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A bargaining model for PLS entrepreneurial financing: A game theoretic model using agent-based simulation

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Abstract

This article aims to use a bargaining power model to reduce moral hazard—in the form of entrepreneurial effort shirking—and derive an optimum sharing ratio of a Profit and Loss Sharing (PLS) contract that involves a Venture Capitalist and an Entrepreneur. The model reveals the following interesting findings. First, under complete information—where the Venture Capitalist has a bargaining power - Venture Capitalist offers the entrepreneur a profit sharing ratio that is less than her capital contribution ratio. Second, in an incomplete information setting, the entrepreneur demands a profit sharing ratio higher than her capital contribution ratio when the sum of the marginal cost (from exercising a higher effort) and private benefits (from exercising a low effort) is greater than the marginal return (from exercising a high effort). In addition, the model is used to derive a span of negotiation about the profit sharing ratio. Finally, an agent based simulation (Netlogo) platform is considered to implement the model, which allows a faster numerical calculations of the profit share and helps decide on the validity of the funding contract.

KEYWORDS

agent-based simulation (Netlogo), finance, moral hazards, optimal profit-sharing, profit and loss sharing contracts

1 | INTRODUCTION

Venture Capitalists (VCs) are known for their role in financing start-up companies that are subject to high risk of failure due to their innovative nature. However, it is shown that they attribute the investment success or failure more to the team involved in running the project than to the business (Gompers, Gornall, Kaplan, & Strebulaev, 2020). Therefore venture capitalists, needs to monitor more intensively as they face a

higher cost of capital (Winton & Yerramilli, 2008). Accordingly, given the high risk of VCs financing many studies have examined the causes of such risks. Some of those studies are empirical in nature; see for example (Cochrane, 2005), (Baierl, Kaplan, et al., 2002), (Hall & Lerner, 2010) (Jain, 2001) Jain (2001), and (Kaplan & Strömberg, 2003) and (Tykvová, 2007) while others are theoretical; see (Casamatta, 2003), (Elitzur & Gavius, 2003), (Keuschnigg & Nielsen, 2003b), (Keuschnigg & Nielsen, 2003a).

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Yet, Despite their different approaches in tackling the causes of the above-mentioned risks, all these studies agree that failure risks are mainly due to agency problems between VC (he) and the financed entrepreneur (she). One substantial example of agency problem is moral hazard, where one party acts in a selfish manner regardless of the actions of their partners, see (Elitzur & Gavius, 2003). Moral hazards manifests in different ways. In our paper, we tackle one specific, yet recurrent, aspect of moral hazards: Entrepreneur's effort shirking. Such shirking behavior is unobservable to the VC and can be interpreted in terms of incomplete information or asymmetric information. In order to reduce this problem of effort shirking, we contribute to literature by introducing a new bargaining model that strive to derive an optimum profit sharing ratio in a Profit and Loss sharing contract involving a VC and an entrepreneur. This model takes into account the capital contribution as well as the expertise level of each participants in determining the extent of each participants bargaining power over the profit share. To provide realism to the model, the optimality of the profit sharing ratio needs to satisfy:

- The VC's and the entrepreneur participation constraints. That is, both participants gaining at least their opportunity cost (reservation utility).
- The entrepreneur's incentive compatibility constraints. That is, the entrepreneur gain more by exercising higher effort than by exercising a lower effort.

In our trial to develop this new model, the rest of the paper will be organized as follows: Section 2 presents a literature review where we put the effort shirking problem in relation to other problems of moral hazards in their current debates context. Section 3 introduces our model and discusses the methodology we adopt. Section 4 presents our main results and their discussion. Section 5 implements our model using an agent based simulation (Netlogo) platform and Section 6 concludes with a summary and possible extensions. The proofs of the main results of the paper can be found in the appendix.

2 | LITERATURE REVIEW

Moral hazards in joint venture such as, Profit and Loss Sharing (PLS) contracts, might manifest in many ways other than the shirking of the entrepreneur in terms of effort. First, moral hazard can be related to the quality and ability of the entrepreneur seeking the financing. Some entrepreneurs exaggerates on their abilities and the way they represents the success prospects of their projects. Evidence shows that exaggerating entrepreneurs

raise more funds in less time(Momtaz, 2020) compared to other entrepreneurs. This, therefore, increases the risk of attracting low ability entrepreneurs in a venture project.

Second aspect of moral hazards is the disagreement between the VC and the entrepreneur that take place after the investment is being made. Under this way, research that are based on control theory - see for example Dessein (2002) who proposes a balanced decision making process where entrepreneur are given decision making power at some stages of the investment process, while the VC are given decision power at other stages.

Third example of moral hazards in VC's financing is profit misreporting. In fact, most existing literature usually focuses on earnings misreporting (Crocker & Slemrod, 2007), (Cornelli & Yosha, 2003), (Merle, Hanlon, & MAYDEW, 2006). It is argued that under equity financing, where the financier is the only shareholder, the entrepreneur can hide part of the sales and, therefore, misreport profits (Fakir, Fairchild, & Tkiouat, 2019). This is also consistent with the findings of Tkiouat, and Allam (2019) where misreporting risk under equity financing such as VCs is more acute than it is under debt financing. While this manipulation of earnings protect, unethically, the entrepreneur against the risk of a low payoff when production is unsuccessful (Miglo, 2010) it does erode the financiers payoffs.

The final example of moral hazards, which is the substance of our paper, is effort shirking. In fact, many researches have tried to solve for the optimal sharing of profits between a VC and an entrepreneur under the risk of entrepreneurs effort shirking. For example it is found that allocation of equity share depends upon the level of complementarity of efforts between a VC and an entrepreneur (Vergara, Bonilla, & Sepulveda, 2016). In our work we argue that studying the effort of the entrepreneur is more important than studying the effort of the VC in making the venture a success. We believe that a VCs have no choice but to provide a high effort. This is due to the fact that they are, usually, the main financiers of the venture while the entrepreneur, usually, contributes to the venture with little to no financial contribution. Similarly another work shows that occupations providing workers with managerial knowledge foster entrepreneurial effort (Laffineur, Barbosa, Fayolle, & Montmartin, 2020). While this is linked to workers rather than entrepreneurs, we can make a concrete analogy here. In fact, the VC contribute to the venture not only financially but also through their expertise and knowledge. Therefore, we argue that this additional expertise could be used as a bargaining power to sustain and induce the entrepreneur to undertake high effort. In the same context, the effort devoted to entrepreneurship depends on individuals' perception that their personal

abilities in a chosen entrepreneurial activity will lead to success (effort-performance relationship) and that the outcome will provide better marginal reward compared to other career choices (Manolova, Brush, & Edelman, 2008), (Renko, Kroeck, & Bullough, 2012) (Thompson, 2009). In our model we make use of this point through the incentive compatibility constraint where exercising of a higher effort should lead to a better outcome than exercising a low effort.

Another research has also tackled the effort shirking of the entrepreneur when choosing between a VC and an Angel (Fairchild, 2011). The entrepreneur may exercise high effort if there is a higher value creating ability from the VC or a strong empathetic relationship with the Angel. This work is different than ours in that we focus on the tangible qualities of the VC such as Capital and expertise rather than behavioral qualities such as empathy. Our work is, also, different as we focus on the bargaining power of each participant in determining the optimum profit share.

In dealing with, moral hazard problem such as effort shirking, many studies argue that the VC should require a certain rate of return, see for example (Mason & Harrison, 2002) and (Manigart et al., 2002). Following this approach, our model suggests that any future investment by the VC will not occur if the VC does not expect the project to yield an expected profit greater than zero. This concept of break even rate of return also reflects a competition among VCs over the funding of the project. Furthermore, VCs face uncertain circumstances called external risks; see (Kaplan & Strömberg, 2004). These external risks include demand for new products, competitors' response to new product, and perception of financial markets when it comes to selling project stake at exit stage. Regarding the first external risk on demand for new products, our model allows for a range of expected revenues. While we believe that future demand cannot be estimated accurately, a hypothetical upper value estimate can be integrated in our model. For the second risk, in our model the competition effect can endogenously be reflected in our estimates of future demand.

Because the VCs would have less information about the likelihood of entrepreneurs shirking, this would create less trust between the parties. This lack of trust would induce the VC to apply a higher risk premium and argue for a larger share of the firm's equity than the entrepreneur deems reasonable (Douglas, Carlsson-Wall, & Hjelström, 2014). Given these facts around the causes of moral hazards in entrepreneurial financing, there is a serious need for tools that can overcome hidden information problems and accurately target capital to entrepreneurs that can use it well (Field, Pande, Papp, & Rigol, 2013). To develop such tools we make recourse to researches that

dealt with bargaining power in reducing the problem of moral hazards with a focus on effort shirking.

Indeed, a number of contributions examine how the relative bargaining power of entrepreneur and VC, for example, proxied by experience, affects contracts and allocation of cash flow as well as control rights (Burchardt, Hommel, Kamuriwo, & Billitteri, 2016). Cumming and binti Johan (2008) show on the basis of their European sample that more experienced entrepreneurs are more likely to finance themselves with common equity and correspondingly less likely to employ convertible preferred equity. VCs represent the mirror image with investment experience being positively related to the use of convertible preferred equity and negatively related to the use of common equity. These results are also consistent with Kaplan, Martel, and Strömberg (2007) and Hsu (2004) who show that investor sophistication leads to more favorable deal terms, which entrepreneurs willingly grant in exchange for their affiliation with more reputed financing partners. In the same line Fairchild (2004) found that welfare is maximized when the venture capitalist has high value-adding capabilities, the market for reputation is informationally efficient, and the manager has bargaining power. Our work gives more flexibility to these findings as both participants have a degree of bargaining power, which is dependent on their expertise and capital contribution in the project. For example the more a VC has an expertise compared to the entrepreneur, the more they are in a comparative advantage to get a higher profit share. Furthermore, the degree of bargaining power also determines the control structure choice between an entrepreneur and investors. For example, it is shown that as the VC's bargaining power increases, the more they are involved in jointly controlling the projects (Wang, Zhou, & An, 2017). This control, in favor of the VC, will even increase when the entrepreneur financing needs increases. In other words, as the VC contributes with more capital, they would gain more control and, hence have more comparative advantage in getting a higher profit share. As a consequence, the control of the VC's over the venture should also increase when the entrepreneurs private benefits increases (Wang et al., 2017). This, in turn, results in higher monitoring costs and, as a consequence, a higher required profit share by the VC.

In our opinion, the fact that there is an increase in the control by the VC in a PLS contract due to an increase in the financing needs and private benefits leads to two main pivot of bargaining:

- More VC's expertise as the VC is involved in the controlling the project. In fact, by jointly controlling the projects VCs still benefit the startup and the

entrepreneur, due to their positive value creation. This, however, shifts the bargaining power to VCs and benefits them at the expense of entrepreneurs (Ewens, Gorbenko, & Korteweg, 2019). In addition The advisory services, which VCs provide become a key factor for the success of a business. As stated by Casamatta (2003), entrepreneurs are endowed with creativity and technical skills in developing innovative ideas, but they often lack business experience and require the assistance that VCs can offer.

- More capital contribution in case the entrepreneur needs more financing (Wang et al., 2017).

Therefore, we focus on the VC capital contribution and expertise as two important determinants in the bargaining over the profit share. Our model is based on sharing profits and losses between the VC and the entrepreneur in a Profit and Loss sharing contract (PLS). In this model, profit is determined based on expected future profits and not on a fixed amount or as a percentage of investment. In other words, in our model there is no guaranteed return to the financier as in the case of debt or standard VC contract, and there is no guaranteed return to the entrepreneur as in the case of fixed wages. Losses are determined according to each participant's share in the project's capital.

3 | THE MODEL

We consider a one period PLS contract between a risk neutral financier VC and an entrepreneur. Both participants agree on a partnership contract that specifies the following elements:

- x : The VC's percentage share of the project capital
- I : The project's required funding
- α : The VC's share of the project's profit
- β : The VC's share of the project's loss

Notice that in this model, VC cannot lose more than his percentage capital contribution x . This a distinguishing feature from the conventional setting where the VC might demand guarantees against losses of more than his capital contribution. Moreover, in running the project, the entrepreneur exercises an effort e_i , where $i \in h = \text{high}, l = \text{low}$. Thereafter, we assume that the project's estimated revenue can take an upper value $\bar{R} = (1 + \bar{r})I > I$ with a probability θ_h under high effort and θ_l under low effort, such that $\theta_h > \theta_l$ and $\bar{r} > 0$. Conversely, this revenue can take a lower value $\underline{R} = (1 + \underline{r})I < I$ with a probability $(1 - \theta_h)$ under high effort and $(1 - \theta_l)$ under low effort. In

this model, the expected project profit under the high and low efforts are given respectively as:

$$\bar{NPV} = \theta_h \bar{R} + (1 - \theta_h) \underline{R} - I > 0 \quad (1)$$

$$\underline{NPV} = \theta_l \bar{R} + (1 - \theta_l) \underline{R} - I < 0 \quad (2)$$

In exercising her effort, the entrepreneur incurs a dis-utility $D(e_i) = d(e_i) (1 - x) I$, which is expressed as a percentage $d(e_i)$ of her investment in the project, $(1 - x) I$, when exercising effort (e_i). Exercising a higher effort results in higher dis-utility compared to exercising lower effort such that $D(e_h) > D(e_l)$. In addition, by exercising a low effort, the entrepreneur enjoys private benefit $S = s (1 - x) I$, which is expressed as a percentage “ s ” of her investment in the project. The entrepreneur also has a reservation utility (opportunity cost) $U = u (1 - x) I$ that we express as a percentage “ u ” of her investment in the project. Using the above elements, we can extract the expected return to the entrepreneur and VC under high and low efforts respectively:

$$E(r_{e/e_i}) = (1 - \alpha) \theta_i \bar{r} + (1 - \beta) (1 - \theta_i) \underline{r} \quad (3)$$

$$E(r_{vc/e_i}) = \alpha \theta_i \bar{r} + \beta (1 - \theta_i) \underline{r} \quad (4)$$

such that:

$$E(r) = \theta_i \bar{r} + (1 - \theta_i) \underline{r} = E(r_{vc/e_i}) + E(r_{e/e_i}) \quad (5)$$

We next discuss the methodology that we follow to solve our model. As we pointed out previously, we aim to reduce moral hazard in a PLS contract involving an entrepreneur and a VC. In our model, Moral hazard manifest itself in the entrepreneur exercising a low effort in running the project. Faced with the situation, VC needs to determine ex-ante the optimal profit-sharing ratio. The latter is derived by following three steps. First, we start by developing a sharing ratio under entrepreneurial observable effort. While this is not going to solve the moral hazard problem, it can serve as basis line for profit share negotiation. Second, we develop a span of profit-sharing ratio in an incomplete information setting where entrepreneurial effort is un-observable. Third, we provide a bargaining setting to determine the optimal sharing ratio from the span provided in the second step. Finally, an Agent based Simulation (Netlogo) platform is used as decision tool to help determine the optimal ratio. The use of the simulation is used to mitigate against the stochastic nature of the project revenues.

4 | RESULTS AND DISCUSSION

To obtain our theoretical results in terms of reducing moral hazard and derive an optimum sharing ratio of a Profit and Loss Sharing contract that involves a Venture Capitalist and an Entrepreneur, we proceed by following two steps. The first step analyses the model under entrepreneurial observable effort. The second step would analyze the model under incomplete information in the form of entrepreneurs unobservable effort.

4.1 | The model under entrepreneurial observable effort

Under this scenario, the entrepreneur cannot deviate from her commitment to provide a high effort. Therefore, the VC is in a comparative advantage in terms of negotiating the profit-sharing ratio. In other words, the objective of the VC would be to minimize the remuneration R_e of the entrepreneur up to a level that equates to her reservation utility and cost of high effort. Formally, we have:

$$R_e - (1 - \beta)I - D(e_h) - U = 0, \quad (6)$$

where

$$R_e = (1 - \alpha)\theta_h \bar{R} + (1 - \theta_h)(1 - \beta)\underline{R}. \quad (7)$$

Or in rate of return terms, we have:

$$E(r_{e/e_h}) = d_h + u. \quad (8)$$

We can now extract the profit share of the entrepreneur that will induce her to participate in the project:

$$(1 - \alpha) = (1 - \beta) \frac{1 + d_h + u - (1 - \theta_h)(1 + \underline{r})}{\theta_h(1 + \bar{r})} \quad (9)$$

or using short notations:

$$(1 - \alpha) = (1 - \beta)M_{(epc)}, \quad (10)$$

where $M_{(epc)}$ represents the entrepreneur's participation multiplier that indicates how many times the entrepreneur's profit-sharing ratio exceeds her capital contribution ratio. We can show that $M_{(epc)} \leq 1$ [see the proof of Proposition 1 in the appendix] leading to the result in the following proposition.

Proposition 1. Under observable effort the VC has a natural bargaining power over the profit share inducing the entrepreneur to demand a profit sharing ratio that is less or equal to her capital contribution ratio.

Proposition 1 can also indicate that, under observable effort, there is no emphasis on the level of expertise of the VC which suggests lower costs for the VC in learning about the venture. This also means that a VC investment in reducing information asymmetries could lead the VC to invest in projects in which they are not expert. The latter, as they are assured of the entrepreneurs high effort ability, would widen the VCs scope of investment.

From another side, the multiplier $M_{(epc)}$ is positively (negatively) related to the entrepreneurs effort disutility, her reservation utility, and low (high) outcomes of the project. This suggest that entrepreneur would be willing to sacrifice profit share if she believe that there is an optimistic project outlook.

Proposition 2. To participate in a PLS contract, the entrepreneur demands a lower profit-sharing ratio when an optimistic profitability outlook scenario exists.

This finding suggests that an entrepreneur demanding of a lower share could also signal a higher ability entrepreneur. Finally, from Equation 10 we can obtain the VC's profit-sharing ratio that induces the entrepreneur to participate in the project as follows:

$$\alpha_{epc} = 1 - (1 - \beta)M_{(epc)}. \quad (11)$$

From Equation(11) and proposition (2), we could see clearly that, under observable effort, the VC would benefit from a higher profit share when there is a positive outlook. As explained in proposition 2, the entrepreneur would be willing to sacrifice more profits, benefiting the VC, when there are positive outlooks ahead. The next section would explore the negotiation over the profit share under entrepreneurial unobservable effort.

4.2 | The model under entrepreneurial unobservable effort

In this case, the VC is unable to observe the entrepreneur's effort after the financing of the project has taken place. Thus, in addition to fulfilling her participation constraints, the VC must incite the entrepreneur to exert a high effort.

4.2.1 | Problem preliminaries

For the entrepreneur to exercise a high effort, she must have a better payoff U_h compared to the payoff under exercising a lower effort U_l . Interestingly we can link the

profit share to the entrepreneur capital contribution through an “entrepreneur’s incentive constraint” multiplier $M_{(eic)}$ leading to the following proposition: [see the proof of Proposition 3 in the appendix].

Proposition 3. To exercise a high effort, there exist an “entrepreneur incentive constraint” multiplier $M_{(eic)}$ such that the entrepreneur demands a profit sharing ratio $(1 - \alpha_{eic}) \geq (1 - \beta) M_{(eic)}$ relative to her capital contribution in the project.

Analysing this multiplier, further, shows that it is not necessarily greater than 1. Which means that it is not always true that entrepreneurs would demand a higher profit share, relative to their capital contribution, when they hold private information about their effort. The following proposition explains why the latter is possible [see the proof of Proposition 4 in the appendix].

Proposition 4. To exercise a high effort, in an unobservable effort setting, the entrepreneur will demand a higher profit sharing ratio than her capital contribution ratio if the sum of the marginal cost (from exercising a lower effort) and private benefits (from exercising a low effort) is greater than the marginal return (from exercising a high effort). Otherwise the entrepreneur accepts a lower sharing ratio than her capital contribution ratio.

This proposition could be helpful to VC as a signaling tool about the entrepreneurs effort ability. In other words, an entrepreneur demanding a higher profit share does not necessarily signals a high quality but could also signal high hidden private benefits that the entrepreneur could get by exercising a lower effort.

Using the above results and solving for α , the profit sharing ratio required by the VC to motivate the entrepreneur to perform a high effort satisfies:

$$\alpha \leq \alpha_{eic} = 1 - (1 - \beta)M_{(eic)} \quad (12)$$

Further analysis of the “entrepreneur’s incentive compatibility” shows a link between the entrepreneur’s reservation utility and the entrepreneur’s private benefits from exercising a low effort. Interestingly, as the private benefits increases above the reservation utility, the more is the entrepreneurs required share of profit. This leads to the following proposition [see the proof of Proposition 5 in the appendix]:

Proposition 5. The entrepreneur will demand a profit-sharing ratio higher than her capital contribution ratio only if her private benefits from exercising a low effort are higher than her reservation utility.

Proposition 5 could also be used as a quick signaling mechanism. In other words, the higher is the profit share demanded by the entrepreneur, relative to her capital contribution, the more is the gap between the private benefits they could get compared to their reservation utility.

From Proposition 5 and Equations (11) and (12), we can infer the VC profit sharing ratio α fulfilling both the incentive and the participation constraints of the entrepreneur:

$$\alpha \leq \min(\alpha_{eic}, \alpha_{epc}). \quad (13)$$

4.3 | Satisfying the VC’s participation constraints

In the case of unobservable effort scenario, the VC works out his payoff taking into consideration two probabilities:

- Certainty probability p_h : The VC estimates a degree of certainty regarding the willingness of the entrepreneur to engage in high effort.
- Effort-based probability: The VC assigns a probability to the success of the project in high and low effort cases, θ_{th} and θ_{tl} , respectively.

Formally, the VC will not enter the PLS contract unless the expected return from the project is positive. i.e.,

$$E(r) = P\bar{r} + (1 - P)\underline{r} > 0, \quad (14)$$

where P is the probability of the success of the project and is given by:

$$P = p_h\theta_h + (1 - p_h)\theta_l. \quad (15)$$

Given the scarcity, the innovative feature of the project, and the unobservable effort scenario, the VC is in a bargaining comparative disadvantage and therefore he needs a sharing ratio α_{vpc} that will enable him to at least break even such that [see the proof of the following derivation in Appendix A]:

$$\alpha_{vpc} = \beta M_{vpc} \quad (16)$$

where

$$M_{vpc} = \frac{1 - (1 - P)(1 + r)}{1 + \bar{r}} \quad (17)$$

is the VC’s participation multiplier that represents what the VC requires as profit sharing ratio relative to his

capital contribution in order to participate in financing the project. We can show that this multiplier is less than 1. This leads to the following proposition [see the proof of Proposition 6 in the appendix]:

Proposition 6. For the VC to participate in financing a scarce project under a PLS contract, the VC is induced to accept a lower share of the profit relative to his ratio of the capital contribution.

This means that the VC might be satisfied with a profit-sharing ratio that is less than his capital ratio contribution as long as he is at least breaking even. This concept shows the impact of asymmetric information, such as unobservational effort, in lowering down the profit share of VCs. The next section shows the minimum acceptable levels of shares that the participants would accept under unobservational effort. These minimum acceptable levels would constitute what we refer to as a 'Span of negotiation'.

4.4 | Span of negotiation

From Equations (13) and (16), we can deduce the range of values of the sharing ratio α that the VC should get and which can satisfy both parties:

$$\alpha_{VPC} \leq \alpha \leq \min(\alpha_{eic}, \alpha_{epc}). \quad (18)$$

From Equation (10), we can establish a span of negotiation (SN) in terms of the profit-sharing ratio such that:

$$SN = \min(\alpha_{eic}, \alpha_{epc}) - \alpha_{VPC}. \quad (19)$$

This span must be positive for the project to be financed. For example, the project will not be financed if the VC profit sharing ratio (α_{epc} required to incite the entrepreneur to participate or to exert high effort α_{eic}) is less than what the VC requires to participate in the project. This leads to the following proposition.

Proposition 7. In PLS contract, a span of negotiation over the profit sharing ratio can be determined. The VC would not finance the project unless this span is positive. Formally:

$$\begin{cases} SN > 0 & \text{Project is financed} \\ \text{Otherwise} & \text{Project is not financed.} \end{cases} \quad (20)$$

This span of negotiation has implications for VCs as it could provide them with more flexibility over the

negotiation of a profit share. By knowing their minimum acceptable level they can negotiate a further increase in that profit share. This will depend on their bargaining power. The latter is the model we try to develop next by starting with the development of a 'Nash equilibrium'.

4.5 | Nash equilibrium

In this section, we consider a strategic game in which VC plays the role of a Stackelberg leader. The latter knows the response function of the entrepreneur and chooses his own action taking into account the effect of his choices on the action of the entrepreneur. Stackelberg's selected actions would elicit a certain desired behavior from the follower (entrepreneur). In our strategic game, the VC decides on the profit-sharing ratio taking into account the self-interest behaviour of the entrepreneur. The VC makes his decision using backward induction. In this approach, players think about the outcomes at the last stage of the game and then roll back through the game tree to the starting stage. For more on this strategic game, the reader can consult (see [Dixit & Skeath, 2015]). Table 1 and Figures 1-4 provide summaries of all stages of the game followed by backward induction to identify the game equilibrium. In the following, we discuss these figures to describe what happens at each stage of the game.

As shown in Figure 1, backward induction is used to solve the strategic game in which the last player in its sequence is the entrepreneur. The latter will compare her payoffs U_e at nodes 5–9 of Figure 1. The payoffs at these nodes are described as follows:

$U_e(5) = (1 - \beta)I$: The entrepreneur gets her reservation utility as the VC does not participate in financing the project. She does not get her minimum required profit share. $U_e(6) = U_e/e_l$, $U_e(7) = U_e/e_h$, $U_e(8) = U_e/e_l$: The entrepreneur participates in the contract, but gets a share less than that required to incentivize her to exert a high effort. She therefore exerts a low effort. $U_e(9) = (1 - \beta)I$: The entrepreneur gets her reservation utility as she does not participate in the project. She does not get her minimum required profit share.

Now since no participation from both parties is expected at nodes (5) and (9), the VC strategies $\alpha < \alpha_{VPC}$ and $\alpha > \alpha_{epc}$ are not feasible and therefore can be eliminated from the tree.

In Figure 2, the payoff of the entrepreneur at node 6 dominates that at node 8. This is because the entrepreneur gets a higher share at node 6 (at which VC gets a lower share with $\alpha \leq \min[\alpha_{eic}, \alpha_{epc}]$) than at node 8 (at which VC gets higher share with $\alpha > \alpha_{eic}$). Therefore, this eliminates nodes 8 and subsequently node 3 and their relevant branches from the tree.

TABLE 1 Summary of the game

Player	Action	Payoff	Profit share	Loss share
Entrepreneur	e_i	$U_{ei} = I[(1 - \alpha)(1 + \bar{r})\theta_i + (1 - \beta)((1 + \bar{r})(1 - \theta_i) - d_i - u + s_i)]$	$1 - \alpha$	$1 - \beta$
VC	α	$V = I[\alpha(1 + \bar{r})P + \beta((1 + \bar{r})(1 - P))]$	α	β

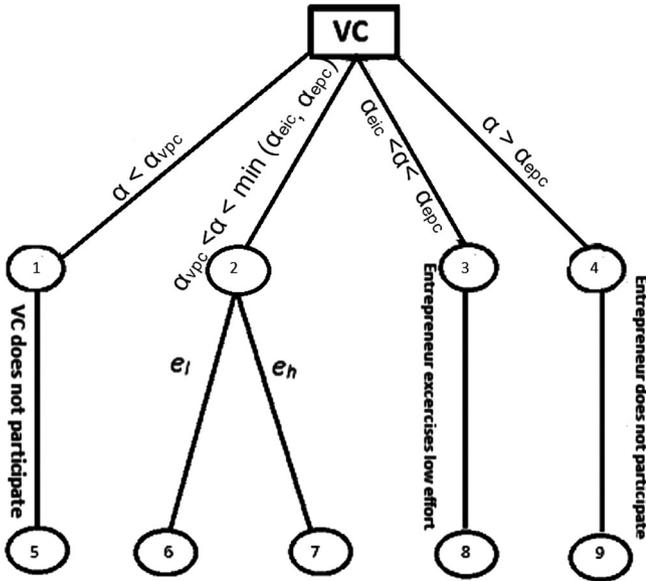


FIGURE 1 Summary of the game: step 1

Using Equation (12), Figure 3 shows that the entrepreneur satisfies her participation and incentive constraints and therefore engages in high effort. Consequently, the payoff to the entrepreneur at node 7 dominates that at node 6.

Finally, Figure 4 shows that the payoff of the entrepreneur at node 2 is represented by that in node 7. Moreover, the payoff to the VC at node 2 is satisfying the VC participation constraint [see Equation (18)], while the entrepreneur is to perform a high effort.

From the analysis above the Nash equilibrium is: VC offering a profit share in the form of a span of negotiation: $\min(\alpha_{eic}, \alpha_{epc}) < \alpha < \alpha_{vpc}$. On the other hand the entrepreneur commits to engaging in a high effort. The next step then is to find the optimal profit share within this span of negotiation. Our bargaining model will help in achieving this objective next.

4.6 | Bargaining power model

From our earlier discussion, we have identified the span of negotiation over the profit-sharing ratio that the VC can get from a PLS contract. We next derive the optimum sharing

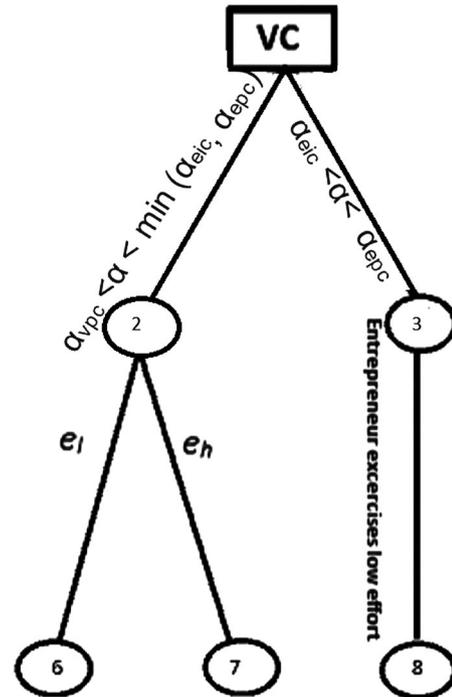


FIGURE 2 Summary of the game: step 2

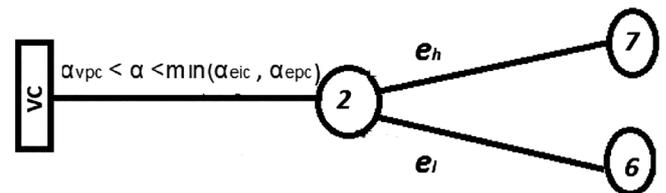


FIGURE 3 Summary of the game: step 3

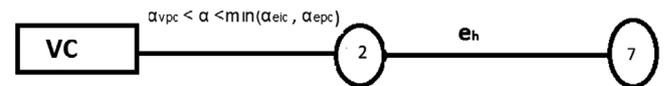


FIGURE 4 Summary of the game: step 4

ratio taking into consideration the bargaining power of both participants in the contract. For this, we assume that the bargaining power depends on three parameters:

- The VC's capital contribution ratio β ;
- The VC's expertise level μ ;

- The weight of importance W attached to β , and $(1-W)$ attached to μ . For example, if $W = 50\%$ then the VC assigns equal importance to the capital and expertise provided to run the project.

We start from the basic point where both participants equally share the project capital and have the same level of expertise (50%–50%); i.e., the span of negotiation is shared equally. In this case, we propose the following contractual agreement:

$$\alpha_{average} = \frac{\min(\alpha_{eic}, \alpha_{epc}) + \alpha_{VPC}}{2}, \quad (21)$$

which constitutes the starting point above which the VC gets more profit share and, hence, has more bargaining power. In the following, a change in the VC's capital contribution is noted as $\% \Delta_B$, and a change in the VC's expertise level is noted as $\Delta_u \in (-0.5; 0.5)$. For example if $\Delta_u = -0.5$, then $u = 50\% - 0.5 = 0$; i.e., the VC has no expertise and therefore it is the entrepreneur who has full expertise. From the above, we can propose an optimum profit sharing α_{opt} to the VC:

$$\alpha_{opt} \begin{cases} \alpha_{average} = W_B \% \Delta_B + W_u \% \Delta_u \min(\alpha_{eic}, \alpha_{epc}) - \alpha_{vpc} & \text{if } \min(\alpha_{eic}, \alpha_{epc}) > \alpha_{vpc} \\ 0 & \text{Otherwise.} \end{cases} \quad (22)$$

The development of this theoretical model would not show its effectiveness without being tested. Ideally this could be done empirically. However, due to novelty of this model, there is a lack of data to test it against. A better alternative, which allows for a large analysis of data is simulation. In our context we use 'Agent Based Simulation' (ABS).

5 | AGENT-BASED SIMULATION

In this section, we create an agent-based simulation (Netlogo) platform to implement the model. This platform will allow for a faster numerical calculations of the profit share and helps decide on the validity of the funding contract. In fact, we would run the simulation for 1,000 times. This will allow us to run the parameters of the same contract to see how many times it is valid. i.e the span of negotiation should be positive. We are using a threshold of 30%. This means if the contract is 70% showing a positive span then it is valid.

TABLE 2 Simulation parameters values

Parameters value	Parameter's description
$I = 100,000$	Project capital
$\beta = 70\%$	VC capital share
$u = 5\%$	Entrepreneur reservation utility
$d_h = 10\%$, $d_l = 5\%$	Entrepreneur's dis-utility of high and low effort respectively
$\bar{R} = 15000 R$ $= 50,000$	Project high and low revenues respectively
$\theta_h = 80\%$, $\theta_l = 40\%$	Probability of project success if a high or low effort is undertaken
$\sigma_h = 10\%$, $\sigma_l = 10\%$	Standard deviation in the estimation of θ_h and θ_l respectively
$p_h = 50\%$	Vc's level of certainty that the entrepreneur would perform a high effort
$\mu = 0\%$	VC expertise level
$W = 50\%$	Weight of importance attached by the VC to her contribution in project β

Otherwise it is not. The simulation also would calculate the optimum profit share out of the span of negotiation.

The parameters considered in our simulation are reported in Table 2.

Taking into account the stochastic nature (as measured by standard deviations) of project revenues, we run our model for 1,000 simulations. This high number of simulations will allow for detecting profit sharing rates, which will not fit the participation constraints and incentive constraints for the participants. The VC determines a threshold, in this case 30% that if exceeded the contract will be void, and therefore would not be financed. Figure 5 shows the results of our simulated model.

From Figure 5, we can see that out of 1,000 simulations, 109 contracts were void. This represents 12% of the contracts, which is below the rejection threshold of 30%. We can also see that the minimum acceptable sharing ratio for the VC is 63.03%, while the maximum sharing ratio for the VC, which is tolerated by the entrepreneur is 71.02%.

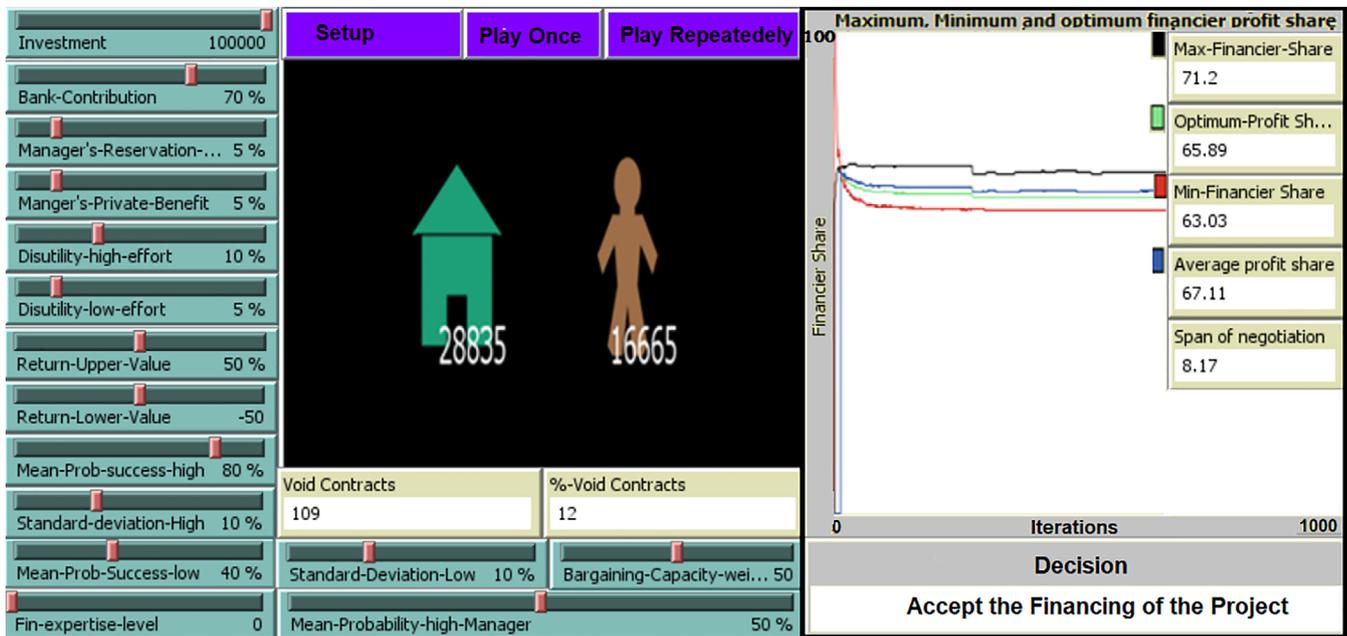


FIGURE 5 The agent based simulation (Netlogo) [Colour figure can be viewed at wileyonlinelibrary.com]

We now consider our bargaining power model. If we start with the initial scenario where both participants have the same capital contribution and expertise level, then the agreed VC profit sharing ratio is $\alpha_{average} = 67.11\%$. Moreover, since $\beta = 70\%$, we have $\% \Delta_B = 20\%$; i.e., the VC is exceeding the average capital contribution by 20%. In addition, since $\mu = 0$, the VC acknowledges that he has no expertise in the project. Therefore, $\% \Delta_u = -0.5$; i.e., the VC is 50% from the neutrality point $\mu = 50\%$ at which both VC and entrepreneur have equal expertise. We also find that capital contribution is given equal importance as the level of expertise in deciding on the optimum profit ratio. Thus, the optimum profit ratio for the VC is given by:

$$\alpha_{opt} = 67.11\% + [50\%10\% + 50\%(-0.5)] \quad (23)$$

$$[71.08\% - 63.03\%] = 65.89\%.$$

If we compare the value of this optimum profit share to the average value of the profit share, we find a difference of 1.22% in favour of the entrepreneur. We also find that the optimum sharing ratio is closer to the VC's acceptable minimal ratio (at which the VC would break even) than it is to the VC's tolerated maximum share (at which the entrepreneur would break even). Both findings indicate that the entrepreneur has some relative bargaining power compared to the VC. This is mainly due to the VC's lack of expertise in this project and also because he is giving a significant importance to the parameter that measures the expertise level (50%).

6 | CONCLUSIONS

This article aims to reduce moral hazard in the form of entrepreneur's exercising a low effort in a PLS contract. The latter contract limits the loss of each participant to their ratio of capital contribution, which differs from the standard VC financing contracts in which the latter might demand extra compensation or collateral in the case of losses. While this limitation in terms of loss adds a security to the entrepreneur in case of liquidation, it can also be a source of extra moral hazard that the VC has to take into consideration when deciding his profit-sharing ratio. To determine the profit-sharing ratio in the context of a PLS contract, we follow four steps. We first identify the profit-sharing ratio under perfect and observable effort information. This case forms the basis for calculating the profit sharing that would allow the entrepreneur to exercise a high effort. In a second step, we consider the case of imperfect and un-observable effort information through which we identified a span of negotiation for the profit sharing. While this step has the advantage of providing flexibility in terms of profit sharing negotiation, it omits the calculation of the optimum profit sharing ratio. This in turn leads to the third step where a bargaining power model is introduced. The latter model depends on three parameters: VC's capital, VC's expertise, and the importance of each of them for the VC. The fourth and final step consists of using an agent-based simulation (ABS) platform to implement our model. The usefulness of the ABS platform is that it allows for faster numerical calculation of the profit share

and helps decide on the validity of the funding contract by using a high number of simulations. There are several ways that one can use to extend the model. First, the model deals with moral hazard from one side (entrepreneur). Thus, the model can be extended by incorporating the shirking of the VC as well (double sided moral hazard problem). The inclusion of the VC's shirking could be interesting in terms of assessing the bargaining power and, as a consequence, calculating the profit share of each participant. A second extension could be the study of the PLS contracts in a multiagent setting. This includes a multitude of entrepreneurs and a multitude of VCs. This would help assess the effect of competition over funding (from VCs side) or financing (from entrepreneurs' side) on the contractual terms. Finally, similar to Fairchild (2011) who compares VC and angel financing, another possible extension could be the comparison of VC's financing under PLS contract and VC's financing under conventional VC contract. This might help the entrepreneur choose which source of financing is adequate and allows PLS contractors and conventional ones to adjust their contractual terms.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this paper as no real data were analyzed or created in this study.

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APPENDIX A:**PROOF OF PROPOSITION 1**

To prove that the ratio M_{epc} is less than or equal to 1, we take the difference between its nominator and denominator. $M_{epc} \leq 1$ only if the difference is less or equal to than 0. Formally, we have:

$$\begin{aligned} & 1 + d_h + u - (1 - \theta_h)(1 + \underline{r}) - \theta_h(1 + \bar{r}) \\ &= d_h + \mu - [\theta_h \bar{r}] + (1 - \theta_h)\underline{r} = d_h + \mu - E(r) < 0. \end{aligned}$$

Thus, the above statement is true since from Equation (8) we have $E(r_{e/e_h}) = (d_h + u)$ and from Equation (5) we have $E(r) > E(r_{e/e_h})$

PROOF OF PROPOSITION 2

We need to fulfill the Incentive compatibility constraint (EIC) of the entrepreneur; that is, $U_h \geq U_l$, where U_h and U_l are the utilities of the entrepreneur under high and low effort, respectively. Using the expectations, rate of returns, percentage dis-utility percentage utilities and percentage private benefits, we have:

$$\begin{aligned} U_h &= \theta_h(1 - \alpha)(1 + \bar{r}) + (1 - \beta)[(1 - \theta_h)(1 + \underline{r}) - 1 - d(e_h) - u] \geq \\ U_l &\theta_l(1 - \alpha)(1 + \bar{r}) + (1 - \beta)[(1 - \theta_h)(1 + \underline{r}) - 1 - d(e_l) - u + s], \end{aligned} \quad (A1)$$

Solving for the share of the entrepreneur $(1 - \alpha)$ we get:

$$(1 - \alpha) \geq (1 - \beta) \frac{\Delta\theta \cdot (1 + \underline{r}) + \Delta d + s}{\Delta\theta(1 + \bar{r})},$$

or

$$1 - \alpha \geq (1 - \beta)M_{eic},$$

where we can name the manager incentive compatible constraint multiplier M_{eic} :

$$M_{eic} = \frac{\Delta\theta \cdot (1 + \underline{r}) + \Delta d + s}{\theta_h(1 + \bar{r})}$$

PROOF OF PROPOSITION 3

To prove that M_{eic} is not necessarily greater than 1, we take the difference between its nominator and denominator:

$$\Delta\theta \cdot (1 + \underline{r}) + \Delta d + s - \theta_h(1 + \bar{r}) = \Delta d + s - \Delta\theta(\bar{r} - \underline{r}).$$

Thus,

$$M_{eic} \begin{cases} < 1 & \text{if } \Delta d + s < \Delta\theta(\bar{r} - \underline{r}) \\ > 1 & \text{if } \Delta d + s > \Delta\theta(\bar{r} - \underline{r}). \end{cases}$$

PROOF OF PROPOSITION 4

The proof of Proposition 4 follows directly from the last result in Proposition 5.

PROOF OF PROPOSITION 5

We must prove that it is not always true that $(1 - \alpha_{eic}) > (1 - \alpha_{epc})$. We have

$$(1 - \alpha_{eic}) = (1 - \beta_{eic})M_{eic} \quad \text{and} \quad (1 - \alpha_{epc}) = (1 - \beta_{epc})M_{epc}$$

Taking the difference, we get:

$$\begin{aligned} (1 - \alpha_{eic}) - (1 - \alpha_{epc}) &= \theta_l[1 + d_h + u + \lambda(s - u)] \\ \text{where } \lambda &= \theta_h/\theta_l > 1 \end{aligned}$$

This difference is not necessarily greater than 0 as it depends essentially on the difference between the private benefit s that the entrepreneur can get from providing high effort and the reservation utility u from not participating in the project. Formally, we have:

$$\alpha_{eic} \begin{cases} < \alpha_{epc} & \text{if } s > u \\ > \alpha_{epc} & \text{if } s < u. \end{cases}$$

PROOF OF PROPOSITION 6

The VC participation constraint is given by:

$$vpc: P\alpha(1 + \bar{r})I + (1 - P)\beta(1 + \underline{r})I - \beta I \geq 0.$$

Solving for α we get:

$$\alpha \geq \alpha_{vpc} = \beta \frac{1 - (1 - P)(1 + \underline{r})}{(1 + \bar{r})P},$$

where

$$M_{vpc} = \beta \frac{1 - (1-P)(1+r)}{(1+\bar{r})P}.$$

This multiplier is less than or equal to 1. This can be proven by taking the difference between the nominator and the denominator of this multiplier and finding that the difference is negative:

$$1 - (1-P)(1+r) - (1+\bar{R})P = -[P + (1-P)r] = -E(r) \leq 0.$$

The last statement is true as the VC cannot enter the contract if the expected return from the project is negative; see Equation (14).