The theoretical research article as a reflection of disciplinary practices: The case of pure mathematics

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Recent years have seen an interest in the generic structure of empirical research articles across a variety of disciplines. However, significantly less attention has been given to theoretical articles. This study aims to begin to address this imbalance by presenting the results of an investigation into the organizational and rhetorical structure of theoretical pure mathematics research articles. The data set combines a close analysis of 22 peer-reviewed articles and semi-structured interviews with their authors. While there is considerable variation in terms of the major section headings and content, the results reveal an overall structure that differs from a typical empirical research article. We argue that this alternative structure is produced by the dual argumentation – mathematical and meta-mathematical – which runs throughout the text. Moreover, triangulation with the interview data indicates that the structural patterns of the theoretical pure mathematics research article can be viewed as a reflection of the research practices and epistemology of the discipline.

INTRODUCTION

The rhetorical and organizational structure of the research article (RA) has been a focus of applied linguistics research for the past three decades. Inspired by Swales' CARS model ('Create-a-Research-Space', Swales 1981, 1990), a great deal of this research has appeared in various applied linguistics journals and has dealt with the analysis of rhetorical moves in the main sections of the empirical research article, most notably the Introduction (e.g. Ozturk 2007; Samraj 2002), Method (e.g. Bruce 2008; Kanoksilapatham 2005), Results (e.g. Bruce 2009; Lim 2010) and Discussion (e.g. Holmes 1997; Peacock 2002). Thus, Swales' approach to genre analysis has been fruitful in enhancing our understanding of specific sections of the empirical research article which was deemed to follow the so-called "conventional" IMRD (Introduction, Methods, Results, Discussion) macro-structure (Lin and Evans 2012). However, it seems that the Research Article as a whole has been less important than its parts, and variation in its overall generic structure has received considerably less attention in applied linguistics research.

In addition to Swales' description of the RA as an academic genre (1990: 110–174), several studies have analyzed structural patterns in RAs in specific disciplines. For example, Wood (1982) examined the rhetorical structure of research articles in chemistry. Posteguillo (1999) attempted to describe a systematic structural model for RAs in computer science but reached the conclusion that no clear pattern could be identified in his selected sample of 40 RAs: his main finding was that the IMRD framework did not apply because most the articles in his corpus were of theoretical nature. Yang and Allison (2004) analyzed the macro-structures of two kinds of RAs in applied linguistics, those based on empirical research and those of an argumentative nature (which the authors labelled 'secondary RAs'). Following the introduction of the genre-based approach, Crooks (1987) undertook a comparative study of structural patterns in the natural and social sciences. Most recently, Lin and Evans (2012) have conducted a larger-scale cross-disciplinary study of the schematic structure of 433 empirical RAs across 39 disciplines, including applied sciences, engineering, social sciences and the humanities. Their article alerts us to the fact that 'our knowledge of RA macro-structures is far from complete' (Lin and Evans 2012: 151) and that the macro-structure of the RA across different disciplines is fluid and context-dependent. Although the "standard" IMRD structure, which dominated scientific texts produced in the 1980s, is still one of the dominant types, it can no longer be regarded as the default option for organizing empirical

RAs. As far as theoretical research articles are concerned, their organizational and rhetorical structure largely remains an unexplored territory.

Apart from the humanities and social sciences, theoretical research articles are common in various scientific fields where empirical research, and experimentation in particular, is not the norm, such as theoretical physics, astrophysics, economics, or engineering; these disciplines publish logical argumentation papers which move from the general to the specific (Swales and Feak 2004: 215). Tarone et al. (1981) were the first to point out the fundamental distinction between empirical and theoretical RAs. According to Swales and Feak (ibid.), mathematics RAs – presumably both pure and applied – belong to the latter type, but, in our view, such generalization is questionable. Due to its unique epistemology, pure mathematics does not easily fit into the established 'hard' and 'soft' dichotomy of academic disciplines (Becher and Trowler 2001). While it shares some common ground with the hard sciences in that knowledge is generated cumulatively and there are 'clear criteria for knowledge verification' (Becher and Trowler 2001: 36), the process of knowledge construction in pure mathematics is different and epistemologically unique since new results are substantiated by proof and logical, mathematical reasoning (Elwes 2010). Natural science is experimentally falsifiable (Popper 1959), but this is not the case with pure mathematics. The outcome of mathematical research is limited to a binary true or false and does not depend on the interpretation of data. The discipline is characterized by the poetic belief that the "purest" mathematics is discovered, and its ultimate aim is to achieve a natural and elegant simplicity. Interestingly, in Bernstein's (1999) classification of knowledge structures, which has become a basis for the study of disciplinary discourses in Systemic Functional Linguistics (e.g. Christie and Maton 2011), mathematics is placed closer to the humanities and is considered to possess a 'horizontal knowledge structure' and 'a set of discrete languages, for particular problems' (Bernstein 1999: 164). Although Bernstein's classification can also be debated, there is little doubt that pure mathematics has a unique epistemology and provides an excellent case to explore the impact of disciplinary knowledge-making practices upon its disciplinary discourse. In this context, our study examines the rhetorical and schematic structure of the pure mathematics RA in the context of its disciplinary practices. We believe that our study will have implications for genre research and academic writing instruction at the postgraduate and professional level.

The knowledge-making practices of specific disciplinary discourse communities which share 'a suitable degree of relevant content and discoursal expertise' (Swales 1990: 27) are fundamental to our understanding of academic genres and the context in which they are created. The research of scholars such as Becher and Trowler (2001), Bernstein (1999), and MacDonald (1994) has shown how epistemological factors shape disciplinary discourses. Hyland's studies (e.g. 1999, 2000, 2005) have described differences in quantitative distributions of lexical items and structural elements in RAs across disciplines, and the findings of this large-scale corpus analysis were supplemented by interviews with writers working in the examined disciplinary fields. Hyland (2006) provides a comprehensive overview of the research on specific disciplinary writing practices, but again, pure mathematics is a notable omission.

Applied linguistics research into mathematical discourse is scarce. For example, Swales *et al.* (1998) have examined the use of imperatives in research level mathematics. O'Halloran's (2005) study has explored the balance between language, symbolism and visual images in mathematical discourse. Shaw (2006) describes how mathematical code is embedded in text across three disciplines, pure mathematics, engineering, and physics. A recent article by Artemeva and Fox (2011) has examined written genres used in the teaching of mathematics. Authors (2012) have applied Hyland's (2005) model of stance and engagement to closely analyze interactional metadiscourse in 25 RAs in pure mathematics. In the field of

mathematics education, Burton and Morgan (2000) have explored the construction of authorial identities in 53 RAs in three branches of mathematics (pure, applied, and statistics). Thus, the connection between the linguistic features of pure mathematical discourse and disciplinary knowledge-making practices calls for further research. In this study, we explore the macro-structure of the RA in pure mathematics with the aim to answer the following research questions:

1) What is the rhetorical and organizational structure of the pure mathematics research articles analyzed in the study?
2) Is there a consistent structure?
3) What connection can be traced between the rhetorical and organizational structure of the research articles and the knowledge making-practices of the pure mathematics academic community?

As demonstrated below, our results suggest that the rhetorical arrangement of the theoretical mathematics article departs significantly from the structure observed in previously studied disciplines, and that the discursive practices of theoretical mathematicians reflect how the field constructs and shares new knowledge.

METHOD

Our study combines the genre analysis of RAs and semi-structured interviews with the lead authors of the same articles. This approach is partly inspired by a number of Hyland's studies (e.g. 2000, 2004, 2005) of disciplinary discourses, although our focus is on rhetorical and organizational macro-structure rather than lexico-grammatical features. A method involving a more sizeable sample and corpus tools was rejected, as our aim was to conduct an in-depth qualitative study. Another difference from Hyland in our method is that all interviewees were both expert informants and authors of the analyzed articles, which is a novel aspect in ESP genre research.

   Thus, our approach encompasses a combination of a close reading of 22 peer-reviewed pure mathematics RAs and transcribed interviews with five lead authors. In our case, this dual approach to collecting both interview and textual data contributed to facilitating not only a mapping of the structure of the articles, but also to providing insights into the reasons behind author's choices in terms of structuring their research-based writing. It should be noted that several previous studies on the rhetorical structure of RAs or RA sections in specific disciplines (e.g. Bruce 2008, Lin and Evans 2012, Ozturk 2007, Posteguillo 1999, Samraj 2002) have focused exclusively on text analysis and do not include any interview data. Thus, by incorporating interviews with the authors of RAs, our approach provides an additional insight into the context of specific disciplinary text production.

The selected research articles
A total of 25 full-length peer-reviewed research articles from the fields of algebraic geometry, ergodic theory and dynamical systems, set theory, and algebraic number theory were collected from the five informants taking part in the study. Of these articles, 22 were identified as being purely theorem-proof based and were therefore classed as theoretical. In other words, the aim of the research being reported in the 22 selected RAs was to establish a mathematical theorem evidenced by proof resulting from logical mathematical reasoning. All the articles were written by the authors interviewed for the study and were published between 1998 and 2011. In order to safeguard the anonymity of the authors, the names of articles are not identified. The impact factors of the journals in which the articles appear range between 0.3 and 2.7 according to the Thompson Reuters database. Impact factors are calculated based on the number of the average number of times articles in the journal are cited within a given time

frame. Therefore, a high impact factor indicates a journal containing oft-cited articles. The relatively small number of articles allowed for a detailed analysis that would have been less practicable with a larger collection. Furthermore, the limited scope of the study facilitated access to all the lead-authors whose work was included in the compilation.

Approach to the analysis
The first stage of the analysis involved identifying the organizational structure of the genre. As mentioned in the introduction, a model for theoretical articles along the lines of IMRD has not been proposed. It has of course been acknowledged that the IMRD structure is inadequate to describe the structure of theoretical research articles and review articles as authors are not reporting on an empirical study (e.g. Arvay and Tanko 2004; Linn and Evans 2011; Swales 2004). Therefore, it is clear that an attempt to simply map these sections onto a pure mathematics article would not have been productive. Furthermore, the argumentation pattern of research-based writing in pure mathematics has the added complication of two interdependent structures. These are 'the *formal* or *logical* structure consisting of definitions, theorems and proofs, and the complementary *informal* or *introductory* material consisting of motivations, analogies, examples, and meta-mathematical explanations' (Steenrod 1973: 1; emphasis added). Thus, the approach outlined below was taken. An initial analysis was carried out on two research papers in collaboration with one of the expert informants whose work is included in this study. Firstly, the headings and sub-headings used in two research articles were identified and classified according to Yang and Allison's (2003: 369, 2004: 270) categories, namely 'major standard headings', 'varied functional headings' and 'content headings'. The major standard headings of RAs across many disciplines are both familiar and self-evident, namely Introduction, Literature Review, Methods, Results, Discussion and Conclusion. Varied functional headings describe the function of the section, but are not one of the standard headings listed above. An example from an article in our study is 'Statement of Results'. Content headings encapsulate and describe the content of the section, for example, 'Generalities on genericity'.

   The content and function of each section was then discussed in detail with the expert informant. Following this collaborative analysis with the informant, the same procedure was carried out independently of the informant on the remaining articles. Particular attention was paid to the introduction sections in order to establish whether the prototypical moves described by Swales' CARS model were present in the pure mathematics articles. The model itself describes an argument structure comprising three main rhetorical moves: 1) establishing a territory, 2) establishing a niche, and 3) occupying the niche (Swales, 1990: 141) and is presented in Appendix A.

   The next stage was to try to depict a prototypical shape to describe the selected articles, along the lines of Hill et al.'s well-known hour-glass representation of an IMRD article (see Figure 1, below)

INSERT FIGURE 1 HERE

Figure 1: Overall organization of the empirical research article based on Hill et al. 1982

This type of article moves from a general contextual discussion to the specifics of the method and results of the particular experiment being reported, and then back to a more general discussion which situates the findings within the wider disciplinary research field and presents the wider implications. This transition from general to specific to general is commonly described metaphorically as narrowing and broadening (e.g. Swales, 1990). Drawing on the evidence from the preliminary analysis of the two articles and the insights provided by the

discussion with the informant, an outline of the shape of the research articles was sketched to provide a model (see Figure 2 in the Discussion section). The applicability of this model was then tested by analyzing the remaining 20 articles in the collection.

Interviews
In addition to the manual analysis of the research articles and initial collaboration with one of the expert informants, semi-structured interviews were conducted with the five lead authors. The authors themselves can be broadly viewed as representative of the pure mathematics research community in that the group comprises two professors (Informants A and D), one honorary research fellow (Informant B), and two post-doctoral researchers (Informants C and E). All are native speakers of English. The interviews were broad in scope: the discussion explored the epistemological and ontological traditions of pure mathematics as an academic discipline, as well as the rhetorical and organizational structure of the research article. Three of the interviews were carried out in person, recorded and transcribed immediately. The remaining two were conducted via Google chat and email. Where necessary, follow-up questions and clarifications were conducted via email. In a follow-up interview, Informant B also gave feedback on the preliminary results of our text analysis and helped to clarify the connection between the rhetorical and organizational structure of RAs and pure mathematics disciplinary practices. Thus, excerpts from the interview data were used to account for the findings of the close text analysis and are intertwined with our presentations of the RA macro-structure in the following section. Before we proceed to the presentation of our findings, it should be pointed out that due to the nature of our data, the analysis below is qualitative.

RESULTS

This section is organized as follows: First, we present the findings of the investigation into the organizational and rhetorical structure of the article, namely the various major sections into which the article is divided and their rhetorical function. We will then discuss the overall shape of the article in the spirit of Hill et al.'s (1982) hour-glass description for empirical articles.

Overview of the results of the structural analysis
The results of the organizational structural analysis are presented as follows: Table 1 presents an overview of the three compulsory stages in the overall organization of pure mathematics RAs, which we labelled "1. Openings", "2. Proofs", and "3. Endings" . Stage 1 also includes an optional element which we labelled "1a. Post-introduction". The second column in Table 1 documents the rhetorical moves and their functions carried out in each of these stages. The bracketed letters C and O show whether the move is compulsory or optional respectively. In the third column, the major standard, functional and content headings used to delineate the sections are provided. The row alignment within stages in columns two and three is coincidental. The numbers in subsequent columns (columns four to seven) show how many of the papers used the relevant heading (listed in column three) in the organization of the discourse. Letters A–E refer to the authors whose articles are included in the study. The last column gives the total number of instances of the heading shown in column three. Note that the percentages in Stage 1a do not total 100% as this is an optional stage.

INSERT TABLE 1 HERE

Table 1: Overview of the organizational and rhetorical structure of the pure mathematics research articles based on the textual analysis of the articles

As can be seen in Table 1, the mapping of the prototypical organizational macro-structure of a theoretical pure mathematics research article based on the sample is not possible, as there is significant variation in the structure. Nonetheless, the summary of the data in terms of frequency of occurrence of headings shown in column three does permit some interesting observations. These are summarized below and will be discussed in more detail, and in the context of the interview data, in subsequent sections.

Articles prototypically begin with 'Introduction'. Following this section, authors may opt to include a discrete section dealing with notational information or to combine notation with other background content under a functional heading; however, this is not the case in the majority of the articles in the sample. More common are background sections providing contextual information, located just before the section dealing with the proofs, and signalled via the use of a content heading. This type of section appeared in 36 percent of the papers. Moving on to the sections which are primarily concerned with the unravelling of the proofs, again the organization appears to be variable. Authors can opt to dedicate a discrete section to the proofs (with an appropriate functional heading) or alternatively, embed the proofs in content-headed sections which also provide some contextual background for the proofs. In our sample, the latter option appears more common, occurring in 68 percent of the articles. Even more variation is apparent in the way authors bring the articles to a close. Prototypically, stage 3 is not introduced via a heading. Nonetheless, the way in which the authors finish their articles reveals a significant variation. This variety will be discussed in more detail in the 'Endings' section. To sum up, the organizational structure of the RA in pure mathematics displays relatively standardized introduction and proofs sections, but an increased variation in the remainder of the article, with endings demonstrating the least standardization, as shown in the 'Endings' section.

Throughout the remainder of this section, examples are given to illustrate the content, structure and rhetorical purpose of the sections in the research articles. Where possible, the text used to introduce and encapsulate the content of a particular section is given, as it is perhaps more informative for a non-member of the pure mathematics academic community than content.

Stage 1. Openings

We begin at the beginning. Twenty of the 22 articles open with the functional heading 'Introduction'. Of the remaining two papers, one article explicitly combines the introduction and results using the heading 'Introduction and statement of results', and another opens with: 'Statement of the main theorem'.

In these sections, some moves identified in Swales' CARS model are evident (see Appendix A). In the case of the articles which used the heading 'Introduction', all contained Move 1 – Establishing territory, and all contained elements of Move 3 – Occupying the niche. However, the concept of 'niche occupation' is problematic in the case of pure mathematics, and will be developed further in the section below.

In the case of the article which begins with 'Statement of results', Moves 1 and 3 of the CARS model are communicated in the subsequent section, and clearly announced at the end of the first section:

> (1) In the next section, we will set Theorem 1.1 in its proper context, as well as outline the structure of the proof.

The role of the introductory section is to convey 'something that will justify someone reading the paper' (Informant A); however, this justification is not necessarily established by rhetorically carving a niche as described by the CARS model, more specifically, Move 2. According to Informant A, this is due to the nature of this discipline (underlined parts show authors' emphasis):

> The niche is implicit because it's a funny shaped discipline. A gap would read very weirdly. You would have to say 'I, uniquely in the world, think it might be interesting to ask…and I know the answer to this question. But I only asked it having answered it. I thought about these things for a bit, and then it occurred to me I know how to prove X. So now it's called X. Well, why bother? We all know that's what's going on. (Informant A)

Where a possible realization of Move 2 was apparent, question raising (Step 1C) was observed in four articles. This function can be a question raised by the authors (Example 2), or indeed an intertextual response to a question raised by another author (Example 3):

> (2) When no universal model for a set of structures exists at a given cardinal, then it is natural to ask how many structures of that cardinality it takes to embed them all.
> (3) Shereshevsky has shown […] and asked if such a homeomorphism can have finite positive entropy.

A further six articles were identified as having some form of Move 2 in the introduction. In these articles, the gap appears to pertain to the limited generalizability of a theorem, which can be extended as a result of the work done by the authors:'

> (4) To our knowledge, until recently, most known non-generic supercuspidal representations were level zero, in particular level zero representations induced from the inflation of a cuspidal unipotent representation of the reductive quotient of a maximal special parahoric subgroup. Our purpose in this paper is to exhaust the non-generic supercuspidal representations of Sp4 (*F*) in odd residual characteristic.

Instead of or in addition to 'filling a gap', all the introductory sections foreground the results of the study, with all but one of the articles 'announcing principal findings' (Swales 1990: 141). For example:

> (5) Here we expand this result to achieve a full version of Global Domination in an inner model while retaining the Sacks forcing at inaccessibles and the method of constructing generics given in (5).

> (6) In this paper we establish new functoriality properties between Bruhat-Tits buildings of a classical reductive group over local fields. More precisely, let *F* be anon Archimedean local field of residual characteristic not 2…

In the remaining article, the aim of the research is stated (Move 3, Step 1 in Swales' 1990 CARS model) followed by generalizations regarding the contribution of the result:

(7) <u>Our purpose here is to show how the methods from (2) extend to *d*>2</u>. This gives sharp information about mixing properties for a distinguished class of examples associated to tight polyhedral.

This early focus on and explicit announcement of results in the opening section of the articles was also observed by informants, as exemplified by the following comments:

> The standard structure of the article enables us to focus on the results. (Informant C)

> The conclusion should be in the introduction, and most of the time it's the main theorem. (Informant E)

To summarize, in pure mathematics, territory is established and results announced towards the beginning of the article. Thus, the meta-mathematical argument structure referred to in the section above, headed 'Approach to the analysis', which provides a context and justification for the research, is introduced. In addition, the results of the second line of argument (mathematical) are given.

Stage 1.a. Post-Introduction

Following the opening stage, the majority of articles continue with sections which set the mathematical context for the subsequent proofs which in some cases include definitions, conditions and notation. Functional headings are used such as 'Mathematical background', 'Notation and preliminaries', and 'Notation'. Five of the 22 papers in the study do not allocate a discrete section for this purpose. Instead, the content is placed in the introductory section. However, given that these papers are all written by the same author in the field of set theory, this could simply be accounted for by personal authorial style or indeed sub-disciplinary conventions.

The next stage in the structure identified is tasked with providing sufficient contextual background to enable the reader to follow the proofs. This stage comprises sections which are titled descriptively, based on their content, such as 'Algebraic background', or 'Generalities on genericity'. Multiple sections are possible, particularly in papers which deal with more than one main result. These sections are tasked with providing sufficient contextual background to enable the reader to follow the proofs.

The content of these 'contextual background' sections is varied. For example, these sections can incorporate methodology, which in the case of pure mathematics pertains to the approach taken to proving the result. The following example in which the author provides an insight into the structure of the proof and mathematical approach was found in a section immediately following the introduction:

(8) <u>The method proceeds</u> in a pincer movement, somewhat similar to that in the two papers (12) and (14). These papers used a good lower bound for the canonical height of a rational point which was obtained in (4). <u>Here, our workhorse is the paper (17), although the height bounds are not stated or used in the same way as (17).</u>

It should be noted that reference to the mathematical approach was also found in the openings.

Parts of these 'contextual background' sections could also be viewed as comparable to the literature review found in empirical papers, particularly in the sense of the broad definition

proposed by Lin and Evans (2012: 153): '[these sections] provide various kinds of background to the study. This background may be (inter alia) contextual, theoretical or methodological in nature'. This content is demonstrated by the following example of introducing one such section:

> (9) 2. Algebraic Zd-actions
> In this section basic <u>results and terminology on algebraic Zd actions are collected.</u>

Nonetheless, these sections do not exclusively deal with background; definitions, the structure of the proof, lemmas, examples and proofs can also be present.

These contextual background sections are broad in scope, and some of the rhetorical work done in the introductory sections found in research articles in other disciplines, such as inter-textual positioning, establishing territory, and engaging with the reader can be present here. This approach perhaps allows for the desired focus on the results in the opening section of the article, as pointed out by one expert informant:

> Often when you read a research article, <u>you want to just sometimes see the results,</u> so you want to see the result and the proof. <u>You don't want to trawl through lots of discursive language and this is why we set up the whole thing in the way we do</u>. (Informant B)

To conclude, the post-introductory stage we have termed 'contextual background' has a broad focus, which differs among the articles. Nonetheless, we can say that these sections are used by the author to provide appropriate content to facilitate the mathematical and/or meta-mathematical argument.

Stage 2. The proofs
The next stage in the article is the presentation of the proofs. The start of the proofs can be signalled via a heading, or announced within a content-headed section once the context has been established, via metatext (e.g. 12) or a sub-heading in bold (e.g. 13-15): example:

> (12) <u>We are now ready to prove</u> Theorem 1.1.

> (13) 2. Socle
> In this section <u>we prove</u> Proposition 1.

> (14) 3. Failure of Poincare duality for M
> In this section <u>we construct counter examples</u> to the Poincare duality part…

> (15) **Proof** It is clear that *P* is …

The approach taken to establishing the proofs can also be announced at this juncture, rather than in the contextual background sections or introductory section, as discussed in the previous section. Below is an example:

> (16) <u>The strategy of the proof is natural</u>: theorem * forces tautological classes of high degree to be supported on strata with many rational components.

The unfolding of the proofs then proceeds through the use of a series of propositions, theorems, definitions, lemmas, examples and remarks.

Stage 3. Endings

Considerable variation was observed in the way the authors end the articles. Among the 22 articles analyzed, 11 have no apparent conclusion in the prototypical sense (i.e. sections containing discussion and conclusion moves summarized by Swales 1990: 172-173, see Appendix B). Of these, four end with QED, the square notation used in mathematical script to indicate the end of a proof, while six simply draw their argumentation to a close textually. The final paper concludes with the statement of a theorem. For example:

> (17) The only thing left to check is that we have not changed the complexity at each X when doing the smaller forcing. This proceeds exactly like the proof of Theorem 4.1. □

> (18) This completes the proof of Proposition 5.1, and hence the proof of Theorem 1.1.

Two papers conclude with the addition of a section titled 'Open questions' or 'Questions' in which questions resulting from the study are embedded in an explanatory paragraph. In one case, the author simply poses a question with no heading or commentary (see below):

> (19) **Question:** Is the map ... (mathematical script follows)

> (20) **7. Open questions**
> We conclude with some possible continuations on the subject.

> (21) **Question 7.1.** Is it possible to extend this type of proof to get the non-existence of a universal graph at the successor of a singular of cofinality $>Nw1$?

> (22) The proof of Theorem 2 constructs a non-ergodic automorphism of a totally disconnected group. Does the same result hold for ergodic automorphisms of connected groups?

In these cases, the discourse is maintained following the completion of the proof in order to indicate future avenues of research, corresponding to 'Recommendation' (Move 8 in Swales' summary of the rhetorical structure of Discussion sections (Swales 1990: 173). Swales refers to a personal communication with Huckin to suggest that this move is in decline in the writing of scientists 'because they do not wish to give advantage to others in an increasingly competitive market for research grants' (Swales 1990: 172). However, in our case, this practice appears to be alive and well in the pure mathematics community, possibly due to very limited and specialized audiences of some journals. In fact, this 'Recommendation' move seems to be a useful rhetorical device in this highly theoretical discipline with very little real world justification for future research, motivating the 'relentless onwards march' (Informant A) that characterizes the discipline. Indeed, as we saw in 'Openings', four of the articles in the sample used this question-raising and responding explicitly to rhetorically justify the research. In addition, two informants assigned these questions a strategic role in increasing the impact of the research:

> The problem here is that no one really knows when the research is enough for a paper. <u>On one hand, the finality of answering all easily-thinkable questions in a particular area shows the confidence (in particular is supposed to convince the referee</u>) that this is a complete paper. On the other hand, <u>open questions inspire interest in the subject</u> and there is some hope that <u>someone else will work on this topic, cite your paper,</u> etc. – which is particularly important as normally only a small handful of people read your article. (Informant C)

> Sometimes these questions are <u>hugely influential</u> and can be <u>fertile ground</u>. (Informant A)

An alternative way to bring the article to a close is 'Remarks'. It has been found that the function of 'Remarks' is varied. In terms of engagement (Hyland 2005), for example, they allow the author to step outside of the logical argumentation of the proof to momentarily engage with the reader (Authors, 2012). In this sense, they belong to the meta-mathematical strand of argument as discussed in 'Approach to the analysis'. In the following example, the reader's attention is drawn by the author to other relevant works and results which are tangentially connected to the author's proposition:

> (23) A similar infinite product construction is used in (4) to exhibit a group automorphism whose periodic points count the Bernouilli denominators. Moss (13) has obtained results showing when a given realizable divisibility sequence can arise from a group automorphism.

Remarks situated at the end of the article can also function to point out any limitations of the study. Below is an example of a terminating remarks section from one of the research articles in our study.

> (24) **Remarks** It will surely not have escaped the reader's notice that the methods in each case are rather similar. It may well be possible to unify the cases into a single proof but we have not been able to do this. We also note that we could not have used (5) Lemma 2.10 here, since the proof given unfortunately does not work. It seems likely that the result there is true, at least in the tame case, as here) but we have not (yet) been able to find a proof.

Thus, no broadening as seen in the Hill et al. hour-glass shape is required in pure mathematics. Nor does the 'general-specific model' (Swales and Feak 2004: 45) seem to fit, as it depicts the macro-structure ending with a broader statement. Rather, the shape remains narrow in focus by either terminating with the end of the proof via the QED symbol or a textual announcement as shown in Example 18. Alternatively, authors can raise questions pertaining to the result (a potential strategy for raising the impact of the research article), or conclude with 'Remarks' to point out limitations of the study, provide any additional information that could be pertinent, or indicate possible future research avenues.

This section has mapped out the rhetorical and organizational structure of RAs in pure mathematics, starting from a broad overview and then zooming into the finer details of text organization and the reasons behind the writers' rhetorical choices. It is now time to return to the research questions posed in the introduction.

DISCUSSION

Summary of the shape, rhetorical and organizational structure

The initial shape, sketched out at the beginning of our genre analysis (see 'Method'), proved to be accurate in its representation of the shape of the pure mathematics research article (Figure 2). The shape is narrow at the start as the specifics of the results of the paper are presented and the mathematical conditions established. Some broadening then occurs as the author works rhetorically to establish territory and in some cases establish a niche. Definitions and notation can be provided in the opening section or in the shaft section (Stage 1a, if present, or Stage 2 onwards), producing a narrowing of the shape. This shaft remains narrow as the proofs are unravelled via propositions, lemmas and theorems, but can broaden in places as the authors supplement the mathematical argument with examples and remarks. This optional broadening is represented by the dotted sections shown in the figure. While an interpretation of some broadening is possible at the end of the papers in the form of concluding remarks or open questions, this cannot be claimed as standard according to the results of our study. Thus, unlike in the case of Swales and Feak's general-specific model (2004: 45), there is no significant broadening at the end of the article in the form of a discussion or conclusion. Nonetheless, recommendations for future research in the form of open questions are possible.

INSERT FIGURE 2 HERE

Figure 2: Overall organization of the theoretical pure mathematics research article Legend: Dotted lines show optional rhetorical elements. The sharpness of the edges in the Figure is for schematic purposes only.

The shape shown in Figure 2 is also produced by the dual argumentation, mathematical and meta-mathematical, which runs through the article (Steenrod 1973: 1, see also 'Approach to the analysis'). On the one hand, following introductory sections, the argument structure is narrow in focus, as the logical mathematical reasoning comprising lemmas, theorems, definitions and propositions are directly contributory to the proof. The broader focus is the meta-mathematical argument incorporating examples, remarks, explanations and motivations. These two distinct rhetorical pathways were observed by Informant A in the following comment:

> There is a very clear distinction between what you are claiming is proved and what is your heuristic view of a subject. (Informant A)

Thus, we can draw some broad generalizations about the overall shape of the theoretical pure mathematics research article. As shown above, the prototypical structure includes three compulsory stages: "Opening", "Proof", and "Ending". In some articles, there is also an optional stage which we termed "Post-introduction". It is more difficult, however, to construct a prototypical structural model to describe the genre in terms of how the content is organized in terms of rhetorical moves. As Table 1 suggests, while all 22 articles have some form of introductory section prototypically announcing principal findings and proceed with the presentation of the proof, the rest of the article is not standardized. For example, all the articles in our collection include notational information; however, the place where this notation is provided (e.g. in the introduction, in a discrete section, incorporated into the main body or referred to at various intervals) appears to be non-standardized.

Connecting shape, structure and knowledge-making practices

Our third research question proposes an epistemological basis for the shape of the research article in pure mathematics. It is therefore useful at this point to recap the epistemological foundations of the discipline.

As pointed out in the introduction, pure mathematics and the hard sciences have much in common. Pure mathematics results on the other hand are verified by means of logical reasoning resulting in proof. This divergence in research methodologies was highlighted by two informants in the study, and is demonstrated by the following description of the research process in pure mathematics provided by Informant B:

> You sit down with a pen and paper and <u>fiddle about with a basic example to see whether it's true or not.</u> You carry out some calculations based on your mathematical knowledge..… These <u>examples, that could be compared to carrying out experiments in the other sciences</u>, that's an important stage in the research, but <u>we don't use that as any kind of empirical evidence. It's something to give you a feeling for</u> what might be true. Then you start working on a theorem if you think it's true. <u>You take the idea and develop it into a theorem to see if it is true in much greater generality and sit down and prove it.</u> I think that part is more comparable to what other scientists do when they are at the research stage. Scientists present the results of their experiments; we never really present the results of our experiments. <u>We develop the results of the experiment into a theorem and then that becomes the result</u>. (Informant B).

This research process is manifest in the shape of the research article shown in Figure 2. The process itself begins with the result, which the researcher intuitively holds to be true. Subsequently, conditions are set in order to explore to what extent the result can be generalized. Notation is fixed to enable the author to demonstrate the stages of the proof which are then unfolded via means of logical argumentation. As no formalized experimentation in the empirical sense is carried out, the narrowing of the hour-glass in a methods and results section is not applicable (see Figure 1).

Nonetheless, as we have observed, the shape is distorted by a broadening of the focus at various points in the argument. Again, we argue that this broadening can be traced to the knowledge-making practices of the discipline. As we have already established, knowledge verification is achieved through the logical reasoning of the author, and of previous scholars (pure mathematics is highly cumulative) as evidenced by the following comments:

> You have other people's results as your starting point and then you've got logical argument. (Informant B)

> You have your logical reasoning and everyone else's logical reasoning. (Informant D)

> You have a pole vaulter's pole and on top of that you balance another one and another one, several hundred of them, and on the top you are trying to do your thing. (Informant A)

Thus, throughout the articles, authors step outside the narrow limits of the proof to call on previous results or examples to provide contextualization for the result. Following Steenrod (1973), this contextualization and exemplification have been described in the 'Method' section as the meta-mathematical argument which runs alongside the formalized logic of the proof. These instances of meta-mathematical argument can be formulated as 'Remarks', a

delineated subsection where the author steps outside of the proof to provide observations or supplementary information. This structural idiosyncrasy can also be explained by the complex nature of the mathematical argument and the need to ensure clarity:

> It's a very detailed argument, you have to choose some constants, you have to define some things, you have to calculate something, then you have to integrate something, there's lots of process happening, rather than a dense piece of text describing what I did in the lab and what I observed. (Informant A).

> You could turn the reader off by having too much discussion […], but we do have a very structured way of having remarks section. You might want to include a remarks section that helps contextualize the theorem and point out surprising things. (Informant B)

It is therefore possible to find a rationale for the schematic shape of the research article in the knowledge-making practices of the discipline. The overall shape reflects the research process itself, while the less standardized organization and content of sections gives the writer the freedom to organize the text in a way that is most appropriate for guiding the reader through the complexities of the results.

CONCLUSION AND FURTHER RESEARCH

It must be pointed out that this study is the first attempt to describe the rhetorical and organizational structure of the research articles in pure mathematics and one of the first in-depth analyses of theoretical research articles. It therefore seems unwise to draw any far-reaching conclusions based on the results of a small-scale genre investigation. Despite the limited scope of our study, this article raises some interesting points concerning previous accounts of the RA macro-structure and offers a useful insight into the academic "tribal" practices in connection to disciplinary rhetoric. This insight was achieved through the combination of methods, including close collaboration with disciplinary insiders.

Based on the results of our analysis, it appears somewhat problematic to label RAs in pure mathematics as belonging to any particular text type and having a clearly defined rhetorical and organizational structure. While the overall shape illustrated in Figure 2 reflects how the disciplinary discourse features are connected to the knowledge-making practices of pure mathematics, the rhetorical structure of the RA is somewhat fluid. This rhetorical flexibility allows writers to adapt the article to the content and reader. Furthermore, pure mathematics is not a homogeneous discipline, and the 22 articles analyzed in this study belong to somewhat different sub-disciplines, each with their own set of ontological assumptions. This variation may largely account for the irregularities in the overall macro-structure, and, at the same time, it also underscores the importance of context in which the academic texts are produced. This finding is in line with Lin and Evans' (2012: 160) conclusion that 'scholars' writing practices are far more complex than might be guessed from the principles set out in many research writing manuals and style guides'. In this context, we believe that our findings provide some relevant insights for academic writing instruction and materials development geared towards specific disciplinary practices, both for native and non-native speakers of English. For example, junior mathematicians could be presented with various simple proofs and asked when and why they would add contextual details in order to support the mathematical argument.

The theoretical RA as an academic genre remains largely under-researched. This is the case for both pure mathematics and other highly theoretical disciplines. Further research should

explore disciplinary discourse features at both the macro- and the micro-levels. In order to avoid unnecessary generalizations and prescription, future studies may consider closer collaboration with disciplinary experts in order to ensure the connection between discourse analysis and knowledge-making practices. This approach will prevent applied linguists from forcing their own epistemological reality upon the discipline under scrutiny.

APPENDIX A

The CARS model for article introductions (Swales, 1990: 141)

Move 1: Establishing a territory   Step 1 Claiming centrality and/or
                                      Step 2 Making topic generalization(s) and/or
                                      Step 3 Reviewing items of previous research

Move 2: Establishing a niche   Step 1A Counter-claiming or
                                      Step 1B Indicating a gap or
                                      Step 1C Question-raising or
                                      Step 1D Continuing a tradition

Move 3: Occupying a niche   Step 1A Outlining purposes or
                                      Step 1B Announcing present research
                                      Step 2 Announcing principal findings
                                      Step 3 Indicating RA structure

APPENDIX B

Prototypical Discussion and Conclusion moves (Swales, 1990: 172-173)

1. Background information
2. Statement of results
3. (Un)expected outcome
4. Reference to previous research
5. Explanation
6. Exemplification
7. Deduction and hypothesis
8. Recommendation

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