

# Design of understeer characteristics through torque vectoring on a lumped-parameter full vehicle model

LENZO, Basilio <http://orcid.org/0000-0002-8520-7953>, PERRELLI, Michele, CARBONE, Giuseppe and MUNDO, Domenico

Available from Sheffield Hallam University Research Archive (SHURA) at:

https://shura.shu.ac.uk/26860/

This document is the Accepted Version [AM]

## Citation:

LENZO, Basilio, PERRELLI, Michele, CARBONE, Giuseppe and MUNDO, Domenico (2020). Design of understeer characteristics through torque vectoring on a lumped-parameter full vehicle model. In: Advances in Italian Mechanism Science -Proceedings of the Third Conference of IFToMM ITALY. Mechanisms and Machine Science book series . Springer, 768-776. [Book Section]

# Copyright and re-use policy

See http://shura.shu.ac.uk/information.html

# Design of understeer characteristics through torque vectoring on a lumped-parameter full vehicle model

Michele Perrelli<sup>1[0000-0001-5677-7861]</sup>, Giuseppe Carbone<sup>1[0000-0002-3228-8040]</sup>, Basilio Lenzo<sup>2</sup> <sup>[0000-0002-8520-7953]</sup> and Domenico Mundo<sup>1[0000-0002-3228-8040]</sup>

<sup>1</sup> DIMEG, University of Calabria, Rende, Italy <sup>2</sup> Dept. of Engineering and Mathematics, Sheffield Hallam University, Sheffield, UK michele.perrelli@unical.it

**Abstract.** Active safety systems play a fundamental role in improving stability and safety performance of modern passenger cars. Within this context, Torque vectoring (TV) represents one of the most promising technologies for the improvement of vehicle dynamics performance. This paper proposes a TV-based Direct Yaw Moment Control (DYC) strategy aimed at designing the vehicle understeering behaviour through a software simulation environment based on an efficient Lumped-Parameter Full Vehicle Model (LPFVM). Simulation results show that the vehicle is able to successfully follow a predetermined understeer characteristic.

Keywords: Torque Vectoring, lumped-parameter modelling, vehicle dynamics.

#### 1 Introduction

Modern vehicular technology puts key attention on safety. Nowadays, drivers and passengers are well aware of the presence of active control systems on their vehicles, including e.g. Anti-lock Braking System (ABS) or Electronic Stability Control (ESC), which enhance vehicle safety [1]. With that in mind, research has kept moving forward and recent years have seen an increasing interest in Direct Yaw-Moment Control (DYC) techniques [2, 3] to further improve vehicle safety. A direct yaw moment can be purposely generated on a vehicle through an uneven left-to-right torque distribution. The ability to distribute desired amounts of torque at different corners/drivetrains is known as Torque vectoring (TV). TV has been widely explored in the literature, mostly to improve vehicle safety and handling [4, 5] as well as performance [6, 7]. TV can also be exploited to enhance energy efficiency [8–10].

The recent market increase of electric vehicles has provoked a boost in TV-related research, nonetheless, TV can also be achieved through appropriate mechanical arrangements, e.g. limited-slip differentials [11].

This paper deals with the design and simulation of a TV-based reference yaw rate control aimed at reshaping the understeer characteristic, a.k.a. cornering response, of an internal combustion engine vehicle with TV-capability at the rear axle. The effects of TV on lateral dynamics of a vehicle is evaluated using a LPFVM implemented with

a PID-based virtual Driver-in-the-Loop (vDiL) scheme. The details of the proposed simulation environment can be found in[12–14].

The outline of the paper is as follows. Section 2 describes the testing manoeuvre and the methodology used to generate different reference yaw rate Look-Up tables (LUT) reshaping the understeer characteristic using a PID controller. Section 3 discusses the simulation results for three different understeer characteristic curves. Section 4 provides discussion on the results and concluding remarks with an overview on future developments.

#### 2 Design of a PID-based controller for TV in a LPFVM

The presented PID-based TV controller aims to modify the lateral dynamics of the vehicle by controlling the torque distribution between rear right and left driving wheels. In the LPFVM described in [12, 13] the torque distribution was modelled with a classical mechanical differential in which the driving torque  $T_d$  is transferred to the driving wheels as follow:

$$T_{rr} = T_d(0.5 + \rho) \tag{1}$$

$$T_{rl} = T_d (0.5 - \rho)$$
 (2)

where  $T_{rr}$  and  $T_{rl}$  are the torque delivered by the differential to the right and left driving wheels respectively. The distribution ratio  $\rho$  is calculated as a function of the angular velocity of the left ( $\omega_{rl}$ ) and right ( $\omega_{rr}$ ) wheels as follows:

$$\rho = -\frac{|\omega_{rr} - \omega_{rl}|}{2 * (|\omega_{rr}| + |\omega_{rl}|) + 1}$$
(3)

To correctly implement a TV controller the above differential model for torque distribution is replaced in the LPFVM by the following distribution relationships:

$$T_{\rm rr} = 0.5 \left( T_{\rm d} - \frac{M_z R}{\rm d} \right) \tag{4}$$

$$T_{\rm rl} = 0.5 \left( T_{\rm d} + \frac{M_{\rm z} R}{\rm d} \right) \tag{5}$$

where  $M_z$  is the yaw moment required by the desired understeer correction, R is the wheel radius and d represents the vehicle half-track width. In the vehicle model used in this work, the values of R and d are equal to 0.3135 m and 0.74 m respectively. The desired yaw moment  $M_z$  is calculated using a classical PID formulation as a function of the error between the desired yaw rate  $\psi^*(\delta(t), S_x(t))$  and the current vehicle yaw rate  $\psi(t)$ , as follows:

$$M_{z}(t) = K_{p} * e_{yr}(t) + K_{i} * \int_{0}^{t} e_{yr}(\tau) d\tau + K_{d} * \frac{de_{yr}(t)}{dt}$$
(6)

where the error function is calculated as:

$$\mathbf{e}_{\mathrm{vr}}(t) = \dot{\boldsymbol{\psi}}^*(\boldsymbol{\delta}(t), \boldsymbol{V}_{\mathrm{x}}(t)) - \dot{\boldsymbol{\psi}}(t) \tag{7}$$

The desired yaw rate value  $\dot{\psi}^*(\delta(t), S_x(t))$  is derived from a Look-Up Table(LUT) as a function of steering angle  $\delta(t)$  and longitudinal speed  $S_x(t)$  at the current simulation time t. To create the LUT, at first, the understeer characteristics of the vehicle was evaluated during a steering manoeuvre performed at a constant speed of 25 m/s using the torque distribution of equations (1) and (2). During the implemented manoeuvre, the sigmoid input signal shown in Figure 1, with continuity of class C2, was used.



Fig. 1: Steering input for LPFVM

Specifically, the manoeuvre is executed by speeding the vehicle up to 25 m/s until a steady-state condition is reached. Afterwards, the virtual driver starts the steering manoeuvre until the target steering wheel input of 110 degrees is reached.



Fig. 2: Understeer characteristic with classical differential: (a)  $\delta_{dyn}(a_y)$  - (b)  $\dot{\psi}(t)$ 

During the steering manoeuvre, the virtual driver uses a PID regulator to keep the car speed around 25 m/s with a maximum error of 0.2 m/s, while the vehicle travels along a counterclockwise spiral trajectory. Fig. 2a and b show the trend of yaw rate  $\dot{\psi}$  and of the dynamic steering angle  $\delta_{dyn}(a_y)$  returned by the LPFVM, equipped with a passive mechanical differential, during the manoeuvre.

The trend of  $\delta_{dyn}(a_y)$  gives information about the understeer characteristic of the vehicle. In this case, the vehicle shows an oversteering behaviour for lateral acceleration value up to 5.5 m/s<sup>2</sup>, then it becomes understeering.

A TV controller allows to change the  $\delta_{dyn}(a_y)$  relationship via DYC. By designing a desired  $\delta_{dyn}(a_y)$  function and calculating the associated yaw rate it is possible to modify the lateral dynamics of the vehicle as explained below.

Given the understeer behaviour of the vehicle equipped with the passive mechanical differential, it is necessary to get a mathematical formulation of  $\delta_{dyn}$  as a function of  $a_y$ . In this study, the numerical  $\delta_{dyn}(a_y)$  curve was interpolated with an 8-degree polynomial function obtaining the interpolating the baseline curve  $\delta^*_{dyn}(a_y^*)$  shown in Fig. 2a.

At first, a vector of steady-state turning radii was computed as follows

$$r(k) = \frac{S_{\chi}(k)}{\dot{\psi}(k)}, \qquad k = 1.....N$$
 (8)

where  $N = \frac{T}{t_s}$  is the total number of data stored for a manoeuvre of duration T equal to 330 s and with a simulation time step  $t_s$  of 0.001 s.

Once known r, it is possible to obtain the dynamic steering angle as:

$$\delta_{\text{dyn}}(\mathbf{k}) = \left(\delta(\mathbf{k}) - \frac{L}{r(\mathbf{k})} \operatorname{St}_{\text{ratio}}\right) * \frac{180}{\pi} , \quad \mathbf{k} = 1 \dots N$$
(9)

where  $\delta(\mathbf{k})$  is the steering wheel angle and St<sub>ratio</sub>, equal to 16.67, is the vehicle steering ratio. By imposing a lateral acceleration  $a_y^*(i)$ , in the range from 0 to 9 m/s<sup>2</sup> with a step of 0.01 m/s<sup>2</sup> and index i varying from 1 to 900, the interpolating curve  $\delta_{dyn}^*(a_y^*(i))$  was calculated using the following polynomial function

$$\delta_{dyn}^{*}(a_{y}^{*}(i)) = 2.9 \cdot 10^{-6} a_{y}^{*}(i)^{8} \cdot 8.62 \cdot 10^{-5} a_{y}^{*}(i)^{7} + 0.001 a_{y}^{*}(i)^{6} + ...$$
  
-0.0065  $a_{y}^{*}(i)^{5} + 0.022 a_{y}^{*}(i)^{4} - 0.04 a_{y}^{*}(i)^{3} + ...$   
+0.034 -0.029  $a_{y}^{*}(i) + 0.00023$  (10)

The coefficients in equation (10) were obtained by using the Matlab<sup>®</sup> polyfit function on the  $\delta_{dyn}(a_y)$  curve. Once obtained the  $\delta^*_{dyn}$  equation, it is possible to populate the reference yaw rate LUT by iterating the following equations for different longitudinal velocity  $S_x(j)$  varying in the range from 1 to 85 m/s with a step of 0.5 m/s and index j varying from 1 to 169:

$$r^{*}(i,j) = \frac{S_{x}^{2}(j)}{a_{v}^{*}(i)}$$
(11)

$$\delta^{*}(\mathbf{i},\mathbf{j}) = \frac{L}{r^{*}(\mathbf{i},\mathbf{j})} \operatorname{St}_{\text{ratio}} + \delta^{*}_{\text{dyn}} \left( a^{*}_{y}(\mathbf{i}) \right)$$
(12)

$$\dot{\psi}^{*}(\mathbf{i},\mathbf{j}) = \frac{S_{\mathbf{x}}(\mathbf{j})}{\mathbf{r}^{*}(\mathbf{i})}$$
(13)

$$LUT(l=1 ... 241, j) = spline(\delta^{*}(l=1 ... 900, j), \psi^{*}(1 ... 900), \delta^{*}_{int}(l=1 ... 241))$$
(14)

5

The variables  $r^*(i,j)$  and  $\psi^*(i,j)$  represent, respectively, the desired turning radius and the desired yaw rate that the vehicle will assume travelling on a curve with a steereing wheel input  $\delta(i,j)$  at a speed of  $S_x(j)$  with the TV controller activated. The variable  $\delta^*(i,j)$ represents the steering wheel input angle. Equation (14) was used to reduce the number of LUT elements by using a spline interpolation of  $\psi^*$  as function of  $\delta^*_{int}(l)$ . The steering input range was chosen between 0 to 120 degrees with a step of 0.5 degrees, so that index 1 varies between 1 and 241. The current reference yaw rate estimation is derived from from the LUT by using a double linear interpolation [15]. After the baseline understeer characteristic of the vehicle is defined, it is possible to modify its lateral dynamics behaviour just by changing the trend of the  $\delta^*_{dyn}(a^*_y)$  curve and calculating the associated LUT as explained above. In this research work, three different driving modes (denoted as "baseline", "mode1" and "mode2") were implemented to test the effectiveness of the proposed TV controller. The following figures show the trend of the curve  $\delta^*_{dyn}(a^*_y)$  and of the yaw rate for each driving mode while fixing the vehicle speed to 25 m/s and using the steering input of Fig. 1.



**Fig. 3:** Desired driving mode: (a)  $\delta_{dyn}(a_y) - (b) \dot{\psi}(t)$ 

The trend of  $\delta^*_{dyn}(a^*_y)$  for mode1 and mode2 was obtained by rotating the baseline  $\delta^*_{dyn}(a^*_y)$  around the point (0,0) to increase the oversteering behaviour up to 6,5 m/s<sup>2</sup> of lateral acceleration (mode1) or to make the vehicle understeering for each value of lateral acceleration (mode2). It is to be noticed that the curve  $\delta^*_{dyn}(a^*_y)$  cannot be changed arbitrarily to avoid negative effects on the overall performance of the vehicle.

Fig. 4a and b illustrate the boundaries for the vehicle, within which the understeer characteristic can be defined. Such boundaries were calculated by manoeuvring the vehicle with two different fixed torque distribution (90% on the right and 10% on the left wheel and vice-versa). The intermediate curves shown in Figures 4a and b refer to a case of even torque distribution between the two driving wheels. The achievement of a characteristic curve falling outside the range above is still possible, but the required values of corrective yaw moment could be generated by the TV controller only by

6

applying a braking torque on one of the wheels. In the latter scenario, vehicle energy efficiency would worsen.



Fig. 4: Boundary and intermediate curves: (a)  $\delta_{dyn}(a_y)$  - (b)  $\dot{\psi}(t)$ 

#### 3 **Test results**

This section shows the results of the tests executed using the TV controller in three different driving modes describe above. In Fig.5, it is possible to observe the trend of M<sub>z</sub> for the analysed driving modes



Fig. 5: : Yaw moment as a function of time for the three understeer characteristics

As expected, using driving mode1 the engine torque is mostly assigned to the right wheel, while in case of driving mode 2 it is distributed mostly to the left wheel. In case of baseline driving mode, the TV controller applies slight torque differentiation between left and right wheels, acting as a classic mechanical differential. The trend of  $\delta_{dyn}(a_y)$  and  $\dot{\psi}(t)$  of the vehicle under TV control (solid lines) are compared with the desired trends of  $\delta^*_{dyn}(a^*_y)$  and of  $\dot{\psi}^*(t)$  (dashed lines) in Figures 6 and 7, which further

7

7

clarify the effectiveness of the implemented TV controller. The latter allows to follow the desired  $\delta^*_{dvn}(a^*_v)$  curve for each driving mode with a maximum error of ±0.5 degs.



Fig. 6:  $\delta_{dyn}(a_y)$  trends (solid line: TV controlled, dashed line: desired) – (a) base, (b) mode1, (c) mode2



Fig. 7:  $\dot{\psi}(t)$  trends (solid line: TV controlled, dashed line: desired) – (a) base, (b) mode1, (c) mode2

#### 4 Conclusion

This paper describes the integration of a TV controller in a platform for simulation of road vehicles for dynamic testing. In the test vehicle model, a classical mechanical differential was replaced by a TV controller for uneven torque distribution. Three different driving modes were designed to test the TV-controlled LPFVM, allowing to verify the reliability of the simulation system, able to produce results in agreement with the expected physical responses. Future developments are planned to implement additional test cases for further validation of the proposed TV controller in driving scenarios where different performance targets (e.g., fuel efficiency, lap time, etc...) are defined.

### 5 Acknowledgement

This work was supported by the project "FASTire (Foam Airless Spoked Tire): Smart Airless Tyres for Extremely-Low Rolling Resistance and Superior Passengers Comfort" funded by the Italian MIUR "Progetti di Ricerca di Rilevante Interesse Nazionale (PRIN) call 2017 - grant 2017948FEN".

References

- 1. Linder A, Dukic T, Hjort M, et al :Methods for the evaluation of traffic safety effects of Antilock Braking System (ABS) and Electronic Stability Control (ESC). (2007)
- 2. Esmailzadeh E, Goodarzi A, Vossoughi GR :Optimal yaw moment control law for improved vehicle handling. Mechatronics.
- Lenzo B, Zanchetta M, Sorniotti A, et al :Yaw Rate and Sideslip Angle Control Through Single Input Single Output Direct Yaw Moment Control. IEEE Trans Control Syst Technol. (2020)
- 4. De Novellis L, Sorniotti A, Gruber P :Driving modes for designing the cornering response of fully electric vehicles with multiple motors. Mech Syst Signal Process.
- 5. Lenzo B, Bucchi F, Sorniotti A, Frendo F :On the handling performance of a vehicle with different front-to-rear wheel torque distributions. Veh Syst Dyn. (2019)
- 6. Ghosh J, Tonoli A, Amati N :A Torque Vectoring Strategy for Improving the Performance of a Rear Wheel Drive Electric Vehicle. In: 2015 IEEE Vehicle Power and Propulsion Conference, VPPC 2015 Proceedings (2015)
- De Pascale, V., Lenzo, B., Farroni, F., Timpone, F., Zhang, X.: Torque Vectoring Control for Fully Electric SAE Cars. In Conference of the Italian Association of Theoretical and Applied Mechanics (pp. 1075-1083). Springer, Cham. (2019)
- Koehler S, Viehl A, Bringmann O, Rosenstiel W :Improved energy efficiency and vehicle dynamics for battery electric vehicles through torque vectoring control. In: IEEE Intelligent Vehicles Symposium, Proceedings (2015)
- Lenzo B, De Filippis G, Sorniotti A, et al :Understeer characteristics for energy-efficient fully electric vehicles with multiple motors. In: EVS 2016 - 29th International Electric Vehicle Symposium (2016)
- De Filippis G, Lenzo B, Sorniotti A, et al :On the Energy Efficiency of Electric Vehicles with Multiple Motors. In: 2016 IEEE Vehicle Power and Propulsion Conference, VPPC 2016 - Proceedings (2016)
- 11. Mircea Gradu :Differential with torque vectoring capabilities. . 1–11 (2005)
- Mundo D, Gencarelli R, Dramisino L, Garre C :Development, validation and rt performance assessment of a platform for driver-in-the-loop simulation of vehicle dynamics. In: Mechanisms and Machine Science pp 130–138 (2018)
- Perrelli M, Cosco F, Carbone G., Mundo D :Evaluation of vehicle lateral dynamic behaviour according to iso-4138 tests by implementing a 15-dof vehicle model and an autonomous virtual driver. Int J Mech Control 20:(2):,pp31–38 (2019)
- Carpinelli M, Gubitosa M, Mundo D, Desmet W :Automated independent coordinates' switching for the solution of stiff DAEs with the linearly implicit Euler method. Multibody Syst Dyn 36:,pp67–85 (2016)
- Molina A, Rajamani K, Azadet K :Concurrent Dual-Band Digital Predistortion Using 2-D Lookup Tables with Bilinear Interpolation and Extrapolation: Direct Least Squares Coefficient Adaptation. IEEE Trans Microw Theory Tech. (2017)