25-0.33, which is close to the modelled value of 0.32. The standard deviation of n for this cohort is 0.031, which is very good compared to the standard deviation for n from Chapter 5 (0.16).

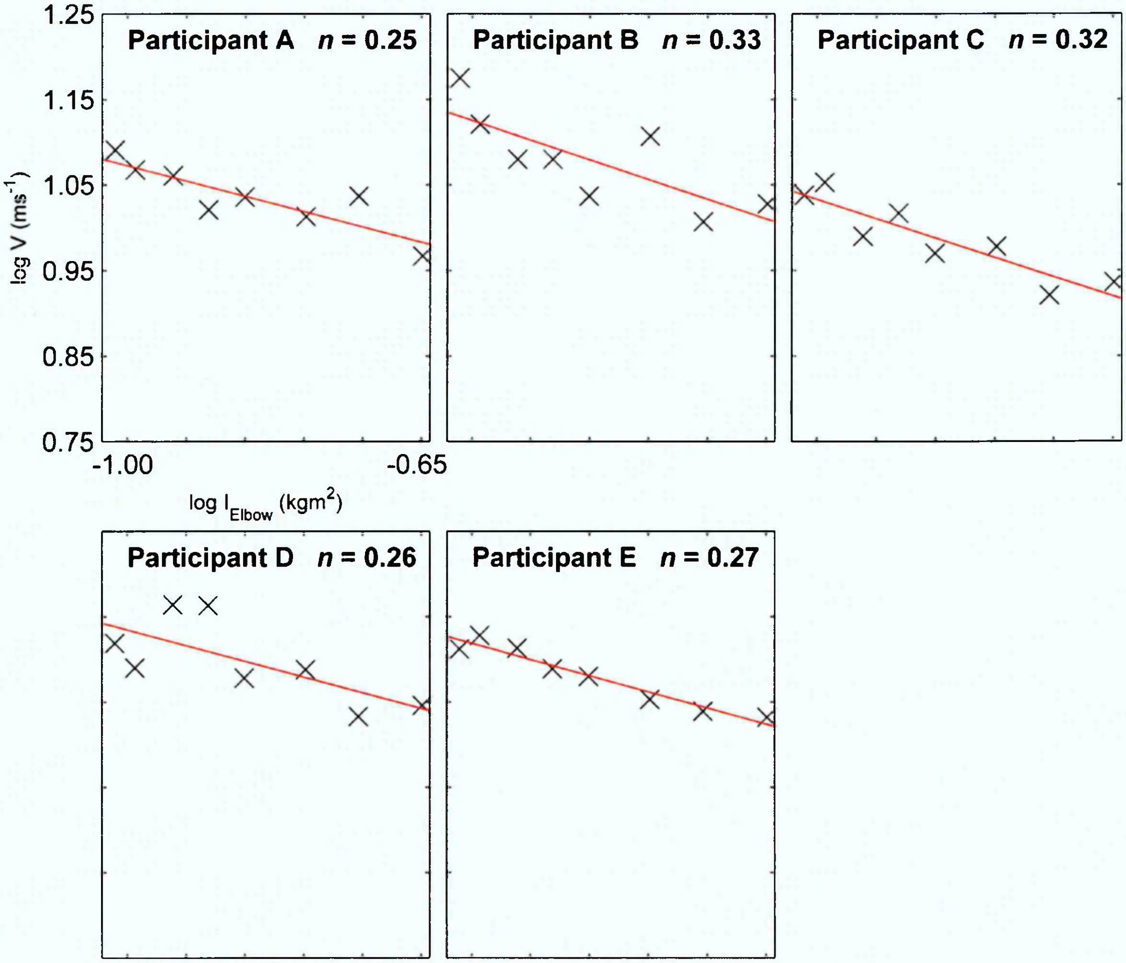

Figure 6.2. Swing speed against moment of inertia for participants A-E on a log scale with a linear best fit.

In order to validate the method of using work done to predict *log(C)*, a ranking analysis was carried out on the new cohort. Participants were ranked from 1 to 5 for work done and mid-range swing speed, with 5 being assigned to the highest value and 1 to the lowest. The rankings are shown in Table 6.2.

The Spearman's correlation coefficient between ranked lists for work done and swing speed is 0.700, which is very strong. With a small number of participants achieving a value close to one is difficult as one participant, whose ranking is not consistent, can reduce the correlation greatly. In this instance Participant A has a much higher ranking for work done than their swing speed rank but the remaining four participants match up well. This can be seen in Figure 6.3, which shows there is a strong relationship between work done and swing speed for this cohort, as there was in the previous data collection.

Table 6.2. Rankings for all participants based on work done and swing speed

Participant	Work Done (J)	Mid-range V (ms^{-1})	Ranked Work Done	Ranked Mid-range V
D	48.4	12.3	1	1
A	43.1	10.7	2	4
E	37.1	11.8	3	2
B	31.3	11.8	4	3
C	30.4	9.52	5	5

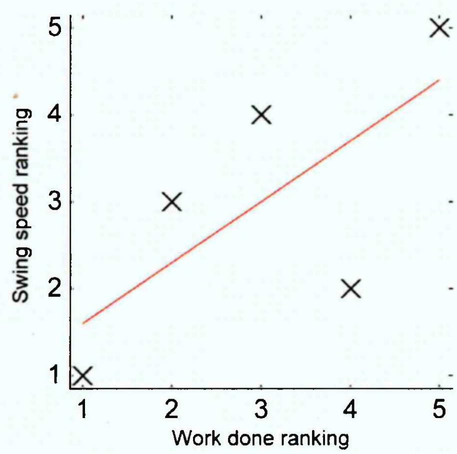

Figure 6.3. Comparison of work done and swing speed rankings for all five participants.

The strength of the relationship in Figure 6.3 is evidence for the relationship between swing speed and work done not being an anomalous finding in Chapter 5. This justifies the decision to use work done as a predictor for swing speed.

6.5 Validating predictions

The plots in Figure 6.4 show the measured data for each participant alongside the predictions made using Equation 6.4. The fit lines through the measured data (red) have the same gradient as the predicted data (n = 0.32) for a true comparison. The line of predicted swing speed is shown in blue and the 95% confidence interval is represented with a blue shaded area bounded by dashed lines at the upper and lower limits.

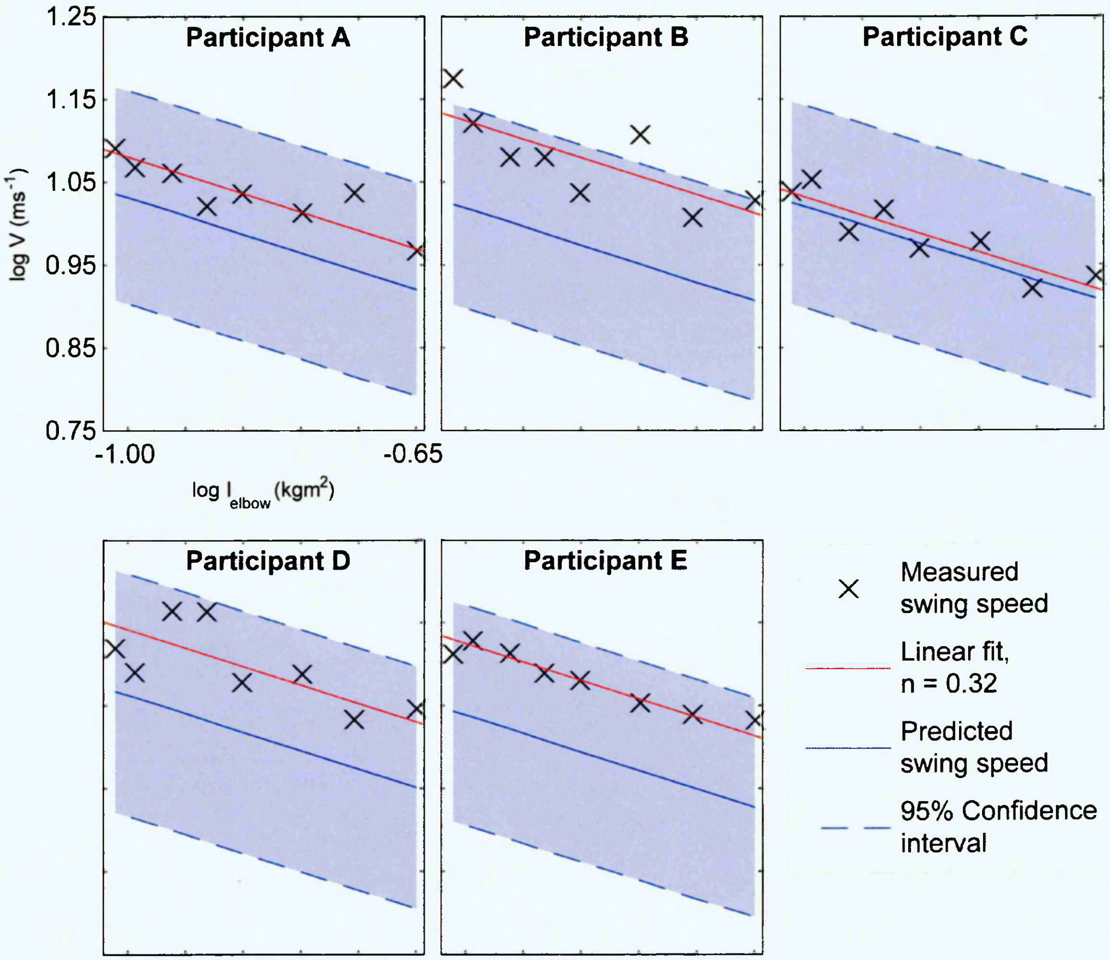

Figure 6.4. Plots of swing speed against MOI on a logarithmic scale, with a forced linear fit of n = 0.32 and the predicted swing speed overlaid, with a 95% confidence interval.

Table 6.3 shows the differences in swing speed between the predicted and measured data sets (V_{diff}) using the measured data as the reference point. The predicted swing speed is consistently an under-estimate here with a mean error of 1.65 ms^{-1} (14%) in the model and a maximum under-prediction of 2.53 ms^{-1}. This is attributed to the strength of the relationship the predictions were based on.

The results in Figure 6.4 show that for participant C, the predicted swing speed very closely matches the measured data, with an under-prediction of 3% at the mid-range swing speed. However, for the remaining four participants there is a larger under-prediction of 11 - 22%.

Even though there is a consistent under-prediction, all of the measured data, with the exception of two points, lie within the confidence intervals of the predictions, as shown in Figure 6.4. The trendlines for the measured data are also inside the bounds of the confidence interval, which shows that the model works quite well.

The model works well within the confidence limits of the predictions. However the accuracy is not high enough to be able to provide useful information to players. This is due to the relationship the model is founded on. For the regression between C and work done, $R^2 = 0.403$, which means that the model is only accounting for 40% of the variation in C (assuming n is constant). Therefore, to get a more consistently accurate prediction, the model will need either additional or replacement predictor variables to improve the strength of the relationship.

The measured data still lies within the confidence bounds for these predictions and there are some very strong results, particularly for Participant C. Therefore, it is worthwhile testing how well the model works for a real sporting task, where predicting performance would be of genuine benefit. If the model can predict swing speed for a more complicated task within the same confidence limits, it will confirm that a model using physical profile and constant n is an appropriate method for performance prediction. If successful, the model could be used as a way for players to determine what implements will be best for them to use in their sport. It could also provide equipment manufacturers with an improved understanding of the range of implement properties that will appeal to players seeking improved performance.

Table 6.3. Differences between predicted swing speed and measured swing speed.

Participant	Difference in swing speeds (ms^{-1})	Difference as percentage of measured V
A	1.14	11%
B	2.54	22%
C	0.26	3%
D	2.05	17%
E	2.13	18%
Mean	1.62	14%

6.6 Summary

A model was used to predict swing speed for a restricted motion using measures of work done and moment of inertia. A common rate of swing speed decay (n) was set in the model. Five new participants were profiled and had swing speeds recorded for a series of weighted rods.

The measured data is of similar appearance to previous findings and falls within the confidence bounds of the model's predictions. The observed values for n are close to the modelled value and have a low standard deviation. The rankings of work done and swing speed agree very well.

Therefore, it is reasonable to accept that the model works for predicting swing speed of an implement with a known MOI in a restricted motion. It is worthwhile exploring whether this same model can be used to predict the swing speed for a more complex sporting motion with the same confidence and accuracy.

Chapter 7: Tennis Swing Speed Analysis

7.1 Introduction

In Chapter 6, it was shown that a player's work done can be used to predict swing speed for a specific implement, in a restricted motion. The aim of this chapter is to assess whether the same predictive model can be used for a more complex task. The following objectives were completed to achieve this aim:

- Carry out a sport specific swing speed analysis.
- Profile all participants with the physical measures used in previous chapters.
- Make predictions of swing speed for each implement using the model from Chapter 6.

The sport selected for this study was tennis. Tennis was very relevant to the previous work as the rods used in chapters 4 and 5 overlap with the range of MOI found in tennis racquets. The forehand groundstroke was chosen as the shot for swing speed analysis. This motion uses the same predominant muscles [subscapularis, pectoralis major] as internal rotation of the shoulder (Chang, Hughes, Su, & An, 2000; Ryu et al., 1988). Internal rotation of the shoulder is also a key component of the latter part of a forehand groundstroke swing (Landlinger et al., 2010b), therefore a forehand groundstroke is very well related to the predictive model and the tests carried out earlier in the thesis.

7.2 Participants

After receiving ethical approval for the study from the faculty research ethics committee, tennis players were recruited from local clubs in Sheffield. It was important that the players involved in the test were able to perform consistently when repeating the same shot. Therefore, only players that played for their club (or at a higher standard) were invited to join the study. A total of ten players were recruited with an age of 43.5 ± 8.3 years old, a height of 1.80 ± 0.06m and a mass of 81.8 ±

13.9kg. Eight of the ten players competed for their clubs regularly in local tennis leagues and one of the ten players is a former professional.

7.3 Racquets

In order to be able to add mass to the racquets while keeping total mass to a realistic level, a light racquet was required. The Dunlop Biomimetic S5.0 Lite was selected as the base racquet for the study. This racquet is one of the lightest available on the market and at the time of the study it was also being used by an ATP tour professional [Nicolas Almagro, ranked 28 by the ATP in August 2014].

The MOI of these racquets was modified using strips of lead tape. Five identical racquets were used and lead tape was added at a different location along the length of each racquet. Tape was added equally on both sides of the racquet's centreline to keep the twist-weight constant (Spurr et al., 2014). Each racquet had a total mass of 0.306kg, 0.033kg of which was lead tape.

To keep the appearance of the racquets uniform, the lead tape was covered with black tape and the four locations without lead tape had a strip of cardboard added, also covered in black tape. Figure 7.1 shows the racquet unmodified (left) and with the added masses, i - v (right).

The MOI of each racquet was measured by timing 50 oscillations of the racquet in a pendulum motion; in the same method as detailed in Chapters 4 & 5. The measures were also verified by measuring swingweight using a Racquet Diagnostics Centre (Babolat, Lyon, France) and converting all of the measurements to be the MOI about the butt end of the racquet.

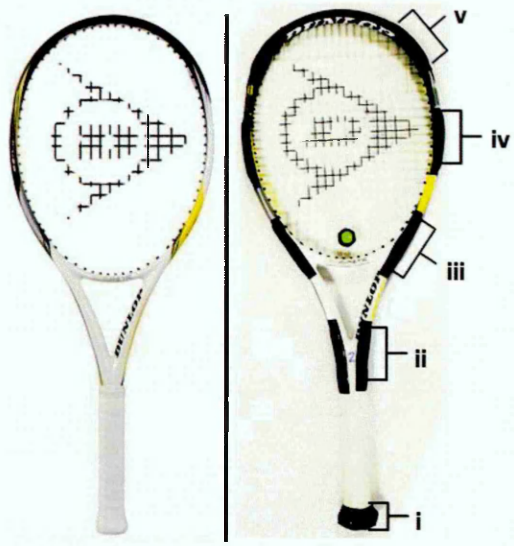

Figure 7.1 - Dunlop Biomimetic S5.0 Lite, unmodified (left) and modified (right).

7.4 Physical profiling

All ten players undertook a physical profiling test, which consisted of measuring height, mass and work done during a restricted motion. The protocol was consistent with that used in Chapter 5 where the players spent five minutes warming up on an upper body ergometer before performing a shoulder internal rotation test on an isokinetic dynamometer, pictured in Figure 7.2. The players produced five maximal efforts, all of which were capped at a rotational speed of 60 ^{o}/s. Work done was calculated as the area under torque - angle plots.

7.5 Physical profile results

The results from the physical profile test are found in Table 7.1. The measured work done for the ten players ranges from 19.9 – 86.2 J.

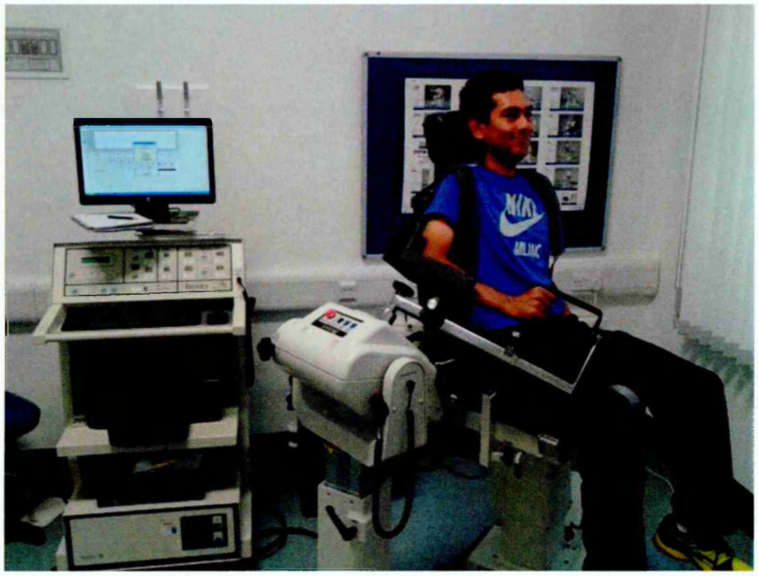

Figure 7.2 - A player performing maximum effort shoulder internal rotation on the Biodex dynamometer

Table 7.1 - Mass, height and work done results for all ten players.

Player	Mass (kg)	Height (m)	Work Done at 60°/s (J)
1	64.4	1.78	36.4
2	70.9	1.77	35.2
3	71.1	1.78	20.6
4	102.0	1.86	46.3
5	80.8	1.87	31.4
6	79.0	1.73	41.1
7	110.0	1.90	86.2
8	72.7	1.70	32.3
9	90.4	1.81	36.1
10	76.6	1.78	19.9

7.6 Swing speed test

7.6.1 Experimental setup

The swing speed analysis experiment took place in a sports hall, which provided more space and a more familiar setting than the laboratory. A tennis net was set up in the centre of the hall and the player stood on one side of the net, opposite a ball bowling machine as can be seen from Figure 7.3. Four motion capture cameras were used to track the racquet and ball and the capture volume surrounding the player was 4.0 x 4.0 m and 1.8 m high. A radar unit was positioned on the other side of the net from the player to measure the outbound ball speed.

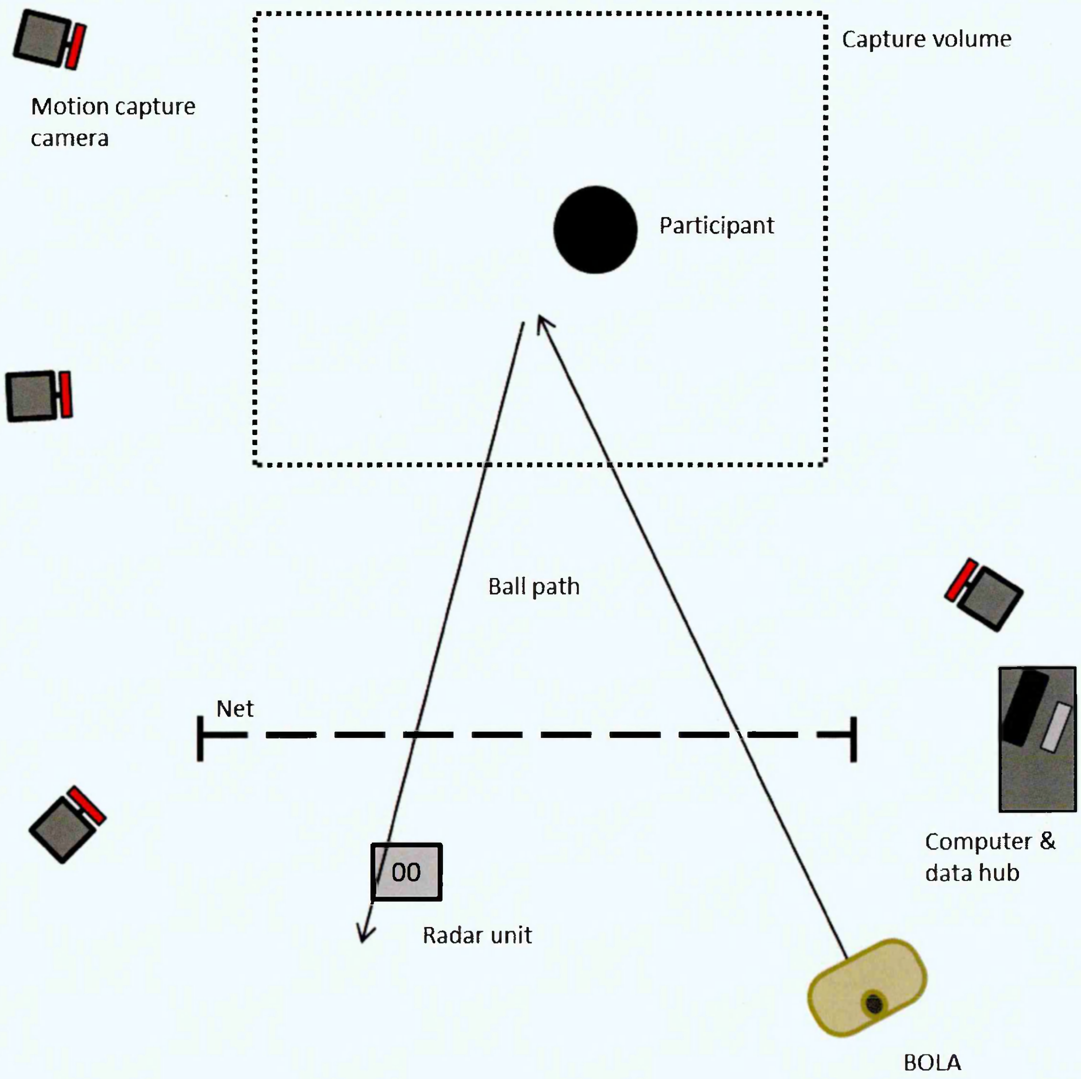

Figure 7.3 - Plan view of experimental setup for swing speed analysis of a right handed player (not to scale).

7.6.2 Motion capture

This experiment used four motion capture (MAC) cameras (Motion Analysis Corporation, Santa Rosa, USA) arranged as displayed in Figure 7.3. The cameras used were Raptor-4 cameras which are able to work in natural light and have a maximum resolution of 2352 x 1728 pixels. These cameras captured data at 300 frames per second with a shutter speed of 1/1000s.

The markers used for this study were a combination of 25mm diameter spheres and 25mm wide strips of retro-reflective tape. The markers were located at the tip, throat and lateral extremities of the racquet head in a kite shape and can be seen in Figure 7.4.

Figure 7.4 - Modified racquet with four reflective markers forming a kite shape.

The tennis balls used for the test were also modified to allow them to be tracked with the MAC system. The balls were covered with retro-reflective tape strips and treated as a large marker in the tracking system, along with the racquet, as can be seen in Figure 7.5.

The addition of reflective tape increased the mass of the balls on average by 4.5g (8%) and changed the surface texture of the balls. However, the players who trialled the ball in pilot testing didn't detect significant changes in terms of 'feel' and this was deemed more important than any changes in ball performance.

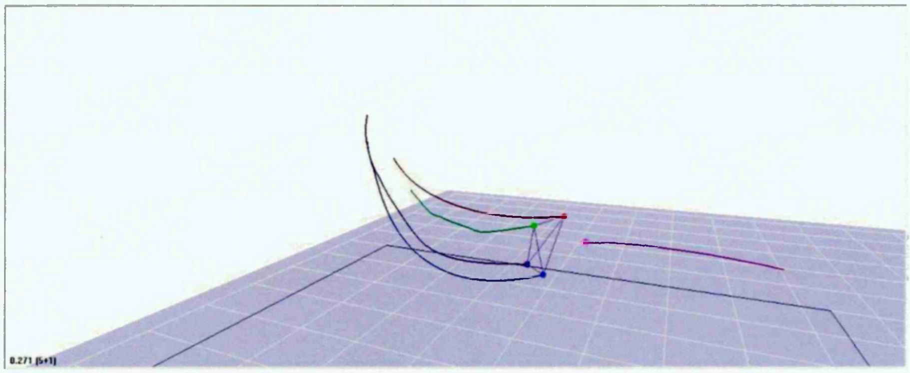

Figure 7.5 - Typical view of a trial through the motion capture software, Cortex.

7.6.3 *Ball delivery*

A tennis ball bowling machine (BOLA, Bristol UK) was used to provide a consistent delivery to the player. The balls were fired at 15.6 ms^{-1} (35 mph) with a top-spin bias and bounced to a mean height of 0.9m above the ground in the capture volume. Five balls were used on rotation throughout the test to prevent excessive degradation of the reflective surface.

7.6.4 *Motion*

The motion being assessed in this study was a maximum-effort forehand groundstroke. This is a fundamental shot in the game of tennis, making up a large proportion of service returns and baseline rallies. The predominant muscles involved in the action of a forehand groundstroke are subscapularis and pectoralis major. These are also with the predominant muscles used in shoulder internal rotation (Chang et al., 2000; Ryu et al., 1988). It was therefore a fair assumption that the model for predicting swing speed, which is based upon internal rotation of the shoulder, could be an appropriate tool to predict swing speed in this tennis specific scenario.

7.6.5 Procedure

On arrival, players were invited to spend approximately ten minutes warming up before starting the test. The warm up consisted of jogging around the hall, shoulder exercises and practice shots but the players were asked to do what they would usually do before a match and were comfortable with, so each warm up was slightly different. The final part of the warm up involved hitting between five and ten shots with the reflective balls delivered from the BOLA. This allowed players to familiarise themselves with the delivery path, the appearance of the ball and the surroundings of the experimental setup.

Immediately following the warm up, the players performed five shots with each of the prepared racquets plus five shots with their own racquet, totalling 30 maximum effort shots per participant. For tracking the player's own racquet, four of the spherical markers were attached at the same locations as in Figure 7.4. The shots were performed with short breaks of around 30 seconds between. In this time the tracking was saved and the player was told their shot speed from the radar unit, to act as encouragement to sustain their effort. There was a slightly longer break after each set of five shots to swap racquets.

7.6.6 Data analysis

The swing speed results were plotted on a logarithmic scale against MOI, as has been done previously. The measure of work done from the physical profiling test was input to the model described in Chapter 6 to generate a predicted swing speed for each of the tennis racquets. Both measured and predicted values were then compared in the same way as in Chapter 6.

7.7 Swing speed test results

Figure 7.6 shows the log-log plots of swing speed against MOI for the five modified tennis racquets and each player's value for n is shown.

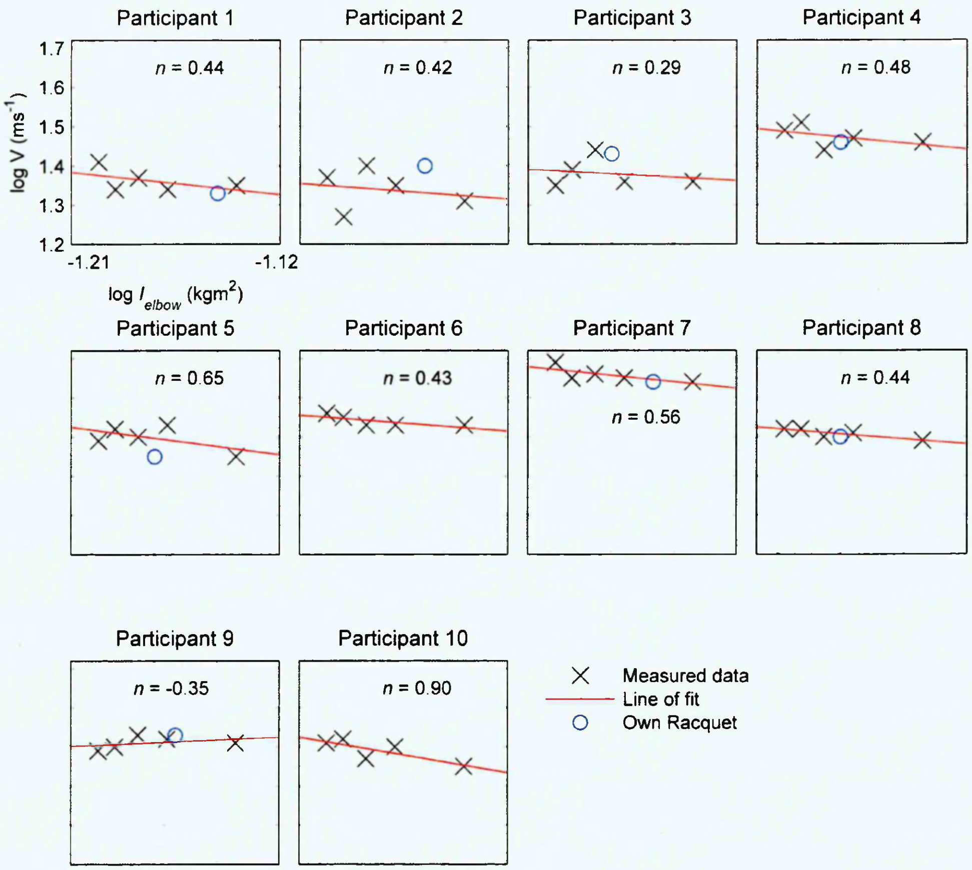

Figure 7.6 - Swing speed against moment of inertia for all ten participants, on a log scale, with natural lines of best fit.

For eight of the ten players, where data was available, swing speed for the player's own racquet is displayed with a blue circle for comparison with the modified racquets. The own racquet swing speeds fit in well with the rest of the measured data, lying on or close to the line of best fit for each player.

This suggests that the players were comfortable in the test scenario and were happy playing with the modified racquets. Furthermore, this means that the swing speed results presented in this chapter can be considered as reliable performance data.

7.8 Model results

7.8.1 Model accuracy

As in Chapter 6, the accuracy of the predictive model can be determined by quantifying the difference in velocity between predictions and measured data.

The swing speed produced by the ten players with the five modified racquets is displayed in Figure 7.7 on a logarithmic scale. The measured data is fitted with red lines where n = 0.318, which is the modelled value from Chapter 5. The predicted swing speeds are shown with blue lines, with the 95% confidence intervals.

Maintaining the same, constant value of n for the lines through the measured data makes the measured data directly comparable with the predictions. The difference between the measured data line of fit and the predicted data line was calculated, denoted V_{diff}.

Table 7.2 - Difference between predicted and measured swing speeds for the ten players in the tennis swing speed analysis.

Participant	V_{diff} (ms^{-1})	Percentage change
1	10.61	46%
2	12.75	50%
3	16.23	59%
4	13.95	51%
5	19.05	61%
6	20.87	62%
7	28.77	63%
8	19.36	61%
9	21.18	63%
10	18.20	61%
Mean	18.10	58%

V_{diff} values for the ten players are shown in Table 7.2. The difference is also shown as a percentage of the measured swing speed. The range in V_{diff} is 10.6 ms^{-1} (46%) to 28.8 ms^{-1} (63%). The predicted swing speed results consistently underestimate compared to the measured data.

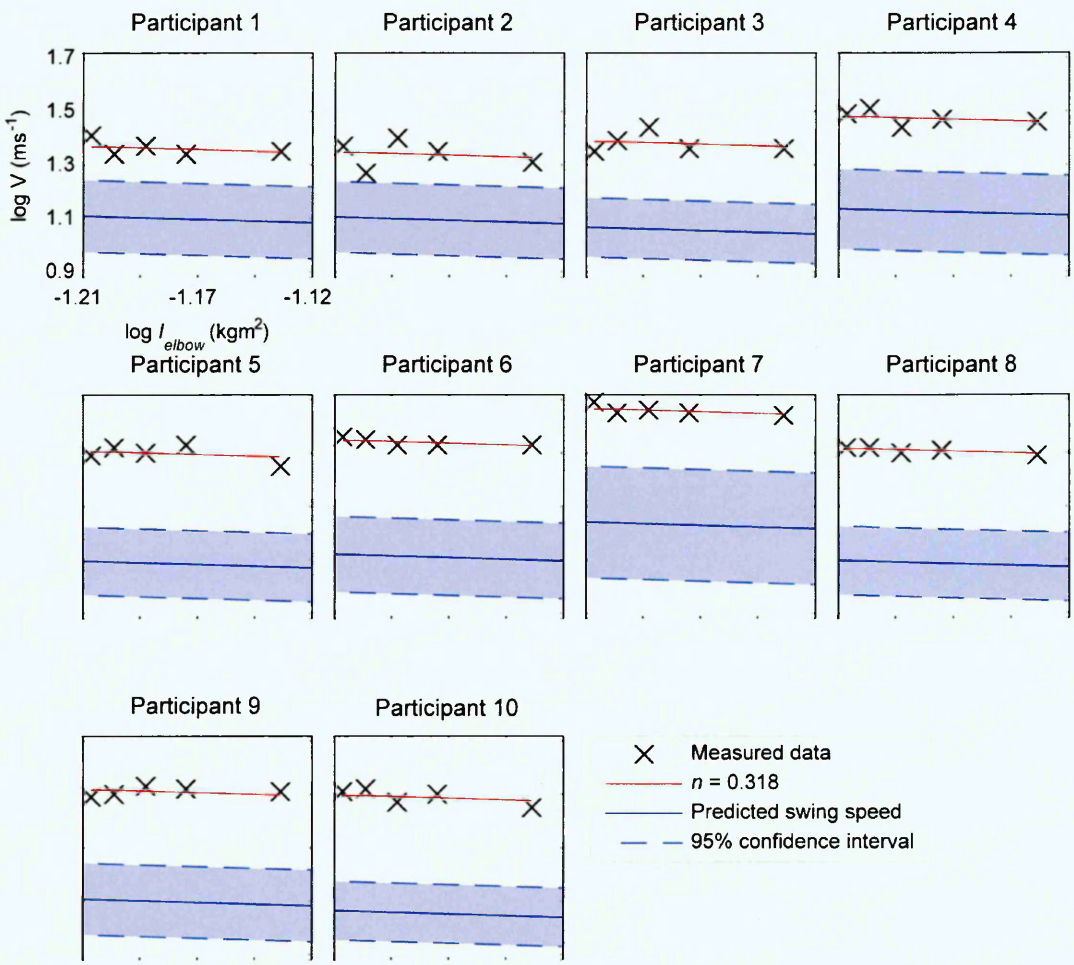

Figure 7.7 - Swing speed against moment of inertia for all ten participants, on a log scale, with a forced fit of *n* = 0.32 and overlaid predictions with a 95% confidence interval.

All predictions underestimate the observed swing speed such that the measured data lies outside of the confidence interval in all ten cases. The mean V_{diff} of 58% is much higher than for the laboratory validation in Chapter 6 where it was 14%. However, this is not completely unexpected and it is understandable that there might be a step difference between the model and measured data. This is because we know that a tennis groundstroke is a much more complex motion than shoulder internal rotation, with a large kinetic chain (Landlinger et al., 2010a, 2010b; Roetert, Kovacs, Knudson, & Groppel, 2009).

The players in this study were not restricted to perform in a set manner but were simply asked to perform a forehand groundstroke. Differences between players in technique, and how well rehearsed that shot is, will have an influence on the resulting racquet tip speed. This contributes to the range of V_{diff} seen in Table 7.2.

7.8.2 Model precision

It is clear from the V_{diff} values that the accuracy of the model is worse than in Chapter 6. However, we can also assess the precision of the model by comparing each player's line of best fit with the modelled lines where $n = 0.32$.

The swing speed data for all ten players is shown in Figure 7.6. Nine of the ten players have the expected trend of decreasing swing speed with increasing MOI, with n ranging from 0.29 to 0.90. Player nine's swing speed increases with increasing MOI, with n equalling -0.35. The mean value for n amongst the ten players is 0.43.

7.8.3 A movement specific model

These results strongly suggest that assuming a constant value of $n = 0.318$ in the model is not acceptable in this case. While this value of n provided a useful model for restricted internal rotation of the shoulder, it seems a different value of n is required for a tennis forehand groundstroke. In order to test this theory, the dataset of ten participants was resampled and a new model was produced.

The data from a random sample of six participants were taken from the initial ten in the tennis study. The prediction model was generated in the same way as in Chapter 6 using the modified least squares method to find the value of n. The new value for n was calculated as 0.314. This is very similar to the previous model where $n = 0.318$.

The measures for work done from the physical profiling test were used to predict the swing speed for the remaining four participants. The results are compared to the new predictions in Figure 7.8.

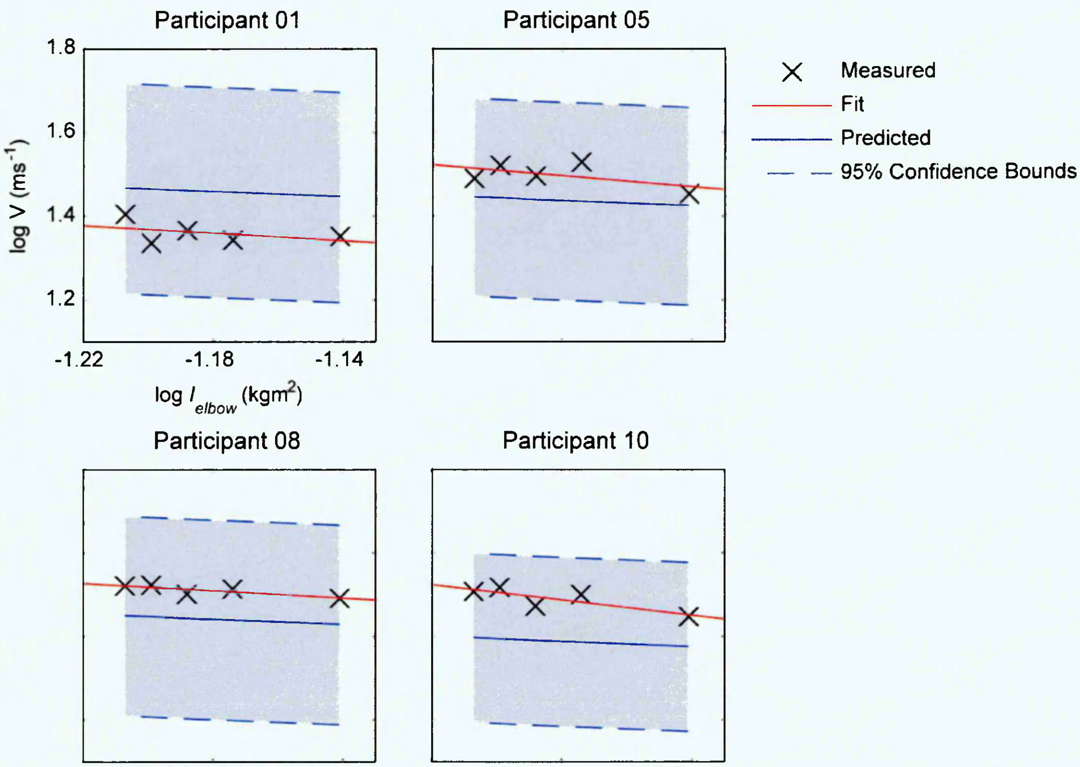

Figure 7.8. Swing speed vs. moment of inertia with prediction confidence interval for the resampling within the tennis study.

One of the most noticeable features of the plots in Figure 7.8 is the large range covered by the prediction confidence interval. This is because the relationship between work done and intercept for this case is much weaker than in the restricted motion model. This generates a large margin of error, resulting in a broad confidence interval.

As a result of the large confidence interval, all of the measured data lies well within the bounds. To get a comparable understanding of the prediction accuracy, we can analyse the difference between the measured and predicted swing speeds at the mid-range, V_{diff}. The V_{diff} data can be found in Table 7.3, where the mean difference between the measured and predicted swing speeds is 4.61 ms^{-1}, or 16% of the measured swing speed. This is a big improvement compared with the restricted motion model, where the mean difference was 58%. However, a difference of 4.6 ms^{-1} is still not accurate enough to be useful. This is especially so when compared to the difference in swing speed from racquet one to racquet five, which across all ten players is a mean of 4.4 ms^{-1}. This means that for a set of implements with an MOI similar to that tested here, the predicted value could be outside the real swing speeds for the whole set of implements. Therefore, these predictions are not useful for a player wishing to quantitatively analyse how their performance might change with different implements.

Table 7.3 - Difference between predicted and measured swing speeds for resampled model.

Participant	V_{diff} (ms^{-1})	Percentage change
1	-5.65	-24%
5	3.80	12%
8	3.93	12%
10	5.07	17%
Mean	4.61	16%

The measured values of n for the remaining four players were 0.44, 0.65, 0.44, and 0.90. This is a very large range of n and all four values are very different to the modelled value of 0.31 as can be seen in Figure 7.8. Such a range of n suggests that a model with a fixed value of n is also not of benefit to players desiring a qualitative analysis of how their performance will change with different implements in a groundstroke motion.

7.8.4 Prediction offset

The accuracy of the swing speed predictions has been shown to be poor for the ten players in this tennis study. The model based on resampling the dataset has a very similar value of n to the initial model and also has significant inaccuracy in the predictions. Given that the motion being predicted here is far more complex than the movement upon which the model is based, the results suggest there is a systematic error in the model that could be accounted for. The model is expressed in Equation 7.1 and the prediction offset was calculated as the difference between the predicted intercept and the intercept for the fixed- n line through the measured data.

$$Intercept = (0.0027 \times Work\ Done) + 0.627 \tag{7.1}$$

The offset ranged from 0.243 – 0.433 for the ten players and the data distribution had a negative skew. Therefore, to best represent the difference across the group, the root mean square offset was calculated. The RMS offset was 0.376 and this was added to the constant in the prediction formula to give Equation 7.2.

$$Intercept = (0.0027 \times Work\ Done) + 1.003 \tag{7.2}$$

The adjusted predictions are shown against the measured data and the initial predictions in Figure 7.9. The mean V_{diff} for the adjusted model was 4.07ms^{-1} (15%).

The adjusted predictions are a great improvement on the initial model, with measured data for eight of the ten players now within the 95% confidence interval. For four of the participants, the adjusted predictions are an overestimate and the predictions underestimate swing speed for the other six. As the systematic difference between the restricted motion and the groundstroke has been accounted for, these remaining differences are likely to be due to the effects of technique. Variations in the path of the racquet or even the timing of the contribution of different body parts to the swing could be responsible.

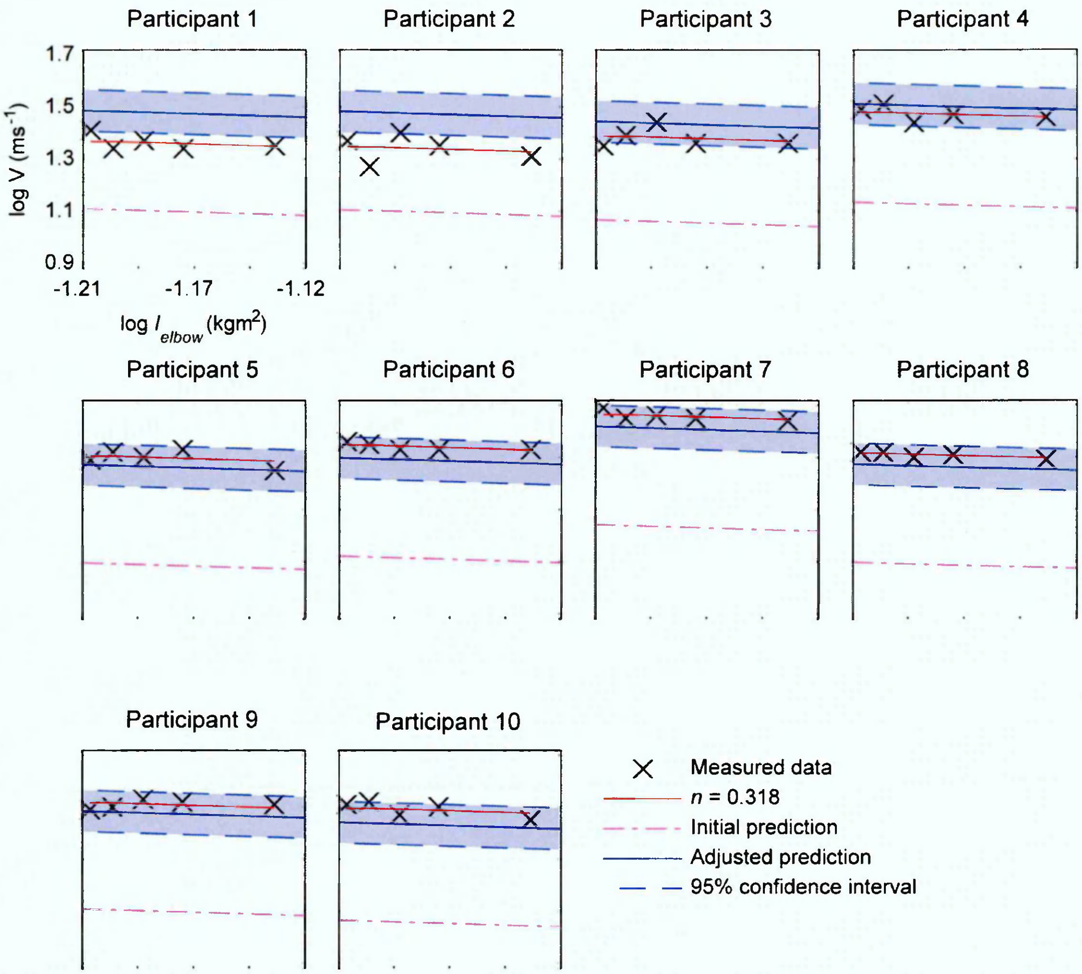

Figure 7.9 - Log plots of swing speed results with initial predictions and adjusted predictions.

7.8.5 Cross-validation

In order to assess the quality of the prediction adjustment thoroughly, a leave-one-out cross-validation (LOOCV) was executed. For this test, the mean prediction offset was calculated a further ten times with one participant omitted on each occasion. The alternate values for the mean offset were compared to the value calculated on the whole data set, as can be seen in Figure 7.10.

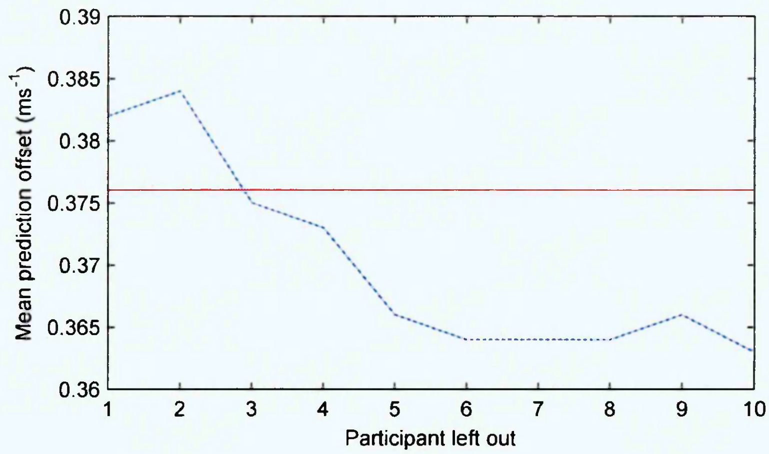

Figure 7.10 - Changes in the prediction offset with each participant's data removed from the model.

The result in Figure 7.10 shows participants one and two lower the mean offset with their inclusion, participants four to ten increase the mean offset with their inclusion and participant three's offset is very close to the mean value for the group. There are no cases where removing a participant's data drastically alters the mean offset. This shows that the adjustment to the model is not dependent on one specific player and is therefore reliable.

However, despite the adjustment to the model being an improvement on the initial model and a reliable alteration, the new, lower error is still a problem. The new value of 4.07 ms^{-1} is very similar to the offset in the resampled model on page 105 (4.61 ms^{-1}). This level of difference is not sensitive enough to be of benefit to players looking for performance predictions. The change in swing speed a player will achieve between two implements of differing MOI is likely to be less than the error in the model. Therefore this model would be more applicable to a situation where a coach or retailer is educating recreational players on the effect of changes in swingweight, where the specific values of the output are not important.

7.8.6 Differing values of n

The final version of the model is a significant improvement on the initial predictions and could be useful for providing approximate information as to how player strength can affect swing speed with differing MOI implements. However, for all versions of the model considered in this chapter, the constant value of n is not representative of the measured data. There is too large a spread in the values to assume that a common value will provide good swing speed predictions for all players in this motion.

In this tennis study there was a range of different techniques used by players to achieve the same outcome, with some shots having very wrist based acceleration, some with much more translation and some with a mixture of the two. This could partly explain differences in n. However, there were also large differences in n in Chapter 5 with a restricted motion analysis. Therefore these results suggest that n cannot be considered a constant and is individual to an athlete. This finding is highly significant as it is in opposition to the work of numerous authors who have found a common value for an entire group (Cross & Bower, 2006; Daish, 1972; Nathan et al., 2011; Smith & Kensrud, 2014; Smith et al., 2003, 2012).

There is one participant who took part in both the restricted motion analysis in Chapter 5 and the tennis specific study in this chapter. This player was Participant 2 in both instances and their measured values of n were 0.48 and 0.42 respectively. In the context of the measured range of n, these values are very close and support the idea that n is an individual term.

7.9 Summary

In this chapter, swing speed was measured for tennis players performing maximum effort shots using a series of racquets with differing MOI. The racquets used were visually identical and with a common mass. Nine out of ten players' swing speed decreased with increasing MOI.

Swing speed predictions were made using data from a physical profiling and an assumed common rate of swing speed decay, n. An offset was applied to the model to account for the change in magnitude of swing speed between restricted shoulder internal rotation and a forehand groundstroke. When compared to the measured data these predictions were a mean of 4.07 ms^{-1} away from the real values. This is not sensitive enough to provide accurate data to a player wanting to know their swing speed with a given implement. Although assuming a common value of n was useful for a restricted motion, it is not satisfactory for this more complex sporting task, as the values for n found in the measured data varied from -0.35 to 0.90.

The results suggest that n is an individual factor and is constant or near to constant for each individual. The implications of a variable n need to be explored to understand how it might affect a player's choice of equipment.

Chapter 8: Practical implications

8.1 Introduction

In Chapter 7 the change in swing speed with variable moment of inertia was analysed for groundstrokes in tennis. The trend of decreasing swing speed with increasing MOI matched what had been seen earlier in restricted motion analyses. However, attempts to predict swing speed based upon physical profile were unsuccessful. The predictions were inaccurate and the assumption that n is constant was found to be invalid for a complex sporting task. The conclusion was that, rather than n being constant, it is more likely that there is a different value for every player. The aim for this chapter is to investigate the implications of changing n via the following objectives:

- Consider what variables might be related to n.
- Use impact modelling to determine the effect of changing n on ball speed.
- Identify whether or not there is an optimum MOI and propose methods for calculating this for players.

The ideas investigated in this chapter are applicable to any motion involving the swinging of an implement but, to continue from Chapter 7, tennis will be used as a case study.

8.2 MOI sensitivity (n) as a variable

In Chapter 5 a decision was made to assume a constant n so that the predictive model could be developed. The subsequent work has shown that a model where n is constant is usable for a restricted motion but it is not sufficient for a full tennis shot. Experimental work found that one value of n does not sufficiently represent the data for all of the participants tested.

Considering n as a variable, we can look at what might affect a player's value of n. The data from the tennis swing speed analysis shows a relationship (R = -0.370) between a player's sensitivity to MOI and the MOI of the racquet they regularly play with. This data is shown in Figure 8.1 with the solid blue line being the trend for all of the data and the dashed black line the trend if Participant 9 is excluded.

The more head heavy (high MOI) a player's chosen racquet is, the lower their sensitivity to MOI and the flatter their swing speed profile. This suggests either that when choosing a racquet players have a good natural sense of their sensitivity to MOI and choose accordingly, or that n is something that can be trained and that regular play with a high MOI implement can decrease a player's n. However, there is evidence to show that training with a high MOI bat does not increase swing speed when returning to play with lower MOI bats, so this may be something that is difficult to achieve or takes a long time (Kim & Hinrichs, 2003; Otsuji et al., 2002).

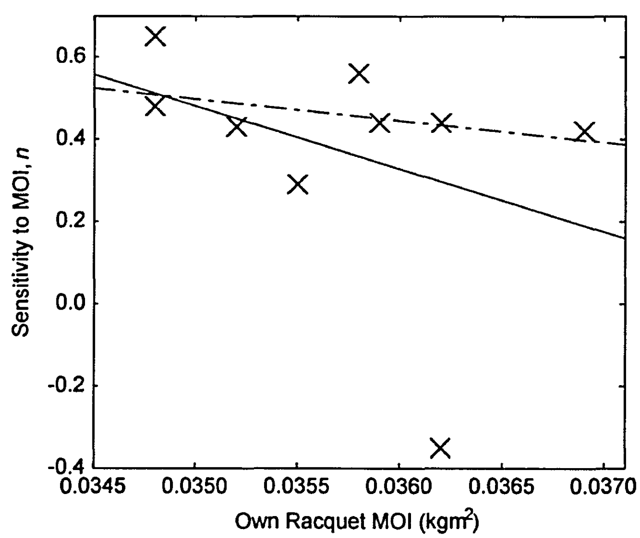

Figure 8.1 - The relationship between n and the MOI of a player's racquet. The solid blue line is a linear best fit through all of the data and the black dashed line is a line of best fit with player 9 excluded.

8.2.1 Swing speed – ball speed trade-off

Swing speed is not the only important parameter in games such as tennis. A player is arguably more interested in ball speed and ball placement than racquet speed. This is where there is a trade-off when altering the MOI of a racquet because (in most cases) an increase in MOI will decrease swing speed, but increasing MOI can also increase outbound ball speed (Cross & Nathan, 2009).

If a player were to choose a head light racquet to improve their swing speed but this actually reduced the speed of their shots, this would be detrimental to their game. Therefore it is important to consider what effect changing MOI will have on the racquet - ball impact. This can be done by modelling the impact using experimental data to set the initial conditions.

8.3 Rigid body impact model

The modelling of racquet and ball impacts was done with a rigid body model as used in previous research (Brody, Cross, & Lindsey, 2002; Brody, 1997; Choppin, 2013). This is a simplified model that assumes the racquet frame does not deform during impact. The model assumes the ball strikes the string-bed on the longitudinal axis of the racquet; the tennis ball translates along an axis normal to the racquet face and the racquet is free to move along one axis and rotate about a second, perpendicular axis.

8.3.1 Model setup

The impact model was run for a set of one-dimensional implements with a broad range of MOI as shown in Table 8.1. The modelled system is shown in Figure 8.2. These implements were calculated as beams of 0.685 m length (l) and total mass (M) of 0.30 kg, comparable to tennis racquets. The beams had an initial mass of 0.1 kg with a moving point mass of 0.2 kg, keeping mass constant but changing balance point (x) and MOI between implements.

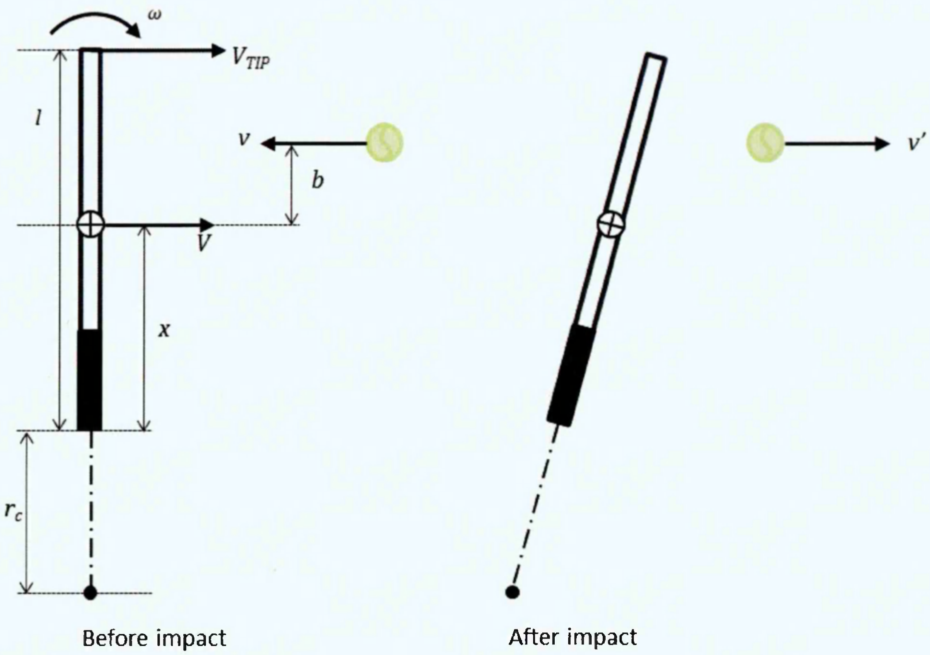

Figure 8.2 - An illustration of the modelled scenario before and after impact.

The impact model starts with the same input as the predictive models used in Chapters 6 and 7, work done. Work done is used to calculate the racquet tip velocity (V_{TIP}) for the MOI of each implement. The MOI here is calculated about an axis through the centre of rotation for a groundstroke and values are shown in Table 8.1. The instantaneous centre of rotation (ICOR) was calculated for all trials in the tennis swing analysis presented in Chapter 7. There were no trends found with the changes in ICOR; therefore a mean was taken, providing ICOR = 0.180 m, measured from the butt end of the racquet, back along the centreline (r_c in Figure 8.2). This is consistent with work by Brody (1997) who found the average ICOR for forehand groundstrokes to be 0.2 m.

Table 8.1 - Physical properties of implements used in impact model.

Mass, M (kg)	Length, l (m)	Balance point, x (m)	MOI about butt, I_{butt} (kgm^2)	MOI about ICOR, I_{CR} (kgm^2)
0.30	0.685	0.114	0.016	0.025
0.30	0.685	0.148	0.016	0.026
0.30	0.685	0.181	0.018	0.027
0.30	0.685	0.214	0.020	0.030
0.30	0.685	0.248	0.024	0.033
0.30	0.685	0.281	0.028	0.038
0.30	0.685	0.314	0.034	0.043
0.30	0.685	0.348	0.040	0.050
0.30	0.685	0.381	0.048	0.057
0.30	0.685	0.414	0.056	0.066
0.30	0.685	0.448	0.066	0.075
0.30	0.685	0.481	0.076	0.086
0.30	0.685	0.514	0.088	0.097
0.30	0.685	0.548	0.100	0.110
0.30	0.685	0.571	0.109	0.119

8.3.2 Equations for the model

The equation to calculate outbound ball speed can be derived using the conservation of momentum, where the combined linear momentum of the racquet and ball will be equal before and after impact (Equation 8.1) as follows (Brody et al., 2002; Brody, 1997; Daish, 1972):

$$M_{IP}V_{IP} + mv = M_{IP}V_{IP}' + mv' \qquad \text{Equation 8.1}$$

where M_{IP} is the effective mass of the racquet at the impact point, V_{IP} is the racquet speed at the impact point before impact, V_{IP}' is the racquet speed at the impact point after impact, m is the ball mass, v is the ball speed before impact and v' is the ball speed after impact. The coefficient of restitution (e), which describes the ratio of the velocity of separation and the velocity of approach, is defined by Equation 8.2:

$$e = \frac{v' - V'}{V - v}$$ Equation 8.2

If Equation 8.2 is rearranged for post impact racquet velocity and substituted into Equation 8.1, the following expression is formed for outbound ball speed:

$$v' = \frac{M_{IP}V_{IP} + eM_{IP}(V_{IP} - v) + mv}{M_{IP} + m}$$ Equation 8.3

The effective mass of the impact point can be described using Equation 8.4 and the impact point velocity can be described using Equation 8.5 (Choppin, 2013).

$$M_{IP} = \frac{I_{COM}M}{I_{COM} + Mb^2}$$ Equation 8.4

where M is racquet mass, I_{COM} is the MOI about the centre of mass and b is the distance between the impact point and the centre of mass.

$$V_{IP} = V + \omega b$$ Equation 8.5

where V is the racquet speed at the centre of mass, ω is the rotational speed of the racquet. Substituting Equations 8.4 and 8.5 into Equation 8.3 provides Equation 8.6 in which swing speed and implement properties were used to calculate outbound ball speed (v').

$$v' = v + \frac{2MI_{COM}(V + \omega b - v)}{m(I_{COM} + Mb^2) + MI_{COM}}$$ Equation 8.6

where v is the inbound ball speed, set to 13.3 ms^{-1} (30 mph), m is ball mass (0.057 kg). In this case, b was set to equal 0.16m from the racquet tip, which is the typical location of the node point on a tennis racquet, which players normally aim to impact (Choppin, 2013; Cross, 2001).

8.4 Optimum moment of inertia

Two scenarios were run through the model. Firstly, n was kept constant at a value of 0.32 as it was in the first version of the predictive model, and the variable input was work done. Secondly, n was input as a variable into the model to reflect the results seen in the tennis analysis. Only one value for work done was required in this second case, so it was set to 38.5 J as the mean of the ten players tested in Chapter 7.

8.4.1 Constant n

In this situation, the distinguishing characteristic for players is their measure of work done, which is used to predict swing speed. Figure 8.3 shows ball speed against MOI for three different values of work done, 15 J, 48 J and 82 J, which covers the range seen during physical profile tests in this thesis.

The curve has a turning point at 0.076 kgm^2 (MOI taken about the butt end of the racquet), which is the optimum value of MOI (I_{OPT}) for this shot. This value is well above the typical range of MOI for tennis racquets, which is 0.03 – 0.04 kgm^2. This maximum point is the same for all three curves and does not change with work done. The value of work done does affect the magnitude of the ball speed throughout the curve, with higher work resulting in faster ball speeds. The curves in Figure 8.3 are very shallow around the maximum point, meaning a small difference in MOI from the optimum point will not greatly affect ball speed.

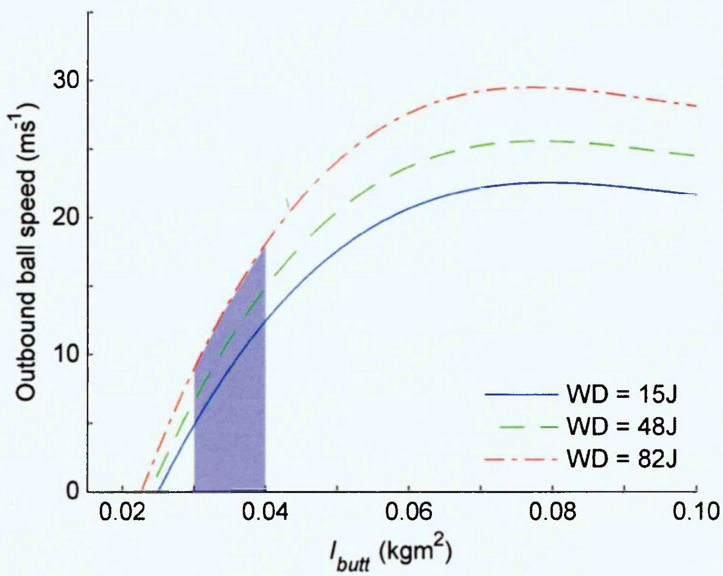

Figure 8.3 - Ball speed against MOI with n = 0.32, for three different values of work done, with a shaded region representing the typical range of tennis racquet MOI.

8.4.2 Variable n

The results in Chapter 7 indicate that the modelled scenario in which we are most interested in is one where n is variable. The output for this case is shown in Figure 8.4 with ball speed curves for seven values of n ranging from -0.3 to 0.9, covering the range observed in Chapter 7. In this case, changing n directly affects the value of I_{OPT}, with high n yielding a lower I_{OPT} and low n yielding a higher I_{OPT}.

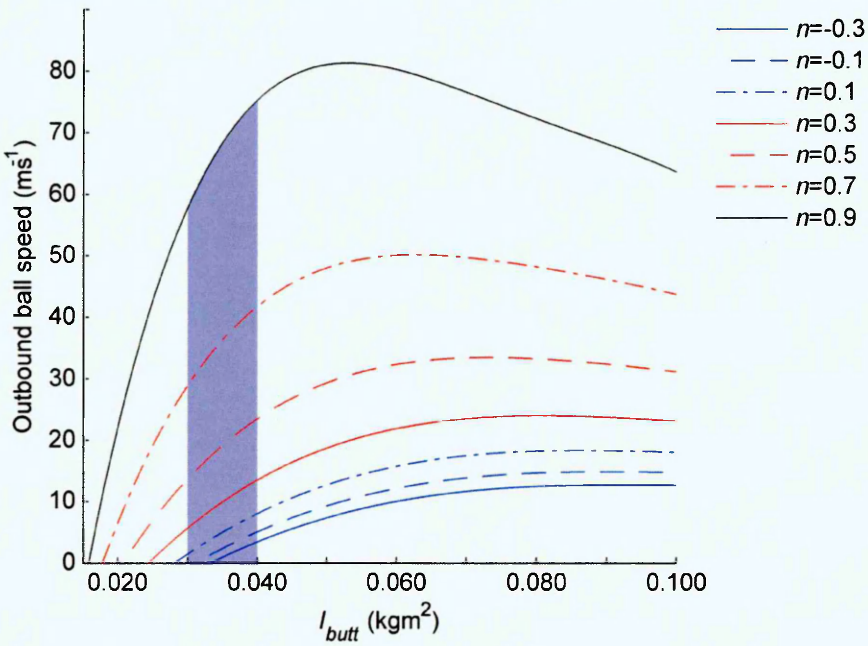

Figure 8.4 - Ball speed against MOI with a variable n. The shaded region represents the typical range of MOI for tennis racquets available on the market.

It can be seen in Figure 8.4 that I_{OPT} changes with n. For low values of n, ball speed is a maximum for high MOI but the peak is very shallow, meaning changes in MOI will not affect ball speed greatly around the maximum point. For high values of n, ball speed is at a maximum at lower values of n and the velocity peak is much narrower, meaning changes of MOI could have a big effect on ball speed.

The relationship between the optimum MOI and n is shown in more detail in Figure 8.5. The data is joined by a quadratic curve, which provided the best fit to the data. Altering the work done input for the variable n model increases the magnitude of the ball speed results but does not affect the trends observed.

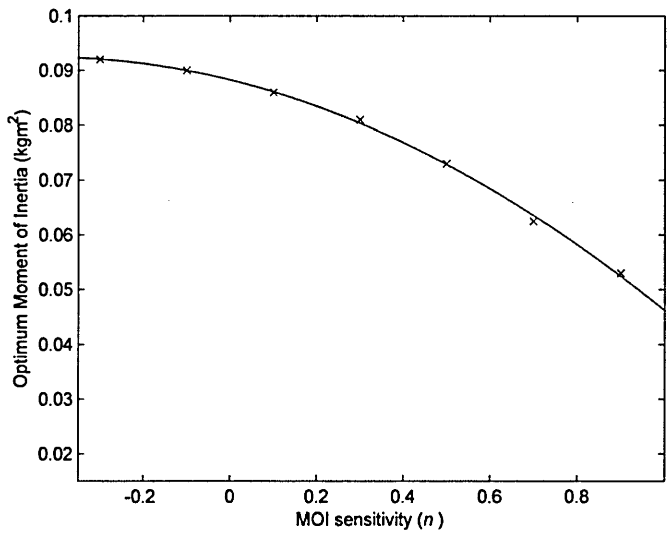

Figure 8.5 - Optimum MOI against sensitivity to MOI.

The idea that a player who is more sensitive to changes in MOI would perform optimally with a lower MOI than a less sensitive player is quite logical. However, it is also worth noting that for very low values of n (< 0.2), there is a large range of MOI either side of the optimum value where there is very little change in ball speed. For example, a player whose n = 0.1 would have an optimum MOI of 0.086 kgm^2 but if they were willing to sacrifice 1% of ball speed they could play with an MOI of 0.076 kgm^2, a decrease of 12% from I_{OPT}. This affords a large amount of flexibility to players with low values of n when seeking to optimise their performance. However, in the case of tennis, these values are far above the range of MOI available. The highest rate of swing speed decay observed in Chapter 7 was n = 0.90, which implies the optimum MOI to use in a groundstroke is 0.053 kgm^2, a 29% increase on the highest MOI racquet currently sold (Tennis Warehouse, 2015). Therefore the model is recommending that for forehand groundstrokes, all players would benefit from adding mass to the tip of their racquets to increase MOI.

Another clear result observed in Figure 8.4 is that the curves of high n have higher ball speeds than the curves of low n. However, this directly agrees with the effect that changing n has on swing speed, as discussed in Chapter 3 (and presented in Figure 8.6). A high value of n will produce a higher swing speed and consequently a higher ball speed, for a given value of C, than a low value of n.

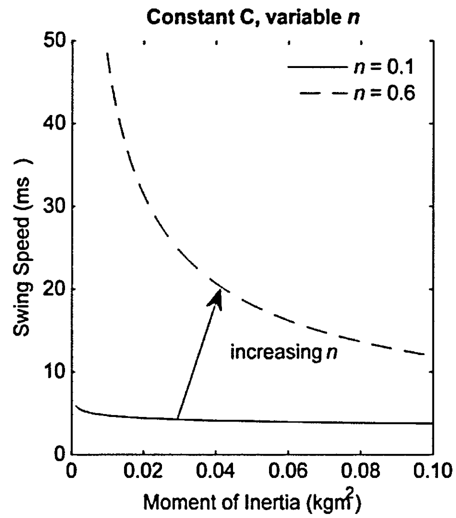

Figure 8.6 – Effect of changing *n* on swing speed.

The optimum MOI will change depending upon a player's n but, according to Figure 8.5, the value of I_{OPT} is going to be beyond the highest end of the MOI range for tennis players. If outbound ball speed is the sole performance metric of importance, this suggests that players will benefit from more head heavy racquets and that, in a tennis test, we should observe increasing ball speed as MOI increases.

The ball speed data from the tennis swing speed analysis in Chapter 7 is displayed in Figure 8.7, plotted against MOI about the butt end of the racquet. For eight of the ten players, ball speed increases with MOI even though racquet speed decreases. This parallels the data in Figure 8.5 and suggests that for eight of these players, their optimum MOI is outside the range tested.

Of the eight participants' data where ball speed increases with MOI, five of these cases have a very strong relationship (r ≥ 0.7). The two cases where ball speed decreases with MOI also have very strong correlations (r ≤ -0.7).

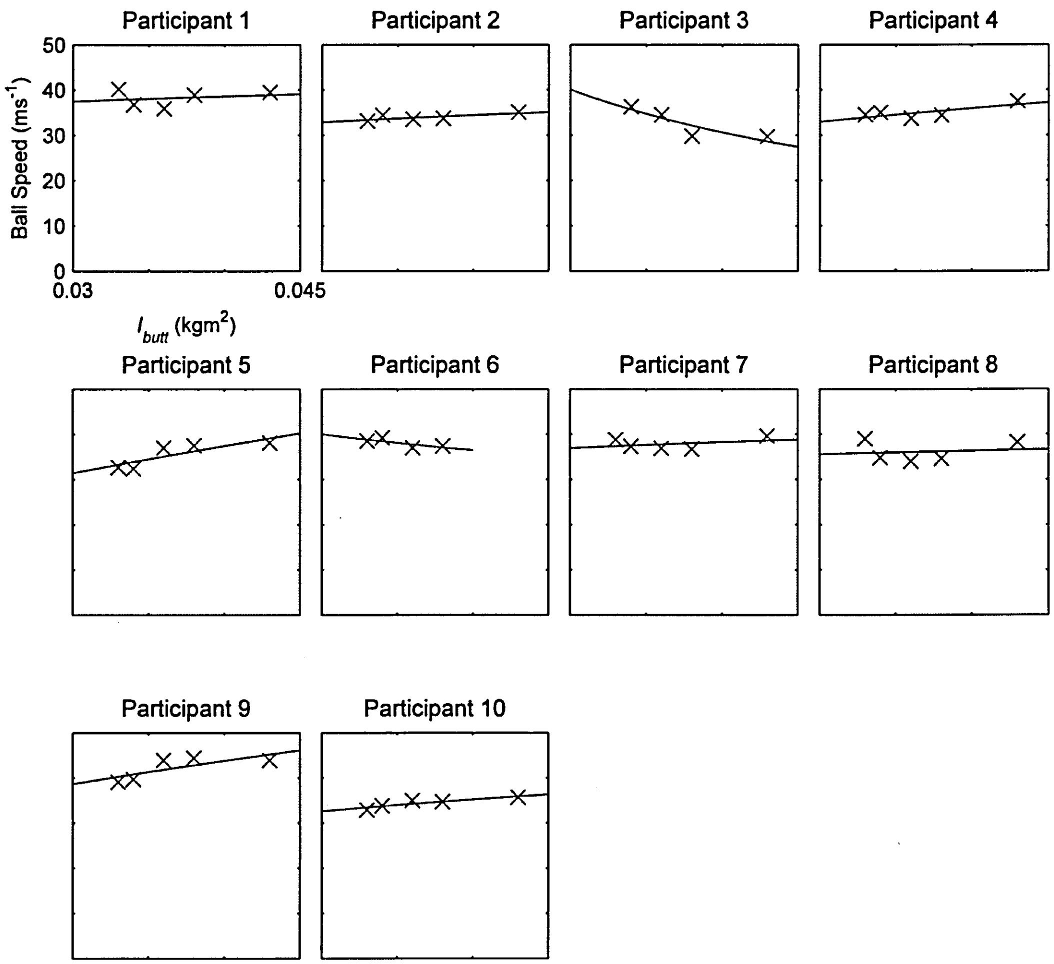

Figure 8.7 – Measured ball speed against MOI for all ten players in the tennis swing speed analysis.

It is thought that swinging higher MOI implements decreases the accuracy of the swing (Bahill & Freitas, 1995; Bahill, 2004). This could be affecting the ball speed of the two players whose ball speed decreases with increasing MOI. Participants were asked to hit maximum effort shots, not to control the strike or ball placement. If they were struggling to control the more head heavy racquets in the same manner as the head light racquets, this would lead to strikes away from the node point and consequently speed would be reduced.

8.4.3 Finding optimum moment of inertia

The results from the impact model show that when changing MOI there is a point where ball speed reaches a maximum value. If the rate of swing speed decay (n) was a constant then this optimal MOI would also be constant. However, the results have shown that n is not constant and changes from player to player, as does the optimum MOI.

This is very significant as it means that the customisation of implements to an individual is possible. While player strength is related to swing speed, it is of more importance to measure the rate at which swing speed decays for a player to understand how their performance changes with MOI and to quantify their optimum MOI.

This places further importance on the need for a simple test to quickly quantify the value of n for a given player. This test would work well taking a similar form to the restricted motion studies in this thesis. A suggested method is as follows; a minimum of three weighted rods, with identical appearance and mass but changing MOI would be swung by the player. An inertial sensor capable of tracking position and orientation would be applied to the base of the rods and the player would swing each rod three times, as hard as they could in the desired motion. The position and angular velocity recorded by the sensor would be used to calculate the maximum swing speed at the tip of the rod, which is of known length. The value of n would be determined for the player by fitting a line through the swing speed and MOI data, as in Chapters 4 – 7. This data would then be input to the rigid body model used in this chapter to identify the optimum MOI for that player and the range of MOI the player could use to achieve 95% of maximum ball speed.

With an appropriate algorithm to analyse the data immediately after the swings, this is a test that could take less than five minutes. The player would be provided with data that could usefully inform choices about what equipment to use and whether they should change from that with which they currently play. Using an inertial sensor would be preferable over motion tracking systems because it would be very compact and so the test could be carried out in a small space. Therefore this test could be performed in sports clubs or equipment retailers, to provide information to players as they require it.

The reality for tennis players is that the best racquet to use for groundstrokes will be the most head-heavy available. The range of I_{butt} for tennis racquets currently on the market is 0.030 – 0.041 kgm^2 (Tennis Warehouse, 2015), which is below the calculated value of I_{OPT} for very high values of n. However, the trends identified in this chapter do not only apply to tennis; they will be relevant to any activity that involves swinging an implement. Therefore, the same modelling methods could be used with data for hockey, baseball, cricket etc. to identify the optimum MOI within each of these sports and assist the player in selecting the equipment that will allow them to achieve maximum performance.

8.5 Summary

This study used a rigid body impact model to simulate the impact between a tennis ball and a series of racquet sized implements with an extreme range of moment of inertia. The results show that there is an optimum MOI that produces a maximum outbound ball speed and the value of this optimum MOI changes with sensitivity to MOI (n). A player with high sensitivity to MOI yields a lower optimum MOI than a player with low sensitivity. This makes the customisation of sporting implements a real possibility and a proposed method to identify optimum MOI in sports clubs and shops is presented.

The optimum MOI values found with the model are all beyond the range currently on offer to tennis players, suggesting that head heavy racquets will be best for all players. This is backed up by data from the tennis swing speed analysis where for eight of ten players, ball speed continues to increase with increasing MOI.

Chapter 9: Conclusions

9.1 Introduction

The aim of this chapter is to summarise the main outcomes from this project. The findings are discussed in the order they are presented in the thesis and are followed by proposed developments and future applications for the work.

9.2 Summary of research

9.2.1 Literature review

After reviewing literature on moment of inertia in sports with a swinging motion, compelling evidence was found to suggest that a strong relationship exists between swing speed and moment of inertia, quantified by the rate of swing speed decay (n). It appeared to suggest that this relationship could be constant across multiple/all motions, with n quoted in a range of 0.20 – 0.29. A meta-analysis was carried out across nine sports comparing swing speed data to MOI. The global relationship across these sports was comparable with the n values reported in literature, suggesting that a common value or range for n might exist.

There was also evidence in the literature to suggest that the physical characteristics of a player, such as height and arm strength, can be linked to performance in sports such as tennis.

9.2.2 Restricted motion analysis (Chapters 4 – 5)

In order to rigorously measure n, a restricted motion analysis was carried out with a series of eight weight rods. The rods had an identical appearance, mass and length, but varied in balance point and MOI. Swing speed was recorded using a motion capture system. A self-organising map method was used to eliminate from the dataset trials with a movement pattern that deviated from the requested restricted motion. A physical profiling test was also developed, measuring work done by participants in the same motion as the swing speed analysis.

This first study had eight participants and measured a large range of values for n (0.19 to 0.79). There was no overall relationship between physical profile and swing speed. However, the swing speed results were more consistent at the higher end of the MOI range tested. This suggested that, at higher moments of inertia, n might be more consistent and work done might be related to swing speed. A large proportion of data in this first study failed to pass the exclusion criteria, meaning that the results were based on 41% of the full data set.

A second restricted motion analysis was carried out, with an additional feedback system informing participants when they deviated from the requested motion. This system lowered data exclusion from 59% to 8%. A set of eight rods, with higher MOI than the first study, were produced. A physical profiling and swing speed analysis was performed for a cohort of 25 participants and, once again, a large range in the value of n was measured (-0.19 to 0.52). A significant relationship was found between work done and swing speed ($r = 0.395$). It was decided that, to develop a model for predicting swing speed, a constant n should initially be assumed. A modified least squares method was used to calculate a constant n for the group, resulting in $n = 0.32$. For the data with an assumed constant n, work done was correlated more strongly with swing speed ($r = 0.64$).

9.2.3 Swing speed model (Chapter 6)

A model was developed that could predict swing speed for a restricted motion based upon inputs of work done and MOI. This model was used to predict the swing speed for five new participants. In most cases there was an under-prediction in swing speed; but the values of n, which range from 0.25 to 0.33, were similar to the modelled value and the range reported in the literature. Despite a mean under-prediction of 1.6 ms^{-1} (14%), data for all five participants fell within the confidence interval, defined by the strength of the work done – swing speed relationship. The model was therefore deemed to work well for predicting the swing speed of a weighted implement in a restricted motion.

9.2.4 Tennis swing speed analysis (Chapter 7)

An experiment was completed with a real sporting action to test the ecological validity of the predictive model. The forehand groundstroke in tennis was analysed because it uses the same dominant muscles as shoulder internal rotation and is mostly also performed in the transverse plane.

Ten club team standard tennis players were profiled to measure work done and then asked to perform maximum effort groundstrokes with a series of five racquets. These racquets had a uniform mass and appearance but different moments of inertia. Swing speed was found to decrease with increasing MOI, as expected, and the value of n ranged from -0.35 to 0.90.

There was a systematic under-prediction of swing speed from the model. This was partly expected due to a tennis groundstroke being a more complex movement than shoulder internal rotation, with more body segments contributing to the swing speed. When an offset was applied to the model to account for this under-prediction, eight out of ten of the predictions fell within the confidence interval and there was a mean error of 4.1 ms^{-1} (15%). However, because of the large range in the value of n, it was inappropriate to continue to assume n was a constant. Hence a model with a variable n was developed, including the impact of racquet and ball.

9.2.5 Impact modelling (Chapter 8)

A rigid body impact model was implemented, using data from the tennis swing speed analysis. The model was used to show that ball speed reaches a maximum value at an optimum MOI. Changes in work done were found to affect the magnitude of outbound ball speed but not the MOI at the peak ball speed. The latter was found to change with the value of n. As n increased, the optimum MOI decreased. The optimum MOI for tennis groundstrokes was found to exist above the range of MOI recorded for racquets available at the time of this research. This finding suggested that ball speed would continue to increase as MOI was increased. Results from the tennis swing speed analysis, where ball speed was recorded, confirmed this relationship, as ball speed increased with MOI for eight of the ten players tested.

For any sport in which an implement is swung, measuring the value of n is important for players who wish to discover the optimum MOI for their equipment. A method is proposed for a test to measure n which could be performed quickly in non-laboratory settings to assist both players and equipment manufacturers.

9.3 Conclusions

The aim of this project was to improve the understanding of the relationship between swing speed and moment of inertia, so that players can make informed decisions when customising or replacing equipment.

This was to be achieved by delivering on the following objectives:

- Carrying out a rigorous experimental analysis with a large group of participants, to quantify n and develop relationships between physical profile and MOI.
- Developing a model to predict swing speed, using MOI and measures for physical profile.
- Measuring the ecological validity of the predictive model.
- Propose a method to determine the optimum value of MOI for an individual.

The objectives were all accomplished through the work summarised above, producing the following conclusions:

- Swing speed decreases as moment of inertia is increased.

- The rate of swing speed decay (n) is not constant between athletes.
- Work done in a movement, as a measure of strength, is significantly correlated to the maximum swing speed a player will achieve for that movement.
- Increasing moment of inertia increases ball speed to a maximum point, at an optimum value for moment of inertia.
- The optimum moment of inertia is dependent upon the value of n for an individual. As n increases, the optimum moment of inertia decreases.
- The optimum moment of inertia for a tennis groundstroke is higher than the moments of inertia of racquets presently available.
- Determining an individual's rate of swing speed decay provides the opportunity to identify the optimum moment of inertia that would afford them maximum ball speed.

9.4 Limitations of the study

There are some aspects of the work presented in this thesis that were not ideal and that will have affected the results and how they are interpreted. This section will discuss them and the potential impact they may have had.

One of the disadvantages of the work has been the number of data points which relationships have been drawn between, particularly for the swing speed analysis in the tennis study of Chapter 7. In this study the values for n were calculated by fitting a line through five data points. It is quite possible in this case that a change in swing speed of up to 5 ms^{-1} for one of the racquets could drastically alter a player's value of n. Subtle shifts in the result for a few racquets could affect the outlook of the set of n values and alter the conclusion of n being a variable, although this is unlikely. A lot of data points would have had to be different for all ten players to have an n value close to the modelled value of 0.32 or some other common value.

The use of five data points was identified as a potential weakness in the planning of the study and that is why there were five trials with each racquet per player, to increase the likelihood of capturing a player's maximum performance with each racquet. Adding extra racquets to the study would have made racquets very similar to one another in MOI because the range of MOI could not be increased with the chosen mass. The time demands placed on the participants would also have increased and this was not desirable.

The cohort of ten players in the tennis study was also low. A much larger tested population would have allowed a more comprehensive analysis of how similar or different players are from one another in their sensitivity to MOI. This larger pool of data would have given much more power to the model and the findings therein and is something to be improved for future work. The intention of the study had been to have a large participant set, with sample size calculations demanding a minimum of 18 participants. Unfortunately it proved very difficult to recruit participants with students not available at the time of the work and with the time constraints of the project the study had to be concluded with only ten participants.

Another potential area of weakness was regression analysis used in Chapters 5 & 6 to create the swing speed prediction equation. This was based upon a cohort of 22 participants and was the largest amount of data collected in this thesis, but adding data from more participants would improve the significance of the relationship and provide more confidence to the value of r.

It is also possible that some participants will respond better than others to being placed in an experimental environment for a test. Even in the study in Chapter 7, where the test was made very similar to a real tennis scenario, players were surrounded by recording equipment and were provided with different visual and kinaesthetic cues to a 'normal' situation. This is a problem that is difficult to work around and requires a measurement system that is less oppressive such as inertial sensors. Alternatively, more work could be done to develop the current system such that 'normal' cues are provided to the participants to allow them to relax and respond as they would away from testing.

9.5 Future developments for the work

The investigations in this thesis have highlighted key areas that would be worthwhile research topics in the near future. These are detailed in this final section.

9.5.1 Development of a tool for measuring n

The logical next step for the research is to develop a working test setup that can be used to measure *n* in a range of environments. To be of most use to players, the test should be capable of being delivered by a non-academic individual and in a location such as an equipment shop or sports club. This would increase the number of potential players that could benefit from the test, as not everybody is able to access laboratory facilities. A method such as the one proposed at the end of Chapter 8 should be developed. The method could then be validated against laboratory measurements and tested for reliability.

9.5.2 Impact location and controllability

The only caveat to the current calculation of optimum MOI would be if the difference between optimum MOI and the MOI of the implement currently used by the player affects shot accuracy. Therefore, a suitable development for this work would be to discover the effect of MOI on shot accuracy. There has been research investigating the ideal impact location on the stringbed in tennis (Choppin, 2013). Rigid body modelling was used to determine the location on the racquet that produces maximum ball speed for simulated shots of differing angular velocity. This principle could be applied to all sports that involve swinging an implement to strike a ball. It would also be beneficial to include this work in the model for optimum MOI, so that the influence of MOI on where the 'sweet spot' lies is accounted for. The impact model could be developed to have an optional input for players who know they regularly strike a ball in the same part of their implement. For example, if a baseball player knows they always strike the ball at the tip of the bat, this could be accounted for in the calculation of their optimum MOI.

A similar area worth researching is how changing the MOI affects the size of the 'sweet spot'. High level players, who can be very accurate and consistent when striking a ball, do not require a large sweet spot; but players who do not have as reliable a technique might prefer a racquet to be weighted such that the sweet spot is enlarged. Weighting the racquet to change this will also affect some or all of the variables previously discussed. Therefore, this is an area that could be researched, so that sweet spot size can be included in the model to calculate optimum MOI.

9.5.3 Testing multiple motions

This project has focussed on two movements: internal rotation of the shoulder and a tennis forehand groundstroke. There was one participant who performed in both tests and the measured values for n were 0.48 for the restricted motion and 0.42 for the tennis shot. Conclusions cannot be drawn from a sample of one, but it is possible that n remains within a small range for each individual. It is also possible that n changes depending on the motion being performed. This could be tested by carrying out a series of experiments for a range of movements, using a similar method to that used in Chapter 5. The results would then determine how specific the optimum MOI is and whether players need to consider what their most common shot type is, when purchasing or modifying equipment. If n is found to be strongly dependent upon the motion being performed, this could still be accounted for. If a player had n measured for each of the important shots involved in their sport and their playing style was quantified, an optimisation model could then be implemented to find the overall optimum MOI based upon the frequency of each shot being used by the player.

The key areas outlined in this conclusion indicate that the research undertaken in this thesis offers several promising avenues for future development.

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Appendices

This section contains evidence of the ethical approval received from the faculty research ethics committee, for each of the studies in this thesis. The first document, shown in Appendix 1, is the ethical approval received for the work in Chapter 4. The document presented in Appendix 2 is evidence of the ethical approval received for the experimental work in Chapter 5 and Chapter 6. Appendix 3 contains the ethical approval document for the experimental work from Chapter 7.

Appendix 1: Ethical approval for Chapter 4

Faculty of Health and Wellbeing Research Ethics Committee
Sport and Exercise Research Ethics Review Group
Report Form

Principal Investigator: David Schoran

Title: Effects of moment of inertia and player strength on swing speed.

Checklist:

Application form	✓
Informed consent form	✓
Participant information sheet	✓
Risk assessment form	✓
Pre-screening form	✓
Pre-screening form (under 18)	n/a
Collaboration evidence/support	n/a
CRB Disclosure certificate	n/a

Recommendation:

Acceptable: ☐

Not acceptable, see comments: ☐

Acceptable, but see comments: ✓

Comments:

Please provide a written response to the attached comments and submit to Sue Wallace.

Signature : Signature : Date: 04.04.12
David Binney
Chair, Sport and Exercise Research Ethics Review Group

Please remember that an up-to-date project file must be maintained for the duration of the project and afterwards. The project file might be inspected at any time.

Note: *Approval applies until the anticipated date of completion unless there are changes to the procedures, in which case another application should be made.*

Comments from the Review Group have been addressed
(Supervisor should check that the above comments have been addressed by the student and sign below)

Signature of Supervisor

.... Date: ....
Name of Supervisor : DavidJames

Appendix 2: Ethical approval for Chapter 5 & 6

Sheffield Hallam University

Faculty of Health and Wellbeing
Collegiate Hall
Collegiate Crescent Campus
Sheffield
S10 2BP

20 May 2013

fao David Schorah

Dear David

Title of research: Effects of moment of inertia and player strength on swing speed

Thank you for informing me of the amendments to your study.

I am pleased to inform you that these are acceptable.

Yours sincerely

David Binney
Chair, Sport & Exercise Research Ethics Review Group

cc David James

Appendix 3: Ethical approval for Chapter 7

Faculty of Health and Wellbeing Research Ethics Committee
Sport and Exercise Research Ethics Review Group
Report Form

Principal Investigator: David Schorah

Title: Effects of moment of inertia and player strength on swing speed.

Checklist:

Application form	✓
Informed consent form	
Participant information sheet	✓
Risk assessment form	✓
Pre-screening form	
Pre-screening form (under 18)	
CRB Disclosure certificate	
Collaboration letter	

Recommendation:

Acceptable: ✓

Revise (see comments):

Resubmit (see comments):

Comments:

Thank you for addressing the comments you received.

Your application is now Acceptable and you may commence your study.

Signature: .... Date: 02/07/2014
Donna Woodhouse
Chair, Sport and Exercise Research Ethics Review Group

Note: Approval applies until the anticipated date of completion unless there are changes to the procedures, in which case another application should be made.

Name of Supervisor: Simon Choppin