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GRAVITY BALANCING OF A SPATIAL SERIAL 4-DOF ARM WITHOUT AUXILIARY LINKS USING MINIMUM NUMBER OF SPRINGS

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Abstract

The principle of gravity balancing has been studied for a long time. It allows a system to be in indifferent equilibrium regardless of the configuration. In the literature, gravity balancing has often been achieved using appropriate combinations of springs and auxiliary links. Some papers address potential layouts without auxiliary links, but limited to planar mechanisms.

This paper proposes a method to passively balance an anthropomorphic arm, with spatial kinematics, avoiding the use of auxiliary links.

The approach used in this paper includes the analysis of all the contributions to the potential energy of the arm. It is shown that they are proportional (according to geometrical and inertial parameters) to scalar products between configuration-dependent unit vectors and/or configuration-independent unit vectors.

Analysing the potential energy contributions for each combination of unit vectors, it is shown how to minimize the number of springs required to balance the mechanism without additional links. As a result, four possible layouts are developed, all of them using only two springs. Features and design issues of the four layouts are discussed. Finally, one of them is chosen for actual implementation.

1 INTRODUCTION

A machine is said to be gravity balanced if no actuator inputs are needed to keep the system in equilibrium, regardless of the configuration. From an Euler-Lagrange point of view the potential energy of the device is invariant, hence there is no force causing the system to change its configuration. The main motivation for studying gravity balancing is that it leads to significant decreases of the required actuator efforts during motion.

Excluding the trivial case in which the global centre of mass is inertially fixed, gravity balancing can be achieved using counterweights [1, 2], with the drawback of adding inertia to the system, or using elastic elements [3, 4, 5], e.g. springs, to compensate the variations of potential energy due to changes of configuration (motion of masses).

In the majority of literature, auxiliary links/parallelograms are needed when springs are used for gravity balancing [6, 7, 8], often making the mechanism bulky and/or reducing its workspace. A recursive method to achieve gravity balancing without auxiliary links is introduced in [9], but it is limited to planar mechanisms.

This paper proposes a simple method to balance a spatial mechanism using springs without auxiliary links. The case-study mechanism features an anthropomorphic (spatial) kinematics.

2 GRAVITY BALANCING PRINCIPLE

Consider a single link, pivoted at point O, having mass m and centre of mass located at distance h from O (Figure 1). The rotation angle θ allowed by the hinge in O is the only degree of freedom (dof) of the system.

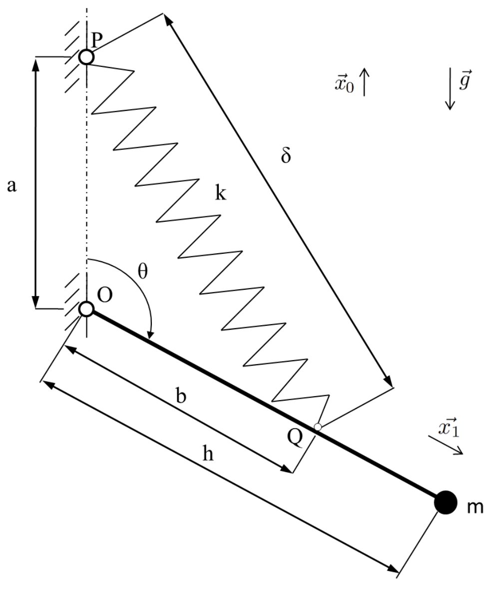

Figure 1: Single link system, 1 dof.

The gravitational potential energy of the system, E_g, is

$$E_g = mgh\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} = mgh\cos\theta \tag{1}$$

being $\overrightarrow{x_1} \cdot \overrightarrow{x_0} = \cos\theta$. To achieve indifferent equilibrium of the system regardless of the configuration (i.e. θ), the overall potential energy of the system must be constant, thus the term multiplying $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$ must be somehow compensated. As well known [3, 7], that can be achieved using a zero free-length spring [10], with stiffness k. The spring needs to be connected between point P - on the fixed frame - and point Q (on the link). Point P is located at distance a from point O, being OP aligned with the unit vector $\overrightarrow{x_0}$, opposite to the gravity acceleration vector $\vec{g}$. Point Q is located at distance b from point O, being OQ aligned with the unit vector $\overrightarrow{x_1}$ defining the orientation of the link.

The elastic potential energy of the spring, E_s, is

$$E_s = \frac{1}{2}k|QP|^2 = \frac{1}{2}k|PQ|^2 = \frac{1}{2}k\; QP \cdot QP \tag{2}$$

Considering the triangle OPQ, it is

$$OQ + QP = OP \tag{3}$$

and therefore

$$QP = OP - OQ = a\overrightarrow{x_0} - b\overrightarrow{x_1} \tag{4}$$

so

$$E_s = \frac{1}{2}k(a\overrightarrow{x_0} - b\overrightarrow{x_1}) \cdot (a\overrightarrow{x_0} - b\overrightarrow{x_1}) = \frac{1}{2}k(a^2 + b^2 - 2ab\,\overrightarrow{x_1} \cdot \overrightarrow{x_0}) = C_1 - kab\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} \tag{5}$$

where $C_1 = \frac{1}{2}k(a^2 + b^2)$ is a constant, hence it does not depend on the mechanism configuration θ. The total potential energy of the system, E, is given by the sum of the gravitational potential energy, E_g, and the elastic potential energy, E_s, as

$$E = E_g + E_s = C_1 + (mgh - kab)\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} \tag{6}$$

To let E be constant independent of the mechanism configuration, E needs to be independent of $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$. This happens if the coefficient of $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$ is zero. As a result, the system is gravity balanced when the following relation is satisfied

$$kab = mgh \tag{7}$$

It is useful to study the extension of this approach to mechanisms with more degrees of freedom. Consider the 2 dof linkage depicted in Figure 2. A subscript i is introduced in the notation, referring to the generic link i quantities. For example, m_2 is the mass of link 2, and $\overrightarrow{x_2}$ is the unit vector defining the orientation of link 2. Also, the quantity l_i is introduced to denote the length of link i. This system can be balanced using two springs, as follows. The gravitational potential energy of the system, $E_{g,2}$, is

$$E_{g,2} = m_1gh_1\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} + m_2g(l_1\overrightarrow{x_1} + h_2\overrightarrow{x_2}) \cdot \overrightarrow{x_0} = \\ (m_1gh_1 + m_2gl_1)\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} + (m_2gh_2)\,\overrightarrow{x_2} \cdot \overrightarrow{x_0} \tag{8}$$

In (8) two configuration dependent contributions appear, i.e. $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$ and $\overrightarrow{x_2} \cdot \overrightarrow{x_0}$. A potential solution is to use two springs, the first to cancel the $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$ contribution, the second to cancel the $\overrightarrow{x_2} \cdot \overrightarrow{x_0}$ contribution. A set of auxiliary links, i.e. a 4 bar mechanism ABCD, is added to the system as shown in Figure 2. As a result, one end of the spring k_2 is aligned with $\overrightarrow{x_0}$, thus generating the required $\overrightarrow{x_2} \cdot \overrightarrow{x_0}$ contribution. The elastic potential energy of the system, $E_{s,2}$, results

$$E_{s,2} = \frac{1}{2}k_1(a_1\overrightarrow{x_0} - b_1\overrightarrow{x_1}) \cdot (a_1\overrightarrow{x_0} - b_1\overrightarrow{x_1}) + \frac{1}{2}k_2(a_2\overrightarrow{x_0} - b_2\overrightarrow{x_2}) \cdot (a_2\overrightarrow{x_0} - b_2\overrightarrow{x_2}) = \\ C_2 - k_1a_1b_1\,\overrightarrow{x_1} \cdot \overrightarrow{x_0} - k_2a_2b_2\,\overrightarrow{x_2} \cdot \overrightarrow{x_0} \tag{9}$$

Similarly to (5), $C_2 = \frac{1}{2}k_1(a_1^2 + b_1^2) + \frac{1}{2}k_2(a_2^2 + b_2^2)$ is a constant. Finally, considering the total potential energy of the system, $E_2 = E_{g,2} + E_{s,2}$, and equating each of the contributions in (8) and (9) as in (6), the system is gravity balanced when the following relations are satisfied:

$$k_1 a_1 b_1 = m_1 g h_1 + m_2 g l_1$$
$$k_2 a_2 b_2 = m_2 g h_2 \tag{10}$$

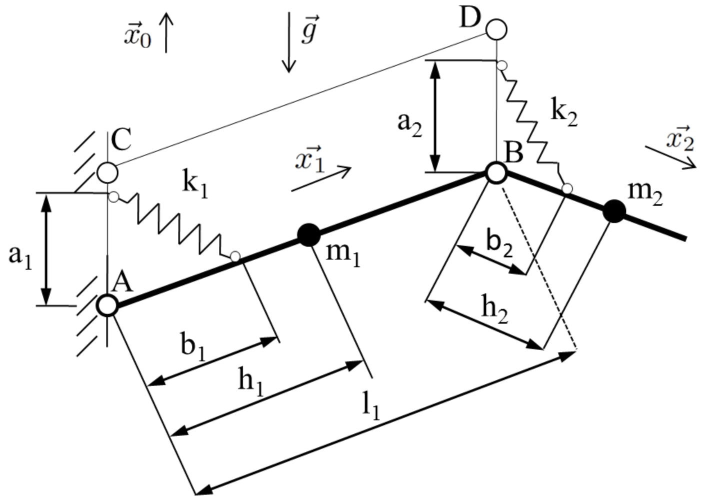

Figure 2: Two links system, 2 dof.

This approach can be further extended to design a gravity balancing system for n dof serial spatial mechanisms as described in [3]. In particular, one spring would be required for each dof, together with appropriate auxiliary links.

3 PROBLEM ANALYSIS

3.1 Case study mechanism

The spatial mechanism studied is a 4 dof anthropomorphic arm including: i) a spherical joint representing the glenohumeral joint, modelling the shoulder adduction-abduction dof, the shoulder flexion dof, and the shoulder rotation dof; ii) a rotational joint, modelling the elbow flexion dof.

That is a 4 dof mechanism, in theory needing 4 springs and relative auxiliary links (as described above). However, because the first three degrees of freedom are obtained by means of a single spherical joint, only one spring is needed for the first link of the kinematic chain. So, only two springs can balance the whole robotic manipulator. Actually, the spherical joint does not make any difference with respect to a rotational joint [3].

The solution already shown in Figure 2 may be implemented. However, the introduction of auxiliary links may be inconvenient for this mechanism, as they: i) increase bulkiness and complication; ii) reduce workspace. Hence, a different approach, averting auxiliary links, is proposed.

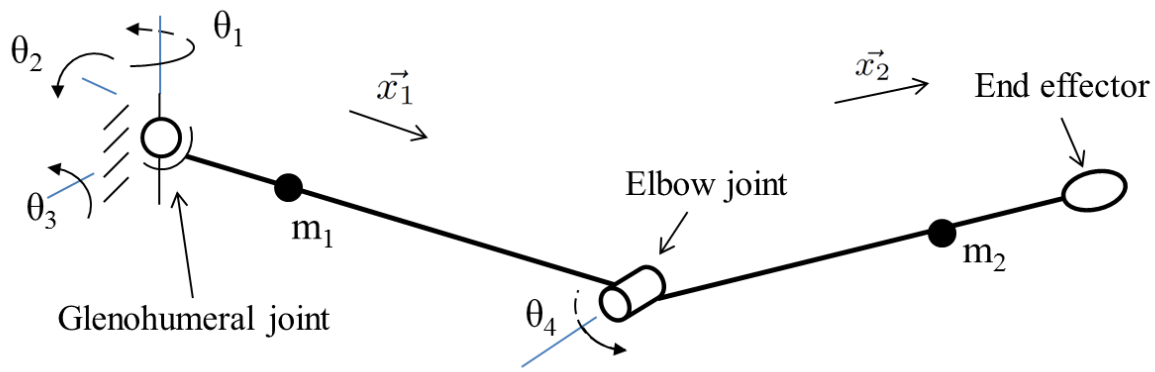

Figure 3: 4 dof system considered: anthropomorphic arm.

3.2 Balancing without auxiliary links and minimum number of springs

The gravitational potential energy of the system, $E_{g,4}$, has the same form of the one shown in (8):

$$E_{g,4} = m_1 g h_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} + m_2 g (l_1 \overrightarrow{x_1} + h_2 \overrightarrow{x_2}) \cdot \overrightarrow{x_0} = (m_1 g h_1 + m_2 g l_1) \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} + (m_2 g h_2) \, \overrightarrow{x_2} \cdot \overrightarrow{x_0} \quad (11)$$

The contribution $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$ can be balanced using a spring (with stiffness k_1) installed as the one with stiffness k_1 in Figure 2. Also a contribution $\overrightarrow{x_2} \cdot \overrightarrow{x_0}$ has to be compensated with a spring, but this task is less trivial without the 4-bar mechanism shown in Figure 2. A first attempt is to attach a spring (with stiffness k_2) from the fixed frame directly to the second link. The resulting layout is shown in Figure 4a.

Geometrically, the spring with stiffness k_2 in Figure 4a is a side of a triangle, being the other two sides $a_2 \overrightarrow{x_0}$ and $(l_1 \overrightarrow{x_1} + b_2 \overrightarrow{x_2})$. The latter is the vector from the first hinge of the mechanism to the attachment point of the spring on the link. The elastic potential energy of the system, $E_{s,4a}$, is

$$E_{s,4a} = \frac{1}{2} k_1 (a_1 \overrightarrow{x_0} - b_1 \overrightarrow{x_1}) \cdot (a_1 \overrightarrow{x_0} - b_1 \overrightarrow{x_1}) + \frac{1}{2} k_2 (a_2 \overrightarrow{x_0} - (l_1 \overrightarrow{x_1} + b_2 \overrightarrow{x_2})) \cdot (a_2 \overrightarrow{x_0} - (l_1 \overrightarrow{x_1} + b_2 \overrightarrow{x_2})) = C_3 - k_1 a_1 b_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} - k_2 (a_2 b_2 \, \overrightarrow{x_2} \cdot \overrightarrow{x_0} + a_2 l_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} - b_2 l_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_2}) \quad (12)$$

As expected there is a contribution $\overrightarrow{x_2} \cdot \overrightarrow{x_0}$, however also an undesired $\overrightarrow{x_1} \cdot \overrightarrow{x_2}$ contribution appears. C_3 is a constant.

The problem of the $\overrightarrow{x_1} \cdot \overrightarrow{x_2}$ contribution is solved adding a third spring (with stiffness k_{12}) connected between the first and the second link, as shown in Figure 4b (where the quantities a_{12} and b_{12} are defined). Following the same method, the elastic potential energy becomes

$$E_{s,4b} = C_4 - k_1 a_1 b_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} - k_2 (a_2 b_2 \, \overrightarrow{x_2} \cdot \overrightarrow{x_0} + a_2 l_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_0} - b_2 l_1 \, \overrightarrow{x_1} \cdot \overrightarrow{x_2}) + + k_{12} a_{12} (l_1 - b_{12}) \, \overrightarrow{x_1} \cdot \overrightarrow{x_2} \quad (13)$$

where C_4 is again a constant.

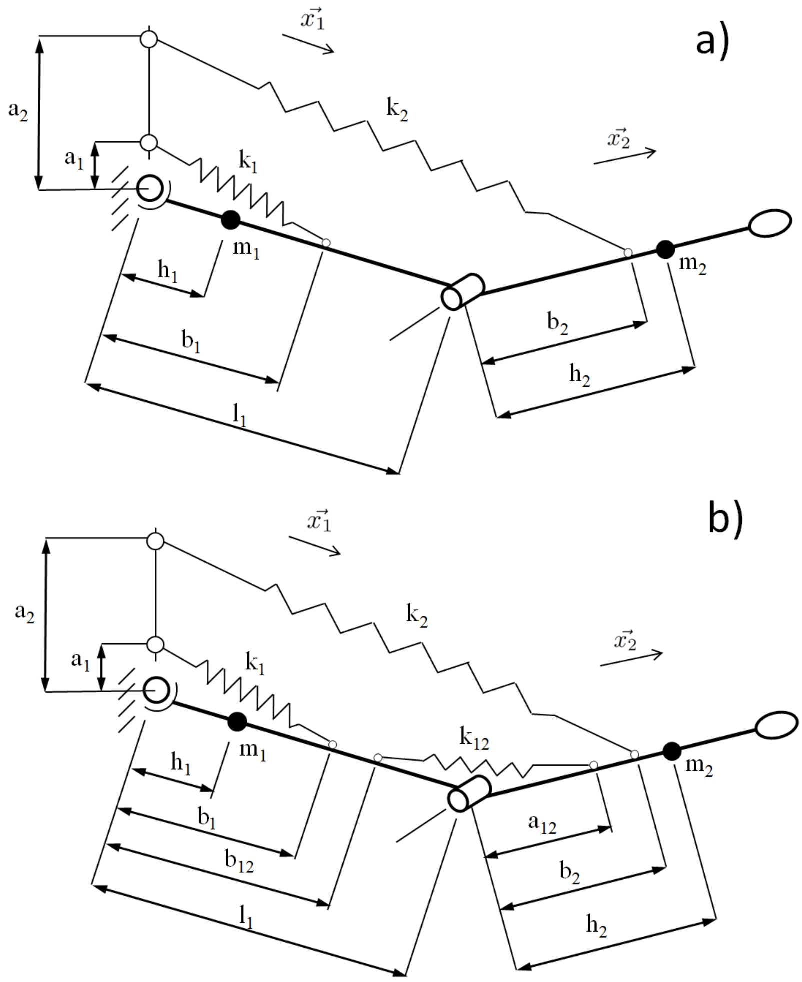

Figure 4: 4 dof system without auxiliary links: a) two springs, system not balanced; b) three springs

Hence, looking at the overall potential energy of the system, collecting all the terms $\overrightarrow{x_i} \cdot \overrightarrow{x_j}$ $(i \neq j)$ in (11) and (13), the system is gravity balanced when the following relations are satisfied

$$(m_1 g h_1 + m_2 g l_1 - k_1 a_1 b_1 - k_2 a_2 l_1)\, \overrightarrow{x_1} \cdot \overrightarrow{x_0} = 0$$
$$(m_2 g h_2 - k_2 a_2 b_2)\, \overrightarrow{x_2} \cdot \overrightarrow{x_0} = 0$$
$$(k_2 b_2 l_1 + k_{12} a_{12}(l_1 - b_{12}))\, \overrightarrow{x_1} \cdot \overrightarrow{x_2} = 0 \tag{14}$$

In (14) the spring with stiffness k_2 introduces a contribution $\overrightarrow{x_1} \cdot \overrightarrow{x_0}$, as the spring with stiffness k_1 does. So, the spring with stiffness k_1 may be removed from the architecture of Figure 4b. This leads to a system with minimum number of springs (i.e. two). At last, to passively balance the system against gravity, it must be

$$m_1 g h_1 + m_2 g l_1 - k_2 a_2 l_1 = 0$$
$$m_2 g h_2 - k_2 a_2 b_2 = 0$$
$$k_2 b_2 l_1 + k_{12} a_{12}(l_1 - b_{12}) = 0 \tag{15}$$

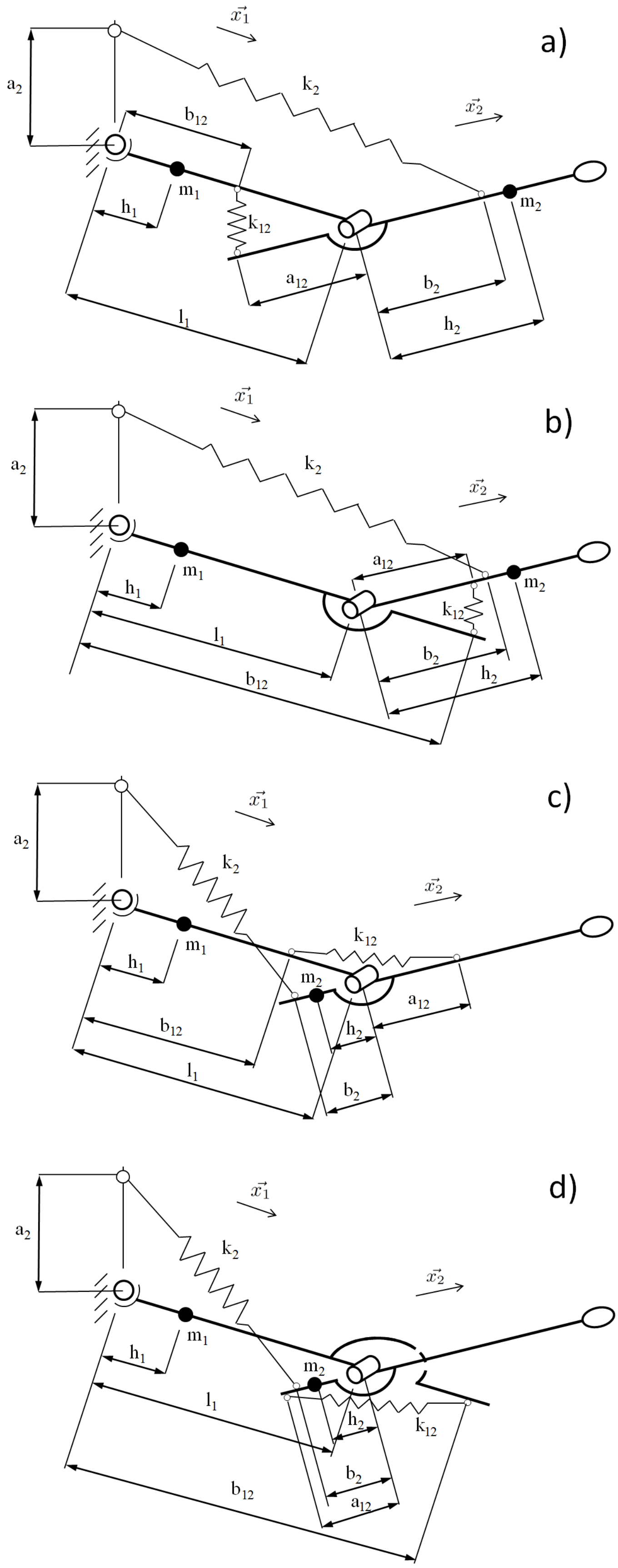

Figure 5: 4 dof system: the four layouts developed

If m_i, l_i, h_i are given, the system (15) consists of three equations in six unknowns $(k_2, a_2, b_2, k_{12}, a_{12}, b_{12})$, allowing some design freedom. It should be considered that the lengths $a_2, b_2, a_{12}, b_{12}, h_1, h_2$ can be negative. The physical meaning is that the denoted quantity has opposite direction with respect to the associated unit vector. For example, $a_{12} < 0$ means that one attachment point of the spring with stiffness k_{12} lays on the extension of link 2 behind the hinge connecting link 1 and link 2 (this is further clarified below as well as in Figure 5). In the remainder, $h_1 > 0$ is assumed.

Four potential layouts are developed, as shown in Figure 5:

- layout a): $a_{12} < 0$, $b_{12} < l_1$, $b_2 > 0$;
- layout b): $a_{12} > 0$, $b_{12} > l_1$, $b_2 > 0$;
- layout c): $a_{12} > 0$, $b_{12} < l_1$, $b_2 < 0$;
- layout d): $a_{12} < 0$, $b_{12} > l_1$, $b_2 < 0$.

From the first equation in (15), $a_2 > 0$ is mandatory, meaning that the attachment point of the spring with stiffness k_2 must be above the first joint of the mechanism. This can be intuitively explained looking at Figure 1: if point P was below point O, the link would be pulled downwards by the spring. The force applied from the spring in point Q would generate a clockwise moment with respect to point O, with the same sign of the moment contribution due to the weight of the link. Thus the system could not be gravity balanced.

According to the second equation in (15), layouts c) and d) work only if $h_2 < 0$, which is possible (e.g. it was chosen in [11]) but inconvenient for the kind of serial mechanism studied. Moreover, layouts c) and d) imply a potential intersection between the spring with stiffness k_2 and link 1. Additionally, layout d) entails an extension of both links, while only one link has to be extended for the other layouts. That makes layouts a) and b) preferable.

Finally, layout b) is chosen because the spring with stiffness k_{12} in layout a), together with the extension of link 2 behind the elbow joint, may reduce the workspace of the arm. In particular, the range of the spherical joint would be affected. On the other hand, the extension of link 1 in layout b) is less troublesome, also because of the limited range of the human elbow joint (the elbow cannot move backwards).

4 CONCLUSION

This paper proposed a gravity balancing method for an anthropomorphic arm, with spatial kinematics, using passive elements (springs) and avoiding the use of auxiliary links.

According to the method proposed, only two springs are needed to balance the mechanism. Still, there are more parameters than equations. Different layouts have been studied, highlighting their potential benefits and drawbacks. A final layout has been identified. Future work will present the implementation and experimental study of the selected configuration.

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