‘Walking in a Foreign and Unknown Landscape’ : studying the history of mathematics in initial teacher education

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Abstract This article develops the argument that students in initial teacher education benefit in terms of who they are becoming from developing awareness of and engagement in the history of mathematics. Initially, current school mathematics practices in the UK are considered and challenged. Then the role of teachers' relationship to mathematical subject knowledge and of teachers' engagement in critical thinking are considered. Connections are made between these concerns and studying the history of mathematics in initial teacher education classrooms. I then draw on the perspectives and practices of the mathematics teacher educators at one institution to understand these connections better and to exemplify them. Issues of equity are threaded throughout.

1. Introduction

Research from a variety of theoretical perspectives indicates that students in initial teacher education gain from developing some awareness of and some engagement with the history of mathematics. (See Kjeldsen and Blomhoj, 2012 and Clark, 2012 for two recent examples and Fauvel and van Maanen, 2000, for an international contribution to the debate.) The study reported here is based on data from one higher education institution where such awareness and engagement has formed part of the students' experience for many years. An earlier research project illustrated the positive effect on the students, as understood by themselves, of such study (Povey, Elliott and Lingard, 2001). This article draws instead on the perspectives of the mathematics teacher educators - their understandings and their descriptions of their practices - to shed further light on the ways in which such benefits accrue, as part of the transformative process of what the students are becoming, with a subtext, considered throughout, which relates to the implications for teaching for equity.

In order to set the context and to see why these matters are of importance, I first consider what school mathematics is currently like and compare this with what research indicates is needed if school mathematics classrooms are to be places where mathematical thinking occurs. Second, I consider what sort of characteristics teachers who promote mathematical thinking for all need to be. Once these are established, we are in a position to make the appropriate set of connections with the value of studying the history of mathematics in initial teacher education classrooms and its role in promoting effective mathematics education for all.

2. Secondary mathematics teaching in the UK: what it is and what it needs to be

That something is currently amiss with secondary school mathematics classrooms in England (and, more generally, in Anglophone countries) is well established. The need for

[1] From Furinghetti, 2007, p133

mathematics teaching to address meaning making by students rather than simply the proficient acquisition of procedural knowledge is long established and well-rehearsed (Brown and Walter, 1993; Cobb *et al*, 1992; Cridland, 2012; Mason, Burton and Stacey, 1982; Schoenfeld, 1988; Watson, 2006; Watson and de Geest, 2005). However, most mathematics classrooms remain places where the emphasis is on learning routines through teacher centred and test dominated practices with a high level of fragmentation in the mathematics presented. Learning mathematics becomes a process of absorbing received knowledge of already existing rules and procedures; and doing mathematics becomes performance. The result is both poor levels of conceptual understanding, creativity and problem-solving skills (Ofsted, 2012) and disengagement and demotivation (Alro and Skovsmose, 2002; Kyriacou and Golding, 2006; Nardi and Steward, 2003).

The issue of motivation, enjoyment and engagement is fundamental and pressing. In general, young people in the UK currently choose not to continue with mathematics as soon as they are able, with less than 2% opting to study the subject at undergraduate level in UK universities (Higher Education Statistics Agency, 2012). In a recent large scale study of 16 year olds, Brown, Brown and Bibby (2008) found that lack of enjoyment coupled with a feeling of boredom with the subject ranked high amongst the reasons given for not continuing.

Table 1 Reasons from boys and girls for not continuing with studying mathematics

Gender	**Male (n=598)**	**Female (n=673)**
Too difficult	37%	66%
Do not enjoy/ like it	24%	35%
Boring	12%	15%
Not needed for future degree/ career	7%	10%
Not useful in life	3%	3%
Prefer other courses	2%	3%
Not continuing with education	7%	3%

Note: these are percentages of those students who gave any reason; some gave more than one reason, so percentages may total to more than 100%.
(adapted from Brown, Brown and Bibby, 2008, p12)

Such disengaged students who are not benefiting from their mathematics schooling are clear about what they want: they do not want to escape from challenge - far from it - but they definitely want their mathematics to be fun.

> When students were asked what sort of things they had enjoyed in their mathematics careers they often used the word ‘fun’ to describe activities. ‘Fun’ often meant different things to

> different students but nearly all students commented that 'fun' activities also helped them remember maths better and reinforced their understanding. (Steward and Nardi, 2002, p4)

Significant, too, about current practices is that the *discipline of mathematics* itself becomes, at best, obscured and, most often, completely disconnected from the experience of the learners.

> Mathematics as a discipline, by contrast to school mathematics, is concerned with thought, structure, alternatives, abstract ideas, deductive reasoning and an internal sense of validity and authority. It is also concerned with uncertainties about ways forward in its own realms of enquiry. (Watson, 2008, p6)

Mathematics in school is experienced as being only a body of already established abstract knowledge, always known and unchanging, a discipline which is 'without fuzziness or debateable results … no experiment, no interpretation of evidence, no comparison of criticisms' (Rodd, 2002, p2): for many learners, such a relationship to the discipline is fraught with difficulty (Boaler, 2002). The argument about whether or not school mathematics can ever be fully like what mathematicians do, can ever fully mirror engagement with the discipline itself, is complex and contested (see Watson, 2008, for a provocative position statement; this is followed by a range of short responses). But some teachers have found the idea of an approach focused on 'becoming a mathematician' (Brown, 2003; Coles and Brown, 1998) and on developing 'mathematical habits of mind' (Watson and De Geest, n.d.) highly productive in generating classrooms in which mathematical thinking occurs. This involves valuing and seeking to inculcate the ways that mathematicians think and work.

> The goal is not to train large numbers of high school students to be university mathematicians. Rather, it is to help high school students learn and adopt some of the ways that mathematicians *think* about problems. In broad strokes … this includes learning to recognise when problems or statements that purport to be mathematical are, in truth, still quite ill-posed or fuzzy; becoming comfortable and skilled at bringing mathematical meaning to problems and statements through definitions, systematization, abstraction, or logical connection making; and seeking and developing new ways of describing situations … [to] give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist. (Cuoco, Goldenberg and Mark, 1996, p376)

It is also well-established that continuing inequities occur in relation to gender, social class and ethnicity in the UK with respect to performance and participation patterns in mathematics (Boaler, Altendorff and Kent, 2011); and there is now a great deal of research evidence that indicates that pedagogies that are designed to promote problem solving and meaning making also promote more equitable classrooms. Such approaches support, for example, 'deep progress' in mathematics with previously low attaining pupils (Watson and de Geest, 2005); they significantly raise attainment in schools where most of the pupils' backgrounds are of low socio-economic status (Watson and de Geest, n.d.); they promote the engagement and participation of girls in mathematics (Boaler, 1997); and, alongside a relational equity approach, successfully counter under-achievement by disadvantaged ethnic groups (Boaler, 2008).

I have argued elsewhere (for example, Angier and Povey, 1999; Povey, 2002; 2003) that classrooms where equity concerns are taken seriously need to be places in which learners set up productive relationships with the process of coming to know. When learners come to see themselves as authoring their own knowledge and therefore as authoritative knowers, not only are they in a position to make considerable gains in understanding; they are also gaining democratic competences (Moreira, 2002) and the sense of themselves as agentic. As Solomon notes '… mathematics can only be made accessible to all in a participatory pedagogy which encourages exploration, negotiation and ownership of knowledge, and the development of the corresponding identity of participation' (2007, p92).

3. Mathematics teachers' relationship to subject knowledge

There is now an extensive literature about the need for teachers of mathematics to have, themselves, good mathematical subject knowledge. More significantly, it has been realised, however, that teachers' relationship to that subject knowledge is also of key importance: it may be such as to offer little support for the sort of pedagogy outlined above. As Silverman and Thompson note, it is now taken as 'axiomatic that teachers' knowledge of mathematics alone is insufficient to support their attempts to teach for understanding' (2008, p499). Prospective teachers may themselves have seldom (never?) previously experienced what has been called the agency of the discipline, with its specific ways of understanding and ways of thinking (Harel, 2008).

Drawing on her seminal work in the United States of America, Ball writes

> long before they enrol in their first education course or a math methods course ...[prospective teachers] have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools. They have already spent years in math classes as students themselves. The experiences they have had with mathematics shape their feelings about the subject and about themselves in relation to it ... prospective teachers do not arrive at formal teacher education "empty headed"; instead they bring with them a host of ideas and ways of thinking and feeling related to math and the teaching of math, drawn largely from their personal experiences of schooling. (1988, p1)

The characteristics of mathematics as a discipline discussed above are likely to be largely absent from that 'host of ideas and ways of thinking and feeling'. If teachers are to teach for understanding and to adopt problem-solving and meaning-making pedagogies, they themselves need to have a productive relationship to mathematics (Povey, 1998; 2002). They need to experience mathematics as a human endeavour undertaken by persons and as a subject to explore, where there is room for fuzziness to be negotiated; for collaborative enquiry; for debate, experimentation and interpretation; for imagination and creativity; and for argumentation and the appreciation of elegance and beauty.

These are all characteristic of mathematicians' ways of working and they are all characteristic of teaching which takes equity concerns seriously - but they are not characteristic of the ways of working of most prospective teachers: 'some of the ideas that students develop about the subjects they study may not accord with the ways in which scholars who work in these fields think about their subjects' (Ball and McDiarmid, 1989, p9).

Understanding, and to some extent experiencing for themselves, how mathematicians come to know mathematics are therefore key components in the development of initial teacher education students for effective participatory teaching for understanding and for equity.

4. Mathematics teachers' critical thinking

If we are seeking an initial teacher education which both supports the challenge to existing, taken-for-granted practices in mathematics education - and one which interrogates the inequities experienced in the *status quo* - we need to develop teachers who are capable of critique, teachers who are critical, reflective, transformative and intellectual. We need teachers who do not simply follow the advice of experts (external authority) nor just do 'what feels right' for themselves (internal authority). Rather they need to be able to weigh up debates, sift evidence, interrogate contradictions and, crucially, produce for themselves and for others well-structured and convincing arguments for the positions and the practices they adopt (Pálsdóttir, Gunnarsdóttir and Kristinsdóttir, 2011), holding their ideas with both provisionality and commitment. They need to develop an "inquiry stance", that is, 'a disposition toward reflective, inquiry-based and analytic thinking' (Darling-Hammond, 2006, p107):

> Preparing teachers to learn from teaching throughout their careers requires a set of tools to develop the skills and practices of systematic, purposeful inquiry and critical reflection. … [It requires helping] student teachers learn critical dispositions and skills that undergird reflective practice, including a commitment to search for answers to problems of practice and the skills of careful observation, data collection, and reasoned analysis. (Darling Hammond, 2006, p106-107)

Developing a 'habit of inquiry' (Darling-Hammond, 2006, p107) is both a personal and an intellectual endeavour, requiring both relevant dispositions and abilities (Hager and Kaye, 1992); and it results in initial teacher education students both recognising and finding their own research-based answers to educational problems. Such critical habits of mind are vital in addressing and overcoming current inequities in schooling (Cochran-Smith, 1991).

5. Studying the history of mathematics in initial teacher education: a local study

As noted above, an earlier research project (Povey, Elliott and Lingard, 2001) explored the impact of studying the history of mathematics as part of initial teacher education from a student perspective. Here, in order to understand better, and also to exemplify, the connections between studying the history of mathematics and the preparation of prospective secondary mathematics teachers, I undertook a small scale study designed to elicit the perspectives, intentions and understandings of lecturers involved in its teaching. The study was based in one UK university where the history of mathematics has been a component of the initial teacher education courses for more than two decades. These courses have included longer routes into teaching (two, three and four year undergraduate routes and a two year post-graduate route where mathematics is studied alongside preparation for teaching) and a one year post-graduate route (where the focus is simply on learning to teach). The data presented below relates to the teaching of the longer routes students who follow a discrete module on the history of mathematics and who are assessed

by an extended essay on a relevant topic of their own choice: I share the view that 'the outputs of [teacher education] programs are not "method-free", that is to say the history of mathematics presented in these courses may have a different influence in teacher education according to the method that has been used to insert it into the program' (Furinghetti, 2007, p133).

A number of staff have taught the history of mathematics over the years but four lecturers in particular, retired and current, have made a major contribution to its development: David, Sue, Jo and Declan. They are all well known to me professionally and all deeply committed to the practice of initial teacher education students studying the history of mathematics. I interviewed all of them for about an hour each, designating the activity as a conversation and using a very loose, semi-structured format. Questions clustered around their own personal engagement with the history of mathematics; what their intentions were and what they valued in their history of mathematics teaching; and what they considered to be the outcomes for the students[2]. I taped the conversations which were personal and informal in tone and listened to each of the tapes three times, over a number of weeks, allowing ideas from the interviews to emerge into consciousness in a way reminiscent of grounded theory (Glaser and Strauss, 1967). Strong personal narratives appeared but here, rather than present these, I have chosen to work thematically. I made extensive notes on each interview and transcribed much of each of them. I then immersed myself in these texts, reading and re-reading them to check out the emergent themes and looking for clarification and refinement. Finally, I had the interviews independently transcribed.

6. Emergent themes

An overarching purpose of the enquiry, shared in the research conversations with the participants, was to address the question: what can studying the history of mathematics with initial teacher education students offer us? No doubt there are many answers to this question[3] (and some of the possible contributions will be by no means unique to such study) but here I present the four answers which emerged most strongly from the interviews: such studies have the capacity:

- to deepen mathematical understanding;
- to broaden and humanise mathematics;
- to develop critical thinking; and
- to provide motivation and fun for learners.

These find echoes in the thinking of Fauvel and van Maanen about the important role of the study of the history of mathematics in the education of teachers:

[2] All of them also had things to say about using the history of mathematics in the school classroom too (as, for example, Rogers,2009; Rogers, forthcoming) but these are not the focus of this article.

[3] A particularly interesting answer is provided by Arcavi and Isoda, 2007. They argue that listening to students in productive ways is at the core of successful mathematics teaching and the "de-centring" needed to engage with historical texts provides valuable experience for teachers in learning how to listen better. This argument has much to offer teacher educators but it did not emerge in my local study.

> There are a number of reasons for including a historical component in such training, including the promotion of enthusiasm for mathematics, enabling trainees to see pupils differently, to see mathematics differently, and to develop skills of reading, library use and expository writing. (1997, p10)

In this section, I draw both on the research literature and also on the observations and reflections of the four lecturers.

Deepening mathematical understanding

We saw above that 'working like a mathematician' and 'developing mathematical habits of mind' are key processes by which leaners deepen their mathematical understanding. Here, I endorse the view of Zazkis that

> … school mathematics and disciplinary mathematics, as well as the intersection thereof, are constantly changing. However, my central claim is that teacher education plays a pivotal role in determining the nature and size of this intersection, in any context. That is, guiding prospective teachers through the experience of "working and thinking like a mathematician" may eventually result in instilling these ways of working in students and thus create a larger intersection between approaches practised in teaching and learning school mathematics and approaches employed in developing disciplinary mathematics. (Zazkis, 2008, p8)

Understanding what it is to 'work like a mathematician' is greatly supported by studying the history of what mathematicians have actually said and done - seeing the stumblings, mistaken arguments, conjectures, experiments and the gradual refinement of ideas as mathematics comes into being. As Laubenbacher, Pengelley and Siddoway note

> Certainly almost every mathematical idea is built upon a succession of preceding ideas. And as one goes back along this chain, the motivation for a problem which started the journey becomes ever clearer, with several works in the chain often standing out as milestones on the road toward our present knowledge. By working through these original sources which discuss and solve, or attempt to solve, antecedent problems, students discover the roots of modern problems, ideas, and concepts, even whole subjects. They also see the obstacles that earlier thinkers had to clear in order to move ahead, and thereby gain insight into current problems and how to approach them. (1994, p2)

Sue describes this process:

> *you can almost relive the pathway to the mathematics, the process that others have taken before so you get more of an understanding of where you're going with it … it's about establishing structure … getting immersed … what were these people, what were they trying to get to grips with*[4]*, you examine that maths then you can make sense of what you're not quite making sense of and don't see the need to make sense of ... so it's a very good way of coming to understand [currently taken-for-granted] structures. (Sue)*

[4] I acknowledge the impossibility of entering the minds of mathematicians of the distant past. As Radford asserts, 'Even the most titanic effort of putting away all our modern knowledge in order to see the historical event in its purity will not succeed: we are damned to bring our modem socio-cultural conceptions of the past with us' (1997, p27). What we are talking about here, however, is the productive results for learning of attempting to do so.

The students, who in Grattan-Guinness' phrase (2004) are certainly "inheritors", are also drawn in to being historians too. Understanding mathematics being supported by, in parallel, understanding what earlier mathematicians needed to understand is also referred to by Jo:

> *understanding the history gives you a much, much richer understanding of the concept than I ever had before so it was a new way of making connections, links between things… one of the things I hugely got for my own mathematical thinking from working with the history of maths is there's something about asking the question "how did people do this before they had X?" gives, actually gives me a better sense of structure and to some extent actually makes me understand X better … how do you think about conics when you haven't got analytic geometry … or cubic equations … if you don't really believe in negative numbers and you definitely don't believe in complex numbers … there are almost stories that have to be told, simplified and not quite true stories, like the story of people who invented imaginary numbers and then they did this and then they did this, I don't think you can really learn about complex numbers without the story about how these were invented at some point to plug some sort of a gap ... the more different points of view you approach something, coming back to my own, my understanding of a lot of concepts has been really deepened by seeing something about where they come from historically. (Jo)*

Declan gave an example about how the study of Egyptian fractions opens up more deeply the mathematics behind the manipulation of rational numbers and David was able to draw on both his own experience and that of the students he worked with on the extended modules as evidence for the way in which such studies deepen understanding:

> *I knew personally from the very little I had read at that time about the history of maths it helped me understand the mathematics, I was a better mathematician because of that … I remember one girl said when they had to choose what they were going to do for their major assessment she wanted to do the calculus - when she handed it in she had an introductory paragraph that she had chosen it because she had never understood any of the calculus at all and she said "I think I now understand what calculus is about, entirely because of my study of the history of it". (David)*

All of the tutors saw the modules not simply as the study of history but equally, if not more importantly, as the study of mathematics as well. This emphasis has grown over the years. The tutors are well aware of the need to avoid the pitfall of some such courses which display a tendency to ' marginalize and eviscerate the very subject matter they champion, generally talking *about* mathematics without actually *doing* mathematics' (Laubenbacher, Pengelley and Siddoway, 1994, p1). This has led to changes not only in the taught components of the course but also what is looked for in the students' extended essay.

> *for me it is a context in which to do mathematics … we've moved much more to the sense that we want them always, the assignment to include actually doing some mathematics … partly, I think because doing the history of maths without getting in there is kind of trivial - can become sort of just scrapbook facts. (Jo)*

> *I thought that they weren't really doing anything any justice just to regurgitate the stories … what Newton had for breakfast! or the influence of his upbringing - I was much more interested in quite why he took that direction in the calculus, for example. I just liked the mathematics and I liked playing around and trying to get to see where that mathematics came from and where it might go to and trying to get inside the heads of mathematicians of*

the time … I wasn't just interested in the historical story, I was interested in the story of the mathematics. (Sue)

As they study the history of what mathematicians have actually said and done, not only are students better able to understand the process of "mathematising", they can also become more forgiving of themselves and their own struggling as they move through 'a sequence of stages: an early intuitive stage and so on, until the mature stage is reached' Bagni, 2005, p1).

certainly with mechanics, I would be talking about Galileo … because you've got to think about the way they would have been thinking to shift a lot of the misconceptions in mechanics. If you're going to shift them, you need to have them out there in order to avoid them. "It's no wonder you feel like this. We were stuck with it for 2000 years, you're not going to get rid of it in a minute, are you? … No wonder it's not making entire sense to you, it hasn't always been this way". (Sue)

In particular, they develop a sense of themselves as also "'making mathematics" and they are able to liken themselves to ' " all those great mathematicians" ' (Povey and Angier, 2007, p1) who struggled in the past to understand mathematics deeply and productively. As Harel points out

> … learning - as a human activity - necessarily involves the construction of imperfect and even erroneous ways of understanding and deficient, or even faulty, ways of thinking. The history of mathematics is rich with cases that attest to this fact. (2008, p10)

The awareness that common, taken-for-granted notions were not always so helps support students in their struggle for understanding.

just by showing that algebra hasn't always been the way you think it is now and drawing attention to the flaws in earlier version of algebra, getting to grips with, you know, it's a <u>decision</u> to use 'x's and 'y's - and there are reasons for it - you know that A did something completely different … it's just a decision - it stops that business of "it's obvious". (Sue)

We saw above that in Brown, Brown and Bibby's study (2008), 66% of girls felt that mathematics was "too difficult" to warrant further study. By studying the history of mathematics and aligning themselves with mathematical struggles through the ages, learners can open up spaces in which to lay claim to a more agentic relationship with the subject and an enhanced sense of self as a knower. This sort of relationship to knowledge is both empowering and less likely to be experienced by those from less privileged groups: such ways of coming to know mathematics and oneself as a knower of mathematics are profoundly important for equity (Povey, 1998; 2002).

In addition, understanding that things have been different in the past will develop and deepen their pedagogic subject knowledge (Shulman, 1986) so that they are able better to see the understandings and misunderstandings of the young people they are teaching and to respond effectively to them.

one of the key things for ITE is realising that the way we do it now is not the only way to do it. So the history of algebra - it's not just history, is it, it's geography as well, different cultures. They have a really strong tendency to think that the way they did it when they were at school

is the method ... to understand that historically that method has been arrived at for all sorts of reasons ... having that wider perspective about the whole notation, language, objects that we talk about ... helps them, the very simple thing, that the realisation that if a child in their room is doing something differently from the method they are going to do on the board that that is worthy of consideration and might be interesting and might make more sense for that child than their way of doing it ... it's also understanding what is arbitrary about our notation and has been an arbitrary historical choice and what has been something that things have converged on because of the important links that something has made with something else. (Jo)

what I was trying to get across was the notion that the algorithms we take for granted are not necessarily the best algorithms you know, historically implanted or whatever - long division ... in Victorian times they used a completely different algorithm ... guess and subtract type algorithm - what was wrong with that algorithm was it was a bit messy and what we sacrificed was what was an intuitive logical method for neatness ... I think you gain insight very often to some of the things we currently do by looking at things that are very, very different. I mean we get our children to do various algorithms almost unquestioningly because there is so much embedded in our own practice and it's quite hard to jump out of them and we do strange things and we say strange things and it isn't until we reflect on them that we realise. (Declan)

Broadening and humanising mathematics

It has long been recognised that studying the history of mathematics helps to 'give mathematics a human face' (Fauvel, 1991, p4); indeed Heiede goes as far as to say that 'a historical creature is not alive if the past is not alive in him or her' (Heiede, 1992, p152). Understanding that mathematics as we know it is a product of human beings, some in distant times and places, some closer to home, has a significant role to play in opening up the accessibility of mathematics as a discipline (Povey, Elliott and Lingard, 2001). What David referred to as 'the people issue' came up in all the conversations. For example, Jo said,

I'm a person who did one year of maths [at university] and then dropped it and did philosophy because of something it didn't give me. I think looking back that one of the things it didn't give me were the stories of where the concepts came from historically ... it felt to me always like there were gaps ... why did people, why those concepts ... at what point do people put these things together to make this the abstraction that they want to do ... why might anyone ever be interested in this ... why were people interested in naming this concept.

... it also brings emotion in, there is emotion in these stories ... I get very emotional responses from people in the room and is also a moral that mathematics is not a subject devoid of emotion. (Jo)

[concerning research on reasons not choosing maths] One quote really stood out and struck home, "Maths is about things. It's not about people". (David)

There's more to mathematics than mathematics which is a strange thing to say. What I mean by that is that mathematics is full of things related to the human condition, personalities, to the time that the mathematics took place ... sometimes in mathematics what we do ... is we strip out that kind of human side to it and I think history of mathematics perhaps more than most has got the human side back in. (Declan)

it's about recognising that there are real human beings having real human struggles - it's really about the nature of mathematics, where did it come from, who it belongs to ... and that it is a human activity. (Sue)

There is an extensive literature related to the issue of to whom mathematics "belongs", a literature informed by both gender (see, for example, Burton, 1995) and ethno-mathematics considerations (see, for example, D'Ambrosio, 2001) and both of these featured in the lecturers' reflections on their practices - 'we're getting a broader cultural approach, it's not all about famous European men' (Declan). All the lecturers were committed to an equity agenda and saw the use of the history of mathematics as contributing to addressing it.

[when I was teaching in school] there was someone in the staffroom who literally said "what has Africa ever given to mathematics?" and I said well, apart from Pythagoras theorem and algebra not very much! this was a mathematics teacher of many years standing ... we spend quite lot of time looking at non-European mathematics and we explicitly talk about what they find in some of the more old fashioned books and what they may have found in school ... there are relatively few images available of women as mathematicians, non-Europeans as mathematicians, that is part of the explicit content of the course, it seems important to me that when people go into initial teacher education that they get that awareness and they can name some women mathematicians and some non-European mathematicians. (Jo)

They talked about the mathematics content of the course - not just the 'really important things mathematically, so calculus obviously, the development of algebra obviously, but Islamic geometry - stuff that people have no idea about at all because of the Eurocentric view' (David) - and they described particular themes pervading it. Crucially, in terms of equity, these were not seen simply as being gender-sensitive or multicultural - though they are both of these - but as opening up a critical sense of both difference and oneself as responsible and agentic.

There are themes that come in, that one can use history to bring in, that are important for mathematics and for politics. We talk about how Descartes was constrained by the religious laws of his time in what he could say, Sophie Germain was constrained by the views on what a woman could do from what she could do and what could happen. But actually these are all stories that are about how are you, here and now, as human beings, constrained in terms of what you would like - it is another way in to that sense of people being political actors. (Jo)

This link between the study of the history of mathematics and the opening up of a space for critique is developed further in the next section.

Developing critical thinking

I argued above that developing a critical frame of mind to mathematics, teaching, learning and schooling was essential for mathematics teachers if they are to be able to successfully interrogate contemporary practices in order to both improve their effectiveness and to make them more just. They need, in Declan's words, to 'think outside the box'. Studying the history of mathematics is one particular way of developing such critical thinking because it helps create distance from our own culturally and temporarily limited experience by engaging us in an awareness of "otherness", what Barbin (1994, cited in Bagni, Furinghetti and Spagnolo, 2000) has called *depaysement*, in which the usual is

displaced by something different, challenging perceptions through 'making the familiar unfamiliar' (Bagni, Furinghetti and Spagnolo, 2000, p3).

> To walk in the foreign and unknown landscape provided by history forces us to look around in a different manner and brings to light elements which otherwise would escape. (Furinghetti, 2007, p133)

Radford, Furinghetti and Katz argue, following Bakhtin, that it is only by experiencing what is foreign and thus seeing how things can be otherwise that closed and one-sided thinking can be overcome.

> … history erects itself as the place where we can surmount the one-sidedness of our particular meanings; it is a place to enter into dialogue with others, and with the historical, conceptual products produced by the cognitive activity of those who have preceded us in the always changing life of cultures. History provides us with a unique experience, one that completes the ephemeral moment in which we live and that reveals to us the depth of the conceptualisations that we share now with our contemporaries. (2007, p109)

As Jo expressed it:

> *… the link between the history of maths and equal opportunities - it's that the history dimension helps you to have the geographical and just the sense that this is like this here and now with these people, it was different some time ago, it could be different somewhere else, it could be different for other people is key - the understanding of difference. (Jo)*

The opportunity for developing critical thinking and the 'habit of inquiry' as cited above is foremost in the minds of the lecturers I interviewed. They see this as threading through the module but particularly enhanced by the form of assessment chosen, the extended essay on a topic of their choice.

> *When it came to how we were going to assess this, there was no decision, no choice about that, it had to be basically we've got to turn these people into researchers. (David)*

Very great emphasis, from the beginning of the module and throughout, is placed on the need for independent research - a bibliography is provided but it is 55 pages long - and in individual tutorials, which form a central element of the support for the module, the lecturers focus sharply on the elements required for critical enquiry

> *we work really quite hard on how do you set a question, how do you structure your essay around why have I chosen these particular facts and what do they tell me about my question, you may start with the facts but at some point you have to choose a question that you feel your facts answer …it's the engagement with that process … in the end it's not that some facts are more relevant ... [it's] not so much which facts you've selected but how well you've told me why those facts are relevant and there may be different stories you tell depending on what you view of history is … I hope what is coming in here is some sense of constructing arguments … you have to ask your own questions … and you have to be critical about them, you have to amass evidence. (Jo)*

The process of framing a question, working out what you need to find out in order to answer it, expecting to interrogate the evidence and to make choices about it and, finally, coming to

a point of view are all activities required of the students by their projects; and these are processes with which a critical, reflective teacher needs to be familiar.

> *looking for evidence, supporting your case, being an academic ...teaching is about ... not making assumptions, about people, about ways of learning, about the way people are thinking, about the way people are feeling, what's going on with either the mathematics or the people you're relating with right now, opening your eyes to different points of view ... just being a bit more flexible and just being prepared to do a bit more listening really. (Sue)*

> *critical thinking is what it comes down to, but the critical thinking that has to have a historical dimension - this is not the only way it could be and that of course is also true of the way schools are set up and everything else ... there is a theme about it being critical thinking which you need as a teacher of mathematics because there is a sort of assumption that you can be a comfortable teacher of mathematics without it. (Jo)*

Providing motivation and fun

The final theme to emerge from the lecturers' interviews was related to engagement and enjoyment of mathematics. As we have seen above, Heiede suggests that mathematics is not alive without its history and indeed he claims that this "deadness" of the subject is one reason why so many people find it 'dull, boring, uninteresting, even hateful' (1996, p232). David had experienced the way in which the history of mathematics motivated school students and found that, as he anticipated, initial teacher education students, in general, found it motivating too.

> *The crucial thing was that it was immediately obvious that the kids were engrossed. Even kids who weren't switched on to maths found it interesting ... the motivation for doing it [at university] was that I just knew how much kids in school had valued this and so I had no reason to suppose this wouldn't be true ... and feedback from the vast majority of students ... "Why didn't we do this in school?" (David)*

Declan drew on his experience with interviewing other undergraduate students to justify his belief that the history of mathematics provides enjoyment for learners:

> *I have interviewed people over the years and very often asked what did you enjoy in your undergraduate degree and frequently they would draw on the history of mathematics, frequently. (Declan)*

He linked this enjoyment of the subject by students themselves with their later capacity to generate pleasurable engagement in their own classrooms when they come to teach.

> *it's not just for the student now it's for the students later, if that student goes on to become an exciting teacher of mathematics, we want them to take a much broader view, we want them to see - hey, look, mathematics can be fun, it's got this going on, it's got that going on and it's just fascinating. (Declan)*

All of them found that the extended essay, although daunting for the students, was very frequently a source of the students' personal and intellectual growth.

> *One of the things I particularly like is that the students can negotiate the areas that they actually do and I think that's good because to give them something which they can actually*

pick up and follow through ... last year [some] ... got completely carried away with the history of maths - it gave them success as well. (Declan)

And it's interesting and fun ... a lot of them find it fun and get sort of wedded to [their enquiry] (Sue)

they do appreciate the opportunity of getting really deeply into something, they appreciated being able to get very much into depth on one thing ... I value so much them having the chance to make their own choice of what they're going to do ... and we do tend to get feedback that that is really valued. (Jo)

8. Conclusion

In this article I have first argued that all is not well with secondary mathematics teaching in the UK and that teachers' engagement with and the depth of their mathematics subject knowledge needs enhancing. I have argued that studying the history of mathematics sets up a productive relationship with the subject and deepens mathematical understanding. I have pointed to research that links the low take-up and alienation from mathematics to its being perceived as a cold and deadening subject, not related to people or culture; and I have suggested that, again, becoming aware of its history opens up and humanises mathematics. I have further argued that both the general state of mathematics teaching, and also the inequities with which it is currently riven, demand teachers who can think critically, who can see beyond the taken-for-granted and envision difference from the here and now. Studying the history of mathematics requires the development of such critical skills and can develop a disposition towards enquiry based on question posing and evidence. Finally I have argued that studying the history of mathematics also provides pleasure. I have illuminated each of these claims by drawing on interview data from lecturers in one institution who have been engaged for many years in teaching the history of mathematics in order to flesh out and make alive what is being asserted.

With Furinghetti, I believe that, 'There are many ways to acquire an aware style of teaching (2007, p142); however, the history of mathematics has clearly much to offer in initial teacher education to the process of becoming a teacher. This is a transformative process and involves the whole person. Echoing Heiede quoted above, Radford, Furinghetti and Katz claim that the interaction, the to-ing and fro-ing, that engagement with past cultures entails not only deepens our understanding of mathematics; it contributes to 'our growing into whole human beings' (2007, p109). That, in turn, contributes to our capacity to know and to act. I leave the last word with Jo:

> [finding out about the history of mathematics] has made me realise that there are many more questions to ask than I ever thought about before and there's probably no end to that, and I think that's a good thing for maths teachers to know. (Jo)

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