

Simone Biles: defying the laws of physics?

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What is the energy Simone needs for the move?

To perform the 'Biles', Simone needs to travel and spin. Her take-off energy therefore needs to incorporate these two elements:

$$[1] E_{k \text{ take-off}} = E_{k \text{ linear}} + E_{k \text{ rotational}}$$

We can break this equation down a bit by considering that she moves up and forward:

$$[2] E_{k \text{ take-off}} = E_{k \text{ vertical}} + E_{k \text{ horizontal}} + E_{k \text{ rotational}}$$

To simplify the problem, we assume that the gymnast is a point mass, i.e. the gymnast is just a point particle. If you wanted to do a more accurate job, you could model each segment of the gymnast separately and combine the values appropriately in a global reference frame (figure 1).

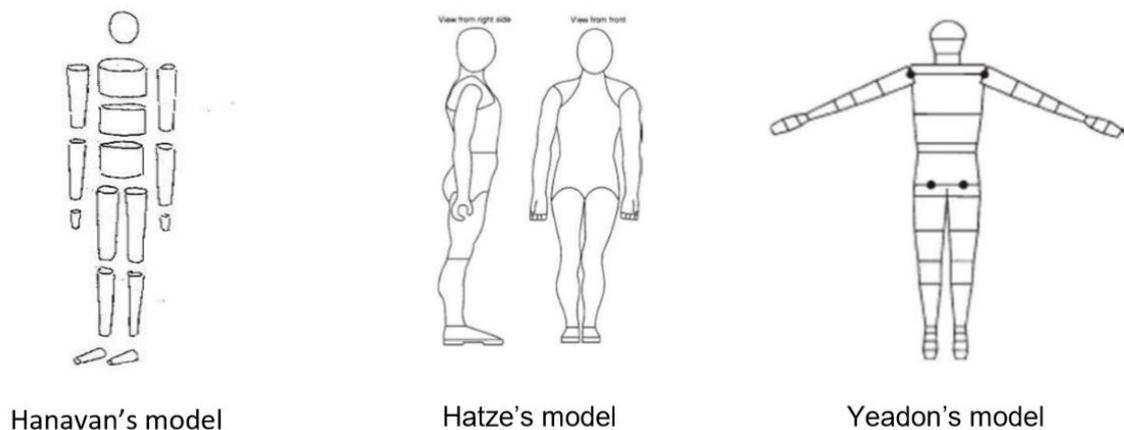


Fig 1 Modes of body segmentation. Taken from Furlong (2010) https://dspace.lboro.ac.uk/dspace-jspui/bitstream/2134/20438/1/2010_Furlong_inertia%20parameters.pdf

Kinetic energy is dependent on the speed and the mass of a body:

$$[3] E_{k \text{ linear}} = \frac{1}{2}mv^2$$

Equation 3 is for linear energy and we will use that equation to calculate $E_{k \text{ vertical}}$ and $E_{k \text{ horizontal}}$. In the 'Biles', there are two rotations about Simone's somersaulting axis and a half rotation about

her twisting axis. A distinction needs to be made between these rotations. The somersaults occur throughout the flight phase but the twist only occurs in the final quarter of the flight phase. The equations that we have discussed so far assume constant motion. They can be applied to Simone's somersaults and her vertical and horizontal travel. From this, we calculate $E_{k \text{ take-off}}$. The twist, however, is not constant – she only twists at the end. The energy required to cause a twist that is not constant over the flight phase must come from work done by Simone during the skill. Practically, this means that when we talk about the twist in the 'Biles', we won't consider the entire flight phase of the skill - just the final quarter. We now have a new structure to our main equation (equation 2):

$$[4] E_{k \text{ total}} = E_{k \text{ take-off}} + E_{k \text{ twist}} = (E_{k \text{ vertical}} + E_{k \text{ horizontal}} + E_{k \text{ somersault}}) + E_{k \text{ twist}}$$

To figure out the rotational equivalent of the linear energy equations, we replace mass with mass moment of inertia and linear velocity with angular velocity:

$$[5] E_{k \text{ rotational}} = \frac{1}{2} I \omega^2$$

A body's mass moment of inertia, I , is an indication of its resistance to a change in its rotation. Think of it like this: the more mass something has, the heavier it is, and heavier things are harder to move than lighter things. Heavier things are also harder to rotate. Additionally, longer things are harder to rotate than shorter things. Mass moment of inertia is a single characteristic that captures a body's resistance to rotation based on its mass and shape. You might already have noticed that a body will have a different moment of inertia depending on the axis about which it is rotated. For example, it is easier to twist a brush handle than it is to swing it. To give a mathematical example, the mass moment of inertia for a cylinder with mass $m = 2$ kg, length $l = 2$ m and radius $r = 0.02$ m, rotating it as a baton twirler might, end over end, is

$$[6] I = \frac{m(3r^2 + l^2)}{12} = \frac{2(3(0.02)^2 + (2)^2)}{12} = 0.6669 \text{ kg}\cdot\text{m}^2$$

But when spinning the cylinder around its radius, as you might a spinning top, it is

$$[7] I = \frac{mr^2}{2} = \frac{(2)(0.02)^2}{2} = 0.0004 \text{ kg}\cdot\text{m}^2$$

We simplified the gymnast as a point mass for linear equations. We will also simplify the gymnast's shape for rotational equations. Let's represent a somersaulting gymnast as a cylinder because it's a simple shape that approximates a standing human.

Even though Simone's mass, m , is the same for the linear kinetic energies, her mass moment of inertia will be different for her somersaulting and her twisting. Her mass moment of inertia for somersaulting, $I_{\text{somersault}}$, is calculated using equation 6 and for twisting, I_{twist} , using equation 7. Her somersaulting energy thus becomes

$$\begin{aligned}
 [8] E_{k \text{ somersault}} &= \frac{1}{2} I_{\text{somersault}} \omega_{\text{somersault}}^2 \\
 &\Rightarrow \frac{1}{2} \left(\frac{m(3r^2 + l^2)}{12} \right) \omega_{\text{somersault}}^2 \\
 &\Rightarrow \frac{1}{2} \frac{(3r^2 + l^2)m \omega_{\text{somersault}}^2}{12}
 \end{aligned}$$

And her twisting energy becomes

$$\begin{aligned}
 [9] E_{k \text{ twist}} &= \frac{1}{2} I_{\text{twist}} \omega_{\text{twist}}^2 \\
 &\Rightarrow \frac{1}{2} \left(\frac{mr^2}{2} \right) \omega_{\text{twist}}^2 \\
 &\Rightarrow \frac{1}{2} \frac{mr^2 \omega_{\text{twist}}^2}{2}
 \end{aligned}$$

When we plug these equations into equation 4 we get

$$[10] E_{k \text{ total}} = \frac{1}{2} m v_{\text{vert}}^2 + \frac{1}{2} m v_{\text{horiz}}^2 + \frac{1}{2} \frac{(3r^2 + l^2)m \omega_{\text{somersault}}^2}{12} + \frac{1}{2} \frac{mr^2 \omega_{\text{twist}}^2}{2}$$

With some tidying up, we get:

$$[11] E_{k \text{ total}} = \frac{m}{2} \left(v_{\text{vert}}^2 + v_{\text{horiz}}^2 + \frac{(3r^2 + l^2)\omega_{\text{somersault}}^2}{12} + \frac{r^2 \omega_{\text{twist}}^2}{2} \right)$$

Now we have the job of estimating the values of the terms. From a quick google, we estimate her height and mass to be height = $l = 1.47$ m and $m = 47$ kg. We use the distance from her spine to her shoulder to represent her radius, approximately $r = 0.25$ cm. For the velocities, we can use the equations of constant motion to help us out. For the vertical velocity, v_{vert} , we can use the equation:

$$[10] v_{vert\ final} = v_{vert\ initial} + at$$

Where $v_{vert\ final}$ is her speed at the instant before the end of flight, $v_{vert\ initial}$ is her speed at the moment of take-off, a is the acceleration that she experiences and t is the duration of flight. We want Simone's energy at the start of the tumble so we need to rearrange equation 10 to make $v_{vert\ initial}$ the subject of the equation.

$$[11] v_{vert\ initial} = v_{vert\ final} - at$$

What terms do we know in equation 11? From looking at her performance, she seems to be in the air for about 1 s, so we'll say that $t = 1$ s. The value of a is $-9.812\text{ m}\cdot\text{s}^{-2}$ because this is the value for acceleration due to gravity (we are assuming that 'up' is a positive direction and 'down' is negative). Once Simone is in flight, there are no other forces acting on her and thus, no other accelerations (we are ignoring air resistance). How do we estimate $v_{vert\ final}$? This is where we have to be a bit clever. Instead of modelling the entire flight phase, we can model just the first half. This is a good idea because we know that her vertical velocity is zero at the middle of the flight because that is the highest point of the flight – what goes up must come down, but it has to stop first. So, if we model half of the flight, then a stays the same, t is halved ($t = 0.5$ s) and $v_{vert\ final} = 0\text{ m}\cdot\text{s}^{-1}$.

$$[12] v_{vert\ initial} = 0 - (-9.812 \times 0.5) \Rightarrow v_{vert\ initial} = 4.6\text{ m}\cdot\text{s}^{-1}$$

To figure out what v_{horiz} is, we can use an even simpler equation:

$$[13] speed = \frac{distance}{time}$$

We can do this because there are no horizontal forces acting on Simone when she is in flight (again, ignoring air resistance). With no forces, there are no accelerations. From viewing her performance, it looks like she travels a horizontal distance of about $distance_{horiz} = 3$ m (the diagonal of the floor is 16.9 m). We said before, that she is in the air for about 1 s, so $time = t = 1$ s. The horizontal speed at which she must be travel is therefore

$$[14] speed_{horiz} = \frac{3}{1} = v_{horiz\ initial} = 3\text{ m}\cdot\text{s}^{-1}$$

We now have everything we need except the rotational velocities, more commonly known as angular velocities. We could decompose the angular velocities into linear velocities applied at some distance from the centre of rotation. We would have to estimate these distances so, instead, it will be easier to estimate angular velocities. She somersaults twice within $t = 1$ s, so

$$\omega_{\text{somersault}} = \frac{2 \times 360}{1} = 720^\circ \text{s}^{-1} = 4\pi \text{ rad}\cdot\text{s}^{-1}. \text{ We said earlier that we would consider her twist only in}$$

the final portion of the skill. She twists half a turn in the final quarter of the skill, so $\omega_{\text{twist}} =$

$$\frac{0.5 \times 360}{0.25} = 720^\circ \text{s}^{-1} = 4\pi \text{ rad}\cdot\text{s}^{-1}. \text{ We have everything we need so we just put the values into equation}$$

11:

$$[15] E_{k \text{ total}} = \frac{47}{2} \left(4.6^2 + 3^2 + \left(\frac{(3(0.25^2) + 1.47^2)(4\pi)^2}{12} \right) + \left(\frac{(0.25^2)(4\pi)^2}{2} \right) \right)$$

$$\Rightarrow E_{k \text{ total}} = 23.5(21.16 + 9 + 30.9 + 4.93)$$

$$\Rightarrow E_{k \text{ total}} = 3.91(65.99)$$

$$\Rightarrow E_{k \text{ total}} = 1,550.77 \text{ Joules}$$

What force does Simone experience on landing?

Now that we know her energy, we can calculate her landing force. Her landing takes about $t_{land} = 0.2$ s, during which time her velocities return to zero. From Newton's 2nd law, we know

$$[16] F = ma = m \left(\frac{v_{final} - v_{initial}}{t_{land}} \right)$$

Where F is a force, m is the mass of the object and a is the acceleration affecting the mass. To calculate the force required to stop her horizontal and vertical velocities in $t_{land} = 0.2$ s, we will first combine the horizontal and vertical velocities into a resultant velocity. This is done using Pythagoras' theorem:

$$[17] v_{initial\ resultant} = \sqrt{v_{initial\ vert}^2 + v_{initial\ horiz}^2}$$
$$\Rightarrow v_{initial\ resultant} = \sqrt{4.6^2 + 3^2} = \sqrt{30.16}$$
$$\Rightarrow v_{initial\ resultant} = 5.49 \text{ m}\cdot\text{s}^{-1}$$

Since the final horizontal and vertical velocities will be zero, $v_{final\ resultant} = 0$. Therefore,

$$[18] F = 47 \left(\frac{0 - 5.49}{0.2} \right) = -1,290.58 \text{ N}$$

That is more force than the bite of an adult American alligator (wiki). The force is negative because it is a retarding force, i.e one that is stopping Simone. The force is equivalent to almost 3 times her body weight. Of course, this calculation does not take into account the forces required to stop her rotations.

Where does this energy come from?

Where does this energy come from? This is where we come to the law of conservation of energy, Newton's third law and Hooke's law. The law of conservation of energy says that energy cannot be created or destroyed. In Simone's case, she converts the energy that she built during her run into the energy needed to perform her tumble. All she is doing is redirecting her horizontal energy from her sprint, and the vertical and rotational energy from her flic. All of this kinetic energy comes from chemical energy released within her muscles, but that is a conversation for another day. It should be noted, that Simone's short stature means that she can include more steps in her run-up than a taller gymnast. Each of these steps adds more and more energy.

Now, Newton's third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body. If this wasn't true, then the weight of your body pushing down on Earth would drill you right through it. After her run, Simone performs a round-off to turn around and a flic to convert linear energy into rotational energy. When Simone pushes into the sprung floor, it is the floor's equal and opposite force that shoots her up, not her muscular force. This might sound weird but try an experiment next time you are in a swimming pool: while floating, try to jump out of the water. You can use all the muscles in your body mimic a jumping movement but you won't leap up like a dolphin. This is because you are pushing against the water and the water isn't as good as a sprung floor at pushing back on you. The sprung floor can't move anywhere so it directs the force back at you. The water is free to flow around the pool so it isn't so concerned with giving you porpoise performance powers.

The sprung floor is key to gymnasts' energy and protection. If they were tumbling on a hard surface like concrete, then the force that they drive into the ground would be shot back at them so quickly that it could be too much for the gymnast to handle. The Gymnova floor that Simone tumbles on is a layer of carpeted foam, on a layer of plywood that is supported by an array of springs. These layers lengthen the duration of

the impact, which means that the total force exerted on Simone is spread over a longer time. Newton's second law pops up again:

$$[1] \quad F = ma = m \left(\frac{v-u}{\Delta t} \right) = m(v-u) \frac{1}{\Delta t} \Rightarrow F \propto \frac{1}{\Delta t}$$

As we can see, if the duration of the impact increases, then the force per unit time must decrease. But, of course, gymnasts don't want their force to be wasted, and it isn't. The force that Simone exerts on the sprung floor is stored as energy potential energy, E_p , within the compliant elements of the floor:

$$[2] \quad E_p = \frac{1}{2} kx^2$$

This is Hooke's law, where k is the spring constant and x is the how far the spring is displaced. During the short time that Simone is in contact with the floor, there will come a point where the energy she is putting into the floor will be less than the energy that the floor is storing. At this point, the sprung floor will start to give Simone her energy back and rocket her up into the air.

How does she make her body spin, and control the spin, in the air?

We've spoken a lot about energy but now it is time to introduce momentum. Here we can bring in Newton's first law: When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a net force. Put simply, things that are moving want to stay moving and things that are stationary don't want to move. The key terms here are 'momentum' and 'inertia': momentum keeps things moving, inertia is what we call a body's resistance to changing its momentum. These ideas are quite intuitive, especially when you think about your own experience of trying to stop moving things and trying to move stationary things. It's safe to say that faster things are harder to slow down and heavier things are harder to move. The formula for linear and angular momentum are:

$$[1] \quad p = mv$$

$$[2] \quad L = I\omega$$

This reflects our experiences because it says that heavier and faster things have more momentum than lighter and slower things. But how does this relate to somersaults and twists?

The law of conservation of angular momentum can be defined in many ways but the gist is that the angular momentum of a body will stay constant provided that it is not acted upon by external forces. For Simone, this means that the momentum that she has when she takes off is all the momentum that can use for her aerial acrobatics. All of our earlier conversations about the energy at take-off explained how she makes sure that she has enough energy for her signature skill: the 'Biles'.

Let's say that I am a gymnast and I need L angular momentum to complete one stretched somersault. If I wanted to do two somersaults with the same angular momentum, then I will need to spin faster. Angular velocity, ω , is a measure of my

spin. According to equation 2, I will need to decrease my mass moment of inertia, I , in order to increase my angular velocity because my momentum, L , stays constant. How do I do that? Well, just like heavier things are harder to move than lighter things, longer things are harder to rotate than shorter things. By tucking into a ball that is half the length of my outstretched body, then I might be able to make it around twice and complete my double somersault.

Simone has oodles of momentum going into her double stretched somersault but she still makes use of the conservation of angular momentum. If you watch her perform the skill, you might be able to notice the two ways that Simone decreases her mass moment of inertia in order to somersault faster. The first is that she pulls herself into an arch. The length between the end points of an arch is always shorter than the length of body that is arching. This does a little but not much. The bigger contributor is when she snaps her arms to her hips just after her first quarter somersault. By snapping her arms down to her hips she now measures from head-to-toe rather than fingers-to-toes. This is a big change in length.

Let's talk about twisting. It seems like out of nowhere, Simone suddenly twists at the end of her double stretched somersault. She takes advantage of a handy characteristic of the conservation of angular momentum: momentum is conserved across all axes. This means that Simone can take momentum from her somersaulting axis and transfer it to her twisting axis. She does this by changing the shape of her body and causing a tilt.

Tilting takes momentum. If Simone tilts during a somersault she must borrow momentum from one of the other axes because the momentum was fixed at take-off. Perhaps she borrowed it from her somersault, but that would slow her somersaulting down – dangerous move. The other option is to borrow it from her twisting axis. But how can she take angular momentum from her twisting axis if she isn't twisting? She goes into negative twisting. 'Positive'; and 'negative' are just opposite directions. A negative twist is just a twist in a different direction to a positive twist. When she tilts, she borrows momentum from her twisting axis so she starts to twist negatively. To stop a twist, she un-tilts, which returns the twisting momentum back to zero.

So, how does she tilt? The answer is asymmetric arm movement. If you hold one arm out to the side you will tilt ever so slightly (hold a weight to feel a bigger tilt). Hopefully, your centre of gravity will stay over your base of support and you won't fall over. As Simone enters her second somersault, you'll see her pull her left arm inward to her belly button. Just like our standing experiment, she has moved one arm so that she will slightly tilt. This little tilt is sufficient for the half-twist that she needs.

The interesting thing about twisting is that somersaults naturally want to twist. Gymnasts put a lot of effort into not twisting. The first reason is that real-life somersaults are not likely to take off perfectly so some of force will be off-axis and cause a turning effect. Another reason is the phenomenon of nutation. In short, the axis of a rotating body wobbles. If Simone kept somersaulting forever you'd see that she would be tilting left and right and she spun. Practically, this means that the gymnast is actually forced into tilting simply because they are somersaulting.

Is there a role for a sports scientist to analyse athletes' moves and use physics to suggest improvements?

Gymnasts are certainly aware of how the orientation of their limbs affects their performance, but they might not use the jargon that I have. From my coaching experience I have found that explaining the underlying physics can be useful for demystifying difficult skills and breaking them down into a manageable sequence of actions. Sometimes, gymnasts just want to know what to do rather than understand why. Studying these things is good though because it means that we can model performance and suggest what is possible but still undone. This idea is especially true in for the kind of tactical modelling done in well-defined events like track cycling, where models take physiological, biomechanical, atmospheric, and mechanical inputs into account. In gymnastics, people like Prof Maurice “Fred” Yeadon have modelled somersaults extensively and concluded that a triple layout somersault is not likely to be possible on current spring floors given the required energy from the gymnast. That said, such energies are possible on a tumbling track and the triple layout is a relatively new skill that was forever thought to be impossible.