A Sheffield Hallam University thesis

THE RESPONSE OF BOX-TYPE STRUCTURES TO VIBRATION

by

Raymond Boon Whee Heng
B.Eng.(Hons.)*

A dissertation submitted to the Council of National
Academic Awards for the award of the degree of Doctor
of Philosophy

Sheffield Polytechnic
Department of Mechanical and Production Engineering

* at present
University of Sheffield
Department of Mechanical Engineering

July 1976

ProQuest Number: 10697086

ProQuest 10697086

THE RESPONSE OF BOX-TYPE STRUCTURES TO VIBRATION

by

Raymond Boon Whee Heng
B.Eng.(Hons.)*

A dissertation submitted to the Council of National
Academic Awards for the award of the degree of Doctor .
of Philosophy

Sheffield Polytechnic
Department of Mechanical and Production Engineering

* at present
University of Sheffield
Department of Mechanical Engineering

July 1976

Acknowledgement

Abstract

The work described in this thesis deals with the vibration
study of an open ended folded plate box type structure leading
up to the prediction of its response to random excitation.
Theoretical and experimental results for the box are presented
as one of the main aims of the work is to predict theoreti-
cally the response of the structure to random excitation
and compare these results with experimentally obtained values
using various methods available.

The determination of the response of a structure to random
excitation depends on the prediction of the response spectral
density. To determine the response spectral density of any
point on a structure a knowledge of its natural frequencies
and mode shapes and modal damping factors is required. In
this work the finite element method of analysis is used to
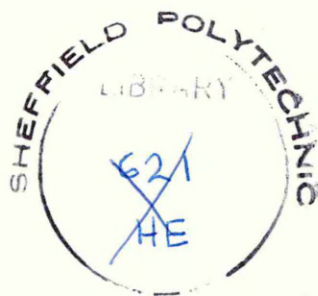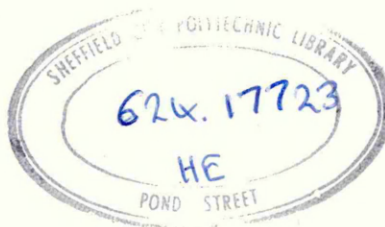determine the natural frequencies of vibration and the
corresponding mode shapes of the box structure and computer
programs were developed to perform this analysis. During
the project, beam and plate structures have also been
investigated to assess the accuracy of the technique and some
results for these are included in chapter 7. The natural
frequencies and mode shapes obtained are compared with
experimental results as well as with those obtained using
other theoretical analyses. For the beam the exact method
was used and for the plate, the energy method using
Warburton's formulae. Computer programs were also developed
to calculate the structural receptance from the natural
frequencies and mode shapes calculated using the finite
element technique and damping factors obtained experimentally.
From this the response spectral density of the structure to
a known excitation spectrum was obtained.

Experimental work include a) sinusoidal excitation tests to
determine natural frequencies, mode shapes and damping factors
and b) random excitation using pseudo-random binary sequence
signals to determine response to random excitation using
narrow band frequency analysis and correlation techniques.

the development of a non-contacting combined exciter pickup probe, are described and discussed. The response to random excitation is obtained experimentally using a pseudo-random binary sequence signal generator and a time domain analyser, giving the cross-correlation function from which the cross spectral density is calculated in a Fourier transform. A Fast Fourier Transform computer program was developed during the work to perform this. The response spectral density is then obtained from a knowledge of the excitation spectral density.

Finally the values of the response spectral density obtained are compared with those obtained using the results of the finite element analysis and using the results of the sine sweep test and narrow band frequency analysis. The technique used in this work has proved satisfactory and the experimental apparatus and computer programs developed, suitable for the investigation.

The work described in this thesis provides the necessary basic requirements for future work in the establishment of suitable experimental apparatus setup and provision of essential computer software. Suggestions as to possible extension of this work are made in the concluding chapter.

CONTENTS

Abstract

Contents   Page

Contents   Page

Contents		Page

List of Figures

List of Tables

List of Tables

List of Plates

Chapter 1.   Introduction

The trend in recent engineering development is towards the
use of light folded-plate boxtype structures as the basic
unit of construction in many fields. In buildings and bri-
dges, box girders figure predominantly and in the transport
industry, motor cars, buses and trucks have always been
basically light folded plate box type structures. This has
stimulated interest in the determination of the response to
vibration stimuli of these structures, hereafter referred to
in this thesis simply as box-structures.. The resulting
reduction in the built-in safety factors as well as in the
damping because of the use of these lighter structural mem-
bers can lead to the build up of serious vibration response
levels.

Ignorance of the vibration characteristics of such struc-
tures is known to account for premature wear and fatigue
failings [1]. The more spectacular catastrophes include the
breaking up of the Liberty ships and the bridge disaster at
Tacooma, in the U.S.A. where, in the latter case, random wind
loading had excited one of the natural vibration modes of the
bridge.

In the field of transportation, broad band vibrations set up
in the vehicle have been shown to affect not only the life
expectancy of the vehicle but also to have undesirable
effects on the commuter. Once again the excitation here is
known to be of a random nature. It is therefore important to
study the vibration response of box structures and in
particular the response of box structures to random excitat-
ion.

This work is an investigation into the vibration characteris-
ics of box structures and ultimately to predict the response
of box structures to random excitation. Robson [2] and
Crandall [3] provide the necessary background knowledge on
random vibration theory and they show that the prediction
of the response of a structure at any point d to a single

random input at p depends on the determination of the receptance α_{dp} (Bishop and Johnson [4]) of the structure at the point under consideration. Robson has shown that in this case the output spectral density Sd(f) of the point under consideration is related to the input spectral density Sp(f) of the exciting force by the expression:

$$Sd(f) = |\alpha_{dp}(if)|^2 \, Sp(f) \qquad (1.1)$$

where $\alpha'_{dp}(f)$ is the receptance of the system from:

$$\alpha_{dp}(if) = \sum_r \mu_r (X_r - iY_r) \qquad (1.2)$$

where $\mu_r = \dfrac{w_r(x_q) w_r(x_p)}{M_r}$,

$$M_r = \int_o^l w_r^2 (x) m dx \; ,$$

$$X_r = \frac{f_r^2 - f^2}{4\pi^2[(f_r^2 - f^2)^2 + \eta_r^2 f_r^4]} \; ,$$

$$Y_r = \frac{f_r^2}{4\pi^2[(f_r^2 - f^2)^2 + \eta_r^2 f_r^4]}$$

$w_r(x_p)$ and $w_r(x_q)$ are deflections of the structure at input and output points p and q respectively when the structure is vibrating in the rth normal mode, f_r is the rth natural frequency, f is the forcing frequency and η_r is the damping loss coefficient for the rth mode.

It is therefore possible to calculate the receptance of any structure given the knowledge of its natural frequencies and mode shapes as well as damping factors.

Any attempt at response work inevitably requires a knowledge of the damping characteristics of the structure. As the mechanism of damping in structures is not yet completely known this work follows a commonly accepted theory for light

structures, that of hysteretic damping (see Crandal [5]).
Experimentally derived values of the loss factors η have been
used as far as possible for the folded plate box configurat-
ion but where this was not possible, extrapolations were used
based on results obtained on plate damping measurements or
unimodal damping assumptions were made.

In using equation (1.2) the natural frequencies of free
vibration and its corresponding mode shapes of the box
structure must also be known. These are obtained by solution
of the equation of motion of the structure. Vibration
analysis of structures can be divided into 'exact' methods
and approximate methods.

The exact method solves the exact equation of motion usually
using an iterative method. This however is only suitable for
beams and plates with simple boundary conditions and becomes
far too complicated for complex structures. It has not
however prevented a recent attempt by Abrahamson [6] in using
the exact method in an analysis of the natural frequencies
and normal modes of a four plate box structure. This exact
method is well documented in standard dynamics textbooks and
will not be gone into here.

Of the approximate methods available the most widely used is
the Rayleigh also known as the Energy method used mainly for
determination of the lower frequencies of vibration. Here
a shape, for example the static deflection curve, is assumed
for the true deflected curve. Using this, the maximum
potential and kinetic energies are calculated and equated.
If the exact shape of the deflected structure happens to be
chosen the calculated frequency corresponding to that shape
will also be exactly correct but if not a close upper bound
approximation will be obtained [4]. This method is also
limited to the analysis of the simpler structures.

For plates the exact method of solution is limited to those
simply supported around their perimeter. For fully fixed
plates Ritz [7] provided probably the earliest analysis using

the Rayleigh method with a series of the various characteristic beam functions. This work was extended by others including Young [8] in 1950. Both the Rayleigh and the Rayleigh-Ritz methods are well documented in standard vibration textbooks e.g. Timoshenko [9] and Bishop & Johnson [4]. Probably one of the best known works in plate vibrations was done by Warburton [10] in which he presented a paper giving approximate formulae for all the twenty one different boundary conditions possible with plates using the Rayleigh method of solution. Recent work on the same subject was done by Leissa [11] who also investivated the effect of changing the values of Poission's ratio and investigated the accuracy of Warburtons formulae.

A theoretical solution to the vibration of box-type structures using a sine series was presented by Dickinson and Warburton [12]. The analysis is however limited to the case in which deflections at the corners of the structure are assumed to be zero. The use of the Rayleigh and Raleigh-Ritz methods is in general not suitable for box structures because of difficulty in satisfying continuity of slope and bending moments at the common edges. The transfer matrix method (see Uhrig [13]) is also not suitable for the box structure because of the geometric difficulties encountered. The Bolotin edge effect method (Dickinson and Warburton [14]) although able to satisfy these continuity conditions does not solve for all the modes of vibration of box structures particularly at the higher modes when modal patterns are not easily represented by lines parallel to the edges.

Of the remaining approximate methods available the advent of the high speed large capacity computer has seen the emergence of the finite difference and finite element methods of analysis. The former (Heng [15]) was possibly until recently the more popular because of its smaller computer storage requirements but convergence especially in higher order equations was slow (Chang [16]). The latter is possibly more appealing to the engineer and has become one of the most important and powerful of the techniques developed recently.

It also appears to be the most suitable method available for the analysis of the box structure and is therefore the method chosen for this work.

One of the earliest references using this method was reported in a paper by Turner et.al. [17] in 1956. Melosh [18] subsequently applied it to the analysis of thin plates in bending. This was followed by Zienkiewicz and Cheung [19] in which they noticed possible application of the method to include vibration and thermal problems. Dawe [20] presents a solution to plate vibration problems and including non-dimensionalised stiffness and mass matrices using rectangular elements.

The rapid increase in the use of matrix methods in structural mechanics led to one of the first major conferences on the subject [21] being held in 1965 at which the finite element method featured predominantly. Since then several textbooks on the method have been published (Przemeiniecki [22] and Zienkiewicz [23]).

Rockey and Evans [24] applied the method to the static analysis of box structures. In dynamic analysis, Handa [25] deals with in-plane vibrations of box structures and Ali, Hedge & Mills [26] approximates the vibrations of an idealised motor car chassis using beam elements only. This project will therefore extend the work to consider the finite element analysis of both in plane and transverse vibration of box structures and to use the results for the determination of the response power spectral density to random excitation. The full potential of the finite element method of analysis is still being constantly increased with new literature on the work being published at the moment in many research establishments.

Use of the finite element method necessitates the availability of some finite element computer package. However, in general, such computer program packages are not always available and it is up to the individual establishments to develop their own.

It is therefore one of the aims of the work undertaken in this project to produce such a computer program package to solve for the natural frequencies and mode shapes of the box structure. These results are then used in the calculation of the receptance of the box structure from equation (1.2). The receptance is used in the prediction of the response power spectral density to any known input from equation (1.1) and its calculation incorporated in the finite element package developed.

Little work, analytical or experimental, appears in the literature on the vibration study of box structures. As a result an extensive experimental as well as analytical program is pursued to obtain.

(a) natural frequencies and mode shapes, with which the accuracy of the finite element prediction of the natural frequencies and mode shapes of the box structure can be assessed, and

(b) the experimental response power spectral density, with which the predicted response power spectral density can be compared.

Although it is not expected that experimentally obtained and analytically predicted results correspond exactly, it is hoped that some degree of collaboration will be achieved.

The experimental work entailed the design and development of suitable experimental apparatus from which the necessary response measurements can be obtained. Because of the lightness of the box structure a non-contacting exciter-pickup had to be developed. The natural frequencies and mode shapes of the box structure were obtained. The response power spectral density was however not obtained by a narrow band frequency analysis of the response itself because of the inaccuracy involved. Instead an alternative method using the cross-correlation of the excitation and response signals is used. Davies [27] and Jones [28] show that if white noise is applied to a linear system, the cross correlation of the input signal to the system with the resulting response of the system gives the systems impulse response function. The pseudo random binary sequence signal is used for determining this impulse response function with considerably less difficulty in

obtaining the delayed signal for correlation and in the operations involving multiplication and integration. Moreover the perfect repeatibility otherwise unobtainable with pure random signals is here possible making it suitable for work of this kind.

A Fourier Transform is necessary to evaluate the frequency response function of the structure from its impulse response. The calculation is found to require very long computational times using the usual method of evaluating the terms. A new technique using the Fast Fourier Transform algorithm (Cooley and Tukey [29], Bingham et.al. [30], Cochran et.al. [31]) is found to reduce this time considerably and forms an important part of the correlation technique used in this work. The theory behind this method of calculating the Fourier transform of a series will be looked into in more detail in the appropriate chapter.

Both the experimental as well as the analytical programme for the box structure has been successfully completed. The necessary apparatus has been developed and the natural frequencies and mode shape of the box structure have been obtained. The response power spectral density of the box structure subjected to random excitation has been obtained experimentally. A finite element computer program package has been developed giving the natural frequencies and mode shapes of the box structure. A further computer program has also been developed to predict the response power spectral density of the box structure subjected to random excitation. Agreement between the experimental and predicted results is found to be satisfactory.

Chapter 2   <u>The finite element method</u>

<u>Summary</u>

The Finite Element method is used to predict theoretically the
mode shapes and frequencies of free vibration of a folded
plate box type structure using rectangular isotropic plate
elements. Both the stiffness and the mass matrices are
derived for the element used and the assembly into the
complete structure explained. The use of various standard
library routines for the solution of the eigenvalue problem is
discussed. A full listing of the complete computer program
developed is included in Appendix 3.

2.1   Introduction

The finite element method is one of the most suitable methods for the analysis of complex structures. In the analysis, whether static or dynamic, the original structure is replaced by an assemblage of small but finite elements which inter-connect with one another only at a finite number of points known as node points. These are usually found at the corners of the element but are also commonly chosen to be along the sides of the element. At each node a number of degrees of freedom are represented usually the deflections and slopes or their derivatives. The theory then is to assume that the dis-placements in any part of the element will be defined in terms of these nodal degrees of freedom via a displacement poly-nomial of the form

$$w = \{P\}[C] \tag{2.1}$$

where $\{P\}$ is a matrix of the coordinates x,y and z,

$[C]$ is a vector matrix of constants,

and the polynomial $w(x,y,z)$ is chosen to satisfy the following conditions:

(a) The number of independent terms must equal the number of degrees of freedom in the element

(b) For convergence to the correct solution it must be complete up to order n where n is the order of the highest derivative appearing in the strain energy integral appropriate to the type of element under consideration.

(c) conforming or compatible elements further satisfy the condition that the (n-1)th derivatives are continuous across the element boundaries.

From this displacement polynomial an elemental 'stiffness' matrix [k] is obtained via strain energy considerations relating the elemental nodal forces and displacements in the form

$$\{F\}^e = [k] \{d\}^e \qquad (2.2)$$

In a static analysis the nodal forces $\{F\}^e$ are due to the applied forces and in a dynamic analysis they represent inertia forces acting on the element. The characteristics of the complete but considerably simplified structure is then represented on the computer by the equation

$$\{F\} = [K] \{d\} \qquad (2.3)$$

where [K] is the stiffness matrix for the complete structure, made up of all the elemental stiffness matrices [k] . This then relates the applied forces $\{F\}$ and the nodal displacements $\{d\}$ of the complete structure and at the same time satisfies the relevant boundary conditions applied.

2.2   The element stiffness matrices

In folded plate structures the plates are subjected to both bending and stretching effects. This is conveniently considered separately as the plate element purely in bending and then in stretching only. The former has three degrees of freedom, w, θ_x, θ_y at each node and the latter, u and v only. However, depending on the orientation of the element in the complete structure, a θ_z component may present itself so that an allowance must be made for this in retaining a total of six degrees of freedom per node, displacements $u,v,w,$ and rotations, $\theta_x, \theta_y, \theta_z$.

2.2.1 The bending stiffness matrix

Figure 2.1 represents one of the four node rectangular elements which is a component idealising a complete side of the box. At each node during bending there are three degrees of freedom, a displacement w, and rotations θ_x, θ_y to which correspond the three bending forces W, M_x, M_y. Thus for the 4 node element with 12 degrees of freedom, a 12 x 12 matrix is required to express nodal forces in terms of the nodal displacements. Other types of elements may also be used e.g. the 3 node triangular element or even the 8 node rectangular element with midside nodes [23]. However, a pair of the former will be necessary to represent each of the 4 node rectangles. The eight node rectangle is used primarily to represent curved geometries and is therefore unnecessary, for the box column with its straight sides, and also has a higher number of degrees of freedom to contend with.

For the 4 node rectangular element chosen, it is assumed that under load the deflected form of the element can be expressed as

$$w = A_1 + A_2x + A_3y + A_4x^2 + A_5xy + A_6y^2 + A_7x^3 + A_8x^2y$$
$$+ A_9xy^2 + A_{10}y^3 + A_{11}x^3y + A_{12}xy^3 \qquad (2.4a)$$

where w is the deflection at the point (x,y)
or in matrix form

$$w = [m]\{A\} \qquad (2.4b)$$

where $[m] = 1,x,y,x^2,xy,y^2,x^3,x^2y,xy^2,y^3,x^3y,xy^3$

and $\{A\}$ is a vector of the constants A_1, A_2, ...A_{12}. This displacement polynomial fully satisfies the necessary requirements discussed in 2.1. The constants of the expression can be evaluated by satisfying the displacement conditions at each node point, i.e. $w = w_i$, $\partial w/\partial y = \phi_i$, $\partial w/\partial x = -\theta_i$ at node point i (i = 1,2,3,4).
The elemental stiffness matrix is then obtained via a strain energy analysis.
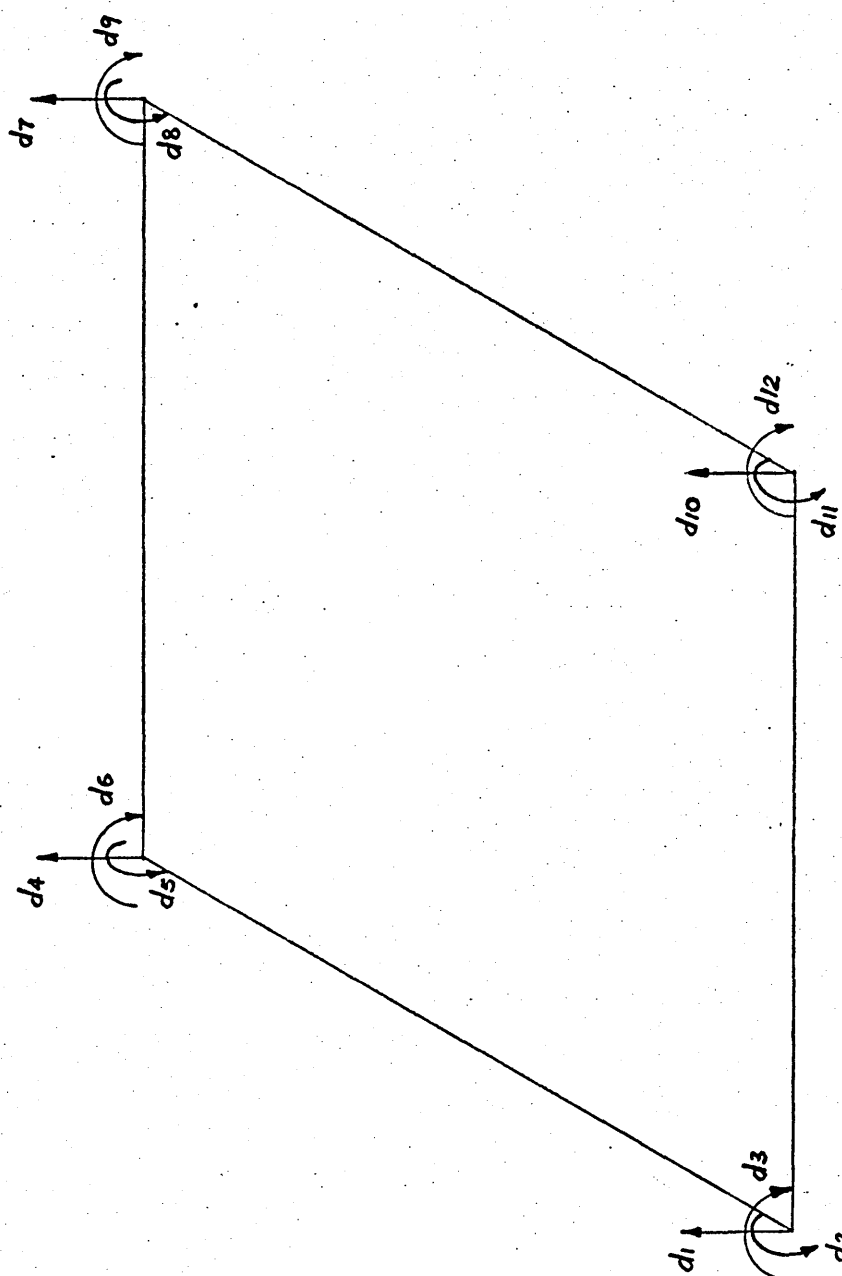
$$\theta_x = \frac{\partial w}{\partial y}$$

$$\theta_y = -\frac{\partial w}{\partial x}$$

Fig 2·1 Element configuration (bending)

Now the bending strain energy U of an isotropic plate of uniform flexural rigidity, D, is given by $\frac{1}{2}\int_V \epsilon^T \sigma \, dV$ where $\sigma = \cancel{} \epsilon$

$$U = \frac{D}{2}\int_{y=0}^{b}\int_{x=0}^{a}\left[(\frac{\partial^2 w}{\partial x^2})^2 + (\frac{\partial^2 w}{\partial y^2})^2 + 2(\frac{\partial^2 w}{\partial x^2})\nu(\frac{\partial^2 w}{\partial y^2}) + 2(1-\nu)(\frac{\partial^2 w}{\partial x \partial y})^2\right]dx\,dy$$

$$= \frac{D}{2}\int_{y=0}^{b}\int_{x=0}^{a} \{C\}^T \left[\cancel{}\right] \{C\} \, dx\,dy \tag{2.5}$$

where $\{C\} = \left\{ \dfrac{\partial^2 w}{\partial x^2}, \dfrac{\partial^2 w}{\partial y^2}, \dfrac{\partial^2 w}{\partial x \partial y} \right\}$

and $[\cancel{}] = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 2(1-\nu) \end{bmatrix}$ \hspace{2em} (2.6)

The curvatures can easily be obtained by differentiating equation (2.4a):

$$\{C\} = \begin{Bmatrix} \dfrac{d^2 w}{dx^2} \\ \dfrac{d^2 w}{dy^2} \\ \dfrac{d^2 w}{dx\,dy} \end{Bmatrix} = \begin{array}{l} 2A_4 + 6A_7 x + 2A_8 y + 6A_{11}xy \\ 2A_6 + 2A_9 x + 6A_{10}y + 6A_{12}xy \\ A_5 + 2A_8 x + 2A_9 y + 3A_{11}x^2 + 3A_{12}y^2 \end{array}$$

i.e. $\{C\} = [E]\{A\}$ \hspace{4em} (2.7)

where $[E] = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 & 0 & 6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 6xy \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2x & 2y & 0 & 3x^2 & 3y^2 \end{bmatrix}$ \hspace{1em} (2.8)

In order to find the matrix of constants $\{A\}$ we return to figure 2.1 considering each node of the element in turn and using equation (2.4a) to determine the deflections at the nodes of the element to obtain

$$d_1 = w_1 = A_1$$

$$d_2 = \phi_1 = A_3$$

$$d_3 = \Theta_1 = -A_2$$

$$d_4 = w_2 = A_1 + A_3 + A_6 + A_{10}$$

$$d_5 = \phi_2 = A_3 + 2A_6 + 3A_{10}$$

$$d_6 = \Theta_2 = -\left[A_2 + A_5 + A_9 + A_{12}\right]$$

$$d_7 = w_3 = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9$$
$$+ A_{10} + A_{11} + A_{12}$$

$$d_8 = \phi_3 = A_3 + A_5 + 2A_6 + A_8 + 2A_9 + 3A_{10} + A_{11} + 3A_{12}$$

$$d_9 = \Theta_3 = -\left[A_2 + 2A_4 + A_5 + 3A_7 + 2A_8 + A_9 + 3A_{11} + A_{12}\right]$$

$$d_{10} = w_4 = A_1 + A_2 + A_4 + A_7$$

$$d_{11} = \phi_4 = A_3 + A_5 + A_8 + A_{11}$$

$$d_{12} = \Theta_4 = -\left[A_2 + 2A_4 + 3A_7\right] \qquad (2.9a)$$

where the subscripts refer to the nodal members.
In matrix notation

$$\{d\} = [B]\{A\} \qquad (2.9b)$$

so that $\{A\} = [B^{-1}]\{d\}$ $\qquad (2.10)$

Substitution of equations (2.7) and (2.10) back into (2.5) and noting that only [E] is a function of x and y gives the strain energy

$$U = \frac{D}{2}\{d\} \left\{ [B^{-1}]^T \left(\int_{y=0}^{b} \int_{x=0}^{a} [E]^T [\chi][E] \ dx \ dy \right) [B^{-1}] \right\} \{d\}$$

or $\quad U = \frac{1}{2}\{d\}^T [K]\{d\}$ $\qquad (2.11)$

Applying the theorem of virtual work $\dfrac{\partial U}{\partial d_i} = F_i$ gives

$$\{F\} = [K]\{d\}$$

where $\{F\}$ is the column matrix of nodal forces

i.e. $\{F\} = \{W_1, M\phi, M\Theta_1, W_2, \ldots\ldots\ldots M\Theta_4\}$

$[K]$ is thus the required stiffness matrix and from (2.11)

$$[K] = [B^{-1}]^T (\int_{y=0}^{b}\int_{x=0}^{a} [E]^T [\%][E] \, dx \, dy)[B^{-1}] \qquad (2.12)$$

where the component matrices $[\%]$, $[E]$ and $[B^{-1}]$ are given in (2.6), (2.8) and the inversion of (2.9a) respectively.
In order to retain the element dimension parameters a and b, the matrix manipulation are carried out by hand giving the matrix $[K]$ (Appendix 1a).

2.2.2   The Membrane stiffness matrix

Figure 2.2   represents the configuration and notation used for the 4 node rectangular element in stretching. The displacements in the x and the y directions, u and v, are independant of each other and so can be considered separately and the analysis is identical in both cases. The displacement polynomial chosen for the displacements in the x direction is

$$u = A_1 x + A_2 xy + A_3 y + A_4$$

and that in the y direction is

$$v = B_1 x + B_2 xy + B_3 y + B_4$$

The assumption for a plane stress distribution is made where $\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0$; this being valid for thin plates.

By following the same steps as illustrated previously for bending, equations (2.5) to (2.12), the stiffness matrix for the element in stretching is obtained (Appendix 1a).

We now have the stiffness matrices for the separate bending and stretching modes of the element which can be used for the solution of problems involving plate structures bending and stretching respectively under static loading.
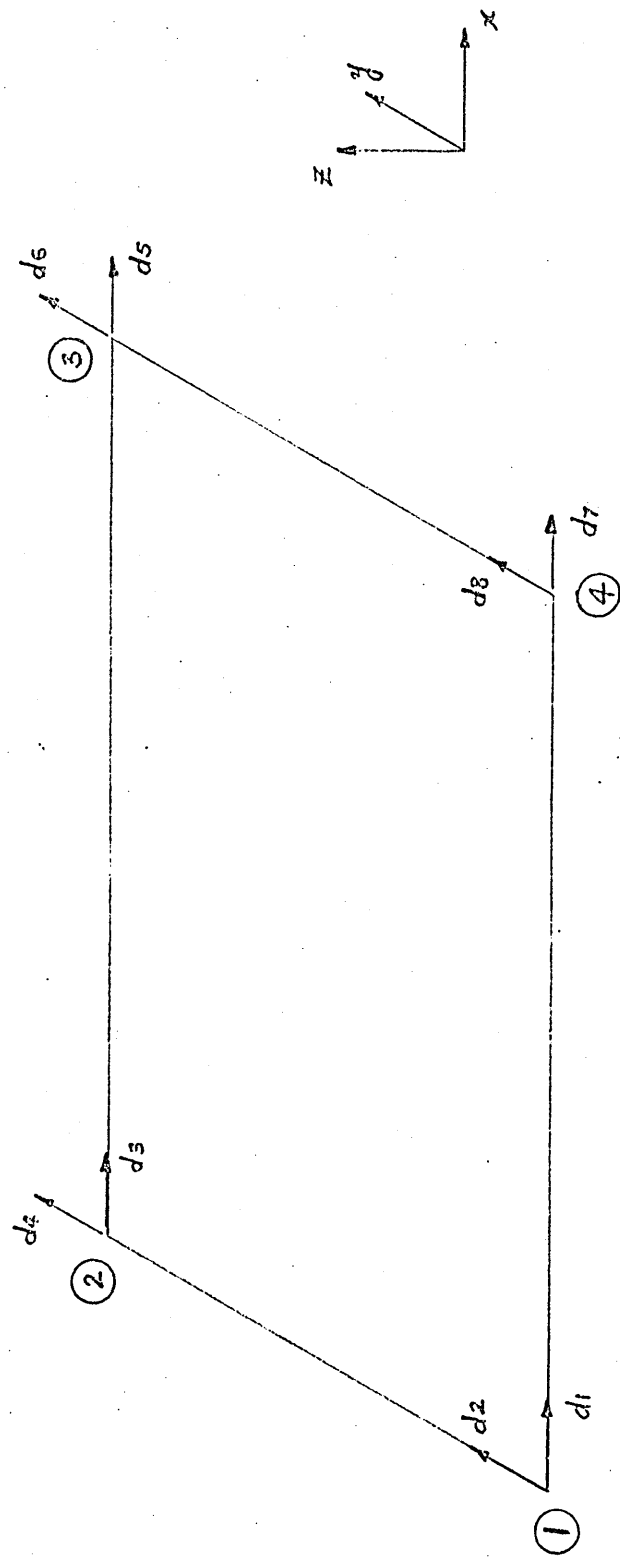
Fig 2.2   Element configuration (Stretch)

2.3   The element mass matrices

In dynamic analysis the nodal forces are induced by inertia loading of the displaced structure. The inertia loading distributed over an element during vibration is here replaced by a system of equivalent nodal forces by means of the formation of a 'mass' matrix. The criterion adopted for such replacement is that the work done by the equivalent nodal inertia forces U_e is equal to that done by the actual inertia forces of the element U_a. The former is given by the equivalent nodal forces $\{F_m\}$ in moving through virtual nodal displacements $\{d_v\}$.

i.e. $U_e = \{F_m\}\{d_v\}$ (2.13)

The latter is given by the actual distributed inertia loading, $[f]$, in moving through a virtual deflection $\{w_v\}$

i.e. $U_a = \{f\}\{w_v\}$ (2.14)

where from (2.4) and (2.10) $\{w_v\} = [m][B^{-1}]\{d_v\}$ (2.15)

The inertia force per unit area when the plate is vibrating sinusoidally with circular frequency p is

$$\{f\} = \rho p^2 w = \rho p^2 [m][B^{-1}]\{d\}$$ (2.16)

where ρ is the density per unit area of the element. Thus, equating the work done in both cases, from equations (2.13) and (2.14)

$$\{d_v\}^T \{F_m\} = \int_{y=0}^{b} \int_{x=0}^{a} \{w_v\}\{f\} \, dx \, dy$$

which from equations (2.15) and (2.16)

$$= \rho p^2 \{dv\}^T [B^{-1}]^T (\int_{y=0}^{b}\int_{x=0}^{a} [m]^T[m] \, dx \, dy) [B^{-1}]\{d\}$$ (2.17)

If each virtual nodal displacement is given the value unity in turn while the remaining displacements are held zero, equation (2.17) becomes

$$\{F_m\} = \rho p^2 [B^{-1}]^T (\int_{y=0}^{b}\int_{x=0}^{a} [m]^T[m] \, dx \, dy) [B^{-1}]\{d\}$$

$$= \lambda [M]\{d\}$$ (2.18)

where $[M] = [B^{-1}]^T (\int_{y=0}^{b} \int_{x=0}^{a} [m]^T [m] \, dx \, dy) [B^{-1}]$ (2.19)

is the required mass matrix of the element as $\{F_m\}$ is the
column matrix of nodal forces representing the inertia loading
on the element. Once again this is derived separately for the
element in bending and in stretching.

2.3.1   Bending mass matrix

From the previous section we have that the mass matrix of the
element is given by

$$[M] = [B^{-1}]^T \iint [m]^T [m] \, dx \, dy \, [B^{-1}]$$ (2.20)

For the element in bending $[m]$ is given by equation (2.4) and
$[B^{-1}]$ by equation (2.9a) so that $[M]$ is easily found
(Appendix 1b).

2.3.2   Stretch mass matrix

Again the element mass mass matrix considering the stretching
case is given by equation (2.20) above. In this case the mass
matrix can be divided into two separate submatrices, one for
each direction as they are mutually independant of each other
(Appendix 1b).

2.4   The combined element stretching and bending matrices

We now have the four bending and stretching rectangular plate
element stiffness and inertia matrices, $[K_B]$, $[K_S]$, $[M_B]$ and
$[M_S]$ of the element. These are combined to form the combined
bending and stretching stiffness and mass matrices $[K]$ and $[M]$
of the element. These are full 24 x 24 matrices with the
complete set of six rotational and linear displacements u,v,w,
$\Theta_x, \Theta_y, \Theta_z$ at each of the four nodes. The matrices available
so far only form a 20 x 20 matrix as the plane rotation Θ_z
had not been taken into account in the derivation of the mass
and stiffness matrices but must be included for rotation into
the three dimensional structure. The method used to overcome
this is simply to include rows and columns of zeros for the
appropriate degree of freedom Θ_z. This method makes the

idealised structure more flexible than the actual but has been successfully used (Rockey and Evans [24], and Clough and Johnson [32]).

The notation (fig. 2.3) used previously was chosen quite arbitrarily and was sufficiently suitable for the purposes of determining the stiffness and inertia matrices of the element. However, it was found that for ease in analysing large and complex structures, a more suitable system of numbering for the element had to be followed so that programming for the computer is simpler. The following system (fig. 2.4) was chosen. This gives a smooth flow in the numbering system, considering first the x axis and then the y axis. In the program this is accomplished by first interchanging rows and columns 2 and 4 in the stiffness and inertia matrices and then doing the same for rows and columns 3 and 4. This procedure is carried out in subroutine REORDER (Appendix 3).

The complete nodal and degree of freedom notation for an element lying in the xy plane is given in figure 2.5 where, considering node 1,

degree of freedom:

$1 = u = $ displacement in x direction
$2 = v = $ displacement in y direction
$3 = w = $ displacement in z direction
$4 = \Theta_x = $ rotation along vector in x direction
$5 = \Theta_y = $ rotation along vector in y direction
$6 = \Theta_z = $ rotation along vector in z direction

2.5 Assembly into the complete structure

The basic element obtained by the previous analysis is a four node plate element which has combined bending and stretching capabilities.

If we inspect the stiffness matrices obtained we see that a column of either matrix is in fact a list of the forces acting at each node when the displacement corresponding to that column is given a unit displacement and all other degrees of

Fig 2.4   Element node renumbered

Fig 2.3   Element node numbering sequence
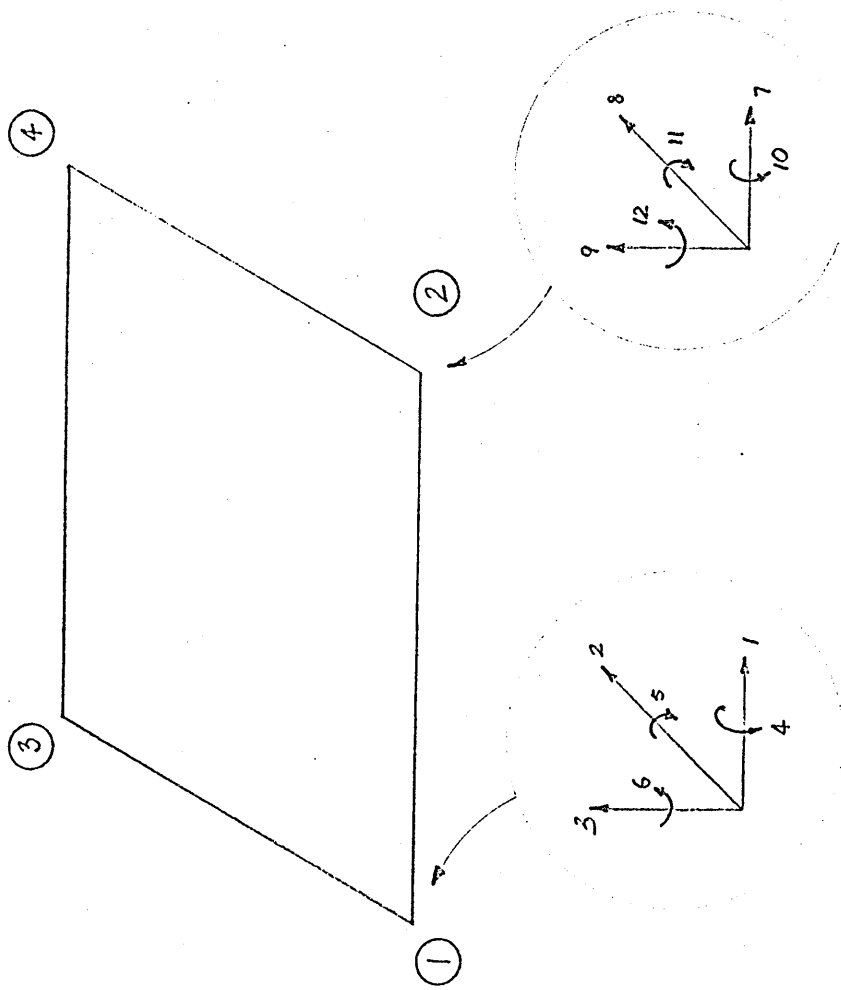
Fig 2.5 Degree of freedom notation for the element.

freedom given the value zero. Thus if two or more elements
have a common node point then the total force acting at that
node point is obtained by addition of forces. The assembly
into the complete structure is then accomplished by the
merging of all the element matrices into that of the complete
structure by a simple point by point addition. The degree of
freedom notation as illustrated in figure 2.5 has been chosen
to facilitate easy assembly into the complete structure.

This merging of the basic element matrices into that of the
complete structure also requires the following

        a)   rotation of the element into the correct plane before
             assembly.

        b)   a systematic assembly of the various elements into
             the complete structure.

        c)   a systematic numbering sequence of the nodes of the
             complete structure to facilitate easy refinement of
             mesh size.

        d)   incorporation of the boundary conditions of the
             complete structure.

2.6   The rotational matrix

So far the stiffness and mass matrices have been found for a
plate element in its local coordinate system (i.e. the stiff-
ness and mass matrices have been calculated for the element
lying in the xy plane). As an element can also lie in the xz
or yz planes or in any inclined plane, its inclination to the
global axis of the whole structure has to be taken into
consideration before it can be assembled into its proper
place in the actual structure (see for example Gere and
Weaver [33]).

To illustrate the procedure, consider a rod of length L lying
on the x axis in the local coordinate system as shown in fig.
2.6. Any point of the rod has components of deflection u,v,
and w relative to the local axes x,y and z. It also has
components of deflection u', v' and w' relative to the global
axes x', y' and z'. These two sets of deflections are related
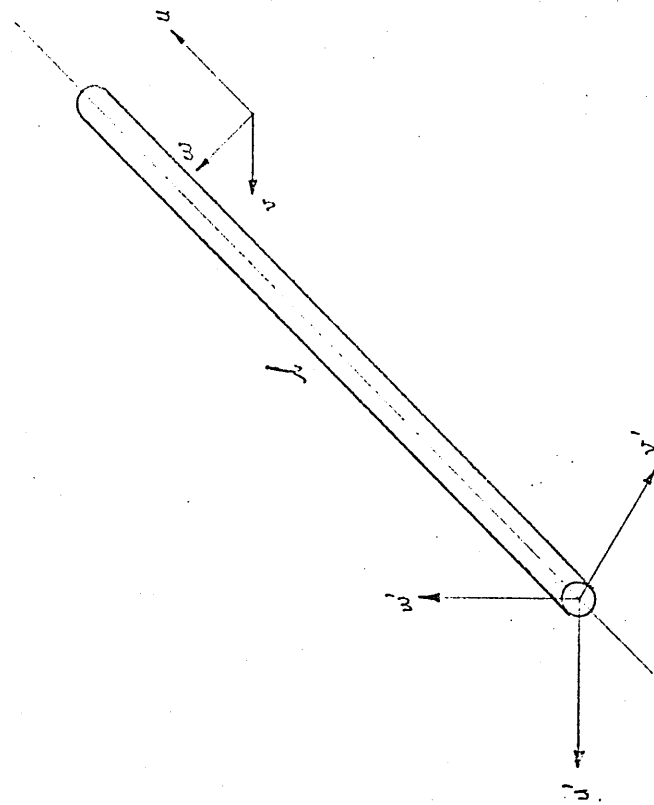
Fig 2.6   Co-ordinate transformation

by the expression

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \lambda \begin{Bmatrix} u' \\ v' \\ w' \end{Bmatrix} \qquad (2.21)$$

From fig. 2.6,

$l(u) = u'\cos u'u + v'\cos v'u + w'\cos w'u$

where $u'u$ etc., refer to the angle turned through from the u' axis to the u axis.

Similarly,

$l(v) = u'\cos u'v + v'\cos v'v + w'\cos w'v$

$l(w) = u'\cos u'w + v'\cos v'w + w'\cos w'w \qquad (2.22)$

For the purposes of this investigation the directional cosines invloved are either 1, 0, or -1 since the sides of the box structure investigated are all at right angles to one another (fig. 2.7).

i) Consider the case of a plate lying in the x'y' plane fig. 2.8). The matrix is simply:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii) For a plate lying in the y'z' plane, (fig. 2.9) it is:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and iii) For a plate in the z'x' plane (fig. 2.10):

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Using these rotation transformation matrices, the stiffness and mass matrices of the plate element in its local coordinate system can be transformed into that of the global coordinate system using the equations

$$\begin{bmatrix} K_G \end{bmatrix} = \begin{bmatrix} R \end{bmatrix}^T \begin{bmatrix} K_L \end{bmatrix} \begin{bmatrix} R \end{bmatrix} \qquad (2.23)$$
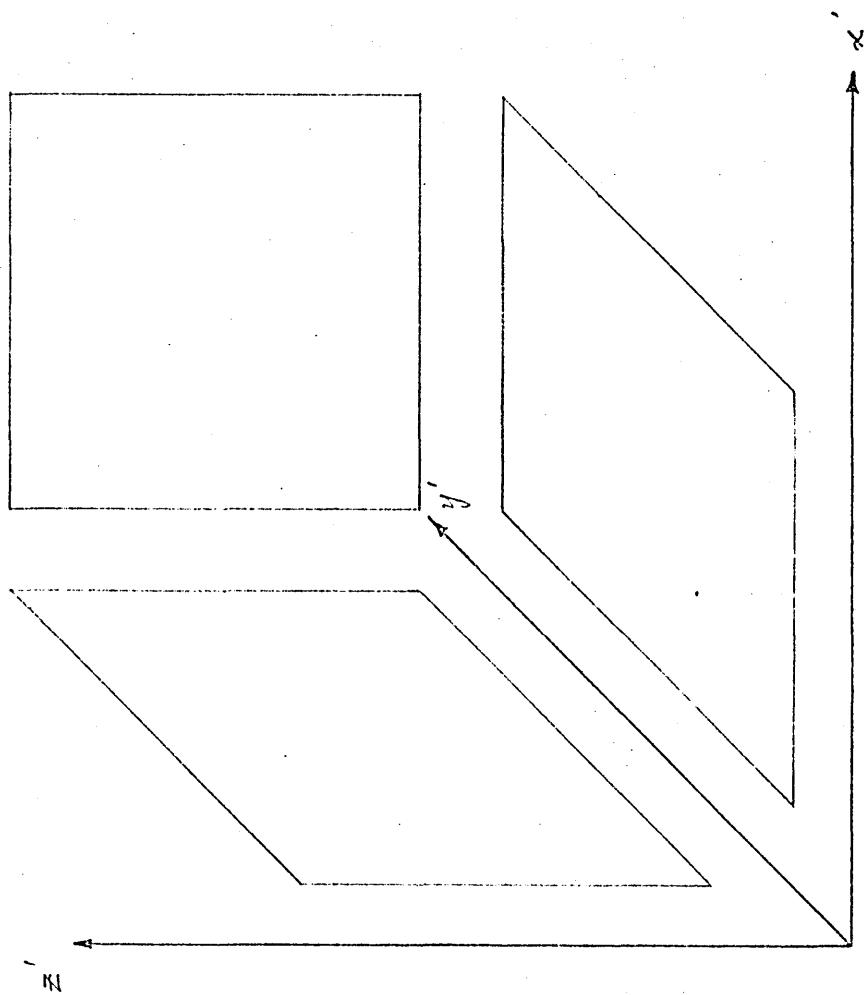
Fig. 2.7 Plate Orientations.
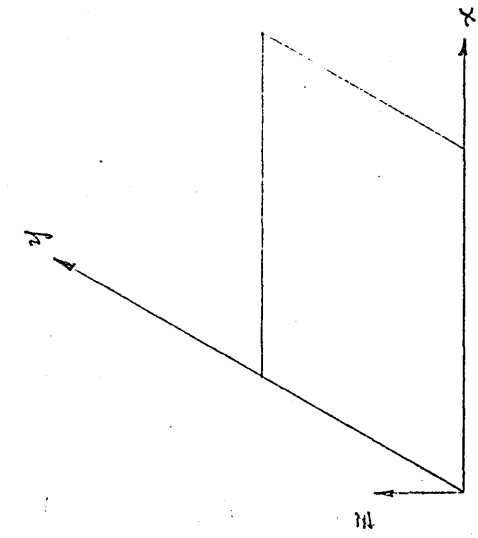
Fig 2.8 Plate in global xy' plane
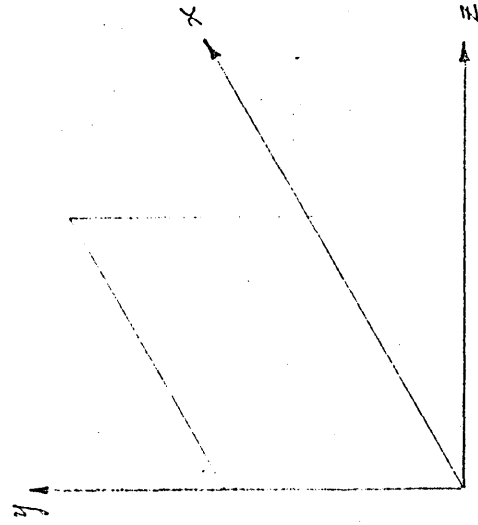
Fig 2.9 Plate in global $y'z'$ plane
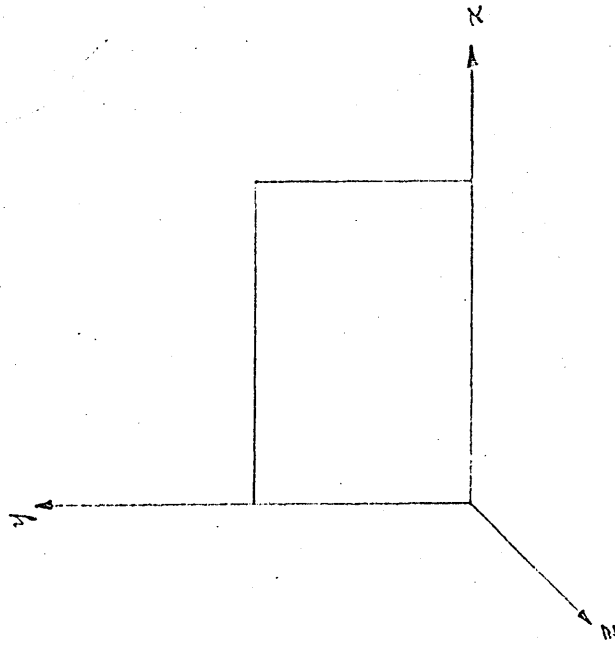
Fig 2.10 Plate in global $z'x'$ plane

and $\qquad [M_G] = [R]^T [M_L] [R]$ $\hfill$ (2.24)

where

$$[R] = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & & \\ \lambda_{21} & \lambda_{22} & \cdot \lambda_{23} & & \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & & \\ \hline & & & & \\ & & & \text{(do)} & \\ & & & & \end{bmatrix}$$

These transformations are performed using the subroutines ROTATE1, ROTATE2, and ROTATE3 for the xy, yz and zx planes respectively (Appendix 3).

2.7   Method of assembly

The priority of assembly of the plate elements into the complete box structure is as follows. The plate elements are first assembled together to form a complete side of the box. The sides in the xy plane are completed first, beginning with that at z = 0. The sides in the yz plane are then built up, starting with that at x = 0 and finally the sides in the zx plane, again beginning with the one in the y = 0 position. The method is illustrated in fig. 2.11 showing the priority in which the sides of the box are built up. This priority in the computer program is indicated by the parameter NA in the calling sequence. The local axes of the elements of each side are as shown previously in fig. 2.8, 2.9 and 2.10. Fig. 2.12 shows the node numbering of the assembled box structure. The above method of numbering of the nodes does not give a narrow band along the diagonal of the matrix in the stiffness and mass matrices. In a dynamic analysis however, this does not present the same advantage it would using a narrow band solution in static analysis.

2.8   Boundary conditions

In a dynamic analysis, a fixed degree of freedom is specified quite simply by eliminating both the row and the column
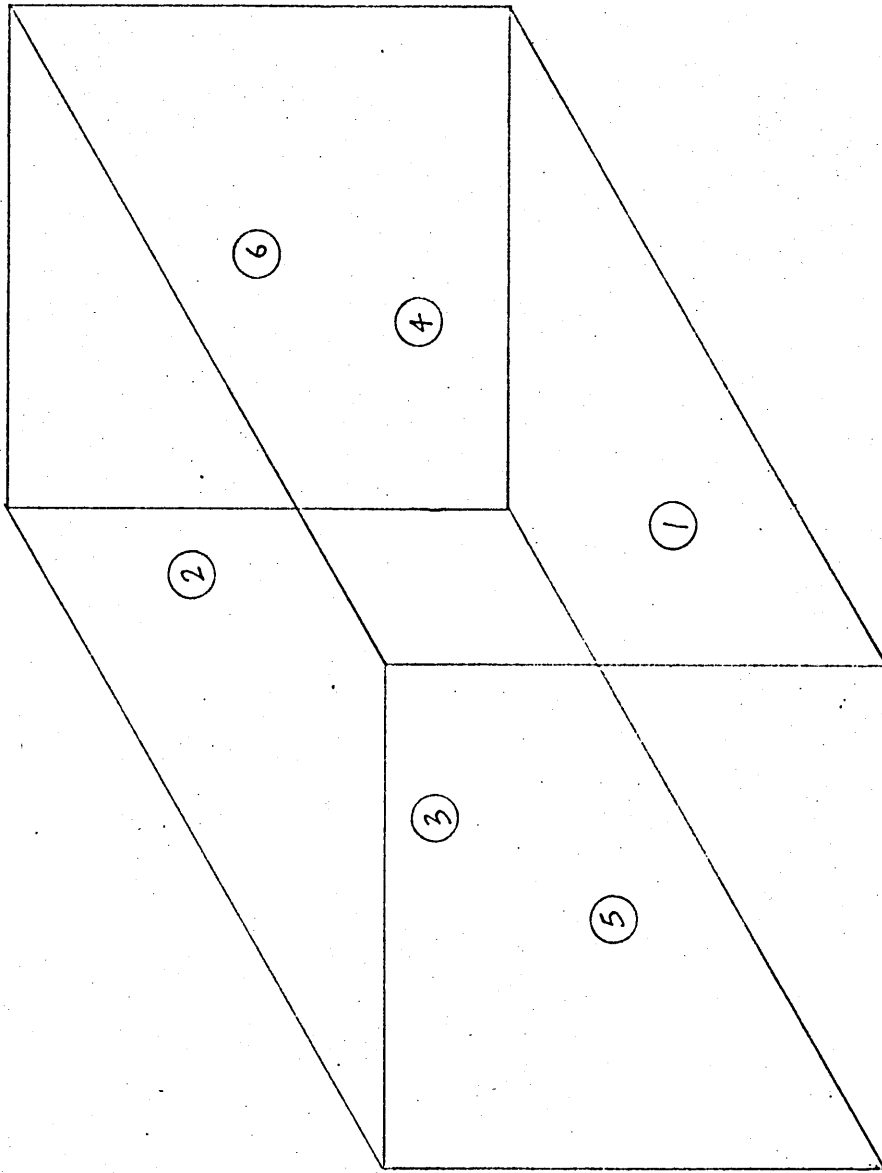
Fig 2.11 Assembly procedure

Fig 2.12   Sketch showing assembled structure
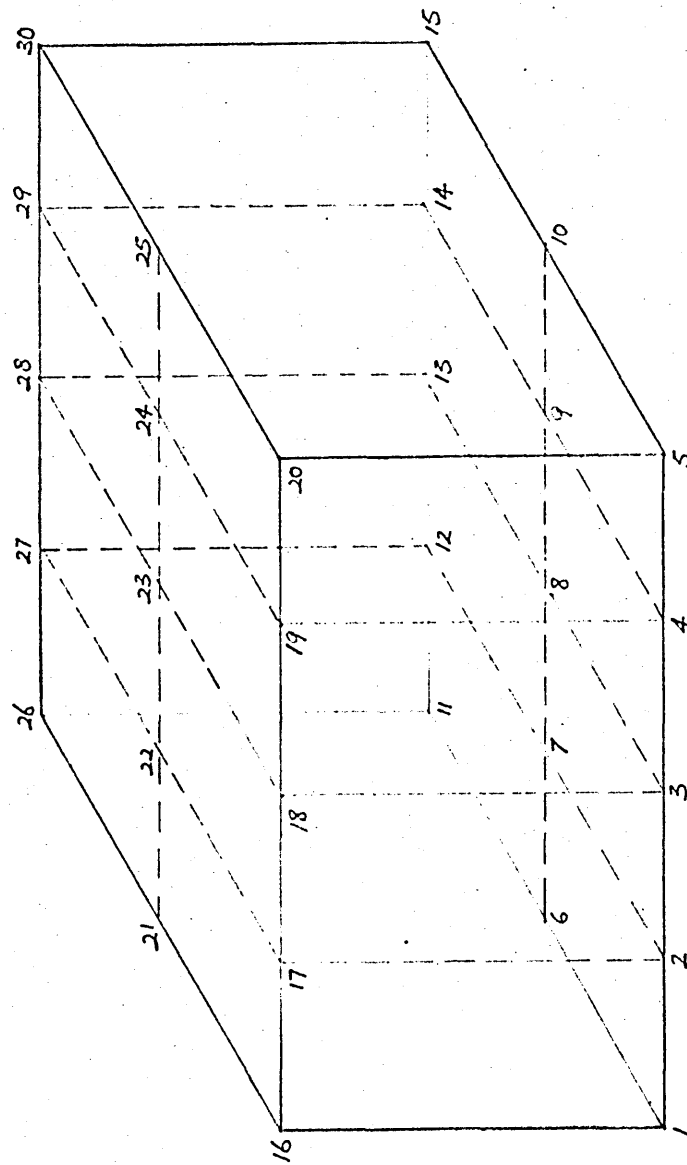
Node numbers of the open ended box structure analysed.

corresponding to that particular degree of freedom. This is because the inertia forces produced in such a case is non-existant since no displacement is allowed at that point. In this way a fully fixed node at the boundary of the structure may have all its degrees of freedom u, v, w, Θ_x, Θ_y, Θ_z eliminated from the system matrices. Similarly a node along a simply supported boundary along the x axis may have rows and columns corresponding to its w and Θ_x displacements removed. Other degrees of freedom that are also reduced out are those of dummy nodes and the dummy degree of freedom Θ_z which had been assigned for ease of assembly. Where there are no contributions to the assembled stiffness and mass matrices the relevant rows and columns (which are all zeros) are removed from these matrices as they would otherwise become singular. The removal of these rows and columns is achieved using a control matrix in which a list is maintained of all the degrees of freedom of the structure, those to be removed represented by zeros and those to be retained by ones.

The subroutine BOUNDARY removes all rows and columns corresponding to the degree of freedom having the value zero, leaving n by n stiffness and mass matrices where n is the total number of degrees of freedom remaining in the structure.

2.9   The solution algorithm

The structural vibration problem (section 2.3) is now reduced to one of solving the generalized symmetric eigenproblem

$$[K] x = \lambda [M] x \qquad (2.25)$$

for its eigenvalues and eigenvectors. From these are obtained the natural frequencies and mode shapes of the vibrating box structure. A short summary of the method used is given here and the reader requiring more detailed information is referred to the computer subroutine manual used [34] and Wilkinson [35].

For the generalised symmetric eigenvalue problem where [K] is a real symmetric matrix and [M] is a real symmetric positive definite matrix i.e. one whose eigenvalues are all greater

than zero, the solution is straightforward. $[M]$ is factorised by Cholesky's method into $[M] = [L][L]^T$ where $[L]$ is a lower triangular matrix. Hence $[K]\{x\} = \lambda[M]\{x\}$ can be written in the form

$$([L]^{-1}[K][L]^{-T})\,[L]^T x = \lambda[L]^T x \qquad (2.26)$$

which is the standard symmetric problem

$$[A]y = \lambda y \qquad (2.27)$$

where $\qquad [A] = [L]^{-1}[K][L]^{-T}$

and $\qquad y = [L]^T x$

The eigenvalues of $[K]\{x\} = \lambda[M]\{x\}$ are the same as those of $[A]$ and if y is an eigenvector of $[A]$ then x, the corresponding eigenvector of the original problem, is obtained by back substitution in the set of linear equations

$$[L]^T x = y \qquad (2.28)$$

Householder's method is then used to tridiagonalise the matrix $[A]$ and the eigenvalues are found using the QL algorithm[35]. The eigenvectors $\{y\}$ of the derived problem are then determined using the QL algorithm and normalised so that $\{y\}^T\{y\} = 1$ and the eigenvectors $\{x\}$ of the original problem are found from $[L]^T\{x\} = \{y\}$ and normalised so that $\{x\}^T[M]\{x\} = 1$. This method can however only be used if $[K]$ is real and symmetric and $[M]$ is positive definite. $[K]$ is checked to be real and symmetric by inspection and in the computer program. The program also has built-in facilities to check the eligibility of $[M]$ using the criterion that its determinant is positive since the determinant is equal to the product of its eigenvalues (which must all be greater than zero).

In some instances due to rounding errors $[M]$ may not be found to be positive definite so that an alternative method has to be used. The procedure commences with the generalised symetric eigenproblem being manipulated into the standard symetric form

$$[B]z = \lambda'z \qquad (2.29)$$

where $\qquad [B] = [K]^{-1} [M]$

and $\qquad \lambda'_n = 1/\lambda_n$

The $[K]$ and $[M]$ matrices are both symmetrical but since $[K]^{-1}$ is not, so neither is the product $[B]$. The problem then becomes one of solving for the eigenvalues and eigenvectors of equation (2.29) where $[B]$ is an unsymmetric matrix. An iterative method applied to this $[B]$ matrix then proceeds to find the lowest natural frequency first directly.

Because the elements of the matrix $[B]$ often varies considerably in size, the process of balancing is used. This is the name given to the rearrangement performed, necessary to obtain maximum accuracy in the subsequent solution for the eigenvalues and eigenvectors. The object of balancing is to make the norm (the sum of the absolute values of the matrix elements in each row or column) the same order of magnitude in the corresponding rows and columns. Error in the calculation of the eigenvalues of the problem is reduced since the eigensolution program used produces results with errors found to be proportional to the norm of the matrix.

The solution then follows with similarity transformation on the balanced matrix $[A]$ so that $[A]$ is transformed into the real upper Hessenberg matrix $[H]$ where $[H] = [S]^{-1}[A][S.]$ An upper Hessenberg matrix is one whose elements h_{ij} are such that $h_{ij} = 0$ when $i - j > 1$. The transformation matrix $[S]$ is built up as the product of n-2 stablised elementary transformation matrices, chosen so that the eigenvalues and eigenvectors of $[H]$ can be more readily determined than those of $[A]$. The eigenvalues of $[H]$ are the same as those of $[A]$ and if y is an eigenvector of $[H]$ then $[S]$ y is the corresponding eigenvector of $[A]$. The transformation process is however considerably simplified because of the balancing already carried out and only part of the matrix is operated upon. There is no simple method for easily calculating selected eigenvalues of an upper Hessenberg matrix and all eigenvalues of $[H]$ are always calculated. A slight saving in computing

time and storage may be made if only at most 25 percent of the
eigenvectors are required. Since more than this is required
it is actually more efficient, both in time and storage, to
use back substitution subroutines which calculate all the
eigenvectors and then to discard those that are not needed.
The QR algorithm [36] is used in the eigensolution subroutine.
This is a very stable method but accuracy is dependant on the
eigenvalues being well spaced out.

A detailed description of all the algorithms and the actual
subroutines used is included in Appendix 2. More details of
the computer program written is given in Chapter 6.

Chapter 3   Random theory

Summary

This chapter describes the basic terms and concepts of
random theory. The significance of the correlation
function of the excitation and the response signals
for the case of white noise excitation is discussed.
The Pseudo random binary sequence signal is a convenient
white noise signal suitable for use in obtaining the
cross correlation function easily and giving the system
impulse response function. The receptance of the system
is obtained by a Fourier transform of the system impulse
response and calculation of the discrete Fourier trans-
form using a new and faster technique is used. It is
also shown that the frequency analysis of any response
to random excitation is not accurate and an alternative
method of determining the response in form of its power
spectral density from a knowledge of the excitation PSD
as well as the receptance of the system is described.

3.1   Introduction

Knowledge of the response of structures to random excitation
of any given type of spectrum is important. This chapter
deals with the background theory relating to random
excitation of a folded plate box type structure which may
represent a car travelling along an uneven road surface, an
aircraft subjected to high speed air turbulence or buildings
and bridges to wind gusts. A brief description of basic random
signal theory is given. The reader requiring a more detailed
knowledge of random vibration theory is referred to standard
text-books e.g. Robson [2].

A truely random process is by definition unpredictable. In
order to make possible any analysis at all a statistical type
solution of the response of structures to random excitation
must be relied upon. Moreover it is difficult experimentally
to generate a true random signal and to be able to reproduce it
again when required. The use of a special type of random signal
overcomes these difficulties. This is known as the pseudo
random binary sequence and will be discussed later in this
chapter.

The response of structures to random excitation is discussed
and the theoretical basis for its experimental as well as
analytical derivation is examined. The method used in this
work to determine this response experimentally is based on the
time domain correlation of the input and response signals. For
the special case of a flat excitation spectrum the cross
spectral density obtained on performing a Fourier transformation
on the cross correlation function also gives the frequency
response function or receptance of the structure. The advantage
resulting in the elimination of interference signals greatly en
-hances this method of systems evaluation. It also gives an
accurate method whereby the response power spectral density
to the white noise excitation is easily obtained.

Robson also shows (eqn. 1.2) that the receptance of a structure can be calculated from a knowledge of its natural frequencies and mode shapes. He provides the theoretical basis for the prediction of the response spectral density from this receptance as well as the knowledge of the excitation power spectral density.

The finite element method of analysis described in the previous chapter enables the calculation of natural frequencies and mode shapes of the box structure. From these the receptance and hence the response power spectral density is predicted.

3.2   Basic Theory

Before going into the realm of random vibration some of the
basic theory and concepts of random processes as given in [2]
are here reviewed, and basic terms peculiar to random theory
are defined.

3.2.1   Statistical approach

Because random signals are by definition unpredictable they
can only be described in statistical terms. The basic
requirement of such a description is an 'ensemble' of time
series records of the signal (fig 3.1) which may represent
force, acceleration, displacement etc.

One common way of describing the signal statistically is by
means of its probability density distribution p(x) which is
defined so that

$$\text{Prob} \left[x \leqslant x(t_0) \leqslant x + dt \right] = p(x)dx \qquad (3.1)$$

where the total area under the p(x) curve is equal to one,
and indicates that the probability of the signal lying be-
tween extreme limits of the curve is 100 %. Many naturally
occuring random signals have a bell shaped probability
density function (fig 3.2) given by

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-m)^2}{2\sigma^2} \right] \qquad (3.2)$$

where σ^2 is the mean square value, also called the variance
of the signal and m is its mean.
The process is then said to have a normal or Gaussian
probability distribution.
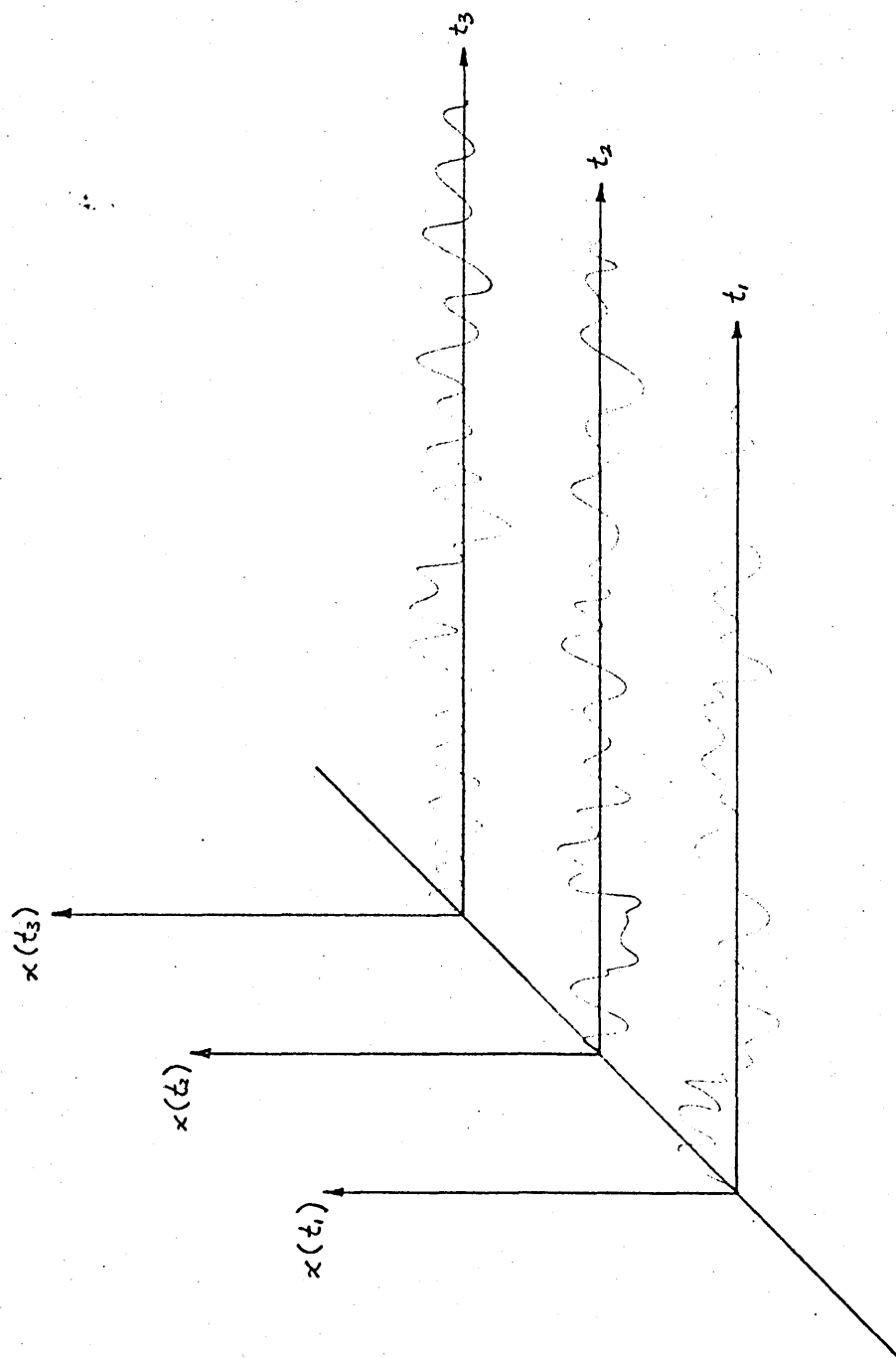
Fig. 3.1 An ensemble of random signals.

$p(x)$

x
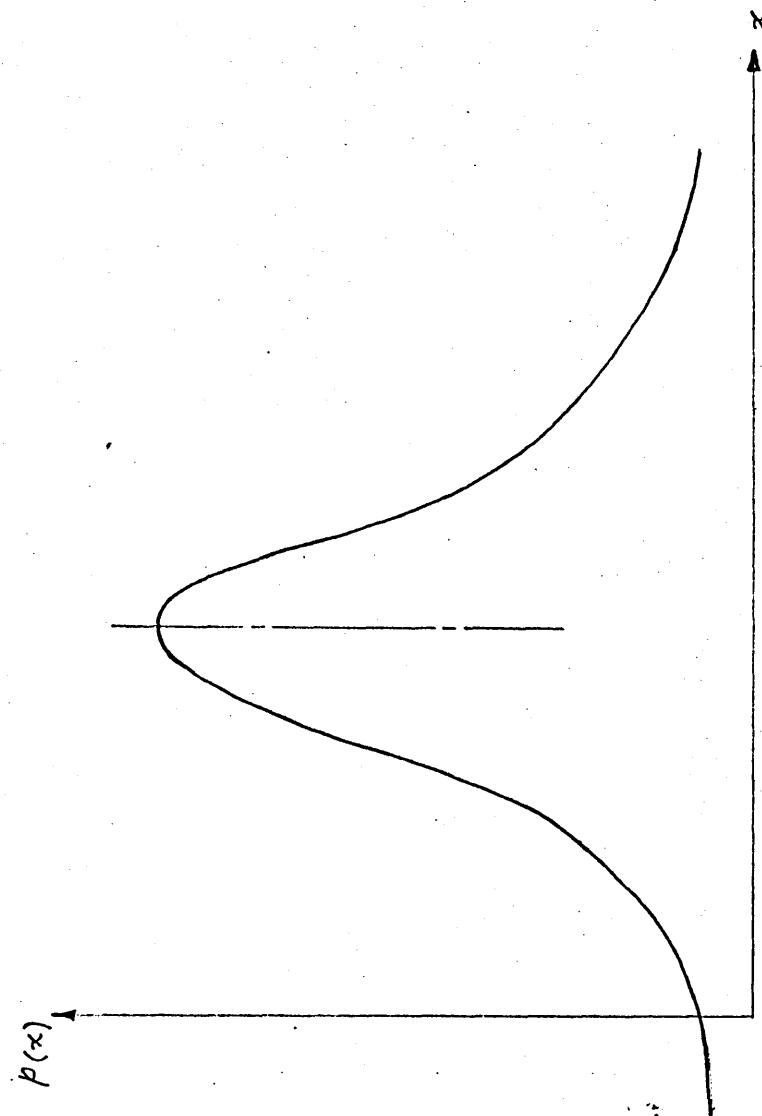
Fig. 3·2   A typical Normal probability distribution.

In practice analysis is simplified by the introduction of the concept of the 'stationary' random process. This is one in which a probability distribution obtained for a particular record is identical to those obtained for all the other records and is therefore independant of the time at which the record is taken. All the statistical characteristics of a stationary random process e.g. its mean, mean square etc. are therefore time invariant. This concept is taken a step further by the introduction of the ergodic process which is defined as not only being stationary but also has its probability distribution taken across the ensemble identical with that taken along any particular record.

3.2.2 The correlation functions

The correlation of the excitation and response signals of the structure will be used in later calculations and provides an important means of signal analysis.

The auto-correlation function Rxx(t,z) is defined as the average over several records of the product of x(t) and its delayed version x(t+z) which for a stationary signal is

$$Rxx(z) = E\left[x(t)x(t+z)\right] \qquad (3.3)$$

where E represents the averaging process.

Similarly the cross correlation function Rxy(t,z) of two stationary signals x(t) and y(t) is

$$Rxy(z) = E\left[x(t)y(t+z)\right] \qquad (3.4)$$

so that in the limit as $z \rightarrow \infty$, $Rxy(z) \rightarrow 0$

This is because there is no correlation between two different signals at that period of delay and this important property makes it suitable for Fourier transformation as will be seen later.

3.2.3 Power spectral densities

Another term commonly used in random signal analysis is power spectral density $S(f)$ or $S(w)$, spectral density for short. The spectral density $S_x(w)$ can be defined in terms of the auto-correlation function $R_x(z)$ of the signal and was first clearly stated by Wiener [37] as

$$S_x(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(z)\, e^{-iwz}\, dz \qquad (3.5)$$

This is an important concept in random vibration analysis since $\int S_x(w)\, dw$ is in fact the mean square $E[x^2]$ of the process i.e.

$$E[x^2] = \int_{-\infty}^{\infty} S_x(w)\, dw \qquad (3.6)$$

For Gaussian processes whose mean is zero, eqn (3.2) becomes

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}} \qquad (3.7)$$

Knowledge of $E[x^2]$ would define the process completely since for a process with a mean about zero

$$E[x^2] = \sigma^2 \qquad (3.8)$$

Hence since $E[x^2]$ can be obtained from eqn 3.6, the determination of the power spectral density is extremely useful in all calculations involving random signals. It should be noted here that $S_x(f)$ is expressed as a function of frequency and 2π times larger than $S_x(w)$. From equation (3.6) the mean square value of $x(t)$ in any given band of frequencies Δf is simply $S_x(f)\, \Delta f$ or $S_x(w)\, \Delta w$. As an example, if $x(t)$ is a varying force expressed in newtons, then $S_x(f)$ would be in newtons^2/Hz and $S_x(w)$ in newtons^{2}sec.

3.2.4   Fourier analysis of random signals

In many cases it may be desirable to express a random signal
in a Fourier series type expansion. However because the signal
is not periodic this is not possible. Moreover since the
signal must be assumed to extend over an infinite period of
time to have stationary properties it cannot be expressed as
a Fourier integral either. A method of measurement is
therefore adopted to overcome the difficulties, using the
Fourier transform $A_T(if)$ of a signal $x_T(t)$ which is identical
to the signal to be analysed over the period $-\frac{T}{2} < t < \frac{T}{2}$ and
zero elsewhere. The spectral density of the original signal
has been shown by Jenkins & Watts [38] to be approximated
by

$$S_x(f) = E\left[\lim_{T\to\infty}\left\{\frac{2}{T}\left|A_T(if)\right|^2\right\}\right] \qquad (3.9)$$

From this it can be seen that the direct Fourier transform
of the response of the structure required, measured within
any finite length of time will at best give only an approx-
imation to the true power spectral density of the response.
A different approach is therefore adopted in this work to
attempt at obtaining a better estimate and the method is
described in section 3.5

The classical method of determining the response of a structure to excitation requires the solution of the differential equation (Newland [39]) relating the excitation x(t) and the response y(t) in

$$\sum_{n=0}^{r} A_n \frac{d^n y}{dt^n} = \sum_{n=0}^{s} B_n \frac{d^n x}{dt^n} + \sum_{n=0}^{t} C_n \frac{d^n x_2}{dt^n} \qquad (3.10)$$

This relationship however cannot be used in random vibration work because

(i)   inadequate data is available to determine the coefficients A, B & C so that the complete differential equation is not available.

(ii)   even if the equation is known, a complete time history of x(t) is not obtainable because of its random nature.

Analysis of the response of any structure to random excitation is therefore best carried out using the form of its response power spectral density, described in the previous chapter. This requires the determination of the structural frequency response curve commonly called the receptance (Bishop & Johnson [4]) of the structure.

The receptance of the structure is defined as the response d of the structure to a unit applied sinusoidal force p of frequency f. It should be noted that the receptance is different at different points of the structure and relates only two specific points on the structure, A the point at which the force is applied and B the point at which the response is measured i.e.

$$d_B = \alpha_{AB}(if) p_A \qquad (3.11)$$

Determination of $\alpha(if)$ will therefore give the response of the structure to any given force p. The receptance and hence the response is complex, with a real and an imaginary part, the physical significance being that there is a time lag between the application of the force p and the occurance of

the response d. For a single degree of freedom structure
with a hysteretic damping factor η the equation of motion
may be written as

$$m\ddot{x} + k(1 + i\eta)x = p\sin\omega t \qquad (3.12)$$

from which the receptance is found to be

$$\alpha(if) = \frac{1}{k - 4\pi^2 mf^2 + i\eta k} \qquad (3.13)$$

Robson [2] shows that the receptance of a multi-degree of
freedom structure can be expressed in terms of its natural
frequencies and mode shapes (equation 1.2). For a structure
with light damping and hence pronounced resonance peaks the
receptance becomes

$$|\alpha(if)|^2 = \sum_r \frac{[w_r(x_A)]^2 [w_r(x_B)^2]}{M_r^2[16\pi^4\{(f_r^2 - f^2)^2 + \eta_r f_r^4\}]} \qquad (3.14)$$

where $w_r(x)$ is the r.th mode shape

f_r is the r th natural frequency

and M_r is the generalised mass at mode r

given by $\int_o^L w_r^2(x)m\,dx$

Here structural damping is taken to be hysteretic only
(applicable to light structures) and any coupling between
the structure and a medium such as air is assumed to be
negligible. η the hysteretic damping factor for the
structure is calculated from its response curves and is
discussed in more detail later.

The natural frequencies and mode shapes of a simple struc-
ture can be determined using the exact method of solving
differential equations or by approximations using various
energy methods. For more complicated structures however the
finite element method which is basically an energy method,
described in chapter 2, is increasingly used because of
difficulties encountered with other methods. In this work
therefore the natural frequencies and mode shapes of the
box structure are predicted using a finite element computer
program package developed by the author and given in Appendix
3. Once the receptance of the structure is known a prediction

of the response spectral density is possible using the relationship given previously in equation (1.1) provided the input spectral density is also known.

$$Sd(f) = |\alpha_{dp}(if)|^2 \, Sp(f) \tag{1.1}$$

Therefore once the receptance of the structure is known, the response spectral density can be obtained for any given excitation spectral density, from which other parameters describing the response such as the mean square value and probability distribution are obtained using equations (3.7) and (3.8).

There are basically two ways of obtaining experimentally the response of structures to random excitation. The first is the direct approach, measuring the frequency content of the response using a frequency analyser. This method however is not accurate in any finite sampling time because of the random nature of the signal and also because of the low signal to noise ratio generally encountered. The alternative method is the synthesis of the response power spectral density from experimentally derived values of both the structural receptance as well as the excitation power spectral density using the equation given previously.

$$Sd(f) \quad = \quad \left| \alpha(if) \right|^2 \, Sp(f) \qquad\qquad (1.1)$$

This method is found to be more accurate than the first since the excitation power spectral density is often known or can be obtained accurately.

There are three main ways of determining experimentally the frequency response or receptance of structures to vibrations. They are (a) sinusoidal frequency testing (b) transient testing using a ramp or impulse input and (c) tests using correlation methods.

The simplest and most commonly used method is sine wave excitation, measuring the amplitude and phase angle of the response over a range of frequencies giving the frequency response curve directly. Disadvantages encountered with this method include inaccuracy as often the response is contaminated with extraneous noise. The excitation cannot be increased since the system will lose its linear behaviour so that an averaging process is often incorporated. It is also very time comsuming to excite each frequency separately and to sweep sufficient frequencies for a complete frequency response curve.

Transient testing involves either step (ramp) response (fig. 3.3a) or impulse response measurements (fig. 3.3b). The former is quick and contains all frequencies in the step and it is therefore possible to obtain the structural frequency response using curve fitting techniques. It is however limited to simple systems and involves a drastic change of initial conditions. The latter retains the steady state initial conditions but is impossible to generate in practice. Both are also susceptible to inaccuracies due to noise interference at low levels of excitation and non linear behaviour at higher levels.

The third method and one of the most important time domain techniques available is the correlation of two signals, giving a measure of the similarity between the two. The input x(t) and output y(t) of a system are related through a finite integral known as the convolution integral described in Robson [2] and elsewhere as

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \qquad (3.15)$$

where h(t) is the impulse response of the structure and τ is the time delay between the application of the impulse and time t at which the impulse was applied. This convolution integral provides a direct method of obtaining the output signal from a knowledge of the input signal and the impulse response of the structure. As it stands this does not mean very much since the impulse response of the structure is unknown but the application of the correlation method opens up an entirely different approach to the problem.
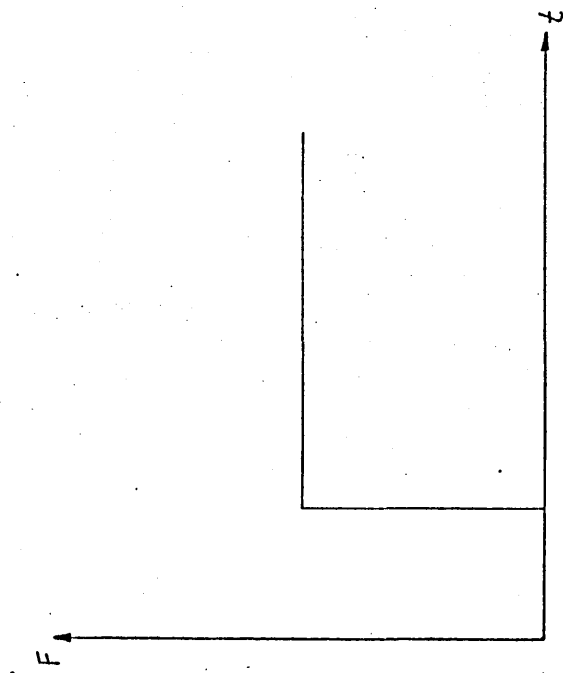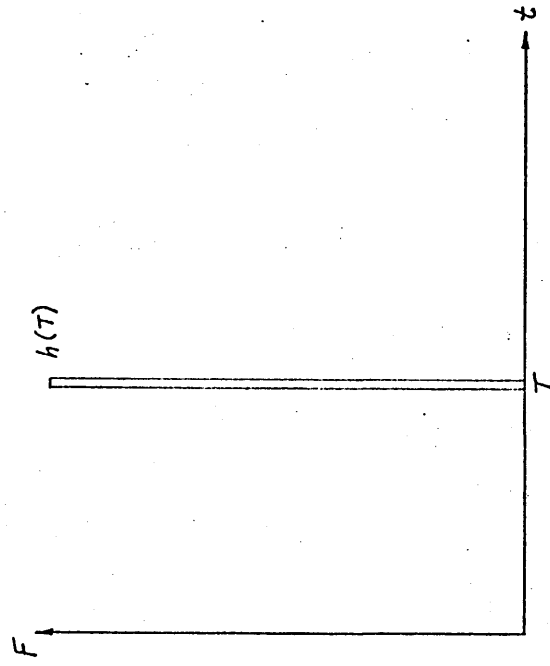
(b) An impulse function

(a) A step function

Fig. 3·3   Transient testing signals

The theory behind the technique lies in the definition of the correlation functions themselves (see section 3.1). The auto correlation function of the excitation $x(t)$ may be expressed as

$$Rxx(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t + \tau)dt \tag{3.16}$$

and the cross correlation function of the excitation $x(t)$ and the response $y(t)$ as

$$Rxy(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t + \tau)dt \tag{3.17}$$

Since the impulse response must be zero before its application $h(t) = 0$ for $t < 0$ and hence (3.15) becomes

$$y(t) = \int_{-\infty}^{\infty} h(s)x(t - s)ds$$

and substituting into eqn. 3.17 ,

$$Rxy(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\left[\int_{-\infty}^{\infty} h(s)x(t+\tau-s)ds \right] dt$$

$$= \int_{-\infty}^{\infty} h(s)\left[\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau-s)dt \right] ds$$

which from 3.16 becomes

$$Rxy(\tau) = \int_{-\infty}^{\infty} h(s) Rxx(\tau - s)ds \tag{3.18}$$

It is immediately apparent that if we choose an excitation signal with a autocorrelation function in the form of a spike or impulse, the impulse response function can be obtained quite simply from the cross-correlation function between the excitation and response signals.

A white noise signal satisfies this requirement. The auto-correlation function of a white noise signal with a flat power density spectrum of K/f is an impulse of magnitude K. Thus for white noise equation 3.18 becomes

$$Rxy(\tau) = Kh(\tau) \tag{3.19}$$

White noise excitation therefore gives a particularly import-ant aspect to correlation techniques as the cross correlation between the excitation and the response of the structure under consideration gives the impulse response function of

the structure multiplied by a constant K. It is therefore
no longer necessary to physically hit the structure with an
impulse of force in order to obtain its impulse response.
The frequency response function or receptance is then quite
simply the Fourier transform of the impulse response function.

$$H(\omega) = \mathcal{F}\left[h(\tau)\right] \tag{3.20}$$

The impulse response function of any structure can therefore
be obtained without the problems associated with sinusoidal
and transient testing and has with it the advantage that noise
originating within the system does not correlate with the
excitation signal and is hence effectively eliminated. Thus
a signal of low amplitude can be used even in a noisy environ-
ment. This is the basis of the so called method of random
systems testing. The method however has several drawbacks.
Firstly the truely random signal is difficult to generate
artifically and impossible to reproduce exactly. The delayed
signal required for calculation of the correlation function
is therefore not available. Furthermore the correlation also
requires multiplication and integration over a theoretically
infinite period, and it would be difficult to obtain a
satisfactory correlation over a finite period of an essentially
unpredictable process. The use of the pseudo random binary
sequence signal, described later overcomes this difficulty.

3.6   The Fourier Transform

The concept of the frequency analysis of a signal of period
T by splitting it into its component frequencies using the
Fourier Series is well known (see Newland [39])

Thus

$$x(t) = \sum_{n=0}^{\infty} A_n \cos \frac{2\pi nt}{T} + B_n \sin \frac{2\pi nt}{T} \qquad (3.21)$$

where

$$A_o = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \, dt$$

$$A_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cos \frac{2\pi nt}{T} \, dt$$

and $$B_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \sin \frac{2\pi nt}{T} \, dt$$

or also expressed as

$$X_n = \frac{1}{T} \int_{0}^{T} x(t) \, e^{-\frac{i2\pi nt}{T}} \, dt \quad \text{where } X_n = A_n - iB_n \qquad (3.22)$$

The limit when T tends to infinity is reached in non-periodic
signals so that the Fourier series becomes a Fourier integral
whose coefficients are the corresponding Fourier transforms.
i.e.

$$x(t) = 2\int_{0}^{\infty} \left[A(\omega) \cos \omega t \, d\omega + B(\omega) \sin \omega t \, d\omega \right] \qquad (3.23)$$

and

$$A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) \cos \omega t \, dt$$

$$B(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) \sin \omega t \, dt \qquad (3.24)$$

with the limitation that $\int_{-\infty}^{\infty} x(t) \, dt < \infty$

This thus provides an indispensible tool which facilitates
signal analysis in transforming a signal from the time to the
frequency domain or vice versa. However, because of the
limitation on the signal x(t) itself, unless special precautions
are taken only a process which in the limit is finite can be
analysed.

In the work a time series which satisfies this requirement of being finite in the limit is used. This is the cross' correlation function of the·white noise excitation and the resulting response of ·the structure. The direct frequency analysis of the response itself is not used as it does not satisfy this requirement however long the record taken may be.

From the previous section (eqn. 3.19), we have seen that for white noise the cross correlation function Rxy(τ) is in fact K h(τ) where h(τ) is the impulse response of the system and K is a constant, given by the power spectral density of the excitation. The Fourier transform of this gives its equivalent in the frequency domain, the frequency response function of the structure, (eqn. 3.20), multiplied by the same constant K. Because the excitation is white noise, this function when multiplied by the constant 2π gives the output power spectral density of the structure as well.

Here a continuous function is to be analysed by taking discrete samples of it at equally spaced intervals of time. Such a series completely represents the continuous waveform providing the waveform is band limited and the samples are taken at a rate of at least twice the highest frequency present in·the original waveform [39]. The samples are then called Nyquist samples and the highest frequency present, the Nyquist frequency. The discrete Fourier transform or DFT of such a time series is then closely related to the Fourier transform of the original continuous waveform and has mathemetical properties analogous with it. If however frequencies in the original signal exist which are higher than half the sampling frequency the Nyquist frequency, a corruption of the graph known as aliasing occurs. In this case, to obtain a true DFT either a higher sampling frequency is required or the existing high frequencies contravening the Nyquist criteria must be filtered out of the signal. In this work the latter method is used.

Thus from eqn. 3.2 2 the DFT of N equally spaced samples taken at intervals of time can be obtained using the formula.

$$X_n = \frac{1}{T} \sum_{r=0}^{N-1} x_r \, e^{-\frac{i2\pi nr\Delta}{T}} \Delta \qquad (3.25)$$

where n ranges from 0 to N - 1 corresponding to harmonics of $\frac{2\pi}{N\Delta}$.

The use of eqn. 3.24 to obtain the DFT is perfectly satisfactory for processing small amounts of data and a computer routine to carry out the algorithm is included in Appendix 4. For large quantities of data however, this conventional method quickly takes up a large amount of computer core and the computing time required becomes prohibitive.

The Fast Fourier Transform or FFT is an algorithm that was developed recently to calculate the Discrete Fourier Transform (DFT) and takes advantage of the fact that the calculation of the coefficients of the DFT can be carried out iteratively resulting in a considerable saving of computing time and increased accuracy. The algorithm used is that reported by Cooley and Tukey [29]. The FFT method is perhaps not as obvious as the DFT and the steps are briefly outlined here.

A time series X_k of N points is divided into two functions Y_k and Z_k where Y_k consists of all the even numbered points and Z_k the odd.

$$\left. \begin{array}{l} Y_k = X_{2k} \\[2ex] Z_k = X_{2k+1} \end{array} \right\} \qquad k = 0,1,2, \ldots \left(\frac{N}{2} - 1\right)$$

Therefore the DFT of Y_k and Z_k will be given by

$$\left. \begin{array}{l} B_r = \sum_{k=0}^{\frac{N}{2}-1} Y_k \, \exp\left(- \frac{4\pi irk}{N} \right) \\[3ex] C_r = \sum_{k=0}^{\frac{N}{2}-1} Z_k \, \exp\left(- \frac{4\pi irk}{N} \right) \end{array} \right\} \quad \text{where } r = 0,1,2 \ldots \left(\frac{N}{2} - 1\right)$$

The DFT of the original series X_k is then

$$A_r = \sum_{k=0}^{\frac{N}{2}-1} \left\{ Y_k \, \exp\left(- \frac{4\pi irk}{N}\right) + Z_k \, \exp\left(- \frac{2\pi ir}{N}(2k+1)\right) \right\}$$

$$= \sum_{k=0}^{\frac{N}{2}-1} Y_k \exp\left(-\frac{4\pi i r k}{N}\right) + \exp\left(-\frac{2\pi i r}{N}\right)\left[\sum_{k=0}^{\frac{N}{2}-1} Z_k \exp\left(-\frac{4\pi i r k}{N}\right)\right]$$

where $r = 0, 1, \ldots \ldots N-1$.

which may be written as

$$A_r = B_r + \exp\left(-\frac{2\pi i r}{N}\right) C_r \quad \text{for} \quad r = 0, 1, 2 \ldots \left(\frac{N}{2} - 1\right)$$

The DFTs, B_r and C_r repeat themselves outside this interval so that

$$A_{\left(r + \frac{N}{2}\right)} = B_{\left(r + \frac{N}{2}\right)} + \exp\left[-\frac{2\pi i}{N}\left(r + \frac{N}{2}\right)\right] C_{\left(r + \frac{N}{2}\right)}$$

$$= B_r - \exp\left(-\frac{2\pi i r}{N}\right) C_r$$

$$\text{since} \quad \exp\left(-\frac{2\pi i N}{2N}\right) = -1$$

Therefore

$$A_r = B_r + W^r C_r \qquad \text{for } r = 0, 1, 2 \ldots \ldots \frac{N}{2}$$

$$A_{\left(r + \frac{N}{2}\right)} = B_r - W^r C_r \qquad \text{where } W = e^{-\frac{2\pi i}{N}}$$

Thus the two halves of the DFT of the N point X_k, A_r and $A_{r + \frac{N}{2}}$ can be obtained from the B_r and C_r, DFT's of Y_k and Z_k each of $\frac{N}{2}$ points. By successively dividing the series into further subseries as long as they are divisible by two, we end up with the DFT of a small series from which the DFT of the original series can be built up. Variations on the method allow the series to be broken up into series of multiples other than two but we shall not look into these here.

In this work, series containing 512 and 1024 points are finally reduced to the single term, the DFT of the single term being the term itself, and the DFT of the series built up from it, and simply divided by the factor 512 or 1024 respectively. Thus it will be seen that a considerable saving in the number of complex multiplications and complex

additions is made using this algorithmn. The resulting saving in the time taken enables this method of calculating Fourier transforms to be run more economically than the normal method especially on the slower computers.

Random-type signals used in the laboratory can be class-
ified into being naturally or artifically generated. Common-
ly used natural sources of random signals include (a) a
tape recordings of a naturally occurring ramdon signal such
as wind noise, road noise etc. (b) the emission of
electrons by a thyratron or zener diode. These have
several disadvantages. The former requires an infinitely
long record to be statistically accurate, the latter is
difficult to control particularly in the low frequency
ranges and moreover is not reproducable whenever required.
To overcome these drawbacks other sources of random signals
have to be used.

Pseudorandom binary sequence signals (referred to as PRBS
signal hereafter) provide artifically generated signals that
are completely random within any specified period of time
but repeat themselves thereafter. Since the signal is
completely random within this interval it meets all the
requirements and has all the properties of random signals
discussed in the previous section. The use of a PRBS signal
as an excitation signal has several advantages over the use
of true random signals.
These include
(a)   repeatibility - it is possible to obtain exact signals
       for repeated tests and these tests do not have to be
       infinitely long. Statistical type results do not
       have to be relied on as in the case with true random
       signals.
(b)   spectrum can be tailored to requirements - the frequency
       of interest, resolution, length of time available for
       each test etc. can be allowed for in each case.
(c)   correlation of excitation and response can be easily
       obtained over a discrete number of signal points.
(d)   for the particular equipment used a time domain correla-
       tor is available as standard equipment so that automatic
       recording of the usually lenghthy correlation function
       . is possible.

A brief description of the signal (fig. 3.4) is now given. For a more detailed account, the reader is referred to the handbook [40].

The PRBS signal is generated using a shift register, the length n of which can be changed at will and which gives it the name of the maximum length sequence of simply m sequence. The outputs from two stages of the shift register are connected to a modulo two adder (exclusive OR gate) whose output controls the state of the first shift register stage. These connections are chosen so that a maximum length sequence is generated of $N = (2^n - 1)$ bits. The necessary delayed signal is obtained simply by dividing the full shift register into two equal parts to form master and slave PRBS generators. The main control logic then arranges for the slave shift to begin its sequence the required number of clock pulses after the master.

It is completely controllable, a considerable advantage over true random signals but unlike random binary signals has the number of positive and negative bits differing by one since the total number of bits in the sequence is given by $(2^n - 1)$. Being a binary signal it has only 2 levels, +1 and -1, which change at intervals of Δ sec given by the clock pulse. Multi-level sequences may however be chosen to assume particular types of probability distributions if necessary.

As mentioned previously the PRBS signal is completely random within the period

$$T = N\Delta$$

where $1/\Delta$ = clock frequency

N = no. of bits in the sequence = $(2^n - 1)$

A comparison of its autocorrelation function (fig. 3.5) reveals that (a) it is a spike which repeats itself at intervals of N (see Newland [39]). (b) a small negative correlation $-a^2/N$ persists. However it approaches that of true white noise when the sequence length N is made large and the clock time Δ small. Within the time interval $N\Delta$ therefore it has all the properties necessary for the random systems testing described in the previous chapter.
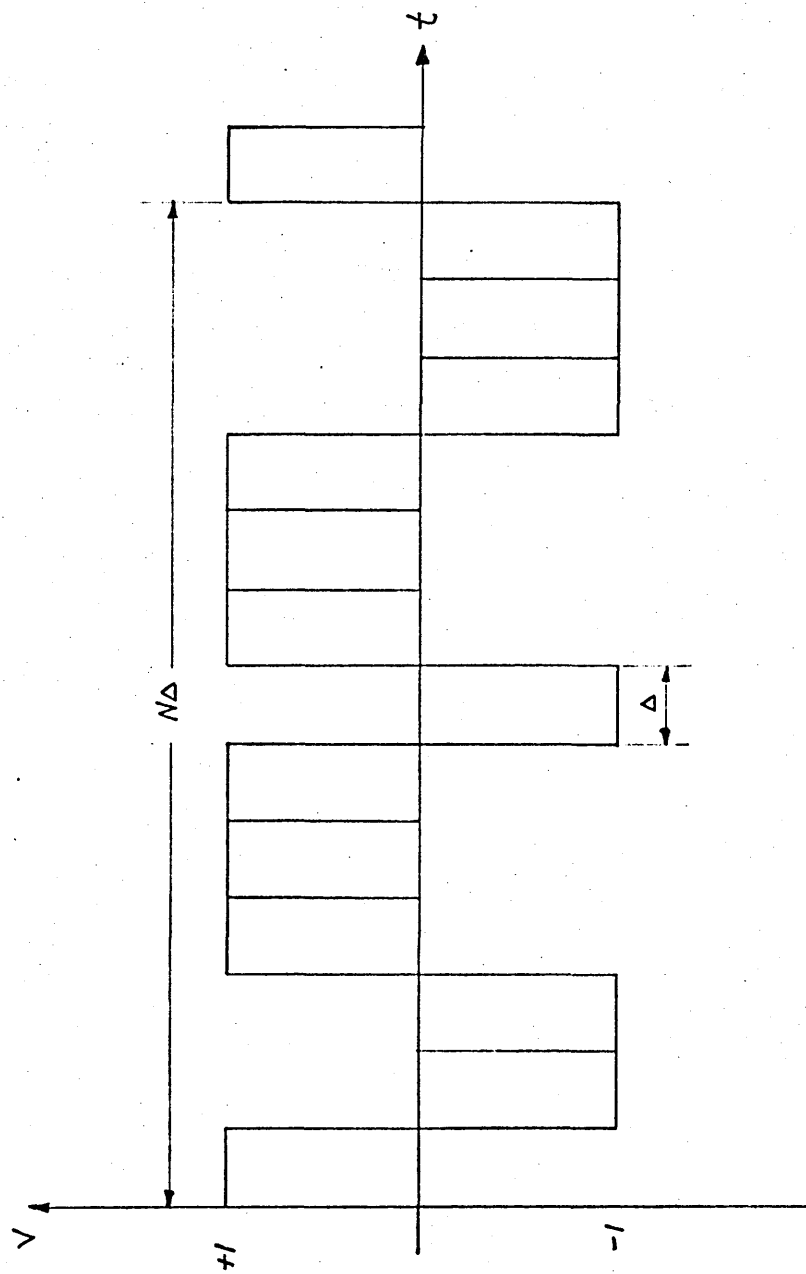
Fig. 3·4 A PRBS signal.
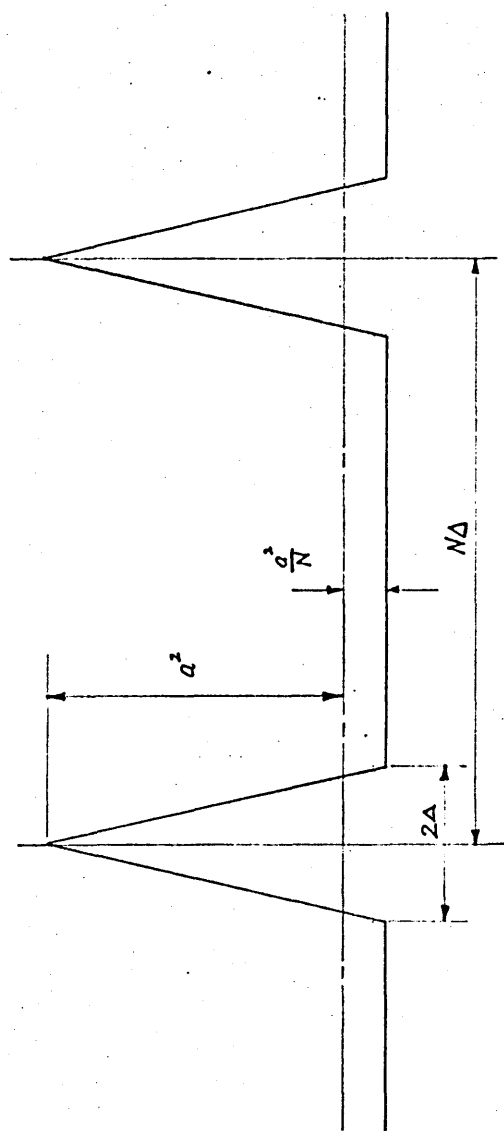
Fig 3.5 . Autocorrelation function of a PRBS signal .
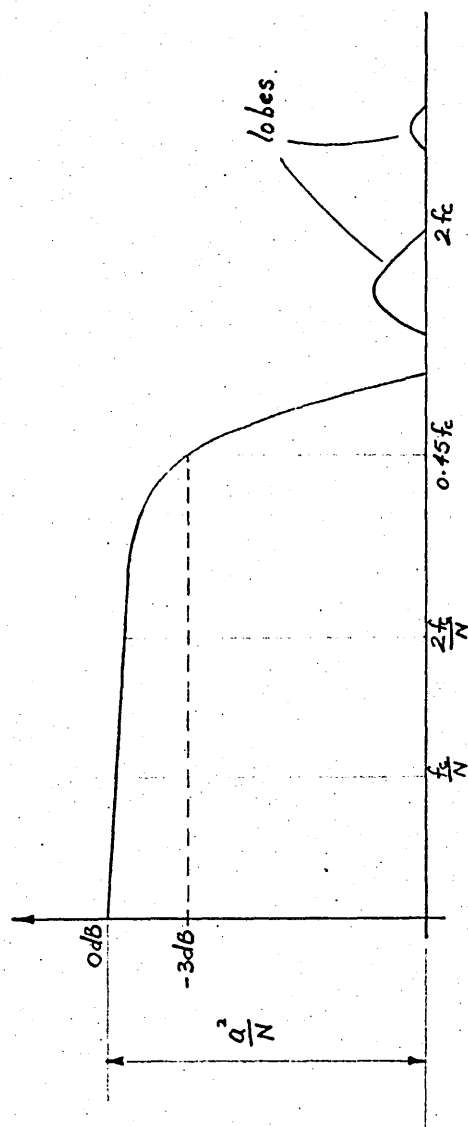
Fig 3.6   Power spectrum of the PRBS signal.

The Fourier transform of the autocorrelation function gives
the graph of relative power plotted against frequency (fig.
3.6). Referring to the figure, because the signal changes
only at discrete intervals of time it will be seen that
this is a line spectrum i.e. one made up of individual lines
instead of the continuous spectrum of true white noise.

These are spaced out at a distance of $\frac{f_c}{N}$ Hz apart where
f_c = clock frequency = $\frac{1}{\Delta}$ and are constant to -3 dB at
0.45 f_c Hz as shown. The shape of the envelope is given
in the manufacturers' specification as being described by
the function $(\frac{\sin x}{x})^2$ where $x = \frac{\pi f}{f_c}$ with a d.c. component of
power = a^2/N since N is an odd number. A closer approxi-
mation to a continuous spectrum is obtained by increasing
the sequence length but reducing the clock frequency.
This is achieved at the expense of amplitude in the former
and the upper frequency in the latter.

The PRBS signal has a probability distribution shown in
fig 3.7. This reveals that the signal occurs only at 2
levels, $\pm$ aV (see fig 3.4) and, of the $2^n - 1$ bits of signal
in a sequence, has one more + a bit than - a bit. This can
however be tailored to approximate the Gaussian distribution
of commonly occuring random signals in a binomial type
distribution(fig 3.8)using multi-level sequences. This is
achieved however at the expense of (a) reducing the amplitude
of the output signal from a^2 to $a^2/(M-1)$,(b) widening the
impulse type spike of its autocorrelation function (fig 3.5)
from 2Δ to $2(M-1)\Delta$ and (c) reducing the highest frequency
(fig 3.6) from f_c to $f_c/(M-1)$ Hz.

It is possible therefore either to approximate a given random
signal or to generate a suitable excitation signal for any
given system and at the same time to be able to repeat it
exactly both for correlation calculations as well as to
repeat a test subsequently. A suitable signal with which
to excite any given structure is governed by the choice of
upper frequency limit, shape of excitation power spectrum,
resolution of component frequencies, probability distribution
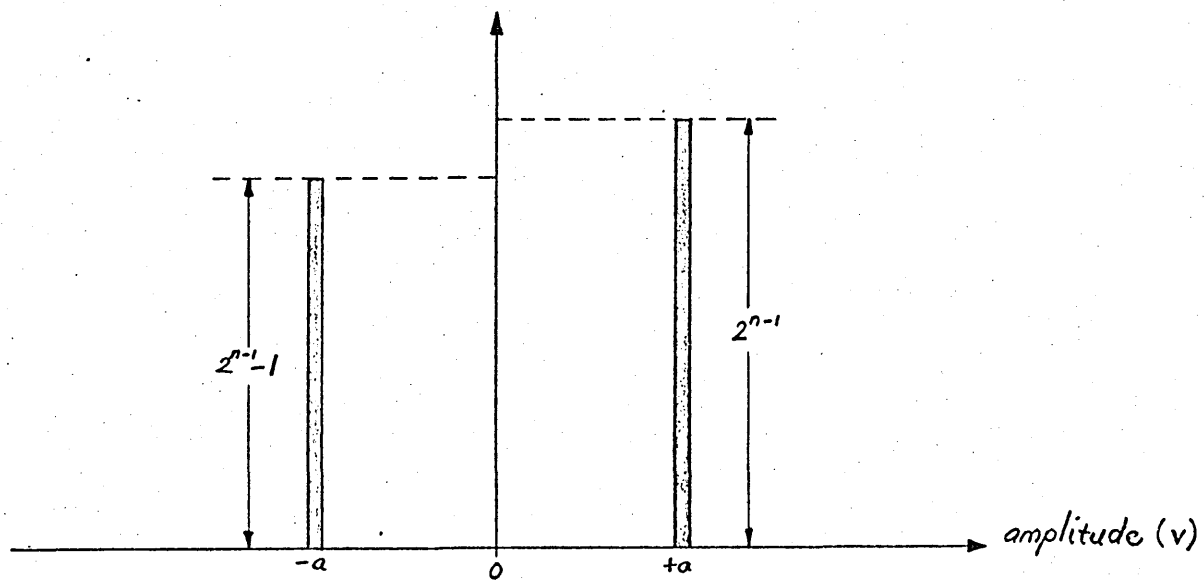
Fig. 3.7   PRBS occurance distribution.
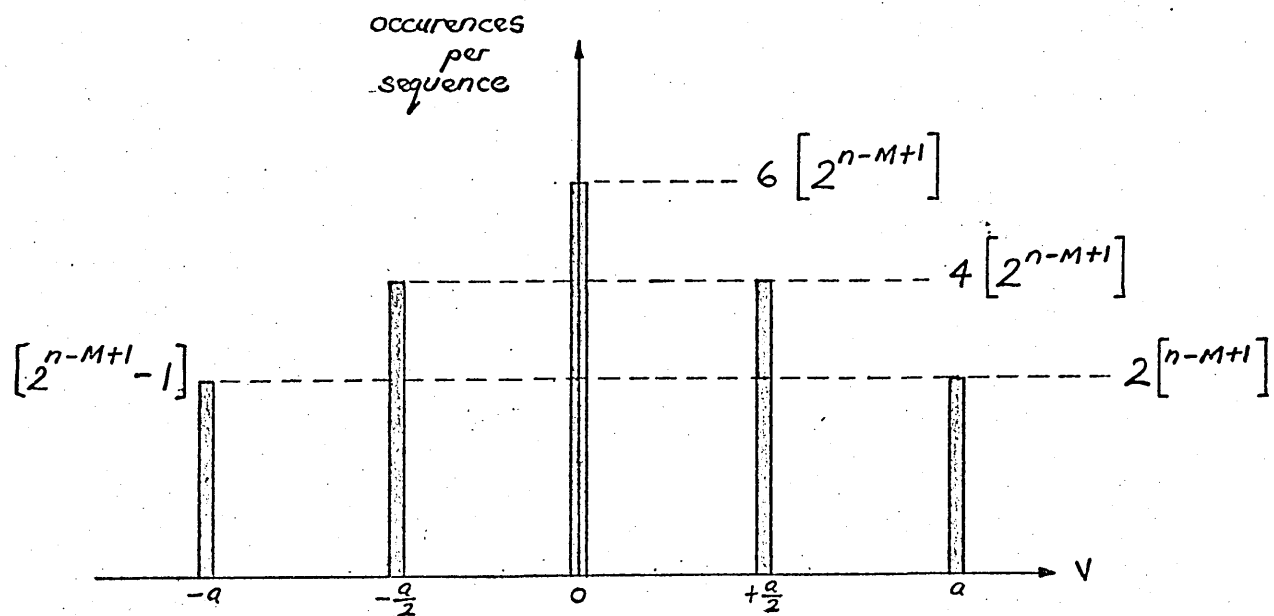
Fig 3.8   Probability distribution of a multi-level sequence.

and so on. All these are easily obtained with a PRBS signal.
The upper frequency limit is given by the clock frequency of
the PRBS generator and is determined by the requirement
to stimulate the structure up to a given frequency. In many
structures the most important frequencies are the lowest few
as the response at higher frequencies tends to be attenuated.
In practice most structures also tend to have the higher
frequencies occurring close together and with the line
spectrum of the PRBS signal there is a danger that a natural
frequency of the structure may not be stimulated.

To summarise therefore the PRBS signal is within its sequence
length completely satisfactory for random testing purposes.
It is easy to generate, reproduce and multiply and is there-
fore particularly suitable for correlation measurements.
Since the correlation is performed over a finite period
however, some degree of correlation with unrelated noise may
lead to a variance in the measurements obtained. By
correlating over a number of complete sequences this effect
is nevertheless considerably reduced.

Chapter 4   The Apparatus

Summary

This chapter describes how the necessary experimental
apparatus was designed, calibrated and used. A vibration
table was built having the necessary characteristics for a
good vibration testing mounting and incorporating an accur-
ate two way traverse mechanism over the horizontal plane.
A novel non-contacting exciter/pickup probe incorporating
an electromagnetic exciter and a capacitance pickup is
described. A brief description of the investigation into
variations in both material and design leading up to the
final probe configuration is included. A fine adjustment
mechanism, linear as well as angular, is incorporated for
accurate adjustment of the air gap between the probe and
the structure. For random vibration work a pseudorandom
binary sequence generator and time domain correlator is used
from which a completely automatic system has been built up.

4.1   Introduction

Since the aim of this investigation is ultimately to be
able to predict the response of box-type structures to
vibration stimuli, a large part of the work necessarily
involves response measurements obtained experimentally
to complement the equally important theoretical analysis.
This basically requires a firm vibration testing table on
which the structure and all the necessary transducers can
be mounted. These exciters and vibration pickup transducers
must be suitable for the structure to be tested.

As light structures are to be tested, in order to reduce
inaccuracies due to the added masses of exciters or pickups,
non-contacting transducers are necessary. Furthermore it
is known that the ideal point at which to excite a particular
mode is its antinode (Plunkett [41] and Dunn [42]).
Exciting the structure at any point other than at a antinode
requires a higher force per unit displacement response.
This is because it can give rise to (a) displacement of a
point which would otherwise be a node (b) rigid body displace-
ment (c) excitation of adjacent nodes or (d) local distor-
tion of the structure at the point of excitation (Pandered
and Bishop [43]). The inherent drawback of a contacting
exciter is immediately apparent since multiple holes must be
made in the structure for excitation of more than one mode.

Non contacting vibration pickups are fairly widely used and
capacitance displacement transducers are well known and
available commercially. In general contacting exciters are
still relied upon and work on non contacting electromagnetic
exciters are fairly poorly documented. Recent work by
Shapiro [44] to produce suitable apparatus for similar
experimental investigations relied on contacting exciters
although non-contacting capacitance displacement transducers
were used. This may be due to difficulties such as a
frequency doubling effect of the original signal in the
force output of an electromagnetic exciter.

This frequency doubling effect and the heat generated in
the exciter during operation were noted in the course of
this work and is discussed later in this chapter. The
development of an efficient non-contacting exciter pickup is
therefore necessary and will be one of the tasks of this
investigation.

4.2   The Vibration testing table

The importance of the vibration testing table is that
it provides a good vibration mounting for the test structure
isolated from interference which is mainly structural
borne. It also provides a stable support for the mounting
of the exciters and vibration pickup transducers with fine
adjustment available between these and the structure. An
accurate traverse mechanism is necessary on which the
vibration pickup probe and the exciter can be suspended
to scan the surface area of the structure locating the
required points.

The work commenced using an existing vibration table
(plate 1). Structural borne interference (the laboratory
was next to a busy main road) was found to be transmitted
straight through the table and picked up at the measuring
transducers. The traverse mechanism holding these trans-
ducers were independent of the table and some floating
relative to the table was detected. A means of fine adjust-
ment of the transducer-plate gap was found to be necessary
because the plates were not absolutely flat and the
exciter-plate gap size was found to be critical. A new
and larger table was made with these requirements in mind.

This is fairly large and heavy and supported on wide base
plates resting on ½" vibration isolation pads to reduce
vibration transmitted through it (plate 2). Structures
measuring up to 1,500 x 900 x 430 mm can be accommodated.
An accurate traverse mechanism is built on to the table
and a screw-type winding, at the ends of which are hand
wheels, drive the exciter-pickup probe in the two perpen-
dicular horizontal directions. The winding mechanism is
supported on special end ball races and phosphor bronze
sliders to reduce friction at the slideways. When the
point to be measured is arrived at, the end clamps are
tightened thus eliminating extraneous vibrations from
the traverse mechanism.

The fine adjustment mechanism is a double screwed device

(plate 3) which allows for accurate changes in linear displacements in the probe-structure gap size. This is necessary since both the exciter as well as the pickup have characteristics which depend on this gap size. The mechanism screws down clockwise at 0.24 mm per turn with a maximum travel of 20 mm. Coarse adjustment is by loosening allen screws holding the whole mechanism in its holder in a vertical sliding arrangement. For angular adjustment three knurled screws on a swivelling ball joint allow for tilt of the transducer relative to the structure (plate 4).

The structures to be tested are held with supports made which would approximate required boundary conditions, e.g. a fully fixed flat plate (plate 5), clamped to the table using large 'G' clamps. A close-up view of the method of fastening of the box structure is shown in plate 6.

Since the box-type structure to be investigated was light,
a non-contacting vibration pickup transducer was used.
Contacting transducers e.g. accelerometers, invariably
implies additional masses attached to the test structures
affecting the frequency characteristics of the structure.

Capacitance probes are part of a large family of non-cont-
acting transducers which are used in vibration studies to
eliminate the possibility of the transducer interfering
with the vibration characteristics of the structure to be
investigated.

The Wayne Kerr capacitance probe was chosen because of its
good characteristics and already widespread use amongst
research workers. It works on the principle that when
placed near to a conducting surface, a capacitance effect
exists between this surface and the face of the probe. The
capacitance so formed is connected to the feedback loop
of a high gain amplifier hence changing its impedance and
causing a voltage output to be formed which is proportional
to the gap distance between the probe and the surface.

The type 'E' probe is most suitable for the purposes of
this investigation, measuring amplitudes of up to 2.5 mm
with a frequency response flat up to 1000 Hz. This is
calibrated together with the associated distance meter,
the 2 channel Wayne Kerr TE 2000 (plate 7 & 8). A full
description of the calibration procedure is given in
Appendix 6.

4.4   The non-contacting exciter

A non-contacting exciter was chosen as being necessary
since the response to excitation at various points of
the structure is required. Contacting exciters would
have contributed additional weight and required holes to
be made in the structure which would have altered the mode
shapes and natural frequencies and hence the overall response
of the structure. This is especially so since light folded
plate structures were investigated.

At the outset of this project a literature survey of current
research work revealed little information on the use of non-
contacting exciters. As a result it was decided to design a
suitable electromagnetic exciter, modifying it from experience
gained as the project progressed. Investigations were carried
out and calibrated using a simple beam experiment (Plate
9) as well as using a sensitive piezoelectric force
transducer in a housing made for the purpose (plate 10). The
complete calibration procedure is described in Appendix 7.
The electromagnetic exciter is basically made up of enamelled
copper wire would on a central core. The number of coils
used is limited by a corresponding increase in the heat
generated and so this was fixed at between 200 and 300 turns
for the investigation. Increasing the gauge thickness of the
windings also resulted in increased heat generation with the
increased current flow.

A search for an optimum exciter design was made into the
effect of changing the material of which the core is made
(plate 11). The first exciter tested was made with a
permanent magnet core. Tests revealed that hysteresis
losses were present in the core and as a result of the
considerable heat generated, further investigation of this
particular exciter configuration was abandoned. In an
attempt to eliminate hysteresis losses and to consider the
effect of omitting the permanent magnet core, a perspex
former was used as the exciter core. The result was a much
lower force output than obtained using the permanent magnet
core and moreover was found to be at a frequency of 60 Hz,

twice that of the 30 Hz input signal. This was attributed
to the absence of the permanent magnet core which provided
a net positive or negative field onto which the exciter
output would be superimposed. Thus in the absence of the
polarising field, a sinusoidal signal produces an attraction
when the signal is positive and yet another attraction even
though the signal goes negative the next time.

To creat a substitute for the effect of the permanent magnet
a second layer of coils was wound on top of the existing
and a d.c. voltage applied to it. This produced a considerable
increase in force output and also eliminated the frequency
doubling effect. However a marked temperature rise for the
higher d.c. voltage ranges in the inner coil caused over-
heating and melting of the enamel coating insulating the
copper coils.

A final investigation was made using a hollow iron bobbin
core. To prevent over-heating due to eddy currents in the
circuit formed by the ferrous bobbin, a slot was machined
completely through it and filled with araldite. This greatly
reduced the heat produced and whatever was produced was
quickly dissipated through the bobbin. Hysteresis losses
were much less that of the permanent magnet exciter and
force output much improved over the perspex core exciter.
This split core design is incorporated into the combined
exciter/pickup (plate 12) described in the next section.

4.5   The combined exciter/pickup probe

In order to obtain a compact transducer as well as to
alleviate the necessity of having separate traverse mechan-
isms for both the exciter as well as the vibration pickup
transducer, it was decided to combine the two into a single
exciter/pickup probe. Excitation and response are thus
measured at the same point on the structure. This has the
added advantage of effectively reducing the impedance to
vibration hence increasing the measurement accuracy.
Furthermore, for excitation at the antinodes there is less
tendency to rigid body motion and minimal excitation of
adjacent nodes (section 4.1).

The pickup transducer was easily incorporated into the
final exciter design in the hollow core of the former on which
the exciter windings were wound. This was made a good fit
and held in place with a grub screw. A cut away view of the
combined probe is shown in fig 4.1. The calibration proce-
dure followed is as carried out previously for the separate
exciter and pickup components and described in Appendices
6 & 7.

From tests made of the separate as well as the combined
transducers it was found that no interaction occurred between
the capacitance type vibration pickup probe and the induc-
tance type electro-magnetic exciter in the combined probe.
The result of the calibration of the combined probe in its
vibration measurement capacity is as follows:

1. The output of the probe in distance measurement is 1.85
    mm per volt displayed on the distance meter.
2. The output of the probe measuring amplitude of vibrat-
    ion is 2.25 mm/volt output (fig 4.2).
3. The effect of the gap distance between the probe and
    the vibrating surface measured is negligible. This
    is true provided the total distance (i.e. the amplitude
    of vibration + the mean distance between the probe
    and the measured surface) is less than 2.72 mm.
4. The frequency response of the probe (fig 4.3) is flat
    and independant of frequency in the range tested from
    30 Hz to 300 Hz.

Bobbin

DC 200 coils

AC 100 coils

Wayne Kerr vibration
pickup probe

Fig. 4.1   A cut away view of the combined probe.

Typical calibration curve of probe measuring
amplitude of vibration. (300 Hz)
Gradient of graph = 2.25 mm per volt outp

Amplitude of vibration (x 10^{-3} mm)

Probe Voltage (mV)
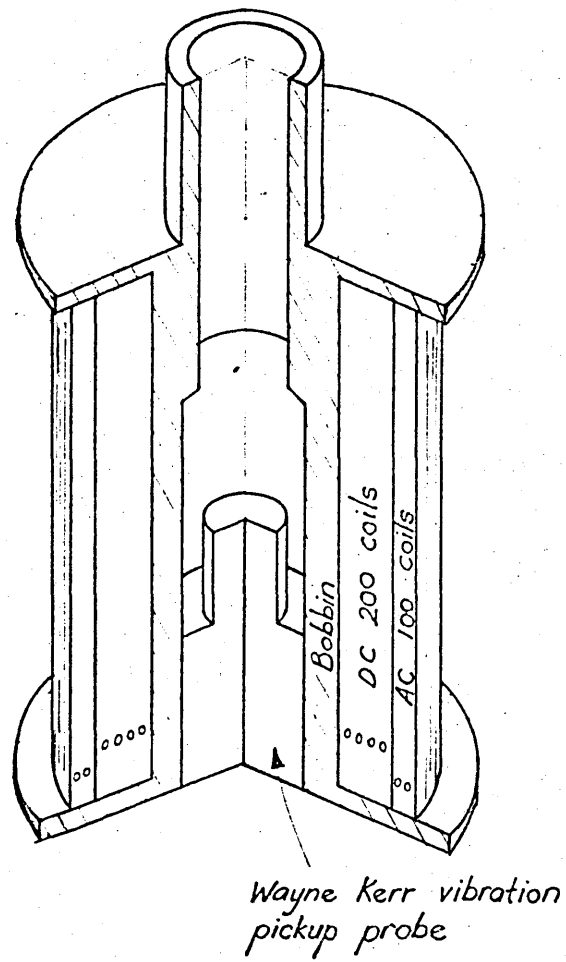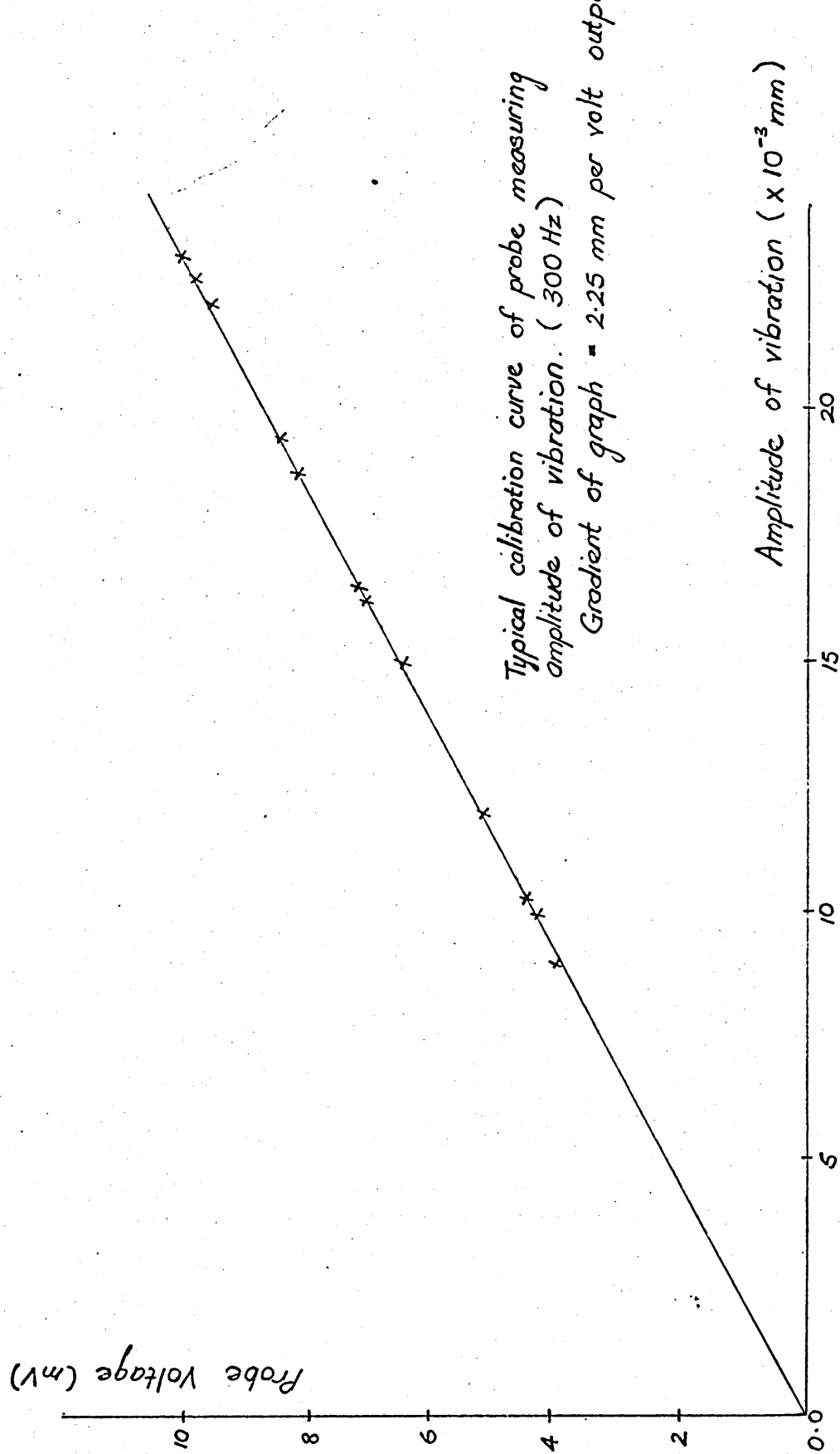
Fig. 4.2   Calibration of vibration pickup transducer.
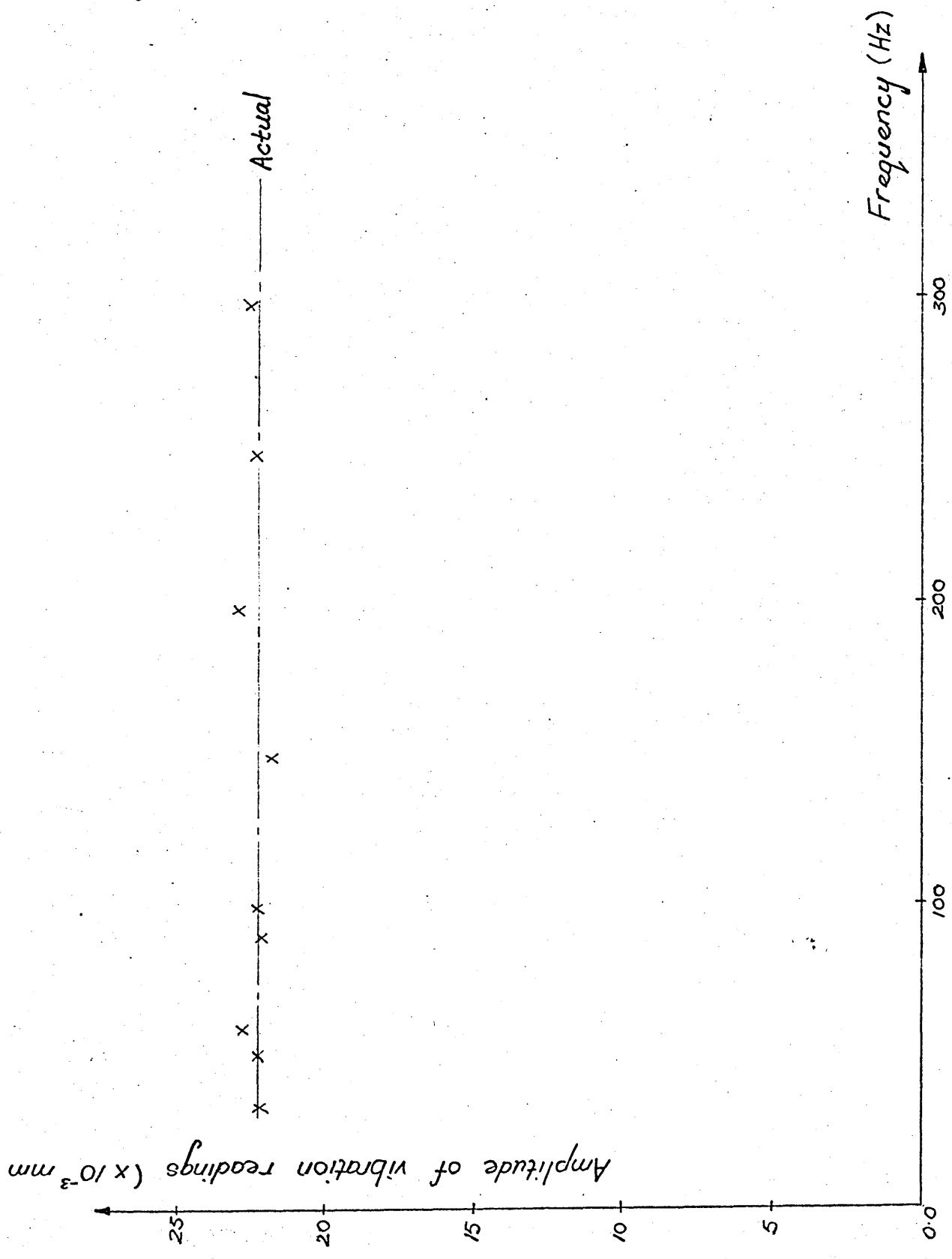
Fig 4.3 Frequency response of vibration measurement transducer.

The calibration of the combined probe in its excitation capacity is as follows:

1. The force output per input volt at a frequency of 100 Hz and probe-to-structure gap of 1 mm is shown in fig 4.4.

2. The change in force output with probe-to-structure gap at a frequency of 100 Hz and excitation voltage of 3 V_{rms} is shown in fig 4.5.

3. The frequency response at a constant excitation voltage and gap size is shown in fig 4.6.

In all these curves the d.c. polarisation voltage/excitation voltage is taken as unity. The effects of variation of this factor is given in McNulty [45]. The output excitation power spectral density to a PRBS signal input was also checked using the correlation method previously described in chapter 3.

Force output (N)

Volts.

$gap = 1 \, mm$

$frequency = 100 \, Hz$
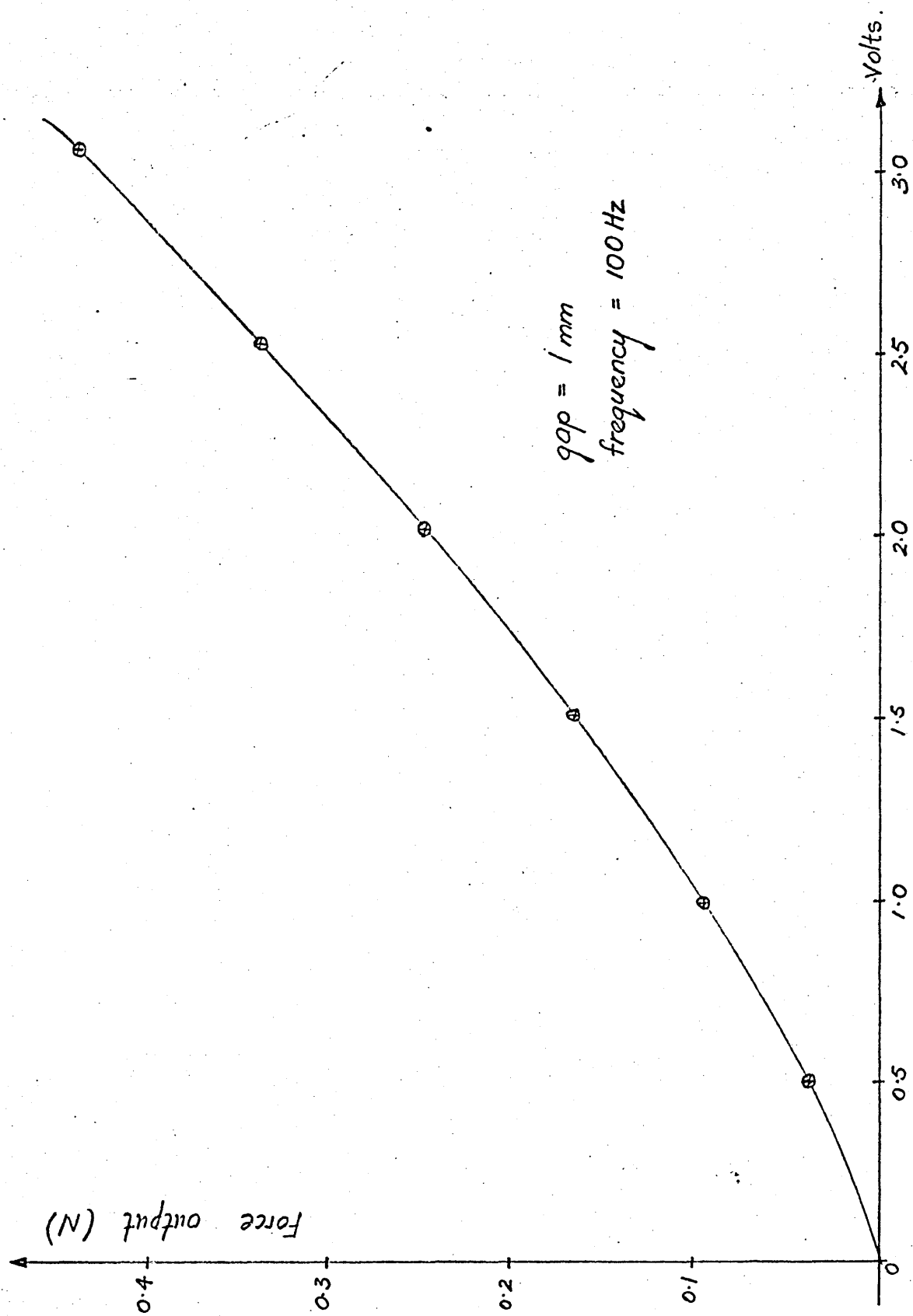
Fig. 4.4 Force output of non-contacting exciter

Force (N)

frequency = 100 Hz
excitation = 3 V rms

gap size (mm)
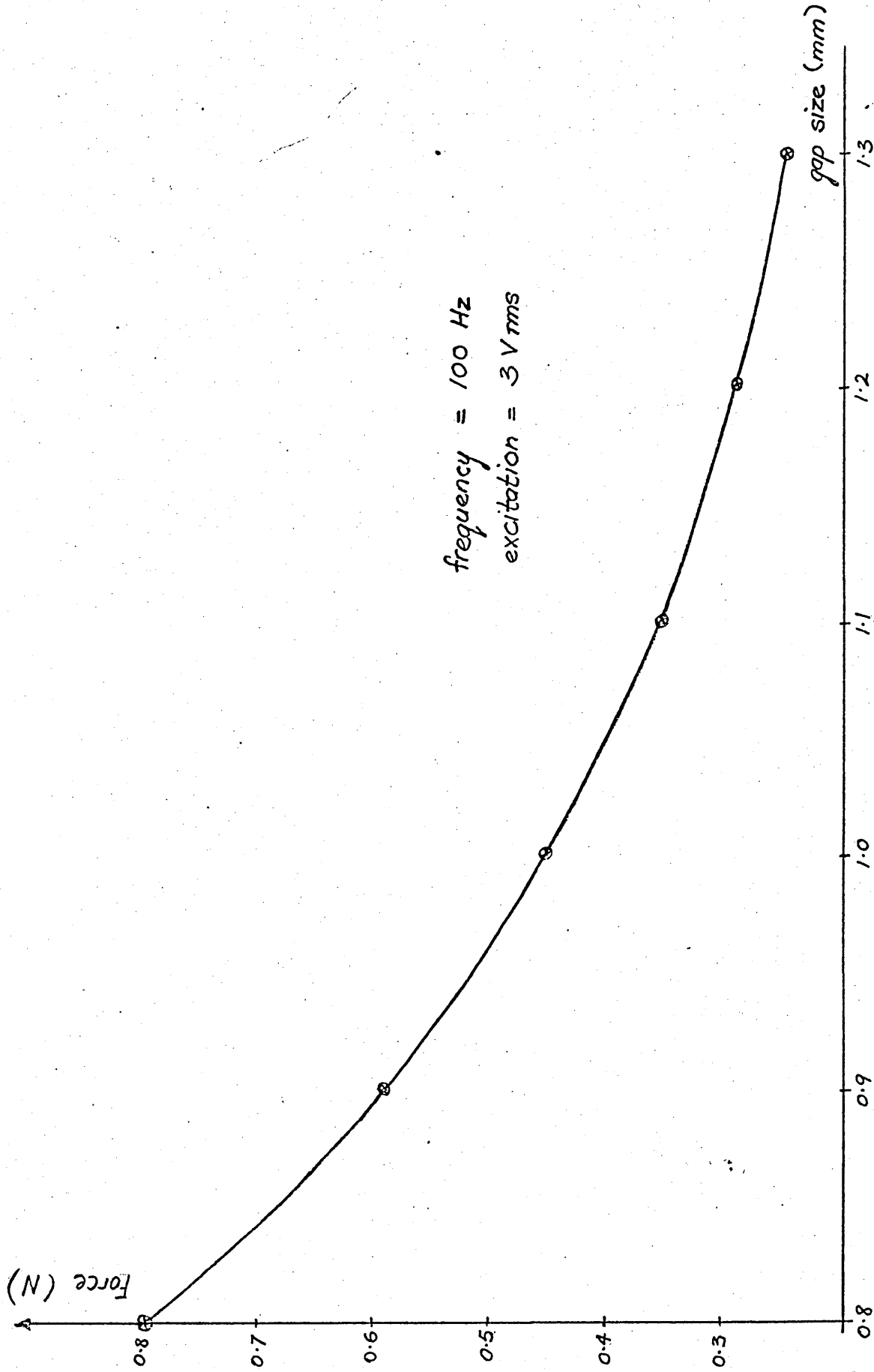
Fig 4.5   Effect of air-gap on excitation output.

excitation = 1 Vrms

gap size = 1 mm
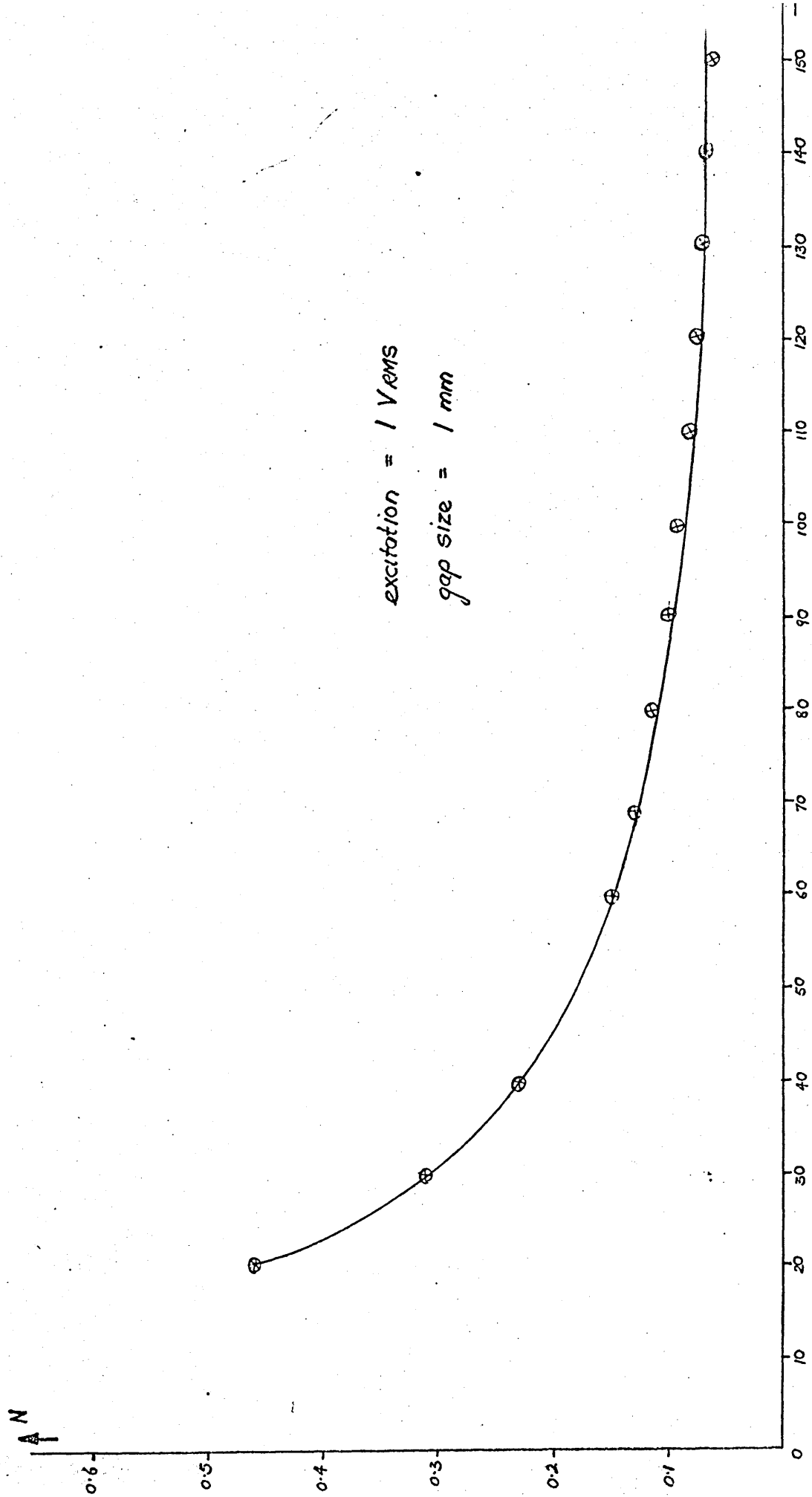
Fig. 4.6   Frequency response of non-contacting exciter.

Chapter 5   <u>Experimental technique</u>

<u>Summary</u>

This chapter describes how the correlation technique
(section 3.5) was applied to obtain the receptance of
the box structure despite the excitation being non-
white noise. It shows how the natural frequencies
and mode shapes of beams, plates and boxes were obtained
experimentally using the apparatus developed and discusses
the choice of the various parameters governing the pseudo
random binary sequence signal used for response measure-
ments of the box structure. It also describes how natural
frequencies and mode shapes obtained for a finite element
analysis of the box structure were used to predict the
response power spectral density using an excitation power
spectral density and damping factors obtained experimentally.
The Fourier transform computer program developed is presented
with flowcharts and descriptions and the advantages using
an optional faster algorithm is illustrated.

5.1   Introduction to the problem

Chapter 3 describes the general theory applicable to random
excitations and how the theory is applied to the special case
when the excitation has a flat spectrum. It is shown that in
this case the impulse response function of the structure can
be easily obtained directly from the cross correlation
between the input and response. (eqn 3.18 & 3.19)
This impulse response function can then be Fourier transformed
to give the receptance of the structure at the point of
excitation (eqn 3.20). Having used this technique to
determine the receptance, the response power spectral density
of the structure at the point of interest can be determined
from eqn (1.1) when the input power spectral density is
known.

In general engineering environments the excitation
experienced rarely has a completely uniform force spectrum
and often has one which tapers off at higher frequencies.
Also the advantages of a non-contacting exciter pickup
entailed the development of an electromagnetic exciter
which, unless sophisticated circuitry was provided, has
this same tapering force spectrum. Although an excitation
signal having a white power spectral density (the PRBS
signal) is applied to the system shown in fig 5.1 the
actual excitation of the structure is no longer white due
to the characteristics of the associated exciter pickup
equipment and the response spectral density at the point
of interest is difficult to obtain.

In this chapter therefore some approximations are made to
take such experimental factors into account by introducing
intermediate stages of analysis which basically depend on
the assumption of linearity in each stage. This is considered
to be valid because of the generally low signal levels used
in all the tests.

Referring to fig 5.1 let p(t) and d(t) be the input and
the response of the system respectively. Within the
system itself let p*(t) and d*(t) be the actual input and

SYSTEM

power amplifier $\xrightarrow{\quad P \quad}$ exciter $\quad H_e(\omega)$

$\downarrow P^*$

box structure $\quad H_B(\omega)$

$\downarrow d^*$

displacement transducer + meter $\quad H_P(\omega)$ $\xrightarrow{\quad d \quad}$

PRBS

Correlator

$p \ (\bar{s}_p = K)$

$K\,\bar{h}(\tau) \Longleftarrow R_{pd}$

$\bar{S}_d(\omega) \overset{2\pi}{\underset{S_p}{\Longleftarrow}} K\,\bar{H}(\omega)^\dagger \longleftarrow$ FFT
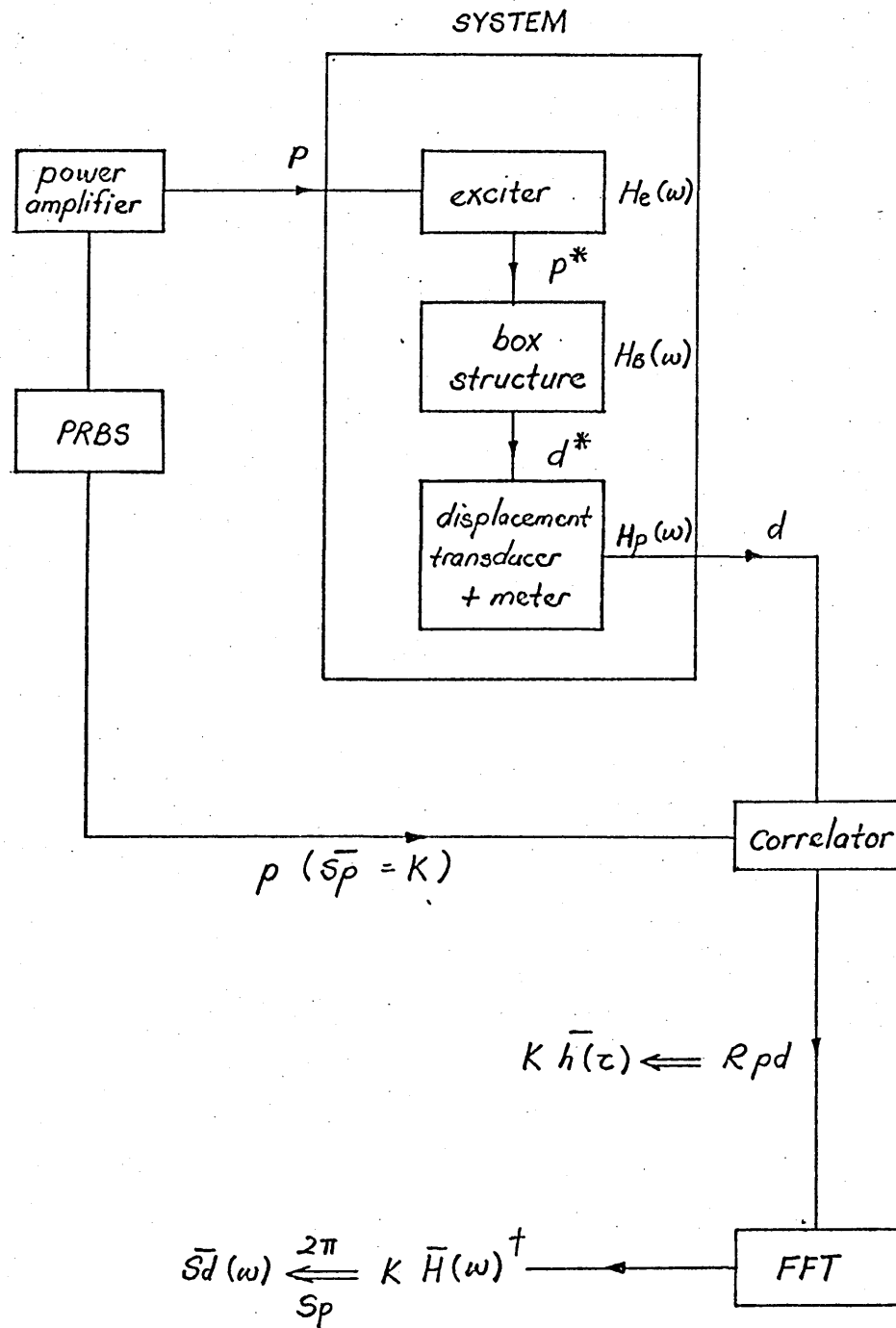
$\dagger$ where $\bar{H}(\omega)$ is the impulse response function of the system shown above.

Fig 5·1 Schematic plan of the method.

response of the box itself. Application of the correlation
method using the correlation of p*(t) and d*(t) would not
give the required impulse response function of the box since
p* is not white noise. However by considering the system as
a whole it will be seen that as far as the system is concerned,
a white PRBS spectrum is being applied to it so that the
necessary requirement for the correlation technique to be
applied to the system is satisfied. Therefore, using a
PRBS signal p(t) with a flat frequency spectrum of mean
square value K, the cross correlation function of the input
and output to the system, Rpd, gives the impulse response of
the system $h(\tau)$ multiplied by the factor K. The Fourier
transform of $h(\tau)K$ gives $H(\omega)K$ the frequency response
function of the system multiplied by K which from above is the
mean square value of the PRBS input. From $H(\omega)$ the system
response power spectral density can be obtained according to

$$Sd(\omega) = |H(\omega)|^2 \, Sp \qquad\qquad (5.1)$$

where Sp is a constant for the PRBS signal and given by

$$\int Sp(\omega) \, d\omega = K \qquad\qquad (5.2)$$

We have therefore obtained experimentally the power spectral
density of system response d. Now in order to obtain the
power spectral density of the response d* of the box itself
we have to go back a stage and consider the characteristics
of the displacement transducer which produces d from d*
(see fig 5.2) The displacement transducer converts displace-
ment d* into voltage d and with its associated equipment
provides the relationship

$$Sd(\omega) = |Hp(\omega)|^2 \, Sd*(\omega) \qquad\qquad (5.3)$$

where $Hp(\omega)$ is obtained from calibration of the displacement
transducer described in the previous chapter (fig 4.2 & 4.3).
This as we have seen is independant of frequency and therefore
a constant, Hp, which is given in units of volts/mm so that

$$Sd*(\omega) = \frac{1}{Hp^2} \, Sd(\omega) \qquad\qquad (5.4)$$

The response power spectral density of the box itself is
therefore obtained.

DISPLACEMENT

d^*

PICKUP

VOLTS

d
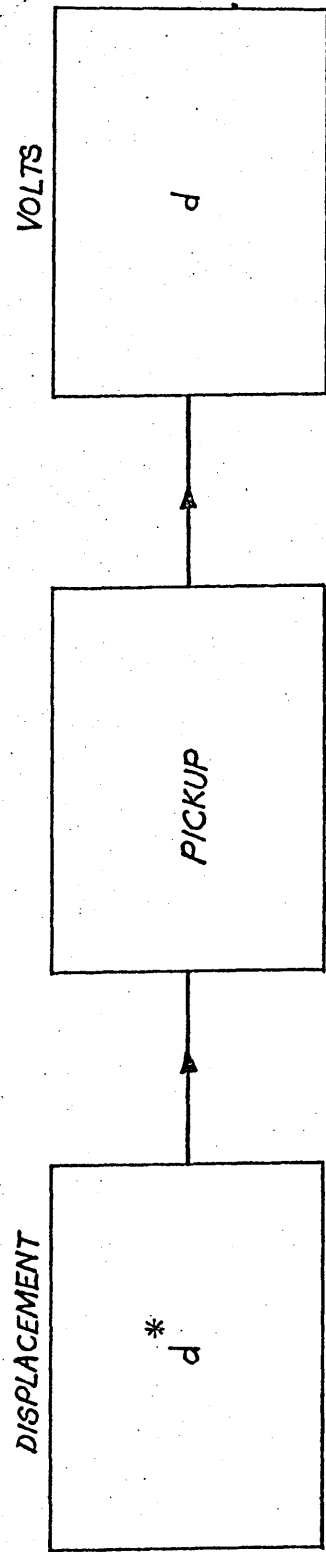
Fig. 5·2   The vibration pickup displacement-volts factor.

The aim of this work is to predict this response from knowledge of the characteristics of the box from its receptance $\alpha(if)$ (see chapter 3). In order to do this the excitation power spectral density to the box must be known. From fig 5.1 we see that although p has a white spectrum p* the actual box excitation does not have the same type of spectrum. In fact $Sp(\omega)$ to the system is modified on passing through the force transducer to become $Sp^*(\omega)$ to the box (fig 5.3). Looking therefore at the stage where the signal is converted into force we see that

$$Sp^*(\omega) = |H_e(\omega)|^2 \, Sp \qquad\qquad (5.5)$$

where Sp is again a constant given by eqn (5.2) and $H_e(\omega)$ represents the output characteristics of the non-contacting exciter at a particular amplifier setting. This is again obtained from the calibration curves given in the previous chapter (fig 4.6) but unlike the displacement transducer is frequency dependant to a considerable extent. Since we now know the power spectral density of the actual excitation to the box we can now attempt to predict the box response using the equation

$$Sd^*(\omega) = |H_B(\omega)|^2 \, Sp^*(\omega) \qquad\qquad (5.6)$$

where

$$|H_B(\omega)|^2 = |\alpha(i\omega)|^2$$

To summarise therefore we have (a) a method using the correlation technique of obtaining the response power spectral density of the structure experimentally and (b) an analytical method of predicting this using the natural frequencies and mode shapes of the box obtained using the finite element program developed. The next sections will discuss the experimental details which arose as work proceeded to determine an experimental response power density as well as a predicted one for comparison.

VOLTS p
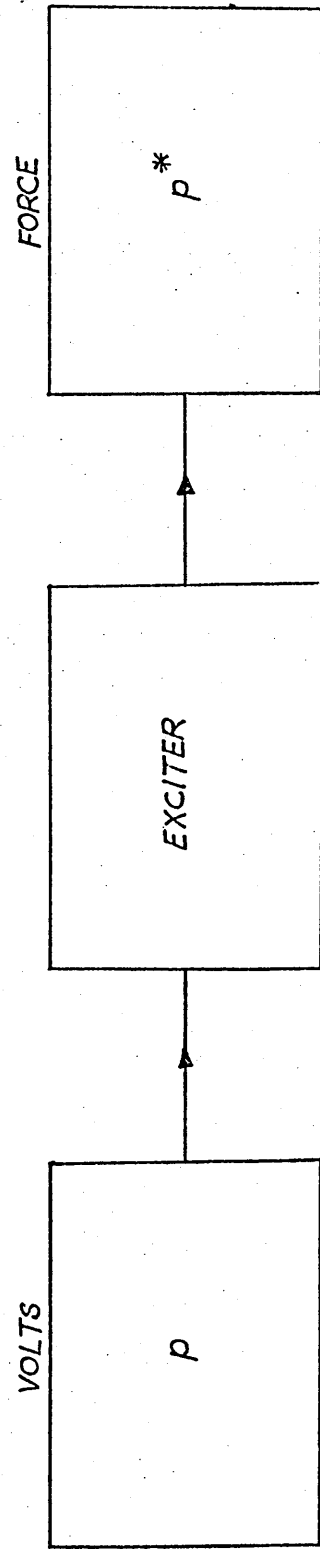
EXCITER

FORCE p^*

Fig. 5·3   The non-contacting exciter force-volts factor.

5.2   The excitation signal

For the random excitation of the box structure considered
in this work, the PRBS signal discussed in section 3.6
is used. The range of frequencies of interest of the box
chosen is from 20 Hz to 100 Hz which contained over 20
natural frequencies. Also it was found that at higher
frequencies the response becomes attenuated and natural
frequencies bunched together, making discrimination
between individual peaks difficult.

The PRBS excitation signal is controlled by several
parameters. The most important being the clock frequency,
the sequence length and the filter cut off frequency.
The choice of these depend on the following considerations.
The clock frequency chosen is not simply that of the
highest of the range of response frequencies required
even though from section 3.6 it could give the highest
frequency in the excitation signal. This is because in
the equipment used, the clock frequency is also the sampling
frequency. The sampling frequency is determined by
considering the spectrum using the Fourier transformation

$$X_k = \frac{1}{N} \sum_{r=0}^{N-1} x_r e^{\frac{-i2\pi k r \Delta}{N}} \Delta \qquad (5.7)$$

The properties of the transformation are briefly summed up
as follows. X_n is limited to the range $n = 0$ to $n = N - 1$
and repeats itself thereafter. This corresponds to the
frequency range 0 to $\frac{N-1}{N} f_c$. Moreover of this the
unique part of the series lies in the range 0 to $\frac{N-1}{2N} f_c$
only, and higher terms are mirror images of these. This
can be expected since the N bits of information can only
give the N/2 real and N/2 imaginary A and B components
of the first N/2 complete Fourier Transform terms. The
frequency $\frac{N-1}{2N} f_c$ is known as the Nyquist frequency or
the 'folding' frequency.

The result of the Nyquist frequency is that for a maximum

frequency of say 150 Hz to be realised in a signal, a
sampling frequency of at least 300 Hz is necessary.
Since the sampling frequency on the PRBS signal generator used
is also the clock frequency, f_c is therefore set at 300 Hz.
Now if frequencies above the Nyquist frequency of 150 Hz
are present in the signal a distortion of the Fourier
transform graph called aliasing is introduced. Setting
the clock frequency at 300 Hz however means that frequencies
up to 300 Hz will be present in the excitation signal
which will inevitably distort the results. Consequently,
to enable the use of a high clock frequency for accurate
sampling purposes and yet retain only those frequencies
in the range of interest, a low pass filter(fig 5.4)
is included. Known as a third order Butterworth filter,
it is flat to -3dB at f_b the upper bound frequency chosen,
then falling off at 18dB/octave, whereas the normal PRBS
spectrum has an envelope given by $(\frac{\sin x}{x})^2$ where
$x = \frac{\pi f}{f_c}$ with -3dB at 0.45 f_c (section 3.6).
Using this filter, clock frequencies of 300 and 1000 Hz
were used. The bulk of the results were however taken from
the former as they contain the main response frequencies and
also the latter gave an inaccurate disperse spectrum.

The accuracy of a spectral measurement depends on the
effective bandwidth Be Hz of the measurement and the
record length T sec. If σ is the standard deviation of
a measurement of spectral density of mean value m, then
Blackman and Tukey [46] shows that

$$\frac{\sigma}{m} \approx \frac{1}{BeT}$$ (5.8)

Thus for a required accuracy of $\frac{\sigma}{m} = 1$ with a frequency
resolution of 1 Hz using a clock frequency of 300 Hz,
a record of 1 second in length is required. i.e. N the
number of samples must be at least 300. For a required
accuracy of $\frac{\sigma}{m} = \frac{1}{2}$ with a resolution of 2 Hz requires a
2 second sample i.e. 600 samples using the same clock
frequency as before. Thus N is chosen to be at least
$2^9 = 512$ in the tests.

GAIN

0

−3dB

cut off at
18 dB/octave

f_b

0
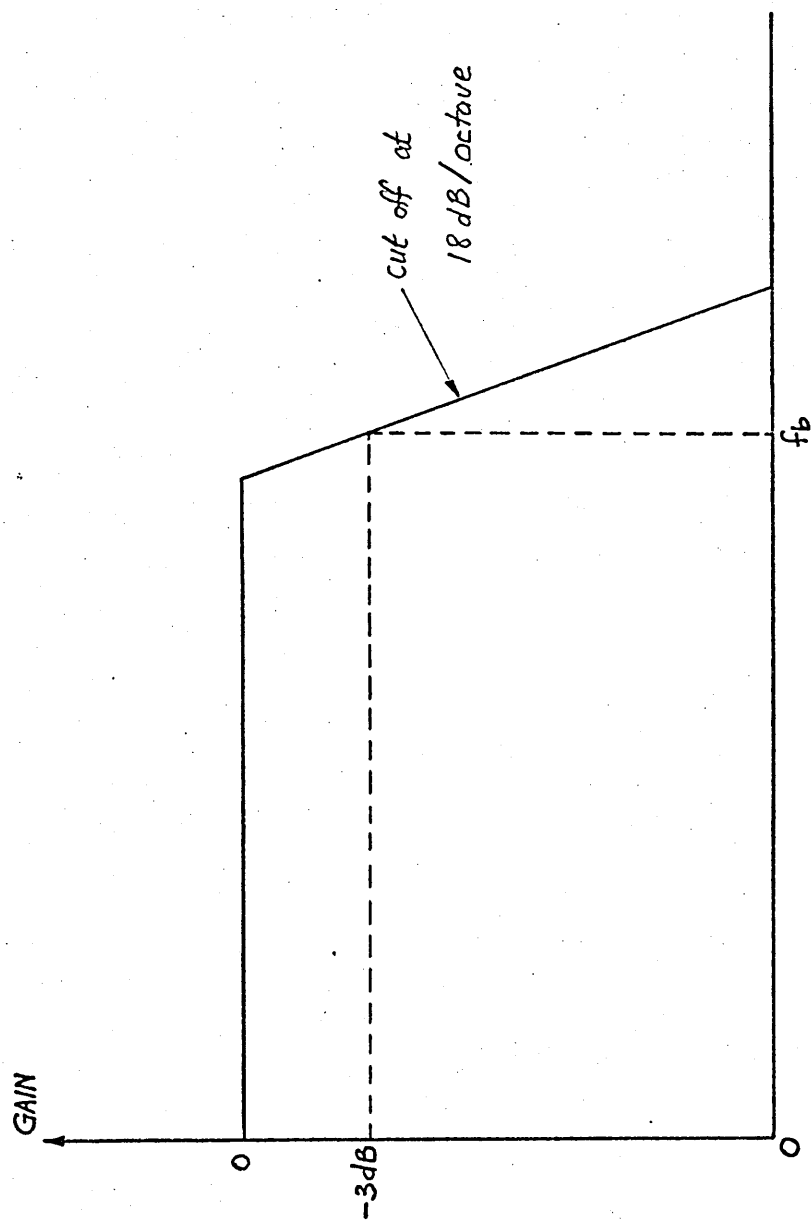
Fig. 5.4   Filter characteristics.

In practice, in order to ensure adequate stimulation at the resonances, the spectral line interval must be such that two spectral lines occur within the half power bandwidth of the resonance.

i.e. $\dfrac{f_c}{N} \leqslant f_h$ where (f_h) = half power bandwidth = (ηf_n)

so that $N \geqslant \dfrac{f_c}{\eta f_n}$ (5.9)

Taking typical values used for the structure

f_c = 300 Hz, η = 0.018, f_1 = 24.6 Hz

$N \geqslant \dfrac{300}{0.45} = 666$

It is evident from (5.9) that only the first natural frequency is critical in determining the minimum number of data N required in the sequence. N is limited physically by the equipment available and on the Solartron equipment used the largest number available with correlation in one sequence is 1023. However, run times increase rapidly as N is increased and a typical time required for a test was approximately $3\frac{1}{2}$ hours using N = 511 and 11 hours using N = 1023 in correlation x 10 mode using fifteen sequences per trigger.

Using the apparatus which was described in the previous
sections, the experimental investigation was carried out
to determine the response of folded plate structures to
random excitation.

An open ended box structure as shown in fig 5.5 was tested.
This was made from four thin plates each of thickness twenty
thousandths of an inch and soldered together along their
edges at right angles. This formed a simple structure which
had a low fundamental frequency and was easy to vibrate.
The box was fastened to supports at the corners of its
lower surface (Plate 6) to approximate simply supported
boundary conditions at the corners. These supports were
themselves attached to the vibration table with large G clamps.
In the preliminary stages of the investigation, work was
also done to determine the natural frequencies, mode shapes
and damping factors of a fully fixed beam, a beam simply
supported at both ends and a flat plate with fully fixed
edges all round (Plate 5). Results for these are included
in chapter 7.

In the course of the experimental investigation the natural
frequencies and mode shapes of the box were obtained.
Difficulty in obtaining the damping factors from the response
curves was experienced (see section 5.3.3). The cross correla-
tion function of the input and response signals were also
obtained experimentally for the box structure when excited
by white noise generated using a P.R.B.S. signal generator.
This gave the impulse response function of the box system
which, when Fourier transformed, gave the receptance of the
box at the point under consideration. The experimental
procedure is discussed in detail in the following sections.

255

255

5/5
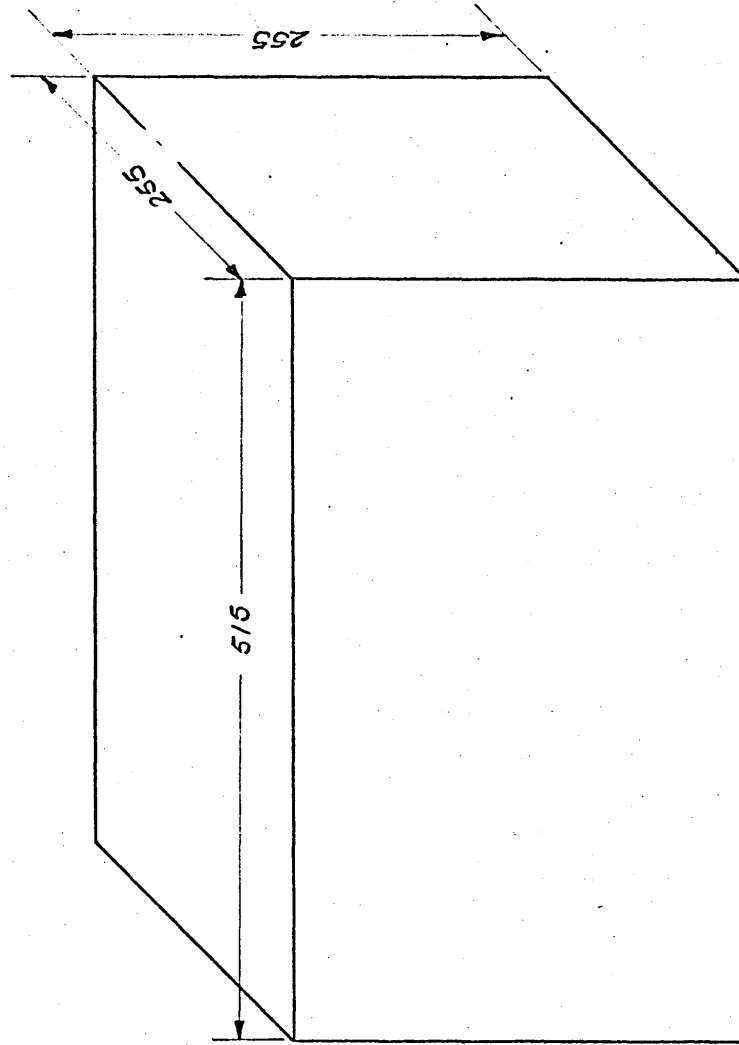
Fig. 5.5   The open-ended box structure used.

5.3.1 Determination of natural frequencies and mode shapes

The natural frequencies and mode shapes were determined using sine wave excitation. The sine wave was generated by a Muirhead decade oscillator which was connected through a power amplifier as shown in figure 5.6 to the combined exciter pickup and a d.c. bias applied as described in section 4.5.

To obtain the natural frequencies, the pickup is positioned at a suitable point (antinode) on the structure and resonant frequencies located by adjusting the excitation frequency from the decade oscillator until the amplitude of vibration is at a maximum (resonance). The excitation frequencies when the vibration amplitudes are greatest are measured using the narrow band frequency analyser and correspond to the natural frequencies of the structure under consideration. Natural frequencies in the range 20 to 100 Hz were obtained for the box structure. The natural frequencies of the beam, plate and box structure analysed are given in tables 7.1, 7.2 and 7.3 in section 7.1.

The mode shape corresponding to each natural frequency was obtained by measuring the amplitude of vibration at a sufficiently large number of points on the structure to define it, keeping the excitation force and frequency fixed. An approximate idea of the mode shape is first obtained using an accoustical method by traversing a hand held microphone across the surface. This was then followed-up using the probe and traverse mechanism using the following technique to give the accurate mode shape.

The combined exciter probe is positioned at the required point using the traverse mechanism and the probe-to-structure airgap checked using the displacement calibration curves for the probe (section 4.5). This is necessary as the force output by the probe is affected by the size of this gap (figure 4.5). The air gap between the probe and
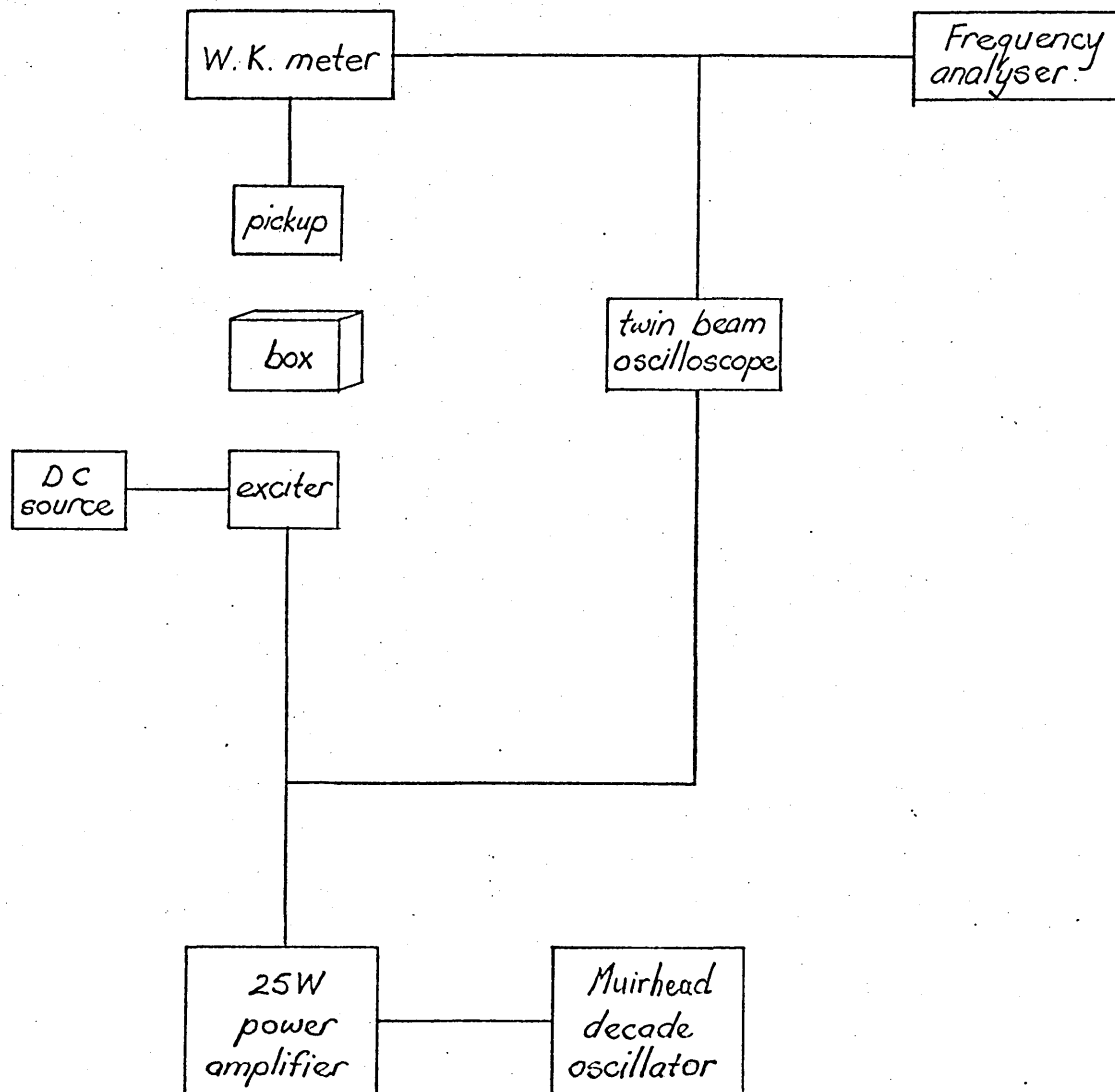
Fig. 5·6   Block diagram of instrumentation for discrete excitation of box structure.

the structure is adjusted to an approximately parallel
position with the fine adjustment mechanism. A block
diagram of the necessary instrumentation used is shown
in figure 5.6. The actual force input to the box is
given by the current into the probe and is read off the
appropriate calibration curves corresponding to that
frequency and probe-to-structure air gap. In determining
the mode shape at each natural frequency the amplitude
of the response signal is measured using a constant
percentage frequency analyser at the low frequency ranges,
shown in Plate 13, and a constant bandwidth frequency
analyser (Plate 14) at the higher frequencies. The
actual response amplitude is then given off the probe
vibration measurement calibration curves. The response
at each of the points chosen were then obtained by
traversing the probe to each of the points in turn
keeping the excitation force and frequency constant,
giving the required mode shape. To plot any particular
mode shape the knowledge of the relative values of the
various amplitudes is sufficient although the receptance
described in the next section requires the absolute
amplitudes to be ascertained.

For the beam structure the exciter probe is simply traversed
along its length and keeping the excitation force constant,
measuring its response at a number of points sufficient
to describe the particular mode shape. Again for the mode
shape only relative values of response are required. These
are illustrated in Table 7.1. The mode shape of the plate
is obtained by dividing the plate into a grid (figure 5.7)
and determining the response at each of the grid points.
The fineness of the grid required depends on the mode
shape to be measured. Some of the lower natural frequencies
and mode shapes of the plate analysed are also indicated
in Table 7.2, described in terms of the number of half
wavelengths along and across the plate. The same procedure
is followed for the determination of the mode shapes of
the box structure since only the top plate of the box
structure was measured using the traverse mechanism. Figures
7.1 to 7.11 show the vibration modes of the box existing
between 20 Hz and 100 Hz.
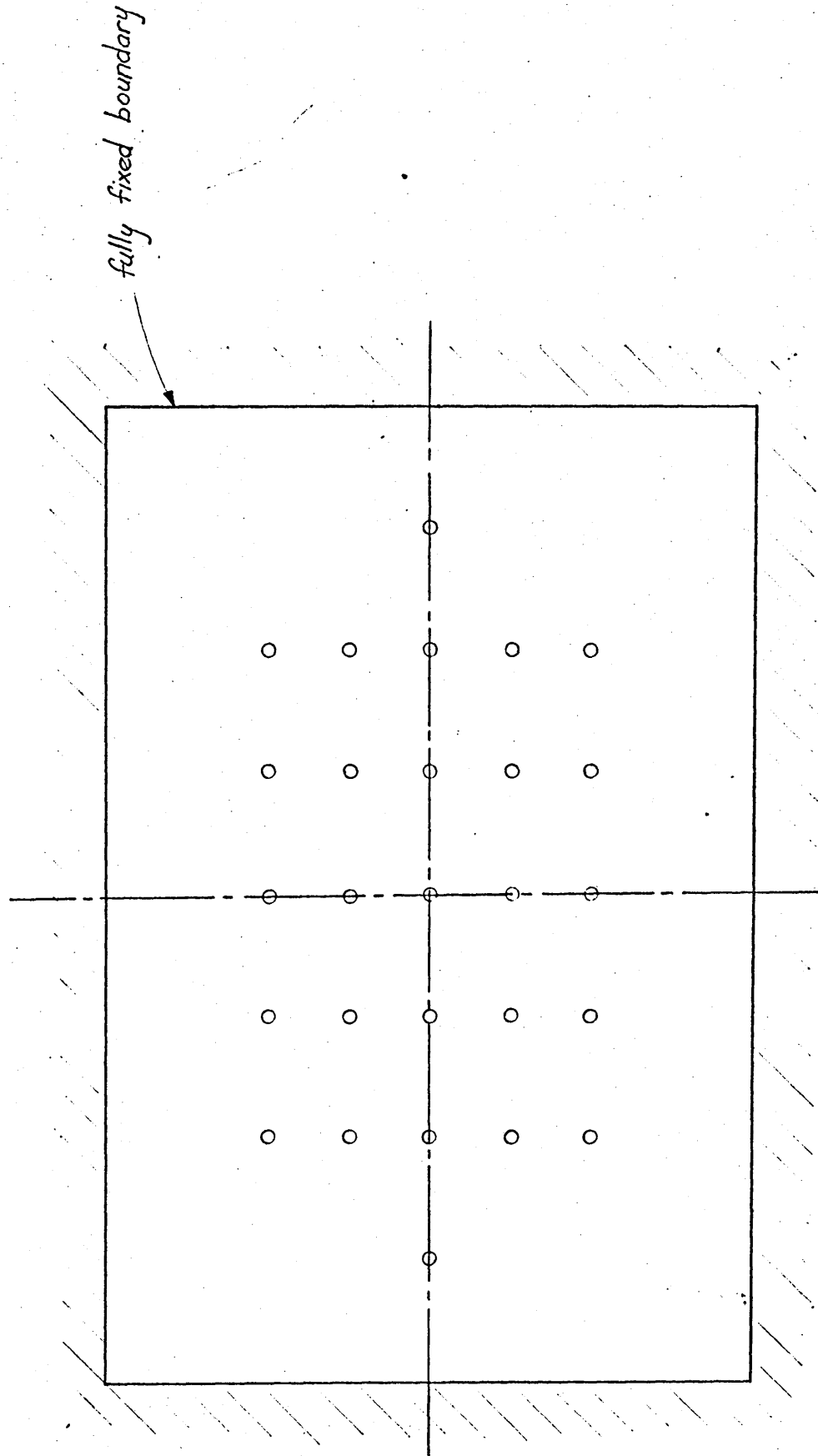
Fig. 5.7 A typical grid for determining mode shape of a fully fixed plate.

Because of the tapering shape of the force spectrum
created by the exciter probe the response to a sweep
test and hence the 'receptance' obtained of the box
tested was not that of the generally accepted response
to a unit force spectrum. A correction to the results
obtained by the sweep test was therefore necessary.
This factor was given by the conversion of the
tapering force spectrum to that of a constant force
spectrum of unit magnitude.

The receptances of the box were determined experimentally
at various points using sine wave excitation from the
Muirhead decade oscillator which was connected as before
through the power amplifier to the combined exciter-
pick-up. The probe was positioned in turn at each point
of interest and the gap between probe and structure was
measured and adjusted to give a suitable force output,
the magnitude of which is obtained from the probe
calibration curves (figure 4.4). Having adjusted the probe-
structure gap, the structure was excited at discrete
frequency intervals of 1 Hz in the range 20 Hz to 100 Hz.
In the region of a resonant, the frequency interval was
reduced to 0.5 Hz. At each of these frequencies the
magnitude of the displacement of the structure was measured
using a frequency analyser as described previously. From
the probe calibration curves the force corresponding to
that excitation signal amplitude, frequency and gap size
was determined. The response amplitude corresponding to a
unit force can then be obtained. This procedure is repeated
over the range of frequencies selected and the receptance of
the box at the chosen points obtained. The receptance of
the box using this technique was obtained at the points
which are indicated in figure 5.8 and from it the response
power spectral density was obtained.
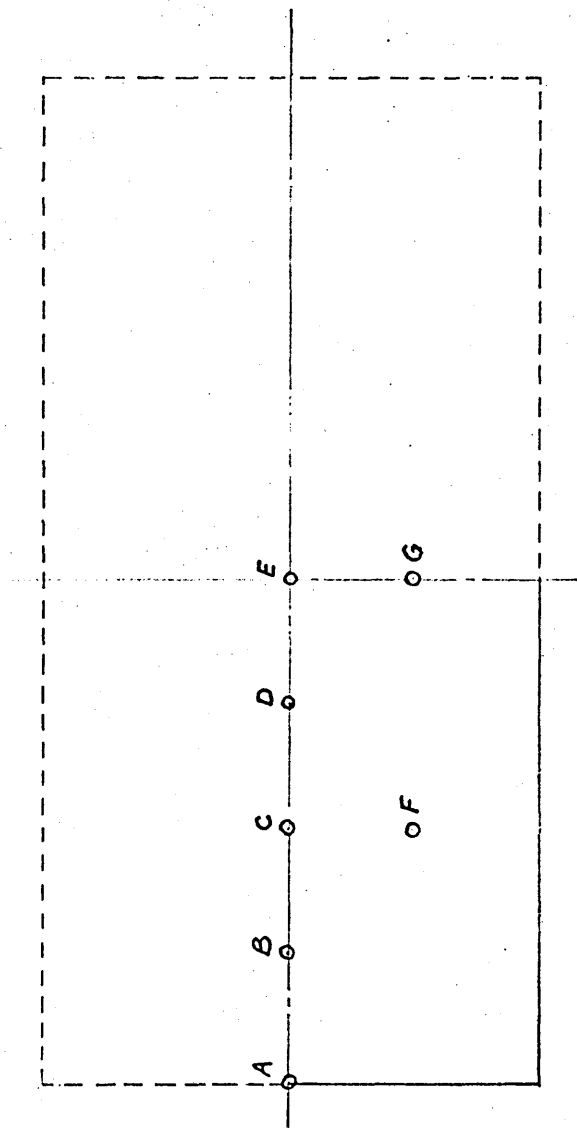
Fig. 5.8   The grid used for determining receptance and response spectral density of the box.

5.3.3   Determination of damping factors.

In this investigation the response of the box structures
to vibration stimuli is required. In the ideal case all
the excitation energy put into the system is converted into
response motion. In practice this is not so.

When the structural material is cyclically stressed,
energy is lost either through radiation to the surrounding
medium or is dissipated in the material. There are various
commonly accepted theories [5] & [47]. In structural damping the
assumption of hysteretic damping has been found to be a
realistic concept where damping is proportional to displace-
ment rather than velocity as in viscous damping. The
equation of motion

$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t$$

then becomes

$$m\ddot{x} + (1 + i\eta)kx = F_o \sin \omega t \qquad\qquad (5.10)$$

where η the non dimensional damping loss factor is defined
as the ratio of the energy loss per cycle to 2π times the
energy stored during that cycle. For linear systems
for small amplitudes, η is usually
taken as a constant, η_r, at the natural frequency
ω_r. Although this is not strictly correct at other
frequencies it is acceptable because the response in a
lightly damped structure is dependant on η_r only at its
natural frequency ω_r and would be virtually insignificant
elsewhere. This holds for multi-degrees of freedom systems
if any possible coupling of modes due to damping is neglected.

From chapter one eqn 1.2 the calculation of the receptance of
the box structure requires the knowledge of this damping
loss factor. This is calculated experimentally using its
frequency response curve (fig. 5.9)
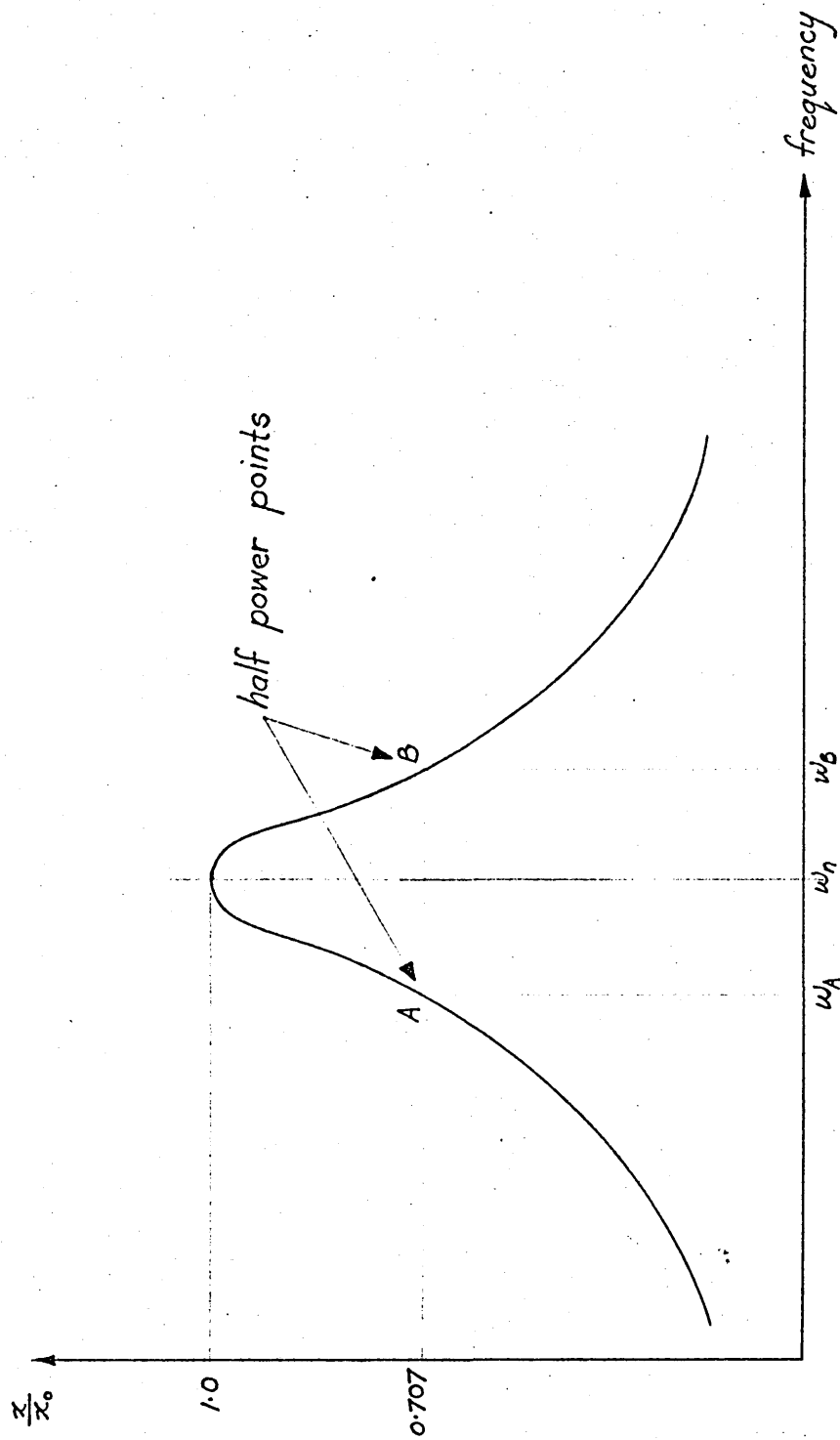
Fig. 5.9 Determination of damping factors using half power points on the response curve.

The separation ($\omega_B - \omega_A$) between the frequencies associated with the points A and B, where $x/x_o = 1/\sqrt{2}$, increases with damping. The non-dimensional ratio ($\frac{\omega_B - \omega_A}{\omega_n}$) is therefore a basis for determination of the damping loss factor η .

The method used, of determining the damping loss factor of a structure from its resonance curve is known as the half power point method. The name originates in the relevant points A and B which are known as half power points since the power during steady state conditions in a linear system is proportional to the square of its amplitude. These points for each mode of vibration of the structure are therefore experimentally obtained and the corresponding frequencies used to calculate the damping loss factor for that particular mode.

This method worked satisfactorily for the preliminary work, where simple beams and plates were used, having their individual resonance peaks well defined and separate. However for the box structure itself, apart from the lowest natural frequencies, some overlapping and coupling of modes occur. As a result approximate values were used, based upon experience gained on the simpler structures and assumption of unimodal damping at the higher frequeucies.

Unfortunately in the event it was found that the method used was not completely satisfactory and possibly an over-simplification and gave rise to inaccuracies in the calculated response values in the higher frequency regions. Other methods of experimentally measuring the damping of structures exist (Kennedy and Pancu [48] , Pendered and Bishop [43] and Bert [49]) which may therefore be more profitably used.

The PRBS signal described in section 3.6 was chosed as a
suitable excitation signal to simulate random excitation of
the box structure. The parameters of the particular PRBS
signal chosen to excite the box was described previously
in section 5.2. In order to ensure adequate stimulation
of close adjacent resonance peaks, sufficient numbers of
spectral lines must be chosen. Thus sequences containing
512 and 1024 samples were used.

Since response frequencies of up to 100 Hz were required the
sampling rate necessary to satisfy the Nyquist criterion,
as described in section 5.2, is at least twice this. On
the equipment available, the sampling rate is determined by
the clock frequency f_c and the nearest frequency available
was 300 Hz. Two complete sets of values were obtained, using
first a clock frequency of 300 Hz and after that the next higher
clock frequency of 1000 Hz. Since the clock frequency is
also the highest frequency of the signal, a low pass filter
was necessary to prevent aliasing (section 5.2).

The points chosen at which the cross correlation measurements
were obtained are shown in fig. 5.8. The exciter probe was
first positioned at one of the points required. The air gap
was then checked using the appropriate calibration curves
and adjusted if necessary to a predetermined value. The PRBS
signal was then applied to be main coils of the exciter via a
power amplifier (and a d.c. bias to the secondary coil) and
displayed on the oscilloscope is shown in the block diagram
(fig. 5.10). A true RMS random noise voltmeter was used to
measure the excitation signal. The response signal picked
up by the displacement transducer is also displayed on the
oscilloscope and fed into the time domain analyser and a
narrow bad analyser. Care is taken to record the position
at which measurements were made as well as all the ampli-
fication factors of both the excitation and response signals
for converting back into the original units later.
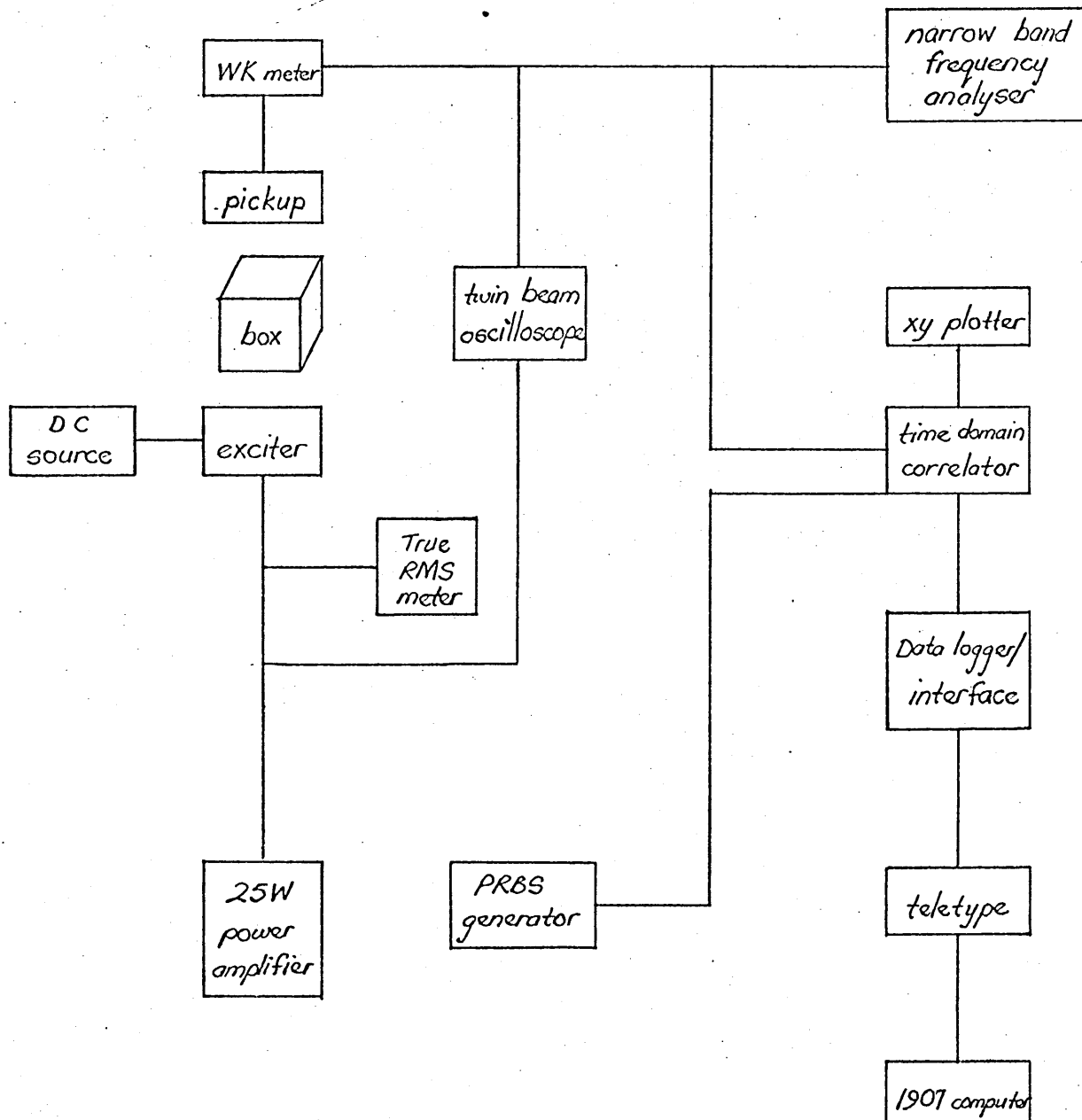
Fig. 5.10 Block diagram of instrumentation for determination of response of box structures to random excitation.

The time domain analyser calculates the cross correlation
function of the excitation and the response signals and
produces it on paper tape via an interface with a teletype.
The paper tape produced is compatible with the paper tape
reader in the ICL 1907 computer where it is to be processed
using the FFT program described in the next section. The
procedure is then repeated at each of the points shown in
fig. 5.8, a different point receptance being obtained at
each respective point as it is position dependant as discussed
in section 3.3. The instrumentation used is shown in Plate 15.

In this work the response spectral density of the box
structure is calculated from the receptance of the box
structure system (section 5.1) as obtained experimentally
by the Fourier Transformation of the cross correlation
function of the PRBS excitation to the box and the resultant
response of the box.

In chapter 3 it was seen that a considerable saving in the
number of complex multiplications and additions is made using
the Fast Fourier Transform algorithmn. The resulting time
saving achieved also enables the Fourier transform to be
obtained much more economically than by the normal straight-
forward operation.

The Fourier transform computer programs developed, MOJFTM1
and MOJFTM2 (fig. 5.11), contains both the FFT and the DFT
algorithms in separate subroutines either of which can be
called up. MOJFTM1 is used for transformation of correlation
functions containing up to 512 points and can be run as a
small express job on the University computer whereas MOJFTM2 has to be
run as a medium priority job and is used for the transform-
ation involving more than 1024 points.

The cross correlation function of the excitation and the
response of the structure is obtained from the time domain
analyser in the form of a paper tape via an interface with
a teletype in a code compatible with the computer. This
program is written to read this data directly off the paper
tape displaying the cross correlation function on a
graphical output (see chapter 7,fig. 7.14) as well as on a
lineprinter output with the exact values obtained, for an
immediate and more accurate output (fig. 7.15). The results
of the Fourier transformation (the receptance) may also be
graphically displayed as well as printed out on the computer
lineprinter output. Finally the response spectral density
is calculated (section 3.2) and displayed in the same manner.
This program also has the facility for producing a linear

Figure 5.11   Computer program MOJFTM flowchart.

```
( Read in data on paper tape )
            |
            v
        / 2^n data \ ----No----> [ Set up zeroes ]
        \          /                     |
            |                            |
           yes                           |
            v                            |
( Display input data on graph plotter )<-'
            |
            v
         / FFT \ -------No----------------------------.
         \     /                                       |
            |                                          |
            v                                          v
     [ Compute FFT ]                          [ Compute DFT ]
            |                                          |
            v                                          |
( Print out result of Fourier transform )<------------'
            |
            v
( Specify Excitation RMS value )
            |
            v
[ Calculate excitation power spectral density ]
            |
            v
[ Calculate K factor ]
            |
            v
[ Calculate receptance of system ]
            |
            v
( Specify amplification of pickup )
            |
            v
[ Calculate box response P.S.D. ]
            |
            v
[ Degree of smoothing required ]
            |
            v
( Print & graph response P.S.D )
            |
            v
        ( End )
```

or a logarithmic graph as chosen.

Also incorporated into the computer program package developed
are subroutines using which smoothing of the coarse peaky
results could be obtained. Figures 7.16-7.21 are typical
traces of the response power spectral density obtained at
the same point but using a pair of PRBS signals of different
sequence lengths and with various degrees of smoothing.
This is done by averaging (2N + 1) adjacent values to give
smoother graphs as N is increased. This, as discussed in
Newland [39], also has the effect of improving the statistical
accuracy of the result (at the expense of frequency
resolution) which now becomes

$$\left(\frac{\sigma}{m}\right)_{new} = \left(\sqrt{\frac{1}{2N + 1}}\right)\left(\frac{\sigma}{m}\right)_{old}$$

so that from eqn (5.8)

$$(Be)_{new} = (2N + 1) \ (Be)_{old} \qquad\qquad (5.11)$$

The superiority of the FFT algorithm is clearly demonstrated
in the following requirements for two jobs one using the
FFT and the other the DFT algorithm on a sample of 512 points.
The former required less than $2 \times 512 \times \log_2(512)$ computer
manipulations compared with 512^2, and took 80 sec and 971 sec
respectively. The saving achieved on a sample of 1024 points
would be even greater.

The results obtained using these computer programs are given
in chapter 7.

Chapter 6   The computer application

Summary

The previous chapter describes the experimental derivation
of the response of the box structure. Here a computer
prediction of the same is described.

6.1   Response prediction

From chapter 3 (equation 3.14) it was shown that the natural
frequencies and mode shapes could be used to calculate the
receptance of a structure from which the response power
spectral density to any known excitation can be predicted.
This section deals with the application of this method to
the prediction of the response power spectral density of the
box structure to the PRBS excitation used. The natural
frequencies and mode shapes of the box in free vibrations
were calculated using the finite element method (chapter 2).
The hysteretic damping loss coefficients were found experi-
mentally (section 5.3.3). The excitation power spectral
density to the box itself was found experimentally (chapter 4).

The receptance of a structure, given in terms of its natural
frequencies and mode shapes, is given by equation 6.1

$$|\alpha_{dp}(if)|^2 = \sum_r \frac{w_r^2(x_d) w_r^2(x_p)}{16\pi^4 M_r^2 \left[(f_r^2 - f^2)^2 + \eta_r^2 f_r^4\right]} \qquad (6.1)$$

In the equation only mode shapes, in relative units, such
as that given by a finite element analysis are required.
The actual units of the calculated receptance are given
by the generalised mass M_r used, by specifying the units
in which the mass of the structure is chosen and will be
discussed later in this section. In this work using the
S.I. units of measurement in M_r the predicted receptance
squared will be in m^2/N^2.

In the experimental work both the excitation and response
measurements were concerned with transverse displacements
of the top surface of the box. In the computation of the
receptance of the structure therefore only the transverse
displacement degrees of freedom are required. In the
finite element idealisation, the open ended box structure
is divided into twenty-four elements as shown in fig. 6.1.
This gives a total of 50 transverse degrees of freedom which
describe the deflected shape of the structure in each
vibration mode. This will also be required in the calculation
of M_r, the generalised mass for the r - th mode of vibration
of the structure. (eqn 6.2)

In the choice of the number of natural frequencies and mode
shapes required to adequately determine the receptance, by
considering its magnitude it will be seen that only frequencies
in the immediate vicinity of the natural frequencies are
important. The frequencies of interest are taken to range
up to 100 Hz, covering some twenty natural frequencies of
the box structure. The computation requires that sufficient
natural frequencies be taken for convergence to the true
value at the upper frequency limit. However, frequencies
much above this becomes redundant as their contribution is
negligible. Also since comparison with experimental results
is one of the main aims of this work, the range of the range
of the instrumentation must also be considered. In this
case only the lower frequency limit of 20 Hz is important as
the upper limit is well above the 100 Hz limit chosen.
Therefore in the computation only the 21 natural frequencies
and their corresponding mode shapes occuring within this band
of frequencies are required.

In the calculation of the generalised mass of the structure
when vibrating in mode r
$$M_r = \int w_r^2(x)m\ dx \qquad\qquad (6.2)$$
where x represents the points at which a transverse degree of
freedom occurs and m is the mass of the structure assumed
to be evenly distributed amongst these points. A summation
is then taken of the product of the square of the mode shape
and the distributed mass over the points on the structure.
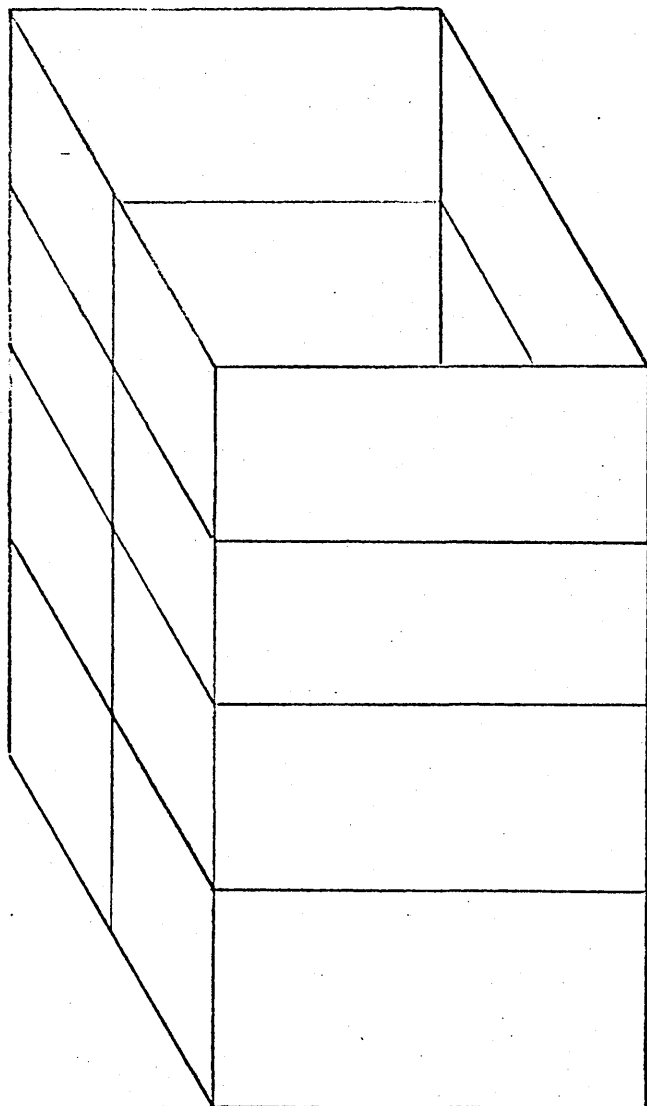
Fig 6.1   The 24 element idealisation of the box structure.

The receptance is then calculated according to equation
6.1 summing over the number of nodes required. Once
the receptance is obtained, the response spectral density
is calculated according to equation 6.3.

$$Sd(f) = |\alpha(if)|^2 Sp(f) \qquad\qquad (6.3)$$

where now both the excitation power spectral density $Sp(f)$
and the receptance $\alpha(if)$ is known.

From chapter 2 it was shown how the finite element method
of analysis was applied to the prediction of the natural
frequencies and mode shapes of a box structure in free
vibration. In the previous section it was further shown that
the natural frequencies and mode shapes could be used to
calculate the receptance of the box and from it to predict
the response power spectral density if the damping and
excitation power spectral density were known. In chapter 5
the experimental derivation of both the damping and the
excitation power spectral density has been described. A
computer based prediction of the response power spectral
density of the box is therefore possible and was carried out.

The suite of computer programs is given in full in Appendix 3
but a brief description of their development and details of
their operation will be discussed here. Because of the size
of the problem the computation was divided into 7 programs
each communicating with the following program via a magnetic
tape or disc peripheral. This strategy also enabled each
program to be developed independantly of the others and in
fact each program was developed and run on different
computers.

The various programs were written and tested on the Sheffield
Polytechnic's IBM 1130 computer, a small machine now
gradually being replaced by a new IBM 360. The IBM 1130 has
a core store of only 10K so that from the start it was
realised that the actual computations would have to be done
on that of the collaborating institution in this project,
the University of Sheffield. This was a ICL 1907 computer,
a medium sized computer with up to 70K of core storage.
The programs were therefore written in a Fortran language
subset that was common to both computers.

In 1974 the University acquired a share of the use of the CDC
7600 of the University of Manchester Regional Computing Centre
with the installation of a satellite terminal. This is a
large and fast machine with up to 124K of storage available

for any job. The programs were therefore modified to run
on this computer to take advantage of the much faster
computing times.

The first program MOJFEM1 fig. 6.2 calculates the stiffness
matrix for the element for the specified element geometry
and the material properties. It then assembles all the
element matrices into their proper places in the system
stiffness matrix for the box structure as shown previously
in fig. 6.1. This is then written to a file for storage.
The same procedure is then followed for the mass matrices
re-using matrices previously used for the stiffness matrices.

MOJFEM2 fig. 6.3 then retrives the stiffness matrix for the
complete box and reduces out the redundant degrees of free-
dom caused by the boundary conditions specified. A check
is then made for the preservation of symmetry in the matrix
after which it is written back to the storage file. This
is repeated for the mass matrix.

MOJFEM3 fig. 6.4 checks that the final reduced mass matrix
is 'positive definite' a necessary condition for the physical
system (see section 2.9).

The problem has by this stage been fully set up and MOJFEM4
fig. 6.5 is the solution program solving the resulting
eigenproblem. The alternative program MOJFEMD fig. 6.6 is
used if the mass matrix is found to be non-positive definite.
As may be expected this phase demands the most core and also
takes the bulk of the total computing time of the whole suite
of programs. Standard library subroutines are used which
perform the various operations described in section 2.9 and
are outlined in Appendix 2. From the solution of the eigen-
value problem the natural frequencies mode shapes of the box
are calculated and written to file storage.

MOJFEM5 fig 6.7 selects the required number of natural
frequencies and the degrees of freedom to give the normal
modes as prescribed in the previous section and creats a
storage file from which a paper tape can be obtained.

MOJFEM6 fig. 6.8 then creats another file from this paper
tape for the final program which is run locally on the ICL

Figure 6.2 Computer program MOJFEM1 flowchart.

Read in box configuration &.mesh specified

↓

Calculate XY plane element bend stiffness matrix ⟶ (Print)

↓

Calculate element stretch matrix & combine ⟶ (Print)

↓

Reorder into element optimum directions ⟶ (Print)

↓

Assemble into complete stiffness matrix ⟶

Calculate element matrix as above,
optimise & rotate into YZ plane ⟶ (Print)

↓

Assemble into complete stiffness matrix ⟶

Calculate element stiff matrix optimise
& rotate into ZX plane ⟶ (Print)

↓

Assemble into complete stiffness matrix ⟶

⟶ (Print)

Write to magnetic disc or tape storage file

↓

Clear all matrices & re-use

```
                ┌──────────────────────────────────────────────────┐        ⇄  ╭─────────╮
                │ Calculate XY plane element bend mass matrix      │───────────│  Print  │
                └──────────────────────────────────────────────────┘           ╰─────────╯
                                     │
                ┌──────────────────────────────────────────────────┐        ⇄  ╭─────────╮
                │ Calculate element stretch matrix & combine       │───────────│  Print  │
                └──────────────────────────────────────────────────┘           ╰─────────╯
                                     │
                ┌──────────────────────────────────────────────────┐        ⇄  ╭─────────╮
                │ Reorder into element optimum directions          │───────────│  Print  │
                └──────────────────────────────────────────────────┘           ╰─────────╯
                                     │
                ┌──────────────────────────────────────────────────┐
                │ Assemble into complete mass matrix               │───────────▶
                └──────────────────────────────────────────────────┘
                ┌──────────────────────────────────────────────────┐
                │ Calculate element matrix as above                │        ⇄  ╭─────────╮
                │ optimise & rotate into YZ plane                  │───────────│  Print  │
                └──────────────────────────────────────────────────┘           ╰─────────╯
                                     │
                ┌──────────────────────────────────────────────────┐
                │ Assemble into complete mass matrix               │───────────▶
                └──────────────────────────────────────────────────┘
                ┌──────────────────────────────────────────────────┐
                │ Calculate element mass matrix optimise           │        ⇄  ╭─────────╮
                │ & rotate into ZX plane                           │───────────│  Print  │
                └──────────────────────────────────────────────────┘           ╰─────────╯
                                     │
                ┌──────────────────────────────────────────────────┐
                │ Assemble into complete mass matrix               │───────────▶
                └──────────────────────────────────────────────────┘
                                                                            ⇄  ╭─────────╮
                                                                               │  Print  │
                                                                               ╰─────────╯
                ┌──────────────────────────────────────────────────┐
                │ Write to magnetic disc or tape storage file      │◀─────
                └──────────────────────────────────────────────────┘
                                     │
                                ┌─────────┐
                                │  Stop   │
                                └─────────┘
                                     │
                                ┌─────────┐
                                │  End    │
                                └─────────┘
```

Figure 6.3   Computer program MOJFEM2 flowchart.

```
┌─────────────────────────────────────┐
( Read K from magnetic tape file )
          │
          ▽
( Set mesh configuration )
          │
          ▽
( Set boundary conditions control matrix )
          │
          ▽
( Print boundary conditions )
          │
          ▽
┌─────────────────────────────────────────────┐
│ Reduce out fixed boundary degrees of freedom in │
│ stiffness matrix using above control matrix     │
└─────────────────────────────────────────────┘
          │
          ▽
       ◇  ←──→  ( Print resultant stiffness matrix )
          │
          ▽
┌──────────────────────────────────────────┐
│ Check symmetry of resultant stiffness matrix │
└──────────────────────────────────────────┘
          │
          ▽
( Write to backing store reduced K )
          │
          ▽
( Read in mass matrix from tape file )
          │
          ▽
┌──────────────────────────────────────┐
│ Reduce mass matrix as per control matrix │
└──────────────────────────────────────┘
          │
          ▽
       ◇  ←──  ( Print out resultant mass matrix )
          │
          ▽
```

```
│
┌─────────────────────────────────────────────────────┐
│ Write reduced matrices to magnetic tape file         │
└─────────────────────────────────────────────────────┘
            │
            ▼
        ┌───────┐
        │ End   │
        └───────┘
```

Figure 6.4   Computer program MOJFEM3 flowchart.

```
┌──────────────────────────────────────────────────────┐
│  Read in reduced matrices from magnetic tape file     │
└──────────────────────────────────────────────────────┘
                          │
                          ▼
┌──────────────────────────────────────────────────────┐
│  Check whether mass matrix is positive definite       │
└──────────────────────────────────────────────────────┘
                          │
                          ▼
        ┌────────────────────────────────────────┐
        │  Check if stiffness matrix is singular  │
        └────────────────────────────────────────┘
                          │
                          ▼
              ┌──────────────────────┐
              │  Print out results    │
              └──────────────────────┘
                          │
                          ▼
                    ┌──────────┐
                    │   End    │
                    └──────────┘
```

Figure 6.5   Computer program MOJFEM4 flowchart.
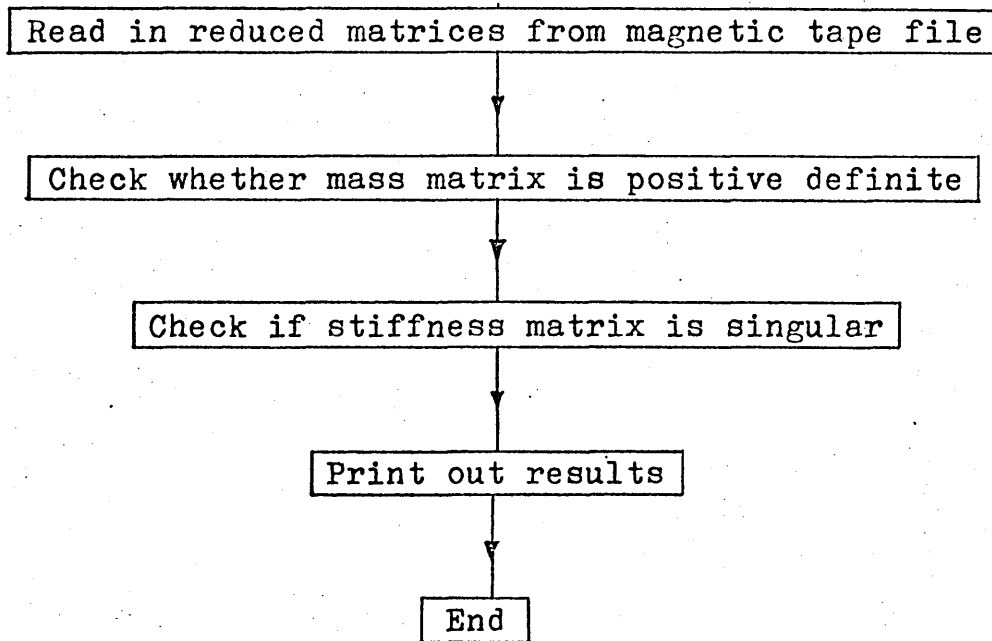
Figure 6.5   Computer program MOJFEM4 flowchart.
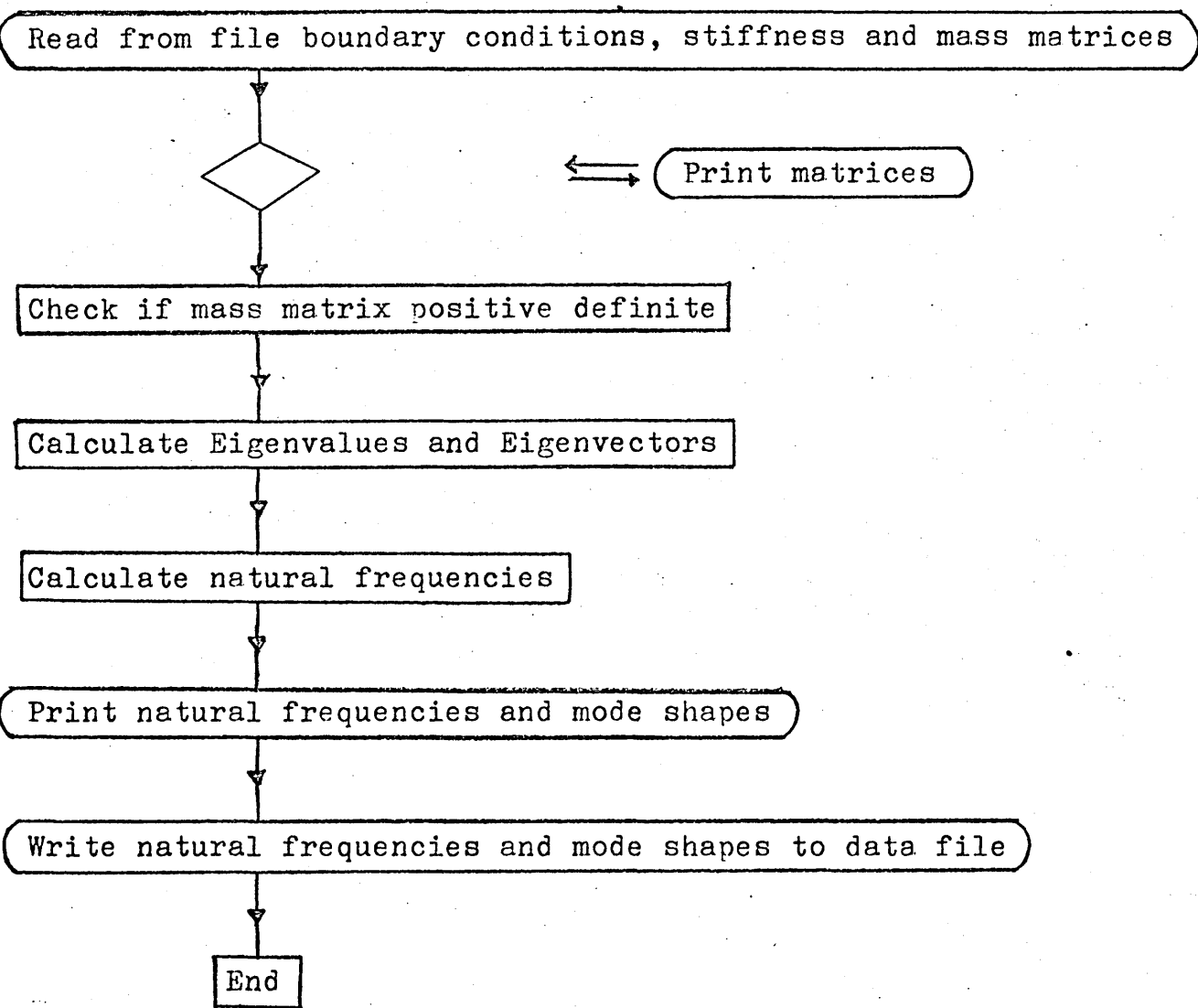
Figure 6.6   Computer program MOJFEMD (optional) flowchart.

Read in boundary conditions, stiffness and mass matrices

← Print matrices

Specify accuracy required in calculations

Check if stiffness matrix singular

→ End

Calculate inverse of stiffness matrix

← Print K^{-1} matrix

Calculate $[K^{-1}][M]$ matrix

← Print resultant unsymmetric matrix

Balance matrix

← Print balanced matrix

Reduce to upper Hessenberg form

Form matrix of accummulated transformations
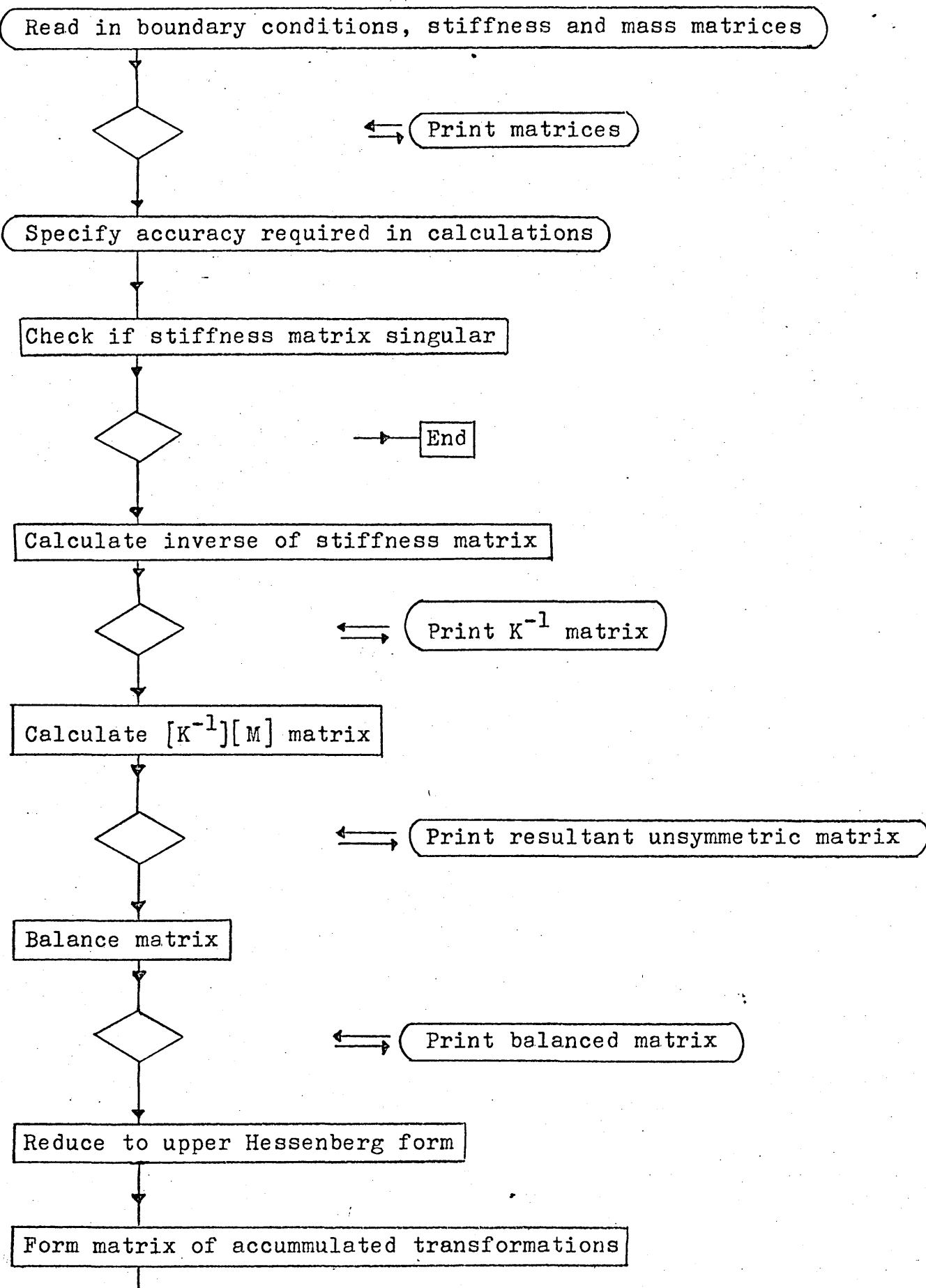
Compute Eigenvalues and Eigenvectors of upper Hessenberg matrix

Calculate natural frequencies from Eigenvalues

Print natural frequencies

Calculate Eigenvectors of original problem

Print Eigenvectors of structure

Write to data file

End

Figure 6.7 Computer program MOJFEM5 flowchart.
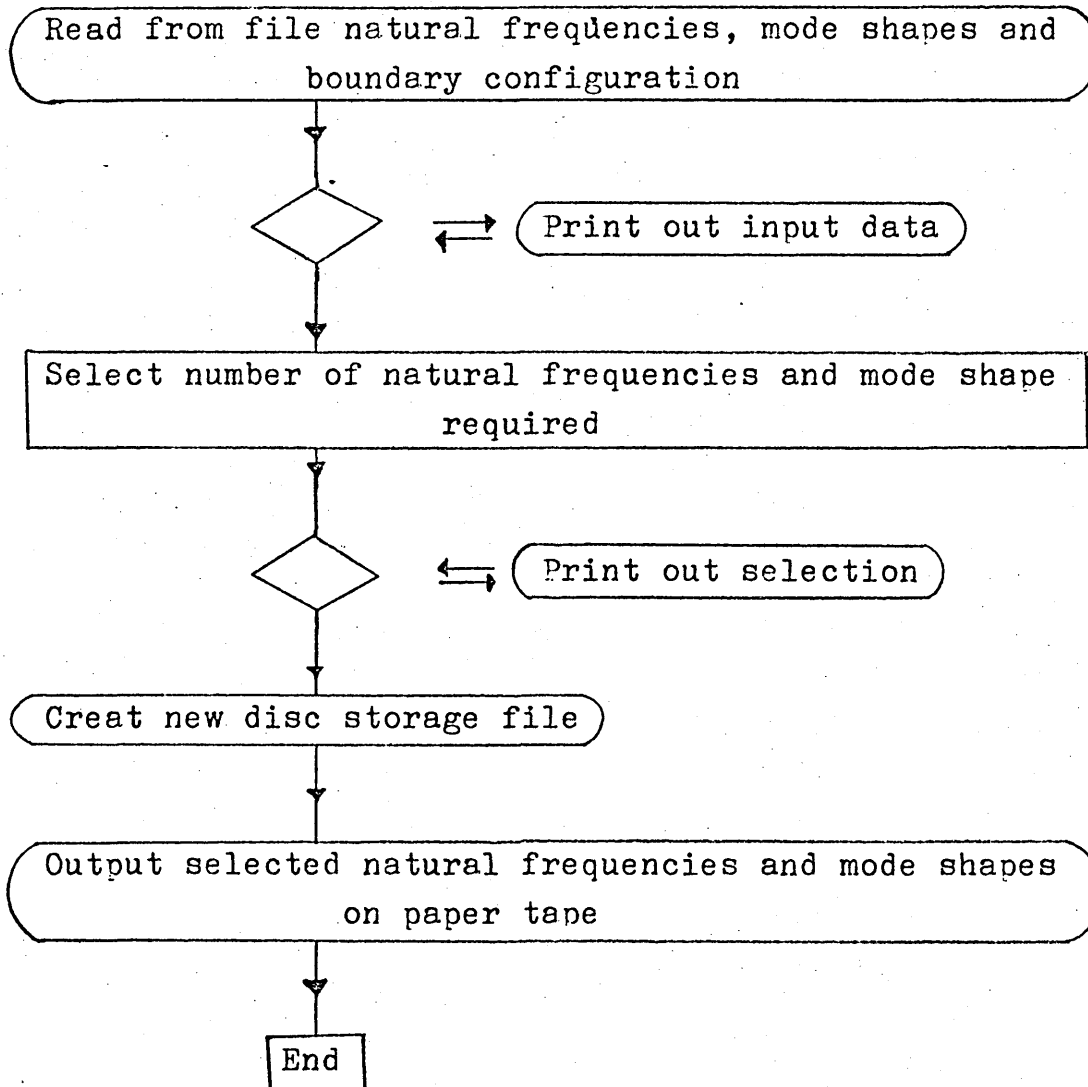
Figure 6.8   Computer program MOJFEM6 flowchart.

Read from paper tape natural frequencies and mode shapes and boundary conditions

Print data read

Creat magnetic tape MOJAMAGDATA 1

End
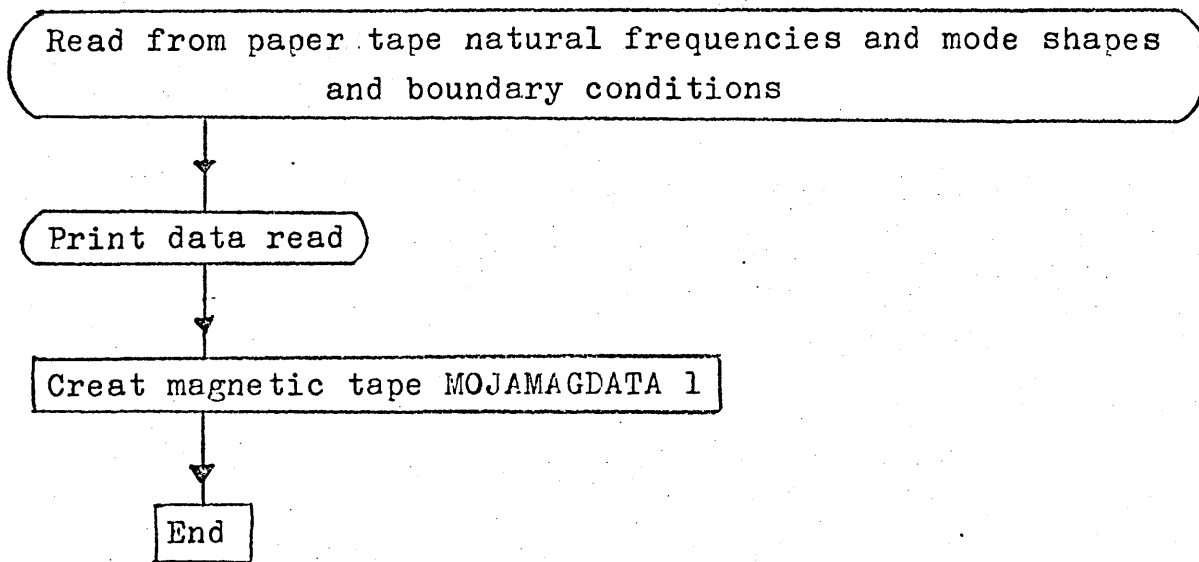
Figure 6.9   Computer program MOJFEM7 flowchart.

```
 ( Read in data from magnetic tape )
                  │
                  ▼
    ( Read in damping factors )
                  │
                  ▼
              ◇ ─────────  Yes ──────▶ ( Print all input data )
                  │         ◀──────
           Check input data?
                  │
                  ▼ No
    │ Calculate mass factor DM │
                  │
                  ▼
    │ Compile list of Degrees of Freedom │
                  │
                  ▼
 ( Print mode shape with transverse Degrees of Freedom only )
                  │
                  ▼
              ◇ ─────────  No ───────▶ │ Interpolate │
                  │         ◀──────
            Major node?
                  │
                  ▼ Yes
    ( Read in input excitation spons )
                  │
                  ▼
              ◇ ─────────────────────▶ ( Print and graph )
                  │         ◀──────
                  ▼
 ( Display excitation spectral density )
                  │
                  ▼
 ( Specify excitation and response positions )
                  │
                  ▼
 ( Display point(s) chosen )
                  │
```

Calculate equivalent masses

No

Completed
number of modes?

Yes

No

Modify mode shape

Major node ?

Yes

Calculate receptance

No

Completed
number of modes?

No

Completed
number of points?

Print and graph

Display receptance

```
┌─────────────────────────────────────────┐
│ Calculate response spectral density     │
└─────────────────────────────────────────┘
                    │
                    ▼
( Print and graph response spectral density )
                    │
                    ▼
( Print maximum frequency )
                    │
                    ▼
              ┌─────────┐
              │   End   │
              └─────────┘
```

1907 computer. This is because having calculated the natural frequencies and mode shapes of the structure, the receptances of the box at a large number of points and calculations of the response spectral densities can be obtained. These are run as small express jobs while can be executed more economically and with faster turn round times on the local computer with graph plots produced.

MOJFEM7 fig. 6.9 computes the response power spectral density at any chosen point from natural frequencies and normal modes of the box given the damping factors and the excitation power spectral density. A peripheral graph plot of the excitation and the response power spectral density is output with an additional line printer output for more accurate checking purposes.

The following is a table of the various programs, the core requirements and the solution times for the twenty-four element-mesh idealisation shown in fig. 6.1.

Program	Core (Kwords)	Time (Sec.)
MOJFEM1	39	4.0
MOJFEM2	57	0.8
MOJFEM3	44	6.5
MOJFEM4	55	17.0
MOJFEMD (optional)	70	88.0
MOJFEM5	24	0.3
MOJFEM6	18	10.0
MOJFEM7	19	72.0

Chapter 7   <u>Results</u>

<u>Summary</u>

In this chapter results of the investigation of natural
frequencies and mode shapes of beams, plates and boxes
are presented. Further results of work leading up to
the response of the box structure to random excitation
are also presented.
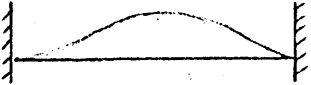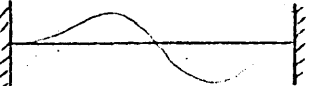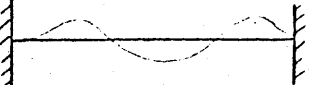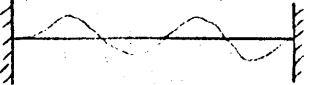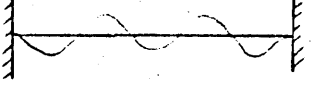
7.1   <u>Natural frequencies and mode shapes</u>

Section 5.3.1 describes how the natural frequencies and mode
shapes of beams, plates and box structures were obtained.
The natural frequencies and mode shapes of a beam simply
supported at both ends and a beam fully fixed at both ends
are presented in Table 7.1. The first five natural frequen-
cies and mode shapes of a plate fully fixed all round are
presented in Table 7.2.

The twenty-one natural frequencies of an open ended folded
plate box type structure, fixed at the four corners of its
base, existing within the frequency range 0 to 100 Hz are
given in Table 7.3. Their corresponding mode shapes are
illustrated in fig. 7.1 to 7.11. The first mode predicted
at 5.6 Hz was not checked experimentally as equipment
available only measured down to 20 Hz. However, a swaying
mode was apparent at a very low frequency which seemed to
correspond to this mode. The other modes have been stimu-
lated experimentally and both the experimental and predicted
results are presented here.

Table 7.1   Results of beam analysis (fully- fixed ends)

Mode shape	Natural frequency (Hz.)			Damping
	Exact	Energy (F.E.)	Expt.	factor (η)
	43.8	43.9	43.2	0.020
	121.0	123.0	117.0	0.007
	238.1	240.0	223.9	0.006
	392.0	397.0	361.0	0.004
	585.0	592.0	545.0	0.003

Results of beam analysis (simply-supported ends)
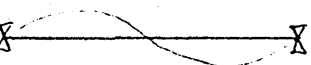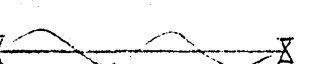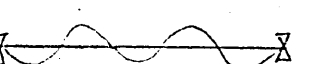
Mode shape	Natural frequency (Hz.)			Damping
	Exact	Energy (F.E.)	Expt.	factor (η)
	19.5	19.5	13.8	0.018
	78.1	78.1	62.0	0.007
	176.0	176.0	145.0	0.008
	313.0	313.0	289.0	0.006
	488.0	488.0	458.0	0.003

Table 7.2   Results of plate investigation (fixed all round)

Length   = 0.78m.
Width   = 0.22m.
Thickness = 0.00061m.

| Mode shape | | Natural frequency | | |
m	n	Expt.	F.E.	Energy
2	2	53.3 Hz	67.2	69.1
3	2	69.5	75.0	74.7
4	2	79.5	84.1	84.9
5	2	92.5	101.4	99.8
6	2	111.0	118.5	120.1

m = Number of half-wavelengths in x direction.
n = Number of half-wavelengths in y direction.

The finite element analysis is made using 8 elements.

The results of Tables 7.1 and 7.2 were obtained principally
to show that the finite element programs were working satis-
factorily.   The close agreement between the finite element
results and other calculated results showed this to be the
case.   The experimentally obtained natural frequencies for
the plate and for the simply supported beam are both low.
This can be attributed to the fact that the required boundary
conditions in these cases were not fully achieved in the
experimental apparatus.   No action was taken to improve
these boundary conditions as these structures did not form a
major part of the work.

Analytical	Experimental
5.6 Hz	–
22.4	24.6 Hz
27.6	27.2
31.8	32.8
36.4	37.0
38.1	42.4
47.0	49.5
48.5	53.1
49.0	55.1
54.3	58.6
59.0	62.4
62.7	66.8
67.9	68.2
69.2	70.8
70.3	72.7
70.9	74.7
75.0	77.6
80.0	84.4
81.0	88.0
91.0	96.5
98.0	105.6

Table 7.3 Natural frequencies of box structure
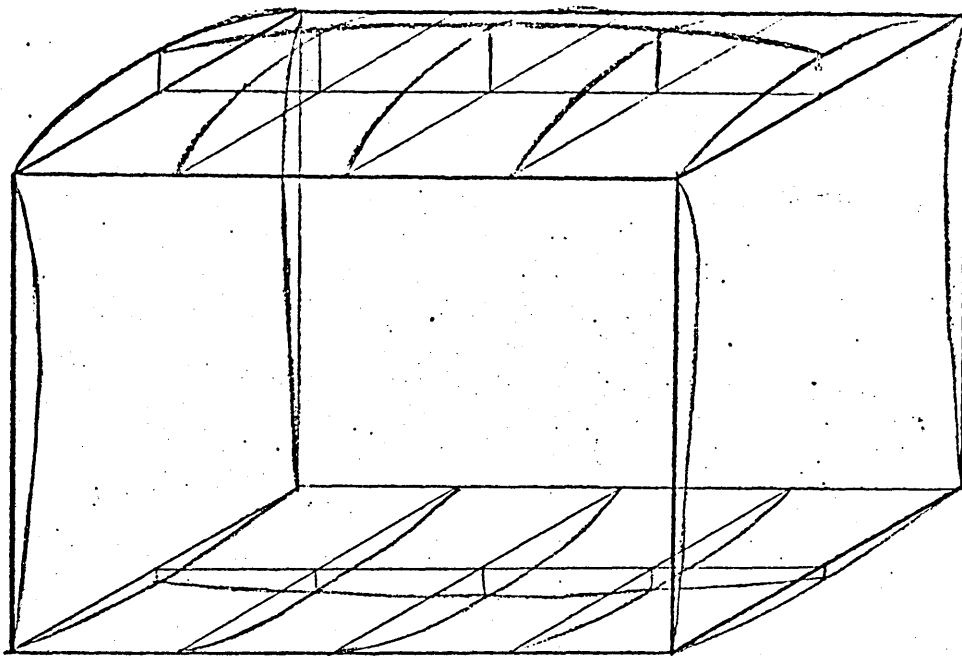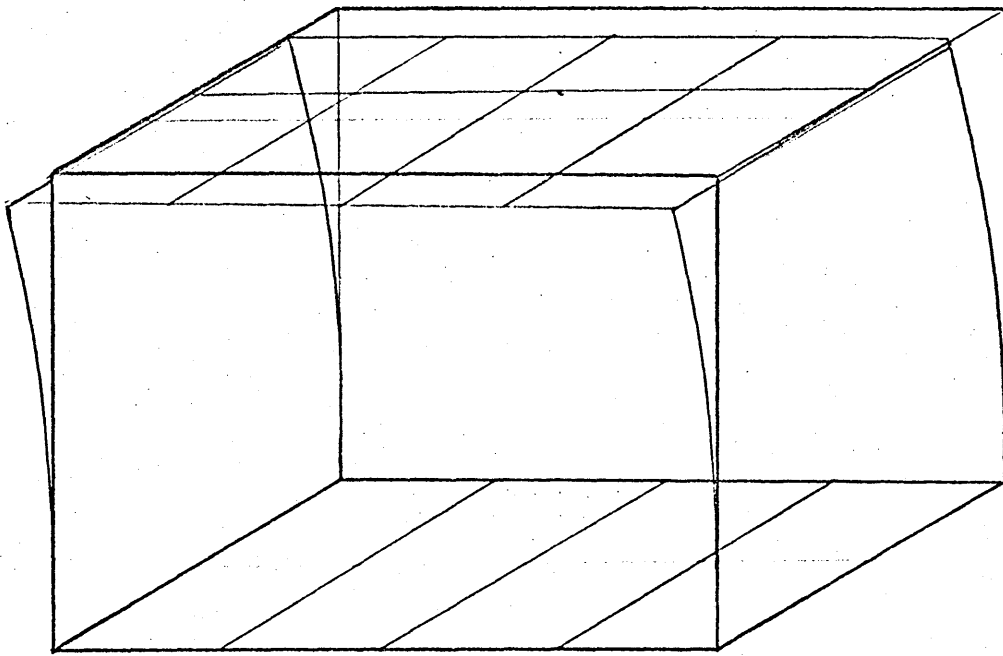
Figure 7.1   1st. and 2nd. mode shapes of the box.
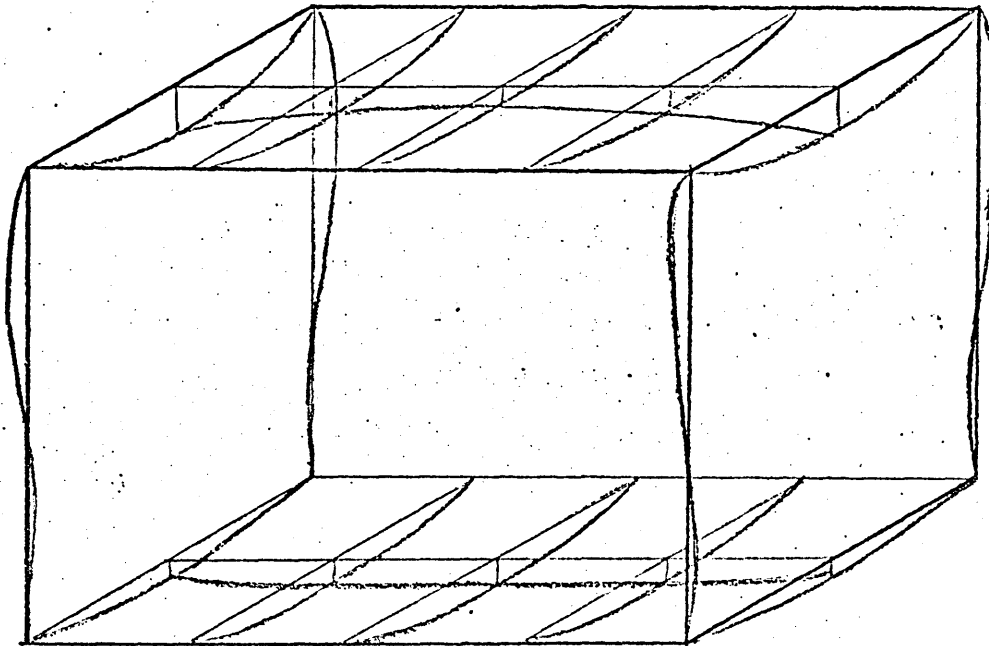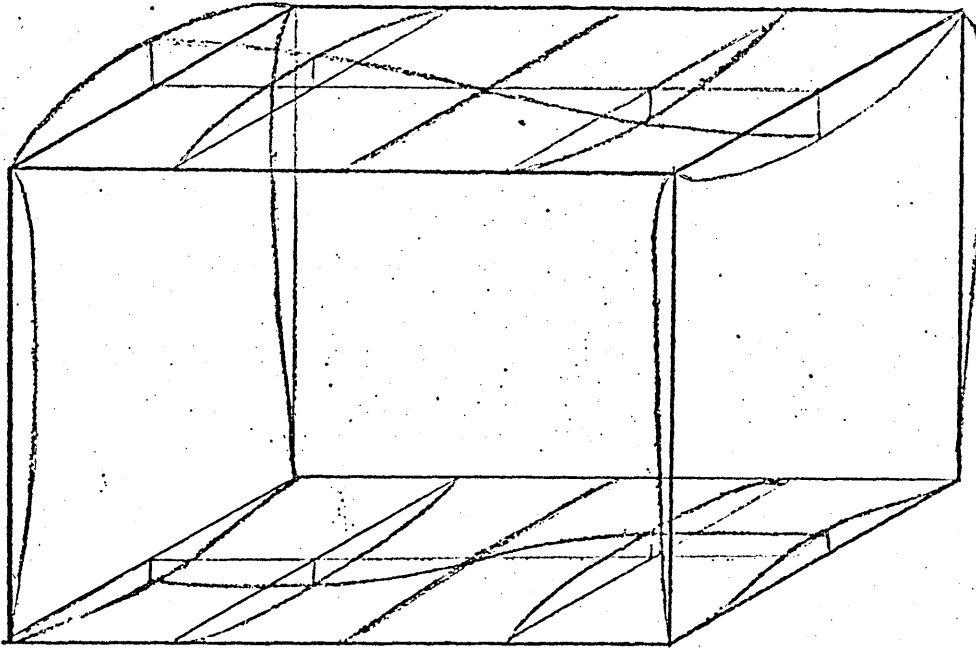
Figure 7.2   3rd. and 4th. mode shape of box.
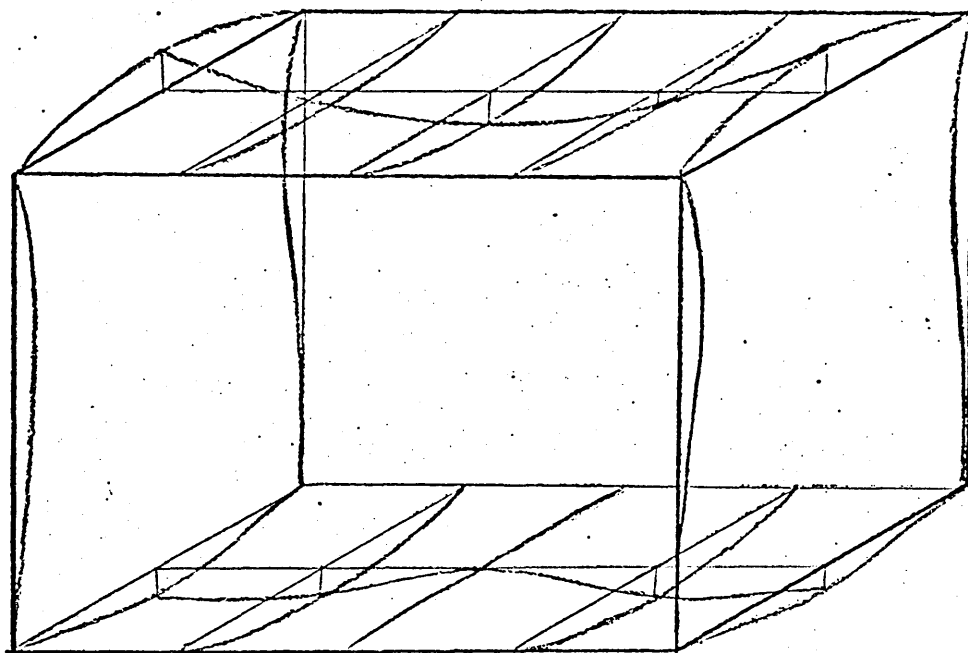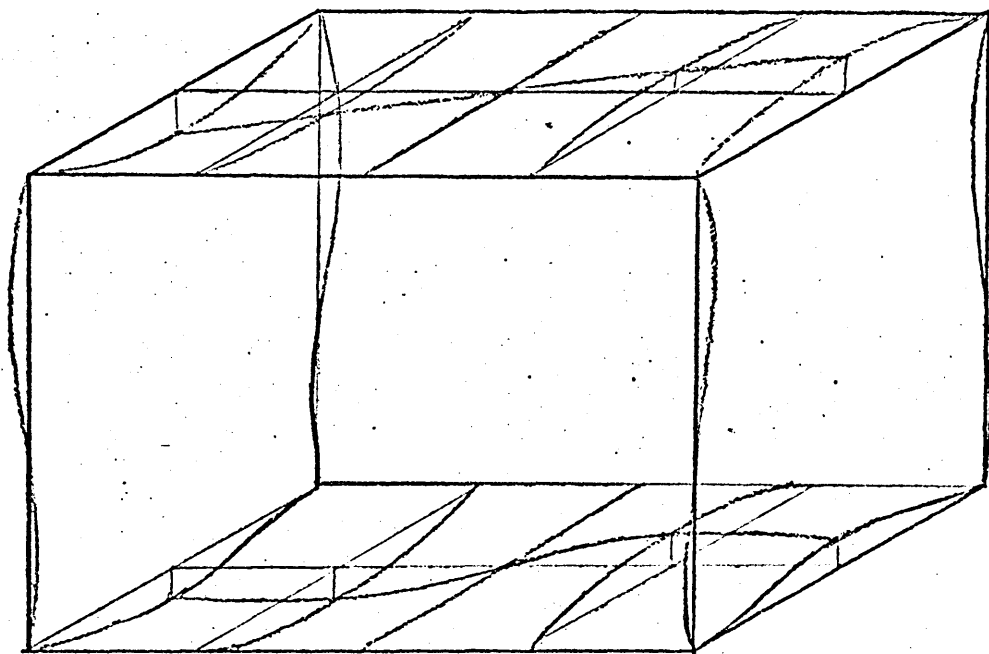
Figure 7.3   5th. and 6th. mode shapes of the box.
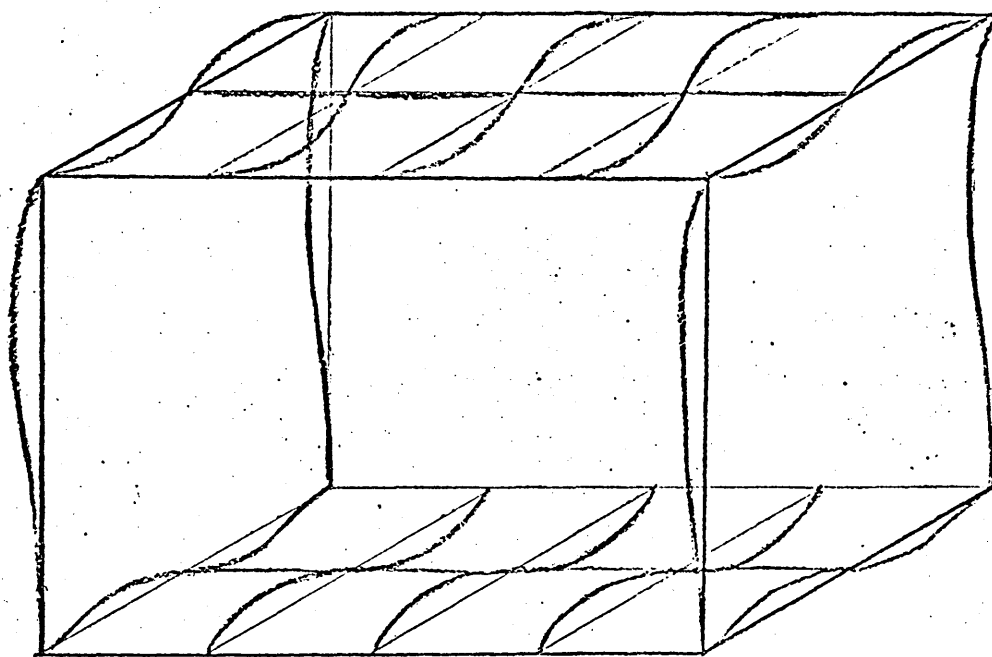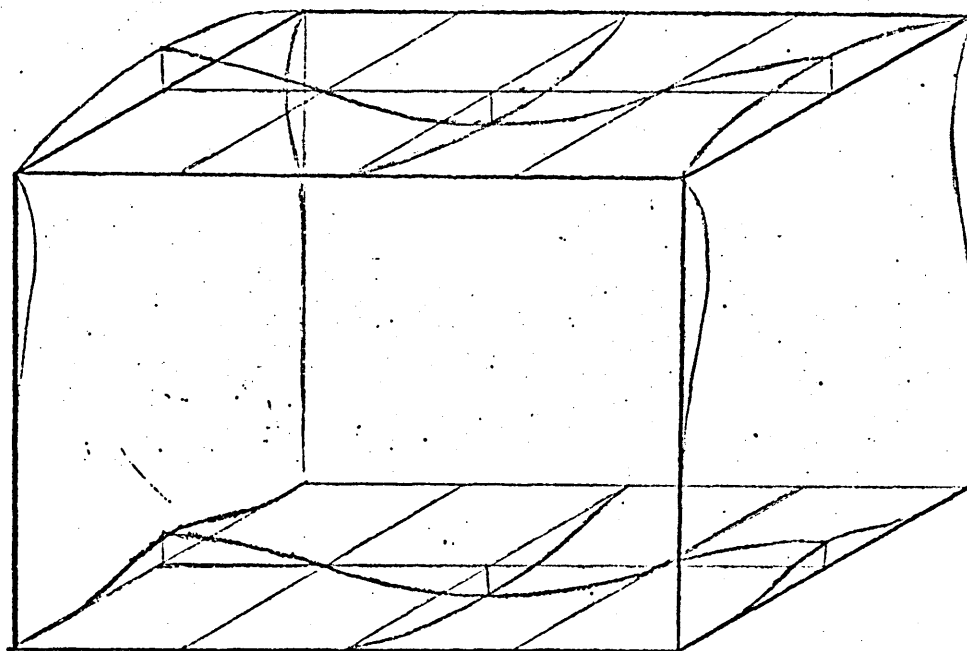
Figure 7.4   7th. and 8th. mode shapes of the box.
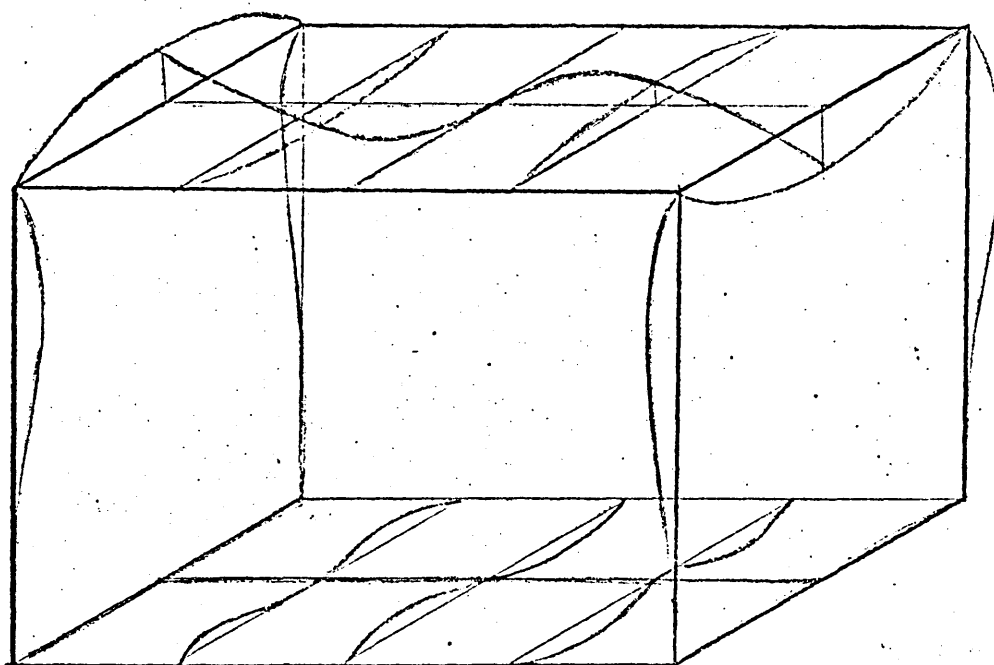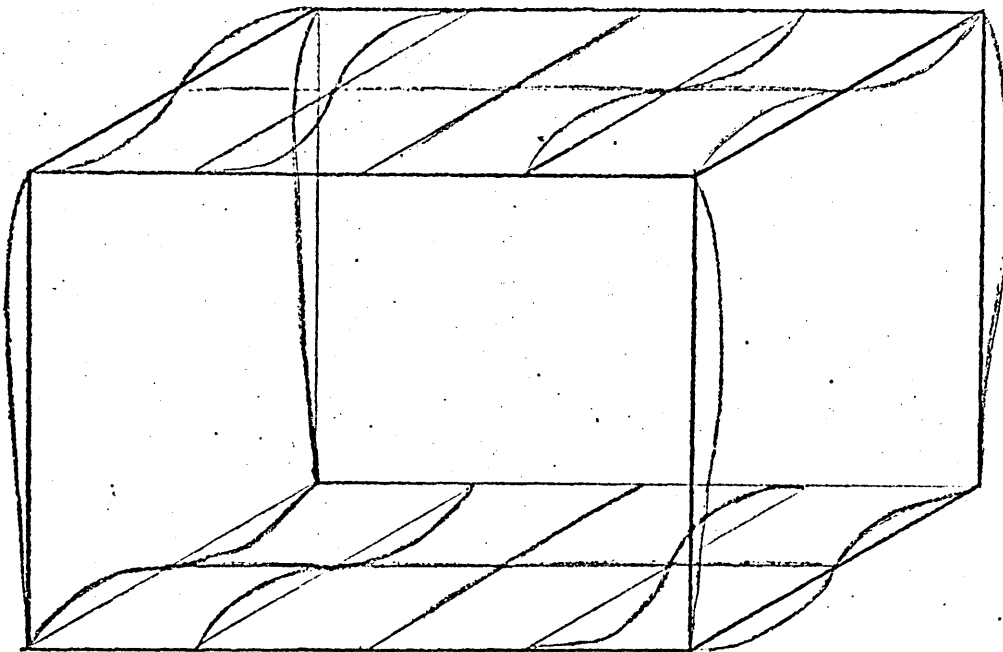
Figure 7.5   9th. and 10th. mode shapes of the box.
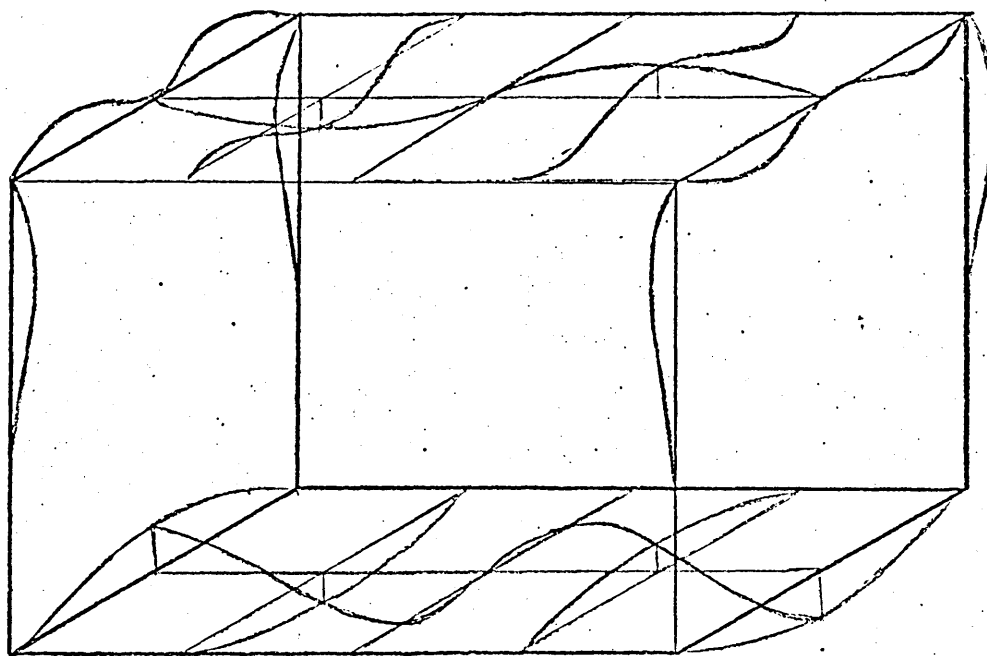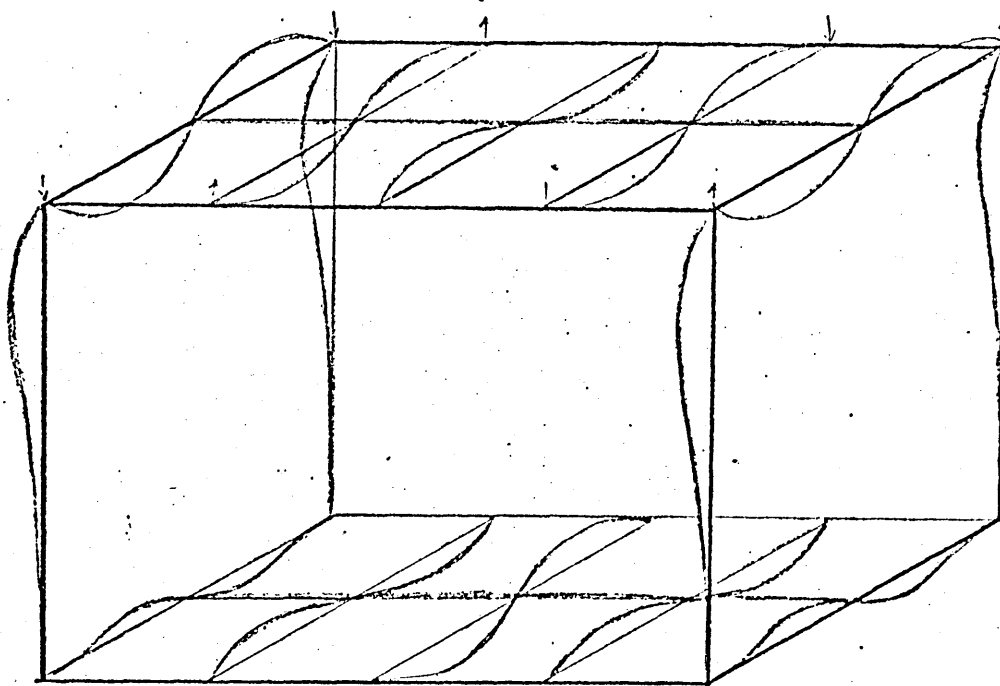
Figure 7.6   11th. and 12th. mode shapes of the box.
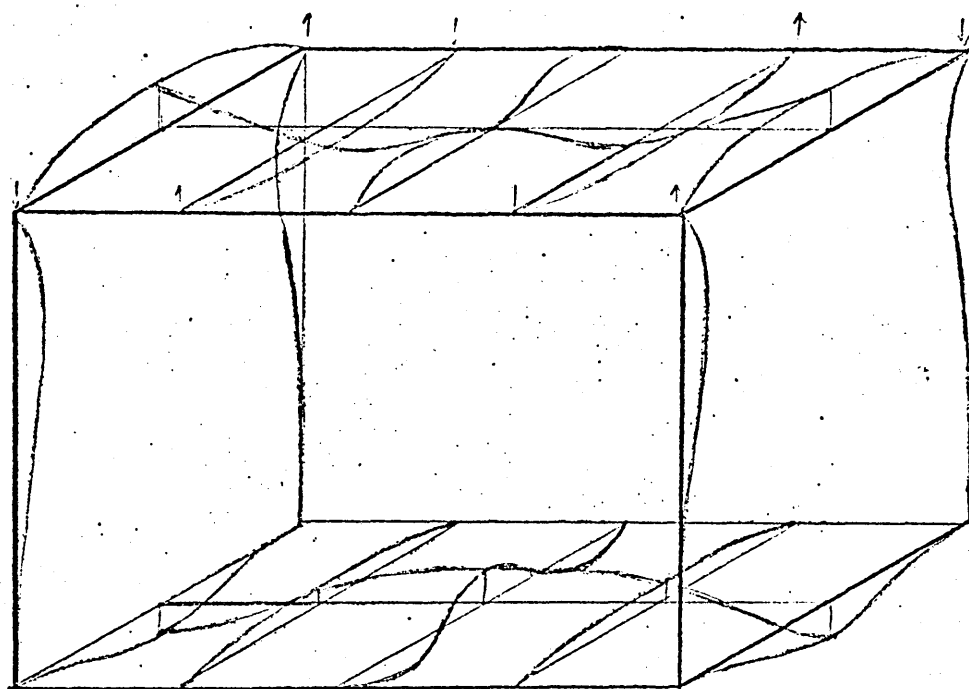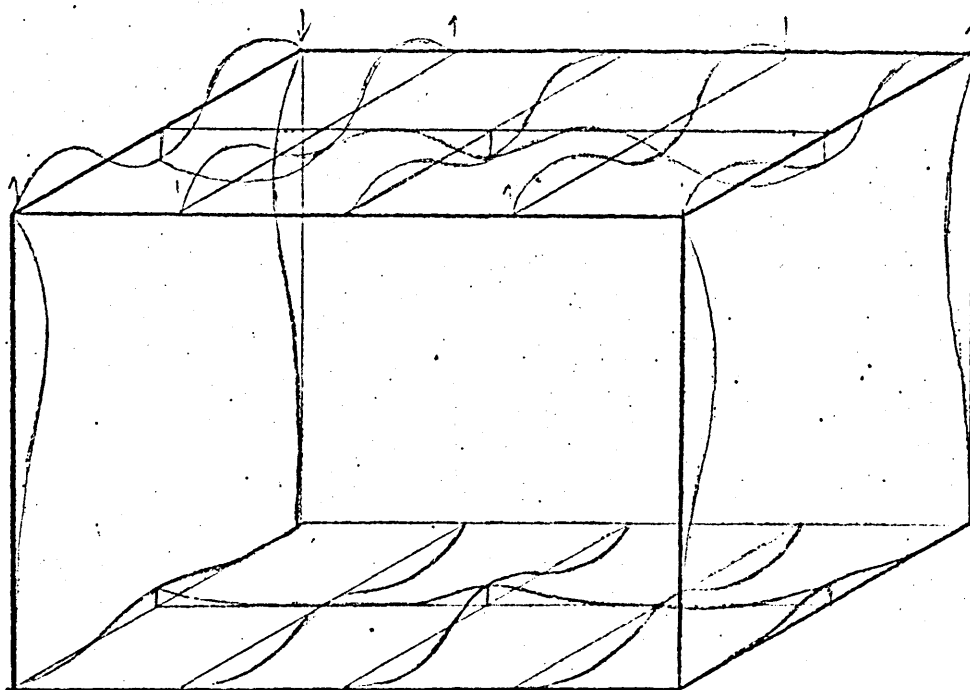
Figure 7.7   13th. and 14th. mode shapes of the box.

θz
also
present.
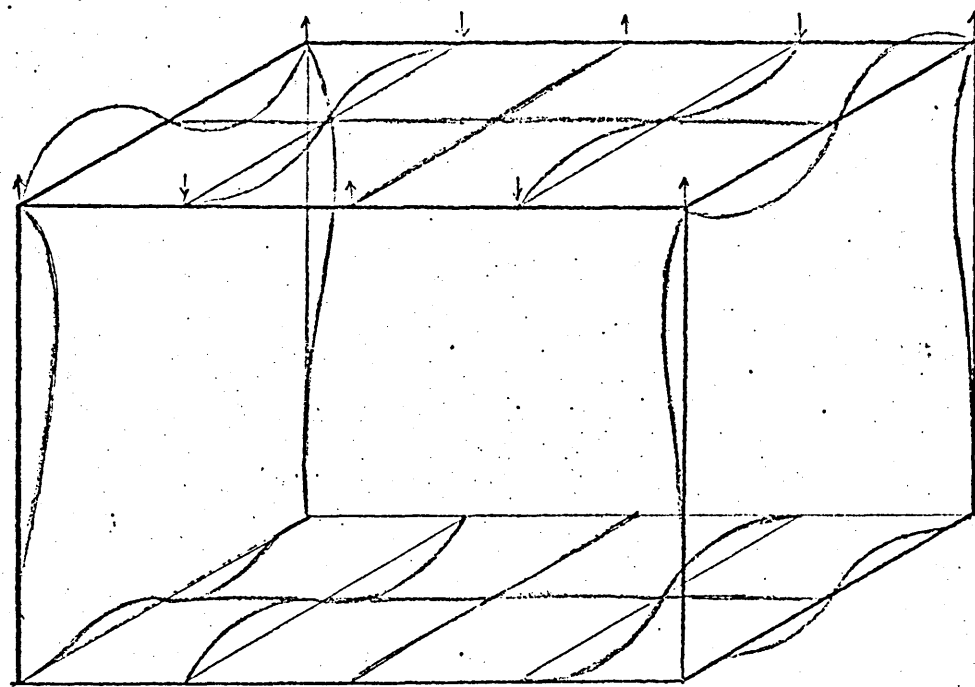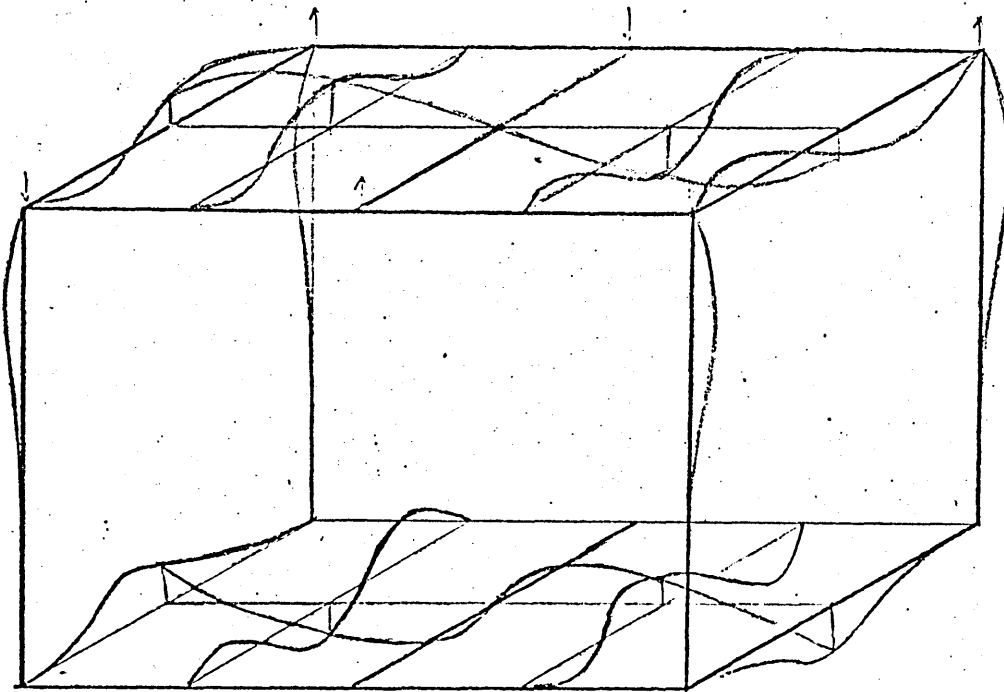
Figure 7.8   15th. and 16th. mode shapes of the box.
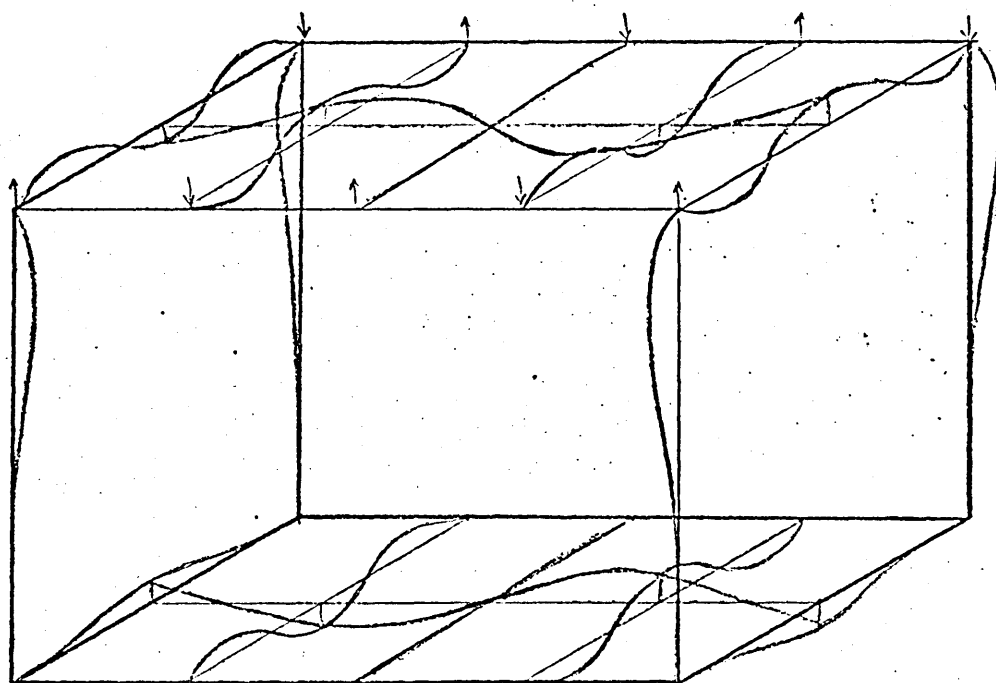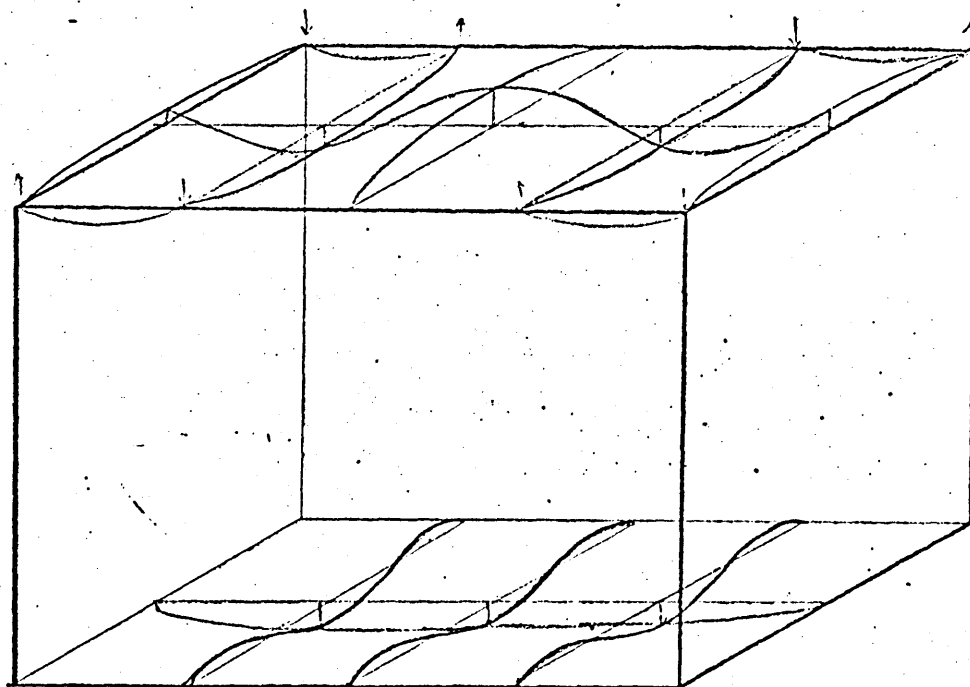
Figure 7.9   17th. and 18th. mode shapes of the box.
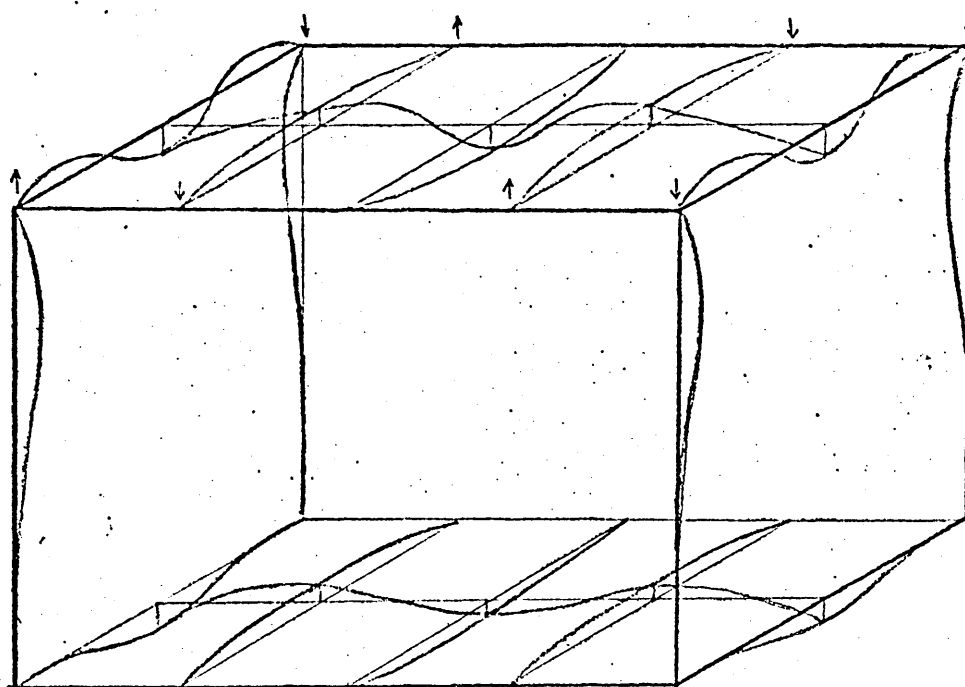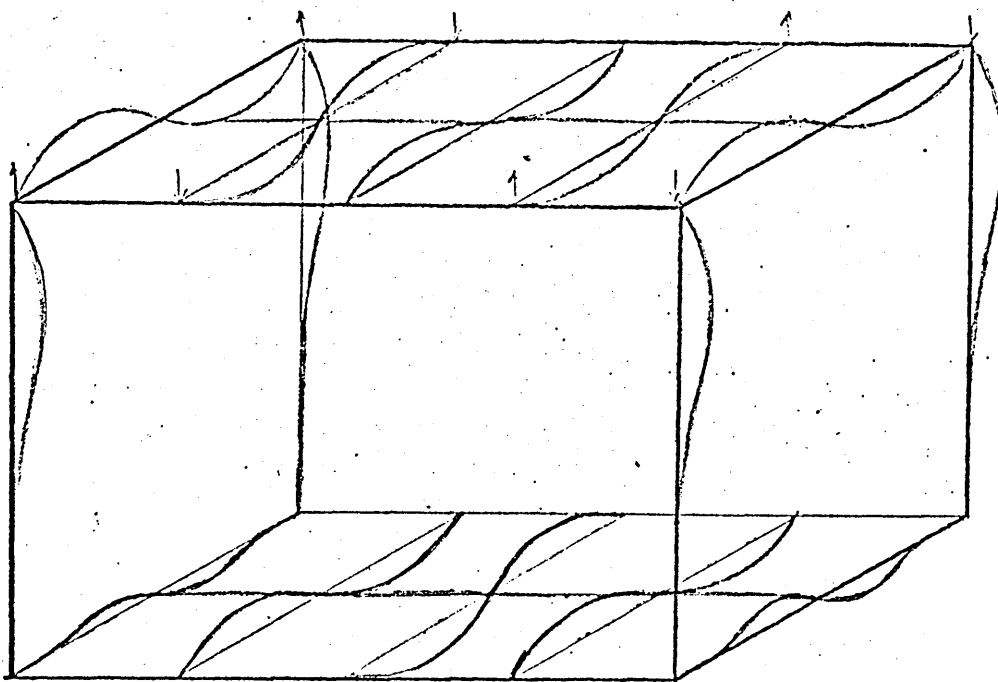
Figure 7.10   19th. and 20th. mode shapes of the box.
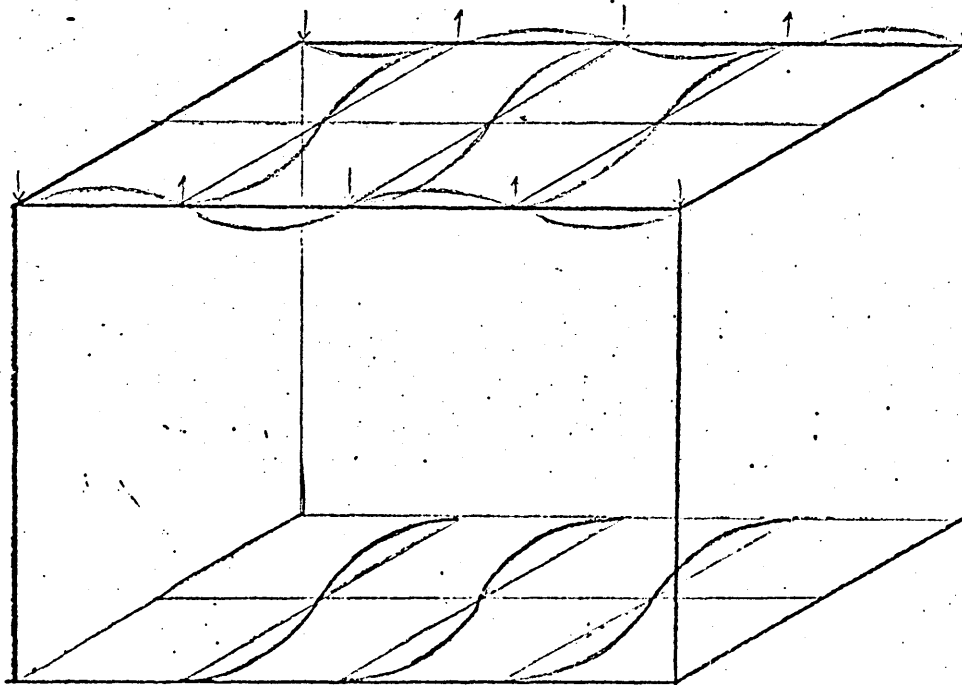
Figure 7.11   21st. mode shape of the box.

2

7.2 Response calculations

The response of the open ended box is presented here in the form of its power spectral density. The excitation used experimentally and simulated on the computer predictions (fig. 7.12) is a filtered PRBS signal applied to the non-contacting exciter probe developed. The results have been obtained using three different techniques for comparison purposes,

(a) using the box receptance as calculated from the natural frequencies and mode shapes predicted by a finite element analysis of the structure, and with the excitation power spectral density and damping factors obtained experimentally.

(b) as obtained using the cross correlation of the excitation and response signals giving the system impulse response function which is Fourier transformed to give the box receptance described in section 3.4. This is then used as in (a).

(c) by narrow band frequency analysis of the vibration response of the box structure using equipment described in chapter 5.

For spectral density calculations because of the symmetry of the problem only a quarter of the top surface of the box is illustrated in fig. 7.13. An average of the results for the other remaining part of the box has been taken because of slight differences obtained possibly due to differences in material composition or incurred during manufacture of the box. The results of the response power spectral density predicted is compared with those obtained experimentally for the seven points on the quarter box as shown in the figure. Less accurate experimental values were obtained along the edges of the box. This may be due to difficulties experienced during the manufacture of the box (a) in the cutting of the plates which make up the sides and in (b) soldering the plates to form the sides at right angles to one another.
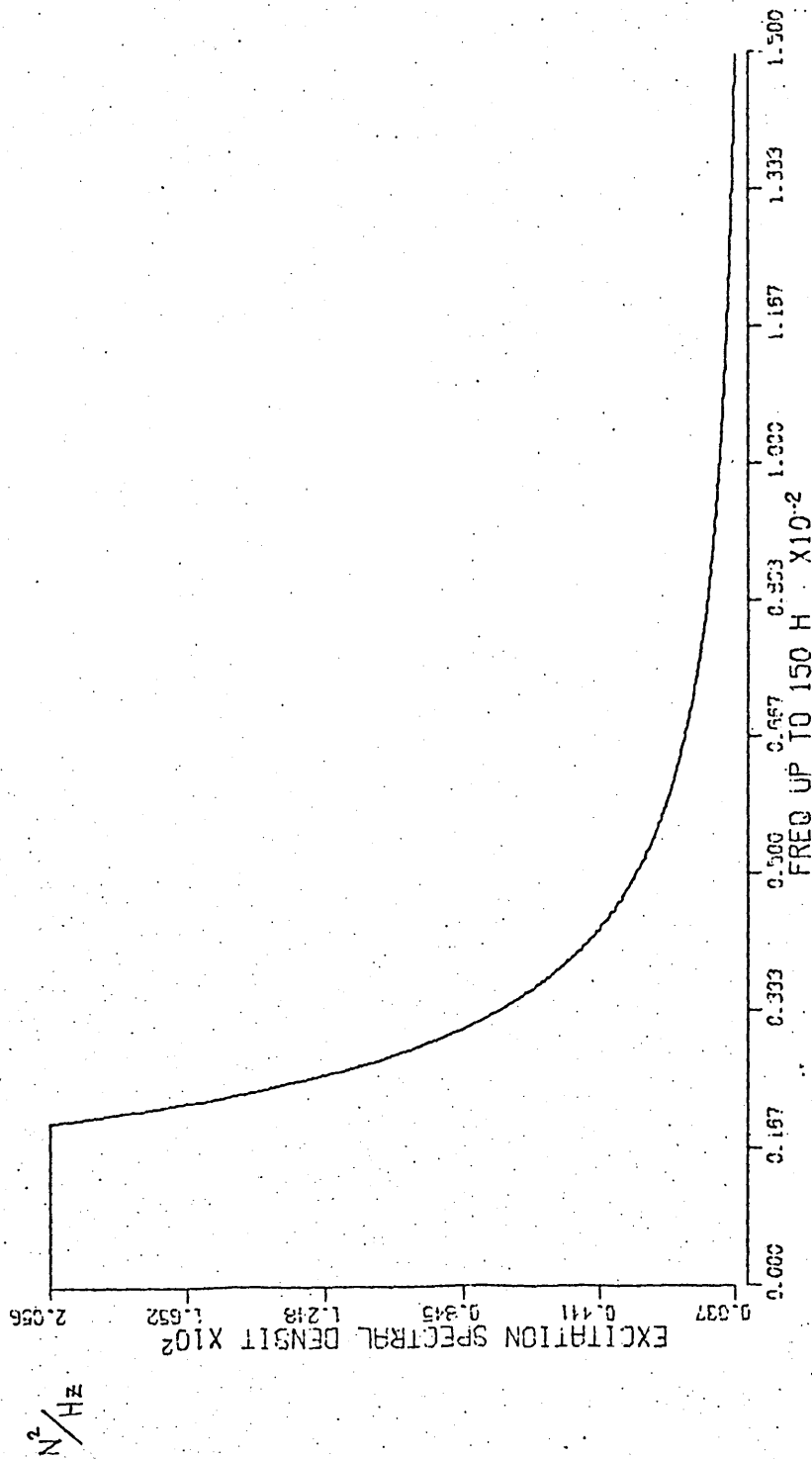
Figure 7.12   The excitation power spectral density used.
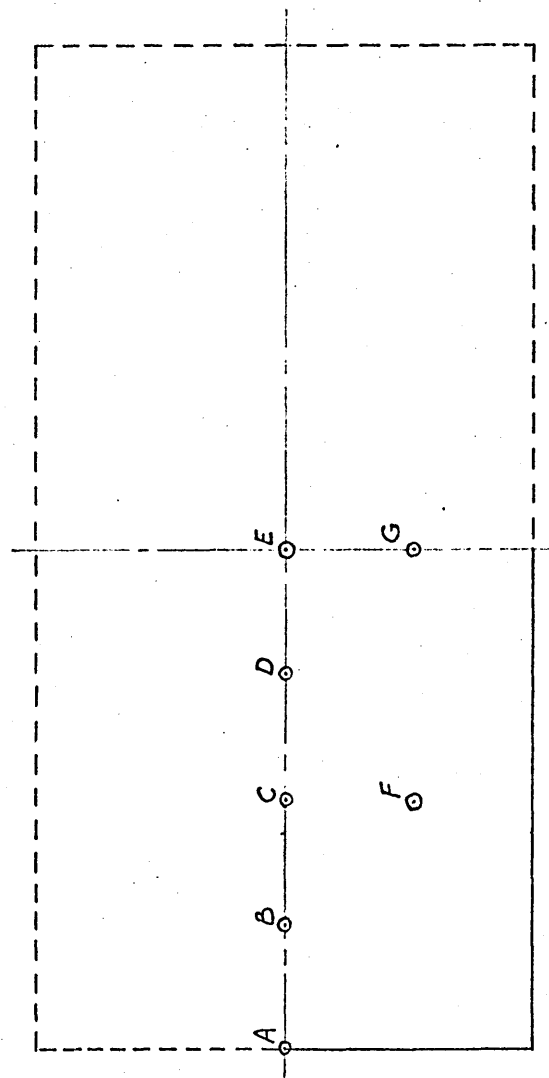
Fig. 7.13   Showing the points for which results are presented.

The excitation signal used in the tests is a PRBS signal
with a true RMS level of 0.3 V. The input-output cross
correlation function was obtained experimentally in the
form of a paper tape to be Fourier transformed on the
computer. A typical trace of the impulse response function
of the box as obtained during the work is plotted by the
Fourier transform program(fig. 7.14 & 7.15).From this the
response of the box computed by the computer program is
obtained and also displayed on a graph plot. Typical
results using various degrees of smoothing for a 512 point
function are shown in fig. 7.16 to 7.18 and those for a
1024 point function, fig. 7.19 to 7.21.

Figure 7.22 is a typical trace of the receptance of the
box structure calculated from the natural frequencies and
mode shapes obtained using a finite element analysis. The
PRBS excitation signal is fed into the non-contacting
exciter probe for which a typical excitation power spectral
density into the box structure as shown in (fig. 7.12) is
obtained. This is used in the calculation of the typical
box response power spectral density (fig. 7.23) from the
receptance obtained.

The response power spectral density of the box to this
excitation obtained using the finite element prediction
of the box receptance is shown in the chain-dotted lines
in fig. 7.24 to 7.30. The full lines in the figures
represent experimental results obtained from the cross
correlation of the excitation and response signals. Poor
results were obtained using direct narrow band frequency
analysis of the response signal and are not included in
the figures.
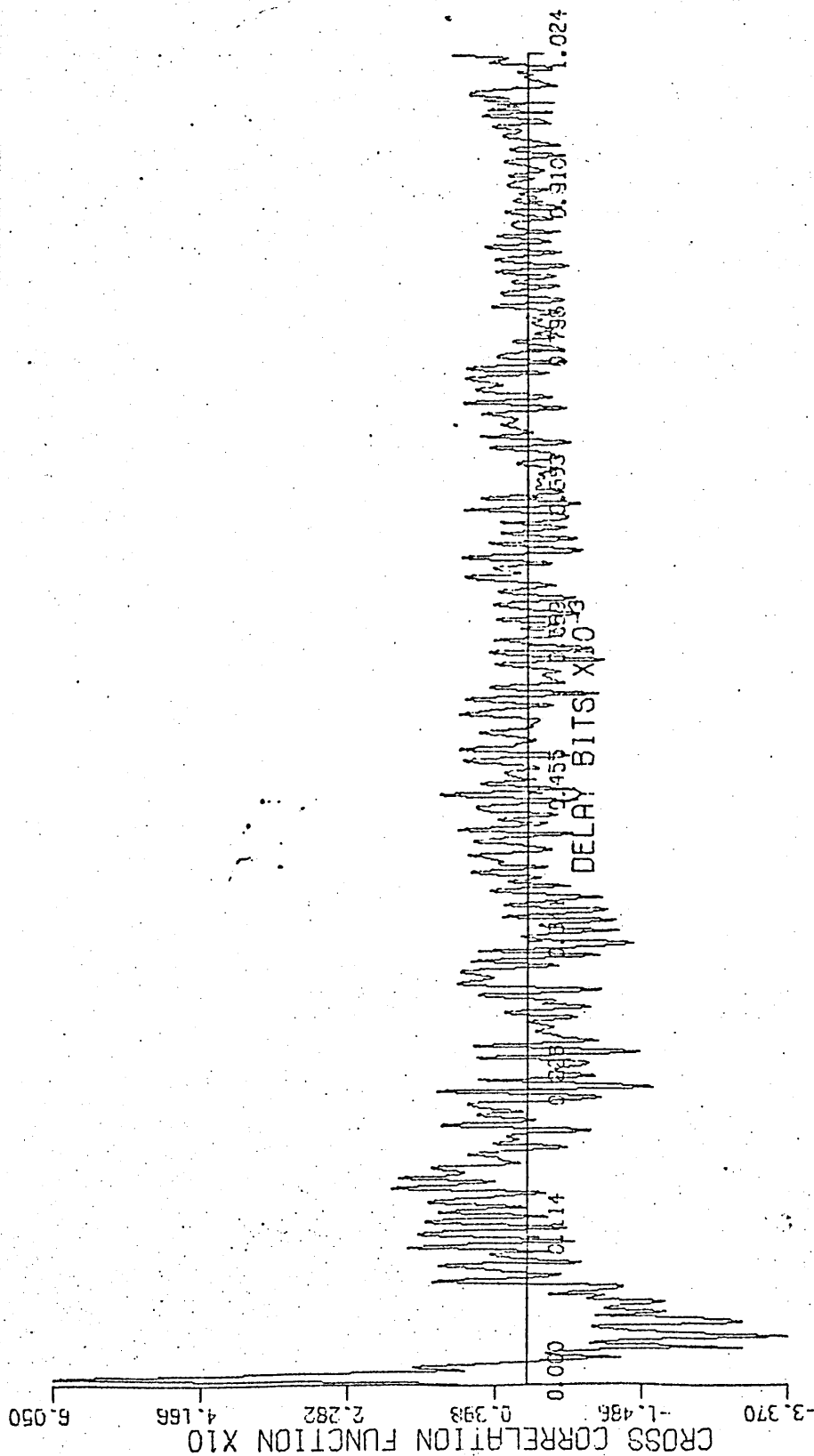
Figure 7.14   A typical graph plot of the cross correlation / impulse response function.

-0.4110   0.0800

0=	2.860F-01
1=	3.150F-01
2=	3.650F-01
3=	2.820F-01
4=	3.330F-01
5=	4.060F-01
6=	4.740F-01
7=	5.650F-01
8=	5.940F-01
9=	6.120F-01
10=	5.790F-01
11=	5.910F-01
12=	6.760F-01
13=	7.690F-01
14=	8.610F-01
15=	9.000F-01
16=	9.290F-01
17=	9.800F-01
18=	9.530F-01
19=	9.590F-01
20=	9.530F-01
21=	9.150E-01
22=	8.600F-01
23=	8.000F-01
24=	6.750F-01
25=	5.800F-01
26=	4.030F-01
27=	4.160F-01
28=	3.310F-01
29=	2.550F-01
30=	1.910F-01
31=	1.110F-01
32=	4.600F-02
33=	-2.000F-02
34=	-8.500F-02
35=	-1.280F-01
36=	-1.350F-01
37=	-1.090F-01
38=	-5.600F-02
39=	-2.100F-02
40=	3.200F-02
41=	8.600F-02
42=	1.440F-01
43=	1.850F-01
44=	1.990F-01
45=	2.440F-01
46=	3.040F-01
47=	3.440F-01
48=	3.680F-01
49=	3.940F-01
50=	3.940F-01
51=	3.770F-01
52=	3.250F-01
53=	2.670F-01
54=	2.060F-01
55=	1.530F-01
56=	1.010F-01
57=	5.300F-02
58=	-1.400F-02
59=	-9.200F-02
60=	-1.650F-01
61=	-2.220F-01
62=	-2.690F-01
63=	-2.880F-01
64=	-2.810F-01
65=	-2.620F-01
66=	-2.300F-01
67=	-1.890E-01
68=	-1.500F-01
69=	-1.170F-01
70=	-1.050F-01
71=	-6.700F-02
72=	-1.100F-02
73=	4.100F-02
74=	7.500F-02
75=	1.100F-01
76=	1.290F-01
77=	1.390F-01
78=	1.390F-01
79=	1.180F-01
80=	6.000F-02
81=	1.000F-02
82=	-4.800F-02
83=	-1.100F-01
84=	-1.830F-01
85=	-2.420F-01
86=	-3.110F-01
87=	-3.550F-01
88=	-3.700F-01
89=	-3.540F-01
90=	-3.150F-01
91=	-2.630F-01
92=	-2.200F-01
93=	-1.850F-01
94=	-1.700F-01
95=	-1.700F-01
96=	-1.660F-01
97=	-1.490F-01
98=	-1.160F-01
99=	-8.200F-02
100=	-4.500F-02
101=	-1.960F-02
102=	1.100F-02
103=	3.100F-02

Figure 7.15   A typical line printer display of the correlation function.
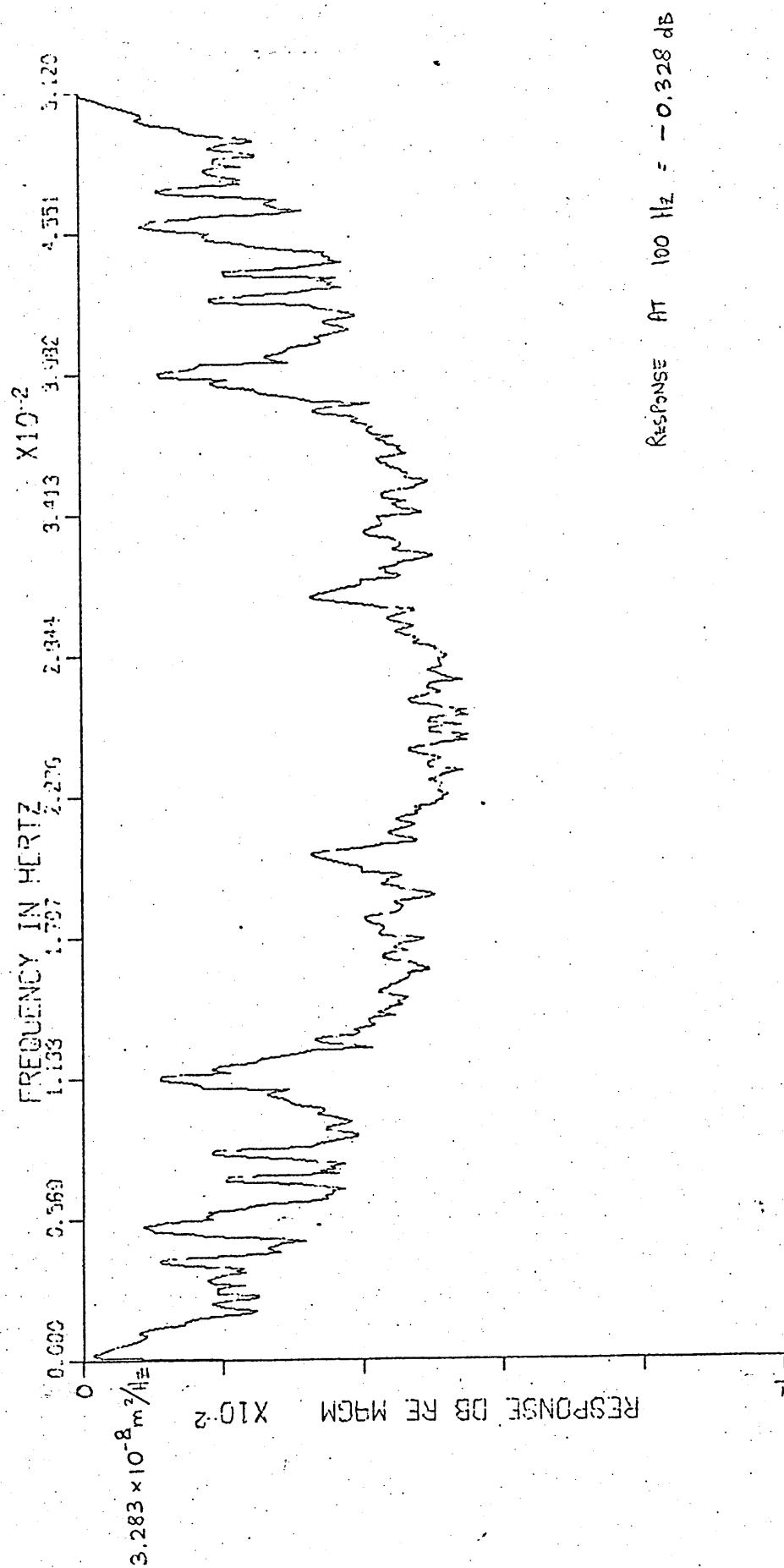
Figure 7.16   Box response to random excitation from the 512 point
correlation function (non smoothed).
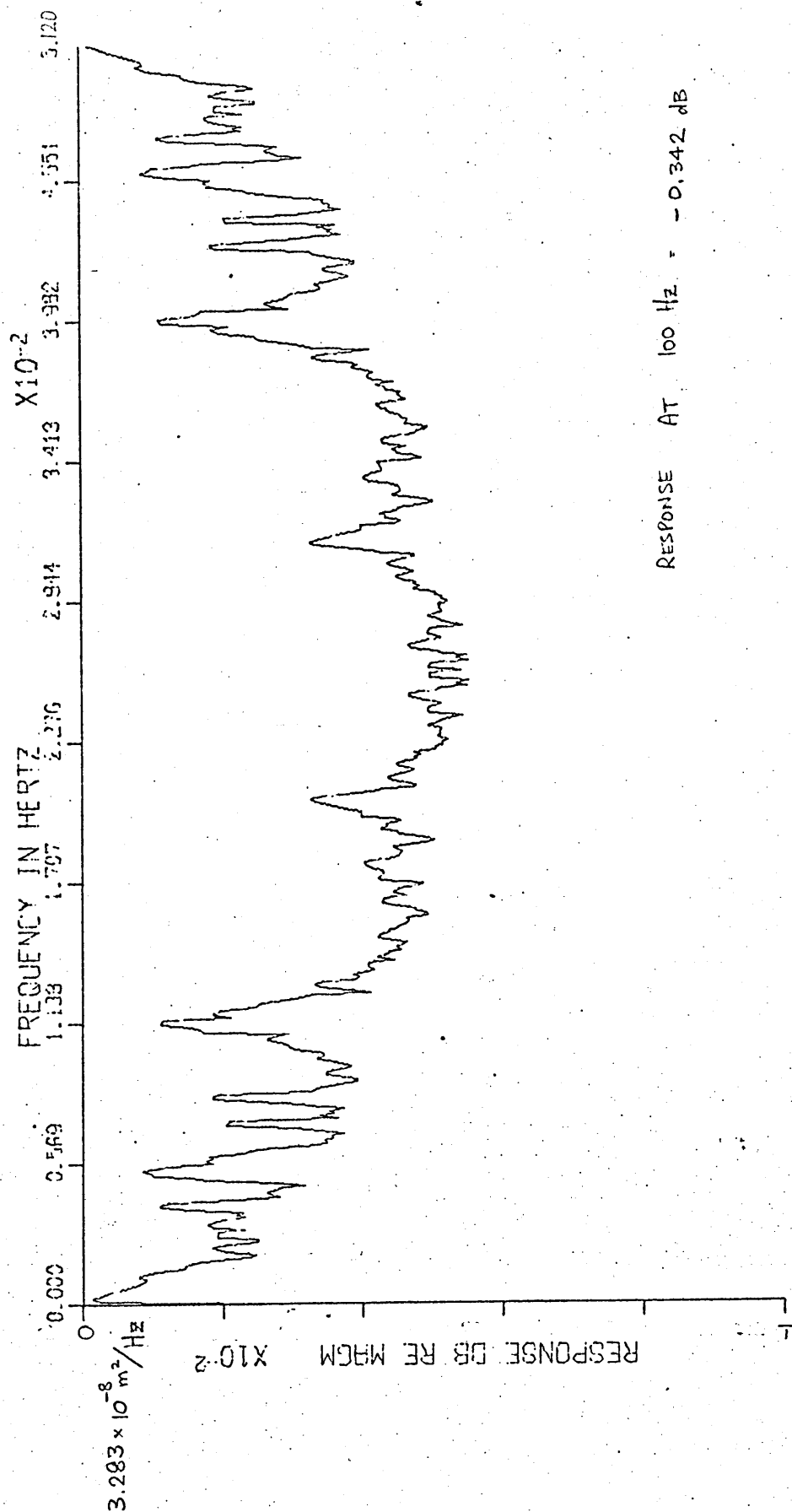
Figure 7.17   The 512 point response PSD curve using a three point smoothing.
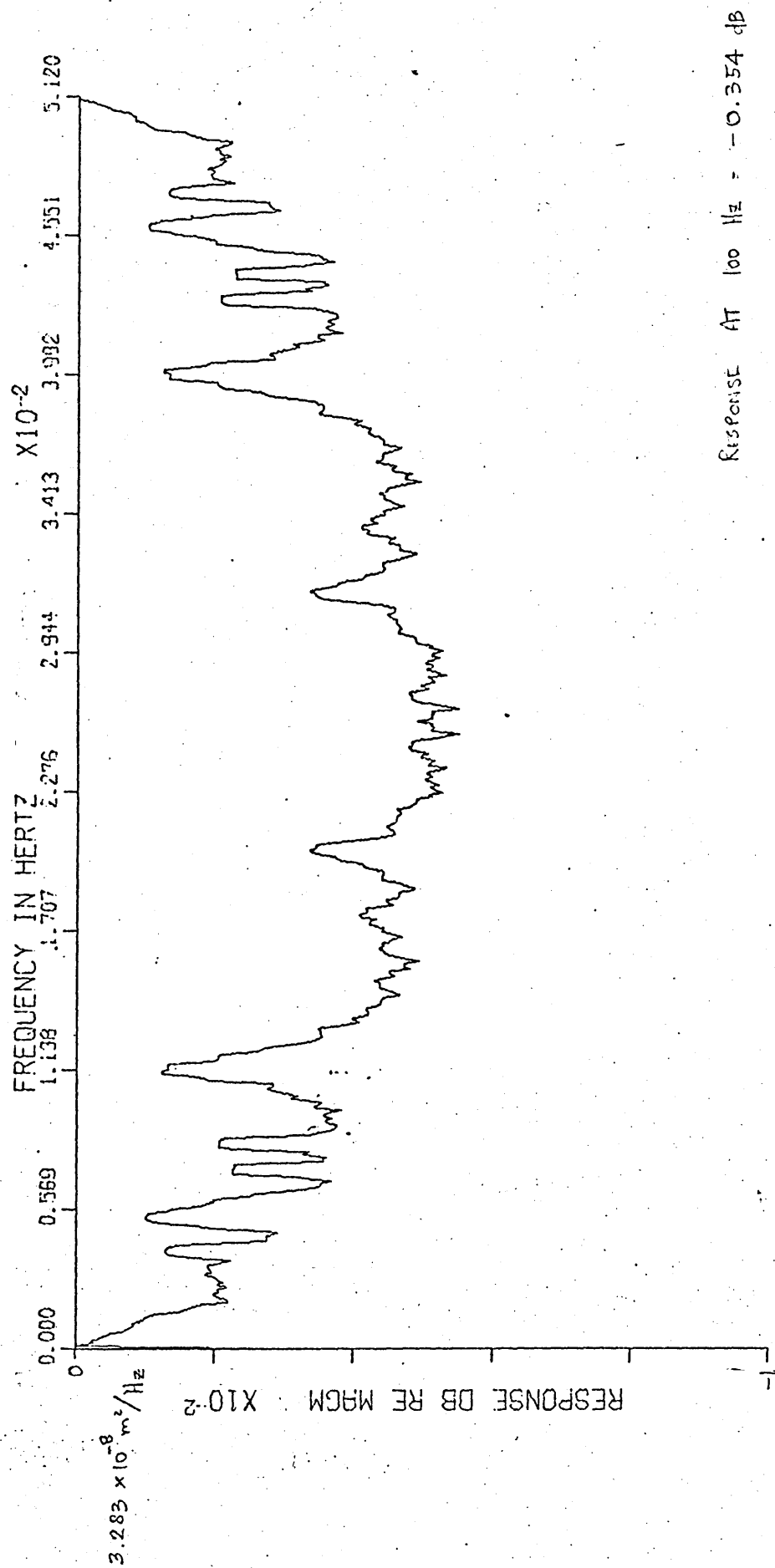
Figure 7.18   The 512 point response PSD curve using a five point smoothing.
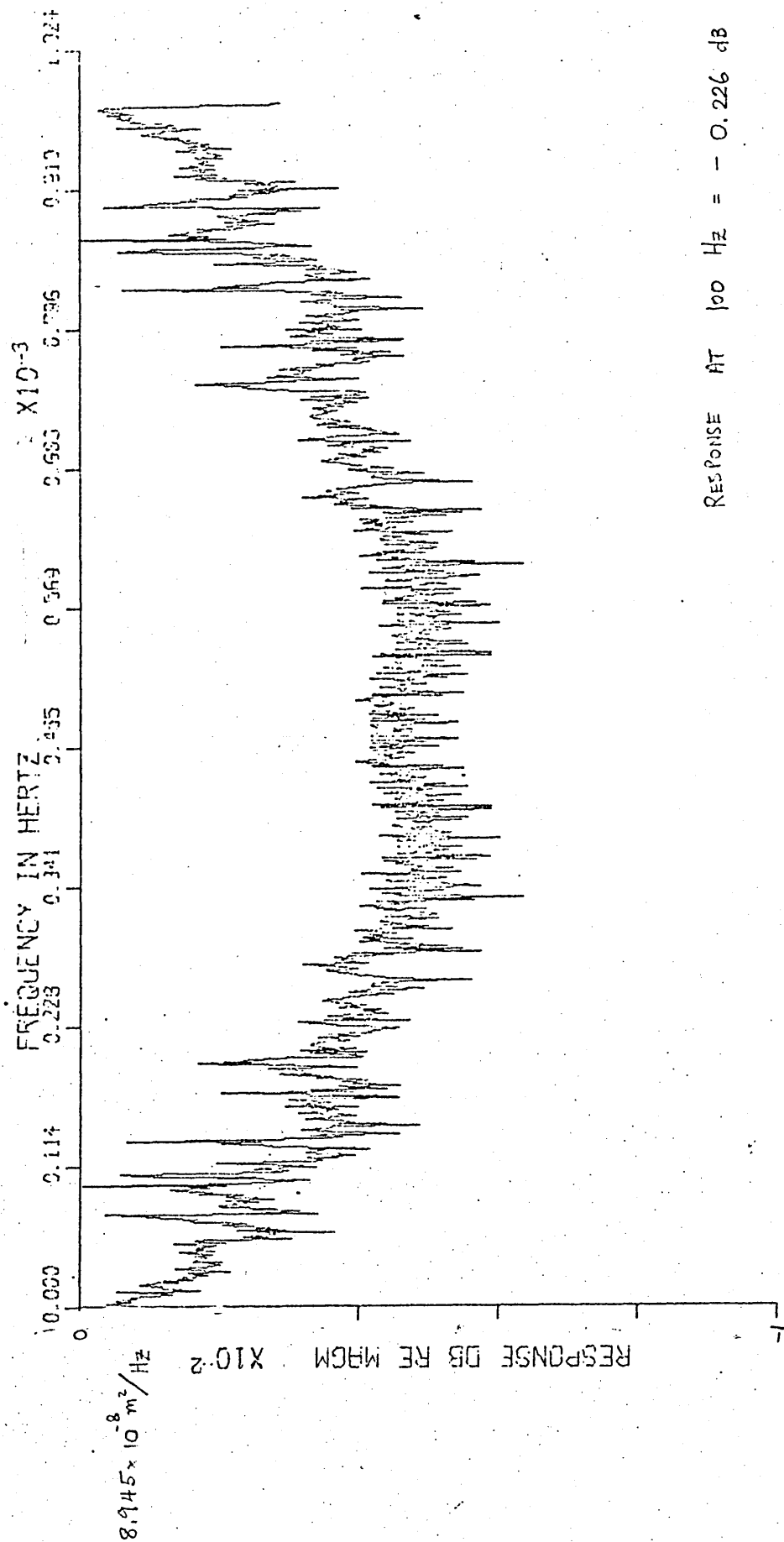
Figure 7.19   A graph of a nonsmoothed 1024 point response power spectral density computed.

The plot axes read:

FREQUENCY IN HERTZ X10⁻³

x-axis labels: 0.000, 0.114, 0.223, 0.341, 0.569, 0.683, 0.796, 0.913, 1.324

y-axis: RESPONSE DB RE M4GM   X10⁻² (0 to -1)

$4.163 \times 10^{-8} \, m^2/Hz$
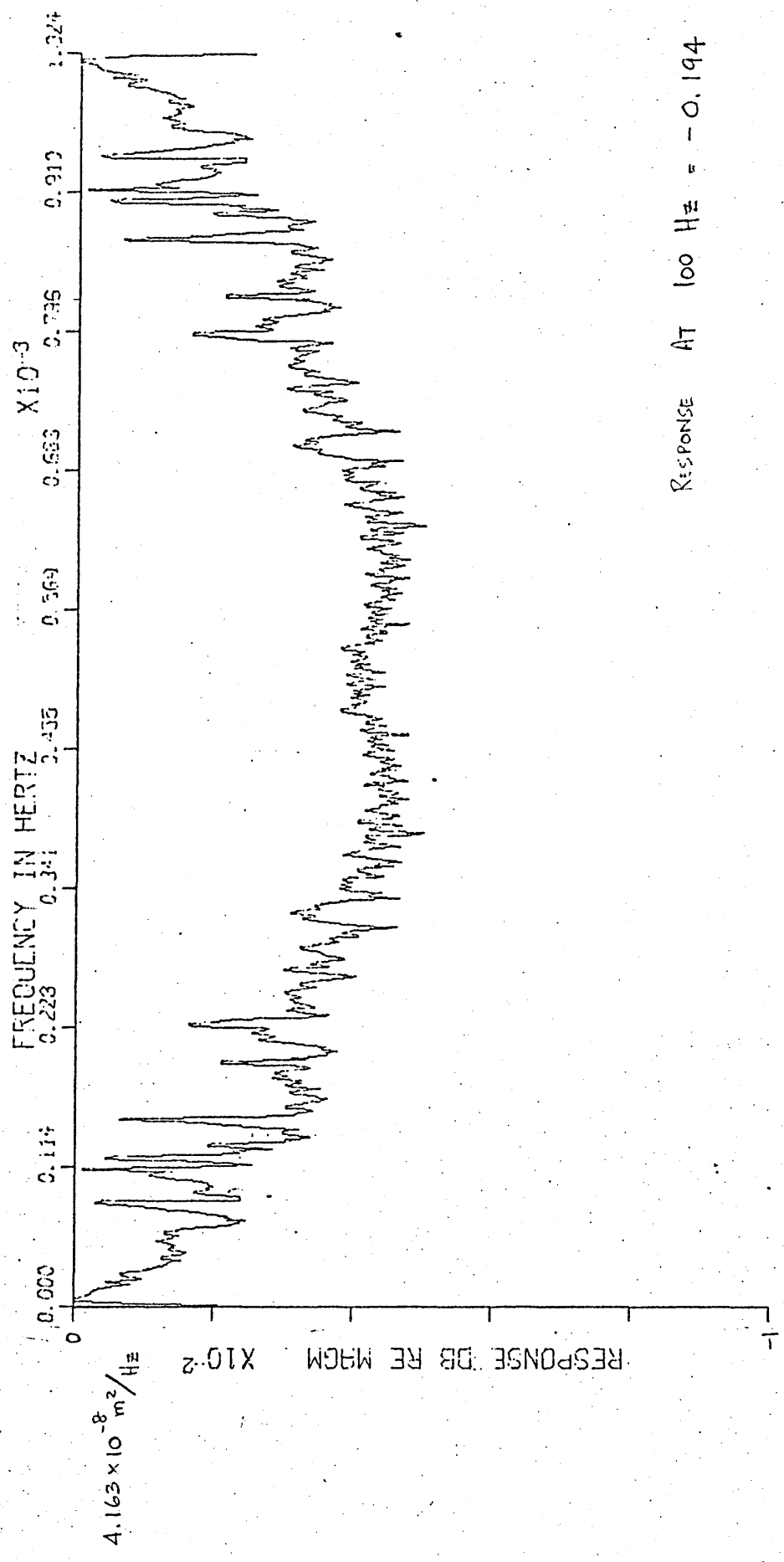
RESPONSE AT 100 Hz = -0.194

Figure 7.20   The 1024 point response PSD curve using a three point smoothing.
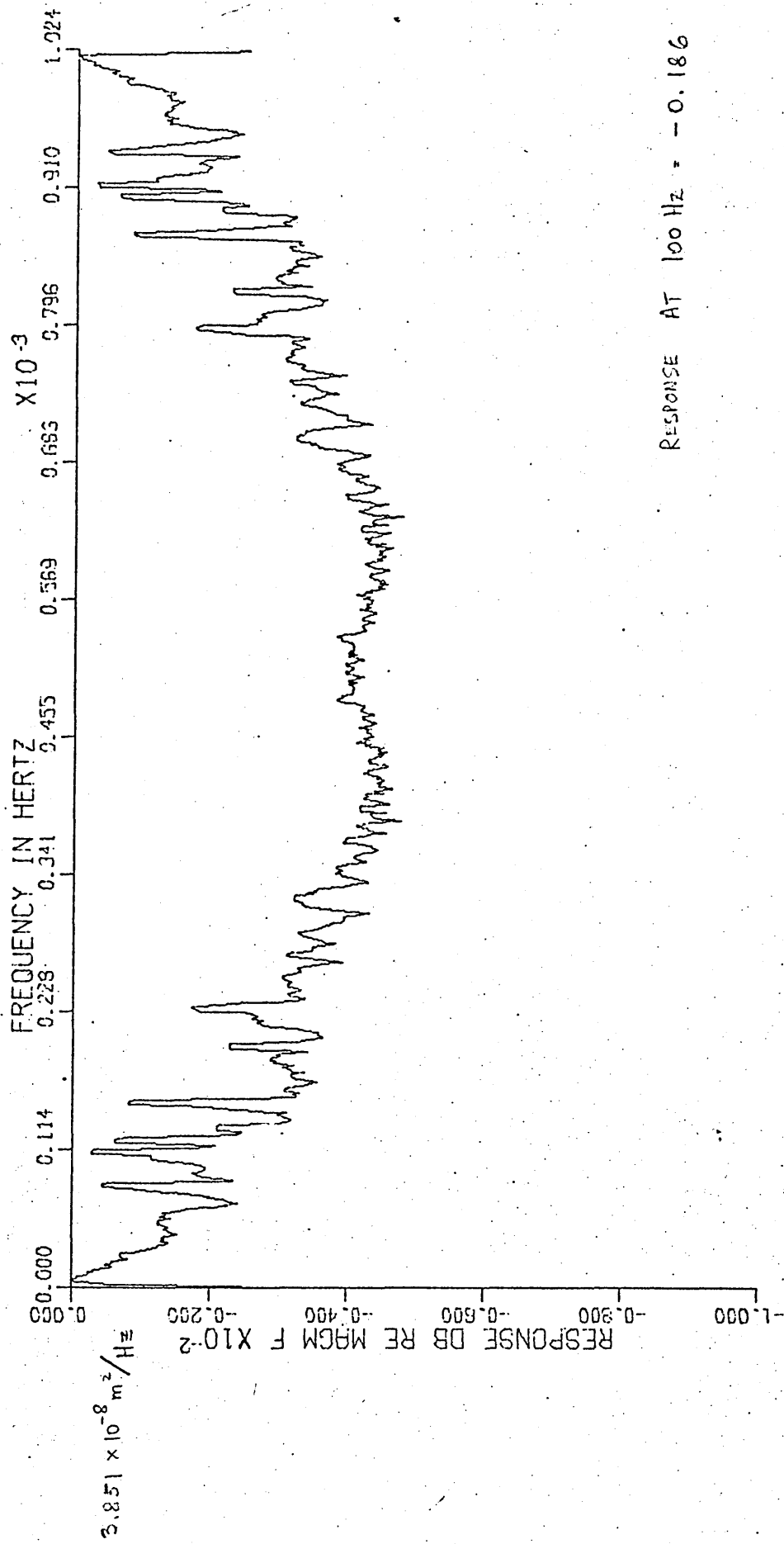
Figure 7.21   The 1024 point response PSD curve using a five point smoothing
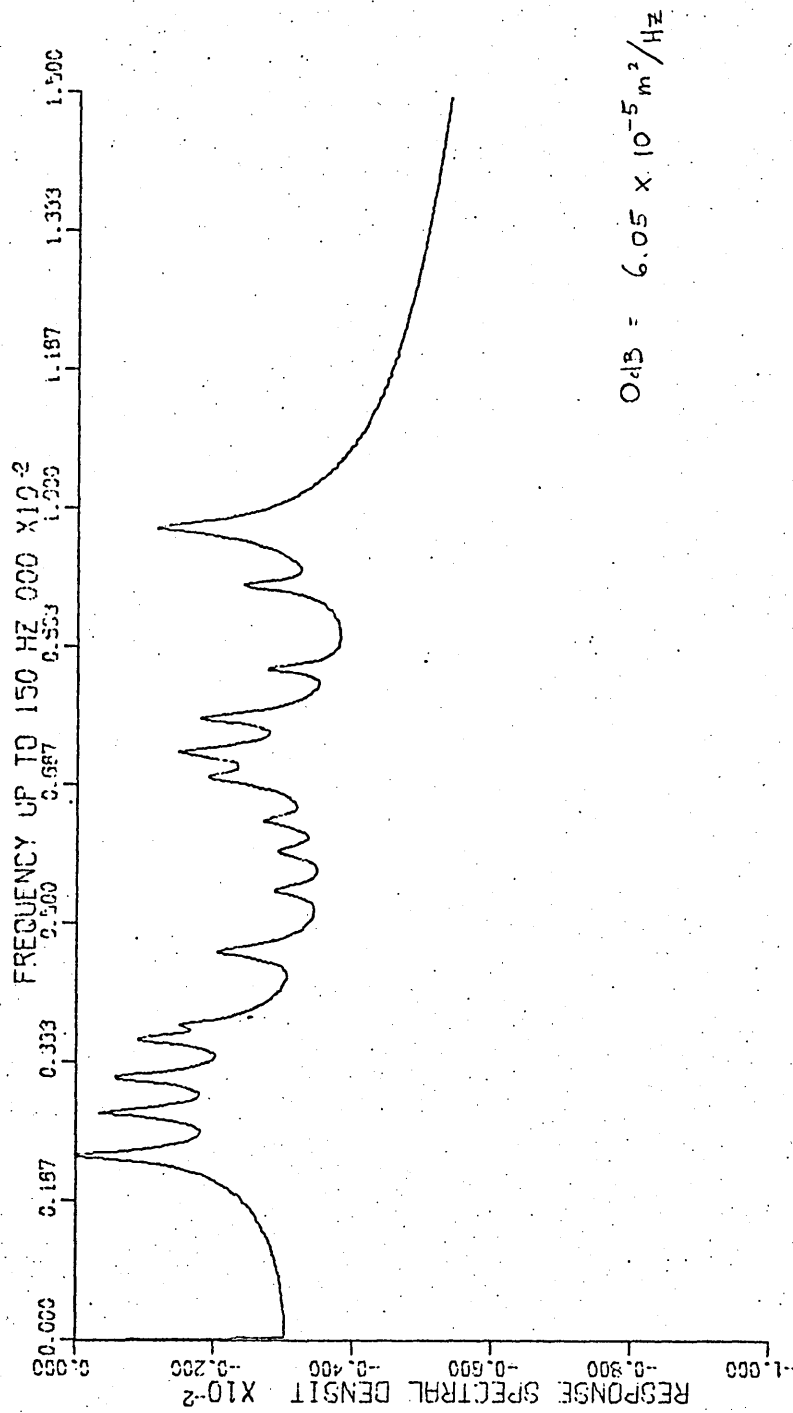
Figure 7.22   A plot of the typical box receptance^2 calculated using finite element results.
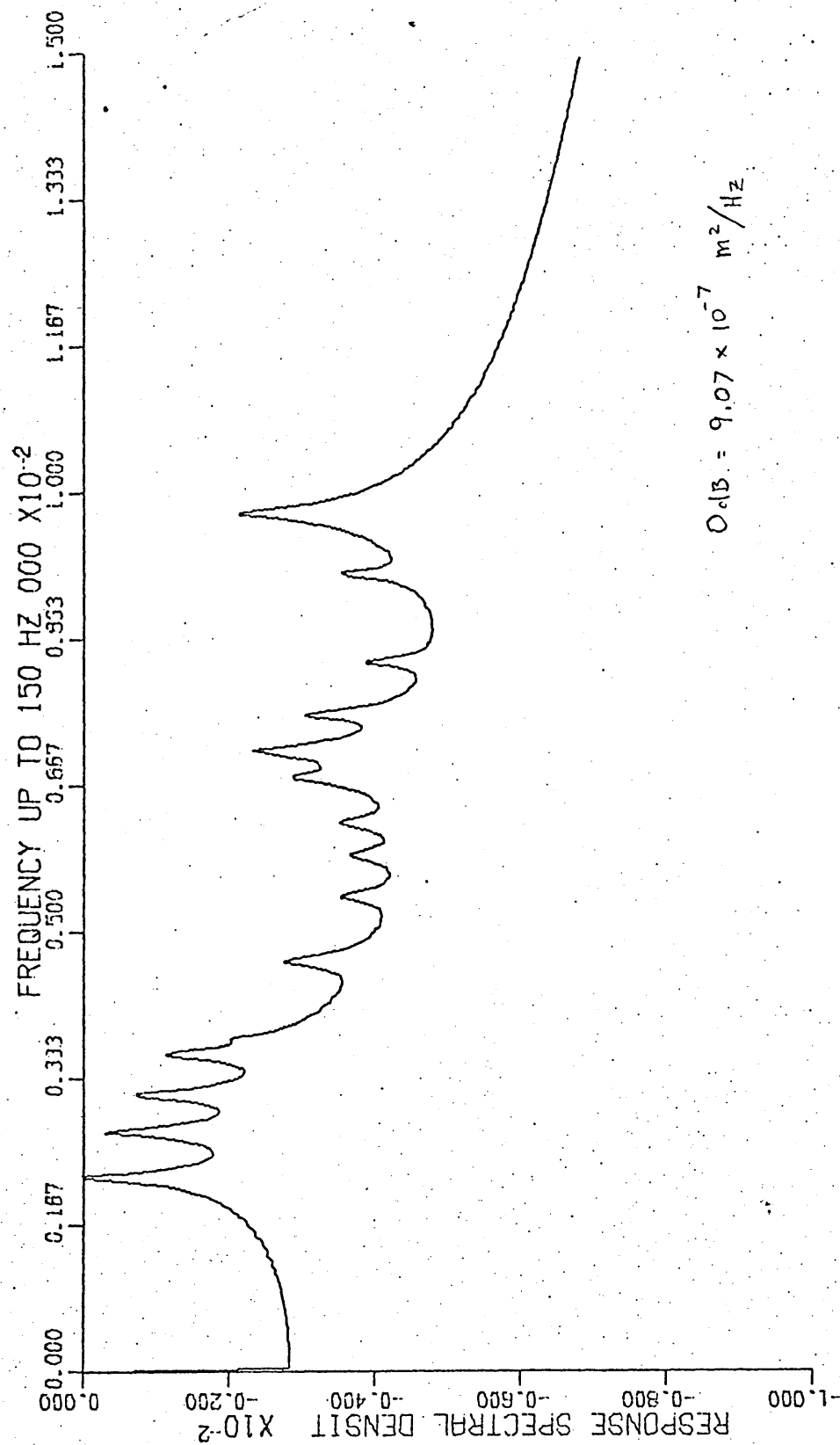
Figure 7.23   A typical response power spectral density calculated using
the finite element results.

$0\,dB = 0.879\,mm^2/Hz$
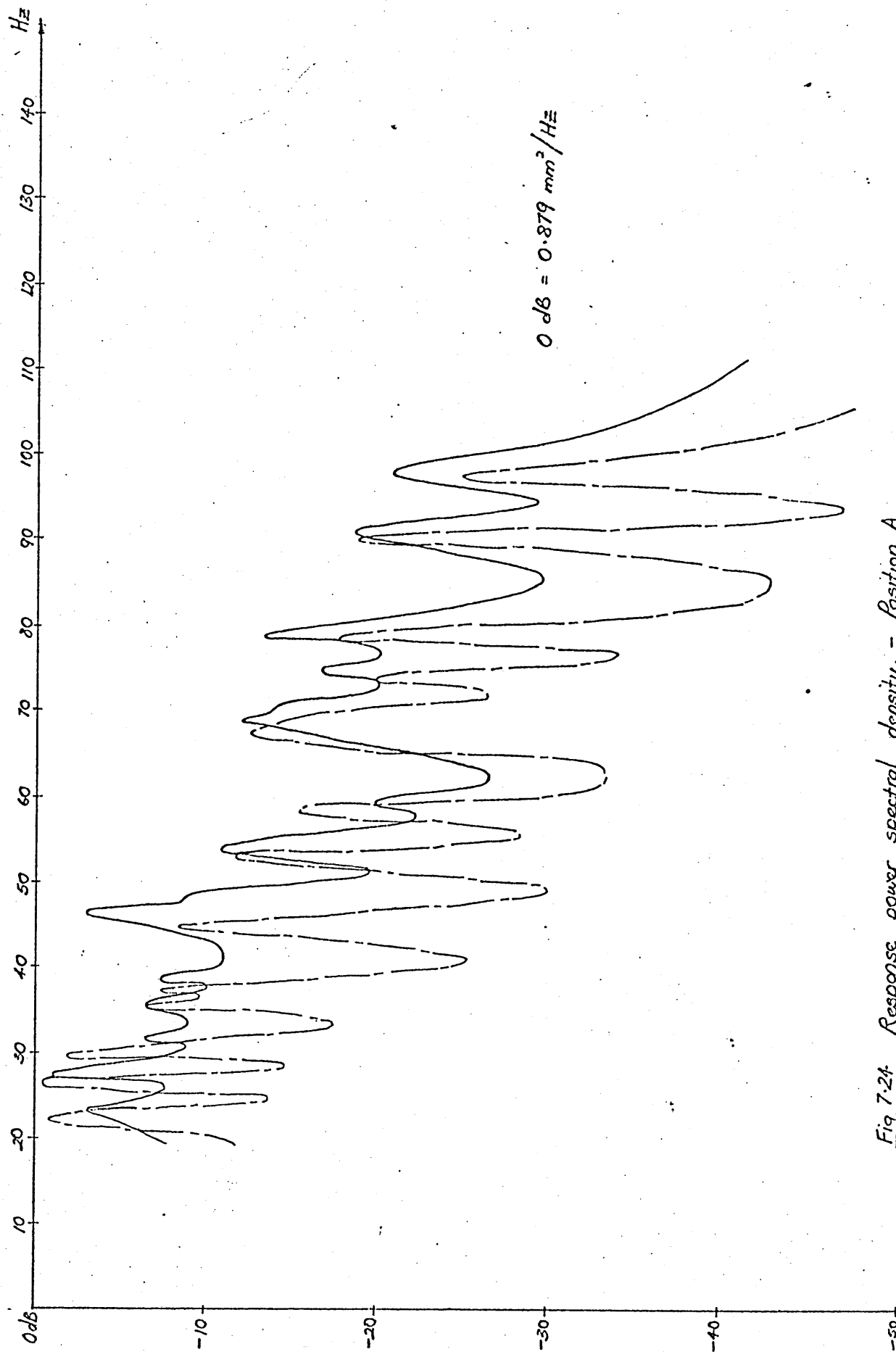
Fig 7.24 Response power spectral density. — Position A

$0 \, dB = 0.932 \, mm^2/Hz$
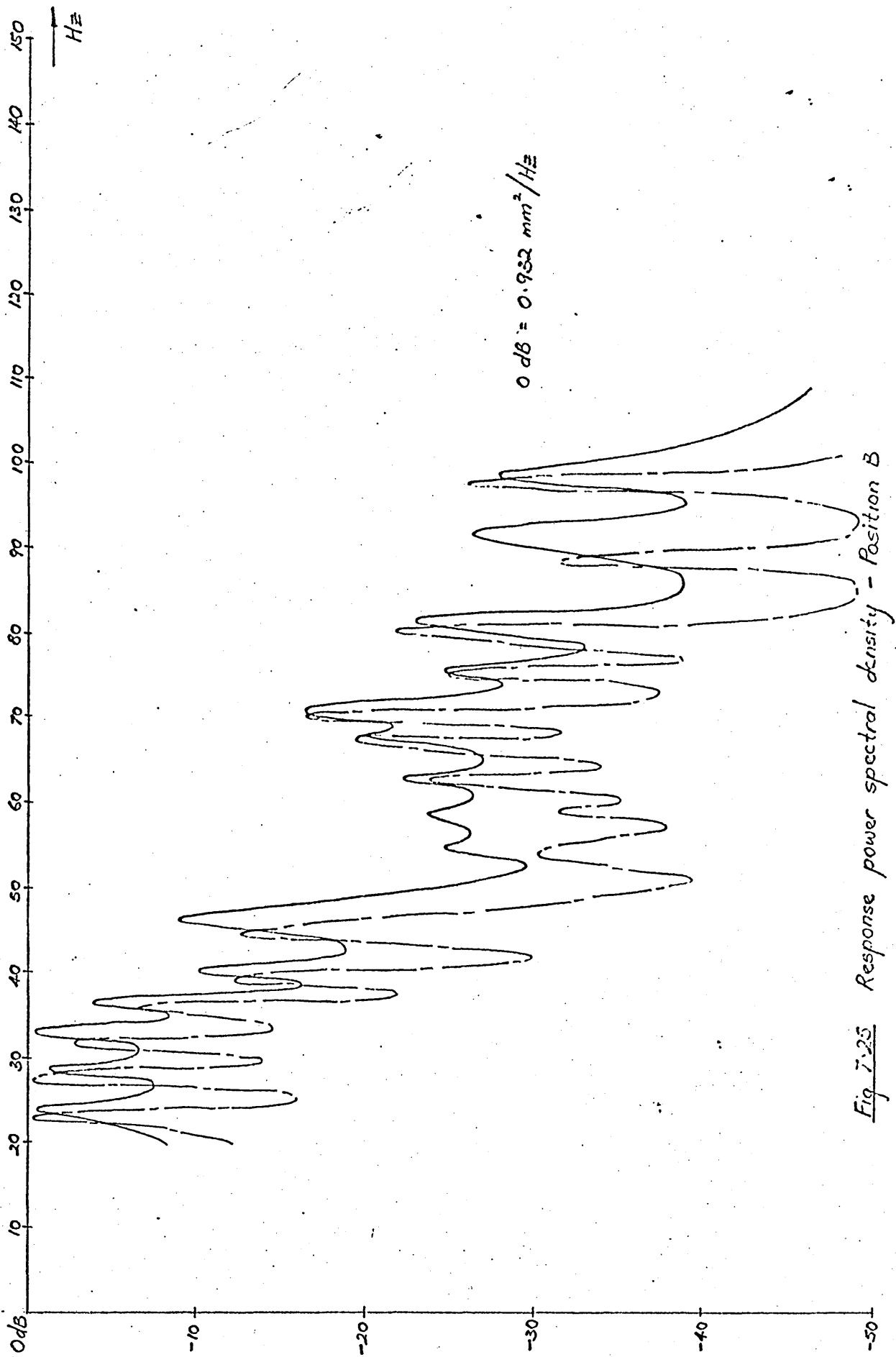
Fig 7.25   Response power spectral density – Position B

$0 \, dB = 0.986 \, mm^2/Hz$
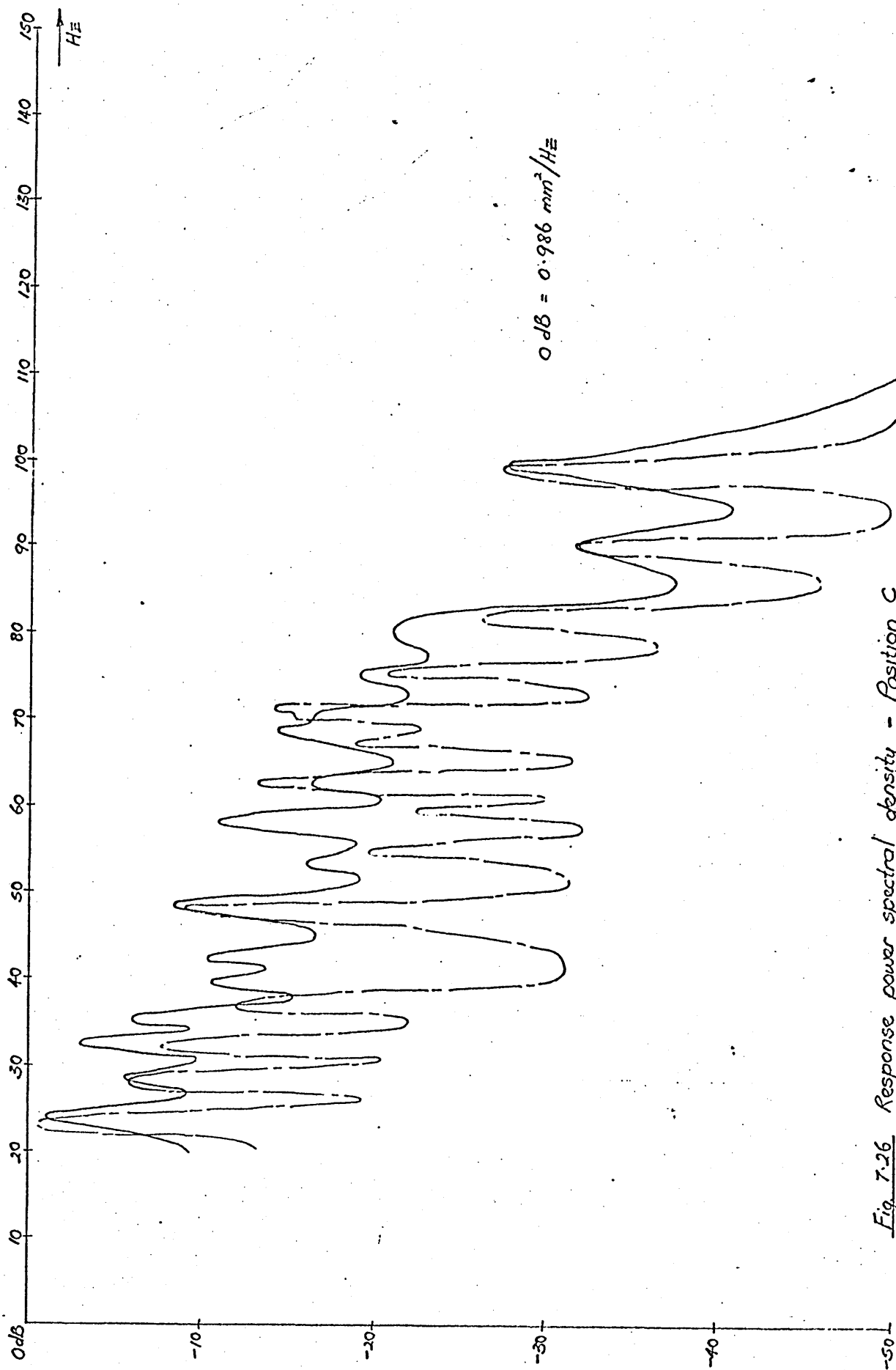
Fig 7.26 Response power spectral density - Position C

$O \, dB = 1.021 \, mm^2/Hz$
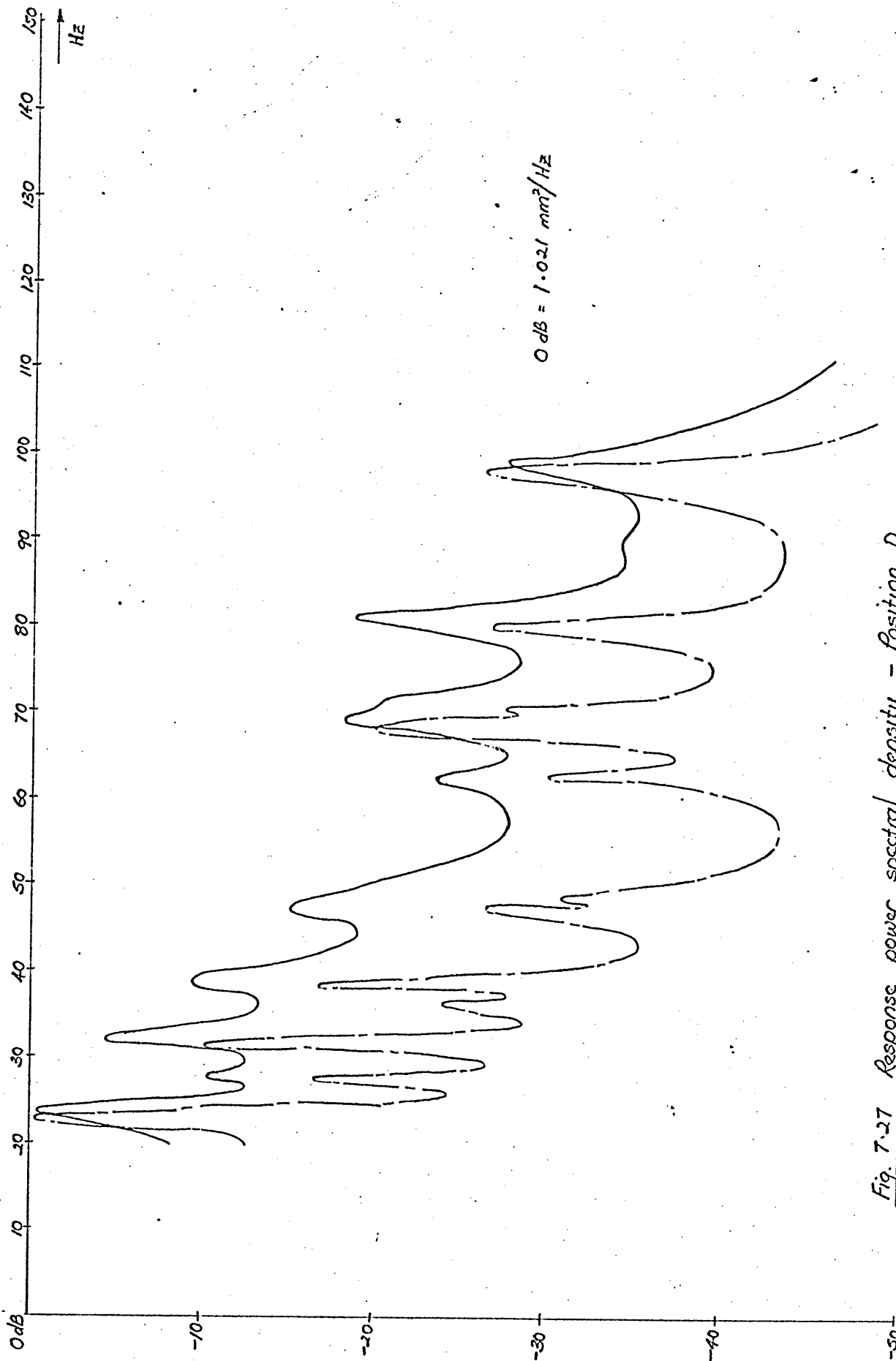
Fig. 7.27 Response power spectral density – Position D

$0 \, dB = 1.146 \, mm^2/Hz$
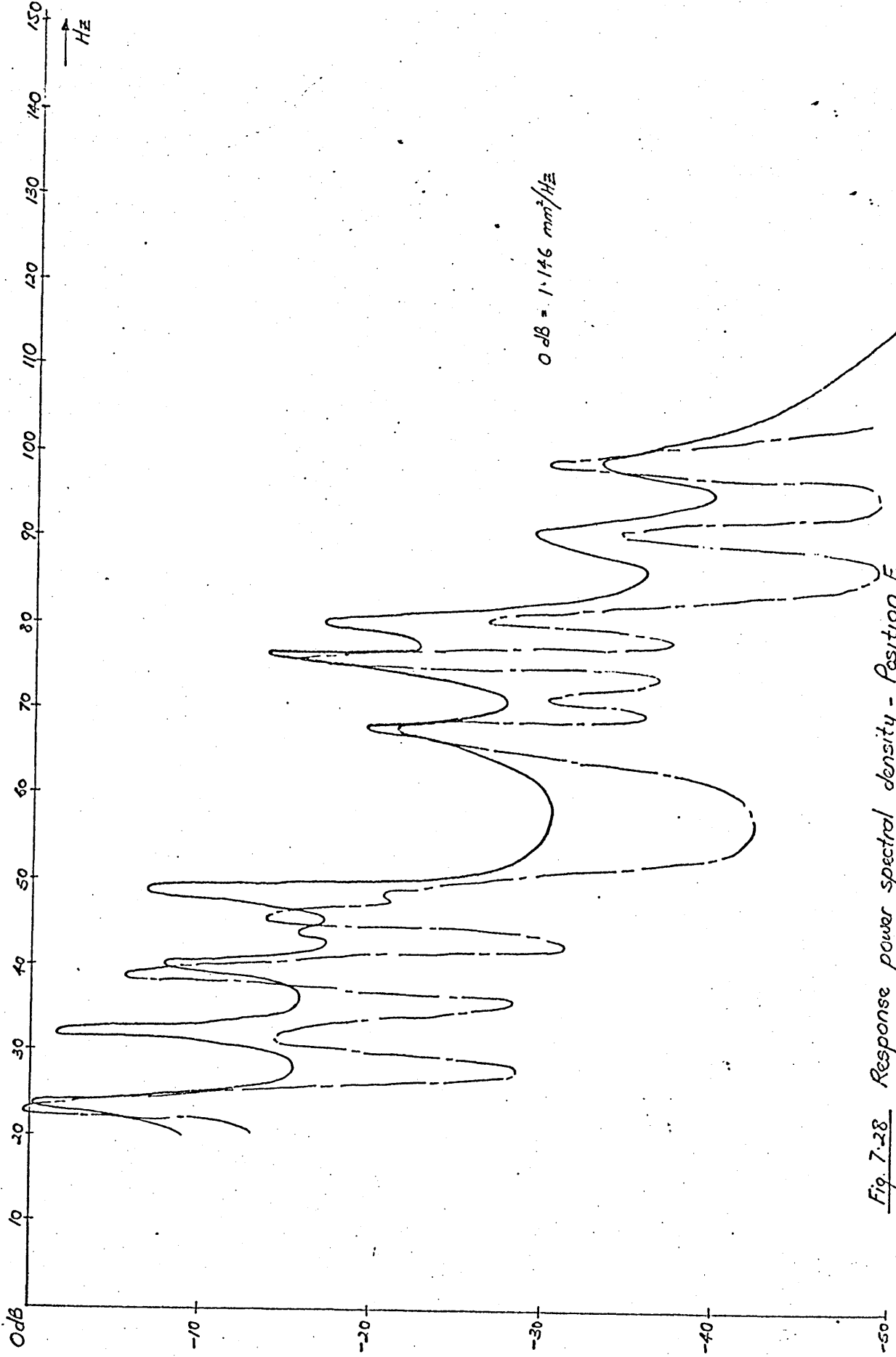
Fig. 7.28 — Response power spectral density – Position E

$0 dB = 0.734 \ mm^2/Hz$
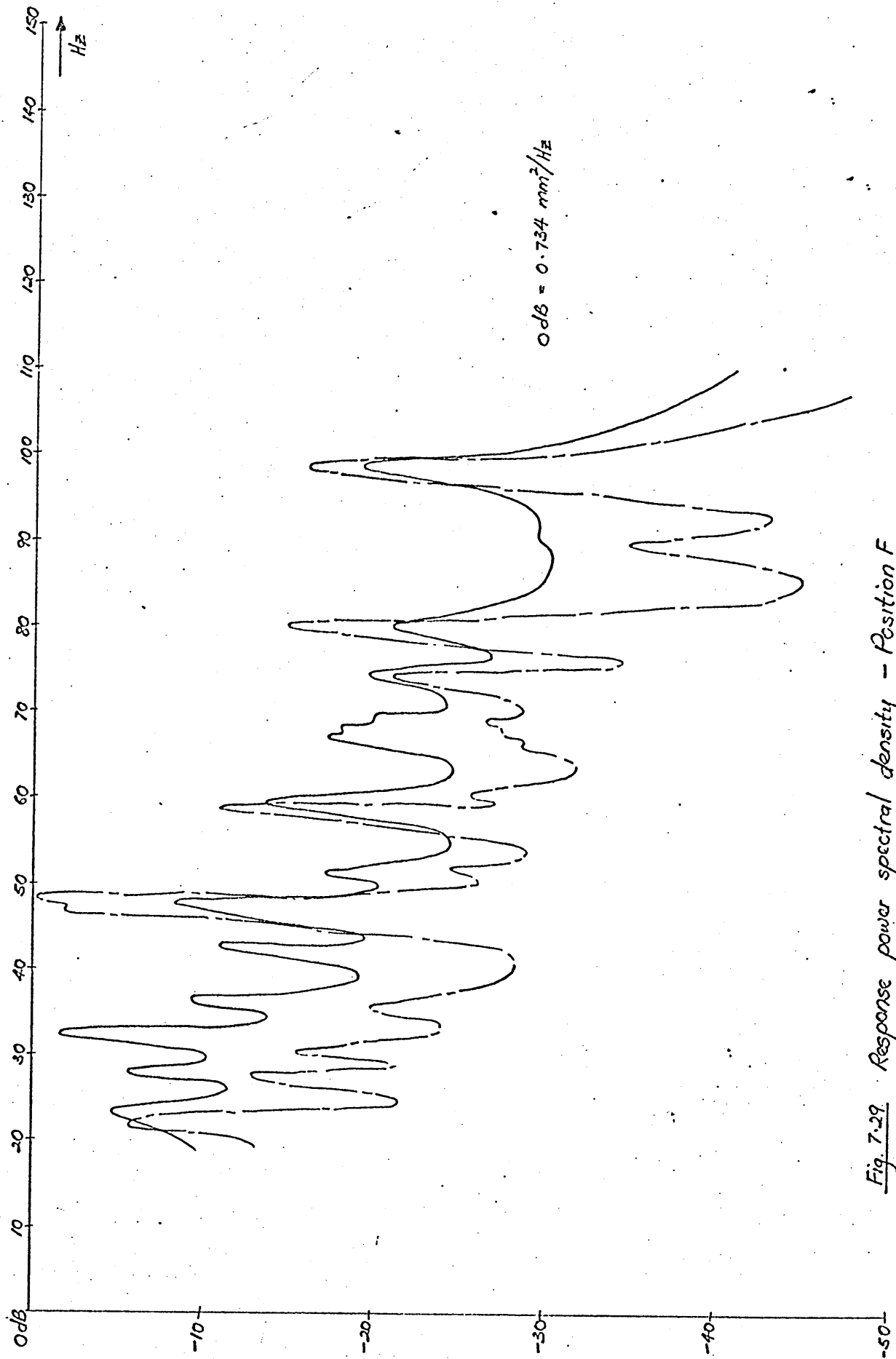
Fig. 7.29. Response power spectral density — Position F
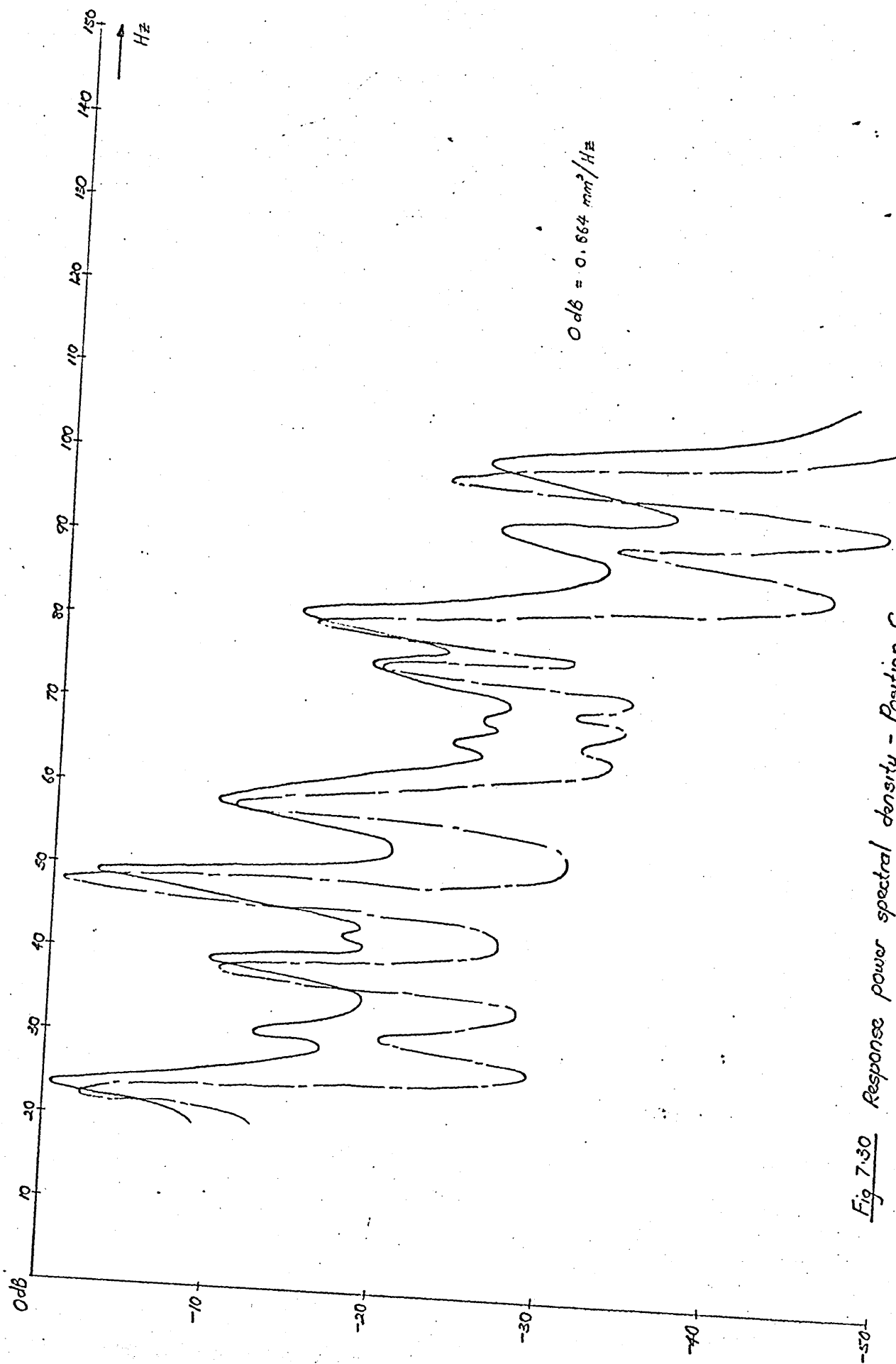
Fig 7.30   Response power spectral density – Position G

$0 dB = 0.664 \ mm^2/Hz$

Chapter 8   Conclusion

From the results given in the previous chapter it will be
seen that the techniques chosen for this work in the analysis
of folded plate box type structures have worked satisfactorily.
The finite element method has been successfully applied to
predict natural frequencies, and mode shapes and used to
predict the power spectral density of the response of the
box structures to random vibration. This is compared with .
actual experimental values obtained. The techniques used in
the experimental investigation using the equipment developed
during this project provides results which form a close basis
of comparison with the predicted values.

Using the finite element method of analysis, the natural
frequencies and the corresponding mode shapes were obtained
for the simple beam, plate as well as the box structure. The
finite element prediction of their natural frequencies were
found to be within 5 % of the experimental values obtained.
The computer program suite developed to analyse the box
structure consists of seven subprograms each reading off
information from the previous from a magnetic disc file,
using it, and then writing the new information back to the
file for the next subprogram. This strategy was found to be
suitable for such a complex structure because of the reduced
computer core requirements and the provision of built in
checks at the end of each stage. Any errors can be detected
and the computation stopped and corrected without the com-
putation going on until the very end.

The analysis of the box structure for its natural frequencies
and normal modes using the finite element method even in the
relatively crude model chosen gave results which compare well
with those obtained experimentally. This shows that the
choice of the method for the analysis of this type of structure
is well justified. Also the simple four node rectangular
element used with its twelve degrees of freedom in bending
and eight degrees of freedom in stretching is shown to be
adequate for the range of frequencies investigated as is

the coarse mesh employed. The finite element method often gives a lower bound solution but in general this is not known unless conforming elements are used [50]. Although theoretically the accuracy obtained in such an analysis is expected to converge to the exact solution as the mesh size is refined, this is limited by the size of the computer storage available. Refinement of mesh size requires increased computer storage that rapidly become prohibitive especially in a dynamic analysis where two large matrices, the mass and the stiffness matrices have to be manipulated simultaneously. Some existing techniques for reducing the size of these matrices are reviewed in Appendix 2. The simplest way of reducing the mass matrix is to use lumped masses but this may lead to some lowering of accuracy (Zienkiewwicz [20]). Techniques such as substructuring and reduction of degrees of freedom by condensation also allow the larger problems to be fitted onto the smaller computers using backing store but at the expense of increased computing time.

Methods considering only the transverse degrees of freedom (Vysloulch et al [51]) have been tried. More recently the use of reduced numbers of degrees of freedom per node eg. by using loof elements (Irons [52]) or relaxed continuity elements (Patterson & Heng [53]) have been developed.

Despite the disadvantage of the tapering shape of the force spectrum produced, the closeness of agreement between natural frequencies and mode shapes predicted and actual experimental values justifies the use of the non-contacting exciter/pickup probe developed since contacting transducers may shift the natural frequencies or distort the corresponding mode shapes obtained. The overheating and frequency doubling effects of the non-contacting exciter have been overcome although the lower force output at the higher frequencies may lead to inaccuracies in that region. It is anticipated that this frequency dependance caused by what is basically an inductance circuit may be overcome using an active resistance-capacitance circuit in parallel with the exciter probe. In the present work this was not done and in the prediction of the response

power spectral density of the box, this was taken into account
in the specification of the input power spectral density. The
non-contacting vibration pickup probe would probably be more
accurate than convectional contacting ones especially on the
light structures investigated. It was found that this capaci-
tance probe was fully applicable over the frequency range,
gap distance and amplitudes of vibration used.

In the use of discrete excitation to measure natural frequen-
cies and mode shapes of the box structure, the closeness of
the higher modes and the necessarily low level of excitation
to maintain linearity allowed only the lower natural frequen-
cies and mode shapes to be easily identifiable. Also because
the tests were preformed in a laboratory situated close to a
busy main road, considerable extraneous noise was detected
which may excite resonance modes adjacent to the one under
investigation. The upper frequency limit chosen of 100 Hz
was found to be sufficient, containing over twenty modes and
furthermore it became difficult to distinguish between several
adjacent modes which were closely bunched together just above
100 Hz. The ascertainment of mode shapes required repeated
measurements taken at a sufficiently large number of points
on the structure to determine the resonance mode. This became
increasingly tedious especially at the higher modes.

The correlation technique has been used to give the impulse
response function from which the receptance of the box system
is obtained. The calculation of the cross correlation func-
tion, although more practicable than measuring the structural
impulse response function, is often both time consuming and
laborious. The use of the PRBS signal considerably eases
the calculations and yet retains the requirement of being
random white noise for the correlation calculations. Also
its parameters are easily set to approximate a random signal
in its frequency content, excitation amplitude and probability
distribution and yet is completely repeatable. The PRBS
correlation method used to obtain the receptance of the box
structure gave frequencies within 5 % of the predicted values.
In some cases especially at the higher frequencies it is
perhaps more reliable than those obtained using sine wave

tests because extraneous noise is eliminated doing the correlation phase. Because of this and despite the generally lower level of the excitation signals used, whilst agreeing with the frequencies obtained by the sine sweep test to within 1 %, the clear superiority of the correlation technique employed is illustrated in its giving consistant results throughout in the calculation of the response power spectral density. The use of the automatically recorded cross correlation function on the PRBS system built up, considerably eased a task which took up to nine hours for a 1024 point correlation. This also enabled better results to be obtained, taken overnight during the quieter late evenings, and early mornings.

The FFT computer program which calculates the receptance of the structure from its impulse response function is completely general and can be used for beam plates or box structures. Data input is chosen to be in the form of paper tape which is easily stored and also is not easily shuffled unlike the more conventional data cards. The provision of output in the form of drawings as well as values on the line printer gives an accurate and fast means of checking both input data as well as calculated values. Graph plots are also produced for ease of storage and display purposes.

The natural frequencies and mode shapes of the box calculated using the finite element method was used to derive the receptance of the box structure using experimentally obtained damping factors. Calculation of receptance from natural frequencies and mode shapes obtained using the finite element method, is carried out using

$$|\alpha(if)|^2 = \sum_r \frac{\left[w_r(x_A)\right]^2 \left[w_r(x_B)^2\right]}{M_r^2 \left[16\pi^4\left\{(f_r^2 - f^2)^2 + \eta_r f_r^4\right\}\right]} \qquad (8.1)$$

Although only the top surface of the box structure is considered, the method is however applicable to all sides of the box taking each side in turn. Only natural frequencies up to 100 Hz were considered and therefore the 21 natural frequencies and mode shapes in that region are used. For the

calculation of α only these natural frequencies and mode shapes play an important part; higher ones have a negligible contribution. This part of the finite element computer program is completely general; beams, plates and boxes may be analysed by the input of its natural frequencies and mode shapes at specified points and corresponding damping factors. The excitation power spectral density produced by the non-contacting exciter probe was not a flat one despite the use of a substantially white PRBS signal. This also had to be taken into account in the computation.

The response of the box type structure is obtained in the form of its power spectral density, using equation (8.2)

$$Sd(\omega) = |\alpha(if)|^2 \, Sp(\omega) \qquad\qquad (8.2)$$

This is commonly used in the calculation of response mean square value, probability distribution, etc. in random work. The response power spectral density of the box structure obtained experimentally compares satisfactorily with the predicted values especially in the lower frequency region. In this region the predicted response power spectral density agrees with the experimental values obtained to within 1 dB (about 25 % difference). At the higher frequencies however the predicted results appear to be attenuated. Average discrepancy between predicted spectral density values an actual values here are in the region of 4-5 dB. This may perhaps be caused by the following:

(a) slight differences (up to 5 %) between computed and actual natural frequencies (since the computed natural frequencies were used in the prediction of the response power spectral densities),

(b) the lower excitation output at the higher frequency region by the non-contacting probe and therefore greater interference effects,

(c) inaccuracies involved in the manufacture of the box, and

(d) a review of the damping assumptions used may also be applicable.

The apparatus, techniques employed and computer software developed during this project forms a convenient point from which extension to a two exciter investigation of random excitation of light folded plate box type structures may be carried out. Although a PRBS signal is used, a true random signal is easily substituted if a suitable correlator is available. Also from equation 8.2 the analysis is not limited to that of a flat excitation spectrum. The design of a compact exciter pickup probe resulted in measurement of excitation and response at the same point of the structure. Again the analysis is also applicable to investigations where the structure is excited at one point and measured at a different point.

Plate 1   The original vibration table.

The original victualling table.

Plate 1

Plate 2   The redesigned vibration table.

Plate 5.

The reassured viridarium rapis.

Plate 3   The linear fine adjustment mechanism.

Plate 3   The linear fine adjustment mechanism.
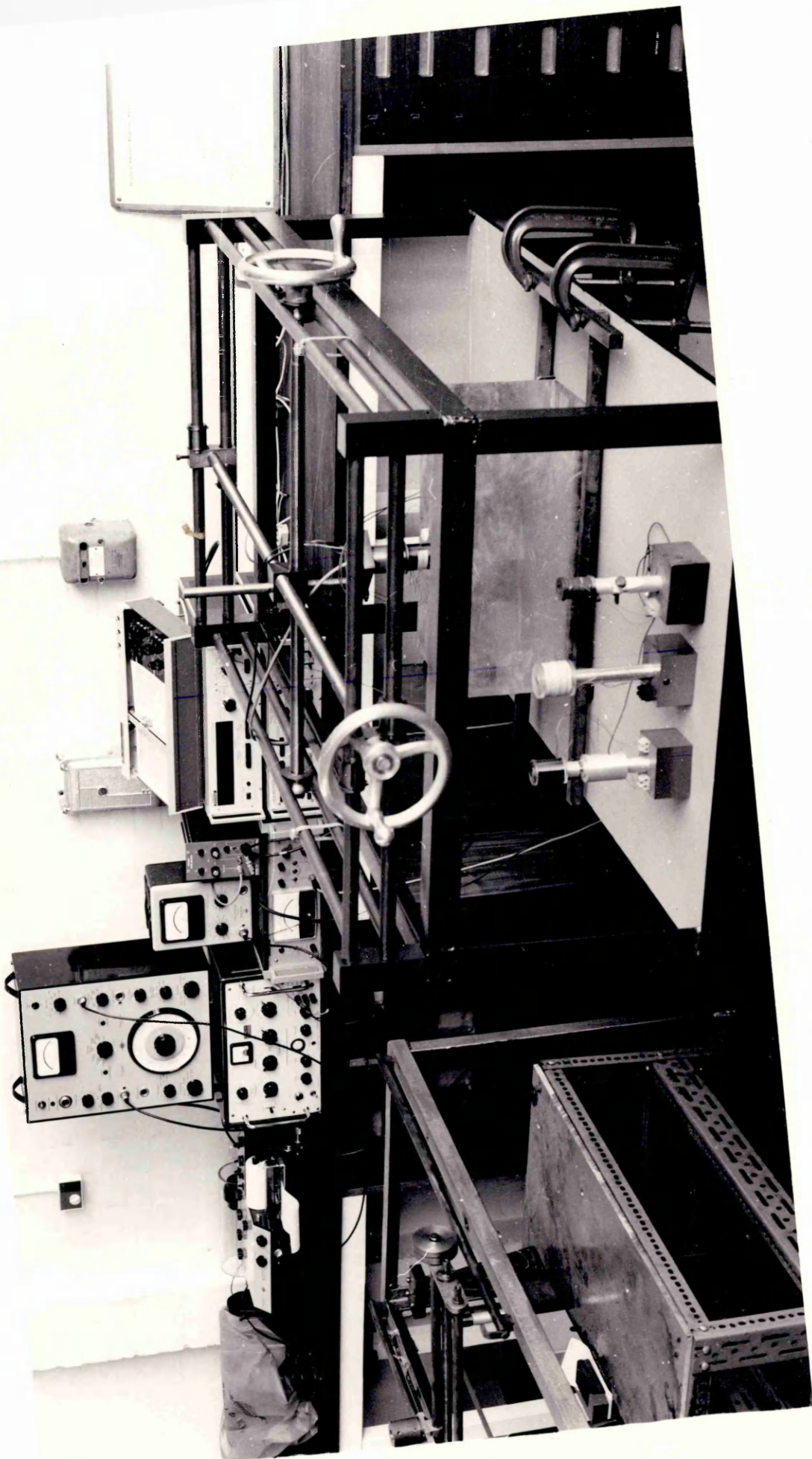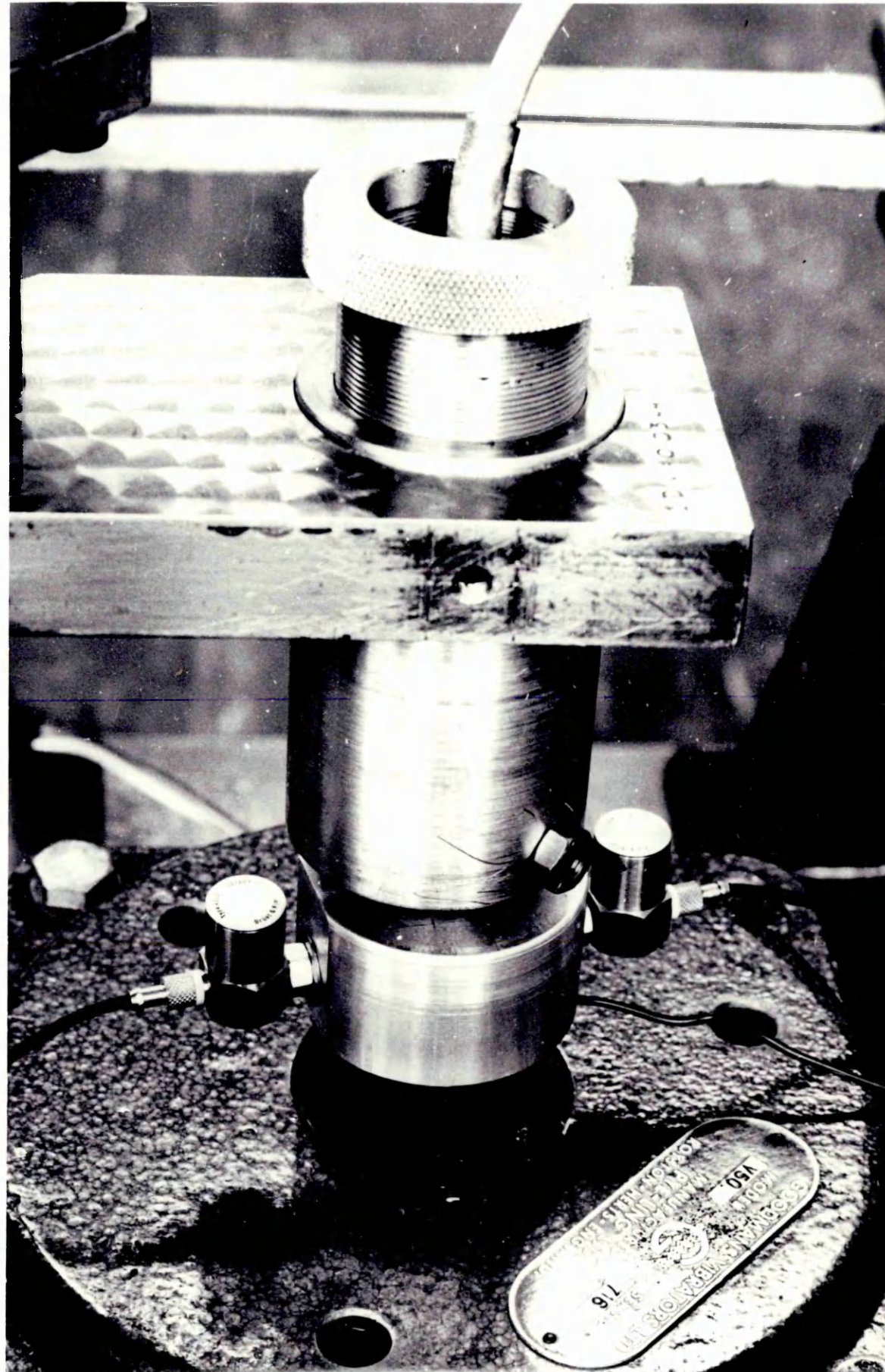
Plate 4   The skew fine adjustment mechanism.

Plate 1.   The skew fine adjustment mechanism.
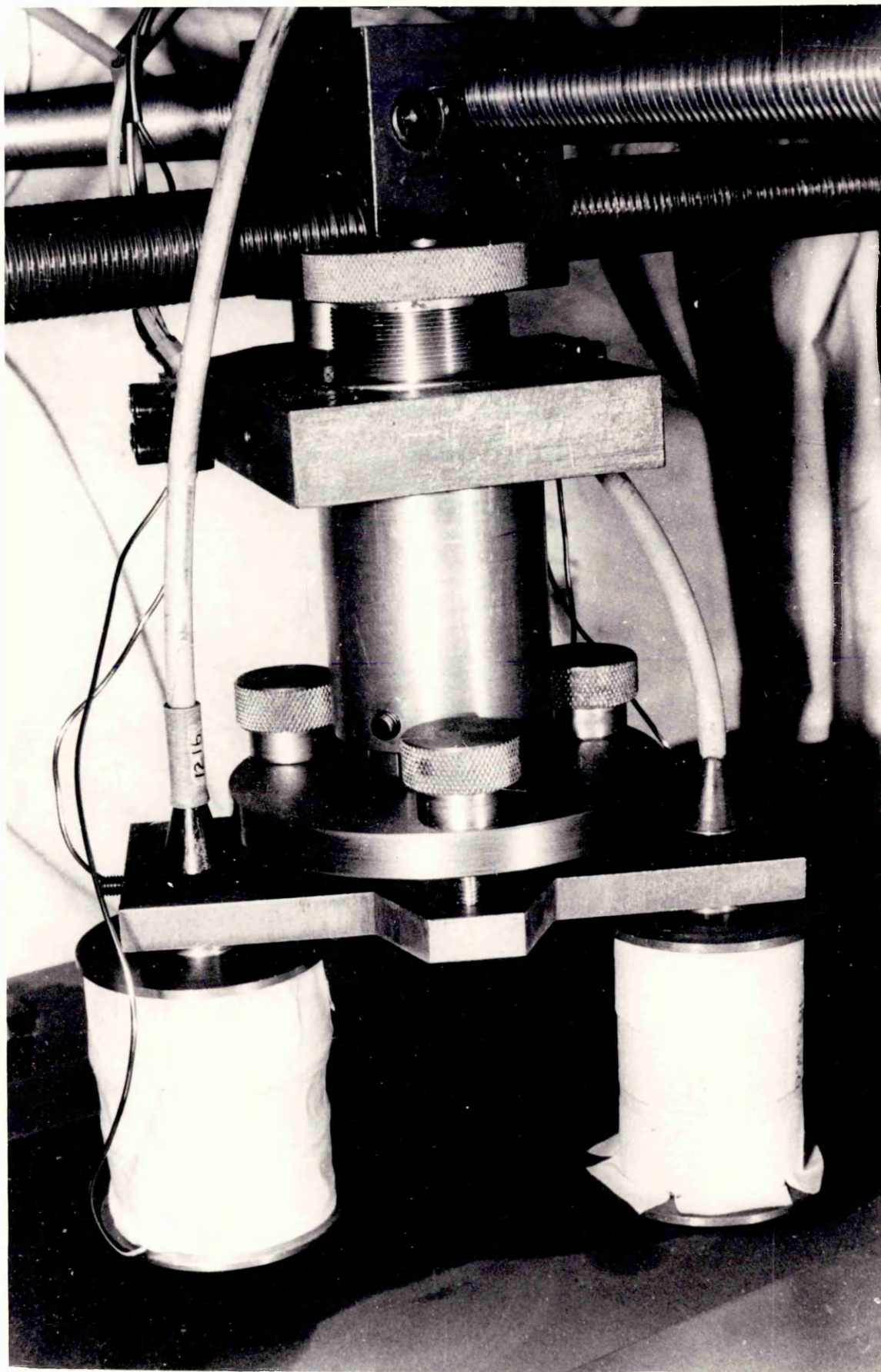
Plate 5   The plate with fully fixed edges.

Plate 5

The plate with truly fixed edges.
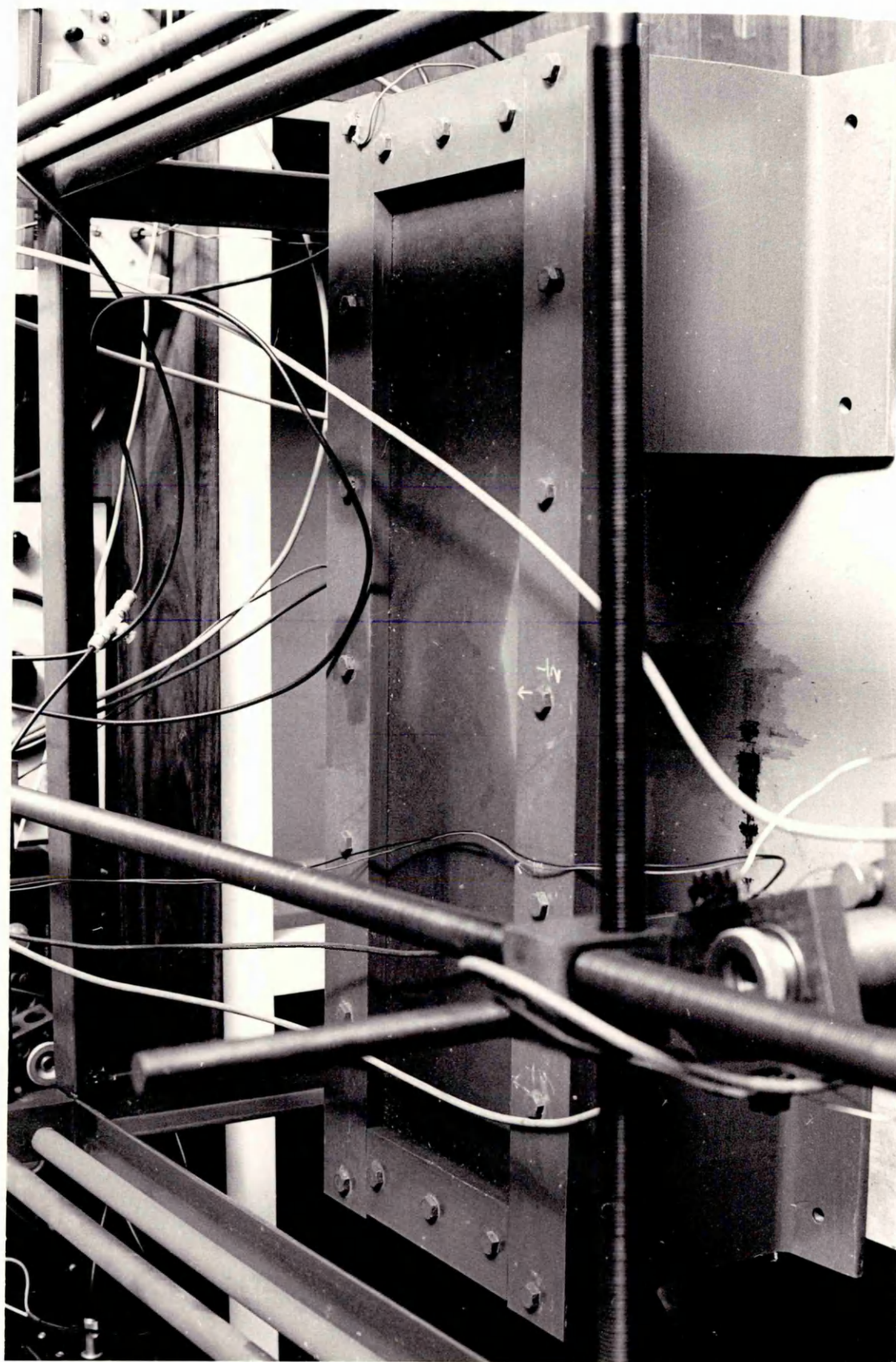
Plate 6   A fixed corner of the box structure.

Plate 7   Close up view of the calibration of the capacitance displacement transducer.

Plate 1   Close up view of, or registration of, the capacitance grainscement transducer.
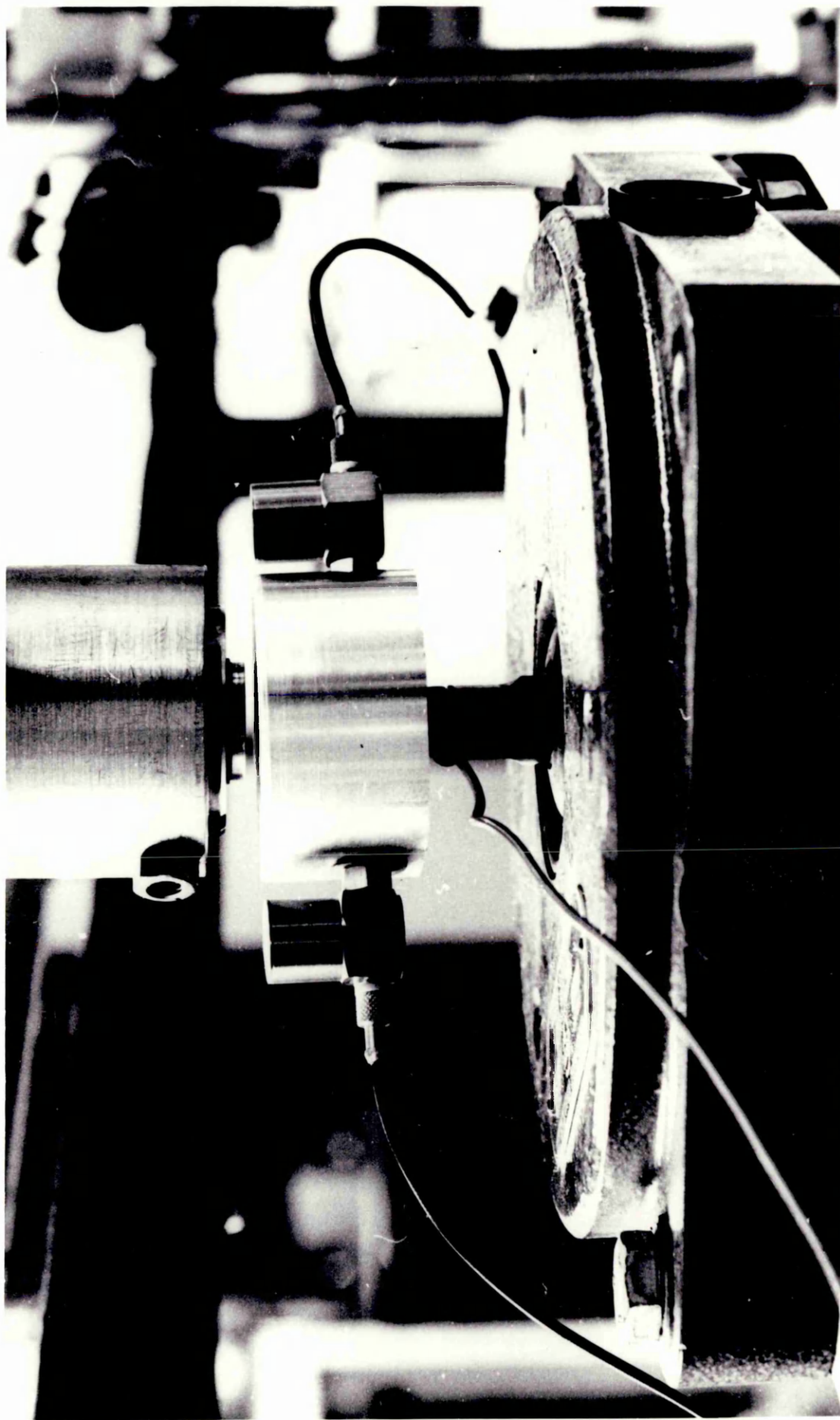
Plate 8   Instrumentation for calibration of the displacement transducer.
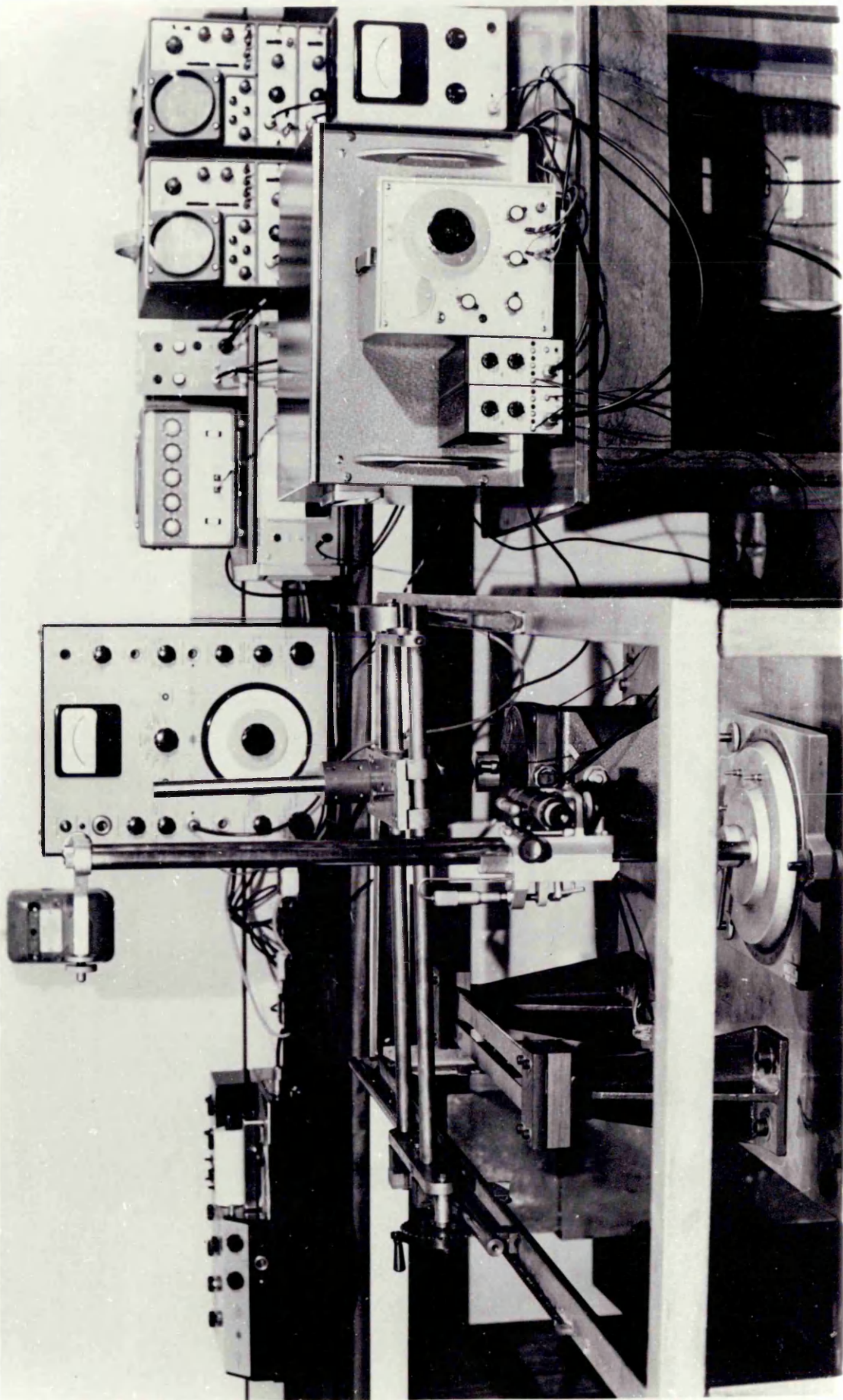
Plate 9   Calibration of noncontacting exciter using a simple beam experiment.
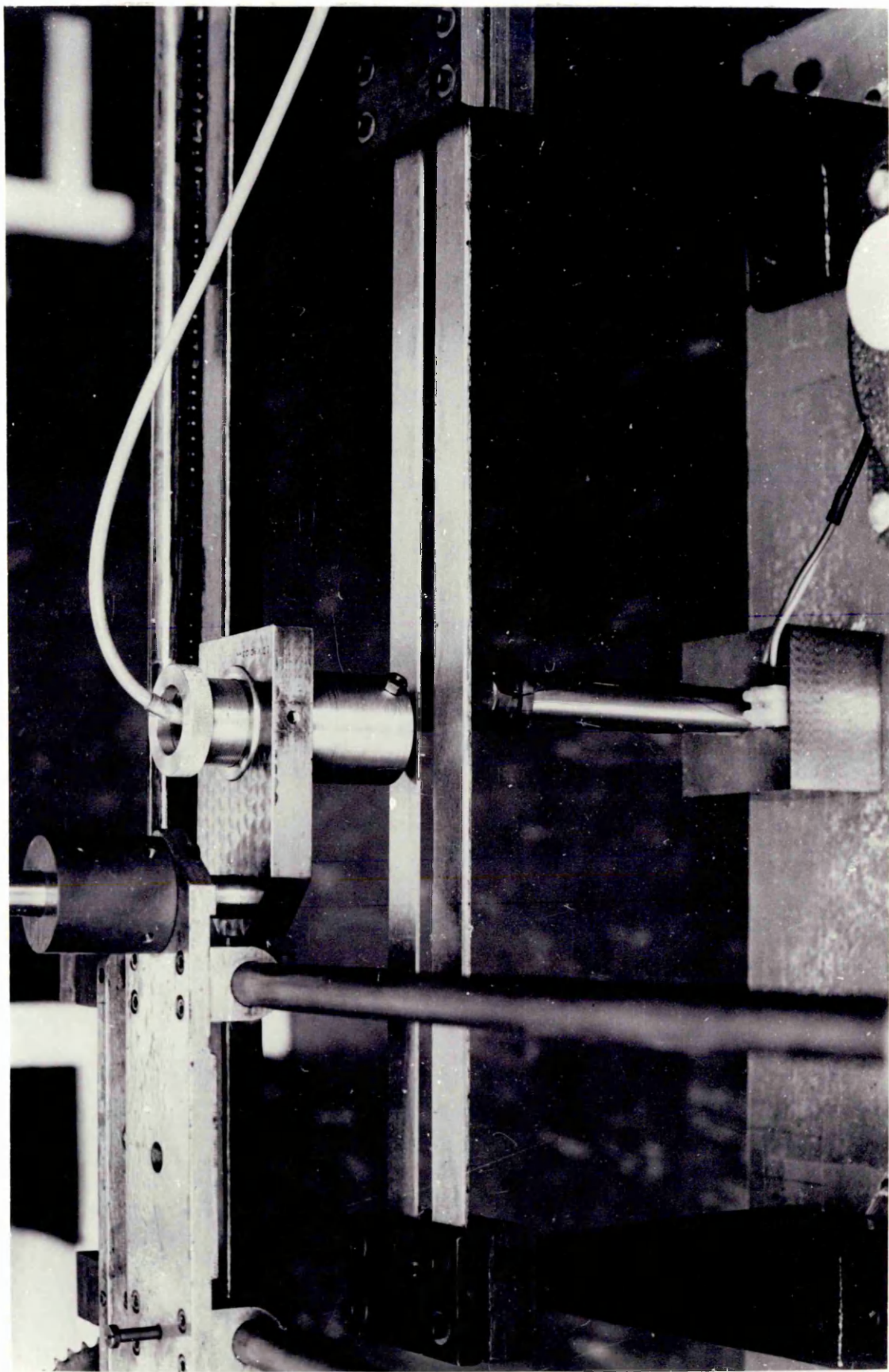
Plate 10   The housing for the piezoelectric force transducer
            used for the calibration of the exciter.

Plate 10   The housing for the piezoelectric force transducer
used for the calibration of the exciter.
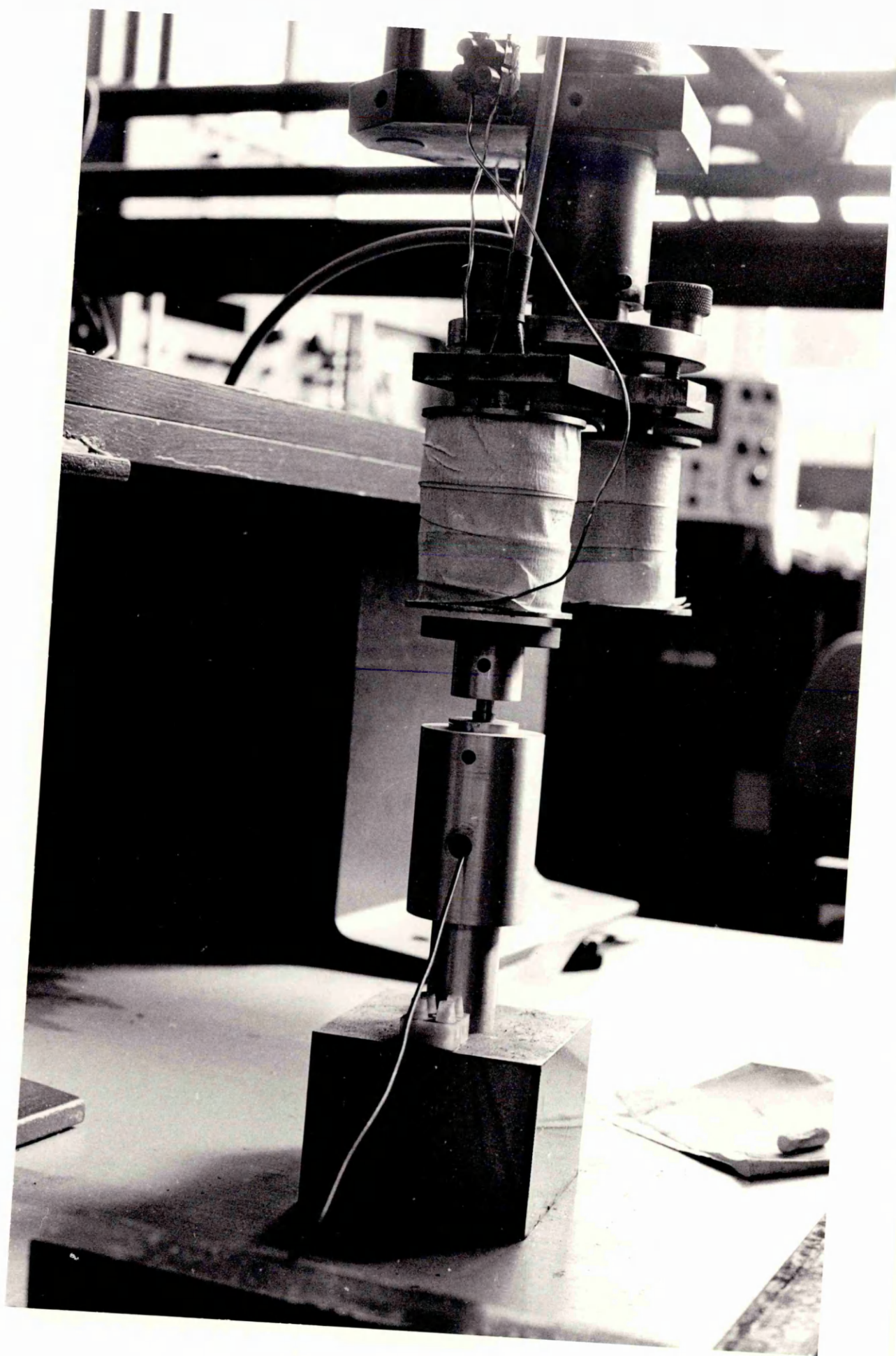
Plate 11   Some of the electromagnetic noncontacting exciters developed.
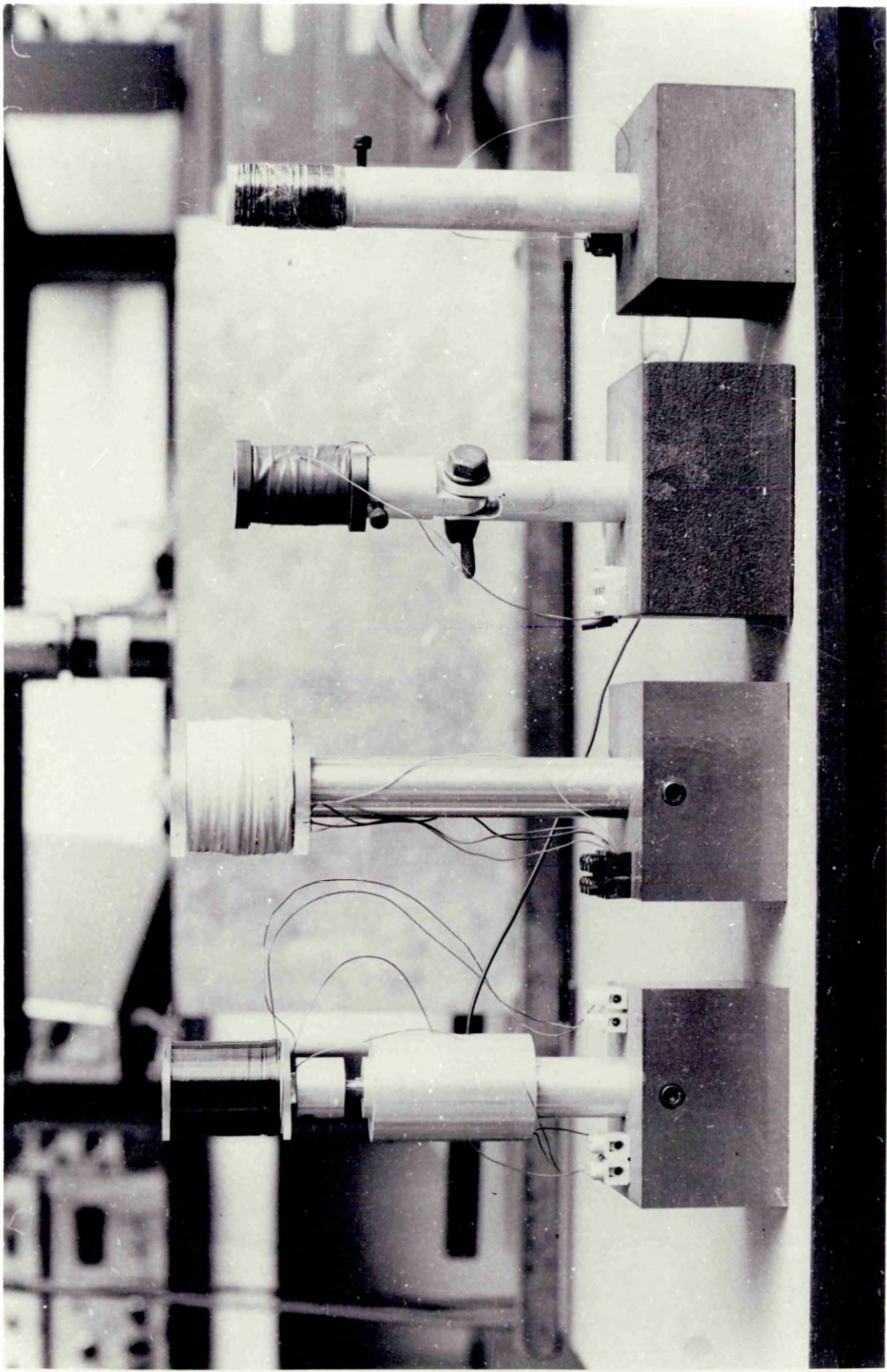
Plate 12   The combined exciter/pickup in position over the box structure.

Plate 15

the combined expressions behind prick\brackets in position over and the box structure.
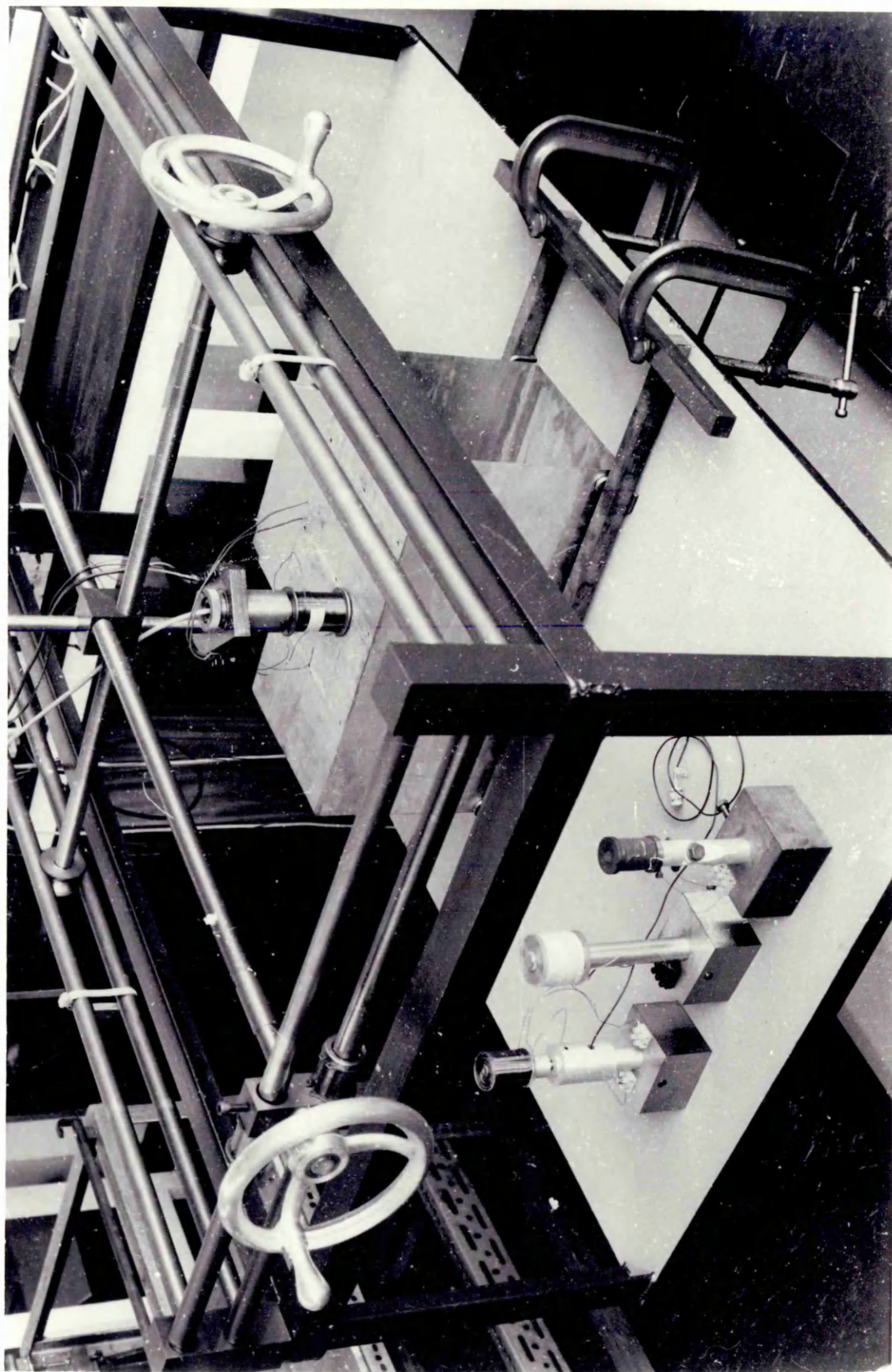
Plate 13   Frequency analysis using constant percentage frequency analyser

Figure 1.1   Predicting analysis using constant percentage treatment assay/ass.
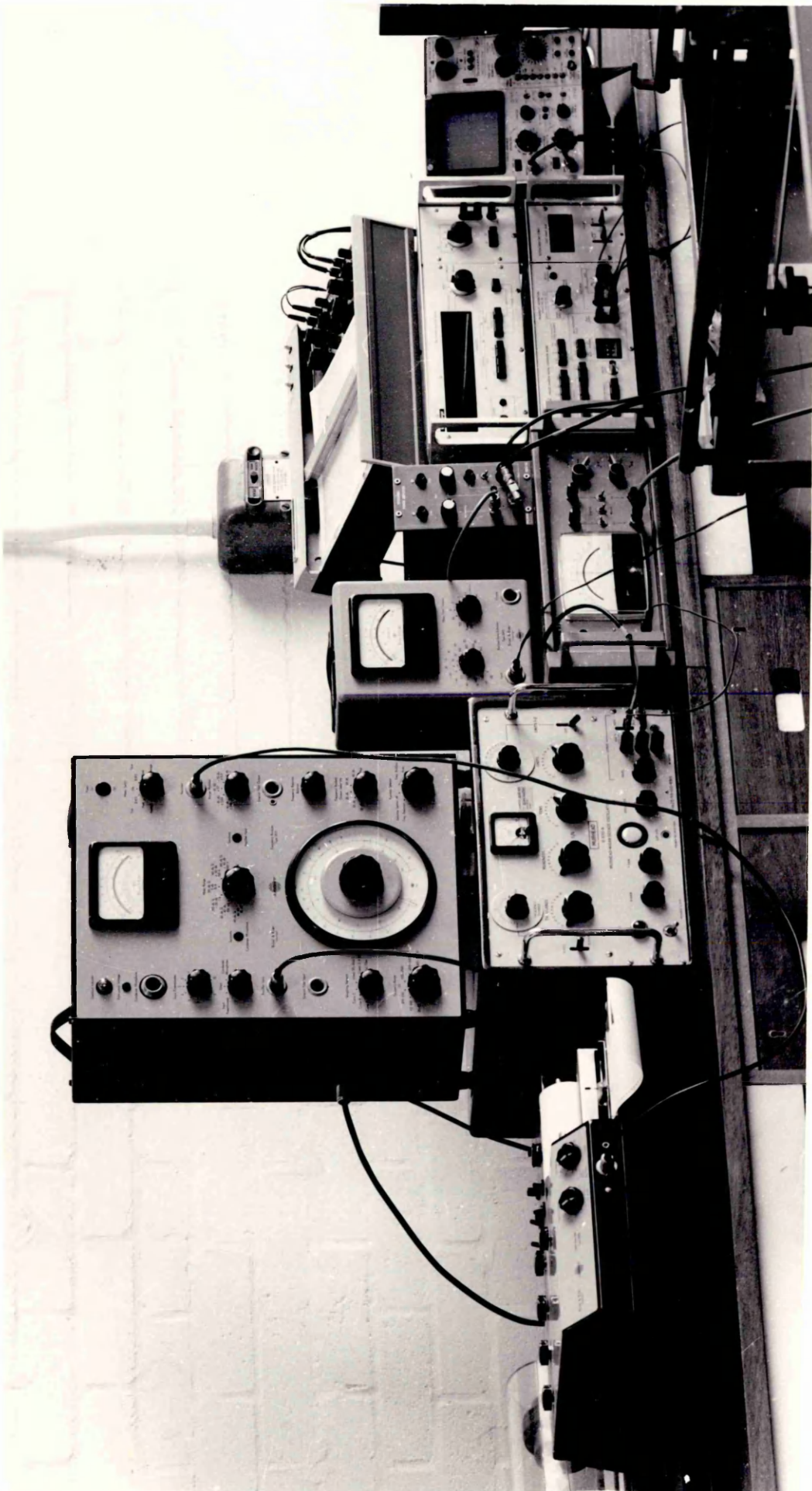
Plate 14   Constant bandwidth frequency analyser

Courants panigheq Lreadpetr uppraphed instance, analysis.
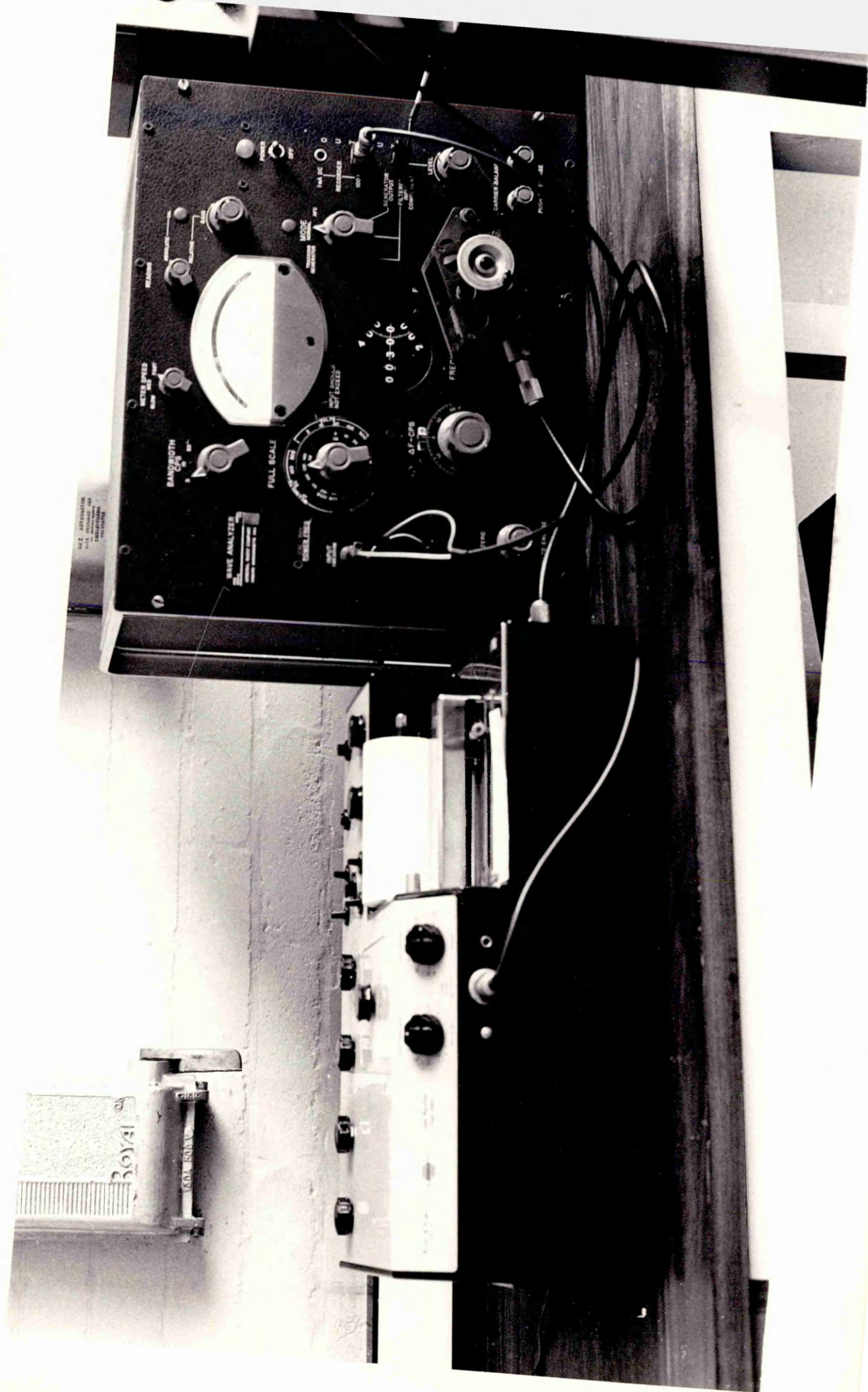
Plate 15   The PRBS instrumentation used for the automatic
            generation of the cross correlation function on
            paper tape.
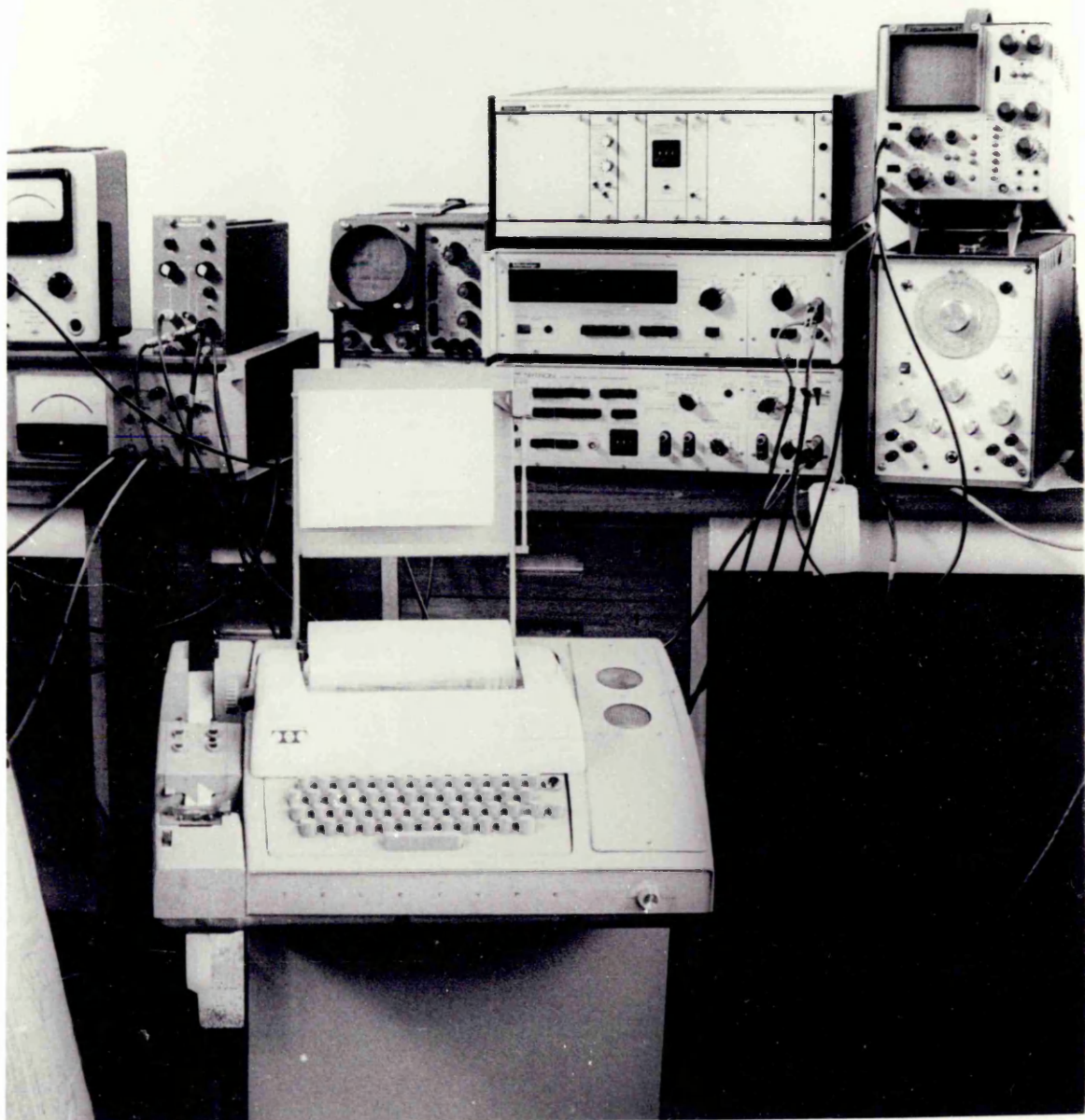
REFERENCES

1. Miles, J.W.,
   On structural fatigue under random loading,
   Journal of Aeronautical Sciences, Vol. 21, 1954

2. Robson, J.D.,
   An introduction to Random Vibrations,
   Edinburgh University Press, 1963

3. Crandal, S.H.,
   Random Vibrations,
   M.I.T. Press, Cambridge, Mass., 1958

4. Bishop, R.E.D., and Johnson, D.C.,
   The Mechanics of Vibrations,
   Cambridge University Press, 1960

5. Crandal, S.H.,
   The Role of Damping in Vibration Theory,
   Journal of Sound and Vibration, Vol. 11, 1970

6. Abrahamson, A.L.,
   Natural frequencies and normal modes of a four plate
   structure,
   Journal of Sound and Vibration, Vol. 28, 1973

7. Ritz,W.,
   Annalen der Physik,
   Vierte Folge, Vol. 28, 1909

8. Young, E.,
   The vibration of rectangular plates by the Ritz Method,
   Journal of Applied Mechanics, Vol. 17, 1950

9. Timoshenko, S.,
   Vibration problems in Engineering,
   3rd Edition D. Van Nostrand 1955

10. Warburton, G.B.,
    The Vibration of Rectangular Plates,
    Proceeding of the Institution of Mechanical Engineers, 1954

11. Leissa, A.W.,
    The Free Vibrations of Rectangular Plates,
    Journal of Sound and Vibrations, Vol. 31, 1973

12. Dickinson, S.M., and Warburton, G.B.,
    Vibration of Box-type structures,
    Journal of Mechanical Engineering Science, Vol. 9, 1967

13. Uhrig, R.,
    The Transfer Matrix Method seen as one method of Structural
    Analysis among others,
    Journal of Sound and Vibration, Vol. 4, 1966

14. Dickinson, S.M., and Warburton, G.B.,
    Natural Frequencies of Plate Systems using the Edge
    Effect Method,
    Journal of Mechanical Engineering Science, Vol. 9, 1967

15. Heng, R.B.W.,
    Prediction of performance of centrally pivotted thrust
    pads,
    Final year project, University of Sheffield, 1971

16. Chang, A.,
    An improved finite difference method for plate vibrations,
    International Journal of Numerical Methods in Engineering,
    Vol. 2, 1972

17. Turner, M.J., Clough, R.W., Martin, H.C., and Topp, L.J.,
    Stiffness and Deflection Analysis of Complex Structures,
    Journal of Aero Space Sciences, Vol. 23, 1956

18. Melosh, R.J.,
    A Stiffness Matrix for the Analysis of Thin Plates
    in Bending,
    Journal of Aero Space Sciences, Vol. 28, 1961

19. Zienkiewicz, O.C., and Cheung, Y.K.,
    The Finite Element Method for analysis of Elastic
    Isotropic and Orthotropic slabs,
    Proceedings of the Institution of Civil Engineers,
    Vol. 28, 1964

20. Dawe, D.J.,
    A finite Element Approach to Plate Vibration Problems,
    Journal of Mechanical Engineering Science, Vol. 7, 1965

21. Proc. Conf. Matrix Methods in Structural Mechanics 1966
    (AFFDL-TR-66-80),
    Wright - Patterson Air Base

22. Prezemieniecki, J.S.,
    Theory of Matrix Structural Analysis,
    McGrw-Hill, 1968

23. Zienkiewicz, O.C.,
    The Finite Element Method in Engineering Science,
    2nd Edition, McGraw-Hill, 1971

24. Rockey, K.C., and Evans, H.R.,
    A finite Element Solution for Folded Plates,
    International Conference on Space Structures,
    Battersea, 1966

25. Handa, K.N.,
    Analysis of In-plane Vibrations of Box-type structures
    by a Finite Element Method,
    Journal of Sound and Vibration, Vol. 21, 1972

26. Ali, R., Hedges, J.L., and Mills, B.,
    Computer Aided Design applied to a model of a chassis
    type structure using Finite Element Techniques,
    Proceedings of the Automobile Division of the Institution
    of Mechanical Engineers, Vol. 184, 1969/70

27. Davis, W.D.T.,
    Random Signal Testing for evaluating System Dynamics,
    Wireless World, August 1966

28. Jones, F.,
    Impulse Testing of Dynamic Systems,
    Electronic Engineering, May 1967

29. Cooley, J.W., and Turkey, J.W.,
    An Algorithm for the Machine Calculations of Complex
    Fourier Series,
    Mathematics of Computation, Vol. 19, 1965

30. Bingham, C., Godfrey, M.D., and Turkey, J.W.,
    Modern Techniques of Power Spectrum Estimation,
    I.E.E. trans, Audio and Elec.acc., Vol. AU-15, June 1967

31. Cochran, W.T., Coolley, J.W., et. al.,
    What is the Fast Fourier Transform,
    I.E.E. trans. Audio and Electroaccoustics, Vol. AU-15,
    June 1967

32. Clough, R.W., and Johnson, C.P.,
    A Finite Element Approximation for the Analysis of
    Thin Shells,
    International Journal of Solid Structures, Vol. 4, 1968

33. Gere, J.M., and Weaver, W.,
    Analysis of framed structures,
    Van Nostrand, 1965

34. Nottingham Algorithm Group Computer Scientific Subroutine
    Manual

35. Wilkinsin, J.H.,
    The Algebraic Eigenvalue problem,
    Clarendon Press, 1965

36. Franklin, J.N.,
    Matrix Theory
    Prentice-Hall Applied Mathematics series, 1968

37. Wiener, N.,
    Random Theory,
    M.I.T. Press, 1958

38. Jenkins, G.M., and Watts, D.G.,
    Spectral Analysis and its Applications,
    Holden-Day Inc., 1968

39. Newland, D.E.,
    An Introduction to Random Vibrations and Spectral
    Analysis, to be published

40. Pseudo-Random Binary Sequence Generator Handbook
    JM 1861
    Solartron Schlumberger

41. Plukett, R.,
    Experimental Measurement Of Mechanical Impedance
    or Mobility,
    J. App. Mechanics, ASME Transaction, Vol. 21/3, 1954

42. Dunn, J.W.,
    Measurement Techniques for the Analysis of Forced
    structural Vibration.
    Proceedings of the Institution of Mechanical
    Engineers, Vol 182/37, 1968

43. Pandered, J.W., and Bishop, R.E.D.,
    A critical introduction to some industrial resonance
    testing techniques (and three other articles)
    Journal of Mechanical Engineering Science, Vol 5
    No. 4, 1963

44. Shapiro, U.,
    An Experimental Investigation into response
    distribution in coupled Plate Systems,
    Ph.D. Thesis, University of Warwick, 1971

45. McNulty, G.J.,
    U.S.A.F. Technical Report, AFSDL / TR / 67 / 97,
    Wright-Patterson Air Force Base,
    Ohio, 1967.

46. Blackman, R.B., and Tukey, J.W.,
    The meansurement of Power Spectra,
    Dover Publications Inc., New York, 1959

47. Myklestad, N.O.,
    The concept of Complex Damping,
    Journal of Applied Mechanics, Vol 19, 1952

48. Kennedy, C.C., and Pancu, C.D.P.,
    Use of vectors in vibration measurement and
    analysis
    J. Aeronautical Science, Vol 14, 1947

49. Bert, C.W.,
    Material damping – An Introductory Review of
    Mathematical Models, Measures and Experimental
    Techniques,
    Journal of Sound and Vibration, Vol 29, 1973

50. Rao, Murty and Rao,
    Bounds for eigenvalues in some vibration problems,
    International Journal of Numerical Methods in
    Engineering, Vol 5, 1974

51. Vysloulch, V.A., Kandidov, V.P., and Chesnokov, S.S.,
    Reduction of the degrees of Freedom in solving
    Dynamic Problems by the Finite Element Method,
    Int. Journal of Numerical Methods in Engineering,
    Vol. 7, 1973

52. Irons, B.,
    The Semi-loof Shell Element,
    Proceedings of Conference on Finite Elements for
    Thin Shells and Curved Members, Cardiff, May 1974

53. Patterson, C., and Heng, R.,
    Finite Elements with Relaxed Continuity for Folded
    Plate Structures,
    Conference on Computing Developments in Experimental
    and Numerical Stress Analysis, Nottingham, Sept 1975

54. Irons, B.M.,
    A Frontal Solution Program for Finite Element
    Analysis,
    Int. Jnl. for Numerical Methods in Engineering,
    Vol 2, 1970

Appendices

1. The element mass and stiffness matrices

2. The solution alogrithms

3. The Finite Element computer program suite

4. The Fast Fourier Transform computer program

5. The experimental apparatus

6. Calibration of vibration pickup transdueer

7. Calibration of exciter

Appendix 1

The stiffness and mass matrices

(a) <u>The 24 x 24 stiffness matrix K</u>

The lower triangle of the symmetrical matrix is given below

where A = length
       B = width

$$P = \frac{A}{B}$$

ν = poisson's ratio
t = thickness

$$C = \frac{Et^3}{12(1 - \nu^2)AB}$$

$$D = \frac{Et}{12(1 - \nu^2)}$$

K(1,1)	= D((4.0/P)+(2.0x(1.0-ν)xP))
K(2,1)	= D(3.0x(1.0+ν)/2.0)
K(2,2)	= D((4.0xP)+(2.0x(1.0-ν)/P))
K(3,1)	= 0
K(3,2)	= 0
K(3,3)	= C((4.0xP^2)+(4.0/P^2)+2.8-(0.8xν))
K(4,1)	= 0
K(4,2)	= 0
K(4,3)	= C(((2.0xP^2)+(0.2)+(0.8xν))xB)
K(4,4)	= C(((4.0xP^2/3,0)+(4.0x(1.0-ν)/15.0))x(B^2))
K(5,1)	= 0
K(5,2)	= 0
K(5,3)	= C(((2.0/P^2)+(0.2)+0.8xν))x(-A))
K(5,4)	= C(-νxAxB)
K(5,5)	= C(((4.0/(3.0xP^2))+((4.0x(1-ν))/15.0x(AxA))
K(6,1)	to K(6,6) = 0
K(7,1)	= D((2,0/P)-(2.0x(1.0-ν)xP))
K(7,2)	= D(-3.0x(1.0-3.0xν)/2.0)
K(7,3)	to K(7,6) = 0
K(7,7)	= D((4.0/P)+(2.0x(1.0-ν)xP))
K(8,1)	= D(3.0x(1.0-(3.0xν))/2.0)
K(8,2)	= D((-4.0xP)+((1.0-ν)/P))
K(8,3)	to K(8,6) = 0
K(8,7)	= D(-3.0x(1.0+ν)/2.0)
K(8,8)	= D((4.0xP)+(2.0x(1.0-ν)/P
K(9,1)	= 0
K(9,2)	= 0
K(9,3)	= C((2.0/P^2)-(4.0xP^2)-2.8+(4.0xν/5.0))
K(9,4)	= C(((2.0xP^2)+((1.0-ν)/5.0))x(-B))
K(9,5)	= C(((-1.0/P^2)+((1.0+(4.0xν))/5.0))xA)
K(9,6)	to K(9,8) = 0
K(9,9)	= C((4.0xP^2)+(2.8)-(0.8xν)+(4.0/P^2))
K(10,1)	= 0
K(10,2)	= 0
K(10,3)	= C(((2.0xP^2)+(1.0-ν)/5.0xB)
K(10,4)	= C(((2.0xP^2/3.0)=((1-ν)/15.0))x(BxB))
K(10,5)	= 0
K(10,6)	to K(10,8) = 0
K(10,9)	= C(((2.0xP^2)+(0.2)+(0.8xν))x(-B))
K(10,10)	= C(((4.0xP^2/3.0)+(4.0x(1.0-ν))/15.0)x(BxB))

```
K(11,1)     = 0
K(11,2)     = 0
K(11,3)     = C(((1.0+(4.0xν))/5.0))xA)
K(11,4)     = 0
K(11,5)     = C(((2.0/(3.0xP²))=(4.0x(1.0-ν)/15.0))x(AxA))
K(11,6) to K(11,8) = 0
K(11,9)     = C(((-2.0/P²)-(0.8xν))xA)
K(11,10)    = C(νxAxB)
K(11,11)    = C(((4.0/3.0xP²))+(4,0x(1.0-ν)/15.0))x(AxA))
K(12,1) to K(12,12) = 0
K(13,1)     = D((-2.0/P)-((1.0-ν)xP))
K(13,2)     = D((-3.0x(1.0+ν)/2.0))
K(13,3) to K(13,6) = 0
K(13,7)     = D((-4.0/P)+((1.0-ν)xP))
K(13,8)     = D((-3.0x(1-(3.0xν))/2.0))
K(13,9) to K(13,12) = 0
K(13,13)    = D((4.0/P)+(2.0x(1.0-ν)xP)
K(14,1)     = D(-3.0x(1.0+ν)/2.0)
K(14,2)     = D((-2.0xP)-((1.0-ν)/P))
K(14,3) to K(14,6) = 0
K(14,7)     = D((3.0x(1.0-(3.0xν))/2.0))
K(14,8)     = D((2.0xP)-(2.0x(1.0-ν)/P))
K(14,9) to K(14,12) = 0
K(14,13)    = D((3.0x(1.0+ν)/2.0))
K(14,14)    = D((4.0x(1.0-ν)/P))
K(15,1)     = 0
K(15,2)     = 0
K(15,3)     = C((-2.0xP²)-(2.0/P²)+(2.8)-(0.8xν))
K(15,4)     = C((-P²+((1-ν)/15.0))xB)
K(15,5)     = C(((-1.0/P²)+((1-ν)/5.0))x(-A)
K(15,6) to K(15,8) = 0
K(15,9)     = C((-2.0x((2.0/P²)-(14.0-(4.0xν))/5.0))
K(15,10)    = C(((-P²+(1.0+(4.0xν))/5.0))x(B))
K(15,11)    = C(((2.0/P²)+((1.0-ν)/5.0))xA)
K(15,12) to K(15,14) = 0
K(15,15)    = C((4.0xP²)+(4.0/P²)+(2.8)-(0.8xν))
K(16,1)     = 0
K(16,2)     = 0
K(16,3)     = C((P²-((1-ν)/5.0))x(B))
K(16,4)     = C(((P²/3.0)+((1-ν)/15.0))x(BxB))
K(16,5) to K(16,8) = 0
K(16,9)     = C((-P²+(1.0+(4.0xν))/5.0)xB)
K(16,10)    = C(((2.0xP²)/3.0)-((4.0x(1.0-ν))/15.0))x(BxB))
K(16,11) to K(16,14) = 0
K(16,15)    = C(((-2.0xP²)-(0.2)-(0.8xν))xB)
K(16,16)    = C(((4.0x P²/3.0)+(4.0x(1-ν)/15.0))x(BxB))
K(17,1)     = 0
K(17,2)     = 0
K(17,3)     = C((1.0/P²)-((1-ν)/5.0))x(-A)
K(17,4)     = 0
K(17,5)     = C(((1.0/(3.0xP²))+((1-ν)/15.0))x(AxA))
K(17,6) to K(17,8) = 0
K(17,9)     = C(((2.0/P²)+((1.0-ν)/5.0))x(-A))
K(17,10)    = 0
K(17,11)    = C(((2.0/(3.0xP²))-((1.0-ν)/15.0))+(AxA))
K(17,12) to K(17,14) = 0
K(17,15)    = C(((2.0/P²}+(0.2)+(0.8xν))x(A))
K(17,16)    = C(-νxAxB)
K(17,17)    = C(((4.0/(3.0xP²))+(4.0x(1-ν)/15.0))x(AxA))
K(18,1) to K(18,18) = 0
```

```
K(19,1)      = D((-4.0/P)+((1.0-ν)xP))
K(19,2)      = D((3.0x(1.0-(3.0xν))/2.0))
K(19,3) to K(19,6) = O
K(19,7)      = D((-2.0/P)-((1.0-ν)xP))
K(19,8)      = D((3.0x(1.0+ν)/2.0))
K(19,9) to K(19,12) = O
K(19,13)     = D((2.0/P)-(2.0x(1.0-ν)xP))
K(19,14)     = D((-3.0x(1.0-(3.0xν))/2.0))
K(19,15) to K(19,18) = O
K(19,19)     = D((4.0/P)+(2.0x(1.0-ν)xP))
K(20,1)      = D((-3.0x(1.0-(3.0xν))/2.0))
K(20,2)      = D((2.0xP)-(2.0x(1.0-ν)/P))
K(20,3) to K(20,6) = O
K(20,7)      = D((3.0x(1.0+ν)/2.0))
K(20,8)      = D((-2.0xP)-((1.0-ν)/P)
K(20,9) to K(20,12) = O
K(20,13)     = D((3.0x(1.0-(3.0xν))/2.0))
K(20,14)     = D((-4.0xP)+((1.0-ν)/P))
K(20,15) to K(20,18) = O
K(20,19)     = D((-3.0x(1.0+ν)/2.0))
K(20,20)     = D((4.0xP)+(2.0x(1.0-ν)/P))
K(21,1)      = O
K(21,2)      = O
K(21,3)      = C(-2.0x((2.0/P²)-P²)-((14.0-(4.0xν))/5.0))
K(21,4)      = C((-P²+((1.0+(4.0xν))/5.0))xB)
K(21,5)      = C(((2.0/P²+((1.0-ν)/5.0))xA)
K(21,6) to K(21,8) = O
K(21,9)      = C(-(2.0xP²)-(2.0/P²)+(2.8)-(0.8xν))
K(21,10)     = C((P²-((1-ν)/5.0))xB)
K(21,11)     = C(((1.0/P²(-((1-ν)/5.0))x(A))
K(21,12) to K(21,14) = O
K(21,15)     = C((2.0x((1.0/P²)-(2.0xP²)))-((14.0-(4.0xν))/5.0)
K(21,16)     = C(((2.0xP²)+((1.0-ν)/5.0))xB)
K(21,17)     = C(((1.0/P²)-((1.0+(4.0xν))/5.0))xA)
K(21,18) to K(21,20) = O
K(21,21)     = C((4.0 x P²)+(4.0/P²)+(2.8)-(0.8xν))
K(22,1)      = O
K(22,2)      = O
K(22,3)      = C(((P²)-((1.0+(4.0xν))/5.0))xB)
K(22,4)      = C(((2.0xP²/3.0)-(4.0x(1.0-ν)/15.0))x(BxB))
K(22,5) to K(22,8) = O
K(22,9)      = C((P²-((1-V)/5.0))x(-B))
K(22,10)     = C((P/3.0)+((1-ν)/15.0))x(BxB))
K(22,11) to K(22,14)= O
K(22,15)     = C((2.0xP²)+((1.0-ν)/5.0))x(-B))
K(22,16)     = C((2.0xP²/3.0)-((1.0-ν)/15.0))x(BxB))
K(22,17) to K(22,20) = O
K(22,21)     = C(((2.0xP²)+(0.2)+(0.8xν))x(B))
K(22,22)     = C(((4.0xP²/3.0)+(4.0x(1-A)/15.0))x(BxB))
```

```
K(23,1)       = 0
K(23,2)       = 0
K(23,3)       = C(((2.0/P²)+((1.0-ν)/5.0))x(-A))
K(23,4)       = 0
K(23,5)       = C((2.0/3.0xP²))-((1.0-ν)/15.0))x(AxA))
K(23,6) to K(23,8) = 0
K(23,9)       = C(((-1.0/P²)+((1-ν)/5.0))xA
K(23,10)      = 0
K(23,11)      = C(((1.0/(3.0xP²))+((1-ν)/15.0))x(AxA))
K(23,12) to K(23,14) = 0
K(23,15)      = C(((1.0/P²)-((1.0+(4.0xν))/5.0))xA)
K(23,16)      = 0
K(23,17)      = C(((2.0/(3.0xP²))-(4.0x(1.0-ν)/15.0))x(AxA))
K(23,18) to K(23,20) = 0
K(23,21)      = C(((2.0/P²)+(0.2)+(0.8xν))xA)
K(23,22)      = C(νxAxB)
K(23,23)      = C(((4.0/(3.0xP²))+(4.0x(1-ν)/15.0))x(AxA))
K(24,1)  to K(24,24) = 0
```

(b) <u>The 24 x 24 mass matrix M</u>

The lower triangle of the symmetrical matrix
is given below where

A = length
B = width
ρ = density / unit area

multiply throughout by $\dfrac{\rho AB}{176400}$

M(1,1)	= 19600
M(2,1)	= O
M(2,2)	= 19600
M(3,1)	= O
M(3,2)	= O
M(3,3)	= 24178.O
M(4,1)	= O
M(4,2)	= O
M(4,3)	= 3227 x B
M(4,4)	= 560 x B x B
M(5,1)	= O
M(5,2)	= O
M(5,3)	= -3227 x A
M(5,4)	= -441.O x A x B
M(5,5)	= 560.O x A x A
M(6,1) to M(6,6)	= O
M(7,1)	= 9800
M(7,2) to M(7,6)	= O
M(7,7)	= 19600
M(8,1)	= O
M(8,2)	= 9800
M(8,3) to M(8,7)	= O
M(8,8)	= 19600
M(9,1)	= O
M(9,2)	= O
M(9,3)	= 8582.O
M(9,4)	= 1918.O x B
M(9,5)	= -1393.O x A
M(9,6) to M(9,8)	= O
M(9,9)	= 24178.O
M(10,1)	= O
M(10,2)	= O
M(10,3)	= -1918.O x B
M(10,4)	= -420.O x B x B
M(10,5)	= 294.O x A x B
M(10,6) to M(10,8)	= O
M(10,9)	= -3227.O x B
M(10,10)	= 560.O x B x B
M(11,1)	= O
M(11,2)	= O
M(11,3)	= -1393 x A
M(11,4)	= -294.O x A x B
M(11,5)	= 280.O x A x A
M(11,6) to M(11,8)	= O
M(11,9)	= -3227.O x A
M(11,10)	= 441.O x A x B
M(11,11)	= 560.O x A x A

```
M(12,1)  to M(12,12) = 0
M(13,1)    = 4900
M(13,2)  to M(13,6) = 0
M(13,7)    = 9800
M(13,8)  to M(13,12) = 0
M(13,13)   = 19600
M(14,1)    = 0
M(14,2)    = 4900
M(14,3)  to M(14,7) = 0
M(14,8)    = 9800
M(14,9)  to M(14,13) = 0
M(14,14)   = 19600
M(15,1)    = 0
M(15,2)    = 0
M(15,3)    = 2758.0
M(15,4)    = 812.0 x B
M(15,5)    =-812.0 x A
M(15,6)  to M(15,8) = 0
M(15,9)    = 8582.0
M(15,10)   = -1393.0 x B
M(15,11)   = -1918.0 x A
M(15,12) to M(15,14) = 0
M(15,15)   = 24178.0
M(16,1)    = 0
M(16,2)    = 0
M(16,3)    = -812.0 x B
M(16,4)    = -210.0 x B x B
M(16,5)    = 196.0 x A x B
M(16,6)  to M(16,8) = 0
M(16,9)    = -1393 x B
M(16,10)   = 280.0 x B x B
M(16,11)   = 294.0 x A x B
M(16,12) to M(16,14) = 0
M(16,15)   = -3227.0 x B
M(16,16)   = 560.0 x B x B
M(17,1)    = 0
M(17,2)    = 0
M(17,3)    = 812.0 x A
M(17,4)    = 196.0 x A x B
M(17,5)    = -210.0 x A x A
M(17,6) to M(17,8) = 0
M(17,9)    = 1918.0 x A
M(17,10)   = -294.0 x A x B
M(17,11)   = -420.0 x A x A
M(17,12) to M(17,14) = 0
M(17,15)   = 3227.0 x A
M(17,16)   = -441.0 x A x B
M(17,17)   = 560.0 x A x A
M(18,1)  to M(18,18) = 0
```

```
M(19,1)    = 9800
M(19,2)    = 0
M(19,3)    to M(19,6) = 0
M(19,7)    = 4900
M(19,8)    to M(19,12) = 0
M(19,13)   = 9800
M(19,14)   to M(19,18) = 0
M(19,19)   = 19600
M(20,1)    = 0
M(20,2)    = 9800
M(20,3)    to M(20,7) = 0
M(20,8)    = 4900
M(20,9)    = M(20,13) = 0
M(20,14)   = 9800
M(20,15)   to M(20,19) = 0
M(20,20)   = 19600
M(21,1)    = 0
M(21,2)    = 0
M(21,3)    = 8582.0
M(21,4)    = 1393 x B
M(21,5)    = - 1918.0 x A
M(21,6)    to M(21,8) = 0
M(21,9)    = 2758.0
M(21,10)   = - 812.0 x B
M(21,11)   = - 812.0 x A
M(21,12)   to M(21,14) = 0
M(21,15)   = 8582.0
M(21,16)   = - 1918.0 x B
M(21,17)   = 1393.0 x A
M(21,18)   to M(21,20) = 0
M(21,21)   = 24178.0
M(22,1)    = 0
M(22,2)    = 0
M(22,3)    = 1393.0 x B
M(22,4)    = 280.0 x B x B
M(22,5)    = - 294.0 x A x B
M(22,6)    to M(22,8) = 0
M(22,9)    = 812.0 x B
M(22,10)   = - 210.0 x B x B
M(22,11)   = - 196.0 x A x B
M(22,12)   to M(22,14) = 0
M(22,15)   = 1918.0 x B
M(22,16)   = - 420.0 x B x B
M(22,17)   = 294.0 x A x B
M(22,18)   to M(22,20) = 0
M(22,21)   = 3227.0 x B
M(22,22)   = 560.0 x B x B
M(23,1)    = 0
M(23,2)    = 0
M(23,3)    = 1918.0 x A
M(23,4)    = 294.0 x A x B
M(23,5)    = - 420.0 x A x A
M(23,6)    to M(23,8) = 0
M(23,9)    = 812.0 x A
```

```
M(23,10)  = - 196.0 x A x B
M(23,11)  = - 210.0 x A x A
M(23,12)  to M(23,14) = 0
M(23,15)  = 1393.0 x A
M(23,16)  = - 294.0 x A x B
M(23,17)  = 280.0 x A x A
M(23,20)  = 0
M(23,21)  = 3227.0 x A
M(23,22)  = 441.0 x A x B
M(23,23)  = 560.0 x A x A
M(24,1)  to M(24,24) = 0
```

Appendix 2

The computer solution algorithms

2.1 Brief notes on current techniques available for
economising on the computer core space required
using the finite element method of analysis.

2.2 Listing of the standard library computer subroutines
used. Acknowledgements are due to the Nottingham
Algorithm Group from whom descriptions of their
scientific subroutines which were used in the
computer analysis performed are here reproduced.

The finite element analysis of the three dimensional folded
plate structure required a very large core storage in order
to solve for the natural frequencies and mode shapes of the
structure. The simple and rather coarse mesh used, shown in
figure 2.12, gave a 146 x 146 matrix for the stiffness as
well as the mass matrices. This required a 70K computer core
storage which is often not available on smaller computers or
even if available, together with a long computer solution
time, is consequently very costly. The results are also
likely to be excessive and is accurate only at the lower
frequencies. In fact only a quarter of the eigenvalues
computed is likely to be accurate enough for engineering
purposes.

Although not used in the program developed, for prospective
students of the finite element technique, savings in core
space may be achieved although often at the expense of
increased computing time by methods,

a)   using the full stiffness matrix - most accurate but very
                                        demanding on core
                                        storages.

b)   banded K - the stiffness matrix is stored banded,
                resulting in a storage saving but at the
                cost of an increased solution time.

c)   frontal solution - reduction or condensation of selected
                        degrees of freedom and solving only
                        the 'front' where unnecessary degrees
                        of freedom at stage are eliminated
                        and put on to a disc backing store
                        to relieve core storage.

d)   substructuring - use of compound elements made up of
                      collections of normal elements.

The first two requires little explanation and core savings
are achieved utilising the matrices' symmetry and sparseness
outside the diagonal band.

Frontal solution

Here only the current active degrees of freedom are kept in the core during the solution [45]. All other degrees of freedom are reduced out of the core after all information relating to them is complete. These are stored in a peripheral storage device such as disc or magnetic tape and the method is especially advantageous for matrices where the bandwidth varies considerably.

The elimination takes place as follows. Basically what we have is a very large linear set of simultaneous equations. When all the information relating to a particular variable is complete then that variable may be eliminated since it can be expressed in terms of the other variables.

$$[K]\{d\} = \{F\} \tag{A.2.1}$$

$$\text{or } \sum_{j=1}^{n} K_{ij}d_j = F_i \text{ for } i = 1 \rightarrow n$$

for $i = k$ (i.e. the kth equation)

$$\sum_{j=1}^{n} K_{kj}d_j = F_k$$

$$\therefore d_k = \frac{1}{K_{kk}} \left[-\left(\sum_{y=1}^{n} K_{ky}d_y \right) + F_k \right] \text{ for } y \neq k \tag{A.2.2}$$

This is then used in all the remaining (n - 1) equations to give a reduced form of (A.2.1)

$$\text{so that } \left(\sum_{j=1}^{n} K_{ij}d_j \right) - K_{ik}d_k = F_i - \frac{K_{ik}}{K_{kk}} \left[-\left(\sum_{y=1}^{n} K_{ky}d_y \right) + F_k \right]$$

$$\text{for } y \neq k$$

on rearranging, for $y \neq k$

$$\sum_{j=1}^{n} (K_{ij}d_j) - \sum_{y=1}^{n} \left(\frac{K_{ky}d_y K_{ik}}{K_{kk}} \right) - K_{ik}d_k$$

$$= F_i - F_k \frac{K_{ik}}{K_{kk}}$$

hence

$$\sum_{j=1}^{n} (K_{ij} - \frac{K_{ik}K_{kj}}{K_{kk}}) \, d_j = F_i - F_k \frac{K_{ik}}{K_{kk}} \qquad (A.2.3)$$

Thus using this method the N equations can be reduced to N - 1 and so on.

Using equation (A.2.2) which is stored on peripheral a back substitution can be performed to determine d_k.
Equation (A.2.3) can be derived from (A.2.1) by replacing

$$K_{ij} \quad \text{by} \quad (K_{ij} - \frac{K_{ik}K_{kj}}{K_{kk}}) \quad \text{and}$$

$$F_i \quad \text{by} \quad (F_i - \frac{F_k K_{ik}}{K_{kk}})$$

where d_k is the degree of freedom to be eliminated.

The mass terms are modified to

$$M_{ij} = M_{ij} - (M_{ik} \frac{K_{kj}}{K_{kk}}) - M_{kj} \frac{K_{ik}}{K_{kk}} + \frac{K_{ik}K_{kj}M_{kk}}{(K_{kk})^2}$$

Thus the net result is that the kth row and column of the stiffness and mass matrices are eliminated in the modified matrices in which terms need only be altered if d_i and d_j are both coupled to d_k (figure A.2.1).

Substructuring

The dynamic analysis of the three dimensional structure requires a very large core storage and sometimes even use of the front solution is not sufficient. Because the problem is so large even the active degrees of freedom themselves require more core storage than is available, substructuring is therefore necessary.

Substructuring is the name given to the reduction of the structure into giant compound elements. Thus only the nodes at which these 'substructures' join are used. In a dynamic analysis the mass matrix is reduced to those at

banded matrix

row and column to be eliminated.

zone affected
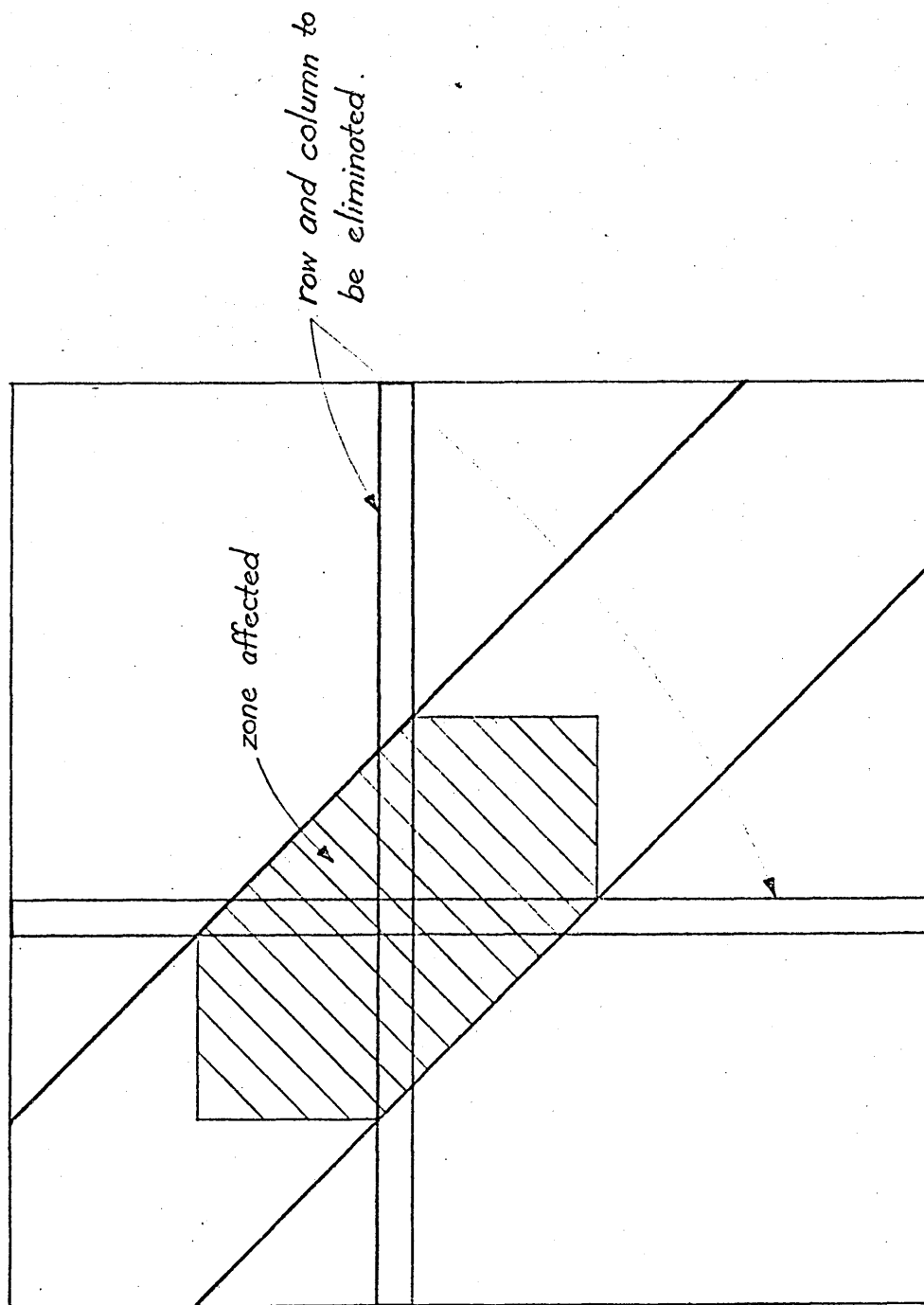
Fig A 2·1

these nodes plus the 'master freedoms' specified.

A backing store is used to store the reduced out matrices
and the substructure matrices are then used to produce the
complete system matrix. The eigenvalue problem of the
master degrees of freedom is then solved and a back sub-
stitution is carried out to determine the displacement of
the 'slave' freedoms 's' on a static basis. The choice
of s is based on the observation that, for the lower modes
of vibration of a structure it is possible to neglect the
effect of local inertia forces. The assumption is made
that s in relation to nearby unknowns d' is given by
static considerations. The inertia associated with s is
not neglected but it is not allowed to affect the relation-
ship between s and d' .

The method introduces errors almost negligible if the
following conditions are satisfied:

1) if rotational rather than translational freedoms are
    eliminated.
2) degrees of freedom close to clamped or hinged boundaries
    and other freedoms having little inertia effects in
    the mode of interest are eliminated.
3) only the first n/3 eigenvalues of the n degrees of freedom
    chosen is required.
4) retention of at least one master freedom to represent
    each of the rigid body modes of the structure.

Here the method is briefly to first partition the stiffness
matrix [K] and the displacements {U} into

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$U = \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}$$

where U_1 contains the retained displacements and U_2 the

remaining. U_2 is then obtained from the static equilibrium eqn. $\{F\} = [K]\{U\}$ by assuming that the external forces F_2 corresponding to the displacements U_2 are all equal to zero.
Therefore $\{U_2\} = -[K_{22}^{-1}][K_{21}]\{U_1\}$
The resulting condensed mass matrix is then

$$[M_c] = [A_c]^T [M][A_c]$$

where
$$A_c = \begin{bmatrix} 1 \\ -K_{22}^{-1} \quad K_{21} \end{bmatrix}$$

Therefore
$$M_c = \begin{bmatrix} 1 & -\dfrac{K_{21}}{K_{22}} \end{bmatrix}[M]\begin{bmatrix} 1 \\ \\ -\dfrac{K_{21}}{K_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\dfrac{K_{21}}{K_{22}} \end{bmatrix}\begin{bmatrix} M_{11} - M_{12}\dfrac{K_{21}}{K_{22}} \\ \\ M_{21} - M_{22}\dfrac{K_{21}}{K_{22}} \end{bmatrix}$$

$$= M_{11} - M_{12}\dfrac{K_{21}}{K_{22}} - M_{21}\dfrac{K_{21}}{K_{22}} + M_{22}\left\{\dfrac{K_{21}}{K_{22}}\right\}^2$$

where $\dfrac{K_{21}}{K_{22}} = [K_{22}]^{-1}[K_{21}]$

The result is considerably reduced stiffness and mass matrices, the dimensions of which correspond to the number of terms retained in U_1.

1. SUBROUTINE F02AEF(A,IA,B,IB,N,R,V,IV,DL,E,IFAIL)

2. Eigenvalues and eigenvectors of $(A-\lambda B)x=0$, where A is real symmetric and B is real symmetric positive definite, by Householder's method and the QL algorithm.

3. Language   FORTRAN IV.

4. Description

The problem is reduced to the standard symmetric eigenproblem using Cholesky's method to decompose B into triangles $B=LL^T$, where L is lower triangular. Then $Ax=\lambda Bx$ implies $(L^{-1}AL^{-T})(L^Tx)=\lambda(L^Tx)$, hence the eigenvalues of $Ax=\lambda Bx$ are those of $Py=\lambda y$, where P is the symmetric matrix $L^{-1}AL^{-T}$. Householder's method is used to tridiagonalise the matrix P and the eigenvalues are found using the QL algorithm. An eigenvector Z of the derived problem is related to an eigenvector x of the original problem by $Z=L^Tx$. The eigenvectors Z are determined using the QL algorithm and are normalised so that $Z^TZ=1$, the eigenvectors of the original problem are then determined by solving $Z=L^Tx$ and are normalised so that $x^TBx=1$.

5. References

MARTIN, R.S., WILKINSON, J.H.   Reduction of the symmetric eigenproblem $Ax=\lambda Bx$ and related problems to standard form. Num. Math., Band 11, 1968, pp 99-110.

MARTIN, R.S., REINSCH, C., WILKINSON, J.H.   Householder's tri-diagonalisation of a symmetric matrix. Num. Math., Band 11, 1968, pp 181-195.

BOWDLER, H., MARTIN, R.S., REINSCH, C., WILKINSON, J.H.   The QL and QR algorithms for symmetric matrices. Num. Math., Band 11, 1968, pp 293-306.

6. Parameters

A   -   the name of a two dimensional REAL ARRAY of at least (N,N)

elements. On entry it should contain the real symmetric
matrix A in $(A-\lambda B)x=0$, the upper triangle only is needed.
On exit the strict upper triangle will be unchanged. The
lower triangle is used as work space. (See section 12).

IA — an INTEGER quantity, the first dimension of A, IA$\geq$N.

B — the name of a two dimensional REAL ARRAY of at least (N,N)
elements. On entry it should contain the elements of the
symmetric positive definite matrix B in $(A-\lambda B)x=0$, the
upper triangle only is needed. On exit the upper triangle
will be unchanged, the strict lower triangle is used as
workspace.

IB — an INTEGER quantity, the first dimension of B, IB$\geq$N.

N — an INTEGER quantity, the order of matrix A.

R — the name of a one dimensional REAL ARRAY of at least (N)
elements, on exit it will contain the eigenvalues of
$(A-\lambda B)x=0$ in order of increasing magnitude.

V — the name of a two dimensional REAL ARRAY of at least (N,N)
elements. On exit it will contain the eigenvectors of
$(A-\lambda B)x=0$ in column order corresponding to the eigenvalues,
i.e. V(I,J), where I=1,N corresponds to eigenvalue R(J).
(See section 12).

IV — an INTEGER quantity, the first dimension of V, IV$\geq$N.

DL — the name of a one dimensional REAL ARRAY of at least (N)
elements used as work space.

E — the name of a one dimensional REAL ARRAY of at least (N)
elements used as work space.

IFAIL — an INTEGER variable. On entry the value of IFAIL
determines the mode of failure in the routine. On exit
IFAIL indicates successful use of the routine or acts as
an error indicator.

If, on entry, IFAIL=0 (hard failure) the program will
terminate with a failure message if any error is detected.

If, on entry, IFAIL=1 (soft failure) control returns to
the calling sequence within the program if any error is
detected by the routine. No failure message will be
printed.

On exit IFAIL=0 for a successful call of the routine.
For other exit values of IFAIL, and their meanings, see
section 7.

It is essential, if the soft failure option is used,
that the value of IFAIL is tested on exit.

7. Error Indicators

IFAIL=1   Failure in FO1AEF, matrix B is not positive definite possibly due to rounding errors.

IFAIL=2   Failure in FO2AMF, more than 30 iterations are needed to isolate any one eigenvalue.

If the hard failure option is employed and the routine fails because of the error labelled by I, the message printed is LIBRARY FAILS IN FO2AEF WITH ERROR I.

8. Auxiliary Routines

This subroutine calls subroutines FO1AEF, FO1AJF, FO2AMF, FO1AFF and PO1AAF.

9. Timing

For a matrix of order 10 the procedure takes 0.1 seconds.
For a matrix of order 20 the procedure takes 0.9 seconds.
For a matrix of order 30 the procedure takes 2.9 seconds.

10. Storage

The compiled subroutine and auxiliary subroutines require 1667 words. There are no internally declared arrays.

11. Accuracy

In general the accuracy of this subroutine is very high. However, if B is ill-conditioned with respect to inversion the eigenvalues and eigenvectors could be inaccurately determined. The answers to the test cases were always accurate to 9 significant figures.

12. Further Comments

If the subroutine is called with the same name for the arrays A and V then the eigenvectors will overwrite the real symmetric matrix A.

13. Example

To find the eigenvalues and eigenvectors of the problem $Ax=\lambda Bx$ where A is the symmetric matrix

```
10.0    2.0    3.0    1.0    1.0
       12.0    1.0    2.0    1.0
              11.0    1.0   -1.0
                      9.0    1.0
                            15.0
```

and B is the positive definite matrix

```
12.0   1.0  -1.0   2.0   1.0
       14.0   1.0  -1.0   1.0
              16.0  -1.0   1.0
                    12.0  -1.0
                          11.0
```

a short program could be

```
      MASTER TESTEIGRV
      DIMENSION A(5,5),B(10,10),EVEC(5,5),EVAL(10),WKS1(5),WKS2(100)
      N=5
      DO 1 I=1,N
      READ(1,100)(A(I,J),J=I,N)
   1  CONTINUE
      DO 2 I=1,N
      READ(1,100)(B(I,J),J=I,N)
   2  CONTINUE
 100  FORMAT(5F0.0)
      I=1
      CALL F02AEF(A,N,B,10,N,EVAL,EVEC,5,WKS1,WKS2,I)
      IF(I.EQ.0) GOTO 10
      WRITE(2,200)I
 200  FORMAT(1H0, 'FAILURE, IFAIL=', I5)
      STOP
  10  DO 3 I=1,N
      WRITE(2,201)I,EVAL(I),(EVEC(J,I),J=1,N)
   3  CONTINUE
 201  FORMAT(1H0, 'LAMDA',I2,'=',1PE16.9//'EIGENVECTOR'//10
     1(1X,1PE16.9/)/)
      STOP
      END
      FINISH
```

Results

```
LAMDA 1 = 4.327872110E-01
EIGENVECTOR
 1.345905740E-01
-6.129472247E-02
-1.579025622E-01
 1.094657877E-01
-4.147301179E-02

LAMDA 2 = 6.636627484E-01
EIGENVECTOR
 8.291980649E-02
 1.531483957E-01
-1.186036679E-01
-1.828130418E-01
 3.561720369E-03
```

```
LAMDA 3 = 9.438590047E-01
EIGENVECTOR
-1.917100316E-01
 1.589912115E-01
-7.483907094E-02
 1.374689295E-01
-8.897789234E-02

LAMDA 4 = 1.109284540E 00
EIGENVECTOR
 1.420119599E-01
 1.424199505E-01
 1.209976230E-01
 1.255310152E-01
 7.692207282E-03

LAMDA 5 = 1.492353233E 00
EIGENVECTOR
-7.638671787E-02
 1.709800187E-02
-6.666453367E-02
 8.604800930E-02
 2.894334142E-01
```

14. Keywords

    Householder
    Eigenvalues
    Eigenvectors
    Standard Eigenproblem

Appendix 3

<u>The finite element suite of computer programs developed</u>.

A listing of the set of computer programs developed to
analyse folded plate box structure.

```
LDSET(PRESET=NGINF,MAP=B/ZZZZMP)
LGO(PL=6000)
CATALOG(TAPE3,MOJDAT1 ,FO=ASIS,ST=S6A)
####S
       PROGRAM STRUCT(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE3)
       LEVEL 2,K
C THIS PROGRAM COMPUTES THE STIFFNESS MATRIX OF A BOX OF NXNYNZ  ELEMENT
C.....FROM 3 STANDARD PLATE ELEMENTS WHOSE STIFFNESS MATRICES ARE COMPUTED
C.....THESE ELEMENTS ARE ROTATED AND MERGED INTO A BOX. THE
C INERTIA MATRIX IS SIMILARLY OBTAINED AND THESE COMPLETED BOX MATRICES
C ARE STORED IN DATAFILE MO//DATA
       DIMENSION K(24,24),Q(24,24)
       INTEGER ORDER
       INTEGER E
       REAL KK(24,24),P,Y,LENGTH,WIDTH,KJ
       REAL K(180,180),K1(24,24)
       COMMON/UPPER/K
       NX=4
       NY=2
       NZ=1
C  ...LENGTH IS LENGTH OF BOX
C......WIDTH IS WIDTH OF BOX
       LENGTH=0.515
       WIDTH=0.255
       HEIGHT=0.255
       A=LENGTH/FLOAT(NX)
       B=WIDTH/FLOAT(NY)
       C=HEIGHT/FLOAT(NZ)
C.....V=POISSON'S RATIO
       V=0.3
C.....H=THICKNESS OF PLATE
       H=0.00054
       Y=(2.06E11)
       V=0.3
C.....RHO=DENSITY OF PLATE
       RHO=7800.0*H
       N=24
       ORDER=(NX+1)*(NY+1)*(NZ+1)*6
       DO 4 I = 1,ORDER
       DO 4 J = 1,ORDER
     4 K(I,J) = 0.0

       CALL BEND1(KK,N,A,B,V,Y,H,RHO)
       CALL CLEAR(K1)
       CALL PLAIN1 (K1,N,A,B,V,Y,H,RHO,KK)
       CALL REORDER(N,KK,KJ)
       CALL ROTATE1 (K1,KK,Q,R)
       CALL COMPLET (K,NZ,ORDER,R,N,NX,NY,1 )
       CALL COMPLET (K,NZ,ORDER,R,N,NX,NY,2 )

C.....CALL ROTATE2 (K1,KK,Q,R)
C.....CALL COMPLET (K,NX,ORDER,R,N,NY,NZ,3)
C.....CALL COMPLET (K,NX,ORDER,R,N,NY,NZ,4)
       CALL BEND1(KK,N,A,C,V,Y,H,RHO)
       CALL CLEAR(K1)
       CALL PLAIN1 (K1,N,A,C,V,Y,H,RHO,KK)
       CALL REORDER(N,KK,KJ)
       CALL ROTATE3 (K1,KK,Q,R)
       CALL COMPLET (K,NY,ORDER,R,N,NX,NZ,5)
       CALL COMPLET (K,NY,ORDER,R,N,NX,NZ,6)
       REWIND 3
       WRITE (3) K


C.....USE K ANDKK MATRIX FOR INERTIA MATRIX
       DO14 I = 1,ORDER
       DO14 J = 1,ORDER
    14 K(I,J) = 0.0
       CALL BEND2(KK,N,A,B,V,Y,H,RHO)
       CALL CLEAR(K1)
       CALL PLAIN2 (K1,N,A,B,V,Y,H,RHO,KK)
       CALL REORDER(N,KK,KJ)
       CALL ROTATE1 (K1,KK,Q,R)
       CALL COMPLET (K,NZ,ORDER,R,N,NX,NY,1 )
       CALL COMPLET (K,NZ,ORDER,R,N,NX,NY,2 )
C.....CALL ROTATE2 (K1,KK,Q,R)
C.....CALL COMPLET (K,NX,ORDER,R,N,NY,NZ,3)
C.....CALL COMPLET (K,NX,ORDER,R,N,NY,NZ,4)
       CALL BEND2(KK,N,A,C,V,Y,H,RHO)
       CALL CLEAR(K1)
       CALL PLAIN2 (K1,N,A,C,V,Y,H,RHO,KK)
       CALL REORDER(N,KK,KJ)
       CALL ROTATE3 (K1,KK,Q,R)
       CALL COMPLET (K,NY,ORDER,R,N,NX,NZ,5)
       CALL COMPLET (K,NY,ORDER,P,N,NX,NZ,6)
C.....STORE MATRIX UNFORMATTED IN MAG TAPE.
       WRITE (3) K
       ENDFILE 3
       REWIND 3
       STOP
       END
```

```
          SUBROUTINE BEND 1   (KK,N,A,B,V,E,T,RHO)
          REAL              KK(N,N)            ,P,A,B,V
C.....CLEAR KK MATRIX
          DO 1  I=1,N
          DO 1  J=1,N
        1 KK(I,J)=0.0
C.....COMPUTE LOWER TRIANGULAR KK MATRIX
C.....N=12 SIGNIFIES BENDING  N=24 FUR BENDING + STRETCHING
          IF(N.EQ.12) GO TO 10
          IF(N.EQ.24) GO TO 20
       10 I=1
C.....THIS GIVES A 12 X 12 MATRIX
          J=3
          WRITE(2,11)
       11 FORMAT(//'THIS IS FOR PLATE BENDING ONLY')
          GO TO 50
       20 I=3
          WRITE(2,21)
       21 FORMAT(//'THIS IS FOR PLATE BENDING AND STRETCHING')
          J=6
C.....THIS GIVES A 24 X 24 MATRIX
          GO TO 50
       50 P=(A/B)**2
          J2=J*2
          J3=J*3
          KK(I,I)=(4.0*P)+(4.0/P)+2.8-(0.8*V )
          KK(I+1,I)=((2.0*P)+(0.2)+(0.8*V))*B
          KK(I+1,I+1)=((4.0*P/3.0)+(4.0*(1.0-V)/15.0))*(B**2)
          KK(I+2,I)=((2.0/P)+(0.2)+(0.8*V))*(-A)
          KK(I+2,I+1)=-V*A*B
          KK(I+2,I+2)=((4.0/(3.0*P))+((4.0*(1-V))/15.0))*(A*A)
          KK(I+J,I)=(2.0/P)-(4.0*P)-2.8+(4.0*V/5.0)
          KK(I+J,I+1)=((2.0*P)+((1.0-V)/5.0))*(-B)
          KK(I+J,I+2)=((-1.0/P)+((1.0+(4.0*V))/5.0))*A
          KK(I+J,I+J)=(4.0*P)+(4.0/P)+(2.8)-(0.8*V)
          KK(I+J+1,I)=((2.0*P)+(1.0-V)/5.0)*B
          KK(I+J+1,I+1)=((2.0*P/3.0)-((1-V)/15.0))*(B*B)
          KK(I+J+1,I+2)=0.0
          KK(I+J+1,I+J)=((2.0*P)+(0.2)+(0.8*V))*(-B)
          KK(I+J+1,I+J+1)=((4.0*P/3.0)+(4.0*(1.0-V))/15.0)*(B*B)
          KK(I+J+2,I)=((-1.0/P)+((1.0+(4.0*V))/5.0))*A
          KK(I+J+2,I+1)=0.0
          KK(I+J+2,I+2)=((2.0/(3.0*P))-(4.0*(1.0-V)/15.0))*(A*A)
          KK(I+J+2,I+J)=((-2.0/P)-(0.2)-(0.8*V))*A
          KK(I+J+2,I+J+1)=V*A*B
          KK(I+J+2,I+J+2)=((4.0/(3.0*P))+4.0*(1.0-V)/15.0))*(A*A)
          KK(I+J2,I)=(-2.0*P)-(2.0/P)+(2.8)-(0.8*V)
          KK(I+J2,I+1)=(-P+((1-V)/15.0 ))*B
          KK(I+J2,I+2)=((-1.0/P)+((1-V)/5.0))*(-A)
          KK(I+J2,I+J)=-2.0*((2.0/P)-P)-(14.0-(4.0*V))/5.0)
          KK(I+J2,I+J+1)=((-P+(1.0+(4.0*V))/5.0))*(B)
          KK(I+J2,I+J+2)=((2.0/P)+((1.0-V)/5.0))*A
          KK(I+J2,I+J2)=(4.0*P)+(4.0/P)+(2.8)-(0.8*V)
          KK(I+J2+1,I)=(P-((1-V)/5.0))*(B)
          KK(I+J2+1,I+1)=((P/3.0)+((1-V)/15.0))*(B*B)
          KK(I+J2+1,I+2)=0.0
          KK(I+J2+1,I+J)=(-P+(1.0+(4.0*V))/5.0)*B
          KK(I+J2+1,I+J+1)=(((2.0*P)/3.0)-((4.0*(1.0-V))/15.0))*(B*B)
          KK(I+J2+1,I+J+2)=0.0
          KK(I+J2+1,I+J2)=((-2.0*P)-(0.2)-(0.8*V))*B
          KK(I+J2+1,I+J2+1)=((4.0*P/3.0)+4.0*(1.0-V)/15.0))*(B*B)
          KK(I+J2+2,I)=((1.0/P)-((1-V)/5.0))*(-A)
          KK(I+J2+2,I+1)=0.0
          KK(I+J2+2,I+2)=((1.0/(3.0*P))+((1-V)/15.0))*(A*A)
          KK(I+J2+2,I+J)=((2.0/P)+((1.0-V)/5.0))*(-A)
          KK(I+J2+2,I+J+1)=0.0
          KK(I+J2+2,I+J+2)=((2.0/(3.0*P))-((1.0-V)/15.0))*(A*A)
          KK(I+J2+2,I+J2)=((2.0/P)+(0.2)+(0.8*V))*(A)
          KK(I+J2+2,I+J2+1)=-V*A*B
          KK(I+J2+2,I+J2+2)=((4.0/(3.0*P))+(4.0*(1-V)/15.0))*(A*A)
          KK(I+J3,I)=-2.0*((2.0/P)-P)-((14.0-(4.0*V))/5.0)
          KK(I+J3,I+1)=(-P+((1.0+(4.0*V))/5.0))*B
          KK(I+J3,I+2)=((2.0/P)+((1.0-V)/5.0))*A
          KK(I+J3,I+J)=-(2.0*P)-(2.0/P)+(2.8)-(0.8*V)
          KK(I+J3,I+J+1)=(P-((1-V)/5.0))*B
          KK(I+J3,I+J+2)=((1.0/P)-((1-V)/5.0))*(A)
          KK(I+J3,I+J2)=(2.0*((1.0/P)-(2.0*P)))-((14.0-(4.0*V))/5.0)
          KK(I+J3,I+J2+1)=((2.0*P)+((1.0-V)/5.0))*B
          KK(I+J3,I+J2+2)=((1.0/P)-((1.0+(4.0*V))/5.0))*A
          KK(I+J3,I+J3)=(4.0*P)+(4.0/P)+(2.8)-(0.8*V)
          KK(I+J3+1,I)=((P)-((1.0+(4.0*V))/5.0))*B
          KK(I+J3+1,I+1)=((2.0*P/3.0)-(4.0*(1.0-V)/15.0))*(B*B)
          KK(I+J3+1,I+2)=0.0
          KK(I+J3+1,I+J)=(P-((1-V)/5.0))*(-B)
          KK(I+J3+1,I+J+1)=((P/3.0)+((1-V)/15.0))*(B*B)
          KK(I+J3+1,I+J+2)=0.0
          KK(I+J3+1,I+J2)=((2.0*P)+((1.0-V)/5.0))*(-B)
          KK(I+J3+1,I+J2+1)=((2.0*P/3.0)-((1.0-V)/15.0))*(B*B)
```

```
                            KK(I+J3+1,I+J3+1)=((4.0*P/3.0)+(4.0*(1-V)/15.0))*(B*B)
                            KK(I+J3+2,1)=((2.0/P)+((1.0-V)/5.0))*(-A)
                            KK(I+J3+2,I+1)=0.0
                            KK(I+J3+2,I+2)=((2.0/(3.0*P))-((1.0-V)/15.0))*(A*A)
                            KK(I+J3+2,I+J)=((-1.0/P)+((1-V)/5.0))*A
                            KK(I+J3+2,I+J+1)=0.0
                            KK(I+J3+2,I+J+2)=((1.0/(3.0*P))+((1-V)/15.0))*(A*A)
                            KK(I+J3+2,I+J2)=((1.0/P)-((1.0+(4.0*V))/5.0 ))*A
                            KK(I+J3+2,I+J2+1)=0.0
                            KK(I+J3+2,I+J2+2)=((2.0/(3.0*P))-4.0*(1.0-V)/15.0))*(A*A)
                            KK(I+J3+2,I+J3)=((2.0/P)+(0.2)+(0.8*V))*A
                            KK(I+J3+2,I+J3+1)=V*A*B
                            KK(I+J3+2,I+J3+2)=((4.0/(3.0*P))+(4.0*(1-V)/15.0))*(A*A)
      C.....MULTIPLY BY FACTOR
                  DO 54 I=1,N
                  DO 54 J=1,I
         54 KK(I,J)=(KK(I,J))*(E*(T**3))/(12.0*(1.0-(V**2))*A*B)
      C.....COMPLETE UPPER MATRIX TRIANGLE
                  NM1=N-1
                  DO 55 I=1,NM1
                  IA1=I+1
                  DO 55 J=IA1,N
         55 KK(I,J)=KK(J,I)
      C.....WRITE COMPLETE ELEMENT STIFFNESS MATRIX
                  WRITE(2,60) ((KK(I,J),J=1,N),I=1,N)
         60 FORMAT (1H1,'KK MATRIX'///(' ',12E10.4))
                  RETURN
                  END



                  SUBROUTINE PLAIN1   (KK,N,A,B,V,E,T,RHO,K1)
                  REAL K1(N,N)
                  REAL KK(N,N)
      C.....N = 8 SIGNIFIES PLAIN STRAIN, N = 24 FOR BENDING AND STRETCHING
                  IF(N.EQ.8) GO TO 10
                  IF(N.EQ.24) GO TO 20
                  STOP
         10 WRITE(2,11)
         11 FORMAT(//'THIS IS FOR PLAIN STRAIN ONLY')
                  J=2
                  GO TO 30
         20 J=6
      C.....THIS GIVES A 8 X 8 MATRIX
         30 J2=J*2
      C.....NOTE THAT P = (A/B)  NOT (A/B**2)
                  P=A/B
                  KK(1,1)=(4.0/P)+(2.0*(1.0-V)*P)
                  KK(2,1)=3.0*(1.0+V)/2.0
                  KK(2,2)=(4.0*P)+(2.0*(1.0-V)/P)
                  KK(1+J,1)=(2.0/P)-(2.0*(1.0-V)*P)
                  KK(1+J,2)=-3.0*(1.0-3.0*V)/2.0
                  KK(1+J,1+J)=(4.0/P)+(2.0*(1.0-V)*P)
                  KK(1+J+1,1)=3.0*(1.0-(3.0*V))/2.0
                  KK(1+J+1,2)=(-4.0*P)+((1.0-V)/P)
                  KK(1+J+1,1+J)=-3.0*(1.0+V)/2.0
                  KK(1+J+1,1+J+1)=(4.0*P)+(2.0*(1.0-V)/P)
                  KK(1+J2,1)=(-2.0/P)-((1.0-V)*P)
                  KK(1+J2,2)=(-3.0*(1.0+V)/2.0)
                  KK(1+J2,1+J)=(-4.0/P)+((1.0-V)*P)
                  KK(1+J2,1+J+1)=(-3.0*(1-(3.0*V))/2.0)
                  KK(1+J2,1+J2)=(4.0/P)+(2.0*(1.0-V)*P)
                  KK(1+J2+1,1)=-3.0*(1.0+V)/2.0
                  KK(1+J2+1,2)=(-2.0*P)-((1.0-V)/P)
                  KK(1+J2+1,1+J)=(3.0*(1.0-(3.0*V))/2.0)
                  KK(1+J2+1,1+J+1)=(2.0*P)-(2.0*(1.0-V)/P)
                  KK(1+J2+1,1+J2)=(3.0*(1.0+V)/2.0)
                  KK(1+J2+1,1+J2+1)=(4.0*P)+(2.0*(1.0-V)/P)
                  J3=J*3
                  KK(1+J3,1)=(-4.0/P)+((1.0-V)*P)
                  KK(1+J3,2)=(3.0*(1.0-(3.0*V))/2.0)
                  KK(1+J3,1+J)=(-2.0/P)-((1.0-V)*P)
                  KK(1+J3,1+J+1)=(3.0*(1.0+V)/2.0)
                  KK(1+J3,1+J2)=(2.0/P)-(2.0*(1.0-V)*P)
                  KK(1+J3,1+J2+1)=(-3.0*(1.0-(3.0*V))/2.0)
                  KK(1+J3,1+J3)=(4.0/P)+(2.0*(1.0-V)*P)
                  KK(1+J3+1,1)=(-3.0*(1.0-(3.0*V))/2.0)
                  KK(1+J3+1,2)=(2.0*P)-(2.0*(1.0-V)/P)
                  KK(1+J3+1,1+J)=(3.0*(1.0+V)/2.0)
                  KK(1+J3+1,1+J+1)=(-2.0*P)-((1.0-V)/P)
                  KK(1+J3+1,1+J2)=(3.0*(1.0-(3.0*V))/2.0)
                  KK(1+J3+1,1+J2+1)=(-4.0*P)+((1.0-V)/P)
                  KK(1+J3+1,1+J3)=(-3.0*(1.0+V)/2.0)
                  KK(1+J3+1,1+J3+1)=(4.0*P)+(2.0*(1.0-V)/P)
      C.....MULTIPLY BY COMMON FACTOR
                  DO 34 I=1,N
                  DO 34 J=1,I
         34 KK(I,J)=(KK(I,J))*(E*T)/(12.0*(1.0-(V**2)))
      C.....COMPLETE UPPER MATRIX TRIANGLE
                  NM1=N-1
                  DO 35 I=1,NM1
                  IA1=I+1
                  DO 35 J=IA1,N
         35 KK(I,J)=KK(J,I)
                  DO 38 I=1,N
                  DO 38 J=1,N
         38 K1(I,J)=K1(I,J)+KK(I,J)
                  WRITE(2,40) ((K1(I,J),J=1,N),I=1,N)
```

```
      I=1
      NO=N
1013  I=I+1
      IF(I.EQ. NO) GO TO 1112
      J=1
1011  IF(K1(I,J).NE.K1(J,I)) GO TO 1111
      J=J+1
      IF(J.LT.I) GO TO 1011
      GO TO 1013
1111  WRITE (2,1012) (I,J,K1(I,J) ,K1(J,I))
1012  FORMAT(////' UNSYMMETRICAL MATRIX'///' ERROR POSITION IS   ',2I3,///' ',
     1' ',2F12.4)
      IF(I.LT.NO ) GO TO 1013
1112  CONTINUE

      RETURN
      END




      SUBROUTINE BEND 2  (MM,N,A,B,V,E,T,RHO)
      REAL                    MM(N,N),P,A,B,V
C.....CLEAR MM MATRIX
      DO 1  I=1,N
      DO 1  J=1,N
    1 MM(I,J)=0.0


C.....N=12 SIGNIFIES BENDING   N=24 FOR BENDING + STRETCHING
      IF(N.EQ.12) GO TO 10
      IF(N.EQ.24) GO TO 20
   10 I=1
      WRITE(2,11)
   11 FORMAT(//'THIS IS FOR PLATE BENDING ONLY')
      J=3
C.....THIS GIVES A 12 X 12 MATRIX
      GO TO 50
   20 I=3
      WRITE(2,21)
   21 FORMAT(//'THIS IS FOR PLATE BENDING AND STRETCHING')
      J=6
C.....THIS GIVES A 24 X 24 MATRIX
      GO TO 50
   50 P=(A/B)**2
      J2=J*2
      J3=J*3
      MM(I,I)=24178.0
      MM(I+1,I)=3227*B
      MM(I+1,I+1)=560*B*B
      MM(I+2,I)=-3227*A
      MM(I+2,I+1)=-441.0*A*B
      MM(I+2,I+2)=560.0*A*A
      MM(I+J,I)=8582.0
      MM(I+J,I+1)=1918.0*B
      MM(I+J,I+2)=-1593.0*A
      MM(I+J,I+J)=24178.0
      MM(I+J+1,I)=-1918.0*B
      MM(I+J+1,I+1)=-420.0*B*B
      MM(I+J+1,I+2)=294.0*A*B
      MM(I+J+1,I+J)=-3227.0*B
      MM(I+J+1,I+J+1)=560.0*B*B
      MM(I+J+2,I)=-1593*A
      MM(I+J+2,I+1)=-294.0*A*B
      MM(I+J+2,I+2)=280.0*A*A
      MM(I+J+2,I+J)=-3227.0*A
      MM(I+J+2,I+J+1)=441.0*A*B
      MM(I+J+2,I+J+2)=560.0*A*A
      MM(I+J2,I)=2758.0
      MM(I+J2,I+1)=812.0*B
      MM(I+J2,I+2)=-812.0*A
      MM(I+J2,I+J)=8582.0
      MM(I+J2,I+J+1)=-1593.0*B
      MM(I+J2,I+J+2)=-1918.0*A
      MM(I+J2,I+J2)=24178.0
      MM(I+J2+1,I)=-812.0*B
      MM(I+J2+1,I+1)=-210.0*B*B
      MM(I+J2+1,I+2)=196.0*A*B
      MM(I+J2+1,I+J)=-1593*B
      MM(I+J2+1,I+J+1)=280.0*B*B
      MM(I+J2+1,I+J+2)=294.0*A*B
      MM(I+J2+1,I+J2)=-3227.0*B
      MM(I+J2+1,I+J2+1)=560.0*B*B
      MM(I+J2+2,I)=812.0*A
      MM(I+J2+2,I+1)=196.0*A*B
      MM(I+J2+2,I+2)=-210.0*A*A
      MM(I+J2+2,I+J)=1918.0*A
      MM(I+J2+2,I+J+1)=-294.0*A*B
      MM(I+J2+2,I+J+2)=-420.0*A*A
      MM(I+J2+2,I+J2)=3227.0*A
      MM(I+J2+2,I+J2+1)=-441.0*A*B
      MM(I+J2+2,I+J2+2)=560.0*A*A
      MM(I+J3,I)=8582.0
      MM(I+J3,I+1)=1593*B
      MM(I+J3,I+2)=-1918.0*A
      MM(I+J3,I+J)=2758.0
      MM(I+J3,I+J+1)=-812.0*B
      MM(I+J3,I+J+2)=-812.0*A
```

```
          MM(I+J3,1+J2+1)=-1918.0*B
          MM(I+J3,1+J2+2)=1395.0*A
          MM(I+J3,1+J3)=24178.0
          MM(I+J3+1,1)=1395.0*B
          MM(I+J3+1,1+1)=280.0*B*B
          MM(I+J3+1,1+2)=-294.0*A*B
          MM(I+J3+1,1+J)=812.0*B
          MM(I+J3+1,1+J+1)=-210.0*B*B
          MM(I+J3+1,1+J+2)=-196.0*A*B
          MM(I+J3+1,1+J2)=1918.0*B
          MM(I+J3+1,1+J2+1)=-420.0*B*B
          MM(I+J3+1,1+J2+2)=294.0*A*B
          MM(I+J3+1,1+J3)=3227.0*B
          MM(I+J3+1,1+J3+1)=560.0*B*B
          MM(I+J3+2,1)=1918.0*A
          MM(I+J3+2,1+1)=294.0*A*B
          MM(I+J3+2,1+2)=-420.0*A*A
          MM(I+J3+2,1+J)=812.0*A
          MM(I+J3+2,1+J+1)=-196.0*A*B
          MM(I+J3+2,1+J+2)=-210.0*A*A
          MM(I+J3+2,1+J2)=1395.0*A
          MM(I+J3+2,1+J2+1)=-294.0*A*B
          MM(I+J3+2,1+J2+2)=280.0*A*A
          MM(I+J3+2,1+J3)=3227.0*A
          MM(I+J3+2,1+J3+1)=441.0*A*B
          MM(I+J3+2,1+J3+2)=560.0*A*A

C.....MULTIPLY BY COMMON FACTOR
      DO 74 I=1,N
      DO 74 J=1,I
   74 MM(I,J)=(MM(I,J))*RHO*A*B/176400
C.....COMPLETE UPPER INERTIA MATRIX TRIANGLE.
      NM1=N-1
      DO 75 I=1,NM1
      IA1=I+1
      DO 75 J=IA1,N
   75 MM(I,J)=MM(J,I)
C.....WRITE COMPLETED ELEMENT INERTIA MATRIX
      WRITE(2,80) ((MM(I,J),J=1,N),I=1,N)
   80 FORMAT (1H1,'MM MATRIX'///(' ',12E10.4))
      RETURN
      END


####S
****



      SUBROUTINE PLAIN2    (MM,N,A,B,V,E,T,RHO,K1)
      REAL K1(N,N)
      REAL              MM(N,N)
C.....N = 8 SIGNIFIES PLAIN STRAIN, N = 24 FOR BENDING AND STRETCHING
      IF(N.EQ.8) GO TO 10
      IF(N.EQ.24) GO TO 20
      STOP
   10 WRITE(2,11)
   11 FORMAT(///'THIS IS FOR PLAIN STRAIN ONLY')
      J=2
      GO TO 30
   20 J=6
   30 J2=J*2
      P=A/B
C.....THIS GIVES A 8 X 8 MATRIX
C.....COMPUTE INERTIA MATRIX
      MM(1,1)=4.0
      MM(2,1)=0.0
      MM(2,2)=4.0
      MM(1+J,1)=2.0
      MM(1+J,2)=0.0
      MM(1+J,1+J)=4.0
      MM(1+J+1,1)=0.0
      MM(1+J+1,2)=2.0
      MM(1+J+1,1+J)=0.0
      MM(1+J+1,1+J+1)=4.0
      MM(1+J2,1)=1.0
      MM(1+J2,2)=0.0
      MM(1+J2,1+J)=2.0
      MM(1+J2,1+J+1)=0.0
      MM(1+J2,1+J2)=4.0
      MM(1+J2+1,1)=0.0
      MM(1+J2+1,2)=1.0
      MM(1+J2+1,1+J)=0.0
      MM(1+J2+1,1+J+1)=2.0
      MM(1+J2+1,1+J2)=0.0
      MM(1+J2+1,1+J2+1)=4.0
      J3=J*3
      MM(1+J3,1)=2.0
      MM(1+J3,2)=0.0
      MM(1+J3,1+J)=1.0
      MM(1+J3,1+J+1)=0.0
      MM(1+J3,1+J2)=2.0
      MM(1+J3,1+J2+1)=0.0
      MM(1+J3,1+J3)=4.0
      MM(1+J3+1,1)=0.0
      MM(1+J3+1,2)=2.0
      MM(1+J3+1,1+J)=0.0
      MM(1+J3+1,1+J+1)=1.0
```

```
              MM(1+J3+1,1+J2)=0.0
              MM(1+J3+1,1+J2+1)=2.0
              MM(1+J3+1,1+J3)=0.0
              MM(1+J3+1,1+J3+1)=4.0
C.....MULTIPLY BY COMMON FACTOR
              DO 50 I=1,N
              DO 50 J=1,I
   50 MM(I,J)=(MM(I,J)*RHO*A*B/36.0)
C.....COMPLETE UPPER TRIANGULAR MATRIX
              NM1=N-1
              DO 60 I=1,NM1
              IA1=I+1
              DO 60 J=IA1,N
   60 MM(I,J)=MM(J,I)
              DO 38 I=1,N
              DO 38 J=1,N
   38 K1(I,J)=K1(I,J)+MM(I,J)
              WRITE (2,70)
   70 FORMAT (' TRANSLATIONAL INERTIA MATRIX COMPLETED')
              WRITE(2,40) ((K1(I,J),J=1,N),I=1,N)
   40 FORMAT (1H1,'MM MATRIX'////(' ',12E10.4))
              RETURN
              END




              SUBROUTINE REORDER (N, K,KJ)
              REAL   K( N,N ),KJ
C     RENUMBER NODAL SYSTEM
C.....INTERCHANGE NODE 2 AND 4 ROWS
              DO 60 I=7,12,1
              DO 60 J=1,24,1
              KJ=K(I+12,J)
              K(I+12,J)= K(I,J)
   60 K(I,J)=KJ
C.....INTERCHANGE NODE 2 AND 4 COLUMNS
              DO 70 J=7,12,1
              DO 70 I=1,24,1
              KJ=K(I,J+12)
              K(I,J+12)=K(I,J)
   70 K(I,J)=KJ
C.....REORDER NODAL SYSTEM

C.....INTERCHANGE NODE 3 AND 4 ROWS
              DO 80 I=13,18,1
              DO 80 J=1,24,1
              KJ=K(I+6,J)
              K(I+6,J)=K(I,J)
   80 K(I,J)=KJ
C.....INTERCHANGE NODE 3 AND 4 COLUMNS
              DO 90 J=13,18
              DO 90 I=1,24
              KJ=K(I,J+6)
              K(I,J+6)=K(I,J)
   90 K(I,J)=KJ
              WRITE(2,100)((K(I,J),J=1,24   ),I=1,24   )
  100 FORMAT('1REORDERED ELEMENT MATRIX',//(' ',10F12.4))

              RETURN
              END




              SUBROUTINE ROTATE1   (R,K,K1,S)
              DIMENSION R(24,24), S(24,24)
              REAL K(24,24),K1(24,24)
              DO 5 I=1,24
              DO 5 J=1,24
              S(I,J)=0.0
              K1(I,J)=0.0
    5 R(I,J)=0.0
              DO 10 I=1,24
   10 R(I,I)=1.0
              WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
C.....POST-MULTIPLY BY R
              DO 50 I=1,24
              DO 50 J=1,24
              DO 50 L=1,24
   50 K1(I,J)=K1(I,J)+(K(I,L)*R(L,J))
              DO 52 I=1,24
              DO 52 J=1,24
              DUMMY=R(I,J)
              R(I,J)=R(J,I)
              R(J,I)=DUMMY
   52 CONTINUE
              WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
C.....PRE-MULTIPLY BY R TRANSPOSED
              DO 55 I=1,24
              DO 55 J=1,24
              DO 55 L=1,24
   55 S (I,J)=S (I,J)+(R(I,L)*K1(L,J))
   40 FORMAT(24F5.2)
              WRITE (2,60) ((S (I,J),J=1, 24),I=1,24)
   60 FORMAT ('1ROTATED ELEMENTAL MATRIX'////(' ',12E10.4))
              RETURN
              END
```

```fortran
      SUBROUTINE ROTATE2 (R,K,K1,S)
      REAL K(24,24),K1(24,24)
      DIMENSION R(24,24), S(24,24)
      DO 5 I=1,24
      DO 5 J=1,24
      S(I,J)=0.0
      K1(I,J)=0.0
    5 R(I,J)=0.0
      DO 10 I=1,24,3
      J=I+1
   10 R(I,J)= 1.0
      DO 20 I=2,24,3
      J=I+1
   20 R(I,J)=1.0
      DO 30 I=3,24,3
      J=I-2
   30 R(I,J)=1.0
      WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
   40 FORMAT(24F5.2)
C.....POST-MULTIPLY BY R
      DO 50 I=1,24
      DO 50 J=1,24
      DO 50 L=1,24
   50 K1(I,J)=K1(I,J)+(K(I,L)*R(L,J))
      DO 52 I=1,24
      DO 52 J=1,24
      DUMMY=R(I,J)
      R(I,J)=R(J,I)
      R(J,I)=DUMMY
   52 CONTINUE
      WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
C.....PRE-MULTIPLY BY R TRANSPOSED
      DO 55 I=1,24
      DO 55 J=1,24
      DO 55 L=1,24
   55 S (I,J)=S (I,J)+(R(I,L)*K1(L,J))
      WRITE (2,60) ((S (I,J),J=1, 24),I=1,24)
   60 FORMAT ('1ROTATED ELEMENTAL MATRIX'///(' ',12E10.4))
      RETURN
      END



      SUBROUTINE ROTATE3 (R,K,K1,S)
      REAL K(24,24),K1(24,24)
      DIMENSION R(24,24), S(24,24)
      DO 5 I=1,24
      DO 5 J=1,24
      S(I,J)=0.0
      K1(I,J)=0.0
    5 R(I,J)=0.0
      DO 10 I=1,24,3
      J=I
   10 R(I,J)= 1.0
C.....POST-MULTIPLY BY R
      DO 20 I=2,24,3
      J=I+1
   20 R(I,J)=1.0
      DO 30 I=3,24,3
      J=I-1
   30 R(I,J)=-1.0
      WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
   40 FORMAT(24F5.2)
C.....PRE-MULTIPLY BY R TRANSPOSED
      DO 50 I=1,24
      DO 50 J=1,24
      DO 50 L=1,24
   50 K1(I,J)=K1(I,J)+(K(I,L)*R(L,J))
      DO 52 I=1,24
      DO 52 J=1,24
      DUMMY=R(I,J)
      R(I,J)=R(J,I)
      R(J,I)=DUMMY
   52 CONTINUE
      WRITE(2,40) ((R(I,J),J=1,24),I=1,24)
      DO 55 I=1,24
      DO 55 J=1,24
      DO 55 L=1,24
   55 S (I,J)=S (I,J)+(R(I,L)*K1(L,J))
      WRITE (2,60) ((S (I,J),J=1, 24),I=1,24)
   60 FORMAT ('1ROTATED ELEMENTAL MATRIX'///(' ',12E10.4))
      RETURN
      END



      SUBROUTINE COMPLET  (K,NZ,NO,KK,NS,NX,NY,NA)
      LEVEL 2,K
      REAL KK(NS,NS),K(NO,NO)
      INTEGER E
C.....WRITE (2,1003) ((KK(J,JJ),JJ=1,NS),J=1,NS)
      I1=(NX+1)*(NY+1)*6
      NK=(NX+1)*6
      N1=(NX+1)*(NZ+1)*6
      IF((NA.EQ.1).OR.(NA.EQ.2))N1=(NX+1)*NZ*6
      NJ=6
      IF((NA.EQ.3).OR.(NA.EQ.4)) NJ=(NZ+1)*6
C.....COMPLETE MATRIX FOR WHOLE PLATE FROM ELEMENT MATRICES
      I=-5
```

```
C.....IT DENOTES STARTING POINT AND INCREMENTAL X.
      IT=1-(NJ)
      NN=NK
      IF(NA.EQ.2) IT=IT+(NX+1)+(NY+1)+NZ*6
      IF (NA.EQ.4) IT=IT+(NZ*6)
      IF (NA.EQ.6) IT=IT+((NX+1)*NZ*6)
   10 I=I+6
      IT=IT+(NJ)
      IF(I.EQ.1) GO TO 100
      IF(NN.GT.(NK))            GO TO 30
C.....DOUBLE POINT FIRST ROW FOLLOWS
      IF(I.LT.(NN-5)) GO TO 200
      IF(I.EQ.(NN-5)) GO TO 300
      IF(I.GT.(I1-NK)) GO TO 40
   25 IF(I.EQ.(NN+1)) GO TO 400
C.....QUADRUPLE POINTS ROWS FOLLOWS
      IF(I.EQ.(NN+7 ))NN=NN+NK
   30 IF(I.LT.(NN-5)) GO TO 800
      IF(I.EQ.(NN-5)) GO TO 500
      IF(I.LT.(I1-NK)) GO TO 25
   40 CONTINUE
      IF(I.EQ.(NN+1)) GO TO 600
C     DOUBLE POINTS ROWS FOLLOWS
      IF(I.LT.(I1-5)) GO TO 700


C.....THIS LEAVES ONLY ROWS OF LAST NODE
      E = 19
      M = IT -NJ-N1
  101 N=1
      L=IT
  102 DO 105 J=1,6
      MM = M+J-1
      K(L,MM) = KK(E,J)+K(L,MM)
      K(L,MM+NJ)=KK(E,J+6)+K(L,MM+NJ)
      K(L,MM+N1) = KK(E,J+12)+K(L,MM+N1)
      K(L,MM+N1+NJ) = KK(E,J+18)+K(L,MM+N1+NJ)
  105 CONTINUE
      L=L+1
      E=E+1
      N=N+1
      IF(N.LT.7) GO TO 102
      GO TO 1000

  100 E=1
      M=IT
C.....ROWS 1 TO 6
      GO TO 101

  300 E=7
      M=IT-NJ
C.....ROWS OF CORNER NODES
      GO TO 101

  600 E=13
      M=IT-N1
C.....CORNER NODES OF LAST NODE
      GO TO 101

  200 E = 7
      M=IT-NJ
  250 L=IT
      N=1
  251 CONTINUE
C.....COMPUTE DOUBLE POINTS ROW
      DO 255 J=1,6
      MM=M+J-1
      K(L,MM)=KK(E,J)+K(L,MM)
      NJ2=NJ*2
      K(L,MM+NJ)= KK(E,J+6)+KK(E-6,J)+K(L,MM+NJ)
      K(L,MM+NJ2)= KK(E-6,J+6)+K(L,MM+NJ2)
      K(L,MM+N1) = KK(E,J+12)+K(L,MM+N1)
      K(L,MM+N1+NJ)= KK(E,J+18)+KK(E-6,J+12)+K(L,MM+N1+NJ)
      K(L,MM+N1+NJ2)= KK(E-6,J+18)+K(L,MM+N1+NJ2)
  255 CONTINUE
      E=E+1
      L=L+1
      N=N+1
      IF(N.LT.7) GO TO 251
      GO TO 1000


  400 L=IT
      M=IT-N1
      E=13
      N=1
C.....ROWS OF LEFT HAND EDGE NODES
  401 DO 420 J=1,6
      MM=M+J-1
      K(L,MM)=KK(E,J)+K(L,MM)
      K(L,MM+NJ) = KK(E,J+6) + K(L,MM+NJ)
      K(L,MM+N1) = KK(E,J+12)+KK(E-12,J)+K(L,MM+N1)
      K(L,MM+N1+NJ) = KK(E,J+18) + KK(E-12,J+6) + K(L,MM+N1+NJ)
      K(L,MM+N1+N1+NJ) = KK(E-12,J+18) +K(L,MM+N1+N1+NJ)
  420 K(L,MM+N1+N1) = KK(E-12,J+12)+K(L,MM+N1+N1)
      N=N+1
      E=E+1
      L=L+1
      IF(N.LT.7) GO TO 401
```

```
                           GO TO 1000

      500 L=IT
          E=19
          M=IT-N1-NJ
          N=1
C.....ROWS OF R,H, EDGE NODES
      501 DO 510 J=1,6
          MM=M+J-1
          K(L,MM)=KK(E,J)+K(L,MM)
          K(L,MM+NJ)=KK(E,J+6)+K(L,MM+NJ)
          K(L,MM+N1) = KK(E,J+12) +KK(E-12,J)+K(L,MM+N1)
          K(L,MM+N1+NJ) = KK(E,J+18)+KK(E-12,J+6)+K(L,MM+N1+NJ)
          K(L,MM+N1+N1)=KK(E-12,J+12)+K(L,MM+N1+N1)
          K(L,MM+N1+N1+NJ) = KK(E-12,J+18)+K(L,MM+N1+N1+NJ)
      510 CONTINUE
          N=N+1
          E=E+1
          L=L+1
          IF(N.LT.7) GO TO 501
          GO TO 1000

      700 L=IT
          E=19
          N=1
C.....COMPUTE DOUBLE POINTS ROWS 88
          M=IT-N1-NJ
          GO TO 250


      800 IF(NN.EQ.(NK+2)) GO TO 801
          GO TO 855
C.....COMPUTE K MATRIX ROWS FIRST OF QUADRUPLE POINTS ROWS
      801 IF(I.GT.(NN-N1+7)) GO TO 861
          M=IT-N1-NJ
          L=IT
          N=1
          E=19
      810 DO 820 J=1,6
          MM=M+J-1
          K(L,MM) = KK(E,J)+K(L,MM)
          K(L,MM+NJ)= KK(E,J+6) +KK(E-6,J)+K(L,MM+NJ)
          N2=NJ+2
      803 K(L,MM+N2)=KK(E-6,J+6)+K(L,MM+N2)
          MM=M+J-1+N1
          K(L,MM) = KK(E,J+12) + KK(E-12,J)+K(L,MM)
          K(L,MM+NJ)= KK(E,J+18)+KK(E-12,J+6)+KK(E-6,J+12)+KK(E-18,J)
         1 +K(L,MM+NJ)
      805 K(L,MM+N2) = KK(E-6,J+18)+KK(E-18,J+6)+K(L,MM+N2)
          MM=M+J-1+N1+N1
          K(L,MM) = KK(E-12,J+12)+K(L,MM)
          K(L,MM+NJ)= KK(E-12,J+18)+KK(E-18,J+12)+K(L,MM+NJ)
      820 K(L,MM+N2) = KK(E-18,J+18)+K(L,MM+N2)
          L=L+1
          N=N+1
          E=E+1
          IF(N.LT.7) GO TO 810

      840 CONTINUE
          GO TO 1000


C     IF NY IS MORE THAN 3
C.....COMPUTE QUAD POINTS ROWS FROM FIRST ROW OF QUAD POINTS
C.....POSSIBLE ERROR BUILD-UP FROM USING THESE ROWS
      855 CONTINUE
          L=IT
          N=1
          IAN1A6 =IT+N1+NJ+5
          ITMN1M6=IT-N1-NJ
      850 DO 851 J=ITMN1M6,IAN1A6
      852 K(L,J)=K(L-N1,J-N1)
      851 CONTINUE
          L=L+1
          N=N+1
          IF(N.LT.7) GO TO 850
          GO TO 1000


C.....COMPUTE QUAD POINTS ROW ONE,  SECOND POINT ONWARDS
      861 CONTINUE
          L=IT
          N=1
          IAN1A6 =IT+N1+NJ+5
          ITMN1M6=IT-N1-NJ
      860 DO 862 J=ITMN1M6,IAN1A6
      862 K(L,J)=K(L-NJ,J-NJ)
          L=L+1
          N=N+1
          IF(N.LT.7) GO TO 860
          GO TO 1000

     1000 IF(I.LT.(NK-NJ))GO TO 10
          IF(NA.LE.4) GO TO 1050
          IF(I.EQ.((NK-NJ+1)*2)) IT =((NX+1)*NZ+6)+IT
          IF(I.EQ.((NK-NJ+1)  )) IT =((NX+1)*NZ+6)+IT
     1050 IF(I.LT.(II-5)) GO TO 10

C.....PRINT COMPLETED K MATRIX
```

```
                              NAUX=NX
                              NX=11
                              IF ((NA.NE.1).AND.(NA.NE.2)) GO TO 1054
                              WRITE (2,1052)
                        1052 FORMAT (///' PLATE IN XY PLANE ONLY'/)
                              GO TO 1065
                        1054 IF ((NA.NE.3).AND.(NA.NE.4)) GO TO 1058
                              WRITE (2,1056)
                        1056 FORMAT(///' PLATES IN XY AND YZ PLANES'/)
                              GO TO 1065
                        1058 WRITE (2,1060)
                        1060 FORMAT (///' PLATES IN XY AND YZ AND ZX PLANES'/)
                        1065 CONTINUE
                              WRITE(2,1001)
                        1001 FORMAT(' COMPLETED MATRIX'////)
                     C.....THIS IS TO REMOVE PRINTOUT INSTRUCTIONS
                     C.....DO 1005 I=1,NO
                              WRITE(2,1002)
                        1002 FORMAT(' ')
                              WRITE(2,1003)(K(I,J),J=1,NO)
                        1003 FORMAT(10E12.4)
                        1005 CONTINUE
                              I=1
                        1013 I=I+1
                              IF(I.EQ. NO) GO TO 1112
                              J=1
                        1011 IF(K(I,J).NE.K(J,I)) GO TO 1111
                              J=J+1
                              IF(J.LT.I) GO TO 1011
                              GO TO 1013
                        1111 WRITE (2,1012) (I,J,K(I,J),K(J,I))
                        1012 FORMAT(////' UNSYMMETRICAL MATRIX'//' ERROR POSITION IS   ',2I3,///' ',
                           1' ',2F12.4)
                              IF(I.LT.NO ) GO TO 1013
                        1112 CONTINUE

                              NX=NAUX
                              RETURN
                              END


                     /*/*
                     JOB MUJFEM2,:ESJJS,CP76(T40,P2000)
                     ATTACH (TAPE3,MUJDAT1  ,FO=ASIS,ST=S6A)
                     ATTACH( AFILE,LIBNAGFTNLCM,ID=LIBAPPL)
                     LIBRARY(AFILE)
                     FTN.
                     LDSET(PRESET=NGINF,MAP=B/ZZZZMP)
                     LGU(PL=6000)
                     REWIND(TAPE3)
                     COPY(TAPE3,TAPE9)
                     CATALOG(TAPE9,MUJDAT1  ,FO=ASIS,ST=S6A)
                     ####S
                              PROGRAM STRUCT(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE3,TAPE4)
                              LEVEL 2,K,S


                     C.....THIS PROGRAM TAKES THE STIFFNESS AND MASS MATRICES AS GIVEN BY MASTER PLAT
                     C     READ IN FROM MAG TAPE. THE INERTIA MATRIX IS SIMILARLY MODIFIED    18
                     C     AND RECORDED IN DISCFILE BOTH TAKING INTO ACCOUNT BOUNDARY CONDITIONS
                              REAL K(180,180),KK(6,6),LENGTH,            PEINT(156)
                              REAL S(146,146)
                              INTEGER ORDER
                              INTEGER W(180)
                              COMMON/UPPER/K,S
                              NX=5
                              NY=3
                              NZ=2
                              READ (3) K
                              ORDER=NX*NY*NZ*6
                              N=ORDER
                              DO 10 I=1,N
                          10 W(I)=1
                              DO 15 I=1,N,6
                              IF(I.EQ.1) GO TO 11
                              IF(I.EQ.25) GO TO 11
                              IF(I.EQ.61) GO TO 11
                              IF(I.EQ.85) GO TO 11
                              GO TO 15
                          11 CONTINUE
                              J=0
                          12 W(I+J)=0
                              J=J+1
                              IF(J.LT.6) GO TO 12
                          15 CONTINUE
                     C.....THIS ELIMINATES THETA FOR EMPTY FIELDS
                              NYM2=NY-2
                              DO 50 JJ=1,NYM2
                              DO 50 J=1,NX
                              JK=(NX*6)+(6*J*JJ)
                              JKK=(NX*NY*6)+JK
                              W(JK)=0
                              W(JKK)=0
                          50 CONTINUE
                              M=0
                              DO 16 I=1,N
                              IF(W(I).EQ.0) GO TO 16
                              M=M+1
                          16 CONTINUE
```

```
                      WRITE(2,17)M
          17 FORMAT(' M = ',I10)
                      WRITE(2,1)
           1 FORMAT(' ')
                      NXM6=NX*6
                      WRITE (2,60)
          60 FORMAT(' DEGREES OF FREEDOM REDUCED OUT'//)
                      DO 19 I=1,N, NXM6
                      II=(NXM6)+I-1
                      WRITE(2,18)(W(J),J=I,II)
          18 FORMAT( 30I4)
          19 CONTINUE
                      WRITE (2,46)
          46 FORMAT('1STIFFNESS MATRIX OF  REDUCED  PLATE')
C.....IW=1 FOR PRINT
                      IW=2
                      CALL  BNDRY  (K,N,W,S,M,1,IW)
                      CALL SYMET(S,M)
                      CALL WRTP(S,1,M)
C       USE K AND KK MATRIX FOR INERTIA MATRIX
                      READ (3) K
                      WRITE (2,64)
          64 FORMAT('1INERTIA MATRIX OF STIFFENED PLATE')
                      CALL  BNDRY  (K,N,W,S,M,2,IW)
                      CALL SYMET(S,M)
                      REWIND 3
                      WRITE (5) W
                      CALL WRTP(S,2,M)
                      ENDFILE 3
                      REWIND 3
                      STOP
                      END




                      SUBROUTINE  BNDRY    (K,N,W,S,A,B,C)
                      LEVEL 2,K,S
                      REAL                S(A,A),K(N,N)
                      INTEGER A,B
                      INTEGER C
                      INTEGER W(N)
C       THIS SUBROUTINE ALLOWS FOR SATISFACTION OF THE BOUNDARY CONDITIONS AND AT
C       THE SAME TIME REDUCES REDUNDANT ROWS AND COLUMNS OF THE MATRIX
                      L=1
C.....DEALS WITH ROWS
                      DO 100 I=1,N
C       W(I) = 0 IF DEGREE OF FREEDOM ELIMINATED
                      IF (W(I).EQ.0) GO TO 100
C         THIS ELIMINATES ROW
                      M=L
C.....DEALS WITH COLUMNS
                      DO 90 J=1,N
                      IF (W(J).EQ.0) GO TO 90
                      S(L,M)=K(I,J)
                      M=M+1
          90 CONTINUE
                      L=L+1
         100 CONTINUE
                      M=M-1
                      IF (A.EQ.M) GO TO 110
                      STOP
         110 CONTINUE
C       COMPLETE LOWER TRIANGULAR MATRIX
C.....BOUND TO BE SYMMETRICAL
                      MM1=M-1
                      DO 120 J=1, MM1
                      JA1=J+1
                      DO 120 I= JA1 ,M
         120 S(I,J)=S(J,I)
                      WRITE (2,140) M
         140 FORMAT (' ',' MATRIX SIZE IS ',I3)
                      IF(C.NE.1) GO TO 143
                      DO 142 I = 1,M
                      WRITE (2,141) ((S(I,J),J=1,M))
         141 FORMAT ('0',(12E10.4))
         142 CONTINUE
         143 CONTINUE
                      RETURN
                      END




                      SUBROUTINE WRTP(S,B,A)
                      LEVEL 2,S
                      REAL S(A,A)
                      INTEGER A,B
C.....THIS SUBROUTINE WRITES REDUCED MATRICES TO DISCFILE
                      WRITE (4) S
                      IF (B.EQ.1) GO TO 200
                      REWIND 4
                      READ (4) S
                      WRITE(3) S
                      READ (4) S
                      WRITE(3) S
                      WRITE(2,89)
          89 FORMAT(//' DATAFILE 'MOJDAT ' HAS NOW BEEN REDUCED BY BOUNDARY')
         200 RETURN
                      END
```

```
      SUBROUTINE SYMET(S,N)
      LEVEL 2,S
      DIMENSION S(N,N)
C.....THIS SUBROUTINE CHECKS MATRICES ARE SYMMETRICAL AFTER REDUCTION
      DO 5 I=2,N
      IM1=I-1
      DO 5 J=1,IM1
      IF(S(I,J).EQ.S(J,I)) GO TO 100
      WRITE (2,10) I,J,S(I,J),S(J,I)
   10 FORMAT(' UNSYMMETRICAL MATRIX AT ',2I5,//' ',2E10.4)
  100 CONTINUE
    5 CONTINUE
      RETURN
      END



      SUBROUTINE DEDET(K,M,I,DET,REINT,IT,S)
      REAL K(I,I),S(M,M),REINT(M)
      LEVEL 2,K,S
      DO 30 II=1,M
      DO 30 J =1,M
      K(II,J)=S(II,J)
   30 CONTINUE
      CALL F03AAF(S,M,M ,DET,REINT,IT)
      WRITE(2,71)DET
   71 FORMAT(' DET = ',E20.10)
      IF(IT.EQ.0) GO TO 85
      IF(IT.EQ.1) GO TO 65
      IF(IT.EQ.2) GO TO 75
   65 CONTINUE
      WRITE(2,70)I
   70 FORMAT(' SINGULAR MATRIX AT',I4)
      WRITE(2,72)(K(1,J2),J2=1,I)
      WRITE(2,72)(S(1,J2),J2=1,I)
      WRITE(2,72)(K(J2,1),J2=1,I)
      WRITE(2,72)(S(J2,1),J2=1,I)
   72 FORMAT(6E15.7)
      STOP
   75 CONTINUE
      WRITE(2,80)I
   80 FORMAT(I4,' VERY LARGE DETERMINANT VALUE DETECTED')
      WRITE(2,72)(K(1,J2),J2=1,I)
      WRITE(2,72)(S(1,J2),J2=1,I)
      WRITE(2,72)(K(J2,1),J2=1,I)
      WRITE(2,72)(S(J2,1),J2=1,I)
      STOP
   85 CONTINUE
      WRITE(2,90)I
   90 FORMAT(I4,' MATRIX IS CONFIRMED NON SINGULAR')
  100 CONTINUE
      DO 200 II=1,M
      DO 200 J =1,M
  200 S(II,J)=K(II,J)
      RETURN
      END
#####S
****


/*/*
JOB MUJFEM3,:ESJJS,CP76(T40,P2000)
ATTACH(TAPE3,MUJDAT1  ,FU=ASIS,ST=S6A)
ATTACH(AFILE,LIBNAGFINLCM,ID=LIBAPPL)
LIBRARY(AFILE)
FTN.
LDSET(PRESET=NGINF,MAP=B/ZZZZMP)
LGU(PL=2000)
#####S
      PROGRAM POSDEF(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE3)
      LEVEL 2,K,M,C
      INTEGER IW(180)
      REAL K(146,146),M(146,146),C(146)
      COMMON/UPPER/K,M,C
      READ(3)IW
      READ(3)K
      READ(3)M
      IF=1
      CALL F01AEF(146,K,146,M,146,C,IF)
      IF(IF.EQ.1) WRITE(2,10)
   10 FORMAT(' M NOT POSITIVE DEFINITE')
      IF(IF.EQ.0) WRITE(2,20)
   20 FORMAT(' M IS POSITIVE DEFINITE')
      STOP
      END
#####S
****
/*/*
JOB MUJFEMD,:ESJJS,CP76(T3//,P0000)
ATTACH(TAPE3,MUJDAT1  ,FU=ASIS,ST=S6A)
ATTACH( AFILE,LIBNAGFTNLCM,ID=LIBAPPL)
FTN(OPT=2)
LIBRARY(AFILE)
LDSET(PRESET=NGINF,MAP=B/ZZZZMP)
LGU(PL=3000)
REWIND(TAPE3)
COPY(TAPE3,TAPE9)
CATALOG(TAPE9,MUJDAT1  ,FU=ASIS,ST=S6A)
```

```
ø##S
      PROGRAM SOLVE(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE3)
      LEVEL 2,K,M,A
      LEVEL 2,BB,BL,W,Z
      COMMON/UPPER/K,M,A
      COMMON/LOWER/BB,BL,W,Z
C.....THIS PROGRAM SOLVES EIGENVALUES AND EIGENVECTORS FROM K AND M MATRICES -
      DIMENSION BB(146),BL(146),W(146),IW(1H0)
      REAL K(146,146),M(146,146),A(146,146)
      INTEGER Z(146)
      N=146
      NE=146
C.....NW EQUAL ZERO FOR NO MATRIX PRINTOUT
      NW=0
      READ (3) IW
      READ (3) K
      READ (3) M
      REWIND 3
      WRITE (2,4)
    4 FORMAT(' K MATRIX'////)
      IF (NW.EQ.0) GO TO 15
    5 FORMAT (///( 9F12.4))
      DO 6 I=1,N
      WRITE(2,5)(K(I,J),J=1,N)
    6 CONTINUE
      WRITE(2,8)
    8 FORMAT(' ')
      WRITE(2,10)
   10 FORMAT ('1M MATRIX'////)
      DO 14 I=1,N
      WRITE(2,11)
   11 FORMAT (' ')
      WRITE(2, 5)(M(I,J),J=1,N)
   14 CONTINUE
   15 CONTINUE
      IF=1
C     EPS GIVES ACCURACY REQUIRED
      EPS=1.0E-11
      DO 13 I=1,N
      DO 13 J=1,N
   13 A(I,J) = 0.0
C.....INVERT K MATRIX
      CALL F01AAF (K,N,N,A,N,BL,IF)
      IF(IF.EQ.0) GO TO 20
      IF (IF.NE.1) GO TO 18
      WRITE (2,17)
   17 FORMAT (' MATRIX SINGULAR ')
   18 WRITE(2,21)
   21 FORMAT('0NO IMPROVEMENT IN ITERATION')
      GO TO 30
   20 WRITE (2,23)
   23 FORMAT ('0SUCESSFUL INVERSION ')
   30 CONTINUE
      IF (NW.EQ.0) GO TO 32
      WRITE (2,31)
   31 FORMAT ('1INVERTED K MATRIX FOLLOWS')
      WRITE(2,5 ) ((A(I,J),J=1,N),I=1,N)
   32 CONTINUE
      IF (NW.EQ.0) GO TO 40
      WRITE(2,8)
      WRITE(2, 5)((K(I,J),J=1,N),I=1,N)
      WRITE(2,8)
      WRITE(2, 5)(BL(I),I=1,N)
      WRITE(2,8)
   40 CONTINUE
      WRITE(2,19)IF
   19 FORMAT(I5)
C.....COMPLETE UPPER TRIANGLE
C.....MULTIPLY 1/K AND M MATRICES
      IF = 1
      CALL F01CKF (K,A,M,N,N,N,BL,N,1,IF)
      IF (IF.EQ.0) GO TO 46
      WRITE (2,45) IF
   45 FORMAT (1H0,' ERROR DETECTED ',I5)
      STOP
   46 CONTINUE
      WRITE(2,48)
   48 FORMAT(' KM MATRIX OBTAINED')
      IF(NW.EQ.0) GO TO 49
      WRITE (2,47) ((K(I,J),J=1,N),I=1,N)
   47 FORMAT (' KM MATRIX'//(10F12.4))
   49 CONTINUE
C.....BALANCE UNSYMMETRICAL MATRIX
      CALL F01ATF(N,2,K,N,IX,IY,BB)
      WRITE(2,8)
      WRITE(2,51)
   51 FORMAT(' BALANCED MATRIX')
      IF (NW.EQ.0) GO TO 57
      WRITE(2, 5)((K(I,J),J=1,N),I=1,N)
      WRITE (2,5) (BB(I),I=1,N)
      WRITE(2,55)IX,IY
   55 FORMAT(2I10)
   57 CONTINUE
C.....REDUCE TO UPPER HESSENBERG FORM
      CALL F01AKF(N,IX,IY,K,N,Z)
      CALL    F01APF (N,IX,IY,Z,K,N,A,N)
C.....COMPUTE EIGENVALUES AND EIGENVECTORS OF UPPER HESSENBERG MATRIX
      IF = 1
      CALL    F02AQF (N,IX,IY,EPS,K,N,A,N,W,BL,Z,IF)
```

```
          IF(IF,EQ,0) GO TO 71
          IF(IF.NE.1) STOP
          WRITE(2,75)
   75 FORMAT('0MORE THAN 30 ITERATIONS NECESSARY IN ONE SOLUTION')
   71 CONTINUE
          WRITE (2,73) (Z(I),I=1,N)
   73 FORMAT ( ' NO OF ITERATIONS =',I5)
          WRITE(2,8)
          WRITE (2,50) NE
   50 FORMAT(1H1,'NO OF EIGENVALUES COMPUTED =',I4//)
          WRITE (2,60) (W(I),I=1,NE)
   60 FORMAT(1H ,'EIGENVALUES'//(1X,10E12.6) )
          IF (NW.EQ.0) GO TO 61
          WRITE (2,60)(BL(I),I=1,NE)
   61 CONTINUE
          DO 63 I=1,NE
          IF(W(I).LT.0.0) GO TO 62
          GO TO 65
   62 WRITE(2,64)I
   64 FORMAT(' NEGATIVE EIGENVALUE AT ',I5)
          W(I)=-W(I)
   63 W(I)=(SQRT(1./W(I)))/(2.0*3.142))
          WRITE(2,65) (W(I),I=1,NE)
   65 FORMAT (1H ,//,' NATURAL FREQUENCIES  (HERTZ)'//(1X,5F12.4))
C......RANSFORM EIGENVECTORS OF BALANCED MATRIX TO ORIGINAL MATRIX
          CALL   F01AUF(N,IX,IY,N,BB,A,N)
          WRITE(2,80)
   80 FORMAT('1EIGENVECTORS')
   74 FORMAT(9F13.6)
          DO 90 J=1,N
          WRITE(2,11)
          WRITE (2,74)(A(I,J),I=1,N)
   90 CONTINUE
  100 CONTINUE
          REWIND 3
          WRITE (3) IW
          WRITE (3) A
          WRITE (3) W
          STOP
          END
####S
****
/*/*
JOB MUJFEM4,:ESJJS,CP76(T40,P2000)
ATTACH(TAPE3,MUJDAT1  ,FO=ASIS,ST=S6A)
ATTACH( AFILE,LIBNAGFTNLCM,ID=LIBAPPL)
FTN(OPT=2)
LIBRARY(AFILE)
LDSET(PRESET=NGINF,MAP=B/ZZZZMP) .
LGO(PL=4000)
REWIND(TAPE3)
COPY(TAPE3,TAPE9)
CATALOG(TAPE9,MUJDAT1   ,FO=ASIS,ST=S6A)
####S
          PROGRAM SOLVE(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE3)
          LEVEL 2,K,M,A
          LEVEL 2,BB,BL,W,Z
          COMMON/UPPER/K,M,A
          COMMON/LOWER/BB,BL,W,Z
C.....THIS PROGRAM SOLVES EIGENVALUES AND EIGENVECTORS FROM K AND M MATRICES
          DIMENSION BB(146),BL(146),W(146),IW(1K0)
          REAL K(146,146),M(146,146),A(146,146)
          INTEGER Z(146)
          N=146
          NE=146
C.....NW EQUAL ZERO FOR NO MATRIX PRINTOUT
          NW=0
          READ (3) IW
          READ (3) K
          READ (3) M
          REWIND 3
          WRITE (2,4) .
    4 FORMAT(' K MATRIX'///)
          IF (NW.EQ.0) GO TO 15
    5 FORMAT (//( 9F12.4))
          DO 6 I=1,N
          WRITE(2,5)(K(I,J),J=1,N)
    6 CONTINUE
          WRITE(2,8)
    8 FORMAT(' ')
          WRITE(2,10)
   10 FORMAT ('1M MATRIX'///)
          DO 14 I=1,N
          WRITE(2,11)
   11 FORMAT (' ')
          WRITE(2, 5)(M(I,J),J=1,N)
   14 CONTINUE
   15 CONTINUE
          IF=1
          CALL F02AEF(K,N,M,N,N,W,A,N,BL,BB,IF)
          IF(IF.EQ.0) GO TO 20
          IF (IF.NE.1) GO TO 18
          WRITE (2,17)
   17 FORMAT (' MATRIX NOT POSITIVE DEFINITE')
   18 WRITE(2,21)
   21 FORMAT('0NO IMPROVEMENT IN ITERATION')
          GO TO 30
   20 WRITE (2,23)
   23 FORMAT ('0SUCESSFUL COMPUTATION')
```

```
      30 CONTINUE
         WRITE(2,80)
      80 FORMAT('1EIGENVECTORS')
      74 FORMAT(9F13.6)
      70 FORMAT (///' ',I5,' MODE SHAPE CORRESPONDING TO NATFREQ'//F10.3)
         DO 90 J=1,N
         W(J) = (SQRT(W(J)))/(6.284)
         WRITE (2,70) J,W(J)
         WRITE (2,11)
         WRITE (2,74)(A(I,J),I=1,N)
      90 CONTINUE
     100 CONTINUE
         WRITE (2,60) (IW(I),I=1,180)
      60 FORMAT(60I2)
         REWIND 3
         WRITE (3) IW
         WRITE (3) A
         WRITE (3) W
         STOP
         END
####S
****
/*/*
JOB MOJFEM5,:ESJJS,CP76(T24,P2000)
ATTACH(TAPE3,MOJDAT1  ,FO=ASIS;ST=S6A)
FTN.
LGO(PL=3000)
REWIND(TAPE3)
COPY(TAPE3,SCRATCH)
CATALOG(SCRATCH,MOJDAT2  ,FO=CONF,ST=S6A)
####S
         PROGRAM SPDNS(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT,TAPE5)
         LEVEL 2,A,W,IW
         COMMON/UPPER/A,W,IW
C.....THIS PROGRAM READS NATURAL FREQUENCIES AND MODE SHAPES OF BOX
C.....AND OUTPUTS SELECTED ONES ON DISC TO PAPER TAPE
         DIMENSION A(146,146),W(146),IW(180)
         READ(3)IW
         READ(3)A
         READ(3)W
         N=146
         NT=180
       1 FORMAT(' ')
       5 FORMAT(' DEGREES OF FREEDOM REDUCED  OUT '//)
       9 FORMAT(//(9E13.4))
      10 FORMAT(//(9F13.6))
      11 FORMAT(30I4)
      70 FORMAT (///' ',I5,' MODE SHAPE CORRESPONDING TO NATFREQ'//F10.3//)
         WRITE(2,1)
         WRITE(2,1)
         WRITE(2,5)
         WRITE(2,11)(IW(I),I=1,NT)
         WRITE(2,1)
         WRITE(2,10)( W(I),I=1,N)
         DO 20 J=2,21
         WRITE (2,70) J,W(J)
         WRITE(2,9)  (A(I,J),I=1,N)  -
      20 CONTINUE
         REWIND 3
C.....SELECT TWENTY NATURAL FREQUENCIES AND MODES SHAPES OF BOX WITHIN
C.....FREQUENCY RANGE 20 TO 100 HZ AND WRITE TO PAPER TAPE FILE
C.....ONLY CONSIDERING UPPER SURFACE OF BOX
         K=62
         DO 50 J=2,21
         WRITE(3,9) (A(I,J),I=K,N)
      50 CONTINUE
         WRITE (3,10) (W(I),I=2,21)
         WRITE (3,11) (IW(I),I=91,NT)
         STOP
         END
####S


****
JOB MOJFEM6,1905M30B,HENG
DISPLAY '4514 MOJAMAGDATA1 33000512 OUT'
FORTRAN 1,CRO(MOJ6PROG),[TRO   (DAT1)],100,1000
****
         DOCUMENT MOJ6PROG
         PROGRAM(MOJ61905MOJB)
         INPUT 1 = CRO
         OUTPUT 2 = LPO
         INPUT 4 = TRO
         USE 3=  MTO(MOJAMAGDATA1)
         TRACE 2
         END
         MASTER TAPE
         DIMENSION A(85,20),W(20),IW(90)
       9 FORMAT((9E13.4),//)
      10 FORMAT    (9F13.6)
      11 FORMAT(30I4)
         REWIND 5
C.....WRITE FIRST TWENTY NAT FREQ AND MODE SHAPE TO PAPER TAPE
C.....ONLY CONSIDERING UPPER SURFACE OF BOX
         N=85
         K=1
         DO 20 J=1,20
```

```
                             READ (4,9) (A(I,J),I=K,N)
                             WRITE(2,9) (A(1,J),I=K,N)
                    20 CONTINUE
                             WRITE (5) A
                             READ (4 ,10) (W(I),I=1,20)
                             WRITE (5) W
                             WRITE (2,10) (W(I),I=1,20)
                             READ (4 ,11) (IW(I),I= 1,90)
                             WRITE (5) IW
                             WRITE (2,11) (IW(I),I= 1,90)
                             ENDFILE5
                             STOP
                             END
                             FINISH
****

#SWITCH

JOB MOJFEM7,1905M30B,HEYG
DISPLAY '4514 MOJAMAGDATA1 33000512 IM'
FORTRAN 1,CRO(MOJ7PROG),CRO(MOJTDATA), 75,1000
****
        DOCUMENT MOJ7PROG
        LIBRARY (FD,SUBGROUPSRGP)
        LIBRARY(ED,SUBGROUPSRF7)
        PROGRAM(MOJ71905MOJC)
        COMPACT DATA
        INPUT 1=CRO
        OUTPUT 2=LP0
        INPUT 3=MTO(MOJAMAGDATA1)
        NO TRACE
        END
        MASTER RESPONSE
        REAL M(20)
        DIMENSION A(512),SX(512),SY(512)
        DIMENSION IW(90),EN(20),F(20),W(90,20),R(102),X(85,20)
        DIMENSION DL(9),V(9),BCA(14)
        DIMENSION WDF(20),WDFCV(20)
        DATA BCA(1)/27HEXCITATION SPECTRAL DENSITY/
        DATA BCA(5)/18HFREQ UP TO 150 HZ /
        DATA BCA(7)/25HRESPONSE SPECTRAL DENSITY/
        DATA BCA(10)/23H FREQUENCY UP TO 150 HZ/
        REWIND 5
C.....15 TRANSVERSE DEGREES OF FREEDOM ON TOP PLATE
        NTRANS=15
        NMODE=20
        N=90
        READ(3)X
        READ(3)F
        READ(3)IW
        NF=512
C.....CALL PRINP(X,F,IW)
        FMAX=150
        FMIN=FMAX/512.0
        READ (1,10) (EN(I),I=1,NMODE)
   10 FORMAT(10F8.4)
        WRITE(2,10) (F(I),I=1,20)
        WRITE(2,10) (EN(I),I=1,NMODE)
        DM = ((0.78*0.22*0.00061)*7800.0)/FLOAT(NTRANS)
        FQ=0.0
        DO 15 I=1,NF
        A(I)=0.0
   15 CONTINUE
        DO 9 J=1,20
        K=1
        DO 9 I=1,90
        W(I,J)=X(K,J)
        IF(IW(I).EQ.1) GO TO 8
        K=K-1
        W(I,J)=0.0
    8 CONTINUE
        K=K+1
    9 CONTINUE
        WRITE (2,4)
    4 FORMAT(/' DEFLECTIONS IN MODES '/)
        WRITE (2,5) ((W(I,J),I=3,N,6),J=1,20)
    5 FORMAT (//(9E13.5))
C......HNODE=0.5 INDICATESHALFWAY  NODE POINTS TO BE INTERPOLATED
C.....FOR ADDITIONAL SPDNS POINTS
        HNODE=1.0
        HNODE=0.5
        IF(HNODE.GT.0.5) GO TO 18

        CALL INTM(W,WDF,WDFCV)
   18 CONTINUE
C.....INPUT SPECTRAL DENSITY
        CALL INSPDNS(SX,NF,FMIN)
        CALL GRAPH (SX,NF,0.0,SY,BCA,1,R,FMAX)
C.....NB VALUE IS CHANGED AFTER PLOTTING
        CALL INSPDNS(SX,NF,FMIN)
C.....RESPONSE AT DEGREE OF FREEDOM K

        K=39
        K=33
        K=27
        K=21
        K=15
        K=9
        K=3
```

```
                            K=87
                            K=81
                            K=75
                            K=69
                            K=63
                            K=57
                            K=51
                            K=45

      C......PLATE EXCITED AT DEGREE OF FREEDOM L
            L=K
         20 CONTINUE
            WRITE (2,200) K,L
        200 FORMAT (' RESPONSE AT NODE',I5,' TO EXCITATION AT NODE',I5)
      C.....I CONTROLS NO OF NAT FREQ AND MODE SHAPES USED
      C.....J CONTROLS NO OF TRANSVERSE DEGREES OF FREEDOM
            DO 1000 IJ=1,512
            FQ=FMIN*(FLOAT(IJ))
            DO 60 I=1,NMODE
            M(I)=0.0
            DO 50 J=3,N,6
         50 M(I)=M(I)+((W(J,1)**2)*DM)
      C......TO TAKE INTO ACCOUNT THE OTHER 3 SIDES
      C....  M(I)=M(I)*3.0
            M(I)=M(I)*4.0
         60 CONTINUE
      C.....THIS CALCULATES MODULUS OF RECEPTANCE
      C.....COMPUTE SQUARE OF MOD OF RECEPTANCE
            IF( NNODE.GT.0.5) GO TO 80
      C.....FOR ADDITIONAL NODES
            DO 70 I=1,20
            W(L,I)=WDFCV(I)
         70 CONTINUE
         80 CONTINUE

      C.....A(IJ) IS RECEPTANCE SQUARED
            DO 100 I=1,NMODE
        100 A(IJ)=A(IJ)   +((((W(L,I)*W(K,I))**2)/((((M(I)**2)*(
           1(((F(I)**2)-(FQ**2))**2)+((EN(I)**2)*(F(I)**4))))))
            A(IJ)=A(IJ)/(16.0*(3.142**4))
       1000 CONTINUE
      C.....CALL GRAPH2(N,DL,NO,V,W,BCD,BCX,IW,R)
            CALL SPDNS (A,SX,SY,NF)
            CALL GRAPH (SY,NF,50,0,SX,BCA,2,R,FMAX)
            WRITE(2,1001) FMAX
       1001 FORMAT (//' FMAX = ',F10.3)
            STOP
            END
            SUBROUTINE INTM(W,WDF,WDFCV)
            DIMENSION W(90,20)
            DIMENSION WDF(20),WDFCV(20)
            ELENG=0.13
            IST=39
            IST=45
            IST=51
            IST=9
            IST=15
            IST=21
            IST=33
            ISTEP=30
            ISTEP=6
            JST=IST+ISTEP
            IF(ISTEP.EQ.30) GO TO 20
            ICR=IST+2
            JCR=JST+2
            GO TO 30
         20 CONTINUE
            ICR=IST+1
            JCR=JST+1
         30 CONTINUE
            DO 40 I=1,20
            WDF(I)=(W(IST,I)+W(JST,I))/2.0
            WDFCV(I)=WDF(I)+(ELENG*(W(ICR,I)-W(JCR,I))/8.0)
         40 CONTINUE
            WRITE(2,10)(WDF(I),I=1,20)
            WRITE(2,10)(WDFCV(I),I=1,20)
         10 FORMAT(' ',//(12E10.4))
            RETURN
            END

            SUBROUTINE INSPDNS(SX,NF,FMIN)
            DIMENSION SX(NF)
            DO 5 I=1,68
            SX(I)=8.40/(((FLOAT(69))*FMIN)**2)
          5 CONTINUE
            DO 10 I=69,NF
            SX(I)=8.40/(((FLOAT(I ))*FMIN)**2)
         10 CONTINUE
            RETURN
            END

            SHORTLIST
            SUBROUTINE PRINP(X,F,IW)
            DIMENSION X(85,20),F(20),IW(90)
            WRITE(2,500) ((X(I,J),I=1,85), J=1,20)
            WRITE (2,600) (F (I),I=1,20)
            WRITE(2,700)(IW(I),I=1,90)
        500 FORMAT (//(9E13.4))
        600 FORMAT (//(9F13.4))
```

```
  700 FORMAT (5014)
      RETURN
      END

      SUBROUTINE SPDNS (A,SX,SY,N)
      DIMENSION A(N),SX(N),SY(N)
C.....THIS COMPUTES THE SPECTRAL DENSITY OF RESPONSE TO INPUT SPD
      WRITE (2,40) (A(I),I=1,N)
      DO 20 I=1,N
      SY(I) = A(I)*SX(I)
   20 CONTINUE
      WRITE(2,30)
   30 FORMAT (' SPECTRAL DENSITY OF RESPONSE')
      WRITE (2,40) (SY(I),I=1,N)
   40 FORMAT (12E10.3)
      RETURN
      END

      SUBROUTINE GRAPH (B,N,LOG,NSKIP,X,BCD,IJ,LN,FMAX)
C.....B IS A POSITIVE REAL ARRAY
C.....LOG IS SET TO 0 FOR LINEAR GRAPH AND 50 FOR LOG GRAPH
C.....NSKIP IS NO OF ENTERIES TO BE SKIPPED
      INTEGER G
      REAL MAGNEG
      REAL MAGM,MAGN,B(N),LN(102)
      DIMENSION X(N)
      DIMENSION BCD(14)
      DATA BL,ST/1H ,1H*/
      IF (LOG.NE.50.AND.LOG.NE.0) GO TO 100
      GO TO 200
  100 WRITE (2,110)
  110 FORMAT (' NO DB SCALE SPECIFIED')
      STOP

  200 WRITE (2,210) N
  210 FORMAT (' N IS HARMONIC NUMBER =',I6/)
      IF (LOG.EQ.0) GO TO 220
      DO 205 I = 1,N

  205 IF(B(I)) 211,213,213
  211 WRITE (2,212)
  212 FORMAT (' ERROR DETECTED   LOG OF NEGATIVE NUMBER')
      STOP

  213 CONTINUE
      WRITE (2,215) B(1)
  215 FORMAT (' A(C) OMITTED FROM GRAPH. A(0) =',1PZE12.3)
      AUX = B(1)
      B(1) =0.0
C.....PUT THE LARGEST POSITIVE OR LEAST NEGATIVE VALUE EQ MAGM
  220 MAGM = B(1)
  230 DO 290 I=2,N
      MAGN = B(I)
  240 IF (MAGM - MAGN) 250,290,290
  250 MAGM = MAGN
  290 CONTINUE
      IF (LOG.EQ.0) GO TO 500
      WRITE (2,300) MAGM
  300 FORMAT (T90,'ZERO DB =',1PE12.3/)
      E=2.0
      IF(E.EQ.1.0) WRITE(2,310)
      IF(E.EQ.2.0) WRITE(2,311)
  310 FORMAT(2X,'-100',7X,'-90',7X,'-80',7X,'-70',7X,'-60',7X,'-50',7X,
     1'-40',7X,'-30',7X,'-20',7X,'-10',9X,'0')
  311 FORMAT (3X,'-50',7X,'-45',7X,'-40',7X,'-35',7X,'-30',7X,'-25',7X,
     1'-20',7X,'-15',7X,'-10',8X,'-5',9X,'0')
      DO 320 I =1,102
  320 LN(I) = BL

      ALMAGM=1.0E-40
      DO 330 I =2,N,(NSKIP+1)
      F= 10.0*ALOG10(MAGM/(B(I)+ALMAGM))
      B(I)=F
      G= 101.5 - (F*E)
      IF (G.LT.1) GO TO 380
      LN(G) = ST
  380 K = I-1
      WRITE (2,340) K,LN,F
  340 FORMAT (I5,102A1,'= -',F10.3)
      IF (G.GE.1) LN(G) = BL
  330 CONTINUE
      B(1) = AUX
      DO 350 K = 1,N
  350 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
      DO 400 I=1,N
  400 B(I)=-((B(I)*5.0)/100.0)
      CALL HGPLOT (0.0,0.0,1,1)
      CALL HGPLOT (-2.0,2.0,1,4)
      CALL HGPAXIS (0.0,-5.0,BCD(7), 25,5.0,90.0,-100.0,20.0)
      AXIL=FMAX/9.0
      CALL HGPAXIS (0.0,0.0,BCD(10),27, 9.0,0.0,0.0,AXIL)
      CALL HGPLOT (0.0,0.0,3,0)
      N=N-1
      CALL HGPLINE(X,B,N,-1)
      CALL HGPLOT (1.0,1.0,1,2)
      RETURN
  500 CONTINUE
C.....PUT LARGEST NEGATIVE VALUE AS MAGNEG.
      MAGNEG = B(1)
```

```
      DO 296 I = 2,N
      MAGN = B(I)
      IF (MAGNEG - MAGN) 296,296,295
  295 MAGNEG = MAGN
  296 CONTINUE
C.....FIND X AXIS
      M = (INT(((ABS( MAGNEG)/(MAGM-MAGNEG))*100.0)+1.0)))
      IF (MAGNEG.GT.0.0) M = 1.0
      IF (MAGM.LT.0.0) M = 101.0
      WRITE (2,505) MAGNEG,MAGM
  505 FORMAT (' THIS MUST BE NEGATIVE',40X,' THIS MUST BE POSITIVE',/
     1'      ',F10.4,80X,F10.4)
      LN(M) = ST
      IF(M.EQ.1.0) GO TO 512
      DO 510 J=1,M-1
  510 LN(J) = BL
  512 CONTINUE
      IF(M.EQ.101.0) GO TO 514
      DO 511 J=M+1,102
  511 LN(J) = BL
  514 CONTINUE
      IF(IJ.EQ.1) GO TO 521
C.....PLOT ON LINE PRINTER
      DO 525 I=1,N,(NSKIP+1)
      F = B(I)
      IF ( M.NE.1.0) GO TO 513
C.....G IS SCALE FACTOR
      G = ((F/MAGM)*100.0)+1.0
  513 IF(M.NE.101.0) GO TO 515
      G = M-((F/MAGNEG)*100.0)
  515 G = (100.0*((F-MAGNEG)/(MAGM-MAGNEG)))+1.0
      LN(G) = ST
      K = I-1
      WRITE (2,520) LN,K,F
  520 FORMAT (102A1,I4,'=',1PE14.3)
      IF (G.NE.M) LN(G) = BL
  525 CONTINUE
  521 CONTINUE
C     INITIALISE PLOTTER
      CALL HGPLOT(0,0,0,1,1)
C     ASSIGN ORIGIN
      AM =((MAGM/(MAGM-MAGNEG))*5.00)+2.0
      CALL HGPLOT (-2.0,AM,1,4)
      AM=AM-2.0
C     PUT IN X AXIS
C     POSITION PEN AT X AXIS
      CALL HGPLOT (0.0,0.0,3,0)
      AXIL=FMAX/9.0
C     DRAW IN X AXIS
      CALL HGPAXIS (0.0,0.0,BCD(5),-18, 9.0,0.0,0.0,AXIL)
      AAM=(MAGM-MAGNEG)/5.0
      AMM=-5.0+AM
      IF(IJ.EQ.1) GO TO 545
      IF(IJ.EQ.2) GO TO 546
  545 CALL HGPAXIS (0.0,AMM,BCD(1),26,5.0,90.0,MAGNEG,AAM)
      GO TO 547
  546 CALL HGPAXIS (0.0,AMM,BCD(7),26,5.0,90.0,MAGNEG,AAM)
  547 CONTINUE
C     SCALE GRAPH
      DO 550 K = 1,N
  550 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
C     SCALE Y COORDINATES
      IF(MAGM.LT.ABS(MAGNEG)) GO TO 555
      AA=MAGM
      GO TO 560
  555 AA=ABS(MAGNEG)
  560 CONTINUE
      DO 600 I = 1,N
  600 B(I)=(B(I)*AM)/MAGM
C     COMMENCE PLOT
      CALL HGPLINE (X,B,N,1)
C     END PLOT
      CALL HGPLOT (1.0,1.0,1,2)
      CONTINUE
      RETURN
      END

      FINISH
****
DOCUMENT MOJTDATA
    .0180    .0150    .0125    .0105    .0100    .0095    .0091    .0085    .0080    .0076
    .0070    .0067    .0064    .0060    .0057    .0044    .0050    .0040    .0030    .0020
```

Appendix 4

The Fourier Transform computer programs

Tha computer programs MOJFTM1 and MOJFTM2, developed to
analyse sets of 512 and 1024 data respectively, Fourier
transforming from the time to the frequency domain using
the discrete Fourier transform or the fast Fourier trans-
form algorithms.

```
JOB MUJFTM1,1905MUJC,HENG
FORTRAN 1,CRO(MUJFPROG),TRO(MUJGDATA),  75,2500
....
DOCUMENT MUJFPROG
      LIBRARY(ED,SUBGROUPSRF/)
      LIBRARY (ED,SUBGROUPSRGP)
      PROGRAM(MUJF1905MUJC)
      COMPACT
      INPUT 1=TRO
      OUTPUT 2=LPO
      NO TRACE
      END
      MASTER FORTFM
C.....THIS PROGRAM COMPUTES THE FOURIER TRANSFORM OF REAL DATA OF IMPULSE
C.....WHICH IS DIVIDED BY THE FACTOR K
C.....FOR THE RECEPTANCE OF THE COMPLETE SYSTEM WHICH IS THEN SQUARED
C.....THIS IS THEN MULTIPLIED BY THE EXCIT PSD   OF THE PRBS SIGNAL
C.....USED AND FROM IT  PSD OF THE SYSTEM RESPONSE IS OBTAINED
C.....THIS IS IN TURN MULTIPLIED BY THE PICK UP FACTOR SQUARED TO GIVE
C.....RESPONSE OF THE BOX STRUCTURE. N IS NO OF DATA
C.....COMPLEX W( 512)
      REAL MAGM,MAGN      ,LN(102)
      DIMENSION BCD(14)
      DIMENSION C( 512),D( 512)
      INTEGER G
      REAL MAGNEG
      COMPLEX X( 512),A( 512)
      NEX=9
      N=(2**NEX)
      DO 1 I=1,N
    1 READ (1,101,END=2) C(I)
      GO TO 3
    2 DO 4 J=1,N-I+1
    4 C(J+I-1) = 0.0
    3 CONTINUE
      WRITE (2,150) I
  150 FORMAT (//' NO OF DATA INPUT = ',I5)
  101 FORMAT (F5.3/)
      DO 200 J=1,N
  200 X(J)=CMPLX(C(J),0.0)
      WRITE (2,210)N
  210 FORMAT (/' PLOT OF DATA INPUT FOLLOWS. N= ',I6)
      CALL GRAPH (C,N,0,0,D)
      CALL FFT(X,NEX,N,A)
C...  CALL      ROOT (W,N)
C...  CALL      DFT (X,W,A,N)
C.....IMPULSE RESPONSE OF BOX PLOTTED
C.....TABULATE FOURIER COEFFICIENTS A
      WRITE (2,220) A(1)
  220 FORMAT (/'1DFT COEFFICIENTS ARE'///' D.C.TERM =',1P2E11.3)
      WRITE (2,230) (A(J),J=2,N)
  230 FORMAT (/4(1PF14.3,1PE11.3))
C.....RMS VALUE OF PRBS SIGNAL
      PRBS=0.8
      PRBS=0.3
C.....EXPSD IS EXCITATION POWER SPECTRAL DENSITY USING 0.45 CLOCK FREQ
      EXPSD = (PRBS**2)/(0.45*500.0)
C.....PLOT SPECTRAL DENSITY OF OUTPUT FROM BOX
C.....EXPSD = 2 PI FACTORK
      FACTORK=EXPSD/6.284
      DO 250 I=1,N
  250 C(I) = CABS(A(I))/FACTORK
      WRITE (2,251) (C(I),I=1,N)
  251 FORMAT (///' SYSTEM RECEPTANCE',//' ',(10E12.4))
C.....PICK UP FACTOR IN MM PER VOLT   INCLUDING OUTPUT AMPLIFICATION
      PF=2.25/100.0
      DO 252 I=1,N
  252 C(I) =       ((PF*C(I))**2 )*EXPSD
      WRITE (2,255)N
  255 FORMAT (/' BOX RESPONSE FOLLOWS. N = ',I6)
      WRITE (2,275) (C(I),I=1,N)
  275 FORMAT (//' RESPONSE',//' ',(10E12.4))
      CALL GRAPH (C,N,50,0,D)
      STOP
      END
      SUBROUTINE GRAPH (B,N,LOG,NSKIP,X)
C.....B IS A POSITIVE REAL ARRAY
C.....LOG IS SET TO 0 FOR LINEAR GRAPH AND 50 FOR LOG GRAPH
C.....NSKIP IS NO OF ENTERIES TO BE SKIPPED
      INTEGER G
      REAL MAGNEG
      REAL MAGM,MAGN,B(N),LN(102)
      DIMENSION X(N)
      DIMENSION BCD(14)
      DATA BCD(1)/2CHCROSS CORRELATION FUNCTION/
      DATA BCD(5)/10HDELAY BITS/
      DATABCD(7)/23H     RESPONSE DB RE MAGM/
      DATA BCD(10)/37HFREQUENCY IN HERTZ(DC COMPONENT=    )/
      DATA BL,ST/1H ,1H*/
      IF.(LOG.NE.50.AND.LOG.NE.0) GO TO 100
      GO TO 200

  100 WRITE (2,110)
  110 FORMAT (' NO DB SCALE SPECIFIED')
      STOP

  200 WRITE (2,210) N
```

```
210 FORMAT (' N IS HARMONIC NUMBER =',I6/)
    IF (LOG.EQ.0) GO TO 220
    DO 205 I = 1,N

205 IF(B(I)) 211,213,213
211 WRITE (2,212)
212 FORMAT (' ERROR DETECTED   LOG OF NEGATIVE NUMBER')
    STOP

213 CONTINUE
    WRITE (2,215) B(1)
215 FORMAT (' A(0) OMITTED FROM GRAPH. A(0) =',1P2E12.5)
    AUX = B(1)
    B(1) =0.0
C.....PUT THE LARGEST POSITIVE OR LEAST NEGATIVE VALUE EQ MAGM
220 MAGM = B(1)
230 DO 290 I=2,N
    MAGN = B(I)
240 IF (MAGM - MAGN) 250,290,290
250 MAGM = MAGN
290 CONTINUE
    IF (LOG.EQ.0) GO TO 500
    WRITE (2,300) MAGM
300 FORMAT (T90,'ZERO DB =',1PE12.5/)
    E = 2.0
    IF(E.EQ.1.0) WRITE(2,310)
    IF(E.EQ.2.0) WRITE(2,311)
310 FORMAT(2X,'-100',7X,'-90',7X,'-80',7X,'-70',7X,'-60',7X,'-50',7X,
   1'-40',7X,'-30',7X,'-20',7X,'-10',9X,'0')
311 FORMAT (3X,'-50',7X,'-45',7X,'-40',7X,'-35',7X,'-30',7X,'-25',7X,
   1'-20',7X,'-15',7X,'-10',8X,'-5',9X,'0')
    DO 320 I =1,102
320 LN(I) = BL
    ALMAGM=1.0E-40
    DO 330 I =2,N,(NSKIP+1)
    F= 10.0*ALOG10(MAGM/(B(I)+ALMAGM))
    G= 101.5 - (F*E)
    IF (G.LT.1) GO TO 380
    LN(G) = ST
380 K = I-1
    WRITE (2,340) K,LN,K,F
340 FORMAT (I5,102A1,I4,'=-',F7.2)
    IF (G.GE.1) LN(G) = BL
330 CONTINUE
    B(1) = AUX
    DO 350 K = 1,N
350 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
    DO 400 I = 1,N
400 B(I) =-5.0*(10.0*ALOG10(MAGM/(B(I)+1.0E-20)))/100.0)
    CALL HGPLOT (0.0,0.0,1,1)
    CALL HGPLOT (-2.0,2.0,1,4)
    CALL HGPAXIS (0.0,-5.0,BCD(7), 25,5.0,90.0,-100.0,10.0)
    AXISL = N/ 9.0
    CALL HGPAXIS (0.0,0.0,BCD(10),37, 9.0,0.0,0.0,AXISL)
    CALL HGPLOT (0.0,0.0,3,0)
    CALL HGPLINE (X,B,N,-1)
    CALL HGPLOT (1.0,1.0,1,2)
    RETURN
500 CONTINUE
C.....PUT LARGEST NEGATIVE VALUE AS MAGNEG.
    MAGNEG = B(1)
    DO 296 I = 2,N
    MAGN = B(I)
    IF (MAGNEG - MAGN) 296,296,295
295 MAGNEG = MAGN
296 CONTINUE
C.....FIND X AXIS
    M = (INT(((ABS( MAGNEG)/(MAGM-MAGNEG))*100.0)+1.0))
    IF (MAGNEG.GT.0.0) M = 1.0
    IF (MAGM.LT.0.0) M = 101.0
    WRITE (2,505) MAGNEG,MAGM
505 FORMAT (' THIS MUST BE NEGATIVE',40X,' THIS MUST BE POSITIVE',/
   1'   ',F10.4,80X,F10.4)
    LN(M) = ST
    IF (M.EQ.1.0) GO TO 512
    DO 510 J=1,M-1
510 LN(J) = BL
512 CONTINUE
    IF(M.EQ.101.0) GO TO 514
    DO 511 J=M+1,102
511 LN(J) = BL
514 CONTINUE
    DO 525 I=1,N,(NSKIP+1)
    F = B(I)
    IF ( M.NE.1.0) GO TO 513
    G = ((F/MAGM)*100.0)+1.0
513 IF(M.NE.101.0) GO TO 515
    G = M-((F/MAGNEG)*100.0)
515 G = (100.0*((F-MAGNEG)/(MAGM-MAGNEG)))+1.0
    LN(G) = ST
    K = I-1
    WRITE (2,520) LN,K,F
520 FORMAT (102A1,I4,'=',1PE14.3)
    IF (G.NE.M) LN(G) = BL
525 CONTINUE
C     INITIALISE PLOTTER
    CALL HGPLOT(0.0,0.0,1,1)
C     ASSIGN ORIGIN
    AM =((MAGM/(MAGM-MAGNEG))*5.00)+2.0
```

```
                CALL HGPLOT (-2.0,AM,1,4)
                AM=AM-2.0
C               PUT IN X AXIS
                AXISL = N/ 9.0
C               POSITION PEN AT X AXIS
                CALL HGPLOT (0.0,0.0,5,0)
C               DRAW IN X AXIS
                CALL HGPAXIS (0.0,0.0,BCD(5),-10, 9.0,0.0,0.0,AXISL)
                AAM=(MAGM-MAGNEG)/5.0
                AMM=-5.0+AM
                CALL HGPAXIS (0.0,AMM,BCD(7),26,5.0,90.0,MAGNEG,AAM)
C               SCALE GRAPH
                DO 550 K = 1,N
    550 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
C               SCALE Y COORDINATES
                IF(MAGM.LT.ABS(MAGNEG)) GO TO 555
                AA=MAGM
                GO TO 560
    555 AA=ABS(MAGNEG)
    560 CONTINUE
                DO 600 I = 1,N
    600 B(I)=(B(I)*AM)/MAGM
C               COMMENCE PLOT
                CALL HGPLINE (X,B,N,1)
C               END PLOT
                CALL HGPLOT (1.0,1.0,1,2)
                CONTINUE
                RETURN
                END
                SUBROUTINE FFT(A,N,NB,B)
                COMPLEX A(NB),U,W,T,B(NB)
                NBD2=NB/2
                NBM1=NB-1
                J=1
                DO 4 L=1,NBM1
                IF(L.GE.J) GO TO 2
                T=A(J)
                A(J)=A(L)
                A(L)=T
      2 K=NBD2
      3 IF(K.GE.J) GO TO 4
                J=J-K
                K=K/2
                GO TO 3
      4 J=J+K
                PI=3.14159265359
                DO 6 M=1,N
                U=(1.0,0.0)
                ME=2**M
                K=ME/2
                W=CMPLX(COS(PI/K),SIN(PI/K))
                DO 6 J=1,K
                DO 5 L=J,NB,ME
                LPK=L+K
                T=A(LPK)*U
                A(LPK)=A(L)-T
      5 A(L)=A(L)+T
      6 U=U*W
                DO 10 I=1,NB
                B(I)=A(I)/(FLOAT(NB))
     10 CONTINUE
                RETURN
                END
                FINISH
****

#SWITCH
```

```
JOB MOJFTM2,1905MOJC,HENG
FORTRAN 1,CRD(MOJFPROG),TRD(MOJGDATA),200,4500
****
DOCUMENT MOJFPROG
      LIBRARY(ED,SUBGROUPSRF7)
      LIBRARY (ED,SUBGROUPSRGP)
      PROGRAM(MOJF1905MOJC)

      COMPACT
      INPUT 1=TRD
      OUTPUT 2=LPO
      NO TRACE
      END
      MASTER FORTFH
C.....THIS PROGRAM COMPUTES THE FOURIER TRANSFORM OF REAL DATA OF IMPULSE
C.....WHICH IS DIVIDED BY THE FACTOR K
C.....FOR THE RECEPTANCE OF THE COMPLETE SYSTEM WHICH IS THEN SQUARED
C.....THIS IS THEN MULTIPLIED BY THE EXCIT PSD    OF THE PRBS SIGNAL
C.....USED AND FROM IT  PSD OF THE SYSTEM RESPONSE IS OBTAINED
C.....THIS IS IN TURN MULTIPLIED BY THE PICK UP FACTOR SQUARED TO GIVE
C.....RESPONSE OF THE BOX STRUCTURE. N IS NO OF DATA
C.....COMPLEX W(1024)
      DIMENSION BCD(14)
      DIMENSION C(1024),D(1024)
      COMPLEX X(1024),A(1024)
      INTEGER G
      REAL MAGNEG
      REAL MAGM,MAGN,          LN(102)
      NEX=10
      N=(2**NEX)
      DO 1 I=1,N
    1 READ (1,101,END=2) C(I)
      GO TO 3
    2 DO 4 J=1,N-I+1
    4 C(J+I-1) = 0.0
    3 CONTINUE
      WRITE (2,150) I
  150 FORMAT (///' NO OF DATA INPUT = ',I5)
  101 FORMAT (F5.3/)
      DO 200 J=1,N
  200 X(J)=CMPLX(C(J),0.0)
      WRITE (2,210)N
  210 FORMAT (/' PLOT OF DATA INPUT FOLLOWS. N= ',I6)
      CALL GRAPH (C,N,0,0,D)
      CALL FFT(X,NEX,N,A)
C...  CALL        ROOT (W,N)
C...  CALL        DFT (X,W,A,N)
C.....IMPULSE RESPONSE OF BOX PLOTTED
C.....TABULATE FOURIER COEFFICIENTS A
      WRITE (2,220) A(1)
  220 FORMAT (/'1DFT COEFFICIENTS ARE'///' D.C.TERM =',1P2E11.3)
      WRITE (2,230) (A(J),J=2,N)
  230 FORMAT (/4(1PF14.3,1PE11.3))
C.....RMS VALUE OF PRBS SIGNAL
      PRBS=0.3
C.....EXPSD IS EXCIT POWER SPECTRAL DENSITY USING 0.45 CLOCK FREQUENCY
      EXPSD =(PRBS**2 )/(0.45*300.0)
C.....EXPSD = 2 PI FACTORK
      FACTORK=EXPSD/6.284
      DO 250 I=1,N
  250 C(I) = CABS(A(I))/FACTORK
      WRITE (2,251) (C(I),I=1,N)
  251 FORMAT (///' SYSTEM RECEPTANCE',//' ',(10E12.4))
C.....PICK UP FACTOR IN MM PER VOLT, INCLUDING OUTPUT AMPLIFICATION
      PF=2.25/31.6
      DO 252 I=1,N
  252 C(I) = ((     PF*C(I))**2 )*EXPSD
C.....PLOT SPECTRAL DENSITY OF OUTPUT FROM BOX
      WRITE (2,255)N
  255 FORMAT (/' BOX RESPONSE FOLLOWS. N = ',I6)
      WRITE (2,275) (C(I),I=1,N)
  275 FORMAT (///' RESPONSE',//' ',(10E12.4))
      CALL GRAPH (C,N,50,0,D)
      STOP
      END


      SUBROUTINE DFT (X,W,A,N)
      COMPLEX X(N),W(N),A(N)
      A(1)=(0.0,0.0)
      DO 10 J=1,N
   10 A(1)=A(1)+X(J)
      DO 20 L=1,(N-1)
      A(L+1)=(0.0,0.0)
      DO 20 M=1,N
      IAR =(((M-1)*L)+1)
      INDEX = IAR-(N*INT((FLOAT(IAR)-0.01)/(FLOAT(N))))
   20 A(L+1)=A(L+1)+X(M)*W(INDEX)
      DO 30 J=1,N
   30 A(J)=A(J)/N
      RETURN
      END

      SUBROUTINE ROOT (W,N)
      COMPLEX W(N)
C......CALCULATE THE COMMON ROOT W(J) WHERE W(J)= CEXP((-2I)*PI*J/N)
```

```
          W(1)=(1.0,0.0)
          C=N
          W(2)=CEXP(CMPLX(0.0,-6.2832/C))
          DO 50 J=3,N
          W(J) =W(J-1)*W(2)
   50 CONTINUE
          RETURN
          END

          SUBROUTINE GRAPH (B,N,LOG,NSKIP,X)
C.....B IS A POSITIVE REAL ARRAY
C.....LOG IS SET TO 0 FOR LINEAR GRAPH AND 50 FOR LOG GRAPH
C.....NSKIP IS NO OF ENTERIES TO BE SKIPPED
          INTEGER G
          REAL MAGNEG
          REAL MAGM,MAGN,B(N),LN(102)
          DIMENSION X(N)
          DIMENSION BCD(14)
          DATA BCD(1)/26HCROSS CORRELATION FUNCTION/
          DATA BCD(5)/10HDELAY BITS/
          DATABCD(7)/23H      RESPONSE DB RE MAGM/
          DATA BCD(10)/37HFREQUENCY IN HERTZ(DC COMPONENT=    )/
          DATA BL,ST/1H ,1H*/
          IF (LOG.NE.50.AND.LOG.NE.0) GO TO 100
          GO TO 200

  100 WRITE (2,110)
  110 FORMAT (' NO DB SCALE SPECIFIED')
          STOP

  200 WRITE (2,210) N
  210 FORMAT (' N IS HARMONIC NUMBER =',I6/)
          IF (LOG.EQ.0) GO TO 220
          DO 205 I = 1,N

  205 IF(B(I)) 211,213,213
  211 WRITE (2,212)
  212 FORMAT (' ERROR DETECTED    LOG OF NEGATIVE NUMBER')
          STOP

  213 CONTINUE
          WRITE (2,215) B(1)
  215 FORMAT (' A(0) OMITTED FROM GRAPH. A(0) =',1P2E12.3)
          AUX = B(1)
          B(1) =0.0
C.....PUT THE LARGEST POSITIVE OR LEAST NEGATIVE VALUE EQ MAGM
  220 MAGM = B(1)
  230 DO 290 I=2,N
          MAGN = B(I)
  240 IF (MAGM - MAGN) 250,290,290
  250 MAGM = MAGN
  290 CONTINUE
          IF (LOG.EQ.0) GO TO 500
          WRITE (2,300) MAGM
  300 FORMAT (1Y0,'ZERO DB =',1PE12.3/)
          E = 2.0
          IF(E.EQ.1.0) WRITE(2,310)
          IF(E.EQ.2.0) WRITE(2,311)
  310 FORMAT(2X,'-100',7X,'-90',7X,'-80',7X,'-70',7X,'-60',7X,'-50',7X,
      1'-40',7X,'-30',7X,'-20',7X,'-10',9X,'0')
  311 FORMAT (3X,'-50',7X,'-45',7X,'-40',7X,'-35',7X,'-30',7X,'-25',7X,
      1'-20',7X,'-15',7X,'-10',8X,'-5',9X,'0')
          DO 320 I =1,102
  320 LN(I) = BL
          ALMAGM=1.0E-40
          DO 330 I =2,N,(NSKIP+1)
          F= 10.0*ALOG10(MAGM/(B(I)+ALMAGM))
          G= 101.5 - (F*E)
          IF (G.LT.1) GO TO 380
          LN(G) = ST
  380 K = I-1
          WRITE (2,340) K,LN,K,F
  340 FORMAT (I5,102A1,I4,'=',F7.2)
          IF (G.GE.1) LN(G) = BL
  330 CONTINUE
          B(1) = AUX
          DO 350 K = 1,N
  350 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
          DO 400 I = 1,N
  400 B(I) =-5.0*(10.0*ALOG10(MAGM/(B(I)+1.0E-20))/100.0)
          CALL HGPLOT (0.0,0.0,1,1)
          CALL HGPLOT (-2.0,2.0,1,4)
          CALL HGPAXIS (0.0,-5.0,BCD(7), 25,5.0,90.0,-100.0,20.0)
          AXISL = N/ 9.0
          CALL HGPAXIS (0.0,0.0,BCD(10),37, 9.0,0.0,0.0,AXISL)
          CALL HGPLOT (0.0,0.0,3,0)
          CALL HGPLINE (X,B,N,-1)
          CALL HGPLOT (1.0,1.0,1,2)
          RETURN
  500 CONTINUE
C.....PUT LARGEST NEGATIVE VALUE AS MAGNEG.
          MAGNEG = B(1)
          DO 296 I = 2,N
          MAGN = B(I)
          IF (MAGNEG - MAGN) 296,296,295
  295 MAGNEG = MAGN
  296 CONTINUE
C.....FIND X AXIS
          M = (INT(((ABS( MAGNEG)/(MAGM-MAGNEG))*100.0)+1.0))
```
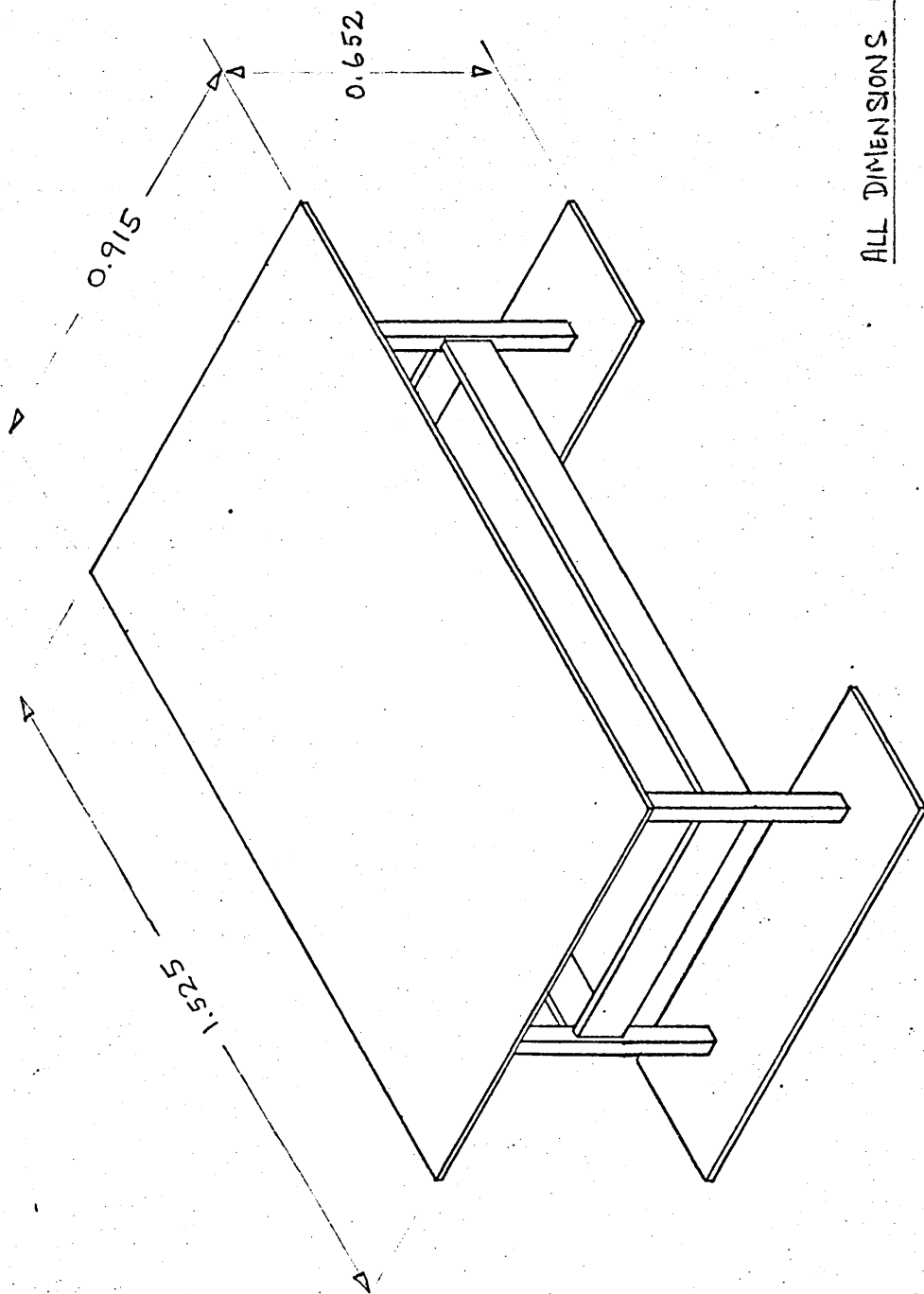
```
      IF (MAGNEG.GT.0.0) M = 1.0
      IF (MAGM.LT.0.0) M = 101.0
      WRITE (2,505) MAGNEG,MAGM
505 FORMAT (' THIS MUST BE NEGATIVE',40X,' THIS MUST BE POSITIVE'/
   1'      ',F10.4,80X,F10.4)
      LN(M) = ST
      IF (M.EQ.1.0) GO TO 512
      DO 510 J=1,M-1
510 LN(J) = BL
512 CONTINUE
      IF(M.EQ.101.0) GO TO 514
      DO 511 J=M+1,102
511 LN(J) = BL
514 CONTINUE
      DO 525 I=1,N,(NSKIP+1)
      F = B(I)
      IF ( M.NE.1.0) GO TO 513
      G = ((F/MAGM)*100.0)+1.0
513 IF(M.NE.101.0) GO TO 515
      G = M-((F/MAGNEG)*100.0)
515 G = (100.0*((F-MAGNEG)/(MAGM-MAGNEG)))+1.0
      LN(G) = ST
      K = I-1
      WRITE (2,520) LN,K,F
520 FORMAT (102A1,I4,'=',1PE14.3)
      IF (G.NE.M) LN(G) = BL
525 CONTINUE
C     INITIALISE PLOTTER
      CALL HGPLOT(0.0,0.0,1,1)
C     ASSIGN ORIGIN
      AM =((MAGM/(MAGM-MAGNEG))*5.00)+2.0
      CALL HGPLOT (-2.0,AM,1,4)
      AM=AM-2.0
C     PUT IN X AXIS
      AXISL = N/ 9.0
C     POSITION PEN AT X AXIS
      CALL HGPLOT (0.0,0.0,3,0)
C     DRAW IN X AXIS
      CALL HGPAXIS (0.0,0.0,BCD(5),-10, 9.0,0.0,0.0,AXISL)
      AAM=(MAGM-MAGNEG)/5.0
      AMM=-5.0+AM
      CALL HGPAXIS (0.0,AMM,BCD(1),26,5.0,90.0,MAGNEG,AAM)
C     SCALE GRAPH
      DO 550 K = 1,N
550 X(K) = ((FLOAT(K-1))* 9.0)/(FLOAT(N))
C     SCALE Y COORDINATES
      IF(MAGM.LT.ABS(MAGNEG)) GO TO 555
      AA=MAGM
      GO TO 560
555 AA=ABS(MAGNEG)
560 CONTINUE
      DO 600 I = 1,N
600 B(I)=(B(I)*AM)/MAGM
C     COMMENCE PLOT
      CALL HGPLINE (X,B,N,1)
C     END PLOT
      CALL HGPLOT (1.0,1.0,1,2)
      CONTINUE
      RETURN
      END

      SUBROUTINE FFT(A,N,NB,B)
      COMPLEX A(NB),U,W,T,B(NB)
      NBD2=NB/2
      NBM1=NB-1
      J=1
      DO 4 L=1,NBM1
      IF(L.GE.J) GO TO 2
      T=A(J)
      A(J)=A(L)
      A(L)=T
2     K=NBD2
3     IF(K.GE.J) GO TO 4
      J=J-K
      K=K/2
      GO TO 3
4     J=J+K
      PI=3.14159265359
      DO 6 M=1,N
      U=(1.0,0.0)
      ME=2**M
      K=ME/2
      W=CMPLX(COS(PI/K),SIN(PI/K))
      DO 6 J=1,K
      DO 5 L=J,NB,ME
      LPK=L+K
      T=A(LPK)*U
      A(LPK)=A(L)-T
5     A(L)=A(L)+T
6     U=U*W
      DO 10 I=1,NB
      B(I)=A(I)/(FLOAT(NB))
10    CONTINUE
      RETURN
      END
      FINISH
****
#SWITCH
```
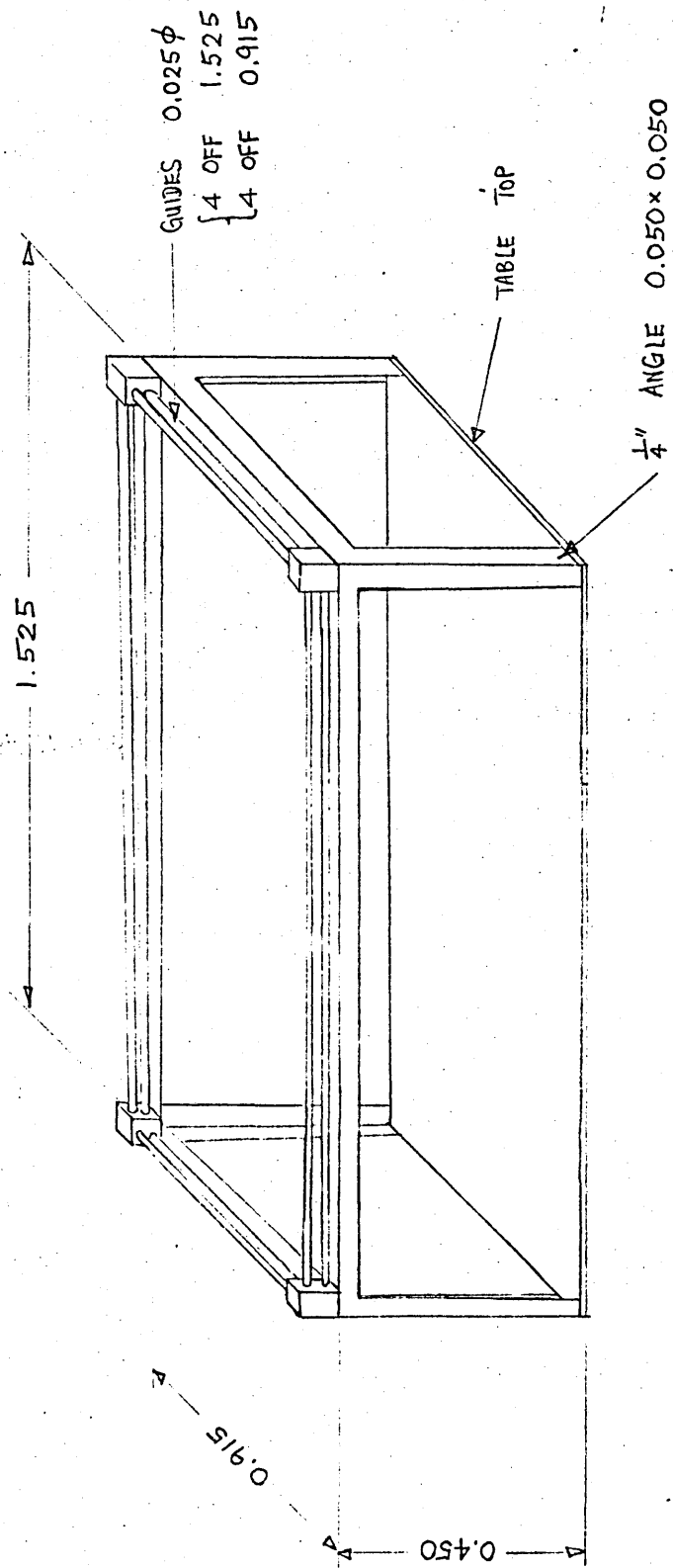
Appendix 5

<u>Diagrams and illustrations of the experimental apparatus</u>
<u>developed</u>

0.652

0.915

1.525

ALL DIMENSIONS IN METRES

FIG A.5.1.   THE VIBRATION TABLE

GUIDES 0.025 φ
{ 4 OFF 1.525
{ 4 OFF 0.915

TABLE TOP

¼" ANGLE 0.050 × 0.050

1.525

0.915

0.450

ALL DIMENSIONS IN METRES

FIG A.5.2.   FRAMEWORK FOR TRANSVERSE GEAR

0.022 DIA

SCREW 10 T.P.I.

0.031 DIA.

0.05

0.250

1.300 *

1.700 *

0.150

* FOR OTHER ROD RESPECTIVELY { 0.700 1.100

ALL DIMENSIONS IN METRES
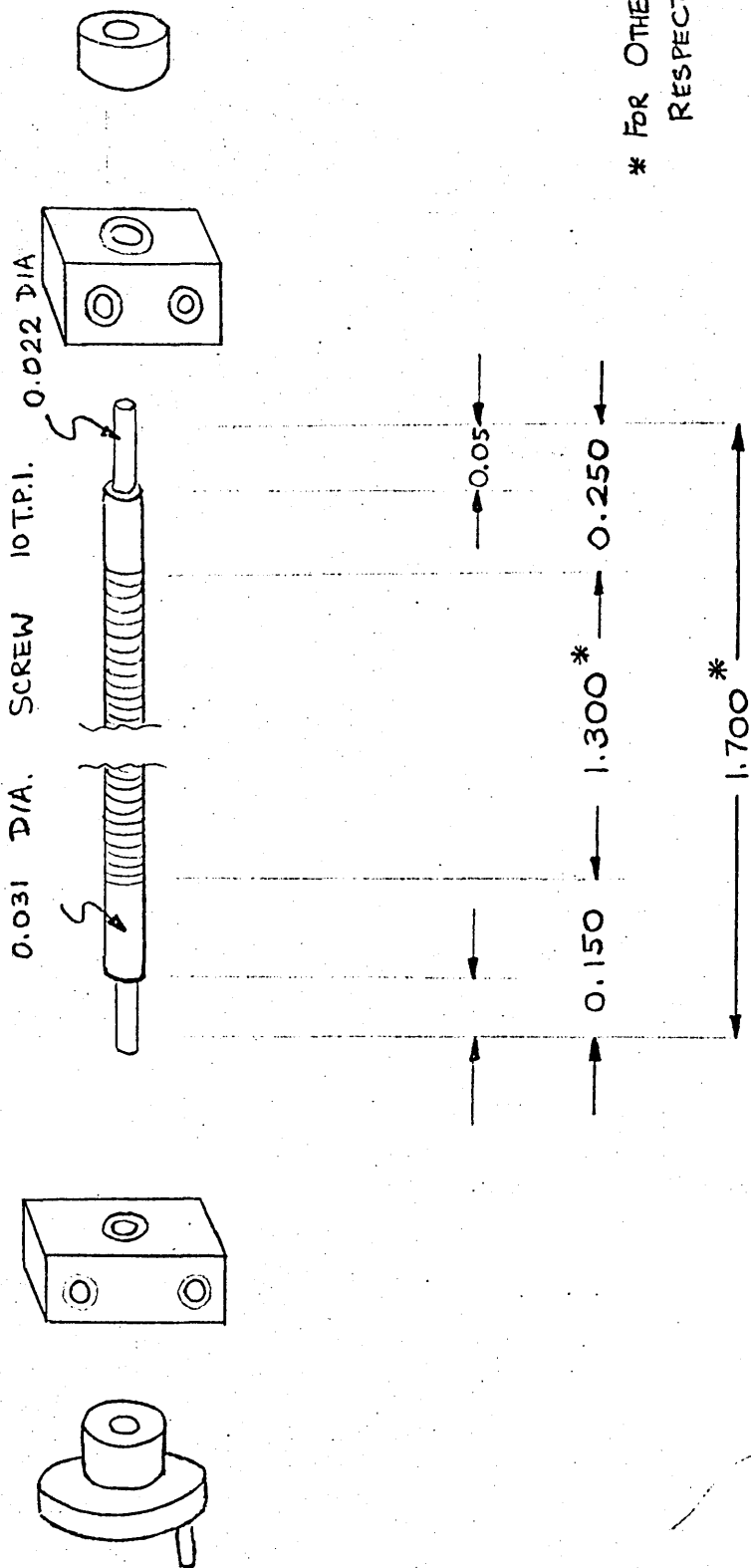
FIG. A.5.3   LEADSCREW + ASSEMBLY   (2 OFF)

0.030

0.040

0.025

GRUB SCREW

0.050

0.080

0.070

0.160

0.200
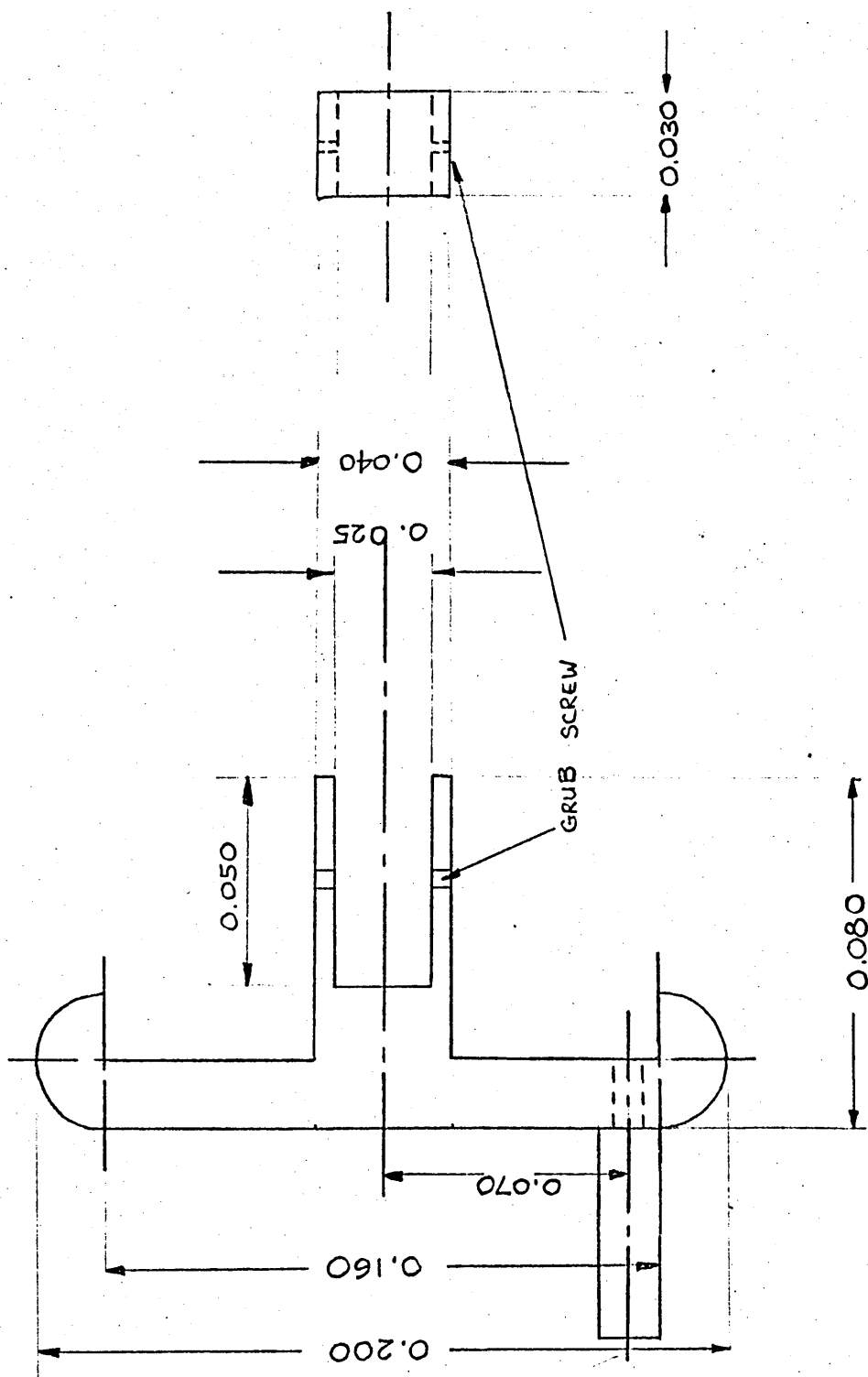
FIG. A.5.4. HANDWHEEL AND ENDSTOP (2 OFF EACH)

SCALE 1:2
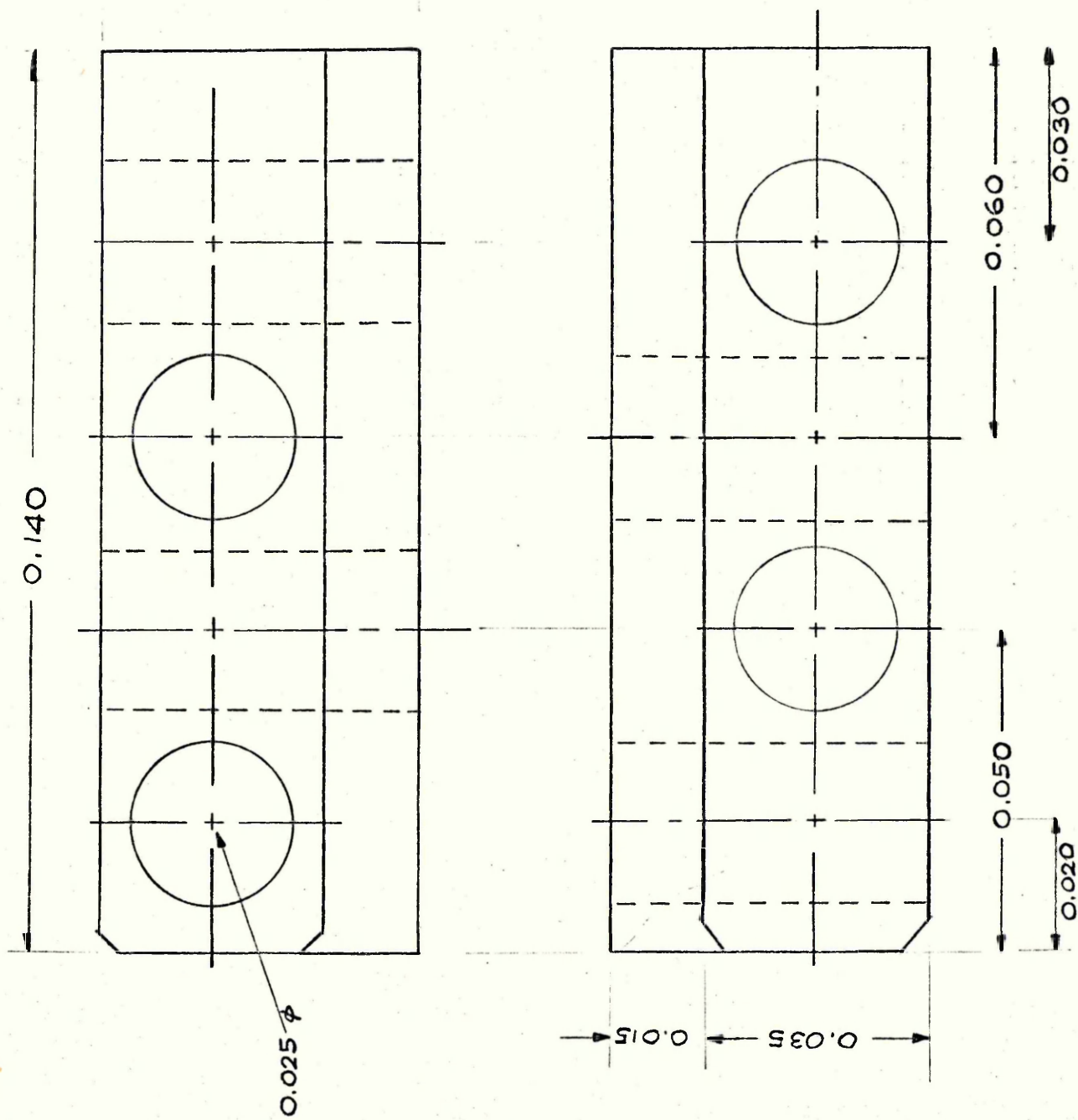
FIG. A.5.5. CORNER SUPPORTS (4 OFF)

FULL SCALE

0.140

0.050

0.025 ∅

0.050

0.015

0.060

0.030

0.050

0.020

0.015

0.035

0.035

0.055

FULL SCALE

0.035

(BALL-
BEARING.)

0.040

BORE TO 0.028 DIA.
BUSH UP TO 0.025 DIA.
(RECYCLING BALL-BEARINGS.)

0.100

DRILL +
TAP

0.022 φ

0.025 φ

0.020

0.050
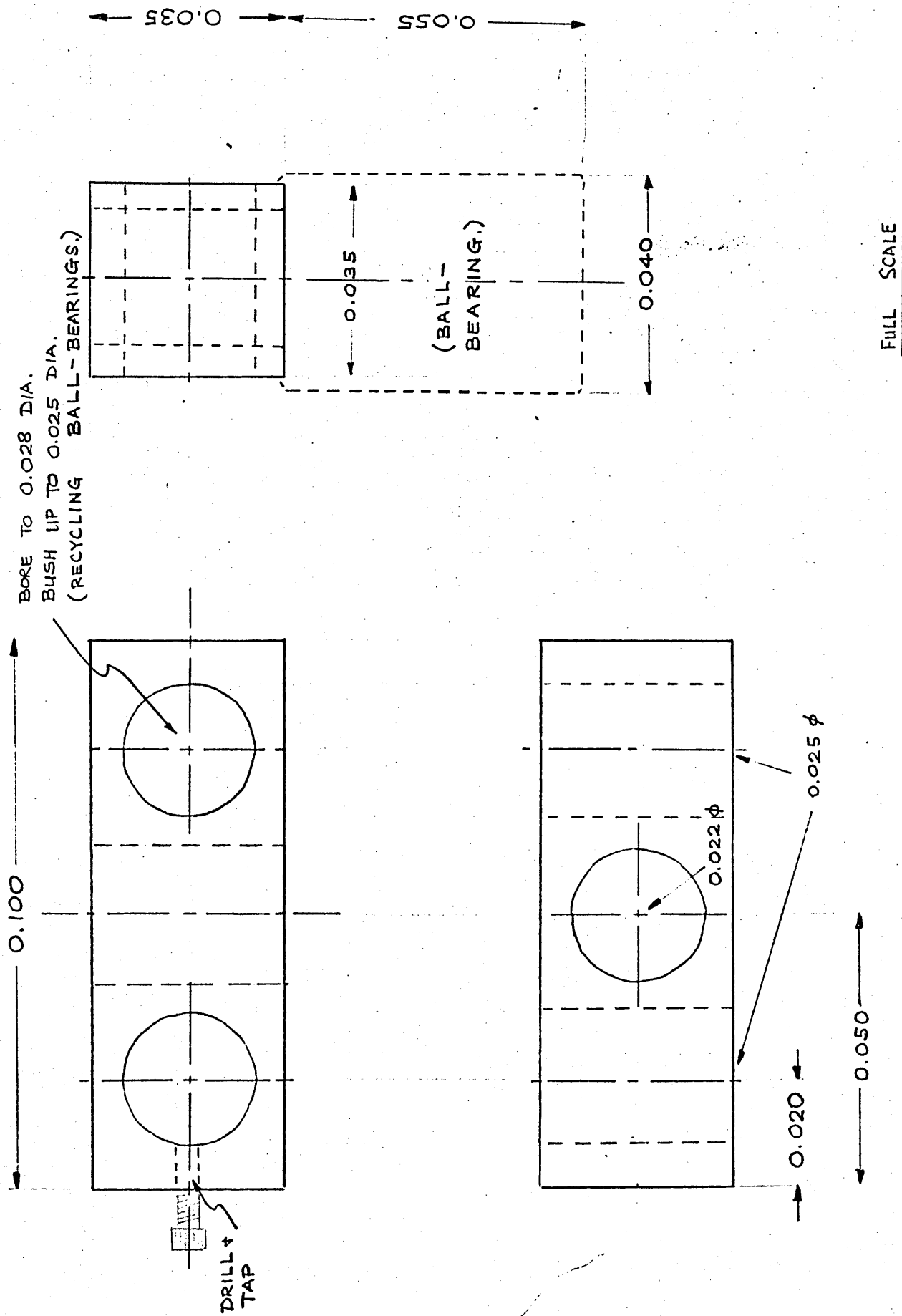
FIG A.5.6.   THE BEARINGS   (2 OFF)

0.025φ THREADED

0.019 φ

0.015 ← 0.030 → 0.015

0.015 → 0.012

0.052

FIG. A.5.7. THE PROBE HOLDER.

TOTAL TRAVEL = 0.25 $^{mm}/t$

INNER THREAD 0.75 $^{mm}/t$

OUTER THREAD
1.0$^{mm}/t$

0.095

0.10

PROBE

0.02063

0.04

0.05

0.08
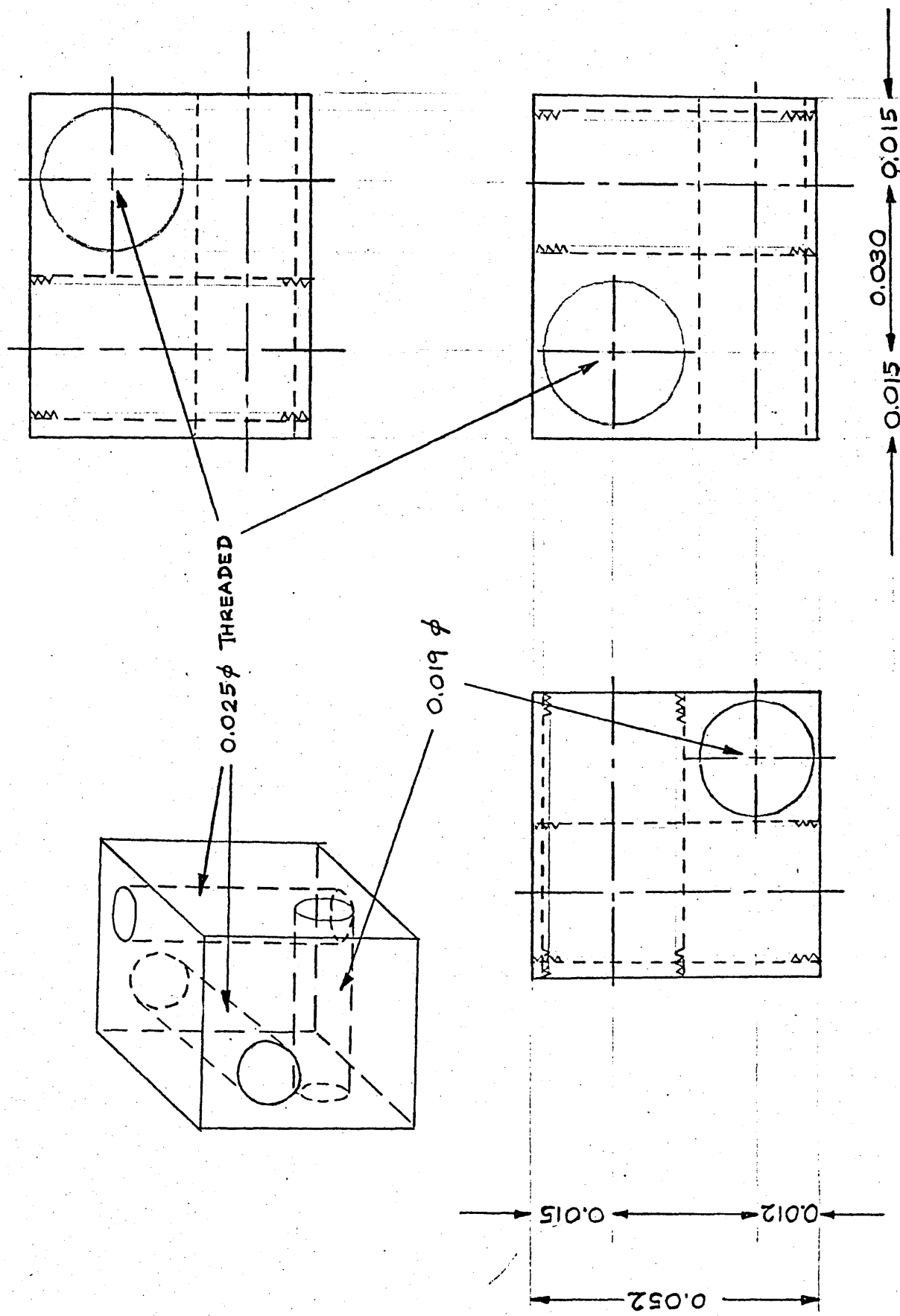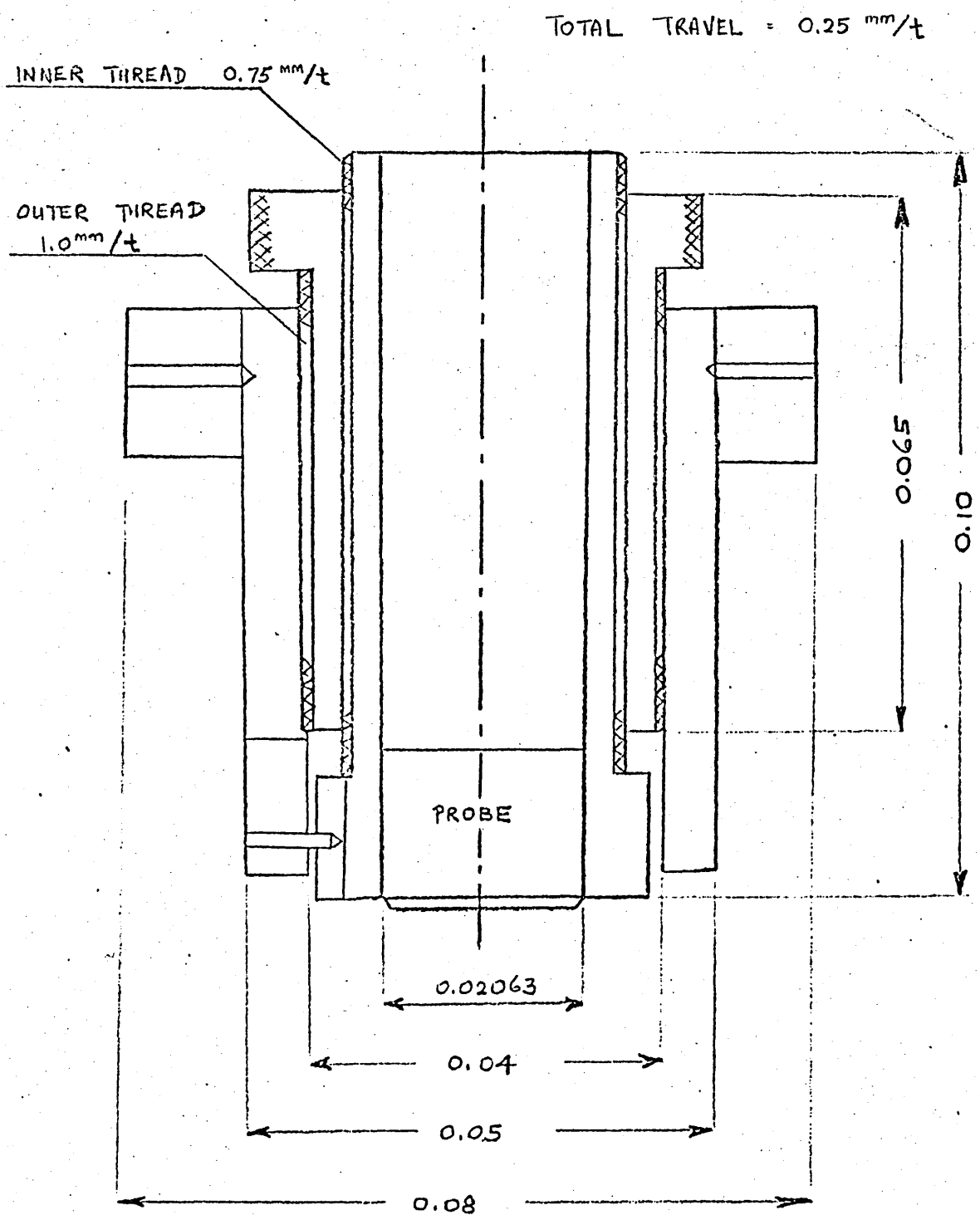
FIG. A.5.8. FINE ADJUSTMENT MECHANISM (GAP SIZE)

3 off.

20
5   5

10
10
15
15

clearance

6 m/m ø THD.
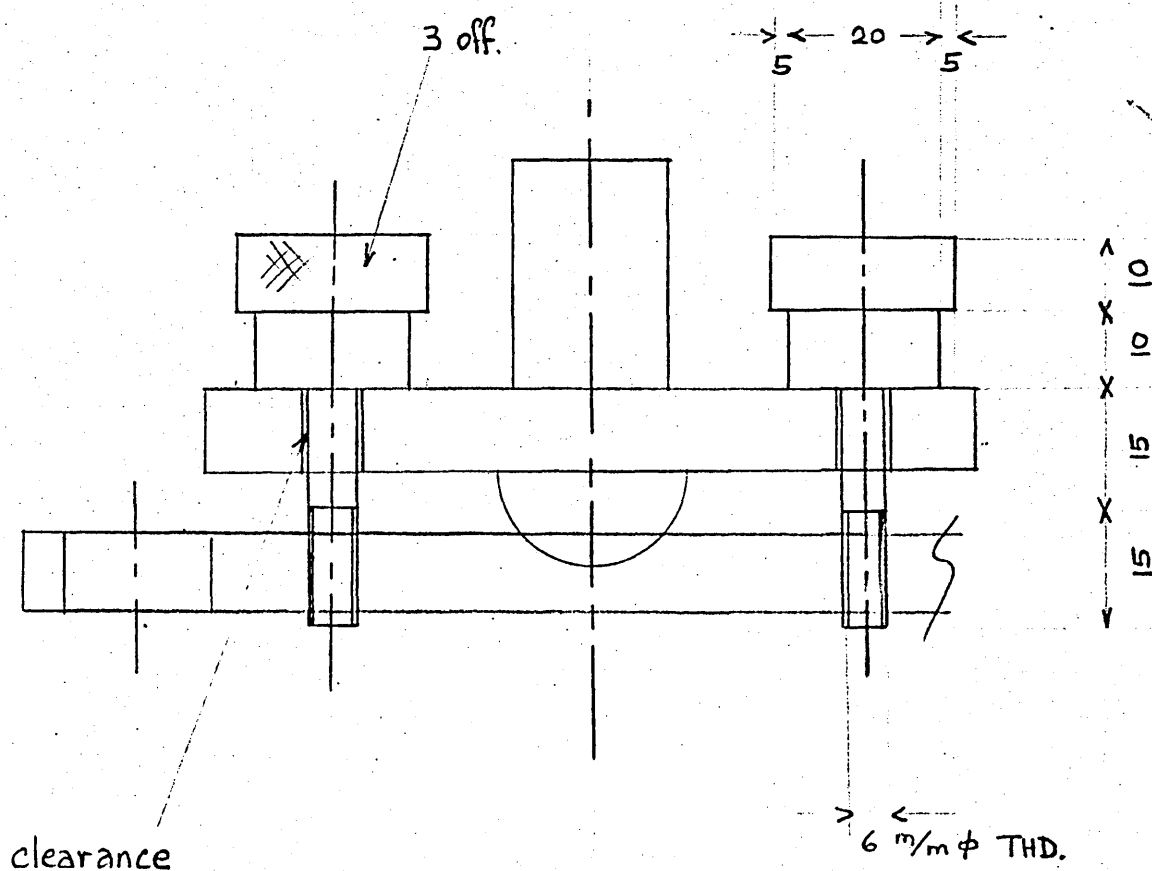
all dimensions in m/m.

FIG. A.5.9.   FINE ADJUSTMENT MECHANISM (TILT)

5 m/m drill & tap.

75   35   25   15

20 m/m clearance

10

20

30

10

10

3 holes 6 m/m drill & tap.

12.5 rad.

7.5

10

150 m/m

all dimensions in m/m
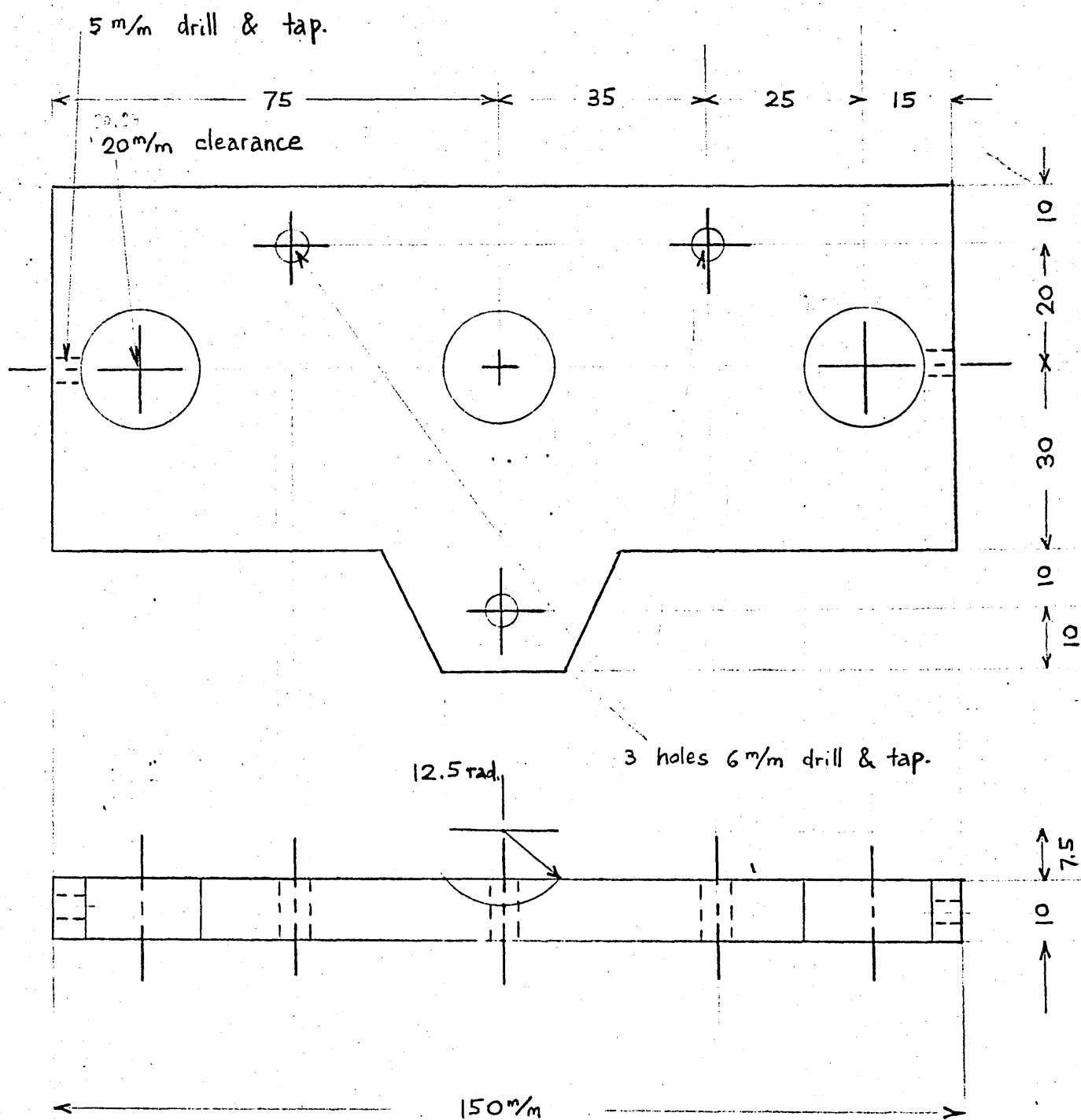
FIG. A.5.10. FINE ADJUSTMENT MECHANISM (BASE PLATE)

40

7 dia.

21

30

10

12.5

12.5 rad.

40

50
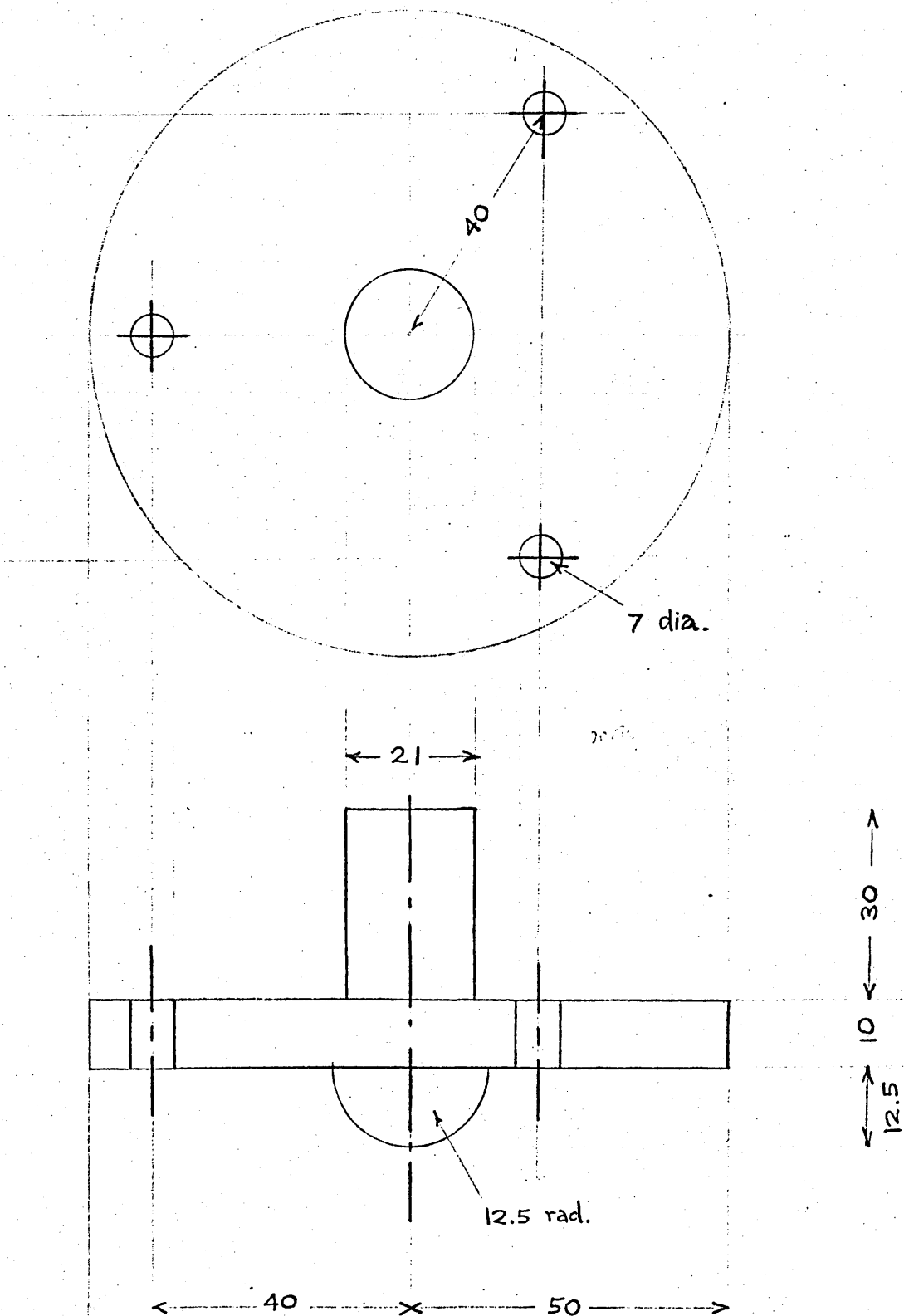
FIG A.5.11. FINE ADJUSTMENT MECHANISM (TOP PLATE)

all dimensions in m/m.

ALL DIMENSIONS IN MM

PROBE (OR REFERENCE PLATE)

M5 GRUB SCREW

13∅

8

25 ∅/D

20

6

M3 THREAD

M5 GRUB SCREW

8

20

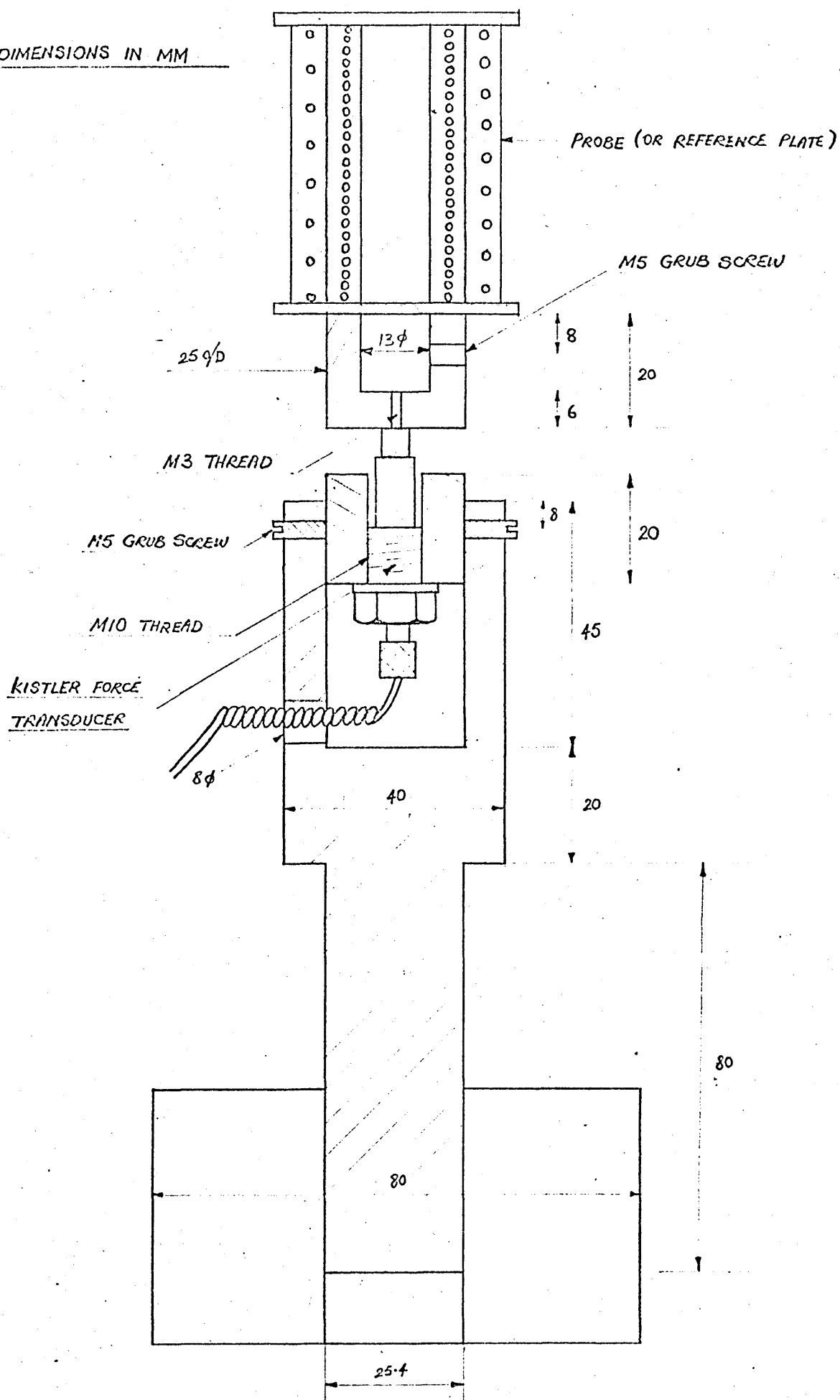M10 THREAD

45

KISTLER FORCE
TRANSDUCER

8∅

40

20

80

80

25.4

FIG. A. 5·12. FORCE TRANSDUCER HOUSING

Appendix 6

Calibration of the vibration pickup transducer

This section describes the method adopted in the calibration
of the proximity probe measuring static displacement as well
as amplitude of vibration.

The probe used is a Wayne-Kerr type E capacitance probe with its associated TE200 transducer equipment. The probe is part of a large family of non-contacting displacement transducers used in vibration studies to eliminate the possibility of the transducer interferring with the vibration characteristics of the structure tested. This is especially evident in the present case where a transducer of considerable mass, e.g. an accelerometer, used to measure the vibration of a thin plate, would drastically alter its vibration characteristics and render them totally false.

Capacitance probes work on the principle that when a probe is placed near to a conducting surface, a capacitance exists between this surface and the face of the probe. The capacitance so formed is made use of in the feedback loop of a high gain amplifier thereby changing its impedance and causing a voltage output to be formed which is proportional to the gap distance between the probe and the surface. The aim of the calibration is to obtain this relationship between the voltage output of the probe and its amplifier equipment and the amplitude of vibration that it measures.

It was decided not to use Wayne-Kerr vibration meters but actually calibrate the probe in preparation for future work on random vibration since Wayne-Kerr vibration meters are only calibrated for sinusoidal wave excitation and are hence valid only for discrete excitation work. The calibration of the probe in terms of R.M.S. volts and R.M.S. amplitudes of vibration would however be equally valid for random excitation work. A block diagram of the apparatus used in the calibration is shown in figure A.6.1.

The probe was positioned above a reference block (plate 7), on opposite sides of which were mounted two accelerometers, and which was fastened on to a vibrator. Adjustment of the probe-to-vibrating-surface gap was possible using the mechanism described in section 4.2. The voltage output of the accelerometer and amplifier system was recorded with corresponding readings of the probe and amplifier output when vibrating
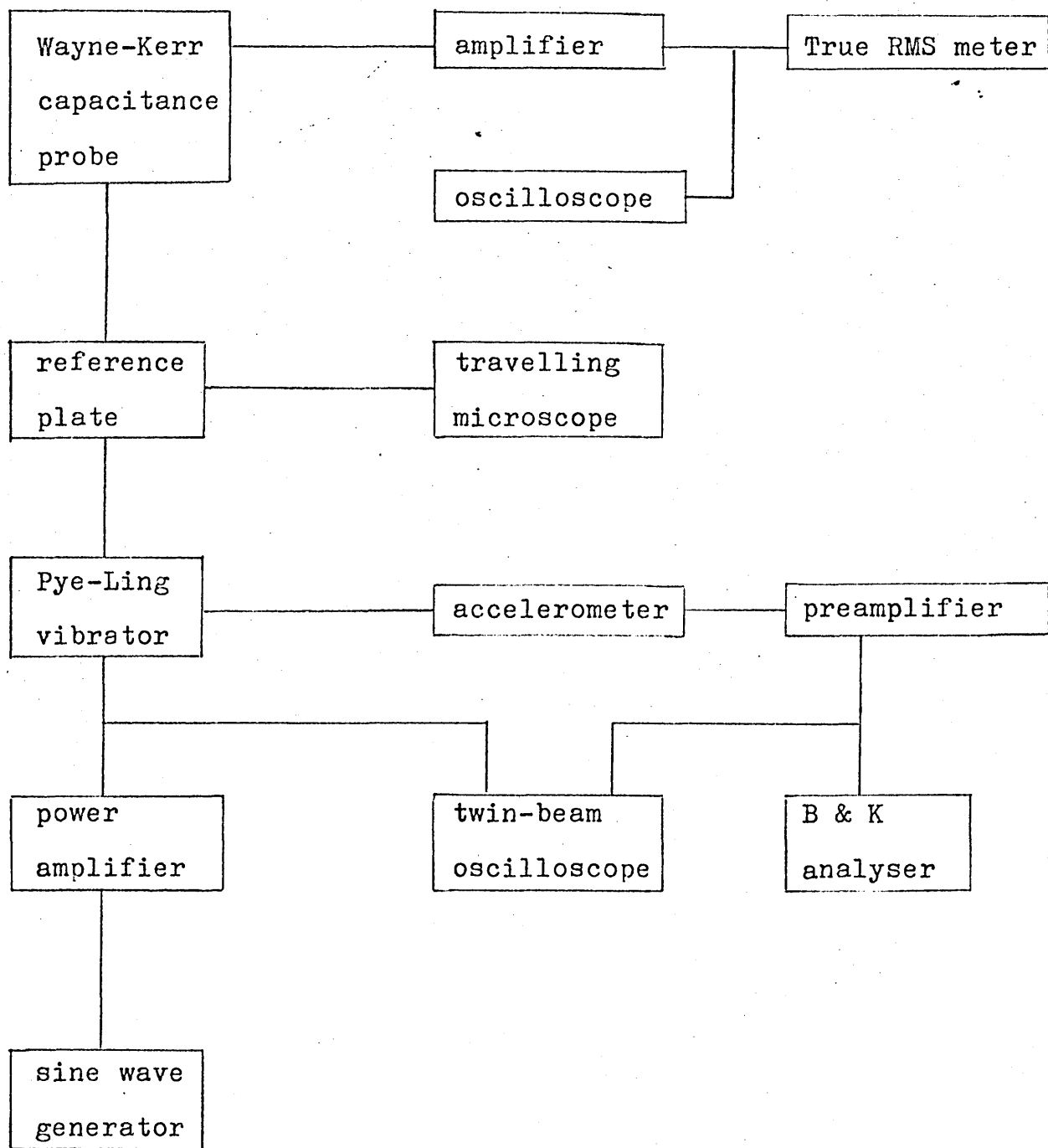
<u>Figure A.6.1</u>   Block diagram of apparatus used for the
calibration of the Wayne-Kerr capacitance
probe.

at a fixed frequency of 30 Hz but with different amplitudes
of vibration. The same procedure was repeated with several
different frequencies but keeping the probe-to-surface
distance constant. Finally the probe-to-surface distance was
varied and the above procedure repeated.

The amplitudes of vibration were calculated from the
accelerometer outputs and graphs of amplitude of vibration
against probe output were plotted. The gradients of these
graphs were calculated for each probe-to-surface distance
and a final graph plotted, showing the effects of the probe-
to-surface distance on these gradients.

Facility for double integration of acceleration by the Bruel
and Kjaer preamplifier to obtain amplitude of vibration was
available but limited to specified frequencies of 1,3, 10,
30, 100 and 300 Hz. As a result this method was only used as
a check for the values of amplitudes of vibration which were
calculated from actual acceleration figures.

A travelling microscope was used to measure static displacements
and low frequency vibrations when a visual measurement of the
amplitude of vibration provides a check on the accelerometer
readings at low frequencies.

The angular fine adjustment mechanism was used to adjust the gap
between the probe and the vibrating surface to ensure that they
were parallel. This was to prevent the surfaces otherwise touch-
ing when the gap size and the amplitude of vibration approach
each other. Apart from this however little effort was necessary
since from the manufacturers specification an eight degree angle
carried an error of only 1% in the distance measurement and no
error at all in the vibration amplitude measurement since only
the difference between the alternating peaks were measured.

Appendix 7

Calibration of the non-contacting exciter

This section describes the method used to calibrate the
non-contacting electromagnetic vibrator developed.

Figure A.7.1 is a block diagram of the instrumentation used
in the calibration of the exciter in which the effect of
signal amplitude, frequency response and gap distance were
investigated.

The exciter was mounted above a reference block supported
by a steel column housing a piezoelectric load sensitive
transducer (plate 10). The force generated by the exciter
would therefore act on the block and be transmitted through
the load cell. A sinusoidal signal was applied to the inner
coil and a corresponding d.c. bias to the outer coil of the
noncontacting exciter which is set at a fixed distance from
the reference block. This produces an alternating force onto
the block which, acting on the load cell, produces a signal
monitoring the force generated by the exciter. This was set
to read directly in Newtons of force using a compatible
factory calibrated charge amplifier. The output was checked
to be of the same sinusoidal waveform as the excitation voltage
to the exciter, both traces being displayed on a twin beam
oscilloscope.

The excitation signal amplitude was then varied but retaining
the same frequency and the variation of force output of the
exciter with input voltage obtained. The whole procedure was
repeated at various signal frequencies and calibrations
corresponding to each obtained. Also by keeping the
amplitude of the input signal constant but varying its
frequency the frequency response of the exciter was obtained.

Finally the effect of variation of the gap between the exciter
and the surface of the structure excited on the force output
was obtained, keeping the frequency and amplitude of the input
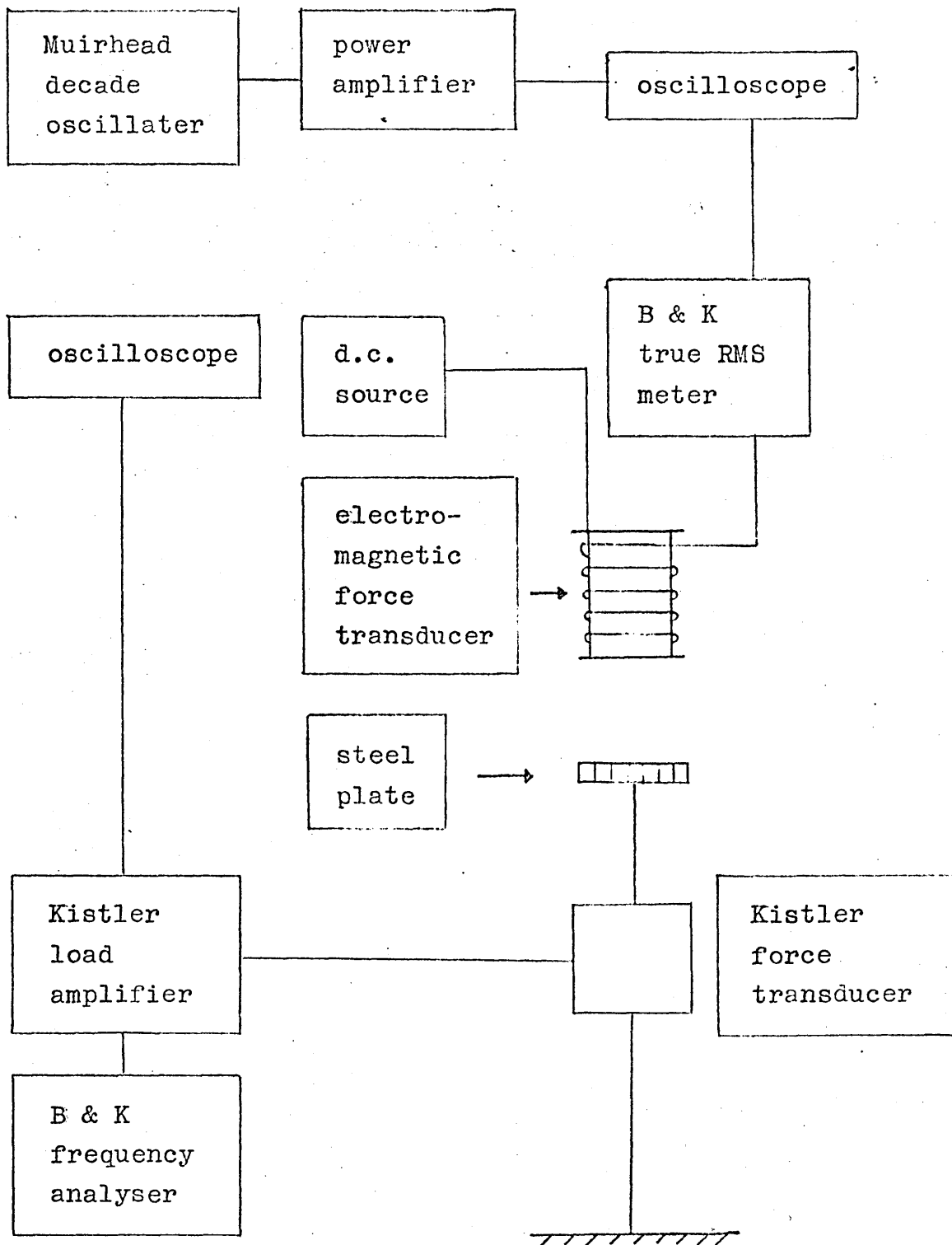signal fixed.

fig. A.7.1 Block diagram of instrumentation used for
calibration of non-contacting force transducer.