A novel approach for transient stability using interval arithmetic and optimization methods

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A novel approach for transient stability using interval arithmetic and optimization methods

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In this paper, a new approach is proposed to include parametric uncertainties in the analysis of the transient stability of power systems by using intervals. Two methodologies based on the interval uncertainties were developed and implemented: i) a robust extension of the PEBS method combined with interval arithmetic and ii) an optimization formulation for the transient stability problem. The interval uncertainties enable one to obtain closed-form expressions for stability assessment of uncertain systems, allowing a faster analysis. The results show that both methodologies can be used to find an accurate estimate of the critical clearing time for uncertain power systems more quickly than using the classical Monte Carlo approach.

KEYWORDS
Robust PEBS, interval arithmetic, optimization methods, transient stability

1 | INTRODUCTION

The use of renewable energy is steadily growing, in the following years alone the global renewable energy market value is expected to reach $1.5 trillion [1]. The European Union, for example, aims to be carbon-free by 2050 [2, 3]. Given this new environment, the grid operator cannot anymore ignore the influence of renewable generation in the control and operation of power systems. These sources of energy, given their intrinsically intermittent and variable nature, contribute to an increasing level of uncertainties in operation. Consequently, it is imperative to consider these uncertainties in the analysis and operation of power systems. In this paper, the problem of considering parametric uncertainties in transient stability analysis of power systems is examined.

The classical method for transient stability analysis of power systems is based on the numerical integration of a set of differential equations to obtain the so-called critical clearing time—the largest time interval over which the protection can act so the post-disturbance system is stable. When dealing with uncertain systems, the classical method is usually combined with a Monte Carlo simulation to obtain the probability distribution for the system stability and critical clearing time. Despite generating complete and precise results, this approach demands a vast amount of information about the system and many iterations for its execution. Thus, with the increasing complexity and size of power systems, its application is restricted and not adequate to assess the transient stability of electric power systems in real-time.

To overcome this challenging issue, in this paper, two methodologies to assess the transient stability of power systems considering uncertainties are proposed. The first is an extension of the direct method PEBS (potential energy boundary surface) [4] presented in [5] with an interval arithmetic formulation for deriving an energy function that includes parametric uncertainties. Interval arithmetic enables one to include uncertainties in the analysis and obtain closed-form expressions for stability assessment, allowing a faster analysis. Further, this formulation allows for a general set of uncertainties to be included in the assessment. Conversely, in [5], a new energy function must be found for each set of uncertainties. The second is the formulation of the stability assessment problem as an optimization problem to obtain an estimate of the critical clearing time considering parametric uncertainties comparable to the value obtained by the classical approach. This second approach formulates the problem of transient stability analysis with uncertainties as a constrained minimisation problem. While both methodologies are faster than the Monte Carlo method, they have advantages and disadvantages. The PEBS extension is faster to analyse the transient stability for a larger set of uncertain parameters, but it can lead to conservative results; the optimization formulation provides less conservative results as compared to the first methodology, but it can be slower to execute.

A research effort has already been made to tackle the problem of stability of uncertain power systems, by using Fuzzy based control [6], selecting the critical parameters in the stability analysis to measure the level of uncertainty in the system [7], robust control based on the change critical clearing time due to a small variation in the output active power [8], or even assessing the stability using the maximum angle difference between two machines in the power system in a probabilistic scenario [9]. In this paper, we differ from these articles as we developed a formulation that can include all possible combination of uncertainties present in a given power system and assess the transient stability in a feasible time. This paper is organised as follows:

- In section 2, a brief literature review of the treatment of uncertainties in transient stability analysis is presented;
- In section 3, the transient stability model of power systems and the PEBS method are reviewed;
- In section 4, the extension of the PEBS method using interval arithmetic is presented;
- In section 5, the proposed optimization scheme for transient stability analysis with uncertainties is presented ;
- In section 6, the methods proposed in this paper are applied to the IEEE 14 bus and 39 bus systems and their

results are compared to the ones obtained using a Monte Carlo simulation;
- Lastly, in section 7, conclusions are drawn from the results.

2 | LITERATURE REVIEW

There are two distinct approaches to deal with uncertainties in transient stability analysis of power systems. The first is denominated probabilistic, in which a probability distribution is calculated for the critical clearing time. The second one is designated deterministic, in which the critical clearing time for the worst-case scenario is calculated and adopted as a measure of robustness for the system.

Following the first approach, Monte Carlo based methodologies were proposed to assess the transient stability of a power system with uncertainties. Despite generating accurate and complete results, these methodologies require a vast amount of simulations [10, 11, 12]. Thus, they are suitable for generating reference results, rather than real-time analysis.

In the same vein, several papers in literature explored statistical properties to study the transient stability problem [13]. In [14], for example, an approximation for the probability distribution of the critical clearing time based on its sensitivity with respect to fluctuations in system load was proposed.

In [15, 16], an analytical formulation for calculating the probability of a power system, with uncertainties in loads, to be stable given the occurrence of a three-phase fault was developed by using a logarithmic approximation between the critical clearing time and the system load. More recently, a polynomial approximation to the boundary of the stability region together with the load covariance matrix and the system stability margin sensitivity to active and reactive powers; angles; and bus voltages was employed to evaluate the risk for a contingency [17]. Also, in [18], a probability measure of transient stability is presented and analytically solved by stochastic averaging, considering that the power system uncertainties are stochastic continuous disturbances.

On the other hand, following the second approach, the papers [19, 20] provided a measure of the robustness of an electric power system via an analysis of the most severe disturbance that can occur on the system, in a particular configuration using a transient stability index.

In [21], a polynomial approximation of the differential equations of the power system together with a sum of squares (SOS) optimization algorithm were employed to find a robust estimate for the critical compensation time. More recently, in [22], the authors have analysed the transient stability problem using a Lyapunov function constructed using an SOS optimization scheme. The main issue of the methods proposed in both papers is that every time a new uncertainty is included, the optimization problem must be executed from the beginning.

This paper follows the second approach since it proposes to find the lowest critical clearing time considering all uncertainties—the worst-case scenario. The main advantage of the methods proposed in this paper is speed. The proposed Robust PEBS method, since it is based on direct methods and interval analysis, can perform a fast analysis without resorting to expensive simulations. Given the conservative nature of the PEBS method, this paper also proposes a model to study transient stability using optimization algorithms (Simulated Annealing and Differential Evolution), which gives the same result as the Monte Carlo method but in a fraction of time.

3 | PEBS METHOD

In this section, we review the formulation of the PEBS method as proposed in [23]. We begin by presenting the mathematical formulation of the transient stability analysis problem.

3.1 | Transient Stability Model

A power system experiencing a disturbance can be modelled by a set of three differential equations:

$$\dot{x}_{prf}(t) = f^{prf}\left(x_{prf}(t)\right) \quad t \in (-\infty, 0], \tag{1}$$

$$\dot{x}_{f}(t) = f^{f}\left(x_{f}(t)\right) \quad t \in (0, t_{cl}], \tag{2}$$

$$\dot{x}_{pf}(t) = f^{pf}\left(x_{pf}(t)\right) \quad t \in (t_{cl}, \infty), \tag{3}$$

where x_{prf}, x_f, x_{pf} and f^{prf}, f^f, f^{pf} are, respectively, the state variables and the dynamical equations of the system at pre-fault; fault-on and post-fault periods; and t_{cl} is the fault clearing time.

The model assumes that the system is operating in an equilibrium (pre-fault period) when at $t = 0$ a fault occurs and the dynamics change from the one driven by f^{prf} to f^f. For $t \in (0, t_{cl}]$, the fault-on period, the system dynamics is ruled by f^f. When protection acts and the faults are cleared, the post-fault period initiates and the system is governed by f^{pf}. Following this model, $x_f(0) = x_{prf}$ since the system is at equilibrium at the pre-fault period, and the initial condition for the post-fault system is $x_f(t_{cl})$.

Now, let x_e be an equilibrium point of the post-fault system (3). A reasonable question then is: will the trajectory $x_{pf}(t)$, with initial condition $x_f(t_{cl})$, converges to x_e as t goes to infinity? The largest value of t_{cl} for which that remains true is denominated the critical clearing time t_{cr}. Finding t_{cr} is, therefore, the main goal of a transient stability assessment.

To precisely define the discussion above, a review the definition of stability regions and the Krasovskii-LaSalle invariance principle [24] must be made:

Stability Region The stability region of an asymptotically stable equilibrium point x_e is the set of states x_0 such that $\lim_{t\to+\infty} x(t) = x_e, x(0) = x_0$. Compactly:

$$A(x_e) := \left\{x_0 \in \mathbb{R}^n : \lim_{t\to+\infty} x(t) = x_e, x(0) = x_0\right\}. \tag{4}$$

Theorem 1 (Krasovskii-LaSalle Invariance Principle) *Let $\Omega \subset D \subset \mathbb{R}^n$ be a compact positively invariant set with respect to $\dot{x} = f(x)$. Let $V : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that $\dot{V}(x) \leqslant 0$ in Ω. Let $E \subset \Omega$ be the set of all points in Ω where $\dot{V}(x) = 0$. Let $M \subset E$ be the largest invariant set in E. Then, every solution starting in Ω approaches M as $t \to \infty$, i.e.,*

$$\lim_{t\to\infty}\left(\inf_{z\in M} \underbrace{\|x(t) - z\|}_{\text{dist}(x(t),M)}\right) = 0, \forall x(0) \in \Omega$$

Note that the inclusion of the sets in the theorem is $M \subset E \subset \Omega \subset D \subset \mathbb{R}^n$

The definition above shows that if a good estimate of the stability region of the post-fault stable equilibrium point x_e is obtained, then t_{cr} can be found as the time when the trajectory of (2) crosses the stability region boundary at x^*. Evaluating the stability region is, however, a difficult process, which requires several simulations of a complex system. Theorem 1 can be employed to find an estimate for the stability region. Let L be a constant, such that the level set $\Omega_L = \{x \in D : V(x) \leqslant L\}$ contains the equilibrium of $\dot{x} = f(x)$ and it is contained in Ω. Then, by Theorem 1, every solution starting in Ω_L approaches M as $t \to \infty$. In particular, if Ω_L is contained in the stability region of the equilibrium point, then Ω_L is an estimate for the stability region. In transient stability, $V(x)$ is called energy function and it is constructed for the post-fault system whose stability region we are trying to estimate and L is known as V_{cr} or critical energy.

3.2 | Power System Model – PEBS Method

The differential equation that models the power system as described in the subsection above, known as the swing equation, for the i-th machine is as follows:

$$M_i\ddot{\theta}_i = P_{m_i} - E_i^2 G_{ii} - \sum_{\substack{j=1 \\ \neq i}}^{n} \left[E_i E_j B_{ij} \sin(\theta i - \theta_j) + E_i E_j G_{ij} \cos(\theta_i - \theta_j) \right] - \frac{M_i}{M_T} P_{COI} \quad i = 1, \dots, n, \tag{5}$$

where n is the number of generators; θ_i and $\tilde{\omega}_i = \dot{\theta}_i$ are, respectively, the generator rotor angle deviation in respect to the centre of inertia angle and frequency deviation in respect to the centre of inertia frequency; M_i is the machine moment of inertia; $M_T = \sum_{i=1}^{n} M_i$; P_{m_i} is the mechanical power injected into the machine; E_i is the machine electromotive force; $P_{COI} = \sum_{i=1}^{n} \left[P_{m_i} - E_i^2 G_{ii} \right] - 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j G_{ij} \cos(\theta_i - \theta_j)$ is the power of the centre of inertia; B_{ij} and G_{ij} are, respectively, the imaginary and real parts of the ij-th entry of the power systems reduced admittance matrix.

The energy function for the power system is as follows:

$$V(\theta, \tilde{\omega}) = \underbrace{\frac{1}{2} \sum_{i=1}^{n} M_i \tilde{\omega}_i^2}_{\text{Kinetic Energy } V_k(\tilde{\omega})} + \underbrace{- \sum_{i=1}^{n} P_i (\theta_i - \theta_i^e) - \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} \left[C_{ij} (\cos\theta_{ij} - \cos\theta_{ij}^e) - \int_{\theta_i^e + \theta_j^e}^{\theta_i + \theta_j} D_{ij} \cos\theta_{ij} \, \mathrm{d}(\theta_i + \theta_j) \right]}_{\text{Potential Energy } V_p(\theta)}, \tag{6}$$

where $\theta_{ij} = \theta_i - \theta_j$; $C_{ij} = E_i E_j B_{ij}$; $D_{ij} = E_i E_j B_{ij}$; and $P_i = P_{m_i} - E_i^2 G_{ii}$. The path-dependent integral of (6) can be approximated by:

$$\frac{\left(\theta_i^u - \theta_i^e\right) + \left(\theta_j^u - \theta_j^e\right)}{\left(\theta_i^u - \theta_i^e\right) - \left(\theta_j^u - \theta_j^e\right)} D_{ij} \left[\sin(\theta_i^u - \theta_j^u) - \sin(\theta_i^e - \theta_j^e) \right]. \tag{7}$$

The procedure for executing the PEBS method is described below. The general idea is to find the first potential energy maximum and considerate it as the critical energy for the system. In sequence, the time at which the total energy, potential and kinetic, equates the critical energy is found. This time is an estimate for the critical clearing time t_{cr}.

- Integrate the fault-on trajectory until V_p reaches a maximum, $V_p^{\max}$ along the fault-on trajectory. Let $V_{cr} = V_p^{\max}$;
- From the fault-on trajectory $[\theta(t), \tilde{\omega}(t)]^\mathsf{T}$, find the time at which $V(\theta(t), \tilde{\omega}(t)) = V_{cr}$. This time is an estimate for t_{cr}.

This classical formulation for the transient stability problem using energy functions does not allow for the inclusion of uncertainties in the analysis. To overcome this limitation, a uniform version of the Krasovskii-LaSalle invariance principle (proposed in [5]) was employed to introduce parametric variables in the stability assessment. This modified version of the PEBS method was named Robust PEBS.

4 | ROBUST PEBS

Let us consider the following autonomous system:

$$\dot{x} = f(x, \lambda), \tag{8}$$

where $\lambda \in \Lambda \subset \mathbb{R}^m$ is a parameter vector and $x \in \mathbb{R}^n$ is the state vector. Set Λ—which models parameter uncertainties—is constructed as being $\Lambda = \left[\lambda_1^{\min}, \lambda_1^{\max}\right]^\top \times \left[\lambda_2^{\max}, \lambda_2^{\min}\right]^\top \times \cdots \times \left[\lambda_m^{\min}, \lambda_m^{\max}\right]^\top$, in which $\lambda_1^{\min}, \lambda_2^{\min}, ..., \lambda_m^{\min}$ and $\lambda_1^{\max}, \lambda_2^{\max}, ..., \lambda_m^{\max}$ represent, respectively, minimum and maximum values of each parameter $\lambda_1, \lambda_2, ..., \lambda_m$.
The Uniform Invariance Principle is defined as:

Theorem 2 (Uniform Invariance Principle [5]) *Suppose $f : \mathbb{R}^n \times \Lambda \to \mathbb{R}^n$ and $V : \mathbb{R}^n \times \Lambda \to \mathbb{R}$ are continuously differentiable functions and $a, b, c : \mathbb{R}^n \to \mathbb{R}$ are continuous functions. Assume that for any $[x, \lambda]^\top \in \mathbb{R}^n \times \lambda$, one has:*

$$a(x) \leqslant V(x, \lambda) \leqslant b(x), \quad -\dot{V}(x, \lambda) \geqslant c(x). \tag{9}$$

For $L > 0$ let $A_L := \{x \in \mathbb{R}^n : a(x) < L\}$. Assume that A_L is non-empty and bounded.
Consider the sets

$$B_L := \{x \in \mathbb{R}^n : b(x) < L\}, C := \{x \in \mathbb{R}^n : c(x) < 0\}, \text{and } E_L := \{x \in A_L : c(x) = 0\}. \tag{10}$$

Suppose now that $\sup_{x \in C} b(x) \leqslant l < L$ and define the sets

$$A_l := \{x \in \mathbb{R}^n : a(x) \leqslant l\} \text{ and } B_l := \{x \in \mathbb{R}^n : b(x) \leqslant L\}. \tag{11}$$

If λ is a fixed parameter in Λ and all the previous conditions are satisfied, then for $x_0 \in B_L$ the solution $\phi(t, x_0, \lambda)$ is defined in $[0, \infty)$ and the following holds:

I. *if $x_0 \in B_l$ then $\phi(t, x_0, \lambda) \in A_l$, for $t > 0$ and $\phi(t, x_0, \lambda)$ tends to the largest invariant set of (8) contained in A_l, as $t \to \infty$;*
II. *if $x_0 \in B_L \setminus B_l$ then $\phi(t, x_0, \lambda)$ tends to the largest invariant set of (8) contained in $A_l \cup E_L$.*

Following the first conclusion of Theorem 2 and the procedure of the classical PEBS, the Robust PEBS is described by the following steps:

- Integrate the fault-on trajectory until a_p reaches a maximum, $a_p^{\max}$ in time. Let $V_{cr} = a_p^{\max}$;
- From the fault-on trajectory find when $b = V_{cr}$. This is an estimate for t_{cr}.

To use the Robust PEBS method for stability analysis, $a(\delta, \tilde{\omega})$ and $b(\delta, \tilde{\omega})$ are needed. In general, for larger systems finding these functions is not an easy task since $a(\delta, \tilde{\omega}) \leqslant V(\delta, \tilde{\omega}, \lambda) \leqslant b(\delta, \tilde{\omega}), \forall \lambda \in \Lambda$. In this paper, the strategy employed to find both functions was to consider that parametric and operational uncertainties can be represented using intervals and to use interval arithmetic to find an interval extension to the energy function.

4.1 | Interval Extension of Energy Function

4.1.1 | Interval Arithmetic

Formally, an interval can be defined as [25]:

$$X = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} : \underline{x} \leqslant x \leqslant \overline{x}\}. \tag{12}$$

The usual sum and difference between two intervals X e Y can be written as follows:

$$X + Y = \{x + y : x \in X, y \in Y\} = [\underline{x} + \underline{y}, \overline{x} + \overline{y}], \tag{13}$$

$$X - Y = \{x - y : x \in X, y \in Y\} = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]. \tag{14}$$

Likewise, multiplication and division are written as follows:

$$\begin{aligned} &X \cdot Y = \{\min \Upsilon, \max \Upsilon\}, \text{ where} \\ &\Upsilon = \left\{\underline{x} \cdot \underline{y}, \underline{x} \cdot \overline{y}, \overline{x} \cdot \underline{y}, \overline{x} \cdot \overline{y}\right\}, \end{aligned} \tag{15}$$

$$\begin{aligned} &X/Y = \{\min \Gamma, \max \Gamma\}, \text{ where} \\ &\Gamma = \left\{\underline{x} \cdot \frac{1}{\underline{y}}, \underline{x} \cdot \frac{1}{\overline{y}}, \overline{x} \cdot \frac{1}{\underline{y}}, \overline{x} \cdot \frac{1}{\overline{y}} \cdot\right\} \text{ e } 0 \notin Y. \end{aligned} \tag{16}$$

Functions of intervals are also defined point-wise [25]:

$$f(X) = \bigcup_{x \in X} \{f(x)\}. \tag{17}$$

Given the necessity of operating function f in the entire interval X to find $f(X)$, it is unpractical to use functions of intervals. Instead, interval extensions of $f(x)$ are usually employed, where the real variable x is replaced by the interval variable X. The interval extension of $f(x)$ is denoted by $\mathcal{F}(X)$.
If $\mathcal{F}(X)$ is obtained simply by substituting all real operations by its correspondent interval operations, then $\mathcal{F}(X)$ is a natural interval extension $f(x)$ and the following condition holds:

$$f(X) \subseteq \mathcal{F}(X). \tag{18}$$

Equation (18) is known as the Fundamental Theorem of Interval Analysis [25].

4.1.2 | Interval Extension Formulation

Consider that the power system has parametric uncertainties that can be represented by intervals. The uncertain system dynamic can be described as:

$$\begin{aligned}
&\dot{X}_f(t) = f^f\left(X_f(t), \Lambda\right) \quad t \in (0, t_{cl}],\\
&X_f(0) = X_{prf},\\
&\dot{X}_{pf}(t) = f^f\left(X_{pf}(t), \Lambda\right) \quad t \in (t_{cl}, \infty),\\
&X_{pf}(t_{cl}) = X_f(t_{cl}),
\end{aligned} \tag{19}$$

where $X_f, X_{pf} \in \mathbb{I}^n$ and $f^f, f^{pf} : \mathbb{I}^n \times \mathbb{I}^m \to \mathbb{I}^n$ are, respectively, the interval state vector and the interval differential equations of the system at fault-on and post-fault periods; $\Lambda \in \mathbb{I}^m$ is the interval parameter vector; $X_{prf} \in \mathbb{I}^n$ is the pre-fault interval state vector; and t_{cl} is the fault clearing time.

In this general model, it is possible to consider the whole set of parameters as uncertain or just a subset of them (since any scalar parameter p can be represented as an interval $p = [p, p]$). Usually, in the transmission system, the line parameters are the most uncertain [26] due to temperature variation [27] or inaccurate specifications. With the increasing use of smart grids, local generation and even electric cars, loads will also contain uncertainties; the reason for this is: loads can represent a system with distributed generation and since this type of generation has low inertia or is controlled by power electronics—which have a faster dynamic than the transient stability analysis—they can be modelled as a static generation with an uncertainty.

Since the power system has uncertainties at all periods, not only the fault-on and post-fault ordinary differential equations became interval differential equations, but also the power flow solution for the pre-fault period is an interval vector. Thus, both arguments of the energy function evaluated at the fault-on trajectory are intervals. Assuming, that $X_f = \left[\Theta, \tilde{\Omega}\right]^{\mathsf{T}}$, the previous statement can be mathematically written as:

$$V(\Theta, \tilde{\Omega}, \Lambda). \tag{20}$$

Thus, $V : \mathbb{R}^n \times \mathbb{I}^m \to \mathbb{I}$ is an interval function and its interval extension is:

$$\mathcal{V}(\Theta, \tilde{\Omega}, \Lambda) = \frac{1}{2}\sum_{i=1}^{n} M_i \cdot \tilde{\Omega}_i^2 - \sum_{i=1}^{n}\left[\underline{P_i}, \overline{P_i}\right] \cdot (\Theta_i - \Theta_i^e) \tag{21}$$

$$-\sum_{i=1}^{n-1}\sum_{j=1+1}^{n}\left[\left[\underline{C_{ij}}, \overline{C_{ij}}\right] \cdot \left(\cos\Theta_{ij} - \cos\Theta_{ij}^e\right)\right. \tag{22}$$

$$\left.-\left[\underline{D_{ij}}, \overline{D_{ij}}\right] \cdot \frac{\Theta_i + \Theta_j - \Theta_i^e - \Theta_j^e}{\Theta_i - \Theta_j - \Theta_i^e + \Theta_j^e} \cdot \left(\sin\Theta_{ij} - \sin\Theta_{ij}^e\right)\right]. \tag{23}$$

Since the expression of (23) was obtained by changing every real variable by an interval variable and every real operation by its equivalent interval operation, function (23) is a natural interval extension of (6), from the Fundamental Theorem of Interval Analysis (18):

$$\underline{\mathcal{V}(\Theta, \Omega, \Lambda)} \leqslant V(\Theta, \Omega, \Lambda) \leqslant \overline{\mathcal{V}(\Theta, \Omega, \Lambda)}. \tag{24}$$

The inequalities in 24 show that $\underline{\mathcal{V}(\Theta, \Omega, \Lambda)}$ can be chosen as a and $\overline{\mathcal{V}(\Theta, \Omega, \Lambda)}$ as b for Theorem 2.

5 | OPTIMIZATION FORMULATION

Considering that the power system parameters are intervals, then it is possible to model the problem of finding the minimum critical clearing time as a minimisation problem. Differently, from the usual optimization problems, the function that is being minimised is not a mathematical function per se, but a computational one that evaluates the critical clearing time given a set of parameters. This formulation aims to provide a balance between the Robust PEBS execution speed and the Monte Carlo precision.
Mathematically, this minimisation problem can be written as:

$$\begin{array}{ll} \underset{\lambda}{\text{minimize}} & f(\lambda) \\ \text{subject to} & \lambda_i^{\min} \leqslant \lambda_i \leqslant \lambda_i^{\max},\ i = 1, 2, \ldots, m. \end{array}, \tag{25}$$

where $f : \mathbb{R}^m \rightarrow \mathbb{R}$ represents the computational function that evaluates the critical clearing time; $\lambda \in \Lambda \subset \mathbb{R}^m$ is the parameter vector; and $\lambda_i^{\min}$, $\lambda_i^{\max}$ are, respectively the maximum and minimum of the i-th parameter.
The function $f(\lambda)$ does not have an analytical derivative, so methods such as Broyden–Fletcher–Goldfarb–Shanno (BFGS) that requires $\nabla f(\lambda)$ cannot be used to find a solution in a feasible time. Conversely, heuristic methods such as Simulated Annealing [28] and Differential Evolution [29] are derivative-free and global, thus are preferable and were adopted in this paper.

6 | RESULTS

The IEEE 14 bus system [30]—illustrated in Figure 1—and the IEEE 39 bus system [4]—illustrated in Figure 2—were employed for testing the proposed methodologies—Robust PEBS and optimization algorithms. In parallel, the critical clearing time was evaluated using a Monte Carlo simulation—the number of iterations were determined as discussed in [31]—and compared the results. All algorithms were programmed in Python using the Numba [32], NumPy [33], and SciPy [34] libraries. The Robust PEBS method used the PyInterval [35] library. All simulations were executed on an AMD Ryzen™ 7 1700 processor.
This section presents the transient stability assessment for the IEEE 14 bus system in three different cases:

1. A small set of transmission line parameters with uncertainties;
2. A large set of transmission line parameters with uncertainties;
3. A small set of loads with uncertainties.

In addition, a transient stability case for the IEEE 39 bus system is also presented.
These cases were purposely chosen to demonstrate that both optimization algorithms and the Robust PEBS method outperform the Monte Carlo method in several situations.

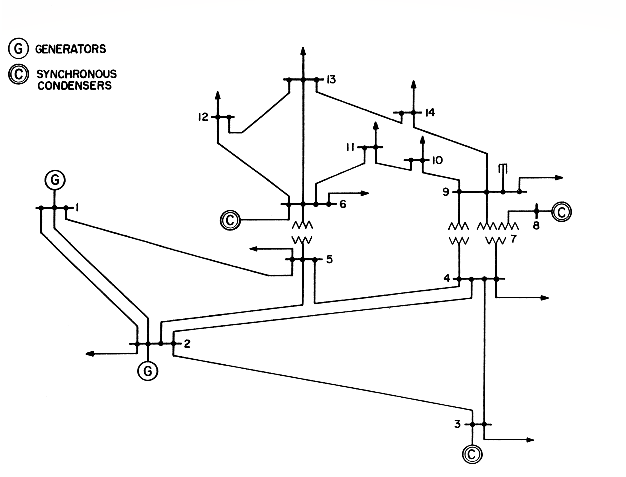

FIGURE 1 IEEE 14 bus system diagram

Source: Retrieved from `https://labs.ece.uw.edu/pstca/pf14/pg_tca14bus.htm`

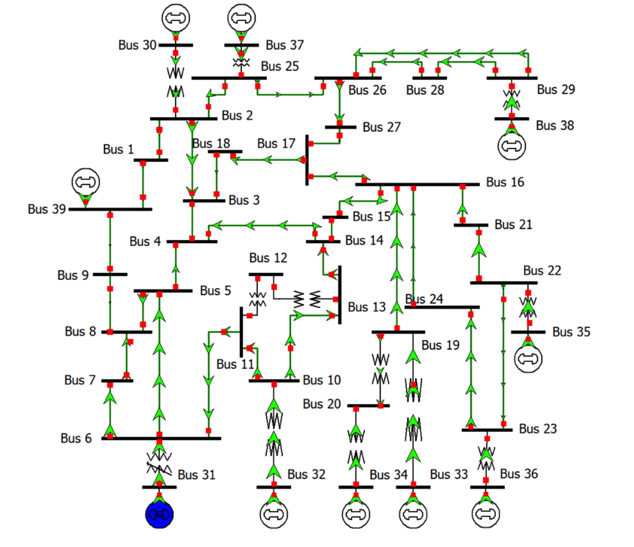

FIGURE 2 IEEE 39 bus system diagram

Source: Retrieved from `https://icseg.iti.illinois.edu/ieee-39-bus-system/`

6.1 | Case 1 - 14 bus

Uncertainties as depicted in Table 1 and a solid three-phase short-circuit occurs at bus 1 and the fault is cleared by opening line 1-5.

TABLE 1 Uncertainties Case 1 - 14 bus

Line		Parameter Uncertainty (%)		
From Bus	To Bus	R	X	B/2
1	2	10	10	1
2	4	0.1	0	0
3	4	1	0	0
6	13	1	0	0
9	10	1	0	0
12	13	0.1	0	0

For the first case, we have obtained the following results:

TABLE 2 Case 1 - 14 bus Results

Method	Critical Clearing Time (ms)
Robust PEBS	296
Simulated Annealing	299
Differential Evolution	292
Monte Carlo	299

TABLE 3 Case 1 - 14 bus Execution Time

Method	Time (min)
Robust PEBS	41
Simulated Annealing	85
Differential Evolution	66
Monte Carlo	283

Table 2 shows that the Robust PEBS method provided comparable results to the Monte Carlo method. Both optimization methods also provided similar results. They, however, performed slower than the Robust PEBS.

6.2 | Case 2 - 14 bus

Uncertainties of the case are depicted in Table 4 and a solid three-phase short-circuit occurs at bus 1 and the fault is cleared by opening line 1-5.

TABLE 4 Uncertainties Case 2 - 14 bus

Line		Parameter Uncertainty (%)		
From Bus	To Bus	R	X	B/2
1	2	10	10	1
1	5	10	10	0
2	4	0.1	0	0
2	5	0	10	0
3	4	1	0	0
4	7	1	0	0
4	9	0	8	5
5	6	0	0	5
6	11	4	1.5	0
6	12	0	1	0
6	13	1	0	2
7	8	0	5	0
7	9	1	0	30
9	10	1	0	0
9	14	0	5	0
10	11	0	5	0
12	13	0.1	0	0
13	14	5	0	5

For the second case, we have obtained the following results:

TABLE 5 Case 2 - 14 bus Results

Method	Critical Clearing Time (ms)
Robust PEBS	281
Simulated Annealing	288
Differential Evolution	285
Monte Carlo	285

TABLE 6 Case 2 - 14 bus Execution Time

Method	Time (min)
Robust PEBS	48
Simulated Annealing	212
Differential Evolution	193
Monte Carlo	848

Looking at Table 5, we can see that the proposed Robust PEBS method was able to provide a result close to the Monte Carlo method benchmark. Likewise, the optimization methods also provided good results, but with a higher computational cost.

6.3 | Case 3 - 14 bus

Uncertainties of Case 3 - 14 bus are depicted in Table 7 and a solid three-phase short-circuit occurs at bus 1 and the fault is cleared by opening line 1-5.

TABLE 7 Uncertainties Case 3 - 14 bus

	Parameter Uncertainty (%)	
Bus	P_L	Q_L
2	50	40
3	25	0

For the third case, we obtained the following results:

TABLE 8 Case 3 - 14 bus Results

Method	Critical Clearing Time (ms)
Robust PEBS	288
Simulated Annealing	297
Differential Evolution	297
Monte Carlo	297

TABLE 9 Case 3 - 14 bus Execution Time

Method	Time (min)
Robust PEBS	51
Simulated Annealing	48
Differential Evolution	204
Monte Carlo	695

Looking at Table 8, we can again verify that our proposed methodology found a close result to the one obtained using the Monte Carlo method, but in a fraction of time. Likewise, both optimization methods also provided good results.

In all cases, the execution time of the Robust PEBS is similar, as this method assumes that all numbers are intervals. Thus, the number of interval operations remains unchanged, from a small set of uncertainties to a larger set. This particularity of the method can be both advantageous and disadvantageous. For a small set of uncertainties, Robust PEBS can be slower than optimization algorithms, making the use of the latter preferable. For a larger set, however, Robust PEBS will be faster, which makes it preferable.

6.4 | IEEE 39 bus

Uncertainties for the 39 bus system are depicted in Table 10 and a solid three-phase short-circuit occurs at bus 2 and the fault is cleared by opening line 2-3.

TABLE 10 Uncertainties - 39 bus

	Parameter Uncertainty (%)	
Bus	P_L	Q_L
3	50	50
16	25	25

For the IEEE 39 bus system, the following results were obtained:

TABLE 11 Results - 39 bus

Method	Critical Clearing Time (ms)
Robust PEBS	293
Simulated Annealing	300
Differential Evolution	300
Monte Carlo	300

TABLE 12 Execution Time - 39 bus

Method	Time (min)
Robust PEBS	67
Simulated Annealing	89
Differential Evolution	252
Monte Carlo	767

As seen Tables 11 and 12, the methodologies proposed in this article surpass the Monte Carlo approach by obtaining a fast, accurate result. Further, looking at Table 12, it can be seen that the methodologies can be used to assess the transient stability of larger systems.

7 | CONCLUSIONS

In this paper, we employed an extension of the PEBS method using interval arithmetic to assess transient stability of a power system considering uncertainties. We also proposed and tested algorithms to find a robust estimate of the critical clearing time of a multi-machine system. The proposed methodologies seem to be very promising in the studies of stability analysis.

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conflict of interest

The author declares that there is no conflict of interest.

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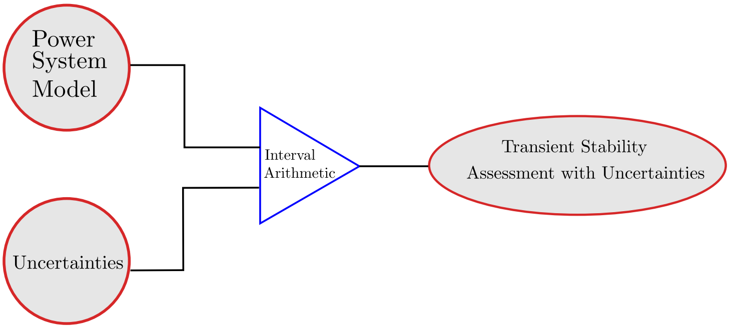
GRAPHICAL ABSTRACT



In this paper, interval arithmetic is employed to include uncertainties in the classic model for power systems transient assessment.