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Dynamic geometry, construction and proof : making meaning in the mathematics classroom

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Dynamic Geometry, Construction and Proof: Making Meaning in the Mathematics Classroom

John Gardiner

A thesis submitted in partial fulfilment of the requirements of Sheffield Hallam University for the degree of Doctor of Philosophy

November 2002

Dynamic Geometry, Construction and Proof: Making Meaning in the Mathematics Classroom

The overall aim of this study was to investigate mathematical meaning making in relation to the areas of construction and proof through the use of a dynamic geometry environment (Cabri II as available on the TI 92 calculator). The experimental work was carried out with 11-14 year old pupils in four schools in the North of England between 1996 and 1999. The research involved working with whole classes and a range of groups of varying sizes. The research methodologies adopted were drawn from various areas (an approach advocated as suitable for classroom research by Klafki, 1998). The researcher acted as both teacher and participant observer. The study was conducted over several cycles, with previous cycles of analysis and reference to the literature being used to inform subsequent stages. After a pilot phase when recording methods and technical approaches were clarified, there were four cycles of investigation. Data collection was by means of participant observation, with audio recording of dialogue. Screens generated by pupils were recorded in field notes.

There was emphasis from the outset of the study to relate the findings to classroom practice. This led to a consideration, as an ongoing part of the study, of ideas of classroom and group dynamics and how these could be combined with, and related to, the use of the technology.

The study illuminated two key areas; the processes of immediate individual and group meaning making and wider aspects of social dynamics in the mathematics classroom.

Socio-cultural analysis of classroom and group discourses identified progression from spontaneous to scientific concepts, illuminating the development of pupils' powers of intuition and sense of conviction. The dynamic geometry environment was used to investigate constructions stable under drag, illuminating the way in which the dynamic aspects afforded by the technology affect pupils' appreciation of the relationship between construction and proof. Various aspects of proof were highlighted and in particular the function of proof as explanation was seen to be an important aspect in the development of pupils' mathematical meaning making. Further analysis illuminated a distinction between the immediate individual sense making of pupils and the way this sense making is brought to social and consensual meaning making.

At the wider classroom level the study identified issues of transparency, the importance of the social use of argumentation to take forward the 'taken as shared' and the development of socio-mathematical norms and whole-class zones of proximal development.

These aspects of individual and group meaning-making and whole class dynamics are advanced as ways of promoting local communities of mathematical practice as advocated by Winbourne and Watson (1998).

Acknowledgements

Learning rests in society. Colleagues and learners in all areas of education have provided me with a background over the years, but the encouragement of those in the research arena has been particularly valuable in forming the ideas expressed here, especially those who have provided such a supportive atmosphere at Sheffield Hallam.
I am indebted to staff and pupils at the schools used in this research, who allowed me to break into their working routines, and to my wife and family for their support.

I am particularly indebted to my supervisors, Brian Hudson and Hilary Povey, who have found time in busy schedules to provide me with unfailing constructive criticism, guidance and encouragement. To them especially I give my thanks.

……………………………………………..

Notes

All through this work pupils are referred to by pseudonyms.

Extracts from pupil diaries are reproduced with the original spelling, and appear in *comic sans ms italic font*.

Pupil diary extracts are referenced as D1, 2, 3 etc to the relevant section in Appendix XIII.

Field Notes, including Scribble Pads are likewise referenced as F1, 2, 3 etc

Transcripts of Audio Recordings are referenced T1, 2, 3 etc

Dynamic Geometry, Construction and Proof: Making Meaning in the Mathematics Classroom

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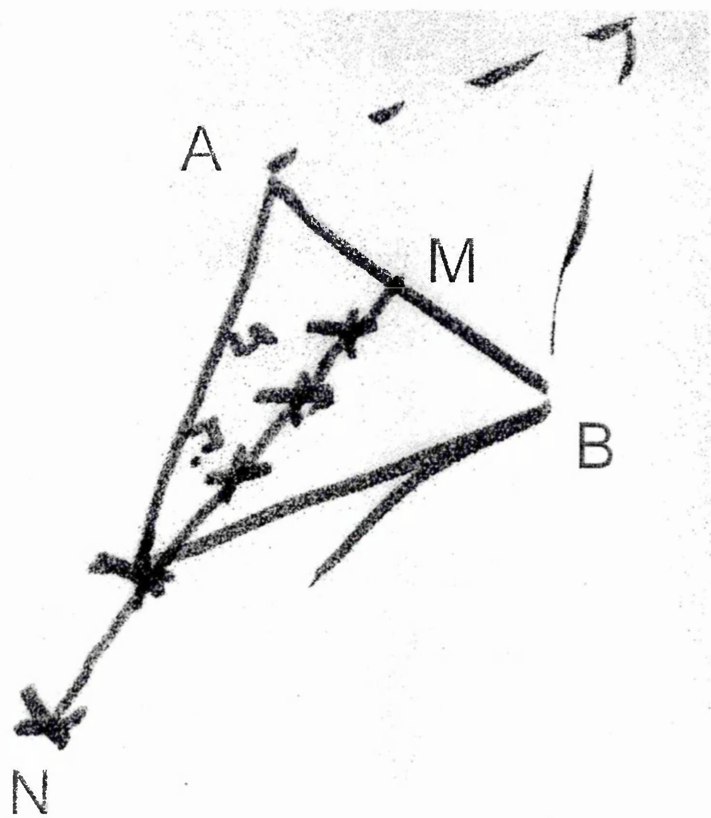
Preface

Geometry as Social

Two pupils had been using dynamic geometry to investigate the possible shapes which can be obtained when a triangle is reflected in one of its sides. After they had looked at the possibilities on the screen I asked them to draw a diagram illustrating the condition for a rhombus to be produced. They identified various points which would produce a rhombus and saw that they lay on a line. I was trying to make them realise something about the properties of the two lines AB and MN in this diagram (letters added later for reference)



Appendix F1

To my question

What can you tell me about these two lines?

I got the reply from one of the two

One is horizontal and the other is vertical

And from the other

Yes that's right

The fact that the two lines should be perpendicular was obviously recognised, but not expressed clearly (mathematically). This incident, showing as it does evidence of a clear mind-picture and social meaning being formed using the flexibility inherent in language, will have echoes with most teachers of geometry.

Geometrical meaning-making will be seen, in this thesis, as being placed firmly in the area of the social. Indeed, meaning-making in a social classroom and the teaching of geometry using a dynamic geometry environment are the two main interwoven strands of this work. The strands will move apart at times and some discussions will be predominantly in the separate fields. The incident described above is used here as the basis for a discussion about the way in which the two strands are linked.

Geometric facts exist in the abstract. Circles and straight lines are only perfect as defined by Euclidean geometry and the strict mathematical terminology used to express them. As soon as we move outside mutually accepted assumptions and terminology, or as soon as a diagram, necessarily imperfect, is produced, the situation changes. Geometry is made social, we move into an area of communication of meaning. Whatever appreciation of the precise fact is formed in the mind, if it is transmitted using language, the accommodation implicit in meaning-making using the spoken word comes into play. Mathematical language is used to codify meaning so that 'square' triggers the same set of properties in the minds of interlocutors, but the problem of the teacher is to arrive at this goal, and the way is through a maze of negotiated meanings, communicated by language and other signs. The pupils in this example have formed ideas about the situation they are addressing, mediated by their own social interaction, the technology and the teacher. They have made their own meaning.

In classroom geometry therefore, the teacher will probably be concerned first with meaning-making and then with formalising this meaning so that it can take its place in the body of mathematical facts. The evident, accurate, meaning made in the example above is valued but the teacher may wish to formalise the fact which is communicated to the whole class. Various viewpoints on this process of informal, individual sense-making being brought to consensual meaning-making are available in the literature. It is the purpose of this thesis to analyse different viewpoints and then relate them to the use of hand-held dynamic geometry in the classroom whilst at the same time seeking insights into successful classroom practice which might have wider application.

Chapter 1

Dynamic geometry and social meaning-making

An introduction

Chapter 1

Dynamic geometry and social meaning-making: An introduction

This chapter provides an overview, in a broadly chronological manner, of the whole project. The origins of the project and its initial aims are discussed. I outline my own background and experience and how I came to the research. The basis of the work in the literature and some of the principles and methodologies adopted are discussed. The chronological progress of the project is summarised and technicalities of hardware and data collection briefly mentioned. An indication is given of the overall direction of the project in terms of theoretical influences, relevant literature and suggestions for the use of dynamic geometry software in the classroom. Reference is made to wider issues of group dynamics in the classroom. Publications arising from the project are listed and briefly discussed.

Initial Influences

This study has been an investigation into the potential of dynamic geometry software to influence pupils' ideas of construction and proof and how these ideas in turn are connected to mathematical meaning-making. The project has been based on the use of the TI 92, a hand-held calculator/computer which has available, among other facilities, a version of Cabri-Géomètre.

The study has investigated individual and group meaning-making and has reached positions in the study of the socio-cultural philosophy of education. However my background as a classroom teacher led to a conviction from the outset that the project should lead to classroom relevance. I was interested in the development of materials and classroom practice by applications of theory. I consider that these principles can be used by teachers in promoting mathematical meaning-making in the local community of practice which is the classroom and all those in it.

Enquiry into a topic such as that under consideration here, the classroom use of dynamic geometry and the attendant processes of mathematical meaning-making, cannot be said to have a particular starting point. This thesis is a description of the work done and the position reached to date. It also details the contribution this project has to make to the discussion on how socio-cultural ideas and ideas on pupils' self awareness of their own learning, can be relevant in the classroom. This project continues and extends the work of previous researchers and has contributed to original knowledge on the topic. However, on a personal level it is also a chronicle of still-continuing development in the way I see myself as a researcher and teacher-practitioner.

With these ideas in mind initial aims and objectives were identified.

Initial Aims and Objectives

The following objectives were identified at the outset, with a main aim to investigate the potential of hand-held dynamic geometry software in the mathematics classroom:

1) To conduct a literature review in the areas of

    i) dynamic geometry use in the classroom

    ii) intuition and proof

    iii) socio-cultural theory in mathematical education.

2) To develop and evaluate classroom materials and approaches using hand-held dynamic geometry software.

3) To investigate the impact of dynamic geometry software on the development of pupils' understanding of construction and proof, through appropriate data collection and analysis.

4) To produce classroom material through development and refinement which realises the potential of dynamic geometry software.

5) To contribute to the debate within the mathematics education community about the potential of hand-held dynamic geometry software to contribute to children's appreciation of construction and proof, through the dissemination of the findings of the study.

The research has also been intended to address the use of readily accessible desktop computers to mediate children's learning. These objectives were reviewed in the light of ongoing progress. In particular, as the project went through successive cycles of development, there was a further element of emphasis, on the dynamics of classroom teaching and methods available for analysis of the interaction between teacher, pupil, group, the whole class and the technology in classroom practice.

The contribution this research makes to the field is founded on an interest in illuminating successful classroom practice, using the knowledge gained in practice and relating this to a background in the literature of socio-cultural theory.

Relevant Literature

Literature referring to the socio-cultural background of the project is discussed in Chapter 2. A wide-ranging review of literature on the use of dynamic geometry in the classroom, on ideas of construction and proof and topics specific to mathematics education is provided in Chapter 4. However it is relevant to include here a summary of literature which has informed and influenced the work and to indicate a theoretical perspective.

This project analyses the development of geometrical ideas in a dynamic geometry environment from a socio-cultural perspective, based on the work of Vygotsky (1962,1978) and the interpretations and extensions of this work by others (Luria 1979, Cole 1979). For wide-ranging reviews of the field see Wertsch (1985), Wertsch and Tulviste (1992), Newman and Holzman (1993) and Daniels (1996).

I have drawn extensively on the work of Lave and Wenger (1991) on the situated nature of learning, which brings forward the idea of legitimate peripheral participation. In more recent work Winbourne and Watson (1998) have pointed to the analysis of classroom activity with hand-held technology in terms of local communities of (mathematical) practice.

Lave and Wenger also refer to the importance of transparency of a resource, such that the resource is a (visible) window on mathematical meaning. The importance of considerations of transparency to aspects of mathematical education is dealt with by Adler (1998). Cobb and Yackel (1996) provide a further way of analysing interactions in mathematics classrooms, in terms of the establishment and maintenance of socio-mathematical norms. They refer to ideas of the use of argumentation, guided by the teacher, to take forward that which is 'taken as shared'.

Various researchers have dealt in particular with the use of dynamic geometry software. Elsewhere in this thesis I have drawn on the work of such writers as Hoyles, Healy and Noss (1995), Healy et al (1994a,b) and Jones (1997) who have dealt in particular with the learning processes which operate in a dynamic geometry environment. Other writers have dealt with the teaching of geometry in general, notably Mason (1991, 1995) and Mason et al (1985). I have drawn on the work of de Villiers (1990, 1991) and Schumann and de Villiers (1993) on the nature and function of proof and have found especially relevant Fischbein's (1982) writing on intuition, conviction and proof. The way these sources and others have provided an initial background and gone on to influence the progress of the project is detailed fully in subsequent chapters.

Influence of Previous Experience

I came to this project after thirty years as a classroom teacher, including fifteen years as a member of various writing groups for major curriculum projects, (SMP 11-16 and SMP Interact). This is not to say that I can supply any answers, rather that some relevant questions can be asked. I adopted the SMP 11-16 course in an inner-city school soon after it was published and used it with an individual learning approach in mixed ability classes. The variety and relevance of the material available was refreshing and the reaction of pupils and staff was encouraging. However, it became clear that this approach could reduce the role of the teacher to that of classroom manager for most of the lesson. An enthusiasm for progress through the scheme at the expense of considered involvement in the mathematics, and a dearth of opportunity for social interaction on the part of the pupils were also apparent.

As an attempt to resolve these difficulties a topic-based approach was adopted with all the class studying the same topic at different levels. This approach enabled social interaction and allowed the teacher to return to leading the class in their learning.

I have seen the advantages of the use of information technology in various forms, whether via a single computer in the classroom, or by moving to a computer suite, or more particularly, with class sets of graphic calculators. This experience has led to an appreciation of the way the use of technology can mediate learning and motivate pupils in the classroom. Children of all abilities can, with suitable material and teaching, use graphic calculators, computers or other technology to great advantage. It must be a particular concern of the teacher, however, to make sure that the objective remains the making of mathematical meaning, not the use of technology for its own sake. I will return to this concern later, in chapter 9 when transparency of a resource is discussed.

An Introduction to Dynamic Geometry

I have seen in my own and other classrooms the power of simple dynamic images and ways of animating diagrams, for example using an overhead projector, or by the teacher using an argument based on a dynamic idea. Ciné and, more recently, computer animations of particular diagrams are available. These devices demonstrate a multiplicity of cases before pupils' attentions are fastened to a static diagram, and it is in this area that the drag function in a dynamic geometry software environment operates. This function allows the independent geometrical entities in the diagram to be dragged on the screen and the consequent, related changes in the diagram to be observed. It is the defining property of a dynamic geometry environment. A diagram can be drawn using the cursor to trace an outline and specify points, all of which are independent of each other. Alternatively it can be constructed by specifying independent points or lengths and using these to define related dependent properties. In each case the independent points can be dragged, by highlighting them and using the drag button.

For instance in figure 1.1 two squares have been drawn, one by eye, one by construction.

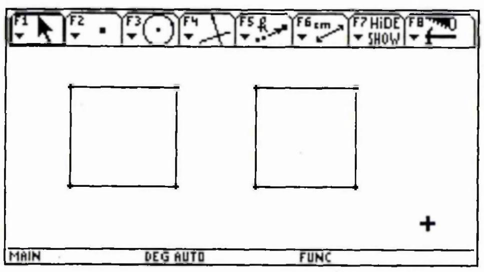


Fig 1.1

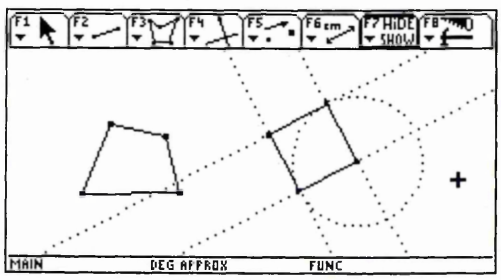


Fig 1.2

Dragging (as in figure 1.2) has different effects on each square. In the case of the drawn square on the left, each corner is an independent point, and as such can be dragged, allowing the square to be distorted. Although the original appearance of this square is exactly the same as that on its right, drawing accurately in this way is only possible because of the coarse nature of the screen graphics on the TI92. It is drawn accurately only in the sense that it is possible to do this 'freehand' on this machine. Each corner can be dragged because it is independent of the other points in the diagram.

In the case of the square on the right, in figure 1.2 the construction method has been shown by using the Hide/Show function. The original circle is defined by its centre and a radius point, and a diameter is drawn. Two perpendiculars are drawn to this diameter, one at the circumference and one at the centre. This last defines a corner of the square on the circumference and a further perpendicular is drawn from this corner to define the square. The independent elements in the diagram are the centre and size of the original circle and the diameter chosen to define the first side of the square. The size of the square can be altered by changing the size of the circle, the orientation of the square changed by moving the line used to define the first diameter. In addition the whole diagram can be moved by dragging the centre of the circle. But as the square is completely defined by the circle and the original diameter it must remain a square under any of these dragging processes.

Relevance to the Classroom

It was a particular concern from the outset to relate this project to classroom practices and to the dynamics available to the teacher in using the technology. The type of software used in classrooms and the way it is used can lead to a wide variety of learning experiences being offered to pupils. Some software, of the type offered for individual use, can lead to closed relationships between single pupils and the screen. In contrast to this, I see the technology as a resource to be used in the classroom to promote social meaning-making and to contribute to the activity of a community of mathematical practice. The use of the technology in this way lays emphasis on the role which all members of the classroom community take. In particular the role of the teacher is more, rather than less important if technology is to be used in a way which promotes social learning. Technology is used by teachers as another tool. Using technology does not diminish the role of the teacher as the guardian of the activity of the local community of mathematical practice, drawing forward the meaning-making of individuals and formalising it, engaging in the dialectic between scientific and spontaneous concepts. Technology use may mean that even more thought should be given to the way the teacher sets the social and socio-mathematical norms of the classroom, chooses appropriately transparent material and seeks to promote its use in a way which encourages the participation of pupils in the activity of social meaning-making within local communities of practice.

Methodology and Data Collection

The methodology adopted in this study has been to use cycles of research, the first emanating from an initial grounding in the literature and previous experience. Each cycle has built on the previous one, with additional input from further reading in the literature. The adoption of such a strategy complements the methodology adopted. Drawing eclectically on various methodologies relevant to the complex social system which is present in the classroom dynamic is an approach advocated as suitable for classroom research by Klafki, (1998). Work was carried out in secondary schools in England, with children in years 7- 9, that is aged from 11 to 14 years. Children from four different schools have been involved. Data collection has been by participant observation by the teacher/researcher, using field notes and audio recorded dialogue. Pupil diaries were also used.

After a short pilot study (chapter 5), work was undertaken with a class of thirty Year 7 pupils (chapter 6). Chapter 7 describes more detailed investigation into meaning-making with dynamic geometry, working with small groups of children. Chapters 8 and 9 deal with work with whole classes following refinement of materials and objectives, chapter 10 discusses issues of classroom and group dynamics and chapter 11 provides a final overview, with some suggestions of future directions.

A further element of the methodology has been dialogue with other researchers. This was promoted in particular by the dissemination of findings in research publications, detailed in appendices I, II, III and IV.

The Equipment

The project used the TI92 hand held computer, and in particular the version of Cabri-Géomètre available on this machine. There is a detailed description of the TI92 in appendix V, but many who know this machine, and have also used Cabri on desktop machines, have expressed reservations about the complication of the keyboard and the slowness of response (the machine has limited memory). I think it is worth saying here that I have found all the pupils I have met to be well motivated to the introduction of the hardware and able to make progress reasonably quickly. I have always tried to make the diagrams used simple, for considerations of principle dealt with elsewhere (see particularly chapter 9), so that dragging works reasonably well. (Trying to drag complicated screens on the TI 92 is not successful.) Whilst slow and coarse in the eyes of those who have used dynamic geometry on a desktop computer, dragging has been accepted by children new to the idea. The problems of a complicated keyboard are not insuperable and there are advantages in small hand-held machines which can be moved to a corner of the desk when the teacher wants the children to use paper. An overhead projector version of the TI92 was available which could be used for class discussion. I consider that the findings of this study are easily applicable to other dynamic geometry environments. Indeed I will seek later to apply some of the findings to broader considerations of technology use in mathematics classrooms and to clarify the socio-cultural principles underpinning them.

Dynamic geometry is based in the formal, Euclidian world. A glance at the language of the drop-down menus, figure 1.3

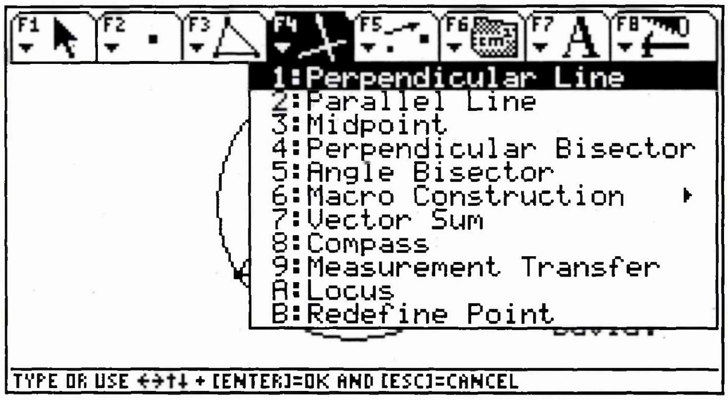


Fig 1.3

shows that successful work with younger (11-14 year old) pupils depends heavily upon teacher intervention, especially at the outset. The language of formal Euclidian geometry used on the menus is not familiar to these pupils, and this must be clarified whilst guiding the pupils around the keyboard. However providing a sheet describing 'Useful Keys' was found to help (see appendix VI), as was a large poster of the keyboard so that keys could be indicated clearly to the whole class. I have been able to use the TI92 with classes and see successful results in terms of mathematical meaning-making well before the end of a first forty minute lesson. Indeed, because of the restricted time available in English schools, where pressure to cover the National Curriculum makes it difficult to gain access for research, it was essential that sufficient mastery of the technology was achieved quickly.

Data Collection Techniques

The recording method used was audio recording of classroom interactions and field notes of the screens used by teacher and pupils. Because the screens used were kept simple for other reasons (transparency and the fact that dragging was faster in less complicated diagrams), it was possible to record them quickly by field notes. Final screens from sessions could be transferred to a desktop computer and an overhead projector version of the machine was available. When working with small groups it was found useful to cover desks with paper and to scribble notes so that pupils' diagrams and diagrams drawn by the teacher were recorded.

Outcomes

There have been many outcomes to this project, some particular, some more general and some of them personal.

- I believe I have illuminated ways in which dynamic geometry software can be used in classrooms in such a way that the meaning-making of the community can be advanced, taking into account the advantages of hand-held machines and considerations of resource transparency.

- I have provided a theoretical analysis, based in the socio-cultural literature, for the advantages of group work and social meaning-making in the use of technology. I feel that this analysis can contribute to a wider debate about good classroom practice.

- Other conclusions are centred around the ways in which teachers and children drive forward learning, in particular the ways teachers can foster an environment in which children are critical of their own learning

- I have been able to review my own classroom practices, some of which developed over a period of years, some of which have recently changed, in the light of reading in the literature and the results of this study.

The current position I have reached is fully dealt with in chapter 11, but it is appropriate to provide a brief overview here. The four constituent elements in the classroom are seen as the subject content, the class pupils, the technology and the teacher. I have looked at the detailed interaction between these elements in five different ways, spontaneous/scientific concepts (Vygotsky, 1962), sense/meaning making (Schultz, 1994), explanation and social proof (de Villiers, 1991), construction and proof (Hoyles et al, 1995) and intuition, conviction and proof (Fischbein, 1982). I see these five factors as acting within an area broadly enclosed by the elements of content, pupils and class, technology and teacher. Outside this area there are more general factors which I believe are relevant to the learning environment in the classroom. These are the ideas of socio-mathematical norms (Cobb and Yackel, 1996), whole-class zone of proximal development (Hedegaard, 1990) and transparency (Lave and Wenger, 1991, Adler, 1998). A further strong influence is the ability of pupils to be constructively self-critical of their own learning (Mercer et al, 1999). As opposed to the immediate nature of the first five detailed interactions mentioned above, these are more long-term considerations, which can be taken up and developed by teachers on a much longer time scale. These short and long-term factors are relevant to the criteria advanced for the development of local communities of practice (Winbourne and Watson, 1998).

In all these areas I consider the input of the teacher to be paramount. The teacher has a place in the immediate, detailed, meaning-making of the community of practice and will be continually involved in ways which can be analysed as suggested above. But the teacher is also able to contribute to the history of that community by developing, over the long term, classroom dynamics which draw the community of practice together in their learning activity.

Publications arising during the project

A major element of this research programme has been the dissemination of findings as an ongoing part of the project, to engage as an active participant in the mathematics education research community, using this experience to feed into the continuing development of the study. To this end, as well as being dealt with in this thesis, this research is reported in four publications which have arisen from the project. In Gardiner and Hudson (1998) the Vygotskian notion of scientific and spontaneous concepts is used to analyse the meaning-making of a group of children using dynamic geometry (see appendix I). In Gardiner, Hudson and Povey (1999) there is a consideration of the significance of local communities of practice as they were observed in class and group work with dynamic geometry (appendix II). In Gardiner, Hudson and Povey (2000) aspects of construction and proof in a dynamic geometry environment are considered (appendix III). In Gardiner (2000) the way my ideas on classroom use of geometry and the technology can be applied to work with younger pupils (9-10 year olds) is reported (appendix IV).

Chapter 2

Theoretical Background

Chapter 2

Theoretical Background

The general socio-cultural literature (that which is not specifically concerned with mathematics) which has been used to guide the project is examined and a theoretical background is developed from this literature. Socio-cultural concepts which are referred to in the rest of the work, such as social learning, the zone of proximal development, spontaneous and scientific concepts, activity and meaning-making are discussed. Whilst chapter 3 deals more fully with the methodology of the project, this chapter mentions aspects of methodology as related to the Vygotskian background of the rest of the work.

Introduction

In this chapter I discuss the theoretical underpinning which is used to provide a foundation to this project. The analysis of classroom research and practice in the rest of the thesis is from the perspective of socio-cultural theory using, as detailed later, the writings and researches of Vygotsky (1962, 1978) and of later workers such as Lave and Wenger (1991), Watson (1998) and Lerman (1998). These writings and my interpretation of them form one of the themes running through the whole of the thesis and it is relevant to discuss them at the outset. The socio-cultural ideas from this chapter have been used to analyse the way the project has used dynamic geometry to illuminate pupils' ideas of construction and proof. This aspect of mathematical meaning-making is a second theme running through the work, but it is appropriate to examine the literature relating to this area separately. Accordingly the literature and background specific to mathematics education and to construction and proof in dynamic geometry are examined later, in chapter 4.

Vygotsky can be seen as a methodologist as well as a psychologist, indeed authorities such as Newman and Holzman (1993) would see the two elements as complementary and inseparable.
This chapter contains a section on Vygotskian methodology and it is appropriate to deal with issues of methodology next, in chapter 3.

This chapter begins by looking at the background in Vygotsky's writing, then looks at developments of these writings by considering the interpretation of the concept of the zone of proximal development within the thinking of various researchers. How the idea of the zone of proximal development can be extended to encompass communities of learners is discussed. After considering Vygotskian methodology, the important ideas of activity and meaning-making are discussed. A discussion of situated cognition within communities of practice precedes a final section dealing with the topic of transparency, before a final summary.

Vygotskian Background

The works of L. S. Vygotsky, largely written in the years just before his early death in 1934, greatly influenced Luria and came to prominence in the West largely as a result of Cole's long collaboration with Luria. Accordingly it was only from the sixties onwards that translations of Vygotsky's works became available (Vygotsky 1962, 1978, Luria, 1979, Cole, 1979). More recent reviews of the field are available, both as original overviews, such as Newman and Holzman (1993) and Wertsch and Tulviste, (1992) and collections of articles (Wertsch, 1985 and Daniels 1996).

Daniels (1996) points out the wide ranging and eclectic nature of Vygotsky's work. He points to the fact that most of his work in educational psychology took place over a relatively short time, and thus was not fully expanded before his death. He suggests that this may have contributed to the breadth of interpretation of this work in the modern educational field and to the richness of the many developments, in their different directions, arising from his thinking. However initially at least it is instructive to retain a broad perspective and to try to look at the wider influence of Vygotsky in the fields of educational and psychological research and research methodology.

Two main aspects of Vygotsky's thinking have influenced later work:

1. Vygotsky formulated the general genetic law of cultural development, emphasising the idea of socialisation before internalisation, the inter-psychological preceding the intra-psychological.

> Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the

> child as an intrapsychological category. This is equally true with regard to voluntary attention, logical memory, and formation of concepts, and the development of volition. We may consider this position as a law in the full sense of the word, but it goes without saying that internalization transforms the process itself and changes its structure and functions. Social relations or relations among people genetically underlie all higher functions and their relationships (Vygotsky, 1981a, p. 163)

Vygotsky defined as 'higher mental functions' volitional functions such as 'voluntary attention, voluntary memory, and rational, volitional, goal directed thought' (Minick, 1987, p. 21). Vygotsky saw these higher mental functions as developed by the intervention of signs or tools, principally speech. These tools allow an intermediate step in the psychological mechanism which leads to the conscious adaptation of behaviour, to meaning-making. Higher mental functions involve the bringing together of the social and the individual, via the mediation of tools.

2. Vygotsky used the idea of a zone of proximal development (ZPD), by which he meant the area where development might take place, the boundaries of the zone being defined, at least in initial work, by a child's unaided ability and the same child's ability when aided by intervention from society, originally by 'adults or more competent peers'.

Vygotsky (1978) originally defined the zone of proximal development in terms of intelligence testing. He posed the question of two children found to have the same mental age in unaided testing, who were then shown to have different potentials after intervention from the teacher.

> Suppose I investigate two children upon entrance into school, both of whom are twelve years old chronologically and eight years old in terms of mental development. Can I say that they are the same age mentally? Of course. What does this mean? It means that they can independently deal with tasks up to a degree of difficulty that has been standardized for the eight year-old level. If I stop at this point, people would imagine that the subsequent course of development and of school learning for these children will be the same, because it depends on their intellect......Now imagine that I do not terminate my study at this point, but only begin it...suppose I show [these children] have various ways of dealing with a task...that the children solve the problem with my assistance. Under these circumstances it turns out that the first child can deal with problems up to a twelve year-old's level, the second up to a nine year-old's. Now are these children mentally the same?
>
> When it was first shown that the capability of children with equal levels of mental development to learn under a teacher's guidance varied to a high degree, it became apparent that those children were not mentally the same and that the subsequent course of their learning would obviously be different. This difference between twelve and eight, or

> between nine and eight, is what we call the zone of proximal development. (Vygotsky, 1978, pp 85-86)

So followers of Vygotsky have considered the development of the mind as happening in the zone of proximal development. However the concept of the zone of proximal development is broad, and different workers have found their own ways of examining the processes operating there. There is almost a chronological drift to the way the zone of proximal development has been regarded and the broadening of the definition over time is perhaps an indication of the wide applicability of the concept, and the way it has been used both as a precise analytical tool and as a useful broad band of ideas. Indeed the various ways in which the zone of proximal development has been defined and the ideas advanced to explain the interactions between society and the individual which happen there form a useful framework for a review of the field.

Definitions of the zone of proximal development have ranged from those related to development and 'scaffolding', (Vygotsky, 1962, Wood, Bruner and Ross, 1976 and Bruner, 1986), to assessment (Vygotsky 1956 pp 446-447), to the distinction and interaction between spontaneous and scientific concepts, (Kozulin, 1987 p xxxiv), to definitions in activity theory (Engeström, 1987 p 174), and to the class and teacher as a whole (Hedegaard, 1990).

Lave and Wenger (1991 pp 48-49) discuss this movement to a broader cultural definition of the zone of proximal development. Following them, in this study it has been found helpful to group the various approaches to the zone into three broad categories, which are, to some extent, in a chronological sequence. Firstly the initial work of Vygotsky and subsequent work by Wood, Bruner and Ross (1976) used the distance between unaided and aided problem solving and proposed the idea of 'scaffolding' to explain the pedagogical approaches used in the zone. In this view it is mediation by tools which leads to development by internalisation. Vygotsky saw mediation of behaviour by sign systems, principally speech, as the way in which the higher voluntary mental functions are changed. Mediation and internalisation are also effected by tools: psychological devices such as mnemonics (Vygotsky, 1981b, Zinchenko, 1985), by speech (seen variously as internal, egocentric, and social, see Wertsch and Stone, 1985), the teacher (Jones, 1997) and the screen (Hoyles et al 1995).

A second approach begins by extending Vygotsky's discussion of scientific and everyday concepts. This cultural interpretation sees development taking place by the dialectical interaction of the socio-historic, scientific concepts, provided by social intervention, and the everyday concepts which form part of the experience of the individual. As development proceeds a dialectic between everyday and scientific concepts will operate, concepts which were held as scientific will become everyday or spontaneous, whilst everyday concepts are themselves instrumental in promoting a framework within which scientific concepts can operate.

> In working its slow way upward, an everyday concept clears a path for the scientific concept in its downward

> development. It creates a series of structures necessary for the evolution of a concept's more primitive, elementary aspects, which give it body and vitality. Scientific concepts in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use. (Vygotsky, 1962 p. 109)

Confrey (1995) discusses Vygotsky's views on development as informed by ideas of dialectical and historical materialism and notes that he refers to development as

> ..... a complex, dialectical process characterised by a multifaceted, periodic timetable ... by a complex mixing of external and internal factors, and by a process of adaptation and surmounting of difficulties. (Vygotsky, 1978, p. 151)

Confrey argues the need for an historical analysis and that one must examine the growth of higher mental functions in order to understand them. She discusses the work of Davydov (1990), who emphasised the importance of labour in cultural development and the way in which labour, by transforming objects using tools, removes incidentals and allows invariant properties to be seen. From this work, she maintains:

> ..... one gets the sense of the centrality of the activity of labor on cognition for Marx, Engels and subsequently Vygotsky. Also one learns that the internal character of an object is not a direct perceptual thing but a mediated relational meaning. Finally, tools, as the means of transformation of labor, possess a central role as both means

> of cultural transmission and as intimately associated with the results of labor. (Confrey 1995 p. 39)

These two classifications of approach then, of mediation and internalisation and of a historico-cultural approach drawing on the idea of scientific and everyday concepts, can be identified. In both of these classifications, there is a somewhat limited function or use for the social in the zone of proximal development. The social element of learning is seen as an almost mechanistic factor in the internalisation of knowledge. There is an element here of Newman and Holzman's (1993) 'tool for result', of the social being lifted down from the toolrack as and when needed, as opposed to 'tool-and-result', the forming of a unity of social meaning-making activity. This critique is developed by Lave and Wenger who argue:

> In these two classes of interpretation of the concept of zone of proximal development, the social character of learning mostly consists of a small 'aura' of socialness that provides input for the process of internalization viewed as individualistic acquisition of the cultural given. There is no account of the place of learning in the broader context of the social world. (Lave and Wenger, 1991 pp 48-49)

More recently, Lerman (1998) makes a similar distinction between developments of Vygotsky's thinking on what happens in the zone of proximal development.

> ….. in the zone of proximal development one can study the mediation of tools but …. activity theory is more fruitful for

> longer-term studies, taking account of goals and needs. There is a dialectical unity in these two methodologies in that, whilst both are rooted in the cultural psychology of Vygotsky, mediation is a generalising principle, looking for similarities, whilst activity theory is a specialising one. (Lerman 1998, p 75)

Thus Lerman acknowledges differing roles in the ZPD for mediation and activity theory, but sees a complementarity between these two approaches.

Workers in activity theory, such as Bakhurst (1988) and Engeström (1987) among others, have used a background of the zone of proximal development. Lave and Wenger (1991) see a societal element to the zone of proximal development leading to social transformation. Hudson (1996) studied group dynamics in the use of technology and Hedegaard (1990) advanced the concept of a whole-class zone of proximal development.
Lave and Wenger (1991) propose the idea of local communities of practice and consider learners as members of those communities, with learning taking its place in the overall cultural activities of the community. Newman and Holzman (1993) see revolutionary activity operating as the unit of Vygotskian analysis, leading meaning-making in the zone of proximal development.

> The significance of the ZPD, in our view, is that it is not premised on the individual-society separation; it is an historical unity. In fact, it methodologically destroys the need for interactionist solutions to the dualism of mind and

> society because it does not accept their ontic separation in the first place! The claim that learning takes place in the ZPD is neither a claim about learning nor a claim about the ZPD. For the ZPD is not a place at all; it is an activity, an historical unity, the essential socialness of human beings expressed as revolutionary activity, as Marx put it.
> (Newman and Holzman 1993 p 79)

Daniels (1996) has pointed out that Newman and Holzman's analysis is essentially political in nature, contrasting as it does pragmatist and dialectical materialist accounts.

> Just as in Lave and Wenger's model of three types of approach to the notion of ZPD, Hood Holzman (1985) and Newman and Holzman (1993) are questioning the breadth of interpretation that is placed on the 'social' in the formation of mind with the additional concern for the mechanism of mediation.
> (Daniels 1996 p 9)

The emphasis in much of the above has been on the individual in the zone of proximal development, albeit on the interactions of that individual with society. I wish to look now at work which advances the idea of a zone of proximal development involving the class and their teacher.

Extending the ZPD

In a long-term study in a Danish school Hedegaard (1990) sought the development of a ZPD which included the classroom as a whole. This was seen to incorporate the teacher, the pupils and their activity rather than the consideration of an individual's learning:

> This activity, in principle, is designed to develop a zone of proximal development for the class as a whole, where each child acquires personal knowledge through the activities shared between the teacher and the children and among the children themselves. (Hedegaard 1990 p 361)

Hedegaard reports in the same paper a motivational shift in children's focus, from an interest in the concrete to interest in the derivation of principles which can be applied to the concrete. She goes on to discuss the evidence she found for the development in children of critical and evaluative attitudes to their own performance and capabilities, and to the content of teaching. The importance of children being consciously involved in and critical of their own learning processes is a theme to which I will return later in this thesis.

More recently Lerman (1998) has pointed to a definition of the ZPD derived from the work of Davydov (1988), seeing the ZPD as

> created in the learning activity, which is a product of the task, the texts, the previous networks of experiences of the participants, the power relationships in the classroom, etc...... The zpd is the classroom's, not the child's. In

> another sense the zpd is the researcher's: it is the tool for analysis of the learning interactions in the classroom (and elsewhere). (Lerman 1998 p 71)

In the same work, Lerman points out that Vygotsky introduced the idea of the ZPD only fifteen months before his death, and reference has been made previously to the wide range of interpretations of the concept. In general however,

> it provides the framework, in the form of a symbolic space ......for the realisation of Vygotsky's central principle of development. (Lerman 1998 p 71)

This central principle is of social learning leading individual learning, of the inter-psychological preceding the intra-psychological. How best to employ this principle in the context of dynamic geometry in the classroom is, of course, the central question of this thesis. In developing a position on this in the rest of the work, I will return to the concept of the zone of proximal development and what happens there.

Vygotskian Methodology

From a Vygotskian perspective methodology should not only be all-pervasive in a study, it should be the study.

> ..the method is simultaneously prerequisite and product, the tool-and-result of the study (Vygotsky 1978 p 65)

Newman and Holzman take up the idea:

> The attempt to categorize Vygotsky, to 'dualize' him as either a psychologist or a methodologist, contradicts, ironically, not only Vygotsky's life-as-lived, but his self-conscious intellectual revolt against dualism. (Newman and Holzman 1993 p 16).

Vygotsky can be seen as a methodologist/ psychologist in the sense that his all-embracing view of the science of learning brings in the Marxist historico-cultural dialectic and the ideas of revolutionary activity and practice. It provides a methodology which informs and pervades a study and is available to constantly influence the conclusions drawn and the direction of future progress. This methodology is echoed in the idea of "tool-and-result" discussed by Newman and Holzman (1993), who point to a distinction between tools such as hammers and screwdrivers (tool for result), and dies and jigs (tool-and-result). Hammers and screwdrivers are bought and used as needed, dies and jigs are tools designed and refined by the worker, the toolmaker. Vygotskian methodology is a 'tool-and-result'. Like the jig, it is bound up in its result. Newman and Holzman say

> The toolmaker's tool is different in a most important way. While purposeful, it is not categorically distinguishable from the result achieved by its use. Explicitly created for the purpose of helping to make a specific product, it has no reified prefabricated social identity independent of that activity. Indeed, empirically speaking, such tools are typically no more recognizable as tools than the product

> (often a quasi-tool or small part of a larger product) itself is recognizable as product. They are inseparable. It is the productive activity which defines both - the tool and the product (the result). (Newman and Holzman 1993 p 38)

The concept of activity has a special significance in the applications of Vygotskian ideas and is dealt with more fully in the next section.

Activity

Activity has been seen as the fundamental unit of Vygotskian psychology, just as behaviour might be seen in the same way in some work in the United States and consciousness in some European work. Zinchenko (1985) points out that:

> ..... the problem of units for psychological research has confronted every school of scientific psychology. In the past a variety of phenomena have been singled out in this capacity. For example sensations (in associationism), figure/ground (in Gestalt psychology), the reaction or reflex (in reactology and reflexology respectively), set (in set psychology), and the behavioural act (in behaviorism) have served as units. (Zinchenko 1985 p. 95)

Kozulin (1986) discusses the role of activity in Soviet psychology, in particular referring to the interpretation of the idea by Leontiev (see for instance, Leontiev, 1981, pp318-349) where activity was used as both a unit of analysis and the subject of that analysis. Kozulin identifies the tautology in this approach, and describes the work of later Soviet workers (Davydov 1981, Lomov 1982, Zinchenko, 1985) and how it began to influence Western thinking. Kozulin summarises:

> As I see it, the real opposition between Vygotsky's theory and Leontiev's thus appears as an opposition of the following two schemas: Vygotsky's theory views higher mental functions as a subject of study, semiotic systems as mediators, and activity as an explanatory principle. In Leontiev's theory, activity, now as activity, and now as action, plays all roles from subject to explanatory principle. (Kozulin 1986 p 271)

Activity then is a two-way, dialectical process, an interaction both between the individual and society and between society and the individual.

> Practical-critical activity transforms the totality of what there is; it is this revolutionary activity that is essentially and specifically human. Such activity 'overthrows' the over-determining empiricist, idealist and vulgar materialist pseudo-notion of particular 'activity' for a particular end - which in reality, i.e., society, is behavior. The distinction between changing particulars and changing totalities is vital to understanding tool-and-result methodology and, therefore, revolutionary activity. (Newman and Holzman 1993 p 41)

The object of activity can become confused. I referred in chapter 1 to my experiences with texts such as the SMP11-16 scheme driving the work of teacher and class in a way which was identified as unhelpful.

Engeström (1991) addresses the problem of the text driving children's learning. He refers to

> the school text as the object of activity instead of being an instrument for understanding the world. When the text becomes the object, the instrumental resources of the activity are impoverished – students are left 'to their own devices'. (Engeström 1991 p 250)

So, to summarise, what is activity, in the sense in which I want to use the term? It is the ability to *engage,* through *social interaction,* with *intention,* in the process of making meaning. The engagement in activity referred to here is a tool-and-result phenomenon, that is the engagement is now as tool, now as result. The social interaction involves the learner drawing from and giving to the social, cultural and historic community, again in a tool-and-result way. The intention points to a conscious involvement of participants. They see themselves as members of a community practising revolutionary-critical activity for the purpose of making meaning. Meaning-making is a term I will use repeatedly in this thesis and as such it too deserves special attention.

Meaning-making

Schultz (1994), in a paper on the hermeneutical aspects of activity theory, offers an interpretation of meaning as a referent property containing overall cultural intention. He distinguishes meaning and sense, seeing sense as being a much more immediate concept. Sense refers to the application of referent meaning made by an individual to the circumstances prevailing at the time for that individual, and as such will depend on an appreciation of meaning but also on emotional, historical and cultural factors. Meaning is seen as an objective function of a phenomenon, independent of the individual. I will refer to this distinction further in Chapter 9, where the distinction between meaning-making and sense-making will be more closely examined.

In the rest of this thesis, except in Chapter 9, I want to use the term meaning-making to signify both the appreciation of the referent, intentional aspects of meaning and the sense-making aspects discussed by Schultz (1994). The distinction between this meaning-making, grounded in the activity of the individual in society and society in the individual, and the positivist idea of understanding is made by Lerman (1998). He speaks of his reasons for avoiding the term 'understanding':

> The term is part of the regime of truth which locates power in the hands of teachers who can say when a child understands or doesn't, independently of what s/he produces, verbally or in writing (Watson, 1995). Its entirely internal nature makes it a rather useless notion (Lerman, 1994), whilst its association with closure places it in a positivist paradigm. (Lerman 1998 p 75)

Thus the term understanding implies a closed, finished process, whilst meaning-making is seen as an on-going, dialectical process of involvement in social activity.

After referring to the work of Schultz (1994), Hudson et al (1997) go on to discuss the work of Crawford (1996) on activity theory.

> She highlights how activity denotes a personal (or group) involvement, an intent, a commitment that is not reflected in the usual meaning of the word in English. She draws attention to the fact that Vygotsky wrote about activity in general terms to describe the personal and voluntary engagement of people in context - the ways in which they subjectively perceive their needs and the possibilities of a situation and choose actions to reach personally meaningful goals. In building upon Vygotsky's work, Leont'ev, Davydov and others made clear distinctions between conscious actions and relatively unconscious and automated operations.
>
> (Hudson et al 1997)

Newman and Holzman (1993, Ch. 6) refer to the English title used for Vygotsky's (1962) work 'Thought and Language' and the subsequent title 'Thinking and Speech'. They see a dialectic between learning and development being complemented by a dialectic between meaning-making and language-making, and propose the title 'Meaning-making/Language making' for Vygotsky's work.

> Long before children do what is recognized as speaking, they are making meaning; they are reorganizing the determining environment, which includes linguistic elements. It is by virtue of children learning to use these elements that they learn the societal use of them (language-

> making/thinking). While sounds and words may be necessary tools for language making, meaning-making is its historical precondition. (Newman and Holzman, 1993 p 113)

These perspectives on activity theory and meaning-making move towards ideas of empowerment, of children taking responsibility for their learning by seeing it as part of the activity of the community in which they find themselves.

> ..'good learning' is and must be learning in advance of development precisely because and as one learns that one is a learner (inseparable from being related to as a learner) through revolutionary activity – making meaning - in the ZPD.
> (Newman and Holzman 1993 p 144)

Situated Cognition within Local Communities of Practice

I have referred earlier to Lave and Wenger's (1991) views on the significance of the idea of the zone of proximal development. The important contribution of Jean Lave to the broad field of Vygotskian socio-cultural studies, in this work (Lave and Wenger, 1991) and others, (Lave, 1988,1993) is now discussed.

Lave decided to move outside the classroom, to look at learning in society, in order to better understand the teaching-learning environment. She refers to research, for example, amongst tailors in Liberia, midwives in Yucatan and quartermaster technicians in the

US Navy. She identifies ‘situated learning’, learning firmly related to the activity of a community of learners.

> In the concept of situated activity we were developing, however, the situatedness of activity appeared to be anything but a simple empirical attribute of everyday activity or a corrective to conventional pessimism about informal, experience based learning. Instead, it took on the proportions of a general theoretical perspective, the basis of claims about the relational character of knowledge and learning, about the negotiated character of meaning, and about the concerned (engaged, dilemma-driven) nature of learning activity for the people involved. That perspective meant that there is no activity that is not situated. It implied emphasis on comprehensive understanding involving the whole person rather than “receiving” a body of factual knowledge about the world; on activity in and with the world; and on the view that agent, activity, and the world mutually constitute each other.
>
> (Lave and Wenger, 1991, p. 33)

In the classroom this thinking will mean that we look at learning activity as situated, not just in the local social interactions of the child, but in the classroom, school and the wider society outside school. It will mean that we must consider school learning as situated cognition, and as such influenced by societal factors, some which we can recognise, some which we cannot know about, and some which, although we can accept their existence, remain beyond our control.

According to Lave and Wenger, situated cognition occurs via legitimate peripheral participation, with learners (newcomers, apprentices) being brought into a local community of practice. The potential difficulties with this approach to school learning are acknowledged by the authors. The community into which newcomers are brought is fairly well defined if one looks at apprentice tailors, but in school society and in the school in society this is not the case. If, as Lave and Wenger hope, their analysis of learning taking place in communities of practice is a model which can be applied to what happens in the school classroom, the idea of community in the classroom needs to be addressed. Lave and Wenger would claim that the classroom needs to be located in the practice of society as a whole. Of Lave's work Watson (1998) says

> ...it is by looking at learning in social contexts in general that we will learn more about learning in schools, and not the other way round. Here is the major attraction in her theories for educators, that when we look at classrooms from her viewpoint we see them as social communities in which all sorts of things are being learnt (how to behave in a way that is valued by the teacher, how to be accepted by one's peers, what writing implements are fashionable....) which are not the focus of the teaching. To describe what goes on in a classroom fully one must consider all the actions, thoughts, feelings and environmental aspects within it. (Watson 1998 p 2)

The curriculum and the teacher, social considerations and peer relationships will make even the identification of which community newcomers are being brought into a difficulty. There will be a tension, for instance, between becoming a member of a community of practitioners of a particular academic subject, a mathematician, say, and becoming qualified, by examination, in this subject. Further tensions will exist between membership of both these communities and membership of the community of learners about the practice of society as a whole. ‘What gets learned is problematic with respect to what is taught’ (Lave and Wenger 1991 p 41). However if we can accept the value of the consideration of learning as a move towards participation in a socio-cultural community of learners and as an all pervasive human activity of legitimate peripheral participation in local communities of practice we need to find ways of applying this analysis to what goes on in school.

Transparency

Taking a socio-cultural perspective on the use of resources, Lave and Wenger (1991 pp 102, 103) address the issue of the transparency of a resource, and this is further examined by Adler (1998 pp 8-11). A resource used in the classroom can be so visible to students that it obscures the topic under consideration and prevents meaning-making. At the same time some visibility is necessary. We want the resource to be visible in the sense that it should direct the gaze of students, so enabling their meaning-making.

> Invisibility of mediating technologies is necessary for allowing focus on, and thus supporting the visibility of, the subject matter. Conversely, visibility of the significance of the technology is necessary for allowing its unproblematic - invisible - use. This interplay of conflict and synergy is central to all aspects of learning in practice: it makes the design of supportive artifacts a matter of providing a good balance between these two interacting requirements. (Lave and Wenger, 1991 p. 103)

Clearly the familiarity of pupils with technology governs its use, in a way which is informed by arguments such as this. As pupils become more familiar with a particular piece of software the teacher will be able to introduce the use of more complicated functions without losing transparency.

A more general consideration is that of the caution which the teacher must employ in selecting ‘real world’ examples to relate a topic to pupils’ personal experience. As soon as appeal is made to experience many different factors are brought into the classroom, the majority of them unknown to the teacher. Examples used by the teacher and in school texts can often be generated for the purpose of illustrating a particular point, but such examples probably bring with them meanings which are different for each individual and are dependent on social class, previous experience and many other socio-historical and socio-cultural factors. 'Real world' examples may well require children to make the effort to divorce the example given from their culture, from their experience of the real world, in order to place it in a school learning context. The teacher can usefully employ the idea of transparency here to consider the suitability of examples as resources.

Language can be viewed as a resource in this way too. Adler (1998) says of language:

> It is a cultural resource in that it includes the main language(s) learners bring to class (and their relation to the language of instruction). It is also a social resource in that it includes learner's verbalisations during class, as well as communication (talk with and between learners). (Adler 1998 p. 7)

In examining language (as a resource) for transparency, we can see that an insistence on formal language related to a topic may interfere with spontaneous meaning-making. Children's informal language, used in sense-making activity, may well not have the precision of more formal language, but it can be used to provide a basis for interaction with the local community of practice, and brought to socially agreed meaning. The teacher will judge when the formal language has become a sufficiently transparent resource to allow its unproblematic use.

A much more challenging aspect of resource use in the socio-cultural view is the generation of a classroom in which pupils are conscious of the whole community, inside and outside school, as a resource. Revolutionary activity, drawing on socio-historical culture, brings the totality of the world into the classroom. In successful communities of practice pupils will see themselves, sometimes as newcomers, sometimes as old-timers, as having a valued part to play, and their place in the community itself as a resource.

Summary

This chapter is an attempt to provide a theoretical backdrop against which the rest of the study can be viewed. Other theoretical aspects, from the specific field of mathematical education, and in particular on construction and proof using dynamic geometry, will be dealt with separately (in chapter 4) and both theoretical aspects will be used to analyse classroom activities. However it is the thinking dealt with in this chapter which will provide the background to the whole project, or rather the various lenses which will be used to examine the classroom use of dynamic geometry. Ideas of social learning, from Vygotsky (1962) and as developed by reference to scientific and everyday concepts by Davydov and Markova (1983), and subsequently the ideas of Engeström (1987) and others on activity theory will be used. From Lave and Wenger (1991) and Winbourne and Watson (1998) ideas of situated cognition in local communities of practice and considerations of transparency of a resource will be applied. The notion of meaning-making will be widely relevant, as will considerations of a whole-class zone of proximal development. However I believe that such a methodology is not at odds with the approach advocated by Dengate and Lerman (1995) who call for the use of the wide-angle lens as well as the microscope.

This last point brings me to an important consideration. This chapter has indeed dealt, at times, with aspects of the theory and philosophy of education in abstract ways, ways which imply close-up examination of the relationships between the individual pupil and the subject content, albeit with emphasis on the social nature of learning. I stressed, in chapter 1, a desire to address classroom practice in the use of the technology. This is still my intention. I believe that the theoretical viewpoints discussed here can be used to analyse successful classroom practice and to develop methods of using technology in a way which fosters whole class and group social learning. As Dengate and Lerman (1995) say in the abstract to the paper cited above:

> The current desire of mathematics educators to devise variations of constructivist models of learning, combined with growing interest in the Soviet school of social constructionist theory, has perhaps clouded a bigger picture regarding the place and role of learning theory, especially as it relates to mathematics classroom practitioners. (Dengate and Lerman, 1995 p.26)

I hope that in the rest of this work, whilst using some of the more close-up and detailed methods of analysis outlined in this chapter, I have been able to keep in mind the need to 'pan-out', to be aware of the classroom and all those present in it, and that classroom's place in a wider society.

Chapter 3

Methods and Methodology

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Methods and Methodology

This chapter discusses the methodological approach adopted in this project and maps out the structure of the work. The socio-cultural approach, founded on the work of Vygotsky, has pointed to an overall view of the methodology being closely bound up in and related to the study. Because this study has sought to relate itself to the classroom, with its many-faceted aspects, an eclectic approach such as that advocated by Klafki (1998) and developed by Hudson (2003) is proposed, also drawing on the writings of Hamilton and Delamont (1974) and Eisenhart (1988), Bassey (1995) and Brown and Dowling (1998) and referring all these to a Vygotskian approach as detailed in Newman and Holzman (1993). The research has been conducted in cycles, with the first being founded in the literature. Subsequent cycles built on previous work, but used additional sources from the literature which were particularly relevant to the approach taken, or in some cases indicated a re-examination from a different standpoint. After some experimentation with video recording the main body of the data was collected by participant observation, using audio recordings of classroom interactions and field notes of the screens produced by pupils. Pupils kept diaries of their reactions to the work. This data was subjected to analysis from various theoretical standpoints as detailed in this chapter and in the relevant areas of the rest of the work.

Introduction

Methodologies for the classroom

This study has been into the way technology, specifically dynamic geometry on the TI 92, can be used with 11-14 year old pupils. There has been emphasis all along on investigating classroom use of this technology. Hudson (2003) has pointed out the need to address the complexity of classroom processes and Watson (1998) emphasises the wide variety of different processes which are taking place in the classroom. Activity within the classroom takes place in many different areas and in ways which are variously important to the people in it. The intended learning programme will be only a subset of these activities. Cultural and societal aspects of the school and society as a whole surround the work which takes place in the classroom and may help or hinder meaning-making in the particular subject area. Hudson (2003) draws attention to European ideas of *Didaktik*, in particular that of *critical-constructive Didaktik.* He suggests that this tradition recognises that the complex processes at work when teaching and learning take place in the classroom are best dealt with by a range of research approaches, which also take into account the wider societal context within which meaning-making is happening. This eclectic approach is intended to 'support pedagogical practice' and 'need(s) to be based on a combination of methods and methodologies'

> The tradition of critical-constructive *Didaktik* offers a distinctive approach to educational research, which

> addresses the complexity of the processes of teaching and learning in the methodologies and methods adopted, whilst maintaining attention to considerations of meaning making within a wider societal context.

Hudson, drawing on Klafki (1998), identifies three method groups/methodologies.

- Historical-hermeneutical methods,
  intended to use scientific method to analyse and deconstruct meaningful phenomena and to relate the didactical process, seen as involving all social aspects, to the wider picture of society and culture.

- Empirical methods,
  which are seen to be necessary when contemporary issues (for instance, in the context of this study, whole class teaching), decoded didactically by a historico-hermeneutic approach are studied in the classroom context.

- Methods of social analysis and ideology critique.
  No pedagogical or didactic province is seen to be outside society. Society directs the direction of educational developments by means of curricula and syllabi, and by assessment procedures. Broader aspects of the school society play their part, such as setting and other organisational arrangements, the attitudes of teachers and other students. The development of social analysis

methods must be tempered by a parallel critique of the ideologies behind the type of education on offer.

The background to this approach is the acceptance that method groups/ methodologies will be found useful in particular areas of a study, but a point will be reached where the preconditions of a particular approach will mean that its usefulness declines and that a different approach will be needed to make further advances.

This eclectic approach has led to reference to a number of methodologies in this study, which are discussed in this chapter.

The view of activity as both a method of research and the object of research, seeing Vygotsky as a methodologist/psychologist, points to a continual dialectic between method and substance, the 'tool-and-result' methodology referred to by Newman and Holzman (1993). Eisenhart (1988) has advocated absorption into the research methods of mathematics education of the approaches used in educational anthropology. Bassey (1995) has presented the idea of the study of singularities. A further influence has been views put forward by Brown and Dowling (1998) on the importance to classroom practitioners of both using and doing research.

In addition this chapter outlines the progress of the study through its different phases and the detailed techniques used at each stage.

Tool-and-Result Methodology

In chapter 2 I introduced the idea of tool-and-result methodology. It is obviously not inappropriate to deal with methodology in a section devoted to theoretical background. Equally, if tool-and-result methodology is used, the methodology should be all-pervasive in the study. Newman and Holzman (1993) use the terms tool-and-result and tool for result. They advance the metaphor of hammers and screwdrivers, which are tools for result, taken from the tool rack, used and replaced to be re-used for a similar purpose. They compare this to the tool-and-result, the toolmaker's tool, a die or jig which has no particular function outside the purpose for which it was created. Vygotskian methodology is tool-and result; it is bound up in the revolutionary activity of meaning-making. Newman and Holzman see Vygotsky as a methodologist/psychologist in the sense that he has an all-embracing view of the science of learning. Revolutionary-critical activity is seen to be the way society acts on the individual in the historico-social dialectic. This activity can be seen to constitute of itself a methodology which is all embracing in a study, which pervades the study and is able to constantly influence its progress and the direction it takes.

Educational Anthropology

Such an emphasis on the location of meaning-making (and consequently of research into it) within the context of society as a whole points firmly to the adoption of an ethnographic approach to inquiry. In a paper which discusses the use of ethnographic methods in mathematics education research Eisenhart (1988) describes herself as an educational anthropologist, using the research tradition of cultural anthropology. Eisenhart points out that ethnography in its basic tenets is grounded in the philosophical position of interpretivism.

> Central to interpretivism is the idea that all human activity is fundamentally a social and meaning-making experience, that significant research about human life is an attempt to reconstruct that experience, and that methods to investigate the experience must be modelled after or approximate to it. (Eisenhart, 1988 p 102)

Eisenhart calls for the adoption of an anthropological approach from mathematics educationalists. Mathematics education researchers can try to embrace more the way ethnographers enter into the lives and activities of those who are the subject of their study. Eisenhart recognises the problematic nature of this approach, in that it can make educational researchers nervous about questions of objectivity. However workers influenced by anthropological considerations (Cole and Means, 1981; Cole and Scriber, 1974; Lave, 1977, 1982, 1985) have shown that there is a way of incorporating the two approaches. Eisenhart emphasises 'these researchers have tried to understand mathematical problem solving *in the same way* as their subjects' (her emphasis). Another difference between the two approaches pointed out by Eisenhart is

> the limited way in which mathematics education researchers have been sensitive to the intersubjective meanings that

> might constitute the schools, classrooms, and instructional dyads they study. (Eisenhart, 1988 p 111)

The knowledge of schools and mathematics brought into the classroom by pupils is perhaps rarely considered by mathematics education researchers, but will be much more in the forefront of the considerations of educational anthropologists. Socio-cultural background will be recognized as much more important by these ethnographers.

On the other hand, if educational anthropologists consider the work of mathematics education researchers, they may wish to give more attention to theories of cognitive development and information processing:

> These two topics are generally outside the scope of socio-cultural theories and, as usually formulated, contradict the thrust of interpretivism; anthropologists resist them because of their acontextual, ahistorical and asocial features. Yet............. cognitive theories might be joined with socio-cultural theories in efforts to create a comprehensive theory of human activity. (Eisenhart, 1988 p. 112)

Eisenhart's final point returns to a theme which has already been addressed and to which I intend to return in this work. She emphasises the need for educational anthropologists to eschew the belief that, because of the necessarily narrow view they take when they are involved with classrooms, they can have nothing to say about the choices which teachers take, 'choices about what to do on

Monday morning, choices no educator can ignore' (Eisenhart, 1988 p. 112). She concludes

> By joining with mathematics education researchers and other educators who, by necessity, must grapple with how to interpret research findings into practice, educational anthropologists could move into a new and potentially fruitful domain of study. (Eisenhart, 1988 p. 112)

This concern, that of relating socio-cultural theory to classroom practice, has been a major guiding principle and is central to this study.

The Study of Singularities

Bassey (1995) has advanced the idea of the study of 'singularities', an another approach which is applicable to this study. This can be seen as the study of something which has occurred at a certain point in space-time. When this event becomes the object of study it is defined as a singularity.

> A singularity is a set of anecdotes about particular events occurring within a stated boundary, which are subjected to systematic and critical search for some truth.

and

> this truth while pertaining to the inside of the boundary may stimulate thinking about similar situations elsewhere.
>
> (Bassey 1995 p 111)

Singularity studies are undertaken on a small scale and are very detailed. The findings are related to (as opposed to applicable to) populations outside the immediate parameters of the study. In this project close studies have been made of classroom episodes, with the aim of relating the findings to more general classroom practice.

This project has also used a technique of revisiting some classroom episodes and re-examining them, using a different background from the literature and a different lens (Lerman, 1998). In chapters 8 and 9 some data is re-examined in this way. This process leads to a degree of triangulation which augments that available from the different approaches employed and from the extension of the study from smaller groups to whole classes.

The Research Model

There were cycles of research, the first using as a starting point and means of location, previous classroom experience and a background in the literature. Subsequent cycles each drew on the previous phase, but were also influenced by further reading of the literature, dialogue with colleagues and researcher introspection. There was also a formal dialogue with the research community, using refereed publications. In the analysis and interpretation of the data gathered in a particular phase there was generally one area of the background literature which was the principal lens used, and this is indicated in the particular chapter. However ideas from the literature were carried forward to other areas and data from previous phases was often re-examined using the lens of a later chapter.

Figure 3.1 illustrates the pattern of progress through the project. After locating the project in current literature and in my previous experience in the classroom, the main path of the research followed successive cycles as shown. Subsidiary features of the progress of the work such as continuing inputs from the literature and from dialogues with research colleagues are shown. Also indicated are inter-phase links and revisits, together with publications produced during the progress of the project.

The Pattern of Research Progress

(Diagrammatic only)

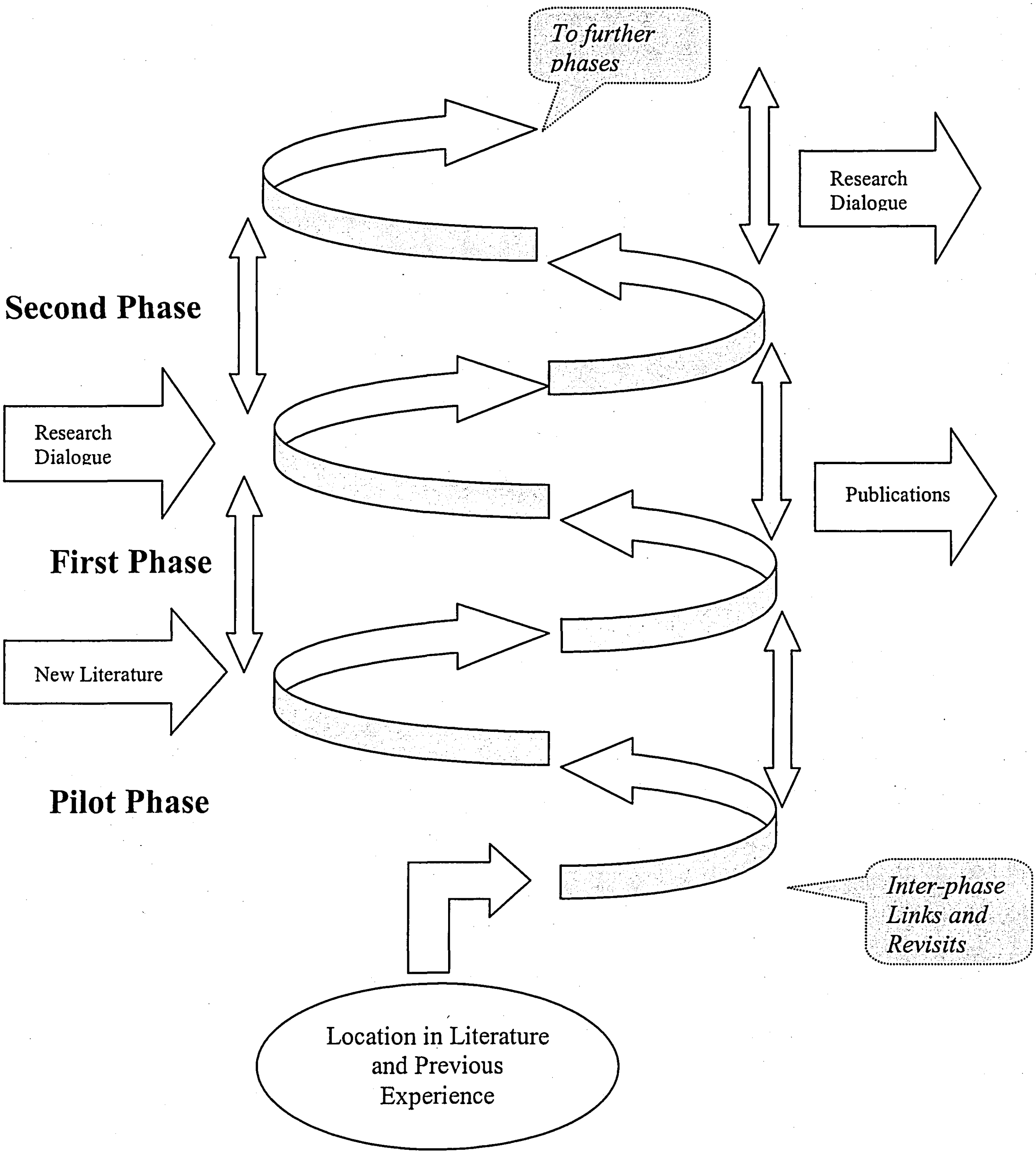


Fig 3.1

Methods of Data Collection

Most of the data collected in this project came from transcription of audio recorded dialogue from groups or classrooms whilst activities involving the TI 92 were taking place, from diaries and notebooks kept by pupils and from field notes recorded in various ways. These are discussed in detail later and their relation to the four main aspects of ethnographic inquiry (Eisenhart, 1988) identified.

However it is felt to be worthwhile recording in a brief aside the other methods which were tried before the final data-recording technique was identified. File handling on the TI 92 could allow pupils to record their screens in the memory of the individual machines used. However the process is far from simple. If the machines were used over a long period with a particular group of children and they could build up their skills it might be possible to use this facility. In the case of this project it became clear very early on that access to secondary classrooms, with departments worried about pressure to cover syllabuses, would be limited. Accordingly it was decided not to use file saving and rather to try to use time with classes concentrating on content. This meant that some other method of recording screens would be needed. In work with dynamic geometry on desktop computers, Hoyles and Noss (1992) used a 'dribble file' which continuously recorded pupils' key-presses and the screens they generated. Individual screens from the TI92 can be captured onto a desktop computer, and this process has been used in providing the figures in this thesis. However the advantage of the TI92 over the desktop is its small size and portability. This would be lost if it were connected in such a way whilst pupils were using it. Video recording of classrooms, by a second observer using a hand-held video camera was tried. There was only limited success in videoing screens. Because of the limited angle of view pupils had to 'show' their screens to the camera, which limited spontaneous observation. The video operator had to be close to the pupils if any conversation was to be picked up and pupils became embarrassed, again limiting spontaneity.

One TI92 was available with a port which allowed linkage to an overhead projector. This was used by me to introduce topics and discuss the pupils' ideas. On one occasion, a group were audio recorded whilst using the projector version, and the projection of their work was videoed. This was again problematic, as another advantage of the TI92, that work can be kept private, was lost, and, whenever I needed the projector version to talk to the class, they had to lose their screen. (Later versions of the TI92 all have the necessary port. This would mean that some of these problems were avoided, and also raises interesting possibilities for classroom practice, since any pupil can show their work to the class.)

In the next stage of the project, working with, at most, two smaller groups, it was found that audio recording of dialogue and the recording of screens by field notes was possible. When subsequent work with classes was undertaken it was decided to rely on this method, with audio recorders on desks for each group. In later stages of the work I was concerned to limit the complexity of the screens used, for reasons of transparency discussed elsewhere, which meant that brief notes could provide a record. It was decided to record these simpler screens by field notes and it was found that a useful way to do this was to cover an area of the table used by each group with a dated scribble sheet, which pupils were free to use, and which I used to illustrate discussions I had with them and to make a record of the screens involved. These methods of recording the activity and dialogue of pupils became the methods relied on for the rest of the project. Together with the collection of pupil diaries they formed the basis of ethnographic methods related to the four aspects detailed by Eisenhart (1988) and discussed above. These can now be related in detail to this project.

Ethnographic Techniques

The four methods of data-gathering commonly used in ethnographic studies (Eisenhart, 1988, p105) are discussed here.

- Participant observation

  Ethnographic research implies the willingness of the researcher to enter into and participate in the activity of the community being studied. Outsiders may not be able to participate in the learning activity. The researcher as participant observer must locate on a spectrum defined by the two ideas of participation and observation. Complete participation in the activity under study will lead to high subjectivity and sympathy with the other participants. A fully observational role, detached from the activity, suggests objectivity. This is not to say that location on this spectrum will be maintained. During the course of this study and in different phases, the position on this spectrum changes as data is collected from small groups or from classrooms. However, in the overall research environment of the present project, where I acted as teacher/researcher within groups and classes, participant observation was largely skewed towards the observational. However, viewing the classroom as a community of practice (Lave and Wenger, 1991) in which class and teacher have 'shared ways of behaving, language, habits values and tool use' (Winbourne and Watson, 1998) indicates the value of a participant role for the researcher. Indeed, such a role is essential if the study is to embrace the ideas of educational anthropology.

- Ethnographic interviewing

Interviews allow the participants, in a way which may be more or less structured, to make their own contribution to the direction of the study. Interviews are the ethnographer's way of finding the subjective views of the participants and may be completely open-ended, almost conversational, or structured questionnaires. In the case of this study, pupils were asked to keep diaries of their impressions of the activities in which they were involved, as an open-ended way of contributing to the study and these are referred to in the relevant sections.

- The search for artefacts

The researcher's view of the context is necessarily constrained, and further data is collected by reference to the literature. This enables a broader view to be taken, which looks at the context from a historical and wider social perspective. Content relevant to this project was used initially to locate the study in the context of the body of knowledge on this topic, providing an initial grounding. My previous classroom experience also contributed to this grounding. Subsequently, in the various phases of the study, there was further input from the literature, but at this stage there began to be a dialogic element, in that the project began, in its turn, to make its own contribution to the body of literature.

- Researcher introspection

In educational anthropology, as opposed to objective research of the hypothesis-test-conclusion type, the researcher will constantly examine the progress of the study and may well revise the nature of it. Reflective introspection on the part of the researcher is therefore an important part of the research process. Researcher sensitivity, recorded in field notes and fostered by the involvement of the researcher in the activity of the study and an introspective approach to it, together with continual referral to the literature typifies this aspect of educational anthropology.

Identification of Aspects of Data Collection

- Participant observation

Participant observation in the classroom, by a teacher/researcher of pupils, as observed above (p 64), is necessarily shifted towards the observational end of the spectrum. In the case of this project participant observation was monitored by audio recording of dialogue between pupils and between teacher and pupils. The collection of field notes, whilst principally intended to contribute to researcher introspection, also contributed to this area, particularly the scribble sheets used to record classroom and group activity.

- Ethnographic interviewing

Rather than using interviews, the views of pupils were gathered by collecting open-ended written impressions of their experiences from pupils, as recorded in diaries. These diaries were found mainly to yield information on affective issues. In many cases audio recording of pupils as they worked on the tasks was carried out without the participation of the teacher, and enabled data collected to be regarded as open-ended.

- Search for artefacts

The literature on the classroom use of dynamic geometry and socio-cultural principles of meaning-making was used to provide a starting location for the study. As it progressed this literature remained important, and further reading in this area was relevant. However there were other aspects of the literature, particularly those relating to classroom dynamics, which were introduced as the project went on. An element of this dialogue with the literature was interaction with colleagues in the field and the publication of works in refereed journals.

- Researcher introspection

Field notes, both from contemporary notes made on scribble sheets while talking to pupils and other notes made before and after sessions, were the immediate input into this part of the ethnographic process. Introspection and examination of the progress of the study and of my personal involvement in it, and the way it was affecting my view of myself as a reflective classroom practitioner provided a way of relating the project to my view of classroom practice.

Calendar of data collection

Figure 3.2 sets out the data collection sessions in chronological order, together with the schools where they were done. It also indicates the chapters which deal with particular sessions and the developing thrust of the work.

Calendar of data collection

Phase	Date	School, Age of pupils	Extent of Involvement	Content/ Direction	Description/ Discussion
Pilot	December 1996	School A 11-12 yrs, mixed ability	Two classes of 25-30 pupils, two 80 minute sessions each	Preliminary classroom work	Chapter 5
Phase 1	June 1997	School A 11-12 yrs, mixed ability	One class of 28 pupils, four sessions of eighty minutes	Classroom exercises in geometry	Chapter 6
Phase 2	October 1997	School B 12-13 yrs	Higher attaining pupils were invited to attend in lunchtime sessions, of about 40 minutes. A total of eight pupils attended for one lunchtime a week, over a period of seven weeks. There was intermittent contact with this school over the course of the project. (See preface)	Meaning-making in smaller groups	Chapter 7, Chapter 8 and Preface
Phase 3	June/July 1998	School C 13-14 yrs	Two classes of 25-30 pupils were involved. One was of high attainment, one a lower set. Each was seen four times, for 75 minutes each.	Classroom work, classroom dynamics	Chapter 10
Phase 4	Summer Term 1999	School D 11-14 yrs	The sessions took place at a lunchtime mathematics club. Attendance was variable, usually about 18 and included children aged between 11 and 14, of all levels of attainment	Sense and meaning making	Chapter 7, Chapter 9

Fig 3.2

The Schools

All the four schools involved in this study are situated in the north of England. Schools were approached and asked if they were willing to take part. Many schools were reluctant to use up curriculum time on a research project, in the case of two of the schools the work was done in out-of-hours sessions. In the other two schools some classroom work was possible.

School A

School A is a large, mixed, oversubscribed, 11-18 comprehensive school in a large city, with a wide range of ability and social groups. About one in seven pupils has a first language other than English.

School B

School B is a mixed 11-16 comprehensive school with 455 students. The school serves a wide rural area. The proportion of students who come from areas suffering significant social disadvantage is small. Standards of attainment of the intake of students are close to the national average.

School C

The school educates about 700 boys and girls in the 11-16 age range. It is designed for 750 pupils and is fully comprehensive, with about six per cent of pupils speaking English as an additional language. The economic, social and demographic characteristics of the school's catchment area are a little less favourable than the national picture.

The number of pupils receiving a free school meal is approximately 26 per cent.

School D

The school is a mixed comprehensive school for pupils aged 11-18, in a semi-rural area. The school draws its pupils from social and economic backgrounds close to the average. The proportion of pupils eligible for free school meals is broadly in line with the national average, as is the level of attainment of pupils on entry. The percentage of pupils speaking English as an additional language is high at 9.5 per cent.

Conclusion

This chapter has developed the overarching methodology of the project. In chapter 2 I dealt with the theoretical background to the study and many of the ideas from that chapter will be applied, using the methodologies discussed here, in a tool-and-result way, to the analysis of singularities in the classroom. In addition the approaches outlined in chapter 4 from the literature concerned with mathematical education, geometry construction and proof and the use of dynamic geometry in particular will be used to inform the analysis of data. In each phase of the project further aspects of the literature and different viewpoints are used, in a process of research sensitivity based on an eclectic approach to methods/methodologies.

Chapter 4

Geometry and Technology

A Review of Literature

Chapter 4

Geometry and Technology: A Review of Literature

After an initial discussion of personal background and the ways in which the literature has been influential in providing starting points for the project, this chapter goes on to review the literature which is specifically relevant to the field of mathematical education in general and dynamic geometry in particular. This is treated in sections:

- *Representation and Technology*
- *Dynamic Geometry*
- *Visualisation, Conviction and the Nature of Proof*
- *Transparency of Resources*
- *Socio-Cultural Dynamics in the Mathematics Classroom*

Literature: Location and Continuing Influence.

Two broad areas of experience in mathematics education, in the classroom and writing for a national mathematics scheme provided a backdrop to this project, but it was with readings in the literature that the project began to take shape. The background in the classroom will be a familiar one to many teachers. A picture of the statutory requirements of the National Curriculum interacting with a necessarily pragmatic approach imposed by time constraints to influence the teacher's belief in the value of an interactive, practical, discovery-based approach to classroom work will be recognised by many. In many ways the same priorities are juggled by writers of classroom texts, with the added complication that texts are expected to be commercial in a limited field and to be applicable to all the different ways schools may use to try to resolve their problems.

Within this background I had used stand-alone computers, computer suites, graphical calculators and other forms of technology. I had collaborated in writing material on the use of technology with classes (Appendix XIII).

At the outset of this project this background was useful, but did not provide a starting point sufficiently grounded in theory. Reference to the literature was needed, as with any study of this kind. The work of many previous authors has made a continuing contribution, both as an initial means of locating the project in current thinking and then contributing to progress over the course of the various cycles of the data gathering. At the outset, reference to the literature enabled the project to be located in the field of study, and provided a starting point, a frame of reference, from which progress could be made. As the work has continued there has been a different, continual involvement in the literature, so that I have felt more in dialogue with the standpoints of other workers.

The cyclical nature of the research and the way different phases were influenced by different areas of the literature is dealt with elsewhere in this work. Particular sources in the literature were relevant at their particular times and these sources and their influences are dealt with in the appropriate chapters. There are, however, large bodies of the literature which are generally relevant to this study. One particular area, that of socio-cultural theory and how it provides a general background to the project, has been dealt with in chapter 2. This chapter will deal with literature specific to mathematics education and the use of technology and in particular sources which deal with the use of dynamic geometry.

I referred in chapter 2 to the idea of transparency developed by Lave and Wenger (1991), which they use to examine resource use. They point out that resources used in learning should be visible enough to direct the gaze of learners, but transparent enough to allow that gaze to see through to significant meaning-making. Along with other researchers I have found this a powerful notion in analysing applications of technology and other resources in the mathematics classroom. In this chapter I will consider the concept of transparency in terms of its application to the topic of mathematics learning generally and dynamic geometry in particular.

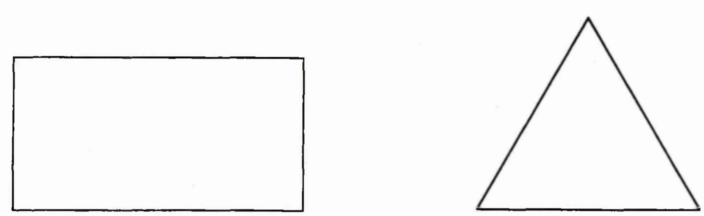
Representation and Technology

There is a large body of work on the importance of representation of mathematical problems in general and the use of diagrams in geometry in particular. In a recent wide-ranging review of the topic by Goldin and Janvier (1998), Mesquita (1998) discusses conceptual obstacles relevant to the use of diagrams and representations in geometry. She points out particularly the distinction between the case where a more experienced student is able to accept a diagram as a representation of a general case, whereas the less experienced will focus on the particular figure given. She refers to this as the 'double status of geometrical representations'.

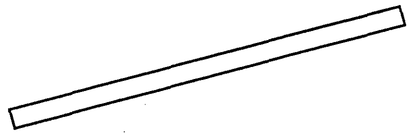
> This means that the same figure can represent either an abstract geometrical object, or a particular concretization. Depending on the problem we are sometimes interested in the first situation, and sometimes in the second. Based on this ambiguity, it is clear that the figurative register, used alone, does not enable one to distinguish between the two cases. The degree of abstraction that is required to deal with a representation varies from one situation to another. (Mesquita, 1998 p. 186)

Mesquita (op cit) goes on to discuss the 'typicality' of diagrams, citing the importance of orientation, particular proportions and preferred shapes. She sees this as connected to the idea of double status, and suggests that preferred orientation of squares, for instance, is due both to a cultural bias derived from architecture and physical constraints and from a readiness to associate orthogonality primarily with horizontal and vertical lines (or lines up and across the page). She goes on to discuss 'prototypical' shapes.

Rectangles and triangles such as



would be prototypical, whereas others of the types

would not, by reasons of proportion and orientation. Mesquita says

> Stability and aesthetic preoccupations may reinforce the perception of these prototypical figures. ......Due to the influence of physical space and other cultural reasons, teachers (and textbooks) tend to privilege prototypical figures, which are more easily used than others. Economy of paper and page setup factors also contribute to the same effect. (Mesquita 1998 pp 189-190)

In working with dynamic geometry, early introduction of the drag function enables pupils to see the general nature of representations. If pupils see that the shape of a triangle they have drawn can be altered by dragging, the generality of the diagram is emphasised in a way which will help to overcome these problems of appreciation of the general applicability of geometric properties. Mason (1991) points out that the indication of generality given by dragging a diagram can itself be used as a stimulus for proof, in that pupils will feel a need to explain why the general case applies.

Berger (1998) makes some relevant general points in a discussion paper on the use of graphical calculators, which can equally be applied to the use of other handheld devices such as the TI 92 in whole class circumstances. After referring to the preference for the availability of hand-held machines in the classroom (as opposed to access to software in a computer suite) she continues

> Unfortunately, despite the potential and actual importance of this tool, there is not much literature dedicated to explaining or understanding how the graphic calculator, specifically, functions in relation to the learner. In fact, much of the literature relating to the graphic calculator is anecdotal or describes evaluative studies which fail to distinguish adequately the role of the tool from that of the instructional process (Penglase and Arnold, 1996 p 53).
> .... I wish to suggest that a Vygotskian approach to learning, with its emphasis on mediated activity within a particular socio-historical context, is appropriate to address the relationship between the mathematical learner and the different sign systems (multiple representations) afforded by the graphic calculator. (Berger, 1998 p 13)

and,

> For internalisation to take place, it is not sufficient that a student is merely exposed to a new technology; rather he/she needs to engage thoughtfully with the technology (Salomon, 1990). In order to interact in such a mindful way, he/she has to use the technology actively and consciously in a socially or educationally significant way. (Berger, 1998 p. 19)

She argues for dedicated research related specifically to the use of graphic calculators.

> ..the learning experience ....is sufficiently different from (that) in a computer environment that it warrants its own dedicated research and interpretation. (Berger, 1998 p. 14)

There is, then, a need to address issues of socio-cultural aspects of learning in relation to the use of handheld calculators such as the TI92. It is hoped that the present study goes some way to answering this need, by looking at the socio-dynamics behind the use of hand-held calculators in general and in the area of dynamic geometry in particular. To this end I now turn to a discussion of the use of dynamic geometry in the classroom.

Dynamic Geometry

Dynamic geometry software has become widely available for desktop computers and networks. Geometry Inventor, Geometer's Sketchpad and Cabri Géomètre are all available commercially and there is at least one source of dynamic geometry software on the Web (appendix VII). All these software suites run on the PC. The present research, using as it does (particularly in the later stages) fairly simple diagrams which would be available on any package, is not specific to any one piece of software. In fact, that used was Cabri II as available on the Texas TI 92.

Although the choice of software is not critical, the hardware used is seen as important to the ideas developed in the project. The TI92 is a hand-held machine available in the normal classroom environment. It can be easily moved to the side of the desk if necessary. The socio-cultural, whole class and group approach which was adopted for the field trials and data collection and which is advocated as a method of classroom use is enhanced by the use of this particular hardware. Features of this machine, which are relevant to this study, are its small size and the possibility of connecting it to a projector for class demonstrations. (See appendix V for details of the TI92). As detailed above, Berger (1998) has referred to the lack of research into the specific socio-cultural implications of such technology.

Cabri Géomètre was developed at the Université Joseph Fourier in Grenoble by a team led by J-M Laborde (1988). Schumann and Green (1994), in their introduction to the use of the software, explain the acronym **CA**hier de **BR**ouillon **I**nteractif (interactive rough book), and Mason (1991) points out that this provides a useful insight into the way in which the authors see the use of the package. In their preface Schumann and Green (1994) quote a passage by Higgo taken from the Mathematical Association pamphlet *Not the National Curriculum* (1992)

> The opportunity to DO, EXAMINE, PREDICT, TEST, GENERALISE should, from an early age, permeate the learning situations pupils are put in. They should be encouraged to question (WHY?) and extend (WHAT IF?) their findings. Geometry should be presented in such a way as to highlight the logical aspects. At appropriate stages

> children should be helped to go on to formulate their own proofs (sometimes as a group).
>
> What is important, however, is that we do not restrict pupils' progress (denying them the opportunity to 'act as mathematicians') and do not try to separate or compartmentalise the stages too much.
>
> (Higgo, 1992)

In all dynamic geometry packages a fundamental feature is the ability to draw a diagram under some geometrical constraint and to investigate the consequences of 'dragging' the independent geometrical entities in the diagram as a means of investigating invariant geometrical relationships. This provides a means of moving from the specific to the general, of considering atypical shapes as well as the prototypical forms of representation discussed by Mesquita (1998). Healy et al (1994a) have indicated that dynamic geometry can be introduced using the drag function to emphasise the difference between drawing and construction. They introduced the software to children without any preliminary work on ruler-and-compass constructions in a deliberate attempt to investigate the way in which pupils could use the software as a starting point. They asked pupils to draw faces and introduced the idea of 'messing up', of allowing the drawings to be checked to see what the effect of dragging was and how and to what extent features were interrelated when independent points were dragged.

> Our purpose was to focus on the difference between constructions which were 'non-mess-up-able' and drawings which could be messed-up in well-defined ways.
> (Healy et al, 1994a, p. 16)

Healy et al report that the idea of whether or not a screen could be messed–up became a powerful image to children, even away from the computer.

In a collection of articles reviewing the field Mason (1995) mentions the opportunities which manipulative software can afford for students

> 'to consider the why, to explore "by hand methods" in order to appreciate what tools are doing'. (Mason, 1995 p. 13)

He acknowledges the importance of pupils becoming involved in the mathematics they are learning, of doing, manipulating, before sense making and articulating. He continues

> But doing is not in itself sufficient, for sense-making does not follow automatically from manipulation. Nor does articulation follow automatically from sense-making. To encourage and support transitions requires the awareness of an expert, a teacher.
> (Mason, 1995 p. 10)

Mason, in the same article, refers to the need for teachers to be 'aware of their own awarenesses' in order to be able to stimulate sense and meaning making in students. The importance of the teacher's contribution to students' work with dynamic geometry is also emphasised by Jones (1997). He draws a distinction between perceiving and specifying geometric relationships. Perceiving is mediated by the dynamic image on the screen, and specifying is the process of expressing this perception in mathematical language. Jones goes on to discuss how this process is mediated by a dynamic geometry environment. Citing Wertsch (1991), Jones sees the mediating artefact and the activity as mutually dependent. In the case of dynamic geometry environments the language of the environment, used in the on-screen menus, leads to the process of specifying relationships.

De Villiers (1995) indicates an approach of allowing generalisation to arise from constructions. An NCET document (Goldstein et al, 1996) has detailed the work done in a pilot study in four secondary schools. The ICMI conference in Pisa (Mammana, 1995) had as its theme the teaching of geometry in the twenty-first century. In their contribution Hoyles et al (1995) considered the interdependence of construction and proof. Healy et al (1994b) refer to the importance of on-screen scaffolding. Healy (2000) reports two distinct types of construction, leading in different ways to mathematical meaning-making. 'Soft' constructions take some of the conditions specified, for instance in attempting to draw congruent triangles. They can be manipulated to allow examination of conditions and the exploration of possibilities. 'Robust' constructions take all the conditions and attempt dragging to assess the soundness of propositions. Reporting this work Hoyle says

> ..our vision was one in which students employ Cabri tools (to) construct, manipulate and check geometrical relationships, receiving computer feedback brought about by their activities that could help towards the proof of any conjectures they formulated. To this end we devised a sequence of activities, each of which included a computer component with a common structure: students were to construct mathematical objects on the computer, identify and describe the properties and relations that underpinned their constructions, use the computer resource to generate and test conjectures about further properties, and make explanations as to why they must hold. (Healy, 2000 p.106)

The drag function and the idea of a construction invariant under drag are central to the use of the software to lead towards an appreciation of geometrical invariants. In introducing the use of Cabri in this project, the classroom material was directed at using the drag function, by showing how diagrams which were not defined by geometrical invariants could be 'messed up', to help pupils to make a distinction between drawing and construction. The materials also sought to promote concepts such as that of using a circle to preserve length (Healy et al, 1994a, 1994b). Pupil fluency with the machines was a further objective of early work (Goldstein et al, 1996). Later work was directed at assessing any particular advantages of the handheld equipment and probing the mediation of learning in the dynamic geometry environment.

The principles adopted were that the materials should allow rich meaning-making to develop from classroom interactions between pupils, teacher and screen. The development of the materials was guided by the ideas of Hedegaard (1990), who advocated the fostering of a 'whole-class zone of proximal development'. Some early thinking by workers in the area of activity theory was sceptical of the use of computers in the classroom, (see, for instance, Engeström et al, 1984) citing Papert's (1980) ideas of the computer and the child forming a private area for learning. Such a union was seen as precluding the social interaction necessary for learning. Such a closed loop is possible with some types of software. However these fears do not seem to be justified provided the possibility is recognised and more recent work has emphasised the advantages of the use of technology in group work (Hoyles, 1985, Hoyles and Noss, 1992, Hudson, 1996).

Visualisation, Conviction and the Nature of Proof

The use and purpose of proof has been the subject of heated debate in the literature. There are those (A. Gardiner, 1995) who insist that a rigorous approach is needed, who value a step by step formal proof as a reproducible and communicable process. There is evidence that at university level, mathematics learners find identification of a logical proof difficult, and are often influenced in their judgement of mathematical arguments by empirical and aesthetic issues, rather than following a chain of reasoning (Finlow-Bates, Lerman and Morgan, 1993). There are others, for example Mason (1991), who call for an acceptance of the ability to test a very large number of examples as a form of proof. According to Mason such a case is provided by dragging in dynamic geometry. Referring to the power of computers to present a dynamic image under the control of the user, he calls for acceptance of a form of proof afforded by such a large number of examples. He writes:

> I predict that one of the long-term effects of computers will be to establish a mode of certainty which lies between the too-easy acceptance of a generalisation from one or two cases and the rigour of mathematical proof. Programs like Cabri-geometry enable the user to experience a huge range of particular cases, and by appeal to continuity, an infinite number of cases. This plethora of confirming instances will be highly convincing for many, if not most, people. I find this entirely reasonable. (Mason 1991 p. 87)

Rotman (1994) gives a further critique of conventional notions of proof and the anticipated impact of computers on our understanding of them.

There is a wide spectrum of opinions between the view of proof as an absolute logical system and a need for conviction and many intermediate positions are taken. But the very fact of this lively debate is evidence of the essentially social nature of proof. We prove to others, proving is about communication, and as such is linked to socio-cultural interpretations of learning. We prove to ourselves as well, but this is an internal dialogue, perhaps analogous to socio-cultural ideas of internal speech and meaning-making. My view of the proof debate is one which draws encouragement from those aspects of it which can, at any particular level, increase opportunities for social interaction. Whatever stance is taken on the importance of proof, it is useful to analyse the processes of proof in more depth, to examine the nature of those processes and the way they can be used in the classroom to assist pupils in the process of deriving mathematical meaning.

The role and function of proof has been discussed by de Villiers (1990). Drawing on work by Bell (1976), he identifies and discusses five components of proof:

- verification (concerned with the truth of a statement)
- explanation (providing insight into why it is true)
- systematisation (the organisation of various results into a deductive system of axioms, major concepts and theorems)
- discovery (the discovery or invention of new results)
- communication (the transmission of mathematical knowledge)

(De Villiers 1990 p. 18)

On the topic of verification, de Villiers points out that this is often an endorsement of an already firmly held conviction. He refers to Polya's (1954) assertion that conviction precedes proof 'When you have satisfied yourself that the theorem is true, you start proving it'. Polya goes on to point out that a theorem, an example of demonstrative reasoning, usually has its genesis in plausible reasoning.

> You have to guess a mathematical theorem before you prove it; you have to guess the idea of a proof before you carry through the details. (Polya 1954 p. vi)

De Villiers notes that empirical examples often lead to conviction/verification; similarly, one of Fischbein's (1982) forms of conviction is empirical conviction arising from a number of practical findings.

Fischbein (1982) identifies three forms of conviction: formal, arising from argument, empirical, arising from a number of practical findings, and an intuitive intrinsic conviction, which he calls 'cognitive belief'. A dynamic geometry environment can echo these ideas in its provision of a climate where argument is fostered, by allowing dragging to provide empirical proof and by triggering with the screen children's intuitive visualisations. There can be classroom interchange which moves between these areas, which can be related to the ideas of spontaneous and scientific concepts offered from socio-cultural theory. There is a place in this dialectic for the way pupils deal with necessary and sufficient conditions in mathematical argument.

Empirical example is a powerful vehicle for conviction and, with dynamic geometry software, may lead to a form of verification; however such verification and conviction do not in themselves lead to meaning-making. Indeed the reaction of pupils to a visual or empirical demonstration intended to convince is often lack of interest. On being presented with a demonstration of a geometrical truth there is a danger that pupils will react with a dismissive 'so what' attitude. De Villiers (1991) claims that, in contrast, it is possible to excite pupils' motivation for and satisfaction from the deductive explanation of a proof, to engage what Mason has called 'this sense of mustness' (Mason 1991, p. 73).

Davis (1993) argues for the interpretation of the word 'theorem' in a sense that "is wide enough to include the visual aspects of mathematical intuition and reasoning". Dynamic geometry can provide a visual trigger to intuition, allowing reasoning and synthesis into formal mathematical language to begin.

Mariotti and Bartolino Bussi (1998) consider the teacher's input into classroom use of dynamic geometry, using a simple construction (of a square given a line segment as a stating point). The authors consider the distinction between accurate concepts and ideas and how pupils represent these on the screen using the technology. Mariotti and Maracci (1999) examine the distinction between argumentation, as a means of reaching personal conviction or of convincing others, and the development of a more formal proof. Boero et al (1999) consider the process of generation of conditionality and give examples of ways of presenting pupils with situations where conditional statements (If .... then) can be encouraged and explored. The drag facility in dynamic geometry allows pupils to explore such situations and test their conjectures.

Visualisation plays a major part in the work of many authors, including Bills (1996) on generic proof and Nelson (1993) on visual proof. Nelson presents proofs entirely visually, via a series of diagrams. Cunningham says

> One of the most remarkable things about visualisation is the amount of mathematics students will learn and the amount of work students will do in order to create images describing a mathematical concept, especially when the computer is used as part of the process. (Cunningham 1994)

Mason (1991) refers to 'inner screens' and makes a plea for visualisation, for 'saying what you see'. He calls for an awareness of 'the fact that there are facts'. He emphasises that pupils, rather than *working through* exercises in geometry, should use dynamic geometry packages in a way that allows them to see themselves as *working on* mathematics. He discusses the conviction available when pupils drag a geometrical diagram and see that in a large number of cases a particular conjecture is correct.

> For many people, this level of convincing will be adequate. But it is more important that people have a well-developed sense of the fact of geometrical facts (together with an underlying scepticism of anything machine generated), than that they are pushed through tedious and for them meaningless computations and reasonings that purport to provide a proof, but which remain mysterious incantations. It is possible to raise the question of proof, of trying to convince yourself that certain facts must hold, and even to be intrigued by the plethora of interconnections between different facts, but this need not be demanded of everyone. (Mason, 1991 p. 72)

Whatever stance is taken on the definition of proof, and as I have outlined these are many, I feel that the importance of proof in the classroom lies in its essentially social nature. Proof in its very nature draws pupils in to a meaning-making activity, into a community whose practice is making mathematics.

Transparency of a Resource

The idea of transparency introduced by Lave and Wenger (1991, pp102, 103) and further examined by Adler (1998, pp 8-11) has been discussed already in Chapter 2. I have found its application to mathematics resources to be relevant and powerful, and discuss it here in relation to mathematics in general and dynamic geometry in particular.

Lave and Wenger use the analogy of a window to explain their thinking on transparency, pointing to the need for a resource to be visible in the sense that a window is visible, drawing the attention, but allowing the gaze to pass through, in this case to allow mathematical meaning-making. Adler (1998) discusses the use of resources in the mathematics classroom, seeing transparency as a useful yardstick by which to consider their use. She points out that the language used in the classroom must be considered for its transparency, and refers to a 'dilemma of transparency' which can arise if the teacher insists on formal mathematical language. Referring to an observed lesson she says

> ..in some moments of practice, explicit focus on mathematical language in fact seemed to obscure mathematical meaning. Instead of mathematical talk being a transparent resource with its dual functions of visibility and invisibility, (visible in that it extends the practice, and invisible in that it enables smooth entry into the practice) explicit mathematical language teaching became opaque. The talk itself became too visible, the object of attention rather than also a means to mathematics. (Adler, 1998 p. 9)

In the same article and in the context of rural schools in South Africa, Adler considers the transparent use of simpler resources, such as the chalkboard and the school's approach to the use of time and the timetable. She suggests that in schools where time is well managed, time itself becomes a transparent resource, leading to economic time use by students.

A further point is made by Adler (1998), which carries significance when brought together with work from other authors who have examined the use of 'real' situations in mathematics classrooms. Adler refers to the use of money as a resource, to the way it is used in the classroom, possibly as a vehicle for the teaching of numeracy. She points out that such examples bring with them, from the rest of the school and above all, from the wider society outside school, a host of socio-historical connections. These will be different for each individual in the class and may be mediated by issues of social class and, in the particular case of money, real buying power. Such considerations will affect the transparency of the resource, and therefore its capacity to allow a way through to mathematical meaning-making.

> This is why drawing on resources from contexts and practices outside of school mathematics creates significant challenges for teachers. .....context crossing can be dangerous and alienating in school, and more so for some learners than others.
> (Adler, 1998 p.11)

Other workers have identified difficulties inherent in the use of examples from the 'real' world in the mathematics classroom. Boaler (1997) has drawn attention to these dangers in the context of female under-achievement in relation to so-called realistic mathematics, and Cooper and Dunne (2000) have looked at issues of social class, gender and equity in relation to UK National Curriculum tests in mathematics. Work such as this emphasises the opacity which can be inherent in examples used in mathematics classrooms in an attempt to relate topics to the 'real world'. In attempting to make the material relevant social and class factors can be introduced at the expense of transparency, of allowing pupils to see through to mathematical meaning-making.

I believe that the idea of transparency is very valuable in assessing the use of resources in the classroom. The teacher must consider carefully the transparent use of resources as they apply to the members of the class and whether or not they allow mathematical meaning-making. Relevant factors include:

- the language used as pupils move from spontaneous sense-making to their own internal meaning-making and on to the use of formal mathematical communication;
- the diagrams employed to support the activity, which should be a window to meaning-making;
- the software used should allow transparent use;
- and the hardware should not dominate the view of the pupil.

This last point indicates a place for portable, hand-held technology such as the TI92 and later I wish to deal specifically with the issue of transparency as it applies to the use of the TI92.

Socio-Cultural Dynamics in the Mathematics Classroom

In chapter 2 I dealt at length with the socio-cultural principles which will be used in the rest of this work. These were general principles of educational psychology, and were not specifically directed at mathematics education. In this section I want to deal with some of the principles I discussed in the previous chapter as they relate in particular to mathematics classrooms and to introduce the ideas of other workers who have developed their own related approaches.
The works of Vygotsky, which emphasise the primacy of the social in learning and the way the zone of proximal development can be seen as the site of learning were a starting point. I discussed the different positions taken on the significance of the zone of proximal development by various workers, as a site for mediation (Bruner, 1986) as the place where there is a dialectic between scientific and everyday concepts (Vygotsky, 1962 p 108) and as the environment

for the practising of revolutionary critical activity (Engeström, 1987). Situated cognition within local communities of practice, as advanced by Lave and Wenger (1991) is developed as a concept with relevance in mathematical education by the work of Winbourne and Watson (1998). A wider definition of the zone of proximal development is adopted in the work of Hedegaard (1990), who sees the idea as embracing the activity of the whole class and the teacher. Lerman (1998) sees the zone of proximal development as including previous experiences of participants and the power relationships in the classroom, and Watson (1998) points out that Lave and Wenger's view of learning in society locates the classroom in the wider community, with all the social and class influences which such location implies.

Activity is seen as the ability to engage, through social interaction, with intention, in the process of meaning-making. Meaning-making is seen as a stage in the developing attitude of learners as they begin to develop a critical awareness of their place in the practical-critical revolutionary activity which is the learning society.

As I have said, many workers in the field of mathematics education have brought their own perspectives to the general socio-cultural background developed from these ideas. The work of Winbourne and Watson (1998) and Cobb and Yackel (1996) in particular have provided insights into factors which influence mathematical meaning-making in a whole-class or group zone of proximal development, indicating socio-cultural vectors which may operate for such meaning-making.

Winbourne and Watson (1998) draw on work by Lave and Wenger (1991) and Lave (1993). They extend the idea of situated cognition within local communities of practice, and the related idea of legitimate peripheral participation, to consider 'local communities of (mathematical) practice'. They identify features of such a local community of (mathematical) practice:

- pupils see themselves as functioning mathematically within the lesson;
- within the lesson there is public recognition of competence;
- learners see themselves as working together towards the achievement of a common understanding;
- there are shared ways of behaving, language, habits, values and tool-use;
- the shape of the lesson is dependent upon the active participation of the students;
- Learners and teachers see themselves as engaged in the same activity. (Winbourne and Watson 1998 p 183)

They examine classroom interaction in terms of such a community and go on to discuss the idea of 'telos', of the meaning-making of the whole class being aligned in directions generated by social interaction. They see telos as a unification of small scale 'becomings' by which many learners join a community of practice. They see:

> a link between our notion of LCP and the situated abstraction of Noss and Hoyles (1996). Just as they claim the computer provides domains which support students' abstraction, so we claim LCPs support students' growing image of themselves as someone who is legitimately engaged in mathematical practice, as someone, in other words, who is becoming a mathematician. (Winbourne and Watson, 1998 p.183)

This approach is echoed in the work of Cobb and Yackel (1996), who have analysed mathematics classrooms in terms of the negotiation and maintenance of social and socio-mathematical norms. Social norms include

- insistence on explanation of answers
- respecting the contribution of others
- making clear agreement as well as disagreement.

Socio-mathematical norms would include

- some notion of what constitutes a valid, complete solution
- agreement on the worth of alternative solutions
- negotiated agreement between teacher and students on the mutual acceptability of solutions.

Social norms will exist in all classrooms, and will bear a direct relationship to the society in which the classroom is situated. Because social norms will affect the negotiation of socio-mathematical norms, Apple (1992) has argued that the classroom is firmly situated in the wider context of the practices of school and society. Yackel and Cobb

(1996) discuss the influence of socio-mathematical norms on argumentation in the classroom. They draw on the ideas of Toulmin (1969) as developed by Krummheuer (1995), seeing argumentation as made up of conclusion, data, warrant and backing. Yackel (1998) says of argumentation:

> it clarifies the relationship between the individual and the collective, in this case between the explanations and justifications that individual children give in specific instances and the classroom mathematical practices that become taken-as-shared. As mathematical practices become taken-as-shared in the classroom, they are beyond justification and, hence, what is required as warrant and backing evolve. Similarly, the types of rationales that are given as data, warrants and backing for explanations and justifications contribute to the development of what is taken-as-shared by the classroom community, that is to the mathematical practices in the classroom. (Yackel 1998 p210)

Thus argumentation is seen as a social, rather than a logical process, a means of establishing that which is held in common about the topic in question and moving forward the 'held in common' by classroom interaction. Voigt (1995) discusses the reflexivity between learning and interaction and speaks of this reflexivity contributing to a classroom microculture which in turn affects the meaning-making which is taking place.

Conclusion

In examining the literature relevant to classroom use of dynamic geometry this chapter has first considered general difficulties which pupils may have with geometrical representations and the way dynamic geometry can be used to overcome them. It has considered the principles adopted by workers in the use of dynamic geometry and drawn on work related to the various functions of proof which can be used in social learning. It has related the general socio-cultural principles outlined previously more specifically to mathematics learning. I suggest that recognition of ideas of a whole class zone of proximal development, of local communities of practice and of negotiated socio-mathematical norms and argumentation have much to offer in looking at how technology, appropriately transparent, can be used to effect social meaning-making in the mathematics classroom. How these considerations from the literature are applied to the various stages of fieldwork and the material and practices used there is dealt with in the chapters which follow.

Chapter 5
Starting Points

This chapter describes a pilot phase carried out at an inner city comprehensive school in the north of England. There were a number of objectives in undertaking the pilot phase and the findings in these areas and in other areas (which were found to be just as valuable) are presented here. In the two weeks (four one hour lessons, two each with two classes) it was intended to introduce the 11-12 year old pupils to the TI92 and to the Cabri Géomètre software, and to observe their reaction to the introduction of dragging as a means of distinguishing between drawing and construction. Methods of data collection were trialled and preliminary observations looked at ways of working with whole classes.
In addition this chapter gives an account, from work at different schools, of other pupils' initial reactions to working with the TI92.

Introduction

As I began to work with children using dynamic geometry on the TI92 for the first time, I had a number of objectives in mind. Among these were:

- an appreciation of the need, on my part, to become familiar with the use of the TI 92 in classrooms;
- an attempt to find out how quickly children could become proficient in the use of the TI 92 and the dynamic geometry software;

- a wish to try to use the machines with whole classes, as well as with smaller groups;
- an early attempt to develop materials and activities which use the software to teach geometry at this level;
- a need to investigate methods of data collection when thirty or so children were each using the calculators independently (albeit on the same or related tasks).

This chapter reports on progress with these early trials. Other initial reactions of children in other schools to the technology are recorded and discussed.

The tasks set for the children were directed at introducing the idea of dragging, and the distinction between a drawing and a construction. These tasks were an adapted version of the work of Healy et al (1994a).
This section will discuss some of the principles behind such work, detail the tasks and the way in which the class responded to them. The problems associated with the collection of data which might throw light on the meaning-making which was taking place are outlined.

First Trials

The first part of this pilot phase was carried out in school A, an inner city 11- 18 comprehensive school. The school has 1600 pupils of the full range of ability, one seventh using a language other than English as their first language. The pupils were two mixed ability Year 7 classes (age 11-12), each of about twenty-seven individuals. Two sessions of one hour each were available with each class for this pilot stage. The pupils had not used dynamic geometry software or the TI 92 before.

The introductory material was intended to provide some familiarity with the software and to look at children's perceptions of the distinctions between drawing and construction. It was intended to investigate if there was a realisation of the possibility of defining a diagram in terms of geometrical constraints, i.e. so that there were geometrical dependencies.

The approach owed much to the ideas used by Healy et al (1994a), who used the notion of 'messing up' diagrams as a way for children to explore geometrical constraints by using the drag function. I also wanted to see how quickly children grew familiar with the TI92 and gain an insight into how the next stage of the research might proceed.

Hoyles, Healy and Noss (1995) discuss the significance of the availability of dragging in a dynamic geometry environment and the possibility that, in such an environment, construction may almost replace proof.

Speaking of dynamic geometry environments in general, they say:

> They provide a model of Euclidean geometry which offers feedback through 'dragging' as to whether constructions or theorems are 'correct'.
> (Hoyles, Healy and Noss, 1995 p. 103)

Healy et al (1994a) describe the introduction of dynamic geometry to Year 8 (twelve year old) pupils by using an exercise designed to highlight the difference between drawing and construction for children who have little formal knowledge of geometry. They asked children to draw a face, with eyes and mouth etc., and to investigate how the face could be 'messed up' by dragging elements of the diagram. They had reservations:

> We were aware that our approach ran some risks. The way students are introduced to a powerful medium inevitably moulds their perception of it and by introducing the element of drawing we were certainly not starting with an approach which encouraged a Euclidean perspective on the activities. We knew from our Logo work that freedom to create one's own goal – degoaling – is double-edged. On the one hand it allows pupils to appropriate the activities, to feel that the work is theirs, rather than the teacher's. On the other hand it sometimes enables pupils to avoid interacting with the mathematics at all! (Healy et al, 1994a p. 14)

Healy and her co-workers worked with one researcher to a pair of pupils and a stand-alone computer, and it was felt in the present project that the slightly more structured approach adopted, with a drawing provided as a file, might be more appropriate when working with whole classes.

Pupils had a TI92 each and worked in pairs. After some explanation and demonstration on the overhead projector version of the machine a worksheet (Appendix VIII) was given out and the pupils called up the first file and began. Two video cameras, operated by volunteers, were used on a roving basis whilst pupils worked.

The file called up by the pupils had been previously loaded on all the machines by linking them one-to-one. This file contained a drawing of a face where parts of the diagram were independent elements and others were dependent. The pupils were asked to explore this drawing by using the drag function to investigate ways in which the drawing could be altered, then exchanging machines and trying to restore their partner's alterations.

Typical results are shown below.

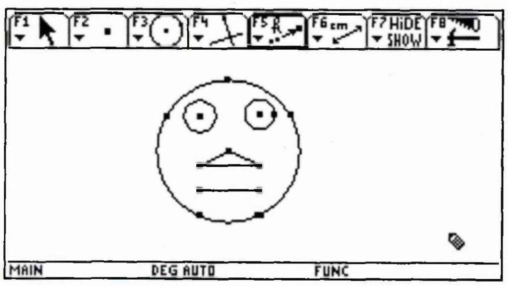


Fig 5.1

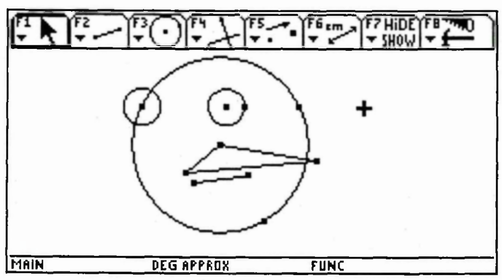


Fig 5.2

Pupils were able to investigate some of the geometrical dependencies in the drawing of the face, but restoring the original drawing proved difficult and there were difficulties in reloading the original file.

A second type of worksheet, which follows the same theme, is detailed in appendix IX. Here the intention was to use a simpler diagram and make the appreciation of methods of construction more accessible. The file as presented to the children (by previously loading it onto the TI92) contained two 'identical' squares, one drawn and one constructed. The children were asked to investigate the effect of dragging on the various elements of the diagrams. The OHP version of the TI92 was used for class demonstration and discussion.

The original file is shown below (Fig 5.3), with, in Fig 5.4, the effect of dragging and using the 'Hide and Show' option to reveal construction details.

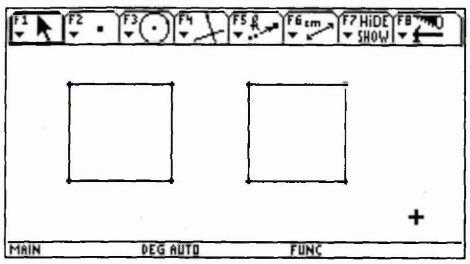


Fig 5.3

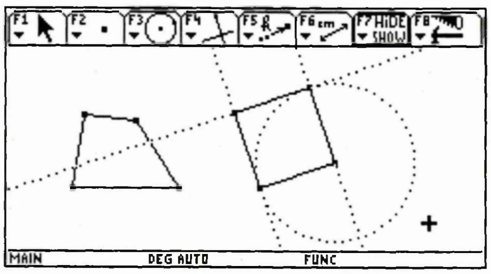


Fig 5.4

After this work some of the children were able to go on to use the TI 92 to construct simple geometrical shapes, particularly an equilateral triangle based on a generating circle.

Data Collection

The intention in the pilot phase was to try different methods of data collection, with an objective of being able to collect information in later trials which threw light on what meaning-making was taking place at the classroom and individual level. The approach consisted of feasibility studies of different methods of collection and an assessment of ways of observing individual meaning-making in the classroom. Video recording of the children and their work was used, using two cameras which were operated by external observers. Additionally children were asked to record their reactions in note-books. It was not possible to record the TI92 screens on the video, and the possibility of gathering individual children's reaction was limited by the need for the operator and camera to be physically close for the microphone to pick up the speech of individuals. Children's reactions were stilted and they were usually reduced to silence. Additionally, in the subsequent stages of the study it was not anticipated that camera operators would be available. Field notes of the sessions revealed an ability on the part of students to become quickly familiar with the software, and to make progress with the tasks set and to move off in their own directions.

> Wednesday 4/12
> Some got on to measuring spontaneously. Good attempts at constructing an equilateral triangle from A, B, C et al. (from field notes) F2

Initial Reactions of Pupils

As in all the schools visited in the project, most pupils' initial reactions to the machines in this trial were favourable.
Typical comments as recorded in note-books were:

I preffer doing maths on the TI92 because its alot more interesting than writing on paper.... I preffer the computers because it saves the bother with compass and rulers and protracts and all the working out.
(Pupil A) D1

I think that maths is better on the TI92 because they are really good (Pupil B*)* D2

I like maths... especially the mini hi-tech computers
(Pupil C) D3

This initial enthusiasm for work with the technology is a valuable asset to the teacher, but there are some areas where thought must be given and caution exercised. For instance there was a realisation on the part of some pupils of the need for familiarity with the software. Typical was pupil A, who continued later in the same account:

But one of the problems is that it takes quite a lot of time to learn how to use it properlly. So it sometimes holds you back but once you learn how to use it it speeds things up considerably. (Pupil A) D4

A further problem was error correction. File handling and re-loading, if the face on worksheet 5.1 was irretrievably distorted, was not easy. These difficulties in error correction were recorded by one pupil:

What I don't like is when a press the wrong botton by accident it makes my work go funny and then I have to start again. (Pupil D)

D5

These comments were informative and valuable in that they pointed to a successful introduction and indicated refinements. However one aspect of the comments which I found particularly striking was the ability from the start of these pupils to examine critically the use of the technology. In the following section I look at the initial reactions of other pupils to the technology and examine them for further insights into this critical faculty of pupils.

Other First Reactions

In a second school, school B, an 11-16 comprehensive with a slightly below average attainment intake, the subjects were pupils aged 12-13 years who volunteered to meet at lunchtime to take part in the project.

There is an advantage inherent in the introduction of new technology in the classroom. Motivation is increased by the use of any new approach. In this school a start was made by letting children take machines away for a time to experiment on their own, with only a brief instruction sheet (Appendix VI) as an introduction. One girl who had done so asked if her friend could also have a machine and if they could stay in at lunchtime so that she could show her friend what she had learnt. The conversation between them (there was no teacher present except to switch on the recorder at the beginning) is transcribed partly here (for a full transcription, see appendix X). There are important points to be made after a detailed examination of the discourse, which will be looked at more closely later. However when considered as a whole it provides a powerful indication of the self-motivation which children can bring to learning with technology and an indication of the potential of this approach. For example, part way through the recording this extract occurs:

18	B	Mmmm yeah	
19	A	Tell me if you don't get it	
20	B	I do get it	
21	A	And then once you've finished your shape you do enter, enter and it draws your shape in bold. Right?	
22	B	Mmmm My rectangle's going to fall apart	
23	A	That's alright, look at mine, it's not even a rectangle, that's just a shape. Right, you can go on F4 and that's all sorts of different lines and points, you can have compass, you can measure, …….	

T1

They had decided to use the polygon option to try to draw a rectangle.
Pupil A is advocating the advantages of the program, and pupil B is responding, being brought into the community of practice. There is evidence (Lines 22 and 23) of spontaneous appreciation of the distinction between drawing and construction. The two devoted half an hour of their own time to this exploration. The commitment shown by these pupils is an example of the motivation which can be generated by using technology.

In another school, school C, an 11-16 community college with a wide rural catchment area and an above average ability entry, lower ability year 9 (13-14 year old) pupils gave their first reactions to using the technology as part of the project.

It was more useful than a blackboard diagram because you were making it happen and you had to understand to do this. If a teacher drew it you could just go along and not learn anything. (Pupil E) D6

I think it is really good because it looks simpler when you do things yourself. It is much easier to understand how things work. Also it makes testing out theorys a lot easier than doing it on paper. (Pupil F) D7

This suggests an opportunity to encourage appreciation of the general nature of constructions and particularly proofs.

One theme running through the initial comments which pupils made about their use of the TI92, which became an important consideration of further work, was that of pupils' consciousness of their own learning processes. Many pupils were critical of their own learning practice and showed an awareness of the implications of the technology. Some of the points made referred to the importance of the drag facility:

The machine is usefull and I think it is good to be able to experiment on your own diagram. (Pupil G) D8

I think its very good, its useful to have a moving image rather than drawing it. (Pupil H) D9

The difficulties which children can experience when presented with a geometrical diagram, which may be intended sometimes to represent a particular situation or at other times a set of general principles is mentioned in chapter 3, where reference is made to the work of Mesquita (1998). Dynamic geometry software, allowing as it does the dragging of independent entities, provides a way of allowing children to see a generalisation before moving to fix that generalisation as a static diagram, perhaps on paper. This suggests an opportunity to encourage appreciation of the general nature of constructions and particularly proofs.

Hoyles, Healy and Noss (1995) discuss the significance of the availability of dragging in a dynamic geometry environment and the possibility that, in such an environment, construction may almost replace proof.

Others wrote about ease of use:

It is easy to clear your screen again if you do make a mistake (Pupil I) D10

The machine is quite easy to use once you have got used to it (Pupil G) D11

There were a number of comments which related to issues of ownership and affect, indicating the particular advantages of the use of hand-held technology such as the TI92:

It is better being able to have your own. (Pupil G) D12

They are good because you can stay where you are, in the classroom. (Pupil J) D13

The TI 92 is a very good and useful machine, powerful and personal because you can have it in front of you and no body can look at it. (Pupil K) D14

Conclusions

Before the pilot study a number of objectives had been identified and information was gained in each area, even if in some cases it indicated that a revised approach would be necessary.

- Personal familiarity with the technology

I had not used the machines in class before. Within the scope of this initial work the lessons were successful. I became confident in the use of the machine, and in the way I was able to demonstrate with the overhead projector version.

- Children's ability to become accustomed to the TI 92.

The TI 92 is a complicated looking machine (Appendix V) and I had some reservations about how children would deal with the complexity of the keyboard, especially in a whole class lesson, where a number of queries could need attention at once. In some of the pilot lessons I was helped by assistants, who could deal with problems which arose, and I chose the type of material with the possibility of this difficulty arising in mind. The children were able to use the files already loaded, and most made good progress. To a large extent they were only using the drag function, and coped with this well. Error correction was one source of problems for pupils and later work used simpler screens so that it was easy to start again.

The initial reactions of pupils in other trial schools were consistently enthusiastic. There was a considerable bonus in pupil motivation with the introduction of the TI92. Pupils of ages between 11 and 14 years, when first presented with the machines, all began work with enthusiasm and made progress when careful consideration was given to the accessibility of the material used. They made cogent initial comments on some points, showing a willingness to be critical of their own learning and thoughtful about the implications of the technology. Particular points made referred to the advantages of the dynamic image, the difficulties of error correction and the need for familiarity with the equipment. There was reference to the advantages of the hand-held technology in matters of affect, children mentioning the possibility of keeping screens private, of not moving from the classroom and a feeling of ownership.

- Use with Whole Classes

I had determined to look at the use of the technology in whole classes and the pilot study was arranged to allow this. As an exercise in teaching this was successful, but from the point of view of research into individual, group and whole class meaning-making it was evident that further development of techniques of data gathering and recording would have to take place. The meaning-making of individuals could not be focussed on by the gathering of data from class teaching, and the material used was not such as could be used to easily align the class in their group meaning-making.

- Development of Materials.

The materials used, which were based on those used by previous workers (Healy et al, 1995) allowed exploration of the drag facility and were designed to indicate the distinction between drawing and construction. They were reasonably successful in this, but as files were provided with the initial screens available (for reasons outlined above), the activity was somewhat limited in the potential it gave for development. Some pupils who finished the activities went on to construct an equilateral triangle which could not be 'messed up', using ideas they adapted from the construction of a square. The square construction indicated the use of arcs to conserve length, and pupils found it relatively easy to use this method to construct an equilateral triangle.

- Data Collection

Data collection was by video recording using hand-held cameras operated by third persons, and as outlined above served to indicate that considerable refinement would be necessary. The screen on the TI 92 was not visible and in order to pick up the voices of individuals or small groups the camera and operator had to be so close as to be intrusive, and this inhibited conversation. The recordings made were not very useful. The children were asked to write down their first impressions of working with the TI 92 in note books, and this proved a more productive source of data.

The results of this pilot study suggested that the use of the technology was not particularly problematic for the teacher or for the children, but that choice of material and methods of data collection would require further refinement. In the next phase of classroom work, described in Chapter 6, material which drew the whole class together in exercises in geometry was used, and a different recording technique trialled.

Chapter 6

Phase One - Classroom Exercises

Chapter 6

Phase One - Classroom Exercises

This chapter describes the first trial of classroom material, which took place in a class of about thirty 11-12 year old pupils (one of the classes from a north of England comprehensive school which had been involved in the pilot study). The theoretical background relevant to this particular work is discussed. There was an attempt to introduce a particular geometrical problem, of translating a line segment parallel to itself. The material used is described and discussed and the method of data gathering is detailed. Data collection was reassessed and refined. Data obtained is discussed and analysed and this phase of the project is critically examined.

Introduction

Following on from the first pilot, one of the two mixed ability year seven (11-12 year old) classes in school A was used for further trialling of material and development of teaching methods and data-recording techniques. More advanced geometry processes were introduced and a coherent theme to the work was developed by looking at problems about comparing lengths and angles. The objective was to gain more insight into the way dynamic geometry could be used in classrooms and the learning processes which were operating within the whole class and smaller groups. It was hoped that using a well-defined problem and developing a solution in the classroom community would provide evidence of the meaning-making which was possible with the topic and the technology. Methods of data collection, which had shown a need for refinement in the pilot phase described in the previous chapter, were reassessed.

Healy et al (1994b) report on using dynamic geometry for constructions, in particular the use of a circle as a length measure using Cabri. Their approach was to ask children to double the length of a line segment. They report on the tendency of pupils to draw a point in the position where they think it should be and discuss the difficulty pupils have with seeing any necessity for construction. They have seen what is required, it becomes a known concept in their minds and they draw it (or rather a representation of it) on the screen. This is essentially what is asked for from pupils if they are asked to sketch on paper, say, an equilateral triangle. They are expected to give the inaccurate drawing they have produced the accurate meaning of 'equilateral triangle'. It is a fundamental element of Euclidean geometry that concepts, firmly held in the mind, are represented in diagrams which are necessarily inaccurate, but which can be used for further meaning-making. However the idea of a construction is the use of firmly held concepts to achieve as accurate a representation as possible. In the classroom the construction may be used to build further meaning-making, such as an appreciation of the locus properties of angle and line bisectors. Outside the classroom, constructions are relevant in accurate drawing (witness the dividers which signify the stonemason).

Dynamic geometry combines elements of both of these ideas. Because the underlying programming distils Euclidean geometry, producing the elements on the screen, constructions on the screen are imbued with an absolute accuracy that they could never have on paper. If a point is drawn on the screen to indicate say, the doubling of a line, it may indicate complete appreciation of what is required, if what is required is a line of a certain length. There is a tendency for pupils to see what they want to see in the diagram. However, what is required here is a construction which cannot be, in the words of Healy et al (1994a), 'messed up'. Pupils need to appreciate that a sufficient construction is one which will survive dragging and still give a line doubled in length. They are required to see beyond the immediate construction task and to appreciate what is needed to find a generally applicable solution, in a way which is closely related to the production of a proof.

Healy et al go on to describe work with intersecting lines, where pupils grasped the idea of a circle as a useful way to transfer length from one line to another. The material used in this phase took up this idea.

The Material

In the classroom work described here, the comparison of length was introduced with rods of slightly different lengths. The first abstract exercise involved comparing the lengths of two line segments with common end points. Estimation of angle was introduced here also, mainly because, in the second exercise, it was intended to look at the parallel translation of a line segment. The problem here was set as that of drawing, on the screen or on paper, two equal line segments which would meet if produced at a given angle, and then checking for equality and the accuracy of the angle.

As an introduction, the lesson started with two plastic rods, each about a metre long, being held up by two pupils on different sides of the classroom. When asked to suggest ways of finding which was longer the children made proposals mainly involving measuring. This perhaps indicates a tendency to use empirical, numerical methods or may be an indication of a primacy of number over shape for these pupils. It was only after some discussion that they decided that a good method would be to stand the two rods next to each other. The translations and rotations of the rods to allow them to be compared by standing next to each other took place in the classroom. This introduced the rotation and translation involved to the pupils, because they could see the sector turned through and the parallelogram swept out by one of the rods as it was moved next to the other (provided they were arranged beforehand to be coplanar). Healy et al (1994b) mention the usefulness of asking pupils to sketch a required construction on the screen before attempting to produce it accurately. This is another way of providing scaffolding to help in an appreciation of the geometry underlying the construction.

This was followed by a discussion of the use of compasses, on paper, and the compass function, on the TI92, to compare the lengths of two line segments which constituted the arms of an angle. The opportunity was taken to introduce estimation of angle, so that pupils were asked to draw two equal line segments which met in a common end point at an angle of (say) 40 degrees, and to find a way of checking accuracy using only compasses and protractor on paper, or the compass (or circle) function and the angle measuring function on the TI92. The version of Cabri used on the TI92 distinguishes between lines, segments and rays, as shown in figure 6.1.

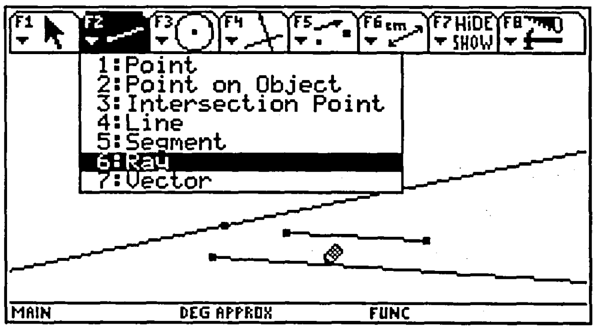


Fig 6.1

It was necessary first of all to ensure that the pupils knew that a segment here meant what they knew as a 'line'. This was an example of the difficulties which can be experienced in using technology such as this with pupils who are not familiar with formal Euclidian terms. Familiarity with the software is soon developed, but such difficulties can obscure the mathematical objective, in this case, the comparison of length. (However it is worth saying that a discussion of the definitions of a ray and a line with this class of 11-12 year olds led to the observation from one pupil that a ray was just as long as a line because each had as many points as you could want.)

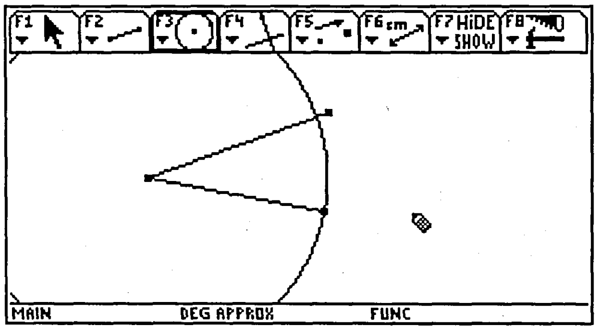


Fig 6.2

The children saw that a circle drawn with the point of intersection of the two line segments as centre, with its radius equal to one of the line segments, was a sufficient device for comparing the length of the two line segments (fig 6.2). The angle measuring facility in Cabri was used to check the size of the angle.

The final task was to draw, by eye, two equal line segments which would meet if produced off the screen or paper, at a given angle, and then to find a way of checking equality and the angle. Again, protractors and use of the angle measuring function were allowed, but not rulers or the length measuring function. There is a compass facility which allows pupils to specify the radius of a circle. The circle shown (fig 6.3) has centre A and radius equal to CD, showing that CD is shorter than AB.

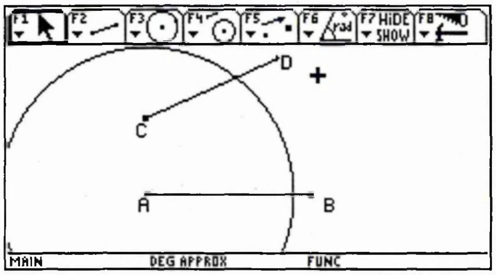


Fig. 6.3

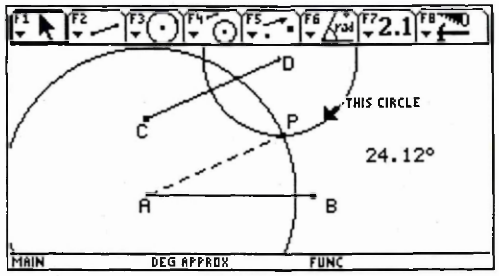


Fig. 6.4

Checking the angle, if producing the line is not possible, presents another problem. A line segment such as CD can be grabbed and translated and C superimposed on A by eye. The angle resulting can be measured, but it will always be the angle subtended by B and D at C or A unless some way is found to combine these two points. The accuracy is in doubt and the value will change if the line segment is translated again.

The solution arrived at involved translation of the line segment CD in a way defined by construction. The need for this translation had been suggested by the use of the plastic rods and the parallelogram swept out in translating the rod indicated to the class a possible successful approach.

In order to measure the angle between AB and CD, AP must be drawn such that ACDP is a parallelogram. The circle with D as centre and radius equal to CA is required. Angle PAB can now be measured, and since AP represents a translation of CD, is the angle required.

The class could see that a parallelogram was needed, but the particular construction outlined above was demonstrated to them.

Data Collection

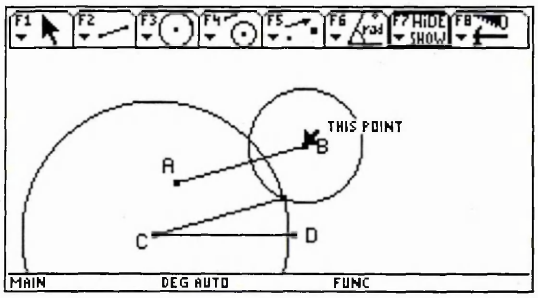
As an attempt to reveal the activity of a group of pupils in the class, they were video recorded from behind whilst using the overhead projector version of the TI92. The projected image of their work on the screen was recorded together with the dialogue of the group of pupils. This was successful to some extent, but there were difficulties caused by the need to use the one overhead projector version of the TI92 additionally with the whole class. (Later versions of the TI92 all have the port needed to connect to the overhead projector. This facility would have helped here, and also opens up the opportunity of any screen generated in the classroom being shown to the whole class.) Further refinements of the recording methods were seen to be needed. Again the children were asked to keep diaries, and this proved to be a useful source of data.

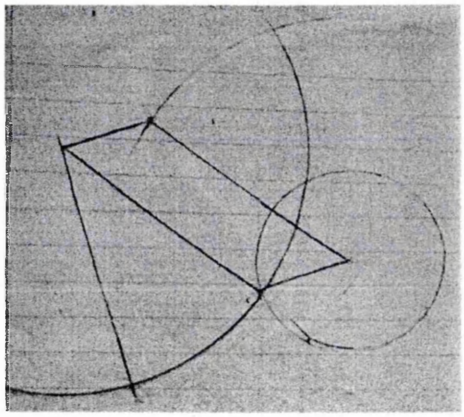
Discussion of Results

Before beginning the second exercise described above Joe wrote in his diary:

I prefer to use the calculators rather than paper because the calculator is more accurate, if you make a mistake you can just *clear it rather than making a mess on paper and it's quicker to use a calculator than a piece of paper, pens and pencils.* D15

However, once embarked on the second exercise difficulties with the screen caused Joe problems in producing the construction on the TI 92.





D16

Fig 6.5

Fig 6.6

Joe's neighbour had produced the screen above (fig 6.5) and Joe had seen the construction was shown to be sound by dragging B. He wrote, after seeing this,

I found it easier to draw the parallelogram using a compass on paper rather than on the T I 92. D17

His construction drawn on paper is shown in figure 6.6.

This episode illustrates an important consideration in the use of hand-held technology such as the TI 92. Aside from the advantages relating to affective issues of privacy and ownership mentioned in the previous chapter, the machine does not dominate the pupil's horizon in the way that a desktop monitor does. It is easily put to one side if work on paper is preferred. (Additionally, as with any portable technology, it is used within a classroom associated with the particular subject area, rather than in a computer suite.) Particular advantage has been found in pupils being able to switch from screen to exercise book, that is to see constructions on the screen, gain an understanding by dragging and then to transfer to paper using a more traditional construction. The use of the concrete example of the rods seemed to allow most members of the class to see what was required, and almost all were able to use a circle to compare the lengths of two line segments which had a common end point.

There appeared to be a general intuitive appreciation of the need to translate the line segment in the second example so that it moved parallel to itself. Some, at least, of the class managed to produce the final diagram on the screen or on paper. However, returning to the considerations raised by Healy et al (1994b), I felt that intuitive understanding had been reached by the majority of the class from the scaffolding provided by the use of the rods. Producing the construction on paper or on the screen was a secondary and probably less important part of the exercise. When pupils showed that the construction was sound by dragging, they were verifying, not making meaning. This construction was too complicated to allow the majority of the class to see through the technology to the geometry beyond. Many who could appreciate the details of the construction, such as the pupil above, preferred to use compasses and ruler to complete the construction on paper. However they had seen that the construction was sound, and seen a dynamic image of it dragged. One advantage of the TI92 is that it does not dominate the view of pupils in the way that a desktop computer monitor does. It is easily put to one side. The pupils who preferred to work on paper were reacting to the obscurity introduced by the technology. Their meaning-making had been stimulated by seeing the dynamic image before attempting the construction. They could, however easily put the TI92 to one side if they wished and work on paper. There is a strong case to be made for the use of dynamic images such as those available from technology such as dynamic geometry in this way. They can be presented to the class before a static diagram, so that the generality of the result is emphasised (Mesquita 1998). Pupils then more easily see a diagram on paper as a representation of a set of general properties.

Conclusions Drawn

- The Material

Some children worked successfully with this material, but it was felt that the intuitive grasp of what was required, generated early in the lesson by the use of the concrete example of the rods, was interrupted in a rather pedestrian way by the use of the technology. This was especially so in the second example, where the line segment had to be translated parallel to itself. The preference of some pupils for drawing with instruments on paper seemed to back this up. In general, meaning-making seemed to come from other sources than the technology, with the software being used to verify, rather than to take forward meaning-making.

- Data Collection

Again the useful data came from pupil diaries. Video recording of the screen used by a group of pupils by arranging for them to use the single TI92 which could be connected to the overhead projector was problematic. The image of the screen was not clear, and the same machine and projector were needed to demonstrate to the whole class. Some refinement of technique was required to enable evidence to be gathered of the meaning-making of individuals and groups.

I felt that this trial had illustrated some physical advantages of the TI 92 and reinforced my ideas of its usefulness as an example of non-intrusive technology. However the content material chosen was too complex to allow the examination of meaning-making, and the recording system used for data collection needed further refinement.

Chapter 7

Zooming In

The second phase involved the study of the mathematical meaning-making of small groups of 12-13 year-old pupils. This was a development from the first phase, to investigate more closely the meaning-making which was taking place when smaller groups used the technology. The material used was more open ended and I acted more as a participator than a leader. Audio-recorded dialogue was transcribed and the screens generated by pupils were recorded in field notes. A theoretical perspective using the ideas of spontaneous and scientific concepts and intuition was applied to the meaning-making which was observed.

Introduction

This chapter reports on further refinement of the research methods in an attempt to narrow the focus onto aspects of concept formation and development which may be operating when groups of children use a dynamic geometry environment. With this in mind, and also bearing in mind experience gained in the pilot trials and phase one, the material was made more open, the recording method was simplified (audio recording of dialogue and field notes of TI 92 screens) and the work was done with smaller groups of children. Groups of up to four pupils were used from school B, aged 12-13 years, and additional results are reported from school D.

The interaction of children and the technology which was observed is examined using, among others, ideas of spontaneous and scientific concepts (Vygotsky, 1962) and intuition, conviction and proof (Fischbein, 1985).

Zooming In

I was concerned from the outset to relate this work to how the technology could be used by the classroom teacher, and previous and later chapters reflect this concern, examine relevant literature and develop this theme. However in planning the work reported in this chapter I decided to investigate meaning-making in small groups, in the belief that this would provide insights into the wider vision of improved classroom use of the technology. This close-up study of the meaning and sense making activity of smaller groups of children was intended to give some insight into detailed interactions between the group, the software and the teacher, which might be relevant to later work with larger classes.

Background Literature

As has been the case all through this study, there was emphasis in this section on the ability of a dynamic geometry environment to allow examination of the stability of the screens produced when geometrically independent entities are dragged. Again the drag function and the idea of a construction invariant under drag were central. The research tasks which were developed were directed at making a distinction between drawing and construction, and at seeking an understanding of concepts such as that of using a circle to preserve length (Healy et al, 1994a, 1994b). Pupil fluency with the technology has been a further central consideration as highlighted by Goldstein et al (1996). They emphasise the importance of allowing pupils to become familiar with the particular dynamic geometry environment by fairly unstructured exploration. In the case of this project the children involved at one school were allowed to take the machines home for a week before the sessions recorded here, and asked to experiment on their own with Cabri.

As discussed previously in chapter four, Fischbein (1982) identifies three forms of conviction: formal, arising from argument; empirical, arising from a number of practical findings; and an intuitive intrinsic conviction, which he calls 'cognitive belief'. I feel that the dynamic geometry environment can reflect these ideas by providing a background for pupil/pupil and pupil/teacher discussion, by allowing dragging to provide empirical proof and also through the way in which the ability to experiment with dynamic screen images triggers pupils' intuitive ideas. Fischbein proposes that

> ...the intuitive and the analytical forms of knowledge are complementary and deeply interrelated. They are two facets of an unique mental productive behaviour. (Fischbein, 1982 p. 11)

Fischbein sees intuitive conviction as intrinsic in character, with no need being felt for formal or factual justification. He sees intuition as triggered 'in the frame of practical situations as a result of the personal involvement of the learner'. When using dynamic geometry with pupils, intuitions can be triggered and it is the concern of this chapter to examine the way in which these intuitions can be used to build more formal reasoning, or, in Vygotskian terms, to move from spontaneous to scientific concepts. In the wider mathematical sense this process is akin to that of moving from intuition to a formal proof. I have tried to use this principle in considering the ability of dynamic geometry to generate intuitive insights by pupils into the geometry they are looking at, and how these intuitions can be brought to analytical knowledge by mediation in the Vygotskian sense.

A Vygotskian analysis would see learning moving from the social to the individual together with the idea of mediation by a variety of tools, the site for learning being the zone of proximal development. There have been many definitions of the zone of proximal development and alternative analysis of the processes of learning which operate there. The present analysis calls on a background of spontaneous and scientific concepts operating in the zone of proximal development. Vygotsky himself advanced the idea of everyday/spontaneous concepts as compared to scientific/systematic concepts. He considered that these were interrelated in a process of development. The growth of scientific concepts supplies a framework which allows everyday, spontaneous concepts to be assimilated into conscious use. At the same time everyday concepts give body and structure to scientific concepts. These two aspects of development occur in a dialectical fashion, as if the two kinds of concept clear paths from scientific to spontaneous and back again. As quoted in chapter 2, Vygotsky saw development as:

> a complex, dialectical process characterised by a multifaceted, periodic timetable … by a complex mixing of external and internal factors, and by a process of adaptation and surmounting of difficulties. (Vygotsky, 1978, p. 151)

The purpose of the analysis of classroom dialogue in this chapter is to examine the way mediation by tools (among them the screen, the teacher and the group) takes place in the context of dynamic geometry in the classroom, looking particularly at this interaction between spontaneous and scientific concepts. From a Vygotskian perspective, emphasis is placed on the idea of mediation by a variety of tools, within the learner's zone of proximal development (Vygotsky, 1962). The contribution of a dynamic geometry environment as such a mediating artefact is highlighted by Jones (1997), who also emphasises the importance of the contribution of the teacher.

The Research Task

In an attempt to set a task which exposed the mathematical meaning-making taking place, the first part of this cycle of the project involved the examination of the pupils' ability to talk about and construct a square. It was hoped that the notion of a square would not be new to the pupils, but that in observing their attempts to construct a square, there might be opportunities for insights into their mental processes and the development of their appreciation of more abstract concepts.

I was present as teacher/researcher during the sessions but tried to take a role which was defined by whatever direction the children moved in their efforts to complete the task. The task itself was simpler than those used in the previous cycle, but at the same time more open ended in that pupils were asked to use their own ideas as to how it might be done. The recording methods were simplified by using audio recording, with field notes to record the screens generated during the discussion. The process of trying to record the screens which pupils generated by video recording had not been very successful, and I found that, with the numbers of pupils involved (never more than four), I could make field notes of the screens in sufficient detail. Pupils also made notes about their work in personal diaries.

These 11-12 year-olds were quite confident about the concept of a square. One said:

> We were probably about five, six maybe seven, no younger than that, we were probably four, when we learnt that it has four sides and corners and that they are all the same but we were probably about six when we learned the word right angle. (Darren)
>
> T2

A preliminary exercise, without the TI92s, was used, with cardboard shapes which were all approximately square, but with only one 'accurate' square. The group (three boys, Darren, Tony and Rob) was given the shapes and asked to use whatever method they liked to identify the 'best' square. The following dialogue indicates an ability to use the equal angle and equal side properties referred to in the writing transcribed above.

1	D	That one's definitely all right angles	*They had used the corners of a piece of paper to check*
2	T	Yeah that one's all right angles	
3	D	But we don't know if it's a square though, right	
4	T	Let's have some more	
5	D	I've tested them, Tony; they're not totally right angles. So now we've got these that are definitely rectangles of some sort but we need something to measure the sides against. We're not allowed to use a pen or anything are we?	
6	R	A ruler	
7	D	I know. This er tape box. So that's the length that way, then these should match up They don't so this is a rectangle but not a square. Right next	*Using a marked length to compare*

T3

By using the right angle properties, (by checking against the corners of a piece of paper) and equal side properties (by offering up sides to a marked length), the group were able to arrive at a consensus about which of the cardboard shapes they considered was a square. There is a systematic use of the equal side and equal angle properties and a sufficient method is found for identifying a square. There was no attempt to move beyond this to look at such things as properties of diagonals and indeed no need to do this to complete the task.

There is social intercourse here, but it is somewhat mechanistic, defined closely by the task set. This is indicated by the language used and the nature of the social interactions present. Darren uses 'definitely' (twice, at 1 and 5) and 'totally' (at 5), and tends to dominate the exchange.

Subsequent work involved using dynamic geometry on the TI 92 to explore ways of constructing a square which was stable when dragged. The pupils had not used the TI 92 before and met with the teacher/researcher in their lunch breaks. After a brief period of familiarisation and demonstration, they were given a TI92 to take home and experiment on, together with an introductory instruction sheet (appendix VI). When they next met they were given a task of constructing a square which was stable under drag. Fragments from the resulting dialogue are presented below. The three pupils involved are Rob and Darren (from the previous transcript) and Janine, and the teacher/researcher is JG.

In his explorations of the machine Rob had seen the 'Regular Polygon' option, which allows 'construction' of a square directly. This extract is from the first session using the TI 92.

1	D	Does anyone know how to draw a square?	
2	R	Polygon, Regular Polygon	*See figure 7.1*

T4

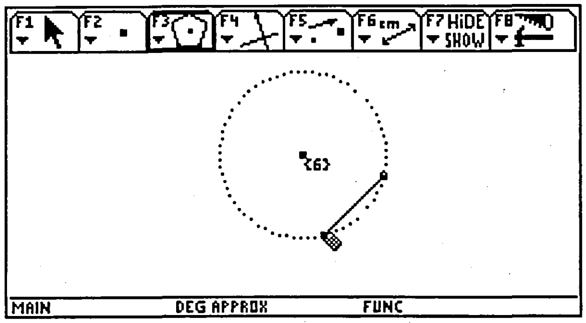


Fig 7.1

The 'Regular Polygon' option offers a hexagon first, as indicated by the (6) in figure 7.1, and it is not immediately evident how to draw a regular polygon with fewer sides. In this case the use of the technology was not very helpful in assisting pupils to develop their ideas about construction. Janine also had taken a machine home and her explorations had led in another direction, towards the 'measuring' menu. This allows the use of tools for measuring lengths, angles and areas. A measuring approach had been introduced in the original squares exercise and it is not surprising that this led to the approach these pupils used. The relatively coarse graphics on the TI92 make it possible to draw a square which is accurate by eye alone.

In the first part of the dialogue which follows a method, using the polygon option which allows free drawing of a polygon is proposed by Janine and accepted (line 6). Then a more formal approach begins to emerge (line 7). The community is beginning to combine a pragmatic approach with a one more based in geometrical principles. Towards the end of the exchange there is an acceptance of the need for the use of abstract definitions rather than practical measurement. The dialogue is reproduced on the next page and further analysed.

3	J	I'm doing it on normal polygon, it's a lot easier and you can always measure your lines.	*This option allows drawing of a polygon by defining its vertices*
4	D	It's hard to get it a proper square.	
5	J	But afterwards you can measure your lines.	
6	R	Yeah, you can, can't you?	
7	D	I know! You could do it two triangles, two right angled triangles next to each other and merge them, then it'd be a proper square.	*Darren is beginning to search for a formalised method*
8	R	I think I've got a perfect square here.	*By using 'polygon' and the rectilinear nature of the screen Rob had drawn an accurate square which would not, however, survive dragging*
9	J	See, I've just figured out mine's not right, cos one of my lines is 1.91 cm and the other is 2.03 cm	*Janine was aware of the necessary conditions and the capacity of the software to check them*
10	J	There's also area; you can do the angle and see if the angle's a right angle, as well.	
11	R	Well you can tell if it's a right angle.	*Rob was still relying on the coarseness of the screen.*
12	J	Yeah but you can't for definite	*Janine is moving to an appreciation of the need for construction.*
13	D	I think it is regular polygon	

T5

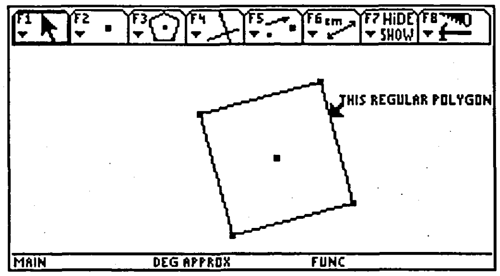


Fig 7.2

All three went on to use Regular Polygon successfully (Fig 7.2) and dragged their squares to confirm that the construction was stable. The end result, of using the rather unproductive Regular Polygon option, does not afford many opportunities for using the ideas of geometrical construction. However the exchange between the three pupils does provide insight into their use of language and the meaning behind that language. The development of ideas of construction as distinct from drawing becomes evident from this interaction. For example, Darren makes reference to a 'proper square' at line 9 and Rob talks about a 'perfect square' at line 10. These examples could be viewed as evidence of these pupils' spontaneous conceptions of the idea of construction (of a square in this particular case). The use of the qualifiers 'proper' and 'perfect' suggests a spontaneous idea of a square as an idealised mathematical object and a readiness to search after a representation of this ideal rather than to draw an approximation to it. Later on, in the continuation of this dialogue, (line 22), the word 'square' is not qualified, possibly because the idealised form is now more deeply embedded in the pupils' conception.

Janine's investigation led her towards attempts at simply drawing the square. It is not difficult to draw by eye an accurate square with the coarse graphics on the TI92 screen. Janine proposed then using the measuring functions and her spontaneous ideas of the properties of a square to check for accuracy. Her ideas on how to complete the task were based in drawing and verification, rather than in construction. Even though the ideas offered for verification are the same ones used for verification of the cardboard 'squares' the discussion is less positive and the need for a definitive construction is indicated (Lines 4, 8, and 12). The discussion at lines 11 and 12 centres on different levels of conviction. For example, Rob suggests that 'you can tell if it's a right angle' which Janine counters with the comment that 'but you can't for definite.

In the second part of the exercise the group were asked to carry out the same task, but not to use Polygon or Regular Polygon. The idea of using a circle came from our discussion of their previous use of the Regular Polygon option, and they were asked to use their own ideas to follow this up. Darren (fig 7.3) and Rob (fig 7.4) both used a circle, a radius and a perpendicular through the centre as a starting point.

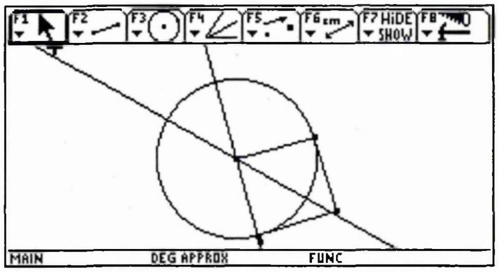


Fig 7.3 (Darren)

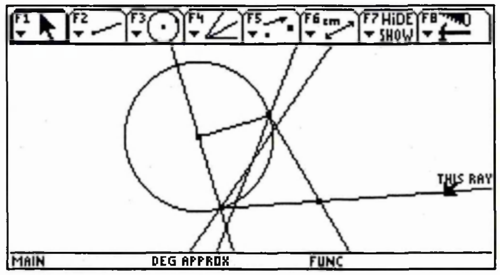


Fig 7.4 (Rob)

Darren had drawn two segments, again by eye, to complete his square. Dragging showed him that the point was not defined. Rob had defined a point where he estimated the other corner of the square to be and drawn two rays through that point. He drew two angle bisectors, which coincided originally, because of the accuracy of his estimation, but separated if he dragged the undefined corner of the square. The scribble sheet used to discuss the exercise is available in the appendix, F3.

This conversation followed.

14	D	I'm trying to do an angular bisector... cos if the angular bisectors make a right angle in the middle then that'll mean it's a square, but I can't get it to do them.	*Darren had followed up Rob's idea of using angle bisectors*
15	JG	How do you know that the angle bisectors will meet in the middle in a right angle?	
16	D	Well I don't know that they will in a right angle.	
17	R	They will.	
18	D	If it's a proper square then it'll be in a right angle because you'd be chopping the square like diagonally.	
19	R	There'd be four triangles.	
20	D	There'd be like four triangles and they'd all be right-angled triangles	
21	R	There'd be two 45° angles	*Darren was shown how to draw an angle bisector*
22	D	Yes!! Now that looks like it's going at a 45° angle right through. That meets in the other corner there, so I think that means it's a square.	

T6

Here the pupils are moving from spontaneous concepts of a square, towards scientific concepts, helped in their meaning-making by the technology. There is interplay between different levels of conviction and mathematical argument. In the passage from lines 14-21 a sufficient definition of a square is arrived at eventually, only to be abandoned at line 22 for the germ of a new approach.

Janine used a different starting point. She began by drawing a line segment and was wondering how to continue.

23	JG	So you've got one line like that.... What would help you to draw a square?	
24	J	It would have to be parallel	
25	JG	So you want to draw a line parallel to this...	
26	J	Yes	
27	JG	And where does it have to be?	
28	J	It has to be the same length as that down	*Defining the square by two opposite sides*
29	JG	Like that? So how would you draw it? What shape would help you draw that down to there?	
30	J	A triangle	*Janine wanted to use the idea of 45° triangles*
31	JG	Look on F3	*F3 offers a circle*

T7

Janine chose the circle option and went on to successfully construct a square (fig 7.5) by drawing two circles of radius equal to her line segment, centred on its ends and two perpendiculars from the ends.

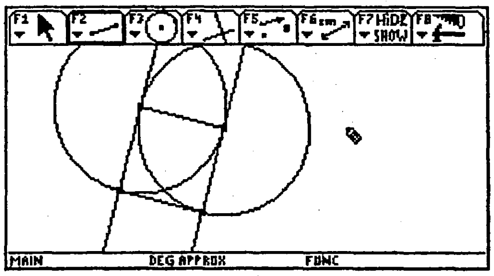


Fig 7.5

There followed an attempt to probe understanding of the independent elements in the diagram. This conversation with Rob refers back to figure 7.4.

32	R	I dunno. If I try dragging this ray, because the ray's not secure at the point, that ray'd drag around wouldn't it? But if that was a perpendicular to that ray,……	*Rob is able to speculate about the effect of dragging*
33	JG	So this circle is a good starting point isn't it? If you have that circle and that ray, how many sizes of square can you draw?	
34	R	Just one	
35	JG	As soon as you've drawn that and that	
36	R	Once you've drawn the circle then you've got the size	*Realised that the ray was irrelevant to the size*

T8

Rob went on to construct a square by drawing a circle (fig 7.6), a ray from the centre and a perpendicular through the centre, followed by two perpendiculars where the first two lines intersected the circle.

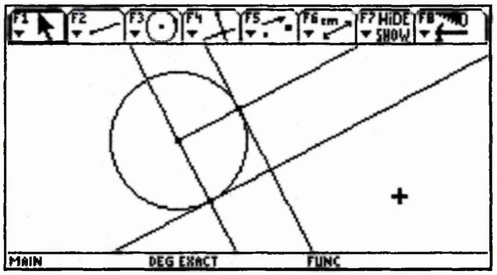


Fig 7.6 (Rob)

Pressing the grab key when the cursor is away from the diagram makes the independent points in the diagram flash. This useful facility allows pupils to explore their diagrams, by finding out the points which can be dragged. Referring to figure 7.6:-

37	JG	What flashes?	
38	R	That corner there. Does that mean that's the only corner that can be dragged?	*The centre of the original circle*
39	JG	That's the only point that can be dragged. Tell me what you drew first.	
40	R	I drew the circle first	

T9

Rob went on to discover that he could grab the circumference of the circle as well as the centre and so alter the size of the square, and alter the orientation of the diagram by dragging the original ray. By a similar process Janine realised that the original line segment in her diagram completely defined her square.

Scientific and Spontaneous Concepts

In observing these classroom activities and in analysing this interaction, there is a clear interplay between ideas of drawing and construction and also between notions of necessary and sufficient conditions (for construction). It is argued that this interplay reflects that between pupils' spontaneous concepts and their developing ideas related to scientific concepts, which in this case are associated with ideas of construction and proof. These pupils can be seen to be operating in a dialectic between their spontaneous conceptions of proof and accurate construction, informed by their ideas and the insights available to them via the mediating role of the dynamic geometry environment and other desk top tools, and the scientific concepts of construction and proof. The second episode in particular provides a rich illustration of how everyday (spontaneous) concepts 'create a series of structures necessary for the evolution of a concept's more primitive, elementary aspects, which give it body and vitality' and hence how scientific concepts 'in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use' (Vygotsky, 1962). By the end of this episode, it is suggested that Rob, Darren and Janine have displayed evidence of an appreciation of the idea of construction and that they have had at least an elementary introduction to ideas associated with geometrical invariants. Spontaneous concepts developed in lines 3-13 are developed and become more scientific by social interaction and by the mediation of the teacher and the technology.

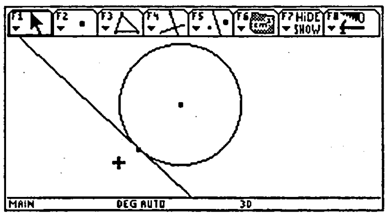
Development over Time

In the course of this project it was not possible to examine the development of particular classes over time. Schools in the UK are pressed for time to cover the curriculum required, and none of the schools which co-operated in the study was prepared to let the research continue over more than two or three sessions with any one class. In the sessions at this school, school B, however, there is evidence of the development of appreciation of geometrical ideas and the use of language with individual pupils. The research was done with volunteers in their lunch hour and so the population was fluid, but as an example, one pupil, Rob, can be traced through several sessions. Rob's first involvement was in the exercise on sorting the 'square' shapes. He was very much on the margins of the discussion, making only one contribution (see the dialogue reported on page ~~138~~, 137 T3). The discussion is about using the obvious square properties, equal sides and equal angles (which were adequate for the task set). Rob was one of the pupils who took a TI 92 away for a time to explore the geometry environment, and his next contribution to the dialogues reported in this session is fuller (see T4 and T5 , pp ~~139~~ 138 and 141). However by the time of the next session he was providing ideas (about angle bisectors, see page 144, scribble pad F3 and transcript T6) which led to much more advanced discussion of the possible ways of defining a square. In a subsequent session he was able to discuss the independent and dependent properties of the diagram he had constructed (see T8 p 147 and T9 p 148). By using the drag function he could identify the elements in his diagram which defined the size and orientation of the square he had constructed. He was talking confidently about the properties of his diagrams. Over a number of sessions he had developed from someone on the margins of discussion with his peers to someone who was able to engage with the problem and whose conversation could be said to reflect an appreciation of rigorous construction and dependent and independent geometrical facts.

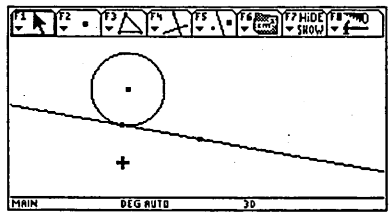
Further Classroom Research

Later in the project, in another school, school D, an 11-18 mixed comprehensive with an intake representing average social and economic background, volunteer pupils met at lunchtimes to use the TI92. Here one of the tasks used was the worksheet below (fig 7.7). The Hide/Show option, which allows construction lines to be hidden, was demonstrated to pupils. They were told that important construction lines on the screens on the worksheet had been hidden in this way. The first two tasks in particular were designed to be simple constructions which would, in two similar diagrams, test pupils' ability to demonstrate their ideas about circles and tangents. The recording methods were similar to those detailed above, with audio recorders on desks to record dialogue and field notes used to record screens.

Try these



1



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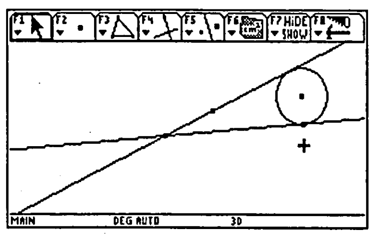


3

1 The line moves round the circle, always touching it.

2 The 'ball' moves down the hill.

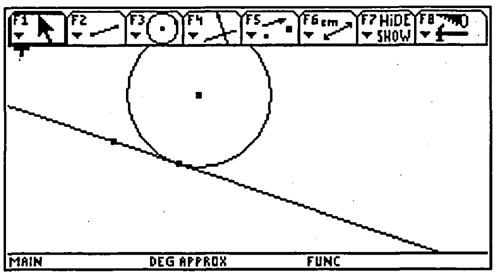
3 The circle always touches the two lines, no matter where they move.

Fig 7.7

This conversation was between two low attainment pupils, Curtis and Dave and the researcher, JG.

1	C	We're doing this one	*Curtis decided to try to draw the circle moving down the slope*
2	D	Go to the centre, we want a line	*They drew a line to represent the slope*
3	C	What did we use? Let's try segment	
4	D	F2 Enter	*F2 reveals the menu which offers 'segment'. The segment was used to define the radius of the circle, ending on the line and drawn perpendicular to it by eye.*
5	C	Then go to the circle	*The circle was drawn using the segment as radius. The segment was then hidden using Hide/Show*
6	D	Watch this see if this does it	
7	C	Ah you've done it You need to draw a segment from the centre	
8	D	I've done it sir. Whoo hoo What! It gets bigger. Why does it do that?	*Circumference point defined as on line, but line not defined as a tangent see figs 7.8, 7.9*

T10



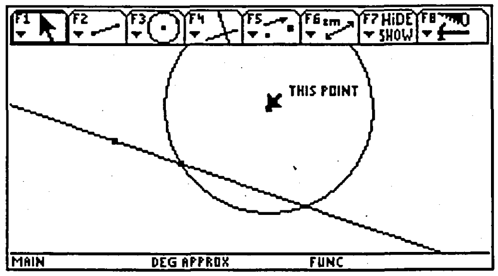


Fig 7.8 Fig 7.9

The pupils had defined their circle with reference to a segment which was originally approximately at right angles to the line (fig 7.8), but attempts to drag the circle by using the centre resulted in the circle crossing the line (fig 7.9).

Curtis and Dave had made a start on the problem by drawing the 'slope' line, (Line 2) and there was a spontaneous realisation that a perpendicular was needed, developed socially at line 7, but it was not defined (Lines 3,4). In the next passage, which followed immediately, mediation is used to make the spontaneous concept of the necessity of perpendicularity more scientific.

9	JG	You've told the circle to go through that point haven't you? How do you make that circle roll? You don't want it to go <u>through</u> that point do you? What do you want to happen?	

10	C	I want it to roll	
11	JG	So it just has to? So you don't want it to go past the line, you want it just to touch the line Can you tell me what would help? Let's just put a point for the centre of the circle. How big do you want the circle to be? Now where will it touch? Centre there, and it's just got to get to......,	*Points on screen*
12	C	That point	*Points to point of contact*
13	JG	Now can you tell me exactly where it should be? Is there a word on the board to help?	*There had been class discussion of important ideas relevant to the topic*
14	C	Pen.......	*Stumbles to read it*
15	JG	That's right, perpendicular. You've got to use the perpendicular	*Went on to a further attempt*

T11

Here there is further evidence of spontaneous concepts (Line 4) being available for development by mediation (for instance line 11). The technology is providing the practical background for intuitive insight and the social intervention of others (Line 5) and the teacher (Line 9).

The possibility of dragging the diagram is also important (refer to figs 7.8, 7.9). This allows consideration of the independent geometrical entities in the diagram on the screen and contributes to the meaning-making activity.

The dialectic between spontaneous and scientific concepts is mediated by social (lines 1-7) and screen interaction (see figures 7.8 and 7.9).

Summary

Fischbein (1982) points to the generation of intuitive insight and its interaction with more analytic processes. Vygotsky (1962) gives us the idea of scientific and spontaneous concepts operating in dialectic in the zone of proximal development, as mediated by social and other factors. The place of the technology and the teacher in this theoretical background is suggested by the analysis presented here.

Fischbein (1982) refers to the importance of intuition and the way this can be triggered by practical interaction, in this case with the screen images. He argues that intuitive intrinsic conviction, or as he calls it 'cognitive belief' is a form of perception. It is a perceived solution to a problem, but will require interaction with its complement, the analytical form of knowledge, in order to lead to meaning-making. This interaction is seen here both as social, with the teacher and other pupils, and also as that provided by the dynamic geometry environment.

From the Vygotskian viewpoint, spontaneous concepts are in a dialectical relationship with scientific concepts, and the interaction between them is mediated by social activity with other pupils, the teacher and the technology.

In considering this process of development, the role of the teacher within the zone of proximal development has been found to be an important element in assisting pupils to move from their spontaneous/everyday conceptions towards more scientific concepts. This echoes the findings of Jones (1996) who argues the need for a significant input from the teacher when pupils are working within a dynamic geometry environment. In later chapters I will refer to these findings and revisit them for examination with a different lens and also examine the way these findings can be applied to whole class and group teaching.

Chapter 8

Construction and Proof, Construction as Proof

Chapter 8

Construction and Proof, Construction as Proof

This chapter considers how the use of dynamic geometry software can contribute to the development of pupils' ideas of construction and proof. Classroom research is reported involving Year 8 pupils (aged 12 to 13) in mixed urban comprehensive schools in the North of England. The previous chapter considered the relationship between scientific and spontaneous concepts and intuition, conviction and proof. A further perspective offered in this chapter considers the elements of proof and concludes that, whilst verification and conviction have an importance, it is in explanation that proof becomes social. Further work examines the relative importance of and the interaction between construction and proof. The findings of these aspects of the study illuminate the potential of the technology in supporting the development of ideas of construction and its relationship to proof.

Introduction

This chapter considers the data analysed in the previous chapter from another theoretical basis, drawing on the work of De Villiers (1991) which proposes different aspects and functions of proof in geometry. The work considered was with secondary school pupils of ages12-13 years, in school B, using Cabri Géomètre on the Texas TI 92 calculator. Classroom activities were audio recorded and the transcripts analysed. Field notes were used to record the screens the pupils had generated.

In the previous chapter the viewpoint of spontaneous and scientific concepts (Vygotsky, 1962, p.109) was combined with reference to the work of Fischbein (1982) on intuition, conviction and proof. In this chapter I use the ideas of de Villiers (1991) on the role and function of proof and follow up the proposal of Hoyles et al (1995) that dynamic geometry constructions can be seen as a form of replacement for proof. De Villiers (1991) has emphasised the diverse nature of proof and it is suggested that explanation and communication, as the social aspects of proof, can be identified in the classroom episodes recorded here. This illuminates how the use of technology combines with social interaction from peers and the teacher to contribute to the development of pupils' understanding of ideas of construction and proof.

Background Literature

There is a strong link between construction and proof in geometry and this relationship is emphasised in the use of dynamic geometry environments. A proof carries authority for mathematicians because of the rigour of the deductive steps which make it up. A further statement of geometric truth is justified by proceeding in logical stages from geometrical truths which the audience can accept. The derived statement becomes a new geometrical truth. When dynamic geometry is used to construct a particular figure, say a square or equilateral triangle, or an angle of 72 degrees, there is an element of proof in the fact that the resulting diagram can be dragged to verify the construction. Just as a reasoned proof gives justification to a statement which may or may not be true, the fact that a construction is stable under drag indicates the steps in its construction are justified. The square is not drawn, but constructed. Whilst it may be possible to change its orientation and size, it will remain a square.

Hoyles, Healy and Noss (1995), writing in a series of discussion papers on aspects of geometry in the twenty first century, (Mammana, 1995), consider the interdependence of construction and proof and suggest the possibility of the replacement of proof by construction in a dynamic geometry environment. They see a need for the use of clearly formulated statements leading to deductions to be augmented by the careful iterative use of empirical evidence, and suggest that dynamic geometry provides an environment in which this can take place.

> Students would *conjecture* about the 'local' relationships between geometrical objects, *construct* these objects and relationships for themselves, and *prove* the truth of their conjectures in ways which are spiral and iterative rather than linear. (Hoyles, Healy and Noss, 1995 p. 104, their italics)

Fischbein's (1982) three forms of conviction: formal, arising from argument, empirical, arising from a number of practical findings, and an intuitive intrinsic conviction, which he calls 'cognitive belief' were referred to in chapter 7. It was suggested that the dynamic geometry environment can echo these ideas in its provision of a climate where argument is fostered, by allowing dragging to provide empirical proof and triggering with the screen children's intuitive visualisations. There is a dialectic of proof (or conviction, which at this level (ages 11 to 14) is seen as almost interchangeable with proof) which moves between these areas. This dialectic is mediated by the technology in ways which can be related to the ideas of the relationship between construction and proof put forward by Hoyles et al (1995) There is a place in this dialectic for the way children deal with necessary and sufficient conditions in mathematical argument. I refer here not to a formal written proof but to the understanding of mathematical necessity which, conceptually, is at the heart of proof.

Various functions of proof were examined by Bell (1976), who studied what he called 'proof explanations' and distinguished between the functions of verification, illumination and systematisation. Bell notes that conviction usually arrives by other means than proof, often by the amalgamation of a number of empirical observations into a judgement. He goes on

> Proof is an essentially public activity which follows the reaching of conviction, though it may be conducted internally, against a potential imaginary doubter. (Bell, 1976 p. 24)

Bell describes the development of a proof from the learner's standpoint, saying that it first grows out of the internal generalisation, which is tried out on other pupils. Contradiction will probably first lead to reassertion, and then to an appeal for evidence. There may be a later recourse to a written statement of the proposition, so that shifts of ground can be prevented and counter examples cited. The final stage will entail an awareness of the need for an argument, probably written, and the formalisation of starting assumptions. Bell points out that this process follows the historical development of the Euclidean model of proof. He notes that such a development of the need for proof in the classroom can start from class activity.

> It follows from the above analysis that pupils will not use formal proof with appreciation of its purpose until they are aware of the public status of knowledge and the value of public verification. The most potent accelerator towards achievement of this is likely to be cooperative, research-type activity by the class.
> (Bell, 1976 p. 25)

De Villiers (1990), drawing on this work of Bell, proposes various further elements of proof. He identifies the areas of

- verification and conviction
- explanation
- systematising
- discovery
- communication.

He notes that empirical examples often lead to conviction/verification; similarly, one of Fischbein's (1982) forms of conviction is empirical conviction arising from a number of practical findings. Mason (1991) refers to the power of computer software to

present a dynamic image. He sees such an image as a 'plethora of confirming instances'. He refers to an intermediate position between generalisation from a few examples and the provision of a rigorous proof, and maintains that conviction arising from the dynamic image afforded by programs such as Cabri Géomètre is justified.
Empirical example is a powerful vehicle for conviction and, with dynamic geometry software, may lead to a form of verification; however such verification and conviction do not in themselves constitute meaning-making. Indeed the reaction of pupils to a visual or empirical demonstration intended to convince is often lack of interest. Pupils do not engage with the geometry if they have not been involved with it via their own activity. De Villiers (1991) claims that, in contrast, it is possible to excite pupils' motivation for and satisfaction from the deductive explanation of a proof, to engage what Mason has called 'this sense of mustness' (1991, p. 86). This partly draws upon the idea of proof operating in a domain 'wide enough to include the visual aspects of mathematical intuition and reasoning' (Davis, 1993, p. 333). Again it is with the involvement of the teacher that this motivation can be brought to meaning-making.

De Villiers points out further that seeing conviction as a function of proof is somewhat difficult. He points out that completely rigorous proofs are very long, citing Renz (1981), who gives an eighty-page proof of Pythagoras' Theorem. He refers to Polya (1954) who said 'When you have satisfied yourself that the theorem is true, you start proving it'. Conviction is needed for proof, rather than proof being a prerequisite of conviction.

De Villiers sees the explanation function of proof as more rewarding. He maintains that explanation, in that it provides an insight into why a proposition is true, can lead to motivation and a much firmer basis on which to base further work. I would argue that another important aspect of explanation is the fact that it has an object. We explain to others or to ourselves. In any case, this makes explanation a meaning-making activity. The importance of the explanation function of proof is discussed more fully later in this chapter.

A further function of proof according to de Villiers is systematisation. He maintains that elaborating the logical steps in a proof helps to give a global perspective, to draw the proof into the body of mathematical knowledge. Proof can also lead to discovery, in that whilst negotiating a proof it is possible to distil the argument to its essentials, and so arrive at limiting conditions. Whilst these aspects of proof are important they are unlikely to occur during the work in dynamic geometry which is described here, depending as they do on argued logical steps.

De Villiers' last function of proof is communication. Here proof is seen as an area of public debate, perhaps in the mathematical community generally or, more relevant to this thesis, between pupil and pupil, pupil and teacher.

> ..proof is a unique way of communicating mathematical results between professional mathematicians, between lecturers and students, between teachers and pupils and among students and pupils themselves. The emphasis thus falls on the social process of reporting and disseminating

> mathematical knowledge in society. Proof as a form of social interaction therefore involves the subjective negotiation of not only the meaning of the concepts concerned, but implicitly also of the criteria for an acceptable argument. In turn such a social filtration of a proof in various communications contributes to its refinement and the identification of errors, as well as sometimes its rejection by the discovery of a counter-example. (de Villiers, 1990, p. 22)

From the socio-cultural stance taken in this thesis, the interesting elements in de Villiers' work are the explanation and communication aspects of proof. If proof is to be an element in the social activity of meaning-making these aspects of it are more relevant than others. If proof is to contribute to social learning in the classroom the teacher will develop the reflective explanation of geometrical truths by pupils and the communication of these explanations to others. In offering a reconsideration of the nature of proof, de Villiers (1991) writes

> not with the intention of sacrificing any fidelity in mathematics merely for pedagogical expediency, but actually the contrary: the encouragement of greater fidelity with respect to the variety of reasons behind proof. (de Villiers 1991 p. 26)

He quotes Chazan (1990) as calling for the

> inclusion of exploration and conjecturing; presentation of demonstrative reasoning as explanatory; treatment of proving as a social activity; and emphasis on deductive proofs as part of the explanatory process, not its end point. (p.9)

It is argued here that consideration should be given to the importance of this explanatory process. With the availability of dynamic geometry software, conviction and verification may often be readily achieved. However, the explanation delivered by a proof brings it firmly into a social dimension, into an area which is open to mediation by others, in a way which the more intuitive functions of conviction and verification do not. Of course, explanation, conviction and verification are often inter-linked. Explanation may lead to conviction or the individual may need first to be convinced in order to be stimulated to produce a deductive explanation; and empirical verification can support the kind of conjecturing required to frame an hypothesis before trying to explain it: 'you have to guess a mathematical theorem before you prove it' (Polya, 1954, p. vi). However, explanation and justification, whether conducted alone or communally, seem inherently social activities, deriving their purpose from the existence of a community of mathematical meaning makers. When explanation in the classroom becomes a social activity, it takes its place in the dialectic of proof and begins to lead to students making mathematical meaning.

In the literature relating to the significance of dynamic geometry environments, Hoyles, Healy and Noss (1995) initially discuss the changing significance of proof, referring to a preference for empirical argument on the part of pupils. They propose that, for many pupils, deductive proof gives only contributory evidence and that proof does not have any significance to such pupils for use in problem solving. Hoyles, Healy and Noss go on to suggest that dynamic geometry environments such as Cabri can provide, if they are used to develop an appreciation of the nature of rigorous construction, a replacement for the need for proof, or at least, an important contribution to a revised view of how proof and construction might be used in the classroom.

Data Analysis and Discussion

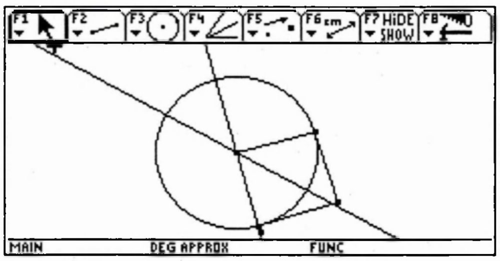
In this chapter some of the dialogue collected in school B, an 11-16 comprehensive is re-examined. Previously it was analysed from the points of view of scientific and spontaneous concepts (Vygotsky, 1962) and intuition and proof (Fischbein, 1982). In this chapter the way in which the social nature of the explanation aspects of proof (de Villiers, 1991) and considerations of the relationship between construction and proof (Hoyles, Healy and Noss, 1995) can be used as new lenses on these observations will be examined. The classroom research has involved the development of materials which have the aim of releasing the potential of the dynamic geometry software and which, at the same time, capitalise on the hand-held nature of the TI 92. This development has been against a backdrop of the desktop nature of the TI92 where a hand-held dynamic geometry environment was used with small groups of pupils in order to stimulate collaboration and interaction. After some time when they had been able to experiment on their own with the TI92, the pupils were asked to construct a square. (See chapter 7 for a more detailed discussion of the background to the task and relevant diagrams. Line numbers in these transcripts are those from chapter 7.)

4	D	It's hard to get it a proper square.	
5	J	But afterwards you can measure your lines.	
6	R	Yeah, you can, can't you?	
7	D	I know! You could do it two triangles, two right angled triangles next to each other and merge them, then it'd be a proper square.	
8	R	I think I've got a perfect square here.	*Drawn using the rectilinear nature of the screen pixels*
9	J	See, I've just figured out mine's not right, cos one of my lines is 1.91 cm and the other is 2.03 cm	
10	J	There's also area; you can do the angle and see if the angle's a right angle, as well.	
11	R	Well you can tell if it's a right angle.	*Again relying on the nature of the screen*

T4

There is evidence here of a distinction between a construction and a drawing and a realisation of the need for logical steps for such a construction, similar to the logical steps in proof (line 14, see below). For example, as noted in chapter 7, Darren makes reference to a 'proper square' at line 9 and Rob talks about a 'perfect square' at line 10. These examples could be viewed as evidence of these pupils' spontaneous conceptions of the idea of construction (of a square in this particular case). It is possible to identify in the above passage elements of the explanation function of proof in lines 7 and 8.

Darren and Rob had gone on to use properties of angle bisectors to help figures



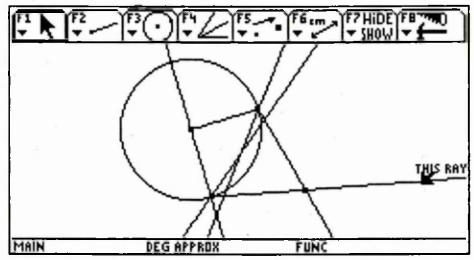


Fig 8.1

Fig 8.2

move towards a construction method (see figures 8.1 and 8.2). It is suggested that there is interplay between different levels of conviction and mathematical argument. The passage from lines 14-21 shows the development through explanation and communication of a sufficient definition of a square. Again, the proving role of explanation and justification can be seen at work, stimulated by the students' social engagement with the process of construction. By the end of this episode, it is suggested that both Rob and Darren have displayed evidence of an appreciation of the idea of construction.

14	D	I'm trying to do an angular bisector... cos if the angular bisectors make a right angle in the middle then that'll mean it's a square, but I can't get it to do them.	

15	JG	How do you know that the angle bisectors will meet in the middle in a right angle?	
16	D	Well I don't know that they will in a right angle.	
17	R	They will.	*Beginning to engage in ideas of construction*
18	D	If it's a proper square then it'll be in a right angle because you'd be chopping the square like diagonally.	
19	R	There'd be four triangles.	
20	D	There'd be like four triangles and they'd all be right-angled triangles	
21	R	There'd be two 45° angles *(in each triangle)*	*Formalising finding*

T5

This development can also be seen to parallel Fischbein's (1982) three forms of conviction: there is a sense, early on, of an intuitive intrinsic conviction; dragging provides experience of a number of practical findings; and finally we see the initial stages of a formal approach arising from argument. This last is not yet a formal proof as accepted within academic mathematics; but we see the beginnings of the use of explanatory chains of reasoning.

Mason, reflecting on aspects of Bruner's (1986) work on Vygotsky, has written of

> 'the role of the teacher as being a vicarious consciousness, able to hold onto global aims and themes when pupils' attention is diverted to detail' (Mason 1991, p. 90).

We see, in the episode above, a complex mixture of elements with mediation by both teacher and technology in the furtherance of pupils' meaning-making. This echoes the findings of Jones (1997) who argues the need for a significant input from the teacher when pupils are working within a dynamic geometry environment.

Explanation as Social

It can be said that whilst conviction and verification have been identified as elements of proof, it is in *explanation* that proof and construction acquire a fundamentally social dimension and begin to impinge on meaning-making. Explanation is the area of proof which is most available for mediation in the Vygotskian sense. Images may be able to convince, dynamic geometry may provide a form of verification, but it is when explanation begins that proof moves into an explicitly social dimension.

However the form of communication which we call explanation has many layers to it and we can identify various shades of meaning. Indeed the imprecise nature of what we understand by explanation suggests that it contributes in various forms to the process of proof. Davis and Hersh (1983, p. 73) describe mathematical argument as 'a human interchange based on shared meanings, not all of which are formulaic.' De Villiers (1990) refers to the unique role of proof in the

> 'explanation, systematisation and verification of results, something which is not possible to the same degree using

> only intuitive and/or quasi-empirical methods' (De Villiers 1990 p. 23).

Explanation as a communication process has an object and it is instructive to analyse the episodes recorded here in a way which identifies these objects. We explain to ourselves (line 32), to others (lines 3-13), to pupils, to teachers. Scaffolding provided by the teacher (lines 32-36), as dialogically pupil and teacher construct a connected chain of reasoning, is often key to building on the conviction which is already present through dynamic geometry or otherwise, with logical deduction leading to richer meaning-making. Because explanation, through communication, draws proof out of the intra-psychological, it can give pupils ownership of their mathematics and provide motivation. Another intuition may lead to further discoveries, which may be verified on the screen, but it is when a deductive explanation can be produced and proof acquires a dialectical social element that it reaches its potential as an important part of meaning-making.

It is possible in these transcripts to trace a path through Rob's progress from empirical drawing (lines 8 and 11), to the beginnings of an appreciation of construction (aided by listening to Darren at line 14), to providing an important piece of the argument at line 21.

32	R	I dunno. If I try dragging this ray, because the ray's not secure at the point, that ray'd drag around wouldn't it? But if that was a perpendicular to that ray,……	*Using formal geometrical terms*

33	JG	So this circle is a good starting point isn't it? If you have that circle and that ray, how many sizes of square can you draw?	
34	R	Just one	
35	JG	As soon as you've drawn that and that	
36	R	Once you've drawn the circle then you've got the size	*Appreciation of geometrical constants in the diagram.*

T7

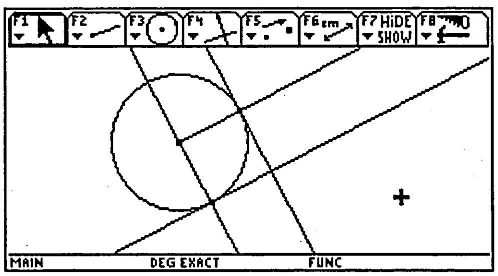


Fig 8.3

Later (lines 32 and 36) Rob is able to discuss the geometrical invariants in his diagram with confidence (see figure 8.3). He has used the social interaction of explanation to make meaning about the problem and is using formal arguments about construction in a way which is not very different from a proof.

Summary

It is proposed that these episodes, when considered from theoretical standpoints such as the dialectic of proof, complemented by explanation as the engine-room of proof, driving forward meaning-making, indicate a framework for a classroom approach. The study points to the potential of hand-held dynamic geometry environments to promote the development of pupils' understanding of notions of construction and proof. The environment provides opportunities for mediation by pupils, the teacher and the technology. It is the explanation aspect of proof which the teacher can use to motivate pupils and help to give them a sense of themselves as the makers of mathematical meaning.

Chapter 9

Making Sense, Agreeing Consensus

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Making Sense, Agreeing Consensus

In this chapter two elements in the process of meaning-making are identified. The sense making and consensus making of children working within a dynamic geometry environment are distinguished and examined. Work with whole classes divided into groups was audio-recorded and is analysed against a background of the work of Schultz (1994) on sense and consensus making in the context of activity theory. The ability of the material used to offer opportunities for these elements to combine in meaning-making, in the context of the ideas of Lave and Wenger and Adler on transparency is discussed.

Introduction

The term meaning-making has been used in this work to signify the appreciation by pupils of a mathematical situation and their ability to move, through social consensus and intervention, to an agreed meaning. The term understanding, with its positivist connotations, has been deliberately avoided.

In this chapter I wish to examine this process of meaning-making more closely, so that the term in this chapter will signify a bringing together of different processes. Schultz (1994) uses the terms 'sense-making' and 'meaning-making', with sense making seen as something which only the individual can do, whilst meaning-making is seen as objective, in that the meaning made is socially agreed and reproducible within the meaning-making practice of the community. I have used the term 'meaning-making' in the rest of this work to signify an area covering both these ideas, and I intend to use the term 'consensus' to cover the socially agreed aspects of the broad topic of meaning-making. Readers of Schultz' work will find his use of the term 'meaning' to be broadly equivalent to my 'consensus' or 'consensual meaning', but I hope that the reader of this work will find the term 'meaning-making' used consistently throughout, and in this chapter, the nature of meaning-making examined and levels within it distinguished.

I wish to look at dialogue recorded during the use of the dynamic geometry environment, and examine it for evidence of geometrical sense-making. I then wish to consider the transfer of this sense into agreed consensus and the level at which this consensual meaning is agreed. Nor will this transfer be one-way. Sense and consensus move backward and forward between individuals and society as they engage in activity. The involvement of the teacher in this activity is also considered. The resources available to help this process are taken to include language, technology and teacher. Furthermore, the transparency of these resources, the way in which they can be used as transparent windows to agreed meaning or become opaque barriers to spontaneous sense making will be considered.

Sense and Consensual Meaning

The word ‘car’ has a consensual meaning which is socially related to the function of the car. It is contained in the idea of what a car is *meant* for, personal transport. The car carries with it this objective meaning. But the sense of 'car' lies in personal perceptions. To different individuals, depending on their personal circumstances and perceptions, the car may signify social privilege, to someone who has no car, status, to someone who has a car they are particularly proud of, or a unit of production, to the workers who produce it. The sense of a car changes from the individual's perspective.

If a pupil interprets consensual meaning, that interpretation can be true or false of itself. An interpretation of sense depends only on the individual who makes it. In the way these terms are used here, and applying them to dynamic geometry environments, we want the spontaneous sense perceptions of pupils to be transformed into agreed mathematical consensus. In a general discussion on the teaching of geometry, Mason (1991) speaks of 'a sense of mustness' and acknowledges that pupils may see no need of justification except to assert that they can see that what they are saying is true.

Children have to be given the opportunity to gain a sense of geometrical truth from their exposure to the resource and that sense has then to be made into agreed meaning. This meaning-making is closely paralleled by, and is a reason for, attempts at the formal justification of the geometrical 'sense' which has been made. In the same work, Mason (1991) suggests that teachers can use pupils' discovery of geometrical facts, and 'their gradual appreciation of the fact that there are facts' to promote learning.

> For me, the real importance of geometry is as a domain in which the fact that there are necessary and inescapable facts can be experienced, developed, manipulated to produce new facts, and, for those who wish, organised into a deductive scheme.
>
> (Mason, 1991 p. 76)

The stimulus of sense-making might be any resource in the classroom; the language which is used there, the technology or other materials. Meira (1995) discusses the importance of representations in sense making activity. He sees it as fundamental that representations or diagrams are seen as 'cultural artifacts, the meanings of which are negotiated and recreated by learners in activity.' He argues that an activity consists of the actions carried out by agents in a specific social setting, involving prior conceptions, interactions with others and the use or production of conventions and artefacts. Sense making activity will be bound up in the development of representations.

Making sense and making consensual meaning are bound up in the dialectical activity of the practice of the classroom. Consensual meaning, as I have mentioned, I see as agreed and objective. It will eventually be expressed socially in the formal language of geometry, perpendicular, mid point, congruent and so on. It will probably lead to further sense making. It might be consolidated by formal argument or proof.

Sense making and consensus making, then, are bound together in an interactive process, the process which in the rest of this work is known as 'meaning-making'. Social intervention by the teacher and others in this dialectic will be at, or towards, the consensus end of this dialectic. Some teachers will be better than others at moving to influence pupils', or a pupil's, sense making. Some teachers will operate more towards the consensus extreme, using formal argumentation. Referent, consensual meaning is the outward sign of classroom mathematical activity, but it is bound up with sense making, and sense making is a delicate process, easily disrupted and much more difficult to promote. Suitably transparent resources, including dynamic geometry, can help the teacher to move to sense-making.

Transparency

As discussed in chapter 4, Lave and Wenger (1991, pp102, 103) address the issue of the transparency of a resource, and this is further examined by Adler (1998, pp8-11). They discuss resource use and point out that whilst a resource has to be visible in order to direct the gaze of pupils, that gaze has to see through the window of the resource to meaning-making beyond.

Adler (1998) develops the ideas of Lave and Wenger on transparency, relating them in particular to mathematics classrooms. She points out that the transparency of many resources used in the classroom can be examined.

> Most of the resources teachers draw on in hybridised school mathematics practice bring the challenge of transparency, that is establishing the balance between visibility and invisibility.
>
> (Adler 1998 p. 11)

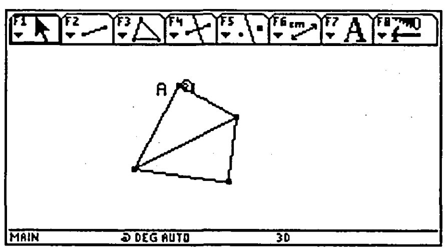
The resource in question, in the context of this project, might be the language used by pupils, the language used by the teacher, the material presented to the pupils, the discussion and interaction going on in the group and the technology itself. Clearly the familiarity of pupils with technology such as the TI 92 governs its transparent use. As pupils become more familiar with the software the teacher will be able to introduce the use of more complicated functions without losing transparency. However Adler (1998) makes further points about the transparent use of resources. She points out that any resource, including examples drawn from outside the classroom, can be critically examined for transparency. She cites as an example the frequently used idea of money in numerical examples, pointing out that the use of money brings with it many social factors from outside the classroom. She points out that the meaning of money in the classroom is often different from its meaning outside, bringing with it ideas of the purchasing power of money in real life which could make its use non-transparent. In general, bringing 'real mathematics', examples from a social context, into the classroom is a process which requires careful thought from the teacher. The resources which are introduced may have different significances for learners than those intended by the teacher, and these significances may be different for different learners. There is a possibility of developing a 'school mathematics' world consisting of frequently used examples, often related to examination questions, which mediates in an unsatisfactory way between mathematical meaning and the real world.

The idea of transparency can be widely applied to many classroom activities; indeed it can be said to contain in it some fundamental aspects of activity theory. Activity involves society and the individual acting in dialectic within the practice of the classroom and society generally, to make meaning from sense. Initial use of a resource is in the area where transparency is needed to allow sense-making activity, but this activity will lead to meaning-making, which will in turn lead to the possibility of more sense-making. Transparency can be seen as an essential part of the process of the use of artefacts to transform the socio-cultural consensus, in moving from sense to meaning.

Language can be seen as a resource which can be analysed in the light of ideas of transparency. Sense making will often take place when the language in use in the classroom, whether by pupils or teachers, is most transparent. Subsequent language, less transparent, is often used by the teacher to guide towards meaning-making.

In the first of the episodes considered here, pupils were asked to draw a triangle and reflect it in one side, and to investigate the shapes which could be made if the other vertex of the triangle was moved. The diagram can be quickly drawn by pupils new to the TI 92 with only a few key strokes, but provides a window which directs the gaze of pupils to allow sense and consensus making to build mathematical meaning-making (see figure 9.1).

Reflecting a triangle



This diagram is made by reflecting a triangle in one of its sides.

As A moves, what quadrilaterals can we get? How many kites?

When do we get a rhombus? How many rhombuses?

How many squares? Any other shapes?

Fig 9.1

The transcript below was recorded in school D, an 11-18 mixed comprehensive with average social and economic background, when volunteer pupils met at lunchtimes to use the TI92. This recording was made of the response at one table, of two pupils Barry and Craig, aged 12-13 from a lower attainment mathematics set. The teacher-researcher is JG. I was directing questions to the whole class of about twenty. They had used the TI92 for about 40 minutes on a previous occasion.

1	JG	I asked you to move this point about. What shape do you all get?	*Developing a community of practice*
2	General response	A kite	
3	JG	I asked you how you could get a rhombus. Look at the screen and tell me when it is a rhombus A rhombus has four sides equal, not just the two pairs	*Using the OHP tablet to display the image to the class*
4	Class	Stop	
5	JG	How should I move it so it is always a rhombus?	
6	B	Turn it around or bring it in to the middle- not right in the middle though	*Sense of perpendicular bisector generated, transparent language*
7	JG	What line will it be?	
8	B	A straight line	
9	C	Symmetry	*Sense of perpendicular bisector*

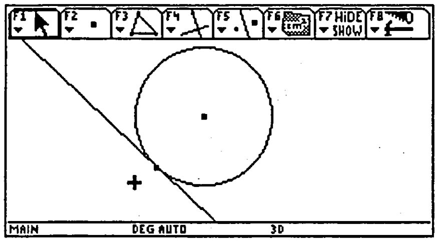
10	B	Down that line, that line like that	*Waving hand in air*
11	JG	What's that angle called	*Prompt to language of consensual meaning*
12	B	What's it called? what's it called? A right angle	*Pupil making consensus*
13	JG	So what's another name for that	
14	B	Ninety	
15	JG	Yes ninety degrees	
16	JG	There's another word for that	*Wanting the response 'Perpendicular bisector'*
17	B	Put a line down the middle	*Returning to transparent language,* *but making sense*

T12

Barry and Craig have made sense of the geometrical invariants in the problem. Their sense-making language is transparent (Line 6 and lines 9,10) to others in the group (it was accepted and unquestioned). Lines 11-17 use more formal language and meaning, in the sense of socially agreed consensus, is being made.

In the next example of classroom observation, using the material first outlined in chapter 7 later in the same session (see figure 9.2), the 'Hide and Show' function was demonstrated to the class and they were asked to choose diagrams from a sheet of examples and to construct them. To do this they needed to decide what construction lines had been hidden.

Try these



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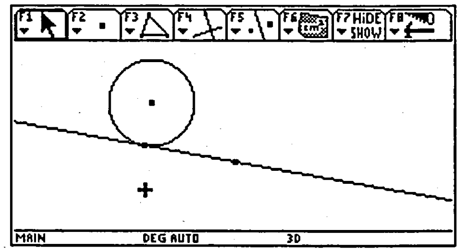

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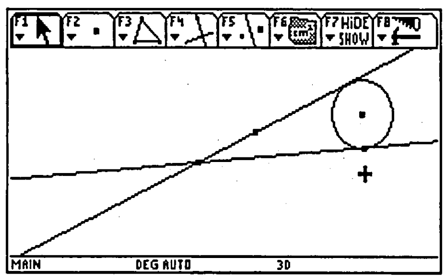

3

1 The line moves round the circle, always touching it.

2 The 'ball' moves down the hill.

3 The circle always touches the two lines, no matter where they move.

Fig 9.2

The following is a transcription of conversation between two pupils and their teacher. The pupils, Andrew and Ben were aged 12-13, again from a low attainment group. They were working in a group with their teacher, TT and in this case decided to try the first example.

18	A	I want to know how to do that one, me	*Line moves round circle, example 1*
19	TT	And how are you going to do that?	
20	A	I know where the line is what he's hiding	
21	A	It's on that big circle there	
22	TT	Go on then	
23	A	Ninety degrees	*Sense of importance of perpendicular radius*
24	B	No he's hiding the line, the line what goes across from its end	*Sense making is happening*
25	TT	The line moves around the circle	
26	B	So it's at a 45 degree, no, 90 degree angle to the line, the line in the circle	*Moving to consensual meaning, with more formal language*
27	TT	Right, go for it, try to draw it. OK escape, clear all your pictures. What do you need first?	*TT has realised that sense has been made.*
28	B	Circle sir, I've done it	*They went on to draw the diagram*

T13

In this example we see how language is used as a transparent resource in the activity of these students. In the extract their teacher, TT, was aware enough to see that sufficient sense had been made to allow progress. The activity of the pupils was then sufficiently engaged by their use of the technology as a transparent resource, so that progress could be made. They were creating cultural artefacts, both at the level of the screen and at a more fundamental level, which were used in a social setting to make first sense, then meaning.

Jones (1997) has pointed out the importance of the role of intervention by the teacher in the use of technology such as this, and I suggest that the analysis above provides a possible structure to the processes behind such intervention. The teacher needs to be sympathetic to the process of sense making, which is sensitive and fragile. The intervention of the teacher in the episode described above is just enough to allow the pupils to move from sense towards some agreed meaning, but still allows them to use the language which they find transparent enough to allow them to make progress. In classroom incidents of this kind, teachers are aware of the need to place their interventions judiciously in the dialectic between sense-making and consensual meaning-making, using just enough formalising language to move forward pupils' meaning-making. They are using themselves and their language as well as the technology as transparent resources, being careful that the window which opens onto meaning-making is able to direct, but not obscure, the gaze of pupils.

Chapter 10

Back to the Classroom

Chapter 10

Back to the Classroom

This chapter looks at the way work on an apprenticeship model of learning has been used in the literature to develop criteria for the promotion of local communities of mathematical practice. The way other theoretical standpoints can be used within these criteria are examined. The use of material developed for classroom use with dynamic geometry software in this project is reported and examined in the light of the same criteria.

Introduction

In previous chapters I have outlined thinking in the literature and evidence from classroom observation of the importance of social meaning-making in the use of dynamic geometry. I now want to look at the way this meaning-making can be fostered by the classroom teacher; the way classroom observation can inform the analysis of social meaning-making.

I referred in chapter 4 to sources in the literature which bear on classroom dynamics in a socio-cultural learning environment. Some of the work in this project has been detailed analysis of the meaning-making of individuals and small groups. Such work is valued for the light it throws on the detail of individual and social aspects of the learning process. However it is equally evident that school based learning will take place by the medium of teachers working in classrooms with (in terms of the size of group often addressed by researchers) large numbers of children. Having developed a view of learning in general as a social phenomenon, it seems that research should have something to say about the way theories of socio-cultural learning are relevant with larger groups. This chapter presents the work of Lave and Wenger (1991), Lave (1996) and Winbourne and Watson (1998) as a framework for analysis of social practice in classrooms, and locates other theoretical standpoints within this framework.

Situated Learning, Legitimate Peripheral Participation

Elsewhere in this work I have referred to bodies of current research which consider, from different perspectives, the dynamics of social meaning-making in classrooms. Lave and Wenger's (1991) monograph, which has the same title as this section heading, sees learning as situated in the wider society and taking place by legitimate peripheral participation in communities of practice.
Lave and Wenger deliberately moved away from the classroom to consider learning in non-school communities. They refer to a wide range of learning communities, tailors in Liberia, non-drinking alcoholics and meat trade operatives in the USA, and midwives in Yucatan. Later work (Lave 1996) refers to Islamic law schools in Cairo. Lave and Wenger see as a common factor the way these learning communities depend on forms of apprenticeship. They see

learning as situated in these communities, as a part of the background which constitutes these societies.

> In the concept of situated activity we were developing ...... the situatedness of activity appeared to be anything but a simple attribute of everyday activity or a corrective to conventional pessimism about informal, experienced based learning. Instead, it took on the proportions of a general theoretical perspective, the basis of claims about the relational character of knowledge and learning, about the negotiated character of meaning and about the concerned (engaged, dilemma-driven) nature of learning activity for the people involved. That perspective meant that there is no activity that is not situated. It implied emphasis on comprehensive understanding involving the whole person rather than "receiving" a body of factual knowledge about the world; on activity in and with the world; and on the view that agent, activity, and the world mutually constitute each other.
>
> (Lave and Wenger, 1991, p. 33)

Lave and Wenger consider learners as **participants** in the society of which they are a part. The participation of members of the society as learners is **legitimate** in the sense that both new-comers and old-timers recognise and accept their place in the learning community. Learning is **peripheral** in the sense that newcomers operate on the edges of the learning process but are gradually drawn in to the community, and begin to see themselves as members of that community, fully participating in it.

In later work Lave develops these ideas further (1996). Lave begins

> Why pursue a social rather than a more familiar psychological theory of learning? To the extent that being human is a relational matter, generated in social living, historically, in social formations whose participants engage with each other as a condition and precondition for their existence, theories that conceive of learning as a special universal mental process impoverish and misrecognize it. (Lave 1996 p. 149)

In this paper Lave advances her studies on forms of apprenticeship (in this paper she refers to communities of Liberian tailors and to an Islamic school in Cairo) as examples of her ideas on social learning. She restates her position that learning is a defining characteristic of society and that decontextualised learning cannot be seen as a sensible goal. She contends that abstract, general knowing cannot be 'powerful knowing', that learning must be constituted in the practice of society. She challenges 'assumptions that decontextualization is the hallmark of good learning' and questions 'the abstract and general character of what constitutes "powerful" knowing' (ibid p.151).

> Whenever people engage for substantial periods of time, day by day, in doing things in which their ongoing activities are interdependent, learning is part of their changing participation in changing practices. This characterization fits schools as well as tailor shops. (Lave 1996 p. 150)

Lave sees learning societies as mutually constituted by teachers and learners, and sees teachers themselves as members of the learning community, learning alongside their pupils. She sees evidence in the communities in which she worked of an exemplar role for teachers, indicating to pupils what it is they will become. She sees this becoming as a fundamental process in social learning and suggests that a *telos,* a direction of change in learning, is present in this becoming.

> The *telos* of tailor apprenticeship in Liberia and legal learning in Egypt was not learning to sew or learning texts, not moving towards more abstract knowledge of the law or separation from everyday life into specialization of production skills or special generalization of tailoring knowledge. Instead, the *telos* might be described as becoming a respected, practicing participant among other tailors and lawyers, becoming so imbued with the practice that masters become part of the everyday life of the *(tailoring)* Alley or the mosque for other participants and others in turn become part of their practice. This might even be a reasonable definition of what it means to construct "identities in practice." It seems that the tailors and law participants, as subjects, and the world in which they were engaged, mutually constituted each other. (Lave 1996 p. 157)

Within schools Lave sees teaching and particularly classroom instruction, as a subsumed and auxiliary part of the learning which is going on and the division and distinction which may exist between teachers and learners as impeding the social learning process. She emphasises the role of teachers as learners:

> Great teaching in schools is a process of facilitating the circulation of school knowledgeable skill into the changing identities of students. Teachers are probably recognized as "great" when they are intensely involved in communities of practice in which their identities are changing with respect to other learners through their interdependent activities.
> (Lave 1996 p. 157)

The significance of Lave's observations for local communities of (mathematical) practice is examined next.

Local Communities of Practice and Telos

The situated nature of learning and the idea of 'local communities of (mathematical) practice' is taken up by Winbourne and Watson (1998). Mentioning further work by Lave (1993), they identify features of a local community of (mathematical) practice, originally quoted in Chapter 3.

- pupils see themselves as functioning mathematically within the lesson;
- within the lesson there is public recognition of competence;
- learners see themselves as working together towards the achievement of a common understanding;
- there are shared ways of behaving, language, habits, values and tool-use;

- the shape of the lesson is dependent upon the active participation of the students;
- Learners and teachers see themselves as engaged in the same activity. (Winbourne and Watson, 1998, p. 183)

Winbourne and Watson use these ideas to examine classroom interactions in terms of local communities of practice and take up the idea of telos. This sees the learning of the community as aligned over time, so that members of the community, coming to the practice from different histories, and leaving it in their own directions, are aligned for a time in their participation in the local community of practice. For Winbourne and Watson telos is a unification of small-scale 'becomings' by which many learners join a community of practice. Telos allows the pupil to be

> 'someone who is legitimately engaged in mathematical practice, as someone, in other words, who is becoming a mathematician.' (Winbourne and Watson 1998, p.183).

Other theoretical standpoints referred to in this thesis can be subsumed into the framework offered by Winbourne and Watson. (1998 p. 183), as can the thinking behind much of the detailed classroom practice. Some of the other work referred to in the thesis is reviewed briefly now, and later related to Winbourne and Watson's six points.

The Zone of Proximal Development

I first want to recall the work of researchers who have applied the idea of the zone of proximal development to the wider classroom. The zone of proximal development has been defined in many ways (see chapter 2), but here I want to look at the way it can be viewed as a space in which the principles of social development proposed by Vygotsky can act (Lerman 1998). Lerman suggests that the zone of proximal development can be seen as belonging to the classroom, or to the researcher, as a 'tool for analysis of the learning interactions in the classroom (and elsewhere)' (p.71).

Whole Class Zone of Proximal Development

Of particular interest here is a definition of the zone of proximal development which includes the classroom as a whole, in this case incorporating the teacher, the pupils and the technology. Hedegaard (1990) has reported in terms of the development of a whole-class zone of proximal development rather than the consideration of an individual's learning. She made a three year study [1]in a Danish elementary school, and developed over this time a teaching method in

[1] This study has sought all along to emphasise the primal importance of teacher involvement in promoting social learning. However it is worth pointing out that this importance is emphasised by the long term nature of many of the research projects referred to (Hedegaard, 1990, Yackel and Cobb, 1996). These workers report on involvement, over time, with particular groups of pupils and teachers. This points to the importance of the long term generation of a learning climate by teachers. Adler (private communication) has referred to the vital importance of teaching and learning styles in school improvement, even in grossly underprivileged rural schools in South Africa. Equally, the successful use of technology is dependent on teaching methods and classroom cultures which may take the teacher considerable time and effort to generate. These will depend on the whole school climate and investment in them, in terms of commitment and effort over time, will be just as relevant as financial investment in technology.

which the children, the resources and the teacher were all seen as part of the same zone of proximal development. She describes the shared activities which enabled personal knowledge to be gained by each child.

In the same paper Hedegaard reports a shift in motivation, with children's focus moving from an interest in the concrete to interest in the derivation of principles which can be applied to the concrete. She acknowledges the individuality of children, but advocates that whatever children have in common should be nurtured in school, together with a willingness to join in interaction and communication. She maintains that 'instruction must be based on development of common knowledge'

> Consequently the zone of proximal development must be used as a tool for class instruction. In our teaching experiment, we saw that it is actually possible to make a class function actively as a whole through class dialogue, group work, and task solutions. The teaching experiment differed from traditional instruction in that children were constantly and deliberately forced to act. .... We can conclude, therefore, that we have succeeded in building a common basis for the children in the class from which future teaching can be developed. (Hedegaard, 1990, p. 192)

This work by Hedegaard suggests the usefulness of the idea of a whole-class zone of proximal development and the way all the activities of the classroom can be incorporated into it. Other workers, considered next, have provided insights into this process.

Socio-mathematical Norms

Cobb and Yackel (1996) have analysed mathematics classrooms in terms of the negotiation and maintenance of social and socio-mathematical norms. Social norms such as expecting and listening to explanations of responses, valuing the contributions of all members of the classroom community and a willingness to enter into discussion are not peculiar to the mathematics classroom and will be promoted in the school culture as a whole.

Socio-mathematical norms including perhaps agreement on whether two proposed solutions are equivalent or whether a proposed solution is complete and valid will be specific to the mathematics classroom. Cobb and Yackel see these norms as negotiated over time by the teacher and the pupils, so that a classroom culture emerges. They distinguish between socio-cultural and emergent perspectives of classroom learning. In their view the socio-cultural approach involves the teacher mediating between pupils' personal meanings and socially established cultural meanings. The emergent perspective takes as its point of reference the local classroom community, rather than the mathematical practice of a wider society, so that classroom norms are more important. Cobb and Yackel acknowledge, however, that their emergent perspective can be said to be better suited to some types of analysis and that it is complemented by socio-cultural approaches which they see as more pertinent when a wider view is taken.

It is difficult to consider the culture of the classroom without seeing it as situated in a wider society and I would argue, with Lave and Wenger (1991) and Apple (1992), that the practice of the classroom must be seen as related to the wider context of the practices of school and society in which the classroom is situated. Negotiation of social and socio-mathematical norms will inevitably be done in this context.

Argumentation

In a related work, Yackel and Cobb (1996) apply the ideas of socio-mathematical norms to the topic of argumentation in the classroom. They refer to the work of Toulmin (1969) as developed by Krummheuer (1995). Krummheuer sees argumentation as a social rather than a logical process, a way of moving forward the body of knowledge which is socially accepted by the classroom practice, the 'taken as shared'. Yackel (1998) sees argumentation as a way of clarifying the relationship between individual pupils and the classroom culture and practice and, by seeing how reference points change over time, as a way of looking at the process of moving forward the 'taken as shared'. Argumentation from class example is seen as made up of conclusion, data, warrant and backing (See Fig 10.1)

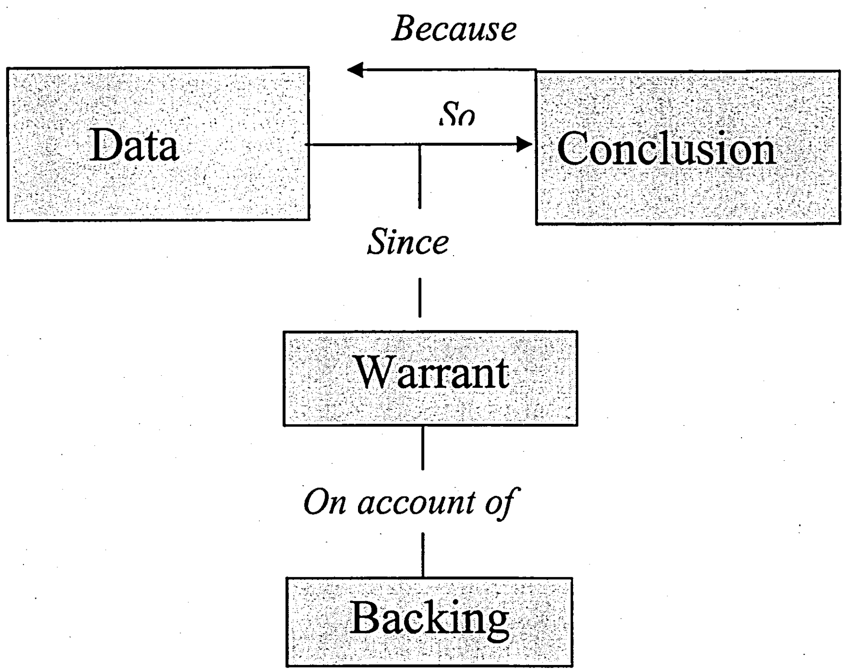

Fig 10.1 (from Krummheuer, 1995)

According to this analysis, data, which is provided by warrant and supplemented by backing, is used to arrive at a conclusion. Yackel (1998) has demonstrated that what constitutes data and warrant changes over time, as the 'taken as shared' advances. This process is not unlike that of the relationship between scientific and everyday concepts advanced by Vygotsky (1962). Conclusions reached, becoming 'taken as shared', will then be available for use as warrant and backing to supply further data.

Thus argumentation is seen as a social, rather than a logical process, a means of establishing that which is held in common about the topic in question and moving forward the 'taken as shared' by classroom interaction. Voigt (1995) discusses the reflexivity between learning and interaction and speaks of this reflexivity contributing to a classroom microculture which in turn affects the meaning-making which is taking place.

Collection and Discussion of Data

The pattern followed was for the class to generate and discuss a simple dynamic image, and to record the result in exercise books as a diagram after the dynamic image had been appreciated. The hand-held nature of the TI92 is particularly suitable for pair discussion and, indeed, as noted previously, for consigning to a corner of the desk when work on paper is preferred.

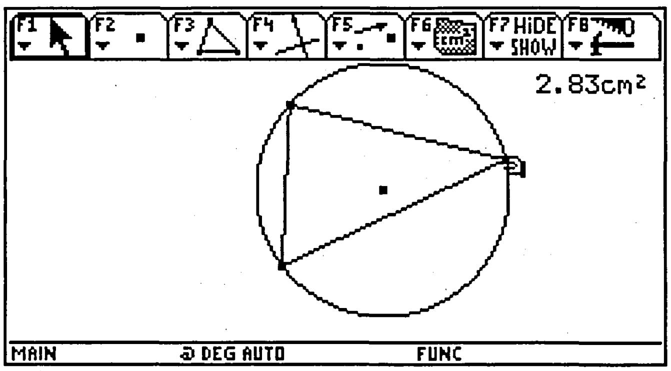

Fig 10.2

The class was a lower attainment year 9 (13-14 year old) group from school C, an 11-16 community college with a wide rural catchment area and an above average ability entry.

I was concerned to present material which was appropriately transparent to these pupils who had not used the TI92 before. The screen used could be generated by these students, helped by worksheet description and overhead projector demonstration, by only a few key-presses. However it led rapidly to an opportunity for spontaneous meaning-making. The class was asked to draw a circle and a triangle with its vertices on the circle, then to measure the area of the triangle. A diagram such as that in figure 10.2 was presented on a worksheet (appendix XI) and the pupils were shown how to draw it on their own machines by using the overhead projector version. They were then asked to investigate the effect of dragging **one** of the vertices, and to look for the maximum area of their triangle. This led to opportunities for class meaning-making about, among others, perpendicular bisectors, isosceles triangles and symmetry. In jointly exploring the same screen in this way, but each on their own machine, a telos is created and students are aligned in the domain provided by the technology. They are operating in a local community of practice.

The classroom interactions between teacher/researcher (JG) and the pupils, Anne, Belle and Charlotte were audio recorded and transcribed.

The following dialogue ensued.

1	JG	What area have you got?	
2	Class	General response	*There was no restriction on the original diagram, a wide range of areas was possible.*
3	JG	Why do we all get different answers?	

4	A	Because we all used different circles	*Use of 'we' suggests the possibility of a local community of practice.*
5	B	And different points	
6	JG	Look at mine while I move the point. Tell me when it will be greatest. What can we all say about our diagrams?	*'What can we all say?' used to promote the local community of practice and the generation of a common direction to learning, telos.*
7	B	It's across from the centre	*Later discussion showed that Belle could demonstrate to the class the idea of the co-linearity of the mid point of one side of the triangle, the centre of the circle and the other vertex. Charlotte pointed out that the triangle was isosceles*
8	JG	Yes, good. Anyone else?	
9	C	It's in the middle	

T14

Here the technology could be said to be driving along the local community of practice. Spontaneous concepts are developed by the participants by looking at the dynamic image, which can then be used by the teacher to interact with scientific concepts (see chapter 7). The use of the resource is sufficiently transparent for mathematical meaning to be made, and expressed in transparent language by the community (lines 7 and 9).

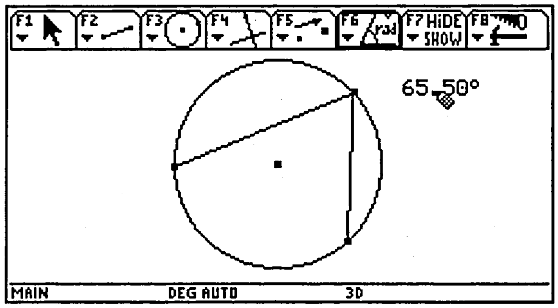

Fig 10.3

Another exercise which is available after only the briefest of introductions to the technology is based on a diagram such as figure 10.3. Here pupils, aged 13-14 years, again from school C, but from a high attainment group, were asked to define and measure an angle in a circle as shown and to investigate the effect of dragging any one of the defining points along the circumference of the circle. Again this was the first time they had used a dynamic geometry environment and it was felt that the screen and the process of generating it was sufficiently transparent to provide a window to mathematical meaning-making. In this case the work demonstrated that the angle on a chord is constant and that opposite angles of a cyclic quadrilateral are supplementary. Moving one of the non-vertex points leads to the observation that the angle in a semi-circle is a right angle. This work was used later with the same pupils to proceed to develop argumentation and explanation about circle theorems, after the angle at the centre had been drawn.

Transcription of classroom audio recordings resulted in the following dialogue. The teacher/researcher is JG and the pupils were Denise, Ellie, Fraser, George and Henry.

10	JG	Does anyone want to tell me what they have found?		
11	D	As you move this down it stays the same angle until you reach this point, then it changes to a completely different angle and stays the same.	*If the vertex is moved round the circle until it passes one of the other points, the angle in the other segment, the supplement of the first, is measured*	*Conclusion*
12	E	Oh yeah!	*Wonderingly*	*Drawn into a community of practice*
13	JG	Will you come and show us?	*In order to demonstrate the OHP version of the machine had to be used.*	*Public recognition of competence*
14	D	It might not work you know… it might just be because of the shape of this one		
15	F	It will work. I got it to work		*Supporting the community of practice*

16	JG	Watch while she drags this. Watch the angle. Moving up …..angle getting bigger	*Dragging one of the non-vertex points*	
17	E	If you change the middle one, watch the middle one, it stays the same and after a certain point it changes		*Now a more assured member of the practice*
18	JG	What's going to happen now?	*Dragging the vertex point*	
19	All	Stays the same		
20	E	Until you pass the point, then it will stay the same again		
21	JG	Look at the angle, it stays at 52.77 degrees. Now changes to……….127.23 Can you make it flip between those two angles? What can you tell me about those two angles?	*to D at front* *to class*	
22	G	Does it add up to 180?		*conclusion*
23	H	Ooo		*realising*

T15

Referring to criteria mentioned earlier for a local community of mathematical practice (Winbourne and Watson, 1998), here pupils can be said to be sharing tool use and purpose by being aligned in the task and their use of the technology. Ellie is drawn into the community of practice and gradually becomes more assured in that community (Lines 12, 17 and 20). Fraser, at line 15, supports the community. All the pupils reported, together with others, are functioning and participating mathematically and recognising the competence of others. There is also, in this dialogue, a sense of telos, in which the pupils are aligned by the technology in a way which drives forward the meaning-making of the community.

Conclusions, Warrant, Data and Backing

In the passage quoted above there is evidence of two 'conclusions' (Yackel, 1998 p210) being reached (as indicated, at lines 11 and 22), without oral evidence of warrant and backing. However it appears that, in this dynamic geometry environment, warrant and backing are supplied by the shared experience of data generated by the technology. The taken as shared of the classroom is moved forward. In follow up work the angle at the centre was drawn and measured, and a worksheet (Appendix XII) used to reinforce argumentation by asking pupils to explain how the other circle theorems already observed and verified can be argued from the angle at the centre theorem. This process of argumentation, used socially in the classroom, is treated by Yackel (1998), when she speaks of

> ...mathematical explanation and justification as an interactional accomplishment and not as logical argument. The focus is on what the participants take as acceptable, individually and collectively, and not on whether an argument might be considered valid from a mathematical point of view. (Yackel, 1998 p. 209,210)

Social Learning

In chapter 2 I described an approach to the analysis of learning based on the work of Vygotsky, placing learning in society. Vygotsky (1962) proposed a social background to learning and formulated the Genetic Law of Cultural Development, with learning moving from the social to the personal. He took up the idea of the Zone of Proximal Development as the area where interaction between the individual and the social leads to development. Within this background, a number of sources were discussed at the beginning of this chapter and it is useful to quickly revisit them here.
Hedegaard (1990), from a background in activity theory and socio-cultural development, sees the possibility of developing over time a zone of proximal development which includes the teacher, the class and the material. Lerman (1998) has emphasised the wide applicability of the concept of the zone of proximal development. In this chapter I show how the viewpoints of these workers can be developed to inform the use of technology in the classroom.

Further, in this study I have identified the social nature of some of the functions of proof and the way in which the sense-making of the individual can be brought to consensual meaning. I have emphasised the social all through the previous chapters. It is important that the principles used so far, and those of researchers who have looked at socio-cultural classroom dynamics, are applied to analyse and inform what happens in the classroom use of technology such as dynamic geometry as available on the TI92.

Cobb and Yackel (1996), whilst acknowledging ideas of socio-cultural learning as influenced by tool mediation (for example Davidov and Radzikhovski, 1985, Leontiev 1978), propose a perspective which takes account of the emergence of a cultural climate in the classroom. They refer to the idea of social and socio-mathematical norms, and suggest (Yackel and Cobb, 1996) the importance of argumentation as a vehicle for learning.
It is helpful here to consider the points suggested by Winbourne and Watson as important in the establishment of local communities of mathematical practice and to identify ways in which the work of these authors can be located within these points.

Criteria for local communities of mathematical practice

1. Pupils see themselves as functioning mathematically within the lesson

There are two parallel aspects to this statement which make this a powerful way of looking at local communities of practice; the idea of pupils 'seeing themselves' and analysis of what might constitute 'functioning mathematically'. Pupils' own examination of the learning community in which they find themselves, and their ability to do this, is a valuable part of the practice of the community. Indeed it can be seen as a responsibility of both teachers and learners to critically examine the learning which is going on in the community. The facility of pupils to do this is fostered by the teacher as guardian of the local community of practice, while at the same time she critically examines her own practice.

An awareness of what constitutes 'functioning mathematically' is also fostered in their pupils by teachers. Geometrical proofs, rigorous constructions, arguing and reasoning are all made available within the medium of dynamic geometry. It is part of the thinking behind the material that mathematics is readily available, that the technology is used to help pupils to function mathematically, rather than as an end in itself. To this end, screens were kept simple, so that the class could draw a simple diagram and then use it to move forward mathematically.

Pupils who critically examine their own and others learning, and see themselves as functioning mathematically then, is the ideal here. The teacher acts as guardian of the practice and develops over time a classroom climate which promotes these values.

2. Within the lesson there is public recognition of competence

Pupils need the opportunity to demonstrate their mathematical competence, to demonstrate their progress to fuller participation in the mathematical community of practice. In the work done in this chapter the pupils were able to demonstrate their ideas to the whole class using the overhead projector version of the TI 92. Pupils could collectively recognise their progress to mathematical meaning-making using the technology, demonstrating to themselves and others their mastery of new ideas and technology. Printouts of screens were made available to teachers for display.

Pupils also need the opportunity to participate in a wider classroom community and to be recognised by their peers and the teacher as socially competent in the community.

3. Learners see themselves as working together towards the achievement of a common understanding

The idea of a telos, of alignment in meaning-making of individual students for a period of time, is advanced by Lave and taken up by Winbourne and Watson. Hedegaard's idea of a whole class zone of proximal development is similar, and Cobb and Yackel describe the way classes can establish shared knowledge by argumentation. They describe the way in which argumentation can be used to move forward the accepted knowledge of the class, the 'taken as shared.' In the example of classroom activity presented above, the way in which Ellie is drawn into the practice, and the promotion of a shared practice by the use of class discussion to take forward the 'taken as shared' promote this approach. This aspect is reinforced by the use of the first person plural in referring to the community.

4. There are shared ways of behaving, language, habits, values and tool-use

Cobb and Yackel refer to the development of norms, social and socio-mathematical, in the classroom. These social norms are developed within and outside the classroom and school, and will be developed over time and often outside the influence of formal education. Watson (1998) has emphasised the need to consider, alongside any mathematical learning, the social learning which is happening in the classroom, about the positioning of individuals in the practice, their relationships with the teacher and their peers. The values of the school and of the society in which it operates will be central in setting social norms. Socio-mathematical norms will also be set over time, but perhaps be more dependent on the work of the mathematics teachers which pupils come into contact with. Within the material used with classes in this thesis, public discussion and argumentation, supported by the use of dragging in dynamic geometry is seen as furthering these aspects of the community of practice. The use of a common tool in the technology also brings individuals into the practice.

5. The shape of the lesson is dependent upon the active participation of the students

Participation in the community of practice is central to the ideas of Lave and Wenger (1991) and Lave (1996). In the communities they investigated they noted the way such participation led to individuals accepting and being accepted to their place in the learning community. The material used in classrooms in this study fostered participation. The work used in classrooms in the project was designed to be easily available to all pupils after a few key-presses and all students had their own machine so that they could generate individual images. Pupils then participated in drawing collective conclusions from their individual work in group and class discussions.

6. **Learners and teachers see themselves as engaged in the same activity**

This last criterion proposed by Winbourne and Watson might seem problematic if teachers see themselves as involved in the transfer of objective knowledge to their pupils. However, if we go back to an apprenticeship model of learning, with pupils aware of their role as becoming mathematicians, it is not difficult to see the practice of the classroom community, pupils and teacher, as that of critically examining the progress of learning and collectively moving forward the 'taken as shared'. If this is seen as an objective of the practice, pupils, individually and in groups, can be encouraged to see their contributions to argumentation and discussion as a valid part of the peripheral participation which is contributing to the learning which is taking place.

Winbourne and Watson (1998) introduce a note of caution into their discussion of the ideas of telos and local communities of mathematical practice. They suggest that local communities of practice, defined by aligned learning of the classroom community, may not occur often. However, with them, I believe that the criteria discussed here are a useful indication to teachers of ways in which such a community can be fostered and monitored. I consider that the nature of the material used in the classroom in this project also works towards the development of a classroom climate which allows the development of the learning community.

Summary

This chapter has looked at how criteria for the establishment of local communities of mathematical practice, as advanced by Winbourne and Watson (1998), can be approached using the classroom material and theoretical background developed in this project. The way perspectives from the literature including ideas of socio-mathematical norms, whole class zone of proximal development, and particularly an apprenticeship model of learning, can be used within a framework of the criteria in the establishment of a local community of mathematical practice is examined. I consider this reflects the concern I have had from the outset to have something to say about what happens in classrooms: about how the teacher can promote a classroom dynamic which uses the resources available, in this case a dynamic geometry environment, to advance the meaning-making of individuals within that classroom. In the next and final chapter I will draw together the strands of this thesis in order to take a view on how the work as a whole has been able to indicate and, to some extent, address the complexity of the classroom.

In this chapter, then, the classroom is analysed from a socio-cultural perspective, making reference to the viewpoints referred to above, and seeking to illuminate the ways in which pupils in classrooms make mathematical meaning in areas such as construction and proof, and the ways in which it is possible for the teacher, as guardian of the local community of mathematical practice, to influence and drive forward the meaning-making of the community at an individual and collective level.

Chapter 11

Local Communities of Practice

An Analysis of Social Learning in the Classroom Use of Dynamic Geometry

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An Analysis of Social Learning in the Classroom Use of Dynamic Geometry

This chapter draws the findings of the study together into a consideration of how the various phases of the work indicate how the development of local communities of mathematical practice can be encouraged, using the approaches developed from the literature. The way in which technology, class pupils, teachers and subject content interplay in the community of practice is modelled.

Introduction

This study into the use of dynamic geometry technology in classrooms has moved from considering individual meaning-making in a social context to the wider study of classroom dynamics from a socio-cultural view. The way in which the various theoretical analyses of classroom learning can be placed into the overall background of a local community of mathematical practice have been identified.

The particular influences on the development of a local community of mathematical practice are divided into two categories, those which have a long-term influence on the practice and those which affect the shorter-term meaning-making of the class and the individuals in it.

1. At the level of the individual situated in society, these are:

    - Spontaneous and scientific concepts
    - Sense-making and meaning-making
    - Proof as social explanation
    - Intuition, conviction and proof
    - Construction and proof

The mediation and intervention of the teacher (assisted by the technology) is seen as vital in all these processes. Most of the categories above are seen as dialectic processes in which teachers can use the technology, together with their own input and the input they can generate from the class, to influence the meaning-making which is taking place as pupils move in these dialectics.

2. At the level of the longer-term development of the classroom community, these are:
    - Whole-class zone of proximal development
    - Negotiated classroom cultures
    - Social argumentation

Here again the role of the teacher is seen as vital, but that role here will be a more long term one, concerned with the generation within the classroom of a dynamic of learning, and within the pupil community of a self-examining atmosphere.

Local communities of practice as introduced by Lave and Wenger (1991) and Lave (1996) can be seen as an over-arching concept into which all these elements can be placed. Winbourne and Watson (1998) provide more detailed analysis of what might constitute a local community of mathematical practice, including *telos*, the way in which more transitory alignments of meaning-making bring together those in the classroom so that together they constitute a community of practice. These ideas were discussed in Chapter 10 and constitute an important background to the findings of this project.

Summary of the Course of the Project

The initial parameters of the project were set as being involved with dynamic geometry use on the TI92. An emphasis on applicability of the results to whole class interactive teaching was brought from the outset, and in developing this aspect it was decided to concentrate on lower secondary age pupils (aged 11-14 years).

The project began with reading in the literature of socio-cultural learning, dynamic geometry use and the nature of proof. This initial reading was needed to locate the project in the literature. As the project developed, these sources were revisited, and others were introduced, but the relation with the literature became more nearly a dialogue with other workers.

After a decision to concentrate on the areas of construction and proof, a pilot study with a class of year 7 pupils (aged 11-12) was carried out, introducing the pupils to the use of the drag function and experimenting with data recording methods. The first phase proper was a further introductory session with the same pupils, introducing whole class geometry exercises. This work was formative, constructive and developmental. However, it was decided to concentrate in the next phase on examining the meaning-making of smaller groups, as an attempt to illuminate processes at this level which might inform a wider view. The third phase looked at considerations of construction and proof, working with small groups and whole classes. The fourth phase involved analysis of further work in classrooms, looked at from the viewpoint of activity theory. The fifth and final phase of data gathering and analysis was concerned with applying ideas from the literature on whole class and group dynamics to previous classroom experiences and to further developments of the approaches and materials used in previous observations.

In each phase of the study, particular areas of the literature were most relevant, and these areas and elements from the general background have been discussed at the beginning of the description of the phase.

Summative Findings: the Classroom in Society

I was concerned from the outset to relate the findings of this project to classroom teaching. Learning is situated in society and much of human activity outside the classroom is learning. Conversely there are many societal influences which impinge on the activity of the learning society which is the classroom. From the point of view of this study, we are looking at ways in which the ideas of a society of learners can be brought to bear on what happens in classrooms. Looked at from this perspective we might welcome the fact that teaching happens in classrooms. Socio-culturally, researchers embrace this, seeing learning as situated in society. But teachers want rather more than this from researchers, and as researchers we need to recognise that some more practical conclusions are demanded. Research into learning is most useful when its relevance to classroom practice is demonstrated. Research into the process of learning which considers the meaning-making of small groups and individuals is valuable. Many of the chapters in this thesis have looked at how technology can take its place in this learning. But researchers need to take account of the fact that children are taught in classes more as a concession to efficiency rather than because we view the classroom as a learning community. Children would be taught in classes whether or not a socio-cultural perspective pointed towards the importance of social learning. Accepting this, we need to consciously apply socio-cultural thinking to the classroom, rather than viewing the classroom as a fortunate opportunity to further our ideas.

I see the school classroom and the learning community situated in it as the predominant element in educating children, with the teacher as an ever more proactive member of this community, using technology to drive forward meaning-making. The teacher must be aware of the place of the classroom learning community within the school and the wider society. However reflective teachers will wish to be thoughtful about the activity of the classroom learning community and researchers need to address the classroom dynamics of this situation.

I brought to this project some thirty years of classroom experience and fifteen years experience of collaboration in writing for a major mathematics scheme. This led to my wish to relate the work to classroom practice from the outset. I had some idea of what I regarded as good practice and had been able to try out this practice and attempt to influence others to adopt it. The ideas of the importance of pupils being brought to making meaning in mathematics by social involvement in what was happening in the classroom and the value of technology in helping with this were parts of the background to these ideas of good practice. In adding the role of researcher to that of practitioner, I have not necessarily set out to answer questions. Rather I have been able to find in the literature descriptions of classroom phenomena and discussions of processes which I consider to be relevant to the illumination of good practice. I hope that I can consider these in my own practice and that other reflective practitioners may find some of the ideas valuable.

The findings of this study are broadly divided into two categories, related firstly to the broad classroom culture and secondly to the closer relationships between teacher, class pupil, the technology and the subject content. We could differentiate them as macro level strategies and micro level techniques, both areas informed by the need to draw all the protagonists in the classroom into the community of practice. The strategic elements of whole-class zones of proximal development, social and socio-mathematical norms and the acceptance of argumentation as a learning tool are ways of looking at the long-term classroom relationships which the teacher will consider. They are ways of developing a long-term learning climate in the classroom. Together with the concept of transparency which is used to look at the way resources are used, these considerations can be thought of as long-term and overarching, strategic. They are methods of looking at the ways teachers locate their classroom within the learning community of the school and society.

Tactically, we can look at ways in which the teacher, the technology and the subject content interact with the class pupils. In this area this thesis offers the ideas of spontaneous and scientific concepts, sense and meaning-making, and, in the specific area of geometry, intuition, conviction and proof, explanation and social proof, and construction and proof. These are concerned with meaning-making at the level of the group and the individual, are involved with the specific subject content being presented. They operate at a specific moment in time but are enabled by the over-arching learning climate generated in the classroom society.

The local community of practice is constituted by the culture of school and society and affects the overall classroom climate. Local communities of practice have also been used in this work and elsewhere to look at the way in which the teacher and pupils develop and respond to more transitory learning alignments in the practice. Both these interpretations are used in this work when considering how various elements of theory can be used to analyse the learning community of the classroom.

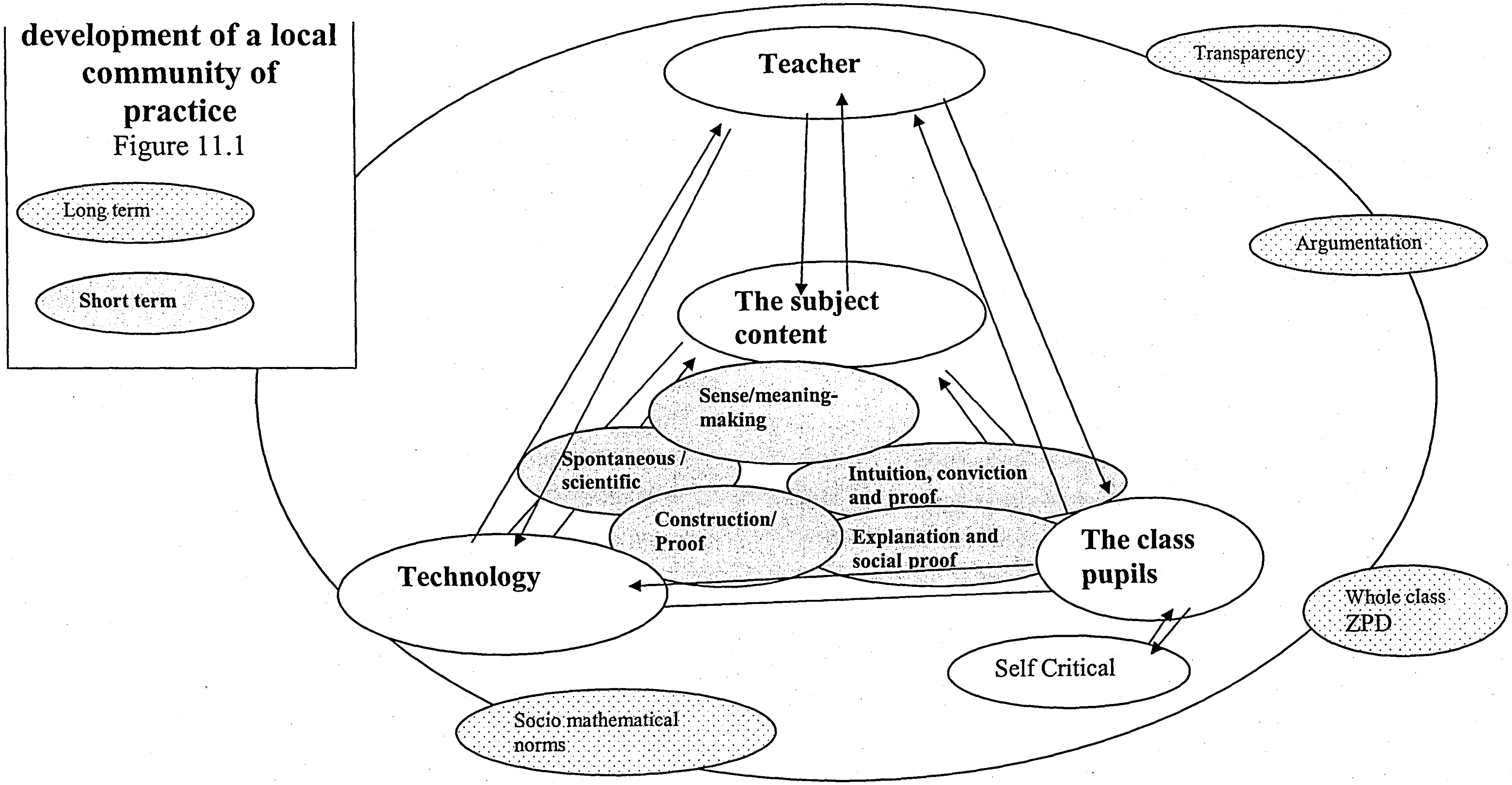
The involvement of the teacher in both the strategic and tactical areas defined above is seen as paramount. Technology in the classroom may change the role of the teacher, but does not diminish it.

Figure 11.1 is intended to illustrate the interrelation of these findings. This represents a tetrahedron, with vertices composed of teacher, subject content, technology and class pupils. These four elements are seen to act on each other within the meaning-making activity of the classroom community. This activity is seen to involve the intentional engagement of the members of the community, teacher and pupils, using social interaction to further meaning-making. The criteria developed for analysing local communities of mathematical practice by Winbourne and Watson (1998) are available to provide further insight into the complex interplay of general factors which operate in the mathematics classrooms and in particular the way in which dynamic geometry can be incorporated into mathematical meaning-making. As discussed in chapter 10, Winbourne and Watson suggest six factors which might affect the establishment of a local community of mathematical practice:

- pupils see themselves as functioning mathematically within the lesson;

- within the lesson there is public recognition of competence;
- learners see themselves as working together towards the achievement of a common understanding;
- there are shared ways of behaving, language, habits, values and tool-use;
- the shape of the lesson is dependent upon the active participation of the students;
- Learners and teachers see themselves as engaged in the same activity. (Winbourne and Watson, 1998, p. 183)

These ideas have been taken forward in the present study and combined with other analysis methods as detailed in previous chapters. They are advanced as a selection of possible tools for the analysis of the complex interaction which is taking place in mathematics classrooms where dynamic geometry technology is being used, with the hope that involved and thoughtful practitioners will find applications outside the immediate subject content.



development of a local community of practice
Figure 11.1

The complexity of the way in which the local community of practice is affected in long and short-term ways is indicated by the nature of this diagram, which can only be said to address some of the issues. School classrooms are complicated social units and mathematics learning may be only one of many things happening there. I have already quoted Watson (1998) in chapter 2, but it is worth recalling her reminder that we can look at classrooms as….

> social communities in which all sorts of things are being learnt (how to behave in a way that is valued by the teacher, how to be accepted by one's peers, what writing implements are fashionable….) which are not the focus of the teaching. To describe what goes on in a classroom fully one must consider all the actions, thoughts, feelings and environmental aspects within it. (Watson 1998 p. 2)

In further applying the ideas used here, seeking to look in more detail at the way they may be applied to the classroom, it is as well to remember that we are seeking to focus on the detailed activity of a community, and a wider view would look at the influence of much wider socio-cultural factors. With this proviso I go on to look in more detail at the areas where some of the methods of analysis used in this thesis may be more relevant than others.

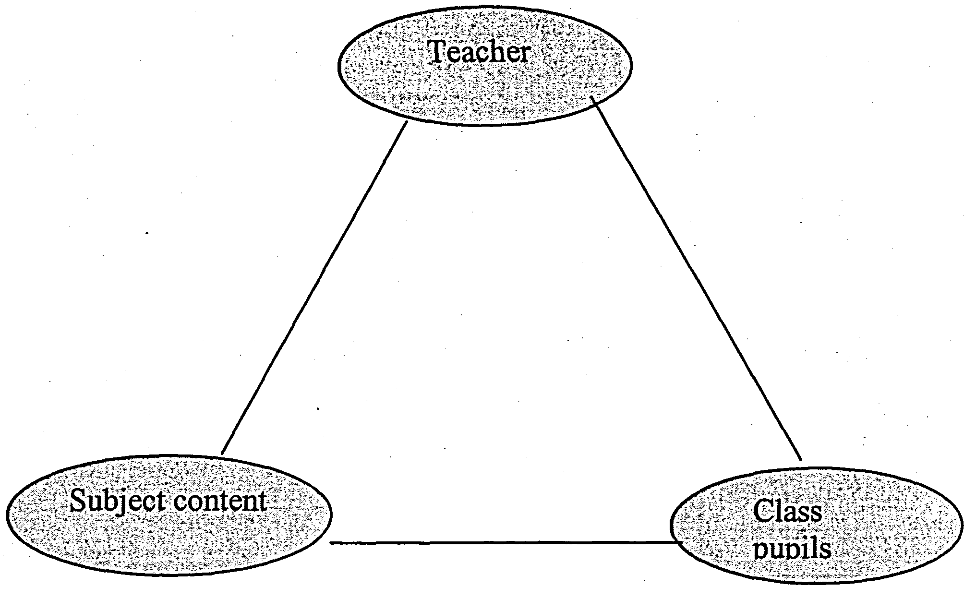


Fig 11.2

One face of the tetrahedron indicated by figure 11.1 is that composed of Teacher, Subject Content and Class Pupils (fig 11.2). Adler (private communication) has remarked on the significance of teaching and learning styles, even in disadvantaged schools in South Africa, and, dealing with UK schools, the Hay McBer report (2000) identified a variable which it defined as 'classroom climate', which led to high expectations and an atmosphere in which they could be met. This thesis, in turn has identified ways in which argumentation, socio-mathematical norms and the development over time of a whole class zone of proximal development can lead to a classroom where individuals feel secure. The development of such a climate, where in the words of Winbourne and Watson (1998) pupils see themselves as 'becoming mathematicians' is a long term process, but the sources referred to in this thesis offer ways of developing such a classroom community. In the immediate meaning-making within the community of practice, Winbourne and Watson also offer ideas which will be more closely related to the subject content in question. Even here they suggest a principle of generating a *telos*, which they describe as a momentary alignment of the meaning-making of the class. This thesis has shown how Winbourne and Watson's principles can be combined with other authors' to offer teachers ideas on how dynamic geometry can be used in the classroom.

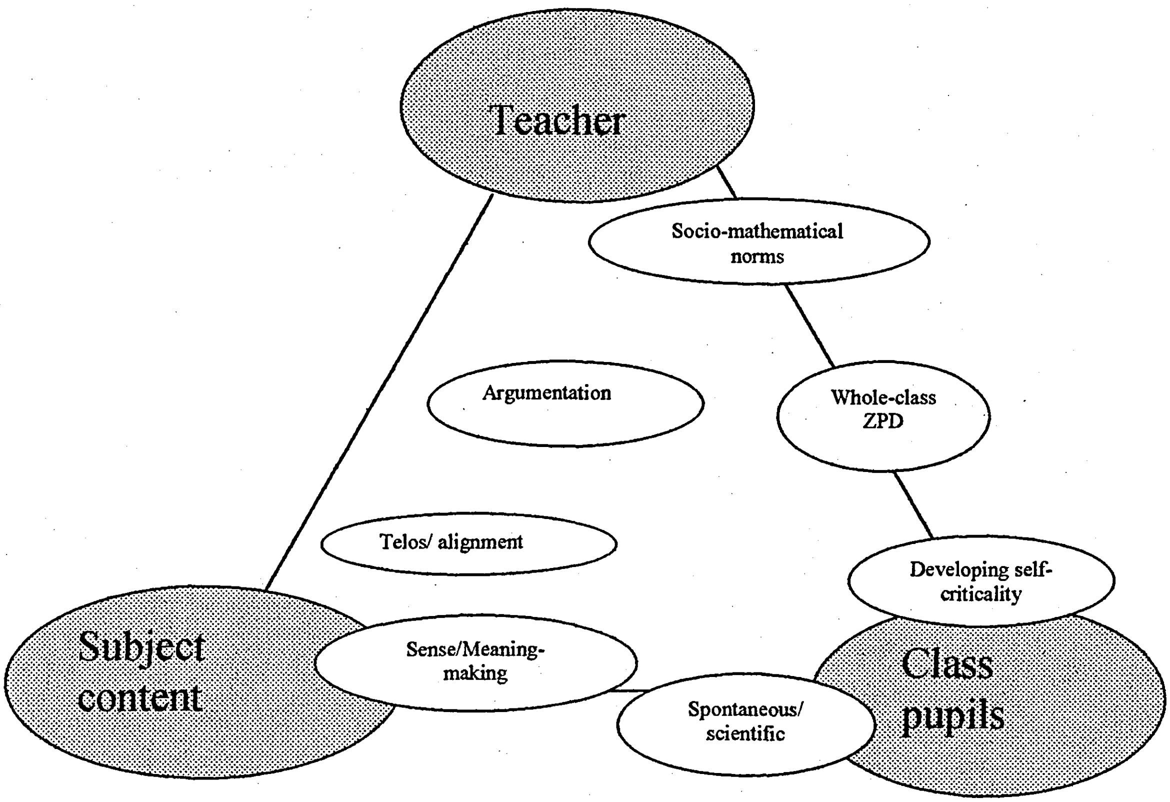
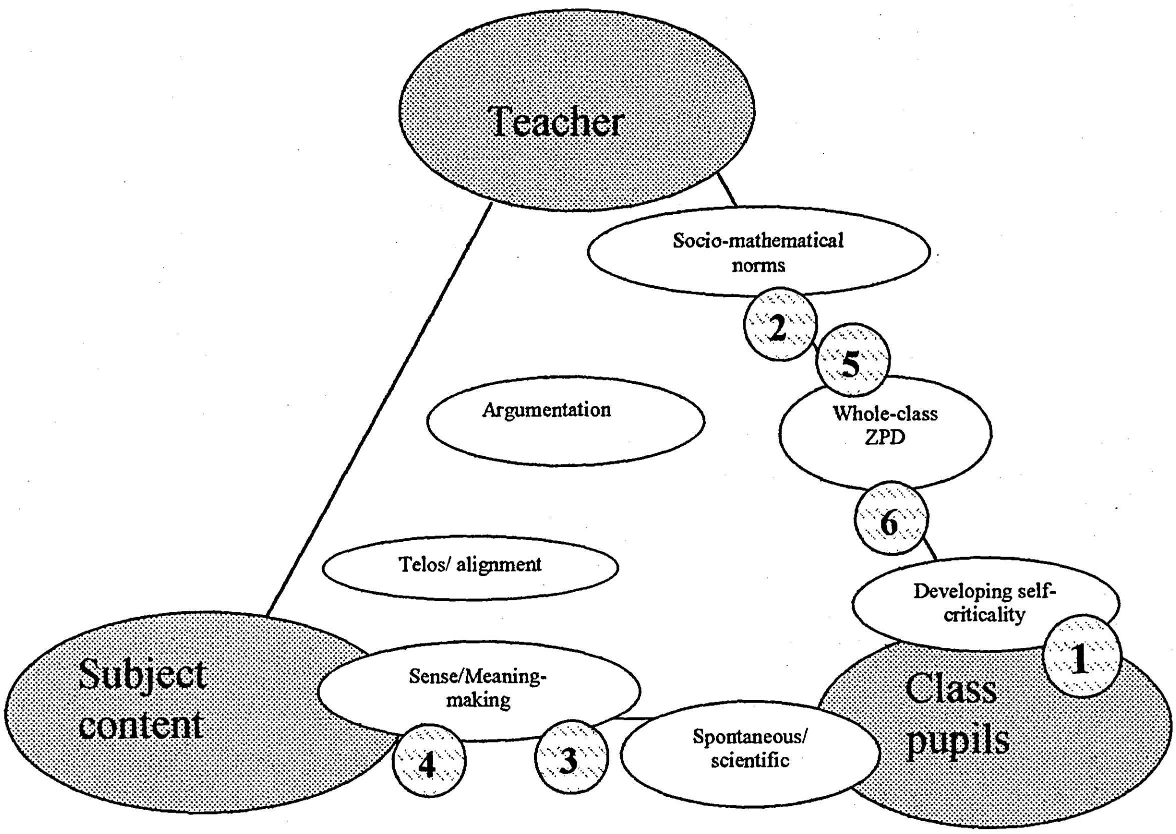


Fig 11.3

Following these ideas we can place the elements indicated in figure 11.3 as shown, with some very much in the area of interaction between teacher and class pupils, independent of subject content, with others involved in the meaning-making of class and pupils about the immediate subject content, but with others operating in the interplay between all three. These placements are arbitrary, and cannot be considered as the only elements affecting the classroom and what is going on in it at any particular time. The importance of these and other elements will depend on emphases and strategies emanating from the teachers, and within their control and many other factors over which they have little or no influence.

The six criteria advanced by Winbourne and Watson (1998) are particularly directed at the long and short-tem development of a local community of mathematical practice, and it is possible to combine them into the picture already developed.

Winbourne and Watson's criteria can be added to the diagram above (figure 11.3) indicating interaction between teacher, class/pupils and subject content. Figure 11.3 might be amended thus, with many of the factors observed more easily in the area of interaction between class/pupils and teacher.



1. pupils see themselves as functioning mathematically within the lesson;
2. within the lesson there is public recognition of competence;
3. learners see themselves as working together towards the achievement of a common understanding;
4. there are shared ways of behaving, language, habits, values and tool-use;
5. the shape of the lesson is dependent upon the active participation of the students;
6. learners and teachers see themselves as engaged in the same activity. (Winbourne and Watson, 1998, p. 183)

(Winbourne and Watson were careful to emphasise that no particular order was implied, the numbers added above are for identification only)

Fig 11.4

In this work the 'subject content' is geometry as available in a dynamic geometry environment, and factors specific to geometry which operate in this area have been identified.

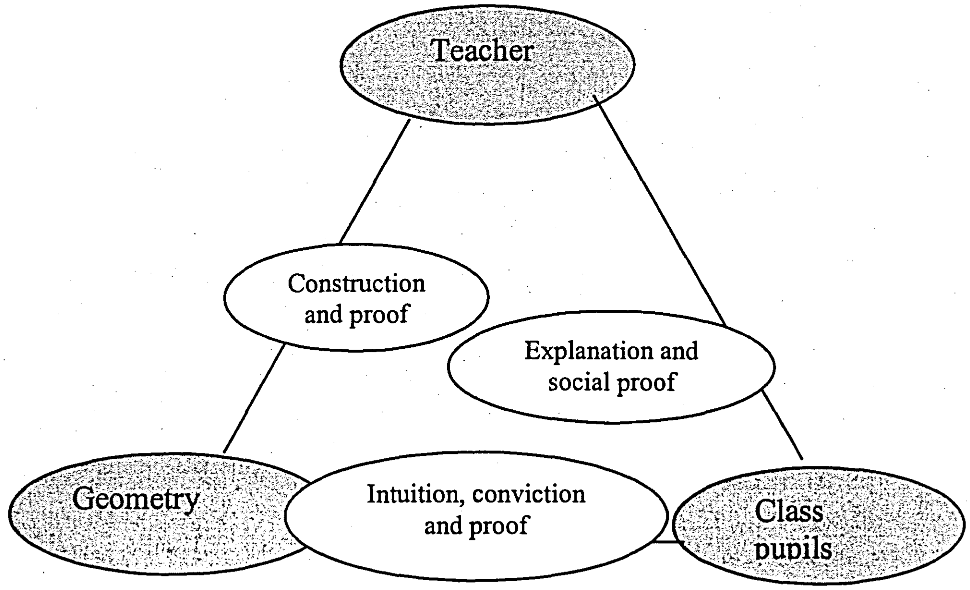


Fig 11.5

These might be incorporated as shown, but again it must be emphasised that placement of these factors is arbitrary, and the intention of this thesis is to identify some of the influences on the local community of practice. Their identification and influence in specific circumstances within particular classrooms when using dynamic geometry, and, perhaps more important, the interplay between them is seen as highly significant. However it is not intended to present this as anything more than an insight into a dynamic situation which, while relevant to general questions about the development of a local community of practice, may have little to say about the relative importance of the various influences in other classrooms, or indeed in the same classroom at a different time.

Whilst many of the factors identified as important in this thesis are independent of the technology used, some specific points involving the use of such resources, and especially a dynamic geometry environment have been identified. They too can be discussed alongside the criteria developed by Winbourne and Watson. (See fig 11.6)

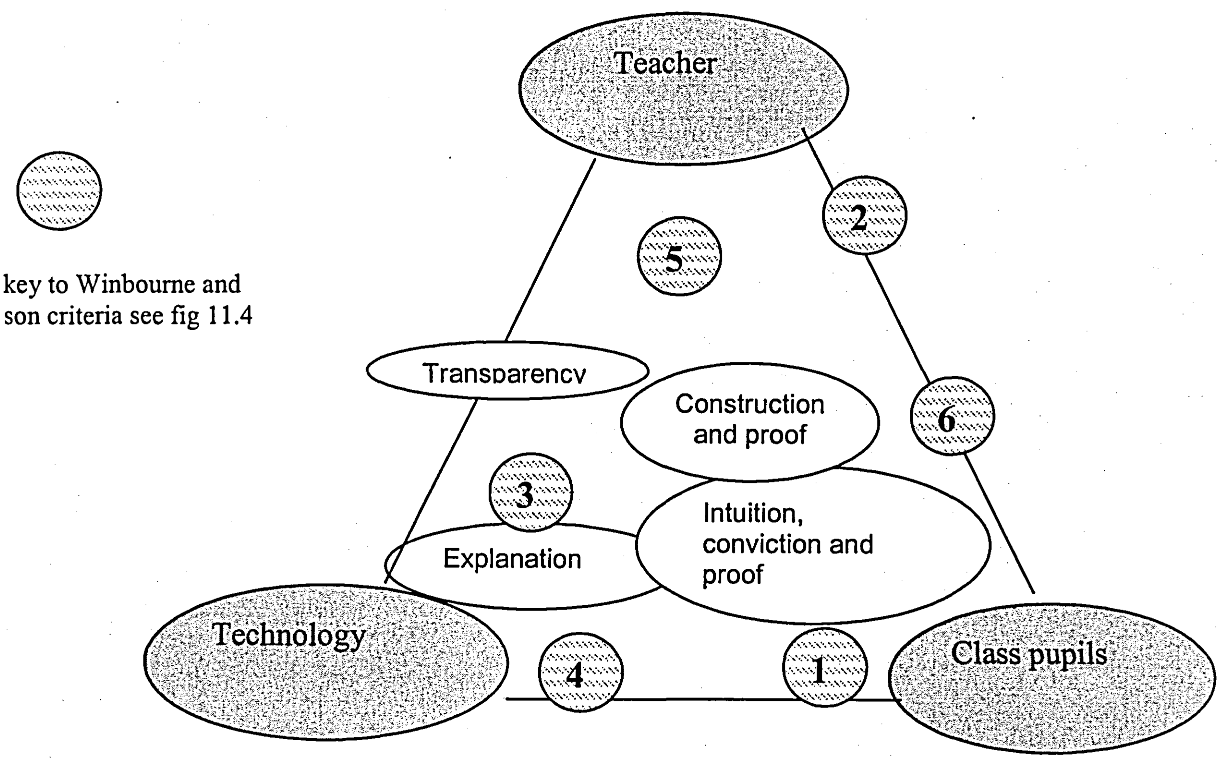


Fig 11.6

In the interaction between class pupils and technology pupils will see themselves as acting mathematically (4) and will be engaging in shared tool-use (1). Explanation will be seen as a movement to a common understanding (3). Teachers will ensure that the use of the technology is transparent. Participation (5) using the technology available to the pupils and discussed in the class is enhanced, and public recognition of competence (2) and the acceptance of the involvement of teacher and pupils (6) is part of the learning climate already established. Again it is relevant to emphasise that these are perceived importances in a particular teaching context, and are suggestions only of a snapshot of the relative importance of these factors within a dynamic.

However it is the contention of this thesis that, accepting the importance of a list of criteria such as that developed by Winbourne and Watson (1998), the various factors identified, both short term and long term, can be used by the teacher/researcher to analyse and inform the development of a local community of practice. Winbourne and Watson have suggested criteria by which a community of practice may be judged. This thesis suggests ways of analysing the community of practice and long and short term strategies by which it may be developed and enhanced. It is useful now to summarise the findings in an overview of the areas covered by this thesis, beginning with the important area of researcher introspection.

Researcher Introspection

The teacher is placed initially at the vertex of the tetrahedron representing immediate learning, but this is done with some caution. Reflective teachers will regard the tetrahedron as regular and remember it can rest on any face as base. At different times the pupils, the content or the technology may be the driving influence in the classroom, but it falls to the teacher to direct this dynamic, to maintain the flow of meaning-making activity along the dialectics defined by the edges of the tetrahedron. One of Eisenhart's (1985) elements of ethnographic research is researcher introspection. Researcher introspection has been a driving force in the development of the ideas of this thesis, and it is researcher introspection as evidenced by reflective practice which drives the dialectics at the edges of the tetrahedron. The reflective teacher is using researcher introspection constantly to analyse, and react to, what is taking place along the dialectical pathways at the edges of the face defined by pupils, content and technology/resources and is, by this introspection, addressing the complexity represented by the tetrahedron indicated in figure 11.1. As ways of analysing the local community of practice constituted by those in the classroom this work offers the ideas of spontaneous/scientific concepts, sense/ meaning making, explanation as social proof, intuition and conviction and construction as proof as ways of analysing the immediate meaning-making when dynamic geometry is used in classrooms. In another content area some of these may be relevant, some not. However that introspective researcher, the reflective classroom teacher, will have other contributions to make to the analysis of the meaning-making activity in this and other content areas, guided not only by readings in the research literature but also by past experience. In this work I have offered five ways to address the interaction between the four elements of pupil/class, technology/resources, content and teacher. They are all dialectical in nature, with the need for cyclical involvement which that implies. The immediate context of their application is work with dynamic geometry technology, but not all of these ideas come from this field.

Two of them, the idea of spontaneous and scientific concepts and that of sense and meaning-making, are generic in nature. Of the other three, all concerned with social aspects of proof, the notion of construction replacing proof comes from applications of dynamic geometry as does work on the explanation aspects of proof. The fifth element, on the significance of intuition and conviction in proof, although concerned with mathematics learning, is not specifically founded in the field of geometry. These viewpoints are derived from various readings in the literature and are dealt with in previous chapters in more detail, as indicated (fig 11.3).

Sense and consensual meaning-making	Schultz (1994)	Chapter 9
Intuition, proof and conviction	Fischbein (1982)	Chapter 7
Spontaneous and scientific concepts	Vygotsky (1962)	Chapter 7
Construction and proof	Hoyles, Healy and Noss (1995)	Chapter 8
Explanation and social proof	De Villiers (1991)	Chapter 8

Factors in the shorter-term alignment of the local community of practice

Fig 11.7

The immediate learning as represented by the area within the tetrahedron is seen as taking place in a learning community influenced by environmental factors as shown in figure 11.1. I have looked at four wider influences on the long-term development of the practice of the classroom in the use of the TI92 and dynamic geometry, which I feel may be of value in other areas of technology use, not necessarily specific to mathematics classrooms. These are consideration of transparency, of argumentation as social, of the generation of a whole class zone of proximal development and the negotiation of social and socio-mathematical norms. Long-term issues such as this will also be dependent on the development of the classroom as part of the learning society in which it is located. These areas and the sources in the literature from which they are derived are listed in the following table and dealt with in the following chapters (fig 11.8).

Transparency	Lave and Wenger (1991) Adler (1998)	Chapter 9
Socio- mathematical norms	Yackel and Cobb (1996)	Chapter 10
Argumentation	Krummheuer (1995)	Chapter 10
Whole class zone of proximal development	Hedegaard (1990)	Chapter 10

Longer-term and strategic issues affecting the community of practice

Fig 11.8

These are longer-term issues, and the original research was longitudinal in many cases (Hedegaard, 1990, Lave and Wenger 1991, Yackel and Cobb, 1996). Such classroom environments are built up on a long-term basis by the teacher, and require a commitment and involvement from the wider communities of the school, the home and society.

The attraction of these schools of thought perhaps lies in their resonance with the way this work has been based on experience and an intention to examine ways of philosophically and scientifically analysing the practicalities of classroom practice and to find ways to work with its complexity. The solution may lie in an integrative holistic approach which touches all aspects of this complexity.

Identified Influences on the Community of Practice

This project has identified the overall idea of local communities of practice, as proposed by Lave and Wenger (1991) and Lave (1996) and as their particular application to mathematics classrooms by Winbourne and Watson (1998) as a background to the application of socio-cultural theory to meaning-making. Against this backdrop, four areas in the practice, class/pupils, teacher, technology and subject content (in this case geometry) have been located at the vertices of a tetrahedron in a model of learning (see figure 11.1).

It is useful here to summarise the findings of the study in the four areas.

- **Geometry**
  - The importance of social results as an indication of generalisations- the use of questions such as 'What can we all say about our diagrams?' in situations where the answer might be 'We all get different triangles, but they are all isosceles.'

The teacher can use such results to move forward the meaning-making of the classroom community.

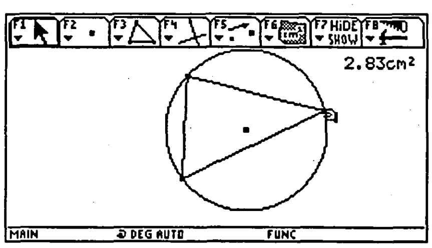


Fig 11.9

What can we all say when the triangle has the biggest possible area?
(See chapter 10)

Once pupils have been convinced of the general significance of the result here, they can move to social explanation and discussion of why the result holds. They are moving in a cycle of specialise, generalise, conjecture and convince (Mason et al, 1985).

- Construction in a dynamic geometry environment requires a previous rigour in the analysis of the geometrical constraints of the diagram required.

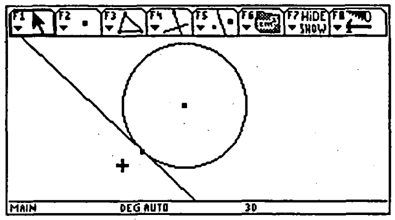


Fig 11.10

The line moves round the circle, always touching it. (See chapter 9)

If pupils produce a dynamic geometry construction which can be shown to fulfil required criteria on dragging, this means that they have applied geometrical rigour to the problem. They may not be operating with formal proof, but they have engaged in geometry as the systematic study of invariance and used the dynamic nature of the environment to consolidate their meaning-making. The technology has been used to produce a construction in a way that offers some rigour, even if it does not constitute proof.

- **Technology**
  - Technology, and indeed any resource used in the classroom, can be usefully examined from the viewpoint of transparency. A resource should be a window to meaning-making, rather than an opaque barrier which stops the gaze of pupils. The exercise used in chapter 6, which led to the screen in Fig 11.11

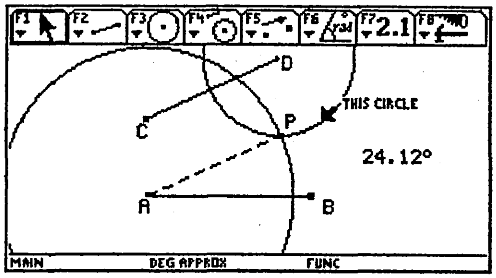


Fig 11.11

was found to be, in the sense of this discussion, too opaque. Later screens, such as that used in chapter 10, were more transparent (fig 11.12).

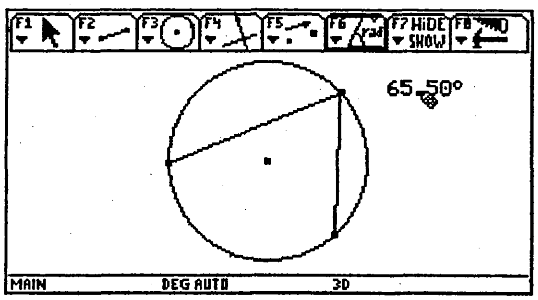


Fig. 11.12

- Hand-held technology such as the TI92 can be used in ways which encourage transparency, by using simple screens and drawing out class meaning-making. The hand-held nature of the technology itself contributes to this, being easily put to one side if another way of working is preferred. One pupil, after seeing a screen demonstration, found the exercise described in chapter 6 easier on paper:

I found it easier to draw the parallelogram using a compass on paper rather than on the T I 92.

(Joe, chapter 6)

Dynamic geometry can be used to introduce pupils to a moving image, avoiding a static representation and encouraging consideration of geometrical invariance. Pupils appreciated this:

I think its very good, its useful to have a moving image rather than drawing it. (Pupil H, from chapter 5)

The hand-held nature of the particular technology used was found to be important in affective issues (again see chapter 5). One pupil wrote

The TI 92 is a very good machine, powerful and personal because you can have it in front of you and no body can look at it. (Pupil M)

- **Pupils and classes**
  - Pupils make sense before meaning. Sense of the geometry behind a diagram is arrived at before consensual meaning is agreed. The sense-making process is delicate and can be stimulated by mediation from the teacher and the technology, but is easily disturbed by non-transparent use of language or resources.

    The following exchange (discussed more fully in Chapter 9) illustrates the way a teacher can respond to sense-making and take it forward to consensual meaning-making.

23	A	Ninety degrees	*Sense of importance of perpendicular radius*
24	B	No he's hiding the line, the line what goes across from its end	*Sense making is happening*
25	TT	The line moves around the circle	
26	B	So it's at a 45 degree, no, 90 degree angle to the line, the line in the circle	*Moving to consensual meaning, with more formal language made*
27	TT	Right, go for it, try to draw it. OK escape, clear all your pictures. What do you need first?	*TT has realised that sense has been made.*
28	B	Circle sir, I've done it	

- Class use of dynamic geometry can stimulate social meaning-making. Asking pupils what they can say about the other diagrams in the class, 'What can we all say?' reinforces the general conclusion, and moves forward the learning of the whole class.
-

From chapter 10:

3	JG	Why do we all get different answers?	
4	A	Because we all used different circles	*The teacher can use 'we' to try to generate a general appreciation of the result.*

- **Teachers**
  - Teachers generate, over time, the learning climate of their classroom and locate it within the school and the community. In doing this the ideas of socio-mathematical norms, whole class zone of proximal development and local communities of mathematical practice may be useful. They choose appropriately transparent resources which allow a window to mathematical meaning-making.
  - In the immediate meaning-making of the class, the reflective teacher is offered ways to monitor the activity of the learning community. There will be interaction between resources, content and class/pupils and the teacher can use sense/meaning making and spontaneous/scientific concepts, together with any factors which are available from the content (such as the opportunity for social explanation in geometry). These will operate within the classroom ethos generated by acceptance, by all those in the classroom, of their place in a wider community of learners.

The Role of the Teacher

I make no excuses for returning at the end of this work to this main theme. The use of technology does not mean that teachers, and class teachers in particular, are redundant. Rather this study has illuminated the role of the teacher as that of fostering and stimulating local communities of practice, whether pupils are working in groups or as a class in discussion with their peers and the teacher. The teacher is seen as the guardian of the local community of practice, with the function of leading that community to meaning-making.

This role of guardian has other aspects. This study has shown the importance of looking at mathematics classrooms in terms of social and socio-mathematical norms. Social norms of respect for others, not shouting out, being prepared to give (and listen to) an explanation exist in classrooms and are fostered by the teacher. In the mathematics classroom agreement on what constitutes a rigorous explanation and a valuing of a sufficient argument will be among the socio-mathematical norms pursued by the teacher. These socio-mathematical norms will be used in the classroom to give direction and thrust to the development of argumentation, seen as a social, as well as a logical, process, in which that which is 'taken as shared' is moved forward.

When children are socially involved in mathematical meaning-making they need to see through the immediate resource to the mathematics beyond. Resources used in the classroom, where the idea of a resource is seen to be very wide, including the language and explanations used as well as teaching materials and technology, can be profitably analysed from the point of view of transparency.

The teacher will be involved in long-term efforts in fostering local communities of practice and developing classroom cultural norms. The reflective, self-critical attitude of pupils to their learning which is developed is seen as important.

The work of this project has placed the teacher firmly, if not physically, at the front of the classroom. It proposes indicators for practice, or at least guidelines for the analysis of practice. The technology can be used transparently to stimulate intuition and to promote the social aspects of proof. The teacher is seen as crucial in setting the social and socio-mathematical norms which prevail. Having chosen an appropriately transparent resource, this role is seen as stimulating and leading local communities of practice, whether group or whole class, where social learning is fostered, where argumentation and discussion, overseen by the teacher, are used to give direction and thrust to whole class meaning-making. Mediation by the teacher is seen to stimulate the dialectics of sense with meaning-making, intuition/conviction with proof and spontaneous with scientific concepts.

So what does my ideal classroom look like, what happens there? The teacher promotes, guides and learns from discussion between pupils and with the whole class, children's work is displayed and practical activities are used to bolster a social dimension to learning. This classroom is a place where the contribution of each child is valued, and where children think about their own learning, about their place in the community of those who are becoming mathematicians.

Successful classroom practice such as this is easily recognised when it is encountered, but is somewhat problematic to a reproduce. I consider that this thesis has shown, in the various chapters, ways in which good practice in the use of dynamic geometry software can be analysed and promoted. It may be that many of the ideas advanced from the literature and identified in the discussions in the work can be found to have application, not only in mathematics education but also more widely.

Future Work

Engeström (1991) has reported on various models for the interaction of community, pupils, content and resources, in a way which suggests some commonality with the notion of the faces of the tetrahedron in figure 11.1. It may be that the development of a synthesis of the ideas of this thesis with some of those of Engeström would provide a basis for addressing ideas relevant to whole-class interactive teaching. The immediate vehicle for this might be an action research based project for serving teachers, using material developed to foster the approaches suggested in this work.

A further area has not been fully developed. Figure 11.1 refers to the promotion of a self-critical attitude in pupils, so that they undertake self-examination in their own learning. The work of Mercer (1998), on exploratory talk and how it can be used in conjunction with the use of technology to develop a self-examining attitude in learners, is relevant here, and would merit incorporation into any future developments.

It has been a theme of this work to relate to classroom practice, and the audience for the findings has been taken to be reflective teachers, as well as members of the research community (as addressed by the papers in the appendix). It is for the writer to be aware of audiences, however, and it is here that work seeking to relate to classroom practice must be carefully weighed. Teachers in England at present have little time to incorporate a multitude of initiatives into their work. They are rightly resistant to formal, academic papers, with their many references. Such papers are, for one reason or another, mainly addressing the research community. Reflective, sympathetic teachers are more likely to respond to a vignette of classroom practice with a lightly sketched background of theory which they can assess for its relevance to their own classroom. However writing such as this, which may not be suitable for submission to academic journals, is not always the first objective of academic researchers. Also, there is a reluctance on the part of schools and teachers with serious time demands to become involved in research. A number of initiatives have addressed this problem. One such (Moseley and Higgins, 1999) uses a collection of vignettes, as outlined above, drawn together and discussed in total as a piece of academic research. The accounts of individual classrooms stand on their own and address the reflective teacher, the whole is seen as a viable entity in the research field.

Trainee teachers, those studying for PGCE and those in their induction year, are a powerful influence for change in schools. Their school based teacher/mentors, encouraged to examine their own practice in applying it to their mentoring, are themselves a powerful catalyst for development. Together these beginning teachers and their in-school mentors constitute a community where small-scale action research projects can be used to develop classroom practice based on the theoretical analyses developed in this thesis and to test classroom resources and materials designed to promote pupils' perceptions of themselves as members of a community of learners of mathematics. An initiative based in a cluster of schools and advised by a university mathematics education department would enable the development of materials and practice in the content area of geometry and elsewhere, calling on both traditional and technology based resources. This thesis has indicated ways in which criteria for the development of a local community of practice can be identified, and ways in which classroom practices based on the research literature can be used to develop these communities. A project which took this forward would see this thesis and the work involved in it impacting on the classroom, which is where teachers, pupils, society and learning come together.

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Appendices

Appendix I

Paper presented at the International Group for the Psychology of Mathematics Education Conference, Stellenbosch, South Africa, 1998

Gardiner, J. and Hudson, B.: 1998, The evolution of pupils' ideas of construction and proof using hand-held dynamic geometry technology, in Olivier, A and Newstead, K, Eds, PME 22: *International Group for the Psychology of Mathematics Education Conference*,2,337-344 University of Stellenbosch, South Africa, July 1998

THE EVOLUTION OF PUPILS' IDEAS OF CONSTRUCTION AND PROOF USING HAND -HELD DYNAMIC GEOMETRY TECHNOLOGY

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Abstract

This paper considers how the use of hand-held dynamic geometry software can contribute to the development of pupils' understanding of ideas associated with construction and proof. In adopting a socio-cultural perspective, the technology is seen as a mediating tool and intellectual development as a complex, dialectical process. Classroom research is reported on involving a group of Year 8 pupils (aged 12-13) in a mixed urban comprehensive school in the North of England during the autumn term of 1997. The data analysis is undertaken with particular reference to Vygotskian notions of 'spontaneous' and 'scientific' concepts. It is suggested that such a perspective helps to illuminate the potential of the technology in supporting the complex and dialectical process of developing ideas of construction and proof.

Introduction

The classroom research reported on in this paper is part of a wider study with the aim of investigating the potential of hand-held dynamic geometry software in the secondary school classroom. The research and development has taken place using Cabri on the Texas TI 92 calculator. The focus of the paper is on how the use of such technology can contribute to the development of pupils' understanding of ideas of construction and proof.

Background literature

Dynamic geometry software, seen as a mediating artifact, provides an environment, which supports mathematics learning as highlighted by Jones (1996). The TI 92 has the particular characteristic of enabling the development of a desktop environment in which the dynamic geometry environment (DGE) can be one mediating artifact used alongside more traditional tools.

Healy et al (1994a) illuminate the way in which dynamic geometry can be introduced using the drag function to emphasise the difference between drawing and construction, and proceed to consider constructions in particular further (1994b). Hoyles et al (1995) consider the interdependence of construction and proof and the replacing of proof by construction in a dynamic geometry environment.

In developing the use of the DGE in this study, the drag function and the idea of a construction invariant under drag have been central. The associated classroom materials, which have been developed are directed at making a distinction between drawing and construction, and at seeking an understanding of concepts such as that of using a circle to preserve length (Healy et al, 1994a, 1994b). Pupil fluency with the technology has been a further central consideration as highlighted by Goldstein et al (1996).

The work of Fischbein (1982) is considered to be relevant to this study. He identifies three forms of conviction; formal, arising from argument, empirical arising from a number of practical findings, and an intuitive intrinsic conviction, which he calls 'cognitive belief'. It is suggested that the DGE can reflect these ideas through dragging to test constructions, dragging to provide empirical proof and also through children's intuitive ideas which are triggered by the use of the DGE.

Theoretical Framework

This study is framed within a Vygotskian perspective. Such a perspective places emphasis on the idea of mediation by a variety of tools, as highlighted by Jones (1996) in a similar environment, within the zone of proximal development (Vygotsky, 1962). Vygotsky originally defined the zpd in terms of development whilst more recent definitions, found to be relevant to this study, have related the zpd to activity theory (Engeström, 1987) and to the teacher and class as a whole (Hedegaard, 1990).

The interplay between *everyday* and *scientific* is also considered relevant:

In considering the notion of development, Confrey (1995) highlights this as follows:

> 'Development conceived of as a complex, dialectical process characterised by a multifaceted, periodic timetable ... by a complex mixing of external and internal factors, and by a process of adaptation and surmounting of difficulties.'

Confrey argues the need for an historical analysis and that one must examine the growth of higher mental functions in order to understand them.

Methodology

From a Vygotskian perspective it could be said that methodology should not only be all-pervasive in a study, it should be the study.

> "The attempt to categorize Vygotsky, to 'dualize' him as either a psychologist or a methodologist, contradicts, ironically, not only Vygosky's life-as-lived, but his self-conscious intellectual revolt against dualism" (Newman and Holzman, 1993, p 16).

Vygotsky can be seen as a methodologist/ psychologist in the sense that his all-embracing view of the science of learning brings in the Marxist historico-cultural dialectic and the ideas of revolutionary activity and practice. It provides a methodology, which informs and pervades a study and is available to constantly influence the conclusions drawn and the direction of future progress.

This methodology is echoed in the idea of "tool-and-result" outlined by Newman and Holzman (1993 p38), who draw a distinction between tools such as hammers and screwdrivers (tool for result), and dies and jigs (tool-and-result). Hammers and screwdrivers are bought and used as needed, dies and jigs are tools designed and refined by the worker. Vygotskian methodology is a 'tool-and-result'. Like the jig, it is bound up in its result.

Found to be consistent with such an approach have been ideas drawn from Grounded Theory (Strauss and Corbin, 1990). This involves the systematic process of review and refinement to allow the simultaneous development of theory and collection of data and for a progessive focussing on the emerging issues.

Data Collection

The classroom research so far has taken place in two phases. In the first phase the teacher/researcher taught a class of 30 Year 7 pupils (aged 11-12). The second phase involved working with a group of Year 8 pupils (aged 12-13). Both phases were carried out in mixed urban comprehensive schools in the North of England during 1997. The classroom research has involved the development of materials, which have the aim of releasing the potential of the dynamic geometry software and which, at the same time capitalise on the hand held nature of the TI

92. The development of the materials was guided by the ideas of Hedegaard (1990) and in particular the notion of a 'whole class zpd' in which the role of the teacher in relation to the class as a whole is emphasised. This development has been against a backdrop of the desktop environment where a hand-held DGE has been shared between pairs of pupils in order to stimulate collaboration and interaction.

Data Analysis

This paper reports on the second phase of the classroom trials. The pupils had not used the T I 92 before and met with the teacher/researcher in their lunch breaks. After a brief period of familiarisation, they were given a task of constructing a square, which was stable under drag. The following fragments from the resulting dialogue are presented below. The three pupils involved are Ryan, David and Joanne and the teacher/researcher is JG.

The pupils had been allowed to take the machines home and Ryan had seen the 'Regular Polygon' option, which allows 'construction' of a square directly. This extract is from the following session.

1. D Does anyone know how to draw a square?

2. R Polygon, Regular Polygon

The 'Regular Polygon' option offers a hexagon first and it is not immediately evident how to draw a regular polygon with fewer sides. In this case the use of the technology was not that helpful in assisting pupils to develop their ideas about construction.

Joanne also had taken a machine home and her explorations had led in another direction, towards the 'measuring' menu.

3. J I'm doing it on normal polygon it's a lot easier and you can always

4. measure your lines.

5. D It's hard to get it a proper square.

6. J But afterwards you can measure your lines.

7. R Yeah you can can't you

8. D I know! You could do it two triangles, two right angled triangles next to each 9. other and merge them, then it'd be a proper square.

10. R I think I've got a perfect square here.

11. J See, I've just figured out mine's not right, cos one of my lines is

12. 1.91 cm and the other is 2.03 cm

13. J There's also area; you can do the angle and see if the angle's a

14. right angle, as well.

15. R Well you can tell if it's a right angle.

16. J Yeah but you can't for definite

17. D I think it is regular polygon.

All three went on to use regular polygon successfully and dragged their squares.

Joanne's investigation led her towards attempts at simply drawing the square. However the development of ideas of construction as distinct from drawing become evident from this interaction. For example, David makes reference to a 'proper square' at line 9 and Ryan talks about a 'perfect square' at line 10. These examples could be viewed as evidence of these pupils' spontaneous conceptions of the idea of construction (of a square in this particular case). The discussion at lines 15 and 16 centres on different levels of conviction. For example, Ryan suggests that 'you can tell if it's a right angle' which Joanne counters with the comment that 'but you can't for definite'.

In the second part of the exercise the group were asked to carry out the same task, but not to use polygon or regular polygon. David (Figure 1) and Ryan (Figure 2) both used a circle, a radius and a perpendicular through the centre, as a starting point. Ryan had defined a point where he estimated the other corner of the square to be and drawn two rays through that point. David had drawn two segments, again by eye, to complete his square. Dragging showed him that the point was not defined. Ryan drew two angle bisectors, which coincided originally but separated if he dragged the undefined corner of the square

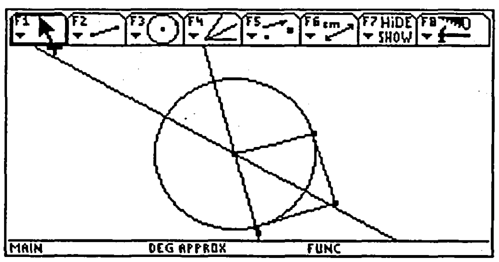

Figure 1 (David)

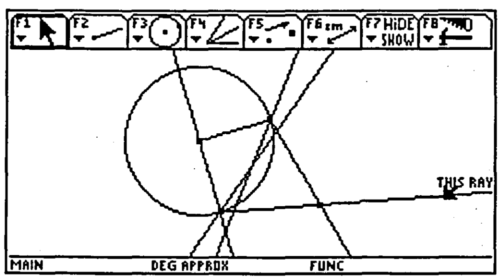

Figure 2 (Ryan)

This conversation followed.

18. D I'm trying to do an angular bisector... cos if the angular bisectors

19. make a right angle in the middle then that'll mean it's a square, but I can't get it to do them.

20. JG How do you know that the angle bisectors will meet in the middle

21. in a right angle?

22. D Well I don't know that they will in a right angle

23. R They will

24. D If it's a proper square then it'll be in a right angle because you'd

25 be chopping. the square like diagonally

26. R There'd be four triangles

27. D There'd be like four triangles and they'd all be right-angled triangles

28. R There'd be two 45° angles

(Shown how to draw angle bisector)

29. D Yes!! Now that looks like its going at a 45° angle right through.

30. That meets in the other corner there, so I think that means it's a square.

Once again, it is suggested, there is an interplay between different levels of conviction and mathematical argument. In the passage from lines 22-28 a sufficient definition of a square is arrived at eventually, only to be abandoned at line 30 for the germ of a new approach.

Joanne used a different starting point. She began by drawing a line segment and was wondering how to continue.

31. JG So you've got one line like that.... What would help you to draw a
32. square?

33. J It would have to be parallel

34. JG So you want to draw a line parallel to this...

35. J Yes

36. JG and where does it have to be?

37. J It has to be the same length as that down

38. JG Like that? So how would you draw it? What shape would help you draw 39. that down to there?

40. J A triangle

41. JG Look on F3

Joanne chose the circle option and went on to successfully draw a square (Figure 3)

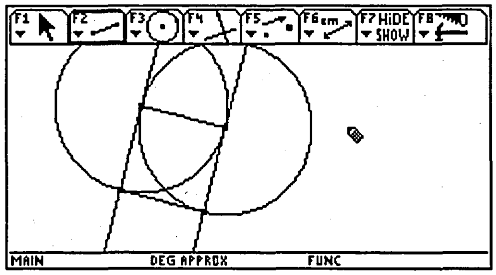

Figure 3 (Joanne)

There followed an attempt to probe understanding of geometrical isometries. This conversation refers back to Figure 2.

42. R I dunno. If I try dragging this ray, because the ray's not secure at the point, 43. that ray'd drag around wouldn't it?

44. But if that was a perpendicular to that ray,……

45. JG So this circle is a good starting point isn't it

46. If you have that circle and that ray, how many sizes of square can you draw?

47. R just one

48. JG as soon as you've drawn that and that

49. R Once you've drawn the circle then you've got the size

Ryan went on to construct a square by drawing a circle (Figure 4), a ray from the centre and a perpendicular through the centre, followed by two perpendiculars where the first two lines intersected the circle.

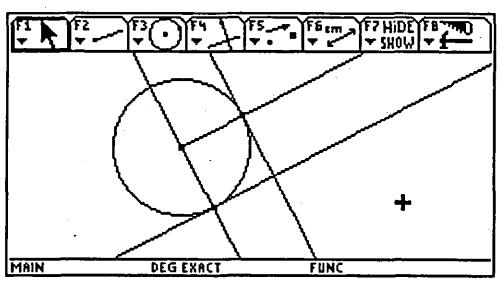

Figure 4 (Ryan)

Pressing the grab key when the cursor is away from the diagram makes the independent points in the diagram flash. This useful facility allows pupils to explore geometrical isometries.

50. JG What flashes?

51. R That corner there. (The centre of the original circle)

52. Does that mean that's the only corner that can be dragged?

53. JG That's the only point that can be dragged. Tell me what you drew first.

54. R I drew the circle first

Ryan went on to discover that he could grab the circumference of the circle as well as the centre and so alter the size of the square, and alter the orientation of the diagram by dragging the original ray. By a similar process Joanne realised that the original line segment in her diagram completely defined her square.

Discussion

In observing this classroom activity and in analysing this interaction, there is a clear interplay between ideas of drawing and construction and also between notions of necessary and sufficient conditions (for construction). It is argued that this interplay reflects that between pupils' spontaneous concepts and their developing ideas related to scientific concepts, which in this case are associated with ideas of construction and proof. These pupils can be seen to be operating in a dialectic between their spontaneous conceptions of proof, informed by their ideas and the insights available to them via the mediating role of the DGE and other desk top tools, and the scientific concepts of construction and proof. The second episode in particular provides a rich illustration of how *everyday* (spontaneous) concepts 'create a series of structures necessary for the evolution of a concept's more primitive, elementary aspects, which give it body and vitality' and hence how *scientific* concepts 'in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use' (Vygotsky, 1962). By the end of this episode, it is suggested that both Ryan and Joanne have displayed evidence of an appreciation of the idea of construction and that they have had at least an elementary introduction to ideas associated with geometrical isometries.

It is further suggested that this development can be seen to parallel Fischbein's (1982) three forms of conviction; formal, arising from argument, empirical arising from a number of practical findings, and an intuitive intrinsic conviction or 'cognitive belief'.

It is also suggested that this interaction is illustrative of development 'conceived of as a complex, dialectical process characterised by a multifaceted, periodic timetable ... by a complex mixing of external and internal factors, and by a process of adaptation and surmounting of difficulties' (Confrey, 1995).

In considering this process of development, the role of the teacher within the zpd has been found to be an essential element in assisting pupils to move from their spontaneous/everyday conceptions towards more scientific concepts. This echoes the findings of Jones (1996) who argues the need for a significant input from the teacher when pupils are working within a DGE.

A further aspect of the wider study, for which there is little room in this paper, to consider in any great depth has been the interplay in the desk-top environment between the DGE and the traditional tools such as pencil and paper.

A final observation in relation to the aims of the wider study is of the undoubted potential of such hand-held dynamic geometry environments to promote the development of pupils' understanding of notions of construction and proof.

Thanks are due to pupils and staff from King Edward VII School, Sheffield and Mossley Hollins School, Tameside.

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Appendix II

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Gardiner, J., Hudson, B. and Povey, H.: 1999, "What can we all say?" Dynamic geometry in a whole class zone of proximal development, *Conference Proceedings of PME 23: International Group for the Psychology of Mathematics Education Conference*, Technion – Israel Institute of Technology, July 1999, 3, 1-8

“What can we all say?” Dynamic geometry in a whole-class zone of proximal development.

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This paper first considers aspects of the literature relevant to class and group teaching in a social context. Ideas of socio-mathematical norms and argumentation, on the significance of local communities of practice and on the development of a whole-class ZPD are examined.These ideas have been used to influence classroom approaches to the use of dynamic geometry (Cabri II on the TI 92 with 11-14 year old pupils in the UK) and analysis of classroom observation is presented. Conclusions are drawn about the interaction of these ideas with the technology and how the alignment of mathematical meaning-making might be promoted.

Introduction

There are bodies of current research which consider, from different perspectives, the dynamics of social meaning-making in classrooms. Cobb and Yackel (1996), Winbourne and Watson (1998), and Hedegaard (1990) and Lerman (1998) all have viewpoints which can be used to inform an analysis of classroom interaction.This paper reports on the development and use of classroom material using dynamic geometry on the TI92 in lower secondary classrooms (age 11-14) in the UK. Classroom dialogue from lessons taught by the researcher was transcribed from audio recordings. This dialogue is analysed from a socio-cultural perspective, making reference to the viewpoints referred to above, and seeking to illuminate the ways in which students make mathematical meaning in areas such as construction and proof.

Literature and Theoretical Background

Vygotsky(1962) proposed a social background to learning and formulated the Genetic Law of Cultural Development, with learning moving from the social to the personal. He took up the idea of the Zone of Proximal Development as the area where interaction between the individual and the social leads to development. Lerman (1998) says of the ZPD ‘it provides the framework, in the form of a symbolic space ,……for the realisation of Vygotsky’s central principle of development.’(p71)

Of particular interest here is a definition of the ZPD which includes the classroom as a whole, in this case incorporating the teacher, the pupils and the technology Hedegaard (1990) has reported in terms of the development of a whole-class ZPD rather than the consideration of an individual's learning:

This activity, in principle, is designed to develop a zone of proximal development for the class as a whole, where each child acquires personal knowledge through the activities shared between the teacher and the children and among the children themselves (p 361).

Hedegaard reports in the same paper a motivational shift in children's focus, from an interest in the concrete to interest in the derivation of principles which can be applied to the concrete. Lerman (1998) takes the discussion further.

The ZPD is the classroom's, not the child's. In another sense it is the researcher's: it is the tool for analysis of the learning interactions in the classroom (and elsewhere) (p 71).

Insights into factors which might influence meaning-making in a whole-class or group ZPD can be drawn from the literature and indicate socio-cultural vectors which may operate for meaning-making. These include Local Communities of Practice, Socio-mathematical Norms and Choice of Materials.

Local Communities of Practice and Telos
Drawing on work by Lave and Wenger (1991) and Lave (1993), Winbourne and Watson (1998) have used the idea of 'local communities of (mathematical) practice'. They identify features of a local community of (mathematical) practice :

- pupils see themselves as functioning mathematically within the lesson;
- within the lesson there is public recognition of competence;
- learners see themselves as working together towards the achievement of a common understanding;
- there are shared ways of behaving, language, habits, values and tool-use;
- the shape of the lesson is dependent upon the active participation of the students;
- Learners and teachers see themselves as engaged in the same activity.'(p 183)

They examine classroom interaction in terms of such a community and go on to discuss the idea of 'telos', of the meaning-making of the whole class being aligned in directions generated by social interaction. They see telos as a unification of small scale 'becomings' by which many learners join a community of practice. They see:

a link between our notion of LCP and the situated abstraction of Noss and Hoyles (1996). Just as they claim the computer provides domains which support students' abstraction, so we claim LCPs support students' growing image of themselves as someone who is legitimately engaged in mathematical practice, as someone, in other words, who is becoming a mathematician. (p183)

Socio-mathematical Norms and Argumentation

This approach is echoed in the work of Cobb and Yackel (1996), who have analysed mathematics classrooms in terms of the negotiation and maintenance of social and socio-mathematical norms. Social norms include

- insistence on explanation of answers
- respecting the contribution of others
- making clear agreement as well as disagreement.

Socio-mathematical norms would include

- some notion of what constitutes a valid, complete solution
- agreement on the worth of alternative solutions
- negotiated agreement between teacher and students on the mutual acceptability of solutions.

Social norms will exist in all classrooms, and will bear a direct relationship to the society in which the classroom is situated. Because social norms will affect the negotiation of socio-mathematical norms, Apple (1992) has argued that the classroom is firmly situated in the wider context of the practices of school and society. Yackel and Cobb (1996) discuss the influence of socio-mathematical norms on argumentation in the classroom. They draw on the ideas of Toulmin (1969) as developed by Krummheuer (1995), seeing argumentation as made up of conclusion, data, warrant and backing. Yackel (1998) says of argumentation:

it clarifies the relationship between the individual and the collective, in this case between the explanations and justifications that individual children give in specific instances and the classroom mathematical practices that become taken-as-shared. As mathematical practices become taken-as-shared in the classroom, they are beyond justification and, hence, what is required as warrant and backing evolve. Similarly, the types of rationales that are given as data, warrants and backing for explanations and justifications contribute to the development of what is taken-as-shared by the classroom community, that is to the mathematical practices in the classroom. (p210)

Thus argumentation is seen as a social, rather than a logical process, a means of establishing that which is held in common about the topic in question and moving forward the 'held in common' by classroom interaction. Voigt (1995) discusses the reflexivity between learning and interaction and speaks of this reflexivity contributing to a classroom microculture which in turn affects the meaning-making which is taking place.

Choice of Material

Lave and Wenger (1991, pp102,103) address the issue of the transparency of a resource, and this is further examined by Adler (1998, pp8-11). A resource used in a mathematics classroom can be so visible to students that it obscures the mathematics and prevents meaning making. At the same time some visibility is necessary. We want the resource to be visible in the sense that it should direct the gaze of students, so enabling their meaning-making.

Invisibility of mediating technologies is necessary for allowing focus on, and thus supporting the visibility of, the subject matter. Conversely, visibility of the significance of the technology is necessary for allowing its unproblematic- invisible -use. This interplay of conflict and synergy is central to all aspects of learning in practice: it makes the design of supportive artifacts a matter of providing a good balance between these two interacting requirements.(Lave and Wenger, 1991 p103)

Clearly the familiarity of students with technology such as the TI 92 governs its use, in a way which is informed by arguments such as this. As they become more familiar with the software the teacher will be able

to introduce the use of more complicated functions without losing transparency.
It is proposed here that these approaches, of a whole class ZPD, of a recognition of local communities of practice, and of negotiated socio-mathematical norms and argumentation have much to offer in looking at how technology, appropriately transparent, can be used in the classroom. In this study such approaches are used, in particular, to analyse social meaning-making in the area of construction and proof using Cabri with the TI 92 hand-held computer with lower school (11-14 years) pupils.

Methodology and Data Collection
A qualitative and ethnographic approach to research has been adopted, with case studies used to provide instances of rich incidents for subsequent analysis. These were subjected to microethnographic interpretive procedures (Erickson, 1986 and Voigt, 1990) Classroom interaction between teacher/researcher and individuals in whole class and group situations was audio recorded and the transcriptions of these recordings analysed. In addition, field notes of memorable incidents were recorded.

Each student had a TI 92 hand-held computer and used the dynamic geometry environment Cabri as available on this machine. An overhead projector version was available for demonstration by pupils and the teacher to the whole class. The following examples were an attempt to set up possibilities for whole class meaning-making with the minimum of previous knowledge of the TI92. The pattern followed was for the class to generate and discuss a simple dynamic image, and to record the result in exercise books as a diagram after the dynamic image had been appreciated. The hand-held nature of the TI92 is particularly suitable for pair discussion and, indeed, for consigning to a corner of the desk when work on paper is preferred.

Collection and Discussion of Data

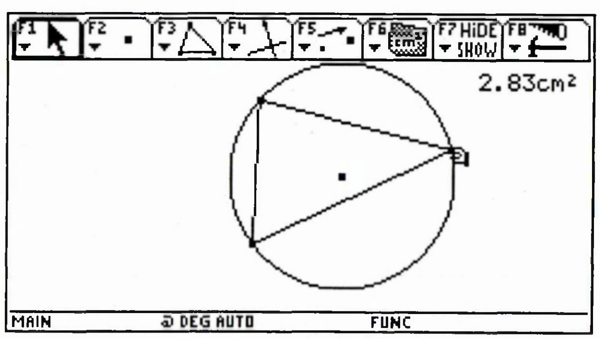

Fig 1

1. The class was asked to draw a circle and a triangle with its vertices on the circle,

then to measure the area of the triangle (Fig 1).They were then asked to investigate the effect of dragging **one** of the vertices, and to look for the maximum area of the triangle. In jointly exploring the same screen in this way, but each on their own machine, a telos is created and students are aligned in the domain provided by the technology.

The following dialogue ensued.[1]

<table>
<tr><td>JG</td><td>What area have you got?</td><td></td></tr>
<tr><td>Class</td><td>General response</td><td>There was no restriction on the original diagram, a wide range of areas was possible.</td></tr>
<tr><td>JG</td><td>Why do we all get different answers?</td><td></td></tr>
<tr><td>Alison</td><td>Because we all used different circles</td><td>Use of 'we' suggests the possibility of an LCP</td></tr>
<tr><td>Barry</td><td>And different points</td><td></td></tr>
<tr><td>JG</td><td>Look at mine while I move the point. Tell me when it will be greatest. What can we all say about our diagrams?</td><td></td></tr>
<tr><td>Barry</td><td>It's across from the centre</td><td rowspan="3">Later discussion showed that Barry appreciated the co-linearity of the mid point of one side of the triangle, the centre of the circle and the other vertex. Leanne had realised that the triangle was isosceles</td></tr>
<tr><td>JG</td><td>Yes, good.
Anyone else?</td></tr>
<tr><td>Leanne</td><td>It's in the middle</td></tr>
</table>

Here the technology could be said to be driving along the LCP. Spontaneous concepts are developed by the participants by looking at the dynamic image, which can then be used by the teacher to interact with scientific concepts (Gardiner and Hudson, 1998), so that that which is 'taken as shared' is moved forward.

[1] Throughout this paper the teacher/researcher is JG and pseudonyms are used for pupils.

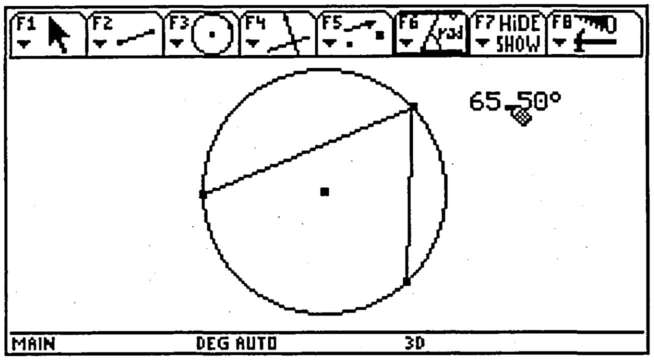

Fig 2

2. Another exercise which is available after only the briefest of introductions to the technology is based on a diagram such as Figure 2. Here pupils were asked to define and measure an angle in a circle as shown and to investigate the effect of dragging any one of the defining points along the circumference of the circle.[2] Transcription of classroom audio recordings resulted in the following dialogue.

JG	Does anyone want to tell me what they have found?		
Sonia	As you move this down it stays the same angle until you reach this point, then it changes to a completely different angle and stays the same.	*In this version of Cabri, if the vertex is moved round the circle until it passes one of the other points, the angle in the other segment, the supplement of the first, is measured*	*'conclusion'*
Nigel	Oh Yeah *(Wonderingly)*		*Drawn into a community of practice*
JG	Will you come and show us		
Sonia	It might not work you know… it might just be because of the shape of this one	*In order to demonstrate the OHP version of the machine had to be used.*	
Tom	It will work.. I got it to work		*Supporting the community of practice*
JG	Watch while she drags this. Watch the angle. Moving up …..angle getting bigger	*Dragging one of the non-vertex points*	
Nigel	If you change the middle one, watch the middle one, it stays the same and after a certain point it changes		*Now a more assured member of the practice*

[2] An idea suggested by Geoff Wake of Manchester University

JG	What's going to happen now?	*Dragging the vertex point*	
Chorus	Stays the same		
Nigel	Until you pass the point, then it will stay the same again		
JG	Look at the angle, it stays at 52.77 degrees, doesn't matter that the line goes through the centre, stays at 52.77. Now changes to..........127.23 *(to Sonia at front)*Can you make it flip between those two angles? *(to class)*What can you tell me about those two angles?		
Anne	Does it add up to 180?		*conclusion*
David	Ooo *(realising)*		
JG	Check that those results are true for your diagram		

Referring to criteria mentioned earlier for an LCP (Winbourne and Watson. 1998), here pupils can be said to be sharing tool use and purpose by being aligned in the task and their use of the technology. They are functioning and participating mathematically and recognising the competence of others. There is also, in this dialogue, a sense of telos, in which the pupils are aligned by the technology in a way which drives forward the meaning-making of the community.

Socio-Mathematical Norms and Argumentaion

In the passage quoted above there is evidence of two 'conclusions' (Yackel, 1998 p210) being reached (as indicated), without oral evidence of warrant and backing. However it appears that, in this dynamic geometry environment, warrant and backing are supplied by the shared experience of data generated by the technology.

Conclusion

This research has indicated how, with a background of individual and class development within a Zone of Proximal Development, the ideas of local communities of (mathematical) practice, telos, socio-mathematical norms and argumentation can be used to indicate how mathematical meaning making in the classroom might be analysed. In particular it demonstrates the benefit of suitably transparent use of technology in promoting alignment of pupil becomings within a whole-class ZPD.

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Appendix III

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ASPECTS OF CONSTRUCTION AND PROOF IN A HAND-HELD DYNAMIC GEOMETRY ENVIRONMENT

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This paper considers how the use of dynamic geometry software (Cabri II on the TI 92) can contribute to the development of pupils' ideas of construction and proof. Classroom research is reported involving a group of Year 8 pupils (aged 12 to 13) in a mixed urban comprehensive school in the North of England. Two perspectives are combined in analysing the classroom activities. Socio-culturally, the technology is seen as a mediating tool and intellectual development as a complex, dialectical process. A second perspective considers the elements of proof and concludes that, whilst verification and conviction have an importance, it is in explanation that proof becomes social. It is suggested that this combination may indicate the potential of the technology in supporting the development of ideas of construction and proof.

Introduction

The classroom research reported on in this paper is part of a wider study[3] with the aim of investigating the potential of the use of dynamic geometry software on hand held machines in the lower secondary school classroom (age 11 to 14). The research and development have taken place using Cabri Géomètre on the Texas TI 92 calculator. Classroom activities were audio recorded and the transcripts analysed. Field notes were used to record the screens the pupils had generated. An attempt is made to combine two approaches in the analysis of the data. From a socio-cultural perspective, the activities of the pupils can be seen as exemplifying ideas of interplay between spontaneous and scientific concepts (Vygotsky, 1962, p.109). De Villiers (1991) has emphasised the diverse nature of proof and it is suggested that explanation, as a social aspect of proof, can be identified in the classroom incidents recorded here. This indicates how the use of technology might combine with social interaction from peers and the teacher to contribute to the development of pupils' understanding of ideas of construction and proof.

Background Literature

Dynamic geometry software, seen as a mediating artefact, provides an environment which can support mathematics learning. The TI 92 has the particular characteristic of enabling the development of a desktop environment in which the use of dynamic geometry software can be one mediating artefact used alongside more traditional tools.

Healy, Hoelzl, Hoyles, and Noss (1994a) illuminate the way in which dynamic geometry software can be introduced using the drag function to emphasise the difference between drawing and construction, and proceed to consider constructions in particular further (1994b). Hoyles, Healy and Noss (1995) consider the interdependence of construction and proof and posit the replacement of proof by construction in a dynamic geometry environment. This partly draws upon the idea of proof operating in a domain 'wide enough to include the visual aspects of mathematical intuition and reasoning' (Davis, 1993, p. 333).

In developing the use of the dynamic geometry environment in this study, the drag function and the idea of a construction being invariant under drag have been central. The classroom materials as developed are directed at making a distinction between drawing and construction and at seeking an understanding of concepts such as that of using a circle to preserve length (Healy et al, 1994a, 1994b). Pupil fluency with the technology has been a further central consideration as highlighted by Goldstein, Povey and Winbourne (1996).

Fischbein (1982) identifies three forms of conviction: formal, arising from argument; empirical, arising from a number of practical findings; and an intuitive intrinsic conviction, which he calls 'cognitive belief'. It is suggested that the dynamic geometry environment can reflect these ideas by providing a background for pupil/pupil and pupil/teacher discussion, by allowing dragging to provide empirical proof and also through the way in which it triggers children's intuitive ideas.

De Villiers (1990) proposes various elements of proof. He identifies the areas of verification and conviction, of explanation, of systematising and communication and of discovery. He notes that empirical examples often lead to conviction/verification; similarly, one of Fischbein's (1982) forms of conviction is empirical conviction arising from a number of practical findings. Mason (1991), referring to the power of computers to present a dynamic image under the control of the user, calls for acceptance of a form of proof afforded by a large number of examples. He writes:

> I predict that one of the long-term effects of computers will be to establish a mode of certainty which lies between the too-easy acceptance of a generalisation from one or two cases and the rigour of mathematical proof. Programs like Cabri-geometry enable the user to experience a huge range of particular cases, and by appeal to continuity, an infinite number of cases. This plethora of confirming instances will be highly convincing for many, if not most, people. I find this entirely reasonable. (p. 87)

Empirical example is a powerful vehicle for conviction and, with dynamic geometry software, may lead to a form of verification; however such verification and conviction do not in themselves constitute meaning-making. Indeed the reaction of pupils to a visual or empirical demonstration intended to convince is often lack of interest. De Villiers (1991) claims that, in contrast, it is possible to excite pupils' motivation for and satisfaction from the deductive explanation of a proof, to engage what Mason has called 'this sense of mustness' (1991, p. 86).

Theoretical Framework

From a Vygotskian perspective, emphasis is placed on the idea of mediation by a variety of tools, within the learner's zone of proximal development (ZPD) (Vygotsky, 1962). The contribution of a dynamic geometry environment as such a mediating artefact is highlighted by Jones (1997), who also emphasises the importance of the contribution of the teacher. Vygotsky originally defined the ZPD in terms of development whilst more recent definitions, found to be relevant to this study, have related the ZPD to activity theory (Engeström, 1987) and to the teacher and class as a whole (Hedegaard, 1990).

The interplay between *everyday* and *scientific* is also considered relevant:

> In working its slow way upward, an *everyday* concept clears a path for the scientific concept in its downward development. It creates a series of structures necessary for the evolution of a concept's more primitive, elementary aspects, which give it body and vitality. *Scientific* concepts in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use. (Vygotsky, 1962, p. 109)

Confrey (1995) discusses Vygotsky's views on development as informed by ideas of dialectical and historical materialism and notes that he refers to development

> as a complex, dialectical process characterised by a multifaceted, periodic timetable ... by a complex mixing of external and internal factors, and by a process of adaptation and surmounting of difficulties. (Vygotsky, 1978, p. 151)

Confrey argues the need for an historical analysis and that one must examine the growth of higher mental functions in order to understand them. This approach can be combined with an analysis of the importance of explanation as the social element of proof.

In offering a reconsideration of the nature of proof, De Villiers (1991) writes

> not with the intention of sacrificing any fidelity in mathematics merely for pedagogical expediency, but actually the contrary: the encouragement of greater fidelity with respect to the variety of reasons behind proof. (p. 26)

He quotes Chazan (1990) as calling for the

> inclusion of exploration and conjecturing; presentation of demonstrative reasoning as explanatory; treatment of proving as a social activity; and emphasis on deductive proofs as part of the explanatory process, not its end point. (p.9)

It is argued here that consideration should be given to the importance of this explanatory process. With the availability of dynamic geometry software, conviction and verification may often be readily achieved. However, the *explanation* delivered by a proof brings it firmly into a social dimension, into an area which is open to mediation by others, in a way which the more intuitive functions of conviction and verification do not. Of course, explanation, conviction and verification are often inter-linked. Explanation may lead to conviction or the individual may need first to be convinced in order to be stimulated to produce a deductive explanation; and empirical verification can support the kind of conjecturing required to frame an hypothesis before trying to explain it: 'you have to guess a mathematical theorem before you prove it' (Polya, 1954, p. vi). However, explanation and justification, whether conducted alone or communally, seem inherently social activities, deriving their purpose from the existence of a community of mathematical meaning makers. When explanation in the classroom becomes a social activity, it takes its place in the dialectic of proof and begins to lead to students making mathematical meaning.

Methodology and Data collection

The methodology of this study is qualitative and ethnographic, using case-studies to provide material for analysis. The classroom research so far has taken place in two phases. In the first phase the teacher/researcher taught a class of 30 Year 7 mixed ability pupils (aged 11to12). The second phase involved working with a group of Year 8 pupils (aged 12 to13). These pupils were volunteers working in their own time, and were from the upper band of two. Both phases were carried out in mixed urban comprehensive schools in the North of England during 1997. The classroom research has involved the development of materials which have the aim of releasing the potential of the dynamic geometry software and which, at the same time, capitalise on the hand-held nature of the TI 92. The development of the materials was guided by the ideas of Hedegaard (1990) and in particular the notion of a 'whole class ZPD' in which the role of the teacher in relation to the class as a whole is emphasised. This development has been against a backdrop of the desktop environment where a hand-held dynamic geometry environment has been shared between pairs of pupils in order to stimulate collaboration and interaction.

Data analysis

This paper reports on the second phase of the classroom trials. The pupils had not used the TI 92 before and met with the teacher/researcher in their lunch breaks. After a brief period of familiarisation, they were given a task of constructing a square which was invariant under drag. Fragments from the resulting dialogue are presented below. The three pupils involved are Ryan, David and Joanne and the teacher/researcher is JG.

The pupils had been allowed to take the machines home and Ryan had seen the 'Regular Polygon' option, which allows construction of a square directly. This extract is from the following session.

1. D Does anyone know how to draw a square?
2. R Polygon, Regular Polygon

The 'Regular Polygon' option offers a hexagon first and it is not immediately evident how to draw a regular polygon with fewer sides. In this case the use of the technology was not very helpful in assisting pupils to develop their ideas.

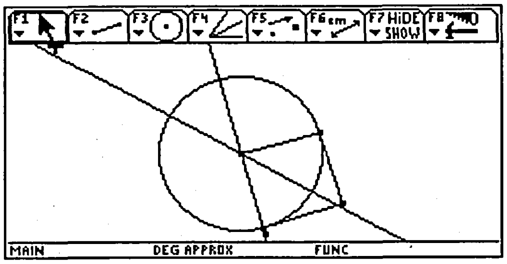
Joanne also had taken a machine home and her explorations had led in another direction, towards the 'measuring' menu.

3. J I'm doing it on normal polygon it's a lot easier and you can always
4. measure your lines
5. D It's hard to get it a proper square
6. J But afterwards you can measure your lines
7. R Yeah you can can't you
8. D I know! You could do it two triangles, two right angled triangles next
9. to each other and merge them, then it'd be a proper square
10. R I think I've got a perfect square here
11. J See, I've just figured out mine's not right, cos one of my lines is 1.91
12. cm and the other is 2.03 cm
13. J There's also area; you can do the angle and see if the angle's a right
14. angle, as well
15. R Well you can tell if it's a right angle
16. J Yeah but you can't for definite
17. D I think it is Regular Polygon

All three went on to use 'Regular Polygon' successfully and dragged their squares.

Joanne's investigation led her towards attempts at simply drawing the square. However the development of ideas of construction as distinct from drawing become evident from this interaction. For example, David makes reference to a 'proper square' at line 9 and Ryan talks about a 'perfect square' at line 10. These examples could be viewed as evidence of these pupils' spontaneous conceptions of the idea of construction (of a square in this particular case). The use of the qualifiers 'proper' and 'perfect' suggests a spontaneous idea of a square as an idealised mathematical object and a readiness to search after a representation of this ideal rather than to draw an approximation to it. Later on (line 30), the word 'square' is not qualified, possibly because the idealised form is now more deeply embedded in the pupils' conception. The discussion at lines 15 and 16 centres on different levels of conviction. For example, Ryan suggests that 'you can tell if it's a right angle' which Joanne counters with the comment that 'but you can't for definite'. Hoyles *et al* (1995) examine the case for construction replacing proof in a dynamic geometry environment and it is possible to identify in the above passage elements of proof as explanation in lines 8 and 9.

In the second part of the exercise the group were asked to carry out the same task but not to use 'Polygon' or 'Regular Polygon'. David (Figure 1) and Ryan (Figure 2) both used a circle, a radius and a line perpendicular to that radius through the centre as a starting point. Ryan had defined a point where he estimated the other corner of the square to be and drawn two rays, back to the relevant points on the circle, through that point. David had drawn two line segments, from the points on the circle, again by eye, to complete his square. Dragging showed him that the point of intersection was not defined as the last corner of a square. Ryan drew two angle bisectors which coincided originally but separated if he dragged what he had intended to be, but was not, a defined corner of his square.



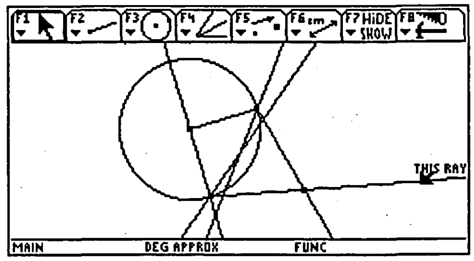


Figure 1 (David) **Figure 2 (Ryan)**

This conversation followed.

18. D I'm trying to do an angular bisector... cos if the angular bisectors make
19. a right angle in the middle then that'll mean it's a square ...
20. JG How do you know that the angle bisectors will meet in the middle in
21. a right angle?
22. D Well I don't know that they will in a right angle
23. R They will
24. D If it's a proper square then it'll be in a right angle because you'd be
25. chopping the square like diagonally
26. RThere'd be four triangles
27. D There'd be like four triangles and they'd all be right-angled triangles
28. R There'd be two 45° angles

(Shown how to draw angle bisector)

29. D Yes! Now that looks like its going at a 45° angle right through
30. That meets in the other corner there, so I think that means it's a square.

Once again, it is suggested, there is an interplay between different levels of conviction and mathematical argument. In the passage from lines 22-28 a sufficient definition of a square is arrived at eventually, only to be abandoned at line 30 for the germ of a new approach. Again, the proving role of explanation and justification can be seen at work, stimulated by the students' social engagement with the process of construction.

Joanne used a different starting point. She began by drawing a line segment and was wondering how to continue.

31. JG So you've got one line like that.... What would help you to draw a square?
33. J It would have to be parallel
34. JG So you want to draw a line parallel to this...
35. J Yes
36. JG and where does it have to be?
37. J It has to be the same length as that down
38. JG Like that? So how would you draw it? What shape would help you
39. draw that down to there?
40. J A triangle

However, Joanne chose the circle option and went on to draw a square successfully (Figure 3).

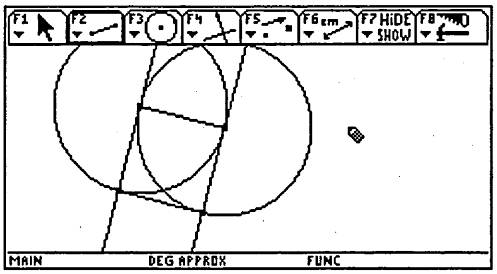


Figure 3 (Joanne)

Ryan was asked to consider more fully the possibilities of dragging elements of his diagram (Figure 2).

42. R I dunno. If I try dragging this ray, because the ray's not secure at the
43. point, that ray'd drag around wouldn't it?
44. But if that was a perpendicular to that ray,......
45. JG So this circle is a good starting point isn't it
46. If you have that circle and that ray, how many sizes of square can you draw?
47. R just one
48. JG as soon as you've drawn that and that
49. R Once you've drawn the circle then you've got the size

Ryan went on to construct a square by drawing a circle (Figure 4), a ray from the centre and a perpendicular through the centre, followed by two perpendiculars where the first two lines intersected the circle.

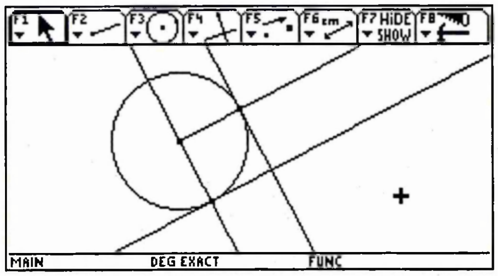


Figure 4 (Ryan)

Pressing the grab key when the cursor is away from the diagram makes the independent points in the diagram flash, allowing further exploration.

50. JG What flashes?
51. R That corner there (The centre of the original circle)
52. Does that mean that's the only corner that can be dragged?
53. JG That's the only <u>point</u> that can be dragged. Tell me what you drew first
54. R I drew the circle first

Ryan went on to discover that he could grab the circumference of the circle as well as the centre and so alter the size of the square and that he could alter the orientation of the diagram by dragging the original ray. By a similar process, Joanne realised that the original line segment in her diagram completely defined her square. The importance of a deductive explanation of the geometrical constants, incorporating 'the fact that there are facts' (Mason, 1991, p87), in the constructions produced by Joanne and Ryan is discussed later.

Discussion

In observing and analysing this classroom activity, there is a clear interplay between ideas of drawing and construction and also between notions of necessary and sufficient conditions (for construction). It is argued that this interplay reflects that between pupils' spontaneous concepts and their developing ideas related to scientific concepts, which in this case are associated with ideas of construction and proof. These pupils can be seen to be operating in a dialectic between their spontaneous conceptions of proof, informed by their ideas and the insights available to them via the mediating role of the dynamic geometry environment and other desk top tools, and the scientific concepts of construction and proof. The second episode in particular provides a rich illustration of how *everyday* (spontaneous) concepts

'create a series of structures necessary for the evolution of a concept's more primitive, elementary aspects, which give it body and vitality' (Vygotsky, 1962, p. 109)

and hence how *scientific* concepts

'in turn supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use' (ibid).

By the end of this episode, it is suggested that both Ryan and Joanne have displayed evidence of an appreciation of the idea of construction and that they have had, at least, an elementary introduction to ideas associated with geometrical isometries.

This development can also be seen to parallel Fischbein's (1982) three forms of conviction: there is a sense, early on, of an intuitive intrinsic conviction; dragging provides experience of a number of practical findings; and finally we see the initial stages of a formal approach arising from argument. This last is not yet a formal proof as accepted within academic mathematics; but we see the beginnings of the use of explanatory chains of reasoning.

This interaction is also illustrative of development understood as subtle and interactive: we see the interplay of external and internal factors as putative solutions are posited, discovered to be flawed and adapted and the difficulties encountered eventually overcome. In considering this process of development, the role of the teacher within the ZPD is observed to be a key element in assisting pupils to move from their spontaneous/everyday conceptions towards more scientific concepts. Mason, reflecting on aspects of Bruner's (1986) work on Vygotsky, has written of 'the role of the teacher as being a vicarious consciousness, able to hold onto global aims and themes when pupils' attention is diverted to detail.' (Mason 1991, p. 90)

We see, in the episode above, a complex mixture of elements with mediation by both teacher and technology in the furtherance of pupils' meaning-making. This echoes the findings of Jones (1997) who argues the need for a significant input from the teacher when pupils are working within a dynamic geometry environment.

It can perhaps be said that whilst conviction and verification have been identified as elements of proof it is in *explanation* that proof and construction acquire a fundamentally social dimension and begin to

impinge on meaning-making. Explanation is the area of proof which is most available for mediation in the Vygotskian sense. Images may be able to convince, dynamic geometry may provide a form of verification, but it is when explanation begins that proof moves into a social dimension.
However the form of communication which we call explanation has many layers to it and we can identify various shades of meaning. Indeed the imprecise nature of what we understand by explanation suggests that it contributes in various forms to the process of proof. Davis and Hersh (1983, p. 73) describe mathematical argument as 'a human interchange based on shared meanings, not all of which are formulaic.' De Villiers (1990) refers to the unique role of proof in the 'explanation, systematisation and verification of results, something which is not possible to the same degree using only intuitive and/or quasi-empirical methods' (p. 23).

Explanation as a communication process has an object and it is instructive to analyse the episodes recorded here in a way which identifies these objects. We explain to ourselves (lines 42, 43), to others (lines 3-17), to pupils, to teachers. Scaffolding provided by the teacher (lines 42-49), as dialogically pupil and teacher construct a connected chain of reasoning, is often key to building on the conviction which is already present through dynamic geometry or otherwise, with logical deduction leading to richer meaning-making. Because explanation, through communication, draws proof out of the intra-psychological, it can give pupils ownership of their mathematics and provide motivation. Another intuition may lead to further discoveries, which may be verified on the screen, but it is when a deductive explanation can be produced and proof acquires a dialectical social element that it reaches its potential as an important part of meaning making.

It is proposed that these episodes, when considered from theoretical standpoints such as the dialectic of proof and the interplay between spontaneous and scientific concepts, complemented by explanation as the engine-room of proof, driving forward meaning making, may indicate a framework for a classroom approach. The study appears to indicate the potential of hand-held dynamic geometry environments to promote the development of pupils' understanding of notions of construction and proof. The environment provides opportunities for mediation by pupils, the teacher and the technology. It is suggested that it is the explanation aspect of proof which the teacher can use to motivate and help give pupils a sense of themselves as the makers of mathematical meaning.

Thanks are due to pupils and staff from King Edward VII School, Sheffield and Mossley Hollins School, Tameside.

Notes

1 A further aspect of the wider study, not discussed in this paper, concerns the interplay in the desk-top environment between hand-held dynamic geometry technology and traditional tools such as pencil and paper.

2 See also Rotman (1994) for a further critique of conventional notions of proof and the anticipated impact of computers on our understanding of them.

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Appendix IV

Article published in Mathematics Teaching, Journal of Association of Teachers of Mathematics, 2000

Gardiner, J.:2000 Area and Shape in Year 5, *Mathematics Teaching* Association of Teachers of Mathematics, 172, 70-72

Area and shape in Year Five

A description of a lesson and some discussion of background reading.

Catherine Gardiner, Ash Tree Primary School, Kippax

John Gardiner, Sheffield Hallam University

Catherine invited me to come to talk to her Year Five class, and suggested that an introductory look at area might be possible. I had been looking at the way resources, including both low tech and the TI92, can be used in class teaching in secondary schools [1] and was interested to get involved at primary level.

- The class of 26 9-10 year-olds were divided into mixed ability groups of four or five. We used a brief warm-up on factors and factor pairs, with answers agreed in groups and written up on boards before being held up for the class to see.

  Catherine writes:

  I had put some thought into the make up of these groups, taking into account both ability and personality. I have tried to promote an atmosphere of helping each other and I was not surprised that all the groups worked well together and were very co-operative about taking turns to record and hold up the boards.

  We had a brief talk about area, using the example of painting surfaces to introduce the idea, before moving to the first activity, which was intended to emphasise the fact that area does not depend on shape.

- Each child had a coloured tissue paper circle, which was cut into two semicircles. (Two acetate semicircles on the OHP showed them what to do.) They had to overlap their semicircles so that they could see two light-coloured and one darker coloured shape, and talk in their groups about the areas they could see. After a time some groups were able to explain to the class that the two light coloured areas in everybody's diagram were equal in area. Children of all abilities joined in finding an explanation of why this had to be so.

Picture here of partly overlapping semicircles of tissue paper.

Sam wrote

The two light areas are the same because the dark bit came from both of the semi circles so the areas of the light bits are the same.

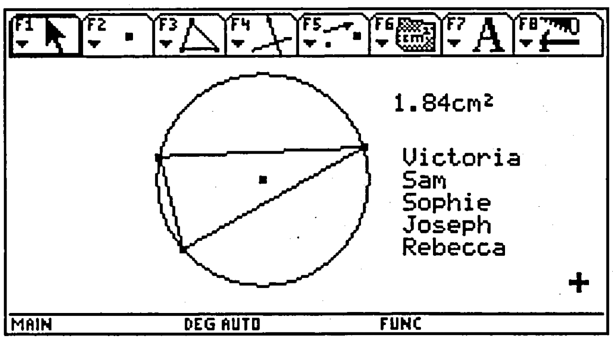
Stuart wrote

The two light areas are equal because the two semicircles are equal. So when you cover up part of the other in a way there both losing the same area

- We moved on to look at what the TI 92 could do to help us with first ideas on area. I drew a circle and a triangle with its corners on the circle on the machine connected to the OHP, and measured its area. Each child had a TI92 and reproduced this diagram, then tried to move one corner of their triangle to make the area as big as possible. We discussed results and I followed their instructions and advice as to where I ought to move one corner of my triangle to get maximum area. I was quickly corrected when I claimed that my answer was right and everyone should get the same, and the question:

All right then, what can we all say about our diagrams?

provoked some useful discussion.



Rebecca wrote about this screen:

`To make the circle press F3 key then we moved the pencil outwards and clicked on the place where the pencil was and it made a circle. To make the triangle we pressed F3 key and made sure that the point of the triangle was exactaly on the edge of the circle. To measure the area press on the F6 key and moved the point of the triangle round so that it made the greatest area. At the end of the lesson my triangle was an isosceles triangle`

Other accounts mentioned mirror lines and right-angles, and the mid point of the 'base'.

Catherine writes about her thoughts on the lesson and what happened later:

John's lesson promoted much valuable discussion the next day. By having the concrete example with the tissue paper, even the children who have SEN in mathematics could explain the concept of the two light areas being equal. I was very impressed with their explanations having previously thought that the thinking required to see area as distinct from shape would be difficult, and appreciating the general nature of the result, how it must apply to everyone's diagram, would also be challenging for the children.

As might be expected, during the lesson some children had 'cottoned on' more quickly than others. The explanations then offered by these children enabled others to build up their meaning-making. I think the group activity here is fostered by the way I have tried to use questions like "Can you explain how you did that?" in numeracy and in all our work.

Each group chose a screen which they thought showed what they had learnt and some days later, using a printout of this screen, prepared a report on what they had done.

The Russian psychologist L S Vygotsky [2] proposed that learning takes place through social interaction and within what he called the Zone of Proximal Development, that space between what the child can achieve unaided and what can be achieved with the intervention of 'adults or more capable peers'. Later Jean Lave and Etienne Wenger [3] saw learning as being situated in social practice, in what they called Legitimate Peripheral Participation. Learning, as opposed to teaching, takes place by newcomers taking their place in a community of practice. Drawing on this work by Lave and Wenger, Peter Winbourne and Anne Watson [4] have used the idea of 'local communities of (mathematical) practice'. They identify features of such a community:

- *pupils see themselves as functioning mathematically within the lesson;*
- *within the lesson there is public recognition of competence;*
- *learners see themselves as working together towards the achievement of a common understanding;*
- *there are shared ways of behaving, language, habits, values and tool-use;*
- *the shape of the lesson is dependent upon the active participation of the students;*
- *Learners and teachers see themselves as engaged in the same activity.*[5]

They examine classroom interaction in terms of such a community and go on to discuss the idea of 'telos', of the meaning-making of the whole class being aligned in directions generated by social interaction. They see telos as a unification of small scale 'becomings' by which many learners join a community of practice. By taking part in the abstract nature of the activities on area, the children in Catherine's class were becoming mathematicians.

The theme of learning being more important than, and perhaps independent of, teaching runs through Lave and Wenger's work. However we consider that this thinking, while subordinating teaching to learning, places more, not less importance on the role of the teacher. It does not mean that, in a more technological classroom teachers, and class teachers in particular, are redundant. Rather it would see the role of the teacher as that of fostering and stimulating local communities of practice, whether pupils are working in groups or as a class in discussion with their peers and the teacher. The teacher is seen as the guardian of the local community of practice, with the function of leading that community to meaning-making

This role of guardian has other aspects. Yackel and Cobb [5] have looked at mathematics classrooms in terms of social and socio-mathematical norms. Social norms of respect for others, not shouting out, being prepared to give (and listen to) an explanation exist in classrooms and are fostered by the teacher. In mathematics in particular, agreement on what constitutes a rigorous explanation and a valuing of a sufficient argument will be among the socio-mathematical norms pursued by the teacher. Catherine had worked to promote these norms in the community of learners we were working with. Yackel and Cobb argue that socio-mathematical norms can be used in the classroom to give direction and thrust to the development of argumentation, seen as a social, as well as a logical, process, in which that which is 'taken as shared' is moved forward.

When children are socially involved in mathematical meaning-making they need to see through the immediate resource, be it a chalkboard, or a computer or an example presented by the teacher, to the mathematics beyond. The idea of *transparency* is seen by Lave and Wenger as valuable when analysing the use of any resource in the classroom. In the work we did the activity with the tissue paper seemed to us to be more transparent than the work with the TI 92 (note Rebecca's preoccupation with button pressing in her account), but this was the first time these children had ever seen the machine! More transparent use of technology follows familiarity.

These ideas from the literature seem to us to place the teacher firmly, if not physically, at the front of the classroom. The teacher sets the social and socio-mathematical norms which prevail, and having chosen an appropriately transparent resource, will be working to stimulate and lead local communities of practice, whether group or whole class, where social learning is fostered, where participation, argumentation and discussion, overseen by the teacher, are used to give direction and thrust to whole class meaning making.

In a review in MT168 Anne Watson pointed out that we could probably foster good practice such as this by videoing our lessons, talking to colleagues or watching them teach. Perhaps teachers will recognise their own successful practice in the outlines of activities and approaches described here. Analysing success is more problematic, reproducing it even more so. Ideas from researchers such as those we have quoted here have helped us to see how we might change our approach or, hopefully, sometimes, why our own practice works.

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2. L S Vygotsky: *Thought and Language,* MIT Press, 1962

3. J Lave and E Wenger: *Situated Learning: Legitimate Peripheral Participation*, Cambridge, CUP, 1991

4. E Yackel and P Cobb: *Socio-mathmatical norms, argumentation and autonomy in Mathematics,* JRME 27, 458-477,1996

5. P Winbourne and A Watson: *Participating in Learning Mathematics Through Shared Local Practices,* PME 22, 4, 177-184, University of Stellenbosch, South Africa, 1998

Appendix V

Details of the TI 92 calculator

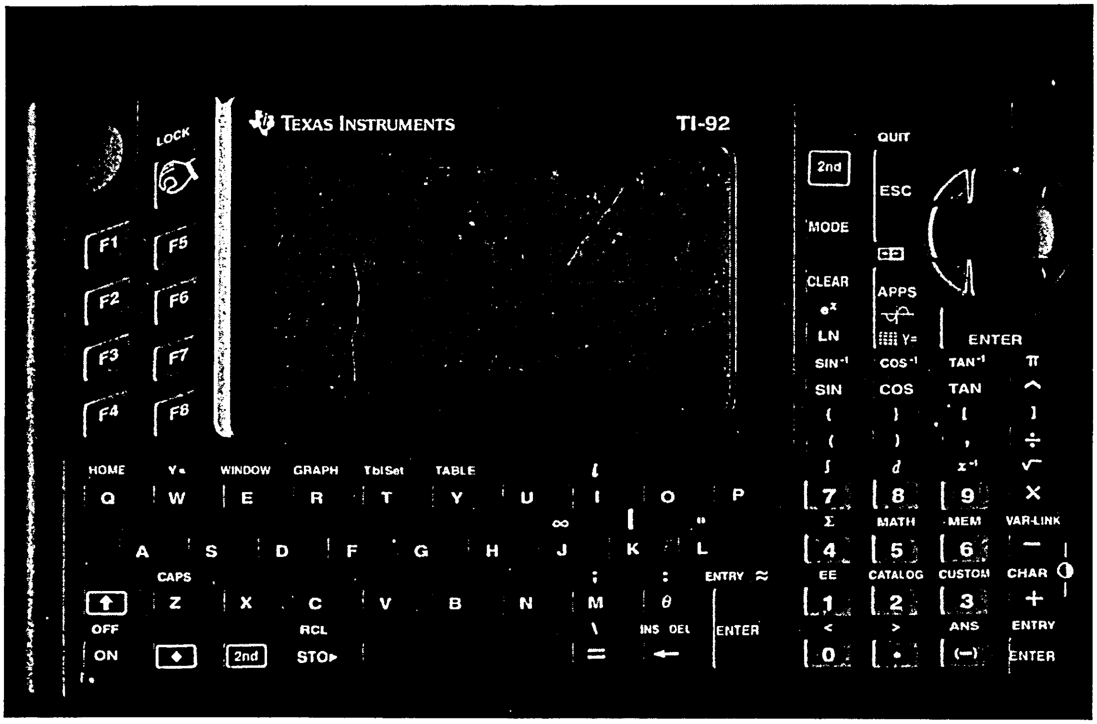


Figure V.1

The TI 92 is a small hand-held computer/calculator. It measures about 21cm x 12cm x 3cm and the screen is 9cm x 5cm. It weighs about 500gm and runs on four AA batteries. It has, in the present version, a 500K memory, although the machines used in this study were an earlier version. To anyone used to running Cabri or other geometry packages on PC the graphics (in monochrome) are coarse and, as soon as the screen is at all complicated, very slow to drag. However, as the students had not met a dynamic geometry package before, and as the diagrams were relatively uncomplicated for reasons of transparency (see the body of the thesis), these considerations did not limit the study.

The keyboard is more like that of a miniature computer than a calculator (figure V.1), but children seem to take to it readily. The dynamic geometry function is controlled by drop-down menus (figure V.2) via the function keys to left of the keyboard, and the cursor is controlled, for moving about the screen and for dragging, by the circular pad at top right.

There is a socket to allow connection of the TI92 to a PC for

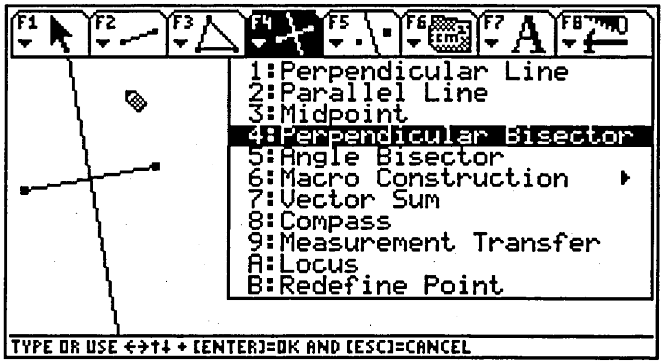


Fig V.2

programming and other functions, used here to download screens. This socket also allows transfer of information between two machines, so that a class set of machines can be pre-programmed. Another socket on one machine allowed connection to a tablet which sits on an overhead projector, allowing the class to see the screen of this machine (newer versions all have this feature, which would enable all children to show the class their individual work).

A large poster of the keyboard was used in class to indicate particular buttons when the machine was introduced, and the 'Useful Keys' sheet (appendix VI) was always available.
In general children of all abilities throughout the age range studied (11-14 years) found little difficulty with the machine and with assistance came to terms with it quickly.

Useful keys

To get to dynamic geometry	*APPS 8*
To start	*New* and type a variable name then *ENTER* twice
Dragging	Use the 'Hand' Key *(lock)* Keep it pressed and when the cursor changes to a hand you can grab and drag
Segment	Is on *F2*
To measure an angle	Use *F6* and *angle*. You tell the machine the angle you want by pressing *enter* on one arm, then the vertex, then the other arm, in that order.
Compass	Is on *F4*. 'Open' your compasses by using the two points you need; then go to the centre point and press enter to draw the circle.
Circle Triangle	)) On *F3*
Hide/Show Thick Label	)) On *F7*)
Perpendicular Perpendicular Bisector Midpoint Angle Bisector	)) On *F4*))
To scroll	2^{nd} and cursor
To correct	*F8 D* undoes the last object created *ESC*ape undoes the last keypress *F8 8* Clears the screen (You lose your work) Highlight an object and *Delete*
To transform	Rotate and dilate are on *F5*. To put a number (angle or dilation factor) on the screen use *Numerical Edit* on *F7 6* Define a vector on *F2*

Appendix VII

Sources of Dynamic Geometry Software (last visited 26^{th} August 2001)

Cabri
http://www.cabri.net/index-e.html

Geometer's Sketchpad
http://www.keypress.com/sketchpad/

Geometry Inventor
http://www.riverdeep.net/math/tangible_math/tm_activity_pages/geometry_inventor/catn.activityi_800174.jhtml#top

Dr Geo
http://ofset.sourceforge.net/drgenius/

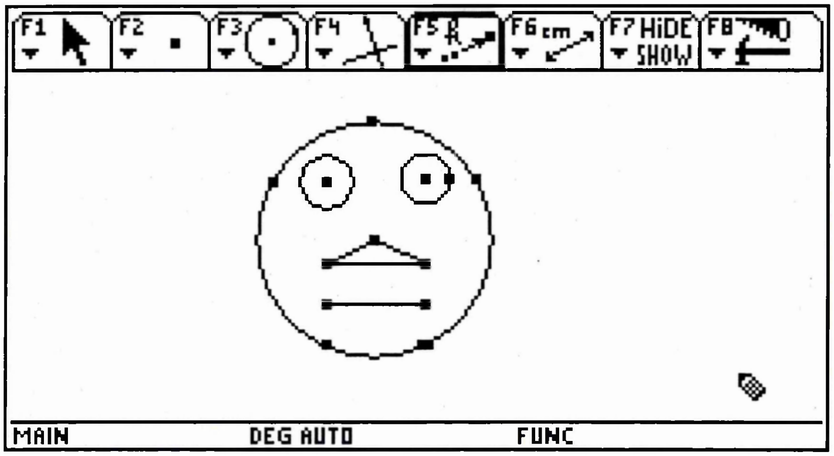
The first three sites provide teaching ideas and extension material, and details of commercial sources and free trials. Dr Geo is a part of Dr Genius and is available for free download.

Appendix VIII

Initial Exercise (see chapter 5)

Pudsey

This is Pudsey. He is in *demo,face2.*



Try to find things on the face that you can drag.

You can use a copy of the diagram to draw on.
You can write a list
You might use a diagram drawn in your book.

Write down anything you notice about what happens when you drag.

Now change your face (remember how you did it!)

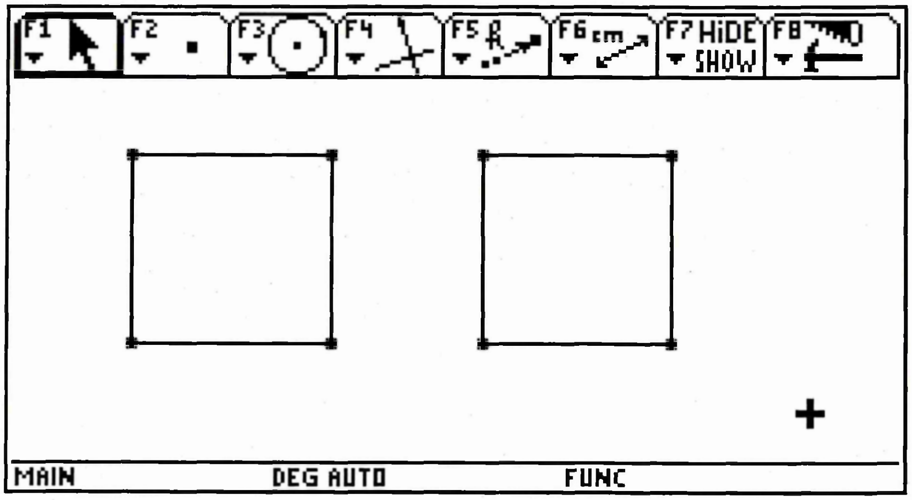
Swap machines with your partner.
Try to change your partner's screen back to a proper Pudsey face.
If you cannot do it, ask them how to, but do not let them do it for you!

Appendix IX

Further Exercise, see chapter 5

Two Squares

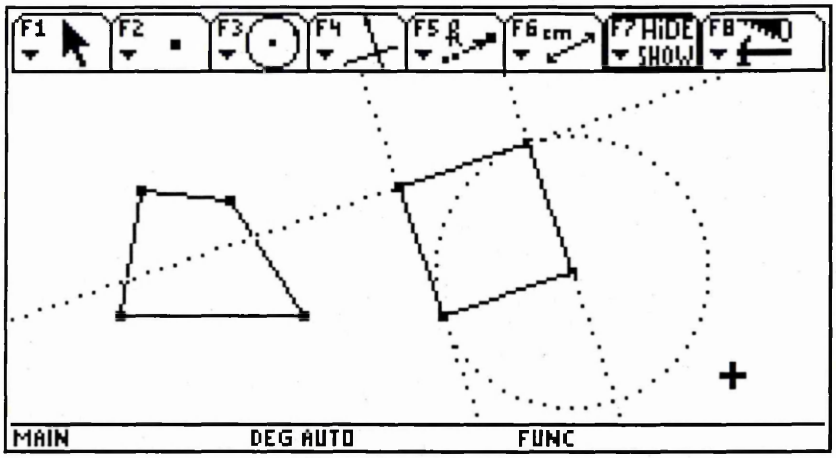
These two squares are in *demo, square*.



Are they drawn in the same way?
Try dragging points on the screen.

Show your partner what you have found, talk about it and record it in your books. You will need to draw diagrams to show what you have found.

The TI 92 can remember how the squares were drawn. Use *F7, Hide/Show.*



Talk to your partner about how the square is drawn.

Draw a sketch in your book of the way the square is drawn using right angles and part of a circle.

Draw a square on your screen, in a *new* file of your own. You might use

F2 Segment F4 Perpendicular line F3 Circle

Remember to show your partner and check theirs by dragging.

Use a setsquare and a pair of compasses to draw a square in your book.

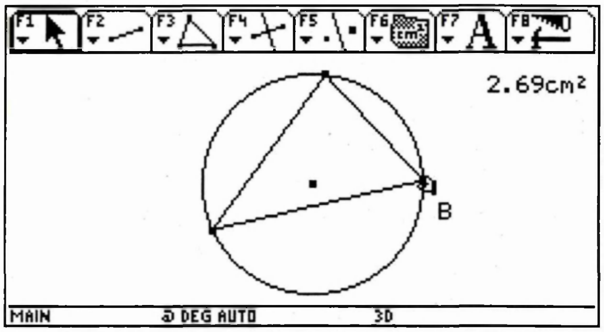
Find out how to use a rope with thirteen equally spaced knots to mark out the base for an Egyptian pyramid. Do it!

Appendix X

Worksheet discussed in chapter 10, page 197 onwards

Triangles in circles

Draw a triangle in a circle and measure its area



Move B round the circle and watch the area of the triangle change.

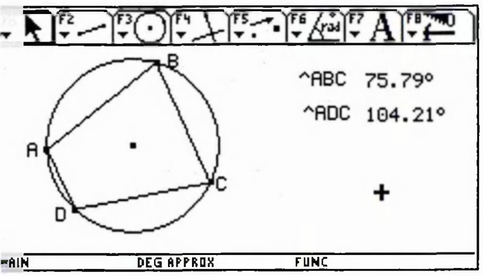
When is it the most? What can everyone say about their triangle when the area is most?

Write about this, and try to explain to each other why this is. Write down the explanation you agree on.

Appendix XI

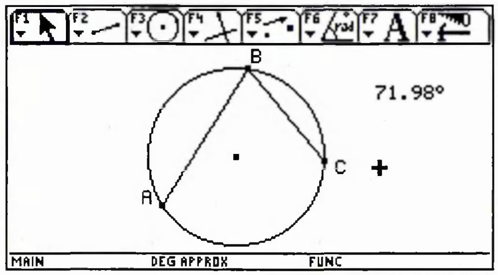
Worksheet discussed in chapter 10, page 202 onwards

Explaining



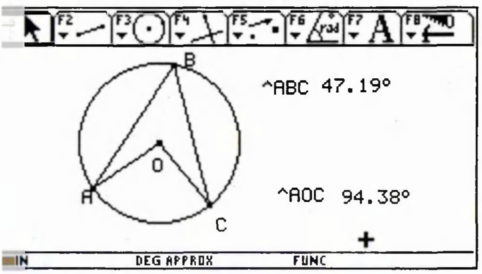
the corners of a quadrilateral on a circle, opposite angles d up to 180°

posite angles of cyclic quadrilaterals supplementary



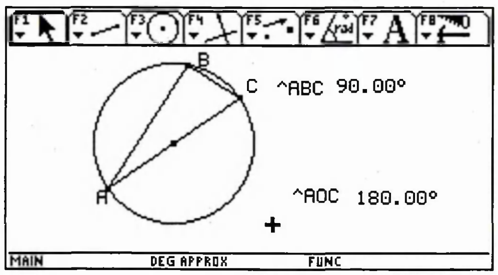
If B stays on one side of AC, angle ABC is always the same size

Angles in the same segment are equal



ngle AOC is always twice angle 3C

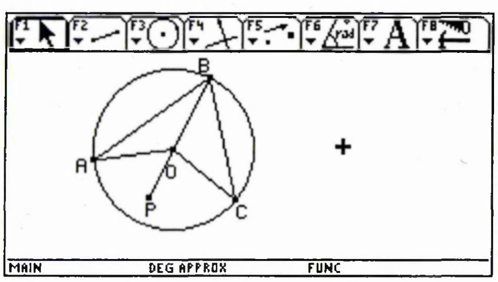
e angle subtended by an arc at the ıtre of a circle is twice the angle ɔtended at any point on the rest of the cumference.



If AC is a diameter, angle ABC is 90°

The angle in a semicircle is a right angle.

Do you think that all these statements are true? If you do, how do you know? You only need to know that **<u>one</u>** of them is true- the others can then be explained. Talk about which diagram is which and explain to each other the steps in the argument.



Now try to use isosceles triangles to explain why angle AOC is twice ıngle ABC

Appendix XII

The researcher has been a member of writing teams involved in the production of the following works, all for SMP and published by Cambridge University Press.

Using Software (1990)

Using BASIC (1990)

G9 Book (1994)

Revised Space booklets to NC criteria (1995)

SMP Interact (2000 and on-going)

Appendix XIII

Collected data gathered in the course of the project.

The data recorded here is referenced in the body of the text as

Pupil Diaries
D1, 2, 3 etc

Field Notes, including Scribble Pads
F1, 2, 3 etc

Transcripts of Audio Recordings
T1, 2, 3 etc

Pupil Diaries

(as scanned)

Scanned extracts from pupil diaries are reproduced here. Where they have been referred to in the text page references are given. Other extracts are given for interest only.

From School A

16/6/97

I preffer doing maths on the TI-92 because its alot more intresting than writing on paper. Personally I like compt and find them very intresting I preffer the computers because it saves the bother with compass rulers and protracts and all the working out. It saves alot of time. But one of the problems is that it takes quite a lot of time to learn how to use it properly. So it sometimes holds you back but once you learn how to use it it speeds things up considerably.

Maths is --one of my favorite subjects Its about--- Addition, subtraction, divides and Multiplication- I think -- that maths is better on the TI92 because -- they are really good. what we did --- we were constucting Angles on the TI92 and trying to do them with straight sides and then measuring the sides and the angles.

Pupil A D1, p 107 D4, p 107

Pupil B D2 p107

16th June 1997

It was today in Maths when Mr Gardener brought the Mini [illegible] computers in school [illegible] it was our second time that Mr Gardeners had brought them before and I like maths [illegible] Mr Gardener especially the Mini [illegible]. On that day we did some angle work and we were trying to find [illegible] on the computer

I've Enjoyed working on the Ti92 because I Enjoy drawing on the and gessing angles:

what i dont like is when a press the WRONG botton by accident it makes my work go funny then I have to start again.

Pupil C D3, p 107

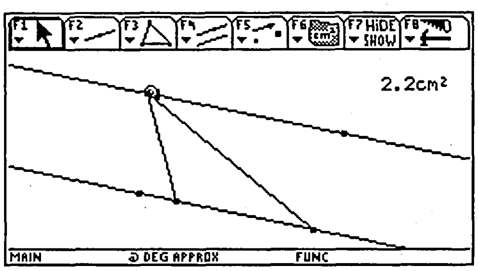
Pupil D D5, p 108

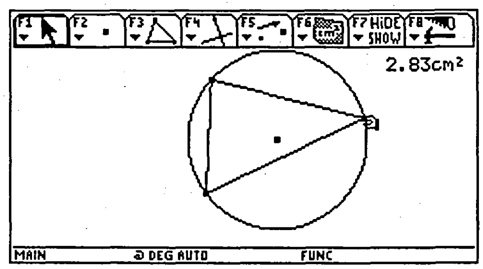
Initial Reactions from School B,

1/7/98, 2/7/98

Pupils' reactions from a lower attaining Y9 group (ages 13-14)

The first exercise involved measuring the area of a triangle drawn with vertices on a pair of parallels, and seeing how the area changed as the vertices were moved. The second involved investigating the change in area of triangle drawn in a circle as one of the vertices was moved round the circle.





Wednesday

I think that the computer is useful, but I would like to know how to operate it a bit better. It was more useful than a blackboard diagram because you were making it happen and you had to understand to do this. If a teacher drew it then you could just go along and not learn anything.

Pupil E D6 p 110

Wednesday	Thursday
I think it is really good because It looks simpler when you do things your self. It is much easier to understand how things work. Also it makes testing out theorys a lot easier than doing it on paper. I feel I learntd quite alot today	~~Plls~~ Today I haven't learned as much as I did yesterday but it was still very interesting & fun. I much prefer using theese computers than the PC's

Pupil F D 7 p 110

Wed

The machine is usefull and I think it is good to be able to [illegible] on your own diagram. The machine is quite easy to use once you have got used to it. It is better [illegible] [illegible] to have your own.

Pupil G D 8, p 111 D11, p 112 D12, p 112

I found this easy by seing whats ~~a~~ on the screne but without instructions I would have got lost.
Ithink its very good. its usefull to have a moving image rather than drawing it.

WED	Thures
we drew parallel lines whith would have been hard to get parralel in our books the area of the triangle would have taken more time without this machine.	

Pupil H D9, p 111

wed

The machine is very useful, and you can try out lots of different Ideas.
It's a bit difficult to get used to and I think it was a good Job we had someone talking us through each procedure. It is easy to clear your screan + start again if you do make a mistake.

Pupil I D10, p 112

Wednesday	Thursday.
On wednesday we used the computers for the first time. I found that it was easy if you listenend out if you did become stuck there was always help. They are good because you can stay where you are, in the classroom. I had a bit of trouble on Wednesday.	Today I was better at working with it because I understood it better. I like them alot and would use them if offered.

Pupil J, p 112

WEDNESDAY

The TI-92 is a very good and useful machine, powerful and personal because you can have it in front of you and no body can look at it

Pupil K, D 14, p 112

Further Extracts from Pupil Diaries

wednesday's lesson

In wednesdays lesson we first used the computer. My first look at these looked as though the computer was amazing. We were told by Mr Gardner ~~the~~ how to draw a line. we then draw another line paralell to that. we then drew a triangle in the lines. we then made the triangle bigger to find the area. we found the area by presing F6.

Thursday lesson

we draw a circle with a triangle in it, found the area of the tringle. and moved it around to find the largest area.

2.7.98 My impressions

TI-92

Wednesday's lesson	Thursday's lesson
On wednesday my first impressions were that they were difficult to use and I did not understand them at all but that may have just been because I hadn't used them before. I think they are a good idea as you can move them from place to place without worring. I found them easier to use when they were plugged in to the over head Projector	Today I found them alot easier to use as I understood more as I had used them before. I think they are brilliant machines. I thought it was good fun playing with those machines and I thought it was good doing the circle and the triangle.
	General comments.
	I thought it was really good fun learning about the TI-92
	thank...

Wednesday

I think using these machines is a good way to explain and under Stand things be cause you can experament. It also helps you under Stand be cause when the teacher (mr gardner) explains and does things you can See this and do this for your Self So it helps you under Stand and learn new things

Wed

I Thought using the Ti-92's Was fun and was a lot easier than doing thing normally like finding the area of a triangle and was interesting to find how things are connected with each other but sometimes you get lost and don't know what to do

Wednesday

I found quite easy to draw the lines and the triangle as the computor did it so fast.
I thought it was good at how fast the computor found the area so fast and could change it very easily. I understood what to do as the computor was clearly laibed.

School A

Joe's work

The episode to which these extracts relate is discussed on page 126

26.9.7 Diary 16th June

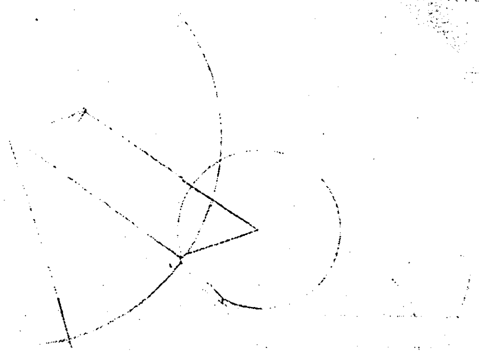
I thought the lesson
was interesting using
the calculators
and I didn't realise
the calculators were
so accurate in measuring
angles. When Mr Gardener
was ~~spe~~ speaking to
us the temptation
is to play with the
calculators.
I prefer to use the
calculators rather
than ~~the~~ paper
because the calculator
is more accurate. If

you make a mistake
you can just clear it
rather than making a
mess on paper and it's
quicker to use a
calculator than a piece
of paper, pen and, rubber.
I find maths quite
straight-forward
as it is quite repetitive
as all the things we
do are similar apart
from using the ~~calc~~
calculators. I enjoy
using the calculators
because it is something
we don't do very often and
it is interesting.

Diary 20th June

I thought the
lesson was quite
difficult as
Mr Gardener didn't
explain what we
had to do ~~the~~ to
try and draw a
Parralelagram.
I found it easier
to draw the ~~parralellog~~
parralellagram ~~on~~ using
a compass on paper
rather than the ~~TI~~
TI-92.
Most of the time I
didn't understand
what Mr Gardener
was talking about.
The compass function
on the TI-92 is
~~a~~ difficult to use
properly.

Joe, D15 (left) and D17 (right) p126



st we drew to ~~4~~ equal lines as eqaul as possible).

n we measured the length of the ~~[illegible]~~ top line with the compass n we draw a circle with the mpass from the top point of the ~~[illegible]~~ 2nd line.

between the top points of the two lines and drew the circle ~~[illegible]~~ on the bottom point of the 1st line.

4. ~~Then we joined up a line betw~~

Then

If you ~~move the~~ press lock on the ends of the ray (second line) the line moves around the centre point like clock hand

If you press lock on the ends of the segment (first line) the ~~line~~ segment and the ray move round the other end of the segment (first line.

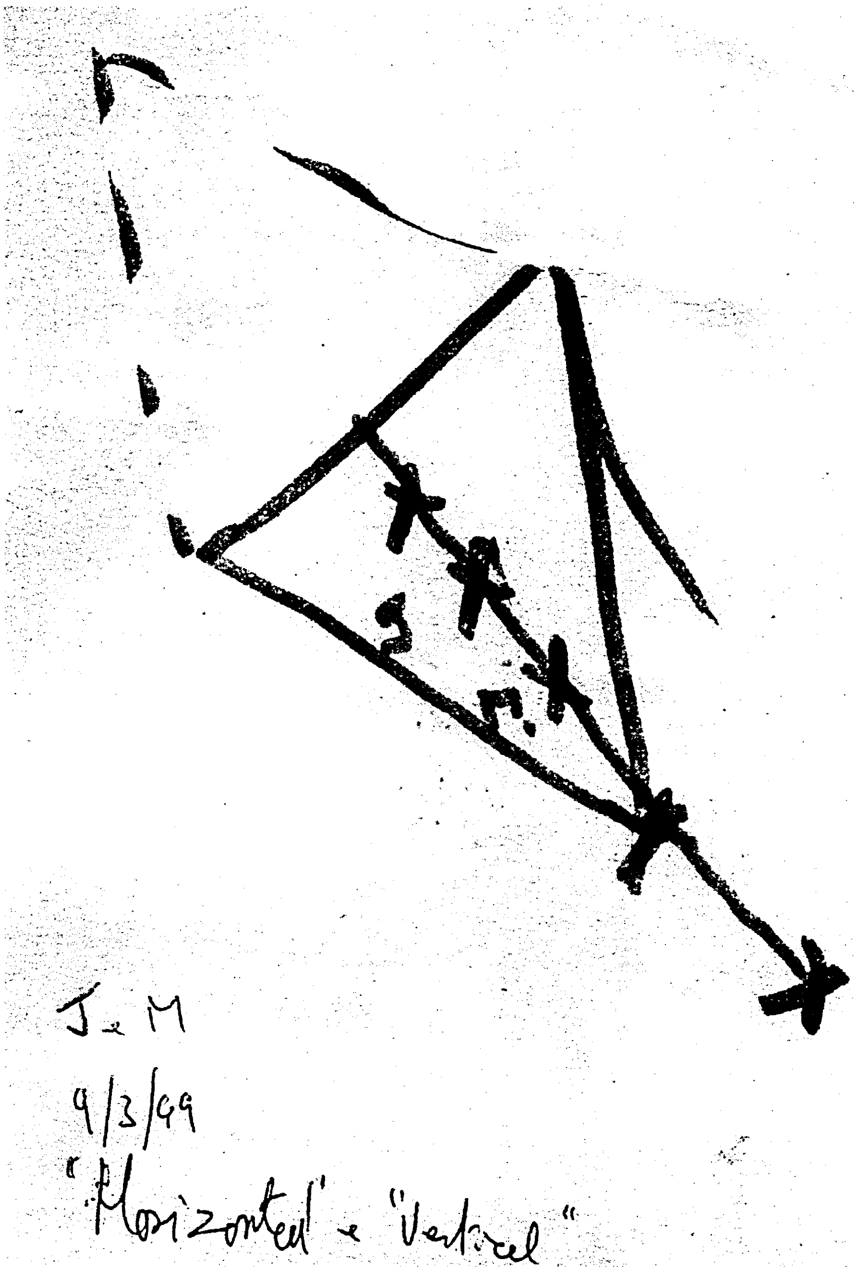
The 4 points always make a parrelelogram when joined together.

Joe's construction D16 p 126

Field Notes and Scribble Sheets

referred to in the text are reproduced here

Field notes and Scribble Sheets



F1
Reflecting a triangle in one side, and discussing the condition for getting a rhombus. See Preface.

Field notes on early pilot, School A, p 107

Wed. 4/12

2nd Pilot Lesson. Went well (Stacey).

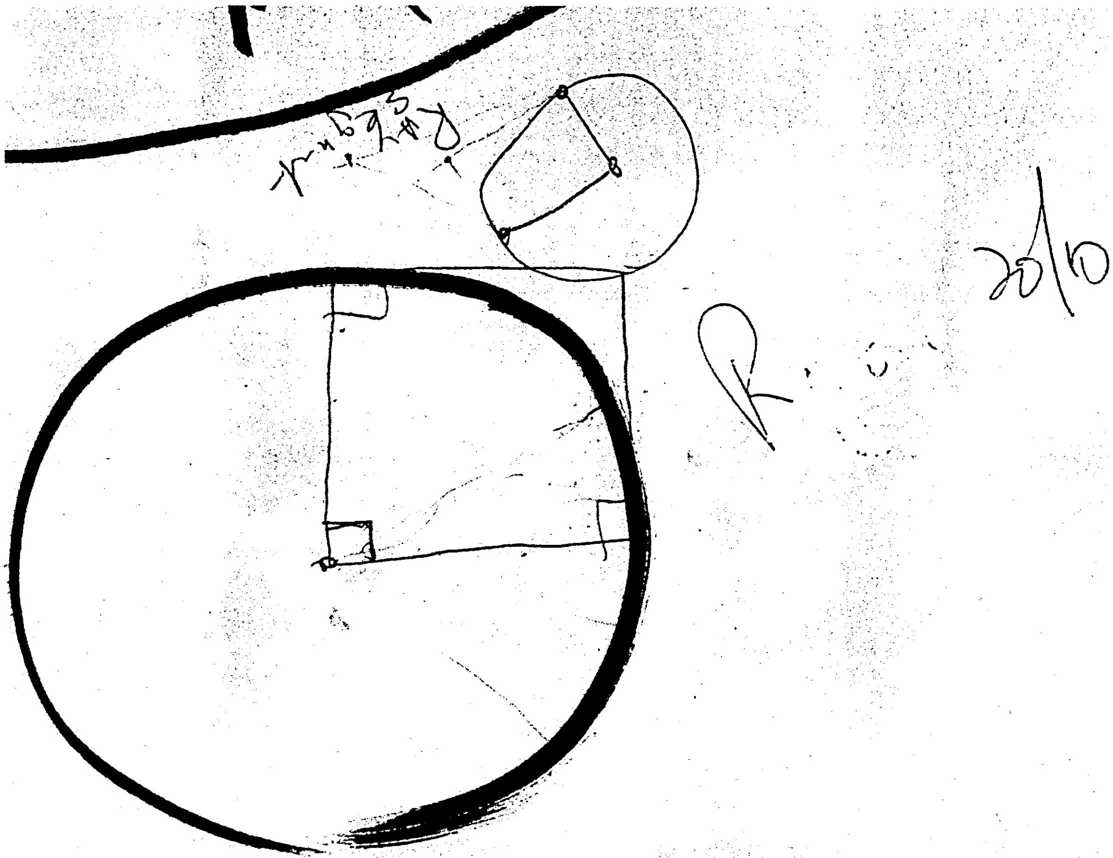
Drawing diagrams good video (Stacey)

Next time constructions from

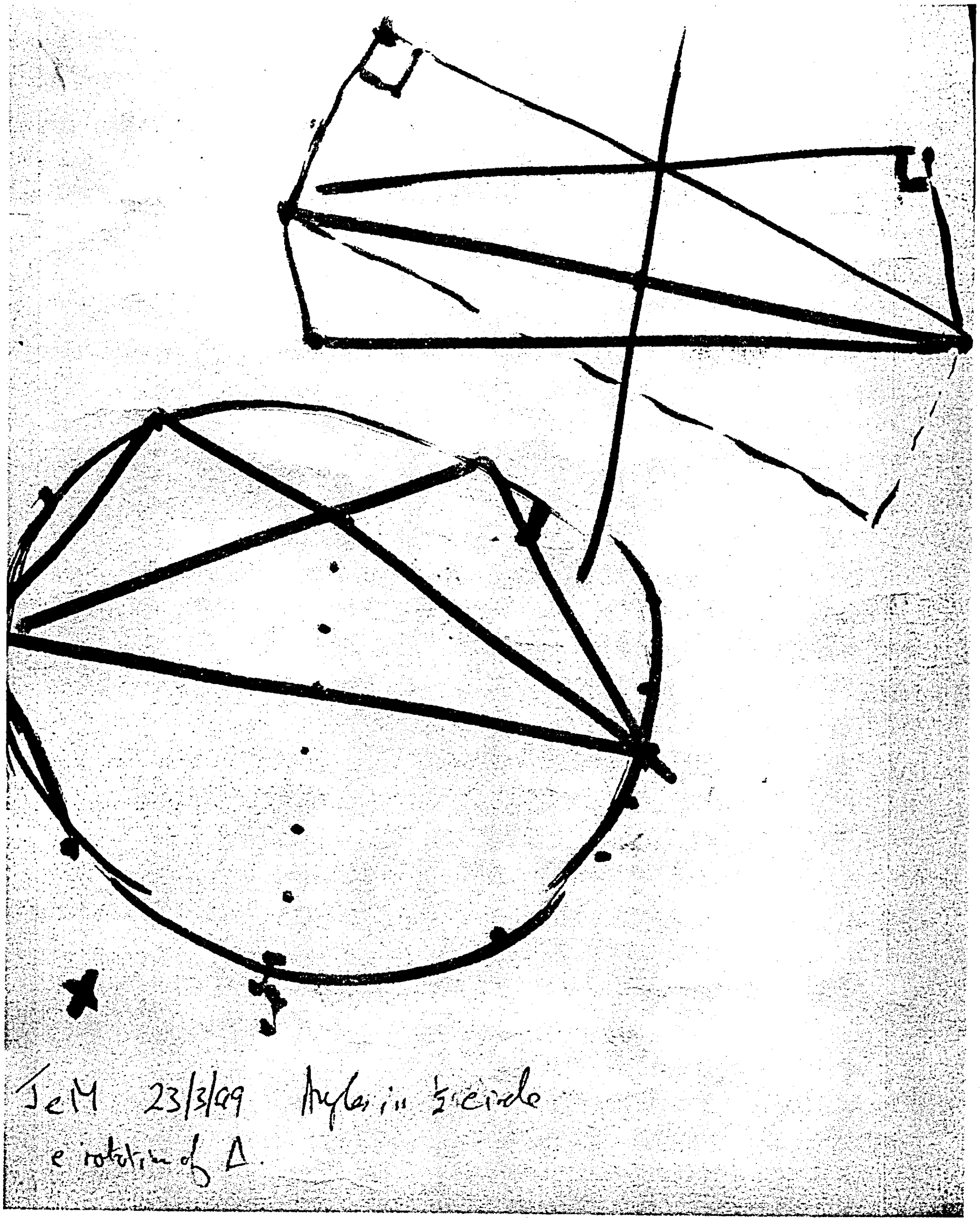
Number line equi Δ, ∠ sum of Δ.

Some got on to necessary spontaneously.

Good attempts at construct an equilateral Δ.



F3
Rob on drawing a square (pupil's name obscured)
Refer to page 144



F4

Rotating Triangles about the mid point of one side. What is the condition for a rectangle?

Annotated File of Transcription Records

This appendix lists passages of transcribed audio-recorded dialogue
Details of the schools involved are noted and reference is made to page numbers in the text where these passages are discussed.

T1 See page 109 School B

Transcription of Pupil/ Pupil instructions

Pupil A had used a brief instruction sheet and taken the TI 92 home to experiment

A Press Apps (Applications) and we're going to go on to Geometry so you press 8

B 8?

A Yeah, and you've already got a file, so you go down to 2 which is open Press enter, enter, right?
And then go down from Main. Oh you've only got one file, I've got two
Right I'm on my file. Now, what do you want to draw?

B Errm, can you only draw shapes or can you draw anything?

A You can only draw shapes.

B Right a circle.

A Circle. So you go onto F3 and it says 'Circle, Arc, Triangle, Polygon or Regular polygon.So you press 1 cos that's what number the circle is.

B Yeah I've got it here

A And then you can draw a circle. So you press enter where you want the middle of your circle to be. You use your arrow keys to come out and draw your circle. It comes out as you bring your arrow key out.

B And then do you press enter when you've drawn your circle?

A Yes press enter.

B Yes I've done

A Right, now what do you want to draw?

B Er. A rectangle

A Right go onto F3 and we'll do polygon cos that's just like a shape with all joined together

B 4

A Yeah 4 enter
And you press enter, and you can go across and you get a line and once you're happy with the line what you've drawn, you press enter again so that means you've got one line and then you can say go across or down and you can draw shapes with these different points what you've got.
Do you get that?

B Mmmm yeah

A Tell me if you don't get it

B I do get it

A And then once you've finished your shape you do enter, enter and it draws your shape in bold. Right?

B Mmmm My rectangle's going to fall apart

A That's alright, look at mine it's not even a rectangle, that's just a shape. Right you can go on F4 and that's all sorts of different lines and points, you can have compass, you can measure, what line you've done, measure, you know the length of your line. On F5 that's translation, rotation, that means you can flip it over, you can make it the same on both sides, and stuff like that.

F6 is mostly for measuring, you can measure the distance and length, you can measure the area.

So do you want to measure the area of your square what you did? So you press F2, 2

B 2, then enter?

A No you just press 2 and it should do it

B Yeah, it's done it.

A And you go onto the shape what you done in the middle like, no,not in the middle, on the line, and it should say this polygon, or this circle or whatever you want to do.

Yeah this circle and when it's got that you press enter and it tells you 4.25 centimetres squared, that's what mine is.

B 8.787 centimetres squared

A Good ennit?

T2 Page 136 recorded comments on properties of squares (Darren)

We were probably about five, six maybe seven, no younger than that, we were probably four, when we learnt that it has four sides and corners and that they are all the same but we were probably about six when we learned the word right angle.

T3 Page 137 School B 30/9/97

The group were given some cut-out cards, which all looked square, but only one of which was a square within the limits of accuracy used to make them. The pupils were asked to identify the square.	
D	Sir is this definitely a rectangle, the big piece of paper?
JG	That's a good question! (retreats)
D	Well because these are rectangular that means the corners are all right angles so we can see these corners…..
T	How do you know that one's a rectangle
D	Cos I've asked him. So we can use the corners
T	Do you mean this one?
D	No this is So we can see… we can rule out some of them by working out that they've not got all right angles. This one isn't. Put the ones that definitely aren't there That's got a right angle This is definitely a right angle.
T	That one is Ah that one's not
D	That one's definitely all right angles
T	Yeah that one's all right angles
D	But we don't know if it's a square though, right
T	Lets have some more
D	I've tested them, T, they're not totally right angles. So now we've got these that are definitely rectangles of some sort but we need something to measure the sides against. We're not allowed to use a pen or anything are we?
R	A ruler
D	I know. This er tape box So that's the length that way, then these should match up they don't so this is a rectangle but not a square Right next

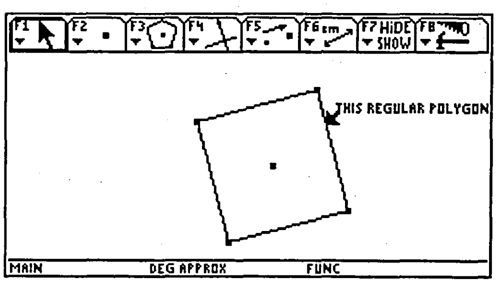
T	It's fairly obvious, it's a rectangle but not a square
D	Well a square is a type of rectangle
T	How? Well yeah, yeah I knew that
D	Right, there's the length, there's the width That is definitely a square. And finally this one, hang on, there's the width, no this one isn't So the only square one is this one.

Drawing a square by regular polygon T4 p 138 and T5 p 140 School B 6/10/97
Also discussed on p 169 and p 171

Square by regular polygon

Rob, Janine, Darren

Draw a square which stays a square when you drag it



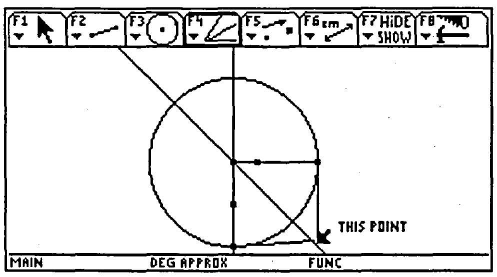
D Does anyone know how to draw a square?
R Polygon, regular polygon
D Regular polygon is ....
R number 5 on F3
J right now just draw it.....no it doesn't work, go on to just polygon.
R Yes it's working
J I'm doing it on normal polygon it's a lot easier and you can always measure your lines
D It's hard to get it a proper square
J But afterwards you can measure your lines
R Yeah you can can't you
D I know! You could do it two triangles, two right angled triangles next to each other and merge them, then it'ld be a proper square.
....

R I think I've got a perfect square here.
J See, I've just figured out mine's not right, cos one of my lines is 1.91 cm and the other is 2.03 cm
...
J There's also area, you can do the angle and see if the angle's a right angle, as well.
R Well you can tell if it's a right angle.
J Yeah but you can't for definite
D I think it is regular polygon.

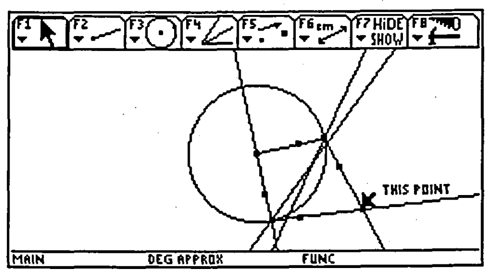
Went on to use regular polygon successfully

Page 145 T6 Discussion of angle bisectors

Transcription
School B 13/10/97 Constructing a square



Darren



Rob

D I'm trying to do an angular bisector ....cos if the angular bisectors make a right angle in the middle then that'll mean it's a square, but I can't get it to do them.

JG How do you know that the angle bisectors will meet in the middle in a right angle?
D Well I don't know that they will in a right angle
R They will
D If it's a proper square then it'll be in a right angle because you'd be chopping the square like diagonally
R There'd be four triangles
D There'd be like four triangles and they'd all be right angled triangles
R There'd be two 45° angles
Shown how to draw angle bisector

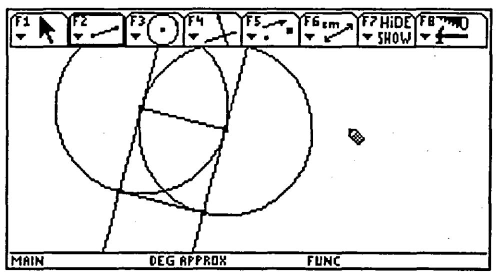
D Yes!! Now that looks like its going at a 45° angle right through.
That meets in the other corner there, so I think that means it's a square.

R I've made an arrow head I know that

P146 T7 School B
Also on p 174

Further transcription from 18/10

JG So you've got one line like that, What would help you to draw a square?
J It would have to be parallel
JG You want to draw a line parallel to this?
J Yeah
JG and where does it have to be?
J It has to be the same length as that down there
JG Like that? So how would you draw it?
What shape would help you to draw that down to there?
J A triangle
JG Look on F3



We eventually decide that a circle can be used and J continues

T8 p 147 School B

Transcription 25/10

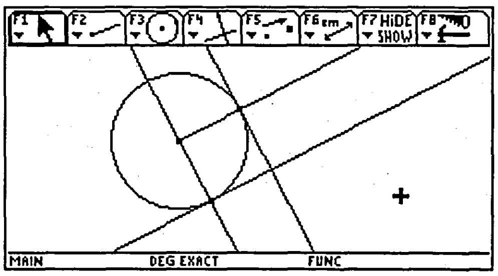
Rob JG

R This is what I'm trying to do right
I've drawn a circle, drawn the centre point, drawn a ray going up there , a ray going there
JG right, so this is a ray is it?
R Yeah and so is that there, but I've just realised I should draw a ray going up there and draw a ray parallel and I've drawn a perpendicular line going up there
So I managed that but it's just one out
JG What do you think will happen if you drag this ray?
R I dunno. If I try draggiing this ray, because the ray's not secure at the point, that ray'd drag around wouldn't it

But if that was a perpendicular to that ray,……
JG So this circle is a good starting point isn't it
If you have that circle and that ray, how many sizes of square can tou draw?
R just one
JG as soon as you've drawn that and that
R Once you've drawn the cicle then you've got the size

T9 p 148 School B

JG What flashes
R That corner there. Does that mean that's the only corner that can be dragged?
JG That's the only *point* that can be dragged. Tell me what you drew first.
R I drew the circle first



T10 p152 School D 29th April

Boys A,B,C,D

	We're doing this one	
D	Go to the centre, we want a line	
C	What did we use? Lets try segment	
D	F2 Enter	
C	Then go to the circle	
D	Watch this see if this does it	

C	Ah you've done it You need to draw a segment from the centre	
D had tried to draw the circle moving down the slope	I've done it sir. Whoo hoo What! It gets bigger .	(circumference point defined as on line, but line not defined as a tangent)
C	Why does it do that?	

T11 See p 154 School D

JG	You've told the circle to go through that point haven't you? How do you make that circle roll? You don't want it to go through that point do you? What do you want to happen?	
C	I want it to roll	
JG	So it just has to ?	
JG	So you don't want it to go past the line, you want it just to touch the line Can you tell me what would help? Let's just put a point for the centre of the circle. How big do you want the circle to be? Now where will it touch?	
JG	Centre there, and it's just got to get to……,	
C	That point	Points to point of contact
JG	Now can you tell me exactly where it should be Is there a word on the board to help?	
C	Pen…….	Stumbles to read it
JG	That's right, perpendicular You've got to use the perpendicular	

T12 Page 184 School D

Speaker		Comment
JG to class	How did you draw a kite?	
General	We drew a triangle and then we reflected it	
JG	I asked you to move this point about What shape do you always get?	
General	A kite	
JG	I asked you how you could get a rhombus Look at the screen and tell me when it is a rhombus A rhombus has four sides equal, not just the two pairs	
General	Stop	
JG	How should I move it so it is always a rhombus?	
A	Turn it around or bring it in to the middle- not right in the middle though	
T	What line will it be?	
A	A straight line	
B	Symmetry	
A	Down that line, that line like that	
T	What's that angle called	
A	What's it called' what's it called a right angle	
T	So what's another name for that	
T	Yes ninety degrees	
T	There's another word for that	

A	Put a line down the middle	
B	It's a square box angle	

T13 p 188 School D

Given intro to hidden lines

A	I want to know how to do that one, me	
TT	And how are you going to do that?	
A	I know where the line is what he's hiding	
A	It's on that big circle there	
TT	Go on then	
A	Ninety degrees	
B	No he's hiding the line, the line what goes across from its end	
TT	The line moves around the circle	
B	So it's at a 45 degree, no, 90 degree angle to the line, the line in the circle	
TT	Right, go for it, try to draw it	
	OK escape, clear all your pictures	
	Clear all your pictures	
	What do you need first	
A	Circle sir, I've done it	

T14 P204 School C

1	JG	What area have you got?	
2	Class	General response	*There was no restriction on the original diagram, a wide range of areas was possible.*
3	JG	Why do we all get different answers?	
4	A	Because we all used different circles	*Use of 'we' suggests the possibility of a local community of practice.*
5	B	And different points	
6	JG	Look at mine while I move the point. Tell me when it will be greatest. What can we all say about our diagrams?	*'What can we all say?' used to promote the local community of practice and the generation of a common direction to learning, telos.*
7	B	It's across from the centre	*Later discussion showed that Belle could demonstrate to the class the idea of the co-linearity of the mid point of one side of the triangle, the centre of the circle and the other vertex. Charlotte pointed out that the triangle was isosceles*
8	JG	Yes, good. Anyone else?	
9	C	It's in the middle	

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Transcriptions and notes from School C, July 1998

JG	Does anyone want to tell me what they have found?	
Pupil 1	As you move this down it stays the same angle until you reach this point, then it changes to a completely different angle and stays the same.	
Pupil 2	Oh Yeah	*Wonderingly*
JG	Will you come and show us	
Pupil 1	It might not work you know… it might just be because of the shape of this one	
Pupil 3	It will work.. I got it to work	
JG	Watch while she drags this. Watch the angle. Moving up …..angle getting bigger	
Pupil 2	If you change the middle one, watch the middle one, it stays the same and after a certain point it changes	
JG	What's going to happen now?	
Chorus	Stays the same	
Pupil 3	Until you pass the point, then it will stay the same again	
JG	Look at the angle, it stays at 52.77 degrees, doesn't matter that the line goes through the centre, stays at 52.77. Now changes to………..127.23 Can you make it flip between those two angles? What can you tell me about those two angles?	
Pupil 4	Does it add up to 180?	
Pupil 5	Ooo	
JG	Check that those results are true for your diagram	
JG	B.., what do we press now?	
Pupil 6	F2	
JG	F2 for?	
Pupil 6	Line err segment	
JG	Yes you need to use segment, it's line segment, but you have to be a bit careful with the word segment when circles are involved	Tails off.. The Explanation Trap
JG *Shows how to draw and measure angle at the centre*	What can everybody in the room tell me about this?	

Much evidence of snatches of conversation of pupils helping each other