Effects of moment of inertia on restricted motion swing speed

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Abstract

In many sports, the maximum swing speed of a racket, club or bat is a key performance parameter. Previous research in multiple sports supports the hypothesis of an inverse association between the swing speed and moment of inertia of an implement. The aim of this study was to rigorously test and quantify this relationship using a restricted swinging motion. Eight visually identical rods with a common mass, but variable moment of inertia were manufactured. Motion capture technology was used to record eight participants' maximal effort swings with the rods. Strict exclusion criteria were applied to data that did not adhere to the prescribed movement pattern. The study found that for all participants, swing speed decreased with respect to moment of inertia according to a power relationship. However, in contrast to previous studies, the rate of decrease varied from participant to participant. With further analysis it was found that participants performed more consistently at the higher end of the moment of inertia range tested. The results support the inverse association between swing speed and moment of inertia but only for higher moment of inertia implements.

Word Count: 184

Key words: Swing-weight, rods, tennis, motion analysis

Introduction

In sports that involve a swinging motion, one of the most important physical properties of an implement is the moment of inertia about an axis normal to the primary swing plane (Cross & Nathan, 2009). Moment of inertia (MOI) is a measure of an object's resistance to angular acceleration about a given axis and is one of the limiting factors for maximum swing speed.

Participants can be highly sensitive to changes in moment of inertia, especially at an elite level (Brody, 2000). It has been shown that people are up to ten times more sensitive to differences of MOI than differences in mass for objects in the range 0.3 to 0.5 kg (Kreifeldt & Chuang, 1979). Multiple studies suggest that the moment of inertia of an implement has an inverse association with swing speed, with evidence in Baseball (Koenig, Mitchell, Hannigan, & Clutter, 2004), Golf (Daish, 1972) and Tennis (Mitchell, Jones, & King, 2000). All of these studies used at least four implements and found that increasing MOI reduced swing speed. However, the range in MOI of the tested implements was relatively small and mass was not kept constant. Smith, Broker and Nathan (2003) undertook similar work in Softball with ten bats of constant mass and ten bats of constant MOI. They showed that swing speed has a high dependence upon MOI but little dependence on bat mass. However, this study also focused on a relatively small range of moment of inertia (0.128 - 0.200 kgm^2).

In previous studies, the participants performed a motion typical to the sport being considered – skill and experience were important factors in the selection of participants in these studies. Each participant performs at different levels of consistency and habitually uses equipment with a specific moment of inertia. Mitchell and co-workers (2000) found that in tennis, individuals performed best with rackets whose MOI (measured 0.1m from the butt end) closest matched that of their own racket.

Cross and Bower (2006) attempted to overcome the potential complicating factors of participant skill and experience by testing a simple restricted motion. Using a one armed, overhead motion in the sagittal plane, participants swung six weighted rods with a large range of mass (0.208 - 0.562 kg) and moment of inertia (0.0103 - 0.1034 kgm^2). A similar trend of decreasing swing speed with increasing moment of inertia was found and a common power law relationship was defined,

$$V = \frac{C}{{I_o}^n} \tag{1}$$

where V is swing speed, C is a participant constant, I_o is the moment of inertia and n is the gradient of the best fit trend line. The n value in literature is found to be between 0.25-0.29 (Daish, 1972; Smith et al., 2003; Cross and Bower, 2006; Nathan, Crisco, Greenwald, Russel and Smith, 2011) and further work in softball found the value of n to lie between 0.20-0.25 (Smith & Kensrud, 2014; Smith, Burbank, Kensrud, & Martin, 2012). These studies have remarkably similar results despite focussing on different sports or movements. However, in work by Cross and Bower (2006), relationships appear to have been established using as few as two or three data points. In addition, implement weighting was not concealed, meaning participants may prepare to perform differently. Furthermore, there is evidence from a meta-analysis to suggest that this relationship may exist between swing speed and MOI across a wide range of sports (Schorah, Choppin, & James, 2012). However, as this work did not involve data collection, more experimental work is required to confirm this.

When swung, an implement's moment of inertia influences the transfer of energy to another object where there is an impact. For a given swing speed, a higher MOI generally produces a higher outbound ball speed (Bahill, 2004). However, because a higher MOI generally results in a lower swing speed, it is likely that this is only true up to a limit. Therefore, it is likely that the optimal implement moment of inertia for an individual will be strongly influenced by their skill level or physical strength. For skilled actions, as used in some of the literature, such as throwing an object or striking an object with a swung implement, there is also a trade-off between the speed of the action and the accuracy of the performed task. The optimum point of this speed-accuracy trade-off was found to be at 70% of maximal throwing speed for baseball players (Freeston & Rooney, 2014) but there is also evidence to suggest that this trade-off only applies when a player is performing with their dominant side (Sachlikidis & Salter, 2007). The kinematics of a swing is also closely linked to resulting swing speeds (Joyce, Burnett, Cochrane, & Ball, 2013) and players have been found to control the variability in their swing to keep the outcome consistent (Tucker, Anderson, & Kenny, 2013).

The aim of this study was to experimentally test the relationship between swing speed and moment of inertia during a restricted motion with a rigorous protocol and a larger number of participants than many previous studies. It is hypothesised that the value for n will be the same for all participants and will lie within the range 0.20-0.29.

Methods

This study used a motion capture system to record the speed of weighted rods swung by participants in a laboratory.

Participants

Once approval was received from the Faculty of Health and Wellbeing Research Ethics Committee at Sheffield Hallam University, eight participants with a range of statures and builds were recruited. The group consisted of six males and two females and all participants were healthy, active individuals with an age of 25.1 ± 5.4 years, mass of 73.3 ± 16.5 kg and height of 1.79 ± 0.27 m. The participants were all active and with no prior experience of performing the motion in the study.

Rods

This study used eight, visually identical rods made from 0.0254 m diameter, hollow aluminium tubing. In order to vary moments of inertia, a solid mass of 0.16 kg was fixed within each rod at varying locations along the length. Each rod was capped at either end, had a length of 0.506 m and a total mass of 0.32 kg (including the additional mass).

Table 1 shows the length (L), mass (M), balance point (L_{cm}) measured from the butt end of the rod, and moment of inertia (I_{rod}) about a perpendicular axis through the butt end of the rod, for the 8 rods. The moment of inertia of the rods was calculated using the method described by Brody (1985), taken about an axis through the butt end, perpendicular to the rod's centreline. Moment of inertia values ranged from 0.0113 to 0.0495 kgm^2 (Table 1), representing the moment of inertia of a typical badminton racket to a typical tennis racket. The moment of inertia measurements were estimated to be subject to error less than 1%, which was deemed acceptable.

Figure 1 shows the bespoke attachment used to fix the rods to the back of the participant's wrist. The attachment for the rods had a mass of 0.16 kg, positioned at the base of the rod and the wrist guard had a mass of 0.076 kg. To account for the effect of this attachment, the rod's moment of inertia was re-calculated about the participant's elbow, and included the mass of the attachment and wrist guard, I_{Elbow}.

The distance from a participant's elbow to wrist was calculated from motion capture data. This method provided a more accurate description of each rod's resistance to angular acceleration, but it also reduced the range in moment of inertia values. Nonetheless, the experimental range still exceeded moment of inertia values typically found in tennis.

The rods were labelled 1 to 8 in a random order and each participant swung the rods in this order. The test was carried out with a double blind protocol, where neither the participant nor observer knew the moment of inertia of the rod being swung.

Swing analysis

Participants performed a maximal, internal rotation of the shoulder, keeping the elbow stationary, with the forearm swinging in the transverse plane. Each of the eight weighted rods was swung three times. Participants had a rest of one minute between swings to eliminate fatigue effects. To add a focal point and reduce unintentional deceleration, participants hit a ball suspended in front of them at the end of each swing. Any swings which did not visibly follow the desired motion were repeated, but these were not always easily identifiable.

To restrict the wrist joint, each participant wore a guard on their swinging arm; the wrist could not rotate to contribute to the movement. Participants also maintained a seated position to limit torso movement.

Tracking

A motion capture system was used to track swing kinematics. Twelve Motion Analysis Corporation Eagle cameras were used, recording at 300 frames per second with a shutter speed of 1 ms. The layout of the cameras with respect to the participant is shown in Figure 2. The system had a residual error of $6.24 \times 10^{-4} m$ in the position of markers in the 3D space.

Eight 12.7 mm spherical reflective markers were used to track the movement of the participant and the rod; their locations are shown in Figure 3. The markers were linked in the software such that the shoulder markers were connected to the Humerus and elbow markers; the elbow marker was connected to the Humerus, wrist and rod base markers and the wrist and rod base markers were connected to the rod tip.

Before swings were recorded each participant stood in a t-pose for a static trail after which the medial epicondyle marker (5) was removed. Maximum resultant velocity of the rod tip was the key variable of interest; other markers were used to review the movement and check adherence to the protocol.

The raw tracking files were initially processed using the Motion Analysis Cortex package. The cubic join function was used to fill in any short sections where the cameras had not seen a marker and the smooth function was used to reduce the noise of a trace. A Butterworth filter was used with a cut-off frequency of 10Hz.

Kohonen self-organizing maps

A self-organizing map (SOM) is an n-dimensional neural network which can be visualised as a 2D map of nodes. A SOM was used to ensure that only swings with good adherence to the desired movement pattern were considered in the analysis. This was necessary to ensure we made a fair comparison between individuals when analysing swing speed. SOM analysis has been used to categorise complex sporting movements in the past (Lamb, Bartlett, & Robins, 2010) and was used in a similar way here. A thorough description on the use of a SOM to investigate player technique is given in work by Lees (2002).

A vector, containing twelve variables, was used as the input to the SOM. These were the x, y and z positions of the shoulder, elbow and wrist joint centres during each swing and the 3 angles between the Humerus and the global coordinate system axes. Each trial was normalised to 10 data points between the start of movement and peak tip velocity. A SOM was initialised and trained using the complete collection of input vectors, producing an 18 x 12 hexagonal map. Each input vector (one for each swing) activated a 'trajectory' of up to ten activated nodes throughout the map (some points activate the same node). These trajectories were used to categorise every swing by dividing the map into sections. In Figure 4a, the divisions of these sections have been overlaid onto a map showing how the Euclidean distance between nodes differs across the grid, with black representing a very short distance and white a large distance. Two nodes that have a short Euclidean distance between them represent a similar magnitude for each variable.

The trajectory analysis produced 2 distinct groups, which are represented in Figure 4b and Figure 4c. Figure 4b shows an example trajectory from group one, travelling from section 4 to section 2, and Figure 4c shows an example trajectory from group two travelling from section 1 to section 5. Group one accounted for 41% of data and group two accounted for 31% of data. The remaining 28% of swings did not fit into either group.

The analysis was carried out using the SOM toolbox for MATLAB (Vesanto, Himberg, Alhoniemi, & Parhankangas, 2000).

A series of new versions of the self-organizing map were plotted, with each map showing the value of a selected variable at each node. As an example, Figure 5 shows a version of the map which has been shaded based upon each node's value of Elbow anterior-posterior position, relative to the direction the participant was facing. The paths of typical group one and group two trajectories were analysed and compared to the values of each variable on these maps. It can be seen on Figure 5 that group one trajectories exhibited very little change in anterior-posterior direction elbow position. Conversely, group two trajectories went through a large change from positive to negative, meaning the elbow was being translated in the posterior direction to help produce rod velocity, rather than just using rotation of the shoulder. After analysing all variables, it was decided that group one trajectories best matched the desired motion and only group one data were used for further analysis.

Statistical tests

The reduced data set produced by the self-organizing map method was plotted on log velocity – log moment of inertia graphs. In order to determine how well related the data were a 2-tailed Pearson correlation was run between the I_{Elbow} values and each participant's velocity data. The residual sum of squares was also calculated to assess the quality of fit in the data and the square root was taken to bring the units back to m/s.

Results

Figure 6 shows logarithmic plots of maximum rod tip velocity against I_{Elbow} for five participants. Almost 60% of the swing data was excluded as a result of the SOM analysis, and furthermore, a participant was only included if there was swing data for at least five of the eight rods. These strict criteria eliminated three participants from further analysis. With linear fits applied to each log-log plot, n values varied from 0.19 to 0.79.

The data in Table 2 shows the outcome of the Pearson's correlation test run between I_{Elbow} and maximum swing velocity and the residual sum of squares for each participant. The five participants have a Pearson's correlation coefficient varying from -0.529 to -0.907 and the rooted residual sum of squares varies from 0.0332 to 0.166. The maximum error in the value of n associated with the accuracy of the tracking system is ±0.08.

Discussion and Implications

The results in Figure 6 clearly show that the relationship between velocity and moment of inertia can be described using the power law in Equation 1. However, the n values are different for each participant in marked contrast with the results from previous studies (Cross & Bower, 2006; Daish, 1972; Smith et al., 2003). In these previous studies, the lines of best fit shown on the logarithmic plots of swing velocity against moment of inertia show near identical n values. Here, the participants have different n values and the velocity data only correlates strongly with moment of inertia for 2 of the 5 participants, as is shown in Table 2. The residual sum of squares values are mostly very low but are variable. Participant 2 in particular has a high SS_R indicating a poor fit to the modelled trend line and this is evident in Figure 6.

The initial aim of this study was to observe whether or not there is a decaying relationship between swing speed and moment of inertia. Initial analysis found that this relationship exists but is different for different participants. As this finding contradicts the work of others, the dataset was further examined to understand whether the inter-participant differences were consistent.

For each rod, participants were ranked in order of their swing velocity. The participant with the highest swing velocity was ranked first, the participant with the second highest swing velocity was ranked second and so forth. The participant rank sets for each rod were then placed in order of their respective moment of inertia (I_{Elbow}).

If the lines of best fit in Figure 6 had similar n values one would not expect the participant rankings to change between rods. Conversely, if the n values were variable (as in this study) one would expect the rank sets to change. A Spearman test was implemented to determine how similar the participant rank sets were as I_{Elbow} increased. The test was run between pairs of rank sets in order of increasing moment of inertia, comparing the rankings for rod 2 with the rankings for rod 1 and so forth. A Spearman's rank correlation coefficient of 'one' indicates that consecutive rank sets are identical; a coefficient of 'zero' indicates that they are unrelated.

In order to confirm that the exclusion method was valid, the ranking analysis was firstly carried out for all eight participants and then repeated for the reduced dataset as specified by the self-organising map method.

Figure 7a shows the Spearman coefficients for all swing trials (no exclusions). It can be seen that there is no clear trend, and the rank sets change in a seemingly random pattern. Conversely, Figure 7b shows the Spearman coefficients the reduced data set.

There are distinct differences between the Spearman's rho values for the full data set and the reduced data set. There is a greater consistency in the reduced data set with the higher moment of inertia rods having values of 1.0. This clearly demonstrates the effectiveness of the exclusion criteria and justifies the decision to only analyse swings that adhered to a consistent technique.

Whilst the Spearman's rank correlation coefficient fluctuates for the low moment of inertia rods, it rests at a consistent value of 'one' for the high moment of inertia rods. This suggests that participants swing a low moment of inertia implement in an unpredictable manner, and it is only at higher moments of inertia where a clear pattern of behaviour becomes established. There can also be interactions with other swing criteria such as swing accuracy and it has been shown that swing accuracy peaks at lower speeds when swinging unfamiliar implements (Beilock, Bertenthal, Hoerger, & Carr, 2008).

It would be of value, for customisation purposes, to be able to predict swing speed for a given MOI. This may be possible if n is constant, which results suggest will be the case for a higher range of MOI. Aside from swing speed, the impact characteristics in racket sports are also important performance parameters influenced by moment of inertia. It has been shown that an increase in moment of inertia can cause an increase in outbound ball velocity (Brody, 1997). This produces a trade-off in performance when changing moment of inertia and should yield an optimum MOI value. It is important to understand this optimum value and whether it changes for individual players of different strengths, as this could allow for customisation.

The consistency of the rankings at higher moment of inertias supports the hypothesis that for these participants, and a higher range of MOI in racket sports, it should be possible to predict a participant's swing speed. This may be achievable using some measures of physical profile, for example joint torque or even standing height, which has been shown to relate well with serve speed in tennis (Vaverka & Cernosek, 2013). Conversely, it may not be possible for lower moment of inertia implements where there is a less consistent ranking of swing speed.

However, this is based on data for only one specific motion and many complex movements take place in all sports. Therefore a larger study with more participants would be required to understand how consistently swing speed is affected by moment of inertia for a higher range of MOI. Work should also be undertaken to look at whether MOI and physical profile can be used to predict an individual's swing speed as this knowledge will be of value to individuals wishing to customise equipment. This work could also be developed to consider a wider range of motions and to explore whether swing accuracy is adversely affected by changing moment of inertia.

Conclusion

This study found that for all participants, swing speed decreased with respect to increases in moment of inertia according to a power law. However, in marked contrast to previous studies, the rate of decrease varied from participant to participant.

It was found that participants swung the high moment of inertia rods in a more consistent manner than the low moment of inertia rods. This suggests that predicting a player's swing speed may not be easily achievable for very low moment of inertia implements common in racket sports but could be feasible for higher moment of inertia implements.

References

Bahill, A. T. (2004). The Ideal Moment of Inertia for a Baseball or Softball Bat. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, *34*(2), 197–204.

Beilock, S. L., Bertenthal, B. I., Hoerger, M., & Carr, T. H. (2008). When does haste make waste? Speed-accuracy tradeoff, skill level, and the tools of the trade. *Journal of Experimental Psychology: Applied*, *14*(4), 340–352.

Brody, H. (1985). The moment of inertia of a tennis racket. *The Physics Teacher*, *23*(4), 213–216.

Brody, H. (1997). The physics of tennis III . The ball – racket interaction. *American Journal of Physics*, *65*(10), 981–987.

Brody, H. (2000). Player sensitivity to the moments of inertia of a tennis racket. *Sports Engineering*, *3*(2), 145–148.

Cross, R., & Bower, R. (2006). Effects of swing-weight on swing speed and racket power. *Journal of Sports Sciences*, *24*(1), 23–30.

Cross, R., & Nathan, A. M. (2009). Performance versus moment of inertia of sporting implements. *Sports Technology*, *2*(1-2), 7–15.

Daish, C. B. (1972). *The Physics of Ball Games*. *The Physics of Ball Games* (pp. 1–178). English Universities Press.

Freeston, J., & Rooney, K. (2014). Throwing speed and accuracy in baseball and cricket players. *Perceptual and Motor Skills*, *118*(3), 637–50.

Joyce, C., Burnett, A., Cochrane, J., & Ball, K. (2013). Three-dimensional trunk kinematics in golf: between-club differences and relationships to clubhead speed. *Sports Biomechanics*, *12*(2), 108–20.

Koenig, K., Mitchell, N. D., Hannigan, T. E., & Clutter, J. K. (2004). The influence of moment of inertia on baseball / softball bat swing speed. *Sports Engineering*, *7*, 105–117.

Kreifeldt, J. G., & Chuang, M. C. (1979). Moment of inertia: psychophysical study of an overlooked sensation. *Science (New York, N.Y.)*, *206*(4418), 588–90.

Lamb, P., Bartlett, R., & Robins, A. (2010). Self-Organising Maps : An Objective Method for Clustering Complex Human Movement. *International Journal of Computer Science in Sport*, *9*(1), 20–29.

Lees, A. (2002). Technique analysis in sports : a critical review. *Journal of Sports Sciences*, *20*(10), 813–828.

Mitchell, S. R., Jones, R., & King, M. (2000). Head speed vs. racket inertia in the tennis serve. *Sports Engineering*, *3*(2), 99–110.

Nathan, A. M., Crisco, J. J., Greenwald, R. M., Russell, D. A., & Smith, L. V. (2011). A comparative study of baseball bat performance. *Sports Engineering*, *13*(4), 153–162.

Sachlikidis, A., & Salter, C. (2007). A biomechanical comparison of dominant and non-dominant arm throws for speed and accuracy. *Sports Biomechanics*, *6*(3), 334–44.

Schorah, D., Choppin, S., & James, D. (2012). Investigating the relationship between swing weight and swing speed across different sports using historical data. In *Engineering of Sport 9, Procedia Engineering 34* (Vol. 34, pp. 766–771).

Smith, L., & Kensrud, J. (2014). Field and laboratory measurements of softball player swing speed and bat performance. *Sports Engineering*, *17*(2), 75–82.

Smith, L. V., Broker, J., & Nathan, A. M. (2003). A study of softball player swing speed. In *A. Subic, P. Travailo, & F. Alam (Eds.), Sports Dynamics: Discovery and application (pp. 12–17). Melbourne, VIC: RMIT University.*

Smith, L. V., Burbank, S., Kensrud, J., & Martin, J. (2012). Field Measurements of Softball Player Swing Speed. In *Engineering of Sport 9, Procedia Engineering 34* (Vol. 34, pp. 538–543).

Tucker, C. B., Anderson, R., & Kenny, I. C. (2013). Is outcome related to movement variability in golf? *Sports Biomechanics*, *12*(4), 343–54.

Vaverka, F., & Cernosek, M. (2013). Association between body height and serve speed in elite tennis players. *Sports Biomechanics*, *12*(1), 30–7.

Vesanto, J., Himberg, J., Alhoniemi, E., & Parhankangas, J. (2000). *SOM Toolbox for Matlab 5*. Retrieved from: http://www.cis.hut.fi/projects/somtoolbox/

Tables

Table 1. Balance point, L_{cm}, and MOI, I_{rod}, I_{Elbow}, of the rods, all of which have length of 0.506m, mass of 0.32kg.

Rod	L_{cm} (m)	I_{rod} (kgm^2)	I_{Elbow} (kgm2)
1	0.129	0.0113	0.0569
2	0.164	0.0130	0.0587
3	0.197	0.0169	0.0626
4	0.233	0.0199	0.0656
5	0.263	0.0261	0.0718
6	0.300	0.0332	0.0789
7	0.340	0.0425	0.0882
8	0.372	0.0495	0.0952

Table 2. Correlation coefficients and residual sum of squares for the five participant's velocity data and rod moment of inertia.

Participant	1	2	5	6	8
Pearson Correlation	-0.629	-0.529	-.907*	-.756*	-0.605
Significance	0.130	0.178	0.033	0.049	0.203
N	7	8	5	7	6
$\sqrt{SS_R}$ (m/s)	0.0878	0.166	0.0548	0.0332	0.0447

*. Correlation is significant at the 0.05 level (2-tailed).

Figure captions.

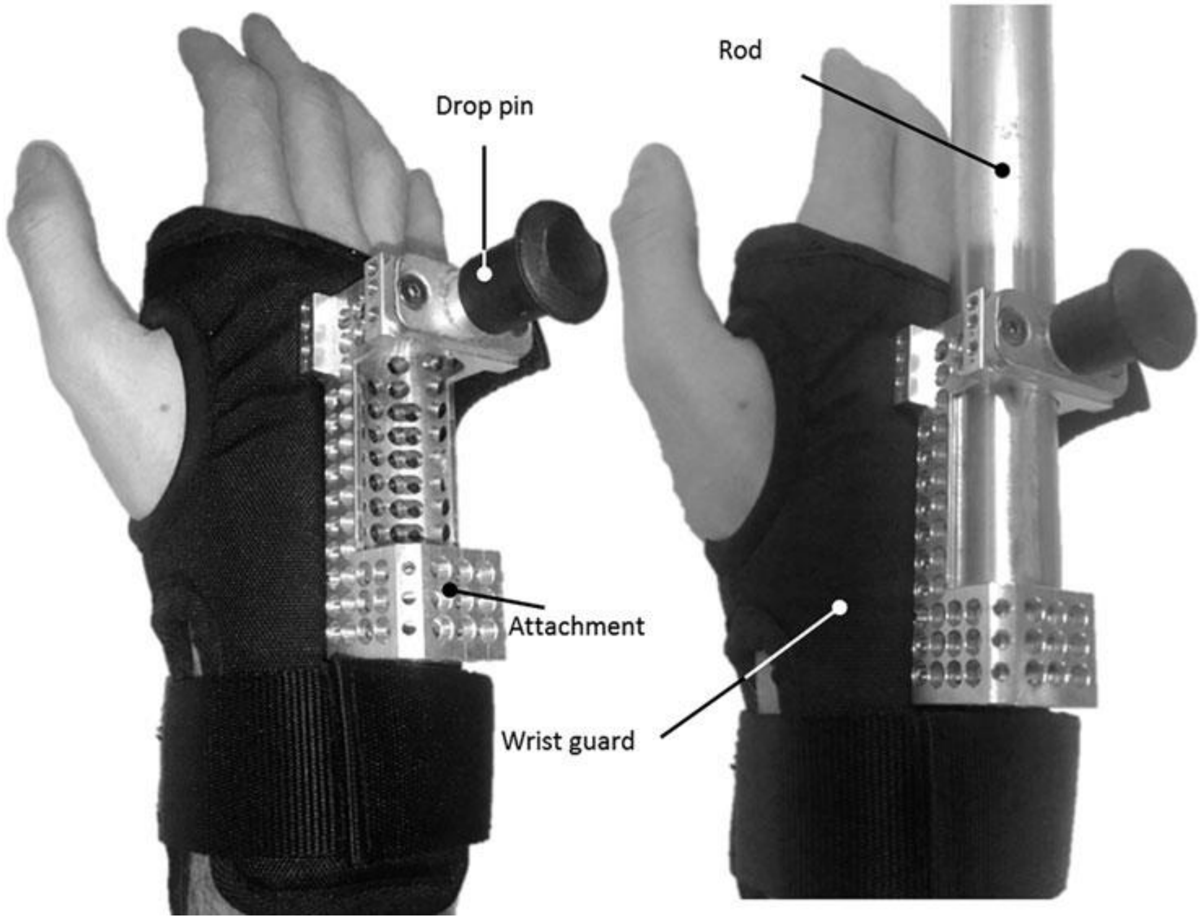

Figure 1. Illustration of attachment mechanism

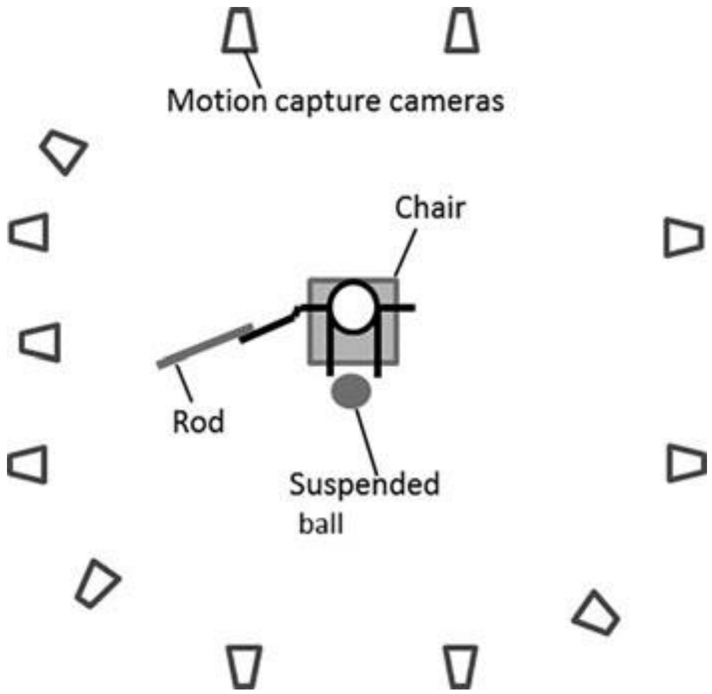

Figure 2. Overhead view of experimental setup

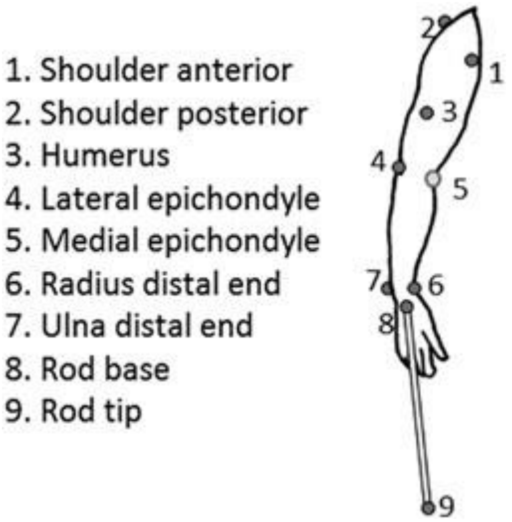

Figure 3. Marker arrangement

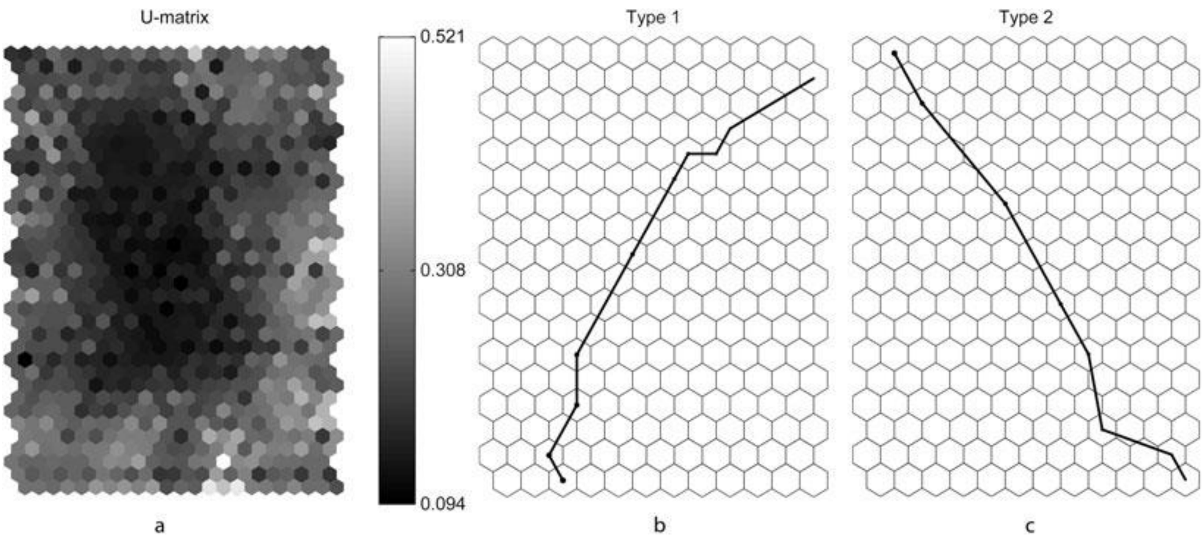

Figure 4. SOM trajectories. ***a*****: U-matrix showing Euclidean distance between nodes with section boundaries overprinted;** ***b*****: example group one trajectory;** ***c*****: example group two trajectory.**

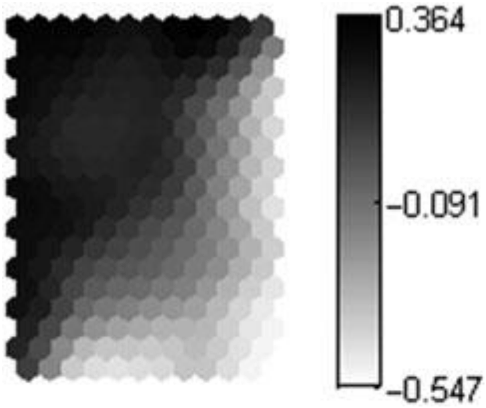

Figure 5. A version of the self-organising map showing values of elbow position anterior-posterior position.

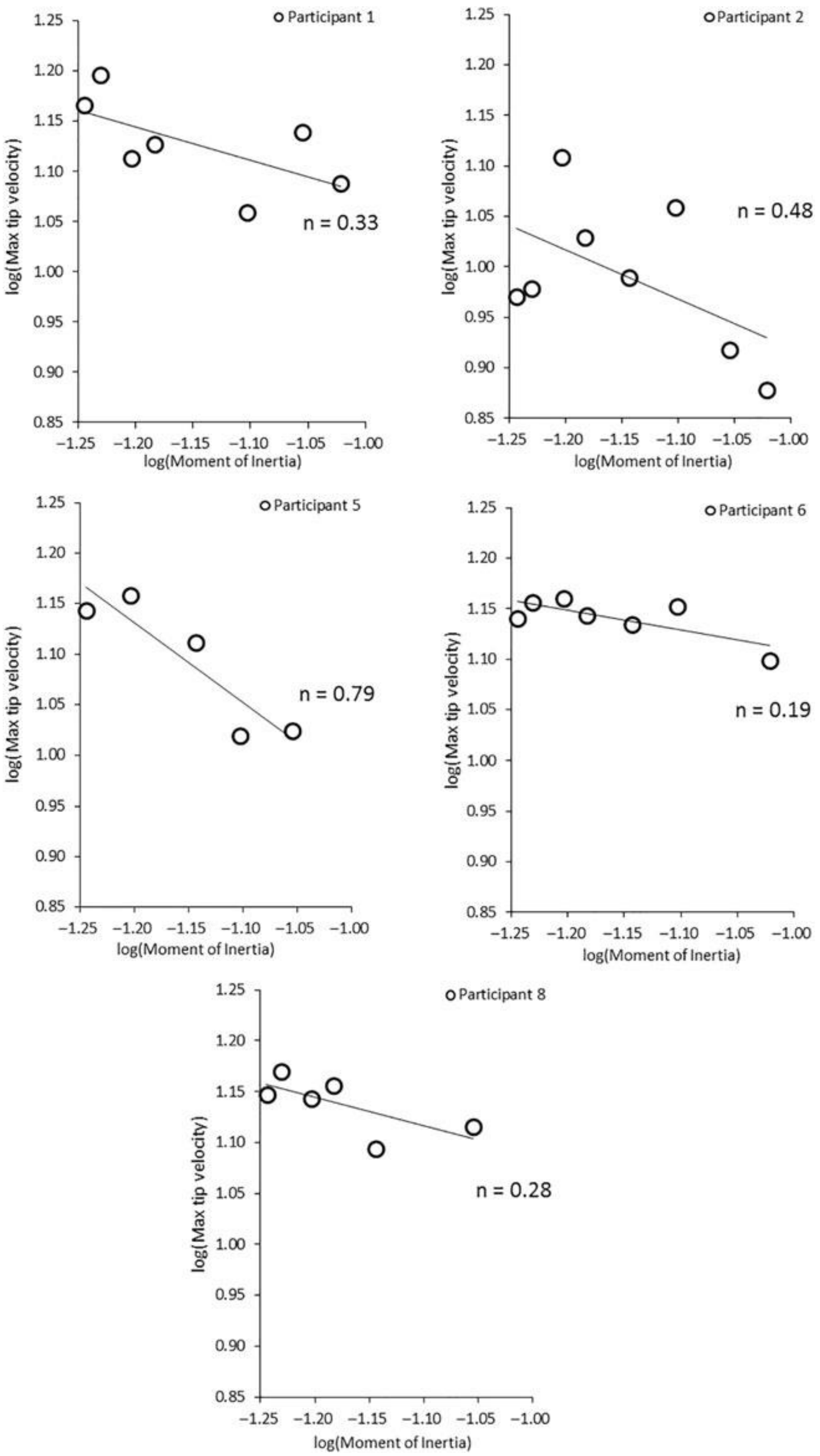

Figure 6. Logarithmic plots of maximum rod tip velocity against moment of inertia.

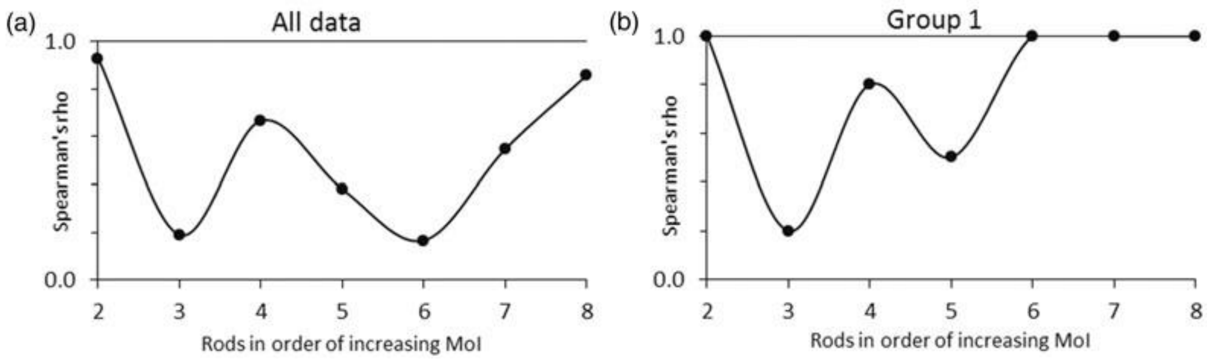

Figure 7. Comparison of participant rank sets between rods of increasing moment of inertia, for all data (a) and the data with exclusions applied (b).