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ANALYSIS OF INTERFACIAL SHRINKAGE STRESSES IN PATCH REPAIRS

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Abstract

The paper presents simple analytical expressions which predict the interfacial shrinkage stresses in a repair patch over time. Four repair materials (L2, L3, L4 and G1) were applied by spraying (gunite) to unpropped compression members of two highway structures and their performance was monitored to approximately six months age. The elastic modulii of all the repair materials, E_{rm}, were greater than the elastic modulii of the substrate concrete, E_{sub}. The mechanics of patch repair interaction with the substrate were established and analytical models, based on an analogy of the bi-metallic strip undergoing a drop in temperature, were developed. Basic properties of the repair material (elastic modulus, shrinkage and tensile creep), substrate concrete (elastic modulus) and geometrical details of the repair patch are required to analyse the interfacial stresses in the repair patch. Verification of the analytical procedures is based on the field data and the results show a satisfactory correlation between the actual and predicted stress redistribution.

1 Introduction

Reinforced concrete is the most widely used construction material due to its relatively low cost and ease of placing. It gives excellent durability when designed, constructed and maintained correctly, justifying the design lives of 60 to 120 years [1]. However, when the

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effects of inadequate detailing, poor workmanship or severe exposure to harsh environments are experienced, the steel reinforcement corrodes which leads to spalling of the concrete. The design life of a structure can be extended by replacing the damaged concrete with a new material. This process can be successful if the repair material is selected on the basis of an adequate understanding of the interaction between the substrate concrete and repair patch. Too often, this is not the case and repair materials are applied with little knowledge of their long term performance under service conditions.

Until recently, the main physical property considered in selecting repair mortars was compressive strength [2]. However, as the understanding of the structural performance of repair patches increases, greater attention is being given to stresses caused by the shrinkage of repair materials [2,3]. For example, modelling of shrinkage stresses in concrete repair has been carried out through finite element (FE) analysis [4,5]. In addition, there are many publications which concentrate on moisture diffusion in concrete [e.g. 6,7] as a way of predicting tensile stress due to shrinkage. It has been established by the authors that properties such as elastic modulus, shrinkage and tensile creep of the repair material are primarily responsible for the structural interaction [8,9,10,11].

In this paper, simple expressions to predict the interfacial stresses in the repair patch due to shrinkage are derived.

2 A theory to predict the structural interaction in repair patches

2.1 Assumptions based on field data

A repair material with an elastic modulus (E_{rm}) greater than that of the substrate concrete (E_{sub}) will transfer a portion of its shrinkage strain to the substrate concrete [8,9]. The amount of shrinkage strain transferred depends on the modular ratio (E_{rm}/E_{sub}), with optimum shrinkage transfer taking place when $E_{rm} > 1.3\ E_{sub}$ [9]. Figure 1 shows the simplified strain distribution with time for a typical repair material of $E_{rm} > E_{sub}$, which is based on wide ranging field data of repair to bridge structures [8,9,10,11]. The interfacial strain in the substrate concrete (and due to strain compatibility, the restrained shrinkage strain in the repair material) was measured by means of a vibrating wire strain gauge (gauge length 140mm). The gauge was located on the cut back substrate concrete as shown in the section through the repair in Figure 1. Extensive details of the highways bridges, strain monitoring equipment employed and repair material properties are given elsewhere [9].

The strain profile presented in Figure 1 is at the repair material/substrate concrete interface (labelled 'subs'). In zone 1, shrinkage strain, due to full bond, is transferred to the substrate concrete (weeks 0 to 11) as the stiffer repair material contracts due to shrinkage. The free shrinkage of the repair material is also shown in Figure 1, zone 1. A virtual tensile strain is present in the repair material at its interface with the substrate concrete which is the difference between its free shrinkage strain and the shrinkage transfer strain measured in the 'subs' gauge. In zone 2, weeks 11 to 25, the strains in the substrate concrete remain constant due to negligible further shrinkage occurring in the repair material in this period.

The transfer of shrinkage strain in zone 1 is shown diagrammatically in Figure 2. Figure 2 (a) shows an oblique view of a compression member repaired in the unpropped state, with $E_{rm} > E_{sub}$. Shrinkage strain of the stiffer repair material is transferred to the substrate concrete during the shrinkage period, weeks 0 to 11. The influence of the steel reinforcement on the transfer of shrinkage strain is omitted for simplification and clarity. Since the substrate concrete has an infinite thickness compared with the thickness of repair patch, the shrinking repair material is assumed only to influence an area adjacent to the interface. This zone of influence, as shown in Figure 2 (b), is assumed to have a depth of

d_{sub}. This depth (d_{sub}) is related to the depth of the repair patch (d_{rm}) and its magnitude is derived in Section 2.3.2.

2.2 *Spring analogy to represent structural interaction in the shrinkage period (zone 1, weeks 0-11)*

When a stiffer repair material shrinks, some compression will be forced into the substrate concrete. Consequently, deformation is evident in both the repair material and substrate concrete, since full bond exists between both materials. Therefore, the simultaneous deformation of the repair material and substrate concrete in the shrinkage period (zone 1, Figure 1) can be represented by the analogy of two deforming springs connected in parallel, as shown in Figure 2 (c). The repair material is represented by a spring of high stiffness whereas the substrate concrete is represented by a spring of lower stiffness. When the stiffer spring compresses, the less stiff spring will provide partial restraint to the deformation. Nevertheless, both springs will exhibit similar strains at the interface due to strain compatibility. The free surface of the stiffer spring will undergo greater compressive strain (equivalent to the net shrinkage of the repair material). This will result in a bending effect in the spring system, as shown in Figure 2 (d). This bending effect is used as a basis to predict the distribution of shrinkage strain in the repair patch as described in the following section.

2.3 *Distribution of shrinkage strain using analogy of bi-metallic strip*

The eccentric effects represented in Figure 2 (d) causes bending in the spring system. Therefore, the deformation in the spring system, and consequently in the repair patch, can be compared to a bi-metallic strip undergoing differential contraction. A bi-metallic strip consists of two dissimilar materials which are perfectly joined at the interface so that they deform together when the temperature is raised or reduced. The coefficients of thermal expansion of the two materials are different, therefore, one metal has the tendency to deform more than the other. An elevation of a bi-metallic strip is shown in Figure 3 (a). The strip consists of two materials, labelled A and B, which exhibit different coefficients of expansion (or in the context of this paper, shrinkage strain). Figure 3 (b) shows an enlarged elevation of a portion of the bi-metallic strip at the datum temperature. Assuming that the materials are not connected at the interface, each would contract freely as shown in Figure 3 (c) when the temperature is reduced. The contraction shown from Figure 3 (c) onwards is exaggerated for clarity. Material A contracts from the datum position (Level 0) to Level 1; material B contracts from Level 0 to Level 2. In reality, due to perfect bond, both materials contract equally at the interface (Level 3), as shown in Figure 3 (d), and a strain gradient will be evident across the strip. Material A will be forced to contract more by a compression force exerted at the interface. Material B, however, is partially restrained by material A and is prevented from undergoing its full free contraction. As a result, a tensile force is exerted in material B at the interface [Figure 3 (d)]. The differential strains in the bi-metallic strip lead to circular arc bending as shown in Figure 3 (e). The radius of curvature of the deflection curve, *R,* is so large compared to the cross sectional dimensions of the strip that it may be taken as the same for both materials A and B. The internal force system for materials A and B is a longitudinal force F_A and F_B respectively and bending moment M_A and M_B respectively, as shown in Figure 3 (f). Material A compresses under the longitudinal force, F_A, and bends under the action of the moment M_A (compression will be induced at the common interface for material A). Material B will exhibit tension under the action of the central force, F_B, and also at the interface due to the action of the moment, M_B [see Figure 3 (f)].

2.3.1 Analysis of shrinkage strain in a repair patch

Figure 4 (a) shows a cross-section through an unpropped compression member, repaired with a material with $E_{rm} > E_{sub}$. The external load remains in place throughout the application of the repair material (unpropped repair). Immediately after application and before shrinkage begins, the repair material extends the full length of the repair patch, labelled Level 0 to Level 1 in Figure 4 (b). Assuming the substrate concrete has a negligible elastic modulus ($E_{rm} \approx 0$), the repair material could shrink freely from Level 0 to Level 2 in Figure 4 (b), displaying a free shrinkage strain, $\varepsilon_{shr(free)}$. In an actual repair situation, the substrate concrete has a stiffness which in the case being considered is less than the stiffness of the repair material ($E_{sub} < E_{rm}$). The repair material, therefore, is prevented from deforming freely due to the partial restraint provided by the substrate concrete. This restraint will be maximum at the substrate concrete/repair material interface, but will gradually reduce as the distance from the interface increases. Bending in the form of a circular arc [Figure 4 (c)] will occur similar to the bi-metallic strip [Figure 3 (e)] due to the restrained shrinkage forces, F_{shr}, at the interface as shown in Figure 4 (c) - tensile in the repair material and compressive in the substrate concrete. The repair material is, therefore, assumed to shrink from Level 0 to Level 3 at the interface [Figure 4 (d)]. The interfacial bond at the interface (assuming no slip) enables the substrate concrete to deform also from Level 0 to Level 3. A strain gradient will be evident across the repair patch and the zone of influence in the substrate concrete. This is similar to the strain gradient across the bi-metallic strip in Figure 3 (d). The internal force system for both the repair material and substrate concrete can be reduced to longitudinal forces acting along each centroidal axis, F_{shr} (tension and compression respectively) plus bending moments ($M_{rm(shr)}$ & $M_{sub(shr)}$), Figure 4 (d). These bending moments will be produced by the eccentric interfacial forces, F_{shr}, acting at $d_{sub}/2$ and $d_{rm}/2$ respectively from the centroidal axis of the substrate concrete and the repair i.e. $M_{rm(shr)} = (F_{shr})(d_{rm}/2)$ and $M_{sub(shr)} = (F_{shr})(d_{sub}/2)$.

2.3.2 Depth of substrate concrete affected by shrinkage of the repair material (zone of influence)

The magnitude of d_{sub}, the depth of the substrate concrete (zone of influence) affected by the transfer of shrinkage strain from the repair material, can be calculated with reference to Figure 4 (c). The radius of curvature of the deflection curve, R, is so large compared to the cross-section dimensions of the zone of influence and repair patch that it can be taken as the same for both, therefore, from elastic theory of bending:

$$\frac{1}{R_{rm}} = \frac{1}{R_{sub}}$$

Equation 1

Equation 1 can be re-written in the form

$$\frac{M_{sub(shr)}}{E_{sub} I_{sub}} = \frac{M_{rm(shr)}}{E_{rm} I_{rm}}$$

Equation 2

Substituting for $M_{rm(shr)} = (F_{shr})(d_{rm}/2)$ and $M_{sub(shr)} = (F_{shr})(d_{sub}/2)$ and expanding the second moment of area terms in Equation 2 gives:

$$\frac{F_{shr}\left(\frac{d_{sub}}{2}\right)}{E_{sub}\left(b\,\frac{1}{12}\,d_{sub}^{3}\right)} = \frac{F_{shr}\left(\frac{d_{rm}}{2}\right)}{E_{rm}\left(b\,\frac{1}{12}\,d_{rm}^{3}\right)}$$

Equation 3

Simplifying Equation 3 gives:

$$d_{sub}^{2} = d_{rm}^{2}\,\frac{E_{rm}}{E_{sub}}$$

Equation 4

Replacing E_{rm}/E_{sub} with m in Equation 4 and simplifying, the depth of substrate concrete, d_{sub}, affected by the transfer of shrinkage strain can be obtained from

$$d_{sub} = d_{rm}\sqrt{m}$$

Equation 5

2.3.3 Equilibrium of forces in the repaired section

Referring to Figure 4 (d), the normal distance between the forces F_{shr} is $\frac{1}{2}(d_{rm} + d_{sub})$. The couple produced by these forces must, for equilibrium, balance the sum of the moments in the repair and substrate materials. Thus

$$\frac{F_{shr}}{2}(d_{sub} + d_{rm}) = M_{sub(shr)} + M_{rm(shr)}$$

Equation 6

From the elastic theory of bending, $M_{sub(shr)} = (E_{sub}I_{sub})/R$ and $M_{rm(shr)} = (E_{rm}I_{rm})/R$, therefore Equation 6 can be written as

$$\frac{F_{shr}}{2}(d_{sub} + d_{rm}) = \frac{E_{sub}I_{sub}}{R} + \frac{E_{rm}I_{rm}}{R}$$

Equation 7

Rearranging Equation 7 gives:

$$F_{shr} = 2\left[\frac{E_{sub}I_{sub} + E_{rm}I_{rm}}{d_{sub} + d_{rm}}\right]\frac{1}{R}$$

Equation 8

The only unknowns in Equation 8 are the force due to shrinkage, F_{shr}, and the radius of curvature due to bending, *R.*

2.3.4 Strain compatibility at the interface

A second relationship can be obtained by considering the strain compatibility of the two materials (repair and substrate) at the interface. These strains are made up of three components, (i) the free shrinkage of the repair material $\varepsilon_{shr(free)}$, (ii) the elastic strains due to the moments, $M_{sub(shr)}$ and $M_{rm(shr)}$, and (iii) the elastic strain due to the longitudinal forces, F_{shr}. Any creep strains in the repair material are neglected at this stage but are considered in Section 2.4.

(i) Strains due to free shrinkage, $\varepsilon_{shr(free)}$

The free shrinkage strains of the repair materials, $\varepsilon_{shr(free)}$, were determined in the laboratory (see Table 1, column 4 [12]).

(ii) Strains due to bending, $\varepsilon_{sub(bend)}$ and $\varepsilon_{rm(bend)}$

Assuming the centroidal axis of the substrate concrete and repair material in Figure 4 (d) to be at half the depth (ignoring the effects of the steel reinforcement), the distance to the interface is $d_{sub}/2$ and $d_{rm}/2$ respectively. Therefore, from the elastic theory of bending, the stress at the interface of the substrate concrete and repair material can be obtained from:

$$f_{sub(shr)} = \frac{d_{sub}E_{sub}}{2R}$$

Equation 9

and

$$f_{rm(shr)} = \frac{d_{rm}E_{rm}}{2R}$$

Equation 10

Dividing Equations 9 and 10 by the elastic modulii of the respective materials gives the strain in each material:

$$\varepsilon_{sub(bend)} = \frac{d_{sub}}{2R}$$

Equation 11

$$\varepsilon_{rm(bend)} = \frac{d_{rm}}{2R}$$

Equation 12

(iii) Strains due to longitudinal force, $\varepsilon_{sub(shr)}$ and $\varepsilon_{rm(tens)}$

The strain at the interface in the repair material due to the axial force, F_{shr}, is given by:

$$\varepsilon_{rm(tens)} = \frac{F_{shr}}{bd_{rm}E_{rm}}$$

Equation 13

Similarly, the strain in the substrate concrete is

$$\varepsilon_{sub(shr)} = \frac{F_{shr}}{bd_{sub}E_{sub}}$$

Equation 14

(iv) Net Strain

At the common interface between the repair material and substrate concrete, the net strain in the substrate concrete is equal to net strain in the repair material. Therefore:

$$\left[\frac{F_{shr}}{bd_{sub}E_{sub}}\right] + \frac{d_{sub}}{2R} = \varepsilon_{shr(free)} - \left[\frac{F_{shr}}{bd_{rm}E_{rm}}\right] - \frac{d_{rm}}{2R}$$

Equation 15

Rearranging Equation 15 gives

$$\frac{F_{shr}}{b}\left[\frac{1}{d_{sub}E_{sub}} + \frac{1}{d_{rm}E_{rm}}\right] + \frac{1}{2R}[d_{sub} + d_{rm}] = \varepsilon_{shr(free)}$$

Equation 16

Equation 15 can be rearranged to give F_{shr} in terms of *R:*

$$F_{shr} = \frac{\left(\varepsilon_{shr(free)} - \frac{1}{2R}(d_{sub} + d_{rm})\right)b}{\left(\frac{1}{d_{sub}E_{sub}} + \frac{1}{d_{rm}E_{rm}}\right)}$$

Equation 17

Substituting for d_{sub} from Equation 5 and simplifying gives:

$$F_{shr} = \frac{d_{rm}E_{rm}b\left(\varepsilon_{shr(free)} - \frac{1}{2R}d_{rm}\left(\sqrt{m}+1\right)\right)}{\left(\sqrt{m}+1\right)}$$

Equation 18

The only unknowns in Equation 18 are the force due to restrained shrinkage, F_{shr}, and the radius of curvature, *R.* Therefore, the simultaneous equations 8 and 18 can be solved to determine the values F_{shr} and *R.* Hence, the compressive stress at the interface of the substrate concrete due to a transfer of shrinkage strain from the stiffer repair material at the end of Zone 1 (Figure 1, week 11) can be determined as:

$$\sigma_{sub(shr)} = \frac{F_{shr}}{b\sqrt{m}d_{rm}} + \frac{E_{sub}\sqrt{m}d_{rm}}{2R}$$

Equation 19

Similarly, the tensile stress in the repair material (at the interface) can be obtained from

$$\sigma_{rm(shr)} = -\frac{F_{shr}}{bd_{rm}} - \frac{E_{rm}d_{rm}}{2R}$$

Equation 20

2.4 Validation of the theory

2.4.1 Analysis of key data

Table 1 shows the repair material properties (further details of the repair materials composition is given elsewhere [9]), repair patch dimensions and calculated values which are needed for determining the *predicted* stresses in the repair patches of materials L4, L3, L2 and G1. Columns 2-6 gives the properties/dimensions that are required for the analysis and columns 7-9 give values obtained from analysis (Equations 5 (d_{sub}) and Equations 8 & 18 (F_{shr}, R)).

The elastic modulus of the repair material (column 2, Table 1) and substrate concrete (column 3, Table 1) were determined by testing 100mm diameter cylinders and cores respectively in the laboratory in accordance with BS 8110 [13] (the analytical procedure employs a straightforward approach where possible, for example, E_{rm} is determined as a compressive value as opposed to a tensile value for simplicity). The magnitude of free shrinkage was originally determined from 500 x 100 x 100mm prisms stored at 20°C and 55% RH in the laboratory. These shrinkage strains were modified by taking into account differences that exist between the controlled conditions in the laboratory and the uncontrolled environment in-situ, such as differing temperature, relative humidity and surface/volume ratios. Details of these modifications are presented in detail elsewhere [12]. The shrinkage of the repair patch determined from the modified laboratory data are given in column 4 of Table 1.

The *actual* stresses in the field were obtained by converting the measured strains (Figure 1) to stresses by multiplying by an effective elastic modulus. The compressive strain in the substrate concrete and tensile strain in the repair material were determined as described in Section 2.1. The basic properties of the repair materials were determined in the laboratory (elastic modulus, shrinkage and creep). However, to meaningfully convert the field strains to stresses, the basic properties were extensively modified to take into account differences that exist between laboratory and field data. For example, the free shrinkage was modified to account for the influences as described above. The elastic modulus was determined at 28 days age under a compressive state of stress in the laboratory but was modified to allow for the influence of developing elastic modulus at earlier ages. The effect of determining the elastic modulus under a compressive state of stress as opposed to a tensile state of stress was also accounted for, as the restraint to shrinkage induces tension in the repair patch in the field. Furthermore, the restraint to shrinkage in the field induces creep in the repair material and this was also taken into account. Details of the procedures adopted are outside the scope of this paper and are presented elsewhere [12].

2.4.2 Creep effects on elastic stress distribution

Throughout this paper, the theory presented is based on an elastic approach for determining the interfacial stresses in a repair patch. In reality, creep will play an important role in the stress distribution since the repair material which is partially restrained from shrinking develops tensile stress and thus undergoes tensile creep resulting in stress relaxation. Therefore, the influence of creep on the stresses developed in the repair material was also determined by employing an effective elastic modulus [12,15], $E_{rm(eff)} = E_{rm}/(1+\phi)$ where ϕ is the creep coefficient. $E_{rm(eff)}$ was substituted in place of E_{rm} in the simultaneous equations 8 and 18 and the results were used to calculate the stresses from equations 19 and 20. The application of the effective elastic modulus incorporating the creep coefficient is justified since it follows a similar approach used in reinforced concrete design in accordance with BS8110 [14] where structural elements are designed using elastic analysis but the effects of creep are accounted for by employing an effective elastic modulus [15]. The stresses calculated using both the elastic approach and the approach taking account of the creep coefficient are presented in Figures 5-8, with the suffix 'model - elastic' or 'model - creep' to distinguish between the two procedures as appropriate. Details of the creep properties of the repair materials and the procedures used to determine E_{rm} and $E_{rm(eff)}$ are given in another paper by the authors [12]. It was assumed that no creep strains occur in the substrate concrete since in a practical repair situation the substrate concrete has been under service load for many years and has already undergone creep. Secondly, the compressive stresses redistributed from the repair patch are too small to cause any significant creep (i.e. the stress/strength ratio is insignificant).

2.4.3 <u>Comparison between field data and predicted stresses</u>

Figures 5 to 8 show the comparison between the actual and predicted interfacial stresses at week 11 (other values within zone 1 are linearly interpolated). The actual stresses in the substrate concrete and repair material are denoted 'subs @ interface (actual)' and 'rm @ interface (actual)' respectively. Two values of the predicted stresses are presented, one based on elastic analysis and the other incorporating the creep coefficient. A summary of the comparisons is also given in Table 2. Referring to Figure 5 and Table 2, the actual compressive stress in the substrate concrete at week 11 after transfer of shrinkage from the repair material is 3.7 N/mm^2. The predicted compressive stress from the elastic analysis in the shrinkage period is 3.0 N/mm^2. When the effect of creep in the repair material is taken into account, the stress in the substrate concrete reduces to 2.5 N/mm^2. The actual tensile stress in repair material L4 at the end of the zone 1 is 1.2 N/mm^2 whereas the predicted tensile stress (elastic analysis) in the repair material is 3.3 N/mm^2. This tensile stress reduces to 1.7 N/mm^2 when the effects of creep are considered. Therefore, the analytical model reasonably predicts the stresses in the repair patch of material L4, although the elastic analysis overestimates the stresses. The actual and predicted stresses at the end of zone 1 are assumed to remain constant in zone 2. Similar presentations of data and analysis for materials L3, L2 and G1 in Figures 6 to 8 and Table 2 show good correlation between the actual and predicted values.

2.5 Design recommendations for patch repairs

The application of the analytical model to real field repairs rather than controlled laboratory experiments provides a critical test for the practical application of the analytical procedure in design. The degree of agreement between the measured and predicted values is likely to be higher in controlled laboratory experiments.

The compressive stresses transferred to the substrate concrete by the shrinking repair material are small compared to typical compressive strengths of substrate concretes e.g. 3 N/mm^2 for a substrate concrete of design strength of 40 N/mm^2 and, therefore, are not a cause for concern. However, the accurate prediction of tensile stresses in the repair material is critical at the design stage of repair to ensure that restrained shrinkage tension does not lead to cracking. The predicted tensile stresses using elastic analysis (Equation 20) are somewhat greater than the actual stresses, whereas the stresses predicted when creep is taken into account are either slightly higher or lower, see Figure 9 and Table 2. On average, the elastic analysis yields tensile stresses greater by a factor of 2.1 than the analysis which include the tensile creep effects (Table 2). In current practice, the creep properties of commercial repair materials are not readily available from the manufacturers. When designing repairs with these materials, it is recommended to use elastic analysis for the estimation of tensile stresses induced in the repair patch by restrained shrinkage whilst recognising that this will provide a significant overestimation by a factor of about 2.1.

In addition, full bond is assumed between the substrate concrete and repair material in both the predicted and actual calculations. However, in reality, slippage may take place soon after application of the repair patch since the repair material has not attained is full hardness and optimum bond is not achieved. This would lead to lower tension in the repair material since full restraint to shrinkage has not yet occurred.

3 Conclusions

The following conclusions are based on the results presented in this paper:

- A repair material that has an elastic modulus greater than the elastic modulus of the substrate concrete ($E_{rm} > E_{sub}$) will transfer a portion of its shrinkage strain to the substrate concrete
- The depth of substrate concrete affected by transfer of shrinkage strain from the repair material can be calculated from:

$$d_{sub} = d_{rm}\sqrt{m}$$

where d_{sub} is the depth of the zone of influence of the substrate concrete

d_{rm} is the depth of the repair material

m is the modular ratio, E_{rm}/E_{sub}

- The moment induced in the substrate concrete by its restraint to the shrinkage in the repair material can be determined from:

$$M_{sub(shr)} = \frac{E_{sub}I_{sub}}{R}$$

- The moment induced in the repair patch by the restraint to shrinkage provided by the substrate concrete can be determined from:

$$M_{rm(shr)} = \frac{E_{rm}I_{rm}}{R}$$

- The compressive stress at the interface of the substrate concrete due to the transfer of shrinkage from the repair material can be determined from:

$$\sigma_{sub(shr)} = \frac{F_{shr}}{b\sqrt{m}d_{rm}} + \frac{E_{sub}\sqrt{m}d_{rm}}{2R}$$

- The tensile stress at the interface of the repair material due to the restraint to shrinkage provided by the substrate concrete can be determined from:

$$\sigma_{rm(shr)} = -\frac{F_{shr}}{bd_{rm}} - \frac{E_{rm}d_{rm}}{2R}$$

- The tensile creep effects in the repair materials can be accommodated in the elastic analysis by replacing E_{rm} with $E_{rm(eff)}$

Application of the analytical procedures presented in this paper will allow concrete repair technologists to better understand the magnitude of interfacial stresses generated in the repair patch, therefore allowing a systematic approach for design of patch repairs to be adopted rather than the ad-hoc method currently being used. Repair materials can therefore be specified with more confidence and the costs associated with repairing failed repair patches will be eliminated

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