The Royal Society Mathematical Futures Programme

Landscaping Mathematics Education Policy: Horizon scanning of international policy initiatives

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Contents

Executive summary

The Royal Society's Mathematical Futures Programme aims to:

a) Understand the mathematical competences that will be needed by students leaving compulsory education and training in the future
b) Consider the implications of reshaping mathematics education for 4–19-year-olds
c) Recognise the skills required for teachers who would teach these curricula

This report contributes to the Mathematical Futures Programme's aim of landscaping mathematics education policy based on a horizon scan of policy and change in international jurisdictions. It complements a study of mathematics education policy in England. It provides an overview of recent developments in mathematical, statistical, and computational thinking and data literacy, informed by views of international mathematics education experts. A combination of desk research, interviews, and workshops with a total of 30 international experts was used to undertake horizon scans of mathematics education internationally. The guiding aim was to use international comparisons and horizon scanning to identify high-performing practices, policy initiatives, and future directions of travel in mathematics education.

Research was organised around the following themes: curriculum and pedagogy; qualifications and assessment; resources and technology; teacher preparation and professional development; and evidence-informed policy development.

Key policy trends and education practices in mathematics education within high-performing jurisdictions

The concept of 'trends' in mathematics education may not be the most appropriate or helpful way of understanding international mathematics education developments as the international picture is diverse, with the exception of a common concern about the implications of an increasingly digital society, economies, and cultures.

'High-performing' for the purposes of this study drew on evidence from international comparative tests. The selection of features of high-performing systems in the report highlights those that contrast with England. Countries with similar PISA outcomes to UK nations were included in horizon scans where relevant international developments were identified.

Curriculum and pedagogy

Although there are common features in East Asian curriculum and pedagogy, practice is not uniform. Developments in both curriculum and pedagogy vary, rooted in social, cultural, and organisational conditions. Common features of curriculum and pedagogy are a focus on coherence, consideration of sequencing of progression, mathematical dialogue, and an integrated teaching approach that addresses conceptual understanding, reasoning, problem solving, and proficiency in routine skills. The curriculum and pedagogy for early years in high-performing systems are generally play-based, with formal schooling and instruction not starting until 7 years old. Generally, in primary and lower secondary schools, the curriculum is accessible to all students.

Qualifications and assessment

In all high-performing countries, certificated mathematics study is compulsory beyond age 16, with many having well-established routes for mathematical study in vocational pathways. Curriculum review, technological developments, and an increasing focus on equity and issues raised during the pandemic have provided opportunities to review assessment design and practices.

Resources and technology

Textbooks are an important resource in nearly all high-performing systems and are embedded in policy. An exception to this is Estonia, where teachers use a wide range of source material. There is more variability in the use of digital technology. In China, the new high school mathematics curriculum includes programming as a core competence; Singapore has recently developed a digital learning strategy; and Estonia has established policy and practice in computer-based mathematics and statistics with widespread use of tablets as a learning platform.

Teacher preparation and professional development

Teacher education is a key factor in system success. In high-performing systems, initial teacher education is longer, university educators and researchers are more influential, and greater prominence is given to subject-specific and pedagogical subject knowledge. Professional development is embedded in teachers' working lives, with entitlements and expectations of engagement often identified in policy. Teachers have greater autonomy about how they use their time to access accredited courses, and as with ITE, the role of universities is prominent. In Singapore, teachers have funded opportunities to engage in masters and doctoral study. A notable feature of high-performing systems is a commitment to collaborative professional development.

Current international trends in mathematics education

Key practice and policy developments (Table 1) are reported, supplemented by case studies, and with consideration of implications for England.

Table 1: Practice and policy developments: summary

Aspects of mathematics education	**Practice and policy developments**
Curriculum and pedagogy	21st Century Skills Interdisciplinary and cross-curricular innovation Mathematical modelling Statistics, data literacy and data science Inclusive curriculum and pedagogy
Qualifications and assessment	Broadening the focus of assessment Mathematical success for all

	Computer-aided assessment
Resources and technology	E-textbooks Digital technology and mathematics teaching Artificial intelligence and computer-aided learning Personal digital devices
Teacher preparation and professional development	Professional learning entitlements Collaborative professional development Distance learning Focusing on content and pedagogical content knowledge

Future possibilities for mathematics education in England

Analysis and synthesis focus on evidence-informed policy development internationally, drawing on features of high-performing systems and examples identified in the research to consider implications for mathematics education in England. Simplified models of the relationship between education innovation and policy development are provided. A range of factors related to context, design, and stakeholders have been identified as either promoting or inhibiting policy development. Features of effective policy development include clarity of purpose, consensus amongst stakeholders, feasibility, coherence, systemic alignment, piloting and sequencing, sustained attention, and collaboration. The models and indicative influences on mathematics education innovation and policy development, together with features of effective policy development and implementation, will inform thinking about future possibilities in England. There is considerable divergence in England from international innovations in education policy recommendations, albeit with some evidence of change more recently that mirrors some international developments.

Conclusion and recommendations

There is considerable innovation and policy development in mathematics education in response to on-going social, economic, and cultural changes and challenges. Mathematics education in England diverges from many features found in high-performing education systems. This divergence is unlikely to change without renewed innovation and policy development in England. We have identified some limited alignment, for example, in areas of curriculum and pedagogy such as problem solving and data science and in the introduction of Core Maths. English policy for mathematics teacher professional development has relatively more features or similarities to those found in high-performing systems or innovations elsewhere. The roles of the NCETM and the Maths Hub network are important here. This is a positive feature for future potential innovation in other areas, given the importance of professional development in other mathematics education policies.

Innovation in England is more likely to be successful if a parallel approach is adopted, i.e., developing innovations in parallel with main policy developments. The existence of already-piloted innovations will increase the chance of policy adoption. Three areas with most potential and need are:

- a coherent and cross-phase approach to the use of digital technology in mathematics
- opportunities for enhancing coding and computational thinking in mathematics
- the integration of data science into the mathematics curriculum

In each of these three areas, it would be fruitful to engage in a more in-depth review of both comparative policy development internationally and promising more localised innovations and programmes. Such reviews could then inform the development and enhancement of collaborations, guide investment in pilot innovations, and support consensus building.

1. Introduction

The Royal Society's Mathematical Futures Programme aims to:

a) Understand the mathematical competences that will be needed by students leaving compulsory education and training in the future

b) Consider the implications of reshaping mathematics education for 4–19-year-olds

c) Recognise the skills required for teachers who would teach these curricula

This report is the first of two to contribute to these aims by horizon-scanning mathematics education policy and change in mathematics education in international jurisdictions.

The report explores innovations and trends in mathematics education policy, providing reflections on **recent developments in mathematical, statistical, and computational thinking and data literacy**. It relates global developments to current innovations and policy trends in England as well as, where appropriate, examples of differences and change in other UK nations. It is **informed by views of international mathematics education experts.** The report aims to inform scenario planning in Mathematical Futures Project 2 with evidence and scenarios for the value of mathematics in the future.

To examine different aspects of mathematics education, research activity was organised in relation to:

- Curriculum and pedagogy
- Qualifications and assessment
- Resources and technology
- Teacher preparation and professional development
- Evidence-informed policy development

These categories structure the main body of the report. For the first four of these categories, each section is organised into three parts. The first part summarises developments in practice and policy in high-performing jurisdictions. A relevant case study and a snapshot of other international developments follow. The identified themes for these four sections are given in Table 2.

The research was undertaken between July 2021 and February 2022.

Table 2: Practice and policy developments and case studies: summary

Aspects of mathematics education	Practice and policy developments	Case study
Curriculum and pedagogy	21st Century Skills Interdisciplinary and cross-curricular innovation Mathematical modelling Statistics, data literacy and data science Inclusive curriculum and pedagogy	Problem solving and curriculum change in Singapore
Qualifications and assessment	Broadening the focus of assessment Mathematical success for all Computer-aided assessment	PISA 2021 mathematics assessment framework
Resources and technology	E-textbooks Digital technology and mathematics teaching Artificial intelligence and computer-aided learning Personal digital devices	Sweden and programming in mathematics
Teacher preparation and professional development	Professional learning entitlements Collaborative professional development Distance learning Important of content and pedagogical content knowledge	Professional Diploma in Mathematics for Teaching (PDMT) in Ireland

The section that follows, on evidence-informed policy development internationally in mathematics education, draws on the previous horizon scans. The structure of this section differs from earlier ones, with sub-sections on

- educational innovation and policy development processes
- policy influences
- features of effective policy development
- case study: The German Centre for Mathematics Teacher Education (DZLM)

In these sub-sections, both features of high-performing systems and examples, including case studies, featured in the report are discussed. Following that, lessons for English mathematics education and implications for future policy planning are considered.

2. Research questions, scope and methods

Research questions

Main research question

How can international comparisons and horizon scanning tell us about high-performing practices, policy initiatives, and future directions of travel in mathematics education?

Contributing questions

a) What are the key policy trends and education practices in mathematics education within high-performing jurisdictions?

b) What are the current international trends in mathematics education, and how are these impacting national mathematics education agendas?

c) How can international comparison and trends inform thinking about future possibilities for mathematics education in England?

From the research undertaken, **the concept of 'trends' in mathematics education may not be the most appropriate or helpful way of understanding international mathematics education developments**. The international picture of innovation in practice and policy is fragmented and not uniform, with complex interactions between global and transnational tendencies and local concerns. The metaphor of horizons is more appropriate than a single geographical horizon. Few developments or concerns are found in all or even a majority of education systems; an exception is a concern **about the implications of an increasingly digital society, economy, and culture.** The term 'trend' in the research questions, as addressed in this report, is better understood as 'developments potentially relevant to mathematics education internationally'.

High-performing systems

The definition of **'high-performing' for the purposes of this study drew on evidence from international comparative tests,** principally PISA (the Programme for International Student Assessment). Therefore, we included East Asian PISA reference societies (Sellar & Lingard, 2013). These education systems are cited as benchmarks due to their success in international comparative tests in mathematics and are influential currently in English mathematics education policy. We also considered European education systems that were comparatively high-performing on PISA, for example, Estonia and Switzerland**. The selection of features of high-performing systems in the report highlights those that contrast with England.**

Systems included in other scans

Countries with similar PISA outcomes to UK nations were included in horizon scans where relevant international developments were identified. These included countries such as New Zealand, Germany, Ireland, and Sweden. Appendix B provides information on PISA 2018 mathematics results by country. Literature examining high-performing education systems (HPES) typically examines the contribution of several explanatory factors: teacher quality, leadership, system characteristics, and reform, while sometimes neglecting cultural and institutional factors (Deng & Gopinathan, 2016); this is a necessary feature of the type of scan undertaken and reported here.

Methods

A combination of desk research, interviews, and workshops with international experts were used to undertake horizon scans of mathematics education internationally. The scope of the initial horizon-scanning activity was initially deliberately broad, considering wide-ranging proposals encompassing future directions in global reform both in and beyond mathematics education. It was informed by an analysis of the call for views and included horizon scans of high-performing systems and education future scans. Key sources included ICMI topic surveys, research reviews, and OECD and TIMMS studies. The initial scan foci were refined through examination of research evidence and consultation with experts in individual interviews and in workshops, with details below.

Table 3: Numbers of interviews and workshops conducted

Focus area	**Number of individual expert interviews**	**Number of workshop participants**
Curriculum and pedagogy	5[1]	6
Qualifications and assessment	1	5
Teacher preparation and professional development	1	6
Technology and resources	2	3
Policy development	1	Theme in all workshops

Further details on methods are given in Appendix 1.

[1] These had specific foci: Curriculum intent and purpose, Data and quantitative literacy, STEAM and applications, modelling, problem solving

3. Curriculum and pedagogy

3.1 Curriculum and pedagogy: policy and practice in high-performing systems

In England, recent policy interest in curriculum and pedagogy in high-performing systems has centred on East Asian practices encapsulated through the concept of mastery and supported through policy in the NCETM's 'teaching for mastery' (NCETM, 2016). **Although there are common features in East Asian curriculum and pedagogy, practice is not uniform**. Curriculum and pedagogy are rooted in social and cultural differences and differences in school organisation and teacher professional conditions (Boylan et al., 2018). For example, in Singapore (see case study below), there is a particular emphasis on metacognitive aspects of problem solving. With these cautions in mind, **common features of curriculum and pedagogy are:**

Table 4: Common features of curriculum and pedagogy in East Asian systems

Curriculum	Coherence Careful sequencing of progression.
Pedagogy	Mathematical dialogue (teacher to pupil and pupil to pupil) is an instructional priority (Clarke et al., 2010) Teaching integrates developing conceptual understanding, mathematical reasoning and problem-solving skills with proficiency in routine skills (Huang & Leung, 2004)

In both East Asian and European high-performing systems, **curriculum and pedagogy for early years are generally play-based, with formal schooling and instruction not starting until 7 years old.** Generally, in primary and lower secondary schools, **the curriculum is accessible to all students.** This contrasts with common practice in England, where students are grouped into sets in secondary education or, in primary education, often either into sets or within-class grouping by prior attainment.

3.2 Curriculum and pedagogy: International developments

21st Century skills

Competences for the 21st century, often referred to as **21st century skills or lifelong learning competencies,** are variously defined in international curricula and research. In a review of key policy frameworks, Voogt and Roblin found considerable consistency, **with ICT literacy, collaboration, communication, and social and cultural skills** common to all, with a majority also **including creativity, critical thinking, and problem solving** (Voogt & Roblin, 2012). The OECD advocates for 21st-century skills in the curriculum[2]. However, international businesses and particularly technology industries were important in the early development of the educational movement for 21st-century skills.

Both the Scottish and Welsh curriculums have been influenced by the 21st century skills agenda. Scotland's Curriculum for Excellence, designed in 2004 following a 'National Debate on Schools in the 21st Century and implemented in 2010, is intended to help 'children and young people gain the knowledge, skills, and

[2] https://www.oecd.org/education/2030-project/

attributes needed for life in the 21st century[3]. The design is underpinned by a set of 'big ideas', and informed by design principles, an acknowledgement that curriculum making is a continuous process, requiring career-long collaborative teacher professional learning, engagement with local communities, and the development of shared aims. The approach is centred on four capacities—**to enable all young people to become confident individuals, successful learners, effective contributors, and responsible citizens—and includes a commitment to interdisciplinary learning**.

Interdisciplinary and cross-curricular innovation

There is increasing acknowledgement that responses to technological and societal changes to prepare students to **meet demands for new skills risk overloading curricula**. To address this, making connections across curriculum areas is one way to embed competencies such as digital skills, environmental awareness, financial literacy, and social and emotional skills. **Increasing interdisciplinarity is found in a number of systems,** for example, embedding computational thinking, digital literacy, and financial literacy in Estonia, and media literacy and global competency in mathematics in British Columbia (OECD, 2020c). Such **interdisciplinarity has been conceptualised through the extension of STEM to STEAM (Science, Technology, Engineering, Arts, and Mathematics**) and, in some cases, the concept of Arts extending to the Liberal Arts to embrace humanities (Colucci-Gray et al., 2017). University researchers have been important in initiating **STEAM projects and have gathered government support in some jurisdictions** (for example, Austria) and EU funding[4].

Linked to this is the recognition that a **focus on well-being** involves 'moving away from focusing solely on academic performance to a more holistic vision of students' and their interests (OECD, 2020a).

Mathematical modelling

Mathematical modelling, **the application of mathematics to real-world problems in order to better understand them,** is increasingly acknowledged as a significant competence in a wide range of employment and in everyday life. The goals of mathematical modelling and applications have been summarised in relation to two perspectives (Stillman, 2019, pp. 4–5).

From **a mathematical perspective,**

- a vehicle to teach mathematical concepts
- a means to teach mathematical modelling as mathematical content
- a way to promote mathematics as a human activity

From an **informed citizenry perspective,** as follows:

- supporting and contributing to social life
- important to question the role of mathematical models in society
- supporting social justice

Mathematical modelling is **required in the study of an increasing range of subjects** and had **increased in prominence in daily life during the pandemic**. Mathematical modelling is included in mathematics

[3]https://education.gov.scot/education-scotland/scottish-education-system/policy-for-scottish-education/policy-drivers/cfe-building-from-the-statement-appendix-incl-btc1-5/what-is-curriculum-forExcellence is
[4] For example, STEAMTeach https://www.steamteach.unican.es/

curricula in many jurisdictions, some with long traditions. There is increasing recognition that students need to be 'familiarized from an early stage of their education with the concept of a mathematical model' (ACME, 2011, p. 3), with, for example**, the recently revised curriculum for Grades 1–8 in Ontario including an expectation that students at all grades engage in mathematical modelling processes** (Suurtamm, 2020).

In Germany, **mathematical modelling has been included as one of six general mathematical competences in the mandatory standards for mathematics in middle schools** since 2003 and later added to the standards for primary and upper secondary schools (see Greefrath & Vorhölter, 2016, pp. 19–20). Although there is a long tradition of modelling in German schools, it remains an issue, with teachers and students finding it challenging. **Opportunities to support them have typically included one-day or week-long modelling projects run by universities for school students, with positive results** (Kaiser & Schwartz, 2006). Blum notes that what is needed is systematic implementation, 'with all system components collaborating closely: curricula, standards, instruction, assessment and evaluation, and teacher education' (Blum, 2015, p. 87).

Statistics, data literacy and data science

An important international development **is responding to the increasing importance of statistics, data literacy, and data science in disciplines, professions, and in citizenship**[5]. In **New Zealand**, the Ministry of Education recently commissioned an expert panel to report on **mathematics and statistics education, which is a single curriculum area** (Royal Society Te Apārangi NZ Expert Advisory Panel, 2021). The report identifies that the awareness of research on statistical education and knowledge is a particular area of concern against the background of general concern with teacher subject and subject pedagogical knowledge in mathematics (see Section 6). In the English National Primary Curriculum, statistics is restricted largely to the interpretation of simple data representations and calculation of averages, although there is also content in other subjects (Pittard, 2018). In **the new California Mathematics Curriculum framework data science is embedded from pre-school and is based from the start on principles of data science problem-solving processes** (including formulation of problems, data collection, analysis and interpretation)[6]. The Californian framework reflects the content of the USA **Common Core standards[7] in which statistics is a more substantial element than in the English National Curriculum**.

Inclusive curriculum and pedagogy

Equity is a concern internationally and, as an example, **features prominently in both the New Zealand and Californian policy initiatives** noted above. In New Zealand, the importance of recognising cultural diversity underpins a recommendation that a group of experts in Māori-medium mathematics and statistics be convened and make independent recommendations to the Ministry of Education. In California, equity is embedded as a central theme of the new curriculum framework. The approach differs from one of intervening to address particular needs or remedy perceived deficits but rather a reformulation of curriculum and pedagogy about **five components of 'equitable and engaging teaching**'[8]:

- Plan teaching around big ideas

[5] The International Data Science in Schools Projects is prominent in the application of this in the school curriculum http://www.idssp.org/

[6] https://www.cde.ca.gov/ci/ma/cf/documents/mathfwchapter5.docx

[7] http://www.corestandards.org/Math/

[8] https://www.cde.ca.gov/ci/ma/cf/documents/mathfwchapter2.docx

- Use open, engaging tasks
- Teach towards social justice
- Invite student questions and conjectures
- Centre reasoning and justification

Global citizenship

Education is included as **one of the United Nations' sustainable development goals, with a specific focus on global citizenship** (Unesco, 2014). This has been contrasted with a focus on the construct of global competence found in OECD texts (Vaccari & Gardinier, 2019). As yet, innovation in this area consists of local programmes as well as some small-scale international collaborations[9]. Given both post-COVID and climate change concerns in education, this focus may potentially become more important in the future.

3.3 Case study: Problem solving and curriculum change in Singapore

Relevance. **Mathematical reasoning and solving problems feature as key components in future visions for mathematics** internationally (OECD, 2019a) and in England (ACME, 2016). At the same time, enacting a problem-solving curriculum is challenging for teachers.

The case. **Problem solving is central to the mathematics curriculum in Singapore,** aligned with an overarching goal to encourage creative, flexible, and critical thinkers able to respond to the challenges of the twenty-first century.

Context and background. Singapore, a highly developed and economically successful city state with a stable education system, is highly centralised, with schools grouped in geographic clusters. **Students have frequently outperformed those across the world in international comparative assessments, including in mathematics.**

Teachers are educated at the National Institute for Education, the only national institution for pre-service education. Education research is well-funded and focused on pedagogy. Following initial teacher training, teachers in Singapore are entitled to 100 hours of professional development every year. The school directs some of this time, but the teacher is largely responsible for choosing the topic.

Innovation. Mathematical problem solving is a central focus of the Mathematics Framework (also referred to as the Pentagon Framework), which has been part of the Singapore curriculum since 1990 and underpins teaching, learning, and assessment from primary to pre-university. This 'pentagon framework' has been retained with periodic minor changes. The centrality of problem solving is evident in the first of three principles for teaching: 'Principle 1: **Teaching is for learning; learning is for understanding; understanding is for reasoning and applying, and ultimately, for problem solving**' (MoE, 2012, p. 21).

Before curriculum reforms, mathematics teaching was usually done through exposition followed by students practising routine exercises, presenting evidence that teachers experience difficulties reconciling the focus on problem-solving approaches with the need to prepare students for tests and national assessments (Foong, 2009, p. 279). Since 1997, a series of policy initiatives, cutting across subject boundaries, have aimed to address such issues, focusing on pedagogical practice through the provision of increased resources for teacher professional learning and the establishment of an independent educational research centre funded by the Ministry of Education. Key initiatives include Thinking Schools,

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[9] http://www.citizenship-and-mathematics.eu/

Learning Nation (1997), curriculum content reduction (1998), a focus on project work, incorporating problem-based learning as part of a move aimed at developing autonomous learners, and providing an integrated learning experience (2000). **In 2005, Teach Less, Learn More (TLLM) was introduced**, continuing this trend and seeking changes to teachers' practice. Reforms to school rankings and a focus on holistic education also aimed to shift the emphasis from examination success at the same time as increasing resource was directed at teacher professional learning (Wong, Kwek & Tan, 2020). Attention is now shifting to teaching mathematics through problem-posing, building on studies in China (see, for example, Zhang & Cai, 2021).

Implications. A **close collaboration among policymakers, education researchers, and teachers** has been important to innovation. This collaboration enables changes to be made in a carefully planned, incremental way. **Teachers' pedagogical practice and students' skills in mathematical problem solving are likely to continue to be refined as the curricula demands are reduced and, more importantly, assessment practices and culture change.**

4. Qualifications and assessment

4.1 Qualifications and assessment: policy and practice in high-performing systems

In all high-performing countries, certificated mathematics study is compulsory beyond 16 years old, and in most cases, a higher proportion study advanced mathematics than in England (Hodgen, Pepper et al., 2010; Hodgen, Marks et al., 2013). High-performing countries tend to have a transparent and simple relationship between school-leaving qualifications and subsequent educational pathways. **Many high-performing systems have well-established routes for mathematical study in vocational pathways**, including Korea, Singapore, Switzerland, and the Netherlands (Hodgen, Wake & Dalby, 2017).

Examinations at the end of school are relatively 'high stakes', with future progression into university or other routes depending on outcomes. Where accountability policies are connected to high-stakes assessment, the assessed curriculum is influential, dictating to varying degrees the enacted curriculum (Suurtamm et al., 2016, p. 5). **Curriculum review, technological developments, and an increasing focus on equity and issues raised during the pandemic have provided opportunities to review assessment design and practices.**

High-performing **systems differ as to the role of assessment at the end of the primary phase,** and there is evidence of policy change, with **Singapore's primary school leaving examination,** a high-stakes exam linked to system success (Schleicher, 2011), **revised in 2021 to reduce competition and stress and move towards a more holistic education** (Wong, Kwek & Tan, 2020).

Compared to curriculum reform, **qualifications and assessment are relatively more stable aspects of educational systems** in general, and this is also the case in high-performing countries. Nevertheless, as described in the next sections, a common feature of high-performing systems is the reform of qualifications and assessment to align with curriculum reform to address current social, economic, and technological developments.

4.2 Qualifications and assessment: International developments

Broadening the focus of assessment

Seminal research by Black et al. (2004) on assessment for learning (AfL) and subsequent outputs from this group continue to influence interventions and classroom assessment practice globally. **Reforms currently being implemented in Singapore and Shanghai place increasing emphasis on assessment for learning**, together with changes in summative assessment policy associated with an **awareness of negative effects of high-stakes assessment on student well-being**. A growing focus on changing assessment practice has resulted in teacher professional development on assessment for learning, a reduction in examinations and high-stakes tests, and a phasing out of streaming in favour of full subject-based banding[10], piloted in Singapore secondary schools from 2020 (Wong, Kwek & Tan, 2020). In **Shanghai,** reforms centre on the *gaokao* (national college entrance examinations) and **the introduction of formative assessment practices**

[10] 'Streaming' allocates students to teaching streams for all or the majority of subjects based on perceived ability. 'Subject based banding', otherwise referred to as setting or tracking is based on attainment in individual subjects. See Francis et al. (2019).

in schools (Reyes & Tan, 2018). In Singapore and Shanghai, there is also a **move towards criteria-referenced assessment rather than norm-referenced assessment**.

Developments attend to more sophisticated assessment design, aligned to curriculum goals and desired mathematical processes, to support teachers to 'deliver a rich and balanced curriculum' (Swan & Burkhardt, 2012, as cited in Suurtamm et al., 2016, p. 21). High-stakes assessment also offers possibilities for teacher learning and classroom practice when linked to professional learning. In **Japan,** for example, **sample assessment materials are shared with teachers, together with an outline of the reasoning that may result in a particular response**, enabling the study of misconceptions (Shimizu, 2011 as cited in Suurtamm et al., 2016).

In **Estonia,** curriculum reforms set out in the *Lifelong Learning Strategy 2021* called for a revision of national assessments to **measure key competencies and problem-solving skills**, integrating key subjects. Here too, there is a focus on formative assessment that supports learning.

There is an awareness that developing practice takes time. Teachers and students need time to experiment with changes and to consider the challenges that changing technologies bring in relation to assessment and questions of equity (Suurtamm et al., 2016, p. 18).

Mathematical success for all

Above, we identified inclusive curricula and pedagogy as important areas of development. This is reflected in **the development of alternative pathways** in college mathematics courses. In the USA, two Carnegie pathways have been developed[11] **the 'quantway' and 'statway'**. These pathways were developed as **alternatives to more traditional pre-calculus and calculus mathematics** courses. The **quantway curriculum focuses on critical thinking and developing quantitative literacy**. **The statway curriculum is centred on statistical skills and decision-making in conditions of uncertainty.** The two pathways are designed to enable more learners to gain mathematics credit at college level. Both **pathways have had positive impacts on student success and retention** (Huang, 2018).

Computer-aided assessment

Technology has an increasing influence on assessment in mathematics education, **facilitating the representation of complex, authentic problems and providing opportunities to practice, develop, and assess higher-order thinking skills**. At the same time, Hoogland and Tout (2018) **note pressures that act against these trends, focusing attention on lower-order skills.** These pressures include views of mathematics exemplified through debates on the use of calculators in school mathematics and a related 'back to basics' perspective where rote learning of mathematical facts and procedures is valued above the development of higher-order skills and 21st century skills**. To maintain focus on assessment of higher-order skills, they highlight the need for connections between curricula and practice and an analysis of mathematical needs to be made in policy, research, and assessment design**.

4.3 Case study: PISA 2021 mathematics framework

The case. **The framework for the mathematics PISA assessment** is the basis for assessment construction for PISA mathematics tests.

[11] https://www.carnegiemathpathways.org/

Relevance. PISA (Programme for International Student Assessment) has influenced the English curriculum and assessment directly, with the last major GCSE reforms involving benchmarking against countries ranked highly on PISA. **The PISA mathematical framework for 2022[12] (OECD, 2018a) will potentially influence curricula and assessment by informing policy development and indirectly, as success on assessments aligned with the framework may influence relative positions of countries.**

Context and background. In each PISA cycle, the principal domains assessed rotate, with mathematics being the last focus in 2012. Previous assessments have emphasised mathematical reasoning, contextualised problem solving with mathematics, and been informed by a mathematical modelling cycle:

- formulate (modelling a problem mathematically)
- employ (mathematics, including strategic choice of mathematics to use)
- interpret and evaluate (within the original context)

Since 2015, the main medium for assessment has been computer-based, with the option of pencil- and paper-based assessment.

Innovation. The framework is designed to compare student performance over time, and so most of the items in the PISA 2021 will be items that have been used in previous PISA assessments. However, the framework also refines and extends what should be assessed given societal change, developments in mathematics education, and educational policies and practices.

Key changes or increased emphasis in the 2021 framework are:

- the **assessment of computational thinking skills** as they apply to mathematics as part of their problem-solving practice
- the need to develop students' capacity **to use mathematics in context** and the need for rich classroom experiences to achieve this
- the prioritisation of **contexts that address important personal, occupational, societal and scientific issues**, including **making judgements about the validity of information** by examining quantitative and logical implications
- mathematical content areas categorised as **quantity, uncertainty and data, change and relationships, space and shape**
- particular emphasis **on non-linear growth phenomena, geometric approximatio**n (e.g., calculating area where simple application of standard formula will not suffice), **computer simulations** to explore complex quantitative problems, and **conditional decision making**
- a selection of 21st Century skills: **critical thinking, creativity, research and inquiry, self-direction, initiative and persistence, information use, systems thinking, communication and reflection**
- reflect the **use of digital tools when using mathematics** in workplaces and learning environments and, in this regard, make more use of the opportunities provided by computer-based assessment

Implications for England. In 2018, in the PISA mathematics assessment, England was ranked 18th, up from 27th, with the increase in score statistically significant (Sizmur et al., 2019). This improvement was due to improvements for lower-attaining students, with the performance of high-attaining students remaining stable. A possible explanation lies in curriculum and examination reform in England, which increased alignment between the secondary curriculum and PISA assessment. This included the introduction in GCSE assessment of a specific problem-solving paper and the change in tiers from three to two, improving

[12] PISA 2021 was delayed due to COVID, hence the reference to the 2021 framework, but will be used in 2022.

access to more challenging mathematics for low-attaining students. However, **England's improvement may be stalled or reversed given changes in PISA assessment, particularly in relation to the application of mathematics and use of digital tools and models.**

5. Resources and technology

5.1 Resources and technology: policy and practice development in high-performing jurisdictions

Apart from teachers, the most **important resources in nearly all high-performing systems are textbooks,** with their development and/or use **embedded in policy** and various processes that ensure quality. Textbooks offer a high degree of curriculum coherence and support effective teaching and professional development (Oates, 2014). One **exception to this is Estonia, where teachers use a wide range of source material**s[13].

There is **more variability in the use of digital technology**. In **China, the new high school mathematics curriculum includes programming as a core competence**; however, implementation of policies related to digital technology use is variable, particularly in rural areas (Jiang, Stephens & Wang, 2020**). Singapore has recently developed a digital learning strategy**. Singapore's strategy builds on attention to the role of technology in learning through the IT masterplan (1997) and masterplan II (since 2003). However, the extent to which policy translates into practice is disputed, with significant differences in enacted curriculum (Deng & Gopinathan, 2016). **Estonia has established policy and practice in computer-based mathematics and statistics with widespread use of tablets as a learning platform** (Hõim, Hommik & Kikas, 2016).

5.2 Resources and technology: international developments

E-textbooks

Innovations in textbooks and technology come together in the development of e-textbooks or digital textbooks (see Pepin et al., 2015). The most common developments so far are digital versions of paper-based textbooks or **integrative e-textbooks,** where publishers offer supplementary digital resources. There have been attempts to develop more **interactive e-textbooks** designed as solely digital artefacts and aiming to make use of embedded interactive learning objects, for example, figures or graphs that can be manipulated. Interactive textbooks may also aim to offer learners and teachers some choice about when materials are accessed. South Korea attempted to develop an interactive e-textbook alongside ones for other subjects; however, this project was paused due to the **lack of an appropriate authoring tool for embedding mathematical learning objects** (Lew, 2020). A third approach is digital versions of teacher-led curriculum development projects, notably the Sésamath system in France[14].

Digital technology and mathematics teaching

Innovations in the use of digital technology relate to how technology may 1) **change interactions** in classrooms and learning spaces between any pair or all of the learner, teacher, and mathematics, and 2) **interrelate mathematics and technology** (Hegedus et al., 2017).

[13] https://www.cambridgeassessment.org.uk/insights/mathematics-education-estonia-style/

[14] https://manuel.sesamath.net/

1) Changing interactions are found in:

- **changes in activity spaces**, e.g., dynamic geometry environments such as geogebra, virtual manipulatives, or games[15] and for calculations[16] using 'content-specific technologies' (NCTM, 2015)
- **classroom connectivity** e.g., SIMCALC[17] and Scratch[18] that change relationship of private (within a class and school) and public mathematics (including with other learners internationally)
- multimodal technologies to augment representations, for example, haptic feedback to bring touch into exploration of shapes, gestures, and motion sensors
- **applications using mobile devices** to collect data from outside the classroom

2) Interrelations between mathematics and technology include:

- **digital tools**: graphing software and apps, CAS for visualisation of relationships, outsourcing repetitive algorithmic tasks and the use of 'content neutral technologies' (National Council of Teachers of Mathematics, 2015)
- **discovering mathematical relationships through simulations and digital models** of a wide range of social and natural phenomena

Artificial intelligence (AI) and computer-aided learning

Computer-aided learning has a relatively **long history of mixed success**. Recent advances in computing power, artificial intelligence, and machine learning have led **to recent innovations, though they have not yet been taken up systematically in policy.** However, outcomes are likely to improve given new applications of AI and machine learning, as well as understanding of the importance of hybrid AI-human tutoring[19].

The Adaptive Learning Lab[20] proposes six levels of personalisation, from teacher fully in control to full automation. Applications have tended to be level 2, which is technological assistance to the teacher. However, **AI has allowed the development of tools with partial automation**. There is **evidence from recent examples of positive impacts on outcomes,** e.g., Snappet in the Netherlands (Fabder, Luyten, & Visscher, 2017) and Cognitive Algebra 1 in the US (Pane, Griffin, McCaffrey, & Karam, 2014).

Computational thinking and coding

Computational thinking is a term first used by Seymour Papert in 1980 in work that led to LOGO programming being incorporated into national curricula and was later discussed as **'thinking like a computer scientist**' (Wing, 2006, p. 35). Weintrop et al. (2016) developed a thorough definition of computational thinking in relation to mathematics education, clarifying the practices involved. This taxonomy comprises **'data practices, modelling and simulation practices, computational problem-solving practices, and systems thinking practices'** (p. 128). They argue that embedding computational practices in mathematics teaching practice requires the collaboration of teachers, researchers, curriculum developers, school leaders, and policymakers.

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15 (e.g. https://www.mathlearningcenter.org/apps and https://www.mathplayground.com/)
16 (e.g. https://www.wolframalpha.com/)
17 https://simcalc.sri.com/
18 https://scratch.mit.edu/
19 https://www.oecd.org/education/oecd-digital-education-outlook-7fbfff45-en.htm
20 https://www.ru.nl/bsi/research/group-pages/adaptive-learning-lab-all/

Several jurisdictions have begun making changes to curricula. **Korea, Taiwan, Hong Kong, and China have launched national curriculum reforms to incorporate computational thinking into the school curriculum**. In Australia, the National Innovation and Science Agenda funded a programme to support the teaching of coding in primary schools, leading to the creation of a Digital Technologies Hub that is valued by teachers as a source of accessible, high-quality resources. **A teaching experiment using Scratch in a Year 2 class indicated the potential for coding contexts to support higher-level thinking in mathematics,** whilst noting the challenge aligning the mathematics and digital technology curriculum statements and planning and teaching coding posed for generalist primary teachers (Miller & Larkin, 2017). This challenge of interdisciplinarity requires clarifying the role of computational thinking in mathematics, digital technology, and other curricula areas.

Personal digital devices

Policy developments regarding access to digital devices offer potential to capitalise on the opportunities afforded by developments in digital approaches to mathematics teaching and learning. **In Singapore, as part of a wider digital literacy policy, by 2021, all learners will own their own school-prescribed personal learning device**. A similar national policy in Estonia aims for access to a computing device as part of a three-strand strategy of:

1) digital culture integration with teaching and learning

2) development and accessibility of digital teaching materials

3) improving access to technology in schools (Lorenz, Kikkas, & Laanpere, 2016).

5.3 Case study: Sweden and programming in mathematics

Relevance. **Sweden has attempted to integrate computing and use of digital tools into the mathematics curriculum**

*The case***.** Sweden introduced a new national IT strategy for schools that includes, starting in 2018, **programming as a core content of the curriculum for pupils of all ages, including in mathematics** and technology subjects. Mathematics, along with other subjects, was 'digitally remodelled'. The professional development of teachers was required to support this change (Heintz et al., 2017; Vinnervik, 2020).

Context and background. The policy context is one where independent government-funded agencies have considerable power over how policy is developed and implemented, but this is mediated by the influence of various groups and organisations that lobby effectively. Sweden's approach to computing in the curriculum was historically similar to England's, with, for example, a specialist computer for schools with a similar role as the BBC micro. Early explorations of computational thinking and coding within subject curricula, led by enthusiasts, were replaced in the nineties with a focus on learning about computers and using digital tools rather than learning with computers. Outside of the formal curriculum, schools, universities, and industry developed voluntary initiatives to offer extracurricular access to computing and support teachers' knowledge and skills**. Explorative projects**, often collaborations between teachers and researchers**, explored possibilities for integration of programming into a wide range of subjects. However, these were localised and fragmented.**

In 2012, inquiries began into educational needs, spurred in part by concerns that the curriculum did not equip all students with skills for an increasingly digitalised world and specifically the labour force's need for programmers. Following a review of the curriculum, **reforms were introduced along with recommendations for professional development for teachers, including mathematics teachers.** The decision to strengthen computing in the curriculum through other curricular areas rather than as a discrete subject, as is the case in England, stemmed from both curricular goals and aims and pragmatic reasons given the lack of teachers with the required skills to teach computer science across grades 1-9. However, this choice still led to significant professional development needs because most Swedish mathematics teachers lack subject and pedagogical knowledge and experience in programming.

Innovation. The new curriculum has four goals focused on the integration of computer science into the other school subjects**:**

1) understanding how digitalisation affects individuals and society

2) understanding and knowing how to use digital tools and media

3) critical and responsible usage of digital tools and resources

4) being able to solve problems and implement ideas in practice.

The focus is not on coding skills but on programming as a pedagogical tool and problem-solving process (Heintz et al., 2017).

In mathematics, coding is mainly located in the Algebra and Problem Solving strands, which provide context for computing across the first nine years of school.

Grades 1-3: Introduction to algorithms and symbolic representation
Grades 4-6: algorithms applied to programming in visual programming environments
Grades 7-9: different programming environments and the creation, testing and iterative improvement of algorithms for mathematical problem-solving

Other curriculum areas, most importantly in technology, complement the integration of programming into the mathematics curriculum.

To support the curriculum reforms, the National Agency for Education, an agency of the Ministry for Education, has developed a 16-hour online introductory course about programming recommended for the whole teaching profession. **Mathematics teachers are expected to take part in more in-depth professional development**. One element of this is an **introductory programming course (7.5 university credits, equivalent to 5 weeks of full-time study**). This course is generally studied online and may be of variable quality. Changes are planned to include such courses in initial teacher education. Further professional development to develop pedagogical subject knowledge and pedagogical skills is available and recommended (Vinnervik, 2020).

The following issues may mean policy goals will not be realised:

- a lack of high-quality appropriate and research-informed curriculum materials available nationally
- the relatively low base in terms of teachers' skills and knowledge to integrate computing into mathematics successfully
- that this area of curriculum or use of digital tools in mathematics is not assessed and is therefore not a priority for teachers

Implications for England. Sweden is taking a different approach from England's focus on computing as a discrete subject and, rather, is integrating programming into subjects. Sweden's curriculum reforms are recent, and it is too soon to judge whether the approach taken will meet the policy goals. However, how the policy is enacted in practice, even by the most enthusiastic teachers, varies a lot. Regardless, Sweden's overall approach of integrating coding into other subjects is unlikely to gain consensus in the UK as a policy change, given the commitment to computing as a subject in the National Curriculum in England and recent investment in the Computing Hubs network. However, **the Swedish model offers an example of how aspects of computer science could be integrated into mathematics in a complementary way to the benefit of both school subjects.**

6. Teacher preparation and professional development

6.1 Teacher preparation and professional development: policy and practice in high-performing systems

Teaching quality and **teacher education are key factors in system success** (Darling-Hammond & Rothman, 2011). Initial teacher education (ITE) varies in high-performing systems in relation to entry requirements and selectivity, length of courses, depth of study, course content, and amount of practical experience. Even in relation to individual variables within ITE for particular phases, comparisons are not easily made given contextual differences and details. However**,** as a general comparison with England**, in high-performing systems, initial teacher education is longer, university educators and researchers are more influential, and greater prominence is given to subject and pedagogical subject knowledge** (OECD 2018b). For example, in Shanghai, primary mathematics teachers are subject specialists.

Considering post-qualification professional development (PD), **entitlements and expectations of engagement are often identified in policy**. However, similarly to ITE, summarising these is inappropriate as, in some contexts, engagement in **professional development is embedded in teachers' regular working lives rather than an additional activity**. Professional development entitlements differ from the statutory five 'in-service training days' in England. Elsewhere, **teachers have greater autonomy about how they use their time to access accredited courses** and often more time available for professional development; for example, in Singapore, teachers have 100 hours of professional development per year, and in Shanghai, teachers report participating in twice as many professional development events per year as in England (OECD, 2019b). **As with ITE, the role of universities is prominent.** For example, the system of teacher research groups that is a feature for all mathematics teachers is linked to university research and researchers (Boylan et al., 2019). The role of higher education is also found in the importance of advanced degrees as part of professional development; for example, **in Singapore, teachers have funded opportunities to engage in masters and doctoral study** (OECD, 2011).

More important than individual components is **a single system approach to recruitment, preparation, induction, professional development, evaluation, progression, and retention success** (Darling-Hammond & Rothman, 2011).

A notable feature of high-performing systems is **a commitment to collaborative professional development,** for example, lesson study in Japan (Fujii, 2014) and teaching research groups and professional learning communities in Shanghai (Zhang et al., 2020).

6.2 Teaching preparation and professional development: international developments

Professional learning entitlement

There is increasing acknowledgement of the role of continuing professional learning opportunities to enable teachers to develop knowledge and skills, learn how to maximise the potential offered by new technologies, respond to new curricula, and engage with emerging research. At the same time, as pathways into teaching diversify, continuing professional learning takes on increased significance (see case study).

A recent OECD policy brief (OECD, 2020b) highlighted the introduction of **entitlement to professional learning, protected time, and finance to support teachers' engagement in professional learning in a number of systems.** To be effective, this was provided alongside professional development opportunities that built on evidence of features of effective professional learning, such as learning that was closely related to practice, focused on specific content, provided coaching and expert support and modelling, and was collaborative and sustained.

Collaborative professional development

In mathematics, significant developments **include lesson study and collaborative professional learning supported by teacher learning communities.** The OECD (2020b) acknowledges that embedding such collaborative professional learning in schools and in wider systems takes time and considerable resources. Examples of recent practices include Communities of Learning (Kāhui Ako) in New Zealand and 'Boost for Mathematics', a successful large-scale professional development programme in Sweden.

Lesson study, originating in Japan over a century ago, has been implemented in mathematics worldwide, though with misconceptions reported in some cases (Fujii, 2014). **In Japan, 'lesson study becomes a lifelong activity as a form of teacher-led professional development'** (Fujii, 2014, p. 14), set in socio-cultural contexts. Teachers and researchers have collaborated in other jurisdictions, developing models that may be more culturally appropriate— for example, collaborative lesson research in the US (Takahashi & McDougal, 2016).

Distance learning

The prevalence of distance and blended learning in teacher professional development has increased (OECD, 2020d). The Australian Government funds professional learning through Mathematics Massive Open Online Courses, providing 'professional development for teachers of foundation to Year 10 students. This is supported by face-to-face professional learning and a repository of teaching and learning resources through an online Mathematics Hub'[21] developed by a national, non-profit ministerial company. In Scotland, all teachers have been required to engage in professional learning since 2014[22]. In addition, blended learning models have been used with some success, for example, in Ireland (see case study below). Access to online or blended courses has been increasing due to the COVID-19 pandemic.

Focusing on content and pedagogical content knowledge

There is policy concern in many jurisdictions about mathematics teachers' content and pedagogical content knowledge (for example, Royal Society Te Apārangi NZ Expert Advisory Panel, 2021). The case study below describes in detail one response to this issue in Ireland to address the professional learning needs of non-specialist teachers of mathematics. Additionally, in Section 7.4, we highlight the case of The German Centre for Mathematics Teacher Education (DZLM) as a policy intervention by a consortium of universities that arose out of a concern that the quality of post-qualification professional learning did not match the quality of initial teacher education. A common feature internationally is university accreditation of professional development through masters-level courses. Although knowledge is an important focus for professional development, teacher enquiry and research are means to develop knowledge and skills, and teachers are seen as researchers.

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[21] (see https://www.dese.gov.au/australian-curriculum/online-teaching-and-learning-resources-support-mathematics-and-numeracy)
[22] https://www.education.gov.scot/Documents/PRDJan14.pdf

As well as concerns with generic teacher knowledge, a specific area of concern is digital content and pedagogical knowledge. In Singapore, an enhanced Professional Development (PD) Roadmap for teachers, "SkillsFuture for Educators," will focus on enhancing teachers' practice of E-Pedagogy alongside other areas of practice.

A second area of specific concern is the professional development needed to support teachers to embrace an increased focus on data science and statistics in mathematics. In the United States, the *American Statistical Association* published comprehensive recommendations for the content needed and ways to enhance the statistical education of teachers (Franklin et al., 2014).

6.3 Case study: Professional Diploma in Mathematics for Teaching (PDMT) in Ireland

Relevance. Issues of teacher supply are particularly acute in mathematics, and in Ireland, as in England, there are significant numbers of non-specialist teachers teaching mathematics.

The case. **The Professional Diploma in Mathematics Teaching (PDMT), funded in Ireland since 2012, develops teachers' mathematics knowledge**. The PDMT was established after a study that found that almost half of those teaching secondary mathematics were not adequately qualified (Ní Ríordáin & Hannigan, 2009).

Context and background. Concerns were raised about student achievement in mathematics in 2008, when the Expert Group on Future Skills Needs identified key issues and made proposals for policy initiatives to address performance, particularly in Leaving Certificate examinations (EGFSN, 2008). **The need to enhance secondary teachers' mathematics knowledge**, was identified given that only an estimated 20% of these teachers had studied mathematics as a major subject beyond the first year of their degrees (2008, 8). Subsequently, Ní Ríordáin and Hannigan (2009) found that of post-primary mathematics teachers sampled, 48% did not have a mathematics teaching qualification. These 'out-of-field' teachers were typically assigned to teach Ordinary level (rather than Higher) classes, generally in non-examination years, and a range of other foundation courses.

In 2012, the Irish Government funded a programme to upskill mathematics teachers, **run by a consortium of universities** under the leadership of the University of Limerick and the National University of Ireland, Galway. **Over six cohorts, 1100 teachers have participated in this programme, all funded by the Department for Education and Skills** (Goos et al., 2020).

Innovation. The PDMT was designed as **a part-time, flexible blended learning programme with a modular structure, run over two years,** with the expectation that teachers would engage in the programme in the evenings, weekends, and during the summer holiday. The programme carries credits, and participants have mathematics added to their Teaching Council registration on completion of the PDMT, but no additional incentives (including time) were made available for teachers to study. The programme aimed for participation from at least one teacher in every post-primary school. A survey of PDMT graduates reported 'a substantial change in their mathematics teaching practices towards the problem-solving orientation promoted by the [...] curriculum' (Goos et al., n.d.).

Alongside this, developments in general education policy focused on teachers' professional learning, introducing Cosán, a framework to guide teachers' learning, in 2016 (The Teaching Council, 2016) after extensive consultation with teachers. This was designed to be a facilitative, flexible framework that acknowledged teachers as autonomous learning professionals. Drawing on literature and consultation with teachers and other stakeholders, the framework recognised a range of dimensions in teachers' learning (formal/informal, collaborative/individual, personal/professional, and school-based/external), together with a range of 'learning processes' and 'learning areas'. Recognising the 'degree of cultural change' both for teachers and the education system, the framework ran in a voluntary development phase from 2016–2019, supported by teacher-led research (The Teaching Council, 2016, p. 25).

Implications. The **PDMT programme, run by a collaboration of universities and funded by the government, is 'unique internationally in its scale, longevity, and education policy alignmen**t, with significant potential for transfer of learning to other countries and disciplines' (Goos et al., 2020).

7. Evidence-informed policy development internationally

In this section, we turn to focus specifically on evidence-informed policy development in mathematics education internationally, and the processes by which innovations influence national policy agendas. To do this we:

- examine the relationship between innovation and policy development, including features of policy development processes in high-performing systems
- summarise current policy influences
- include a case study of policy influence, The German Centre for Mathematics Teacher education (DZLM)

7.1 Education innovations and policy development processes

The relationship between education innovation and policy development processes is complex. Figure 1 offers four simplified models to support comparison of processes internationally.

Education innovation systems

Figure 1 presents elements of educational innovation systems found internationally. Each of these five elements may be used, alone or in combination, for various purposes:

- **the origin of innovations and change**
- **the propagation**
- **and/or innovation influencing policy**

These components may extend beyond a single educational system, as exemplified by transnational educational businesses. Similarly, the scale of actors can vary considerably. The category of third sector not-for-profit educational organisations spans from small educational charities and associations with relatively few employees or members to large, powerful organisations such as the OECD. To add to the complexity of educational innovation systems, such third sector organisations may be sponsored by or even be the arms of private sector businesses—for example, the Gates Foundation in the USA and its role in the development of the common core standards.

Further, **in one system, an element may be important to the emergence or creation of an innovation, but in another system, a different element may be important to its introduction**. An example of this is lesson study, which originated and was rooted in practice in Japan but was introduced into other systems often by university researchers, and, in some cases, propagated or encouraged by a variety of third sector organisations. In relation to different aspects of mathematics education (as in the categories comprising previous sections above), different elements may be more important; for example, innovation in digital technology often results from educational businesses or university research.

The model presents types of potential elements that vary in their importance in different educational systems, both generally and specifically in relation to each educational innovation. This complexity means that **the concept of 'trend' is problematic when applied to the relationship between education innovation and policy, as the landscape has more of a mosaic than uniform quality.**

Table 5 below provides examples of the role of different elements in innovation in digital technology:

Table 5: Innovation in digital technology

Element	Example
Practice (schools, school leaders, teachers)	Individual enthusiast teachers taking up support to offer codedojo, codeclubs or engaging in Bebras (international computational thinking competition)
Educational business R&D	Movement for digital and ICT as part of 21st Century skills curriculum; development of educational software
Third sector not-for-profit educational organisations	Sweden's policy to introduce programming builds on the international movement to provide access to coding through after school and out of school clubs
State organisations	South Korea's development of e-textbooks; personal learning device policies
University research	At different scales, from local small collaborations with groups of teachers as 'proof of concept' to international infrastructure such as Scratch

Innovation policy systems

Figure 1 also depicts three models of the relationship between education innovations systems and policy[23]:

a) **linear innovation policy system**

b) **parallel innovation policy system**

c) **embedded innovation policy system**

The linear innovation policy system implies a policy cycle where policy needs and aims drive innovation through mobilisation of the innovation system. This model simplifies important features of policy innovation in Singapore and China, for example, in relation to Singapore's problem-solving based curriculum. Similarly, the Irish PDMT arose from identification of a policy need, and a consortium of universities was commissioned to develop the programme. Motivations for policy change or formulation of policy needs in such cases may arise from external events and changes, and educational research. In the case of New Zealand, results from PISA were influential. In Ireland, research on teacher qualifications was important.

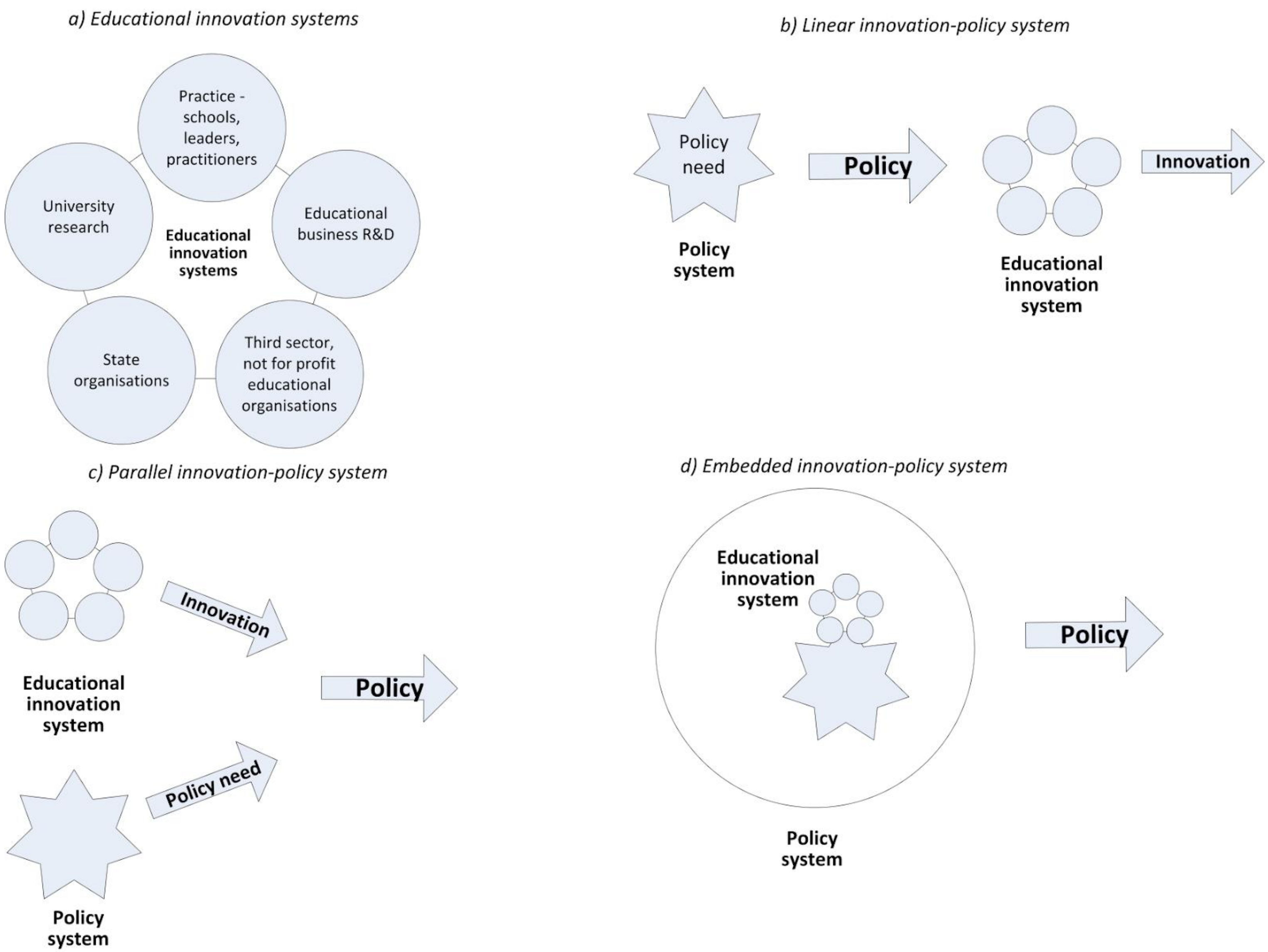
Specific educational innovations aside**, a common feature of high-performing systems is a set cycle for policy development,** with Estonia, Japan, South Korea, Singapore, and Ontario having a 5-to-10-year cycle. Central to policy development cycles is the curriculum, with other system features being reviewed, developed, or reformed to align with or support curriculum development**. Hong Kong is the only high-performing system that has a similar approach to UK nations in that it undertakes reforms as appropriate** (OECD, 2020c).

[23] These three models are informed by and simplify generic models and analyses of policy development, including policy cycles, multiple streams models, and the advocacy coalition framework (see Cairney, 2019).

A parallel innovation policy system represents features of Sweden's programming curriculum developments, where **independent organisations and bodies, including university researchers, developed local projects and national extra-curricular initiatives in parallel with policymakers identifying the need for curriculum reform. T**hese two strands then came together, resulting in the new policy.

A third model posits the educational innovation system as being part of the policy-innovation system and the various possible elements within the innovation system, including 'advocacy coalitions'. The Advocacy Coalition Framework (Sabatier & Weible, 2019) **models the importance of networks and interest groups in the development of policy.** The current reform processes in New Zealand (see Section 6.2, above) might appear to correspond to a linear pathway in which the ministry of education commissioned a report on mathematics and statistics education curriculum reform. However, it also had features of the embedded model, at least considering two aspects of both the commission and the aim of the expert group's task. The first of these is the importance and prominence of the development of addressing issues of equity and the need for a further independent study of the Māori-medium pāngarau education curricula. Here, the importance of coalitions that seek to address the particular historical legacy of the treaty settlements that followed the arrival of Europeans into Māori lands is highlighted. The second aspect is the profile and the long-standing importance of statistical education in New Zealand, in part due to the policy influence of the New Zealand Statistical Association.

Figure 1: Models of education innovations and policy systems



7.2 Policy influences

In the interviews and workshops, participants **identified forces and concerns influencing mathematics education innovation and policy development. Further influences were identified from horizon scans of particular innovations and policies**. Given the limitations of the research undertaken, the identified influences should be read as indicative rather than a comprehensive audit of relevant factors globally. They are organised below thematically in relation to policy context, design, and stakeholders (see Table 6). A model of policy implementation informs these policy aspects (Viennet & Pont, 2017). They are further categorised as influences that either promote or inhibit innovation and change.

Table 6: Influences that promote or inhibit policy development and implementation

Policy aspect	Promote	Inhibit
Context	Societal and economic change	Frequent policy changes
	Technological developments in education	Performativity and accountability measures
	Increasing concerns with global issues such climate change, decolonisation, global citizenship	Marketisation of education
	Influence of the OECD	
Design	Increasing teacher professionalisation in some systems; teachers as partners in policy development	Benchmarking against past as **the** 'standard'
	Remote education due to the pandemic	National assessment determining curriculum, pedagogy and professional learning
	Research innovation and development	Deprofessionalisation of teaching with teachers as technicians, training models and alternative certification routes
Stakeholders	Equity commitments	Politicisation of mathematics education (e.g., 'math wars')
	Collaboration of a range of stakeholders in education policy development	'Back to basics' influences

7.3 Features of effective policy development and implementation in mathematics education

Drawing on the horizon scans and expert contributions, we turn to factors that are important in successful policy development and implementation in mathematics education (and where they are absent, it may help to explain the lack of success).

The following features are associated with successful policy development:

Purpose

Clear vision of policy purpose, rooted in an articulated purpose and nature of mathematics education.

Consensus

Through dialogue that involves stakeholders, particularly around ensuring concerns are addressed about the role of basic mathematical skills in relation to mathematical thinking.

Feasibility

This is particularly relevant to transnational policy movements—what is often referred to as policy borrowing or contextualised adaptation where feasibility might be appropriate in a given context.

Coherence

Multiple simultaneous innovations can lead to a lack of coherence between innovations in mathematics education and other educational policies, or between mathematical educational innovations.

Systemic alignment

Alignment has two aspects:

- between curriculum, pedagogy, assessment and teacher professional development and how lack of such alignment can stifle innovation
- with wider system issues such as teacher professional conditions, accountability measures and marketisation

Piloting and sequencing

Piloting an initiative, depending on scale and governance structures, before wider changes. Professional development is taking place in parallel with or even leading changes to curriculum, pedagogy, qualification, and assessment.

Sustained attention

As noted in discussion of policy development, high-performing systems tend to have longer policy cycles in recognition that change takes time. There are positive examples of long-term policy commitment to developing practice, initiated by the state and policymakers, for example, Singapore problem solving (see case study, section 3.3, Ireland 'out of field' teacher development).

Collaboration and relationships

This relates to the importance of creating opportunities for dialogue between policymakers, mathematics education researchers, teachers, and other stakeholders. There is a growing acknowledgement that recognising teachers as important actors in policy development and enactment brings multiple benefits, including greater ownership of policy. Successful local, regional, and national innovations build on collaborations between teachers and mathematics education researchers, bridging gaps between formulated policy and enacted policy.

7.4 Case study: The German Centre for Mathematics Teacher education (DZLM)

Relevance. DZLM aims for mathematics education policy in Germany to be based on research and evidence. It works in a complex policy environment, **seeking to develop national consensus among disparate systems** at state level.

The case. The DZLM was established in 2011 as a consortium of nine universities with initial funding from a telecoms company. Subsequent funding has been through research grants and for specific programmes.

Context and background. The DZLM was initiated out of a concern that in-depth, up to 5 years, initial teacher education was not matched by professional development of a similar standard. A particular concern was the variability across Germany, with education policy in Germany being the responsibility of each of the 16 states rather than a federal responsibility.

Innovation. The **DZLM undertakes research on mathematics teaching and subject-specific professional development and makes its results available for practical use.** Its scope encompasses early, primary, and secondary education and cross-phase foci on research, development, and digitisation. DZLM organises its offers as PD materials (ready-to-use PD materials for their PD course); teaching materials; meetings with a main focus on support for PD leaders; courses for those responsible for preparing PD facilitators, for example from state institutes; a master's programme for current or future PD facilitators; PD courses, and programmes for teachers, often undertaken in collaboration with a state institute and tailored to a state's needs.

The DZLM has **engaged with policymakers at the state level, seeking to directly influence policy and indirectly influence it through specific programmes of work**. Policy engagement built on and broadened relationships between policymakers and a small number of eminent mathematicians and mathematics educators. The DZLM **has initiated a 'State Advisory Council' with representatives from ministries of education and federal state institutes.** The State Advisory Council both influences the work of the DZLM to address issues important to policy and acts as means for DZLM to influence policy. The orientation towards policy dialogue means that the DZLM seeks to develop programmes that meet policy priorities, for example, issues of inclusion in mathematics learning and students for whom German is not their home language. Considering the three models of the relationship between educational innovation and policy development presented above, the DZLM has, over time, embedded itself within the policy development processes.

Implications for England and the UK

The DZLM's vision and aims are long term to develop research informed practice and to influence policy and practice at the state level. The move to develop policy originated outside the policy and governmental fields in the research and practitioner communities**. DZLM suggests that sustained engagement with policy processes over time can be fruitful.**

8. Lessons for future possibilities for mathematics education in England

Table 7 below summarises an assessment of the extent to which the innovations and developments identified above are present or absent, and if present, whether this is to a limited extent in mathematics education in England compared to international innovation and developments. Whether either convergence or divergence is desirable is a separate issue that relates to an overarching vision for mathematics education.

There is considerable divergence in England from international innovations in education policy, albeit with some evidence of change more recently that mirrors some international developments. The analysis of policy development presented in Section 7 emphasises the complexity of policy development and intervention. Nonetheless, the features associated with successful policy development can inform thinking about possibilities for mathematics education in England.

Teacher preparation and teacher development are two areas where policy in England has some convergence with international developments. However, this is in relation to the form and infrastructure to support professional development rather than necessarily to the content. Resources and technology are the areas with the most divergence.

Table 7: Comparing international developments and innovations to England

Mathematics education developments and innovations	Similarity to England	Details and examples of policy and practice in England
		Curriculum and pedagogy
21st Century skills	No	Current antipathy to skills – dominant focus on knowledge-rich curriculum
Interdisciplinary and cross-curricular innovation	No	Reduced focus on STEM in compulsory schooling, emphasis on individual subjects
Mathematical modelling	Limited	Introduction of core maths
Statistics, data literacy and data science	Limited	Reduction in statistics in KS1 and KS2, opportunities in Core Maths
Inclusive curriculum and pedagogy	Limited	Teaching for mastery in primary learners progressing together
Global citizenship	No	No, but recent govt commitments on education on climate change
		Qualifications and assessment
Broadening the focus of assessment	Limited	At KS1 & KS2 narrower focus; In KS4 GCSE, some focus is on problem solving also in Core Maths
Mathematical success for all	Limited	Move to two tiers in GCSE, however, GCSE resits policy development was unsuccessful
Computerised assessment	Limited	The times table test will be online
		Resources and technology
E-textbooks	No	A general issue in England is the multiplicity of curriculum resources and of varying quality
Digital technology and mathematics teaching	No	Policy movement away from technology use towards 'back to basics' traditional teaching
AI and computer aided learning	No	Post-COVID England has taken an approach of remote human tutoring
Computational thinking and coding	No	Computing policy is separate from mathematics education policy
Personal digital devices	No	Discouraged – mobile phone bans encouraged in schools
		Teacher preparation and professional development
Professional learning entitlement	Limited	ECF, but subject-specific content is low
Collaborative professional development	Limited	NCETM and the Maths Hub Network
Distance learning	Limited	Through the NCETM portal – post-COVID more blended PD approaches used
Professional learning to develop subject ad pedagogical subject knowledge	Yes	NCETM and Maths Hub network, but countervailing influences of generic PD in the ITT Core Content Framework and Early Career Framework.

9. Conclusion and recommendations

In many education systems, particularly those that are high-performing, there is considerable innovation and policy development in mathematics education. These developments are responses to ongoing social, economic, and cultural changes and challenges. **Mathematics education in England already diverges from many features found in high-performing education systems. This divergence is unlikely to change without renewed innovation and policy development in England.** We have identified some limited alignment in relation to some areas of curriculum and pedagogy, such as problem solving and data science. However, even here, innovation is marginal to the main mathematics national curriculum and policy. Recent innovation in post-16 with the introduction of Core Maths is one area of greater alignment. **English policy for mathematics teacher professional development has relatively more features or similarities to those found in high performing systems or the focus of innovation elsewhere**. The roles of the NCETM and the Maths Hub network are important here. This is a positive feature for future potential innovation in other areas, given the importance of professional development in policy development and implementation.

Given the nature of educational policy making processes in England, innovation is more likely to be successful if a parallel approach is adopted: i.e., developing innovations in parallel with main policy developments. As and when external influences, including long term relationships, and potentially shocks (Viennet & Pont, 2017) lead to policymakers' identification, the existence of already piloted innovations will mean a greater chance of policy take up. Three areas with the most potential and need are:

- a coherent and cross-phase approach to the use of digital technology in mathematics
- opportunities for enhancing coding and computational thinking in mathematics
- the integration of data science into the mathematics curriculum

In each of these three areas, it would be fruitful to engage in a more in-depth review of both comparative policy development internationally and promising, more localised, innovations and programmes. Such reviews could then inform the development and enhancement of collaborations and guide investment in pilot innovations. In addition, they could support consensus building across stakeholders.

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Appendix 1: Methods

Identification of foci for horizon scanning

The scope of the initial horizon-scanning activity was initially deliberately broad, considering wide-ranging proposals encompassing future directions in global reform both in and beyond mathematics education. It was informed by an analysis of the call for views, the analytic frame for WP1, and included horizon scans of high-performing systems and education future scans. Key sources included ICMI topic surveys, research reviews, and OECD and TIMMS studies. The initial scan foci were refined through examination of research evidence and consultation with experts.

Individual interviews

International experts in mathematics education were identified for each of the initial themes, drawing on a range of sources, including ICMI topic surveys, research reviews (including from the OECD and TIMMS), and consultation with the Royal Society's MFP team. Experts were invited to participate in individual interviews. Prior to the interview, participants were sent an overview of the project, the initial themes arising from the horizon scanning, and an outline of the interview focus. A total of 10 interviews were conducted in 2021.

Workshops

Workshops, informed by initial horizon scanning activity and individual interviews, were conducted to identify key issues, conflicting perspectives, and the implications of initiatives for England. Experts in international mathematics education were identified based on the mathematics educational framework categories: curriculum and pedagogy, qualifications and assessment, teacher preparation and professional development, technology, and resources. Each workshop also considered two further cross-cutting categories in relation to the central foci - purposes, values, and systems, and structures, incentives, and drivers. Prior to the workshops, participants were sent an overview of the project, an outline of areas arising from the horizon scans and initial analysis of interview data, and details of key questions. Twenty experts were consulted (3-6 in each workshop).

Analysis

Data from interviews and workshops with experts were analysed thematically. Together with the analysis of documentary evidence, this informed case studies of identified innovations.

Appendix 2: PISA 2018 Mathematics results by country[24]

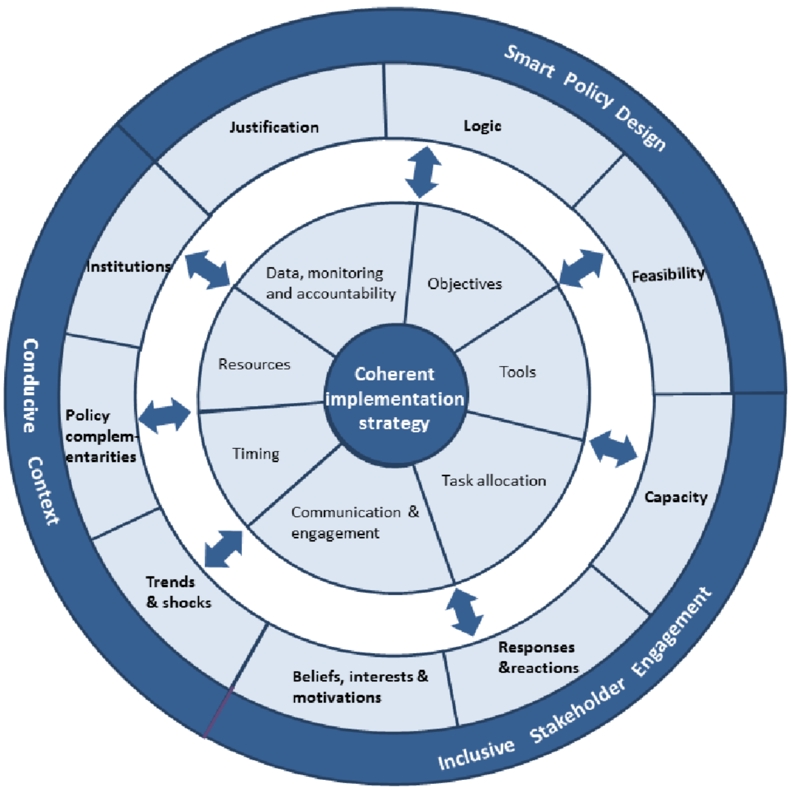
1.	China (Beijing, Shanghai, Jiangsu, Zhejiang)	591
2.	Singapore	569
3.	Macao	558
4.	Hong Kong, China	551
5.	Taiwan	531
6.	Japan	527
7.	South Korea	526
8.	Estonia	523
9.	Netherlands	519
10.	Poland	516
11.	Switzerland	515
12.	Canada	512
13.	Denmark	509
	Slovenia	509
15.	Belgium	508
16.	Finland	507
17.	Sweden	502
	United Kingdom	502
19.	Norway	501
20.	Germany	500
	Ireland	500
22.	Austria	499
	Czechia	499
24.	Latvia	496
25.	France	495
	Iceland	495
27.	New Zealand	494

[24] Available from: https://www.oecd.org/pisa/data/

Appendix 3: Policy development and implementation

As noted, and used to inform 7.2, the OECD promotes a framework of three dimensions to support a coherent implementation strategy: smart policy design; inclusive stakeholder engagement; and a conducive institutional, policy, and societal context, represented in Figure 2 (Viennet & Pont, 2017). Figure 2 shows factors that influence, and are influenced by, policy implementation. The process highlights the 'specificity of policy, stakeholders, and local context' (Viennet & Pont, 2017, p. 44). This model has the potential to inform future policy development and address the concerns raised above.

Figure 2



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