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An options-pricing approach to election prediction

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Abstract

The link between finance and politics (especially opinion polling) is interesting in both theoretical and empirical terms. Inter alia the election date corresponds to the effective price of an underlying at a known future date. This renders a derivative pricing approach appropriate and, ultimately, to a simplification of the approach suggested by Taleb (2018). Thus, we use an options-pricing approach to predict vote share. Rather than systematic bias in polls forecasting errors appear chiefly due to the mode of extracting election outcomes from the share of the vote. In the 2016 US election polling results put the Republicans ahead in the electoral college from July 2016 onwards. In the 2017 UK general election, though set to be the largest party, a Conservative majority was far from certain.

Keywords: Behavioural Finance; Complexity in Finance; Econophysics; Forecasting Applications; Real Options
JEL classification: C53, D72, E17, G10

1 Introduction

The interplay between finance and politics is interesting (Prechter, 1999; Wang et al., 2009). Of particular interest is the analogy between opinion polling and behavioural finance (Fry and Brint, 2017; Wu et al., 2017). This is given added prominence by recent concerns raised over behavioural effects in polling and apparent forecasting inaccuracies (Hopkins, 2009; Coppock, 2017; Brownback and Novotny, 2018; Kimball, 2019). Moreover, financial tools and techniques are well-suited to operational aspects of political prediction problems (Taleb, 2018).

As prototypical examples of complex social systems finance and politics share complementary features (Prechter, 1999; Sornette, 2003). Behavioural aspects (Forbes, 2009) underpin both settings. A common concern with opinion polls is Socially Desirable Response bias. Documented examples include the Bradley or Whitman effect, where polls exaggerate the level of support for black and ethnic minority candidates (Hopkins, 2009), a Whitman effect similarly biased against female candidates (Hopkins, 2009), and the Le Duc law whereby risk-averse voters are more likely to support the status quo (Clarke et al., 2017). The effect holds across different countries and includes concerns over a Shy Trump effect in the US (Coppock, 2017; Brownback and Novotny, 2018; Kimball, 2019) and a Shy Tory (Conservative) effect in the UK (Whiteley, 2016). Socially desirable responses may also mean survey respondents exaggerate their likelihood

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of voting (Whiteley, 2016) and bias polls on issues such as immigration (Janus, 2010), same-
sex marriage (Powell, 2013) and votes involving liberal/conservative attitudes (Funk, 2016). A
further stylised empirical fact (in contradistinction to e.g Cont, 2001; Chakraborti et al. 2011) is
that polls typically over-state the levels of support for leading candidates (Eriksson and Weizizen,
2008).

This paper adds financial derivatives to the range of different interdisciplinary techniques
used to forecast elections – a link previously made in Taleb (2018). Typically, political applica-
tions use prediction markets (Rhode and Strumpf, 2004) or opinion polls (Eriksson and Weizizen,
2012). Opinion polls have also been variously combined with de-biasing techniques (Eriksson
and Weizizen, 2008), individual constituency-level information (Fisher, 2016) and other political
and economic data (Campbell et al., 2006). Data availability and theoretical complexity mean
that economic models of voting behaviour are relatively under-explored (Leigh and Wolfers,
2006). Hummel and Rothschild (2014) combine economic data with data on political funda-
mentals. Financial data used in political prediction problems include stock markets (Prechter
et al., 2012), currency markets (Wu et al., 2017; Auld and Linton, 2019) and derivatives markets
(Clark and Amen, 2017).

The contribution of our paper is threefold. Firstly, we use a financial options-pricing method-
ology to predict the share of the vote. Secondly, we develop constrained regression models to
extrapolate election outcomes from the share of the vote. This exploits the known mathematical
structure of the underlying problem, offers an important simplification over existing approaches
(Taleb, 2018) and may, at least at an aggregate level, address concerns over forecasting perfor-
ence. Thirdly, we present novel empirical applications to the 2016 US presidential election
(Coppock, 2017) and to the 2017 UK general election (Heath and Goodwin, 2017).

The layout of this paper is as follows. An options-pricing model for predicting the share of
the electoral vote is outlined in Section 2. Section 3 introduces a constrained-regression approach
to extrapolate election outcomes from the share of the vote. This exploits some of the problem’s
known mathematical structure and is a simpler alternative to the sigmoidal transformation
approach employed by Taleb (2018). Sections 4-5 examine empirical applications to the 2016
US presidential election and the 2017 UK general election respectively. Section 6 concludes and
discusses the possibilities for future work.

2 An options-pricing model for political predictions

Let $P_{1,t}$ denote the subjective probability of event $E_i$ at time $t$. For example $E_i$ might represent
the event that a randomly chosen constituent votes for the Republicans in the US election.
The subjective probability $P_{1,t}$ can be thought of as representing the price of a wager at time $t$
that pays $1 if event $E_i$ occurs and 0 otherwise (see e.g. Lad, 1996). This enables us to make
an explicit link between financial and political data in terms of the proportion of the share of
the vote. This philosophical link between estimated probabilities and financial betting is also
significant (Taleb, 2017; 2018).

Consider, first, the univariate problem of predicting the vote share for one party with $P(t) =
P_{1,t}$. Here, we will estimate the relevant probabilities by appealing to the Binary Options pricing
formula (Hull, 2008: Chapter 24 with $Q = 1$) in a standard Black-Scholes setting:

$$\begin{align*}
\text{Binary call option Payoff : } & I(P_T>K) \text{ Price : } e^{-r(T-t)}\Phi(d_2) \\
\text{Binary put option Payoff : } & I(P_T<K) \text{ Price : } e^{-r(T-t)}\Phi(-d_2),
\end{align*}$$

where $\Phi(\cdot)$ denotes the standard normal CDF, $I(\cdot)$ denotes the indicator function, $T$ is the
expiration date, $K$ is the strike price and $d_2 = (\ln(P_t/K) + (r - \sigma^2/2)(T - t))/\sigma \sqrt{T - t}$. As is common in empirical options-pricing applications (see e.g. Dowd et al., 2019) $r$ is the risk-free interest rate. The purpose of this formulation is thus to make an explicit link between (subjective) probabilities and the price of an associated bet – a link that has been vociferously argued for (Taleb, 2017; 2018). In empirical applications in Sections 4-5 we follow Dowd et al. (2019) in assuming a risk-free interest rate of 1.5% per annum.

Conventional probabilities must satisfy

$$0 \leq P_{i,t} \leq 1.$$  

(2)

Tractability is an important consideration in options-pricing applications (see e.g. Dowd et al., 2019 for an application to equity-release mortgages). Partly for these reasons we do not rigorously enforce the constraint (2). However, standard Dutch-book arguments (see e.g. Lad, 1996) mean that empirical subjective probabilities must satisfy (2). Moreover, the Gaussian nature of the model means that the probability that the $P_{i,t}$ violate the constraint (2) must be vanishingly small. There are also similarities with conventional applied modelling where, for example, assets such as currencies satisfy constraints that are not rigorously enforced. One recent example includes how the value of the GBP/USD series is tied to the implications for international trade as revealed by the recent Brexit referendum (Wu et al., 2017; Auld and Linton, 2019).

Suppose we want to estimate the vote share at time $t < T$, where $T$ is the known election date. Suppose we use the median price of the bet to estimate the probability. Using the binary call option price in equation (1) gives

$$e^{-r(T-t)}\Phi(d_2) = \frac{1}{2}; \ln K = \ln P_t + \left( r - \frac{\sigma^2}{2} (T - t) - \sigma \sqrt{T - t} \Phi^{-1}\left( \frac{1}{2} e^{r(T-t)} \right) \right),$$

$$K = P_t e^{\left( r - \frac{\sigma^2}{2} (T - t) - \sigma \sqrt{T - t} \Phi^{-1}\left( \frac{1}{2} e^{r(T-t)} \right) \right)}.$$  

(3)

Equation (3) therefore means that $K$ would be the estimated vote share, where $P_t$ is the polling percentage at time $t$, $r$ is the interest rate and $\sigma$ is an estimate of the underlying volatility associated with polling numbers. In Taleb (2018) the drift in a probability model such as this can be interpreted as a bias towards one of the candidates. In Fry and Brint (2017) the volatility can be interpreted in this context as the market risk or fundamental polling uncertainty. In empirical applications in Sections 4-5 we estimate $\sigma$ using the historical volatility. This can be computed as the residual mean square of a weighted linear regression model without an intercept term (see e.g. Bingham and Fry, 2010).

Mathematically speaking, in equation (3), $K$ is the median vote share conditional on the information available at time $t$. This conditional median contrasts slightly with the time-$t$ forecast probability that the vote exceeds $K$ as presented in Taleb (2018). Other quantiles of the vote share can be calculated in a similar manner. Values of $K$ corresponding to probability values other than $\frac{1}{2}$ in the above may also be interpreted as conditional quantiles.

Suppose that instead of the above the intention is to estimate the probability that one party achieves a higher share of the vote than another. This may be of particular interest in binary elections (Fry and Brint, 2017) but may actually have less importance in determining the outcome of elections in the US and in the UK where the relationship between electoral outcomes and the share of the vote is complex (see Section 3).

Let $P_{i,t}$ denote the vote share of Party $i$ at time $t$ and let $P_t = P_{1,t}/P_{2,t}$. The vote share of Party 1 exceeds that of Party 2 precisely when $P_t > 1$. This means that we can estimate the

$$3$$
probability that Party 1 finishes ahead of Party 2 by letting the exercise price $K = 1$ in equation (1). This gives

$$\text{Estimated Probability} = e^{-r(T-t)} \Phi \left( \frac{\ln(P_t) + \left( r - \frac{\sigma^2}{2} \right) (T-t)}{\sigma \sqrt{T-t}} \right), \quad (4)$$

where $T$ is the known election date, $r$ is the interest rate and $\sigma$, as discussed above, is an estimate of the volatility.

### 3 Converting between election outcomes and the share of the vote

Neither the US presidential election nor the UK general election employ proportional representation. This is an important complication. Inter alia it is perfectly possible to win the popular vote but lose out in terms of the electoral college votes (US) or the number of parliamentary seats (UK). In the sequel we estimate the relationship between vote share and electoral outcomes on a semi-empirical basis. This leads to a constrained regression problem and an important simplification of the sigmoidal transformation approach used in Taleb (2018). Whilst complex the relationship between vote share and election outcomes does contain some known mathematical structure. This structure can be exploited in empirical applications to yield non-trivial insights beyond simple predictions of the share of the vote. See Sections 4-5.

Consider a two-party election system. Let $x$, $y$ denote the proportion of the share of the vote won by parties $X$ and $Y$ respectively. Consider the proportion of parliamentary seats won by party $X$ as a function of $x$ and $y$ only:

$$X\% \text{ seats} = f(x, y).$$

Clearly, we must have

$$0 = f(0, y) \& f(1, 0) = 1. \quad (5)$$

Expanding in a two-dimensional Taylor series up to second order gives that $f(x, y)$ is equal to

$$\alpha_0 + \alpha_1 x + \alpha_2 y + \beta_1 x^2 + \beta_2 xy + \beta_3 y^2. \quad (6)$$

However, imposing the constraints shown in equation (5) leaves us with the following formulation:

$$f(x, y) = x^2 + \alpha_1 (x - x^2) + \beta_2 xy. \quad (7)$$

Equation (7) can thus be estimated as a regression model with an offset term in $x^2$ and no intercept (Bingham and Fry, 2010) in order to give an empirical description of the relationship between vote share and the number of parliamentary seats or number of electoral college votes won that obeys the constraints shown in equation (5).

The above model can be extended to estimate the difference, $g(x, y)$ say, in the number of parliamentary seats held by two major parties. Keeping the general form given in equation (6) it follows that $g(x, y)$ must satisfy

$$g(1, 0) = 1 \& g(0, 1) = -1. \quad (8)$$
Plugging these constraints into equation (6) it follows that

\[ g(x, y) = 2x - 1 + \alpha_2(x + y - 1) + \beta_1(x^2 - x) + \beta_2 xy + \beta_3(x + y^2 - 1). \]  

(9)

Equation (9) again suggests a regression model with an offset term (this time in the variable \(2x - 1\)) without an intercept term (Bingham and Fry, 2010). Using the above explicit forecasts can be made as follows. From equation (7) the probability that \(X\) wins a parliamentary majority can be estimated using

\[ Pr(X > 0.5) = \Phi \left( \frac{\hat{\mu}_T - 0.5}{\hat{\sigma}_T} \right), \]  

(10)

where \(\Phi(\cdot)\) is the CDF of a \(N(0, 1)\) random variable, \(\hat{\mu}_T\) is the linear predictor and \(\hat{\sigma}_T\) is the prediction error (Bingham and Fry, 2010). Similarly, using equation (9), the probability that \(X\) wins more parliamentary seats than \(Y\) can be estimated using

\[ Pr(X - Y > 0) = \Phi \left( \frac{\hat{\mu}_T}{\hat{\sigma}_T} \right). \]  

(11)

4 Application to the 2016 US presidential election

Following Silver (2012), polling data for the 2016 US presidential election is obtained from fivethirtyeight.com. Following the standard approach (see e.g. Leigh and Wolfers, 2006) we remove the “don’t know” responses. Predictions obtained for the share of the popular vote are as follows. An estimate of the share of the vote according to equation (3) is shown below in Figure 1. The probability of winning the popular vote according to equation (4) is shown below in Figure 2. Figure 1 shows that the gap between the Republicans and the Democrats seems to be decreasing as the election day approaches. This notwithstanding the Democrats are consistently shown to be ahead in Figures 1-2. Whilst the Democrat lead, in terms of the share of the popular vote, seems well established, the implications for the actual election outcomes are more complex (Taleb, 2018).

In order to convert between electoral outcomes and the estimated share of the vote a fit of the model shown in equation (7) is shown below in Table 1. The relationship between vote share and election outcomes is complex (Taleb, 2018). The implication of this model is that a plausible election scenario is that the Republicans may lose the popular vote yet still win the electoral college.

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Table 1: Results for equation (7) applied to post-war US election data (\(R^2\) value=0.8872).

Using the model shown in Table 1 we try to predict the number of electoral college votes based on the estimated vote proportions shown in Figure 2. To guard against an over-fitting problem discussed in Hummel and Rothschild (2014) we have re-fit the model omitting the last data-point corresponding to the 2016 election. The results are shown below in Figure 3. Contrary to the perceived wisdom this places the Republicans ahead in the electoral college in the first half of 2016 and from July 2016 onwards.
5 Application to the 2017 UK general election

Following Pack and Maxfield (2016) polling data for the 2017 UK general election is obtained from markpack.org.uk. Following the standard approach (Leigh and Wolfers, 2006) we remove the “don’t know” responses.

An estimate of the share of the vote according to equation (3) is shown below in Figure 4. The probability of winning the popular vote according to equation (4) is shown below in Figure 5. Results shown in Figure 4 do suggest a significant narrowing in the share of the popular vote during polling. However, it appears almost certain that the Conservatives will both win the popular vote (Figure 5) and be the largest parliamentary party (Figure 7).

In order to estimate election outcomes given the share of the vote equation (7) estimates the number of seats won by the Conservatives and Labour. Equation (9) estimates the difference in the number of seats won. Results for both models are shown below in Table 2. Based on these models, but omitting the 2016 values in the estimation so as to avoid data snooping (Hummel and Rothschild, 2014), a plot of the predicted number of parliamentary seats based on the opinion-poll data is shown below in Figure 6. This shows the dramatic narrowing in the polls, with the inescapable conclusion that although certain to be the largest party winning a Parliamentary majority is by no means a foregone conclusion. Whilst the probability appears to be very high that the Conservatives will be the largest parliamentary party (Figure 7) as polling closes there is a non-trivial probability (roughly 0.4) that the Conservatives will fail to win a
Figure 2: Real-time estimates of the probability that the Republicans win the popular vote in the 2016 US presidential election according to equation (4). Points above the line indicate points at which the Republicans are favourites to win the popular vote.

parliamentary majority (Figure 8).

6 Conclusions and further work

There is renewed interest in the analogies between financial and political systems (Taleb, 2018; Wu et al., 2017; Wang et al., 2009). The options-pricing methods developed here add to a wide array of different approaches that have previously been used for political prediction problems (Leigh and Wolfers, 2006). As far as finance is concerned the link between vote share and subjective probabilities obtained via a betting argument is important philosophically (Taleb, 2017; 2018). Further, recent concerns over Socially Desirable Response Bias in opinion polls (Brownback and Novotny, 2018; Coppock, 2017; Kimball, 2019) may be analogous to bubbles and mis-pricing in financial markets (Fry and Brint, 2017).

The contribution of our paper is threefold. Firstly, we develop a method for calculating vote-share projections using an options-pricing argument. Secondly, we develop constrained regression models to extrapolate election outcomes from the share of the vote. This exploits the underlying mathematical structure of the problem, simplifies previous approaches (Taleb, 2018) and appears to resolve the apparent forecasting errors made. Thirdly, political forecasting remains an active subject of academic debate (Vaughan Williams and Reade, 2016) and the
Figure 3: Real-time predicted Republican college votes during the 2016 US presidential election based on polling data coupled with the model shown in Table 1 (re-fitted without the 2016 values). Points above the horizontal line indicate levels of support required for the Republicans to win the electoral college.

empirical applications of our model to recent US and UK elections are interesting and important in their own right.

The empirical application of our models produces some results similar to those reported elsewhere (see e.g. Taleb, 2018) but contain a mix of intuitive and counter-intuitive findings. In the 2016 US election the Democrats are generally ahead in an admittedly close popular vote. In the 2017 UK election the Conservatives are comfortably ahead both in terms of the popular vote and the relative size of the two main parliamentary parties. However, for much of 2016 the Republicans are ahead in terms of the electoral college. In the UK as polling develops a relatively high probability emerges (roughly 40%) that the Conservatives will fail to win a parliamentary majority.

Both political forecasting (Vaughan Williams and Reade, 2016) and the interdisciplinary application of financial tools and techniques (see e.g. Lumberas et al., 2016; Prechter, 1999) remain extremely interesting. Social Media usage is likely to play a key future role in both areas. Social Media usage has been found to have some predictive ability for stock markets (Sprenger et al., 2014). Of particular interest here are ideas related to speculative bubbles, social media and over-confidence in complex social systems. This is especially true with respect to echo chamber effects (Barberá et al., 2015) amid deep-seated concerns for the stability of our existing democratic institutions (Wiesner et al., 2018). Other social-science applications of
physics-based models remain extremely interesting (Haven and Khrennikov, 2013).

Allied to the above the original motivation behind this work was to try and model bias in opinion polls using a jump-process model of the form of Johansen et al. (2000) or Fry (2012). For an early attempt at work along these lines see Fry and Brint (2017). However, as helpfully pointed out by a reviewer, in this case the time of the jump, the known election date, is deterministic meaning that arbitrage-free pricing approaches do not apply. In this case the asset price becomes deterministic after a certain period of time. This has important ramifications for notions like risk, return and liquidity. Future work will extend the risk-return framework of Fry (2012) and related models to this case. SDE models for the Brownian Bridge (see e.g. Calin, 2015) rather than Geometric Brownian Motion form the natural reference point here.

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Figure 5: Real-time estimates of the probability that the Conservatives win the popular vote in the 2017 UK general election according to equation (4).

References


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Table 2: Results for equations (7) and (9) applied to post-war UK elections.


Figure 6: Real-time predicted numbers of UK parliamentary seats won in the 2017 UK general election based on polling data coupled with the model shown in Table 2 (re-fitted without the 2016 values). The horizontal line indicates the number of seats needed to win a parliamentary majority. Solid Line: Labour. Dashed line: Conservative.


Figure 7: Real-time estimates of the probability of being the largest parliamentary party in the 2017 UK general election according to equation (11). Solid Line: Labour. Dashed line: Conservative.


Figure 8: Real-time estimates of the probability of winning a parliamentary majority in the 2017 UK general election according to equation (10). Solid Line: Labour. Dashed line: Conservative.


