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Topical Review

Mechanical metamaterials for sports helmets: structural mechanics, design optimisation, and performance

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Abstract
Sports concussions are a public health concern. Improving helmet performance to reduce concussion risk is a key part of the research and development community response. Direct and oblique head impacts with compliant surfaces that cause long-duration moderate or high linear and rotational accelerations are associated with a high rate of clinical diagnoses of concussion. As engineered structures with unusual combinations of properties, mechanical metamaterials are being applied to sports helmets, with the goal of improving impact performance and reducing brain injury risk. Replacing established helmet material (i.e. foam) selection with a metamaterial design approach (structuring material to obtain desired properties) allows the development of near-optimal properties. Objective functions based on an up-to-date understanding of concussion, and helmet testing that is representative of actual sporting collisions and falls, could be applied to topology optimisation regimes, when designing mechanical metamaterials for helmets. Such regimes balance computational efficiency with predictive accuracy, both of which could be improved under high strains and strain rates to allow helmet modifications as knowledge of concussion develops. Researchers could also share mechanical metamaterial data, topologies, and computational models in open, homogenised repositories, to improve the efficiency of their development.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Sporting concussions are prevalent and recognised as a public health concern [1–5]. Mechanical metamaterials are engineered structures with combinations of mechanical properties that are not possible in the individual materials they are made from [6–15]. They are suggested as options to improve helmet impact performance (e.g. [16–33]). In helmets, mechanical metamaterials can be tailored to reduce linear and rotational acceleration, thought to be associated with the clinical diagnosis of concussion [16–33].

Helmets are an established mechanical intervention for reducing head injury risk. They are considered effective at preventing severe head injury (e.g. skull fracture), but less so for concussion [34–42]. While there are many mechanical metamaterials reviews [14, 43–49], including those on protective equipment [50, 51], as well as some on general helmet materials (e.g. [52]), there is not a published review of mechanical metamaterials for sports helmets. Here, we formulate the current challenges relating to helmet development, and summarise the breadth of relevant mechanical metamaterials research. Finally, we include perspectives on opportunities and requirements for helmet development, focusing on reducing concussion risk.

Concussions can cause short-term functional impairments and long-term health problems [53, 54]. They are typically considered a mild traumatic brain injury (TBI) [55–57], and are part of the larger family of TBI. Concussion injuries can be caused either by a direct blow to the head, or an impact to the body that causes head acceleration (e.g. via. whiplash) [2, 58]. Symptoms of concussion, such as those related to physical, cognitive, and emotional health, usually resolve within two weeks [2, 3, 53, 59–61], but can last longer [62, 63]. Concussions do not typically cause detectable structural damage to the brain [2, 62, 64], so they are challenging to diagnose and manage.

A history of concussions [3, 54, 62, 65] or repetitive subconcussive head impacts [53, 62, 66] are associated with microstructural changes in the brain, and short or long-term functional, physiological, and neurological changes. Reported consequences include reduced quality of life [67], and increased risk of psychiatric disorders [4, 53, 60, 66, 68, 69], neurodegenerative disorders [53, 60, 66, 70], and suicide [71].

Rugby, American football, and ice hockey have the highest reported concussion rates in mainstream sports [41, 72]. Concussions are also of concern in other sports, including association football [73–76], lacrosse [72, 77], snow-sports [34, 78–82], cycling [27, 83–85], water-sports [86], and rock climbing [87]. Strategies to reduce concussion risk, such as rule changes and helmet developments, have been introduced to various sports, with limited success [35, 37, 38, 40, 42, 83, 88, 89]. There has also been notable investment by governmental agencies [90, 91], and charitable organisations [92, 93], in concussion research and related technology development over the past two decades.

Team sports are often played in environments that can be controlled and regulated [41, 72], unlike outdoor sports such as cycling, snow-sports, water-sports, and climbing [27, 34, 78–87]. In many mainstream sports, strategies such as promoting helmet use have had limited effects on concussion rates [34, 37, 39–42, 72, 79, 82, 94]. Factors affecting reported concussion rates are multifaceted, so identifying the effect of interventions is challenging.

Risk factors for concussion include impact surface shape and stiffness, and impact speed, energy, direction, and location [35, 37, 38, 40, 42, 83, 88, 89, 95]. Ice hockey presents an interesting case for helmet development, as it includes various diverse impact types (e.g. high-speed puck, rigid ice and boards, and collisions between players and their equipment—which are considered compliant) [96, 97]. The introduction and regulation of ice hockey helmets have helped to nearly eliminate serious head injuries, particularly skull fractures [37, 38]. Despite these developments, and as with other mainstream sports [37, 39–42, 72], concussion rates in ice hockey have been steadily increasing [37, 39–42, 72]. Most ice hockey concussions (93%) are caused by collisions between players (i.e. compliant surfaces) [37, 63, 88, 96–99], while the remaining 7% are from falls onto ice [97]. About two-thirds of players indicate they would continue to play even if they thought they had sustained a concussion [100]. This attitude to concussion likely results in underreporting [37, 39, 41, 101].

There is an ongoing debate over different possible concussion mechanisms [89, 102–105]. It is generally agreed that the clinical condition resulting from an injury associated with diagnosed concussion is caused by excessive, or overly rapid, tissue deformation [102]. Such tissue deformation can be caused by skull deformation [89, 106, 107], movement of the brain within it [102, 104, 106, 108], and by pressure gradients [106]. Most closed head injuries (non-fracture) follow head accelerations that damage brain tissue [104, 109–112]. During linear (radial) impacts, injury can be caused by the brain being forced against the faster-moving skull [89, 113, 114]. During head rotation, loose coupling can damage connective blood vessels and neurons [102, 104, 106, 114]. Linear and rotational head accelerations are likely to be present during head impacts [103, 110, 115, 116], and helmets should aim to limit both.

2. Measures of concussion risk

Helmets are typically designed to decrease the various measures thought to contribute to head injury risk. Peak linear acceleration (PLA) is thought to contribute to severe injuries such as skull fractures, and concussions [109, 115, 117–121]. The Wayne state tolerance curve, derived from animal and cadaver tests, combined linear acceleration and duration when assessing injury risk [122, 123]. Further threshold curves (e.g. Gadd severity index [124] and head injury criterion [125]) integrate acceleration over a portion of the impact duration, with a weighting factor for high accelerations [95, 109, 125–142].

Peak rotational acceleration (and velocity) are commonly considered as measures of concussion risk [109, 120, 127, 137, 143–156]. Various measures of head injury risk use rotational kinematics (e.g. the rotational injury criterion [157] and brain injury criterion (BrIC) [158, 159]). The generalized acceleration model for brain injury tolerance [160] and head impact power [161–163] combine linear and rotational kinematics, while the weighted principal component score also includes impact location [55, 164].

Numerical brain trauma models have been developed (e.g. [114, 165–168]). These models use measured kinematics as input variables to predict brain deformation metrics, such as principal strain, cumulative strain damage, or pressure [169–172]. Modelling the material properties of the brain is challenging, and care must be taken to ensure meaningful results [173].

In-field measurements with sensors, following validation (typically against video footage), can detect and characterise actual sporting head impacts [73, 174]. These sensors can be attached to the skin [73, 175–178] or helmet [73, 176, 179–184] or embedded within mouthguards [73, 185–188]. Collected sensor data, along with subsequent clinical diagnosis, are helping to develop our understanding of concussion [73, 174], as are mechanical tests [189, 190], numerical simulations [173], and measurements from cadaver [70] and animal testing [191]. Findings from such work indicate benefits to (i) minimising peak linear and rotational accelerations; (ii) minimising the duration over which these values remain elevated; and (iii) shifting focus from PLA to also include rotational kinematics and duration. These measures, thought to increase concussion risk, are associated with impacts with compliant bodies, such as collisions between ice hockey players [37, 63, 88, 96–99]. Validated test methods, representative of conditions in the field of play, as well as brain models and biofidelic (similar to a biological system) headforms [192, 193], help further our understanding of concussion mechanisms.

3. Helmet testing

There are many reviews on helmet testing, and Whyte *et al's* is particularly comprehensive [95]. As such, only key points related to helmet development are summarised here. Helmets are typically fitted to a headform when tested [95]. Most helmets certification tests within standards include a drop test onto a fixed anvil [126–130, 132, 194–216], with some exceptions [126–130, 200]. None of these tests cover the full range of impact types a helmet may experience during use [95, 143, 190, 217, 218]. Certification tests within helmet standards are typically designed to ensure a minimum level of protection from a severe head impact (e.g. skull fracture, rather than concussion) [95].

Standards typically use centric impact vectors that cause predominantly linear acceleration [126, 127, 195, 197, 199, 213–215, 219–221], and a rigid anvil [95, 217, 222]. Tests using non-centric impact vectors are more common in research publications than in standards, following growing recognition that few actual sporting collisions and falls cause centric impact vectors (e.g. [143–145, 170, 223–228]). Such non-centric impact vectors can be imparted using drop tests onto oblique anvils (e.g. [32, 179, 229, 230]), pneumatic rams [151, 190, 224, 231], pendulums and impulse hammers [77, 153, 226–228, 232, 233], or projectiles [234].

Energy is the typical metric used to classify impact tests and, for helmet testing, is usually between 18 and 150 J (depending on the sport [95]). Where rigid anvils are used for testing, energies may be lower than those expected during actual sporting collisions and falls, with a view to maintaining similar severities, and acceleration vs. time profiles, to the actual collisions and falls [190, 227, 229]. Wider ranges of impact velocities, energies, and anvil compliances are used in research studies than in standards [85, 95, 189, 229, 234]. Measures of magnitude, and sometimes duration, of linear and rotational acceleration, are used to define injury risk (to prevent varying helmet mass from affecting perceived performance). As covered in section 2, there is ongoing discussion around the acceleration magnitudes and time profiles that are associated with clinical diagnosis of concussion, which should be resolved before updating standards [95]. As such, metrics are often compared to in-field measures for actual sporting collisions and falls, and those collected with an un-helmeted headform [190, 226–229].

Various standardised headforms, with limited biofidelity [95, 234–241], are used to test helmets [95, 234–244]. Attempts have been made to use low friction covers to improve the biofidelity of the headform and helmet interface, with clear differences in rotational acceleration [32, 136, 245, 246]. Attempts have also been made to develop more biofidelic headforms [192, 193]. Neckforms [145, 153, 167, 170, 179, 222, 223, 239, 246–249], including biofidelic ones [95, 143, 146, 156, 218, 238, 250–254], are sometimes attached to headforms to achieve more realistic post-impact behaviour.

Sports concussions are typically caused by impacts with compliant surfaces [37, 63, 88, 96–99], inducing long-duration impacts (noted as high risk in section 2). These compliant surfaces may also increase friction, and rotational head acceleration, during oblique impacts [189, 190] thought to cause concussion. Such oblique impacts with compliant surfaces are not tested for in certification tests within standards and are rarely used in peer-reviewed studies on new helmet technologies. Further, headforms with low biofidelity may cause unrealistic coupling with the helmet, potentially introducing errors while measuring rotational kinematics in the laboratory [32, 136, 245, 246]. Mechanical metamaterials, offering greater control over effective properties than conventional materials, could be used in helmet development efforts focused on reducing measures of concussion risk, while maintaining protection against skull fracture.

Figure 1. (A) Vinyl nitrile (VN) foam, (B) Expanded polypropylene (EPP_ foam and slip plane, and (C) a shear-thickening polymer (STP) pad as parts of an ice hockey helmet liner (authors' own images).

4. Helmet design

4.1. Established concepts

The idealised goal of impact protective equipment is to absorb induced impact energies without exceeding measures associated with injury risk. For a consistent impact scenario, like a certification test, the selection process for an energy-absorbing material is established. The challenge with helmets is the diverse variety of impact scenarios. There are various helmet designs available for different sports [35, 36, 217], with two main categories. The first category is single-impact helmets, which crush under impact and are designed to protect the head against one severe (high energy) impact. These include motorsports, cycling, and alpine sports helmets. After such an impact, these helmets should be replaced as they are damaged, and offer limited protection [35, 217]. The other category is multi-impact helmets, which are designed to maintain their impact performance over a (typically) long service life, e.g. several years. These are used for American Football, ice hockey, and lacrosse [35, 217], to name a few.

Helmets typically have at least three layers. A stiff (polymer or composite) outer shell (figure 1(A)) prevents penetration [35, 36, 84, 217, 255, 256], absorbs the initial shock [84, 257], and helps to hold the helmet together during or after an impact [84]. A compressible foam or lattice liner absorbs energy through deformation (figure 1, [217, 256]). Most single-impact helmets, particularly those for cycling, use crushable, expanded polystyrene (EPS) foam [36, 258]. Vinyl nitrile (VN) (figure 1(A)) and expanded polypropylene (EPP) (figure 1(B)), are often used in multi-impact helmets [35, 151, 259–261]. Many helmets also include a comfort liner, often a compliant foam [262], as shown in figure 1(A). New materials and components are also being added to helmets, generally intended to exceed minimum requirements in certification tests (e.g. [30, 31, 33, 263–269]).

Inspecting example compressive stress (σ) vs. strain (ε) relationships (figure 2), the area under the curve is the energy absorbed per unit volume (W [270]). The compressive stress vs. strain relationships of cellular materials, such as foams, can often be divided into three sections: (i) linear elasticity, up to ~5% strain; (ii) plateau, elastic or plastic buckling of the cell walls; and (iii) densification, where cell walls self-contact and the constituent material is compressed [270].

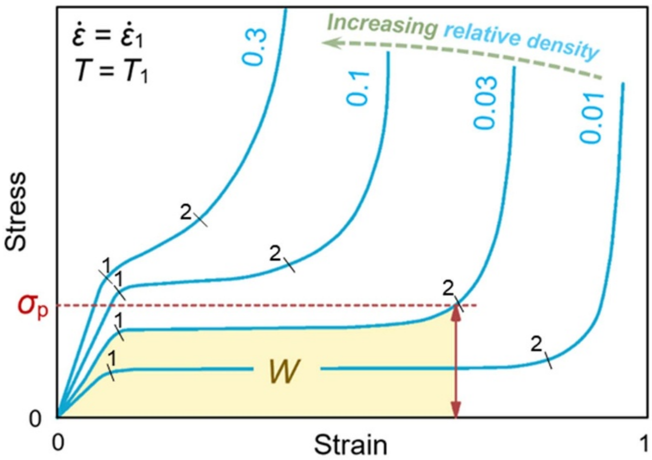

Figure 2. Example compression stress vs. strain for different relative densities of a foam at equal strain rates (arbitrarily using engineering strain values for simplicity). Area W under the curve illustrates the absorbed energy. The start (1) and end (2) of the stress plateau are marked for each foam. Reproduced with permission from [270].

Energy absorption efficiency (W/σ) is highest during the plateau region [271], which can be tailored by modifying the constituent material or foam relative density [270]. An ideal foam for a given impact (e.g. curve 0.03 in figure 2) absorbs the induced energy during the plateau region, without densifying [35, 270]. Energy absorption before densification increases with liner thickness. However, overly large helmets are uncomfortable [35, 270] and can increase rotational accelerations by increasing torque applied to the head [35]. As such, helmet liner thickness is generally limited. So, combining layers of foam of varying relative density, and hence stiffness, may broaden the range of manageable impacts, but will give lower maximal efficiency [36, 272].

4.2. Emerging developments

Various approaches have been taken to make helmets more effective over a wider range of impacts. Shear-thickening fluids (STFs) and polymers (STPs) are non-Newtonian (figure 3). The viscosity of these materials increases with shear strain rate [273–276]. STFs include suspensions [273] and gels

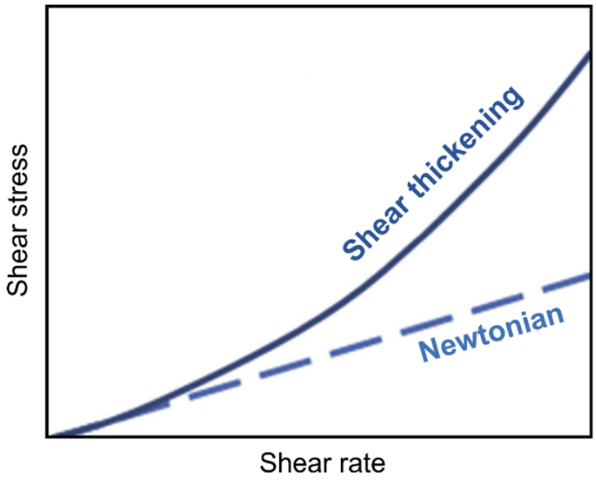

Figure 3. Behaviour of Newtonian and shear-thickening fluids (where gradient increase with shear rate).

[277], while STPs (which are more commonly used in consumer products) are viscoelastic solids [275, 276, 278]. STPs adapt to impact severity [279, 280]; they can be flexible and elastic during normal use and minor impacts, or stiffen and increase damping during severe impacts [273, 281]. The viscosity change is reversible, providing an alternative to crushable foam over multiple impacts [282, 283], with slow recovery potentially reducing rebound, impact duration, and various measures of injury risk [125, 133–136]. So, foamed STPs are used in PPE [273, 275, 277, 279], including helmets liners (figure 1(C)) [34]. STPs can also be formed into a structure and used within helmets to reduce rotational kinematics [18, 25].

A low-friction layer, placed between the helmet's liner and shell (figure 2(B) [21, 284]), or between layers of foam [265], allows relative rotation between components. This relative rotation has been shown to reduce the rotational kinematics of headforms during oblique impact tests [32, 230, 245, 284–288]. A well-known example of this technology is the multi-directional impact protection system (MIPS) [264]. The inclusion of anisotropic helmet liners has also been shown to reduce rotational acceleration during certain oblique impacts [19–21, 289]. Such liners may have fibrous columns [21], or elongated cells [20], making them stiff through thickness but transversely compliant, lowering shear stiffness [290, 291]. Salomon's EPS 4D helmet liner uses a similar principal, whereby columns of EPS foam appear to be designed to shear during oblique impacts [268]. These examples are relatively standard, using conventional materials and manufacturing methods.

Patents have been filed featuring concepts related to the application of mechanical metamaterials in helmets. These include helmets, or helmet liners, based on structured polymers such as lattices (e.g. [292–297]), sprung/suspended inserts (e.g. [298]), modular/custom fit structured components (e.g. [299, 300]), foamed/structured shear thickening materials (e.g. [301, 302]), bulk shear thickening materials (e.g. [303]), and fluidic properties (e.g. [304, 305]). Many of these innovations feature in commercially available helmets (e.g. [30, 31, 263, 269]).

5. Mechanical metamaterials

Mechanical metamaterials can be made in various ways and have unique properties that could improve sport helmet performance. They can be fabricated from conventional materials such as foam [306, 307] or textiles [51, 308], or designed as periodical/graded cellular structures [12–14, 309]. Mechanical metamaterials can also be made from sheets of material by folding (known as origami) [310–327], or by folding, cutting, and joining (known as kirigami) [17, 328–344]. With high levels of control over end properties—given the additional degrees of design freedom afforded by controlling topology and base material—mechanical metamaterials are well suited to addressing complex engineering problems, like impact protection [17, 50, 262]. The common forms of unusual mechanical properties are auxetic (negative Poisson's ratio) behaviour (covered extensively in various reviews [46, 50, 307, 309, 345, 346] and textbooks [308, 347, 348]), negative stiffness [349–356], shape morphing [337, 357–359], force/torque coupling [360–364], active/adaptive behaviour [351, 365–367], or programmable properties that are tuned to a specific application [17, 26, 28, 341, 361].

5.1. Auxetic metamaterials

Poisson's ratio is the negative product of the ratio of lateral to axial strain. Auxetic materials undergo transverse expansion when stretched axially, and contract transversely in compression [11, 285, 306]. So, they form a dome shape under bending, and are used in a helmet liner for this reason (i.e. flat sheets can fit into domed helmets [32, 263]).

Poisson's ratio is one of the basic elastic constants, and (with Young's modulus/stiffness) affects shear modulus, bulk modulus, and indentation resistance [368]. As detailed elsewhere [368], Poisson's ratio increases resistance to penetration by concentrated loads [369–373], and shear modulus [374–378]. The increased tendency of materials with a low or negative Poisson's ratio to deform volumetrically, rather than in shear (figure 4), may also increase strength. With lower shear strain, the likelihood of failure close to a crack tip or an ellipsoid reduces, according to Von-Mises, Tresca, and crack propagation theories [379, 380]. Without the presence of stress concentrations caused by a crack or ellipsoid, Von-Mises and Tresca criterion are unaffected [368]. So, auxetic helmet sections may fail less readily, reducing waste and severe head injury risk.

The re-entrant-like cellular structure of auxetic foams is imparted by compressing conventional foam to buckle cell ribs [306, 307, 381]. So, while there is some uncertainty over whether these foams meet the requirement for the precisely defined topology of some metamaterial definitions [7–9], they are still a related medium. Readers interested in auxetic foam manufacturing are referred to Jiang *et al*'s review [307].

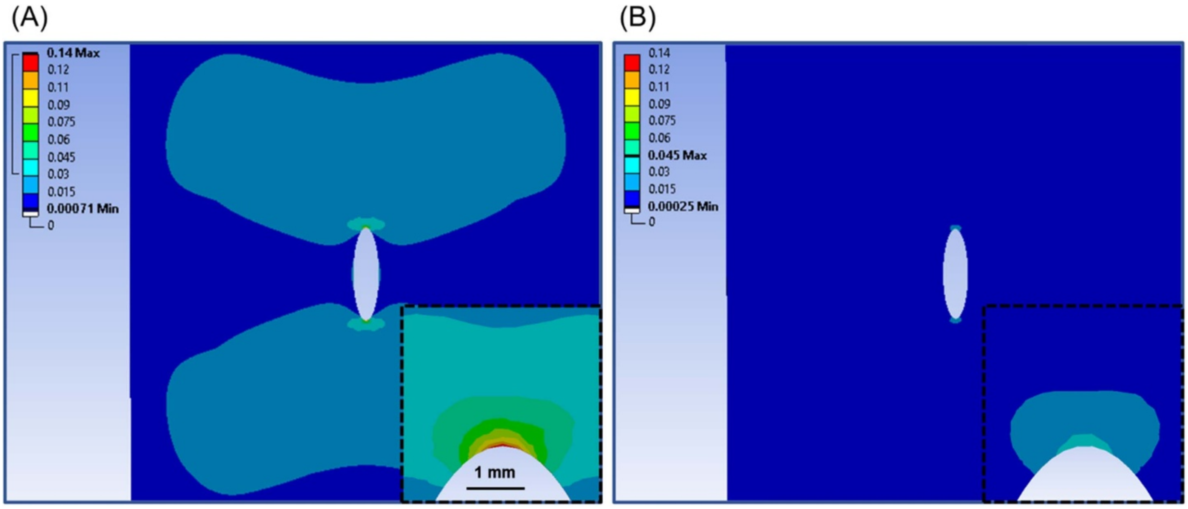

Figure 4. Contour plots of maximum shear (engineering) strain in 100 × 100 mm thin plates loaded parallel to the short axis of a 5 × 20 mm central ellipsoidal hole, with arbitrary, equal tensile loads and moduli, but Poisson's ratios of (A) 0.5 and (B) −0.5. Static structural simulations were undertaken in Ansys Mechanical to demonstrate this concept.

Auxetic foams have been shown to increase vibration damping [382, 383], and to exhibit peak impact forces up to ten times lower than their conventional counterparts [291, 384–388]. It should be noted that peak force during the typically stiff anvil and impactor impacts is not a scalar measure. Indeed, peak force increases exponentially as the foam densifies (see figure 2) and 'bottoms out' under impact. Further, auxetic foams made from expanded foam, as typically used in helmet liners, have not been reported.

Auxetics may provide benefits in helmet liners: (i) The ability to adapt to the shape of impacting bodies—remaining soft when impacting a relatively flat surface, but effectively stiffening under concentrated loads. (ii) High vibration damping, redistributing vibrations transversely. (iii) A tendency to bulk over shear deformation, reducing the likelihood of failure. Conversely, the early densification strain caused by the tendency to bulk deformation may shorten the stress vs. strain plateau in cellular solids—causing densification at lower strain ([291], figure 2). With the ability to include a stiff shell, the benefit of the high indentation resistance possible for auxetics is unclear and has not been empirically demonstrated in helmets. Flexible shell helmets (e.g. [227, 228]) may, however, benefit from the increased indentation resistance of auxetic materials. Further, the increase in shear modulus with negative Poisson's ratio goes against the broad strategy of reducing liner shear stiffness to reduce rotational acceleration [25, 32, 263, 264, 284, 286–288]. So, the application of auxetic materials in helmet liners requires careful design based on justifiable benefits, such as increasing indentation resistance to facilitate lower stiffness liners.

An unstudied, potentially useful topic is auxetic helmet shells. Fibre-polymer composites can be auxetic, with the negative Poisson's ratio achieved by fibre alignment [389, 390]. Due to the use of conventional fibres and pre-preg, auxetic fibre-polymer composites can be made with standard composite manufacturing methods [391]. These auxetic composites have been shown to resist back face damage under impacts [392]. Such increased resistance to back face damage could increase the lifespan of multi-impact helmets featuring composite shells, particularly those with flat sections that are more susceptible to back face damage.

5.2. *Periodic structures*

Advances in additive manufacturing, and moulding methods [25], have allowed mass production of lattice and honeycomb mechanical metamaterials (e.g. [31, 33, 263, 393, 394]). These methods allow precise manufacturing of complex geometries [16, 395–397], expanding the range of available properties to meet complex requirements, such as those seen in helmets. 2D extruded cellular structures, such as honeycombs (figure 5(A)), repeat periodically in two directions [398]. Honeycomb and tubular structures are studied frequently as energy-absorbing elements in sports equipment and helmets [23, 399–403], such as in Koroyd's helmet liner [31]. 3D periodic cellular structures, such as lattices, consist of unit cells repeating in three directions, increasing degrees of freedom during design, but also increasing manufacturing complexity and hence costs [395, 397, 398, 404–409].

Exemplary unit cell designs include hexagonal/re-entrant (figure 5(A)) [410, 411]), square/cubic (figure 5(B)) [412]), or chiral/antichiral (figure 5(C)) [413–417]. Unit cell design degrees of freedom include; varying rib orientation (figures 5(A) and (B)), length, or thickness—slender ribs are often less stiff, varying rib form (figure 5(B))—Eigenmode example); varying the number of ribs (figures 5(A) and (B)), or adding and combining shapes and features (figure 5(C)). Unit cell patterning also affects topology, and so metamaterial properties; unit cells can be mirrored (figures 5(A-i) and (C)), linearly repeated (figures 5(A), (B) and (C-iii)), or rotated (figure 5 (C-v)).

When creating honeycombs, some variation can be applied in the extruded direction, as is the case of Miura-ori-inspired structures (figure 5 (B-ii)) [418, 419]. Gradient metamaterials

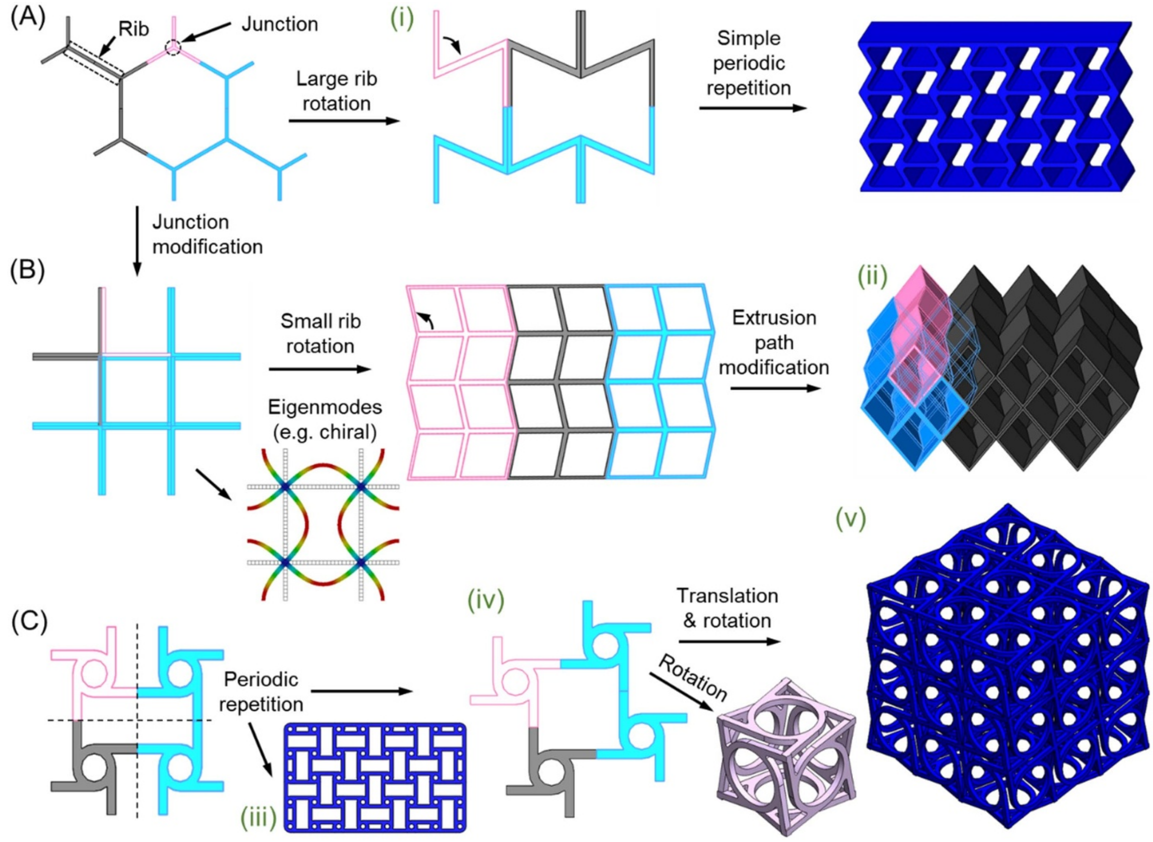

Figure 5. Some notable mechanical metamaterial design degrees of freedom: (A) Honeycombs, with large angle rib modifications, becoming (i) re-entrant. (B) Quadrilateral honeycomb, with Eigenmode rib tessellation to form an auxetic unit cell, or small rib rotations and extrusion path modification, forming a (ii) Miura-ori inspired metamaterial. (C) Various periodic rotational and translational repetitions of a chiral unit cell, forming (iii) antitetra chiral, (iv) 2D chiral and (v) 3D chiral metamaterials. Pink wireframe notes the simplest repeating unit cell, subsequent repetitions are shown in grey or blue.

can also be developed by spatially varying unit cell parameters and properties [402, 403, 420, 421]. These variations can be continuous or discrete (i.e. gradual or abrupt change) [422–424]. 3D periodic cellular structures are being used in helmets (e.g. ice hockey [33] and American football [393]). Such liners, or inserts, can also feature some through thickness variation, and can be made from STPs [30].

Concerning some common topologies, hexagonal honeycombs are relatively stiff, with low density, for compression parallel to their extruded dimension [402, 425–428]. During impacts in the extruded direction, cell walls crumple and buckle [31, 426], with densification occurring at ∼75% compression [402]. When impacted or compressed perpendicular to their extruded dimensions, honeycomb stiffness is lower [28, 402, 427, 429, 430]. Re-entrant hexagonal honeycombs can be more compliant than equivalent density regular hexagonal ones (at low strains), due to the extra junction for rib hinging to occur around [423, 429, 431, 432]. Buckling may not occur with these re-entrant unit cells, causing an almost linear compressive stress vs. strain response, i.e. a less pronounced plateau and densification region [369, 423, 429]. Such re-entrant unit cells may not be optimal within a target impact severity (e.g. as in figure 2), but are less prone to stark peak force increases during severe impacts [29, 291, 385]. It is possible to design tall, slender stiff re-entrant cells that undergo buckling [271, 422].

Structures made of solid rotating shapes are often stiff in compression, as the internal shapes undergo self-contact/densification at low strains, making them less suited for sporting impacts [433–436]. These rotating shapes have been made as lattices with hollow cells [437], or cut from foams, to tune out of plane properties while relying on foam characteristics through thickness [438, 439]. High energy absorption, low initial crushing peak forces, large densification strain, and low strain rate sensitivity can be achieved with folded (kirigami or origami) structures [332, 333, 335, 440, 441]. By including such folds through the thickness of these structures, it is possible to tune buckling regions, and the length of the compressive stress plateau [17] (figure 2). Kirigami structures made from paper have been used in a recyclable cycling helmet [442].

For application in helmets, periodic structures must be patterned to fill an often complex/nearly spherical space. Such patterning, using conventional computer-aided design software, can be time-consuming and inefficient. Algorithmic-based design software, such as those marketed by nTopology [443], Hyperganic [444], or Rhino3d [445] can make the process of generating such geometries more efficient. Noting

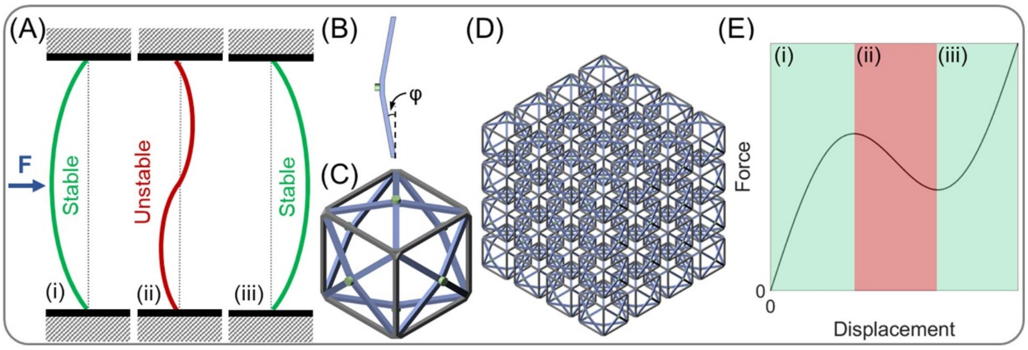

Figure 6. (A) Stages of snap through in a buckling beam. (B) Snap through element. (C) Inclusion in a cubic unit cell and (D) metamaterial, recreated from ref [353]. (E) Example force displacement (arbitrary values), including stages from (A). Reprinted from [353], © 2019 Elsevier Ltd.

that impact vectors are usually non-centric, further challenges arise. Where there are enough unit cells, the response at various angles can be calculated based on orthotropic, strain-dependent properties, using standard elasticity tensor transformation [423, 446, 447]. So, response to off-axis impacts can be designed by tuning the out-of-plane properties. Where there are too few unit cells to homogenise the material properties, as is often the case for periodic structures, the off-axis response must be obtained by higher order material approximations [448], microstructurally faithful simulations [25], or experimentally [25, 32].

5.3. Force torque coupling

Advances in fabrication methods have allowed realisation of a wide range of unusual and potentially beneficial properties. Mechanical metamaterials with force–torque coupling (known herein as twist) have seen increasing interest, since their rational design was shown in 2017 [363]. Like (negative) Poisson's ratio, twist translates axial deformation to transverse deformation—increasing resistance to indentation [360]. So, twisting mechanical metamaterials may resist penetration by concentrated loads, without shortening the stress plateau under compression (as seen for auxetics—see section 5.1).

The development of these metamaterials could also facilitate more efficient analysis and design of lattices. The twisting response is not included in classical (Cauchy) continuum mechanics, but it is in micro-morphic continuum mechanics (where a uniform load causes internal strain gradients [448]). Eringen presented some special cases in micro-morphic theories [448]. These include micropolar—where the gradients occur by rotation of rigid unit cells (sand provides an intuitive example), and micro-stretch—where the gradients occur by unit cell volume change, without shape change (picture the bronchi). Each of these allows some simplification (over the micro-morphic continuum)—reducing the amount of information required to approximate the response of a metamaterial—but may also cause some loss in precision.

Clearly, in the case of foams and lattices, which have relatively large internal features (when compared to the scale of external loads), micro-morphic continuum theories often apply [360, 449, 450]. Where classical continuum theories cannot be used, typical approaches to designing mechanical metamaterials for impact protection are experimental, or by using microstructurally faithful numerical models [17, 26, 28, 29, 415, 416, 419, 451], which are less efficient. So, developing and applying these micro-morphic continuum theories, including their viscoelastic and visco-plastic formulations, could facilitate more efficient mechanical metamaterial analysis, design, and application in single or multi-impact helmets [448].

5.4. Negative stiffness

Snap-through elements cause negative stiffness behaviour, corresponding to a drop in force as applied deformation increases [349–355]. Negative stiffness can be achieved by the buckling of an end constrained/preconditioned beam (figure 6). The beam snaps from one state of equilibrium to the next following the application of a perpendicular load (often via a connecting rib) [349, 350, 355]. The negative stiffness region is present for a segment of the force vs. compression relationship, corresponding to when the beam snaps through (figure 6(E)). Increasing the diagonal angle of the symmetric beam (φ, figure 6(B)) increases the onset, and length, of the region [353]. Making the beam less slender increases the critical buckling load and hence the magnitude of both positive and negative stiffness [353]. Negative stiffness has been shown to improve protection during impacts [452], balancing the positive stiffness of neighbouring unit cells to flatten and elongate the stress plateau (figure 2). Designing and manufacturing relatively unstable negative stiffness inclusions within helmets could bring added complexity, and further work applying these concepts to helmets is needed.

5.5. Topology optimisation

Precise control over topology allows the design of a desired response to various loading conditions. An efficient approach to topology optimisation is to optimise a unit cell, and homogenise (expand to an effective bulk material) using a set of boundary conditions (i.e. periodic symmetry [451, 453–458]). Readers interested in homogenisation theory could refer to refs [459–461]. Such an approach is not widely applied during

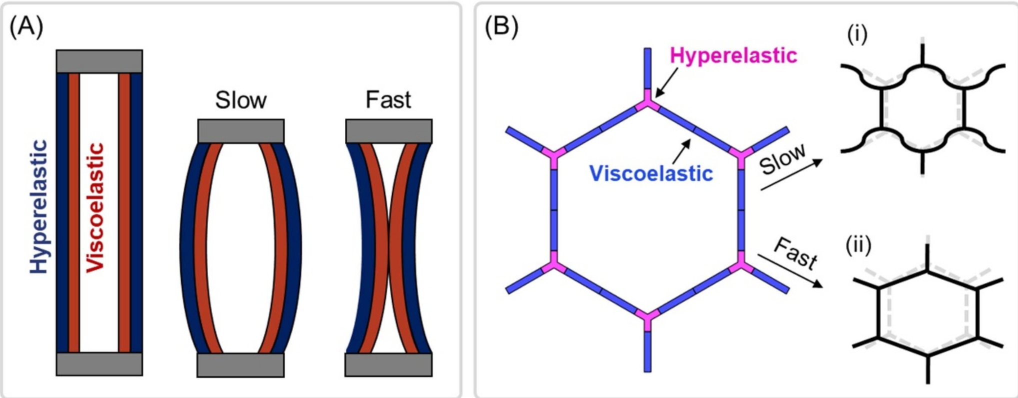

Figure 7. (A) Single bi-beam design and buckling direction for different axial compressive strain rates. (B) Multi-material hexagonal cell causing a switch in the dominant deformation mode.

sporting impacts, causing high strains and strain rates, meaning degrees of freedom surrounding unit cell boundaries are influential and challenging to predict. For example, end loading for buckling beams, such as cell ribs, can vary with neighbouring cell deformation, meaning a multi-cell optimisation is needed [462–464]. Instead, whole metamaterial samples are often optimised or iteratively improved [17, 22, 26, 32]. Developing and applying micro-morphic theories [448] to lattices under impact may facilitate the prediction of rib constraints, and efficient (unit-cell) topology optimisation [465]. Open data approaches (such as the meta-genome [466]) could help develop these new homogenisation methods.

5.6. Adaptive metamaterials

While shear thickening polymers can adapt their stiffness to various collision types, more degrees of design freedom, such as deformation mode switching, are possible. Deformation mode switching can be achieved by including an adaptive material, such as a viscoelastic one, that activates a topological instability [14]. For example, a beam's buckling mode (e.g., direction) can be designed to switch at a given strain rate. When two laterally connected beams of different stiffness (i.e. bi-beams) are axially compressed (figure 7(A)), the stiffer one drives the buckling direction while the other provides support, causing buckling towards the stiffer side. So, in figure 7(A), the deformation direction will switch at the strain rate when the viscoelastic beam becomes stiffer than the hyperelastic one. Bi-beams positioned like those in figure 7(A) will buckle away from each other at low compression rates, and towards each other, causing stiffening via self-contact, at high compression rates. Negative effective viscoelasticity can also be achieved with such a system of bi-beams [365], if the order in figure 7(A) is reversed, so bi-beams effectively soften by buckling away from each other at high compression rates. Obtaining negative effective viscoelasticity with highly viscoelastic materials demonstrates the level of control available via topology that could be useful when designing helmets. The response of these bi-beams, while previously shown to be retained for off-axis deformation angles of $\sim 10°$ [365], is unknown during oblique impacts. Flexing of the beams may provide a desirable low shear modulus, as in similar tests of long-cell anisotropic foam liners [20, 289], and should be studied further.

More stable adaptive metamaterial systems can also be designed. The dominant deformation mode in hexagonal honeycombs and lattices is cell rib flexure [429, 447] (figure 7(B-i)). Such flexure reduces the distance to the neighbouring junction, respectively reducing and increasing the magnitude of positive and negative compressive Poisson's ratios. Placing viscoelastic material in the cell ribs could switch the dominant mode, increasing the magnitude of positive Poisson's ratio, and hardness [368], during more severe impacts (figure 7(B-ii)). Conversely, placing viscoelastic material in the junction of auxetic, re-entrant honeycombs or lattices could have a similar effect, by amplifying the dominance of rib flexure to draw neighbouring junctions inward. These concepts have been demonstrated using dual materials of different stiffness, but not viscoelastic ones, and present notable options for future research, particularly related to applications in helmets [467, 468]. Interestingly, such switching mechanisms and changes to Poisson's ratio can be achieved with one material, based on local changes in strain rate and stiffness; shifting the point of deformation [469–471].

The use of embedded electronics as active adaptive mechanisms is emerging [14, 472, 473]. These include piezoelectric inclusions, which stiffen when an electrical field is applied [472], or embedded electromagnets [352]. Such systems allow a controlled response, with the potential for embedded electronics to sense environmental changes, like impact severity or strain rate, fall initiation, temperature or relative humidity, and micro-controllers to define a programmed response or adaption [14, 472, 473]. Challenges to application are associated with the manufacture of sufficiently small and robust components [14, 352].

6. Discussion

Some commercial, or mid- to high- technology readiness level mechanical metamaterials, show promise to reduce concussion risk when applied to helmets

Table 1. Summary of key metamaterial types, properties, benefits, and challenges for application in helmets.

Metamaterial	Potential benefit	Potential application	Challenges
Periodic structures	Tuneable response	Compliant/crushable liners	Efficient design and manufacturing, particularly during non-centric impacts
Auxetics	Domed curvature	Helmet liner manufacturing solution	Already established
	High indentation resistance	Compliant/crushable liners, particularly of soft-shell helmets	Early densification, and increased shear modulus
	High toughness	Compliant/crushable liners	Making crushable auxetic foam
		Helmet shells, particularly fibre-polymer composites	Demonstrating the requirement
Force–torque coupling	High indentation resistance	Compliant/crushable liners, particularly of soft-shell helmets	Cannot be simulated as bulk solids using the Cauchy continua
Negative stiffness	Extension of stress vs. strain plateau region	Compliant liners	Cost-effective design of such unstable mechanisms
Adaptive metamaterials	Improved performance across various impact types	Compliant liners	Cost-effective manufacture and design—often featuring multiple materials deformation mechanisms

[18, 21, 25, 30, 32, 263–265, 393]. Separately, these metamaterials appear to have sufficient degrees of freedom to reduce shear and compressive response, and rotational and linear measures (e.g. [18, 21, 25, 30, 32, 263–265, 268, 284, 393]), reduce the duration of high accelerations via crushable or viscoelastic liners (e.g. [18, 25, 30–32, 263, 393]), and adapt to surface compliance, or impact severity, via rate dependency (e.g. [18, 30, 266]). For commercial helmets [30–33, 263–266, 268, 284], independent, peer-reviewed, analysis of such functions is rarely published. These systems, along with [18], have demonstrated commercial viability of mechanical metamaterials (e.g. use of single material injection moulding or additive manufacturing). So, they reduce barriers to entry and raise awareness, facilitating continuous improvements and further application of mechanical metamaterials. Table 1 summarises the key mechanical metamaterial types covered in this review.

With the greater degrees of freedom afforded when designing mechanical metamaterials come some additional challenges. Firstly, metamaterials can be more expensive to manufacture than established helmet materials, so currently tend to only be used in high-end helmets [30, 31, 33, 263, 269]. Secondly, the effect of increasing, or changing, the pool of materials used in helmet design needs to be considered; particularly susceptibility to environmental considerations such as temperature, relative humidity, contaminants, and ultra-violet radiation [474]. Conversely, the greater design affordances associated with mechanical metamaterials could reduce susceptibility to environmental effects; with potential to achieve the required performance using only materials that resist degradation due to environmental conditions. The sports market may again be an important early adopter; providing long term, in-field (user) testing in variable environments before uptake by more conservative sectors such as aerospace.

Peer-reviewed publications noting tests of new helmet technologies rarely use biofidelic anthropomorphic test dummy heads or necks, nor impacts onto compliant anvils. Unrealistic coupling between helmet and head may affect rotational accelerations [32, 136, 245, 246], while impacts with compliant surfaces are amongst the most common causes of sporting concussion [37, 63, 88, 96–99]. Mechanical metamaterial design streams that reflect these high injury risk scenarios could be developed, to ensure mechanical metamaterials are designed and implemented in helmets based on up-to-date measures of concussion risk. An extensive range of mechanical metamaterial properties has been demonstrated (e.g. [18, 21, 25, 32, 284]), so such design streams appear feasible. Funding calls, challenges, and open data approaches that promote collaborations and knowledge exchange between groups with state-of-the-art test methods, and those developing metamaterials and helmets, could be beneficial. Such initiatives could be sport specific (e.g. [475]), or broader (e.g. [90–93, 466]).

A metamaterials approach to helmet design: unit cell optimisation and patterning, based on an objective function of measures of concussion risk and manufacturing constraints, could be used to improve helmet impact performance [456, 462]. Such an approach requires some form of rate dependency or adaption, justifying research developing ranges of viscoelastic materials for additive manufacturing. We note two forms of unusual property that are of prominent interest in helmet development: (1) negative stiffness inclusions, to extend or flatten the stress plateau (figure 6), and (2) adaptive metamaterials (figure 7). Switching of deformation mechanisms may provide options to increase rate dependence without the presence of extreme viscoelasticity. Efficient topology optimisation for such systems also requires some development, to apply periodic constraints that reflect rib buckling with single/minimal unit cells [456, 462]. With such methods, helmet manufacturers could adapt to developing knowledge of concussions, or design affordances offered by new manufacturing methods.

7. Conclusions

Mechanical metamaterial design affords degrees of freedom that could allow helmets with an impact response that adapts between severe impacts that cause skull fracture and those that might lead to clinically diagnosed concussion. The objective functions that mechanical metamaterial helmet liners are designed or optimised for could be modified, by testing and refining the designed helmets on biofidelic headforms under representative test conditions to better understand the required effective properties. As such, efforts encouraging collaborations between those developing helmets, mechanical metamaterials, and test methods, could improve helmets. Epidemiological studies may help identify the effect of such interventions over time. Improving the efficiency and availability of topology optimisation tools at high strains and strain rates, would allow helmets to be updated as knowledge of concussion improves. Here, open data and opensource software initiatives will be beneficial. To increase options for mechanisms of adaption to impact severity, researchers could focus on increasing options to print or mould viscoelastic materials or developing topological approaches to tune effective viscoelasticity.

Data availability statement

The data cannot be made publicly available upon publication because they are not available in a format that is sufficiently accessible or reusable by other researchers. The data that support the findings of this study are available upon reasonable request from the authors.

Conflict of interest

Dr Greenwald is the founder of Simbex, Lebanon, New Hampshire, USA, and is involved in the development of head impact exposure monitoring technology used commercially by Riddell, Inc. All other authors declare no conflict of interest.

Author contributions

Mr Haid drafted the review, while Drs Duncan, Allen, Sareh, and Greenwald edited it. Drs Hart and Foster provided feedback.

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