Stochastic Modelling of the Banking Sector of the Nigerian Stock Market

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DECLARATION

I certify that the substance of this thesis has not been already submitted for any degree and is not currently being submitted for any other degree. I also certify that to the best of my knowledge any assistance received in preparing this thesis, and all sources used, have been acknowledged and referenced in this thesis.

ABSTRACT

Stochastic Time Series Modelling of the Banking Sector of the Nigerian Stock Market

We investigate empirical finance issues: stylized facts, market efficiency, anomaly, bubble and volatility, characterizing stock prices of sixteen (16) Nigerian banks in the Nigerian stock exchange (NSE) from June 1999 to December 2014, encompassing periods of financial and banking reforms by the Central Bank of Nigeria (CBN) and the 2007-2009 global financial crisis witnessed by the Nigerian financial system. Both daily and monthly returns are examined and compared. Various financial and stochastic time series methods are applied to these series. These include a variety of initial plots, tests and models. The tests include: Jarque-Bera and a host of other normality tests; Ljung-Box (Q) test of autocorrelation; Augmented Dickey Fuller (ADF), Phillip-Peron, and KPSS tests; variance ratio test, BDS tests, runs test for Random Walk, unit root and market efficiency tests; Duration dependent test and appropriate GARCH families of models. The results are compared to the existing literature for other countries and also other studies in Nigeria but at the market index level. The results largely reveal that while in some cases about 90% of the banks behave uniformly with respect to some of the concepts, in most other cases their behaviour differs significantly depending on the concepts investigated. Also, it is found that while the results of this study agree in a few cases with some of the outcomes of the overall market level - for example, the banking industry is largely weak-form inefficient in most other circumstances, there are marked differences. Specifically, unlike at the overall market level, bubbles were identified in some of the banks and only two anomalies such as January-holiday and turn-of the-month were found with most of the banks. Therefore, a good understanding of how each bank reacted to different scenarios is identified. This should form a basis upon which good investment decisions could be made. This also provides a good understanding of which bank is performing well or at risk, so that appropriate decisions that would enhance the performance of the banking market are made by market regulators.

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and

My father, Mr. Raheem Abdul-Salaam.

List of Publications from the Thesis

1. Raheem, M.A. and Ezepue, P.O. (2017): A Three-State Markov Model for Predicting Movements of Asset Returns of a Nigerian Bank. CBN Journal of Applied Statistics, 7(2), 77-99.
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3. Raheem, Maruf A.,Alboul,Lyuba and Brunsdon, Teresa (2017):Volatility Modelling of Global Financial Crises Effects on theNigerian Banks (Submitted to The Journal of African Economics).
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6. **Raheem, M. A.** and Ezepue, P. O (**2014**): Three-State Markov Model for Predicting Asset Returns Behaviour of the Nigerian Bank. In Proceedings of International Forecaster Society; Rotterdam, the Netherlands.
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ABBREVIATIONS	MEANINGS
A.G.	Accountant General
ACF	Autocorrelation Function
ADF	Augmented Dickey Fuller
AfBC	African Banking Corporation
APARCH	Asymmetric Power Autoregressive Conditional Heteroscedastic
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive moving average
ASEA	African Securities Exchange Association
ASI	All Share Index
BDS	Brock, Dechert and Scheinkman
BIAO	Banque Internationale Pour L'Afrique Occidental
CAPM	Capital Asset Pricing Model
CBN	Central Bank of Nigeria
CDF	Cumulative Density Function
CRSP	Center for Research in Security Prices
Dago	D'Agosto
DF	Dickey-Fuller
DGP	Data Generating Process
DJIA:	Dow Jones Industrial Average
ECDF	Empirical Cumulative Density Function
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
EMH	Efficient Market Hypothesis
ETFs	Exchange Traded Funds
EWMA	Exponentially weighted moving average
FCMB	First City Monument Bank
GARCH:	Generalized Autoregressive Conditional Heteroscedasticity
GDP:	Gross Domestic Product
GED:	Generalized Error Distribution
GJR-GARCH	Glosten, Jaganathan and Runkle GARCH
GNP:	Gross Net Product
GTB:	Guaranty Trust bank
i.i.d	Independently and identically distributed
IOSCO:	International Organization of Securities Commissions
IPO	Initial Public Offer
IQR:	Interquartile Range
ISA:	Investments and Securities Act
ISE:	Istanbul Stock Exchange
IUS	Information Unavailable from Source
JB	Jacque-Bera
KPSS:	Kwiatkowski–Phillips–Schmidt–Shin
LM	Lagrange Multiplier
LSE	London Stock Exchange
MAD	Mean Absolute Deviation
MC	Market Capitalisation
N/A	Not Applicable

MEMART	Memorandum and Articles of Association
NAICOM	National Insurance commission
NASDAQ	National Association of Securities Dealers Automated Quotations
NDIC	National Deposit Insurance Corporation
NIC	News Impact Curve
NoB	Number of Business
NSE	Nigerian Stock Exchange
NSM	Nigerian Stock Market
NYSE	New York Stock Exchange
OTC	Over-the-Counter
Own.Typ	Ownership Type
Own.Typ	Ownership Type
P/E	Price-Earnings ratio
PACF	Partial Autocorrelation Function
Pdf	Probability density function
PENCOM	National Pension Commission
PP	Phillip-Perron
RW	Random Walk
RWH	Random Walk Hypothesis
S&P:	Standard and Poor
SE:	Standard Error
Sec. Typ:	Security Type
SEC:	Securities and Exchange Commission
SES	Standard Error of Skewness
SSMC	Systematic Stock Market Characterisation
SW	Shapiro-Wilk
TGARCH	Threshold Generalized Autoregressive Conditional Heteroscedasticity
TS	Time series
U.S.	United States
UBA	United Bank for Africa
UK:	United Kingdom
VaR:	Value-at-Risk
WFE	World Federation of Exchanges
Yr. Inc:	Year of Incorporation
Yr. Lstd:	Year Listed

Parameters Used in the Thesis	Meanings
Uppercase letters **X** and **Y**. E.g $\boldsymbol{X_i}$	Are used for random variables is the i^{th} random variable, for $i = 1, 2, \ldots, N$
Lower case x and y E.g. x_i	Are the non-random scalars or the realisation for X and Y respectively. is the i^{th} sample values drawn from a sample
$\boldsymbol{X_{(i)}}$:	is the i^{th} ordered random variable drawn from the population
. $\boldsymbol{x_{(i)}}$:.	is the i^{th} ordered sample values
$\bar{x}$: called x-bar,	representing the mean value obtained from a sample
$\bar{r}$:	is the mean for log returns
$\check{r}$:	Median for return series
$\boldsymbol{R_t}$:	is the simple return at time t
$\boldsymbol{r_t}$:	is the log return or compounded return at time t
r_t^2:	is the squared returns (squared log-returns) at time t
$\lvert r_t \rvert$	is absolute returns or absolute log-returns at time t
$log(x)$ or $ln(x)$.	is the natural logarithm of x
N:	Population size; that is, the total number of units in the population. $\boldsymbol{n}$: Sample size; that is, is the total number of units in a sample
p-value	It is the attained level of significance. It is the smallest level of significance for which the observed sample statistic tells us to reject the null hypothesis.
A hat over a parameter, e.g. $\boldsymbol{\hat{\theta}}$,.	Denoting an estimator of the corresponding parameter. This example goes for any parameter
Q_1 or $Q_{0.25}$	Is the first quartile, representing the median of the lower half of the data falling below the median.
Q_2 or $Q_{0.5}$:	It is the second quartile or median, representing the central value of an ordered data.
Q_3 or $Q_{0.75}$:	is the third quartile, representing the median of the upper half of the data falling above the median
σ^2: Sigma square-	It is generally used to represent population variance in statistics. However in this study, it is mostly called unconditional variance.

σ_t^2	It is called conditional variance or conditional volatility at time t in this study.
σ:	Population standard deviation or unconditional standard deviation for the random variable X
μ: "mu",	representing unconditional mean or expected value of the random variable X
ρ_k: Rho at lag k-	It is the autocorrelation function (ACF) at lag k
γ_k : Gamma-	It is the auto covariance function at lag k
ϕ_{kk}: Phi-	Partial autocorrelation function at lag k
$\hat{S}_{kr}$:	representing estimated sample returns' skewness
$SE_{\hat{\rho}}$:	Standard Error for ACF
$Z_{S_{kr}}$:	Z-score test statistic for sample skewness
T:	representing last trading period or total number of trading period
AR(p):	Autoregressive model of order
MA(q):	Moving Average of order q
ARMA(p,q):	autoregressive moving average of order (p,q)
$Q(m)$:	Ljung and Box test statistic for ACF
$\propto$:	Representing the coefficient of lagged squared residual, which is a function of historic information in ARCH/GARCH models.
$t_{(\varphi=1)}$:	The Dickey-Fuller test statistic for random walk
φ:	unit root parameter between the two series $(\mathrm{r_t} \& \mathrm{r_{t-1}})$
ε_t:	Independently and identically distributed random error terms
$\chi^2_{(h)}$:	Chi-squared distribution with h degrees of freedom, where h is any integer number
$\hat{q}_{0.75}$: Q-hat-	Is the estimated sample third quartile (at 75th-percentile)
$\hat{q}_{0.25}$: Q-hat-	It is the estimated sample first quartile (at 25th-percentile)
$Cov(r_t, r_{t-k})$:	Covariance between returns at time t and time $t-k$
$Y_{i's}$:	ordered set of random variables
ψ:	is the coefficient of the deterministic trend parameter
$t_{(\tau=0)}$:	The Augmented Dickey-Fuller test statistic for random walk
H_0:	Null hypothesis
$V(2)$:	Two-period variance
$\widehat{VR}(N)$: variance	N-period estimated
∀:	Used as "For all"
Σ:	Sigma- Sum of the data
T_{st}:	Student –t distribution
P_t:	Closing price of stock returns at the current time (t).

p_t:	Logarithm of the price at the current time; i.e. $\ln(P_t)$.
P_{t-1}:	Closing price of stock returns at immediate previous time.
p_{t-1}:	Logarithm of the price at immediate previous time; i.e. $\ln(P_{t-1})$.
φ_i	is the i^{th} parameter of the autoregressive model in [**4.34**], for $i = 1, 2, \dots, p$
$\mathfrak{I}$:	drift parameter of the random walk model in 4.40
$\exp[.]$:	Exponential function
ϕ_i:	is the i^{th} coefficient of the squared residual autoregressive model in equation [**4.74**], for $i = 1, 2, \dots, p$
υ_1: :	is the i^{th} parameter of the autoregressive model in equation [**4.75**], for $i = 1, 2, \dots, p$
$C_{\varepsilon,m}$:	Spatial correlation's measure between points m and ε
h_{it}:	representing hazard function
D_t:	Dividend at time t [**5.9; 5.10**]
e^{r_t}	: Exponential rate of returns at time t in [**5.12**]
α_0 / ω:	Long run volatility or unconditional variance
ε_{t-p}^2:	squared residuals for time t-p in the GARCH model
BS:	Black-Scholes
$h(t, X)$:	Cox-proportional hazard function
σ_{t+1}:	Conditional volatility at time t+1
$S(t)$:	Survivor function, representing conditional density function for duration of time length *t*,
ν:	is the coefficient for auto regression of order 4 {AR (4)} in [**8.6**]
ξ:	regression coefficient of Cox-model in [**8.4**]
W_E:	Length of the estimation window in EWMA model
δ:	representing measure of long memory in APARCH model in [**9.29**; **9.30; 9.31**]
$E[.]$:	Expected value of a function
$S_n(x)$:	It is the empirical cumulative frequency of ordered series of random samples of x-variable from 1 to n
$\boldsymbol{F(x)}$ or $\boldsymbol{F_x}$	It is the cumulative distribution or distribution function of the population.
$f(x)$ or f_x:	probability density(or mass) function for random X
$\boldsymbol{D_n}$:	is the test statistic for the Kolmogorov Smirnov (KS)
$S(\check{x}_i) = \frac{n}{N}$:	is the empirical cumulative distribution of the generated ordered random sample
A^2:	Representing Anderson-Darling test statistic.

$\underbrace{sup}_{\{x\}}$:	Supremum restricted to random sample x
a_i:	represents constants generated from the means, variances and covariance of the ordered statistics
Z: Z-score:	Standard normal variable (Normal variable with mean = 0 & SD = 1).
Δ:.	differencing parameter in the Augmented Dickey Fuller model
W:	representing Shapiro-Wilk test statistic
s_r	Sample standard deviation for return distribution
γ_0: -	auto covariance function at lag 0 / representing variance of the autocorrelation
$\widehat{K}_r$:	representing estimated sample returns' kurtosis
$SE_{\hat{\phi}_{kk}}$:	Standard Error for the estimated partial autocorrelation function (PACF)
Z_{K_r}:	Z-score test statistic for sample kurtosis
$(\widehat{K}_r - 3)$= 0	excess kurtosis
μ_t: -	Conditional mean-return at time t
$Q_*(m)$:	Box and Pierce test statistic for ACF
$\widetilde{W}$:	Doornik and Hansen test statistic for normality test
$\wp_1^2$:	representing finite-sample squared skewness
$\wp_2^2$:	representing finite-sample squared kurtosis
$\hat{s}^2$:	Estimated sample variance
$V(r_t)$:	Variance for the returns at time t
$n(i)$:	is the ith number of ordered random variable before Y_i
τ :	measure of correlation between the two the series in ADF (Δr_t, Δr_{t-1}&Δr_{t-i})
I_t :	An information set, sometimes called the history of prices up till time t
H_1:	Alternative hypothesis
α_i:	mean effect of a i^{th} return anomaly
ρ_1:	First lag autocorrelation of one-period return
ϑ_1:	is the j^{th} parameter of the moving average model in [**4.35]**, for $j = 1, 2, \dots, q$
$SE(\hat{\varphi})$:	Standard error of the estimated parameter, $\hat{\varphi}$ of the random walk
z_N:	N-period standardized distribution of the sample variance ratio
$\exp(r_t)$:	Exponential/ continuously-compounded returns at time t
ε_t:	random error or shock or residual term
u_t:	Sub-error term
w_i:	Weight of an i^{th} asset in the portfolio of assets

B_t: t	Bubble parameter at time
$\mathcal{F}_{t-1}$:	Information set at time t-1
$I_{[\varepsilon_{t-i}<0]}$:	Parameter representing asymmetric effect of both negative and positive news
ℓ:	coefficient of the regression of the Sign bias joint model
β_1:	Shocks to lagged conditional volatility at time 1 or shocks due to GARCH effect
Z_t:	Standardized residual at time t
Þ	Parameter representing anomaly effect of a given period-day, month, year, etc.
$tr[.]$	Is called “trace” of a function
$\sigma^2_{\mathcal{E}_t}$	Is the variance of residuals at time t

1 CHAPTER ONE: INTRODUCTION

1.1 General Overview

Financial time series is a fast-developing field of applied statistics, essentially because it deals with analysis of financial data such as stock prices, stock market index values, currency exchange rates, inflation rates, electricity prices, and interest rates. Understanding stock price behaviour would greatly help corporate and private investors, businessmen, international or transnational traders, brokers, bankers, and financial analysts to adopt the best possible strategies for optimising their investment returns and hedging likely risks associated with fluctuations in the returns. The market makers or regulators, and policy makers, could be guided by such knowledge to suggest appropriate policies and regulations aimed at further deepening the market and ensuring high level of sanity is maintained among relevant interacting market agents. In addition, the ability of the research community to access updated information and furnish same to the public is enhanced, such that the source of most risks, typically measured by variances of individual asset returns and/or portfolios of asset returns, is identified.

Moreover, since asset prices and returns are time-varying, the approach to understanding their behaviour is to estimate the basic data generating processes (DGPs) which underpin them, fit appropriate statistical distributions to the observed data, and use the statistical properties to make related investment and risk management decisions.

Even though in financial markets prices are being observed, most empirical studies deal mostly with price changes (known as returns) simply because returns measure the relative changes in stock prices, and hence the performances of the assets over time.

Considering these facts, this study uses stochastic time series techniques to investigate the stock market characteristics of the banking sector in the Nigerian Stock Market (NSM) based on the stock returns from 1999-2014, a period which encompasses the 2007 global financial crisis, banking and financial reforms in Nigeria starting from 2004. The study will among other objectives examine the stylized facts of bank returns, investigate some key features of the banking sector such as efficiency, anomalies, bubbles and volatility. The implications of the research findings for investments in bank shares and the development of the Nigerian Stock Market (NSM) shall be examined. This is especially important given the fact that the banking sector is a key driver of economic development of a country.

1.2 Research Aim and Objectives

1.2.1 Aim

As noted in Subsection 1.1, the research is aimed at applying appropriate (financial) time series techniques to investigate if the markets for bank shares in the Nigerian Stock Exchange (NSE) are efficient and the implications of the results for stock market performance of different banks and development of the Nigerian Stock Market (NSM). For this, the research explores five key empirical finance issues which relate to and include market efficiency, namely stylized facts, anomalies, bubbles, market efficiency, and volatility of the banking sector in the NSM, using the stock returns from 1999-2014. The research extends the results obtained by Omar (2012) at the overall stock market level, by using bank returns instead of the All Shares Index of the NSM. The specific research objectives and questions are as follows.

1.2.2 Research Objectives (ROs):

1. To explore the empirical distributions and stylized facts of bank stock returns in the NSM, for different sub-periods of the study determined by the 2007 global financial crisis and financial/bank reforms in Nigeria;
2. To investigate key stock market characteristics of the banking sector of the NSM across the study periods, notably efficiency, anomalies, bubbles and volatility;
3. To, where appropriate, discuss the implications of the research results for investments in banking shares and development of the NSM.

1.2.3 Research Questions (RQs):

1. What are the stylized facts of bank returns in the NSM and how do they compare with known results in other financial markets? In other words, are the stylised facts of bank returns in the NSM similar or different from those of other countries such that analysis of investment risk and return in the banking sector could be handled differently?
2. How efficient is the banking sector of the NSM in different study periods?
3. Are the Nigerian banks' stock returns characterised by anomalies or bubbles for the study periods?
4. How volatile are the bank returns and which volatility models are most suitable for describing the volatility behaviours of the returns in different study periods?

5. How do the research results compare across the study periods and what are the significance of the research findings for relative performance of Nigerian banks and development of the NSE in the different periods?

Besides this, the main purpose of this research is to fill the gaps in Omar (2012)'s work requiring in-depth study of the Nigerian stock exchange through a sectoral approach of investigating the behaviour and the challenges that have characterised, particularly the Nigerian banks within the NSE from 1999 to 2014 using individual bank's stock returns as against the overall average index (e.g. All share index (ASI) used by Omar, 2012), that was used in other studies across different assets and markets. Moreover, our choice of the data periods (June, 1999-December, 2014) was informed by the fact that these were the periods of: (i.) Transition from military to civilian government in Nigeria; (ii.) banks' recapitalisation and stock market reforms (2004-2006); (iii.) the global financial crisis (2008-2009) and (iv.) banking reforms (June, 2009-December, 2010).

1.3 Rationale for the Research

During the **2008-09** global financial crises, the Nigerian stock market (NSM) and banking sector witnessed near collapse manifested by near bank collapse that was stopped by timely interventions by the Central Bank of Nigeria (CBN) and a sudden decline (more than 60%) in the market capitalization[1] of the NSM**.** This experience necessitates the study of risks and returns associated with fluctuations in stock prices, most importantly as they relate to systematic stock market characterisation (SSMC), which is meant to capture the market dynamics for informed investment, risk management (Yahaya, 2012), financial policy (Musa et al., 2013), economic and equity market growth, linked to the GDP growth, competitiveness, market microstructure, and macroeconomic, monetary and fiscal policies (Ezepue and Omar, 2012; Omar, 2012; Ezepue and Solarin, 2009; African Development Bank [AFDB], 2007; Aliyu, 2012; Alade, 2012; Osinubi, 2004).

Despite negative effects of the global financial meltdown in various financial markets, lessons learnt from the crisis are yet to be incorporated into a plan for stock market research, characterisation and development, which will guide policy makers in (developing) economies

[1]**Market Capitalization:** represents market value of a company's paid-up capital, obtained by multiplying the current quoted price by the total number of shares outstanding. The market capitalization of a securities exchange is the aggregate market capitalization of all its quoted securities.

to be knowledgeable about the systemic interdependencies among different sectors of the financial systems. For the NSM, the only attempts at systematic stock market characterisation were initiated by Ezepue and Solarin (2009) and only recently developed in Ezepue and Omar (2012). The latter proposes that a robust characterisation of financial markets should consider the six main issues in empirical finance which underpin market dynamics and performance – anomalies, bubbles, efficiency, volatility, predictability and valuation – as positioned within the macroeconomic situations (Cuthbertson and Nitsche, 2005).

Ezepue and Omar (2012) also suggest the need to conduct such theoretical and empirical financial studies at overall and sub-market levels, to map vital linkages and effects which may manifest differently at these levels, and to provide micro- and macroeconomic market information to market participants (households, firms and governments), that is relevant for their investment and policy making objectives.

As noted earlier, the need to characterise the banking sector within the NSM became evident from the outcomes of Omar (2012) and Ezepue and Omar (2012) at the overall market level of the NSM, they observed that in order to capture the behaviour of this market as it relates to the identified market characteristics, it is better to examine the market subsectors so that any characteristics, such as bubbles, which are not easy to unpick at the market level due to confounding effects of data agglomeration across market sectors, may be explored without such effects. With this approach, it is envisaged that well-informed interpretations of the effects of various characteristics of investment returns and risks, capital growth, and stock market performance against changing policy context would be better addressed. Hence, this study examines four market issues - anomalies, bubbles, efficiency and volatility for the banking sector of the NSM to extend the results obtained in Omar (2012) to such sector-specific levels. Using bank stock returns in the study period 1999-2014 will enable the study to explore the relative effects of the 2008-2009 global financial crises and different financial and banking reforms started in 2004 on the banking sector.

1.4 Contributions to Knowledge and its Suitability for PhD Work

This study presents in-depth analysis of share price behaviour in the banking sector of the NSM, which according to a report in July, 2017 (Olarinmoye, 2017), is among the five largest stock markets in Africa where foreign investors are often focused for investments. Moreover, it gives an exposition of how stochastic modelling techniques can be used to analyse emerging market characteristics.

Addressing Objectives 1 and 2 represents, for the first time, an in-depth study of the stylized facts of the NSM and four of the six key market characteristics typically studied in financial economics - market efficiency, anomalies, bubbles, volatility, predictability, and valuation will be investigated in a key sector of the NSM.

Work on Objective 3 will provide useful new knowledge about relative performance of different bank stocks, hence their risk and return characteristics and stock market development in the Nigerian financial system.

The results will strongly inform similar research in other sectors (e.g. the energy, telecommunications, agriculture, and manufacturing sectors) of the NSM as well as emerging (African) markets which have similar characteristics to NSM.
Previous work such as: Ezepue and Omar (2012); Ezepue and Solarin (2009); Nnanwa et al., (2016); and Urama et al., (2017) indicate that western markets have statistically similar stylised facts like emerging markets but are deeper in the sense of having a greater number of financial assets traded. This feature makes the markets more liquid, structurally more robust in their trading and regulatory frameworks, and able to support a wider range of financial transactions across equity, bond, commodity, foreign exchange, financial derivatives (Urama et al., 2017), and real estate asset classes than is the case in emerging markets. This point suggests that the envisaged results from this research will provide additional contributions to knowledge compared to those explored in the literature review on developed markets, because (i) African emerging markets, particularly the Nigerian market, has not been researched in-depth regarding the key market issues examined in this thesis; (ii) the concentration on the banking sector has rarely been covered at most markets; and (iii) the use of the individual bank's returns as against the market index used across different markets make this research to be unique.The theoretical, research, practical portfolio management (Nnanwa et al. 2016), and policy making implications of the results will therefore be explored in this light.

1.4.1 Benefits of the Research to Stakeholders in the Nigerian Financial System

As summarised in the above paragraph, knowledge of the underlying characteristics of the banking sector of the NSM across different periods determined by the global financial crisis and banking reforms will help the market makers and the investors as follows.

1. This will encourage investors to enhance their investments and improve on their risk management plans on the banking stocks.

2. It will help policy makers to overhaul the operations of the NSM, by initiating strategies that will make every key sector, including banking industry to be more informationally efficient, less exposed to bubbles and volatility.

1.5 Distinguishing Features of this Research

This section distinguishes this research from related work such as Omar (2012) which it extends.

1. While Omar (2012) focuses on the entire NSM, this study is targeted at the banking sector. Our choice of the banking sector was based on the outcomes of both Ezepue and Solarin (2009) and Omar (2012), which suggest that for better and in-depth understanding of the dynamics of the NSM, there is a need to additionally analyse data at firm-or sector-specific levels, to obtain a better understanding of the stock market dynamics. Consequently, the returns of individual banks will be used in this research instead of the All Share Index (ASI) used in Omar (2012), and other studies [such as: Mecagni and Sourial (1999); Jefferis and Smith (2005); Appiah-Kusi and Menyah (2003); Omran (2007); Alagidede and Panagiotidis (2009), etc.], where stock index is used. Focusing on individual asset return series eliminates the confounding effect of using the overall market index, whereby some market characteristics may not be evidenced at this aggregate level because of the off-setting effects of information from different sectors.

   Moreover, the banking sector predominantly drives the performance of modern capitalist economies, including the Nigerian economy. Hence, results in this sector combine with those at the overall market level to provide deeper insights into the stock price dynamics of the NSM, and their policy making implications for the management of the Nigerian Stock Exchange (NSE) and economy (Ezepue and Omar (2012)).
2. This research fills some gaps in Omar (2012) by conducting a more detailed study of the stylized facts of the bank returns, which strongly inform better choice of various models and techniques used in the research.
3. Also, while Omar (2012) used ten years of data, 2000 to 2010, this research examined fifteen years data, ranging from June 1999 to December 2014.

1.6 Thesis Structure

The thesis will be comprised of 10 chapters as follows.

Chapter 1: Introduction.

Chapter 2: General Background.

Chapter 3: Literature Review.

Chapters 4: Methodology.

Chapter 5: Stylized Facts of Asset Returns in the NSM.

Chapter 6: Stock Market Efficiency Models and Tests.

Chapter 7: Market Anomalies.

Chapter 8: Rational Speculative Bubbles.

Chapter 9: Volatility Modelling.

Chapter10: Main Results-Summary, Interpretation and Further Studies.

In Chapter 2, general and historical background knowledge about the financial system, Nigerian Stock Market (NSM), and the Nigerian banking sector are discussed.

Chapter 3 presents the overall literature review of different aspects of the study related to the objectives. This chapter surveys the stochastic and applied statistical models used in financial market analysis. Further literature on the models used to study specific characteristics of stock markets are given in the specific chapters which look at the issues. Also, a brief review on the banking reforms initiated by the Central Bank of Nigeria (CBN), and their impact on the Nigerian banks and their performance are given.

Chapter4 presents the research methodology including the data coverage, choice of software and summary of methods used subsequently in Chapters 6-9.

Chapter 5 presents the descriptive statistics and the general stylized facts of the returns as a prelude to identifying appropriate model(s) that would be most suitable for describing and evaluating the riskiness of stocks of these banks within the NSM.

Chapters 6-9 apply appropriate statistical tests and models to the specific issues and characteristics of the NSM such as market efficiency, stochastic volatility, anomalies and speculative bubbles.

Chapter 10 summarises the main results, their implications for the NSM and the Nigerian financial system and recommends future work.

1.7 Summary and Conclusion

This chapter provided a general background to the significance of the research, justification for the focus of the research as well as motivation, aims and objectives for this research. It also summarised the expected contributions to knowledge by research objectives. These are followed up on in chapter 10. The chapter distinguished this research from related work by Omar (2012), and finally outlined the structure of the thesis.

2 CHAPTER TWO: GENERAL BACKGROUND

2.1 Introduction

This chapter briefly discusses essential issues touching on: the financial system, the financial market, Nigerian capital market creation, the Nigerian stock exchange and regulatory institutions, the Nigerian banking industry and the rationale for choosing the sector, the 2007-2009 global financial crisis and its impacts on the Nigerian banks and economy, the listed Nigerian banks currently on the exchange market, and banking reforms.

2.2 Financial System

A financial system facilitates more efficient financial transactions by investors, financial analysts, enables economic policy makers to regulate and reform a country's economic system. Without an effective financial system, a party to a transaction with superior information than the other party may cause what is termed the information asymmetry problem, which encourages inefficient allocation of financial resources. To overcome this problem, the financial system strikes a balance between those with funds to invest and those needing funds.

Structurally, the financial system is made up of three components, namely financial markets, financial intermediaries and financial regulators (see Figure 2.1 below), with each component playing a certain role in the system. While the financial markets enable the flow of funds to finance investments by corporations, governments and individuals, financial institutions perform the role of intermediation, thus determining the flow of funds, and financial regulators function as the monitoring and regulatory body in the financial system.

The Nigerian financial system is made up of banks and non-bank financial institutions such as insurance, pension, mortgage, capital market, and the regulatory bodies, namely: the CBN, National Deposit Insurance Corporation (NDIC), Security and Exchange Commission (SEC), National Insurance Commission (NAICOM) and the National Pension Commission (PENCOM)

The diagram below presents the interrelationships among the three agents which play significant roles at promoting economic growth in the financial system. The three basic agents are: firms/companies, financial markets and intermediaries such as banks, and government departments.

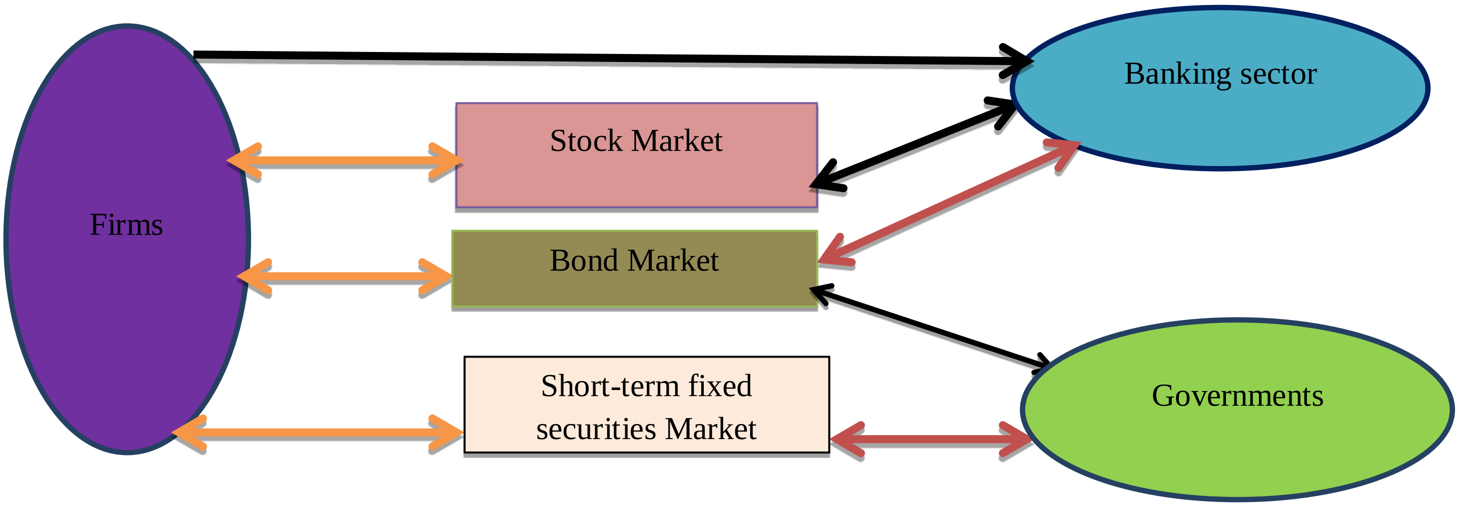

Figure 2.1: The Financial System Structure

2.2.1 Financial Markets

A financial market is an environment where various types of financial instruments, such as equities, currencies and debt securities, are bought and sold, according to a set of rules:

- Financial markets serve to transfer funds from lenders to borrowers.
- For lenders, the markets provide a platform for their excess liquidity[2] and a way to store wealth.
- For borrowers, the markets provide credit to finance their consumption and investment.

Figure 2.2 below (Financial System Architecture) highlights how funds are transferred in the financial system from the lenders or surplus channels to the borrowers or the spenders through financial intermediaries such as banks, financial market, and brokers. When money moves from a lender through the financial market to the borrowers/users, such movement is called "Direct financing". When such transfers occur through banks, finance houses, and brokers, this constitutes "Indirect financing".

[2]**Liquidity:** is the extent to which a security or financial asset can swiftly be traded in an exchange market at a stable price; thus, in liquid markets investors become more active and can design different arbitrage strategies.

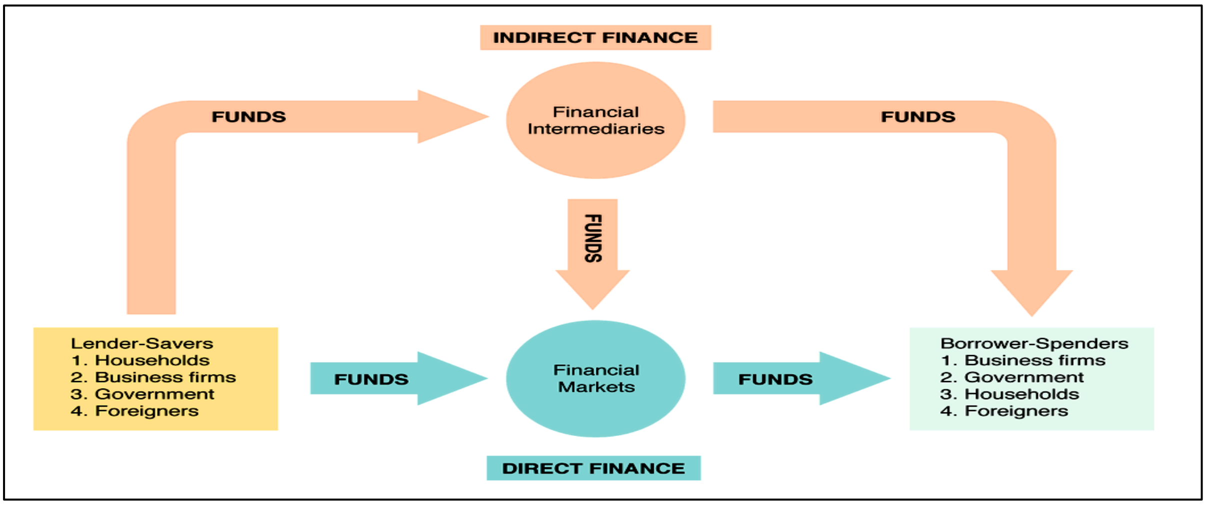

Figure 2.2: Financial System Architecture: Flow of Funds through the Financial System (Sakanor, 2015)

Financial markets are classified into five categories based on:

- The **Nature of Claim**:
  - a) **Debt Market**: e.g. Bonds;
  - b) **Equity Marke**t: e.g. Common stock.
- The **Period of Claim**:
  - a) **Primary Market**: markets for Initial Public Offer (IPO); i.e. where new securities are issued and purchased by initial buyers;
  - b) **Secondary Market**: markets where the existing securities are issued, bought and sold.
- **Market Structure**:
  - a) **Exchange Market**: markets where trading activities are conducted in central locations e.g. stock exchange;
  - b) **Over-the-Counter (OTC) Market**: market where dealers at different locations buy and sell securities.
- **Period of Delivery**:
  - a) **Cash or Spot Market:** this is a market where quick delivery of traded assets(securities) is made, mostly within two trading days;
  - b) **Derivative Market:** this is futures or forward market where contracts (claims) are made for future delivery of securities.
- **Period of Maturity**:
  - a) **Money market:** is the market where short-term assets of one year or less maturity is traded;

b) **Capital Market**: the market where long-term securities of more than one-year maturity are traded.

Brief Account on the Creation of the Nigerian Capital Market

Dougall and Gaumnitz (1980), Srinivasan (2010) and Okafor and Arowoshegbe (2011) define capital markets as mechanisms and institutions through which intermediate term funds such as loans of ten-year maturity, and long-term funds such as loans with longer-term maturity and corporate stocks are raised. It is also a market where the outstanding financial instruments are made available to business, governments, and individuals. A well-functioning capital market serves as a fulcrum for any financial system because it facilitates fundraising needed to finance both business, and other economic institutions and government programmes (Osaze and Anao, 1999).

2.2.2 Nigerian Capital Market

In Nigeria, the decision to establish an indigenous capital market was made in the early 60s due to the lack of channel through which both long term and short-term funds could be raised locally. This was because at that time the only existing banks were owned by foreign investors. To manage the debt incurred by the federal government, the office of the Accountant General (A.G) of the federation then was given the responsibility to create the Central Bank of Nigeria (CBN) in 1959. The CBN was to manage the debts and generally take over management of Nigerian government finances.

Despite the establishment and commencement of operation by the CBN, there was no institution to discharge the regulation and oversight functions of the capital market. This prompted the then government to establish the Lagos Stock Exchange with an Act enacted in 1961 for full management and regulation of the Nigerian capital market. Even though the exchange market was incorporated on 5th September 1960, it did not begin full operation until 5th June 1961. By 1977, the market changed its name from "Lagos Stock Exchange" to the "Nigerian Stock Exchange" – a decision that put a stop to attempts at establishing other regional based exchange markets named after a state of the federation, such as the Kaduna and Port Harcourt Stock Exchange.

Objectives for the Creation of the Nigerian Capital Market

The intents behind the establishment of the Nigerian Capital Market according to Osinubi and Amaghionyeodiwe (2003), Osinubi (2004), and Okafor and Arowoshegbe (2011) include:

- To ensure even distribution of wealth;
- To decentralize the ownership of assets and create a healthy private sector;
- To discourage excessive concentration of economic power in the hands of a few private individuals;
- To mobilise savings for economic growth;
- To promote co-operation between local and foreign investors leading to economic enhancement;
- To strengthen the banking industry and reduce government dependence on taxation for economic development;
- To diversify capital from less productive sectors like real estate to more productive sectors such as industry.

The Nigerian Stock Exchange

The Nigerian Stock Exchange (NSE) was created in 1960 under a limited guarantee, licensed under the Investments and Securities Act (ISA), governed by a national council, known as Board of Directors and regulated by the Nigerian Securities and Exchange Commission (SEC). The exchange has 13 branches across Nigeria, with its head office in Lagos. It services the second largest financial centre in Sub-Saharan Africa, is a founding member and executive committee member of the African Securities Exchange Association (ASEA), and an affiliate member of the World Federation of Exchanges (WFE), which is also an affiliate member of the International Organization of Securities Commissions (IOSCO).

The Nigerian Stock Exchange, as with other exchanges, represents a market place providing a fair, efficient and transparent securities market to investors. It currently has 191 companies listed, operating three major types of market, namely: The Equity market, the Bond market and the Exchange Traded Funds (ETFs) market. Its daily trading activities run as follows:

(1.) A Pre-Opening Session, between 09h30 and 10h15 daily. (2.) Opening Auction Trading from 10h15 to 14h30. (3.) Market Closes by 14h30.The trading activity is either "Quote-Driven" or "Order-Driven" in the NSE.

(a.) By Quote-Driven, we mean the phenomenon whereby the market makers are allowed to provide two-way quotes and licensed brokers (or dealers) of exchange are given the right to submit orders.

(b.) Order-Driven makes it possible for all the orders made by the buyers and sellers to be displayed in the market, with such orders revealing the quantity and price a buyer or seller is willing to purchase or sell (respectively), his/her asset(s).

With this form of trading (either "Quote-Driven" or "Order-Driven"), NSE is said to be operating as a “Hybrid Market".

Its national council has fourteen (14) individual members charged with the responsibilities of:

- Directing exchange business and financial affairs, strategy, structures and policies
- Monitoring to ensure that delegated authority is exercised
- Handing challenges and issues bordering on corporate governance, corporate social responsibility and ethics.

The council, given its roles, comprises of seven committees which include:

1) **Audit and Risk Management Committee**: takes care of financial reporting, internal controls, and risk management systems amidst other relevant roles.
2) **Demutualization Committee**: handles issues relating to the exchange structure, legal, regulatory and financial strategy relative to the demutualization of the exchange.
3) **Disciplinary Committee**: in charge of hearing and adjudicating on disciplinary matters in respect of dealing members, and sets rules governing dealing members.
4) **Governance and Remuneration Committee:** ensures compliance with corporate governance policies and practices and provides oversight functions of the exchange's human resources among other functions.
5) **Rules and Adjudication Committee:** ensures continuous review of the exchange's rules and regulations.
6) **Technical Committee:** in charge of information systems management and technical issues including associated risks.
7) **MEMART Committee** is an ad-hoc committee to ensure that the exchange business operation is in line with its Memorandum and Articles of Association ("MEMART") and review of the articles.

2.2.2 Securities and Exchange Commission (SEC)

SEC is the main regulatory body of the NSE under the supervision of the Federal Ministry of Finance. The commission is charged with the responsibility of maintaining surveillance over the NSE with a view to ensuring orderly and equitable dealings in securities and protecting the market against insider trading abuses.

The commission was an ad hoc, non-statutory committee established in 1962 from the Central Bank of Nigeria (CBN), which later became the Security Exchange Commission in1973. In1979, the commission was chartered under Decree 71 and subsequently was chartered by the Investments and Securities Act No 45 of 1999, which is the current and major legal instrument empowering SEC to regulate the affairs of the NSE.

2.2.3 The Nigerian Banks

The three largest banks that could equally be categorised as first-generation banks due to their existence from the colonial period are: First Bank, which used to be British Bank for West Africa, and was incorporated in 1894; Union Bank, formerly known as Colonial Bank and later acquired by Barclays, which came to be in 1917; and United Bank for Africa (UBA), which used to be the British and French Bank, which was established in 1949. The three banks, which were originally foreign banks were acquired by the Nigerian government through shares in the mid '70s. During the colonial era, these banks faced serious competition from local investors before the enactment of the first banking legislation in 1952.

The foremost bank to operate in Nigeria, named the African Banking Corporation (AfBC), was established in 1892 with the primary role of facilitating transfer of cash from the colony, later named Nigeria, to the then home country, United Kingdom. In 1894, Bank for British in West Africa, now called "The First Bank of Nigeria" was created and subsequently acquired the AfBC. Many other banks owned by foreign banking organisation were later established in the country (see Uzoaga, 1981, p. 66).

The fourth largest bank, Afribank, a second-generation bank established at the time of independence, was also a foreign bank previously named Banque Internationale Pour L'Afrique Occidental (BIAO). Later, around mid '70s, many more commercial banks were established by the state governments in Nigeria; thereafter, merchant banks, which were joint ventures between foreign investors and the Nigerian government, were established.

A review of the present structure of the Nigerian banking sector by the Central Bank of Nigeria in September 2009 revealed that there are 1014 community and 24 commercial banks in Nigeria, which combined account for 93 percent of private asset values of the entire financial sector, indicating a large banking sector relative to the financial sector. According to the Central bank of Nigeria’s draft Annual Report for the year ended 31 December 2008 (Alzahrani, Gregoriou and Hudson, 2013), the Nigerian banking sector is dominated by four banks, which account for 58 % of the value of assets and 65% of the total deposits held by the banking sector. Only two of Nigeria's banks have some measure of control by foreign banks with only a 4% stake of total assets.

According to Sanusi (2010), the significant growth and expansion recorded within the Nigerian banking sector has been characterized by series of challenges, though there was a remarkable increase in the overall total of Nigerian banks from 41 banks before 1986 to 120 by 1994. However, stiff competition and other associated challenges facing the sector brought about a sharp drop in the number of these banks from 120 banks to 89 banks by 2004 and later to 24 by 2009.While some of these changes were market instigated and due to the bank consolidation policy of the then CBN governor, Prof Chukwuma Soludo in 2004, others were outcomes of the federal government's earlier proposals to introduce an indigenisation policy in the sector, allowing Nigerian citizens to have total control of the sector.

The financial sector boom, therefore, was, however, accompanied by financial disintermediation. "This tragic situation led to the continued foreclosure and technical insolvency of many banks and finance houses" (Eke 2003, p. 4) and "the latest assessment shows that while the overall health of the Nigerian banking system could be described as generally satisfactory, the state of some banks is less cheering” (Soludo 2004, p. 5). Access by the Nigerian banking sector to the stock market as a channel for raising long term capital to finance its activities has been and may continue to be a major catalyst for any future growth of the banking sector.

2.3 Global Financial Crisis/Meltdown: The Nigerian Experience

In this section, a brief account of what went wrong in the global market as well as the impacts on the Nigerian financial system are discussed. The focus will be on the 2007-2009 crisis, which according to some studies (Reinhart and Rogoff (2008) and Eigner and Umlauft (2015)), is the worst financial meltdown in the twenty first century, and indeed since the Great Depression of 1929, based on its impacts on the global economy.

To start with we observe that, financial meltdown represents a situation when financial markets and financial networks become evidently strained or unstable to the point of near collapse (Sanusi, 2011a). Some of its characteristics include sudden changes in expectations, declining prices, recurrent bankruptcies and speculative bubbles (Sanusi, 2011a). The Global financial crisis served as a major restriction to development and growth in most nations, aggravated by the banking system crisis, currency crisis as well as a foreign debt crisis. Financial institutions such as banks, or assets, e.g. stocks, bonds and currencies instantly lose most of their value during such a crisis.

2.3.1 How the Crisis Happened

Right from 2002 to early 2007, there were decreases in volatility in the global economy and financial markets; this led to a significant reduction in investment risks, thereby encouraging most companies to become less risk-averse and invest more capital in the markets.

In August 2007, there were reported cases of constrained liquidities, whereby financial institutions were confronted with challenges in raising funds in the United States of America (USA) (Sanusi, 2011a). Towards the end of 2007, several American and European banks declared massive losses in their end-of-year financial reports.

Theearly signals of a crisis observed in January 2008, starting with a sharp drop in the profits of the Citigroup bank, leading to a sharp fall on the New York Stock Exchange. Thereafter, there were spectacular drops in share prices in all major world markets, leading to series of collapses. By March 2008, there was shocking and unusual credit shrinkage or crunch as the financial institutions tightened up credit in the U.S; the credit crunch rose to become a full-blown crisis by mid-2008 such that by July 2008, the crisis found its way into other sectors of the economy.

According to Reinhart and Rogoff (2008), the he crisis became pronounced because of the failures of multinationals such as Goldman Sachs, Barclays and Deutsche Bank, Merrill Lynch and Morgan Stanley in the US; the spill-over effects of which caused the failures of many European banks, thereby transforming into global crisis. Precisely by15th September 2008, the biggest bankruptcy in the world started when Lehman Brothers Bank failed with liabilities of

US $600 billion (Edey, 2009) and thereby filed for Chapter 11 of the bankruptcy protection (Schwarcz, S. L. (2008); Shah (2009); Miller and Stiglitz, (1999); Wiggins, Piontek and Metrick (2014)). According to Gokay (2009), "the last months of 2008 experienced what is called the worst financial crisis since the Great Depression of 1929-30".

Consequently, most commercial banks in the US suffered a great setback such that most financial institutions lost substantial part of their worth within a short time period due to sudden and panicked withdrawals by the depositors - a situation called a bank run, leading to recessions across the global economy.

Meanwhile, Figure 2.3 below provides a summary of the origin of the 2007-09 financial/ economic crisis, which had its root in excessive leverage in financial institutions to housing and stock market bubbles, leading to a series of crashes that developed to a full-blown financial crisis, which eventually led to the global recession of 2008-09.

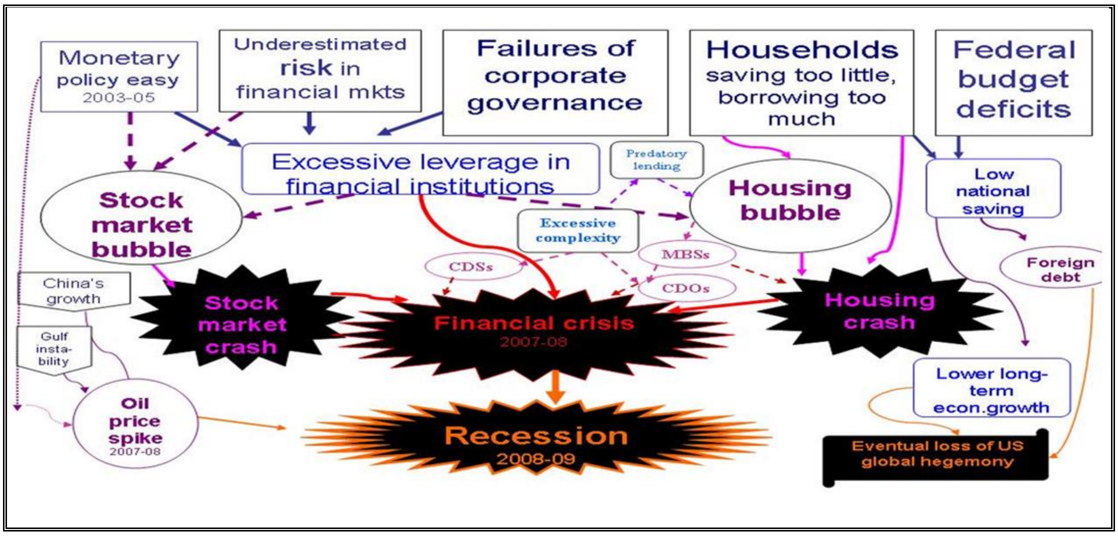

Figure 2.3: Diagram Showing Origin of the 2007-2009 Financial/Economic Crisis (DeMichele, 2016)

However, during those periods when the US and other industrialized economies were experiencing losses due to the crisis, the Nigerian market was immune from the crisis because of the strict financial controls imposed on the market by the CBN and SEC.

2.3.2 Nigerian Experience

The unprecedented effects of the 2007-2009 economic and financial crises on the world economy led to global recession and the collapse of many giant financial institutions such that many nations went bankrupt (Sanusi, 2012). The Nigerian economy was immune to the first-round effects of the crises due to strong monetary and fiscal policies, but the second-round effects hit the economy such that the stock market collapsed by 70% between 2008 and 2009, with many banks recording huge losses due to their exposure to the financial market and oil and gas industry.

This prompted the CBN to rescue eight banks found to be deeply affected, by injecting capital and liquidity into the banks, sacking the banks' executives and subsequently punishing those found culpable, with a view to restoring public confidence and sanity into the industry (Sanusi, 2011a; 2012).

2.3.3 Effects on the Nigerian Stock Market (NSM)

According to Sanusi (2011a), the greatest impact was felt in the nation's capital market. There was:

a) Extended recession in the market triggered by huge divestment by foreign investors;
b) Lingering liquidity tightness;
c) Declining public confidence; and
d) Panic selling by domestic investors, which led to significant losses being recorded by investors.

The stock market, which was bullish[3] between December 2005 and early March 2008 to the tune of hitting equity market capitalization of N12.64 trillion by March 11, 2008, instantly became bearish[4] in April 2008 and remained so with only minor recovery by 2010. The market which recorded about 14.45% increase in its All Share Index (ASI) between December 31, 2007 and March 2008 (the peak of the bull-run), suddenly experienced a sharp decline of nearly 45.8% in ASI and 32.4% in market capitalization by the end of 2008. The capital market recession had significant negative impact on banks' balance sheets via rise in provision for bad debts and lower profitability.

2.3.4 Impacts on the Banking Sector

Between 2004 and 2008, excess liquidity was recorded in the NSM as reflected in the unprecedented rally in stock prices on the Nigerian Stock Exchange (NSE) from 2006 to March 2008 (Sanusi, 2011b). This enabled banks to raise capital to the tune of N1.603 trillion. However, during the financial crisis, banks were grossly negatively impacted due to spill over

[3]Stock or commodity market witnessing a general rise in prices
[4]A security market where a general decline in prices of securities or commodities is being experienced

effects from the oil and gas sector as well as the stock exchange market where Nigerian banks were a principal actor. There was a collapse of confidence in the banking system, de-leveraging, and banks' incapability to enhance capital adequacy, poor consumer demand and a drop in global output, which impacted the country through financial and real (trade and remittances) channels, Sanusi (2010).

With this experience, many of the banks were distressed, which caused the CBN to initiate another round of rigorous reforms in the banking sector by June 2009 after the first one introduced in 2005. Due to the new reforms, 9 banks were found to be in a grave situation and were unable to meet minimum 10% capital adequacy ratio and 25% minimum liquidity ratio. The said banks were not only principally dependent on the stock market as well as oil and gas, but also engaged in unethical and unprofessional practices.

However, the fact that the Nigerian banking system was partially integrated into the global market, coupled with strong macroeconomic policies implemented by the government, assisted the country to hedge the effect of the crisis.

2.4 Nigerian Banking Reforms

The Nigerian banking system has witnessed a series of reforms or restructuring in the last fifty-eight years since independence. The sector was moved from the dominance in the 1960s by a small number of foreign-owned banks into public ownership from the1970s through to the 1980s in which Nigerian private investors have become major stakeholders since the middle of 1980s.

Based on the initial intention behind the creation of banks as a major source of fundraising for government projects, government policies have significantly shaped the industry since its inception. Continuous intervention by the government in terms of financial sector policies right from the 1960s through to the 1970s with the aim of promoting indigenisation and influencing resource allocation has helped to galvanise liberalisation and prudential regulation and has saved the industry from avoidable collapse.

According to Balogun (2007), banking industry reform in Nigeria has remained a focal point of economic reforms right from the '80's when it was first introduced through the popular structural adjustment programs initiated around 1986 by the then military government. Then, four phases of reforms aimed at strengthening the economy and the banks, were introduced. The first phase, targeted at restructuring the banks, was tagged "financial systems reforms", and was initiated to deregulate the banking sector; it included credit rate, foreign exchange rate and interest rate policy adjustments. These, according to the CBN's report in 1993, brought about a deepening of the sector such that the number of banks in the country was raised from forty commercial and merchant banks with over 1,655 branches across the nation to 121 with about 2385 branches in 1992.

In the second phase, when a series of regulations and controls were re-introduced, the sector was deeply hit such that the country witnessed near total collapsed of the sector between 1993 and 1998 due to serious financial distress experienced by most of these banks and instability in the political space trailed by inconsistent policies of the then military government, which greatly affected the entire economy. Thus, another round of reforms was initiated with a view to salvaging the sector from total collapse, (Balogun, 2007).

In 2004 the then CBN governor, Prof Charles Soludo, observed in the banking system several weaknesses that brought about a reduction in number of banks in the nation from 89 to 24 through what were then referred to as re-capitalization (Soludo, 2004). Banks were requested to increase their minimum capital requirement from N5bn to N25bnanduniversal banking was introduced so that the banks could diversify their portfolio to cover all aspects of retail banking.

However, on his assumption of office as the CBN governor in June 2009, Sanusi (2010) found the banks in a serious state of financial weakness despite the initial capitalization introduced by his predecessor. The problems found with the system were traced to:

1) Weak corporate governance;
2) Operational indiscipline and
3) The effects of global financial crisis.

These inadequacies in the system caused Nigerian banks to:

a) Be disconnected from the rest of the economy and their core mandate as channels for transmitting the monetary policies of the CBN;
b) Be unable to commit themselves to transparent and responsible investments and
c) Lack the ability to guarantee credit flow to the real sector (such as manufacturing, agriculture, as well as small and medium enterprises that are critical to employment and income generation).

Consequently, the following measures were taken to salvage the system:

- A critical redefinition of banking architecture, corporate governance, bank operations and system ethics;
- The tackling of issues of excess risks with depositors' funds via the instigation of a specialized risk buffer system against "toxic assets" in the system.

The aim of the reform according to Sanusi (2012) was to establish a reliable and efficient banking system for the nation's economy to achieve the objective of being one of the 20 largest world economies by 2020. He observed, however, that one of the challenges of the regulatory agencies is how often these kinds of reforms should be carried out in the financial system.

2.4.1 The Need for the Reforms

The recent global financial crisis has further emphasised the importance of regular banking reforms throughout the world. The banking industry is expected to be able to efficiently perform its function of intermediation in the financial system and achieve global competitiveness in the international financial markets. For these to be achieved, there is need for close monitoring. Moreover, given the fact that banks receive deposits from the public, it is imperative that periodic reforms are initiated to ensure that they gain public confidence and are financially stable.

2.5 Rationale for Focussing on Banking Sector

In this section, we briefly highlight the roles banks play in the financial system and the rationale behind focusing on the banking sector of the NSM.

2.5.1 Functions of Banks in the Financial System

In theoretical economics as well as finance, one of the issues of concern is the study and understanding of the functions of banks within the financial system.

Allen and Carletti (2008) observe that banks are very important in the financial system given the significant functions they perform in a nation's economy. These functions, according to them, are as follows:

- Amelioration of the information issues between borrowers and investors by tracking the former closely to ensure the funds are utilised judiciously;
- Provisions of inter-temporal smoothing of undiversified risk at a given point in time, these they achieve by ensuring depositors are protected against consumption shocks;

- They contribute to economic growth;
- They help to overcome asymmetric information challenges by developing long-lived connections with companies;
- Performance of an important role in corporate governance;
- Provision of financial assistance to companies, thereby strengthening the economy.

Sanusi (2012) summarises the functions expected of banks in the economy as being:

a) To mobilise savings for investment purposes;
b) To provide credit for the real (productive) sector of the economy;
c) To serve as a financial intermediary between the financial markets and investors;
d) To serve as a medium of fund raising for government to finance its developmental programs and strategic objectives.

According to Allen and Gale (1997; 2000a, Chapter 6), banks are expected to make enough savings in times of high yields on their assets so that they can be sustained by this in times of fall in returns. They further maintained that for banks to play their critical role of risk sharing, they are expected not to be subject to serious competition from the financial markets.

To further emphasise the significance of banks in the financial system, Allen and Carlleti (2008) note that banks are always at the centre of every financial crisis and thus can easily spread crises if there is contagion to such an extent that slight or small shocks on them can result in a huge impact on the financial system and the nation's economy in general.

Further, Castren, Fitzpatrick and Sydow (2006) state that studying bank' stocks behaviour is important because:

i. A bank's stock price may fully summarise all the public information available from the bank, including potential risks, in a single Figure.

ii. Under the efficient-market hypothesis, banks' securities' prices incorporate expectations of both positive and negative future earnings prospects.

iii. Banks' stock price information is available at higher frequency compared to accounting information.

iv. A financial crunch in one bank is easily spread through various channels, which may be reflected in stock markets. Thus, knowing the extent to which the variability in individual banks' stock returns are driven by common versus bank specific components is imperative.

Finally, considering the intertwined nature of the financial system, which is comprised of the financial market and banks (intermediaries), coupled with the series of challenges the system had been bedevilled with across the global economy, owing to the spill-over effects of the global financial crises, in-depth study of the entire financial system has become a common theme globally in recent years. These studies have yielded many research outcomes targeted at unravelling the genesis of the crises to prevent their re-occurrence and to strategize on how to shield the system from all forms of financial crisis in the future.

However, despite significant efforts aimed at exploring the financial system, especially stock markets, globally through research in the developed markets, African emerging markets, particularly the Nigerian market, have not been researched in-depth with regards to the key market issues examined in this thesis. Moreover, given the significant roles played by banks in the stock market and the nation's economy in general, including the series of restructuring and reforms that Nigerian banks have undergone, this study will be the first time (in so far as we know) that this important sector will be methodically studied in detail across different periods determined by bank reforms and the global financial crisis.

In summary, given the above reasons, understanding the share price dynamics of the Nigerian banks would help their management, investors and policy makers to address many issues that would make these banks function more effectively as financial intermediaries. Hence, the points outlined above form the basis for our choice of this sector as the subject of study.

The remaining sections of this chapter summarise some characteristics of the Nigerian banks further and indicate typical stock market indicators used to assess their financial performance as well as that of other stocks traded in the NSM.

2.5.2 The Nigerian Banks on the Exchange Market

In this section, brief details on the banks listed on the Nigerian Stock Exchange are discussed. Information on the year of incorporation (Yr. Inc), merger and acquisition experience, years of listing (Yr. Lstd), ownership type, nature of business (NoB), security type (Sec. Typ) and security name (Sec. Name) are summarised below. There are eighteen banks currently on the NSE list altogether (see table 2.1). The banks’ characteristics will inform interpretations of different results on them presented in subsequent chapters of the thesis. These details were extracted from the official page of the Nigerian Stock Exchange (http://www.nse.com.ng/)

From the table below, we see that all the banks trade on ordinary shares, with more than 50% are owned by the public. Seven of the banks: Access, First, GTB, Sterling, Union, UBA and WEMA banks were listed before 1999, which is the start date for the data for this research.

Table 2.1: Showing Relevant Details of the Listed Banks on the NSM

Bank Name	Previous Name	Nature of Business	Ownership Type	Year of Incorporation	Year Listed	Security Type	Security Name
Access bank Nigeria Plc	N/A	Commercial	Private	1989	1998	Ordinary	Access
Afribank Nigeria plc	International bank for West Africa (IBWA)	Commercial	Public	1969	IUS	Ordinary	Afribank
Ecobank Transnational Incorporation	N/A	Wholesale and retail	Banking and financial services	1985	2006	Ordinary	ETI
Ecobank Nigeria Plc	N/A	Wholesale and retail	Independent and Regional bank	IUS	IUS	IUS	ECOBANK
Fidelity Bank plc	FSB International Bank Plc	Universal banking		2005	2005	Ordinary	FIDELITYBK
First Bank Nigeria Plc	N/A	Commercial	Public	1894	1971	Ordinary	FBNH
FCMB	First City Merchant Bank	Commercial	Private	IUS	2004	Ordinary	FCMB
First Inland Bank Plc	Inland Bank Plc First Atlantic Bank Plc IMB International Bank Plc NUB International Bank Ltd	Commercial	IUS	2006	2006	Ordinary	FIRSTINLND
Diamond Bank	N/A	Commercial	Public	1990	2005	Ordinary	DIAMONDBNK
Guaranty Trust Bank Plc	N/A	Commercial	Public	1990	1996	Ordinary	GUARANTY
SKYE Bank Plc	Prudent Bank, EIB International Bank, Bond Bank, Reliance Bank and Cooperative Bank	Commercial	Public	2006	2006	Ordinary	SKYEBANK
Spring bank Plc	ACB International Bank Plc Citizens International Bank Plc Fountain Trust Bank Plc Guardian Express Bank Plc	Commercial	Public	2006	2006	Ordinary	SPRINGBANK

	Omega Bank Plc Trans International Bank Pl						
STANBIC IBTC Bank plc	IBTC Chartered Bank plc STANBIC Bank Nigeria Limited	Commercial	Public	IUS	2005	Ordinary	STANBIC
Sterling Bank Plc	Nigeria Acceptances Limited (NAL) Bank Plc, **Indo-Nigeria Merchant Bank (INMB) Ltd**, Magnum Trust Bank Plc, **Trust Bank of Africa Ltd** and **NBM Bank Ltd**	Retail/Commercial	IUS	IUS	1993	Ordinary	STERLNBANK
Union Bank of Nigeria plc	N/A	Commercial	Public	1969	1970	Ordinary	UBN
United Bank for Africa plc	N/A	Commercial	Public	1961	1971	Ordinary	UBA
Unity Bank plc	N/A	Commercial	Public	1987	2005	Ordinary	UNITYBNK
WEMA Bank plc	N/A	Universal	Public	1987	1990	Ordinary	WEMABANK
Zenith Bank Plc	N/A	IUS	IUS	IUS[5]	2004	Ordinary	ZENITHBANK[6]

[5]**IUS-Information Unavailable from Source (NSE website- http://www.nse.com.ng/)**: No available information as at the time of the research for cells with the coloured acronyms
[6]Further details on the banks are presented in **Appendix 2A**

2.6 Rationale for the Choice of the African Emerging Market

This section highlights the reasons for restricting the focus of our study to the African emerging market, particularly the Nigerian market. The benefits of investing in these markets as well as the progression witnessed so far by some selected African markets are outlined.

Several years ago, the investment community was in search of markets to divert their investment opportunities to, in order to earn high profits. This requires good knowledge of how stock prices behave at different potential markets of choice. The statistical measures obtained from stock returns such as the mean, kurtosis, skewness, variance, generating distribution, correlation coefficients, the presence of autocorrelation in returns and in squared returns (Panait and Constantinescu, 2012), are vital characteristics inherent in asset returns useful for investors or investment manager to understand how best to create optimal portfolios.

Investigating the statistical behaviour of stock market returns within the developed markets has for some time now preoccupied researchers' attention with little concentration on emerging markets. The emerging African markets are of interest to this research because results from the NSM will support ongoing efforts to systematically characterise and develop the NSM and apply similar insights to other developing African markets for example the SANE markets of South Africa, Algeria, Nigeria, and Egypt, alluded to in Ezepue & Omar (2012) as being important to the African Development Bank Group (Ezepue & Omar, 2012; AfDB, 2007).

It is reiterated that this research is part of a line of work in Systematic Stock Market Characterisation and Development (SSMCD) started in the Statistics and Information Modelling Research Group at Sheffield Hallam University; the topics cover empirical finance modelling and applications in the NSM (Raheem & Ezepue, 2016, 2017, 2018; Ezepue & Omar, 2015; Ezepue and Omar, 2012; Omar, 2012; Ezepue & Solarin, 2009), and related work in mathematical finance and investment theory (Urama, Ezepue & Nnanwa, 2017; Nnanwa, Urama & Ezepue, 2016).

According to Alagidede and Panagiotidis (2009), with the rising level of globalisation as well as financial market integration, interest has been directed towards investment in emerging African stock markets because of their low correlations with developed markets, high correlations among themselves and their potential to enhance and encourage optimal portfolio diversification (Harvey, 1995). For example, in 1994, emerging markets recorded the highest profits in U.S. dollars compared to their developed market counterparts, with Kenya (75%), Ghanaian stocks (70%), Zimbabwe (30%) and Egypt (67%) maintaining the lead. By 1995, African stock markets recorded gains of average returns of about 40%, with the stock values of the Nigerian Stock Markets and Côte d'Ivoire registering over a 100% rise in dollars (Alagidede and Panagiotidis, 2009). By 2004, emerging African markets' average returns hit about a 44% increase from the periods before the 2004 Nigerian bank reforms.

Again, most of the research has however been channelled towards underpinning the dynamics of advanced equity markets with little attention paid to the emerging African markets, except for few studies (see, Koutmos 1997). Thus, any investor hoping to maximize opportunities in emerging markets will have a better knowledge of how these markets behave if similar studies as investigated in this research are replicated in those markets, compared to when such research insights were not available (Koutmos, Pericli, and Trigeorgis, 2006). A good example of these markets, which has so far not been characterised in the SSMCD way with a core focus on the banking sector, is the NSM. The NSM with its position and potential in African and World markets has progressed tremendously in recent years, especially after surviving the 2007-2009 financial crisis. Thus, a sound knowledge of its asset price dynamics should capture the attention of both local and foreign investors. It would be valuable to investigate to see if those stylized facts which characterise the behaviour of most of the advanced markets can also be linked to that of the NSM.

More significantly, NSM returns have been discovered to be poorly correlated with the returns of most advanced markets, thereby yielding opportunities for foreign investors to diversify their investments and associated investment risks (Harvey, 1995). Therefore, exploring the dynamics of the NSM becomes imperative for both domestic and foreign investors, with a view to helping them to identify areas of differences and similarities between developed markets and the NSM. As noted earlier, scientific research that systematically investigates the characteristics of asset returns in the NSM is very scant even with the prospective significance of this market to shareholders (Koutmos, Pericli, and Trigeorgis, 2006).

2.7 Summary and Conclusion

This chapter presented a general background for this research, including notes on financial systems, financial markets, the Nigerian capital markets, the NSM, NSE and SEC, the Nigerian banking system, the global financial crisis, the characteristics of the Nigerian banks listed in the NSM, and some justification for our choice to investigate African emerging markets. The notes will inform subsequent discussions of the research findings as relates to the different banks, especially considering the banking services they offer, namely commercial, wholesale, retail, or universal banking.

3 CHAPTER THREE: LITERATURE REVIEW

3.1 Introduction

Levine (1997) suggests that a financial system symbolises a group of institutions, marketplaces and rules facilitating timely allocation of resources through a generally acknowledged medium. Financial markets and financial institutions interact and merge in different forms to represent a nation's financial system. Financial intermediaries interrelate in the financial markets either between themselves or with other public agents, such as the government, regulatory bodies, consumers and non-commercial companies. Moreover, because the stock market connects producers (or firms) and consumers (or investors) together for economic benefits (Stiglitz, 1981a); thus, the developments of financial markets and a stock market in a nation are, according to Chen, Roll and Ross (1986), most critical elements in inducing enhanced investment, which may also be strong contributory factors towards a nation's economic advancement.

Moreover, stock markets are increasingly becoming a significant channel for generating long-term capital (Khambata, 2000). Cho (1986) suggests that credit markets should be complemented by strong asset markets, given the fact that equity finance is in most cases insulated from hostile selection and ethical risks to a similar degree to which liability finance is exposed to in the presence of asymmetric information. Levine and Zervos (1995, 1998) maintain that the two key networks of financial intermediation (banks and the equity market), are expected to supplement each other (Alagidede and Panagiotidis, 2009). Hence, the availability of stock markets would enable risk diversification and increased the capital allocation by the investors to favourable assets with higher returns on their investments.

Further, the efficiency of stock markets is essential for the functioning of the exchange markets, especially when it comes to capital allocation and pricing as well as investment risk diversification. Hence, all available information should be fully incorporated into the prices of assets in an efficient market, (Fama, 1965; 1970). The liquidity that an exchanged market offers investors enables them to sell their assets easily and without delay (Ma, Anderson and Marshall, 2016). Thus, one might argue that enhanced liquidity remains an impressive advantage of investing in equities relative to other investments, which are less liquid, with some firms even actively contributing towards arise in the liquidity by trading in their own share (Simkovic, 2009).

Stock prices, as well as the prices of other assets, have overtime been an essential aspect of the dynamics of economic activity, which may either impact or indicate social mood. According to De Cesari, et al. (2012), an economy in which the equity market seems to be rising is considered as an up-and-coming economy; thereby making the stock market a vital indicator of a nation's economic development and strength.

In financial markets, short-run price changes are influenced by the trading decisions of market participants; hence, shifts in their confidence and preferences generate a feedback effect from such beliefs to the real experience in the market. Research has shown that individual asset prices (such as stock prices) are prone to the influence of both economic news and various unexpected circumstances (Chen, Roll and Ross; 1986; Ezepue and Omar, 2012). Knowing the endogenous features of market dynamics requires a better appreciation of the inherent complex connections between the market participants' beliefs, their actions, and the effects they generate.

Mapping systematic characteristics of stock markets across key market sectors, in a manner akin to meteorological weather maps, will facilitate the detection of early warning signals which relate observed shifts in the beliefs of the market participants to feedback effects from the beliefs and the actual market outcomes, as determined by macroeconomic variables in an economy. This indicates that understanding how bubbles (for example), arise and their negative impact on investment due to associated risk(s) involved when they burst could serve as early warning signals for investors and market participants to check any irrationality in their behaviours with respect to sudden and persistent rise in security prices. Thus, with the obtained results, identifying banks which have previously been exposed to bubble-prone assets would help relevant stakeholders in proper monitoring so that investments are protected from avoidable risks that could in turn impacts negatively on the economy.

Empirical evidence on the stochastic behaviour of stock returns has produced an essential stylised fact - the distribution of stock returns appears to be leptokurtic (Mandelbrot, 1963, Fama, 1965 and Nelson, 1991). This enables us to determine a suitable probability distribution that best captures stock returns dynamics. Further, short-term stock returns exhibit volatility clustering. These processes that have been modelled successfully by autoregressive conditional heteroscedasticity (ARCH)-type models (Engle, 1982; Bollerslev, 1986; Ezepue and Omar, 2012), which shall further be discussed in the subsequent section and implemented in chapter nine of this thesis. Moreover, changes in stock prices tend to be inversely related to changes in volatility (Black, 1976, Christie, 1982, and Bekaert and Wu, 2000). This study will examine to what extent these facts characterise the banking sector of the NSM given that they generally obtain at the overall market level (Ezepue and Omar, 2012; Omar, 2012).

Most of the empirical studies on stylised facts have focused primarily on developed economies and the emerging markets in Asia and Latin America. With regards to African markets, there are only a few studies on the behaviour of stock returns (Omran, 2007; Mecagni and Sourial, 1999; Appiah-Kusi and Menyah, 2003; Smith and Jefferis, 2005; Ezepue and Omar, 2012). That said, related stock market characterisation at deepening these lines of investigation have only been outlined in the recent literature (Ezepue and Solarin, 2009; Ezepue and Omar, 2012).

In this study, we intend to rigorously study the behaviour and performance of the banking stocks within the NSM in relation to the stylised facts of asset returns and four but interdependent financial econometric issues such as Anomalies, Bubbles, Efficiency and Volatility. Interestingly, these issues have been explored in the literature to describe the dynamics of a stock market and stock returns across the developed markets (Islam and Oh 2003; Mills 1999; Cuthbertson & Nitsche, 1996) and few emerging economies (Bhattacharya, Bhattacharya and Guhathakurta, 2018). However, no research had examined the Nigerian market and indeed the Nigerian banking sector, holistically in relation to the issues being investigated in this research before until that of Omar (2012), which was the foundational research for our research, which we seek to extend.

Thus, this research, having identified this gap in knowledge, concentrates on the banking sector of the NSM as an extension of the pioneering study initiated by Omar (2012) at the overall market level. This we believe will not only provide leverage for would-be investors (domestic or foreign) in any sector of interest of the Nigerian market but will also help the market regulators and indeed the government to implement appropriate checks and controls (or reforms when necessary), that would shield the market from avoidable risks that could negatively impact the nation's economy.

3.2 Application of Non-linear Stochastic Time series modelling to Financial Data

This section presents our motivation for our choice of theoretical research domain and research direction. The reasons for the determination of the procedure towards achieving the overall aim and objectives of this study are also highlighted. Finally, drivers for choice of the model to be used in this study are elucidated.

In conventional time series, assumptions of linearity and homogeneity in the series is integral to building relevant models that could be used to describe the behaviour of the series and to make necessary forecasts. However, this is not the same with the returns obtained from speculative prices, as these are known to be uncorrelated and hence white noise.

Modelling and statistical analysis of financial time series analysis and its applications in engineering, physical sciences as well as earth sciences has often been based on the second-order properties of the data as described by their mean and covariance functions, with the assumption that the observations come from a normal distribution. Thus, from a second-order perspective, one might say there is no need for further modelling. On closer inspection however, one observes that the sample autocorrelations of both the absolute and the squares of returns do not seem to disappear no matter the lag size. The slow decay of the autocorrelations is sometimes termed long memory or long-range dependence coming from the volatility sequence of the data (Andersen, et al., 2009). The non-zero autocorrelation in absolute and squared log returns represents another vital stylized fact, which is an indicator of the existence of serial correlation beyond the level found in the log returns, Sewell (2011). This calls for the building of nonlinear models that will reproduce these facts.

Another common feature of the returns has been that most of them are concentrated in a small neighbourhood of zero, giving rise to the leptokurtic shape of their marginal distributions, which is an indication that these distributions cannot be well described by a normal distribution. Mandelbrot (1963), Fama (1965) and Mandelbrot and Taylor (1967) focused on this stylised fact of the returns. Since in the 60s quite a few heavy-tailed distributions were identified, stable Paretian distributions were assigned to return data. Although this modelling technique generated divergent views, given the fact that the non-normal stable distributions are characterised by infinite variance, the consolation remains that there is a consensus on the fact that financial series possess unusual heavy tails, even though the degree of heaviness remains a subject of concern.

The analyses conducted on heavy-tailed distributions include risk calculations and associated methods using tail indices (Hols & DeVries, 1991; Koedijk and Kool, 1992; and Loretan & Phillips, 1994). Raheem and Ezepue (2018) note that it is important to examine in future how all such stylized facts of bank returns are linked to the business models of different banks, and hence their relative financial performance in different sub-periods of the study.

This line of work belongs to comprehensive studies in bank financial management and empirical finance support for such management, which are outside the scope of this research. Indeed, Raheem & Ezepue (2018), which is derived from this research, further suggest the need for: 1) more detailed analysis of returns distributions of Nigerian banks, using suitable univariate and multivariate probability models and their mixtures, linked to tail behaviours of returns distributions and risk factors; 2) exploring the implications of these results for portfolio and risk management involving bank assets; and 3) linking the results to further empirical analyses of the banking sector (efficiency, volatility, bubbles, anomalies, valuation, and predictability), for different periods of bank and financial reforms and global financial crisis.

Two other features commonly exhibited by stock prices are both time-variation and non-stationarity (Fasen, 2013 and Andersen et al., 2009). By non-stationarity we mean inability of the returns to revert to a common and central value called the mean of the series, while time-varying volatility represents a tendency for values of the same magnitude to follow one another. For ease of modelling, therefore, there is a need to seek a transformation that would enable modelling of the return series by a stationary process. Thus, the thought of representing speculative prices by return and log-return came up among the analysts. One could contend that the return series obtained from the corresponding prices are white noise, which is believed to be better in modelling; however, there remains the time-varying volatility issue to be addressed.

Engle (1982) was the foremost researcher who published a paper to consider a parametric model for volatility to account for the time-varying component of the return series and named the model the Autoregressive Conditionally Heteroskedastic (ARCH) model. ARCH models are an improvement of linear time series models, used to model time-varying squared volatility as a moving average of past squared returns. This technique exploits the stationarity of returns, for which the volatility is taken as a conditionally time-varying (heteroskedastic) series based on the past observations. This innovation was welcomed by the financial time series community, given the wide acceptability of the conditional method to modelling volatility as the standard approach to solving the problem of the time-varying properties of the returns (or volatility).

Given the emphasis of the research on stylised facts and stock market characteristics of bank returns in the NSM, the following sections of the general literature review in this chapter will be focused on the stylized facts characterising the behaviour of stock returns and other market issues, including anomalies, bubbles, efficiency and volatility; which help to determine the level of risks involved in stocks and in particular the Nigerian banks' stocks. Additional references are included for each market characteristic in the specific chapters of the thesis which investigates the characteristic.

3.3 Stylised Facts of the Emerging Markets

Some distinct features were identified with the stock returns behaviour of emerging markets. These characteristics amongst others include high volatility, little or no correlation with developed and between emerging markets, long-term high yields in returns, higher predictability potentials than could be recorded with the developed markets. This is because; emerging markets are prone to the influence of external shocks such as political instability, changing economic and fiscal policy or the exchange rate (Bekaert et al., 1998). Bekaert and Harvey (1997, 2017) examine the causes of varying volatility across emerging markets, especially as concerns the timing of reforms of the asset market. They observe that capital market liberalisation, which is always responsible for a high correlation between local market returns and the developed market, has been unable to trigger local market volatility.

Meanwhile, Bekaert, et al., (1997a, b and 1998) observed the following fundamental features, which are peculiar to the emerging markets' returns:

- Lower Market Capitalisation increases the chance of positive skewness in the returns;
- Skewness is positively correlated with the inflation rate, a book-to-price and beta, which is a coeffient of the Capital Asset Pricing Model (CAPM) and measures the relative sensitivity of an asset to market movements;
- The negative correlation between skewness and GDP growth rate;
- The negative correlation between Kurtosis, Market Capitalisation and GDP growth.

3.4 Stylised Facts of Asset Returns

In this section, the statistical properties of asset returns, otherwise known as stylised facts, and the literature on some selected works by other researchers across different disciplines, especially the ones found relevant to our study, are presented.

Several studies have been directed towards investigating the dynamic nature of major stock markets (at both the developed and emerging levels), with the discovery of quite a significant number of stylised facts. A stylised fact is a statistical property that is expected to be found in any series of observed stock prices or returns across many financial assets and markets (Taylor, 2011; Cont, 2001). These features, some of which have briefly been mentioned in previous studies, to be discussed subsequently, include: lack of autocorrelations in returns, high probabilities for extreme events (or thick tails of the distribution: "heavy tails”), asymmetry, volatility clustering, positive autocorrelation in squared returns and variance, leverage, correlation dependence time (Panait and Constantinescu, 2012), aggregation Gaussianity and slow decay of autocorrelation in absolute returns, and volume/volatility correlation (Cont, 2001).

Studies suggest that one of the primary purposes for modelling stock market data lies in determining the nature of the unobservable data generating process (DGP) that determines observed stock prices. The process of examining how fit this DGP is to the data leads to identifying the “stylised facts” of stock returns (Thompson, 2011). Hence, if a model is to approximate the behaviour of asset returns, then it should capture these facts. In other words, the stylised facts underpin the choice of appropriate statistical models for later modelling of the market features such as volatility, for example use of normal- or non-normal-based error distributions.

Below is the review of current literature on the presence and identification of some of these stylized facts.

3.4.1 Distribution of Asset Returns Is Leptokurtic and Non-Normal

Studies suggest that the empirical distribution of asset returns seems to be leptokurtic, thereby making them able to be described only via non-normal distributions; see Mandelbrot (1963), Fama (1965), Nelson (1991), and Booth et al., (1992, 2000), Koutmos, Pericli and Trigeorgis (2006).Mainly, the empirical distributions of most daily stock return series tend to be leptokurtic and more skewed than would be a normal distribution (Pagan, 1996; Taylor, 2011; R Cont, 2001). That is, daily returns series are known for having heavy tails and peaked centres. However, non-normality appears to be less pronounced in the distributions of monthly returns series.

Fama (1965a) examined the daily returns data spanning from 1957 to 1962 on 30 stocks obtained from DJIA (Dow Jones Industrial Average) and found that the returns’ distribution for each stock is leptokurtic. He then concluded by saying that daily stock returns are undoubtedly leptokurtic, with the same view shared by Pagan (1996) and Cont (2001). The same findings were made by Ding, et al. (1993) and Ding and Granger (1996) using a more extended period data set (63 years and 22 years respectively). It is believed that the distributions of stock returns are leptokurtic and skewed, yet there is no unanimity regarding the best stochastic return generating model to capture these empirical characteristics of asset returns (Corhay and Touran, 1994).

Other researchers who obtained similar conclusions about returns data having higher kurtoses compared to the normal distribution include Taylor (2011), Karoglou (2010), Andersen et al. (2001), and Yu (2002). The same observations were made by Aggarwal et al. (1999) and Youwei et al. (2010) in their works on daily index returns from emerging markets of Latin America, Asia and Africa, respectively. It was however found that the degree of leptokurtosis in monthly returns series is weaker than those of daily returns (Richardson and Smith, 1993; Cont, 2001; Taylor, 2007).

Many studies have observed that the distribution of asset returns is non-normal. For example, Praetz (1972*)* examined weekly returns of 17 share-price index series of the Sydney Stock Exchange and arrived at the same conclusion as Officer (1972) that the stock returns distribution is non-normal. Laopodis (1997) finds with the weekly stock returns of the Athens Stock Exchange, an emerging capital market, a lack of independence and normal distribution. DE Santis and İmrohoroğlu (1997) observed weekly stock returns of some selected emerging markets to have higher kurtosis compared to the developed markets.

The daily stock index of Nigerian stock market (NSM) was suspected to be highly skewed and non-normally distributed (Ayadi, Blenman and Obi, 1998). Bekaert, Erb, Harvey and Viskonto (1998) report that the monthly returns of most emerging markets are highly skewed and leptokurtic; and are far from being normally distributed. Ozer (2001) finds the daily and weekly returns of the ISE to be skewed and leptokurtic and thus non-normally distributed. (Taylor, 2007) also suggested that a suitable probability distribution for daily returns should be leptokurtic and symmetric.

According to Taylor (2007), the following distributions have been suggested suitable for describing behaviour of asset returns: (i.) mixture of normal distributions with the assumption of constant mean and variance by Praetz (1972), Clark (1973) and some other studies; (ii.) lognormal-normal distribution by Clark (1973); (iii.) Conditional t-distributions for the ARCH framework by Bollerslev (1987); (iv.) Normal Inverse Gamma (NIG) distribution by Madan and Seneta, 1990 and Barndorff-Nielsen and Shepard, 2001; and (v) Generalized error distribution (GED) by Nelson (1991)

Meanwhile, despite the general believe that the distributions of stock returns are leptokurtic and skewed, no unanimity regarding the best stochastic return generating model to capture the empirical characteristics of the asset returns (Corhay and Tourani, 1994). However, considering the above, we shall be exploring alternatives to the normal distribution, such as: Student-t and generalised error distributions (GED), which are frequently applied in the literature to describe the behaviour of asset returns across different markets.

3.4.2 Absence of Autocorrelation in Daily Returns

Lack of serial correlation or linear autocorrelation has remained a significant observed characteristic of asset returns across many markets in both developed and developing economies, except for a few markets where a certain degree of linear autocorrelations has been recorded with asset log returns. For example, it is often claimed that daily returns for liquid stocks exhibit non-significant (linear) autocorrelation at various lags (Pagan, 1996; Taylor, 2005; Ding et al, 1993; Cont, 2001). Taylor (2011) and Cont. (2001) argued that intra-daily returns of liquid stocks for periods more than 20 minutes do not show signs of significant autocorrelation, whereas those of shorter periods reflected negative autocorrelation. Such a conclusion, according to them, might be attributed to market microstructure, which refers to a combination of management, regulation and macroeconomic influences that determine how a market generally performs, thus making it difficult to specify these effects, as market structure analysis is outside the scope of this thesis.

However, Aggarwal et al. (1999) observed that daily returns of some selected markets of Latin America and Asia with illiquid stocks from 1985 to 1995 are significantly auto-correlated. Fama (1970) observed that out of 30 daily stocks of the Dow Jones Industrial Average (DJIA) examined, 22 were positively as well as serially correlated. Lo and MacKinlay (1988) noted that both the weekly and monthly stock indices exhibited significant positive serial correlation, with the weekly individual asset returns rather containing negative serial correlation. Ball and Kothari (1989) discovered negative serial correlation in 5-year asset returns. Lo and MacKinlay (1990) noticed that while the weekly returns of the individual stocks were negatively and serially correlated, the weekly portfolio returns were rather positively auto-correlated.

According to Jegadeesh (1990), there was highly significant negative autocorrelation in the monthly returns of individual stock returns and strong positive autocorrelation in the twelve months returns of any of the market whose asset price make up the Center for Research in

Security Prices (CRSP)'s market index used in the study. Zhou (1996) observed that high frequency returns were strongly negatively correlated at lag 1. Longin (1996) found positive autocorrelation in the daily stock index. According to Campbell et al. (1999), while the autocorrelation of daily, weekly and monthly index returns is positive, the weekly stock returns are weakly negatively correlated.

Ahn et al. (2002) observed that while daily indices contain positive autocorrelation, the futures could not provide any significant autocorrelation. In a year of portfolio stock returns, there was negative autocorrelation (Lewellen, 2002). Bianco and Reno (2006) observed negative autocorrelation in the returns of the Italian stock index futures in less than 20minutes periods. Lim et al. (2008) found that all the returns of the 10 emerging Asian stock markets exhibit no significant autocorrelation.

The daily returns on 4 US stock market indices showed evidence of mean reversion, which implies a tendency for the indices to return to a generalised mean over time, considering absence of long-term upward or downward trends in the trajectory of the indices (Serletis and Rossenberg, 2009). From this, it could be inferred that weekly and monthly returns are weakly negatively correlated while daily, weekly and monthly index returns are positively correlated. Also, high-frequency market returns exhibit negative autocorrelation, while the absolute and squared returns are always positive, significant and slow-decaying, with absolute returns generally displaying a higher autocorrelation compared to the squared returns (Taylor's effect) (Sewell, 2011; Dosi and Staccioli; 2015).

Fama (1965a) stated that when all publicly available information is immediately and completely incorporated into the stock prices, it is expected that insignificant autocorrelation in the daily returns would be observed. Slow response in prices to the information would lead to positive autocorrelation, whereas the quick response to the newly arrived information is responsible for negative autocorrelation.

Meanwhile, a lack of significant autocorrelations in returns indicates some scientific support for 'random walk' models of prices wherein the returns are taken to be independent random variables. It is good to understand that failure of the returns to be auto correlated is not an indication for the series to be independent because independence means that any higher order form (absolute and squared returns) of returns will contain no autocorrelation, which obviously is not the case as far as the return's series, is concerned (Taylor, 2007 and Thompson, 2011)

Thus, with respect to this property of lack of significant autocorrelations in returns as established in the existing literature, we shall be examining presence of significant autocorrelation in the daily return of the respective bank in NSM for us to have a clearer picture of the behaviour of the stock prices of the banking industry with the periods covered by this study.

3.4.3 Presence of Autocorrelation in the Squared and Absolute Returns: Volatility Clustering and Long Memory

Studies such as Cont (2001), Taylor (2005) and Thompson (2011) have shown that significant linear autocorrelation in the daily log returns does not make the returns independent; the reason for this is that both absolute and squared returns are significantly auto-correlated in most daily stock returns. Also, the presence of significant autocorrelation in the absolute and squared returns poses a challenge to the validity of random walk theory, which supposes statistical independence in the return series.

Ding et al. (1993) found positive autocorrelation in the returns of both squared and absolute returns for lags running to thousands – a situation termed "Volatility clustering". Specifically, short-term stock returns (e.g. intra-daily, daily and weekly) exhibit volatility clustering, which, according to Mandelbrot (1963, p. 418), implies that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". This feature of volatility clustering has successfully been modelled with ARCH-type models (see: Engle, 1982; Bollerslev et al., 1994; Koutmos and Knif, 2002; Moschini and Myers, 2002; Scruggs and Glabanidis, 2003; and Zhou, 2002).

Apart from significant autocorrelations, also observed is the slow decay of autocorrelations in absolute and squared returns over time. Studies such as: those of Ding and Granger (1996), Taylor (2005), Cont (2001), Pagan (1996) and McMillan and Ruiz (2009) have also established that some stocks on NYSE, LSE and Nikkei exchange markets show evidence of significant and slow decay in autocorrelations of both absolute and squared returns. Aggarwal et al. (1999) also found the evidence of such persistence in the squared returns for index returns of some selected emerging markets.

Though the level of decay of the autocorrelation of absolute and squared daily stock returns is well established (Thompson, 2011), the implications of this on the stock returns series create divergent views within the research community (see: Diebold and Inoue, 2001; Granger and

Hyung, 2004; Banerjee and Urga, 2005). While some believe that the slow decay of the autocorrelation of absolute returns could be consistent with stock returns following a stationary process that exhibits long memory, others think the non-stationarities in the process for stock returns could be responsible for a slowly decaying autocorrelation function of absolute returns. Thus, non-stationarities imply the presence of spikes and sustained upward and downward trends in a time series, with lack of mean reversion and volatility clustering. Hence, such fluctuations will inherently decay slowly.

3.4.4 The Taylor Effect

Another fact of interest as observed by Granger and Ding (1993) which has subsequently enjoyed wider acceptability and been classified as a stylised fact (Cont, 2001; Taylor, 2005) is the "Taylor effect" based on the findings made by Taylor (2007). The Taylor effect is a term used to represent the circumstance when the autocorrelation of the absolute returns tends to be higher than the autocorrelation of the squared returns, indicating there is higher tendency for predicting absolute returns compared to squared returns (Thompson, 2011). More generally, the Taylor effect comes into play when the k^{th}-order autocorrelation of absolute returns$|r_t|^k$ is maximised at the point when the k-exponent is equal to 1.

According to Granger and Ding (1993), the autocorrelation of $|r_t|^k$reachesits maximum for values of k falling between 0.75 and 1.25. Ding, Granger, and Engle (1993) examined the returns of the S&P500 index and observed that the sample autocorrelations of $|r_t|^k$ for various values of 'k' tend to be highest when$k = 1$. Ding and Granger (1996) also noticed the presence of this effect in several daily and intra-daily returns and found that it was more pronounced in the intra-daily series than either daily or higher frequency data.

3.4.5 The Leverage Effect

Black (1976) first identified the leverage effect, which is defined as the tendency for most measures of the volatility of returns such as the sample variance, absolute or squared returns tend to increase when asset price decreases (Cont, 2001), thereby implying that stock price changes are often negatively correlated with changes in volatility (see Black, 1976; Christie, 1982; and Bekaert and Wu, 2000). More precisely, it means that negative returns (falls in price) are trailed by higher volatility compared to positive returns (increases in prices) of the same magnitude. In some contexts, this is referred to as the "Asymmetric effect".

First-order autocorrelation of stock index returns is also observed to be negatively correlated with high volatility (e.g. LeBaron, 1992; Campbell et al., 1993 and Sentana and Wadhwani, 1992). One might then conclude that both positive and negative returns impose an "Asymmetric effect" on most measures of volatility (Taylor, 2007). Negative returns, which indicate reductions in stock prices, tend to be correlated with an increase in volatility measures, whereas positive returns, which reveal a rise in stock prices, tend to be correlated with declines in volatility (Thompson, 2011). It is said that the lack of identification or investigation of the presence of asymmetry in volatility results in an underestimation of the Value-at-Risk, which is a vital tool in portfolio selection and risk management.

It is also maintained that whenever there is a rise in the leverage effect, the riskiness of assets of a firm increase, thereby leading to higher volatility. Black (1976) and Christie (1982) note that there is a likelihood of a reduction in equity value, accompanied by a rise in debt-to-equity ratio whenever stock prices experience negative shock. Consequently, investors are thrown into undue panic over the future value of their securities. Many models for stock returns have been designed to handle the asymmetric effect in volatility (Taylor, 2005).

Meanwhile, according to Oskooe, and Shamsavari, (2011), authors such as Rousan and Al-Khouri (2005), Brooks (1996), Mun, Sundaram and Yin (2008), Bahadur (2008), Alagidede and Panagiotidis (2009), Jayasuriya et al. (2009) and Cheng et al. (2010) argue that there is no evidence to conclude that the asymmetric effect is more pronounced in the emerging stock markets than in the developed markets. Other notable works in this direction include: Bollerslev, Chou and Kroner (1992), Brock and Lima (1995), Campbell, Lo and McKinlay (1997), Gourieroux and Jasiak (2001), Maddala and Rao (1997), Pagan (1997) and Shephard (1996).

3.4.6 Implications of the Stylized Facts for this study

This research will, among other issues, examine whether the stylised facts found in the developed equity markets also characterise the behaviour of returns in the NSM, particularly the banking stock returns.

Given the findings as established in the literature with respect to the observed concepts, we hope to examine the behaviour of Nigerian banks' stock prices using daily, weekly and monthly returns for each bank to ascertain to what extent the returns reflect the listed stylized facts that have been noted to characterise stock behaviour across various markets. The outcome of this will be compared with the findings of other researchers and will also help to indicate appropriate models to describe the risk associated with the returns across different periods of interest in this study. Understanding these features in this study data would be beneficial for the following reasons: (i.) To see how well the data behave as expected of financial data; (ii.) To be able to understand our data and identify appropriate model(s) that best fit for describing the data behaviour; (iii.) To see how many of the empirical finance issues such as: anomalies, bubbles, efficiency and volatility due to trading activities in the market, are reflected in the data. For instance, investigating efficiency helps us to understand the nature of the market and how well the trading information is reflected in the prices. Knowledge of these stylized facts will guide investors and investment managers on how best to optimize their investments or create optimal portfolios. It will also help the regulator to understand the behaviour of the markets at different periods and in different scenarios which may impact the nation's economy so that the right measures can be instituted to ensure market stability.

3.5 The Vital Stock Market Characteristics

This section briefly presents the six empirical financial issues influencing the behaviour of asset prices in the financial markets. The literature on each of these issues and findings from the relevant works to our study that are previously investigated across different markets of both developed and developing economies are examined.

There is a dense literature on the four stock market characteristics studied in empirical finance, with most of the studies conducted in developed markets and a few in emerging markets. These characteristics include Efficiency, Anomaly, Bubbles, and Volatility. We discuss them briefly below and indicate some selected literature relevant to the objectives of this research. Following this, we provide further details on the characteristics in subsequent chapters of this thesis.

3.5.1 Market Efficiency

In early 2000, a significant number of studies examined how efficient financial markets are with regards to the flow of relevant information into the markets. According to Taylor (2007), the efficient market hypothesis (EMH) is conceptualized in line with the theory of random walks first introduced by a French broker, Jules Regnault in 1863 and French mathematician Louis Bachelier in 1900. The concept was, however, abandoned because it was initially seen as being irrelevant to the theory of financial markets, since efforts at establishing random walks in financial data proved unsuccessful before the mid-20th century. In financial econometrics, efficient-market hypothesis (EMH) is a theory stating that asset prices are full reflection of all available market information; indicating that it is impossible to beat the market by any participant using risk-adjusted method (Fama, 1965). This is because, according to Delcey (2018), changes in price not only that they are nearly random in the financial markets, prices are reflection of economic fundamentals.

An efficient weak-form market as defined by Fama (1965) is a market where a good number of market makers (or participants) meet to trade in assets with one another, such that the market forces would be the determinant of the future market values[7] of their assets, because the latest and most vital information is easily accessible to every participant at the same time. The act of competition is expected to enable the real price of each asset to reflect the impact of information from historic and current events as well as the anticipated happenings, which implies that an efficient market paves the way for the actual security prices to serve as a good estimate of the securities' intrinsic value. According to Stglitz (1981b), "if markets were perfectly efficient in transmitting information from the informed to the uninformed, informed individuals would obtain no return on their investment in information; thus, the only information which can, in equilibrium, be efficiently transmitted is costless information". The implication of this statement is that for a market to be regarded as efficient, well conveyed information should instantly and fully be incorporated into the market prices of the securities.

According to Fama (1965, 1970), a stock market is said to be efficient subject to how the market transmits information to the market participants. He refined and extended the EMH by classifying financial markets into three forms of market efficiency as presented below:

a. **Weak-form**: current stock prices reflect historical prices only, meaning that it is not possible to earn superior risk-adjusted profits which are based on past prices (Shleifer, 2000). By this definition, it means that past data on stock prices are of no use in predicting either current or future stock returns. This leads to the random walk hypothesis (RWH) (Dupernex, 2007).
b. **Semi-strong form**: current stock prices reflect historical prices and all publicly available information. Any price anomalies are quickly detected, and the stock market adjusts. Hence, only traders with additional inside information could earn excess profit.
c. **Strong form**: current prices reflect all available information, be it private or public.

[7]**Market value** is the **value** of an **asset** as currently priced in the marketplace

Further, when a market is efficient, the activities of the contending participants are expected to force real asset prices to vary arbitrarily about their fundamental values. Thus, prices often totally incorporate the available information with little or no room for anyone to take the market by surprise based on the information through trading (Lo and MacKinlay, 2000). According to Allen, Brealey and Myers (2011), an **efficient market** is a market where it is not possible to earn excess returns higher than the market returns. This results in a random walk, wherein the more efficient a stock market is, the return series becomes more random because the trading information not only is instantaneously reflected in the current prices and available to every participant, exploiting such information to earn abnormal gains by any market participant becomes almost impossible (Dupernex, 2007).

A market where successive returns on every asset are independent is termed a random walk market (Kendal and Hill, 1953; Fama, 1965), meaning that the past movement or trend of a stock return or market cannot be used in predicting its future movement. This is an indication that series of changes in stock prices lack memory, meaning that the historic price level is not useful in predicting the current stock price. This assumption of independence of the random walk remains intact so long as information on the previous returns' behaviour could not in any way be exploited to earn excessive profit.

The notion of stock prices following a random walk path could be linked to the EMH (Poshakwale, 1996); which according to Samuelson (1965; 1970) is the proposition that stock markets are efficient, with the prices of stocks reflecting their true economic value. Malkiel, (1992) also interprets the concept of EMH in finance to mean that asset prices are fair, that every relevant market information is available and is spontaneously incorporated into the prices; and that the traders are rational; indicating that, the price $P_t$incorporates all relevant market information historically, up to time t, and no trader has comparative privileges in acquiring information over the other. The phenomenon indicates that price change (or the return) is *only* subject to the news arrival from time, $t-1$ to t,thereby, giving no room for arbitrage opportunities among the participants, particularly in earning investments with return higher than a fair payment for undertaking riskiness of the asset.

Meanwhile, since Bachelier (1900) formulated the random walk hypothesis, research has been geared towards examining whether the theory can be used in describing stock price distribution and to see if the return series are serially correlated. Where stock prices have independent increments, a mathematical concept of prices was developed for this case and applied to the

French Bond Market, with the discovery that asset returns follow the random walk. Most of the developed equity markets, especially the New York stock exchange (NYSE) and the London stock exchange (LSE), have shown evidence of being efficient at least in the weak-form. US stock markets have widely been studied in the past.

Cootner (1962) focused on the weekly returns of 40 stocks drawn from the New York Stock Exchange and found a small level of dependence. Fama (1970) examined the daily returns of 30 common stocks drawn from the Dow Jones Industrial Average (DJIA) and applied both auto-correlation coefficients and run tests. He found evidence for presence of positive serial correlation in the returns. Solnik (1973) in his research to determine if the samples of 234 stock returns drawn from across the 8 prominent European stock markets - French, German, British, Italian, Dutch, Belgium, Swiss, and Swedish - follow a random walk theory, confirmed that the European stock returns deviate from random walk theory more than American stocks. Lo and Mackinlay (1988), however, rejected the random walk hypothesis using a variance-based test for weekly stock returns calculated from the Cyprus daily stock (CRSP) returns indices.

In an efficient market the price instantly adjusts to its new equilibrium level. If the market is inefficient, the market may underreact or overreact to new information. If there is under-reaction, the price adjustment is gradual; whereas, if there is an overreaction, the market price overshoots the new equilibrium value (see Figure3.1 below). If the market is inefficient, then between the time of the news revelation and the adjustment to the new equilibrium value, informed investors would be able to profit at the expense of less sophisticated investors.

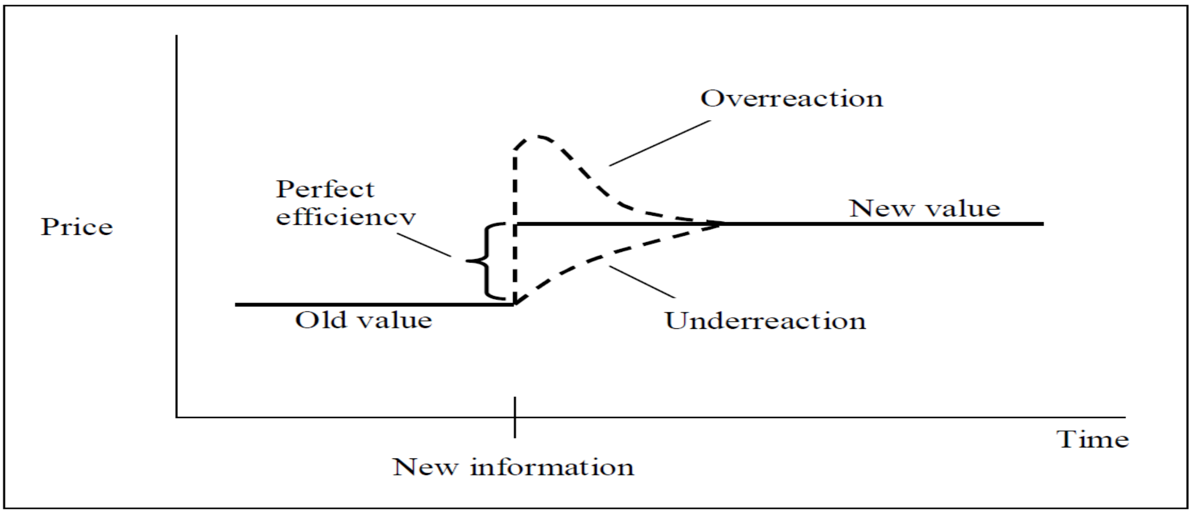

Figure 3.1: Market Reaction to New Information. Source: Darškuvienė (2010)

From the Figure above, one could suggest that prices could adjust to the unexpected news in three possible ways as indicated below:

a. An efficient Market Reaction, whereby the price instantaneously adjusts to the new information path;
b. Under-reaction: in this context, the price partially adjusts to the new information;
c. Over-reaction and Correction: in this circumstance, the price over-adjusts or reacts too fast to the new information, but eventually drops to the appropriate price level.

Brown and Easton (1989) establish the presence of weak-form market efficiency in the London Stock Exchange market using 3 per cent consoles obtained from the daily closing prices of stocks running from1821 to 1860. Comparing their results with the same tests run on data obtained from the present-day markets, they found that the outcomes are the same. This shows that weak-form market efficiency is an enduring feature of markets, not limited to particular periods in the past.

Sharma and Kennedy (1977) show that the developed equity markets are efficient, well organised and consistent with the random walk hypothesis (RWH). Poterba and Summers (1988) carried out studies on the transitory components of equal-weighted and value weighted NYSE returns, with data covering 59 years, and found that the stock returns exhibited positive serial correlation over a short horizon, and negative correlation over the longer time.

Alparslan (1989*)* adopts two forms of weak-form efficiency tests such as the tests of independence versus autocorrelation and runs tests, and filter rules (trading rules) tests, to investigate weekly adjusted price data from the Istanbul Stock Market. Alparslan (1989*)* argues that both autocorrelation and runs tests jointly support weak-form efficiency, whereas the filter tests reject the efficiency of Istanbul Stock exchange (ISE), suggesting that some investors could take advantage of the market to earn an excess gain on some stocks. Findings of Unal (1992) from daily adjusted closing prices of 20 major securities also support that the ISE is not weakly efficient by adopting the same methods used by Alparslan (1989). Huang (1995) examined whether the Asian stock markets follow random walk theory with the use of weekly stock returns obtained from the Morgan Stanley Stock Index Database. The findings show that out of all examined markets, only in the Korean and Malaysian stock markets is the random walk hypothesis (RWH) is rejected at different holding periods.

Further, the RWH was equally not supported by the Hong Kong, Singapore and Thailand markets applying variance ratio estimator that is heteroscedasticity-consistent. Balaban (1995) uses both parametric and non-parametric tests (Mobarek, Mollah and Bhuyan, 2008) to examine the extent to which the Turkish Stock Market supports the random walk theory in daily, weekly and monthly returns data. It is found that while daily and weekly series reject random walk theory, monthly returns agree with the theory. Dockery and Vergari (1997) discovers that the Budapest Stock Exchange (BSE) is weakly efficient. Blasco et al. (1997) found Spanish stock market to be weakly inefficient but attributed this to the theory of time-varying volatilities.

Karemera et al. (1999) present empirical evidence supporting the null hypothesis of weak- form market efficiency in the Turkish stock market. In Africa, Dickinson and Muragu (1994) find that the Nairobi market is weakly efficient. Olowe (1999) and Mikailu & Sanda (2007) report that Nigerian stock market is weakly efficient, but Omar (2012) and Ezepue and Omar (2012) found the Nigerian market to be weakly inefficient.

Mobarek and Keasey (2000) discovered that the daily stock returns for every listed security on the Dhaka Stock Exchange (DSE) reject the random walk theory and that the presence of significant autocorrelations at various lags indicates the inadequacy of the weak-form market efficiency. Ozer (2001) confirms a lack of support for RWH in daily, weekly and monthly stock prices in the ISE, thereby concluding that the market cannot be taken to be efficient. Worthington and Higgs (2003) applied multivariate statistical tests to the daily returns of 16 advanced and 4 emerging markets and found that only 5 out of 16 developed markets supported the RWH.

Gu and Finnerty (2002) in their study of stocks from the Dow-Jones Industrial average (DJIA) for the period 1996 to 1998, reveal that market efficiency is better viewed as a dynamic issue rather than a fixed principle. A market can progressively be efficient with time subject to a few factors which include enhancing trading through a reduction of transaction costs; improvement in the economy due to information technology; the influx of more experienced investors; and general economic prosperity. These factors in turn increase trading volume and reduce volatility rate. With these findings, one could argue that market efficiency is positively correlated with market development.

Ezepue and Omar (2012), using All share index data from 2000 to 2010 established that the Nigerian market is not weakly efficient - a finding which contradicts that of Mikailu and Sanda (2007), reported earlier. Other relevant studies on the concept of market efficiency are: Huang, 1995; Karemera et al., 1999; Olowe, 1999; Shleifer, 2000; and Omar, 2012. These studies portray different degrees of market inefficiency in emerging markets prior to the study conducted by Ezepue and Omar (2012) and Omar (2012). Meanwhile, Schleifer (2000) explores in detail the links between market inefficiency and behavioural finance, which will be useful for interpreting investor behaviours amid market movements in contexts of market inefficiency that suggest existence of valuable market signals.

Given the preceding, it could be argued that majority, i.e. about 60%, of the reviewed studies, especially those dealing with the emerging markets, reject the random walk hypothesis and the weak-form market hypothesis. However, there is no consensus on the efficiency of the Nigerian exchange market; for example, while, Olowe (1999) and Mikailu and Sanda (2007) observe the presence of weak-form market efficiency, Omar (2012) and Omar and Ezepue (2012) have opposing views on the efficiency of the Nigerian market.

Meanwhile, considering the conclusion by Gu and Finnerty (2002) that market efficiency is dynamic and not fixed, meaning that market efficiency varies with time depending on certain factors that could enhance liquidity and reduce risk in the market. Further, Ibikunle, et al. (2016), state that an asset market being efficient over a given period (e.g. daily, weekly, etc.), does not entirely means that such market becomes efficient at all times, even during such indicated period. With these two views, one could argue that efficiency of any market is not perpetuity; indicating that concept of market efficiency is dynamic and not static. Consequently, we believe that examining the efficiency of a sector such as the banking industry within periods which are slightly different from those of the previous studies may help us to further deepen existing findings on the concept of market efficiency. A more detailed account of this and the empirical findings on this concept will be discussed in chapter 6 of this thesis.

In reference to the approaches common to the existing literature for investigating the level of market efficiency across various markets, we hope to test for the weak-form efficiency of the Nigerian banks across the indicated study periods of interest (see **Section 4.3**). Specifically, the relevant popular EMH tests we intend to apply in this study are: The Augmented Dickey fuller (ADF) test of RW or non-stationarity, the Ljung Box Q test for serial correlation, the runs test for randomness and the variance ratio (VR) test. Again, pertinent results of these tests are presented in Chapter 6 of this thesis. The key insight from these results is how efficient the banking sector is in different sub-periods of the research determined by bank reforms and the global financial crisis.

3.5.2 Anomalies

Market anomalies are irregular behaviours or empirical findings characterising stock market trading, contrary to the EMH. They are features that cast doubts on a market's efficiency, leading to mispricing of securities. They are caused by the reactions of traders to the environmental changes or the quest to exploit favourable circumstances to earn an abnormal profit by investors at the expense of the market. Thus, the presence of anomalies indicates a breakdown of market equilibrium and the efficient allocation of resources in any economy (Watanapalachaikul and Islam, 2006)

It is believed that some abnormal returns are noticeable within the stock market (Guo and Wang, 2008). Some studies define anomalies under three main forms, which are: calendar anomalies, fundamental anomalies and technical anomalies (Srinivasan and Kalaivani, 2013).

Calendar anomalies occur because of deviation of returns from normal movement over time. These are categorised into different forms such as: turn-of week/weekend, turn-of-the-month, turn-of-the-year and January effect. Some of the likely causes of this strange behaviour include cash flow adjustments, different tax treatments, the market's failure to adjust to new information in a timely fashion, and restrictions due to investors' behaviours. Ritter (1988) established that the ratio of share purchases to sales of individual investors attains an annual low by December ending and a yearly high in early January (Guo and Wang, 2008). The day-of-the-week effect reveals that Monday generally records the lowest stock returns, with the highest returns on Wednesdays and Fridays (Ross, Westerfield and Jaffe, 2002). The semi-month effect reveals that returns are higher in the first half of the month compared to the second half (Guo and Wang, 2008).

Fundamental anomalies occur when stock prices fail to fully reflect their intrinsic values. For example, the price-earnings ratio (P/E)[8] anomaly, dividend yield[9] anomaly, value versus growth anomaly, low price-sales anomaly and over-reaction anomaly. Value strategies outperform growth stock due to market overreaction, but growth stocks are more influenced by market down movement. A dividend yield anomaly occurs when high dividend yield equities

[8]**Price/earnings ratio (P/E ratio)** is the ratio of the stock market price to the earnings per share. It is also called the earnings multiplier. It is a measure of the price being paid by investors for given earnings of a company and shows the time it would take an investor to recoup his or her investment in a company when profit and distributed income are fixed.
[9]**Dividend yield** is the annual dividend per share as a percentage of the stock's actual price.

outperform the market. Stocks with low price to earnings ratio (P/E) outperform stocks with high P/E (Aronson, 2007).

Technical anomalies deal with the historical prices and trends in the stocks. For example, the momentum effect wherein investors can beat the market by buying past winners and selling past losers. Technical analysis also involves trading strategies such as moving averages and trading breaks (see: Latif et al., 2012).

The Day-of-the-Week Effect

The day-of-the week effect is related to patterns exhibited by the stock market on Friday and Monday trading days. For this, the mean returns on each day of the week vary, and particularly for Mondays, the average returns have been claimed to be negative and lower than any other week day, while being positive on the last trading day of the week before the weekend. This circumstance primarily occurs because firms prefer to release good news as it happens and going forward to the middle of the week so that such news can positively impact on the returns. However, bad news is released at the close of trading on Fridays so that investors will digest the news over the weekend (Holden, Thompson and Ruangrit, 2005). This behaviour is therefore responsible for negative returns on Mondays due to the impact of the bad news that was received over the weekend and whose effect is being adjusted to.

Studies have shown that there is evidence of the presence of the day-of-the-week effect anomaly in stocks across different markets of both developed and developing economies. For example, according to Taylor (2005), there is a high likelihood for stock prices to increase on Fridays and drop on Mondays. It has been generally noted that the day of the week effect is no longer restricted to Mondays and Fridays in the global markets. Cross (1973) found that stock market returns tend to rise on Fridays and fall on Mondays. Gibbons and Hess (1981) and Smirlock and Starks (1986) observed that while the stock returns on Mondays tend to be lower; those on Fridays are most likely to be higher than any other days from the US stock market they studied from 1962 to 1978. Keim and Stambaugh (1984) reported that while Monday's stocks exhibit negative returns, Friday’s yield positive returns from US stock data examined for 55 years.

Jaffe and Westerfield (1985) which studied the daily asset returns from Australia, Canada, Japan, UK and the US, indicates the presence of a weekend effect in each country and moreover observed that Tuesday average returns were lowest for both Japan and Australian markets. Tuesday's average return was also found to be lowest in the Paris market by Solnik and

Bousquet (1990) and Baron (1990). Harris (1986) examined intra-daily and weekly patterns in stock returns and discovered prices drop on Monday mornings, but rise on every other day's mornings. Lakonishok and Smidt (1988) examined daily stock data on the DJIA running for 90 years and established that Mondays' rate of return was significantly negative (Sewell, 2011).

Agrawal and Tandon (1994) examined returns from 18 countries and established that there exist large positive returns on Fridays and Wednesdays in most of the countries, while most countries revealed lower or negative returns on Mondays and Tuesdays (Guo and Wang, 2008). Berument and Kiymaz (2001) observed that the S&P 500 was characterised by day-of-the-week anomalies between 1973 and 1997, with Monday returns found to be the lowest while Wednesday witnessed the highest returns. Mitra and Khan (2014) also found that Monday an average return was lowest in the Indian market between 2001 and 2012.

One might argue that the presence of the day-of-the-week effect in any market means that investors attempt to manipulate the market by purchasing assets on days when there are negative abnormal returns and then resell them on days with positive returns. The main explanation offered by Rahman (2009) for this behaviour is that most firms usually release positive news relating to trading activities as the week progresses; which in turn psychologically encourages investors to exploit such news to make favourable investment decisions. However, by the close of market on Friday, negative news is released for investors to digest over the weekends, thereby negatively impacting on the Monday returns. As for Tuesdays, the most reasonable explanation responsible for negative returns is that the negative shocks of the weekend characterising the US market on Mondays tend to persist and thus spill-over to Tuesday, simply because investors are yet to get over such shocks (Tachiwou, 2010). Thus, the consensus across most advanced markets is that while Mondays and probably Tuesday’s trading tend to end low on average, the other days and especially those of Fridays end high on average (Tachiwou, 2010).

Meanwhile, a few other works present dissenting results against this effect. Abraham and Ikenberry (1994) observed that Friday and Monday's returns were related in that for negative Friday returns, Monday returns were almost 80% of time negative and whenever Friday returns were positive, Monday returns were positive close to 56% of the time (Mittal and Jain, 2009). Connolly (1989) used some US indices from 1963 to 1983, and submitted that the weekend effect was smaller than earlier observed, thereby suggesting that the effect might have disappeared by the mid-1970s. Jaffe, Westerfield and Ma (1989) studied indices from US, UK, Japan, Canada and Australia, and they observed that the low or negative Monday effect has almost disappeared (Guo and Wang, 2008).

However, some of the findings arising from the Spanish markets show no sign of any day-of-the-week effect (see: Santemases, 1986; Pena, 1995; Gardeazabal and Regulez, 2002).

The Month of the Year Effect (or January effect)

The month of the year effect is defined as the existence of patterns in stock returns in a given month of the year. The most mentioned effect is the January effect. The January effect is concerned with the higher average stock returns in January compared to any other months of the year (Guo and Wang, 2008). In a study of six Asian stock markets - Korea, Hong Kong, Taiwan, Philippines, Malaysia and Singapore - using daily stock returns from 1975 to 1987, Ho (1990) observed higher average returns for January and February than for any other months of the year. It was noticed that a monthly effect is present in the US and some other developed countries, while December's returns were generally smaller; January's returns were higher compared to those of other months.

Jaffe and Westerfield (1989**)** examined four markets of Japan, Canada, Australia and the UK, and found that returns for the 1st- half of the month were higher than returns for the 2nd-half for Canada, Australia and the UK. Ziemba (1991) presented evidence of a turn-of-the-month effect in Japan running through the last 5 and first 2 trading days of the month. Kok (2001) in another study on the turn-of-the-month effect in the Asia-Pacific stock markets confirmed that this effect is most prevalent across those stock markets, with half-month effect noticed to be unstable as well as weak within the markets (Mittal and Jain, 2009)**.**

For the January effect, studies such as those of Henke (2001), Nassir and Mohammad (1987), Roll (1983), Chen and Singal (2001), and Cabello and Ortiz (2003) found that average monthly returns in January were higher than the average returns in any other month. This may be attributed to investors in small-cap stocks selling their stocks at the end of the year, in December, only to rebound in early January as they repurchase these stocks to sustain their investment positions (Schwert, 2003), but this is not applicable to the Australia market (Watanapalachaikul and Islam, 2006). If indeed the effect exists, it would suggest that the market lacks efficiency; otherwise, the effect should disappear. Essentially, the effect states that stocks, especially small firms' stocks, tend to rise in the first five days in January (Reinganum, 1983).

However, according to Keim (1983), it is possible that the effect may not have an economic basis, indicating that it may be subject to spurious causes such as the concentration of listings and delisting at year-end and the presence of outliers or errors in the data base. Further, as cited in Rathinasamy and Mantripragada (1996), studies such as: Blume and Stambaugh (1983), Roll (1981), Reinganum (1981, 1983), Ritter (1988) and Stoll and Whaley (1983) believe that the following reasons could be responsible for the January effect: Tax loss selling [10]; High transaction cost, especially with respect to the small firm; buying and selling behaviour of individual investors at the beginning of the year; upward biased estimates of average returns for small firms as a result of daily portfolio rebalancing; and downward biased estimates of beta for small firms. Keim (1983), however, observes that most prominent among these reasons are the tax-loss selling and the information hypotheses. Meanwhile, it is important to emphasise that all these studies and others that have investigated this effect assert that none of the highlighted reasons has any plausible impact or could empirically or theoretically be held responsible for the higher returns in January (see Rathinasamy and Mantripragada, 1996; Keim, 1983; Thaler, 1987).

According to Keim (1983) and Reinganum (1983), most of the abnormal returns earned, especially by small firms, often occur during the first two weeks in January, thereby making higher average returns within these periods in January possible compared to the rest of the days of the month. Such an anomaly is called a turn-of-the-year effect. Abnormal pre-holiday returns were first observed in the US stocks a long time ago. For example, Merrill (1966) found unexplained rises in the DJIA on the trading day preceding the holidays for the period from 1897 to 1965. Also, Fosback (1976) observed high pre-holiday returns in the S&P 500 index.
Considering the above, one could suggest that the January and weekend effects are more common in small-cap stocks, compared to big-cap stocks (Kim and Park, 1994) and that the mean returns are different on the day before a holiday and the day after; this is attributed to the fact that higher than average returns occur before a holiday, because of increased activity and lower returns after the holiday. Further evidence for this effect can be found in Pettengill (1989), Ariel (1990) and Vergin and McGinnis (1999).

The Holiday Effect

The holiday effect is the circumstance whereby the market performs well in terms of increase in sales and profitability on a day prior to a holiday. Fields (1934) discovered that the DJIA exhibited a substantial level of improvements just a day before a holiday start. Lakonishok and Smidt (1988) observed that the rate of returns before the holiday was 23 times higher than on

[10]. Sales of securities with declining values typically at the end of a calendar year in order to use the realised losses in reducing taxable incomes

any other day, with holiday returns accounting for nearly 50% of the total rise in price for the DJIA. Ariel (1990) established that the average rate of returns on days before holidays is often about 9 to 14 times higher than that realised on days far from holidays.

Cadsby and Ratner (1992) discovered that while ahead-of-holiday effects were noticed in the US, Canada, Japan, Hong Kong and Australia's market, none were noticed in the UK, Italy, Switzerland, West Germany and France markets. Liano and Lindley (1995) investigated the behaviour of daily returns on the daily value-weighted and the equally-weighted return indices of over-the-counter (OTC) stocks for the period ranging from1973 to 1989 and found evidence of unprecedented high and low returns on pre-holiday and post-holiday trading days respectively.

Further, according to Kim and Park (1994), there was evidence of holiday effects in three major stock of USA: NYSE, AMEX and NASDAQ, the UK and Japan. Arsad and Coutts (1997) confirmed the presence of a holiday effect in the FT 30 index. Brockman and Michayluk (1998) examined the holiday effect in securities traded on the NYSE, AMEX and NASDAQ exchanges for 1987–93, finding that pre-holiday returns are substantially higher than those of other days without a holiday. Vergin and McGinnis (1999) suggested that for ten years, i.e. between 1987 and 1996, holiday effects disappeared for large companies, and diminished significantly for small firms.

A study by Meneu and Pardo (2004) confirmed the presence of a pre-holiday effect in the frequently traded stocks of the Spanish exchange. Keef and Roush (2005) found that there was a substantial pre-holiday effect in the S& P 500 investigated until 1987, but that this vanished after1987. McGuinness (2005) observed a strong pre-holiday effect in Hong Kong returns. Chong et al. (2005) investigated the pre-holiday effect in the US, UK and Hong Kong markets. Their findings showed evidence of a decline in returns across the three markets, with only the US market where the pre-holiday effect anomaly was slightly high. Other studies where the preholiday effect was significant to include those of Sifeng (2008), Marrett and Worthington (2009) and Dzhabarov and Ziemba (2010) (see also Sewell, 2011).

Turn-of the-Year effect

This states that the returns, especially for small firms, are higher in the first two weeks of January than those of other days of the month. Meanwhile, for the month effects, where the average returns are believed to vary depending on the month, some of the following empirical studies (Thaler, 1987; Mills and Coutts, 1995; Cheung and Coutts, 1999) have established that average return in January is often higher than in any other months. The justification for this is traced to the practice of payment of tax bills in the US every December, after which the left-over funds are planned for investment in January (Holden, Thompson and Ruangrit, 2005).

Further, according to Rozeff and Kinney (1976), in the US, ‘January marks the beginning and ending of several potentially important financial and informational events. It is the month marking the start of the tax year for investors, and the beginning of the tax and accounting years for most firms.’ Also, it is the period when the preliminary announcements of the previous calendar year’s accounting earnings are made, thereby making January the month of high expectations, full of uncertainties and anticipation as a result of the impending release of important information (Keim, 1983).

Traditionally in Nigeria, the yearly national budget is read on the 1st of January, and this has a lot of implications for investment/trading activities, such that most investors hold their funds to observe relevant economic indices contained in the budget and how favourable the government policies for the New Year will be for their investment. Moreover, the first two weeks of January in Nigeria is when many investors try to purchase more assets for them to resell later when the market value of such assets will have risen. Thus, studying this effect would help determine if this behaviour of the Nigerian investors has significantly impacted on the average return for the month over the years.

October-March Seasonality Effect: This states that the average returns from October to March of every year are always higher than those of April to September.

Meanwhile, of the three forms of anomalies highlighted, we hope to concentrate more on the calendar anomalies in this research, which is why we present some of the findings from previous studies in this direction. Our choice of the type anomaly is informed by the nature of the data available at our disposal as well as the general understanding that the presence of calendar anomalies signals the presence or absence of the other two forms of anomalies, and that it is the most researched of the three, based on the existing studies on anomalies. Moreover, in the case of Nigeria, it is common that people rush to the market to buy (or sell or trade) their assets ahead of the weekends, most public holidays or the expectation of changes in the price of goods or change in the political structure of the country. Recall that the presence of anomalies in stock returns indicates the violation of market efficiency and reflects the current state of the entire market.

Thus, from most of the studies reviewed so far, the following observations are made: (1) for the day-of-the-week effect, Monday average returns are found not only to be the lowest, but also mostly negative, whereas Friday average returns are known to be highest and mostly positive; (2) for the month-of-the-year effect, January average returns are reported to be higher than any other month, and generally, the first half month's average returns of any month are found to be higher some studies than those in the second half of the month; (3) as regards the holiday effect, the average rate of returns on a day (or a few days) preceding a holiday have been reported to be higher than the average returns of a day (or a few days) after the holiday across different markets.

Further, most of the studies reported so far focus more on the developed markets, with few focusing on the emerging markets. Also, the literature on anomalies in emerging African market is very scant, particularly the Nigerian market, before Omar (2012) where a search light was shone on the NSM at the overall level. Hence, investigating anomalies in a key sector of the NSM like banks in this study extends the findings of Omar (2012).

To complement the existing studies, therefore, this research investigates whether the returns of individual bank trading in the NSM are characterised by any possible calendar anomalies across the various study periods of interest (see Section 4.1**)**, with further details to be provided in **Chapter 7** of this thesis.

3.5.3 Bubbles

In the equity markets, the systematic deviation of the underlying price of an asset from its fundamental value[11] is known as a speculative price bubble (Garber, 1990; Homm and Breitung, 2012). It is an observed phenomenon whereby equity market prices continuously and significantly rise above the economically justifiable level (fundamental level) for quite a length of time (Anderson, Brooks and Katsaris, 2010). The circumstance often happens when the purpose for holding an asset by investors is to possibly sell it at a higher price to some other investors not minding if such an asset's price exceeds its rational valuation of the securities being traded. Shiller (1989a) suggested that during price bubbles, rises in price lead continuously to higher increases in asset prices such that a point of unsustainability in demand

[11] "The Fundamental value" of an asset is the present value of the stream of cash flows that its holder expects to receive. These cash flows include the series of dividends that the asset is expected to generate and the expected price of the asset when sold (http://www.econlib.org/library/Enc/Bubbles.html). Fundamental value of an asset may also be defined as the price at which investors could buy the asset given that they cannot resell it later. See Camerer (1989) and Brunnermeier (2008) for surveys on bubbles

is reached wherein no further rise is possible, then the bubble bursts and the price drops sharply (Harman and Zuehlke, 2004).

The Figure below present a graphical look at asset bubbles. The peaks in each of the diagrams indicate the collapse (or "burst") of a bubble.

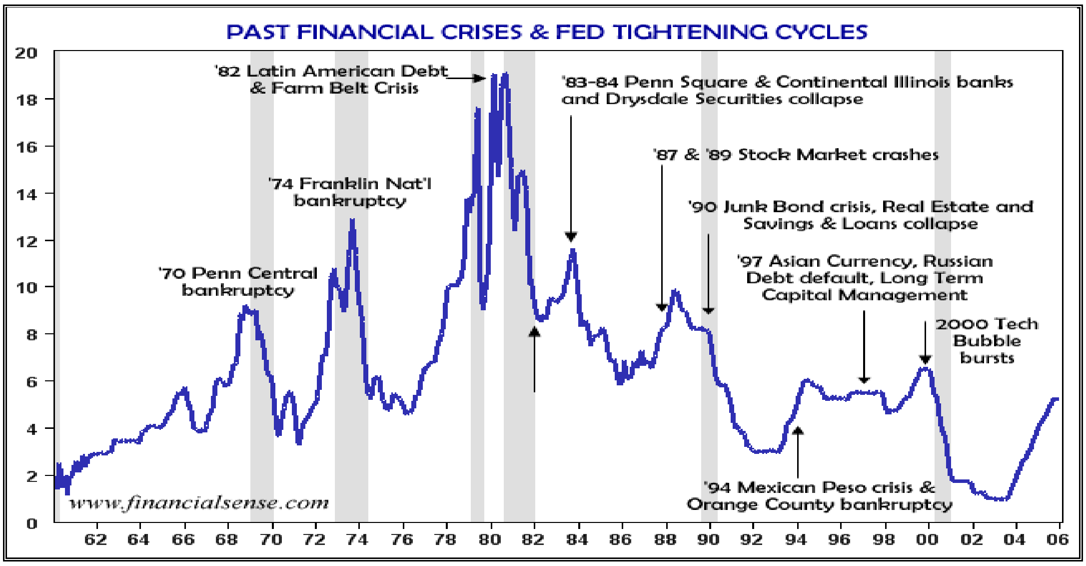

Figure 3.2: Previous Financial Challenges across various Developed Markets

Figure 3.3: Japan's Nikkei Stock Bubble

A bubble (sometimes called a "speculative bubble") in another context is defined as a spike in asset values within a certain sector, asset class, or commodity. Bubbles are generated in economies, equities, stock markets or particular sectors due to changes in the way market participants trade their assets. The situation that is often triggered by exaggerated expectations of impending growth, price appreciation, or other scenarios capable of causing an increase in asset values, thereby driving trading volumes higher. As more investors develop heightened expectations, buyers outnumber sellers, thus causing a shift in the asset price beyond its fundamental value. The fall in price back to its normalized levels makes the bubble complete in a sense. This usually involves a period of steep decline in price during which most investors resort to panic sales of their assets. This may also be referred to as a "price bubble" or "market bubble”.

McQueen and Thorley (1994) note that rational speculative bubbles allow assets prices to diverge away from their fundamental value without assuming irrational investors. In this case, investors, though aware that prices exceed fundamental values, nevertheless believe that there is a likelihood of the bubble expanding and yielding a high return, and furthermore that the probability of a high return cancels out the risk of a crash. However, there are divergent views as to the circumstances that can lead to such episodes. A bubble is destructive because many bad investments are made along the way as it grows over time but when bursts, this results in crisis. The fact that asset price significantly impacts the economy makes it useful to understand any possible phenomenon leading to deviation of asset price from its fundamental value.

One of the circumstances likely to lead to a significant change in the financial system is as seen in the case of Japan in 1980 when a bubble was experienced in the entire economy, which led to the introduction of partial deregulation in banks, or a paradigm shift as experienced during dotcom in the late 90s and early 2000s. During the dotcom boom for instance, people rushed to buy tech stocks at high prices with the expectation of reselling the stocks at higher prices in the future until there was a loss of confidence such that an unexpected crash was witnessed in the market.

Bubbles in stock markets and economies often cause resources to be diverted to areas of rapid growth, and upon the disappearance of such bubbles, resources are moved, thus, leading to price deflation. This indicates that bubbles can only result in a small long-term return on an asset. According to Blanchard and Watson (1982) and Campbell, Lo and MacKinlay (1997) the fundamental price of the asset is obtained from the present value of all expected pay-outs. Martin et al. (2004) provide evidence that increasing rises in an asset price pave the way for bubbles, which is then followed by a substantial fall in the asset price. Diba and Grossman (1988) found evidence against the existence of bubbles in the S&P 500 stocks.

Meanwhile, many studies have attempted to detect bubbles in equity markets. A strand of the literature that regards equity bubbles as deviations of actual prices from their fundamentals develops a variance bounds test to detect the bubbles, e.g. Shiller (1989a) and LeRoy and Porter (1981). However, the variance bounds test relies on a linearity assumption that relates all the observations to the value of prior observations. Gurkaynak (2008) suggests that bubbles demonstrate nonlinear patterns in returns, and one cannot necessarily attribute the violation of the variance bound in the data to the existence of a bubble. Roodposhti et al. (2012) used the variance ratio test and reliability cost-benefit test to explore the presence of informational efficiency and rational price bubbles in the Tehran Stock Exchange and found that the market is not only weakly inefficient but also shows signs of bubbles.

Phillips et al. (2007) apply the Dickey-Fuller unit-root method to detect the date of the emergence of a bubble in the Nasdaq stock index by estimating the date of a regime switch from a time series behaviour which is integrated of order one (i.e.I (1))to an explosive state. Nunes and Da Silva (2007) used the conventional models of integration and co-integration to examine the presence of rational bubbles in the selected emerging stock markets of Chile, Indonesia, Korea, and the Philippines; finding evidence of explosive bubbles, and in the stock markets of China, Brazil, Venezuela, Colombia, Indonesia, Korea, and the Philippines, where collapsing bubbles were found.

Kim (2000) and Busetti and Taylor (2004) proposed procedures seeking to test the hypothesis that time series is integrated of order zero, $(I(0))$, across the entire sample against the alternative, which is atransition of the series from integration of order 0 to order 1, that is, $I(0)\ to\ I(1)$, or vice-versa. Bhargava (1986), investigates whether a time series is integrated of order 1 ($I(1)$) as against an explosive alternative hypothesis of it having an order above one.

McQueen and Thorley (1994)'s theoretical models of rational speculative bubbles suggest that every bubble process results in explosive price change such that the bubble grows each period that it survives. As the bubble component grows, it begins to dominate the fundamental component, which is that aspect of the stock price determined by the discounted value of the future cash flows.

In Nigeria, according to the former Central Bank of Nigeria (CBN)'s governor, Sanusi (2010b), market capitalization of the Nigerian stock market rose by 5.3 times between 2004 and 2007 (when it reached its peak), with the market capitalization of bank stocks rising to 9 times its fundamental value within this period, leading to a financial asset bubble in bank stock prices. Also, of significant note is that the 2008 banking crisis witnessed in Nigeria was caused by

large and sudden capital inflows (amongst other interrelated macroeconomic factors), which led to the introduction of financial and banking reforms in the country (Sanusi, 2012).
Consequently, in this research we intend to critically investigate the presence of bubbles in Nigerian bank stocks across the study period and within the crisis period, using several techniques which we have cited from the relevant literature. Moreover, Ezepue and Omar (2012) examined bubbles based on the overall market data and surprisingly could not find evidence of bubbles, even though there was significant volatility in bank stocks between 2009 and 2010.

Our study thus considers this absence of bubbles in the overall market to possibly be traceable to the compensating effect of bubbles in different sectors cancelling each other out. Therefore, the analysis of sectoral bank data in this research aims to probe the phenomenon of bubbles more deeply, compared to Ezepue and Omar (2012), and with relatively more approaches to detecting bubbles. In other words, detecting bubbles at a sectoral level may be more promising than at the overall market level.

This section has presented a brief account of what bubbles in returns mean, given some examples of bubbles and presented a few findings as contained in the literature on approaches to detecting bubbles. Further relevant details on this concept shall be discussed in Chapter 8 of this thesis.

3.5.4 Volatility

Volatility is a measure of price variability overtime, which in most cases is described as the standard deviation of returns. It serves as a signal for determining the extent to which prices fluctuate such that if prices fluctuate significantly, volatility is known to be high, but the degree of fluctuations cannot be ascertained precisely. The reason for this is that no one can determine whether a considerable shock to prices is permanent or transitory (Danielson, 2011, p.31); certainly, this confirms that market volatility is not directly observable. Typically, volatility is in practice estimated from the stock prices, or derivatives or both (Tsay, 2012, p.177).

Volatility has become such an integral concept in trading that its importance to investment analysis, security valuation, risk management, and monetary policy making cannot be over-emphasized. It is the most important concept in pricing derivative assets, whose trading volume, according to Poon and Granger (2003), is on the increase in recent years in most developed markets especially, and there is the intent to introduce it into most African markets, for example the NSM (Urama et al., 2017). Before an option is priced, one needs to keep studying how volatile the underlying asset is until the option expires.

Since the 2008 financial crisis, volatility has received further increased attention from many studies in financial econometrics. Crucial to the belief of relevant stakeholders is that understanding volatility in stock markets can help to determine the cost of trading, to assess investment strategies as well as performance, and to create awareness that high volatility may result in illiquid financial markets, which pave the way for poor economic performance as well as a weakened financial system (Yildirim, 2013).

Essentially, it is to be noted that though volatility serves as a significant tool for evaluating the unavoidable risk of investing in financial assets, it is not the same as risk itself. It also measures likely size of errors while modelling returns and other financial variables such as interest rates, economic growth rates, and GDPs (Engle, Focardi and Fabozzi, 2008). When viewed as uncertainty, it constitutes a vital input to many investment decisions and portfolio creations, thereby indicating that investors and portfolio managers are prone to certain levels of bearable risk (Poon and Granger, 2003).

Thus, knowledge about risk generally helps to reshape and reposition the market, the participants and relevant stakeholders. According to Tsay (2005), volatility possesses a number of characteristics which include: (a) clustering, a phenomenon whereby at varying periods, low or high volatility values occur together; (b) that volatility evolves overtime in a continuous manner, indicating that volatility jumps are rare; (c) volatility does not diverge to infinity, meaning that it fluctuates within some fixed range; and (d) volatility reacts to big price increases and big price drop with the latter impacting volatility more than the former(the leverage effect).

Volatility increases during crises and then decreases during tranquil periods. The stock market crash of October 1987 globally, which was because of the financial crisis at that time, led to previously un-imagined high levels of volatility (Schwert, 1990). The economic crisis generally referred to as the Great Depression gave rise to fear, panic and social instability (Voth, 2003). There was a high level of volatility before the terrorist attacks on September 11, 2001 and it was far higher after the reopening of the markets six days later in the US (Taylor, 2005). In Nigeria, the effect of the September 2008 financial meltdown led to a crash of the stock market due to high volatility (Sanusi, 2010). Consequently, in this work we hope to study how volatile the banking sector stocks were during and after this period, and at various intervention periods initiated by the regulatory agency, the CBN, to rescue the banks.

Volatility is said to be positively correlated with trading volume but does not imply that changes in volume cause changes in volatility, or vice versa (Karpoff, 1987; Gallant, Rossi, and Tauchen, 1992; Longstaff and Wang, 2008). Meanwhile, the serial correlations found in daily return series are traceable to several market dynamics or idiosyncrasies, which include a possible common trading norm such as thin (nonsynchronous) trading in few assets, information incorporation and processing speed by market participants (Yildirim, 2013).

Epps and Epps (1976) and Perry (1982) list changing volatility as being attributable to information arrival rates, level of trading activity, and corporate financial and operating leverage decisions, as a few of the factors responsible for the linear dependency in asset returns. Clark (1973), Oldfield, Rogalski and Jarrow (1977), and Kon (1984) model changing variance with returns distributions defined as either mixtures of distributions, or as distributions with stochastic moments (Akgiray, 1989). Also, for a vast class of models, the average volatility size keeps changing with time and is predictable (Engle, Focardi and Fabozzi, 2008).

Meanwhile, volatility has been extensively studied in many contexts across various developed and emerging Asian markets, with little attention to emerging African markets, especially the NSM. According to Shiller (1990), most literature examines the volatility of equity returns from the developed exchange markets, especially those of industrialized nations (Green, Maggioni and Murinde, 2000). Despite most emerging markets still lacking some sophisticated financial instruments, the priority towards developing these markets lies in exploring the dynamics as well as the distribution of their asset prices (De Santis and Imrohoroglu, 1997; Hatgioannides and Mesomeris, 2007).

Understanding volatility is integral to examining the dynamics of stock returns and, by extension, to the study of financial market data. For example, according to Engle (2003), while trying to hedge risk, Markowitz (1952) and Tobin (1958) succeeded in deriving optimizing portfolio and banking behaviour in their study on the associated risk with the variance in the value of a portfolio.

Several studies have been carried out in financial econometrics with respect to volatility and its effects on the expected returns across various assets and markets. For example, Eun and Shim

(1989) examined daily stock market returns from Australia, Hong Kong, Japan, France, Canada, Switzerland, Germany, the US and the UK, wherein they find a considerable level of interdependency among the national exchange markets of the US to be the most prominent ones (see Bala and Premaratine, 2004). Whilst these findings are not technically focused on volatility, the phenomenon of interdependence in financial markets is subtly linked to underpinning market factors which affect different markets, including the six fundamental market features studied in empirical finance, namely: efficiency, volatility, bubbles, anomalies, valuation, and predictability (Ezepue and Solarin, 2009).

Investigating daily and intra-day stock prices, Hamao, Masulis and Ng (1990) discovered that there was significant spill over effects from the UK and the US stock markets to the Japanese market, but not vice versa (Siddiqui, 2009). Park and Fatemi (1993) investigate the relationships between the equity markets of the Pacific-Basin countries and those of the US, UK and Japan. The US market is noticed to be dominant compared to those of the UK and Japan, with the Australian market found to be the most exposed to the US markets. The Hong Kong, New Zealand and Singapore markets are moderately linked with that of the US, whereas Korea, Taiwan and Thailand show little or no linkage with any of these markets (Bala and Premaratne, 2004).

Lin, Engle and Ito (1994) present findings on how foreign returns can considerably impact local returns as it is with the Japan and US markets (see: Bala and Premaratne, 2004). In this research however, within the NSM, since the banks are interconnected and interact among themselves, with other sectors and the entire stock market, we hope to see the level of co-movements and influences in the volatilities in their equity returns (or sectors) influence one another within the NSM.

The ARCH model introduced by Engle (1982) was the foremost model used to capture the time-varying second-order moment component of financial data, and has been extensively explored in financial economics, primarily for the modelling of stock returns, exchange rates, interest rates and inflation rates. Bollerslev, Chou and Kroner (1992) present a detailed appraisal of various studies applying ARCH models to different financial markets. Apart from this, the model, with its many other extensions based on the associated stylized facts of the stock returns, see Baillie, Bollerslev and Mikkelsen (1996), for example, has been adopted in different contexts to assess the term structure of interest rates, create optimal dynamic hedging strategies, price options (Yildirim, 2013), and describe in general the data generating processes which engender financial market data. Their popularity stems from the fact that these models are sophisticated enough to capture many empirically observed stochastic behaviours such as the thick tail of finite dimensional marginal distributions and volatility clustering of the many economic and financial variables.

Empirical evidence also shows that daily stock returns exhibit substantial levels of dependency and that conditional heteroscedastic procedures enable autocorrelation between the first and second moments of the return distribution to be captured over time (Yildirim, 2013). For instance, French, et al. (1987) use daily data from the S&P index for 1928 to 1994 and find evidence of conditional volatility in returns (see: Poshakwale and Murinde, 2001). Pindyck (1984), Chou (1988), and Baillie and DeGennaro (1990) report time-varying interdependency between expected returns and volatility in the US stocks (Li, 2002). In this research, within the NSM, since banks are interconnected and interact among themselves, with other sectors and the entire stock market, it will be of interest to explore the level of co-movements and influences in the volatilities among their equity returns.

Bollerslev (1986), while extending the ARCH model to the generalised autoregressive conditional heteroscedasticity (GARCH) model, introduced lagged conditional variances as predictor variables in the conditional variance equation (Yildrim, 2013). The ARCH-GARCH models remain the most widely used econometric models to describe the unique features of financial market data, namely volatility clustering, leptokurtic and symmetric distributions of returns.

Some of the later extensions of the ARCH model proposed and applied in the literature are based on some underlying stylized facts which the two baseline models would not be able to capture including: the exponential GARCH (EGARCH) model by Nelson (1991); the non-linear ARCH (NARCH) model by Higgins and Bera (1992); the GJR-GARCH by Glosten, Jaganathan and Runkle (1993); the asymmetric power (APARCH) model by Ding, the Granger and Engle (1993); and the threshold ARCH (TARCH) model by Zakoian (1994); see Yildirim, 2013 for details and Omar (2012) for applications of five of these core models to the All Share Index (ASI) returns data in the NSM – results which the banking sector research in this thesis extends.

We reiterate that most of these extensions were proposed to correct the weaknesses of the ARCH/GARCH models due to their inability to capture some of the outlined characteristics.

For instance, the EGARCH and TGARCH models were proposed to capture the asymmetry in volatility, which surfaces due to leverage effects associated with large negative and positive returns.

Indeed, plausible directions in ARCH-GARCH modelling of volatility and related empirical finance issues in specific markets, such as the NSM include: a need to constructively elicit particular models in the family which are best-fit data generating stochastic processes for key market data; an examination of the four market features in key sectors of the market; and using the combined insights from the research results to systematically characterise and develop the market (Ezepue and Omar, 2017, Urama et al., 2017 and Nnanwa et al., 2016). We mentioned earlier that this research contributes to the banking and financial sector aspects of the overall systematic stock market characterisation and development (SSMCD) of the NSM.

Thus, in this research, candidate ARCH/GARCH models will be explored using the stock returns of Nigerian banks, with a view to fitting appropriate models that explain the volatility dynamics of the banking sector of the NSM. By doing this, suitable ARCH/GARCH models shall be fitted to the returns across the intended sub-periods of the study based on the inherent stylized facts identified with the return series of each of the banks. For example, in situation (s) where the returns are asymmetric, it has been found that EGARCH performs better than the ordinary GARCH model.

Chapter 9, which is dedicated to discussing volatility, will present more detailed findings as contained in literature on volatility, as well as different candidate GARCH models that have previously been applied across exchange markets of developed and emerging economies. The details will highlight the models found relevant to this study based the identified stylized facts characterising the research data.

3.6 Summary and Conclusion

In this chapter, we have examined various issues, which form the core of our research. These include general background, factors responsible for the dynamic behaviour of stock returns, and stock markets in general. We also discussed: the rationale for the choice of NSM and emerging African markets as our research domains; our motivation for applying non-linear time series models for modelling the volatility; the stylized facts of both emerging markets and those of stock returns; and four of the six empirical finance issues - anomalies, bubbles, efficiency, and volatility.

It was noted that while the related studies are predominantly concentrated on developed markets, there are relatively fewer studies on emerging African markets, including the NSM. Hence, there is a need to conduct such studies for the key sectors of these stock markets, as we do in this research.

Our discussion was focussed on the findings from various literature which were found to be relevant to our studies. We hope that, by this review, the direction of our research subject to our research topic, objectives and the gaps observed in current knowledge shall be filled as much as possible, and especially linked to the wider SSMCD goals reiterated above.

4 CHAPTER FOUR: METHODOLOGY

4.1 Data Presentation and Coverage

The proposed data for this research, which was obtained from the NSM, runs from 1st June 1999 to 31st December2014, encompassing the scenarios experienced by the Nigerian banks. The dataset is the daily closing stock prices of the sixteen banks being considered. From these, we shall obtain monthly closing prices and compute corresponding returns series for the daily and monthly data. The dataset is expected to cover the overall period, ranging from June 1999 to December 2014 and the sub-period of the financial crisis, ranging from July 2007- June 2009. **Table 4.1** below presents the total number of observations per bank for the overall and financial crisis periods.

Table 4.1: Data Size for the Respective Banks at the Overall and Financial Crisis Period

Bank	Period (years)	T (Overall Trading days)	T (Fin. Crisis)
Access	June,1999-Dec,.2014 (15.5years)	**3869**	**517**
Diamond	May 2005-Dec 2014(9.5years)	**2368**	**517**
Ecobank	Sept 2006-Dec 2014(9.25years)	**2050**	**517**
First	June 1999-Dec 2014(15.5years)	**3869**	**517**
FCMB	Dec 2004-Dec 2014(10years)	**2474**	**517**
Fidelity	May 2005-Dec 2014(9.5years)	**2376**	**517**
STANBIC	April 2005-Dec 2014(9.67years)	**2391**	**517**
GTB	June 1999-Dec. 2014(15.5years)	**3869**	**517**
Skye	Nov 2005-Dec.,2014 (9.083years)	**2391**	**517**
Sterling	June 1999-Dec. 2014(15.5years)	**3869**	**517**
UBA	June 1999-Dec. 2014(15.5years)	**3869**	**517**
Union	June 1999-Dec 2014(15.5years)	**3869**	**517**
Unity	Dec 2005-Dec 2014(9 years)	**2223**	**517**
WEMA	June 1999-Dec 2014(15.5years)	**3869**	**517**
Zenith	Oct 2004-Dec 2014(10.17years)	**2516**	**517**

Detailed information on the return's computation will be presented in chapter 5, which deals with the stylized facts of asset returns.

4.1.1 Choice of Software Program (s)

Many software programs are available for analysing quantitative data. Specifically, those that have statistical functions, either already written or requiring programming through coding include: VIEWS, STATA, SAS, R, Ox-metrics, RATS, SPSS, MATLAB, and EXCEL.

However, in this research, many packages are applied but **R** is the main one, complemented by **EVIEWS**, **SPSS** and **EXCEL**.

Essentially, **R** to many analysts, is next to SAS. It is open source programming software with many batched programs. Its graphics are attractive and applicable to different fields of data analytics. It is employed to obtain about 70% of the results in this research.

EVIEWS, called Econometric Views, as the name implies, is dedicated to time series and econometric analyses such as cross-sectional and panel data analyses, and with lower sophistication compared to R. Its contribution to the results is about 10%.

SPSS- (Statistical Package for Social Sciences**),** as the name implies, is a statistical program predominantly used in the social sciences. Though applicable to several fundamental statistical analyses, its level of sophistication is incomparable to R. It contributes not more than 5% to the results of this research.

Microsoft Excel is a spreadsheet program. It is employed as foundational tool to compute returns and their derivatives (such as: adjusted, absolute and squared returns) from the closing prices, as well as, for coding (with the dummy variables) and other basic data manipulation. It contributes about 15% to the analyses of this research.

4.2 Investigating Stylised Facts of Asset Returns

This section summarises the research methods, models and indicative data for the different RQs. For example, the approaches for RQ1 are stated below. The same returns data used for the question are analysed for other research questions. This section and the subsequent ones (**4.3 and 4.4)** present specific approaches to adopt in addressing **RQ1**, essentially on stylized facts of asset returns obtained from banks share prices. The data set for this part are the bank returns from the NSM.

4.2.1 Methods and Designs for Investigating Stylized Facts of Asset returns

In this section, the following steps will be taken to explore RQ1, which deals with stylized facts of asset returns:

- To present time plots of daily closing prices for each of the sixteen banks for the overall data and financial crisis periods;
- To present time plots of the monthly closing prices for the sixteen banks;

- To present time plots for the daily log returns across the sixteen banks for the overall and financial crisis periods;
- To present time plots for the monthly log returns across the sixteen banks;
- To present time plots for the daily absolute log returns across the sixteen banks for the overall and financial crisis periods;
- To present time plots for the monthly absolute log returns across the sixteen banks;
- To present time plots for the daily squared log returns across the sixteen banks for the overall and financial crisis periods;
- To present time plots for the monthly squared log returns across the sixteen banks;
- To present Autocorrelation (ACF) and Partial Autocorrelation Function (PACF) plots for the log returns, absolute and squared returns for the daily and monthly data;
- To produce histograms and Quantile-Quantile (Q-Q) plots for the log returns for the daily, monthly and financial crisis data;
- To compute summary statistics for the daily overall, financial crisis and monthly data across the sixteen banks;
- To conduct relevant normality tests for each of the data categories. Some of the tests of interest being proposed are- Kolmogorov-Smirnov (KS), Sharpiro-Wilk (SW), D'Augusto, (Dago) Anderson Darling (AD) and Jarque-Bera (JB);
- To conduct diagnostic ACF tests for the log returns, absolute and squared returns for the overall data using the Box Ljung (Q) test;
- To obtain diagnostic correlation tests between the squared and lag-1 returns to identify the presence or absence of a leverage effect.

4.2.2 Normal Distribution (Definition)

A random variable X following the normal distribution with mean μ and variance, σ^2 is denoted as $N(\mu, \sigma^2)$ and given by:

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad -\infty \leq x \leq \infty, \qquad \textbf{(4.1)}$$

$$-\infty \leq \mu \leq \infty;\ \sigma^2 > 0.$$

A standardized normal random variable such that the mean and variance are respectively zero and one is defined as:

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad -\infty \le z \le \infty. \qquad \textbf{(4.2)}$$

Remarks

i. The normal distribution is bell-shaped and symmetric about its mean, meaning the normal distribution shows no skewness.
ii. The skewness and kurtosis coefficients for the normal distribution are respectively 0 and 3.

4.2.3 Student T- Distribution (Definition)

It $X \sim N(0,1)$ and $Y \sim \chi^2(m)$ are independent, then the random variable:

$$T_{st} = \frac{X}{\sqrt{Y/m}}; \sim (distributed\ as)\ t(m)$$

And m is the degree of freedom

Then for, $T_{st} \sim t(m)$, the probability density function $(p.d.f.)$ is:

$$f(t) = \frac{\Gamma((m+1)/2)}{\sqrt{\pi m}\Gamma(m/2)} \frac{1}{(1+t^2/m)^{(m+1)/2}}, \qquad \textbf{(4.3)}$$

$for\ all\ -\infty < t < \infty.$

4.2.4 Moments of the Returns Distribution

Let $\{r_t : t = 1,2,\dots,T\}$ be a time-series of log-returns that we assume to be the realizations of a random variable.

1. **Measures of Location**:
   a. The sample mean (or average return) is the simplest estimate of location:

$$\bar{r} = \frac{1}{T} \sum_{t=1}^{T} r_t. \qquad \textbf{(4.4)}$$

It is important to note that while the mean is generally sensitive to outliers, the median is robust to outliers and it is the middle most value of a distribution; the observation falling within the 50^{th}quartile $(Q_{0.5})$ of the distribution, computed using the formula

$$\check{r} = \left(\frac{T+1}{2}\right)^{th} \text{term.} \qquad \textbf{(4.5)}$$

Where T is the total number of observationsin an ordered odd-number series.

2. Measures of dispersion:

b. Sample standard deviation (square root of variance) is the simplest measure of dispersion, obtained as:

$$s_r = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}. \qquad \textbf{(4.6)}$$

Note that the standard deviation is very sensitive to outliers, whereas the median absolute deviation (MAD) is robust to outliers; the MAD is given by

$$MAD = med(|r_t - \check{r}|). \qquad \textbf{(4.7)}$$

where $\check{r}$ is the median and under a normality assumption, the standard deviation (s_r)

$$s_r = 1.4826 \times MAD$$

The Inter-Quartile range (IQR) is robust to outlier and is defined as:

$$IQR = Q_{0.75} - Q_{0.25}, \qquad \textbf{(4.8)}$$

where $Q_{0.75}$ is called 3rd or 75th quartile and $Q_{0.25}$ is the 1st or 25th quartile

Under normality, we have

$$s_r = \frac{IQR}{1.34898}.$$

3. Skewness:

c. The sample coefficient of skewness is the simplest estimate of asymmetry and is computed as:

$$\boldsymbol{\hat{S}_{kr} = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{r_t - \bar{r}}{s_r}\right]^3.} \qquad \textbf{(4.9)}$$

Remarks:

i. If $\hat{S}_{kr} < 0$, the distribution is skewed to the left, meaning that the distribution has a long-left tail. This implies that the left tail is long relative to the right tail.

ii. If $\hat{S}_{kr} > 0$, the distribution is skewed to the right, indicating that the distribution has long right tail. The skewed right indicates that the right tail is long relative to the left tail.

4. Kurtosis

The sample kurtosis coefficient is the simplest estimate of tail thickness and it is estimated as:

$$\hat{K}_r = \frac{1}{T}\sum_{t=1}^{T}[\frac{r_t - \bar{r}}{s_r}]^4. \qquad \textbf{(4.10)}$$

Note that:

If $\hat{k}_r < 3$ the distribution has thinner tails than the standard normal distribution

If $\widehat{K}_r > 3$ the distribution has thicker tails than the standard normal distribution

Three general forms of kurtosis are: Leptokurtic (Sharply peaked with fat tails, far above the Normal curve and less variable), Mesokurtic (Medium peaked, same level with Normal) and Platykurtic (Flattest peak, far below the Normal curve and highly dispersed).

Under normality, the following results holds as $T \longrightarrow \infty$,

$\sqrt{T}(\hat{\mu} - \mu) \sim N(0, \sigma^2)$;$\sqrt{T}(\hat{\sigma}^2 - \sigma^2) \sim N(0, 2\sigma^4)$; $\sqrt{T}(\hat{S}_{kr} - 0) \sim N(0, 6)$ and

$\sqrt{T}(\widehat{K}_r - 3) \sim N(0, 24)$ (see Snedecor and Cochran (1980, p. 78); Hamilton (1994); Tsay (2005)). These asymptotic results for the sample moments can be used to perform statistical tests about the distributions of the returns.

4.2.5 Normality Tests

Various tests of normality have been developed based on a) moments of the return series; b) the density function of the distribution; or c) some properties of ranked series explained below.

Tests based on Moments:

Meanwhile, the most widely used in financial time series is that from Jarque and Bera (1980) and Bera and Jarque (1981), which relies on the fact that for a normal distribution both the skewness (S_{kr}) and excess kurtosis $(\mathrm{K}_r - 3)$should be equal to zero. It is important to note that test statistic is mostly applicable to financial/econometrics data

To conduct the normality tests, two approaches are adopted in the literature, namely the use of statistical tests and graphical methods. We shall explore both methods in this study.

Statistical Tests:

These are tests based on the “Returns Moments" and the “Returns Density Function".

Tests Based on Returns Moments

Consider a series of asset returns $\{\mathrm{r_t} \,:\, 1 \leq \mathrm{t} \leq \mathrm{T}\}$, where T is the maximal time point.

To test whether the skewness is significant relative to the normal distribution levels in the return’s series, the following hypotheses are set:

$\mathrm{H_0}: S_{kr} = 0$, versus $\mathrm{H_1}: S_{kr} \neq 0$. **(4.11)**

The Test Statistic to ascertain this claim is given as:

$$Z_{S_{kr}} = \frac{\hat{\mathrm{S}}_{kr}}{\sqrt{6/\mathrm{T}}} \sim \mathrm{N}(0{,}1). \quad \textbf{(4.12)}$$

The decision rule (DR) is to Reject H_0 when the p-value is$<\alpha$ (which is commonly set to be either 5% or 1%).

Note: Skewness is a measure of asymmetry of the probability distribution of a random variable. If a set of data is either normally or approximately normally distributed, its skewness is expected to be close to zero.

To see if the return series has significantly different Kurtosis from that of a normal distribution which is 3, we set the hypotheses:

$H_0: (K_r - 3) = 0$ versus $H_1: (K_r - 3) \neq 0$. **(4.13)**

The Test Statistic required here is also expressed as:

$$Z_{K_r} = \frac{\hat{K}_r - 3}{\sqrt{^{24}/_T}} \sim N(0,1). \quad \textbf{(4.14)}$$

DR: Reject $H_0$when the p-value is $<\alpha$.

The Jarque-Bera (JB) test combines the above test statistics to test whether the series is normally distributed, i.e. the distribution of r_t is such that S_r= 0 and $(\hat{K}_r - 3)$ (excess kurtosis) = 0, with the hypotheses:

H_0:Data come froma Normal distribution, versus

H_1: Data do not come from a Normal distribution. **(4.15)**

The JB test statistic is given by

$$\textbf{JB} = \frac{T}{6}\left(\hat{S}_{kr}^2 + \frac{(\hat{K}_r - 3)^2}{4}\right) \text{ (Asymptotically)} \sim \chi^2_{(2)}, \quad \textbf{(4.16)}$$

where $\hat{S}_{kr}$, $\hat{K}_r$ and T are respectively the estimated sample skewness, the estimated sample kurtosis and the total number of observations

Although this (JB) is most widely used in finance, it does have two limitations. First, it only holds for very large samples; to correct this bias, Doornik and Hansen (1994, 2008) provide an "Omnibus" test (i.e. test statistic that can detect deviations from normality due to either skewness or kurtosis), for normality. Second, the empirical skewness and kurtosis are computed for given values of the mean and variance, which are both subject to sampling errors. Doornik and Hansen (1994) first obtain approximations for the finite sample distributions of skewness and kurtosis under a normality assumption, coupled with the assumptions that the kurtosis follows a Gamma distribution and that $\hat{k} > (1+\hat{s}^2)$. They show that with the normality assumption

$$\tilde{W} = \wp_1^2 + \wp_2^2 \sim \chi^2_{(2)}. \quad \textbf{(4.17)}$$

where $\wp_1$ and $\wp_2$ are respectively denoted as the finite-sample skewness and kurtosis, and computed as:

$$\wp_1 = \frac{1}{\sqrt{\log(\omega)}} \log\left(g + \sqrt{1 + g^2}\right). \quad \textbf{(4.18)}$$

$$\wp_2 = \left[\left(\frac{\chi}{2\alpha}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right]\sqrt{9\alpha}. \quad \textbf{(4.19)}$$

where $g = \hat{s}^2\sqrt{\frac{\omega^2-1}{2}\frac{(T+1)(T+3)}{6(T-2)}}$; $\omega^2 = -1 + \sqrt{2(b_0 - 1)}$; $\chi = 2b_1\left(\hat{k} - 1 - \hat{s}^2\right)$ and

$\alpha = b_2 + b_3\hat{s}^2$**, w**hile the correction factors for finite samples are defined as:

$b_0 = \frac{3(T^2+27T-70)(T+1)(T+3)}{(T-2)(T+5)(T+7)(T+9)}$; $b_1 = \frac{(T+5)(T+7)(T^3+37T^2+11T-313)}{12\tau}$

$b_2 = \frac{(T-2)(T+5)(T+7)(T^2+27T-70)}{6\tau}$; $b_3 = \frac{(T-7)(T+5)(T+7)(T^2+2T-5)}{6\tau}$, and

$$\tau = (T-3)(T+1)(T^2 + 15T - 4)$$

Remarks:

$\widetilde{W}$ in **(4.17)** is very similar to D'Agostino and Pearson (1973), an "Omnibus" test, proposed to correct the limitations of the JB test.

Tests based on the Density function

Some of the relevant methods, which compared the empirical cumulative density function (CDF), $F_R(.)$ of the returns with the CDF of a normal distribution or any other assumed distribution, $F^*(.)$ are briefly discussed below.

Kolmogorov-Smirnov (K-S) Test

The K-S testis used in determining whether a sample comes from a population with a specific distribution. It is a test based on the empirical cumulative distribution function (ECDF). Given N ordered data points: $Y_{(1)}, Y_{(2)}, Y_{(3)}, \ldots, Y_{(N)}$, the ECDF is defined as:

$E_N = {n(i)}/{N}$, where $n(i)$ is the number of points less than or equal to $Y'_{(i)}s\ \forall\ i = 1,2,\ldots,N$, which are ordered from smallest to largest value. The distribution of the K-S test statistic does not depend on the underlying cumulative distribution function being tested. It is an exact test, rather than a chi-squared goodness-of-fit test that depends on an adequate sample size for the approximations to be valid. The limitations of this test are that it is applicable only to continuous distributions; it is more sensitive to near centre of the distribution than at the tails; and that the distribution must be fully specified.

Definition

Assume that $x_1, x_{2,} x_{3,} \dots, x_n$ is arrayed as ordered sample with $x_{(1)} \leq\ x_{(2)}\ \leq\ x_{(3\,)} \leq \cdots \leq x_{(n)}$ and define $S_n(x)$ as:

$$S_n(x) = \begin{cases} 0, & x < x_{(1)} \\ k/_{n}, & x_{(k)} \leq x \leq x_{(k+1)}. \\ 1 & x \geq x_{(n)} \end{cases} \quad \textbf{(4.20)}$$

Consider the hypotheses:

H_0: The data follow a specified distribution, versus

H_1: The data do not follow a specified distribution.

The K-S test statistic could be defined as: $D_n = \max_x |F(x) - S_n(x)|$ or

$$D_n = \max_{1\leq i\leq N}[F(Y_i) - \tfrac{i-1}{N},\ \tfrac{i}{N} - F(Y_i)]. \quad \textbf{(4.21)}$$

where $F(x)$ is the theoretical cumulative distribution of the population being investigated, which must be a continuous distribution.

Practically, this test is implemented as follows:

a. The dataset is sorted in ascending order as stated above, from which the empirical cumulative density function (CDF), $S\big(x_{(i)}\big) = \frac{n_i}{n}$ is generated.
b. We then generate the assumed $CDF, F^*\big(x_{(i)};\ \vartheta\big)$ for every value of $x_{(i)}$. If the referenced distribution is normal, then we assume that the mean μ and standard deviation σ are known.
c. Finally, the KS test statistic is computed as:

$$\mathrm{KS} = \underbrace{sup}_{\{x\}} \left| F^*\big(x_{(i)};\ \vartheta\big) - (\tfrac{n_i}{n}) \right| \quad \textbf{(4.22)}$$

The major limitation of the KS test, however, is that the mean and standard deviation are unknown, and must be estimated from the sample, thereby leading to sampling errors. To adjust for this, the modified Lilliefors KS_L test is applied (see: Adhikari (2014); Adhikari and Schaffer (2015)).

The Anderson-Darling Test

There is also a test to examine if a sample comes from a specified distribution. It is a modification of the K-S test whereby more weights are given to the tails of the distribution than does the K-S test; thus, it is an alternative test to both Kolmogorov-Smirnov and chi-squared goodness-of-fit tests. It is a distribution free test because the critical values do not depend on the specific distribution being tested. It makes use of the fact that, when given a hypothesized underlying distribution and assuming the data come from this distribution, the cumulative distribution function (CDF) of the data can be assumed to follow a Uniform distribution. The data can then be tested for uniformity with a distance test (Shapiro, 1965).

Definition: Suppose we define the hypotheses:

H_0: The data follow a specified distribution, versus

H_1: The data do not follow a specified distribution.

The test statistic A to assess whether the ordered series $\{Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}\}$ comes from a given CDF, F(x) is

$$A^2 = (-N - S). \tag{4.23}$$

where: N is the total sample size, S = $\sum \frac{(2i-1)}{N}\left[ln\ \mathrm{F}\ (Y_i) + \ln\left(1 - \ \mathrm{F}\ (Y_{N+1-i})\right)\right]$ and F is the cumulative distribution function of the specified distribution; which is an ordered random sample drawn from the specified population.

Stephens (1974) found A^2 to be one of the best empirical distribution function statistics for detecting most departures from normality. Empirical testing has found that the Anderson–Darling test is not quite as good as the Shapiro-Wilk test but is better than other tests (Razali and Wah, 2011).

The Shapiro-Wilk (W) Test

The Shapiro-Wilk test, proposed in 1965, calculates a W statistic that tests whether a random sample, $x_1, x_{2,}\ x_{3,} \ldots, x_n$ comes from a normal distribution. The W statistic is calculated as follows:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i}^{n}(x_i - \bar{x})^2}. \tag{4.24}$$

Where the $x_{(i)}$is the ordered sample values starting from $x_{(1)}$, the smallest observed value, and the a_i are constants obtained from the means, variances and covariance of the ordered statistics of a sample of size n from a normal distribution (See: Shapiro and Wilk, 1965; Pearson and Hartley, 1972, Table 15)

4.2.6 Autocorrelation Tests

In this section, the autocorrelation function (ACF), partial autocorrelation function (PACF) and the Ljung-Box (Q) test statistic for testing autocorrelation functions are presented.

Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF)

The Autocorrelation Function (ACF): Autocorrelation also called serial correlation is the linear dependence of a variable with itself at two points in time. For stationary processes, autocorrelation between any two observations only depends on the time lag, k between them.

Define $Cov(r_t,\ r_{t-k}) = \gamma_k$.Then the kth lag autocorrelation is given by

$$\rho_k = Corr(r_t, r_{t-k}) = \frac{Cov(r_t,r_{t-k})}{V(r_t)},$$

$$\rho_k\ = \frac{\gamma_k}{\gamma_0}, \qquad \textbf{(4.25)}$$

where the numerator, γ_k is the covariance between the return r_t at time (t) and return (r_{t-k}) at time $(t-k)$ and the denominator, γ_0, is the variance of the autocorrelation for series,

The autocorrelation function (ACF) (also known as the correlogram) is a standard graphical technique for exploring predictability in statistical data and is obtained by plotting the lag-k autocorrelations against the lags. It measures how returns on one day are correlated with the returns on previous days. If such correlations are significant, there is strong evidence for predictability (Danielson, 2011). ɤ

The Theoretical ACF and PACF

The autocorrelation function (ACF) for a time series, r_t, t $=\ 1, \ldots, \mathrm{T}$, is the sequence ρ_k; for k $=\ 1, 2, \ldots,$ T$-$1. The partial autocorrelation function (PACF) is the sequence, ϕ_{kk},

k $=\ 1, 2, \ldots,$ T$-$1, where ϕ_{kk} is defined below.

The Sample ACF and PACF

The sample autocorrelation and sample partial autocorrelation are statistics that estimate the theoretical autocorrelation and partial autocorrelation. As a qualitative model selection tool, one can compare the sample ACF and PACF of the data against known theoretical autocorrelation functions. For the series, $r_1, r_2, \dots, r_T$, suppose that the sample mean is denoted as $\bar{r}$. Then the sample lag-k autocorrelation is given

$$\hat{\rho}_{k,r} = \frac{\sum_{t=k+1}^{T}(r_t-\bar{r})(r_{t-k}-\bar{r})}{\sum_{t=1}^{T}(r_t-\bar{r})^2}, \qquad k > 0 \tag{4.26}$$

with $\bar{r}$ representing the overall sample mean for all T observations and estimating the expected returns ($E[r_t]$), which is assumed to be constant.

The standard error for testing the significance of a single lag-k autocorrelation $\hat{\rho}_{k,r}$ is approximately normally distributed. It is important to note that, as opposed to prices, sample autocorrelations of returns are generally close to zero, irrespective of the time lag.

The Standard Error (SE) for ACF

According to Tsay (**2014, p. 45-46**), the standard error and the respective confidence interval for ACF are as presented below

$$SE_{\hat{\rho}} = \sqrt{(1 + 2\sum_{i=1}^{k-1}\hat{\rho}_i^{\,2})/T}. \tag{4.27}$$

The 95% Confidence interval for ACF is:

$$\hat{\rho}_{k,r} \pm 1.96 SE_{\hat{\rho}}. \tag{4.28}$$

The Partial Autocorrelation Function (PACF)

PACF is the correlation between $r_t$and r_{t-k} less the part explained by the intervening lags defined mathematically as

$$\phi_{kk} = Corr\,[r_t - E^*(r_t \backslash r_{t-1,} r_{t-2}, \dots, r_{t-k+1}), r_{t-k}]. \tag{4.29}$$

where $E^*(r_t \backslash r_{t-1,} r_{t-2}, \dots, r_{t-k+1})$ stands for the minimum mean least-squared predictor of r_t by $r_{t-1,} r_{t-2}, \dots, r_{t-k+1}$.

The standard error for testing significance of a single lag partial autocorrelation is given as:

$$SE_{\hat{\phi}_{kk}} = \frac{1}{\sqrt{T-1}}. \qquad \textbf{(4.30)}$$

The 95% Confidence interval for the PACF is:

$$\hat{\phi}_{kk} \pm \frac{1.96}{\sqrt{T-1}}. \qquad \textbf{(4.31)}$$

The autocorrelation function (Box and Jenkins, 1976; Box, et al, 2015) can be used for the following purposes. To determine stationarity and seasonality, to detect non-randomness in data and to identify an appropriate time series model if the data are not random.

Note that, as a qualitative model selection tool, one can compare the sample ACF and PACF of the empirical data against known theoretical autocorrelation and partial autocorrelation functions. For example, **Table 4.2** (see **Choi, (2012)**), below summarises the criteria used in identifying a suitable model using the ACF and PACF's (Das, 1994; Box, et al. 2015).

Test Statistics for ACF

Recall that the sample ACF presented in equation (**4.25)** is used in linear time series modelling to specify a suitable model that can capture the dynamic dependence of the data (see Tsay, 2014, **p**. **47-48**). Thus, in many areas of applied finance, the focus is on checking to determine if many autocorrelations of say, r_t are jointly equal to zero.

To achieve this, the following hypotheses are defined:

H_0: $\rho_1 = \rho_2 = \cdots = \rho_m = 0, \forall\, i = 1,2, \dots, m$

Versus

H_1: At least one of the $\rho_{i's}$ is non-zero

The proposed test statistic by **Box and Pierce (1970)** is given as:

$$Q_*(m) = T \sum_{k=1}^{m} \hat{\rho}_k^2. \qquad \textbf{(4.32)}$$

However, the modified form of the statistic commonly used to increase the power of the test in finite samples as proposed by **Ljung and Box (1978)** is given as:

$$Q(m) = T(T+2) \sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{T-k}. \qquad \textbf{(4.33)}$$

Table 4.2: Time Series Model Selection Criteria based on ACF and PACF's behaviour

Models	ACF	PACF
AR(p)	Tails off gradually as exponential decay or damped sine wave	Cuts off after p lags
MA(q)	Cuts off after q lags	Tails off gradually as exponential decay or damped sine wave
ARMA(p, q)	Tails off gradually after lag $(q - p)$	Tails off gradually after lag $(p - q)$

The autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) time series models stated above with indicated lags are specified as followed:

ARIMA model provides one of the basic tools in time series

Given that the autoregressive of order p, **AR**$(\boldsymbol{p})$:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + \varepsilon_t. \quad \textbf{(4.34)}$$

The moving average of order q **MA**$(\boldsymbol{q})$:

$$x_t = \varepsilon_t + \vartheta_1 \varepsilon_{t-1} + \vartheta_2 \varepsilon_{t-2} + \cdots + \vartheta_q \varepsilon_{t-q}. \quad \textbf{(4.35)}$$

Combining the two equations to have a general **ARMA**$(\boldsymbol{p}, \boldsymbol{q})$ model:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + \varepsilon_t + \vartheta_1 \varepsilon_{t-1} + \vartheta_2 \varepsilon_{t-2} + \cdots + \vartheta_q \varepsilon_{t-q}. \quad \textbf{(4.36)}$$

4.3 Examination of Market Efficiency

Several tests have been proposed to examine randomness, stationarity, independence and volatility, all of which serve as proxies for the efficiency of a stock market. Specifically, auto-correlation, runs, co-integration, unit root, and variance ratio tests as contained in Lo and Mackinlay (1988), Chow and Denning (1993), Ozer (2001), Taylor (2005, 2011) and Ezepue and Omar (2012), will be adopted in this research.

Below are the various tests that are focused on achieving the research aim in Chapter 6 of investigating market efficiency.

Methods for Chapter 6

1. To test for stationarity using the KPSS, Philips-Peron and ADF (level) tests;
2. To test for Random Walks using
   i. the Variance Ratio test, and
   ii. the Unit Root test using ADF, both with and without drift and trend tests;
3. To test for non-linear independence using the BDS (Brock, Dechert and Scheinkman) test;
4. To test for Randomness using the Runs tests, including the parametric tests based on mean and median and the non-parametric form of the test;
5. To obtain the ACF test on log returns at lags 1, 5, 10 and 20.

It is important to emphasise that among the listed test statistics, the variance ratio test has gained prominence in empirical finance, given its wider acceptability, and is the most powerful in detecting the presence of random walk (RW) (see Taylor, 2005 & 2011).

4.3.1 Tests for Unit Roots

In financial time series, attention is primarily focused on series that are often non-stationary. Some of these include: foreign exchange rates, inflation rates, interests rate and security prices.

For example, in stock price series, non-stationarity is strictly since there is no fixed level for the price, and in time series such a series is classified as a unit-root non-stationary time series. Meanwhile, the best-known example of unit-root non-stationary TS is the random walk model, discussed briefly in the previous chapter.

Remarks

According to Engle and Granger (1987):

1. A series without deterministic component and has a stationary, invertible, ARMA representation after differencing **d** times, is said to be integrated of order **d**, denoted as $X_t \sim I(d)$. Thus, for **d** = 0, X_t will be stationary and for d = 1 the change is stationary.
2. If a time series, X_t is distributed of order zero (i.e. $X_t \sim \mathrm{I}(0)$) with zero mean, any of the following holds

(i) The variance is finite

(ii) an innovation (or the associated error) has only temporary effect on the series

(iii) the spectrum (or distribution) of the series, $f(X_t)$ has the property such that $0 < f(X_t) < \infty$

(iv) the expected length of times for the series to degenerate, that is for $X_t = 0$ is finite

(v) The autocorrelations of the series ρ_k decreases steadily in magnitude for large enough k, such that it sum is finite.

3. If a time series, X_t is distributed of order one (i.e. $X_t \sim \mathrm{I}(1)$) with zero mean, any of the following holds:

(i) The variance of X_t is infinite as t, time becomes large

(ii) an innovation (or the associated error) has a permanent effect on the series, as the series is the sum of all previous changes

(iii) the spectrum (or distribution) of the series, $f(X_t)$ has approximate shape such that $f(X_t) = \infty$

(iv) the expected length of times for the series to degenerate, that is for $X_t = 0$ is infinite

(v) The autocorrelations of the series $\rho_k \to 1$ for all k as time becomes large, that is, $t \to \infty$

Additionally, according to Hendry (1995, p. 43), a finite (non-zero) variance stochastic series, X_t that does not accumulate past errors is said to be integrated of order zero ($X_t \sim \mathrm{I}(0)$)

In summary,
If a time series is characterised by unit roots, a series of successive differencing of such series, d, can transform series to a stationary series. These differences are represented as, $I(d)$, where d is the order of integration.
Thus, a non-stationary time series that is transformable in such manner is called time series *integrated of order k*; and it is either of order 0 ($I(0)$, where the series is stationary in its original form and does not require differencing) or of order one ($I(1)$, where the series is non-stationary and does require differencing)

Autoregressive ($AR(p)$) Unit root Tests

This is a test used to confirm whether a time series is consistent with Random Walk theory. It serves as proxy for stationarity or a random walk in time series data. The common approach used to achieve this was proposed by Dickey and Fuller (1979) and was later extended to the Augmented Dickey Fuller (ADF) test.

Dickey-Fuller (DF) Test

Consider the time-varying AR (1) model

$$r_t = \mu + \varphi r_{t-1} + \varepsilon_t \quad \text{for } \varepsilon_t \sim N(0, \sigma^2). \tag{4.37}$$

where r_t is the return series at time t, φ is an autoregression coefficient between the two the series (r_t and r_{t-1}), μ is the mean of the series, r_t and ε_t, is a series of independently and identically distributed random error terms, which behaves like white noise. A white noise is a stochastic process that is a set of independent normally distributed residuals when all systematic effects are removed from the process, for example trends and other factor effects in a time series. The test hypotheses here are

$$H_0: \varphi = 1 \text{ vs } H_1: \varphi < 1. \tag{4.38}$$

The three possible cases are:

a. If $|\varphi| = 1$, then there is a possibility for the presence of a unit root in the series which requires special treatment by differencing the series;
b. If $|\varphi| < 1$, then the series is covariance stationary;
c. If $|\varphi| > 1$, then the series is said to be boundless (or explosive), indicating the likely presence of bubbles.

Note that if φ=1 in **(4.39)**, it means that r_t is non-stationary.

The test procedure here is as follows:

1. Fit an AR (1) model by least squares

$$r_t = \varphi r_{t-1} + \varepsilon_t \text{ (RW without drift)} \tag{4.39}$$

Or

$$r_t = \mathfrak{I} + \varphi r_{t-1} + \varepsilon_t \text{ (RW with drift,} \mathfrak{I}) \tag{4.40}$$

Or

$$r_t = \mathfrak{I} + \zeta t + \varphi r_{t-1} + \varepsilon_t \text{(RW with drift and trend),} \tag{4.41}$$

where the error term, $\varepsilon_t \sim WN(0,\ \sigma^2)$, with WN denoting a white noise Wiener distribution[12].

2. Set the hypotheses (see **4.38**)

3. Define the test statistic proposed by Dickey-Fuller (1979):

$$t_{(\varphi=1)} = \frac{\hat{\varphi}-1}{SE(\hat{\varphi})}. \tag{4.42}$$

where $\hat{\varphi}$ is the least squares' estimate of, $\varphi = \frac{\sum_{t=1}^{T} r_t \times r_{t-1}}{\sum_{t=1}^{T} r_{t-1}^2}$, $SE(\hat{\varphi}) = \frac{\sum_{t=1}^{T}(r_t - \hat{\varphi}_1 r_{t-1})^2}{T-1}$ is the least squares estimates of the standard error of $\hat{\varphi}$, and T is the sample size.

Remarks

Related to the **DF** test is the Augmented Dickey-Fuller, ADF, test, which is based on ordinary least square (OLS) regression:

$$\Delta r_t = \Im + \tau r_{t-1} + \sum_{i=1}^{m} \tau_i \,\Delta r_{t-i} + \varepsilon_t. \tag{4.43}$$

Where, the: ε_t are independently and identically (iid) normally distributed, $NID(0, \sigma_\varepsilon^2)$

Thus, the null hypothesis of non-stationarity is: $H_0 : \tau = 0$ versus $H_1 : \tau < 0$, while the test statistic is

$$t_{(\tau=0)} = \frac{\hat{\tau}}{SE(\hat{\tau})}. \tag{4.44}$$

Inability to reject null hypothesis of a unit root indicates that the series may wander off to plus or minus infinity, meaning that the series has unconditional variance proportional to time (see Cuthbertson and Nitzsche, 2005, pp. 37). This, indicates that, the effects of any shock to the series remains permanent or persists for a long time. Rejecting the null hypothesis, however, would mean that the series is covariance stationary and thus would not 'explode'.

Unit Root Tests ($\mathbf{H_0}$: $\mathbf{Non-stationary}$)

The Phillips and Perron (1988) (PP) test is another test to be compared with the above ADF test discussed above

[12] **Wiener's process, also called Brownian motion** is a simple continuous stochastic process that is widely used in physics and **finance** for modelling random behaviour that evolves over time. Examples of such behaviour are the random movements of a molecule of gas or fluctuations in an asset's price (https://www.glynholton.com/notes/brownian_motion/) (see further: Billingsley (1986))

Phillips - Perron (PP) Unit Root Tests

The PP (Phillips and Perron (1988)) unit-root test investigates the null hypothesis that a TS is integrated of order 1 (i.e. $I(1)$). It builds on the Dickey–Fuller test which fits the regression model:

$$\Delta r_t = \varphi r_{t-1} + (constant, time\ trend) + \varepsilon_t. \quad \textbf{(4.45)}$$

With "Δ" being the first difference operator

However, in (**4.45)** serial correlation may pose a problem. Considering this, the augmented Dickey-Fuller test's (ADF) regression includes lags of the first difference of r_t.

The Phillips-Perron test involves fitting a slightly different model to (**4.45**), as presented below:

$$r_t = \varphi r_{t-1} + (constant, time\ trend) + \varepsilon_t. \quad \textbf{(4.46)}$$

Note that in (**4.46)**, ε_t is integrated of order zero ($I(0)$) and may be heteroscedastic. The PP test, however, corrects for any serial correlation and heteroscedasticity in the error term ε_t non-parametrically through modification of the Dickey Fuller test statistic. This means that the Phillips-Perron (PP) unit root tests differ from the DF tests mainly in how they deal with serial correlation and heteroscedasticity in the errors, and for the PP test, one only needs to decide whether to include a constant and/or time trend. The modified statistics applied are denoted by z_1 and z_2 are given as:

$$z_1 = \sqrt{\frac{\hat{\sigma}^2}{\hat{\lambda}^2}}\, t_{\hat{\varphi}_0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right)\left(\frac{T\left(SE(\hat{\varphi}_0)\right)}{\hat{\sigma}^2}\right). \quad \textbf{(4.47)}$$

$$z_2 = T\hat{\varphi}_0 - \frac{1}{2}\frac{T^2\left(SE(\hat{\varphi}_0)\right)}{\hat{\sigma}^2}\left(\hat{\lambda}^2 - \hat{\sigma}^2\right). \quad \textbf{(4.48)}$$

The terms $\hat{\sigma}^2$ and $\hat{\lambda}^2$ (https://www.bauer.uh.edu/rsusmel/phd/ec2-5.pdf) are consistent estimators of the variance parameters:

$$\sigma^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E[\varepsilon_t^2], \quad \textbf{(4.49)}$$

$$\lambda^2 = \lim_{T\to\infty} \sum_{t=1}^{T} E[T^{-1} \sum_{t=1}^{T} \varepsilon_t^2]. \quad \textbf{(4.50)}$$

where T is the total length of the series, φ is an autoregression coefficient between the two the series ($\mathrm{r_t}$ and $\mathrm{r_{t-1}}$) and $t_{\hat{\varphi}_0}$is the test statistic for a unit root.

Under the null hypothesis, $H_0: \varphi_0 = 0$, the PP modified statistics z_1 and z_2 are asymptotically distributed as the DF t-statistic and normalized bias statistics. Note further that the PP tests tend to be more powerful than the ADF tests but are subject to severe size distortions especially when autocorrelations of ε_t are negative and they are more sensitive to model misspecification

(That is, the order of the ARMA model; see https://www.bauer.uh.edu/rsusmel/phd/ec2-5.pdf).

Advantages of the PP tests over the ADF tests:

1. They are robust to general forms of heteroscedasticity in the error term ε_t.
2. There is no need to specify a lag length for the ADF test regression.

Unit Root tests ($\boldsymbol{H_0}$: $\boldsymbol{stationary}$)

The Kwiatkowski, Phillips, Schmidt and Shin (1992) KPSS test is used and will be compared with the PP and ADF test statistics.

KPSS Test

The Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test (Kwiatkowski, Phillips, Schmidt, and Shin, (1992); Kokoszka, and Young (2016)), is used to test whether a time series is stationary around a mean or linear trend, or is non-stationary due to a unit root. Note that, a stationary time series has constant mean and standard deviation overtime. The KPSS test is a regression-based test with three components, namely a deterministic trend (ψt), a random walk (p_t), and a stationary error (ε_t), and is given by:

$$r_t = p_t + \psi t + \varepsilon_t. \tag{4.51}$$

Setting the hypothesis

The null hypothesis for the test is that the data are stationary versus a non-stationary alternative hypothesis.

Stationarity can be viewed in two senses, namely stationarity about a constant mean level or intercept (Wang, 2006; p.33) or about a linear trend. The least-squares results for the test differ slightly depending on whether level stationarity or trend stationarity is intended (Kocenda & Cerný, 2015). When testing for level stationarity, the trend linear trend component is removed, and this simplifies the test. The data are normally log-transformed before running the KPSS test, to turn any exponential trends into linear ones.

A disadvantage of the KPSS test is that it has a high rate of Type-I-error rate which means that it tends to reject the null hypothesis too often. Using smaller p-values to control the error reduces the power of the test. A good way to overcome this limitation is to combine the KPSS with an ADF test and then to check whether the two results suggests that the time series is stationary.

4.3.2 The Variance Ratio (VR) Test

The VR test was proposed by Lo and Mackinlay (1988). It is otherwise called the variance of multiple period returns test and is defined as the sum of the single-period variances when the random walk hypothesis (RWH) is true.

The underlying assumption of this technique is that the variance of the increments in the random walk series is linear in the sample interval. In other words, if the logarithms of the stock prices are generated by a random walk, the variance of the returns should be proportional to the length of the sample interval. If a returns series follows a random walk process, the variance of its q-differences would be q times the variance of its first differences (Abbas, 2015). Meanwhile, **VR** is widely used in finance, and has previously been applied in studies such as: Olfield and Rogalski (1980), French and Roll (1986), Jones, et al. (1994), Ronen (1997) and Lee and Mathur (1999).

Derivation of the VR Test

Suppose the stochastic process generating the returns is stationary, then with the variance of single period return being $V(1)$, this is said to be equal to$V(r_t)$; that is, V (1) =$V(r_t)$. Thus, the variance of the two periods returns according to Taylor (2005, 2011)

$$V(2) = V(r_t + r_{t+1}) = V(r_t) + V(r_{t+1}) + 2Cov(r_t, r_{t+1}) \quad \textbf{(4.52)}$$

$$= V(1) + V(1) + 2\rho_1 V(1) \quad \textbf{(4.53)}$$

$$= 2V(1) + 2\rho_1 V(1) \quad \textbf{(4.54)}$$

$$=2(1 + \rho_1)V(1), \quad \textbf{(4.55)}$$

where ρ_1 is the first lag autocorrelation of one-period return.

Thus, the two-period variance ratio is defined as:

$$VR(2) = \frac{V(2)}{2V(1)} = 1 + \rho_1. \qquad \textbf{(4.56)}$$

The autocorrelation term $\rho_1$becomes zero when the RWH is true such that the variance ratio equals 1, but in circumstances when the RWH is false, then the variance ratio is not equal to one.

Meanwhile, for higher period or N-period returns, that is for $N \geq 2$, and when the RWH is true, such that,

$$V(N) = var(r_t + r_{t+1} + \cdots + r_{t+N-1}). \qquad \textbf{(4.57)}$$

$$= NV(1) \qquad \textbf{(4.58)}$$

That is, N times variance of a one-period return Thus the N-period returns variance ratio is expressed as:

$$VR(N) = \frac{V(N)}{NV(1)} = 1. \qquad \textbf{(4.59)}$$

For further details on this derivation see Taylor (2005, 2011)

The VR Test Statistic

According to Taylor (2005, 2011)**,** the choice of N in (**4.59**) is arbitrary. Thus, assuming a set of *N's* observed returns $(r_{t1,}r_{t2}, \ldots, r_{tN})$ with mean $\bar{r}$ and variance

$$\hat{V}(1) = \sum_{t=1}^{n-1} \frac{(r_t - \bar{r})^2}{n-1}, \qquad \textbf{(4.60)}$$

the corresponding estimate for V (N) is:

$$\hat{V}(N) = \frac{n}{(n-N)(n-N+1)} \sum_{t=1}^{n-N+1} (r_t + r_{t+1} + \cdots + r_{t+N-1} - N\bar{r})^2. \qquad \textbf{(4.61)}$$

Then the sample variance ratio test statistic is:

$$\widehat{VR}(N) = \frac{\hat{V}(N)}{N\hat{V}(1)}. \qquad \textbf{(4.62)}$$

The RWH should be rejected if the sample variance ratio is significantly greater than 1. Meanwhile, according to Taylor (2011), determination of what is significant is possible only when the distribution of $\widehat{VR}(N)$ under the hypothesis that the RWH is true is known, and the assumption that the returns process is stationary is not necessary. This is because the variance splits in the variance ratio tests can be around a constant mean or a trend.

Considering the test hypotheses are:

$H_0: VR = 1 \; vs \; H_1: VR \neq 1.$

The standardized distribution of the sample variance ratio can be obtained as:

$$z_N = \frac{\widehat{VR}(N)-1}{\sqrt{\upsilon_N/n}} \, is \; asymptotically \; N(0,1), \quad \textbf{(4.63)}$$

where $\upsilon_N = \frac{4}{N^2}\sum_{k=1}^{N-1}(N-k)^2 b_k$; $b_k = \frac{n\sum_{t=1}^{n-k} s_t s_{t+k}}{\left(\sum_{t=1}^{n} s_t\right)^2}$ and $s_t = (r_t - \bar{r})^2$.

4.3.3 RUNS Test

A run is a sequence of observations of the same sign. Its associated test is nonparametric, applied to check for possible presence of randomness in the stock returns or data generating process of a series. That is, it is intended to explore if the observed sequence of a series is randomly generated, or in other words, if the process of obtaining any succeeding observation is independent of the preceding value. The runs test, sometimes called the Geary test, is a test wherein the number of sequences of successive negative and positive returns are ordered and compared with its sampling distribution under the random walk hypothesis (Campbell et al. 1998; Gujarati, 2003; Ananzeh, 2014).

The test obtains the total number of times a run type (negative, positive or zero) occurs successively in the series and uses the runs data to test for randomness of the series. The logic of the test is that having two few or too many runs contradicts randomness.

Assumptions

It is assumed that the data consist of observations recorded in the order of occurrence that can be categorised into two mutually exclusive classes.

Test Hypotheses

The null hypothesis is that the series is random, and the alternative is that the series is non-random. Suppose *n* is the total number of runs in a series. The test compares this observed number of runs with the expected number of runs under the null hypothesis. The usual two-sided hypothesis in statistics applies if there is no knowledge about the direction of non-randomness in the series. The usual one-sided hypotheses of the left- and right-tail kinds apply if there is a reason to think that the series is non-random of these kinds. Hence, the test criteria are as follows.

The Hypotheses:

H_0:The returns are generated through a random process, versus

H_1:The returns are not generated through a random process.

Decision Rule: reject the null hypothesis when the p-value is less than **0.05**, indicating that the return series is likely not generated by random process.

It is known in nonparametric statistics with the null hypothesis that sequential outcomes are independent, that the total expected number of runs is approximately normally distributed with the following mean and standard deviation:

$$\mu = \frac{2k_+k_-}{k} + 1 \text{ and } \sigma = \sqrt{\frac{2k_+k_-(2k_+k_- - k)}{(k-1)k^2}},$$

where k_+ and k_- represent the total numbers of runs of positive returns (+) and total number of runs of negativities returns (-) regarding to a sample with k observations respectively, with k= $(k_+ + k_-)$ (Fisher and Van Belle, 1993, p. 333).

Thus, the test statistic is given as:

$$z = \frac{k-\mu}{\sigma} \sim N(0,1). \qquad \textbf{(4.64)}$$

4.3.4 The Brock, Dechert and Scheinkman (BDS) Test of Non-linear Independence

The BDS test developed by Brock, Dechert and Scheinkman (1987) (and later published as Broock, Dechert, Scheinkman and LeBaron, 1996) is a powerful tool for testing for nonlinearity or serial correlation in time series. It was originally developed to test the null hypothesis of independence and identical distribution (iid) of a time series; that is, to investigate the presence of any non-random and dynamic behaviour in a series. Rejection of such a hypothesis indicates that there remains some non-linearity or non-stationarity structure in a (de-trended) series.

According to several studies, the BDS test is robust against a spectrum of linear and nonlinear alternatives (see Brock, Hsieh and LeBaron, 1991 and Barnett et al., 1997). It is also used as a portmanteau test or misspecification test when applied to the residuals from a fitted linear time series model; in this case, it could be used to identify the remaining dependence and the presence of any omitted nonlinear structure, such that rejecting the null hypothesis means that the fitted linear model is miss-specified and thus could be taken for a test of non-linearity.

The BDS Test Statistic

The BDS test uses the concept of a correlation integral, which measures the frequency with which temporal patterns are repeated in the data.

Procedures for its Computation

1. Suppose $\{r_t\} = \{r_1, r_2, \dots, r_N\}$ is a returns series of a stock prices.
2. Choose a value of m (the embedded dimension) and embed the time series into m-dimensional vectors, by taking each m successive points within the series. By this, the series of scalars are converted into vector series with overlapping entries. That is:

$$r_1^m = (r_1, r_2, \dots, r_m)$$
$$r_2^m = (r_2, r_3, \dots, r_{m+1})$$
$$r_3^m = (r_3, r_4, \dots, r_{m+2})$$
$$.$$
$$.$$
$$.$$
$$r_{N-m}^m = (r_{N-m}, r_{N-m+1}, \dots, r_N).$$

3. Compute the correlation integral, $C_{\varepsilon,m}$, which measures the spatial correlation among the points, by adding the number of pairs of points $(i,j), \forall\ 1 \le i \le N; 1 \le j \le N,$ in the m-dimensional space which are "near" because the points fall within a radius or tolerance ε of each other. This is given by:

$$C_{\varepsilon,m} = \frac{1}{N_m(N_m - 1)} \sum_{i \neq j} I_{i,j;\varepsilon}, \qquad \textbf{(4.65)}$$

where

$$I_{i,j;\varepsilon} = \begin{cases} 1\ if\ \left\| r_i^m - r_j^m \right\| \le \varepsilon \\ 0 \qquad otherwise\ , \end{cases}$$

with the standard Euclidean norm used in specifying the indicator function's value.

4. Brock, Dechert and Scheinkman (1987) establish that if the series is independently and identically distributed (iid), then the relation below holds:

$$C_{\varepsilon,m} \approx \left[C_{\varepsilon,1}\right]^m \qquad \textbf{(4.66)}$$

If the ratio $\frac{N}{m} > 200$, then the value of $\frac{\varepsilon}{\sigma}$ ranges from 0.5 to 2 (Lin, 1997), while the values of m fall between 2 and 5 (Brock et al, 1988); thus, the quantity $[C_{\varepsilon,m} - \left(C_{\varepsilon,1}\right)^m]$ is asymptotically normally distributed with mean zero (0) and variance, $V_{\varepsilon,m}$, which is defined as:

$$V_{\varepsilon,m} = 4\left[K^m + 2\sum_{j=1}^{m-1} K^{m-j}\, C_\varepsilon^{2j} + (m-1)^2 C_\varepsilon^{2m} - m^2 K C_\varepsilon^{2m-2}\right] \qquad \textbf{(4.67)}$$

where $K = K_\varepsilon = \frac{6}{N_m(N_m-1)(N_m-2)} \sum_{i<j<N} h_{i,j,N;\varepsilon}$, and

$$h_{i,j,N;\varepsilon} = \frac{[I_{i,j;\varepsilon} I_{j,N;\varepsilon} + I_{i,N;\varepsilon} I_{N,j;\varepsilon} + I_{j,ij;\varepsilon} I_{i,N;\varepsilon}]}{3}.$$

5. The BDS test statistic is thus stated as:

$$BDS_{\varepsilon,m} = \frac{\sqrt{N}\, [C_{\varepsilon,m} - \left(C_{\varepsilon,1}\right)^m]}{\sqrt{V_{\varepsilon,m}}} \qquad \textbf{(4.68)}$$

The test statistic is a two-tailed test, and the null hypothesis is rejected if the test statistic is greater or less than the usual standard normal critical values.

In summary, the BDS test examines the spatial dependence in the observed series, and to achieve this, the series is embedded in m-space and the dependence of such a series is examined by counting what are called "near" points. That is, points for which the distance is less than epsilon are called "near". The test statistic is asymptotically Standard Normal.

The Hypothesis commonly set:

H_0: the returns are independent, **versus** H_1: the returns are not independent.

4.4 Testing for Anomalies in Returns

Recall the research question **RQ3** from chapter 1 of the thesis**:** is there any evidence of anomalies or speculative bubbles in the banking sector of the NSM for the study periods?

This section **provides** relevant methods as adopted in other studies found suitable for this research and considers their application to RQ3. We hope to follow the approaches used in those studies to investigate anomalies and bubbles in the bank data obtained from the NSM across the study periods.

Planned Methods for Anomalies in Returns in Chapter 7

1. To fit regressions of log returns on the Day-of-the-week;
2. Plotting of the Day-of-the-week effect;
3. To fit regression of returns on Holiday and January days;

4. Plotting of holiday and January effects;
5. To obtain October-March Seasonality effects;
6. Plotting of the October-March seasonality effects;
7. To obtain Turn-of-the-year effects and plot the corresponding series;
8. To obtain Turn-of-the-month effects and plot the corresponding plot series;
9. To obtain Turn-of-the-year effects and plot the corresponding plots series.

The approaches discussed in Taylor (2008), Schewert (2003), and Archana, Safeer and Kevin (2014), have been found suitable to be followed in testing and identifying the presence of our choice of anomalies in stock returns.

4.4.1 Testing for the Day of the Week Effect

For any of the above anomalies, the regression equation stated as below is commonly applied in the literature:

$$\mathrm{r_t} = \text{þ}_0 + \sum_{i=1}^{4} \text{þ}_i D(1) + \varepsilon_t \;. \qquad \textbf{(4.69)}$$

where$\mathrm{r_t}$ represents daily stock returns and þ_i $\forall$ $\mathrm{i} = 1,2,3,4$ are mean daily returns (or effects) for each trading day of the week and the ε_t is the random error . For example, if the Tuesday effect anomaly is being investigated, $D(1)$ is the dummy variable for Tuesday, thus $D(1)$ = 1 if Tuesday or $D(1)$=0 if any other day. The same goes for dummy variable for every other day, apart from Monday which without loss of generality is treated as the reference category and is represented by þ_0.

Alternatively, **(4.69)** can be set for Monday effect as:

$$\mathrm{r_t} = \text{þ}_0 + \text{þ}_1\mathrm{T} + \text{þ}_2\mathrm{W} + \text{þ}_3\mathrm{Th} + \text{þ}_4\mathrm{F} + \varepsilon_t \qquad \textbf{(4.70)}$$

Where in this case, þ_0 represents Monday effect in the model.

Note: for the method to be consistent with weekend effect, þ_1 in **(4.69) or** þ_0 in **(4.70)** should be significantly lower (and is sometimes 'negative') than the associated parameter for any other day, while þ_5 should be higher (and positive) (see Taylor, 2005, 2011; Sewell, 2011).

4.4.2 Half of the Month Effect

We have

$$r_t = \text{þ}_0 + \text{þ}_1 D + \varepsilon_t, \qquad \textbf{(4.71)}$$

where r_t is the daily returns and D=1 represents the dummy variable for the first fifteen calendar days of the month and D = 0 otherwise with$\varepsilon_t$an error term.

Note: for this to hold it is expected that þ_1, representing mean returns for the first fifteen days, should be higher than that of the remaining days.

4.4.3 Turn-of-the Month Effect

We have

$$r_t = \text{þ}_0 + \text{þ}_1 D_1 + \text{þ}_2 D_2 + \text{þ}_3 D_3 + \text{þ}_4 D_4 + \text{þ}_5 D_5 + \text{þ}_6 D_6 + \text{þ}_7 D_7 + \varepsilon_t, \qquad \textbf{(4.72)}$$

where D_1, D_2, and D_3respectively represent dummy variables for the first, second and third days preceding the last trading day of the month, $\mathbf{D_4}$ for the last day of the month and D_5, D_6, D_7 represent dummy variables for the first, second and third days succeeding the last day of the month respectively.

It is also expected that the mean returns for the first three days before and after the last day of the month should be higher than those of the other days.

4.4.4 Holiday Effect

Given that

$$r_t = \text{þ}_0 + \text{þ}_1 D_1 + \text{þ}_2 D_2 + \varepsilon_t, \qquad \textbf{(4.73)}$$

where D_1 is the dummy variable for a day before a holiday; and $D_2$is the dummy variable for a day after a holiday. Also, for this effect to hold it means that the returns on a day before a holiday should be higher.

4.5 Determining the Presence of Bubbles in Returns

Many tests have been proposed for detecting the presence of bubbles, these include: (i) Variance bounds or Volatility tests as proposed by Shiller (1981), and LeRoy and Porter (1981); (ii) Test for premium bubbles, introduced by Hasdouvelis (1988); (iii) Tests based on Cointegration analysis as proposed by Diba and Grossman (1988) and a host of other authors (iv) Tests based on Regime Switching Models as applied by Van Norden (1996); and (v) the duration dependence method proposed by McQueen and Thorley (1994), which uses log-logistic regression.

While the fifth of these has been observed to be a direct test for presence of bubbles, the other four are indirect tests applied to determining the distributional properties of prices and fundamental measures.

However, in this research our focus will be on the fifth approach, considering the nature of the available data for this research. This is because the focus of this research is on the empirical analysis of observed data on bank stock returns, not the distributional historical prices of bank stocks. Moreover, Omar (2012) which this research extends uses the fifth approach for the same reason and it is easier to compare our results with that of Omar (2012).

Methods for Determining Presence of Bubbles in returns

1. For each bank, we shall obtain the summary statistics, particularly the length of the bubble (T), along with the mean, standard deviation, skewness and kurtosis, for the returns corresponding to any bubble episodes.
2. We shall compare the mean, standard deviation, skewness and kurtosis credited to bubble episode(s) to the overall respective attributes for each bank. We shall further test for the presence of positive autocorrelation for each bank at lag one.
3. We shall test for the hypothesis of a unit root as against the alternative of "explosiveness" for the corresponding price series for the bubble episodes within each bank.
4. We shall then proceed to set up the return series for each respective bank for the adopted duration dependence tests.

Setting the objectives for Chapter 8, we have the following:

1. To see if there is skewness, kurtosis and positive autocorrelation in the daily returns within the identified bubble regions;
2. To test if rational speculative bubbles exist in the Nigerian banks' stocks by applying a simple diagnostic test for the rational speculative bubble model;
3. To apply the duration dependence test using the Cox-proportional hazards model to detect the presence of rational speculative bubbles in the Nigerian banks' stocks.

In applying the unit root tests discussed so far, the following null and alternative hypotheses are often set.

H_0 : The variable is stationary or there is no price bubble.

H_1: The variable is explosive or there is a price bubble.

Details of the Cox-proportional hazard model will be provided in Chapter 8 of the thesis.

4.6 Fitting Appropriate Volatility Models to the Returns

Recall that RQ4 states: How volatile are the bank returns and which volatility models are most suitable for describing the volatility behaviours of the returns in different study periods? Section 9.2provides the methods and models needed to address RQ4 and examine the presence of volatility in the banks' returns of the NSM**.** We shall aim to appropriately model the returns across different study periods to describe the level of volatility in the banks' data. The relevant data for this investigation is the system date obtained from the NSM and Cash Craft.

In this research, various univariate GARCH modifications will be explored to model daily stock return volatility of Nigerian banks trading at the NSM, during both the overall and financial crisis periods. For example, we intend to explore models such as $\text{GARCH}\,(\text{p},\text{q})$, $\text{EGARCH}\,(\text{p},\text{q})$, $\text{GJR-GARCH}\,(\text{p},\text{q})$, $\text{TGARCH}\,(\text{p},\text{q})$ and $\text{PGARCH}\,(\text{p},\text{q})$; with $\text{p} = \text{q} = 1,2, \dots$. The functional forms of these models are further explicated in chapter 9.

4.6.1 Planned Procedures in Fitting and Selecting Volatility Models

1. Having confirmed that the returns series for each of the banks is stationary in chapter six, we shall check for the presence of an ARCH effect/heteroscedasticity in the return series of each bank using the Lagrange Multiplier (LM) test statistic proposed by Engle (1982) or Breusch and Pagan (1979) (these shall be discussed below).

2. We then fit different possible candidate GARCH family models -both symmetric and asymmetric forms.

3. We group the fitted models based on two error distribution assumptions - Normal and Student-t distributions.

4. Selection of the best models is done, first at the error distribution level, and then at the overall model level using AIC (or BIC).

5. The final model is then chosen by further examination of the extended family of the selected candidate model based on (I) the lag levels and/or (ii) the use of the skewed form of the error distribution for the model fitted in (4) above. This is done to see if a better fit could be achieved.

6. Checking the goodness-of-fit of the fitted model by examining presence of: (i)anasymmetric effect (ii) serial correlation in the standardized and squared standardized residuals (iii) any ARCH effect in the standardized residuals and (iv) stability of the estimated parameters using the Nyblom test.

7. Computation of the persistence rates, unconditional variance and half-life for the fitted model for each bank.

8. Finally obtaining the plots for the fitted series, the standardized and squared standardized residuals, and the News Impact Curve (NIC) for each of the banks.

4.6.2 Test of Heteroscedasticity

One major preliminary check prior to applying the Autoregressive Conditional Heteroscedasticity (ARCH)/Generalized Autoregressive Conditional Heteroscedasticity (GARCH) family of models is to look for the presence of an ARCH effect in the returns series using the residual series $\{\varepsilon_t\}$, also called innovations. The Langrange Multiplier (LM) test of Engle (1982) is found suitable.

Detecting Heteroscedasticity

There are two basic methods to detect heteroscedasticity in time series, including stock returns, namely the Graphical Method and econometric tests for heteroscedasticity, which may be either of the following:

1. The Breusch-Pagan LM Test;

2. Engle's ARCH Test.

The Breusch-Pagan LM Test

The Breusch-Pagan test is a formal way to test whether the error variance depends on the observed series

Procedures

Step 1: Set the following hypotheses:

$H_0: \upsilon_0 = \upsilon_1 = \upsilon_2 = \cdots = \upsilon_p = 0$, versus H_1: At least one of the parameters is not 0

Step 2: Run a linear regression of the returns series of the form:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_1 r_{t-1} + \cdots + \phi_1 r_{t-l} + \varepsilon_t. \quad \textbf{(4.74)}$$

Estimate the model parameter using Ordinary Least Square OLS, and then obtain the residualsresiduals, $\hat{\varepsilon}_t$

Step 3: To test if the error variance depends on the series, we regression the squared residuals on the lagged return series as follows:

$$\hat{\varepsilon}_t^2 = \upsilon_0 + \upsilon_1 r_{t-1} + \upsilon_2 r_{t-2} + \cdots + \upsilon_p r_{t-p} + u_t. \quad \textbf{(4.75)}$$

where u_t is the error term

Step 4: Compute the Lagrange Multiplier statistic

$$\text{LM} = \text{T}R^2 \sim \chi_{p-1}^2. \quad \textbf{(4.76)}$$

where T is the number of observations and R^2(the coefficient of determination) is estimated from the regression (**4.75**).

Step 5: If LM-stat > χ_{p-1}^2,critical value corresponding to a specified significance α-level, reject the null hypothesis and conclude that there is significant evidence of heteroscedasticity, at the α-level of significance.

Engle's ARCH Test

In the case of Engle’s test, heteroscedasticity is supposed to characterise the error term's variance rather than the error term itself. The underlying concept is that the error term's variance is assumed to depend on the squared lagged error terms, that is:

Procedures

Steps 1 and 2 are the same as above

Step 3: Square the residuals obtained from (**4.75**), and regress them on p own lags to test for ARCH of order p as follows:

$$\hat{\varepsilon}_t^2 = \upsilon_1 + \upsilon_2 \varepsilon_{t-1}^2 + \cdots + \upsilon_p \varepsilon_{t-p}^2 + u_t. \quad \textbf{(4.77)}$$

Obtain R^2 from (**4.77**)

Step 4: The test statistic is the Lagrange Multiplier of Engle (1982), defined as TR^2 (the number of observations multiplied by the coefficient of multiple determination, R^2), and is distributed as a χ_p^2 **(** Brooks (2019, p.390)).

4.7 Summary and Conclusion

This chapter has reviewed briefly the data intended to be used in this research and some selected methodologies to be adopted to address the research questions presented in chapter one of this thesis. Essentially the concepts discussed so far include tests of skewness, kurtosis, Normality, unit roots, autocorrelation and partial autocorrelation, calendar anomalies, and heteroscedasticity/ARCH effect tests. Further details of the choice of methodologies that will fully address the research objectives will be presented in detail within the respective chapters dedicated to each of the issues this study examines.

5 CHAPTER FIVE: Stylized Facts of Asset Returns in the NSM

5.1 INTRODUCTION

The underlying aim of investing in financial assets such as stocks, bonds, bank deposits, futures or derivatives is mainly to earn profits after holding them for a reasonable period with little or no risk. Positive income or returns are earned if the price of a holding asset at the end of the holding period is higher than it was at the time of acquisition. The amount of income realized at the end strictly depends on three components, which are: (i) the initial capital, which translates to the volume of assets purchased, (ii) the length of the holding period, and (iii) the returns on the asset over the holding period. Thus, a return could simply be defined as the dividend (or interest) plus the change in current asset price.

In financial time series, asset return has remained the variable of interest instead of price, for two simple reasons, which according to Campbell, Lo, and Mackinlay (1997), are: (1) to the investors, asset returns are completely a scale-free summary of any investment prospect; (2) asset returns are generally known to be stationary and easier to handle, due to having attractive statistical properties, unlike the asset price itself which, empirically, is non-stationary, and successive prices are highly correlated, with time-varying variances, thus obtaining meaningful statistical analyses becomes difficult.

Definition (Asset Returns)[13]

A. **Single-period Simple Returns:** this is divided into the Simple Gross return and the Simple Net Return, otherwise called the Simple Return.

Holding an asset from time $t-1$ to t, is the price of the asset changes from P_{t-1} to P_t , such that we assume no payment of dividend, then:

a. The Simple Gross Return is defined as the ratio of the new market value at the end of the holding period over the initial market value, which mathematically is defined as:

$$1 + R_{t,1} = \frac{P_t}{P_{t-1}}. \quad \textbf{(5.1)}$$

b. The Simple Return is derived as:

[13](see Tsay, 2002; p. 2-5)

$$R_{t,1} = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}. \quad \textbf{(5.2)}$$

where $R_{t,1}$ is the profit rate of holding an asset from time $t - 1\ to\ t$, which sometimes is written as $100R_{t,1}\%$, the percentage rate of change in price within the holding price, representing the percentage gain in price relative to the initial capital, P_{t-1}.

In terms of high frequency data, such as daily or hourly returns, $R_{t,1}$ is usually very small, with the returns of less risky assets such as bonds taking even smaller values over a short horizon.

B. **Multi-period Simple Return**

Suppose an asset is held for a single period between times$t - k\ and\ t$, then the k −period **Simple Gross Returns**, also called **Compound Return,** is derived as:

$$1 + R_{t,k} = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_t}{P_{t-2}} \times \dots \times \frac{P_{t-k+1}}{P_{t-k}} = \left(1 + R_{t,1}\right) \times \left(1 + R_{t-1,1}\right) \times \dots \times \left(1 + R_{t-k,1}\right).$$

The $k -$ period simple return therefore equals:

$$R_{t,k} = \frac{P_t - P_{t-k}}{P_{t-k}}. \quad \textbf{(5.3)}$$

Annualized Returns-

For the **geometric mean,** this is obtained as:

$$\text{Annualized } \{R_{t,k}\} = [\textstyle\prod_{i=0}^{k-1}(1 + R_{t-i})]^{\frac{1}{k}}. \quad \textbf{(5.4)}$$

For the logarithmic mean

$$\text{Annualized } \{R_{t,k}\}=\exp[\tfrac{1}{k}\textstyle\sum_{i=0}^{k-1} ln(1 + R_{t-i})] - 1. \quad \textbf{(5.5)}$$

For the mean by one-period

$$\text{Annualized } \{R_{t,k}\} \approx \tfrac{1}{k}\textstyle\sum_{i=0}^{k-1} ln(1 + R_{t-i}). \quad \textbf{(5.6)}$$

The **Continuously Compounded Return,** which is the natural logarithm of the gross return of a security, **log return** is obtained as**:**

$$\boldsymbol{r_{t,1}} = \boldsymbol{ln}\left(\boldsymbol{1} + \boldsymbol{R_{t,1}}\right) = \boldsymbol{ln}\frac{P_t}{P_{t-1}} = lnP_t - lnP_{t-1}, \quad \textbf{(5.7)}$$

where $\mathrm{P_t}$ is an asset's closing price at a period t and $\mathrm{P_{t-1}}$ is the price a prior period before, or on a particular period $\mathrm{t} - 1$, noting that "t" may be "days," weeks," "months," or "years".

The concept of a **continuously compounded** return is closely related to the concept of compound rates or interest rates. For instance, for a bank deposit account, the interest rate is usually quoted as simple interest.

C. **Portfolio Return**

The simple net return of a portfolio is comprised of N assets and is a weighted average of the simple net returns of the assets involved, where the weight (w) on each asset is the fraction of the portfolio's value investment in that asset. That is:

$$R_{p,t} = \sum_{i=1}^{N} w_i R_{it} \,. \quad \textbf{(5.8)}$$

D. **Dividend Payments**

Suppose that an asset pays dividends (D_t) periodically, then the definition of asset returns needs to be adjusted. In this case, the return is defined as follows:

For the simple returns

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1. \quad \textbf{(5.9)}$$

For the compounded return

$$r_t = \ln(P_t + D_t) - lnP_{t-1}. \quad \textbf{(5.10)}$$

E. **Excess Return**

The excess return of an asset at a time t is the difference between the asset's return and the return of a riskless asset such as a short-term U.S. Treasury bill return, bank interest rates and LIBOR (London Interbank Offered Rate) rates. That is, the Excess returns is

$E_t = R_t - R_{0t}$, and the compounded excess return is

$$e_t = r_t - r_{0t}, \quad \textbf{(5.11)}$$

where R_{0t} and r_{0t} are respectively the simple and log returns of a riskless asset (Tsay, 2002; p. 5)

Excess return may be defined as the payoff on an arbitrage portfolio that goes long in an asset and short in the riskless asset with no net initial investment.

For bonds, *yield spread* is an excess yield defined as the difference between the yield of a given bond and the yield of a reference bond such as a US treasury bill with a similar maturity

Remarks

The financial econometrics literature generally employs the log-return formulation (also known as the log-price relative) for two key reasons:

1. Log-returns can be interpreted as continuously-compounded returns. This is easily seen by exponentiation to yield:

   $$\exp(r_t) = \frac{p_t}{p_{t-1}} \equiv p_t = p_{t-1}e^{r_t}, \qquad \textbf{(5.12)}$$

   which is easily recognised as the growth of the value of the Investment (p_{t-1}), over the time interval $(t-1,\ t)$ when compounding continuously at the rate r_t.
2. Continuously-compounded returns are time additive. To compute a weekly returns series from daily log returns, it is valid simply to add up the daily returns.

5.1.1 Empirical Properties of Financial Returns

a. Daily returns of the market indices and individual stocks tend to have higher excess kurtosis than do monthly returns. For monthly series, the returns of market indices have higher excess kurtosis than individual stocks.
b. The mean of the monthly return series is slightly higher compared to that of the daily returns, which is always approximately zero.
c. The standard deviations of daily returns are smaller compared to those of the monthly returns.
d. The standard deviations of daily market indexes are smaller than those of the individual stocks.
e. The empirical difference between simple and log returns is not significant.
f. The empirical distribution of asset returns has a higher peak (leptokurtic) around its mean, but fatter tails than that of a normal distribution. This means that asset returns have a distribution that is taller, and with thinner tails but with a wider support, than that of a normal density (Taylor, 2005 & 2011).

5.1.2 Empirical Peculiarities of Daily Financial data

In this section, we summarise some of the findings on the empirical characteristics which have been established by the previous studies, to have characterised daily financial data across different exchange markets

a. Daily returns appear to have a much higher kurtosis than is consistent with the normal distribution (McNeil, Frey and Embrocates, 2015).
b. The distribution of daily returns is said to be leptokurtic, meaning narrower in the centre, but longer with heavier tails compared to the normal distribution. The measure of thickness of tail of a distribution, called Kurtosis, measures the mass in the centre relative to the non-centre part of the distribution (Danielson, 2011). Skewness, on the other hand, is a measure of how symmetric the distribution of a variable is relative to the mean. Note that both skewness and kurtosis are sensitive to outliers such that when skewness is negative for instance, it implies the returns is more left-tailed than right-tailed, meaning that extreme losses are more probable than the extreme gains and vice-versa.
c. The tails of daily or short time interval returns decay slowly according to the power law.
d. The Leverage Effect is a negative correlation between past returns and future volatility of returns. It could be defined as the ratio of debt to equity of a company or firm such that the higher the leverage effect, the greater the risk or volatility of the firm. High leverage occurs due to negative returns, which translates to low equity prices, meaning, a higher debt-equity ratio of a firm (Black, 1976; Christie, 1982; Henry, 1998). Negative leverage means that a positive shock has less effect on the volatility compared to a negative shock/news. When there is a high volatility in returns, the risk of doing business goes up; thus, investors move their funds to a less risky investment
e. A sudden rise in returns, identified by a positive value of the residual, ε_t, indicates the arrival of good news, whereas a negative value of ε_t implies bad news.
f. Daily asset returns mostly have negative or zero autocorrelation at lag 1 especially. A lack of autocorrelation corresponds with weak-form market efficiency, indicating that returns are non-predictable.
g. The significant autocorrelation in absolute returns $|r_t|$ or their squares with a positive and slowly decaying autocorrelation function such that $corr(|r_t|, |r_{t+k}|) > 0$ for k ranging from a few minutes to several days or weeks, indicates substantial volatility clustering in returns (Mandelbrot, 1963; Cont., 2005). Significant autocorrelation in squared returns implies that there is presence of **Autoregressive Conditional Heteroscedasticity** or ARCH in the returns. A lack of ARCH effects in the returns series however, is an indication that the series is dependent (Danielson, 2011).

In the meantime, considering previous studies, such as: Granger and Ding (1995), Campbell, Lo, and Mackinlay (1997), Granger, Spear and Ding (2000), and Cont (2011), the focus of this study will primarily be on the following properties of financial asset returns as they relate to Nigerian banks.

Stationarity: That the prices of a financial asset traded over times are usually not stationary, likely due to the steady growth recorded in the economy, increased productivity levels due to technological advancement, and economic recessions or financial meltdown. On the other hand, the returns normally fluctuate around a constant level over a period.

Heavy Tails: The probability distribution of the returns r_t often exhibits heavier tails than those of a normal distribution. A frequently used tool for checking the tail-heaviness is the Quantile-Quantile (Q-Q) plot, which will be discussed in further detail in the later sections of this chapter.

Asymmetry: The distribution of the returns r_t is usually negatively skewed, indicating that downturns in financial markets are often much steeper than recoveries. Investors' reaction to negative news compared to positive news appears to be stronger.

Volatility Clustering: This concept indicates that returns of equal magnitude and sign occur in clusters. That is, large price changes are trailed by large price changes, and periods of calm are followed by small changes in price, thereby producing high volatility.

Aggregational Gaussianity: In this case, a returns series over *m* number *of* days is simply the aggregation of *m* daily returns. This means that when the time horizon increases, the central limit theorem comes into play such that the distribution of the returns over a long time-horizon (such as a month) tends towards a normal distribution (Rogers and Zhang, 2011).

Long Range Dependence: The returns themselves rarely exhibit any serial dependence, which does not imply that they are independent. Practically, however, both daily absolute and squared returns often show small and significant autocorrelations. The autocorrelations are more persistent for absolute returns than for squared returns, indicating that the signs of long-memory are stronger in the former than in the latter, but these autocorrelations gradually become weaker and less persistent when the sampling interval is increased from a day, to a week, to a month(Cont., 2001; Thompson, 2011).

Leverage effect: according to this concept, asset returns tend to be negatively correlated with changes in volatility (Black 1976, Christie 1982), meaning that as asset prices drop, firms become more leveraged- their debt to equity ratios increase, hence it becomes riskier to trade in such assets, because their stock prices become more volatile. However, when stock prices become more volatile, investors push for high returns, leading to a price drop. Also, volatilities due to price decline are usually higher compared to rises in price due to reduced volatilities.

5.2 Graphical Presentation of Data

In this section, we briefly discuss the various methods adopted to graphically summarise our data with a view to identifying various stylized facts characterising the returns of the banks under study. Specifically, we define a time plot, a histogram, a quantile-quantile plot and outliers.

5.2.1 Time Plots

A graphical depiction of time series data is called a time plot. This is simply a line plot with the time series data on the y-axis and the time index on the x-axis. Time plots are useful for a quick examination of the inherent features (such as trend and patterns), of time series data.

5.2.2 Histograms

A histogram of returns is a graphical summary used to describe the general shape of the unknown probability density function (pdf) of a return, $f_R(r)$. It is constructed by ordering the returns from smallest to the largest, dividing the data range into N equally sized bins, and plotting the frequencies of the bins as areas of bars in the histogram. Histograms are useful in visually assessing the normality of a dataset and are distribution-free.

Moreover, since histograms approximate the shape of the population distribution through the frequency polygon, all the empirical statistical measures such as sample means, standard deviation, skewness and kurtosis used to describe the stylized facts of returns apply to histograms of the returns.

5.2.3 Quantile-Quantile (QQ) Plots

In addition to visualising the shape of the histograms of the returns and error terms associated with different models explored in the thesis, we will use QQ plots to determine whether the returns are random samples from a specified probability distribution, typically a normal distribution, by comparing the empirical quartiles of the observed returns and error terms to those from a reference probability distribution. If the plots closely follow the 45-degree line, then this provides strong evidence that the reference distribution appropriately describes the observed data. Technically, the QQ-plot is composed of the reference distribution quartiles on the x-axis and the empirical quartiles on the y-axis. If the quartiles do not match up, then the shape of the QQ-plot indicates which features of the data are not captured by the reference distribution.

5.2.4 Outliers

We will detect outliers in the bank stock returns using the standard approaches based on box plot limits implemented in mainstream statistical software. Any outliers that are due to data entry error will be removed from the data. Genuine outliers provide important information about the behaviour of bank returns in the different sub-periods of the study and should not be removed from data sample. For financial market data, outliers are generally extremely large or small values that could be attributed to data entry error (e.g. price entered as 10 instead of 100) or a valid outcome associated with some unexpected bad or good news.

On the assumption that a series say, $X \sim N(\mu, \sigma^2), \Pr(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 0.99$.

However, since the mean μ and the standard deviation σ are not robust against outliers simply because they become larger in the presence of outliers, we then replace them with median and interquartile ranges (IQR), which are very robust to outliers. Thus, a moderate outlier in the right tail of the distribution may be determined by box plots to fall outside the limits (https://faculty.washington.edu/ezivot/econ424/descriptivestatistics.pdf)

$$\hat{q}_{0.75} + 1.5 * IQR < x < \hat{q}_{0.75} + 3 \times IQR. \quad \textbf{(5.13)}$$

If the data were normally distributed, then

$$\hat{q}_{0.75} \approx \hat{\mu} + 0.67\hat{\sigma}, \text{ and } IQR = 1.349\hat{\sigma}$$

And **(5.13)** becomes: $\hat{\mu} + 2.67\hat{\sigma} < x < \hat{\mu} + 4.67\hat{\sigma}$. The same applies to a moderate outlier falling in the left tail of the distribution (below the median), and it is a data point falling in the range:

$$\hat{q}_{0.25} - 3 * IQR < x < \hat{q}_{0.25} - 1.5 * IQR \quad \textbf{(5.14)}$$

The extreme outliers are found outside of the tails of the distribution, defined as:

$$x < \hat{q}_{0.25} - 3 * IQR \text{ and } x > \hat{q}_{0.75} + 3 * IQR \quad \textbf{(5.15)}$$

Note: The relatively large proportion of outliers in return series may be due to the presence of bubbles, anomalies and pronounced market volatility.

5.3 Results and Discussions

In this section, the time series plots for the daily, monthly and financial crisis periods shall be presented and briefly discussed, including discussions on the distributions and the autocorrelations of the sixteen Nigerian banks (Access, Afribank, Eco bank, Diamond, First bank, First City Monumental bank (FCMB), Fidelity, STANBIC IBTC, Skye, Sterling, United Bank for Africa (UBA), Guaranty Trust bank (GTB), Union, Unity, WEMA and Zenith banks). Further discussions will focus on the summary statistics and relevant tests such as: skewness, kurtosis, normality and ACF tests, leverage diagnostic tests, and long memory tests. Essentially, by the end of our presentations and discussions, questions about the presence or absence of the highlighted stylized features of various banks' stock data will have been addressed. We note that except for Afribank, the remaining fifteen banks are among the top active and top traded stocks in the financial services industry, as listed on the Nigerian Stock Exchange (NSE).

5.3.1 Discussion of the Time Plots for the Daily Data

Specifically, this subsection presents and discusses time plots of the closing stock prices, log returns, absolute (log) returns and squared (log) returns recorded daily across the sixteen (16) banks of the Nigerian stock markets (NSM) listed above. In each of the Figures to be presented across the 16 banks is a 2 x 2 matrix of time series plots. Starting from the upper left panel is the price, log returns, absolute returns and squared returns (the 4^{th} on the bottom right) series, respectively.

To start with, **Figure 5.1** has the four series of the Access bank from June 1999 to December 2014. From the price series we can see a gradual but slow increase in price from June 2002 up to the period of the bank consolidation program of the CBN between August 2004 and December 2005 till around August 2006 before a sudden boom that was sustained till January 2008, when the series attained an unprecedented height before a sharp decline around April 2008, and it then dropped to its lowest point around January 2009. From then onward, attempts at rising again were characterised by ups and down till the end of 2014.

The returns series shows spikes and non-constant oscillation around a constant level close to zero as against the price which shows trend. Moreover, high oscillations not only persist but tend to cluster together, reflecting more volatile markets. The length of persistence in volatility close to the end of 2007 till the end of 2014 is much longer than those from June 1999-Sept. 2001 and June 2002 to June 2004. The interplay of tranquil periods from Oct. 2001 to May

2002 and Sept. 2004-Sept. 2005, with the highly volatile (turbulent) periods, indicates evidence of volatility clustering.

Unlike the log returns which oscillate around a constant level, both the absolute and squared returns series, which generally represent volatility, fluctuate positively. From the Figure, there are clear signs of sharp jumps in volatility (more pronounced in absolute than squared returns), at three major points of around September 2001, August 2004 and October 2006 respectively, with that of around October 2006, which coincides with a positive spike in log returns, being the longest.

We can see that following the 2007 positive spike in returns, there was a decline in returns between 2008 and 2009, which may be associated with the effects of the 2007-09 global financial crisis. There was a negative spike in 2004 which suggests that the bank was negatively affected by the CBN bank reforms of 2004. These facts indicate how to generally relate fluctuations in bank returns presented in this subsection to different financial policy and crisis periods, albeit visually. The point of detailed modelling of the empirical financial features manifested by these returns in subsequent chapters of the thesis is to explore possible significant differences in the stochastic models that describe the fluctuations, for example the different types of ARCH-GARCH models that explain return volatilities.

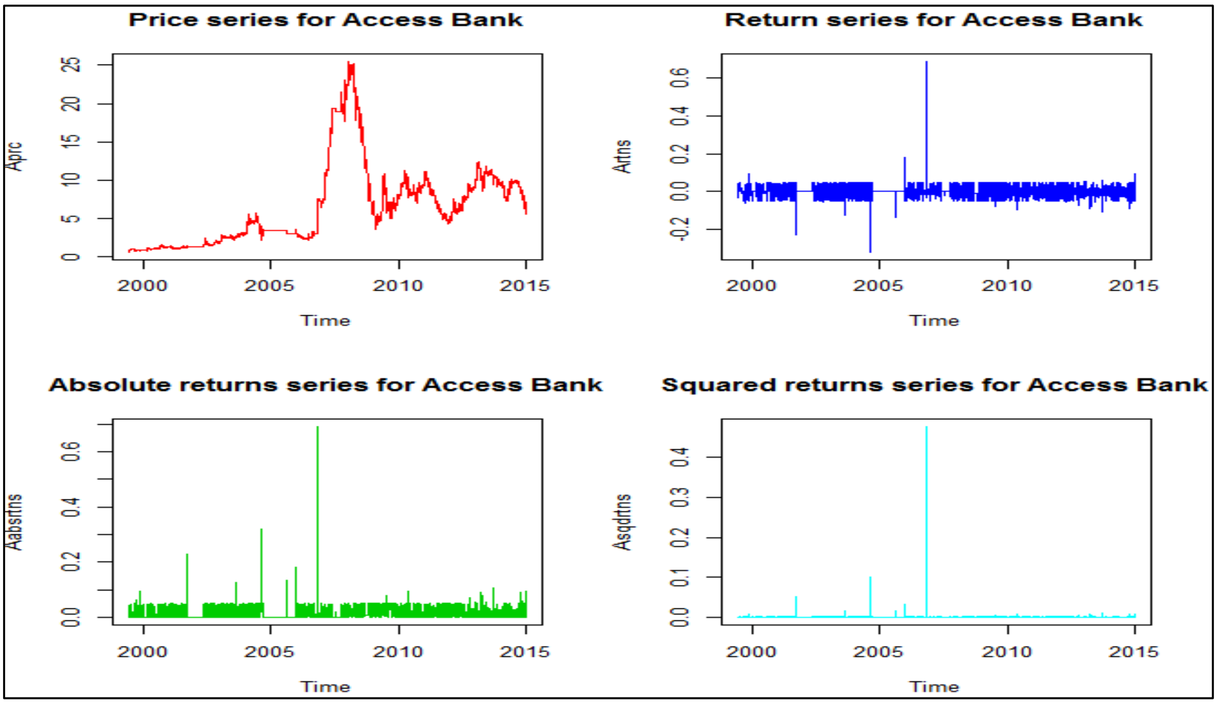

Figure 5.1: Access bank daily Closing Price, log Returns, Absolute Returns and Squared Returns (June 1999-Dec 2014)

Note that the presentation and interpretations of the above plots for the remaining banks follows the same patterns and these are presented in Figure 5.2a to 5.16a (see Appendix **5A**). However, to compare time plots for the daily closing prices and the return series among the 16 banks, **Figure 5.2, 5.3, 5.4** and **5.5** are presented below.

Comparative Price-Return Plots and Bank Characteristics

Given the foregoing notes on the observed differences in banks' returns profiles across different periods related to financial reforms and crisis, Table 5.1 below classifies the banks into two groups, namely: A:"those whose trading activities predate 2004 bank consolidation" and B: "those starting during and after the bank consolidation in 2004**".**

Table 5.1: Classification of Nigerian Banks into Groups A and B (T is number of days)

Grp A- Trading before 2004	**Periods (T)**	**B: Started Trading during and 2004**	**Period (T)**
Access	June, 1999 – Dec 2014 (15.5 years) **3869**	**Diamond**	May 2005-Dec 2014 (9.58 years) **2368**
First	June 1999 – Dec 2014 (15.5 years) **3869**	**Ecobank**	Sept 2006-Dec 2014 (9.25 years) **2050**
GTB	June 1999 – Dec 2014 (15.5 years) **3869**	**FCMB**	Dec 2004-Dec 2014 (10 years) **2474**
Sterling	June 1999-Dec. 2014 (15.5 years) **3869**	**Fidelity**	May 2005 – Dec 2014 (9.5 years) **2376**
Union	June 1999-Dec 2014 (15.5 years) **3869**	**Skye**	Nov. 2005-Dec. 2014 (9.083 years) **2391**
UBA	June 1999-Dec 2014 (15.5 years) **3869**	**STANBIC**	April 2005-Dec 2014 (9.67 years) **2391**
WEMA	June 1999-Dec 2014 (15.5 years) **3869**	**Unity**	Dec 2005-Dec 2014 (9 years) **2223**
Afribank	June 1999-Sept 2011 (12.2 years) **3046**	**Zenith**	Oct 2004-Dec 2014 (10.17 years) **2516**

Now we present the following **Figures 5.2, 5.3 5.4 and 5.5** to give clearer pictures of the comparative price and log returns behaviours of the sixteen banks. From **Figures 5.2 and 5.3**, a critical assessment of the price plots across the sixteen banks shows that the 2007-2009 global financial crises had significant impacts on Nigerian banks, given the fact that virtually all the banks price series experienced boom-bust periods from 2008 to 2009. Banks such as STANBIC, Union, Unity, WEMA and possibly GTB, however, recorded at least two episodes of boom-bust behaviour that could be a bubble, which we will investigate in chapter 8 of this thesis.

A further in-depth look reveals that the price series of **Diamond, FCMB** and **Fidelity** banks behave almost the same way. Banks such as **Diamond** and **Skye** witnessed a slight increase in price for at least three years after the reforms. **WEMA** and **Unity** banks experienced a second rise in price mid-2014.**GTB** responded positively to the second reforms after the crisis in 2009 showing sustained increase in stock prices till close to the end of 2014.Other banks such as **ECO, Afribank, FCMB, Fidelity, First, Sterling, UBA, Union**, and **Zenith** banks did not experience such significant rises in price of their stocks after the financial crisis and the initiated reforms, with **Eco Bank** being worst off due to the crisis.

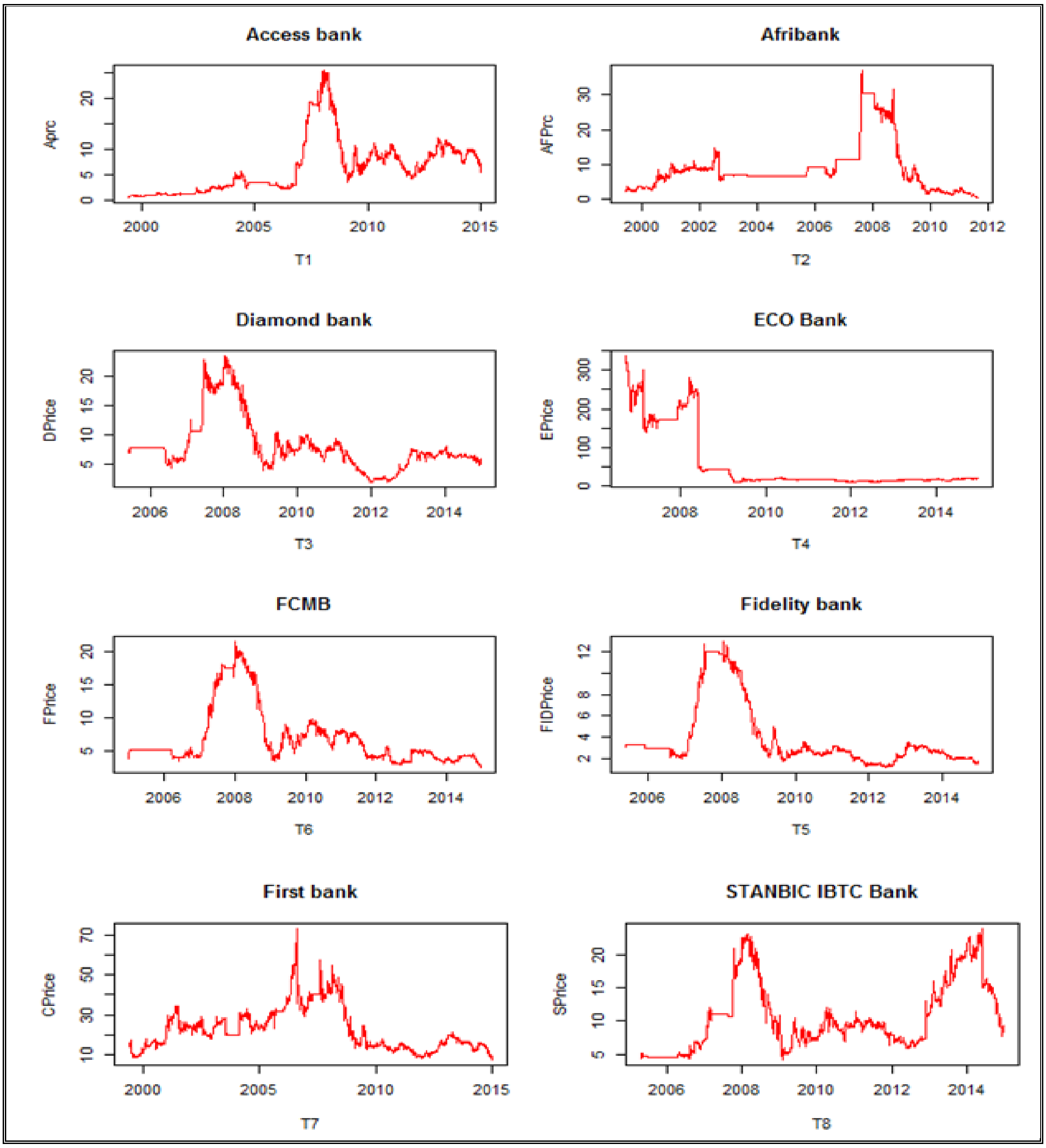

Figure 5.2: Time Plot series for Daily Closing Prices across the Nigerian Banks for the Overall Period

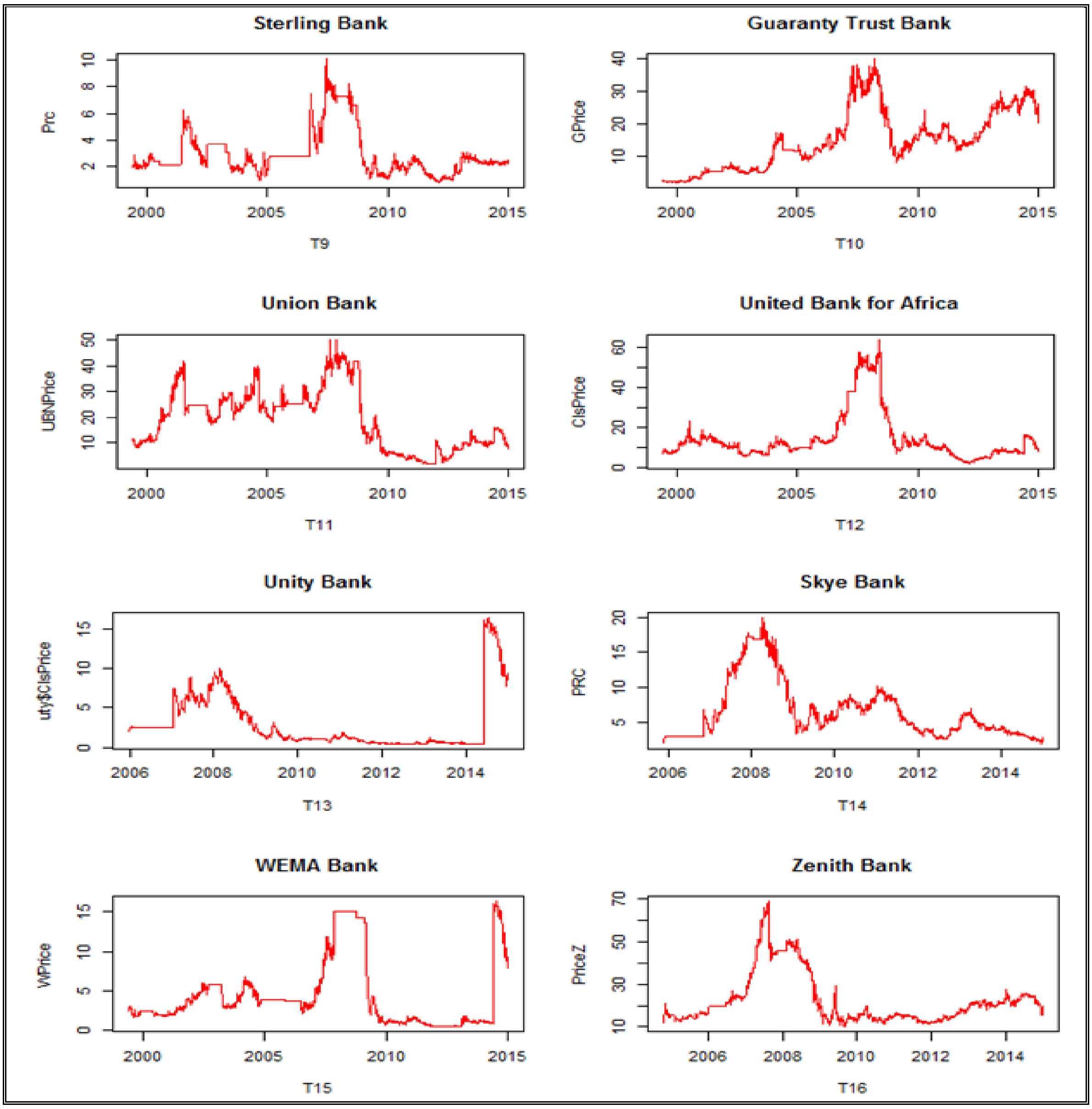

Figure 5.3: Time Plot series for Daily Closing Prices across the Nigerian Banks for the Overall Period

Moreover, while the return series of **Diamond, FCMB and Fidelity** banks are highly volatile and persistent (**see Figure 5.4**), indicating that for the periods under investigation, investments in the three banks are associated with high levels of risk, while those of **First GTB and Zenith** banks are dominated by negative spikes and are fairly volatile. **STANBIC, Union, WEMA and Unity** banks, however, behave the same way with a low level of volatility (See Figure**5.3** below).

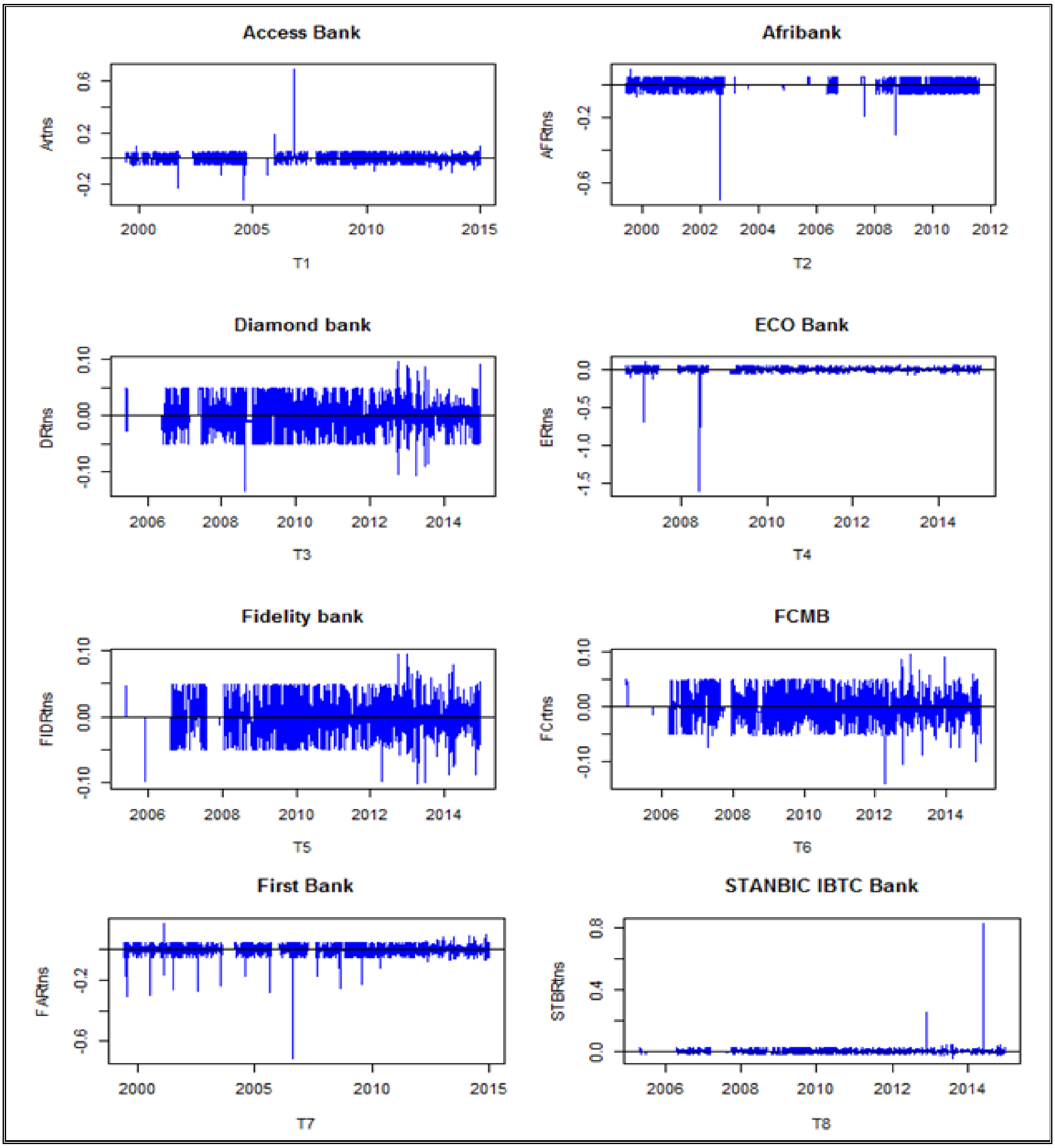

Figure 5.4: Time Plots for Daily Log Returns of the Nigerian Banks at Overall Periods

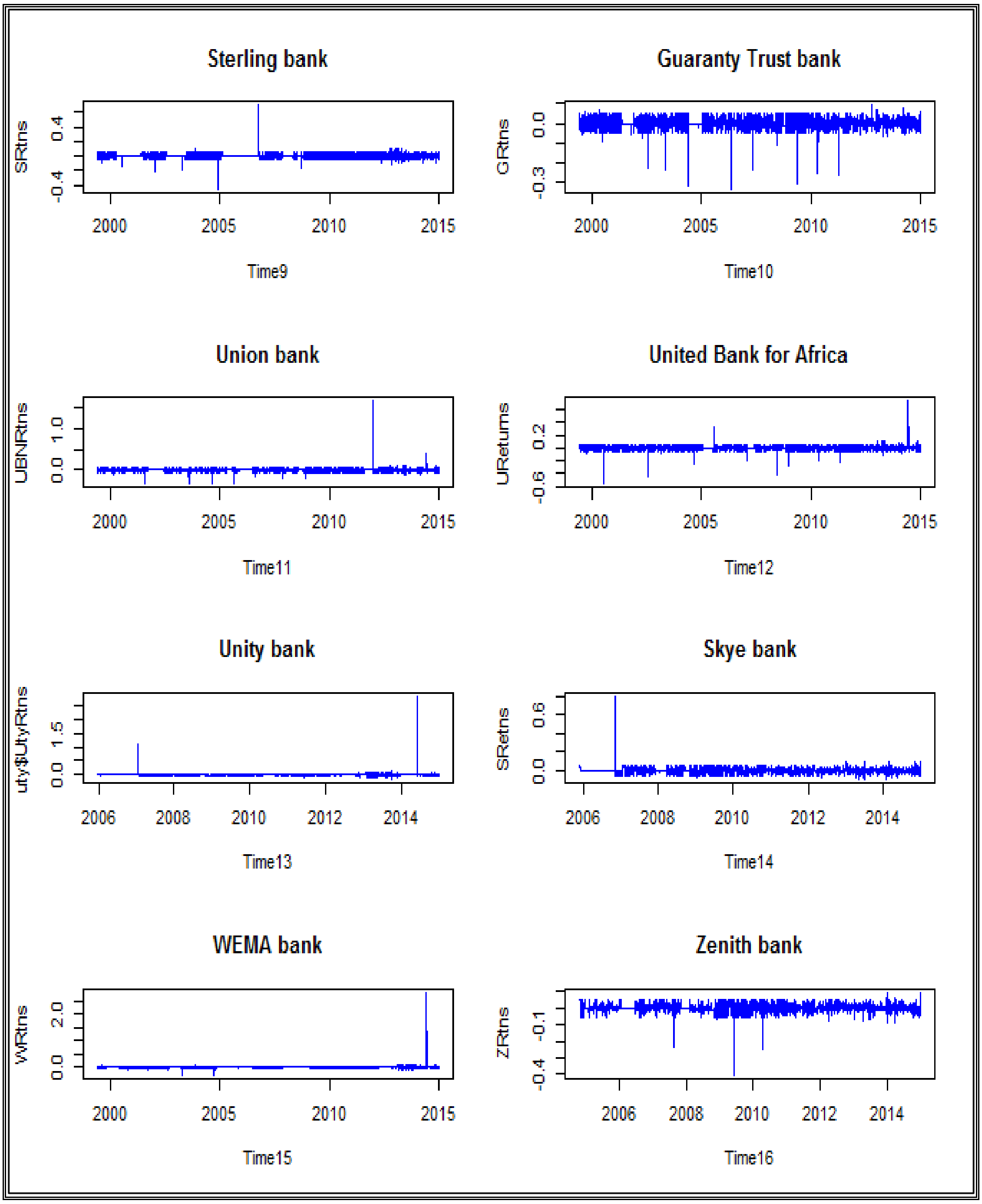

Figure 5.5: Time Plots for Daily Log Returns of the Nigerian Banks at Overall Periods

Summary, the findings obtained up to now could generally be summarised as follows.

1. The price series across the 16 banks exhibit random walk like behaviour. That is, there is no tendency to revert to a time-independent mean so that the series non-stationary.
2. All the banks show one large, boom-bust behaviour in their prices during the run-up to the financial crisis.
3. STANBIC, Unity, WEMA and GTB show second signs of boom-bust behaviour after the second bank reforms initiated by the CBN in the aftermath of the financial crisis.

4. Due to the oscillating movements of log returns around a constant mean, which is approximately equal to zero, there is evidence of mean-reversion behaviour through time across the 16 banks; thus, the return series appear to be stationary. The constant mean value of 0 is an indication that the common mean value assumption of covariance stationarity seems to be satisfied. This is a major stylized fact of asset returns as discussed earlier and to will be explored further through appropriate test statistics in the subsequent section.
5. The fluctuations of the returns about the mean across the banks appear to change overtime confirms the time-variation or volatility of the asset returns. However, visually, with the nature of the return series we have, one cannot be sure if there is evidence of systematic time dependence (at least in a linear form), in the returns; this shall be investigated by examining if the autocorrelations of the log returns are approximately zero.
6. The volatility level in absolute returns is more pronounced and persistent through time than the squared returns across all the banks, and this is visual evidence of long-range dependence or predictability in volatility - another major stylized fact of volatility across different markets, as noted in the literature (see, Danielson, 2011; Tsay, 2002).

5.3.2 Discussions of the Monthly versus Daily Time Plots

It is important that prior to discussing the respective bank's time plots, the following are noted:

1. To obtain the monthly closing price series, we concentrate only on the last trading day of the month's closing price from the daily closing price of each bank.
2. The monthly return series for each bank is obtained from the monthly price series generated in (1) above, after which we obtained both the absolute and squared returns from the monthly returns.

As argued in Ezepue and Omar (2012), empirical finance typically explores differences in series behaviour across different time epochs, for example daily, monthly and yearly, since different short-, medium- and long-term investment decisions depend on such differences. Figure 5.6 presents the time plots for the monthly stock prices, returns, absolute and squared returns for Access bank, while those for the remaining 15 banks are presented in Appendix 5B**(ii).**

The following are the major observations made across the 16 banks' series:

1. The monthly closing stock price series for the respective banks generally resemble their daily closing series, but those of the former are more revealing and well spread than those of the latter.
2. The spikes in the monthly returns are not as clustered as those in the daily returns; their oscillations are rather smoother and better spread than those of the daily returns.
3. The monthly returns series fluctuate more like Gaussian white noise compared to the daily returns, and the inherent heteroscedastic component of the monthly returns is easier to visualise compared to the daily return series.
4. In effect, the volatility level in the monthly series is far more pronounced and spread out and not as clustered as those of the daily series. This can be confirmed from both the monthly absolute and squared returns across all the banks.
5. The spikes in the monthly volatility are longer and better spread compared with those of the daily volatility (see **Figure 5.6**).

Meanwhile, we shall further examine the distribution of the monthly series through histogram and QQ-plots in the relevant section to investigate the level of normality of the monthly series as against the daily series.

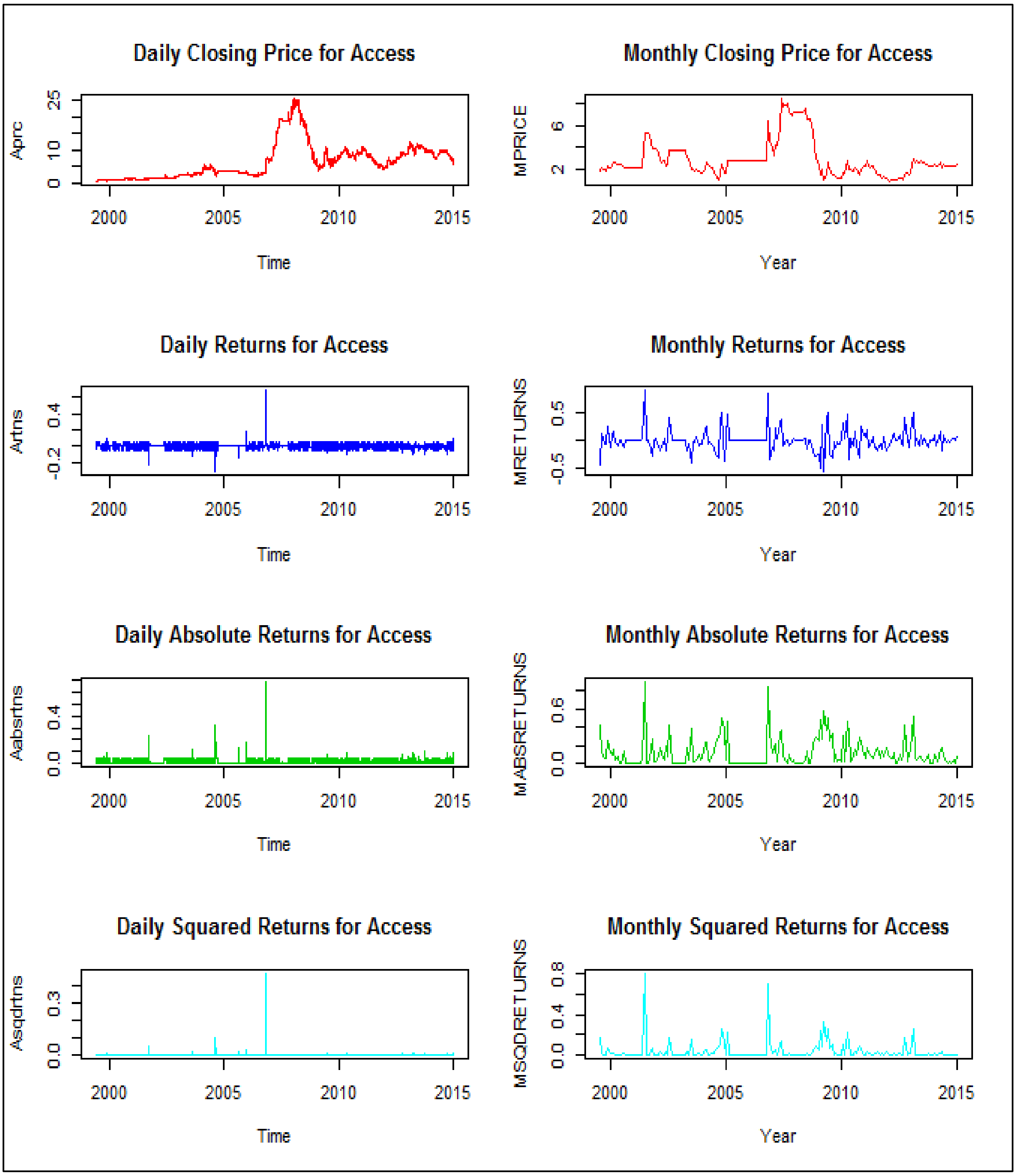

Figure 5.6: Access Bank-Daily versus Monthly Time Plots for Price, Log Returns, Absolute and Squared Returns (June 1999- Dec. 2014)

5.3.3 Discussion of the Time Plots for the Banks during Financial Crisis

Table 5.2 below presents the length of time taken by the 2007-2009 financial crisis across the sixteen Nigerian banks. From the table, Afribank and Zenith experienced the crisis for the longest periods of 18 and 17 months (see column 3) respectively. The banks with the shortest lengths of **4 and 3** months are respectively Union and WEMA banks.

Table 5.2: Length of Financial Crises across the Sixteen Banks

Bank	Downturn Periods	Length in months
Access	April 2008-Feb., 2009	11
Afribank	August 2007-Feb., 2009	18
Diamond	Jan 2008-Jan, 2009	12
ECO	May 2008-Feb, 2009	10
FCMB	Jan 2008-March, 2009	15
Fidelity	Feb. 2008-April, 2009	14
First	Feb. 2008-April, 2009	14
STANBIC	March 2008-Jan, 2009	11
Skye	April 2008-Jan, 2009	10
Sterling	May 2008-May 2009	12
UBA	May 2008-Jan, 2009	9
Union	Oct 2008-Jan, 2009	4
GTB	March 2008-Jan, 2009	11
Unity	Feb. 2008-March, 2009	11
WEMA	Feb. 2009-April, 2009	3
Zenith	August 2007- Jan, 2009	17

From **Figure 5.7**, we see that while some banks are faced with a gradual decline in their daily stock prices for the crisis period, the **ECO** and **WEMA** banks are characterised by sharp declines. Unlike other banks, which attempted to readjust after the downturn and the introduction of the second reforms in June 2009, **ECO** bank was unable to get out of the crisis despite the second reforms.

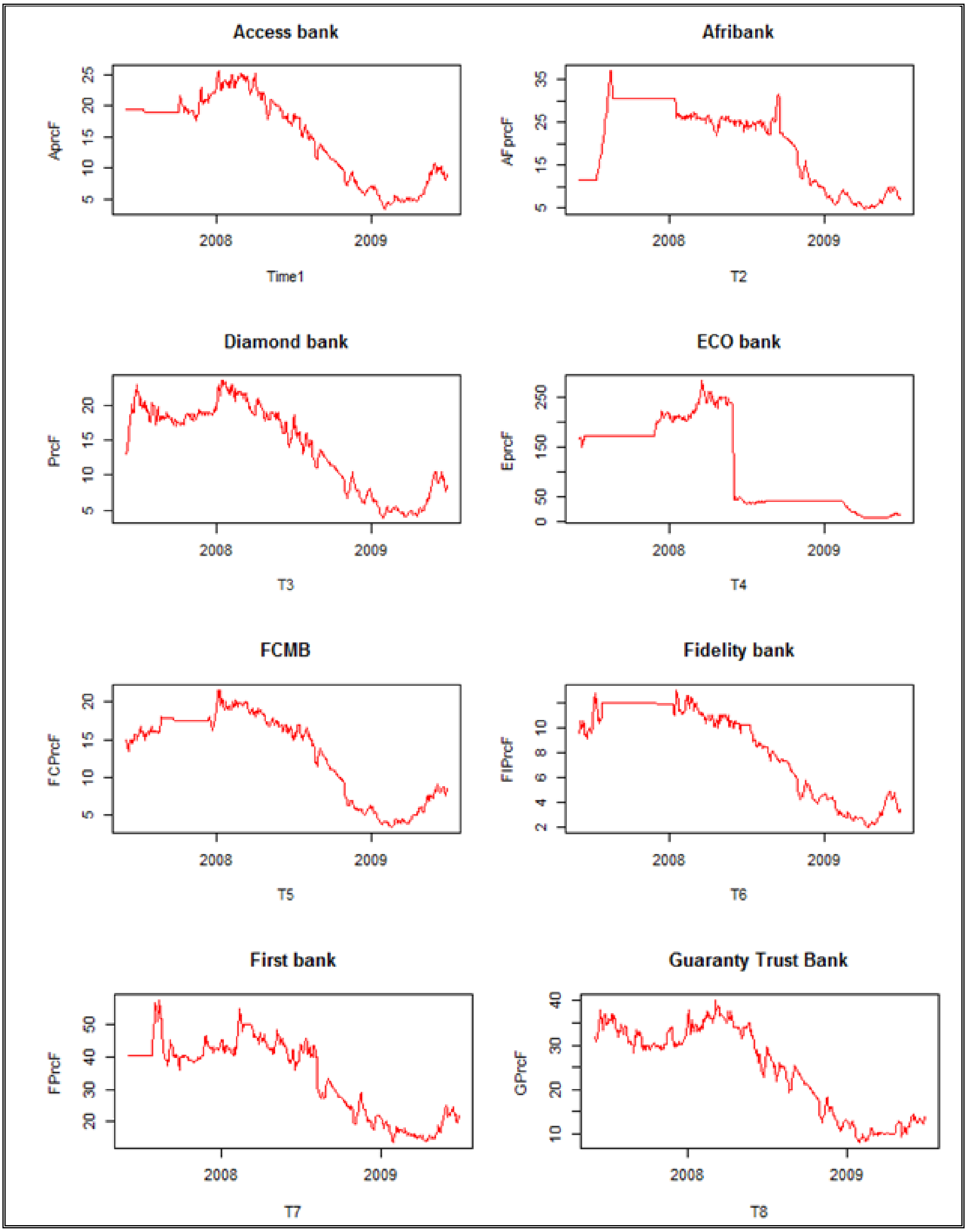

Figure 5.7: Price Series across Eight of the Nigerian Banks for the Periods of Financial Crisis

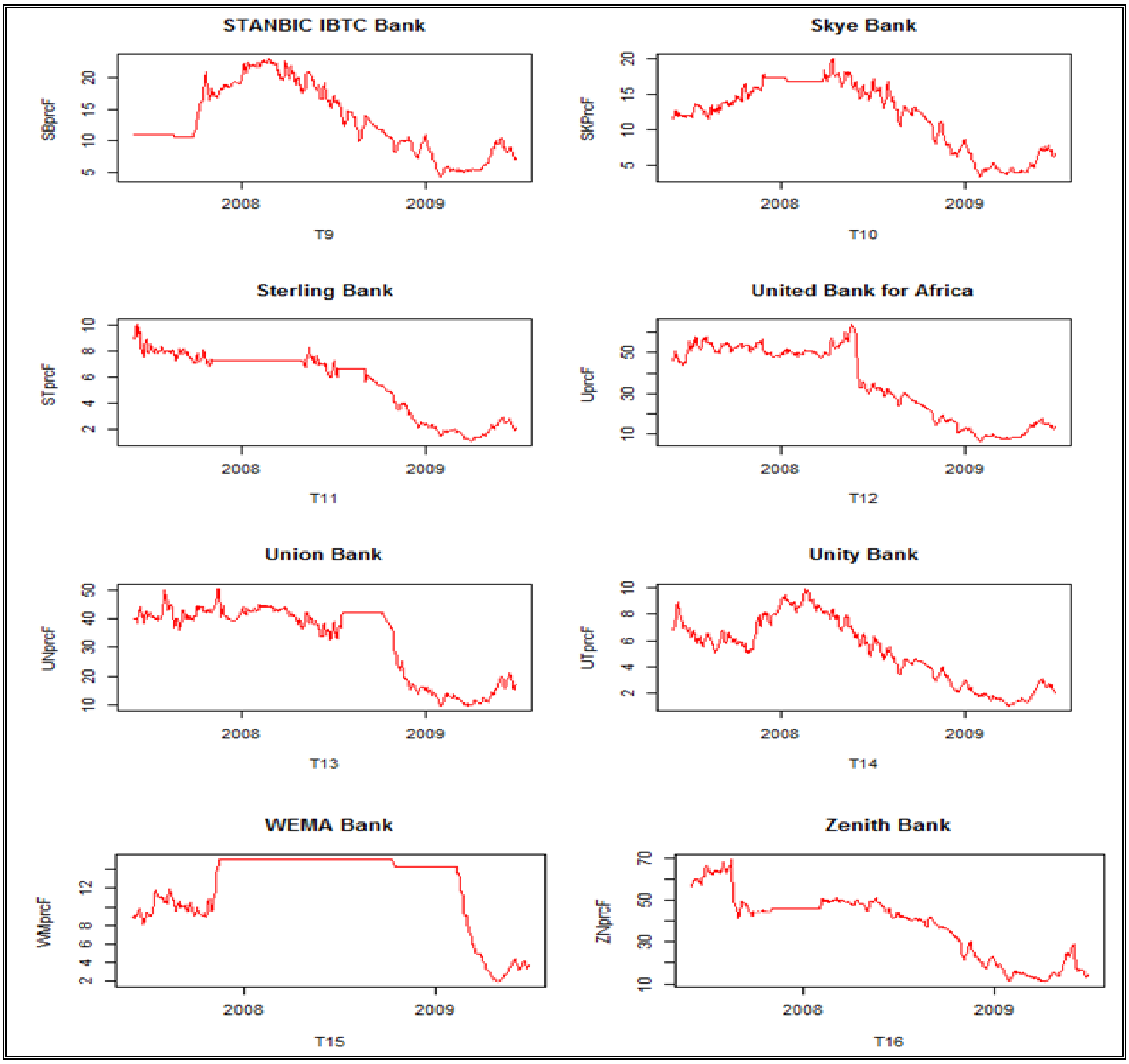

Figure 5.8: Price Series of the Nigerian Banks for the Periods of Financial Crisis (Contd)

Visual assessment of the returns series as presented in Figures **5.9 and 5.10** reveal that **Unity bank** is most volatile, followed by **FCMB** and **Fidelity** banks (to nearly the same degree). Next are **Access** and **STANBIC** (to a similar degree), then Diamond and Skye. All the banks exhibit long-range volatility persistence. While WEMA Bank experienced along-range level of calmness in between two extreme volatile periods, ECO remained the least volatile during the crisis.

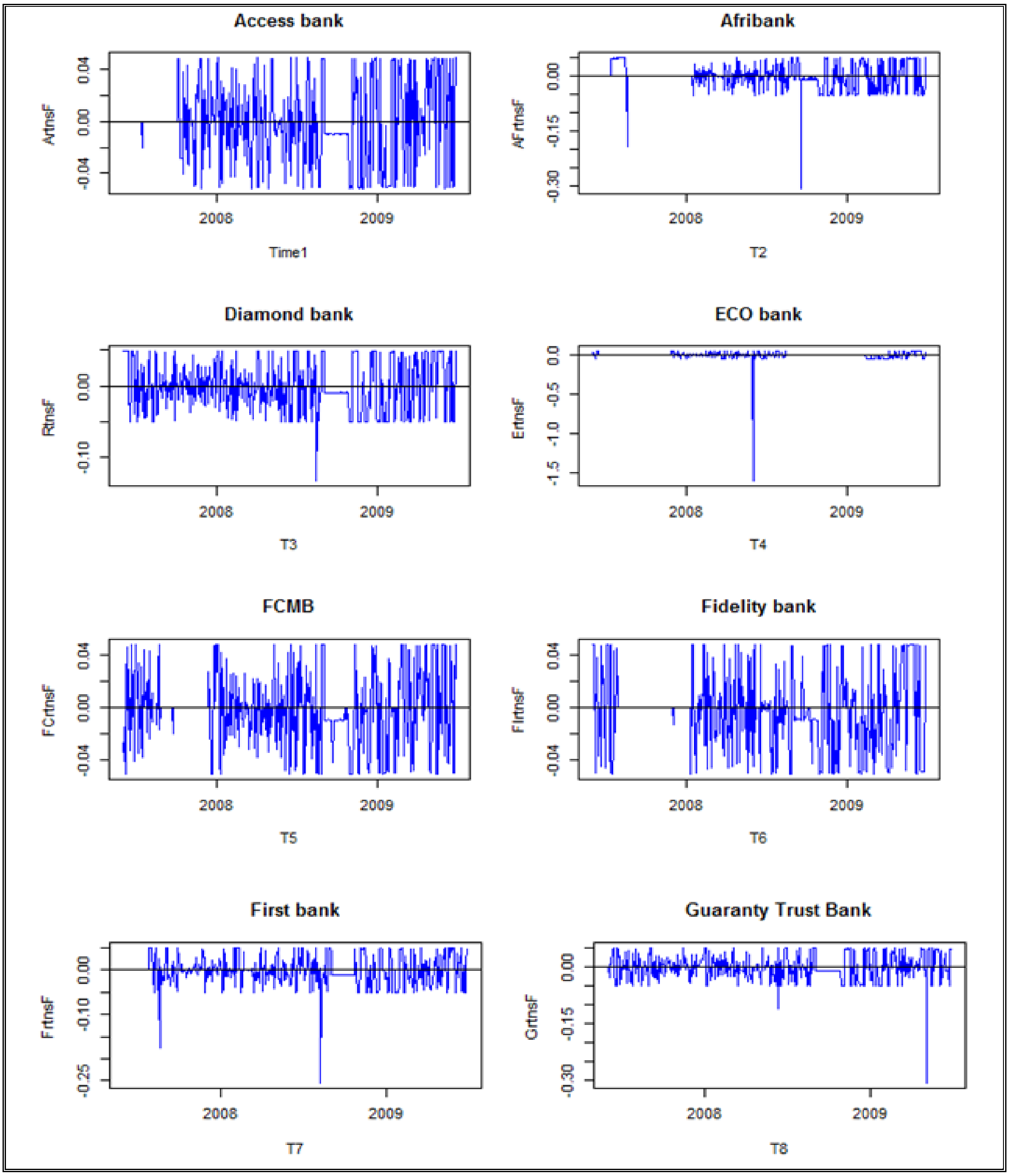

Figure 5.9: Time Plots for Daily log returns of the Nigerian Banks during the financial crisis

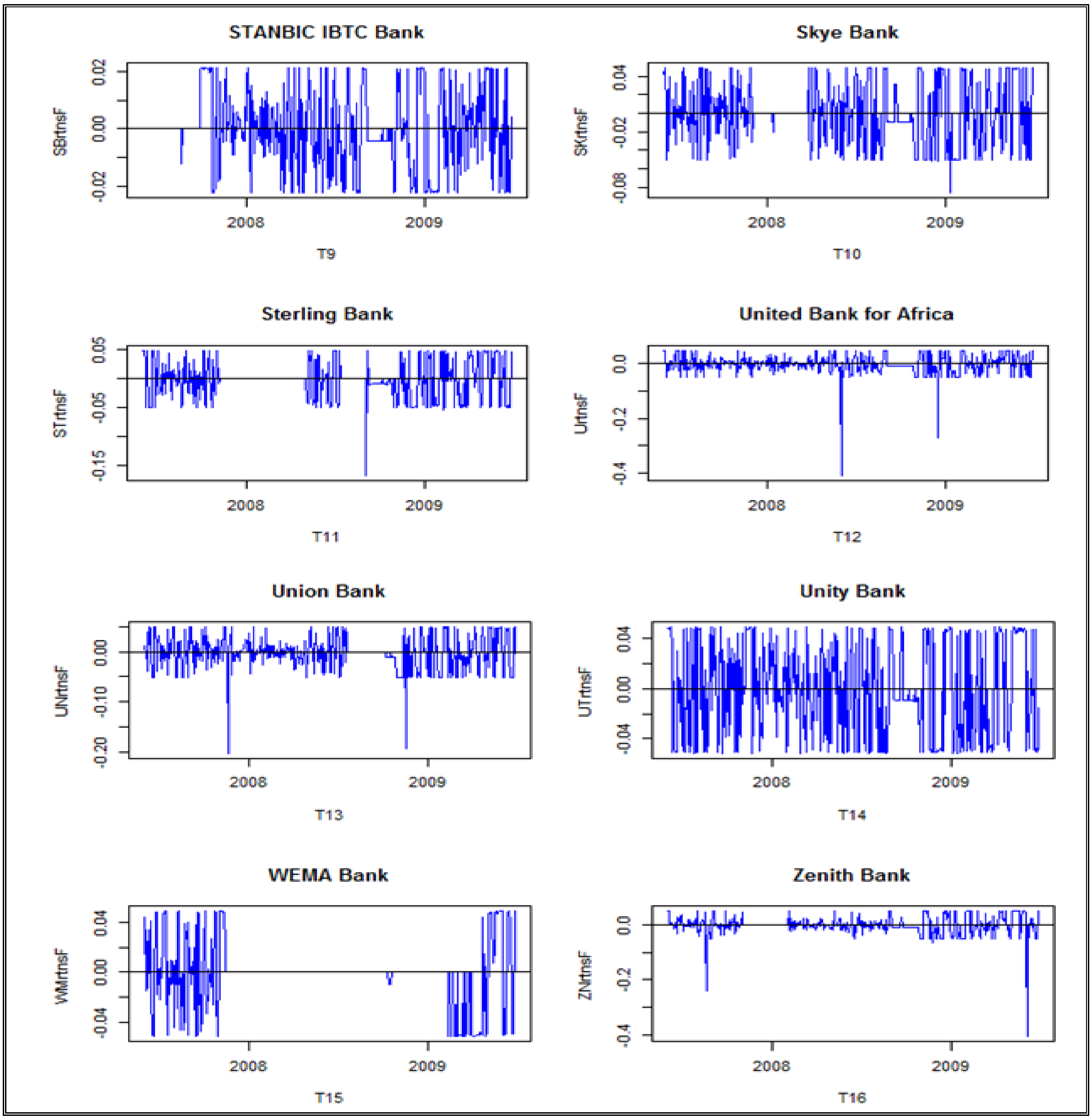

Figure 5.10: Time Plots for Daily log returns of the Nigerian Banks during the financial crisis (Contd)

Table 5.3 provides a subjective assessment of the observed degrees of volatility in the log returns plots across the 16 banks for the overall and financial crisis periods, based on the above visual assessments, which will be explored in more detail in Chapter 9.

Table 5.3: Summary of Visual Degrees of Volatility across the 16 Banks for the Overall and Financial Crisis Periods

Degree of Volatility	**Bank: Overall**	**Bank: Financial Crisis**
Low	STANBIC, Unity, WEMA, ECO, Union	ECO and Zenith
High	Access, First, Skye, Sterling, Afribank and UBA	Sterling, Union, First WEMA, GTB and Afribank
Higher	GTB and Zenith	Diamond and Skye

Highest	Diamond, FCMB and Fidelity	Unity, STANBIC, Fidelity, FCMB and Access

Notice that some banks have moved columns from different degrees of volatility across the overall or financial crisis periods. For instance, STANBIC Bank has overall a low volatility but has one of the highest volatilities during the financial crisis. These findings have subtle implications for investment decisions regarding the potential risks associated with investing in different banks' stocks across different investment horizons. For long-term growth investment objectives, the lower-risk bank assets could be of interest while for short-term speculative trading, the higher volatilities in both periods may be of interest. Such investment decisions must consider the differential dynamics of the returns examined above, and financial analysis of the business models, management effectiveness, and fundamental valuations of the different banks which these imply. For portfolio construction and management involving Nigerian banks, a correlation matrix of the bank assets will enable investors to choose optimally diversified portfolios. These investment considerations which may be linked further to SSMCD within the banking sector of the NSM are outside the scope of this research.

5.3.4 Discussion of Summary Statistics for the Daily and Monthly Data for the Overall and Financial Crisis Periods

Table **5.4** presents summary statistics for daily overall data with indicated total daily observations (T). Seven banks, namely Access, First, Guaranty Trust, Sterling, UBA, Union and WEMA banks have the highest sample size (T) of 3869 observations, and ETI, also known as Ecobank, has the least data size of 2050 observations.

With respect to their expected returns, the following banks: Afribank, Diamond, ETI, FCMB, Fidelity, First, and Union banks were found to have negative means, while the remaining nine are positive, and they are all approximately zero. However, the median return is zero for all banks. The standard deviation is far higher than the mean across the banks, thereby causing the coefficient of variation (CV) to be very high for each of the banks. For example, for Access bank, the standard deviation (0.029) is about 48 times higher than the mean (0.0006), a behaviour common to asset returns across most global markets (see Zivot, 2009).

Regarding skewness, seven of the bank's returns are negatively skewed, namely: Afribank (-4.375), Diamond (-0.043), ETI (-23.2311), FCMB (-0.057), First (-5.0916), GTB (-2.216) and Zenith (-2.280). Hence, their returns have longer left tails, meaning that extreme losses are more probable than gains, while the remaining nine banks' returns, being positively skewed with long right tails indicate more probable gains than losses. Of all the banks however, ECO, with skewness -23.2311, is found to be the most negatively skewed, while WEMA, with a skewness of 36.1655, is the most positively skewed. The above implications of negative or positive skewness hold even more for these two banks.

While the least excess kurtosis of 0.2706 is attributed to **Diamond** bank, the following banks have kurtosis above 500; **WEMA** (1882.011), with the highest kurtosis, **Unity** (1085.073), **STANBIC** ((1065.405), **Union** (744.061) and **ETI** (783.7068). From our results, however, few of the banks has excess kurtosis of nearly zero (0), which is expected of a normally distributed variable; only three have excess kurtosis close to 0 (Diamond, FCMB and Fidelity). Thus, most of the banks are leptokurtic with fat tails. This implies that for a large part of the time, the banks' returns fluctuate in a range smaller than a normal distribution and assuming normality in stock returns would underestimate the impact of shocks on the daily stock returns of these banks.

Table 5.4: Summary Statistics on Daily Returns of the Banks (Overall Period)

BANKS	T	Mean	Median	Min	Max	Std Dev	Variance	Skewness	Kurtosis
ACCESS	**3869**	0.0006	0.0000	-0.3192	0.6898	0.0290	0.0008	2.9845	86.8983
AFRIBANK	**3045**	-0.0005	0.0000	-0.7036	0.0953	0.0305	0.0009	-4.3754	94.7174
DIAMOND	**2368**	-0.0001	0.0000	-0.1343	0.0953	0.0280	0.0008	-0.0430	0.2706
ETI	**2050**	-0.0014	0.0000	-1.6094	0.0976	0.0455	0.0021	-23.2311	783.7068
FCMB	**2474**	-0.0002	0.0000	-0.1400	0.0953	0.0252	0.0006	-0.0572	0.8811
FIDELITY	**2376**	-0.0003	0.0000	-0.1017	0.0946	0.0266	0.0007	0.0047	0.2950
FIRST	**3869**	-0.0001	0.0000	-0.7070	0.1643	0.0296	0.0009	-5.0916	97.7673
GTB	**3869**	0.0006	0.0000	-0.3309	0.0913	0.0269	0.0007	-2.2160	23.1739
SKYE	**2243**	0.0001	0.0000	-0.1004	0.8029	0.0330	0.0011	6.4348	155.7539
STANBIC	**2391**	0.0006	0.0000	-0.0413	0.8234	0.0206	0.0004	27.2745	1065.4050
STERLING	**3869**	0.0001	0.0000	-0.4478	0.6931	0.0324	0.0011	1.6371	63.1626

UBA	**3869**	0.0001	0.0000	-0.5698	0.7340	0.0342	0.0012	0.1967	89.7707
UNION	**3869**	-0.0001	0.0000	-0.3394	1.6743	0.0407	0.0017	17.6137	744.0610
UNITY	**2223**	0.0004	0.0000	-0.1886	2.8439	0.0725	0.0052	28.6789	1085.0730
WEMA	**3869**	0.0003	0.0000	-0.3134	2.8439	0.0547	0.0030	36.1655	1882.0110
ZENITH	**2516**	0.0002	0.0000	-0.0580	0.0969	0.0254	0.0006	-2.2796	32.3184

Table 5.5 presents summary statistics on the monthly data of the respective banks at overall period. The banks with the highest sample of 186 each are: **Access, First, Guaranty Trust, Sterling, UBA, Union** and **WEMA** banks, whereas **ECO** has the lowest sample size of 99. Nine (9) of the banks: **Access, Afribank, Diamond, ETI, FCMB, Fidelity, First, Sterling and Union** have a negative mean while the remaining seven are positive. Comparisons between the mean returns in the daily and monthly data shows that those of the monthly data are largely and relatively higher than those the daily data in general; this is in line with this established stylized fact of asset behaviour, (Zivot (2009)).

Unlike the daily returns, five of the banks: ETI, First, GTB, UBA and Zenith have a median monthly return slightly different from zero. The standard deviations are also found to be much higher than the means for the monthly returns of each bank, as was found with the daily data.

Both the degrees of skewness and kurtosis in the monthly data are drastically lower than for the daily data, due to the smoothing down of the noise in monthly returns compared with the daily returns. For example, WEMA bank with the highest skewness value of 36.1655 in the daily data reduced to 5.7294 in the monthly return. Also, the same bank with highest excess kurtosis of 1882.011 in the daily returns reduced to 59.6458 in the monthly returns, a behaviour which supports the stylized facts of asset returns that monthly financial data are closer to a normal distribution compared to daily data. Furthermore, six banks: ETI (-5.1584), First (-0.3327), GTB (-0.3554), Skye (-0.7237), STANBIC (-1.666) and Zenith (-0.9269) are negatively skewed, while the remaining 10 are positively skewed. None of the banks was attained the excess kurtosis of zero expected of a normal variate indicating that our monthly returns series is still leptokurtic.

Table 5.5: Summary Statistics on Monthly Returns of the Banks (Overall Period)

Banks	T	Mean	Med	Min	Max	Std Dev	Var.	Skew.	Kurt.
ACCESS	186	-0.0006	0.0000	-0.5738	0.9016	0.1931	0.0373	1.1358	4.4862
AFRIBANK	147	-0.0118	0.0000	-0.6686	0.6783	0.1838	0.0338	0.2056	3.2612
DIAMOND	115	-0.0023	0.0000	-0.5889	0.7186	0.1715	0.0294	0.5550	3.6408
ETI	99	-0.0281	0.0039	-1.8331	0.3914	0.2332	0.0544	-5.1584	35.1432
FCMB	120	-0.0061	0.0000	-0.3732	0.3930	0.1409	0.0199	0.0931	1.0079
FIDELITY	116	-0.0059	0.0000	-0.3865	0.6510	0.1405	0.0197	1.0543	4.4468
FIRST	186	-0.0027	0.0007	-0.4376	0.4376	0.1257	0.0158	-0.3327	3.0332
GTB	185	0.0123	0.0031	-0.3985	0.2953	0.1135	0.0129	-0.3554	1.6129
SKYE	109	0.0004	0.0000	-0.9151	0.6008	0.1727	0.0298	-0.7237	7.8071
STANBIC	117	0.0047	0.0000	-2.2930	1.8718	0.3167	0.1003	-1.6660	31.8347
STERLING	186	-0.0006	0.0000	-0.5737	0.9016	0.1931	0.0373	1.1358	4.4862
UBA	186	0.0000	-0.0031	-0.6788	0.7100	0.1663	0.0277	0.2109	3.3004
UNION	186	-0.0006	0.0000	-0.9052	1.6237	0.2035	0.0414	2.0168	22.7054
UNITY	108	0.0126	0.0000	-0.4818	2.8198	0.3508	0.1231	4.9406	35.9688
WEMA	186	0.0058	0.0000	-0.8104	2.8198	0.2723	0.0742	5.7294	59.6458
ZENITH	122	0.0015	0.0082	-0.6140	0.6084	0.1440	0.0207	-0.9269	6.5598

In **Table 5.6**, we present descriptive statistics on the stock data of the banks during the financial crisis, July 2007-June 2009. All the banks' series are of equal length; none of them has a positive mean, and five banks: Diamond, First, GTB, UBA and Unity banks, have a median below zero. Also, the standard deviations are all higher than the mean returns.

Only five of the banks: Access (0.0346), FCMB (0.0108), Skye (0.00425), STANBIC (0.00687) and Unity (0.03347), constituting 31.25% of the banks, are positively skewed but the values are close to zero (0), while the remaining 11 banks are negatively skewed. While seven banks, constituting 43.75% of the banks, attain negative excess kurtosis and are thus, platykurtic (i.e. have thinner tailed than normal distribution), the remaining nine (with ETI having the highest excess kurtosis of 404.3597) are heavy or fat-tailed and leptokurtic compared to the normal distribution. Note that the returns distributions with thinner tails than the Normal distribution indicate lower chances of extremely small or large returns, and vice versa. The changes in the return's profiles of banks during the financial crisis show that the crisis distorts the economic environment in which banks trade in the NSM.

Table 5.6: Summary Statistics on Daily Returns of the Banks (Financial Crisis Period)

Banks	T	Mean	Median	Min	Max	Std Dev	Variance	Skewness	Kurtosis
ACCESS	517	-0.0015	0.0000	-0.0513	0.0488	0.0299	0.0009	0.0346	-0.7279
AFRIBANK	517	-0.0008	0.0000	-0.3040	0.0488	0.0338	0.0011	-1.6597	12.8694
DIAMOND	517	-0.0007	-0.0011	-0.1343	0.0488	0.0336	0.0011	-0.0509	-0.7470
ETI	517	-0.0047	0.0000	-1.6094	0.0488	0.0749	0.0056	-19.0046	404.3597
FCMB	517	-0.0012	0.0000	-0.0513	0.0488	0.0272	0.0007	0.0108	-0.3752
FIDELITY	517	-0.0019	0.0000	-0.0513	0.0488	0.0276	0.0008	0.1230	-0.3137
FIRST	517	-0.0012	-0.0003	-0.2538	0.0488	0.0320	0.0010	-1.0765	7.8447
GTB	517	-0.0016	-0.0020	-0.3066	0.0181	0.0334	0.0011	-1.4398	12.1239
SKYE	517	-0.0010	0.0000	-0.0864	0.0488	0.0320	0.0010	0.0043	-0.8883
STANBIC	517	-0.0004	0.0000	-0.0223	0.0212	0.0130	0.0002	0.0069	-0.6497
STERLING	517	-0.0027	0.0000	-0.1683	0.0488	0.0306	0.0009	-0.2516	0.7709
UBA	517	-0.0023	-0.0010	-0.4099	0.0488	0.0361	0.0013	-3.4967	35.5618
UNION	517	-0.0016	0.0000	-0.2026	0.0488	0.0328	0.0011	-0.7720	3.7275
UNITY	517	-0.0023	-0.0033	-0.0513	0.0488	0.0371	0.0014	0.0335	-1.4704
WEMA	517	-0.0016	0.0000	-0.0513	0.0488	0.0241	0.0006	-0.1102	0.7930
ZENITH	517	-0.0026	0.0000	-0.4058	0.0488	0.0333	0.0011	-4.0211	44.7612

5.3.5 Discussions of Tests of the Mean, Skewness and Kurtosis for Daily and Monthly Data, for the Overall and Financial Crisis Periods

In this section, test results on the mean, skewness and kurtosis across the sixteen banks are discussed for daily, monthly and financial crisis periods respectively. This will help to confirm whether the banks' asset returns satisfy the stylized facts relating to asymmetry, leptokurtosis and indeed the distributional assumptions of each bank's returns across the three periods of interest. **Test Results on the Mean, Skewness and Kurtosis for Daily Data**

Table 5.7 displays the test results on the mean, skewness and excess kurtosis obtained with respect to daily stock returns in the overall period. In this case, student-T test statistic (for skewness) and Z-test statistic (for the mean and kurtosis) are employed for testing the following hypotheses:

1. $H_0: mean\ returns, \mu = 0\ vs\ H_1: mean\ returns,\ \mu \neq 0$
2. $H_0: skewness, S_{kr} = 0\ vs\ H_1: skewness, S_{kr} \neq 0$
3. $H_0: excess\ kurtosis, (K-3) = 0\ vs\ H_1: excess\ kurtosis, (K-3) \neq 0$

The test statistic values relating to the first hypotheses are contained in **column 2** of the table, with the p-value in brackets. Considering the p-values, at any possible significance level (**5%, 1% or 10%**), we cannot reject the null hypothesis, indicating that the mean daily return for any of the banks is not significantly different from zero.

For the skewness test results in **column 3**, it is in the case of only three banks whose p-values: 0.9317 (Diamond), 0.2444 (FCMB) and 0.9254 (Fidelity) are higher than 0.05 that we cannot reject our null hypothesis; indicating that their skewness is not significantly different from zero. At 5% level, the null hypothesis is rejected for excess kurtosis across all the banks, and at the 1% level as well for all banks apart from Diamond bank.

Table 5.7: Mean, Skewness and Kurtosis Tests for the Banks Daily Stock Returns (overall)

Banks	Mean	Skewness	Kurtosis
Access	1.2851 (0.1988)	41.7742 (2.2e-16)	38.6144 (2.2e-16)
Afribank	-0.8651 (0.3871)	-43.9973 (2.2e-16)	34.6632 (2.2e-16)
Diamond	-0.1614 (0.8718)	-0.0857 (0.9317)	2.4831 (0.013)
ETI	-1.3838 (0.1666)	-61.4830 (2.2e-16)	33.1810 (2.2e-16)

FCMB	-0.2987 (0.7652)	-1.1641 (0.2444)	6.4524 (1.1e-10)
FIDELITY	-0.4589 (0.6463)	0.0937 (0.9254)	2.6760 (0.007451)
First	-0.2856 (0.7752)	-52.6550 (2.2e-16)	39.0966 (2.2e-16)
GTB	1.3631 (0.1729)	-35.8670 (2.2e-16)	31.8540 (2.2e-16)
Skye	0.1515 (0.8796)	43.8922 (2.2e-16)	31.3150 (2.2e-16)
STANBIC	1.3059 (0.1917)	68.6900 (2.2e-16)	36.2100 (2.2e-16)
Sterling	0.1160 (0.9077)	30.0771 (2.2e-16)	37.2168 (2.2e-16)
UBA	0.1268 (0.8991)	4.9592 (7.08e-07)	38.7493 (2.2e-16)
Union	-0.0952 (0.9242)	78.3902 (2.2e-16)	45.0762 (2.2e-16)
Unity	0.2669 (0.7896)	67.3227 (2.2e-16)	34.9827 (2.2e-16)
WEMA	0.3756 (0.7072)	93.3703 (2.2e-16)	46.7330 (2.2e-16)
Zenith	0.3735 (0.7088)	-29.4246 (2.2e-16)	27.4034 (2.2e-16)

Test Results on the Mean, Skewness and Kurtosis for the Monthly Data

Table 5.8 shows the mean, skewness and excess kurtosis tests for monthly data. It is obvious from the results below that only Afribank has a mean monthly return that is significantly different from zero. Afribank, FCMB, UBA and First banks are the four banks whose skewness is not significantly different from zero at the 5% level, given that their respective p-values: 0.286, 0.6572, 0.2242 and 0.05865 are higher than 0.05, and thus their monthly returns are approximately symmetric. However, none of the banks has excess kurtosis that is approximately zero as expected of a normal random variate.

Table 5.8: Mean, Skewness and Kurtosis Tests for the Banks Monthly Stock Returns (overall)

Banks	Mean	Skewness	Kurtosis
Access	-0.0411 (0.9673)	5.4580 (4.82e-08)	5.1294 (2.907e-07)
Afribank	9.1400 (4.884e-16)	1.0654 (0.2867)	4.1354 (3.544e-05)
Diamond	-0.0514 (0.718)	2.4558 (0.01406)	4.0195 (5.833e-05)
ETI	-1.1979 (0.2338)	-9.6329 (2.2e-16)	7.2521 (4.103e-13)
FCMB	-0.4746 (0.636)	0.4438 (0.6572)	2.0984 (0.03587)
FIDELITY	-0.4560 (0.6493)	4.2061 (2.599e-09)	4.3767 (1.205e-05)
First	-0.2965 (0.7672)	-1.8908 (0.05865)	4.3337 (1.466e-05)
GTB	1.4710 (0.143)	-2.0086 (0.04458)	3.1298 (0.00175)
Skye	0.0256 (0.9796)	-3.0263 (0.00248)	5.2514 (1.509e-07)
STANBIC	0.1593 (0.8738)	-5.8043 (6.465e-09)	7.4835 (7.239e-14)
Sterling	-0.0411 (0.9673)	5.4579 (4.817e-08)	5.1294 (2.907e-07)
UBA	0.0000 (1.000)	1.2155 (0.2242)	4.5037 (6.678e-06)
Union	-0.0388 (0.9691)	7.9429 (1.998e-15)	8.1846 (2.22e-16)
Unity	0.3720 (0.7106)	9.7894 (2.2e-16)	7.4602 (8.638e-14)
WEMA	0.2898 (0.0703)	12.9745 (2.2e-16)	9.5550 (2.2e-16)
Zenith	0.1159 (0.9079)	-3.8896 (0.0001004)	5.1302 (2.894e-07)

Test Results on the Mean, Skewness and Kurtosis for the Financial Crisis Period

Table 5.9 displays the results on the mean, skewness and kurtosis tests during the financial crisis. From the table, only in the case of Sterling bank do we have evidence of the mean not being equal to zero at the 5% (but not at the 1%) level. As for skewness, the following 8 banks' returns appear to be symmetric: Access, Diamond, FCMB, Fidelity, Skye, STANBIC, Unity and WEMA, whereas the remaining 8 banks are non-symmetric. For kurtosis, only FCMB and Fidelity appear to show no evidence of having a non-zero kurtosis.

Table 5.9: Mean, Skewness and Kurtosis Tests for the Banks Financial Crisis Stock Returns

Banks	Mean	Skewness	Kurtosis
Access	-1.1170 (0.2644)	0.3262 (0.7443)	-5.2494 (1.526e-07)
Afribank	-0.5466 (0.5849)	-11.3193 (2.2e-16)	10.8455 (2.2e-16)
Diamond	-0.4777 (0.6331)	-0.4805 (0.6308)	-5.4920 (3.97e-08)
ETI	-1.4286 (0.1537)	-30.0144 (2.2e-16)	16.9705 (2.2e-16)
FCMB	-0.9644 (0.3353)	0.1024 (0.9185)	-1.9495 (0.05124)
FIDELITY	-1.5398 (0.1242)	1.1571 (0.2473)	-1.5289 (0.1263)
First	-0.8348(0.4042)	-8.4402 (2.2e-16)	9.4280 (2.2e-16)
GTB	-1.0829 (0.2794)	-10.3368 (2.2e-16)	10.6811 (2.2e-16)
Skye	-0.6762(0.4992)	0.0401 (0.968)	-7.6412 (2.154e-14)
STANBIC	-0.6369 (0.5245)	0.0649 (0.9483)	-4.3435 (1.402e-05)
Sterling	-2.0279 (0.04308)	-2.3399 (0.01929)	2.8943 (0.0038)
UBA	-1.4332 (0.1524)	-16.8270 (2.2e-16)	13.3220 (2.2e-16)
Union	-1.1287 (0.2595)	-6.5146 (7.289e-11)	7.1425 (9.166e-13)
Unity	-1.3881 (0.1657)	0.3159 (0.7521)	-6.8710 (2.2e-16)
WEMA	-1.4786 (0.1399)	-1.0370 (0.2997)	2.9518 (0.00316)
Zenith	-1.7431 (0.08191)	-17.8980 (2.2e-16)	13.7955 (2.2e-16)

Meanwhile, using the values of skewness and kurtosis, across the two periods as contained in 9th and 10th columns of Tables 5.4, 5.6 and 5.8 respectively, we ranked the banks to determine the degrees of skewness and kurtosis as displayed in Tables **5.10** and **5.11** below.

In **Table 5.10** below, the ranks for positively skewed banks appear in brackets ( ), in green while those with negative skewness appear in red. Those without any rank are those that are approximately symmetric. The squared brackets [ ] contain the standard errors for skewness in each cell (where the sizes, T, differ) and in the column heading (where the size is the same).

To determine the groupings into "Positively skewed", "Negatively skewed" and "Approximately symmetric", we use the following rules:

a. If the test statistic values, **t** (see **column 3** of Tables 5.10 and 5.11) < -2, the series is likely negatively skewed;
b. If the test statistic values, **t** falls in the interval -2 < **t** < 2: the series is approximately symmetric;
c. If the test statistic values, **t > 2,** the series is likely positively skewed (**Cramer, 2003, p. 89**)

where the **t-** test statistic is given by **t** = **skewness-0/SES** (standard error of skewness).

Subject to the above, for daily data, eight banks are **positively skewed**, with WEMA bank taking the lead and UBA being the least; five banks are **negatively skewed,** with ECO (or ETI) taking the lead and GTB being the least and only three (3) banks- **Diamond, FCMB** and **Fidelity** are approximately symmetric.

For monthly data, six banks are **positively skewed** (with a tie- **Access and Sterling**), WEMA takes the lead again and Fidelity, the least; four banks are **negatively skewed** with ECO also leading and Skye being the least**;** and six banks- **Afribank, Diamond, FCMB**, **First, GTB** and **UBA** are approximately symmetric.

For the financial crisis periods, while eight banks are negatively skewed with ECO taking the lead and Sterling being the least, eight banks are approximately symmetric, and none is positively skewed.

Implications:

For positively skewed banks, their returns have a longer right tail indicating that extreme gains are more probable than extreme losses. For the negatively skewed banks, their returns have a longer left tail, implying that large negative returns are more probable than large positive returns (see **Danielson, 2011**). These remarks may partly explain the different experiences of the banks in managing the effects of the financial crisis and related bank reforms. Thus, during the financial crisis, eight banks experienced negative returns.

Table 5.10: Degree of Skewness across the Banks for the three Periods

Bank	Daily		Monthly		Fin. Crisis	
	Skewness	Deg. Skew.	Skew.	Deg. Skew.	Skewness	Deg. Skew.
ACCESS	2.9845 **(6th)** **[0.039]**	Positively skewed	**1.1358 (4th)** **[0.178]**	Positively skewed	0.0346 **[0.107]**	Approx. Symmetric
AFRIBANK	-4.3754 **(3rd)** **[0.044]**	Negatively skewed	**0.2056** **[0.2]**	Approx. Symmetric	-1.6597 **(4th)** **[0.107]**	Negatively skewed
DIAMOND	-0.0430 **[0.05]**	Approx. Symmetric	**0.5550** **[0.226]**	Approx. Symmetric	-0.0509 **[0.107]**	Approx. Symmetric
ETI	-23.2311 **(1st)** **[0.054]**	Negatively skewed	**-5.1584 (1st)** **[0.243]**	Negatively skewed	-19.0046 **(1st)** **[0.107]**	Negatively skewed
FCMB	-0.0572 **[0.049]**	Approx. Symmetric	**0.0931** **[0.221]**	Approx. Symmetric	0.0108 **[0.107]**	Approx. Symmetric
FIDELITY	0.0047 **[0.05]**	Approx. Symmetric	**1.0543(5th)** **[0.225]**	Positively skewed	0.1230 **[0.107]**	Approx. Symmetric
FIRST	-5.0916 **(2nd)** **[0.039]**	Negatively skewed	**-0.3327** **[0.178]**	Approx. Symmetric	-1.0765 **(6th)** **[0.107]**	Negatively skewed
GTB	-2.2160 **(5th)** **[0.039]**	Negatively skewed	**-0.3554** **[0.179]**	Approx. Symmetric	-1.4398 **(5th)** **[0.107]**	Negatively skewed
SKYE	6.4348 **(5th)** **[0.052]**	Positively skewed	**-0.7237 (4th)** **[0.231]**	Negatively skewed	0.0043 **[0.107]**	Approx. Symmetric
STANBIC	27.2745 **(3rd)** **[0.05]**	Positively skewed	**-1.666 (2nd)** **[0.224]**	Negatively skewed	0.0069 **[0.107]**	Approx. Symmetric
STERLING	1.6371 **(7th)** **[0.039]**	Positively skewed	**1.1358 (4th)** **[0.178]**	Positively skewed	-0.2516 **(8th)** **[0.107]**	Negatively Skewed
UBA	0.1967 **(8th)** **[0.039]**	Positively skewed	**0.2109** **[0.178]**	Approx. Symmetric	-3.4967 **(3rd)** **[0.107]**	Negatively skewed
UNION	17.6137 **(4th)** **[0.039]**	Positively skewed	**2.0168 (3rd)** **[0.178]**	Positively skewed	-0.7720 **(7th)** **[0.107]**	Negatively skewed
UNITY	28.6789 **(2nd)** **[0.052]**	Positively skewed	**4.9406 (2nd)** **[0.233]**	Positively skewed	0.0335 **[0.107]**	Approx. Symmetric
WEMA	36.1655 **(1st)** **[0.039]**	Positively skewed	**5.7294 (1st)** **[0.178]**	Positively skewed	-0.1102 **[0.107]**	Approx. Symmetric
ZENITH	-2.2796**(4th)** **[0.049]**	Negatively skewed	**-0.9269 (3rd)** **[0.219]**	Negatively Symmetric	-4.0211 **(2nd)** **[0.107]**	Negatively skewed

Standard errors are in squared brackets

Also in **Tables 5.11 and 5.12 below,** the degrees of kurtosis are ranked, with ranks displayed in brackets (), coloured red, and standard errors appear in square [ ] brackets as defined above.

The classifications according to Westfall (2014) and Ghasemi, and Zahediasl (2012), are based on:

a. If excess kurtosis (**kurtosis-3** as displayed) ≈ 0, the distribution is mesokurtic (at the same level as the Normal distribution);
b. If excess kurtosis (kurtosis-3, as displayed) < 0, the distribution is platykurtic (lower or shorter than Normal distribution, hence thin tailed);
c. If excess kurtosis (**kurtosis-3,** as displayed) > 0, the distribution is leptokurtic (higher than Normal distribution, hence fat tailed) (see https://brownmath.com/stat/shape.htm and Wright and Herrington (2011):

Furthermore, in Table 5.11 excess kurtosis, the standard errors and the confidence interval for the excess kurtosis of each bank returns, are provided. According to the rule, once the interval contains zero it means the kurtosis is not significantly different from zero; the distribution with such kurtosis is said to be mesokurtosis. However, when the confidence interval does not include zero, the returns with such distribution is either leptokurtic or platykurtic depending on the classifications b and c above.

Consequently, all the banks are apparently leptokurtic for both daily and monthly data, with WEMA having the highest excess kurtosis in these two periods, while Diamond has the least kurtosis with respect to daily periods. Nine of the banks however are leptokurtic with ECO bank taking the lead; the remaining is platykurtic compared to a normal distribution. For financial crisis period, we found that the confidence intervals around FCMB and Fidelity banks, contains zero, making the excess kurtosis for the respective banks to be insignificantly different from zero; thus making them to be mesokurtic (see Figures 5.11 and 5.12). Since kurtosis and corresponding fat-tailed distributions are associated with outliers, this shows that higher kurtosis indicates that more of the variance is due to the infrequent extreme deviations, as opposed to frequent modestly sized deviations. Further, while higher values for kurtosis mean a higher and sharper peak, a lower value implies a lower and less distinctive peak (**Cramer, 2003).**

Furthermore, by implication, according to **Danielson (2011**), for fat-tailed series, it means that extreme values occur more often than implied by a normal distribution. It further shows that for a large part of the time, financial asset returns fluctuate in a smaller range than a normal distribution.

Table 5.11: Excess Kurtosis and Confidence Interval across the 16 banks for the Three Periods

Bank	Daily		Monthly		Fin. Crisis	
	Excess Kurtosis	**Confidence Interval [LL, UL]**	**Excess Kurtosis**	**Confidence Interval [LL, UL]**	**Excess Kurtosis**	**Confidence Interval [LL, UL[14]]**
ACCESS	86.8983 **[0.079]**	[86.7403, 87.0563]	4.4862 **[0.355]**	[3.7904, 5.1820]	-0.7279 **[0.214]**	[-1.1473, -0.3085]
AFRIBANK	94.7174 **[0.089]**	[94.5430, 94.8918]	3.2612 **[0.397]**	[2.4831, 4.0393]	12.8694 **[0.214]**	[12.4500, 13.2888]
DIAMOND	0.2706 **[0.101]**	[0.0726, 0.4686]	**3.6408** **[0.447]**	[2.7647, 4.5169]	-0.7470 **[0.214]**	[-1.1664, -0.3276]
ETI	783.7070 **[0.108]**	[783.4953, 783.9187]	35.1432 **[0.481]**	[34.2004, 36.0860]	404.3597 **[0.214]**	[403.9403, 404.7791]
FCMB	0.8811 [**0.098]**	[0.6890, 1.0732]	1.0079 **[0.438]**	[0.1494, 1.8664]	**-0.3752** **[0.214]**	**[-0.7946, 0.0442]**
FIDELITY	0.2950 **[0.1]**	[0.0990, 0.4910]	4.4468 **[0.446]**	[3.5726, 5.3210]	**-0.3137** **[0.214]**	**[-0.7331, 0.1057]**
FIRST	97.7673 **[0.079]**	[97.6125, 97.9221]	3.0332 **[0.355]**	[2.3374, 3.7290]	7.8447 **[0.214]**	[7.4253, 8.2641]
GTB	23.1739 **[0.079]**	[23.0191, 23.3287]	1.6129 **[0.355]**	[0.9171, 2.3087]	12.1239 **[0.214]**	[11.7045, 12.5433]
SKYE	155.7540 **[0.103]**	[155.5521, 155.9559]	7.8071 **[0.459]**	[6.9075, 8.7067]	-0.8883 **[0.214]**	[-1.3077, -0.4689]
STANBIC	1065.4050 **[0.1]**	[1,065.2090, 1,065.6010]	31.8347 **[0.444]**	[30.9645, 32.7049]	-0.6497 **[0.214]**	[-1.0691, -0.2303]

[14] LL: Lower Limit; UL:Upper Limit

STERLING	63.1626 **[0.079]**	[63.0078, 63.3174]	4.4862 **[0.355]**	[3.7904, 5.1820]	0.77085 **[0.214]**	[0.3514, 1.1903]
UBA	89.7707 **[0.079]**	[89.6159, 89.9255]	3.3004*)* **[0.355]**	[2.6046, 3.9962]	35.5618 **[0.214]**	[35.1424, 35.9812]
UNION	744.0610 **[0.079]**	[743.9062, 744.2158]	22.7054 **[0.355]**	[22.0096, 23.4012]	3.7275 **[0.214]**	[3.3081, 4.1469]
UNITY	1085.0730 **[0.104]**	[1,084.8692, 1,085.2768]	35.9688 **[0.461]**	[35.0652, 36.8724]	-1.4704 **[0.214]**	[-1.8898, -1.0510]
WEMA	1882.0110 **[0.079]**	[1,881.8562, 1,882.1658]	59.6458 **[0.355]**	[58.9500, 60.3416]	0.7930 **[0.214]**	[0.3736, 1.2124]
ZENITH	32.3184 **[0.098]**	[32.1263, 32.5105]	6.5598 **[0.435]**	[5.7072, 7.4124]	44.7612 **[0.214]**	[44.3418, 45.1806]

Table 5.12: Degree of Kurtosis across the 16 Banks for the 3 Periods

Bank	**Daily**		**Monthly**		**Fin. Crisis**	
	Excess Kurtosis	**Deg. Kurt.**	**Kurtosis**	**Deg. Kurt.**	**Kurtosis**	**Deg. Kurt.**
ACCESS	86.8983 **(10th)** **[0.079]**	Leptokurtic	4.4862 **(8th)** **[0.355]**	Leptokurtic	-0.7279 **(4th)** **[0.214]**	**Platykurtic**
AFRIBANK	94.7174 **(8th)** **[0.089]**	Leptokurtic	3.2612 **(12th)** **[0.397]**	Leptokurtic	12.8694 **(4th)** **[0.214]**	Leptokurtic
DIAMOND	0.2706 (16th) **[0.101]**	Leptokurtic	**3.6408 (10th)** **[0.447]**	Leptokurtic	-0.7470 **(3rd)** **[0.214]**	**Platykurtic**
ETI	783.7070 **(4th)** **[0.108]**	Leptokurtic	35.1432 **(3rd)** **[0.481]**	Leptokurtic	404.3597 **(1st)** **[0.214]**	Leptokurtic
FCMB	0.8811 **(14th)**	Leptokurtic	1.0079 **(15th)**	Leptokurtic	**-0.3752**	**Mesokurtic**

	[**0.098]**		**[0.438]**		**[0.214]**	
FIDELITY	0.2950 **(15th)** **[0.1]**	Leptokurtic	4.4468 **(9th)** **[0.446]**	Leptokurtic	**-0.3137** **[0.214]**	**Mesokurtic**
FIRST	97.7673 **(7th)** **[0.079]**	Leptokurtic	3.0332 **(13th)** **[0.355]**	Leptokurtic	7.8447 **(6th)** **[0.214]**	Leptokurtic
GTB	23.1739 **(13th)** **[0.079]**	Leptokurtic	1.6129 **(14th)** **[0.355]**	Leptokurtic	12.1239 **(5th)** **[0.214]**	Leptokurtic
SKYE	155.7540**(6th)** **[0.103]**	Leptokurtic	7.8071**(6th)** **[0.459]**	Leptokurtic	-0.8883 **(2nd)** **[0.214]**	**Platykurtic**
STANBIC	1065.4050 **(3rd)** [**0.1]**	Leptokurtic	31.8347**(4th)** **[0.444]**	Leptokurtic	-0.6497 **(5th)** **[0.214]**	**Platykurtic**
STERLING	63.1626 (11th) **[0.079]**	Leptokurtic	4.4862***(8th)*** **[0.355]**	Leptokurtic	0.77085 **(9th)** **[0.214]**	Leptokurtic
UBA	89.7707 **(9th)** **[0.079]**	Leptokurtic	3.3004***(11th)*** **[0.355]**	Leptokurtic	35.5618 **(3rd)** **[0.214]**	Leptokurtic
UNION	744.0610 **(5th)** **[0.079]**	Leptokurtic	22.7054 **(5th)** **[0.355]**	Leptokurtic	3.7275 **(7th)** **[0.214]**	Leptokurtic
UNITY	1085.0730**(2nd)** **[0.104]**	Leptokurtic	35.9688 **(2nd)** **[0.461]**	Leptokurtic	-1.4704 **(1st)** **[0.214]**	**Platykurtic**
WEMA	1882.0110**(1st)** **[0.079]**	Leptokurtic	59.6458**(1st)** **[0.355]**	Leptokurtic	0.7930 **(8th)** **[0.214]**	Leptokurtic
ZENITH	32.3184(12th) **[0.098]**	Leptokurtic	6.5598**(7th)** **[0.435]**	Leptokurtic	44.7612 **(2nd)** **[0.214]**	Leptokurtic

Standard errors are in square brackets. For the Fin. Crisis period, there is only one standard error since the sample sizes are the same across the banks

5.3.6 Discussions of Tests of Normality for Daily Data, Monthly and Financial Crisis Periods

Having tested for the significance of the first, third and fourth moments (mean, skewness and kurtosis) for the various banks' returns across the three periods, we now proceed to testing to determine the conformity or otherwise of the banks' returns distributions to normality.

Results of the Normality Tests for Daily Data

Table 5.13 below presents test results of the normality tests carried out on all the banks' daily stock returns for the overall period. Five different relevant tests were carried out with their results (and p-values in bracket) presented in the table. The tests are: Kolmogorov-Smirnov (KS), Shapiro-Wilk (SW), D'Agostino (Dago) and Jarque-Bera (JB) tests. However, the most widely used in financial time series, especially for series that are leptokurtic, is the JB test and for us to reject H_0 the test statistic should be greater than 6 or the p-value should be less than 0.05. Thus, using the JB test in column 6 of the table, given that none of the statistic is equal to 6 or less and since none of the p-values (in parentheses) are greater than **0.05**, we reject the null hypothesis in all cases.

1. H_0:The respective bank's daily returns series follow a Normal distribution,

   versus

   H_1:The respective bank's daily returns series do not follow a Normal distribution

Finally, considering the other test statistics, none support a normal distribution for any of the banks' daily returns. This behaviour is in line with established stylized facts of high frequency financial data in the literature, as discussed in Section **3.5** of the thesis.

Table 5.13: Normality Tests for All the Banks Daily Stock Returns (Overall)

Banks	KS	SW	Dago	JB[15]
Access	0.1529 (2.2e-16)	0.8168 (2.2e-16)	3236.1550 (2.2e-16)	1224384 (2.2e-16)
Afribank	0.2356 (2.2e-16)	0.7526 (2.2e-16)	3137.3064 (2.2e-16)	1149515.3946(2.2e-16)
Diamond	0.1216 (2.2e-16)	0.9585 (2.2e-16)	6.1732 (0.04566)	7.3808(0.02496)
ETI	0.1933 (2.2e-16)	0.3496 (2.2e-16)	4881.1075 (2.2e-16)	52750147.9442 (2.2e-16)
FCMB	0.4753 (2.2e-16)	0.9395 (2.2e-16)	42.9885 (4.6e-10)	81.9560 (2.2e-16)

[15]Kolmogorov-Smirnov (KS), Sharpiro-Wilk (SW), D'Augusto (Dago) and Jarque-Bera (JB). P-values in parentheses

FIDELITY	0.1537 (2.2e-16)	0.9546 (2.2e-16)	7.1696 (0.02774)	8.7887 (0.01235)
First	0.1188 (2.2e-16)	0.7596 (2.2e-16)	4301.1250 (2.2e-16)	1559273.6415 (2.2e-16)
GTB	0.1043 (2.2e-16)	0.85417 (2.2e-16)	2301.1179 (2.2e-16)	89844.1042 (2.2e-16)
Skye	0.1241 (2.2e-16)	0.7693 (2.2e-16)	2907.1609 (2.2e-16)	2286858.5800 (2.2e-16)
STANBIC	0.1965 (2.2e-16)	0.3351 (2.2e-16)	6066.6427(2.2e-16)	113569853.6270 (2.2e-16)
Sterling	0.1877 (2.2e-16)	0.8233 (2.2e-16)	2289.7203 (2.2e-16)	645570.2481 (2.2e-16)
UBA	0.0954 (2.2e-16)	0.7621 (2.2e-16)	1526.0984 (2.2e-16)	1300556.2606 (2.2e-16)
Union	0.4741 (2.2e-16)	0.5383 (2.2e-16)	8176.8855 (2.2e-16)	89542173.2762 (2.2e-16)
Unity	0.2307 (2.2e-16)	0.2731 (2.2e-16)	5756.1384 (2.2e-16)	109557197.8524 (2.2e-16)
WEMA	0.1987 (2.2e-16)	0.3316 (2.2e-16)	10901.9795 (2.2e-16)	572429930.5313 (2.2e-16)
Zenith	0.1082 (2.2e-16)	0.8592 (2.2e-16)	1616.7541 (2.2e-16)	111868.3710 (2.2e-16)

Results of the Normality Tests for the Monthly Data

Table 5.14 presents the normality test results for monthly returns, and by examining the p-values under the JB test, only **FCMB** is approximately normally distributed at the 5% level, whereas the rest of the banks are non-normally distributed. Though this contradicts the general axiom that monthly data are approximately normal, comparing the resulting JB test statistic values for the daily and monthly data, however, we see that those of monthly are greatly lower than those of the daily data.

Table 5.14: Normality Tests for All the Banks Monthly Stock Returns (Overall)

Banks	KS	SW	Dago	JB
Access	0.3394 (2.2e-16)	0.8724 (1.93e-11)	566.0990 (6.579e-13)	202.3080 (2.2e-16)
Afribank	0.2060 (2.2e-16)	0.8703 (5.048e-10)	18.2360 (0.0001097)	69.6840 (7.77e-16)
Diamond	0.3620 (1.7e-13)	0.9241 (6.38e-06)	22.1872 (1.521e-05)	73.7245 (2.2e-16)
ETI	0.2690 (2.2e-16)	0.5210 (2.2e-16)	145.3860 (2.2e-16)	5776.5630 (2.2e-16)
FCMB	0.1149 (0.00052)	0.9567 (0.00069)	4.6000 (0.1002)	5.9616 (0.05075)

FIDELITY	0.1750 (2.453e-09)	0.8934 (1.36e-07)	36.8500 (9.98e-09)	123.3031 (2.2e-16)
First	0.1070 (2.115e-05)	0.9267 (4.71e-08)	22.3560 (1.398e-05)	77.8950 (2.2e-16)
GTB	0.0840 (0.002775)	0.9603 (4.39e-05)	13.8301 (0.0009928)	25.2820 (3.24e-06)
Skye	0.1336 (5.686e-05)	0.8610 (1.04e-08)	36.7360 (1.05e-08)	301.0376 (2.2e-16)
STANBIC	0.2338 (2.2e-16)	0.5337 (2.2e-16)	89.6918 (2.2e-16)	5185.0989 (2.2e-16)
Sterling	0.1923 (2.2e-16)	0.8724 (1.93e-11)	56.0990 (6.58e-13)	202.3080 (2.2e-16)
UBA	0.1107 (8.39e-06)	0.9440 (1.27e-06)	21.7610 (1.88e-05)	89.3500 (2.2e-16)
Union	0.1418 (9.43e-10)	0.7580 (3.35e-16)	130.0780 (2.2e-16)	4222.2148 (2.2e-16)
Unity	0.2323 (9.212e-16)	0.6000 (1.079e-15)	151.4870 (2.2e-16)	6512.9853 (2.2e-16)
WEMA	0.2028 (2.2e-16)	0.5740 (2.2e-16)	259.6360 (2.2e-16)	29237.1700 (2.2e-16)
Zenith	0.1558 (1.072e-07)	0.8522 (1.08e-09)	41.4480 (9.992e-07)	247.3544 (2.2e-16)

Note: p-values in parentheses

Test Results of the Normality for the Financial Crisis Data

The normality test results for daily stock returns during the financial crisis are presented in **Table 5.15** below. Based on the p-values (in parentheses), and the test statistic values across the four normality tests presented, especially JB and Dago test statistics, which are mostly applied in financial time series and econometrics (see Ruppert (2011); Tsay, (2014)), only two banks' returns, **FCMB and Fidelity**, are approximately normally distributed. However, for the remaining 2 test statistics, KS and SW, none of the banks' series seems to follow a Normal distribution.

Table 5.15: Normality Tests for All the Banks' Daily Stock Returns during the Financial Crisis

Banks	KS	SW	Dago	JB
Access	0.1090 (4.156e-16)	0.9350 (3.02e-14)	27.6600 (9.844e-07)	11.2414 (0.003622)
Afribank	0.4810 (2.2e-16)	0.8350 (2.2e-16)	245.7530 (2.2e-16)	3840.7500 (2.2e-16)
Diamond	0.4805 (2.2e-16)	0.9307 (9.85e-15)	30.3944 (2.512e-07)	11.9646 (0.002523)
ETI	0.2676 (2.2e-16)	0.1903 (2.2e-16)	1188.8617 (2.2e-16)	3581098.3309 (2.2e-16)

FCMB	0.1291 (2.2e-16)	0.9420 (2.171e-13)	3.8110 (0.1487)	2.8799 (0.2369)
FIDELITY	0.4805 (2.2e-16)	0.9137 (2.2e-16)	3.6760 (0.1591)	3.2927 (0.1928)
First	0.4805 (2.2e-16)	0.8887 (2.2e-16)	160.1150 (2.2e-16)	1440.3296 (2.2e-16)
GTB	0.4805 (2.2e-16)	0.8854 (2.2e-16)	220.9351 (2.2e-16)	3376.7667 (2.2e-16)
Skye	0.4805 (2.2e-16)	0.9287 (5.691e-15)	58.3893 (2.094e-13)	16.6881 (0.0002378)
STANBIC	0.4915 (2.2e-16)	0.9250 (2.164e-15)	18.8700 (7.988e-05)	8.8425 (0.01202)
Sterling	0.4805 (2.2e-16)	0.8977 (2.2e-16)	13.8524 (0.0009817)	18.7751 (8.376e-05)
UBA	0.0898 (1.022e-10)	0.7770 (2.2e-16)	460.6193 (2.2e-16)	28531.7949 (2.2e-16)
Union	0.4805 (2.2e-16)	0.9055 (2.2e-16)	93.4551 (2.2e-16)	355.1623 (2.2e-16)
Unity	0.4805 (2.2e-16)	0.8815 (2.2e-16)	12.3860 (2.2e-16)	46.2966 (8.848e-11)
WEMA	0.4805 (2.2e-16)	0.7771 (2.2e-16)	9.7885 (0.00749)	15.1041 (0.000525)
Zenith	0.1544 (2.2e-16)	0.7370 (2.2e-16)	510.6568 (2.2e-16)	44919.5254 (2.2e-16)

Note: p-values in parentheses

Fundamentally, the distributions of the Nigerian banks' daily returns are non-normal, as established in other markets; therefore, they satisfy the stylized fact for daily data across most global markets. Non-normality indicates the presence of non-random influences in the dynamics of financial markets. These influences support the belief that markets are not usually efficient, so that opportunities for predicting the direction of returns and possibly making money exist in such inefficient markets. This is, however, a simple way of describing the effects of non-normality. It is known that further examination of the influences on market dynamics are needed as investment analysts explore those opportunities through stylised facts and related market features, namely efficiency, bubbles, anomalies, volatility, predictability and valuation (Ezepue and Omar, 2012).

As for the monthly data, though the asset returns are largely non-normal, the degrees of non-normality are far lower than those of the daily returns across the banks. During the financial crisis, **FCMB** and **Fidelity** are approximately normally distributed at the 5% level. The difference in degrees of normality between daily and monthly returns suggests the presence of agglomeration effects whereby financial data behave differently over different granularities of the datasets -namely daily, monthly, yearly or even seconds used for algorithmic trading (Ezepue and Omar, 2012). The implications of these differences in distribution are such that investment decisions across the short-, medium- and long-term horizons are treated differently (Raheem and Ezepue, 2018).

To further visualise the distributions of these returns, we present histogram and Q-Q plots in the next section to further establish our findings.

5.3.7 Discussions of the Histograms and Quantile-Quantile (Q-Q) Plots

In this section, we present both histograms and Q-Q plots for the 16 banks for their daily, monthly and financial crisis data, to visually strengthen the results presented in the previous section and to further identify the nature of the distributions of the various banks' returns.

Discussions of the Overall Daily Returns Distribution using Histogram and Q-Q plots

Figure 5.11 and 5.12 respectively display normalized histograms (with superimposed kernel densities) and Q-Q plots for the daily stock returns for the sixteen banks for the overall data. Visually, it can be confirmed that eight banks, namely: **WEMA (1st), Unity (2nd), STANBIC (3rd) , Union (4th), Skye (5th), Access (6th), Sterling (7th) and UBA (8th)** are respectively rightly skewed with longer positive tails than would be accommodated by a normal distribution; five banks-**ECO (1st), First (2nd), Afribank (3rd), Zenith (4th) and GTB (5th)** are respectively negatively skewed with longer negative tails than would be a normal distribution; and only three banks-**Diamond, Fidelity and FCMB** are approximately symmetric (see **Figure 5.13**).This fact further confirms our previous findings as presented in **Table 5.16.**

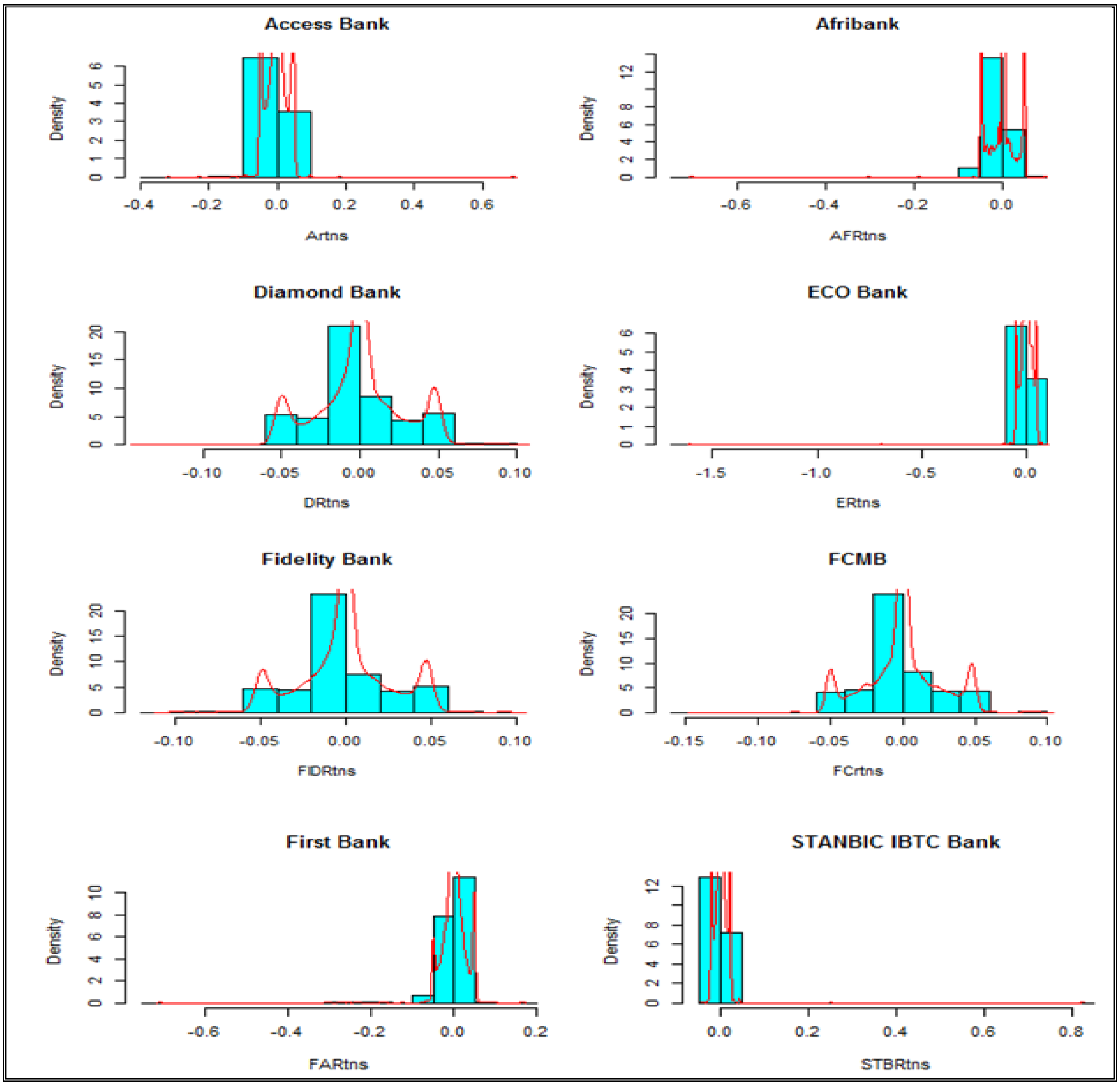

Figure 5.11: Daily Normalized Histogram Plots for the Nigerian Banks' Returns (for the overall period)

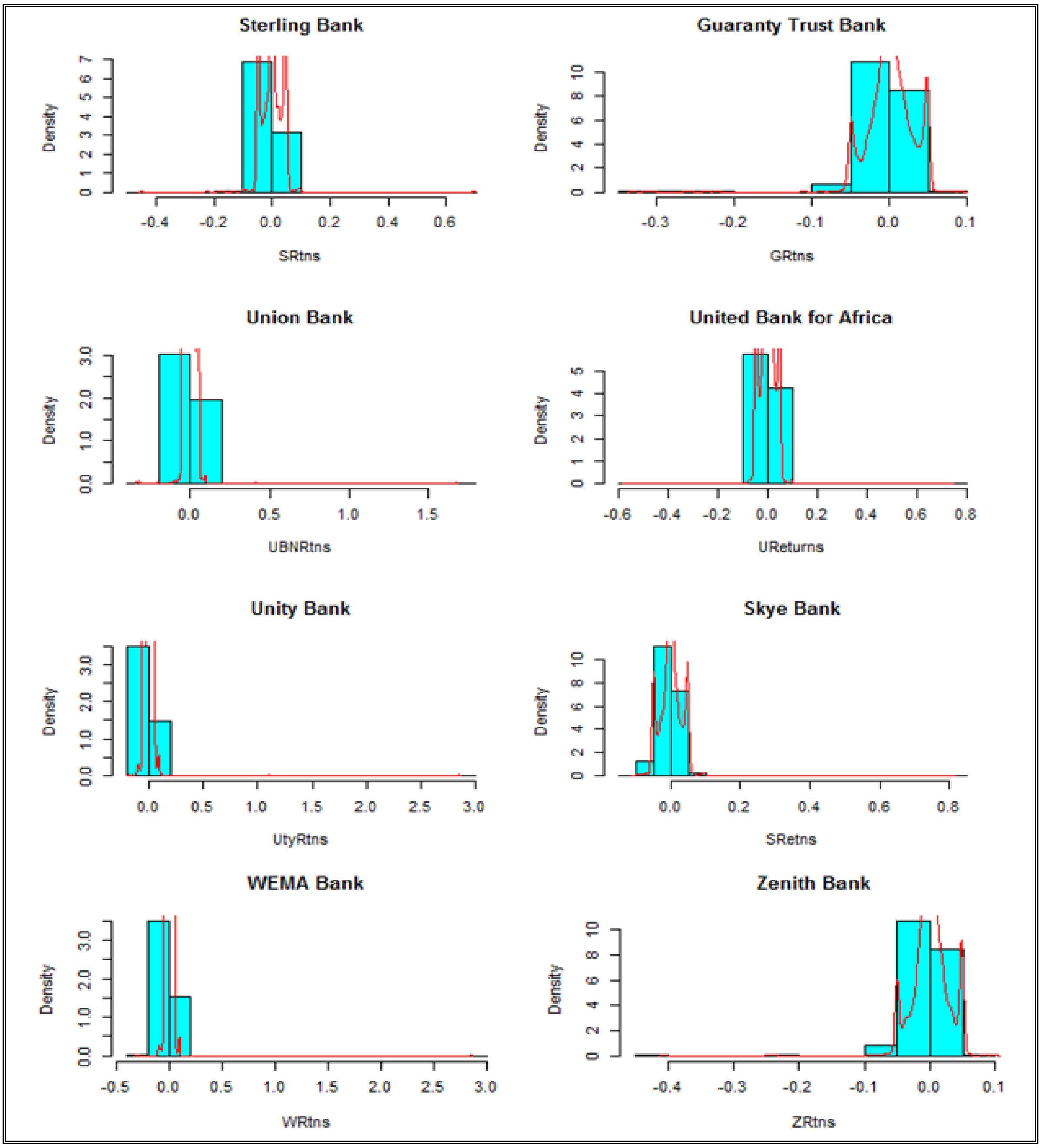

Figure 5.12: Daily Normalized Histogram Plots for the Nigerian Banks' Returns for the overall period (Contd)

Further examination of Figures **5.13 and 5.14** below helps us to visualise the tails of each distribution as compared to a normal distribution with the same mean and variance. The black points represent the empirical distribution for each series, whereas the superimposed straight line in red is for the normal distribution. Our observation reveals that all the eight banks that are positively skewed are above the red lines, whereas those that are negatively are the red lines. For all the cases where the empirical data deviated from a normal distribution, we observe that the deviation of one or more points away from the other points was responsible for the increased kurtosis such that the farther away a data point is from the straight line, the higher the kurtosis. For example, **WEMA** bank which is the most leptokurtic has just one point (an outlier) farther away from the rest of the points, which are almost perfectly aligned with the normal; this fact confirms the earlier submission that kurtosis is sensitive to outliers.

To conclude, with these plots we have been able to graphically establish that the Nigerian banks are largely non-normally distributed with very high kurtosis, especially for daily data. However, with the positively skewed bank returns outnumbering the negatively skewed ones, the views expressed by **Zivot (2009)** about the general behaviours of daily returns that "the distribution of daily returns is clearly non-normal with negative skewness and pronounced excess kurtosis", is partly demonstrated by our findings.

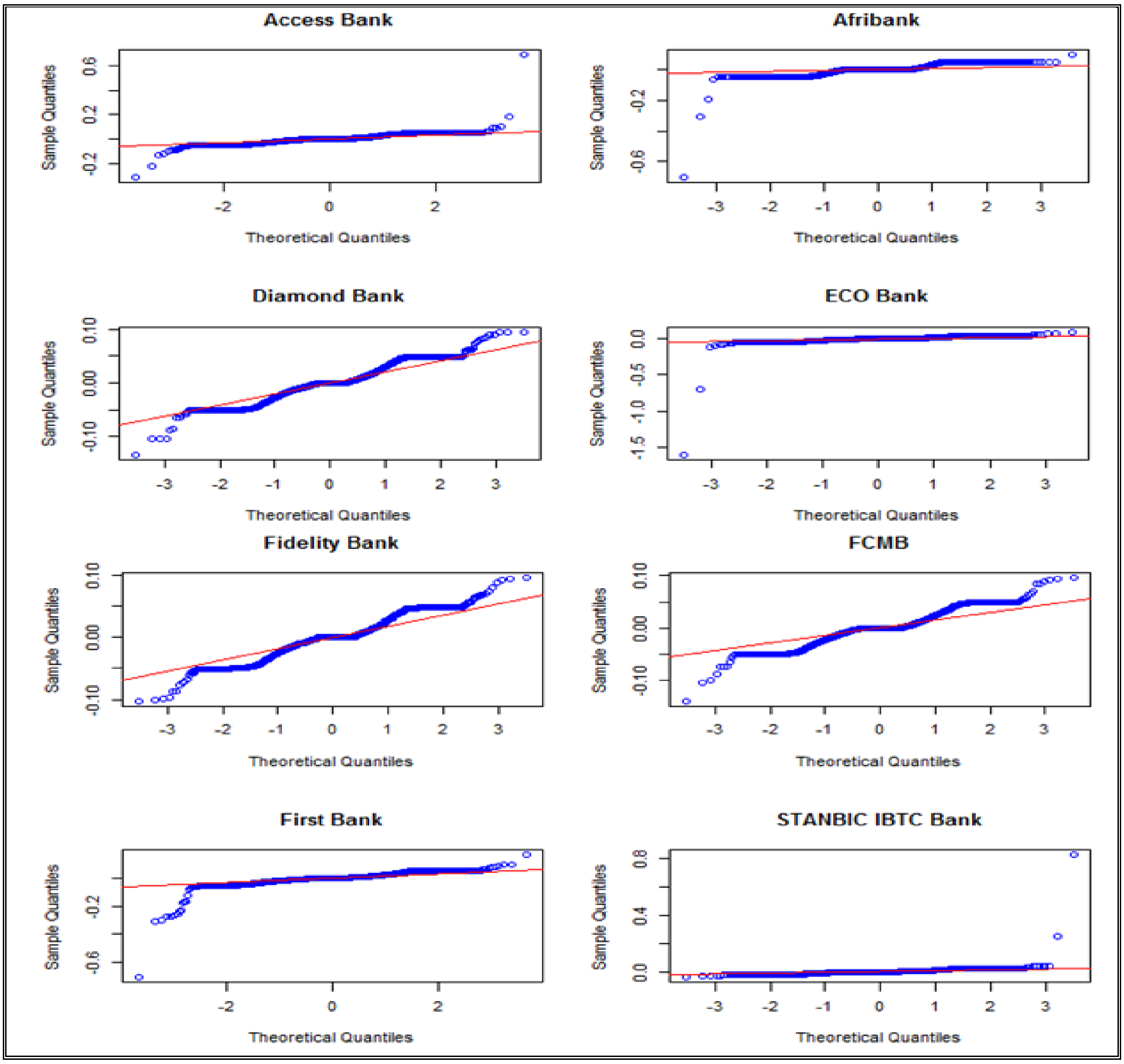

Figure 5.13: Q-Q Plots for the Daily Returns for Nigerian Banks for the Overall period

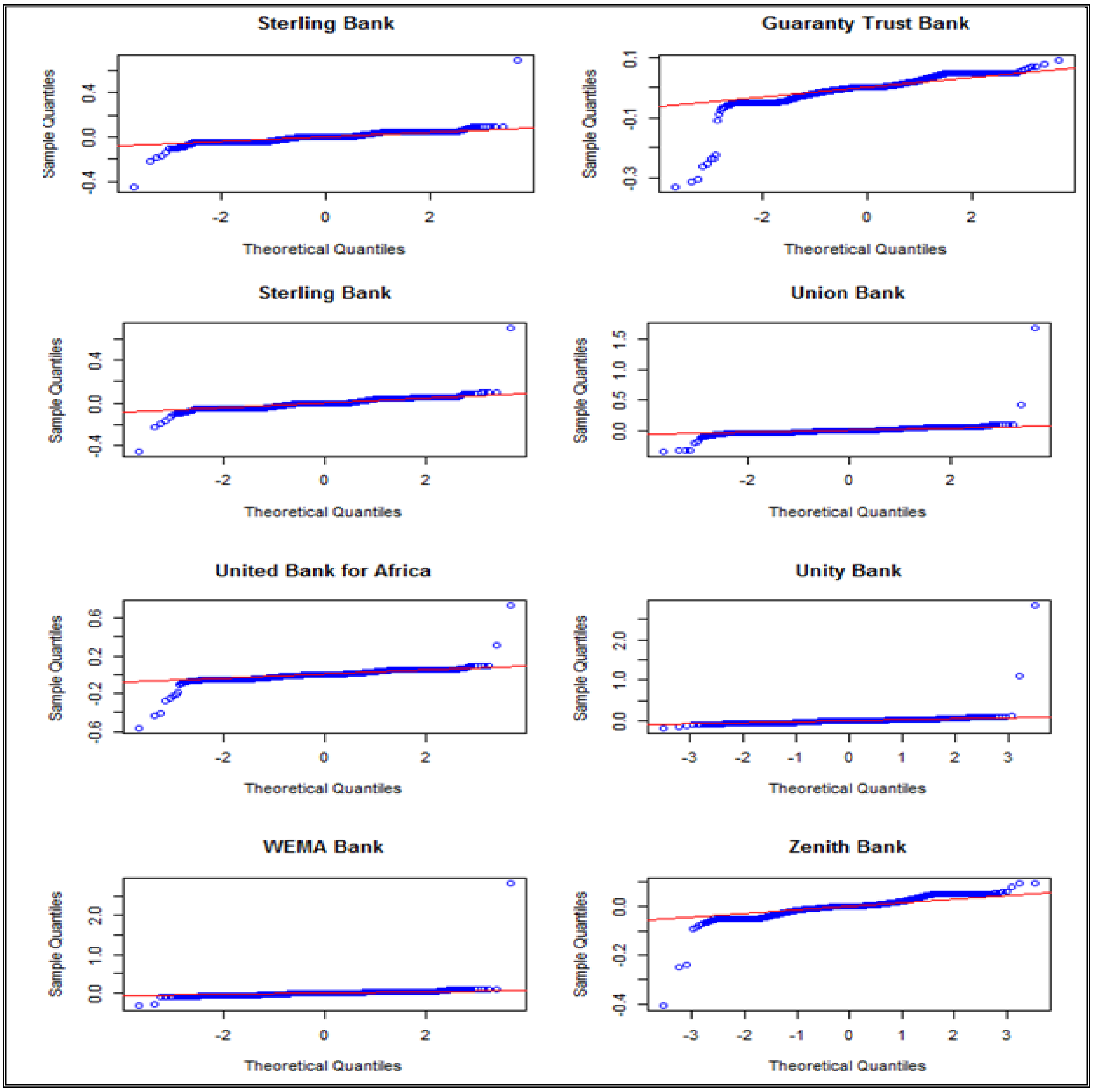

Figure 5.14: Q-Q Plots for the Daily Returns for Nigerian Banks for the Overall period (Contd)

Discussions of Histograms and Q-Q plots for Daily and Monthly Returns

In this section, both histograms and Q-Q plots for the 16 banks are presented and discussed. However, only plots for Access are only displayed in **Figure 5.15** below, while those for the rest of the banks can be found in Appendix **5C**.

Figure **5.15** below presents the histograms and Q-Q plots for Access Bank's daily and monthly data for comparison. It is apparent that the histograms for daily returns (in red) and monthly returns (in green), are both skewed to the right. However, that of the monthly data is less skewed than that of the daily data. The same goes for their Q-Q plots; for the daily data, it vividly shows that while one of the points is farther away from other points at the upper right part of the normal line, two points are however away from the rest of the points at a relatively smaller distance, but with more points tailing off at upper right part of the normal line (in red).This goes to confirm the degree of skewness and kurtosis presented in the previous sections.

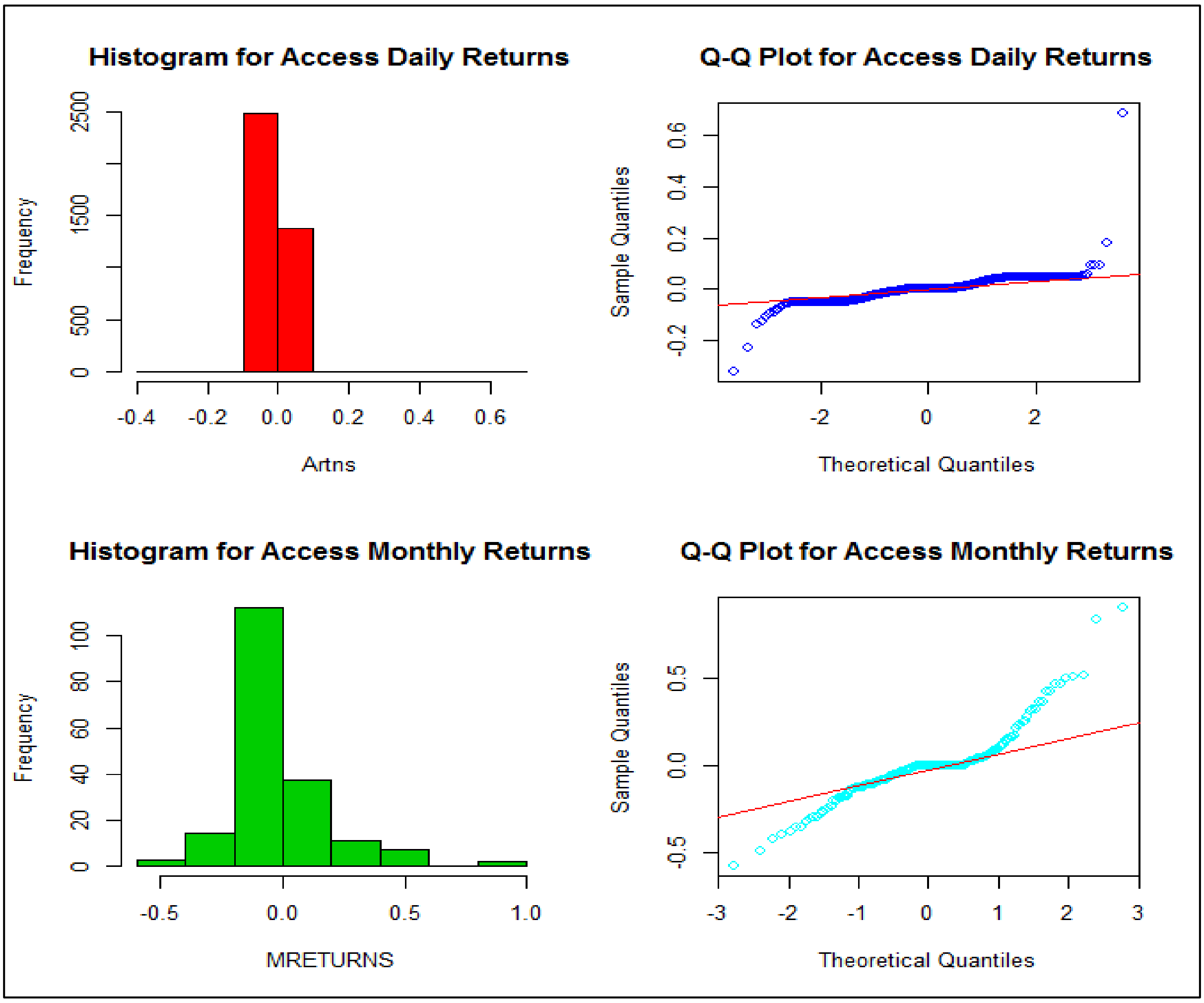

Figure 5.15: Histograms and Q-Q Plots for Daily (above) and Monthly (below) Returns for Access Bank

Generally, the degrees of skewness and leptokurtosis in the daily series for each bank are largely reduced in the monthly plots. However, the distributions are still mostly non-normal across the sixteen banks examined.

Discussions of Histogram and Q-Q Plots for the Financial Crisis Periods

Below in Figure **5.16** and **5.17** are the histograms and Q-Q plots respectively, across the sixteen banks for the financial crisis period. In Figure **5.16**, we have the histograms with superimposed kernel densities; from the plots, while eight (8) of the banks: **ECO (1st), Zenith (2nd), UBA (3rd), Afribank (4th), GTB (5th), First (6th), Union (7th) and Sterling (8th)** are negatively skewed, the remaining eight are, however, approximately symmetric.

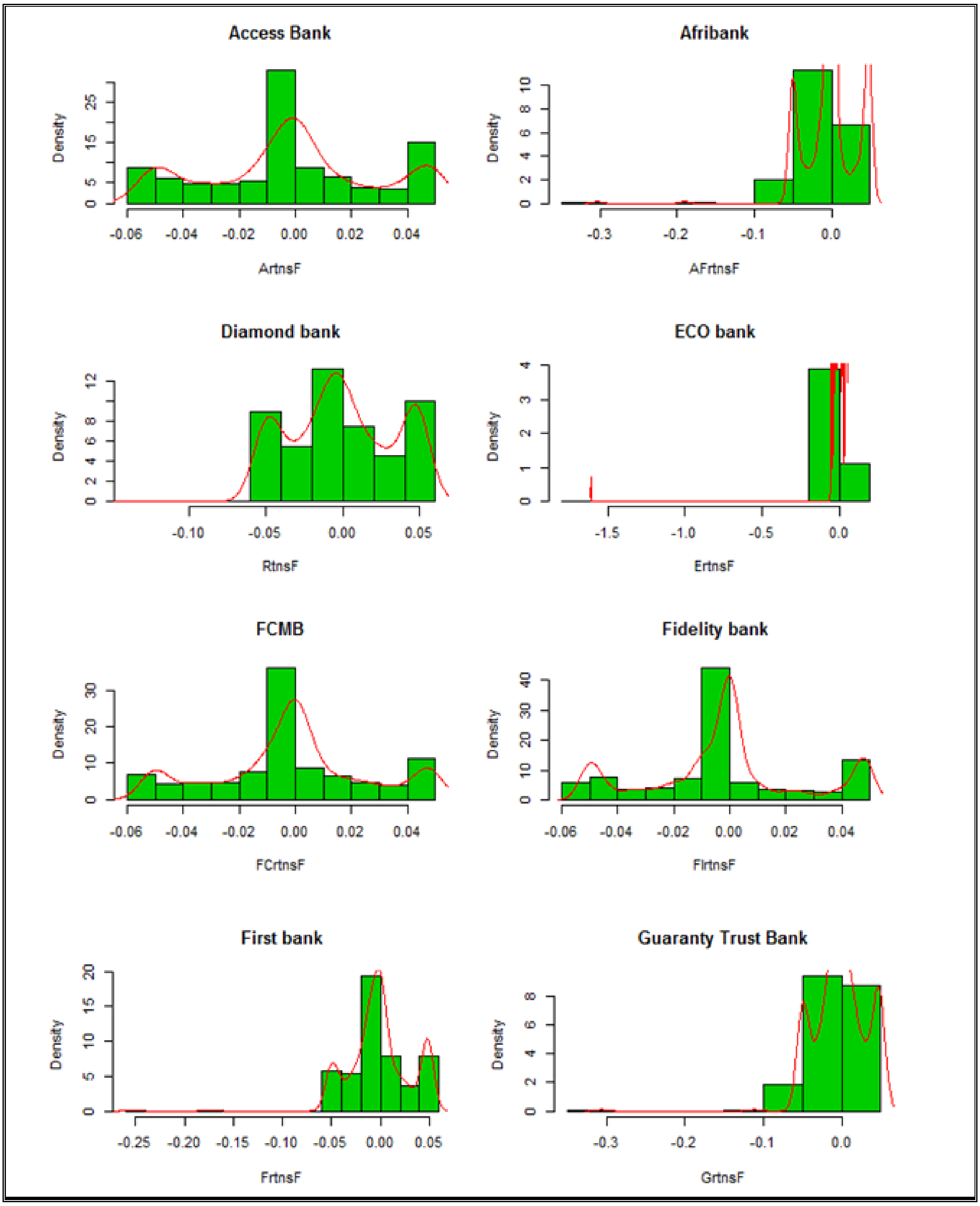

Figure 5.16: Histograms for Daily returns of the Nigerian Banks during Financial Crisis

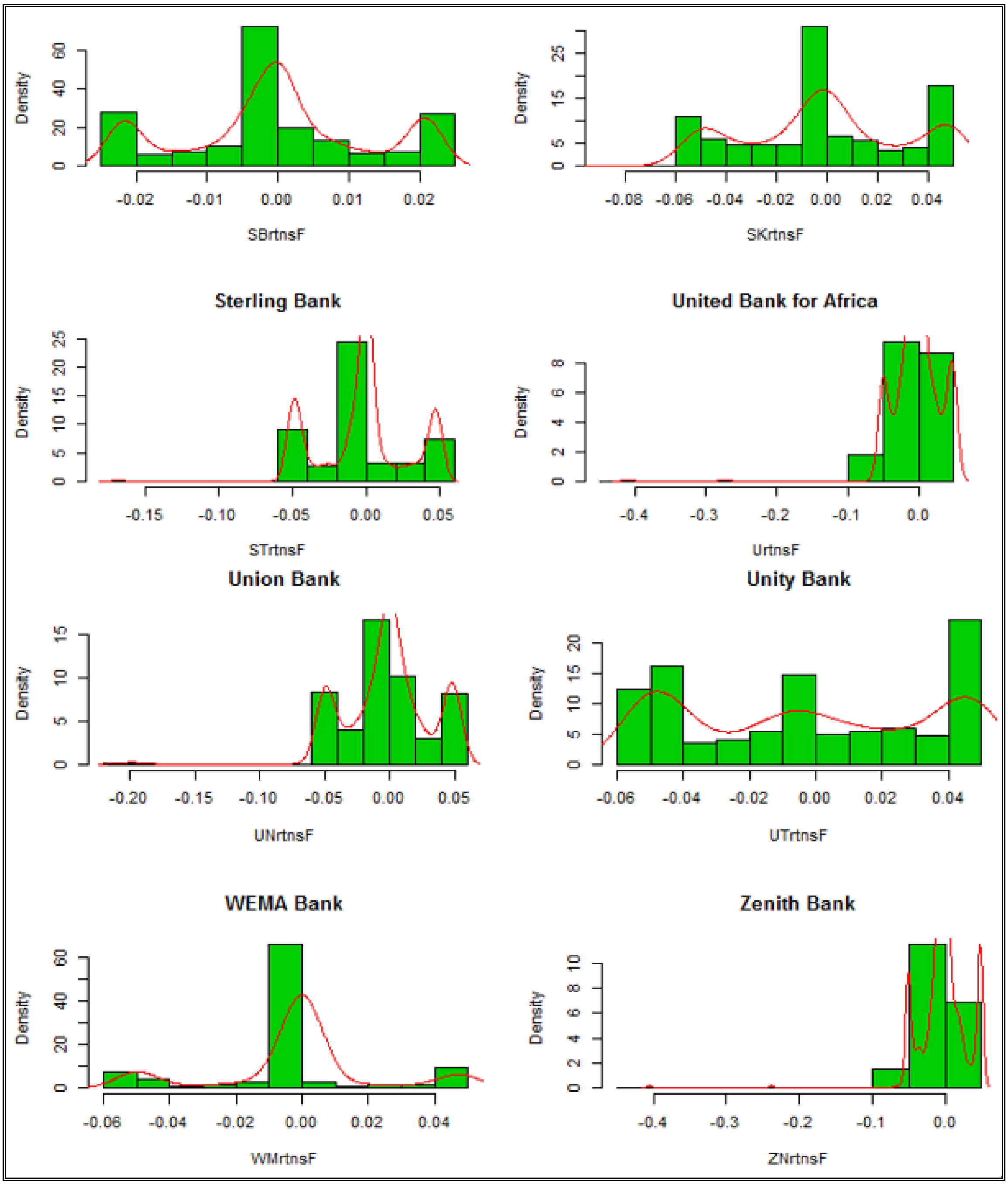

Figure 5.17: Histograms for Daily returns of the Nigerian Banks during Financial Crisis (Continued)

In Figures **5.18 and 5.19** below, we observe nine banks having long tails, with eight of them having some points below the normal line, with ECO bank having the longest negative tail. Five banks-Unity, Skye, Diamond, Access, and STAMBIC banks are thin-tailed, while the remaining two- FCMB and Fidelity are approximately normal (**see** Tables 5.11 and 5.12).

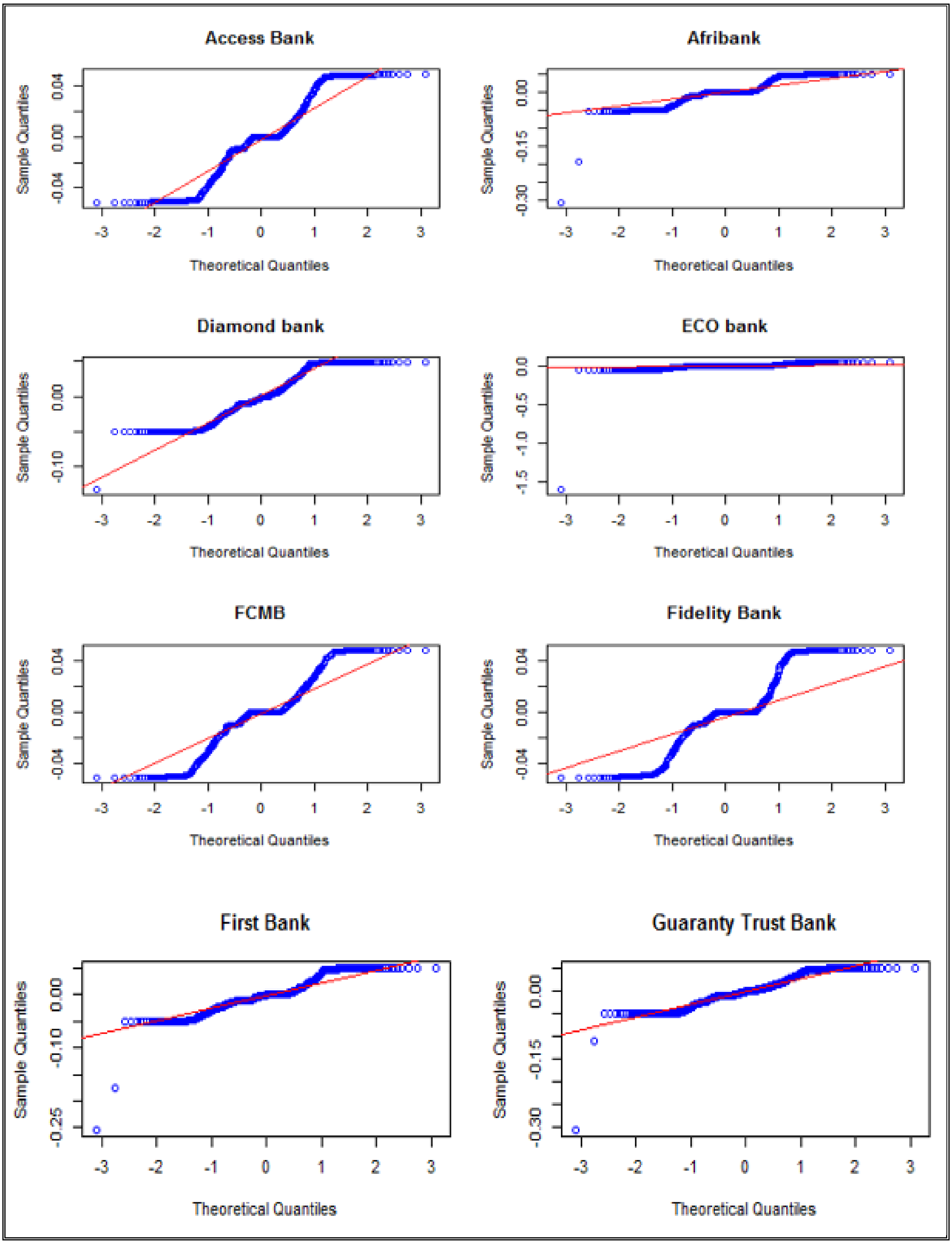

Figure 5.18: Q-Q Plots for Daily returns of the Nigerian banks during the Financial Crisis Period

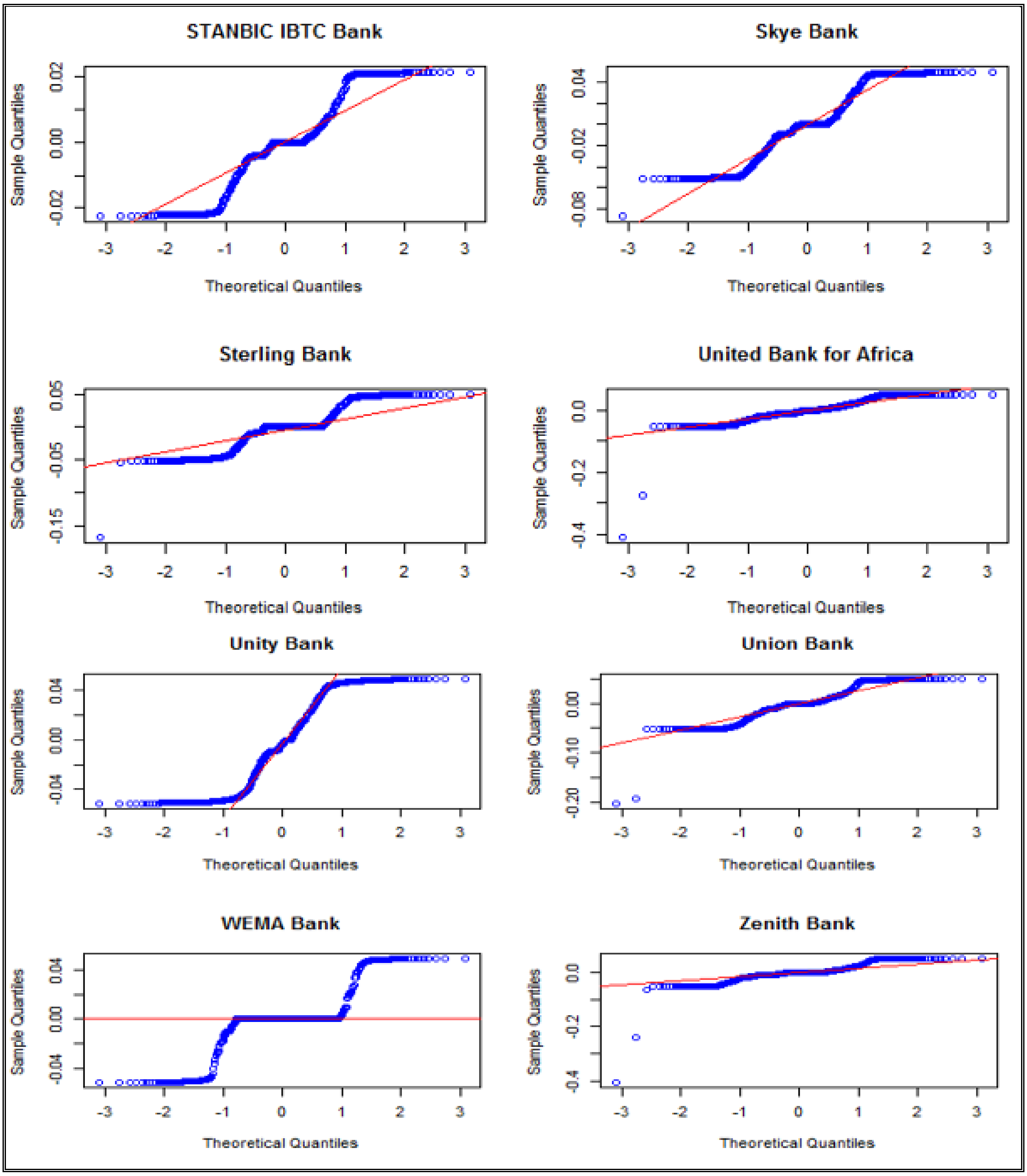

Figure 5.19: Q-Q Plots for Daily returns of the Nigerian banks during the Financial Crisis Period (Continued)

Generally, the returns of the sixteen banks are significantly non-normal - behaviour which agrees with the established stylized facts of asset returns across global financial markets. Meanwhile, the summary of the banks' situations with respect to symmetry and normality of the distributions of their returns series, during the financial crisis is presented in Table 5.16 below. From the table, it is seen that ten banks- Afribank, Diamond, Eco, First, GTB, Skye, UBA, Union, Sterling and Zenith - are negatively skewed, whereas the remaining six banks are symmetric. This shows that the banks' investments were characterised by negative shocks during the crisis, with only very few of them managing to surmount the pressure posed by the crisis in the market. This reveals the extent of loss recorded in the Nigerian market during the financial crisis by investors in the banking sector. This experience is also a confirmation of the observation of the CBN as referenced in Section **2.3.** Generally, the Nigerian experience revealed by this result shows the negative impacts of the financial crisis on the global economy, particularly, the Nigerian economy, because more than half (10) of the 16 banks were negatively impacted.

Table 5.16: Showing Banks Symmetry and Normality Status

Bank	**SKEWNESS STATUS**	**NORMALITY STATUS**
Access	Positively (or Right) Skewed	Non-normal
Afribank	Negatively (or left) Skewed	Non-normal
Diamond	Fairly symmetry	Non-normal
Eco (ETI)	Negatively (or left) Skewed	Non-normal
FCMB	Negatively (or left) Skewed	Non-normal
Fidelity	Negatively (or left) Skewed	Non-normal
First	Negatively (or left) Skewed	Non-normal
GTB	Negatively (or left) Skewed	Non-normal
Skye	Positively (or Right) Skewed	Non-normal
STANBIC	Positively (or Right) Skewed	Non-normal
UBA	Fairly Symmetric	Non-normal
Union	Positively (or Right) Skewed	Non-normal
Sterling	Fairly Symmetric	Non-normal
Unity	Positively (or Right) Skewed	Non-normal
WEMA	Positively (or Right) Skewed	Non-normal

Zenith	Negatively (or left) Skewed	Non-normal

5.3.8 Results and Discussion of the Auto-correlation functions (ACFs) and Tests

In this section, brief interpretations of the estimated autocorrelation functions (ACFs), $\hat{\rho}_k$ plotted against the time lag k for the compounded (log) returns, absolute and squared returns for each of the banks are rendered. It is to be noted that the two dashed horizontal lines in each plot are $\pm 1.96/\sqrt{T}$, the bounds representing the 95% confidence limits for ACF (ρ_k) if its true value, $\rho_k = 0$; ρ_k is adjudged to be non-significant if the estimator, $\hat{\rho}_k$ is between the two dashed lines.

Discussion of ACF for Daily Log Returns, Absolute and Squared returns

Figure 5.20 presents the ACF plots for Access Bank. It is seen that the log return autocorrelation is significant at lag 1 since the autocorrelation function at this lag falls outside the 95% confidence limit lines but dies off quickly after the lag. This negates the random walk theory and contradicts this common stylized fact of daily returns. Meanwhile, the ACF plots of the absolute returns are significant even at long lags and die off slowly at long lags - a behaviour which confirms the fact that the daily returns are non-linearly correlated. This level of persistence is also an indication of long memory in the series of the bank. However, none of the ACFs of the squared returns is significant even at lag 1. Although this behaviour appears to be at variance with the stylized facts of squared returns as established in the literature, it may be attributed to the fact that individual banks' returns series are being studied as against the market index examined in other global markets.

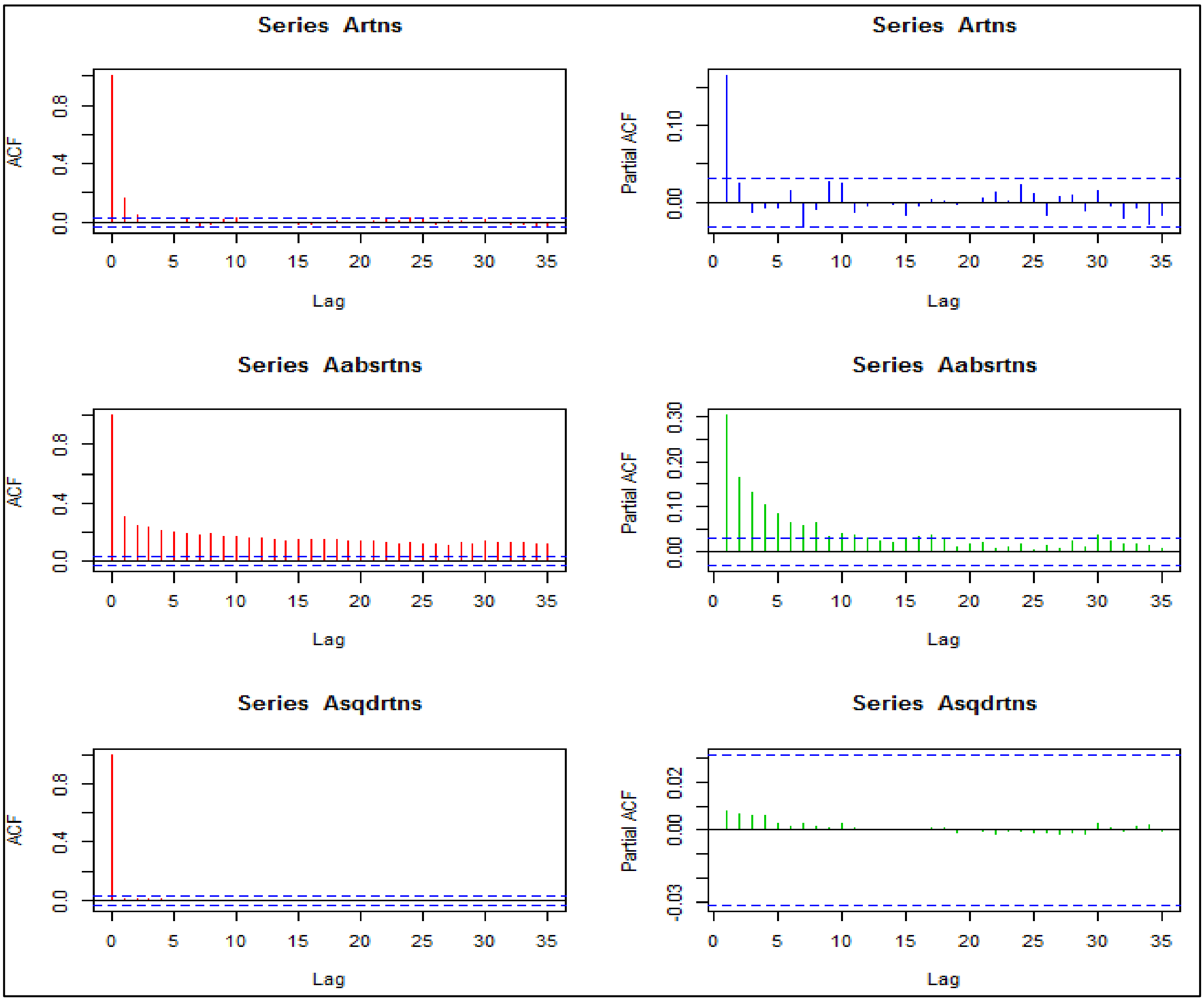

Figure 5.20: Autocorrelation Plots for Log, Absolute and Squared Returns of Access Bank (For Overall period)

The results for all the banks are summarised in **Table 5.17** below. For the details on these banks see, Appendix 5C**.**

Table 5.17: Summary Table of the ACF/PACF Plots on Log, Absolute and Squared Returns for the 16 Banks

Banks	**Log returns**	**Absolute Returns**	**Squared Return**
Access (**Figure 5.27**)	Significant at lag 1, then dies off	Significant at several lags and exponentially decay slowly; an indication of long memory	None is significant
Afribank (**Figure 5.30**)	Significant till lag 2, then dies off	Significant at many lags and dies off at slower rate than Access; an indication of long memory	None is significant
Diamond (**Figure 5.31**)	Significant at lag 1, then dies off	Significant at several lags and dies off slowly; an indication of long memory	Significant at some lags but with slower decay rate compared to Absolute plot

Ecobank (**Figure 5.32**)	None is significant	Significant at few lags with faster decay rate	None is significant
First bank (**Figure 5.33**)	Significant at lag 1, then dies off	Significant at several lags and dies off a bit slowly	Significant at No lag
FCMB (**Figure 5.34**)	Significant at lag 1, then dies off	Significant at several lags and dies off slowly; an indication of long memory	Significant at some lags but with slower decay rate compared to Absolute plot
Fidelity (**Figure 5.35**)	Same as above	Significant at several lags and dies off slowly; an indication of long memory	Significant at some lags but with slower decay rate compared to Absolute plot
GTB **(Figure 5.36)**	Same as above	Same as above	None is significant
Skye **(Figure 5.37)**	Same as above	Same as above	None is significant
STANBIC (**Figure 5.38**)	Same as above	Significant at few lags with faster decay rate	None is significant
Sterling(**Figure 5.39**)	Significant till lag 2, then dies off	Significant at several lags and dies off slowly; an indication of long memory	None is significant
UBA(**Figure 5.40**)	Significant at lag 1, then dies off	Significant at few lags and dies off faster	None is significant
Union (**Figure 5.41**)	Same as above	Significant at several lags and dies off slowly	None is significant
Unity (**Figure 5.42**)	None is significant	Significant till lag 2, then dies of f	None is significant
WEMA (**Figure 5.43**)	Significant at lag 1, then dies off	Significant at many lags and dies off slowly	None is significant
Zenith(**Figure 5.44**)	Same as above	Significant at many lags and dies off slowly	None is significant

Table 5.18 below gives summary of our observations with regards to the ACF plots for all the banks considered in this research. **Columns 2, 3 & 4** summarise the level of significance observed in the log returns, absolute returns and squared returns for the daily data for the overall period. Apparently, it is only with Eco (ETI) and Unity banks that the daily log returns are linearly independent, while the remaining banks are linearly dependent, in contrast to the established popular stylized fact that daily returns are expected to have negative or non-significant ACF at lag 1 to be linearly independent. By this finding, the conclusion could be drawn that only two banks conform with the random walk hypothesis, while the rest contradict it. Again, the implication of these stylised facts for investment decision making linked to the different types of distributions that describe observed bank returns are discussed in Raheem and Ezepue (2018). The gist of the paper is that stylised facts of asset returns generally underpin the way investments are structured over different investment horizons (short, medium and long-term) and the underlying distributions determine the nature of the financial risk calculations involved.

Furthermore, we also observe that all the absolute returns are significant and persistent at long lags with slower decay rate, except for a few banks (such as: ETI, STANBIC, UBA and Unity), which die off faster. This is an indication of non-linear dependence/volatility clustering and long memory in the returns of the banks. However, except in the case of three banks (**Diamond, Fidelity and FCMB)** where the squared returns ACF was significant at some lags with faster decay and slower persistent rates compared to that of absolute returns, the remaining 13 banks did not show any sign of significance of their ACF in their squared log returns.

Table 5.18: The Results of the ACFs of Log Returns, Absolute and Squared Returns across the Banks

Banks	**Log Returns**	**Absolute returns**	**Squared Returns**
Access	Significant at lag 1	Significant at several lags	None appears significant
Afribank	Significant at lag 1	Significant at several lags	None appears significant
Diamond	Significant at lag 1	Significant at several lags	Significant at many lags
ETI	Not significant at lag 1	Significant at some lags	None appears significant
FCMB	Significant at lag 1	Significant at several lags	Significant at many lags
Fidelity	Significant at lag 1	Significant at several lags	Significant at many lags

First	Significant at lag 1	Significant at several lags	None appears significant
GTB	Significant at lag 1	Significant at several lags	None appears significant
Skye	Significant at lag 1	Significant at several lags	None appears significant
STANBIC	Significant at lag 1	Significant at some lags	None appears significant
Sterling	Significant at lag 1	Significant at several lags	None appears significant
UBA	Significant at lag 1	Significant at some lags	None appears significant
Union	Significant at lag 1	Significant at several lags	None appears significant
Unity	Not significant at lag 1	Significant at few lags	None appears significant
WEMA	Significant at lag 1	Significant at few lags	None appears significant
Zenith	Significant at lag 1	Significant at several lags	None appears significant

Presentation and Discussions of Test Results of ACFs for the Daily Returns

To further strengthen the observations made through the correlograms (ACF plots) presented above, **Table 5.19** presents the respective ACF values for the returns at lag 1 across the banks, and the results of the test statistics for the series. The ACFs for Eco bank (0.02) and Unity bank (0.038) in column 2, could be said to be approximately equal to zero, which is one of the criteria to conclude that the returns of the two banks are linearly uncorrelated, which conforms with the random walk theory, so that they can satisfy one of the stylized facts of stock returns.

Also, comparing the values obtained for absolute and squared returns, it is obvious that those of the absolute returns are higher than those of the squared returns; this is expected according to **Taylor effect theory**, discussed in the literature, that the values of the former should be higher than those of the latter. Thus, from these results we see that the returns of these banks satisfy another stylized fact of assets result. Meanwhile, the significance of the ACFs for absolute returns is a confirmation of the presence of **volatility clustering** in the returns of all the banks, and the slow rate in the decay of this series (absolute returns) is an indication for the presence of **long memory** in the banks returns.

The table further contains the Box Ljung (Q) test statistic results for the returns and its multiples at lag 1, with the p-values in parenthesis. The results of the tests show except for ECO bank, all the banks are significantly auto correlated at lag 1 a sign of predictability, which sometimes implies that those stocks were bearish **(Danielson, 2011)** for the periods of the research.

From the table further, it could be confirmed that for squared returns, the six banks (uncoloured ones), - Access, **Diamond, FCMB, Fidelity, First and Skye** are the only ones significant only at the 5% level, while the rest are not; but for the absolute returns, all are significant (see **columns 3 and 4**).

Table 5.19: ACFs of Log Returns, Absolute and Squared Returns at lag 1

Banks	Log Returns	Absolute Returns	Squared Returns
Access	γ_1 =0.1640; $Q_{(1)}$ =104.1600 (2.2e-16)	γ_1 =0.3030; $Q_{(1)}$ =356.1900 (2.2e-16)	γ_1 =0.0080; $Q_{(1)}$ =104.1600 (2.2e-16)
Afribank	γ_1 =0.2320; $Q_{(1)}$ =163.9100 (2.2e-16)	γ_1 =0.3030; $Q_{(1)}$ =356.1900 (2.2e-16)	γ_1 = 0.0110; $Q_{(1)}$ = 0.3660 (0.5451)
Diamond	γ_1 =0.2840; $Q_{(1)}$ =190.5800 (2.2e-16)	γ_1 =0.4830; $Q_{(1)}$ =552.6600 (2.2e-16)	γ_1 =0.4230; $Q_{(1)}$ = 425.0100 (2.2e-16)
ETI	γ_1 =0.0200; $Q_{(1)}$= 0.8549 (0.3552)	γ_1 =0.0780; $Q_{(1)}$ = 12.3760 (0.00043)	γ_1 =0.0000; $Q_{(1)}$ = 1.099e-05 (0.9974)
FCMB	γ_1 =0.1800; $Q_{(1)}$ =80.5700 (2.2e-16)	γ_1 = 0.4440; $Q_{(1)}$ = 1421.1000 (2.2e-16)	γ_1 =0.3660; $Q_{(1)}$ =332.3900 (2.2e-16)
FIDELITY	γ_1 =0.2340; $Q_{(1)}$ =130.3800 (2.2e-16)	γ_1 = 0.4570; $Q_{(1)}$ = 1613.8000 (2.2e-16)	γ_1 =0.3710; $Q_{(1)}$ =327.9100 (2.2e-16)
First	γ_1 =0.1280; $Q_{(1)}$ =63.8980 (1.332e-15)	γ_1 = 0.2970; $Q_{(1)}$ =341.5800 (2.2e-16)	γ_1 =0.0160; $Q_{(1)}$ =341.5800 (2.2e-16)
GTB	γ_1 =0.1460; $Q_{(1)}$ =82.6460 (2.2e-16)	γ_1 = 0.2970; $Q_{(1)}$ =340.6300 (2.2e-16)	γ_1 =0.0300; $Q_{(1)}$ =3.4441 (0.0635)
Skye	γ_1 =0.1800; $Q_{(1)}$ =72.5760 (2.2e-16)	γ_1 = 0.2710; $Q_{(1)}$ =494.1800 (2.2e-16)	γ_1 =0.0040; $Q_{(1)}$ = 494.1800 (2.2e-16)

STANBIC	γ_1 = 0.0740; $Q_{(1)}$ =13.249 (0.000273)	γ_1 = 0.1080; $Q_{(1)}$ = 28.0880 (1.169e-07)	γ_1 =0.0000; $Q_{(1)}$ =0.0003 (0.9853)
Sterling	γ_1 =0.1980; $Q_{(1)}$ =151.7455 (2.2e-16)	γ_1 = 0.3800; $Q_{(1)}$ =3648.4090 (2.2e-16)	γ_1 = 0.0060; $Q_{(1)}$ = 0.15397 (0.9665)
UBA	γ_1 =0.1620; $Q_{(1)}$ =101.3200 (2.2e-16)	γ_1 =0.2250; $Q_{(1)}$ =449.3400 (2.2e-16)	γ_1 = 0.0090; $Q_{(1)}$ = 0.3075 (0.5792)
Union	γ_1 = 0.1120; $Q_{(1)}$ =48.7590 (2.895e-12)	γ_1 =0.1730; $Q_{(1)}$ =115.6000 (2.2e-16)	γ_1 =0.0000; $Q_{(1)}$ =0.0005 (0.9821)
Unity	γ_1 =0.0380; $Q_{(1)}$ =3.2053 (0.0734)	γ_1 = 0.0600; $Q_{(1)}$ = 8.1251 (0.004366)	γ_1 =-0.0010; $Q_{(1)}$ =0.0006 (0.9798)
WEMA	γ_1 = 0.0610; $Q_{(1)}$ = 14.5360 (0.0001375)	γ_1 = 0.1220; $Q_{(1)}$ = 56.0070 (7.216e-14)	γ_1 = 0.0000; $Q_{(1)}$ = 4.5240e-05 (0.9946)
Zenith	γ_1 = 0.2410; $Q_{(1)}$ = 146.4600 (2.2e-16)	γ_1 = 0.2740; $Q_{(1)}$ = 313.9800 (2.2e-16)	γ_1 = 0.0370; $Q_{(1)}$ = 3.4527 (0.06315)

p-values in parenthesis.

Comparisons between ACFs of Daily and Monthly Returns

This section is intended to briefly compare autocorrelations between daily and monthly log, absolute and squared returns; it is expected that while the autocorrelation functions (ACFs) for monthly data will die off faster, decay rates for daily data will be slower and more prolonged, especially those of absolute returns

Figure **5.21** displays ACF plots for the log, absolute and squared returns for **Access bank**, and from the Figure, it appears that (1) while ACF for daily log returns is significant at lag 1, that of the monthly log returns is non-significant at the same subsequent lags; (2) while absolute returns for daily returns persist (for several lags) and die off slowly, those of the monthly are non-persistent and die off faster; and (3) the ACFs for the squared returns of both daily and monthly data are insignificant.

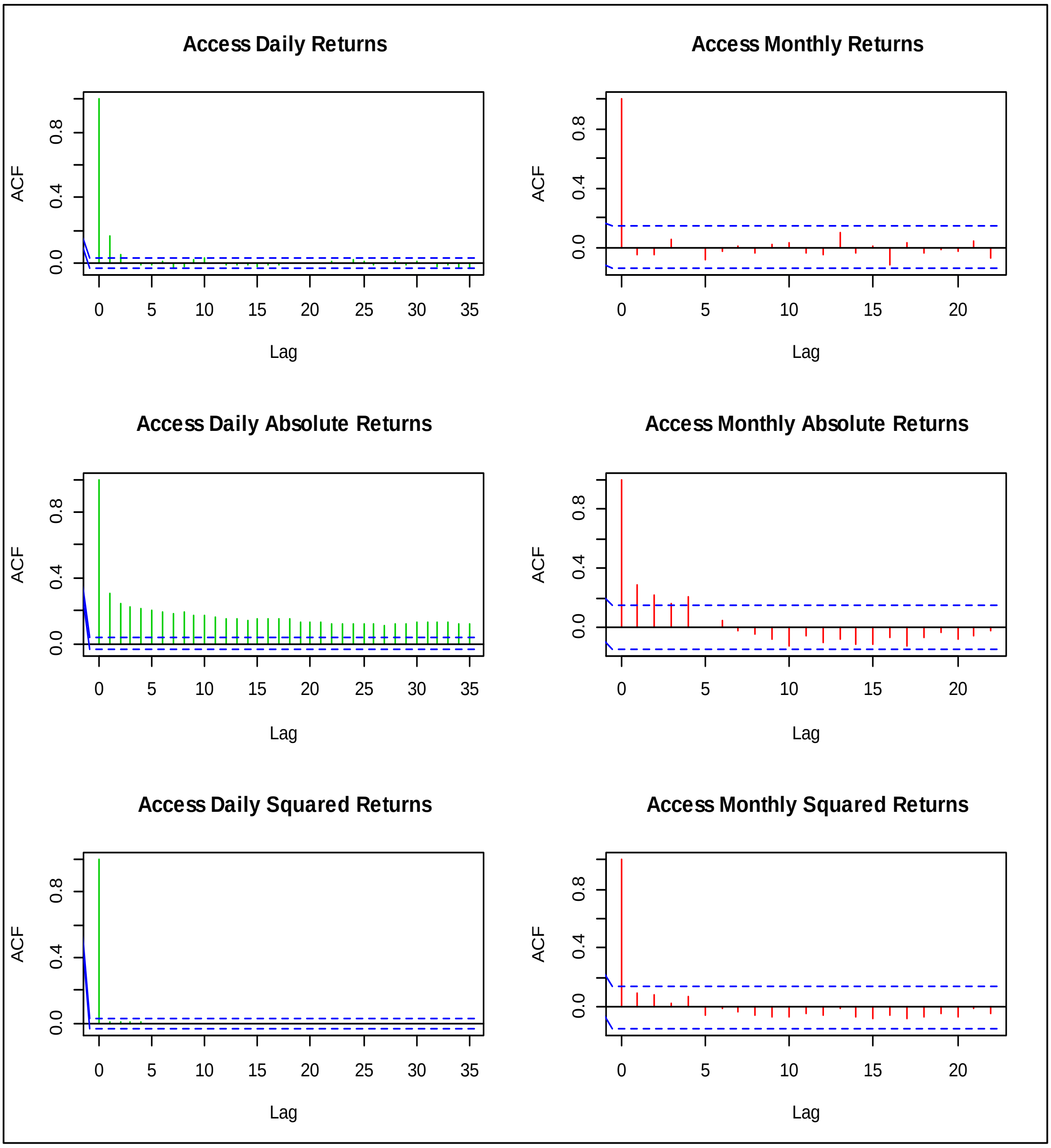

Figure 5.21: Daily versus Monthly ACFs for Log returns, Absolute and Squared Returns for Access Bank

For further emphasis, we hereby present only monthly absolute returns across the sixteen banks in Figure **5.22** below to visually observe the decay rates for monthly returns. From the plots it can be seen that they all die off faster compared to those of the daily absolute returns discussed at the beginning of Section 5.3.8 above; this behaviour is common to monthly ACFs across different markets (**See Taylor, 2011**).

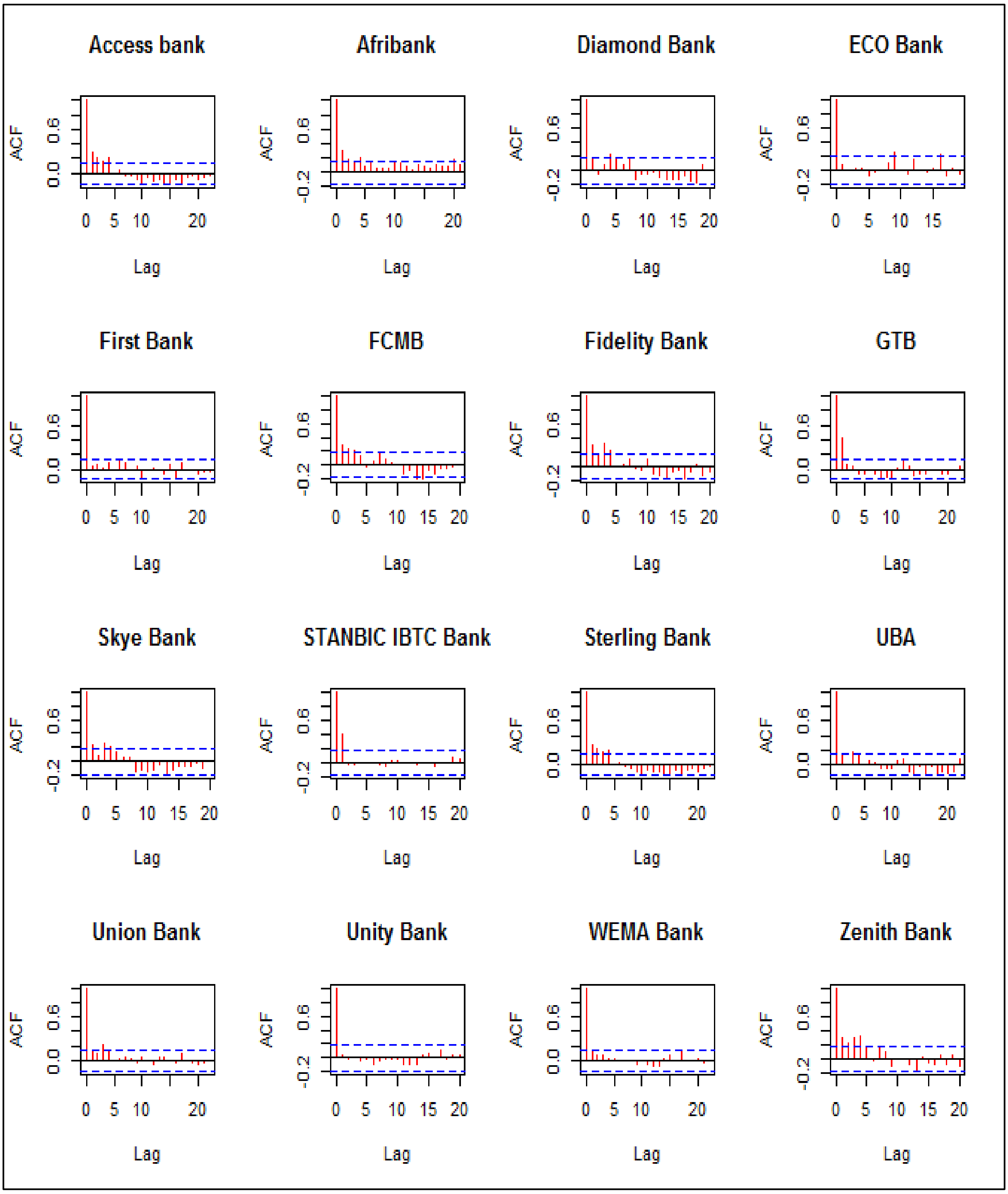

Figure 5.22: ACFs for Monthly Absolute Returns across the Sixteen Banks

5.3.9 Discussion of Tests of Leverage or Asymmetric Effects

We have in the tables below correlation test results between the respective banks' squared returns, as a proxy for volatility (see: Taylor, 2011; Zivot, 2008; Tsay, 2005), and a one-day lagged continuously compounded returns series; this serves as a preliminary or diagnostic test to identify if there is possibility of leverage or an asymmetric effect on an asset return (Zivot, 2009). While **Table 5.20** contains the results and remarks on the daily and financial crisis stock returns of the sixteen banks at the overall level. Apparently, five banks: ETI (for Ecobank), Guaranty Trust, UBA, WEMA and Zenith banks are negatively correlated; indicating the possibility of negative uncertainty or bad news dominating the returns of the banks.

During the financial crisis, the number of leveraged banks increased to 8 (see columns 4 and 5); namely: Access, Ecobank, FCMB, Fidelity, Sterling, UBA WEMA and Zenith banks. They are all negatively correlated with their respective volatility, which indicates that negative uncertainty or bad news impacted greatly on the returns of the concerned banks, compared to good news or positive returns.

Meanwhile, when the stock of a firm is leveraged it shows that their debt equity ratio rises, leading to more losses by the investors trading in such stocks. Thus, to adequately describe the volatility behaviour of stocks with potential signs of leverage via GARCH family models, those with the capacity to incorporate a leverage component would be preferred.

Further to our findings, four banks, namely: Ecobank, UBA, WEMA and Zenith have negative correlations both at the overall level and during the financial crisis; Guaranty Trust with negative correlation overall fails to show any sign of leverage during the crisis.

Table 5.20: Correlation between Squared and Lagged Log Returns for Daily (Overall) and Financial. Crisis Data

	Overall Period		**Financial Crisis Period**	
Bank	**Corr.** (r_t^2, r_{t-1})	**Remarks**	**Corr.** (r_t^2, r_{t-1})	**Remarks**
Access	0.0087**(0.587)**	NSLE	-0.0149**(0.7349)**	**SLE**
Afribank	0.0267**(0.1415)**	NSLE	0.0079**(0.8577)**	NSLE
Diamond	0.0261**(0.2049)**	NSLE	0.1177**(0.0074)**	NSLE
ETI	-0.0127**(0.5655)**	**SLE**	-0.0134**(0.7618)**	**SLE**
First	0.0181 **(0.2593)**	NSLE	0.0058**(0.8958)**	NSLE
FCMB	0.0522**(0.0094)**	NSLE	-0.0291**(0.5093)**	**SLE**
Fidelity	0.0614**(0.0027)**	NSLE	-0.0113**(0.7983)**	**SLE**
GTB	-0.0159**(0.324)**	**SLE**	0.0110 **(0.8034)**	NSLE
Skye	0.0034**(0.874)**	NSLE	0.0280**(0.05956)**	NSLE
Sterling	0.0111**(0.49)**	NSLE	-0.0830 **(0.5259)**	**SLE**
STANBIC	0.0192**(0.232)**	NSLE	0.0344**(0.4351)**	NSLE
UBA	-0.0048**(0.764)**	**SLE**	-0.0610 **(0.1666)**	**SLE**
Union	0.0017**(0.914)**	NSLE	0.0533 **(0.2272)**	NSLE
Unity	0.0002**(0.992)**	NSLE	0.0336**(0.4461)**	NSLE
WEMA	-0.0123**(0.444)**	**SLE**	-0.0866**(0.04923)**	**SLE**
Zenith	-0.0392**(0.049)**	**SLE**	-0.0764**(0.0828)**	**SLE**

Note: p-value in parenthesis; **NSLE**: No Sign of Leverage effect; **SLE**: Sign of Leverage Effect

5.3.10 Discussions of Long-Range Dependencies/ Long Memory in Returns

Table 5.21 below presents results of Ljung-Box tests on both the absolute and squared returns for Access bank for the other fifteen banks see Appendix 5C (ii). The tests are intended to show the existence of long-range dependence or long memory in the daily returns of the banks.

To begin with, it is noted that the test values of absolute returns are persistently higher than those of the squared returns across the banks, and the tests show that while the absolute returns continue to be significant across all the lags considered, the squared returns are non-significant. This implies that the decay rates of the return are very slow; this is indeed a sign of long memory in the returns, and it also implies that the series for each bank is neither independently nor identically distributed. Long memory in returns, indicates that predictability is possible and that any shock to the returns persists for a reasonably long time. This behaviour also points to the presence of a high level of volatility in the returns.

Table 5.21: Ljung Box Tests for Access Bank Daily Absolute and Squared Returns (Overall)

Returns\ Lags	5	10	20	30	50
Absolute	1121.1000 (2.2e-16)	1735 (2.2e-16)	2568 (2.2e-16)	3140 (2.2e-16)	4169 (2.2e-16)
Squared	0.7190 (0.982)	0.8060 (0.9999)	0.8174 (1)	0.89344 (1)	1.7172 (1)

Note: p-value in parentheses

5.4 Summary, Conclusion and General Interpretation of the Results

This section provides a synopsis of all the concepts and findings obtained so far by applying appropriate test statistics as discussed earlier. Finally, conclusions shall be drawn based on those findings.

5.4.1 Summary

In this Chapter we have: (1.) briefly introduced FTS, its objectives, its common and distinguishing features compared to conventional TS (see **Section 5.0**); (2) introduced Asset returns, their various types such as- single and multi-periods simple returns, portfolio and excess returns in **Section 5.1**; (3) provided further details on stylized facts as they relate to financial returns generally, and their peculiarities to daily and monthly data in Sub-s**ections 5.1.1** and **5.1.2**; (4) the graphical data presentations, where time plots, histograms and Q-Q plots are briefly discussed in Sections 5.2 and 5.3; (5) presented results on the four moments of returns distribution in **Section 5.3**; (6) discussed distributional properties of financial asset returns in **Section 5.3** and (7) presented results on both normality and autocorrelation tests, where for normal test, both single tests based on moments and joint tests were focused on, and for the autocorrelation test, discussion centred on the both ACF and PACF for log returns, absolute and squared returns in **Section 5.3**.

Meanwhile, while presenting results, time plots for price, log returns, absolute and squared returns are presented in a 2 by 2 matrix across the 16 banks (see Figure 5.1 and Appendix 5A**).** From the plots, the following observations were made regarding the distributions of the four series:

1. The prices exhibit random walk like behaviour and appear to be non-stationary across the sixteen banks a behaviour common to asset prices at various time frequencies (either daily, weekly or monthly) and across various markets of both developed and developing economies. Log returns unlike price series show clear mean-reverting behaviour by oscillating around a common mean value, which is approximately zero. The near common mean values across the banks conform to the assumption of covariance stationarity expected of asset returns across markets of different economies.
2. All the banks show one large boom-bust period in their prices during the run-up to the financial crisis (see Figures 5.2 above; and 5.17 in Appendix 5b (i)).
3. Four banks STANBIC, Unity, WEMA and GTB had two episodes of boom-bust periods the run-up to financial crisis and after the second reform initiated by the CBN aftermath of the financial crisis (see Figures 5.2; 5.18 in appendix 5b (i)).
4. The Fluctuation of the returns about the mean across the banks appear to change overtime and this is a confirmation of time-variable behaviour in the volatility of asset returns. However, visually, with the nature of the returns series we have, one cannot be sure if there is evidence of systematic time dependence (at least in linear form), in the returns. This will be investigated by examining if the autocorrelations due to log returns are approximately zero.
5. The volatility level in absolute returns is more pronounced and persistent through time than it is in the squared returns across all the banks. Visually, this is an evidence of long-range dependence or predictability in volatility-another major stylized fact of volatility across different markets, as contained in the literature.

6. The Fluctuation of the returns about the mean, otherwise known as the volatility, tends to change over time. This is an indication of possible non-stationarity in volatility. Clustering of periods of high and low volatility is more pronounced in the daily returns than in the monthly returns (see Figure 5.6).

Further, visually, fourteen banks were highly volatile during the financial crisis as against the eleven that were observed to be highly volatile in the overall daily returns.

Skewness-For the daily overall data: while eight banks are positively skewed, five are negatively skewed; for the monthly data: the same number of banks (five each), are positively and negatively skewed; and for the financial crisis data: while eight banks are negatively skewed, the remaining eight are approximately symmetric.

Kurtosis-Virtually all the banks are positive and highly leptokurtic for daily data, with WEMA bank being the most leptokurtic. The degree of leptokurtosis is much lower for the monthly data than it is for the daily data, though WEMA still leads here. However, only nine banks, with ECO bank leading (with the highest value in the degrees of kurtosis presented in **Table 5.12**), are positively leptokurtic, five are platykurtic, while two are mesokurtic for financial crisis data, indicating that the Nigerian banks are highly leptokurtic and that the financial crisis negatively impacted on the Nigerian banks.

The **histograms** also reveal visually that for the daily data, eight banks are positively skewed with a longer right tail, five are negatively skewed with longer left tails while the remaining two-FCMB and Fidelity are approximately symmetric (**see** Figures 5.11 and 5.12). For the financial crisis data, while eight banks are visually negatively skewed with a longer left tail, the remaining eight are approximately symmetric (**see** Figures 5.13 and 5.14).

The **Q-Q plots** which determine the heaviness or length of the tails clearly reveal how kurtosis is prone to outliers such that the farther away a point is (or some points are), from the straight lines (or other points), the heavier or longer the tails. Thus, by this, the closeness of data points to the straight line representing the referenced distribution, which in this case is the normal distribution, is a determinant of the tails thickness. For example, for the overall data, the following banks with at least one point farther away from the rest of the points in Figures 5.13 and 5.14-**WEMA (1882.011), Unity (1085.073), STANBIC (1065.405), ETI (783.707) and Union (744.061)** are the first five with thickest and longest tails and are thus, the most leptokurtic respectively (see their kurtosis values in parenthesis).

On the **normality tests** presented in Tables 5.13, 5.14 and 5.15, considering the Jarque-Bera test results in the sixth column of the tables, for the overall daily data, all the banks' returns series are non-normally distributed (see Table 5.16); for the monthly returns, except FCMB that is approximately normal, the rest are non-normally distributed (Table 5.17); and for the financial crisis returns data, all the banks are non-normal except for FCMB and Fidelity banks (**see** Table 5.16).

Regarding dependence in returns, none of the banks except for ECO and Unity banks is linearly independent as shown in Table 5.17 (Figures 5.32 and 5.42 in Appendix 5B**).** The sixteen banks are however non-linearly dependent as observed in their absolute autocorrelation functions (ACFs), especially for overall daily data. Additionally, it was found that in accordance with the **"Taylor effect**", absolute returns always produce the highest ACFs across the 16 banks compared to other "degrees" of the log returns for daily overall data (see Table 5.18).

Diagnostic tests to examine possible presence of a "Leverage effect", as proposed by Zivot (2009) reveal that (1) for the overall data, 5 banks- ECO, GTB, UBA, WEMA and Zenith show signs of a leverage effect; and (2) for the financial crisis data, 8 banks-Access, FCMB, Fidelity, ECO, Sterling, UBA, WEMA and Zenith show signs of a leverage effect (**see Table 5.20**).

Finally, it can be confirmed from Table 5.21 (and Tables 23 to 36 in Appendix 5C **(ii))** that all the banks could be said to show signs of **long-range dependence (or long memory)** because the ACFs of the absolute returns are significantly different from zero at long lags (**5, 10, 20, 30 and 50**).

5.4.2 Conclusion

In light of the above, the following conclusions are hereby drawn:

1. As expected, while the price series across the sixteen banks appear to be random walk non-stationary, the log returns series appear to show signs of stationarity since they oscillate around the common constant mean zero.
2. The changes of returns volatility overtime are an indication of **time variation** in the returns and the persistent alternation in varying levels of the returns is a confirmation of volatility **clustering** across the 16 banks; that is, "a volatile period tends to be followed by another volatile period of equal magnitude" (Mandelbrot, 1963).
3. The daily returns are more (positively) skewed and highly leptokurtic with longer tails compared to the monthly returns, and the rejection of the normality of the returns for

virtually all the banks indicates that a normal distribution does not generally match the empirical distributions of the banks' returns.

4. During the financial crisis, more than half of the banks (10) were negatively skewed. This means that, investors in ten of the sixteen banks experienced losses due to the 2008 global financial crisis that characterised the Nigerian economy. This is a confirmation of the impacts of the global financial crisis on the Nigerian economy as highlighted in Section **2.3.**
5. No evidence of linear dependence in the returns, at least at lag 1, was noted behaviour that Danielson (**2011, pp.12-13**) interpreted as evidence of predictability of volatility, which is a possible violation of market efficiency.
6. There is evidence for presence of a "Taylor effect" across the banks; and a "Leverage effect" in some banks' returns.
7. Finally, virtually all the stylized facts outlined in **Section 5.3** are found in the Nigerian banks' returns.

5.4.3 General Interpretation of the Results

It could be deduced that the behaviour of the data used in this study is a confirmation of how financial series behave across different markets. The possible lack of market efficiencies identified is also a reflection of lack of discipline leading to arbitrageur opportunities, which were found to have characterised the NSM both before and during the financial crises. The results also identified the possible reasons for the level of losses recorded in the market by the investors, culminating in near collapse of the Nigerian banks in 2008-2009 - an event which then prompted the CBN to initiate the second banking reforms in July 2009. These findings could significantly help the market regulators to ensure that a minimum level of discipline and control are maintained among the market participants. The findings could also suggest that periodic checks should be undertaken to track market movements and behaviour overtime by relevant stakeholders such as market makers and investors, with a view to making riskless investment decisions.

6 CHAPTER SIX: Market Efficiency Models and Tests

6.1 Introduction

This chapter discusses some stock returns properties, such as stationarity, random walks, and the Efficient Market Hypothesis (EMH), along with market efficiency and relevant test statistics found appropriate to investigate if the Nigerian banking industry could be said to be at least weak-form efficient, a result found not to hold at the overall market level (Ezepue and Omar, 2012). The results of our analyses will be presented, discussed, summarised and conclusions will be drawn.

6.2 Stationarity in Stochastic Processes

A stochastic process is a time-dependent sequence of random variables, and it is sometimes referred to as either a data generating process or a model. Various types of stochastic processes include the strictly stationary process, the stationary process, the uncorrelated process, the auto-correlated process, White noise, strict white noise, the Martingale process, the Martingale difference, the Gaussian process, and the linear process.

Meanwhile, the basis for time series (TS) analysis is stationarity. A TS is said to be stationary when there is absence of a systematic trend, no systematic change in variance, and neither is there a presence of periodic (cyclic) variation nor seasonality. There are essentially two forms of stationarity, namely **strict stationarity** and **weak stationarity**. A time series, say, r_t is strictly stationary only if the joint distribution of any sub-series, say $r_{t_1,}$ r_{t_2},…, $r_{t_n,}$ is exactly or approximately the same as that of any other sub-series of the same order, say $r_{t_{1+k}}$, $r_{t_{2+k}}$,…, $r_{t_{n+k}}$, for any time lag (k), where n and k are arbitrary positive integers, and $t_1, t_2, \dots, t_n,$ is the series of n positive integers. Thus, a strictly stationary TS is the one where the joint distribution of (r_{t_1},r_{t_2},…,r_{t_n}) is time-invariant. A time series is said to be weakly stationary if both the mean μ of the series, r_t and the covariance of (r_t, r_{t-k}), γ_k, say, are time-invariant.

Thus, for a **weakly stationary** series:

(1) $E(r_t) = \bar{r}$, a constant and

(2) $Cov\ (r_t, r_{t-k}) = \gamma_k$, a function depending on k (which is the lag) only.

A weakly stationary series tends to fluctuate with a constant variation around a constant level, whereas a strictly stationary series is difficult to verify using empirical data. In financial time series, therefore, the assumption of weak stationarity is commonly made about stock returns, and this can be confirmed given enough historical data.

6.3 The Efficient Market Hypothesis and Statistical Models for Returns

Assumptions of an Efficient Market

The following reasons account for why a market should be efficient (see Malkiel (1989; 1992)).

1. Price-takers: in an efficient market, one participant alone cannot influence the price of an investment, meaning that there is no room for arbitrageur opportunities.
2. Information is costless and widely available to all market participants at approximately the same time.
3. Information is generated in a random fashion such that announcements are independent of each other.
4. Investors react quickly and fully to the new information, with asset prices adjusting accordingly.

Implications of Market Efficiency

1. **Diversification**: when a market is efficient, the true value of investments and that of the market is obvious to every participant, thereby encouraging investors to take well informed decisions on how to spread their investments such that they do not put all their eggs in one basket.
2. **Portfolio Risk**: efficiency enables minimization risks by spreading wealth in various efficient market forms.
3. **Fees and Costs**: to hedge the asymmetric effects of shocks on investment, investors pay higher fees for more reliable and relevant market or trading information.
4. Only unexpected information impacts security prices.
5. Security selection becomes less useful simply because prices of assets are fairly determined.
6. The professional money managers have little or no role to play in determining security prices.
7. Timing the market or deciding when to trade to make abnormal gains is almost impossible.

8. Available Information is already incorporated into stock prices.

Reasons for Studying Market Efficiency

Degutis and Novickytė (2014) listed the first four of the following five reasons for studying market efficiency.

1. Inefficient markets accommodate higher risk-weighted returns. This is important for investors of both private and public firms.

2. Market efficiency helps corporate executives to determine the perceived value of companies.

3. The EMH helps market operators or planners to monitor or model stock market development.

4. The EMH serves as an underlying assumption in multiple financial models.

5. Market efficiency helps to determine the level to which market prices reflect the information of informed traders (Stiglitz, 1981a)

Meanwhile, there are some conditions according to Stiglitz (1981b) that an asset market needs to meet for it to be information efficient, these include:

i. Firms are expected to transmit information efficiently regarding their prospects to prospective investors.
ii. The market should provide the right incentives for gathering accurate and relevant information
iii. The market prices should reflect the available information to every market participants

6.3.1 Tests Related to Efficient Market Hypothesis

In financial econometric research (such as explored in Cuthbertson and Nitzsche, 2005), the objective is to determine if the efficient market hypothesis is consistent with the empirical market data. As noted earlier, to achieve this requires investigating if asset returns are predictable. We now discuss the two popular statistical tests applied to do this.

Tests for white noise

It is well established that if the returns are unpredictable, they should be white noise and may not necessarily be entirely independent. A process is white noise if it is stationary, uncorrelated and has zero mean (That is, if the returns at time $t\{r_t\}$ is a white noise process, it is expected that autocorrelation at lag, $k, \rho_k = 0, \forall\ k \neq 0$). To test for white noise simply indicates that the returns are linearly independent (especially at lag 1) but may depend on each other at higher magnitudes (other levels). Though there exists a range of test statistics in the literature used in testing for white noise, the simplest and most often applied is the omnibus test called the$Q(m)$Ljung-Box [16]portmanteau test presented in (**4.32**).

Tests for RWH

A**RW** is a special case of a unit root process, used in identifying the forms of shocks driving stock prices to make successive independent price changes (Lam, Wong and Wong, 2006).
To test for a **RW**, we investigate the presence of serial correlation and the common test statistic for this is the unit root test developed by Dickey and Fuller (1979, 1981). According to Lo and Mackinlay (1988), this test, though necessary, is not a sufficient condition for showing that a RW is a unit root process; this is because while a **RW** indicates that the returns must be uncorrelated, the unit root test allows for predictability. Thus, the test statistics for a RW are: (1) the Box-Ljung test and (2) the Dickey-Fuller unit root test. Taylor (2011) however states that variance ratio (**VR**) test is a better alternative to these tests and is more often used to test the hypothesis of RW processes for stock prices. If the **VR** test is accepted, it means that the market is efficient. The power of the available tests of the RWH depends on the test statistic and the alternative hypothesis.

Statistical Tests for Weak-Form Market Efficiency

Statistical tests such as the runs test, unit root test, serial correlation tests, and spectral analysis are some of the popular techniques used for testing for weak form efficiency. Most studies on the weak form of the EMH in emerging stock markets have applied the runs test and/or the unit root test as a principal method for detecting a random walk, which is a necessary and sufficient condition for market efficiency in the weak form.

[16]"m" is the maximum number of lags of autocorrelations

The runs test has found favour in the hands of authors such as: Barnes (1986), Dickinson and Muragu (1994), Sharma and Kennedy (1977), Karemera et al. (1999), Wheeler et al. (2002), Abraham et al. (2002), while the unit root test has received the attention of authors such as: Groenewold et al. (2003), and Seddighi and Nian (2004). Authors such as Fawson et al. (1996), Mookerjee and Yu (1999), and Abeysekera (2001) applied both methods to investigate the presence of weak-form efficiency in their work.

Other statistical methods adopted previously by researchers include the serial correlation test, including the correlation coefficient test, the Q-test, and variance ratio tests. Researchers such as: Dickinson and Muragu (1994), Fawson et al. (1996), Dockery and Vergari (1997), Alam, Hassan and Kadapakkam. (1999), Karemera et al. (1999), Mookerjee and Yu (1999), Abeysekera (2001), and Groenewold et al. (2003) applied both the correlation coefficient test and the Q-test to investigate efficiency in various markets.

The Variance Ratio test was applied by: Chang and Ting (2000), Cheung and Coutts (2001), Abraham et al. (2002), and Lima and Tabak (2004) to examine weak-form market efficiency in their studies.

Some researchers applied other techniques, such as spectral analysis (Sharma and Kennedy, 1977 and Fawson et al., (1996), the fractional integration test (Buguk and Brorsen, 2003), and the autoregressive conditionally heteroscedasticity (ARCH) test (Seddighi and Nian, 2004) to investigate evidence for market efficiency, especially at its weak-form level.

Given the foregoing and the nature of the available data for this study, the following methods (discussed in Chapter 4), are to be applied in this research: parametric tests (the variance ratio test and the autocorrelation test) and non-parametric tests (the runs test and the BDS test). The choice of the variance test in this study is informed by the fact that it is regarded as the most powerful parametric test statistics for market efficiency compared to any other (see Taylor (2005; 2011) and Campbell, Lo, and MacKinlay (1997)). Lo & MacKinlay (1989) and Faust, (1992) note that using the VR statistic has optimum power against alternatives to the random walk model can be advantageous when testing against several interesting alternatives such as those hypotheses associated with mean reversion. Also, given the fact that the distribution of the financial assets is non-normal, using distribution free statistics such as the runs and BDS test to investigate randomness is most appropriate.

6.4 Results and Discussions

This section presents the results and brief discussions of the outcomes of the various analyses carried out in this chapter. Following the procedures outlined in Section **4.3**, we shall be presenting results and discussions on the following tests considered so far, namely: stationarity tests using three different statistics; the variance ratio test; the unit root/ random walk test; BDS tests of non-linear dependency; the runs test of randomness; and the Ljung-Box tests of linear independence.

6.4.1 Discussions on Stationarity Tests on Daily Data at Overall level

To be able to check if the returns series for each of the banks is stationary or not, three different testing methods were adopted, namely: the Phillips-Perron (PP) Unit root tests (of Non-stationarity) by Phillips and Perron (1988), the Augmented Dickey-Fuller (ADF) test (of non-stationarity) by Said and Dickey (1984) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (for stationarity) by Kwiatkowski, Phillips, Schmidt and Shin (1992), the results of which are presented in **Table 6.1** below. It is obvious that the outcomes of the three different tests confirm that the series for each of the banks is stationary at the 5% level and thus could be used for further time series analysis without a need to transform (by way of differencing) the original series.

It is important to note that while for the KPSS test, a null hypothesis of stationarity is set; whereas for the PP and ADF tests, a null hypothesis of non-stationarity is set. On this note, it is expected that for each case, the null is rejected only if the p-value (in parenthesis) is less than 0.05, the level of significance selected. Thus, looking at the table, it is only with Afribank where the p-value is 0.0237 under KPSS that hypothesis of stationarity seems to be rejected; whereas in the remaining cases, the stationarity of the series is confirmed.

Table 6.1: Stationarity Tests on Daily Stock Returns across the Bank

Bank	**KPSS**	**Phillips-Perron**	**ADF (Level)**
Access	0.1918 (0.1)	-3237.9000 (0.01)	-52.7150 (0.01)
Afribank	0.5888(0.0237)	-2696.9000 (0.01)	-13.1260 (0.01)
Diamond	0.0900 (0.1)	-1627.5000 (0.01)	-36.3380 (0.01)
Ecobank	0.3474 (0.0998)	-2067.8700 (0.01)	-31.3901(0.01)

FCMB	0.3028 (0.1)	-2056.4400 (0.01)	-12.0090 (0.01)
Fidelity	0.1691(0.1)	-1744.4000 (0.01)	-12.4740 (0.01)
First	0.1640(0.1)	-2958.9000 (0.01)	-54.6450 (0.01)
GTB	0.1217 (0.1)	-2944.7000 (0.01)	-53.6550 (0.01)
Skye	0.3432 (0.1)	-1783.7000 (0.01)	-13.0810 (0.01)
STANBIC	0.1372 (0.1)	-2247.4000 (0.01)	-13.5540 (0.01)
Sterling	0.0414(0.1)	-3277.6660 (0.01)	-15.1424(0.01)
UBA	0.1102 (0.1)	-3050.6000 (0.01)	-52.8120 (0.01)
Union	0.0951 (0.1)	-3360.2000 (0.01)	-15.3520 (0.01)
Unity	0.1307 (0.1)	-2166.2000 (0.01)	-12.7880 (0.01)
WEMA	0.1049(0.1)	-3757.2000 (0.01)	-15.0470 (0.01)
Zenith	0.1511(0.1)	-1783.7000 (0.01)	-14.0420 (0.01)

Notes: p-values are in parentheses. Note

6.4.2 Discussions on the Variance Ratio Tests across the Banks

Table 6.2 below displays the results of the variance ratio tests which as previously stated, is a parametric test that is used in checking for the conformity of the financial series to the Random walk Hypothesis and thus weak-form market efficiency. From the results, it is noted that only in five banks: Ecobank, STANBIC, Union, Unity and WEMA, the hypothesis of Random Walk (RW) cannot be rejected, whereas the remaining twelve reject the RW hypothesis, which implies weak-form inefficiency subject to the hypotheses

$$H_0: VR = 1 \ vs \ H_1: VR \neq 1$$

Campbell, Lo and MacKinlay (1997, p.69) interpret that increasing variance ratios with lag values (especially for equally weighted returns/index) suggest a positive serial correlation in multi-period returns.

We apply variance ratios to test the Random Walk 3 model (or RW3) in Campbell, Lo and MacKinlay (1997, p.33). In RW3, the null hypothesis of constant mean returns and uncorrelated residuals is given as:

H_0: $r_t = \mu + \varepsilon_t$, versus an alternative hypothesis with trend component, with the residuals ε_t satisfying the constraint: $Cov[\varepsilon_t, \varepsilon_{t-k}] = 0 \ \forall\, k \neq 0 \ and \ Cov[\varepsilon_t^2, \varepsilon_{t-k}^2] \neq 0 \ \forall\, k = 0$, which are at least approximately satisfied by the stationarity of the returns series.

Table6.2: Summary Results of the Variance Ratio Tests

Possible Lags	2		4		8		16	
Banks	Variance Ratio (VR)	z-statistic	Variance Ratio	z-statistic	Variance Ratio	z-statistic	Variance Ratio	z-statistic
Access	0.5677	-4.9200 (0.000)	0.3013	-5.2500 (0.000)	0.1518	-5.3700 (0.000)	0.0757	-5.2600 (0.000)
Diamond	0.6321	-13.3100 (0.00)	0.3603	-12.7900 (0.00)	0.1771	-10.9800 (0.000)	0.0835	-8.5980 (0.00)
Ecobank	0.5065	-1.5400 (0.125)	0.2505	-1.5530 (0.112)	0.1303	-1.5400 (0.123)	0.0636	-1.5500 (0.122)
FCMB	0.5564	-14.2940 (0.00)	0.3103	-12.8600 (0.00)	0.1476	-10.7470 (0.00)	0.0760	-8.3020 (0.00)
Fidelity	0.6086	-14.3280 (0.00)	0.3396	-13.7100 (0.00)	0.1671	-11.4517 (0.00)	0.0809	-8.7710 (0.00)
First	0.5797	-3.9820 (0.0001)	0.3067	-4.3504 (0.00)	0.1440	-4.5350 (0.000)	0.0712	-4.4500 (0.00)
GTB	0.572	-8.3739 (0.00)	0.3139	-8.6924 (0.00)	0.1497	-8.6971 (0.000)	0.0738	-7.9330 (0.00)
Skye	0.587	-2.3920 (0.017)	0.3154	-2.6330 (0.009)	0.1532	-2.7745 (0.006)	0.0796	-2.7820 (0.005)
Stanbic	0.5407	-1.7940 (0.073)	0.2934	-1.8370 (0.066)	0.1446	-1.8996 (0.058)	0.0738	-1.9090 (0.06)
Sterling	0.5621	-5.1287 (0.00)	0.3075	-5.3526 (0.00)	0.1563	-5.4670 (0.000)	0.0783	-5.3500 (0.00)

UBA	0.5656	-4.7029 (0.00)	0.3124	-4.9140 (0.00)	0.1552	-5.0880 (0.000)	0.0733	-5.0600 (0.00)
Union	0.5424	-1.7930 (0.073)	0.2894	-1.8550 (0.064)	0.1396	-1.9238 (0.054)	0.0716	-1.9340 (0.053)
Unity	0.5128	-1.3160 (0.1881)	0.2601	-1.3320 (0.183)	0.1329	-1.3381 (0.1809)	0.0652	-1.3460 (0.178)
WEMA	0.5167	-1.2580 (0.208)	0.2658	-1.2740 (0.203)	0.1316	-1.2912 (0.197)	0.0670	-1.2940 (0.196)
Zenith	0.6227	-5.4425 (0.000)	0.3376	-6.1313 (0.00)	0.1639	-6.2051 (0.000)	0.0847	-5.7010 (0.00)

Notes: p-values are in parentheses

6.4.3 Discussions on the Unit Root (Random Walk) Tests across the Banks

Table 6.3 below presents the results of the Unit root non-stationary test via the Augmented Dickey Fuller test of random walk without drift subject to the hypothesis $H_0: \varphi = 1\ vs\ H_1: \varphi < 1$ (hypothesis of covariance stationarity, see (**4.38**))

The results show that each bank's series has no unit root since the φ value linked to each bank is less than 1. This means that there is significant evidence that their returns series are stationary, and thus rejects the random walk hypothesis or possibility of a unit root. Alternatively, we say that all the banks' returns are (covariance) stationary, implying that the series can be used in further analysis without any need for differencing.

Table 6.3: Unit Root Tests on Daily Returns across the Banks

Bank	**ADF (Without Drift Parameter)**	**Test statistics** (p-values in parentheses)
Access	$\varphi = -0.8146$	$t = -39.1730\ (2e - 16)$
Afribank	$\varphi = -0.6758$	$t = -30.2980\ (2e - 16)$
Diamond	$\varphi = -0.7299$	$t = -25.5600\ (2e - 16)$
Ecobank	$\varphi = -0.9697$	$t = -31.3540\ (2e - 16)$

FCMB	$\varphi = -0.7730$	$t = -30.0210\ (2e - 16)$
Fidelity	$\varphi = -0.75602$	$t = -29.7730\ (2e - 16$
First	$\varphi = -0.8950$	$t = -42.1850\ (2e - 16)$
GTB	$\varphi = -0.8946$	$t = -35.6990\ (2e - 16)$
Skye	$\varphi = -0.8158$	$t = -30.1270\ (2e - 16)$
STANBIC	$\varphi = -0.9059$	$t = -32.5560\ (2e - 16)$
Sterling	$\varphi = -0.7522$	$t = -36.9710\ (2e - 16)$
UBA	$\varphi = -0.8158$	$t = -39.1820\ (2e - 16)$
Union	$\varphi = -0.8655$	$t = -40.3920\ (2e - 16)$
Unity	$\varphi = -0.9497$	$t = -32.2560\ (2e - 16)$
WEMA	$\varphi = -0.9136$	$t = -41.4700\ (\ 2e - 16)$
Zenith	$\varphi = -0.7600$	$t = -31.0180\ (2e - 16)$

Note: φ is the unit root parameter

6.4.4 Discussions on the Non-linear Independence (BDS) Tests across the Banks

In this test, both the dimension and the epsilon tolerance (are arbitrary values), for close points were chosen for the respective banks automatically by the R-software used to analyse the data. From the results, the significance of the p-value across the banks is an indication for rejection of the null hypothesis of linear independence and leads us to conclude that the returns for each bank are non-linearly dependent, which is a good indication of non-random walk behaviour. This means that there is an element of predictability, which contradicts weak-form market efficiency (see **Table 6.4** and Appendix **6A**).

Table 6.4: BDS Test Results for Access Bank

Dimension\Epsilon for close points	0.0145	0.029	0.0435	0.058
2	34.8877 (0.00)	30.4356 (0.00)	28.6468 (0.00)	29.0610 (0.00)
3	51.3960 (0.00)	36.9197 (0.00)	32.3018 (0.00)	32.7991 (0.00)

Notes: p-values are in parentheses

6.4.5 Discussion of the RUNS Test

Table 6.5 below presents the results of the tests of randomness in the data generating process for the daily returns of the banks using the non-parametric test called the runs test. In a bid to achieve this, we chose the mean and median returns and whether the returns are randomly assigned below and above zero point as our target for the runs test. **Table 6.5** shows the results where the runs were classified based on the mean. For those based on the median see Appendix 6b. Thus, **Table 6.5** tests if the returns are randomly assigned below and above the mean. From the p-values (in the 8^{th} column of Table 6.5), the null hypothesis of randomness (or linear independence) is rejected in favour of the alternative of non-randomness - an indication of violation of the random walk hypothesis and thus a sign of rejection of weak-form market efficiency across the sixteen banks.

Table 6.5: Summary Results of The Run Tests based on Mean

Bank	**Test Value (Mean)**	**Cases < Test Value**	**Cases >=Test Value**	**Total Cases**	**No of Runs**	**Z statistic**	**p-value**
Access	0.0060	2499	1370	3869	1351	-14.7560	0.000
Afribank	-0.0005	823	2222	3045	737	-21.3740	0.000
FCMB	-0.0002	896	1578	2474	948	-8.5310	0.000
Fidelity	-0.0003	923	1453	2376	900	-9.9290	0.000
First	-0.0001	1662	2207	3869	1544	-11.5850	0.000
GTB	0.0006	2256	1613	3869	1585	-9.8250	0.000
Diamond	-0.0001	974	1394	2368	881	-11.3220	0.000
Eco bank	-0.0014	706	1344	2050	847	-3.9000	0.000
Skye	0.0001	1397	846	2243	794	-11.7250	0.000
STANBIC	0.0001	2400	1469	3869	1444	-12.9540	0.000
Sterling	0.0001	2658	1211	3869	1138	-19.7010	0.000
UBA	0.0001	2228	1641	3869	1604	-9.4460	0.000
Union	-0.0001	1494	2375	3869	1404	-14.6250	0.000
Unity	0.0004	1558	665	2223	618	-15.9450	0.000
WEMA	0.0003	2694	1175	3869	1082	-21.1130	0.000
Zenith	0.0002	1446	1070	2516	1062	-6.8900	0.000

6.4.6 Discussions on ACF Tests of the Log Returns across the Banks

Table 6.6 below displays the Autocorrelation test for linear independence of the return series at different lags of orders 1, 5, 10 and 20.

Table 6.6: Log Returns ACF Tests across the Banks

Bank\ Lags	1	5	10	20
Access	104.1600 (2.2e-16)	114.6900 (2.2e-16)	123.8100 (2.2e-16)	125.6500 (2.2e-16)
Diamond	190.5008(2.2e-16)	219.1000 (2.2e-16)	222.7600 (2.2e-16)	233.3200 (2.2e-16)
Ecobank	0.8549(0.3552)	2.9914 (0.7013)	7.0563 (0.8539)	19.0244 (0.5202)
Afribank	163.9100 (2.2e-16)	287.4800 (2.2e-16)	310.9200 (2.2e-16)	330.2500 (2.2e-16)
FCMB	80.5700 (2.2e-16)	106.1300 (2.2e-16)	117.0300 (2.2e-16)	134.6500 (2.2e-16)
First	63.8980 (1.332e-15)	112.8000 (2.2e-16)	113.4000 (2.2e-16)	117.2400 (2.2e-16)
UBA	101.3200 (2.2e-16)	128.1500 (2.2e-16)	139.6500 (2.2e-16)	148.2100 (2.2e-16)
Unity	3.2053(0.0734)	3.7771 (0.5819)	5.0360 (0.8888)	5.4417(0.9995)
GTB	82.6460 (2.2e-16)	114.4600 (2.2e-16)	136.2500 (2.2e-16)	144.8200 (2.2e-16)
Skye	72.5760 (2.2e-16)	78.8850 (1.443e-15)	82.1780 (1.876e-13)	96.2050 (5.98e-12)
STANBIC	13.2490 (0.000273)	16.4470 (0.005677)	18.3980 (0.0486)	24.3050 (0.2293)
Sterling	151.7455(2.2e-16)	194.7000 (2.2e-16)	195.7000 (2.2e-16)	203.8000 (2.2e-16)
Union	48.7590 (2.895e-12)	60.5200 (9.492e-12)	66.3990 (2.186e-10)	70.5900 (1.457e-7)
Fidelity	130.3800 (2.2e-16)	145.1600 (2.2e-16)	151.6000 (2.2e-16)	163.6600 (2.2e-16)
WEMA	14.5360 (0.0001.375)	19.2130 (0.00175)	20.2020 (0.0274)	23.0980 (0.284)
Zenith	146.4600 (2.2e-16)	159.2100 (2.2e-16)	167.5800 (2.2e-16)	180.2400 (2.2e-16)

Notes: p-values are in parentheses

The results show that all the banks except for two - Eco and Unity banks - where the correlations are not significant at any of the indicated lags, whereas STANBIC and WEMA banks became insignificant at lag 20. Meanwhile, the significance of the fourteen other banks shows that these banks' returns are not Random Walks, indicating that the bank returns are do not have unit root.

6.5 Summary and Conclusion

In this chapter, the following three concepts of stock return properties bordering on market efficiency of the banking sector of the NSM were discussed: stationarity; random walks (RW); and the efficient market hypothesis (EMH). Under these concepts, we briefly examined strict and weak stationarity and unit-root non-stationarity, an example of which is the random walk hypothesis. Further discussion was therefore focused on random walk theory 'without drift'.

The efficient market hypothesis (EMH) and different categories of innovations in a mean equation, namely martingale differences, white noise, and independently and identically distributed (iid) innovations, were discussed. Tests Related to the EMH were also discussed. Various forms of market efficiency –the perfect, strong, semi-strong, and weak forms were also discussed. The relevant test statistics used in examining weak-form market efficiency found in the literature were surveyed. Finally, test statistics such as the variance ratio, BDS, runs, ADF, KPSS and PP tests found relevant to our research objectives were discussed and applied.

Having applied the various methods, the following findings were made. From the KPSS, PP and ADF tests on the stationarity of the banks' return series, it was found that only with Afribank was the null-hypothesis of stationarity rejected in favour of the non-stationarity alternative, while for the remaining 15 banks the null hypothesis of stationarity was not rejected using the KPSS test. However, for the PP and ADF tests, we rejected the null-hypothesis of non-stationarity (or unit root) across all 16 banks in favour of the stationarity alternative (that is, no unit root; see **Table 6.1**). Hence, the banks' returns were overall stationary and do not require differencing for further empirical financial analysis using appropriate time series techniques. The random walk and weak-form market efficiency tests using variance ratio tests showed that while the hypothesis of a random walk and related market efficiency was not rejected only in four banks' returns, namely **Eco, STANBIC, Union and WEMA** banks. The rest rejected the RW hypothesis, meaning that these banks' returns are not-weakly efficient (see **Table 6.2**).The results on the random walk without drift using the ADF test show that the series are stationary, and not random walks, which indicates that the returns are not-weakly efficient (**see Table 6.3**).

From **Table 6.4**, it is apparent that the null hypothesis of no non-linear independence is rejected, which indicates that the banks' returns are non-linearly dependent, and this again contradicts weak-form market efficiency.

The runs tests displayed in **Table 6.5**, where the hypothesis of randomness was set, however, rejected randomness across the 16 banks, which is a violation of the random walk and linear independence properties of the returns.

Complementary ACF tests for the log returns for lags (**1-10**), intended to determine linear independence in the returns, were conducted, and the results revealed that the hypothesis of independence is rejected for all the banks except for ECO and Unity banks.

In light of the above findings, we can conclude that the Nigerian banks could not be said to be weak-form efficient – findings which are in line with those of Omar (2012) and Ezepue and Omar (2012) on the All Share Index (ASI) of the NSM at the overall market level. As noted in Chapters 1 and 2 of the thesis, these initial studies on systematic stock market characterisation (SSMC) of the NSM suggested complementary sector-level studies of the six main empirical market features (anomalies, bubbles, efficiency, predictability, valuation and volatility). What this chapter contributes additionally to the overall market knowledge of the dynamics of the NSM, as elucidated in Chapter 1 of the thesis, is a demonstration that the weak-form market inefficiency in the NSM found in these studies applies to the banking sector.

The Chapter also expanded on the different tests of market efficiency-related characteristics of the NSM, beyond the extent achieved in Ezepue and Omar (2012) and Omar (2012). The implications of these results together with the stylised facts of banks' returns in the sector which were explored in Chapter 5 of the thesis were also foreshadowed in Chapter 1 in relation to systematic stock market characterisation and development (SSMCD) (Raheem and Ezepue, 2018). These references further indicated ways in which the stylised facts of the kinds obtained in Chapter 5 and the six empirical market features noted above could inform investors' strategies, NSM policy directions and market development moves, as well as wider economic management of the Nigerian economy by the CBN and other financial agencies in the country. We also noted that the banks' returns stylised facts reinforce these ideas for such a dominant part of the Nigerian economy. It was mentioned in Chapter 1 that this is the first time known to the researcher that this in-depth a characterisation of the empirical financial features of Nigerian banks has been examined (using individual returns series for the sixteen banks considered).

Particularly regarding the efficiency-related tests in this chapter, the lack of weak-form efficiency of the banking sector of the NSM was found to be associated with evidence of nonlinearity and predictability in returns. This implies the possibility of making money (arbitrageur opportunities) in the sector a quest that requires not only knowledge of SSMCD and related algorithmic investment but also portfolio management of the kind explored in Dalio (2017). The next chapter explores market anomalies in the banking sector of the NSM.

7 CHAPTER SEVEN: MARKET ANOMALIES

7.1 Introduction

In this chapter, the attributes of, reasons for studying, and implications of stock anomalies, together with the specific objectives of our study and the descriptive statistics on the yearly average bank returns will briefly be introduced. Following that, we apply the various methodologies relevant to our objectives (as presented in Section **4.4**). The results of these will be presented and discussed in the subsequent sections of this chapter.

7.2 Attributes of Market Anomalies

1. They occur outside of the relevant market information;
2. They are expected to be consistent over long time periods;
3. They are well researched and are not due to data related error(s) such as missing data, data imputation errors or small sample sizes.
4. According to Schwert (2003), anomalies are strange and difficult to predict because they are bound to fade and resurface (**see** Ullah, Ullah and Ali, 2016).

7.3 The reasons for studying Stock Market anomalies

Stock market anomalies are studied for several reasons which include:

1. It helps investors to understand how a market reacts to the variations caused by changing patterns in investments related to days, weekdays, months, seasons, years, government policies and general activities in the economy, both at local and international levels. Knowing these enables them to design favourable trading strategies that factor such predictable dynamics into their investment decisions.
2. Market makers and regulators also require knowledge of such regular shifts to be able to ensure continuous liquidity in the market and to maintain balanced checks that permit little or no room for arbitrage opportunities by any participant.

7.4 Implications of Stock Anomalies for Investment Strategies

There are implications of the anomalies for relevant stakeholders and the market; these include:

1. It is difficult in most cases to benefit from the anomalies because they tend to disappear with time such that prices adjust back to the rational level.

2. The fact that some anomalies are not violations of the efficient market hypothesis, but rather they are due sometimes to the statistical methods applied in detecting the anomalies and the data available.

Thus, an investor who wants to earn abnormal profit using the anomalies may over time be disappointed, especially in a well-regulated market with little or no room for arbitrage opportunities.

Specific objectives of this chapter

1. To determine the yearly averages to see the differential patterns across the years for all the banks;
2. To investigate the presence of Day-of-the-week Effect Anomalies in the Nigerian Banks' stock returns;
3. To determine if there exist January/Holiday Effect Anomalies in the Nigerian banks' stock returns;
4. To see if Nigerian banks' stocks are characterised by an October-March Seasonality Effect Anomaly;
5. To investigate the presence of a Turn-of-the-year Effect Anomaly in the Nigerian banks' stock returns.

Descriptive Statistics on the Yearly Average Banks' Returns

In this section, the yearly average returns across the sixteen banks are presented in an abridged **Table 7.1** below, where only the years with the highest and lowest average returns are shown, followed by their respective bar plots in Figures 7.55g-7.68g (see **Appendix 7G**).

Table7.1: Yearly Average Returns across the Banks

Bank	Period (years)	Highest	Lowest
Access	June1999-Dec.2014 (15.5years)	**2007 (0.28%)**	**2008(-0.45 [17]%)**
Afribank	June 1999-Sept 2011(13.25years)	**2000(0.49%)**	**2011(-0.66%)**
Diamond	May 2005-Dec 2014(9.58years)	**2005(0.63%)**	**2011(-0.53%)**
Ecobank	Sept 2006-Dec, 2014(9.25years)	**2013(0.15%)**	**2008(-0.66%)**
First	June 1999-Dec 2014(15.5years)	**2000(0.31%)**	**2008(-0.30%)**
FCMB	Dec2004-Dec2014(10years)	**2004(4.48%)**	**2008(-0.46%)**
Fidelity	May 2005-Dec 2014(9.5years)	**2007(0.74%)**	**2008(-0.37%)**

[17] The red color in the Table 7.1 is used to identify 2008 as the year with the highest frequency in terms of drop in the average returns of 11 out of 16 banks.

STANBIC	April 2005-Dec 2014(9.67%)	**2014(0.53%)**	**2008(-0.65%)**
GTB	June1999-Dec.2014(15.5years)	**2003(0.31%)**	**2008(-0.39%)**
Skye	Nov. 2005-Dec.2014 (9.083years)	**2005(2.84%)**	**2011(-0.33%)**
Sterling	June 1999-Dec., 2014(15.5years)	**2007(0.28%)**	**2008(-0.45%)**
UBA	June 1999-Dec. 2014(15.5years)	**2007(0.28%)**	**2008(-0.53%)**
Union	June 1999- Dec 2014(15.5years)	**2011(0.38%)**	**2008(-0.43%)**
Unity	Dec 2005 -Dec 2014(15.5years)	**2005(3.44%)**	**2009(-0.76%)**
WEMA	June 1999 - Dec 2014(15.5years)	**2014(0.78%)**	**2009(-1.08%)**
Zenith	Oct 2004- Dec 2014(10.17years)	**2004(1.66%)**	**2008(-0.29%)**

Notes: Number of years in parentheses in column 2; Yearly averages (in %) in parentheses for columns 3 and 4

Meanwhile, for further understanding, **Figure 7.1** below presents the bar plot for Access Bank, wherein the bank was mostly bullish in 2007 with average returns of 0.28% but became most bearish in 2008 as shown in Table 7.1 above.

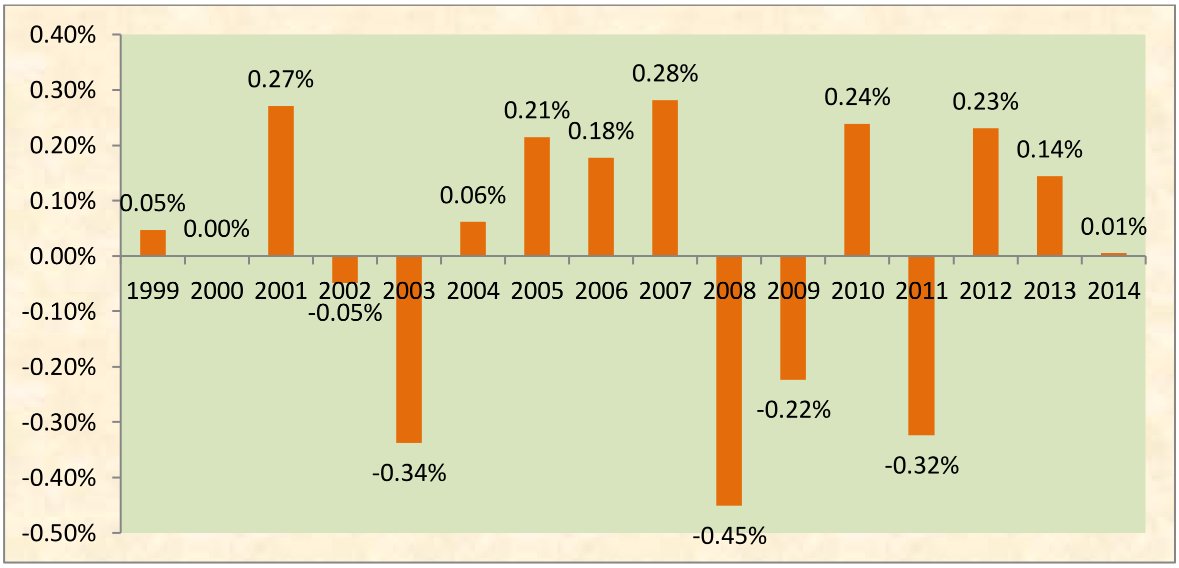

Figure 7.1: Yearly Average for Access Bank

From Table 7.1, apparently 2008 (in red) is the year when 11 out of 16 banks, representing approximately 69% of all the banks examined, experienced the lowest average stock returns. Combined with the bearish nature of Access Bank's stock from 2008-2009, the results demonstrate the negative effect of the 2007-09 global financial crunch on Nigerian banks. For example, three banks- Afribank, Diamond and Skye became bearish in 2011, while two banks, Unity and WEMA were bearish in 2009. Again, these findings show that 2008-2009 largely impacted negatively on the Nigerian banks such that 13 out of 16 became bearish, representing about 82% of all the banks investigated.

Given the nature of the data available for this research and the nature of the Nigerian financial market, which is still nascent, this study will be focused on few of the calendar anomalies that we believe are likely to characterise the Nigerian market, and have empirically been studied across different markets, especially developing markets. Some of these calendar anomalies include, but are not limited to, the following: Day-of-the week and Monday effect anomalies; a January effect; a Holiday effect; a Turn-of-the-year effect, and the October-March seasonality effect. Thus, the effects listed above will be explored in the context of recent banking and financial reforms in Nigeria and the 2008-2009 global financial crises.

7.5 Methodologies

This section discusses the statistics analysing anomalies. As mentioned earlier, the analysis will be focused on Day-of-the-Week / Monday Effect, Holiday effect, Turn-of-the-year Effects and October-March Seasonal Effects. The approaches and dummy regression models (equations **4.69 to 4.73**) for the analyses are presented in Section **4.4** of Chapter 4 of this thesis. Note that other anomaly effects models than those presented in that section follow the same dummy variable regression technique aimed at isolating the effects of the days, months, holidays, or other factors like financial reforms and crises.

7.6 Results and Discussions

It is important to note that while presenting the results, both tables and bar graphs shall be used; given the small magnitude of the values, percentages shall be used. The results are now presented below for all the banks.

7.6.1 Day-of-the-week/Monday effect Results

Table 7.1 presents the results for this anomaly. Generally, seven banks-Access, Afribank, Ecobank, First, FCMB, Union and Sterling banks - are found to have negative average returns on Monday, while the remaining nine bear positive average returns on Monday. Similarly, except for Union Bank where the lowest average returns were recorded on Tuesday, the remaining six banks with negative Monday returns produced the least average returns on Monday. Further, Ecobank and Fidelity recorded negative average returns on Fridays as against what is contained in the literature that Friday average returns are mostly positive and the highest. Also, while every other bank has its highest average returns on Friday, Ecobank, and Fidelity, with negative Friday average returns, have their highest average returns on Wednesday. It is important however to note that based on the p-value, there is no significant difference in the average returns except for only four banks, namely: Diamond, Skye, UBA and Union banks.

For a more visual understanding, Figure 7.1 (below) and 7.2a-7.16a (in **Appendix 7A**) provide a better assessment of Access bank and others, with respect to this type of anomaly. From the results, Access, Afribank, First, and Sterling banks could be said to reflect both a Monday and Day-of-the-week effect. Ecobank and FCMB reflect a Monday effect, and Union shows a Day-of-the-week effect, at least to an extent. Diamond and Union have the lowest average returns on Tuesday, as was found in Smirlock and Starks (1986) and Barone (1990).

Table: 7.2: Day-of-the-Week/Monday Effect Table across the Banks[18]

BANK	**WEEKDAY**	**DAILY**	**Remarks**
Access	**Monday**	**-0.07%(0.568)**	**Lowe[19]st (Negative)**
	Tuesday	-0.05%(0.6715)	
	Wednesday	-0.04%(0.685)	
	Thursday	0.04%(0.874)	
	Friday	**0.16%(0.370)**	**Highest**
Afribank	**MONDAY**	**-0.08%(0.637)**	**Lowest(Negative)**
	TUESDAY	-0.01%(0.5635)	
	WEDNESDAY	-0.02%(0.575)	
	THURSDAY	-0.14%(0.5445)	
	FRIDAY	**0.02%(0.422)**	**Highest**
Diamond	**MONDAY**	**0.10%(0.3881)**	**(Highest)**
	TUESDAY	-0.27%(0.02335)	**Lowest**
	WEDNESDAY	-0.03%(0.8616)	
	THURSDAY	0.08%(0.4537)	
	FRIDAY	**0.09%(0.4057)**	
Eco	MONDAY	-0.34%(0.3379)	Lowest
	TUESDAY	-0.19%(0.8079)	
	WEDNESDAY	**0.05%(0.3365)**	**Highest**
	THURSDAY	-0.08%(0.756)	
	FRIDAY	**-0.16%(0.9277)**	
First	**MONDAY**	**-0.19%(0.0693)**	**Lowest**
	TUESDAY	-0.06%(0.6306)	
	WEDNESDAY	0.03%(0.6315)	
	THURSDAY	0.01%(0.8182)	
	FRIDAY	**0.14%(0.1189)**	**Highest**
FCMB	MONDAY	-0.21%(0.0649)	Lowest
	TUESDAY	-0.01%(0.9661)	

[18] **p-values in parentheses for test based on equation 4.69 in Chapter 4**

[19] Colour green is used to identify when Monday's effect is lowest and negative; whereas colour yellow is applied whenever Friday's effect is the highest par bank in **Table 7.2**.

	WEDNESDAY	**0.09%(0.3107)**	**Highest**
	THURSDAY	0.02%(0.7491)	
	FRIDAY	**0.03%(0.664)**	
Fidelity	MONDAY	**0.17%(0.9142)**	
	TUESDAY	-0.19%(0.2309)	Lowest
	WEDNESDAY	**0.68%(0.5741)**	**Highest**
	THURSDAY	0.42%(0.6221)	
	FRIDAY	**-0.40%(0.2466)**	**Lowest**
GTB	MONDAY	0.09%(0.7571)	
	TUESDAY	0.05%(0.9102)	
	WEDNESDAY	-0.09%(0.0907)	Lowest
	THURSDAY	**0.17%(0.2149)**	**Highest**
	FRIDAY	**0.08%(0.7919)**	
Skye	**MONDAY**	**0.38%(0.01009)**	**Highest**
	TUESDAY	-0.11%(0.4015)	
	WEDNESDAY	**-0.15%(0.2347)**	**Lowest**
	THURSDAY	-0.13%(0.2929)	
	FRIDAY	**0.09%(0.5829)**	
STANBIC	MONDAY	**0.23%(0.624)**	**Highest**
	TUESDAY	0.05%(0.4575)	
	WEDNESDAY	**-0.25%(0.5359)**	**Lowest**
	THURSDAY	0.00%**(0.9524)**	
	FRIDAY	**0.08%(0.3469)**	
Sterling	MONDAY	-0.07%(0.4662)	Lowest
	TUESDAY	-0.05%(0.6051)	
	WEDNESDAY	-0.04%(0.6244)	
	THURSDAY	0.04%(0.7702)	
	FRIDAY	**0.16%(0.1503)**	**Highest**
UBA	MONDAY	0.21%(0.06349)	Highest
	TUESDAY	-0.06%(0.5149)	
	WEDNESDAY	0.01%(0.9532)	
	THURSDAY	**-0.21%(0.0441)**	**Lowest**
	FRIDAY	**0.09%(0.4371))**	
Union	MONDAY	-0.07%(0.656)	
	TUESDAY	**-0.31%(0.0215)**	**Lowest**
	WEDNESDAY	-0.08%(0.5543)	
	THURSDAY	0.14%(0.2613)	
	FRIDAY	**0.28%(0.02721)**	**Highest**
Unity	MONDAY	0.31%(0.6385)	Highest

	TUESDAY	**-0.33%(0.06883)**	**Lowest**
	WEDNESDAY	0.12%(0.3313)	
	THURSDAY	-0.04%(0.3785)	
	FRIDAY	**0.14%(0.1492)**	
WEMA	MONDAY	**0.26%(0.1972)**	**Highest**
	TUESDAY	-0.11%(0.4249)	
	WEDNESDAY	-0.14%(0.3144)	**Lowest**
	THURSDAY	0.03%(0.9995)	
	FRIDAY	**0.13%(0.5926)**	
Zenith	MONDAY	0.03%(0.9211)	
	TUESDAY	**-0.08%(0.3277)**	**Lowest**
	WEDNESDAY	0.02%(0.9826)	
	THURSDAY	**0.07%(0.6347)**	**Highest**
	FRIDAY	**0.06%(0.7033)**	

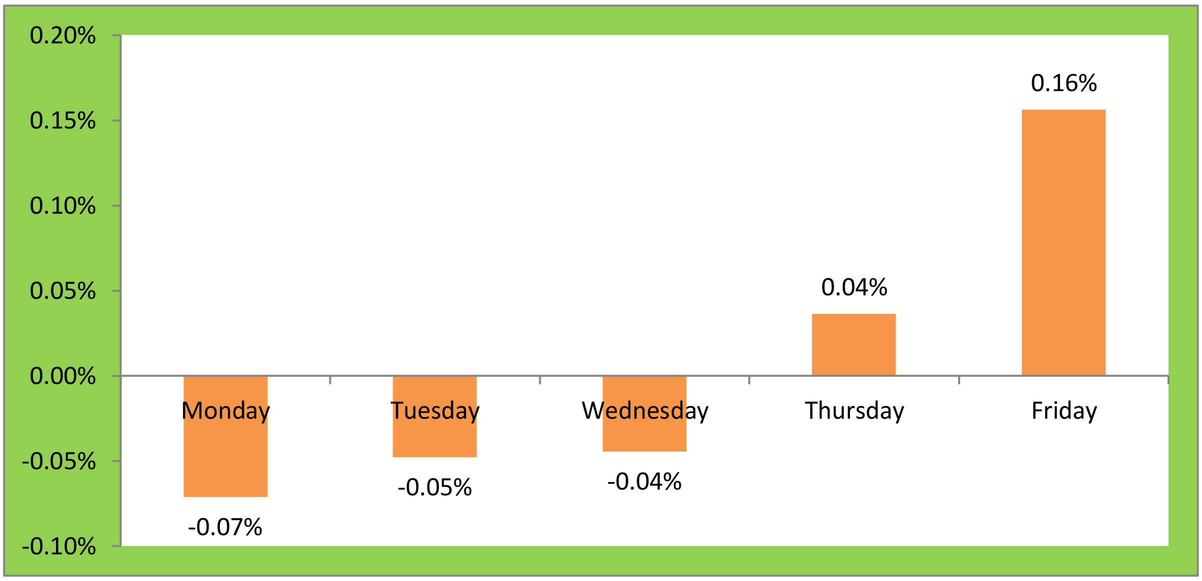

Figure 7.2: Day-of-the Week/Monday Effect Plots for Access

7.6.2 Holiday/January Effect Results

For these effects, it is expected that a day preceding a holiday will give the highest average returns, for which the second to fifth day of January should give high returns put together for a January effect to hold in the case of Nigeria, as the general and regular holiday days include 1st May, 1st October, 27th May, 25th and 26th December and 1st January.

For Access Bank

There is an indication of a holiday effect with the returns on the 31st December producing higher effect returns compared to any other days preceding it. However, there seems to be no significant evidence of a January effect in the bank's returns because, apparently, the p-value does not show a significant difference from 2nd to 5th January (see **Table 7.2** below). For other banks, only four - First, Union, Unity and WEMA have no evidence of either of the two effects; two banks- FCMB and GTB exhibit significant presence of the two effects, while the remaining 9 exhibit either a holiday or a January effects (see Appendix 7B for the details).

Table 7.3: Holiday and January Effects for Access Bank

DAY	Month	þ(effect)	t-statistic	P-value
24	December	0.003	0.324	0.747
27	December	0.000	-0.045	0.964
28	December	0.010	1.032	0.303
29	December	-0.006	-0.678	0.498
30	December	0.008	0.904	0.367
31	**December**	**0.022**	**2.576**	**0.010**
2	January	[20]0.010	0.857	0.392
3	January	0.001	0.065	0.948
4	January	0.014	1.365	0.173
5	January	0.000	-0.040	0.968

Figure 7.2 below displays the Access bank plots for holiday or January effects. It can be confirmed that 31st December, a day prior to 1stJanuary, a public holiday, has the highest bar, showing that there is an evidence of a holiday effect (see 31st December in Table 7.3). As noted above, for other banks' plots, see **Appendix 7C**.

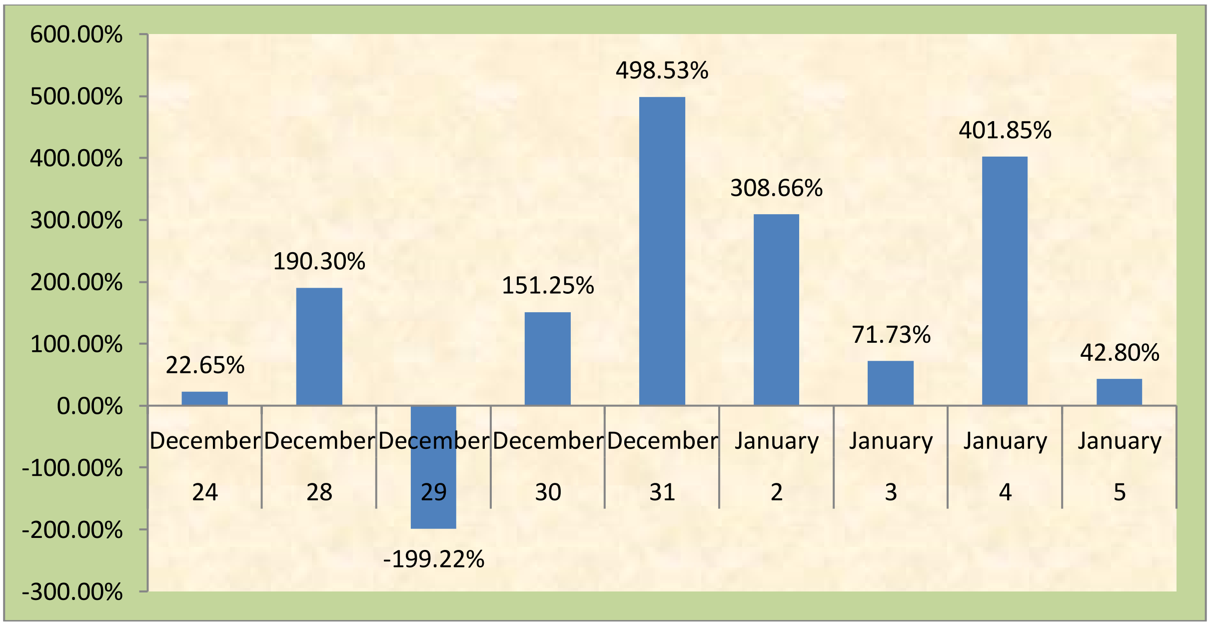

Figure 7.3: Holiday and January Effect Plot for Access Bank

[20] Green colour is used to identify the effects of the first five working days of January.

7.6.3 October-March Seasonality Effect Results

The condition for this effect is that the average returns from October to March should be higher than those of April to September of every year. Table 7.3 below presents the empirical results for this effect for all the banks. From the table, we see that none of the test statistics was significant, meaning that average returns between the two periods was not significantly different from each other for any of the 16 banks. Thus, this implies that there is largely a lack of October-March seasonality effect across the 16 banks for the periods investigated. Figure 7.8d to 7.23d (in the **Appendix 7D**) present the plots for this effect across the sixteen banks, while Figure 7.3 and 7.4 below display those of Union and Zenith banks. Note that columns 2 and 3 contain daily average returns across the banks as percentages.

Table 7.4: Results of October-March Seasonal Effects Anomalies across the Banks[21]

Banks	Mean effects		Test Statistic	P-value
Access	0.06% (M1)	-0.05%(M2)	1.001	0.317
Afribank	-0.04% (M1)	-0.06%(M2)	0.133	0.894
ECO	-0.08%(M1)	-0.20%(M2)	0.618	0.536
Diamond	0.01%(M1)	-0.03%(M2)	0.288	0.774
First	0.08%(M1)	-0.10%(M2)	1.867	0.062
FCMB	0.01%(M1)	-0.04%(M2)	0.491	0.623
Fidelity	0.02%(M1)	-0.07%(M2)	0.829	0.407
GTB	0.13%(M1)	-0.01%(M2)	1.569	0.117
Skye	0.10%(M1)	-0.08%(M2)	1.329	0.184
STANBIC	0.06%(M1)	-0.02%(M2)	0.471	0.638
Sterling	0.06%(M1)	-0.05%(M2)	1.001	0.317
UBA	0.02%(M1)	0.00%(M2)	0.159	0.874
Union	-0.03%(M1)	0.01%(M2)	-0.282	0.778
Unity	0.17%(M1)	-0.10%(M2)	1.109	0.267
WEMA	0.11%(M1)	-0.04%(M2)	0.892	0.372
Zenith	0.02%(M1)	0.02%(M2)	-0.073	0.942

[21]**M1**: October-March Average returns; **M2**: April-September Average returns

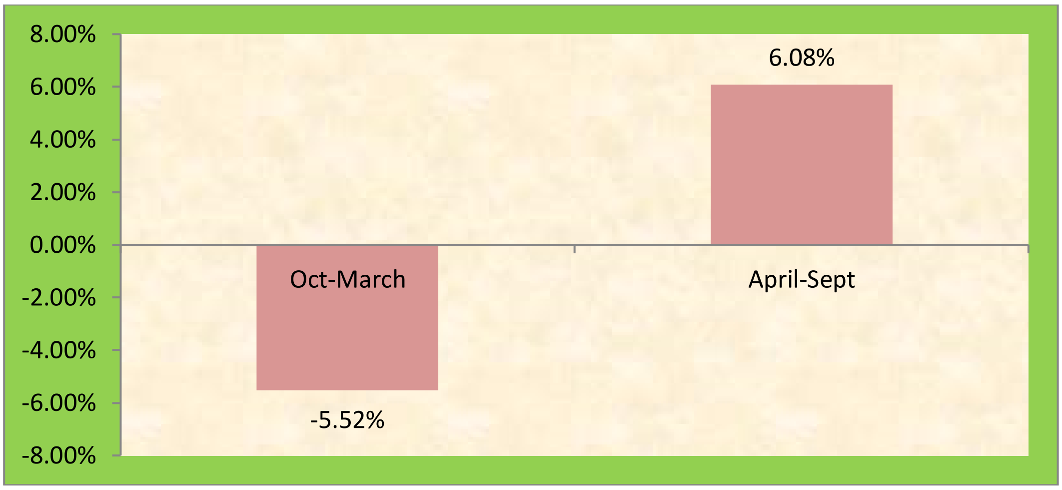

Figure 7.4: Seasonality Effects Plot for Union Bank

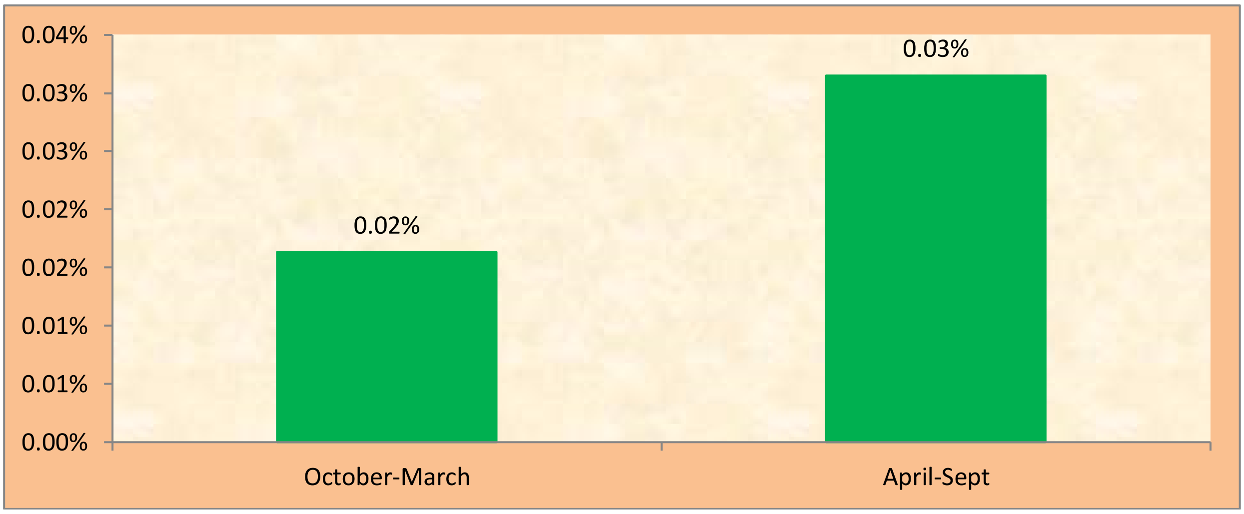

Figure 7.5: Seasonality Effects Anomalies Plot for Zenith Bank

7.6.4 Turn-of-the-year Effects

The approach for this anomaly is descriptive comparison and visualisation of average returns in the first and second halves of January. **Table 7.4** displays the empirical results for this effect. According to this effect, it is expected that average returns from 2nd to 15th January of every year should be higher than those of the remaining days of the year. Since there are generally higher January returns than the rest of the year by the January effect, comparing the two halves of the year more directly explores the turn-of-the year effect as the year goes from December of a previous year to January of the following year. From Table 7.4, only Afribank is significant while the rest of the banks are not, indicating there is largely a lack of turn-of-the year effect within the Nigerian banking industry.

However, for further visual assessment, while Figure 7.5 below presents the plot of turn-of-the-year effect for Access Bank, Figures 7.24e-7.38e (in Appendix 7E) are shown for the rest of the banks. From the plot, the average abnormal returns of the first two weeks of the year seem to be higher than the rest of the trading days in January.

Table7.5: Results of Turn-of the-year/January Effects across the Bank

Banks	Test Statistic	P-value
Access	0.004	0.626
Afribank	2.570	0.011
ECO	0.783	0.435
Diamond	1.474	0.142
First	1.242	0.215
FCMB	0.518	0.605
Fidelity	1.096	0.275
GTB	1.826	0.069
Skye	1.101	0.272
STANBIC	1.267	0.206
Sterling	0.488	0.626
UBA	0.923	0.367
Union	1.779	0.086
Unity	0.980	0.328
WEMA	0.486	0.627
Zenith	1.873	0.062

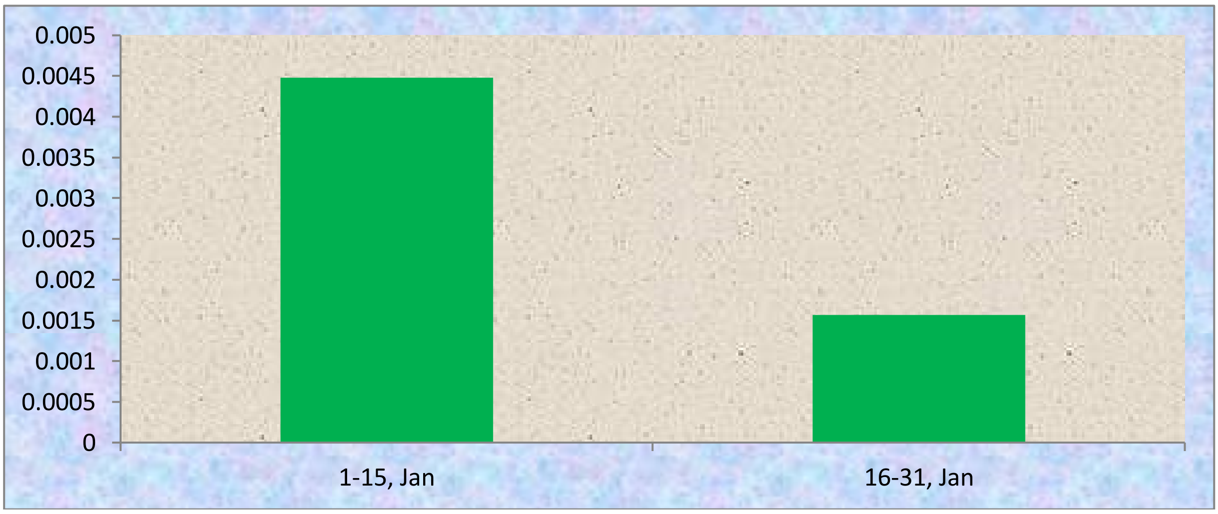

Figure 7.6: Turn-of-the-year Plot for Access Bank

7.6.5 Turn-of-the-Month Effect Results

For this effect to hold, the average returns across the months of the year should differ. In the case of the Nigerian banks considered in this research, the results are presented in the **Tables**7.6 and 7.7 below. From **Table 7.6**, only with 8 banks that we have at least one turn-of-the-month effect, with Union bank (see **Table7.6**), having the highest number of 6 months (March, July, August, September, November and December) whose effects are significantly different; however, in the remaining 8 banks, there is no significant difference in the monthly effects. While we present analysis results for Access banks in Table 7.7, the results for the remaining banks are in Appendix 7F of the thesis**.** Apparently, there no evidence to support the general belief that January's average return is usually higher than that of any other month of the year as found with the US markets, especially for small-cap stocks (**see**: Cabello and Ortiz 2002; Schwert, 2003).

Further, Figures**7.39f to 7.53f** (see **Appendix 7F**) show graphically the differential patterns in the average returns per month across the remaining fifteen banks. For example, Union Bank's results are visually presented in Figure 7.6, revealing that the month of October witnesses the highest average returns, while December has the lowest average returns.

These results have implications for investment strategies on the part of speculative or long-term investors. For speculative investors the monthly swings in returns will help them with more effective market timing of portfolios that include bank stocks. Long-term investors will concentrate rather on the overall trend of bank returns and the underpinning fundamental analysis, including management quality and track records of different banks' performance.

Our conclusion is that there is evidence of a Turn-of-the-month effect in only half of the Nigerian banks.

Table7.6: Summary Results on the Turn-of-the-Month Effects across the Banks

Banks	**Number**	**Month**
Access	0	None
Afribank	0	None
ECO	0	None
Diamond	2	March and August
First	2	July and August
FCMB	2	August and October
Fidelity	0	None
GTB	1	May
Skye	1	August
STANBIC	0	None
Sterling	2	July and November
UBA	0	None
Union	6	March, July, August, September, November and December
Unity	0	None
WEMA	0	None
Zenith	1	August

Table 7.7: Results on the Turn-of-the-Month Effect for Union Bank

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	.005	.002		2.274	.023
	D1	-.005	.003	-.031	-1.421	.155
	D2	-.005	.003	-.031	-1.446	.148
	D3	-.009	.003	-.060	-2.747	.006
	D4	-.004	.003	-.024	-1.110	.267
	D5	-.001	.003	-.009	-.397	.691
	D6	-.003	.003	-.024	-1.099	.272
	D7	-.007	.003	-.051	-2.310	.021
	D8	-.007	.003	-.052	-2.339	.019
	D9	-.007	.003	-.045	-2.054	.040
	D10	-.006	.003	-.042	-1.907	.057
	D11	-.008	.003	-.056	-2.532	.011

a. Dependent Variable: Rtns

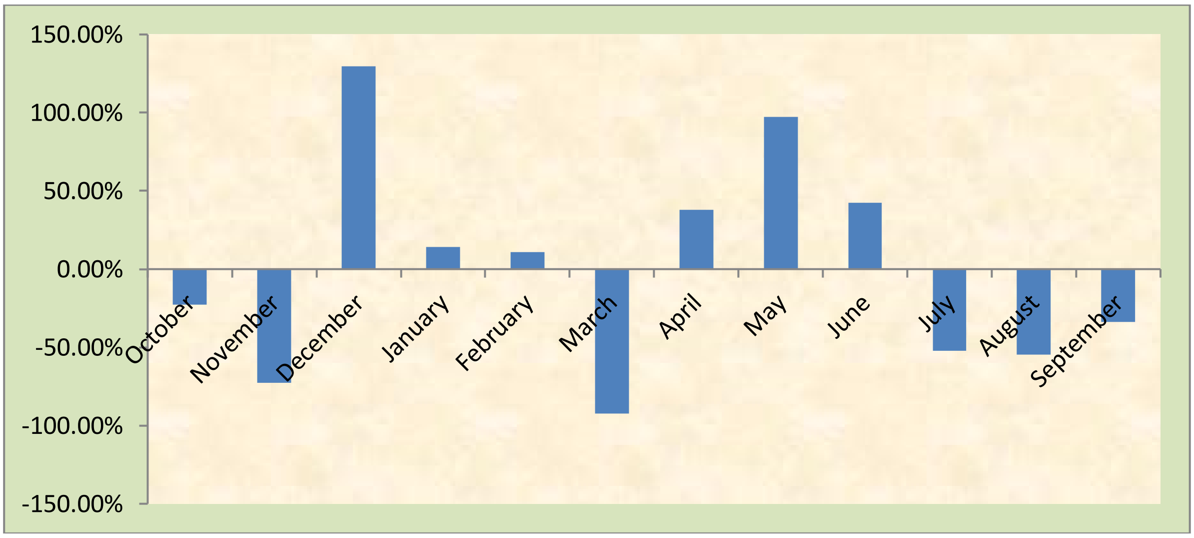

Figure 7.7: Month-of-the-year Effects Plot for Union Bank

7.7 Summary and Conclusion

In this chapter, we explored different types of market anomalies. We briefly discussed their attributes, with more emphasis on the Time series / calendar anomalies as the major focus of the chapter. Comparative studies related to the calendar anomalies in different countries in the literature were reviewed. The implications of the anomalies for stock investment strategies were highlighted. The need for studying these and the specific objectives of the chapter were examined in the introduction section. The chapter then recalled the techniques for analysing anomalies that were stated in Chapter 4 of the thesis and applied some of these techniques to the banks' returns.

The yearly averages reveal that in the year 2008, 69% of the banks were bearish, and between 2008 and 2009, approximately 82% of the banks were bearish, indicating that more banks were negatively impacted as the crisis persisted.

Meanwhile, we looked at the following anomalies: The Day-of-the-week/Monday effect, the Holiday/January effect, the Seasonality effect, with a focus on the October-March effect, the Turn-of-the-year effect and the Month-of-the-year effect.

While presenting results on each of the effects, we observed that only four banks: **Diamond, Skye, UBA and Union**, show some signs of a Day-of-the-week effect; with seven-Access, Afribank, Ecobank, First, FCMB, Union and Sterling –showing evidence of negative(and the least average) returns on a Monday. All the banks had their positive and highest effects on Fridays, except for Eco and Fidelity banks, as noted in Table 7.1 above. However, largely, based on the test results, one could not conclude that any Day-of-week effect could be exploited to earn abnormal profit in the market.

Table 7.2also displays results on the holiday/January effect for Access bank, while those of the remaining banks are presented in Appendix 7B. The findings reveal the following: for Access and other 6 (Afribank, ECO, Skye, STANBIC, Sterling and UBA), there is evidence of a holiday effect. For the other banks, only four - First, Union, Unity and WEMA show no significant evidence of either of the two effects; two banks- FCMB and GTB exhibit significant presence of the two effects, while the remaining three (Diamond, Fidelity and Zenith), only exhibit signs of a January effect. Thus, it could be concluded that there is, for the most part, evidence of both effects characterising the Nigerian banking industry.

For the October-March effect, we found that none of the banks show significant evidence of this effect even though there seem to be marked difference in the average returns between the two periods (see Table7.3, Figures 7.3 & 7.4).

With respect to the turn-of-the year effect, whereby we expect that the average returns in the first two weeks of January of every year should be higher than the average returns of the remaining days of the month, it is evident from our findings that only one bank (Afribank) conforms to this effect.

As expected for the turn-of-the-Month effect, the effect for each month should be different from the rest of the months in the year. Our key findings show that none of the banks has its highest average return in January, while 8 banks (**Diamond, First, FCMB, GTB, Skye, Sterling, Union** and **Zenith**) have at least one of the months significantly different from the others, but for the remaining 8 banks there is no evidence of significant difference in the monthly effect even though there are graphical signs of difference in average returns across the months for all the banks (See for example Figure 7.6).

In summary, a Day-of-the week effect was largely not found in the banks. The holiday effect was found in some and but was absent in others, but only in five of the banks there is some evidence of the January effect. The October-March effect does not impact on any of the 16 banks and neither does, the turn-of-the year effect except for Afribank bank. The month of August was when majority of the banks witnessed their lowest returns, but only 8 of the banks reveal some evidence of the presence of a turn-of-the-month effect.

With these findings, we have succeeded in identifying how the market and investors (in the Nigerian banking industry) react to environmental changes or turn-of-events in the Nigerian society. Investors (either local or foreign), with a good understanding of the reactions of the Nigerian banks to the anomalies discussed so far would be properly guided on favourable investment decisions based on their disposition to risks.
However, it is important to note that since each country has its own environmental/turn-of-the event peculiarities, investors aiming at diversification would require a better understanding of the reactions of the respective market/sector of the market to the established anomalies within such contexts. For example, the findings made in this research regarding market anomalies are unique to the Nigerian banking industry and do not apply to any other sector or market/economy. It is in this sense that systematic stock market characterisation and development (SSMCD) research, of which this thesis is a part, requires researchers to replicate the methodology in this thesis in other sectors of the NSM and emerging African markets.

8 CHAPTER EIGHT: Rational Speculative Bubbles

8.1 Introduction

Speculative excesses, also called speculative bubbles, play a significant role in economic history. They are attributed to periods (episodes leading to persistent overvaluation of an asset price) when financial asset prices peak unsustainably beyond their real face values subject to the prevailing economic realities. A bubble is a long run-up in price (or a long run of positive abnormal returns) followed by a crash or market collapse. The cause of unusual rises and falls in prices, whether there were bubbles, and whether the bubbles were behavioural or rational, have been issues of concern in finance and macroeconomics in recent years. Essentially, bubbles occur when assets consistently sell at prices in excess of their fundamental values[22]. This distorts efficient allocation of capital to investments in a way that maximises the overall well-being of households, firms and governments (Roubini and Setser, 2004).

It is noted in Ezepue and Solarin (2009) that the genesis of the 2007-2009 global financial crisis was a bubble that built up in the US subprime market due to banks' securitisation of house loans and the selling of these to borrowers who could not continue to afford the payments. In other words, the buyers of the loans were accumulating repayment debts that exceeded their incomes. The original ideas in Ezepue and Solarin (2009) were developed in a three-part paper (Ezepue and Solarin, 2012 a-c) that addresses more fully the lessons from the credit squeeze for modelling emerging financial markets (Ezepue and Solarin, 2012a), applications of the ideas to some topics in investment and financial risk management (Ezepue and Solarin, 2012b), and a rejoinder to the then Central Bank of Nigeria (CBN) Governor, Professor Soludo, inaugural lecture of the Professor of Economics, which linked the financial crisis to financial globalisation and financial engineering research (Urama and Ezepue, 2018) that would best prepare emerging markets for such crises in the future (Ezepue and Solarin, 2012c). The Ezepue and Solarin (2012a-c) papers inform this research in the banking sector of the NSM and prior to that Omar's (2012) thesis at the overall NSM level.

[22] "The Fundamental value" of an asset is the present value of the stream of cash flows that its holder expects to receive. These cash flows include the series of dividends that the asset is expected to generate and the expected price of the asset when sold see (Steimetz (2008); http://www.econlib.org/library/Enc/Bubbles.html). The Fundamental value of an asset may also be defined as the price at which investors could buy the asset given that they cannot resell it later". See Camerer (1989) and Brunnermeier (2008) for surveys on bubbles.

Characteristics of bubbles

Some of the common features with which bubbles may be identified include:

1. **Self-Reinforcement**: meaning that the higher prices become, the more there is a justification for even higher prices and higher demand beyond available supply;
2. They collapse or burst suddenly and swiftly (**see Figures 3.1 and 3.2**);
3. They are roughly symmetrical in both time and price. This implies that however long it takes to create the bubble, it will take roughly the same amount of time to unwind the bubble, though it tends to deflate faster than it is formed, and prices often tend to fully adjust.

Though outside the scope of this research, the above features of bubbles suggest that a mathematical approach to modelling bubbles could involve the notions of dynamical systems (Kreyszig, Kreyszig and Norminton, 2011; Brannan and Boyce, 2011 and Beltrami, 2014) that are subject to chaos effects, avalanches and crashes, such as explored in the literature of statistical physics, complexity science and critical mass ideas (Mandelbrot and Hudson, 2004; Ball, 2004).

Ezepue and Solarin (2012c) discuss further links between bubbles, financial crises, and monetary policy, and advanced empirical finance and risk management research - ideas that are further explored in (Raheem and Ezepue, 2018; Sollis, 2012; Alexander, 2008 (Volumes I-IV); Cuthbertson and Nitzsche, 2005; McNeil, Frey and Embrechts, 2015; and Ezepue, 2017). Related global economics constructs are explored in (Miles, Scott and Brendon, 2012; Griffiths and Stuart, 1999 eds.; Greenspan, 2008; Roubini and Setser, 2004; Sharma, 2016; Acharya and Richardson, 2009; Dalio, 2017; and Ezepue, 2017). These references will inform the lines of future work in SSMCD research outlined in Ezepue (2017), some of which are currently discussed in Ezepue and Ekahguere (2016) eds., Nnanwa, et al. (2016), Urama et al. (2017). The above references show the multidisciplinary character of SSMCD research which this chapter and thesis contribute to, primarily from an empirical finance perspective.

Future work on bubbles, anomalies and volatility following this research will explore some of these lines of work, with a view to mapping the DNA of the NSM and using the results to enhance investment management strategies within the NSM (Ezepue, 2017). We note that this chapter takes a more in-depth look at bubbles than is provided in Omar (2012) – the first PhD thesis in the SSMCD line of research. The reason for this is to provide stronger guidelines for replicating such results in other market sectors. The above references, as highlighted, will help to ground such research in meaningful applications to investment theory and their global economic ramifications. Such extensions of current knowledge are suggested in Dalio (2017), Raheem and Ezepue (2018), and Ezepue (2017).

Indeed, Raheem and Ezepue (2017) show how the stylized facts of banks' returns explored in this thesis can be used to enhance technical trading strategies related to portfolios that include such returns. The paper develops a three-state Markov chain model for this.

The promise of the additional work on bubbles discussed in this chapter is that, when combined with ideas revealed in the anomalies chapter (see Chapter 7 of this thesis), we have more results that characterise the dynamics of bank returns in the NSM. It is envisaged that these results when pooled across all six empirical market features including volatility modelling (Chapter 9 of this thesis), and anomalies (Chapter 7), market efficiency (Chapter 6), and the fundamental stylised facts results in (Chapter 5) will arguably provide the deepest empirical finance-based characterisation of the banking sector of the NSM available in the literature.

Fuller details of the SSMCD remits involved in these characterisations are presented in Ezpue (2017) generally and Urama and Ezepue (2018) with respect to the development of novel derivative products in the NSM. The latter paper articulates how such stylized facts researched in this thesis aid the intended derivatives pricing models and provides support for the contributions of knowledge made by the results in this thesis, including this chapter on bubbles. Also, Dalio (2017) as argued in Ezepue (2017) shows how all the results can be used in computational schemes to better manage complex investment portfolios, including the differential impacts of unforeseen economic events such as that of the 2007-2009 global financial crisis.

Impacts of Bubbles

1. They have significant negative impacts on the real and financial sectors;
2. They can be destructive to the wider economy especially if it is a major market, such as the stock or housing market;
3. A stock market crash can cause a loss of confidence and lower spending.

A few examples of historical circumstances where bubbles were experienced in the past include: The Tulip Mania (Holland, 1630s), and the South Sea Bubble (London, 1720). In the twentieth century, the most famous bubbles are the Wall Street Crash (the US, 1929), the Japanese bubble (1980s), the Asian Tigers bubble (mid-1990s), and the technology mania (late 1990s).

Causes of Bubbles

Generally, bubbles start for some good economic reasons. For example, in the early 2000s, low interest rates and economic growth encouraged people to buy a house in the US. In the 1990s internet stocks did offer good potential growth for this new business. However, increasing prices and rising demand can create a dynamic where good news inspires people to take more risks, thereby leading to an increase in prices beyond what they should reach. Some factors that can cause bubbles as found in the following literature such as: Shiller (2000), Brunnermeier (2008), Stiglitz (1990), Thornton (2009) and McQueen and Thorley (1994), include:

1. **Irrational exuberance**: In certain situations, investors buy assets due to strong psychological pressures, which encourage them to ignore the fundamental value of the asset by believing that prices will keep rising.
2. **Herd behaviour**: People usually presume that the majority can't go wrong. If banks and well-established financial leaders are buying an asset, they assume it must be a good investment (the economics of herding and irrationality).
3. **Short termism**: The situation where people's decisions are largely focussed on the short term rather than the long-term.
4. **Adaptive expectations:** People usually see or judge the state of a market and economy subject to the recent past happenings.
5. **Hope they can beat the market**: This is a situation where people believe that they can beat the market and get out before the bubble bursts.
6. **Cognitive dissonance**: The act of filtering out bad news and looking for views which strengthen their beliefs in the market.
7. **Financial instability hypothesis**: The theory that a period of economic boom makes investors increasingly reckless in trading, thereby generating financial instability.
8. **Monetary policy**: Sometimes bubbles occur as an indirect consequence of monetary policy. For example, the FED's decision to keep interest rates in the US low encouraged the credit bubble of the 2000s. Excess liquidity may easily generate bubbles because people are simply desperate to invest their money.

9. **Global imbalances**: Instability experienced in a particular market has the potential to cause panic in another market, thereby leading to systemic risk across different markets. For example, there are strong arguments that the US financial bubble of the 2000s was caused by an inflow of currency from abroad, simply because the US ran a trade deficit and attracted hot money inflows, leading to a higher demand for US securities. This kept interest rates low and values of US financial assets higher than they otherwise would have been.

8.2 The Rational Speculative Bubble Models

Various models have previously been proposed by a few authors with closed objectives relevant to our study. The models, which range from simple to general models for investigating rational speculative bubbles in equity returns across the exchange markets of both developed and emerging economies, and their implications, are to be examined alongside the relevant test statistics, such as the Augmented Dickey-Fuller (ADF), Phillip-Peron (PP) and Duration dependence test with various extensions, as contained in the literature.

Brooks and Katsaris (2003) group the various methods for investigating rational bubbles into three major groups such as: tests for the co-integration of dividends and prices, tests for bubble premiums and tests for excess volatility in returns. They also state that tests of stationarity and co-integration are the best analytical tools available to identify the presence of a long-term relationship between actual prices and fundamental variables. The presence of a long-term relationship between dividends and prices can indicate the absence of bubbles, but the tests greatly depend on the method employed to construct the fundamental values (Brooks and Katsaris, 2003: 331). Gurkaynak (2008), however, classifies the econometric tests of rational bubbles as “variance bounds tests”, “West’s two-step tests”, and “integration/co-integration-based tests”.

The co-integration method is criticized as it is based on the joint test of a null hypothesis of no bubble and no model misspecification. Gurkaynak (2008: 182)points out that the econometric bubble detection tests impose very little structure on the bubble process, simply because those tests do not produce a time series of the bubble component, so it is difficult to evaluate whether the implied properties of the bubble are reasonable or not.

Many other studies developed a new approach, which has become popular over the last decade. These investigate the presence of speculative bubbles[23] in asset prices. The method is the one applied by McQueen and Thorley (1994), derived from the statistical theory of duration dependence. The technique ideally assumes that if an asset price series is characterised by bubbles, then runs of positive abnormal returns will exhibit negative duration dependence, which is indicated by a decreasing hazard rate. This implies that the conditional probability of a run terminating, given its duration, is a decreasing function of the duration of the run. The findings of the study established evidence for the presence of negative duration dependence in the runs of positive abnormal monthly returns for both equal-weighted and value-weighted portfolios of the New York Stock Exchange (NYSE) traded securities.

McQueen and Thorley (1994) classify bubble tests into two basic categories. One category of bubbles tests compares actual prices to fundamentals, which are believed to determine price. A second category of bubble tests examines returns for empirical attributes of bubbles such as autocorrelation, skewness, and kurtosis, which result from the two characteristics of bubbles-namely, extended runs of positive abnormal returns and crashes; that is, a long-run up in positive abnormal returns representing a bubble, followed by a collapse in prices (a crash) (Martin et al., 2004 and Dou, 2010). This indicates that a speculative bubble occurs with a gradual upsurge in prices but ends with sharp and rapid declines, which results in an asymmetric pattern in the asset returns (Jirasakuldech and Zorn, 2002; Dou, 2010).

The above points are such that diagnostic tests for bubbles could potentially be based on these stylised facts, but in fact such tests are inconclusive, even if the stylised facts are significant, because fundamental price movements and other stylised facts of equity returns can also be associated with bubbles (Chan, McQueen and Thorley, 1998). For example; time-varying risk premiums (Fama and French, 1988), fads (Poterba and Summers, 1988) and non-synchronous trading (Lo and MacKinlay, 1990), could all induce autocorrelation; Skewness could result from asymmetric fundamental news, while leptokurtosis could be a consequence of the batched arrival of information (Tauchen and Pitts, 1983). Hence, the duration dependence tests discussed below are more appropriate for detecting bubbles compared to the stylised facts-based tests (Chan, McQueen and Thorley (1998)).

[23]A **speculative bubble** is a spike in asset values within a particular industry, commodity, or asset class, which is often caused by exaggerated expectations of future growth, price appreciation, or other events that could trigger an increase in asset values. http://www.investopedia.com/terms/s/speculativebubble.asp

Implications of the Bubble Model

If prices contain bubbles, then runs of observed positive abnormal returns will exhibit duration dependence with an inverse relation between the probability of a run ending and the length of the run (McQueen and Thorley 1994). The fact that this can be investigated without the need to observe fundamental values directly makes it easier to use duration dependence tests, compared to tests that estimate fundamental values).

Duration Dependence Tests

Duration dependence tests deal with the fact that the probabilities of ending a run - a sequence of abnormal returns of the same sign - vary subject to: (1) the length of the run and, (2) whether the run is of either negative or positive abnormal returns. Thus, returns are converted into series of run lengths of positive and negative observed abnormal returns. Negative abnormal returns then become less likely and are only generated when the bubble bursts.

As noted above, the test is different from other methods because it is flexible and has no requirement for identifying fundamental factors and no requirement that the time series must be normally distributed (Haque, Wang and Oyand, 2008; Jaradat, 2009; Jirasakuldech, Emekter and Rao, 2008). It is a joint test for the presence of bubbles where misspecification of the problem and model does not matter (Blanchard and Watson, 1982; Shiller, 1981; West, 1987). McQueen and Thorley (1994), proposed a duration dependence model, based on the log transformation of logistic regression, and called it the log-logistic function, to investigate the presence of rational speculative bubbles. Their model predicts that the hazard function for a run of positive abnormal returns is a decreasing function of the length of the run.

Alternative Tests

a. To compare actual prices to fundamentals: Shiller (1981) used variance bounds, whereas West (1987) used regressions.
b. To examine returns for empirical attributes of bubbles, such as skewness, which result from the two characteristics of bubbles - extended runs of positive abnormal returns and crashes, Evans (1986) applies the “median test” of (skewness).
c. Taylor and Peel (1998) applied non-cointegration tests of stock prices and dividends.
d. Markov-switching methods are a useful tool for modelling stock-market fluctuations and bubbles that switch between two or more states- Homm and Breitung (2012) proposed several tests in this regard.

As stated by Gurkaynak (2008), many existing econometric bubble tests have one thing in common: they are not very good at detecting bubbles. However, the duration dependence test can overcome most of the criticisms laid against traditional bubble tests. Its advantages are that it is unique to bubbles, it addresses nonlinearity and it does not require correct identification of the observable fundamental variables. It works based on the principle that if the price is characterised by bubbles, then the trend of its positive (and not negative) abnormal returns will exhibit negative duration dependence. This means that the probability of bubble burst is expected to fall with the length of the time the bubble lasts, given that the duration time is finite. This follows the assumption that if the investors are rational, then they do not sell assets whose prices are far above their intrinsic values because the returns of holding such assets can adequately compensate for the risk of the bubble bursting. Part of the stylized facts of rational speculative bubbles is that of a long run-up in price, followed by a crash.

However, one disadvantage of using the duration dependence test is that it is sensitive to the data set used for example weekly versus monthly returns. Harman and Zuehlke (2004) found inconsistencies in their results by using weekly and monthly data, which shows the sensitivity of the duration dependence test results to the data set.

Table 8.1: Summary of few Studies on Bubble and their Findings across different Markets

Author	Year	Data	Methods	Findings
Chan et al.	1998	Monthly and Weekly data (1975-1994)	Duration Dependence Test	Evidence of bubbles found
Ahmed et al.	1999	Daily-1986-1996	Markov-Switching Model	Evidence of bubbles found
Watanapalachaikul and Islam	2007	Daily and Monthly-1992-2001	Duration dependence test based on Weibull hazard model	Evidence of bubbles present
Jirasakuldech Emekter and Rao	2008	Daily and Monthly-1975-2006	Multivariate Co-integration Test with duration dependence test	Evidence of bubbles found
Haque, Wang and Oyang	2008	Weekly real returns-1991-2207	Log-logistic and Weibull Hazard	Rational speculative bubbles found
Mokhtar, Nassir and Hassan	2006	Monthly real returns: 1994-2003	Log-logistic and Weibull hazard	Presence of bubbles

				pre-crisis (1994-1996) and post-crisis (1999-2003) periods
Zhang	2008	Weekly and monthly excess returns	Log-logistic	Speculative bubbles found

8.3 Methodology Adopted in this Study

In Section 4.5, we presented the procedures to be adopted in determining the presence of bubbles in the respective banks' stock returns.

8.3.1 Tests of Hypothesis for Explosiveness

In applying the unit root test discussed so far, the following null and alternative hypotheses are often set: $H_0: \varphi = 1$ vs $H_1: \varphi > 1$, whereφ, is the unit root parameter in both the ADF and PP tests. Recall that:

a. If $|\varphi| = 1$then there is the possibility for the presence of a unit root in the series, which requires special treatment by differencing the series;
b. If $|\varphi| > 1$, then the series is said to be boundless (or explosive), indicating the likelihood of the presence of bubbles.

For the duration dependence test, this study tests the presence of rational speculative bubbles in the Nigerian banks' stocks for the sample period ranging from June 1999 to December 2014.

8.3.2 Benefits of Studying Asset Bubbles

Understanding how bubbles are generated over time and their early warning signs will enable investors to be cautious of the negative impacts their behaviour and trading activities of some of their mischievous members may have on their asset returns, thereby enabling them to avoid actions (such as herding, exuberance, etc.) that could expose their assets to avoidable or "self-inflicted" risk. With such knowledge, the market regulators would be able to initiate rules that would serve as check and balances to avoid bubble circumstances and subsequent spread.

Knowledge about bubbles would help investors to conduct due diligence with respect to the sustainability of their future investments and how accurate the valuation of the intended company of interest for investment is. For instance, in the case of prospective investors into the

Nigerian banking industry, understanding the historic behaviour of each bank's prospects and bubble history would help them to be cautious about investment decisions in the respective banks, rather than merely following the hype.

8.3.3 Justifications for the Choice of Duration Dependence Model

Note that the inspiration for the methods used in the study was largely drawn from McQueen and Thorley (1994) who invented the dependence model. However, for the choice model, the following reasons are the motivating factors:

1. Recall that duration dependence is different from other methods because it is flexible and does not require identifying fundamental factors and there is no need for the series to be normally distributed (Haque, Wang and Oyand, 2008; Jirasakuldech, Emekter and Rao, 2008; Jaradat, 2009). Analogously, the cox-proportional hazards model shares the same characteristics of robustness with other duration dependence models given that it does not require information on the underlying distribution of the data set.
2. The Cox model is semi-parametric, unlike other duration dependence models, such as log-logistic and Weibull hazard models that are parametric.
3. The estimated abnormal returns (representing the independent variable) could either be positive or negative and this shows that it is time independent. This is an important assumption for the explanatory variable in the Cox-model.
4. The estimated hazards are non-negative.
5. The Cox-model is preferred to a logistic model because it provides information about survival time and censoring information unlike the logistic model.
6. The model has been applied in many studies, especially in health to determine time to developing a hazard or survival rate of a subject. In financial research, studies such as: He, et al (2019); Anderson and Brooks (2014), Menezes and Bentes, (2016); Hossain (2004), Shumway (2001) and Ni (2009) are good inspiration for this study.

Cox-Proportional Hazard Model

Cox's regression compares the hazards (as ratios) of the two treatment groups (positive and negative returns) and allows several variables to be considered. The hazard is the risk of reaching the endpoint (e.g. death), at time point, i given that the individual has not reached it up to that point.

Hazard rate (HR) is the probability that the event will occur at time t, given that the individual is at risk at time, t and usually varies over time. The dependent variable is the duration (time to event or time to being censored, see notes below); it is a combination of time and the event/censoring.

Survival, Hazard and Cumulative hazard functions

In medical statistics, survival data relate to the time taken for an individual to witness or experience a certain event or scenario. This event is not necessarily death, and neither would all subjects in a study have witnessed the event by the end of the study. A circumstance when an individual has not experienced or did not pass through the event by the end of the study is called **censoring**, which might be due to his/her withdrawal from the study or death. In this research, a bubble is the event of interest, while censored observations are likened to those returns falling outside the "bubble region" as described earlier.

The dependent variable, duration, is assumed to have continuous distribution, $f(t)$ with the cumulative distribution function and defined as:

$$F(t) = P(T \leq t) = \int_0^t f(s)ds. \tag{8.1}$$

The survivor function, which is the probability that the duration will be at least t, is defined mathematically as:

$$S(t) = 1 - F(t) = P(T \geq t) \tag{8.2}$$

As earlier, the hazard rate (HR), which is the rate that the duration will end at time, t given that it has lasted until time t is

$$h(t) = \frac{f(t)}{S(t)}. \tag{8.3}$$

The HR in this research is the relative risk of the bubble associated with positive returns ending at a given time, t, given that the bubble had lasted up to that time.

The Cox-proportional hazards model is

$$h(t, x) = h_0(t)\exp[\sum_{i=1}^{p} \xi_i x_i]. \tag{8.4}$$

Where x = $(x_1, x_2, \ldots, x_p)$ are the explanatory/predictor variables, and $h_0(t)$ is the baseline hazard - that is, the hazard function when all the explanatory variables are set to 0. The $\xi's$, are regression coefficients.

Characteristics of the Cox Proportional Hazard Model

1. The baseline hazard $h_0(t)$ is independent of x but depends on *t*,
2. The x's are time independent,
3. The distribution of the survival time is unknown,
4. The hazard function is unspecified.

Reasons for the Method's Popularity

1. Robustness: estimated hazards are always non-negative.
2. It is possible to estimate the $\xi's$ and the hazard rate function is unspecified.
3. $h(t, X)$ and $S(t, X)$ can be estimated for a Cox model with minimal assumptions.

Relative Hazard Rates (Hazard Ratio)

$$HR = \frac{\hat{h}(t,x)}{\hat{h}(t,y)} = \frac{e^{\xi x}}{e^{\xi y}} = e^{(x-y)\xi}, \quad \textbf{(8.5)}$$

Where: x = $(x_1, x_2, \ldots, x_p)$ and y = $(y_1, y_2, \ldots, y_p)$. Thus, the HR is the hazard rate for (bubble) in returns series x divided by hazard for a returns series y. Again, in our study, the (abnormal) returns series for each bank is grouped into "negative" (coded as "1" when returns are too high) and "positive" (coded as "2" when returns are not excessive) during the analysis, with the reference group being the "negative" returns. For interpretation, the HR should be equal or greater than 1; that is, $\hat{h}(t, x) \geq \hat{h}(t, y)$. It can however be interpreted as a percentage change in the risk of negative returns.

The Model Decision Rules

Table8.2: Summary of Possible Outcomes of the Hazard Function Coefficient, Hazard rate and Conclusion

Coefficient	Hazard rate (HR)	Conclusion
Positive	HR>1	Higher duration of positive returns; higher hazard rates. That is, it is more likely for bubbles to occur.
Negative	0<HR<1	Higher Duration of negative returns, lower hazard rates. That is, it is less likely for bubbles to occur.

Independence Assumptions of the Cox Model

1 The ratio of the hazards between positive and negative returns (i.e. hazard rates) is **constant** over time; meaning, hazard rates are proportional across the banks

2. The length of the series is large enough to achieve convergence.

To structure our data to fit into the methods used, the following procedures were used:

1. The periods suspected to be bubbles in the price series are coded as "1" and “2” which represent "events", whereas other periods which do not fall within the bubble region, representing "censored", are coded "0". All these periods are visualised from the plots of price series and vary for each bank.
2. From the returns series, we generate the runs of "negative" and "positive" returns, which are based on the realizations ("-" or "+") of the returns series for each of the bank's stocks. The negative series are those returns that are less than zero, and are coded "1", whereas, the positive series are those returns that are greater than or equal to zero and are coded “2”.
3. In line with McQueen and Thorley (1994), in considering rational speculative bubbles by applying duration dependent methods, we generate "abnormal returns". However, given the lack of relevant data to obtain inflation rates, dividend yield, and term spread included in the regression model from which the abnormal returns by McQueen and Thorley (1994) were generated, we decide to follow Chan et al. (1998) and use a fourth-order autoregressive model of daily returns to determine abnormal returns[24]. Abnormal returns, then which are taken to be the residuals from the autoregressive model of order 4, AR (4), which are fitted to the returns for each bank as follows

$$r_t = \nu_0 + \nu_1 r_{t-1} + \nu_2 r_{t-2} + \nu_3 r_{t-3} + \nu_4 r_{t-4} + \varepsilon_t. \qquad \textbf{(8.6)}$$

Where r_t is the returns series, $\nu_i \ \forall \ i = 0, 1, 2, 3, 4$ is the AR’s coefficients (parameters) and ε_t is the error term. Generally, an estimate of ν that is negative and significantly different from zero for positive runs, in conjunction with a non-significant estimate of the parameter, ν for negative runs, is considered evidence of speculative bubbles (Harman and Zuehlke, 2004).

[24]According to Chan et al. (1998), an AR (4) model is favourable to impose a common mean, asdoing so would help to control for short-term sources of autocorrelation; e.g. non-synchronous trading.

Motivation

Although the duration dependence test is unique to bubbles, due to its sensitivity, the fact remains that our results could be impacted by the choice of sample period, the choice of model, and the type of data used, for example by choosing daily or monthly data. We therefore distinguish between our approach and techniques adopted in earlier studies as follows.

The previous investigation on the reference of bubbles in the NSM by Omar (2012) and Omar and Ezepue (2016) used the All-Share Index (ASI) at the overall market level and found no evidence of bubbles in the NSM. This result was considered counterintuitive since it was felt that the banking and financial services industry witnessed periods of boom after the 2004 financial reforms, followed by bank failures and later near bank failures that forced the CBN to recapitalise some of the banks. For example, Bank PHB plc, Oceanic Bank plc and Intercontinental Bank plc all collapsed, which could be attributed to the presence of bubbles in the sector's asset returns and mismanagement. The referenced researchers noted that using ASI data might have masked the existence of bubbles in a sector by mutually offsetting bubbles in different sectors. This study, therefore, implements their suggestion to perform sector-specific analyses in sectors such as banking, telecommunications, oil and gas. Hence, we use the banks' returns which better reflect the stylised facts of stock prices in the banking sector, creating an additional contribution to knowledge compared to the above overall market level study.

In addition, the data period for the previous study by Omar (2012) the bubble-related aspects of which were published in Omar and Ezepue (2016) was between 2000 and 2010 (10 years), whereas this study extends the data period from 1999 to 2014 (15 years). While the above authors used the basic stylised facts tests – series dependence using autocorrelations along with skewness and kurtosis – and a version of the duration dependence tests based on positive and negative counts and the discrete log-logistic model, this study uses the Cox Proportional Hazards model, which is robust against distributional assumptions and has a number of desirable properties as mentioned earlier. This model has rarely been applied to financial data, particularly to detect rational speculative bubbles in the NSM and its banking sector.

Moreover, our use of daily data provides a large amount of observations to increase the statistical power of the procedures we use. More importantly, compared to the weekly and monthly data used by Omar (2012) and Omar and Ezepue (2016), daily data track the more volatile nature of emerging markets, given that daily changes in stock prices are higher compared to developed markets. Hence, our results replicate and extend the approach in Omar and Ezepue (2016).

8.4 Presentation of the Results and Discussions

In this section, the results on the summary statistics and results of the unit root and duration dependent tests are presented (in Tables 8.3-8.10) and discussed.

8.4.1 Summary Statistics

Sixteen banks are examined, out of which five: STANBIC, Sterling, Unity, GTB, and WEMA, have two suspected episodes of bubbles unlike the remaining eleven, with one episode each, as reflected in their price series (see Table 8.3 and Figure 8.1). In Table 8.3, we have the mean returns, bubble length (T), standard deviation, skewness, kurtosis and autocorrelation function (ACF) for each bank's returns for the identified bubbles' periods.

From the table, the following banks have positive skewness: Access (9.9349), GTB_1 (0.2657) GTB_2 (0.1644), STANBIC_1[25] (8.2388), Sterling_2 (8.2388), Unity_1 (12.0994, Unity 2 (12.9654,) and WEMA_2 (13.5745). With the bubble's attributes described above, these six banks do not show evidence of a bubble in terms of skewness.

Also, in terms of kurtosis, the following banks are not leptokurtic as could be expected during a bubble: Diamond, GTB_ (1, 2), FCMB, Fidelity, Skye, STANBIC_2 and WEMA_1. Finally, Eco bank, GTB_2, STANBIC_2 and Unity_2 are characterised with a non-significant ACF - a behaviour that does not conform to the presence of a bubble. From the listed banks, while only GTB_2 fails to possess the three attributes expected of a bubble, the following seven banks: **Afribank, *Sterling[26], UBA, Union, *WEMA, Zenith and First banks** appear to possess all 3 attributes expected of a bubble: "negative skewness", "high kurtosis" and "significant ACF at lag one".

Figure 8.1 below presents a combined plot of the price series for seven banks with the same length, indicating plausible bubble periods around 2006 to 2009, with some possibly having more than one episode, for example Sterling and GTB.

[25]Note that the number with underscores (1 or 2) in front of some banks indicate the number of episodes of bubble experienced by the respective bank within the referenced periods. We use the number with banks that have more than one episode. And where the number of episode is just one, we do not use any number with underscore. For example, GTB with two episodes, we present each episode as GTB_1 for the first episode and GTB_2 for the second episode.

[26]The asterisks on Sterling and WEMA banks indicate that just one of their two episodes were found to possess all the indicated three attributes

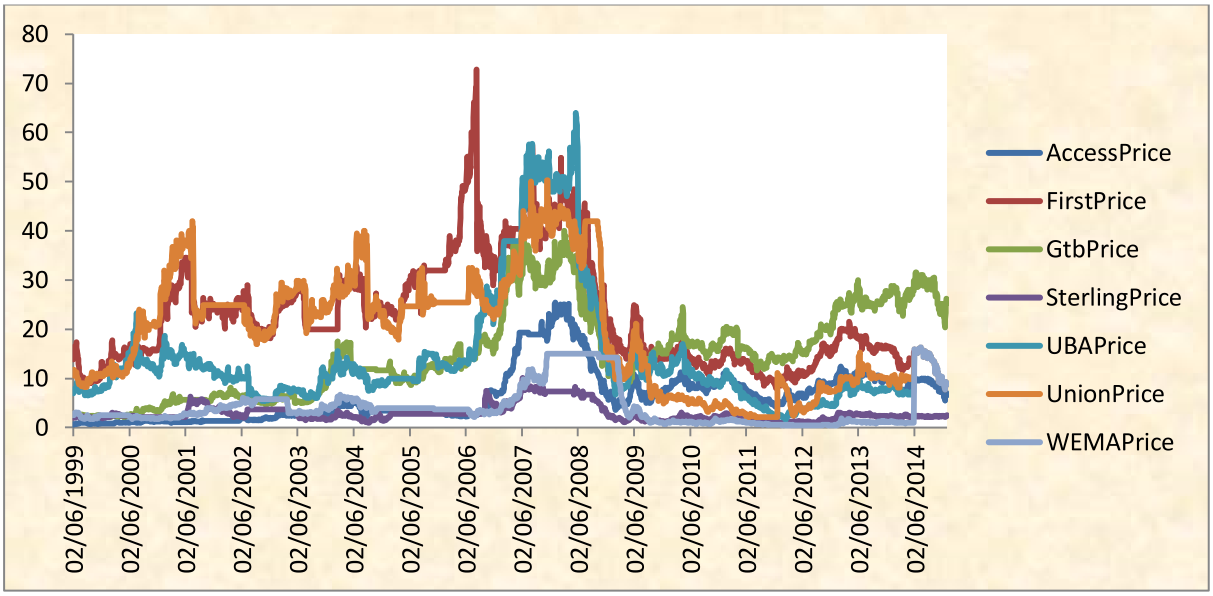

Figure 8.1: Combined Series of some Nigerian banks with Suspected Bubble Episodes

Table 8.3: Summary Statistics of the Returns for Periods Identified as Bubble for Each Bank

Banks	Period(T)	Mean	Std. Dev	Skewness	Kurtosis	ACF
Access	Oct ‘06 – Jan ‘09 (553)	0.0008	0.0390	9.9349	173.8108	18.0690 (2.13e-05)
Afribank	May ‘07–Sep ‘09(572)	-0.0016	0.0338	-1.4900	11.4475	69.9850 (2.2e-16)
Diamond	Oct '06-Apr'09 (621)	-0.0003	0.0305	-0.0755	-0.3291	85.0240 (2.2e-16)
ECO	Oct'07-June'08 (149)	-0.0089	0.1341	-11.3998	133.1542	0.0296 (0.8633)
GTB-1	June'05-Jan'09 (892)	22.0284	9.1950	0.2657	-1.4644	47.5400 (5.39e-12)
GTB-2	Sep '09-Dec '14 (810)	0.0009	0.0190	0.1644	1.9943	0.0020 (0.9642)
FCMB	Dec'06-Jan'09 (522)	13.5328	5.0734	-0.5590	-1.1027	38.1800 (6.45e-10)
Fidelity	Nov'06-Feb'09 (551)	8.3665	3.3669	-0.4489	-1.2811	59.1150 (1.49e-14)
Skye	Dec'06-Jan'09 (514)	-0.0001	0.0342	-0.0342	-0.8254	70.0200 (2.2e-16)
Stanbic-1	June'06-Jan'09 (637)	-0.0007	0.0397	8.2388	144.9833	23.1410 (1.51e-06)
Stanbic-2	Dec '12-Dec '14 (562)	15.3655	4.3267	-0.1868	-0.8198	0.0438 (0.8342)
Sterling-1	May '01-May '02 (243)	0.0005	0.0371	-0.8873	3.6482	5.5825 (0.0181)
Sterling-2	June'06-Jan'09 (637)	-0.0007	0.0397	8.2388	144.9833	23.1410 (1.51e-06)
UBA	Dec'05-Jan'09 (748)	-0.0002	0.0324	-3.7691	41.7315	38.2260 (6.30e-10)
Union	Feb'00-March'09(2237)	0.0001	0.0285	-3.1097	33.6428	74.9680 (2.2e-16)
Zenith	Feb'05-Jan'09 (976)	-0.0001	0.0218	-1.3012	15.0893	117.1500 (2.2e-16)
Unity-1	Nov '06-Mar'09 (571)	-0.0008	0.0577	12.0994	228.4125	7.1657 (0.00743)
Unity-2	Apr'14-Dec'14 (180)	0.0125	0.2134	12.9654	169.0254	0.0483 (0.826)
WEMA-1	Aug'06-Oct'09 (781)	-0.0008	0.0290	-0.0220	-0.4730	227.6600 (2.2e-16)
WEMA-2	Mar'14-Dec'14 (200)	0.0105	0.2031	13.5745	186.4299	0.1885 (0.6642)
First	Jan'05-Feb'09 (1009)	-0.0003	0.0356	-8.6118	158.8920	979.7100 (2.2e-16)

Further, **Tables 8.4** and **8.5** display the comparisons of mean and standard deviation, skewness and kurtosis of the suspected bubble episodes with the overall mean, standard deviation skewness and kurtosis, for the whole study period respectively. From the results in **Table 8.4**, it is apparent that the mean and standard deviation for the suspected bubble periods across the banks are generally far higher compared to the overall-period a reflection that the periods of bubbles are characterised by many abnormalities in stock prices.

Further, seven banks (Access, Diamond, Fidelity, First, FCMB, Sterling_2 and UBA), have a skewness during the suspected bubble periods that is far above 100% of their respective overall period skewness, with six of these banks (Access, Diamond, Fidelity, First, FCMB, Sterling_ (1, 2) also experiencing more than a 100% rise in kurtosis during suspected bubble episodes, compared with their respective overall kurtosis (**see Table 8.5**).

Table 8.4: Comparisons Overall Mean Returns with Standard deviation of the Bubble Period across the 16 Banks

Banks	**Overall Mean**	**Mean**	**%Mean of the Overall**	**Overall Std. Dev.**	**Std. Dev**	**%Std. Dev of the Overall**
Access	**0.0006**	0.0008	139.83%	**0.0290**	0.0390	134.45%
Afribank	**-0.0005**	-0.0016	324.17%	**0.0305**	0.0338	110.79%
Diamond	**-0.0001**	-0.0003	311.11%	**0.0280**	0.0305	108.96%
ECO	**-0.0014**	-0.0089	635.71%	**0.0455**	0.1341	294.73%
GTB-1	**0.0006**	22.0284	3733627.12%	**0.0269**	9.1950	34182.16%
GTB-2	**0.0006**	0.0009	152.54%	**0.0269**	0.0190	70.63%
FCMB	**-0.0002**	13.5328	-9021866.67%	**0.0252**	5.0734	20132.54%
Fidelity	**-0.0003**	8.3665	-3346600.00%	**0.0266**	3.3669	12657.52%
Skye	**0.0001**	-0.0001	-49.52%	**0.0330**	0.0342	103.58%
Stanbic-1	**0.0006**	-0.0007	-117.00%	**0.0206**	0.0397	192.72%
Stanbic-2	**0.0006**	15.3655	2560916.67%	**0.0206**	4.3267	21003.40%
Sterling-1	**0.0001**	0.0005	463.00%	**0.0324**	0.0371	114.35%
Sterling-2	**0.0001**	-0.0007	-700.00%	**0.0324**	0.0397	122.53%
UBA	**0.0001**	-0.0002	-210.00%	**0.0342**	0.0324	94.65%
Union	**-0.0001**	0.0001	-60.00%	**0.0407**	0.0285	69.93%
Zenith	**0.0002**	-0.0001	-64.74%	**0.0254**	0.0218	85.83%
Unity-1	**0.0004**	-0.0008	-190.00%	**0.0725**	0.0577	79.59%
Unity-2	**0.0004**	0.0125	3125.00%	**0.0725**	0.2134	294.34%
WEMA-1	**0.0003**	-0.0008	-246.06%	**0.0547**	0.0290	52.93%
WEMA-2	**0.0003**	0.0105	3181.82%	**0.0547**	0.2031	371.30%
First	**-0.0001**	-0.0003	222.14%	**0.0296**	0.0356	120.17%

Table 8.5: Comparisons between overall Skewness and Kurtosis with those of the Bubble Period across the Banks

Banks	Overall Skewness	Skewness	%Skewness of the Overall	Overall Kurtosis	Kurtosis	%Kurtosis of the Overall
Access	**2.9845**	9.9349	332.88%	**86.8983**	173.8108	200.02%
Afribank	**-4.3754**	-1.4900	34.05%	**94.7174**	11.4475	12.09%
Diamond	**-0.0430**	-0.0755	175.58%	**0.2706**	-0.3291	-121.62%
ECO	**-23.2311**	-11.3998	49.07%	**783.7068**	133.1542	16.99%
GTB_1	**-2.2160**	0.2657	-11.99%	**23.1739**	-1.4644	-6.32%
GTB_2	**-2.2169**	0.1644	-7.42%	**23.1739**	1.9943	8.61%
FCMB	**-0.0572**	-0.5590	977.27%	**0.8811**	-1.1027	-125.15%
Fidelity	**0.0047**	-0.4489	-9551.06%	**0.2950**	-1.2811	-434.27%
Skye	**6.4348**	-0.0342	-0.53%	**155.7539**	-0.8254	-0.53%
Stanbic_1	**27.2745**	8.2388	30.21%	**1065.4050**	144.9833	13.61%
Stanbic_2	**27.2745**	-0.1868	-0.68%	**1065.4050**	-0.8198	-0.08%
Sterling_1	**1.6371**	-0.8873	-54.20%	**1.6371**	3.6482	222.85%
Sterling_2	**1.6371**	8.2388	503.26%	**1.6371**	144.9833	8856.11%
UBA	**0.1967**	-3.7691	-1916.17%	**89.7707**	41.7315	46.49%
Union	**17.6137**	-3.1097	-17.66%	**744.0610**	33.6428	4.52%
Zenith	**-2.2796**	-1.3012	57.08%	**32.3184**	15.0893	46.69%
Unity_1	**28.6789**	12.0994	42.19%	**1085.0730**	228.4125	21.05%
Unity_2	**28.6789**	12.9654	45.21%	**1085.0730**	169.0254	15.58%
WEMA_1	**36.1655**	-0.0220	-0.06%	**1882.0110**	-0.4730	-0.03%
WEMA_2	**36.1655**	13.5745	37.53%	**1882.0110**	186.4299	9.91%
First	**-5.0916**	-8.6118	169.14%	**97.7673**	158.8920	162.52%

8.4.2 Unit Root Tests Null versus Explosive Alternative

Table 8.6 below presents Augmented Dickey-Fuller (ADF) and Phillip-Peron (PP) test results to confirm the presence or otherwise of bubbles in the stock prices of the banks' stocks. The ADF test was applied and complimented by the PP test, which is non-parametric and more flexible, and the results show that in most cases the two statistics agree, except in one (Fidelity), where there is a discordant outcome. However, we have in nine banks: **Access, Ecobank, Fidelity (PP), Skye, UBA, WEMA_1, Zenith, GTB_1, and FCMB** that the "explosive" alternative hypothesis takes precedence, (see the null and alternative hypotheses in Section 8.3.1 above), indicating that there is likely to be a presence of bubbles in those banks' stocks.

Another critical test or signal for the presence of a bubble is to see if there is an element of trend in the price series. Our results reveal that there is evidence of a trend in ten banks: Access, Diamond, Eco, Fidelity, Skye, Sterling-1, Unity-1, WEMA-1, Afribank and FCMB. Interestingly, it is evident that six (shown in green) of the nine banks that are shown to be prone to explosiveness show a trend under the ADF and PP, confirming that the identified banks are likely be characterised by a bubble.

Table 8.6: Unit Root Tests for Stationarity versus Explosiveness

Banks	**Price: Unit Root**	
Access	DF = -0.0811 (0.01)	PP = -1.8160 (0.026)
Diamond	DF = -1.3535 (0.148)	PP = -5.7130 (0.813)
Ecobank	DF = 1.3196 (0.01)	PP = -1.2410 (0.003)
First	DF = -1.7667 (0.323)	PP = -10.4800 (0.4748)
Fidelity	DF = -1.0165 (0.064)	PP = -1.5480 (0.021)
Skye	DF = 0.0216 (0.01)	PP = -0.4465 (0.01)
Stanbic_1	DF = -0.3348 (0.01067)	PP = -0.3681 (0.01)
Stanbic_2	DF = -0.9598 (0.055)	PP = -3.4440 (0.086)
Sterling_1	DF = -4.1370 (0.99)	PP = -8.7320 (0.380)
Sterling_2	DF = -1.1860 (0.091)	PP = -2.9730 (0.065)
UBA	DF = -0.0079 (0.01)	PP = -0.8058 (0.01)
Union	DF = -2.2570 (0.531)	PP = -12.4500 (0.585)
Unity_1	DF = -2.0770 (0.4543)	PP = -7.9620 (0.334)
Unity_2	DF = -1.7390 (0.315)	PP = -4.8400 (0.1625)
WEMA_1	DF = -0.5143 (0.01883)	PP = -0.1266 (0.01)
WEMA_2	DF = -1.3301 (0.143)	PP = -3.8960 (0.1433)
Zenith	DF = 0.7195 (0.01)	PP = 1.3357 (0.01)
Afribank	DF = -3.0420 (0.863)	PP = -8.0671 (0.3403)
GTB_1	DF = -0.0394 (0.01)	PP = -0.6218 (0.01)
GTB_2	DF = -1.7440 (0.3135)	PP = -8.9980 (0.3922)
FCMB	DF = -0.4280 (0.015)	PP = -0.7295 (0.01)

Null=Unit root (or Non-stationary); Alternative=Explosive; DF: Dickey-Fuller; PP=Phillip- and p-values in parentheses

8.4.3 Duration Dependent Test Results

In this section, we present the outcomes of fitting a Cox-Model based on the cases we considered as follows:

Case 1: Fitting Runs of positive (coded 2) and negative (coded 1) returns on the event with zero returns (and coded 0) as censored observations, and time as trading days of the month.

Our emphasis here is that instead of using "Abnormal" returns as in previous studies, we use "Returns" series and the results are presented in **Table 8.7** below.

Also, to note is that the model concerns "positive returns", simply because "negative returns" was set as the reference category; the runs of the returns serve as the explanatory variable, while "event", coded as 1 is "bubble" and "censored" data are coded as 0.

The Relevant Hypotheses are: $H_0: \xi = 0$ versus $H_1: \xi < 0$.

Decision Rule: for the runs of positive returns to end, ξ should be negative and the probability that positive runs end with time ($(exp(\xi))$ should be greater than **0.5** (**see** McQueen and Thorley, 1994).

From **Table 8.7**, we found eight banks (in green) - Afribank, FCMB, Skye, STANBIC, UBA, Union WEMA and Zenith banks that show significant, ξ at the 5% level. We also found that all the parameter values are negative and that none of the hazard rate levels are less than 0.5, indicating the presence of bubbles in the stock prices of those banks. Also, the effect parameter for Union bank is positive with the HR > 1, which shows that the hazard rate of positive runs increases with time; indicating likelihood for bubbles' presence in the series (see Table 8.2).

Table 8.7: Summary Results on Cox-Model on the Runs of the Returns with Time as the Trading Days of the Month

Bank	**Coefficient/ effect parameter (ξ)**	**Hazard Rate/Ratio ($exp(\xi)$)**
Access	**-0.115 (0.188)**	**0.751**
Afribank	**-0.380 (0.000)**	**0.684**
Diamond	**-0.037 (0.650)**	**0.823**
Ecobank	**-0.130 (0.444)**	**0.631**
FCMB	**-0.182 (0.039)**	**0.833**
Fidelity	**0.018 (0.830)**	**0.830**
First	**0.093 (0.142)**	**1.098**
GTB	**0.114 (0.091)**	**1.120**
Skye	**-0.196 (0.013)**	**0.822**
STANBIC	**-0.192 (0.001)**	**0.734**
Sterling	**-0.016 (0.819)**	**0.984**
UBA	**-0.172 (0.020)**	**0.842**
Union	**0.191 (0.000)**	**1.210**
Unity	**0.065 (0.321)**	**1.067**
WEMA	**-0.392 (0.000)**	**0.675**
Zenith	**-0.348 (0.000)**	**0.706**

p-values in parentheses

Case 2: Fitting Runs of positive (coded 2) and negative (coded 1) "abnormal returns" on the event with zero returns (and coded 0) as censored observations, and time as trading days of the month.

From our results, we see that there are only five banks whose effect parameters (**Eco, FCMB, GTB, STANBIC and Unity**) were found not to be significant at the 5% level, meaning that their prices series do not contain a bubble (see Table **8.8**). Also, while the hazard coefficients (ξ) of Afribank, Diamond, First, Fidelity, and Union banks are positive, indicating that their hazard increases with time (this contradicts McQueen and Thorley, 1994), those of the remaining six banks -**Access**, **Skye, Sterling, UBA, WEMA** and **Zenith -** are negative, which is in conformity with McQueen and Thorley (1994)'s proposition. Hence, these banks do not seem to show evidence of a bubble in their returns.

Table 8.8: Summary Results on Cox-Proportional Hazard Model with Days as Time across the Overall Periods

Bank	Hazard coefficient (ξ)	Hazard Rate (exp(ξ))
Access	-0.336 (0.000)	0.715
African bank	0.086(0.000)	1.569
Diamond	0.213(0.008)	1.238
Eco bank	-0.152(0.091)	0.859
First	0.230(0.000)	1.259
FCMB	0.121(0.168)	1.128
FIDELITY	0.290(0.001)	1.337
GTB	-0.011(0.871)	0.989
SKYE	-0.263(0.003)	0.769
STANBIC IBTC	-0.098(0.101)	0.907
STERLING	-0.339(0.000)	0.713
UBA	-0.192(0.009)	0.825
UNION	0.272(0.000)	1.313
UNITY	0.110(0.145)	1.116
WEMA	-0.553(0.000)	0.575
ZENITH	-0.367(0.000)[27]	0.693

P-values in parentheses

Case 3: Fitting Runs of positive (coded: 2)/negative (coded: 1) "Abnormal Returns" on the event (1)/censoring (0) observation and Time (as trading days of the month)

[27] Colour green is used for banks with negative hazard coefficient

Table 8.9 below displays the results from which we see that only six banks: Afribank, GTB, UBA, Union, WEMA and Zenith banks are significant, with **GTB** and **Union** banks having significantly negative hazard coefficient, indicating the absence of a bubble in their stock returns.

Table8.9: Summary Results on Cox-Proportional Hazard Model with Monthly Trading Days as Time for each Bank

Bank	Hazard coefficient (ξ)	Hazard Rate
Access	**-0.115 (0.188)**	**0.751**
African bank	**-0.380 (0.000)**	**0.684**
Diamond	**-0.037 (0.650)**	**0.964**
Eco bank	**0.130 (0.444)**	**1.139**
First	**0.058(0.362)**	**1.060**
FCMB	**-0.156 (0.076)**	**0.855**
FIDELITY	**-0.035 (0.686)**	**0.966**
GTB	**0.160 (0.018)**	**1.029**
SKYE	**-0.087 (0.330)**	**0.917**
STANBIC IBTC	**0.003 (0.962)**	**1.003**
STERLING	**-0.070 (0.321)**	**0.933**
UBA	**-0.192 (0.009)**	**0.825**
UNION	**0.177 (0.000)**	**1.193**
UNITY	**0.110 (0.145)**	**1.116**
WEMA	**-0.345 (0.000)**	**0.708**
ZENITH	**-0.227 (0.000)**	**0.797**

Case 4: Fitting Runs of positive (coded: 2)/negative (coded: 1) "Abnormal Returns" on the event (1)/censoring (0) observation and Time (as the months across the periods)

From **Table 8.10,** seven banks: Afribank, FCMB, UBA, **Union, Unity**, WEMA and Zenith have significant negative hazard coefficients, except for Union and Unity banks which have significant positive coefficients. Hence, five of the banks do not show evidence of a bubble while the rest do so, particularly Union and Unity banks.

Table 8.10: Summary Results on Cox-Proportional Hazard Model with Months as Time for each Bank

Bank	Hazard Coefficient (ξ)	Hazard Rate ($exp(\xi)$)
Access	**0.006 (0.944)**	**1.006**
African bank	**-0.443 (0.000)**	**0.642**
Diamond	**-0.002 (0.981)**	**0.981**
Eco bank	**-0.169 (0.319)**	**0.844**
First	**0.093 (0.142)**	**1.098**

FCMB	**-0.182 (0.039)**	**0.833**
FIDELITY	**-0.007 (0.935)**	**0.993**
GTB	**0.114 (0.091)**	**1.120**
SKYE	**-0.027 (0.761)**	**0.973**
STANBIC IBTC	**-0.021 (0.729)**	**0.979**
STERLING	**-0.016 (0.819)**	**0.984**
UBA	**-0.172 (0.020)**	**0.842**
Union	**0.191 (0.000)**	**1.210**
Unity	**0.266 (0.000)**	**1.305**
WEMA	**-0.392 (0.000)**	**0.675**
Zenith	**-0.204 (0.000)**	**0.816**

P-values in parentheses

8.5 Summary and Conclusion

In this chapter, we explained the meaning of bubbles, as well as their characteristics, consequences, impacts and causes. We proceeded to review some relevant literature in this regard, both in the context of what has been reported concerning bubbles and the findings from various studies using the different methods we intended to adopt and we outlined our proposed methods. Then, we discussed models of rational speculative bubbles, both at the "simple" and "general" levels, which use stylised facts of returns, through to the duration dependent models which are directly relevant for our objectives.

We also presented a summary of some related work that has used these models in different contexts. We then stated the specific objectives of this bubbles chapter, our choice of models, and our motivations for adopting duration dependent models and unit root tests.

While presenting our results, we first obtained the summary statistics (on the mean, standard deviation, skewness, and kurtosis, including the results of the ACF tests), for the price series of the respective banks particularly for the suspected "bubble periods" for each bank. With this basic exploratory approach, we observed that five banks - Sterling, STANBIC, Unity, GTB and WEMA - have two suspected episodes of bubbles based on the bubble characteristics highlighted (see Table8.3). For the specific summary statistics, we compared each of them to their respective overall period values as presented in Chapter 5 of the thesis and obtained percentages showing how they compare to the overall values (see Tables8.4 and 8.5).

With these preliminary results, we suspect the presence of bubbles, based on the bubble attributes that have been discussed in previous studies, some of which were discussed earlier. However, since some of these attributes can also be a signal for other stylized facts of stock returns, we conducted unit root tests using the ADF and PP test statistics, which are the popular tests for identifying presence of a unit root against the explosive alternative (which indicates the presence of a bubble). The results of these tests are presented in Table 8.6. From our results, nine banks - Access, Ecobank, Fidelity, Skye, UBA, WEMA-1, Zenith, GTB-1, and FCMB - were found to conform to the explosive alternative hypothesis of non-stationarity, meaning that they are likely to be characterised by bubbles.

For further investigation, we applied Cox-proportional hazards regression - a duration dependent model not previously used this way to analyse the NSM (Omar, 2012; Omar and Ezepue, 2016) - and considered four different cases subject to: (1) "Time" or data set durations (day or month) and the use of original returns and (2) Using "Abnormal" returns, which is the residual series derived from fitting an autoregressive model of order 4, as used in other studies. These two types of tests led to four specific tests and altogether there were six tests of bubbles used in the chapter.

Under case 1, we obtained runs of "positive" and “negative” returns from the original returns series and "days of the month" served as the Time variable. The results revealed that in seven banks- **Afribank, FCMB, UBA, Union, Unity, WEMA and Zenith -** the hazard coefficients were significant.

For case 2, where we obtained the runs of both positive and negative returns from the estimated "Abnormal returns" from **(8.6),** and "days" across the data period served as the Time variable, the results showed that while in six banks - **Access**, **Skye, Sterling, UBA, WEMA** and **Zenith –** had significant and negative hazard coefficients (ξ) (as observed by McQueen and Thorley, 1994), the remaining five banks - Afribank, Diamond, First, Fidelity, and Union banks produced significant but positive**ξ,** indicating that the hazards of runs of positive returns increased rather than decreased. Thus, the latter set of banks show evidence of bubbles and the former set of banks show no evidence of bubbles.

In case 3, where Abnormal returns were considered and days of the month served as Time variable were used, the results show that while ξ **for Afribank, UBA, WEMA** and **Zenith** banks are significantly negative; for GTB and Union banks, the coefficient ξ was also significant, but positive.

Finally, for the fourth case where "Abnormal" returns were again used, but "months" were the Time variable, the results indicate that **Afribank, FCMB, UBA**, **WEMA and Zenith** have significant negative coefficients, while Union and Unity banks have significant positive coefficients.

Table 8.11 below has the details on the number of banks in which a bubble was detected using the six different methods. Please see a summary of bubble characteristics which informs this table in the footnotes.

Table 8.11: Summary of Bubble Results from the Six Methods

Bank	Bubble Characteristics[1](7)	Unit Root Explosiveness (9)	Case1(7)	Case 2 (6)	Case 3 (4)	Case 4 (5)
Access		YES (trended)		YES		
Afribank	YES		YES		YES	YES
Diamond						
Eco bank		YES (trended)				
First	YES					
FCMB		YES (trended)	YES			YES
FIDELITY		YES (trended)				
GTB		YES				
SKYE		YES (trended)	YES	YES		
STANBIC IBTC			YES			
STERLING	YES			YES		
UBA	YES	YES	YES	YES	YES	YES
Union	YES					
Unity						
WEMA	YES	YES (trended)	YES	YES	YES	YES
Zenith	YES	YES	YES	YES	YES	YES[28]

Generally, from our findings we have been able to establish that: (1) a bubble is present in some of the Nigerian banks, though this is not shown through all the methods; (2) virtually all the banks which were found to be characterised with explosiveness under unit root tests were also

[28]**Bubble Characteristics**-Negative. Skewness, positive kurtosis and Significant ACF; **Case 1**: Fitting Runs of positive (coded: 2)/negative (coded: 1) Returns on the event (1)/censoring (0) observation and Time (as trading days of the month). **Case 2**: Fitting Runs of positive (coded: 2)/negative (coded: 1) "Abnormal Returns" on the event (1)/censoring (0) observation and Time (as days across the fifteen years). **Case 3**: Fitting Runs of positive (coded: 2)/negative (coded: 1) "Abnormal Returns" on the event (1)/censoring (0) observation and Time (as trading days of the month). **Case 4**: Fitting Runs of positive (coded: 2)/negative (coded: 1) "Abnormal Returns" on the event (1)/censoring (0) observation and Time (as the months across the periods)

identified under the Cox proportional hazards model, which confirms the power of the model in such empirical finance work and increases the evidence in favour of bubble' presence in the returns series; and (3) the duration dependent methods are sensitive to data periods, be it days, weeks or months, given that the results from different cases of duration dependence models do not identify the presence of a bubble in the same banks.

We note that detecting bubbles is somewhat tricky as most bubble characteristics are embodied in traditional stylised facts of asset returns, as explored in Chapter 5 and recalled in this chapter as appropriate. Hence, this chapter adds to existing knowledge of bubbles in the literature by demonstrating how to use a triangulated approach involving six methods to build a stronger evidence base for concluding whether a bubble exists in asset data (namely returns series). This extends the approach in the related studies of bubbles at the overall NSM market level (see: Ezepue and Omar, 2012; Omar and Ezepue, 2016). The fact that the methods provide stronger evidence of bubbles in the banking sector of the NSM supports the suggestion by these authors that bubbles are best studied within sectors of a market using asset returns, to avoid the masking effect of using the ASI at an overall market level.

Additionally, the triangulated bubbles tests indicate a consensus approach to finally deciding on which banks show bubbles in their returns. This is done by checking how many of the approaches are positive for bubbles. For example, seven banks –First, Fidelity, Eco, GTB, Union, STANBIC and Sterling banks [29]have only one or two methods indicating presence of bubbles, compared to, say, WEMA, UBA and Zenith banks that were identified by all six methods applied to be positive for bubbles. We can therefore state that perhaps the first seven banks are not severely susceptible to bubbles, while WEMA, Zenith and UBA clearly are.

The implications of these results for investment decisions by potential investors in the different banks' stocks is that the banks' stocks should be treated differently in portfolios containing such stocks. Investors with this knowledge of the variations in bank asset dynamics and stylised facts, including bubbles, will be prompted to ask more questions of possible causes of such differences, including quality of management and other fundamental measures of bank financial performance. Indeed, these results reinforce the need for deeper empirical analysis and characterisations of key sectors of a financial market, as suggested by Omar and Ezepue (2012) and Ezepue (2017).

[29]Five of which, namely: First, Fidelity, Eco, GTB and Union were identified by only one method; the remaining two- STANBIC and Sterling, were detected by two methods to be positive for bubbles

In conclusion, this chapter extended substantially the approach to modelling bubbles presented in previous work by using six approaches to build a stronger evidence base for confirming the existence of bubbles in Nigerian banks' returns. The implications of this consensus approach for investment purposes and stock market analyses have been noted.

9 CHAPTER NINE: VOLATILITY MODELLING

9.1 INTRODUCTION

Volatility measurement and modelling is the basis for financial econometrics, and time-varying volatility is a common feature of financial time series, such that most asset returns series are characterised by it. A major challenge in modelling volatility is that, market volatility being a latent variable, it is not directly observable. Thus, volatility is obtained by making inferences on the movement of prices in the market. For instance, when there is an increase in market prices, volatility rises, but to what extent remains a subject of investigation. This is simply because whether a shock to prices is permanent or transitory is unknown.

The latent nature of volatility implies that it must be forecast by building a statistical model subject to making relevant assumptions. Thus, modelling volatility is highly demanding due to the challenges posed by certain factors, such as non-normality, structural breaks and volatility clusters, which characterise financial volatility. The presence of volatility clusters especially indicates that the best option to predict volatility is to consider either the most recent observations or to assign a higher weight to the most recent observations.

9.1.1 Importance of Volatility in Financial Markets

According to Engle and Patton (2007), predicting asset **volatility** is useful in risk management, derivative pricing and hedging, market making, market timing, portfolio selection and many other financial activities. A risk manager needs to know today whether his or her portfolio will decline (or rise) in the future. The same applies to an option trader who will be seeking to understand the level of volatility likely to characterise the future life of the contract, and to hedge such contract, he or she will also want to know how volatile the projected volatility is. A portfolio manager may want to sell a stock or a portfolio before it becomes too volatile. Understanding volatility helps to assess riskiness of investment and the level of risk characterising the stock market (Foucault et al., 2011). It helps to determine how the overall economy is impacted, especially during financial crisis; if there is any change in volatility, it immediately impacts the economy as well as the investors.

Further, according to Daly (2008), the following six fundamental reasons make studying volatility rewarding:

1. Sharp fluctuations (or high volatility) in asset prices over a short horizon, say intraday or daily, due to fundamental economic factors may lead to lack of confidence in stock markets by investors and a drop in capital flow into the market.
2. For small firms, volatility is an important measure of the chance of bankruptcy; thus, the higher the volatility of a capital structure, the higher the likelihood of default.
3. Volatility is a major determinant of bid-ask spread, such that the spread becomes wider during high volatility, thus indicating that a market's liquidity is impacted by its volatility
4. Hedging techniques such as portfolio insurance are affected by the volatility level. Price of insurance therefore rises when there is high volatility.
5. For risk-averse investors, increasing volatility due to trading activity reduces trading volume, which adversely affects further investment in the stock market.
6. During periods of high volatility, the regulatory agencies may force financial intermediaries such as banks to reallocate capital to cash-equivalent investments, thereby reducing the allocation efficiency and value of the assets of such banks.

9.1.2 Some Stylized Facts about Volatility

Miron and Tudor (2010) note the following stylised facts about volatilities.

1. **Volatility clustering**; meaning that a slight increase (decrease) in today's asset returns may be followed by a slightly increase (decrease) in such returns tomorrow. That is, a volatile period tends to be followed by another volatile period, or volatile periods are usually clustered, which is an indication of persistence in past shocks.
2. Volatility spikes up during crises but falls back to approximately the same level it was at before the crisis as soon as the crisis disappears, such that the price movements are negatively correlated with volatility, with negative returns generating higher volatilities than positive returns **(*Leverage effects*).**
3. The empirical distribution of financial time series (or returns) exhibit excess kurtosis (or fatter tails) relative to a normal distribution.
4. ***Long memory-***This property means that volatility is highly persistent and that there is evidence of near unit root behaviour of the conditional variance process.
5. ***Co-movements in volatility-*** a circumstance when time series from different markets move together such that large movements in one financial time series are matched by large movements in another time series from a different market.

Why Statistical Models?

We fit models in statistics for reasons including the following:

1. To control a physical process;
2. To understand a data-generating process;
3. To predict a future outcome of a data-generating process.

In this chapter, we describe different models for measuring different types of volatility and explore particularly the ARCH-GARCH family of volatility models of stock returns, which are then applied to the Nigerian bank returns.

9.2 Modelling Volatility

From among the six most commonly applied models for volatility forecasting listed below, emphasis in this chapter shall be placed on the ARCH/GARCH family models. As argued in Omar (2012, Chapter 9) and section 3.3 of this thesis, ARCH/GARCH are known to be suitable for many financial time series studied in empirical finance because the data exhibit conditionally heteroscedastic errors. Hence, since the research objectives in this study are similar to and extend those in Omar (2012) to the banking sector of the NSM we also make use of these models. The six volatility models alluded to above are:

1. Moving Average (MA) models;
2. Exponentially weighted moving average (EWMA) sometimes called Risk Metrics models;
3. Implied volatility models;
4. Realized volatility models;
5. Stochastic volatility models;
6. GARCH family models.

We shall briefly discuss each of these but with more emphasis placed on the different types of ARCH/GARCH models that we will use to try to explain and forecast Nigerian banks' returns data and their time-varying conditional volatilities

9.2.1 Volatility Models

The easiest way to forecast volatility is simply to obtain the sample standard error from a sample of returns, and then to keep the sample size constant and overtime add the newest returns and drop the oldest; this method is known as the moving average (MA) approach. This method is, however, known to perform badly, but can be improved by exponentially weighting returns such that the most recent returns are assigned the highest weight in forecasting volatility; thereby making Exponentially Weighted Moving Average (EWMA) a better model.

Moving Average Models

According to Danielson (2011), the simplest volatility forecast model is defined as

$$\hat{\sigma}_t^2 = \frac{1}{W_E}\sum_{j=1}^{W_E} r_{t-j}^2. \quad \textbf{(9.1)}$$

Where$r_t$is the observed return on day t, $\hat{\sigma}_t$ is the volatility forecast for day t, and W_E is the length of the estimation window, that is, the number of observations used in the calculation, usually chosen arbitrarily. It is important to state that in forecasting volatility, lagged observations are used in the MA model.

Its major limitation is that observations are equally weighted, which is a substantial issue in the face of volatility clusters, given the fact that the most recent observations are more indicative of whether we are in a high or low volatility cluster. The method is also sensitive to the length of the estimation window and has the possibility of generating volatility forecasts that jump around and are generally systematically too high or too low.

The Exponentially Weighted Moving Average (EWMA) Model

The moving average can be improved by assigning higher weights to the more recent observations, and one of such way to achieve this is the use of the EWMA model. Thus, EWMA is a modification of MA so that weights decline exponentially historically:

$$\hat{\sigma}_t^2 = \frac{1-\lambda}{\lambda(1-\lambda^{W_E})}\sum_{j=1}^{W_E} \lambda^j r_{t-j}^2. \quad \textbf{(9.2)}$$

Here 0 <λ< 1 is a parameter whose successive powers generate the exponential weights in the model. The first part of the equation automatically ensures that the sum of the weights is equal to 1.

Rewriting the model as the weighted sum of the previous period's volatility forecast and squared returns gives conditional volatility of the model as:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, 0 < \lambda < 1. \quad \textbf{(9.3)}$$

This model was proposed by J. P. Morgan (1993), withλestimated to be equal to 0.94 for daily returns, and because the exponential weights decline to zero very quickly, the model is inappropriate in calculating value at risk (VaR).

The main limitation of the EWMA model is that λ is a constant and identical for all assets; this establishes the fact that the model is not optimal for any asset, unlike GARCH models which are explained above to be typically suitable for analysing time-varying and heteroscedastic volatilities. Consequently, volatility forecasts based on EWMA are inferior and unreliable compared to GARCH models (Danielson, 2011; p.).

The major strength of the model lies in its ease of implementation as against most of its counterparts, and that its multivariate forms can easily be applied without any modification.

Note that EWMA is a special case of a GARCH model that is covariance non-stationary. Thus, its unconditional variance cannot be calculated because it is undefined, meaning that if allowed to run for a while, the model will explode.

The Implied volatility (IV): is the model used in the Black-scholes (BS) model to predict option prices traded on the exchange market based on the current prices (rather than historical prices) of the option. For example, the Black Scholes pricing model relates the price of a European call option to the current price of the underlying, the strike, the risk-free rate, the time-to-maturity and the volatility $BS(S_t, K, r, t, \sigma_t) = C_t$, where C_t is the price of the call. The challenges with this model are that it accuracy depends on that of the BS equation, and with the assumption of constant conditional volatility and normal error distribution, the IV model is thus unsuitable for time-varying volatility.

The Realized Volatility (RV) model: is a volatility model used in predicting future volatility from historical data based on the intraday data, obtained at regular intervals of time. It is data driven and does not rely on parametric models. Implementation of the **RV** model is challenging due to the availability of data, simply because intraday data are hard to obtain, usually not very clean and costly.

Stochastic Volatility (SV) Model: in this case, the volatility model is a function of an exogenous shock and past volatility, thereby making the conditional volatility random with unknown innovation term at time t. In the **SV model**, there are two innovation terms - one for the returns and one for the conditional variance itself. This additional innovation makes estimation of the model parameters to be more difficult compared to GARCH family models. This is simply because the volatility equation follows a separate distribution which cannot be estimated by the historical returns data. Though the model is more advantageous than GARCH models, it is not as common as the latter in forecasting and there is no evidence that SV produce better forecast than GARCH models.

GARCH FAMILY MODELS

The GARCH family models are members of the conditional volatility family of models, which define return volatilities at given times t as functions of previous errors and volatilities up to time t-1.

The first of this family is the autoregressive conditional heteroscedasticity (ARCH) model proposed by Engle (1982), while its generalized form, GARCH, proposed by Bollerslev (1986) extends the ARCH model to accommodate different volatility behaviours. The GARCH models are therefore more commonly used to analyse volatilities because they are more suitable in capturing time-variation and most of the stylized facts of the financial data better than their contemporary models (see: Tsay (2005, 2014) and Taylor (2005; 2011) and Danielson (2011)).

9.2.1.1.1 ARCH Models

ARCH models were introduced to accommodate the possibility or likelihood of time dependence in volatility. ARCH models are specified relative to the discrete-time process for the price of a security/portfolio: $\{ P(t), t = 1,2, \dots \}$. Engle (1982) models the discrete returns of the process as

$$r_t = \mu_t + \varepsilon_t \,. \tag{9.4}$$

In the mean equation **(9.4)**, μ_t, is the mean return at time t,conditional on $\mathcal{F}_{t-1}$, the information available up to time $(t-1)$, and $\varepsilon_t = Z_t \sigma_t$ where Z_t $is\ i.i.d$ normally distributed, with $E[Z_t] = 0, var\ [Z_t] = 1$. The model assumes that the processes σ_t and Z_t are stochastically independent.

In this model, the residuals do not have constant variance but are multiples of white noise (Z_t) (with mean zero and unit variance), and, σ_t defined as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2. \tag{9.5}$$

with the constraints: $\alpha_j > 0,\ j = 0,1, \dots, p$; where $\sigma_t^2 = var(r_t | \mathcal{F}_{t-1})$ is the conditional variance of returns at time t given previous return errors and available information up to time (t – 1), which is denoted by $\mathcal{F}_{t-1}$. The key feature of this model is that the variance of the shock, ε_t, is time-varying and depends on the past p shocks, ε_{t-1}, $\varepsilon_{t-2}, \varepsilon_{t-2}, \dots, \varepsilon_{t-p}$through their squares.

The volatility model states that the volatility is a weighted average of the squared residuals over the last p lags. Thus, if there is a large residual, then that could persist such that the next observation has a large variance, indicating that there is an element of time dependence. The constraints ensure that the alphas are positive so that the volatility will be positive, otherwise the latter will be negative, which is not possible.

Alternatively, the model can be defined as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2$$

$$\varepsilon_t = r_t - \mu_t \,;\, \forall\, r_t | \mathcal{F}_{t-1} \sim N(\mu_t, \sigma_t^2). \qquad \textbf{(9.6)}$$

Which reads as: "$r_t$given the information set at time $t-1$ is conditionally normal with mean $\mu_t$and varianceσ_t^2".

To estimate the model's parameters, recall that:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2, \qquad \textbf{(9.7)}$$

which is simply an autoregressive (AR) model in squared residuals (ε_t^2) as shown below.

Suppose $\tau_t = (\varepsilon_t^2 - \sigma_t^2)$ is added to both sides of the model **(9.7)**, we have:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \tau_t, \qquad \textbf{(9.8)}$$

so that $\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \varepsilon_t^2 - \sigma_t^2,$ **(9.9)**

where $E[r_t|\mathcal{F}_{t-1}] = \mathrm{E}[(\varepsilon_t^2 - \sigma_t^2)|\mathcal{F}_{t-1}] = 0$ and $var[r_t|\mathcal{F}_{t-1}] = \mathrm{var}\,[\varepsilon_t^2] = 2\sigma_t^4$.

Here, τ_t is almost white noise as it has mean zero but is not a standard white noise because its variance may change with time, thus making it a conditionally independent function (but with some variability). This implies that the model is an autoregressive model with time-varying variances in the process. Based on this, one may then need to check if there is an ARCH structure in the data and this is achieved using Engle's method, utilizing Lagrange multipliers (see Tsay, 2005; 2014).

Deriving the Unconditional Variance of the ARCH (p) model

Given that:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p,}^2 \qquad \textbf{(9.10)}$$

Taking the expectation of both sides of $(\mathbf{9.10})$**,** we have

$$E[\sigma_t^2] = \omega + \alpha_1 E[\varepsilon_{t-1}^2] + \alpha_2 E[\varepsilon_{t-2}^2] + \cdots + \alpha_p E[\varepsilon_{t-p}^2]. \qquad \textbf{(9.11)}$$

Recall that

$$\varepsilon_t = \sigma_t, \tag{9.12}$$

and thus substituting (9.12) into (9.11), we have

$$E[\sigma_t^2] = \omega + \alpha_1 E[\sigma_{t-1}^2 z_{t-1}^2] + \alpha_2 E[\sigma_{t-2}^2 z_{t-2}^2] + \cdots + \alpha_p E[\sigma_{t-p}^2 z_{t-p}^2],$$

which under our assumption of the independence of $\sigma_t$and z_t gives:

$$E[\sigma_t^2] = \omega + \alpha_1 E[\sigma_{t-1}^2]E[z_{t-1}^2] + \alpha_2 E[\sigma_{t-2}^2]E[z_{t-2}^2] + \cdots + \alpha_p E\left[\sigma_{t-p}^2\right]E[z_{t-p}^2]. \tag{9.13}$$

since $E[z_t^2] = 1$, **(9.13)** becomes:

$$E[\sigma_t^2] = \omega + \alpha_1 E[\sigma_{t-1}^2] + \alpha_2 E[\sigma_{t-2}^2] + \cdots + \alpha_p E\left[\sigma_{t-p}^2\right]. \tag{9.14}$$

Collecting like terms in (**9.14**), we have

$$E[\sigma_t^2] - \alpha_1 E[\sigma_{t-1}^2] - \alpha_2 E[\sigma_{t-2}^2] - \cdots - \alpha_p E\left[\sigma_{t-p}^2\right] = \omega$$

The ARCH model assumes that the conditional volatilities are covariance stationary with a common unconditional variance, so that

$$E[\sigma_t^2] = E[\sigma_{t-p}^2] = \sigma^2.$$

Thus,

$$\sigma^2\left(1 - \alpha_1 - \alpha_2 - \cdots - \alpha_p\right) = \omega, \text{ and} \tag{9.15}$$

$$E[\sigma_t^2] = \sigma^2 = \frac{\omega}{\left(1 - \alpha_1 - \alpha_2 - \cdots - \alpha_p\right)}$$

The condition needed to ensure that the unconditional variance is finite is:

$$1 - \alpha_1 - \alpha_2 - \cdots - \alpha_p > 0.$$

The properties of the ARCH family models applied in the derivation include:

1. The definition of the shock $\epsilon_t^2 \equiv z_t^2 \sigma_t^2$ is used in separating the independently and identically distributed $(i.i.d)$ normal innovation (z_t) from the conditional variance (σ_t^2).

2. z_t and σ_t^2 are independent since σ_t^2 depends on $\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-p}$ (and $z_{t-1},\ z_{t-2},\ \dots\ ,\ z_{t-p}$), and, $z_t$is an $i.i.d$innovation at time *t*.

One limitation of ARCH models is that they require higher lags, say **5-8** lags of the squared shock to adequately model conditional variance. Tsay (2005) discusses the following further weaknesses of ARCH models.

Weaknesses of ARCH models

1. The model assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. Practically, however, the price of a financial asset responds differently to positive and negative shocks. This asymmetric or leverage effect is therefore not adequately explored by ARCH models hence the importance of GARCH models, which generalise the ARCH models.

2. The ARCH model is rather restrictive, and this gets more complicated as the order of the model increases, thereby limiting the ability of ARCH models to capture excess kurtosis, especially when innovations with non-normal distributions are assumed.

3. ARCH models are likely to over predict the volatility because they respond slowly to large isolated shocks to the returns series.

4. ARCH models often requires many parameters and a high order p to capture the volatility process (Alberg, Shalit and Yosef, 2008).

9.2.1.1.2 GARCH Models

The Generalized ARCH (GARCH) process, introduced by Bollerslev (1986), improves the ARCH specification by including lagged conditional variances in the present conditional variance at time *t*. GARCH models typically fit as well as a high order ARCH model and yet remain parsimonious, in the sense that lower-order GARCH models have been shown to successfully model returns volatilities which may rather require higher-order ARCH models (see Danielson, 2014).

Definition

A GARCH (p, q) process is defined as:

$$r_t = \mu_t + \varepsilon_t$$

Such that

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{9.16}$$

where and each $\varepsilon_t = \sigma_t z_t$ where $z_t \sim N(0,1)$.

The GARCH(p, q) model builds on the ARCH(p) model by including q-lags of the conditional variance: $\sigma_{t-1}^2, \sigma_{t-2}^2, \dots, \sigma_{t-q}^2$.

Now, consider the simplest form model, the GARCH$(1,1)$ model, where the conditional mean is assumed to be zero. This is given by: $r_t = \varepsilon_t$,

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{9.17}$$

The model states that the future variance will be an average of the current shock, ε_{t-1}^2 and the current variance, σ_{t-1}^2 plus a constant, with constraints on the parameters that ensure positive variance and stationarity as follows: $\omega > 0$, $\alpha_1 \geq 0, \beta_1 \geq 0$ and$\alpha_1 + \beta_1 < 1$.

In the simple GARCH (p, q) model, the impacts to conditional variance of positive and negative shocks are symmetric. Thus, a GARCH model is unable to express the Leverage Effects, a limitation that is overcome by some asymmetric GARCH models discussed later in this chapter.

Remark

In a complete GARCH(p, q) model the parameter restrictions are difficult to satisfy. For example, in a GARCH$(2, 2)$, one of the two $\beta's$ (say, β_2) may be slightly negative while ensuring that all conditional variances are positive (see Nelson & Cao, 1992).

Now, consider the **Unconditional Variance** for GARCH (1, 1)

In the GARCH (1, 1) model there exists a finite unconditional variance

$$E[\sigma_t^2] = \sigma^2 = \frac{\omega}{1 - \alpha_1 - \beta_1}, \tag{9.18}$$

if and only if $(\alpha_1 + \beta_1) < 1$.

Rewriting$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ using the unconditional variance equation, we have

$$\sigma_t^2 = (1 - \alpha_1 - \beta_1) E[\sigma_t^2] + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{9.19}$$

where the coefficient of the lagged conditional volatility term is, β_1, α_1 is the parameter for the ARCH effect while the sum $(\alpha_1 + \beta_1)$ is a measure of volatility persistence, which is the rate at which the volatility reverts towards the unconditional variance. This indicates that the next period's conditional variance is a weighted combination of the unconditional variance of returns, $E[\sigma_t^2]$, the last period's squared residuals ε_{t-1}^2 and the last period's conditional variance, σ_{t-1}^2 with weights $(1-\alpha_1-\beta_1)$, α_1 and β_1 which sum to one.

When $(\alpha_1+\beta_1)=1$, there is a unit root in the GARCH process, thereby resulting in a new process called the Integrated Generalized Autoregressive Conditional Heteroscedastic (IGARCH(1,1)) model, wherein the GARCH (1, 1) automatically becomes:

$\sigma_t^2 = \alpha_0 + (1-\beta_1)\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$. In this case, squared shocks are persistent such that the variance follows a random walk with drift, μ (see **4.36**).

Limitations of GARCH Models

The correlation between stock returns and changes in returns' volatility is negative; that is, volatility tends to increase due to "bad news"- a situation where there are negative returns and decreases during "good news", when returns are positive. The simple GARCH (p,q) models, however, assume that only the magnitude and not the sign (either positivity or negativity) of unanticipated excess returns determine the level of σ_t^2. If the distribution of z_t is symmetric, the change in variance tomorrow is conditionally uncorrelated with excess returns today (Nelson, 1991). Suppose σ_t^2 is written as a function of its lags and that of z_t, noting that

$$\varepsilon_t^2 = z_t^2\sigma_t^2\text{: } \sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i z_{t-i}^2\sigma_{t-i}^2 + \sum_{j=1}^{q}\beta_j\,\sigma_{t-j}^2$$

Obviously, the conditional variance is invariant to changes in the sign of the, $z_t's$ and the innovations are $i.i.d$ (Rossi, 2004).

In GARCH models, the non-negativity constraints are imposed to ensure that σ_t^2 remains positive at any time t with probability one. These constraints imply that increasing z_t^2 in any period increases σ_{t+k}^2 $for\ all\ k \geq 1$, cancelling out any possibility of random oscillatory behaviour in the σ_t^2 process.

The GARCH model is not capable of explaining the observed covariance between, ε_t^2 and, ε_{t-i}. This is possible only if the conditional variance is expressed as an asymmetric function of ε_{t-i}.

In the GARCH (1, 1) model, shocks may persist in one period and die out in another, and the conditional moments of GARCH (1, 1) may explode even when the process is stationary.

9.2.1.1.3 EGARCH MODEL

The Exponential GARCH (EGARCH) model of Nelson (1991) is a major shift from the ARCH and GARCH models previously discussed, whereby rather than modelling the variance directly, the natural logarithm of the variance is modelled, and thus there is no need for parameter restrictions to ensure that the conditional variance is positive.

Definition

An Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) process is mathematically defined as

$r_t = \mu_t + \varepsilon_t$, and

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} g(\varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j \ln\left(\sigma_{t-j}^2\right). \quad \textbf{(9.20)}$$

where

$$g(\varepsilon_{t-i}) = \alpha_i \left(\frac{\varepsilon_{t-i}}{\sqrt{\sigma_{t-i}^2}}\right) + \gamma_i \left[\left|\frac{\varepsilon_{t-i}}{\sqrt{\sigma_{t-i}^2}}\right| - E\left[\left|\frac{\varepsilon_{t-i}}{\sqrt{\sigma_{t-i}^2}}\right|\right]\right]. \quad \textbf{(9.21)}$$

with $\varepsilon_t = \sigma_t z_t,\ where\ \sigma_t > 0$ and $z_t \sim N(0,1)$ (see Tsay, 2014, Danielson, 2014 and Taylor, 2005; 2011).

Here, γ_k is the leverage parameter to be computed along with the, $\alpha_i's$ and β_j's.
Consider now a simpler form EGARCH (1, 1), with a constant mean:

$\boldsymbol{r_t = \mu + \varepsilon_t}$, withln$(\sigma_t^2) = \omega + g(\varepsilon_{t-1}) + \beta_1 \ln(\sigma_{t-1}^2)$,

$$\text{so}\ln(\sigma_t^2) = \omega + \alpha_1 \left(\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}\right) + \gamma_1 \left[\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - E\left[\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|\right]\right] + \beta_1 \ln(\sigma_{t-1}^2).$$

This can be rewritten more simply, using standardised innovation termsas:

$$\ln(\sigma_t^2) = \omega + \alpha_1(z_{t-1}) + \gamma_1\left[|z_{t-1}| - E\left[|z_{t-1}|\right]\right] + \beta_1 \ln(\sigma_{t-1}^2) \quad \textbf{(9.22)}$$

$\varepsilon_t = \sigma_t z_t,\ where\quad z_t \sim N(0,1)$.

As noted in Malmsten and Teräsvirta, (2004), $E[|z_{t-1}|] = \sqrt{\frac{2}{\pi}}$, which is equal to 0 iff $z_t \sim N(0,1)$.

$$E[|z_{t-1}|] = \frac{2\sqrt{ʚ-2\Gamma\left(\frac{ʚ+1}{2}\right)}}{\sqrt{\pi(ʚ-1)\Gamma\left(\frac{ʚ}{2}\right)}}, \text{ where } z_t \sim t(\nu), \text{ and} E[|z_{t-1}|] = \Gamma(2\eta^{-1})/\{\Gamma(\eta^{-1})\Gamma(3\eta^{-1})\}^{1/2}$$

where $z_t \sim GED(\eta)$, is the Generalised Error Distribution with parameter η and ʚ is the degree of freedom. The term $\alpha_1(z_{t-1})$ produces a symmetric increase in the log variance, while the term: $\gamma_1[|z_{t-1}| - E|z_{t-1}|]$ gives an asymmetric effect in the volatility. Thus, γ_1 is the asymmetric parameter, and isexpected to be less than zero to indicate that the volatility rises more in response to anegative shock than toa positive shock.

In the usual case where γ_1<0, the magnitude of the shock can be divided into two based on the sign of z_{t-1}. Let the shock coefficient be $\mathfrak{I}$. Then

$$\mathfrak{I} = \begin{cases} \alpha_1 - \gamma_1 & when \quad z_{t-1} < 0, \\ \alpha_1 + \gamma_1 & when \quad z_{t-1} > 0. \end{cases} \quad \textbf{(9.23)}$$

Since both shocks have mean zero and the current log variance is linearly related to the past log variance through β_1, the EGARCH (1, 1) model is an AR model (Taylor, 2005)

Properties of EGARCH Models

1. The function $g(\varepsilon_{t-i})$ is linear in ε_{t-i} with the coefficient $(\alpha_i + 1)$ if ε_{t-i} is positive, but it is linear with the coefficient$(\alpha_i - 1)$ ifε_{t-i} is negative.
2. If α = 0, large innovations increase the conditional variance if and only if $(|z_t| - E\ [|z_t|]) > 0$ and decrease the conditional variance if $(|z_t| - E\ [|z_t|]) < 0$.
3. Suppose that $\alpha < 1$; the function $g(\varepsilon_t)$ is positive if the innovations ε_t are less than $\sqrt{2/\pi}/(\alpha_i - 1)$ thus, negative innovations ε_t in returns cause the innovation to the conditional variance to be positive if $\alpha < 1$ (see Taylor, 2005 and Tsay, 2014).

EGARCH models usually provide superior fits over the standard GARCH models. The presence of the asymmetric term is substantially responsible for the superior fit since many assets' returns series have been confirmed to exhibit a "leverage" effect and the use of standardized shocks (z_{t-1}) in the log-variance tends to reduce the effect of large shocks.

9.2.1.1.4 GJR-GARCH Model

This model was named after its proponents - Glosten, Jagannathan and Runkle (1993). It extends the standard GARCH (p, q) model by including asymmetric terms that capture the leverage (or asymmetric) effects, which are the tendency for the volatility to rise more in response to large negative shocks than to large positive shocks.

Definition

A GJR-GARCH (p, k, q) process is defined as:

$r_t = \mu_t + \varepsilon_t,$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{k} \gamma_i \, \varepsilon_{t-i}^2 I_{[\varepsilon_{t-i}<0]} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \qquad \textbf{(9.24)}$$

$$\varepsilon_t = \sigma_t z_t, \;\; where \quad z_t \sim N(0,1),$$

where $I_{[\varepsilon_{t-i}<0]}$ is an indicator function taking value 1 if $\varepsilon_{t-i} < 0$ and 0 otherwise.

The parameters of the GJR-GARCH model like the standard GARCH model, must be restricted to ensure that the volatility is always positive. This seems difficult to describe for a full GJR-GARCH (p, k, q) model, but for the simple GJR-GARCH $(1, 1, 1)$, we have that:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \varepsilon_{t-1}^2 I_{[\varepsilon_{t-1}<0]} + \beta_1 \sigma_{t-1}^2, \qquad \textbf{(9.25)}$$

where $\omega > 0,\; 0 \leq \alpha_1,\; \beta_1 < 1,\; (\alpha_1 + \beta_1) < 1$ and $(\alpha_1 + \gamma_1) \geq 0$. If the innovations are conditionally normal, a GJR-GARCH model will be covariance stationary and strictly stationary once the constraints are satisfied and the persistence rate$\left(\alpha_1 + \beta_1 + \frac{\gamma_1}{2}\right) < 1.$

The model explores the impact of ε_{t-1}^2on the conditional variance, σ_t^2.It also confirms that bad news $(\varepsilon_t < 0)$ and good news $(\varepsilon_t > 0)$ impact the conditional variance differently. If the leverage effect exists, γ_1 is expected to be positive. The leverage effect is observed as the impulse $(\alpha_1 + \gamma_1)$ of negative shocks, which is higher than the impulse (α_1) of positive shocks (Taylor, 2011).

Unconditional Variance of the GJR-GARCH $(1, 1, 1)$ model is given by

$$E[\sigma_t^2] = \sigma^2 = \frac{\omega}{1-(\alpha_1 + \beta_1 + \frac{\gamma_1}{2})}. \qquad \textbf{(9.26)}$$

9.2.1.1.5 TARCH/AVGARCH/ZARCH

The Threshold ARCH (TARCH) model (also called AVGARCH and ZARCH) is slightly different from the GJR-GARCH model in the volatility equation, whereby rather than modelling the conditional variance directly using squared innovations, the conditional standard deviation is modelled as a function of the lagged absolute value of the innovations. It also captures asymmetries using an asymmetric term in the same manner as the GJR-GARCH model (see: Taylor, 2008 and Zakoian, 1994).

Definition

A Threshold Autoregressive Conditional Heteroscedasticity (TARCH) process is defined mathematically as:

$$
\begin{aligned}
&r_t = \mu_t + \varepsilon_t, \\
&\sigma_t = \omega + \textstyle\sum_{i=1}^{p} \alpha_i |\varepsilon_{t-i}| + \sum_{i=1}^{k} \gamma_i \, |\varepsilon_{t-i}| I_{[\varepsilon_{t-i}<0]} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}, \qquad \textbf{(9.27)} \\
&\varepsilon_t = \sigma_t z_t, \;\; where \quad z_t \sim N(0,1).
\end{aligned}
$$

TARCH models are also known as ZARCH due to Zakoian (1994) or AVGARCH when no asymmetric terms are included (*i.e.* $\gamma_i = 0$, Taylor, 2008).

The TARCH $(1, 1, 1)$ Model is given by

$$\sigma_t = \omega + \alpha_1 |\varepsilon_{t-1}| + \gamma_1 |\varepsilon_{t-1}| I_{[\varepsilon_{t-i}<0]} + \beta_1 \sigma_{t-1}, \; with \, (\alpha_1 + \gamma_1) \geq 0, \qquad \textbf{(9.28)}$$

where $I_{[\varepsilon_{t-i}<0]}$, is an indicator variable that takes the value 1 if $\varepsilon_{t-1} < 0$ and zero otherwise.

Note that models of the conditional standard deviation often out-perform models that consider the conditional variance directly, and this is because absolute shocks are less responsive than squared shocks to volatility.

9.2.1.1.6 APARCH Model

This is another asymmetric model that seeks to extend the TARCH and GJR-GARCH models by directly introducing different orders of nonlinearity in the conditional variance. For example, where the GJR-GARCH model uses 2 and the TARCH model uses 1 for the non-linearity in the conditional variance, the Asymmetric Power ARCH (APARCH) of Ding, Engle & Granger (1993) parameterizes this value directly using the parameter δ, providing greater flexibility in modelling long-memory volatility while still maintaining model parsimony.

Definition

An Asymmetric Power Autoregressive Conditional Heteroscedasticity, APARCH (p, k, q), process is defined byDing, Granger and Engle. (1993), as:

$r_t = \mu_t + \varepsilon_t,$

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{max(p,k)} \alpha_i \left(|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}\right)^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}, \qquad \textbf{(9.29)}$$

$$\varepsilon_t = \sigma_t z_t, \; where \quad z_t \sim N(0,1),$$

where "$p \geq k$", and when,$p > k,\ \gamma_i = 0$ only if $i > k$. For the conditional variance to be non-negative, we must have $\omega > 0, \alpha_k \geq 0$ and $-1 \leq \gamma_i \leq 1$.

Consider the APARCH $(1, 1, 1)$ model.

$$\sigma_t^{\delta} = \omega + \alpha_1 \left(|\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1}\right)^{\delta} + \beta_1 \sigma_{t-1}^{\delta}, \qquad \textbf{(9.30)}$$

where $\delta \geq 0, \quad \omega > 0, \quad \alpha_1 \geq 0, \quad \beta_1 \geq 0 \quad 0 \leq \alpha_1 + \beta_1 \leq 1$ and $-1 < \gamma < 1$.
A positive γ indicates that a negative shock has a stronger impact on the stock's volatility than a positive shock.

This model nests some of the other volatility models including those discussed so far in different and special ways. For example, we derive from the model:
ARCH (of Engle), when: $\delta = 2;\ \gamma = 0\ and\ \beta = 0;$
GARCH (of Bollerslev), when: $\delta = 2;\ \gamma = 0;$
GJR-GARCH (of Glosten, Jagannathan, and Runkle), when: $\delta = 2;$
TGARCH (of Zakoian), when: $\delta = 1;$
NARCH (of Higgens and Bera) when: $\beta = \gamma = 0$; and
Log-ARCH (of Geweke and Pentula), when: $\delta = \infty$.
See: Ding, Granger and Engle (1993); Wurtz, Chalabi and Luksan (2006); Ding (2011) and Danielson (2011)), for further details.

The APARCH model exhibits several stylized properties of financial time series. For example, its unconditional volatility has large kurtosis. It also captures asymmetry in returns like the GJR-GARCH model, and at the same time it captures the long-memory property of the returns.

The Unconditional Variance for the APARCH (1, 1, 1) model is

$$\sigma_t^{\delta} = \frac{\boldsymbol{\omega}}{1-\boldsymbol{\alpha_1}(\mathbf{1}-\gamma_1)^{\delta}-\boldsymbol{\beta_1}}. \tag{9.31}$$

Error Distributional Assumptions

The probability distribution of asset returns often exhibits fatter tails than the standard normal distribution. The presence of heavy-tailedness is largely due to volatility clustering characterizing stock data- especially daily data. A further reason for this heavy-tailedness could be sudden changes in stock returns, which are typically negatively skewed and leptokurtic. Thus, for us to capture this phenomenon of heavy-tailedness, the student-t-distribution is also explored in our analysis.

Error Distributions

The two error distributions to be adopted in this research are the Gaussian and student-t error distributions and are defined as follow:

Normal Error distribution

$$f(\varepsilon_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}}\, exp - \frac{1}{2}\left(\frac{\varepsilon_t}{\sigma}\right)^2 \tag{9.32}$$

Student-t- distribution by Bollerslev (1987)

$$f(\varepsilon_t) = \frac{\Gamma((r+1)/2)}{\sqrt{\pi r}\Gamma(r/2)} * \frac{1}{\left(1+{\varepsilon_t^2}/{2}\right)^{(r+1)/2}} \quad \forall -\infty < \varepsilon_t < \infty \tag{9.33}$$

Mean- $E(\varepsilon_t) = 0$; Variance- $Var(\varepsilon_t) = \frac{r}{r-2}\ \forall\, r > 2$; Skewness- $Sk\ = 0\ for\ r > 3$ and kurtosis-

$$kurt = \frac{6}{r-4}\ \forall\, r > 4; for\ 2 < r \leq 4 =\ \infty\ and\ undefined\ otherwise.$$

Where r and ε_t are the degree of freedom and error term respectively.

Note that the two (error) distributions, **(9.32) and (9.33)** are premised on equations **(4.2)** and **(4.3)** respectively.

The News Impact Curve (NIC)

In the asymmetric volatility models, good news and bad news have different impacts on future volatility. The news impact curve characterizes the impact of past returns shocks on the returns volatility, which is implicit in a volatility model.

Holding constant the information dated, $t-2,\ t-3,\ t-4, \ldots$, according to Engle and Ng (1993), we can estimate the implied relation between information available in the next period $(t-1)$, summarised by ε_{t-1}, and the conditional variance σ_t^2, with $\sigma_{t-i}^2 = \sigma^2\ for\ i = 1, 2, 3, \ldots, p$. The curve thus generated, with all lagged conditional variances evaluated at the level of the unconditional variance of the stock returns is called the news impact curve (NIC), because it relates past returns shocks (news) to current volatility. The curve measures how new information is incorporated into volatility estimates.

The NIC is therefore a graphical representation of how investors tend to forecast the market and how they react to positive and negative shocks characterising their investments such that when there is good news, volatility does not rise because there is some level of confidence among the investors in the market.

For the symmetric GARCH model, the News Impact Curve (NIC) is both centred at and symmetric about the point where $\varepsilon_{t-1} = 0$, whereas in the case of EGARCH, the curve is centred at the point $\varepsilon_{t-1} = 0$ and for GJR-GARCH it is centred at $\varepsilon_{t-1} = -\gamma_1$. However, while the NIC for the EGARCH (1, 1) model which is exponentially increasing in both directions but with different slopes, has a steeper slope for points where $\varepsilon_{t-1} < 0$, the GJR-GARCH model's NIC has different slopes for both its positive and negative parts (Henry, 1998).

For the GARCH (1, 1) model, we have

$\sigma_t^2 = \ \omega + \ \alpha_1\varepsilon_{t-1}^2 + \ \beta_1\sigma_{t-1}^2$, and the news impact curve is presented as follows:

$$\sigma_t^2 = \ A + \ \alpha_1\varepsilon_{t-1}^2, \qquad \textbf{(9.34)}$$

where: $A = \ \omega + \ \beta_1\sigma_{t-1}^2$.

For the EGARCH $(1, 1)$ model with

$\ln(\sigma_t^2) = \ \omega + \ \alpha_1(z_{t-1}) + \ \gamma_1[|z_{t-1}| - E|z_{t-1}|] \ + \beta_1\ln(\sigma_{t-1}^2)$, where $z_t = \ {}^{\varepsilon_t}/_{\sigma_t}$, the NIC is

$$\sigma_t^2 = \begin{cases} Aexp\left(\frac{\alpha_1+\gamma_1}{\sigma}\varepsilon_{t-1}\right), \ \text{for } \varepsilon_{t-1} > 0, \\ Aexp\left(\frac{\alpha_1-\gamma_1}{\sigma}\varepsilon_{t-1}\right), \ \text{for} \varepsilon_{t-1} < 0, \end{cases} \qquad \textbf{(9.35)}$$

where $A \equiv \sigma^{2\beta_1}\exp\left[\omega - \ \gamma_1\sqrt{2/\pi}\right]$ with $\alpha_1 < 0$ and $\alpha_1 + \gamma_1 > 0$.

Remarks

1. The EGARCH model allows good news and bad news to have different impacts on volatility, while the standard GARCH model does not.
2. The EGARCH model allows important news to have a greater impact on volatility than GARCH model.
3. EGARCH imposes no constraints on the model parameters to ensure the non-negativity of the conditional variance, unlike the symmetric GARCH model.

NIC for TARCH (1, 1, 1) Model:

$$\sigma_t^2 = (\alpha_1 + \gamma_1 I_{[\varepsilon_t<0]})^2 \varepsilon_{\mathrm{t}}^2 + (2\omega + 2\beta_1\sigma)(\alpha_1 + \gamma_1 I_{[\varepsilon_t<0]})|\varepsilon_{t-1}|. \quad \textbf{(9.36)}$$

NIC FOR GJR-GARCH (1, 1, 1) Model

$$\sigma_t^2 \ = A + \alpha_1\varepsilon_{\mathrm{t-1}}^2,\ \forall\, \varepsilon_{t-1} > 0,$$

$$\sigma_t^2 = A + (\alpha_1 + \ \gamma_1)\varepsilon_{\mathrm{t-1}}^2\,, \forall\, \varepsilon_{t-1} < 0. \quad \textbf{(9.37)}$$

where, $A = \ \omega + \ \beta_1\sigma^2$

Testing for (G) ARCH Effects

Although conditional heteroscedasticity can often be identified by graphical inspection, a quantifying test of conditional homoscedasticity is rather more reliable. The popular standard method to test for ARCH effect is the Lagrange multiplier (LM) test, called the ARCH-LM test of Engle (1982), which is implemented as a regression of *squared* residuals $\hat{\varepsilon}_t^2 = (r_t - \mu)^2$ on lagged squared residuals $\hat{\varepsilon}_{t-i}^2$ and it directly exploits the autoregressive (AR) representation of an ARCH process. The test is computed by estimating:

$$\varepsilon_t^2 = \ \upsilon_1 + \ \upsilon_2\varepsilon_{t-1}^2 + \cdots + \ \upsilon_p\varepsilon_{t-p}^2 + \ u_t. \quad \textbf{(9.38)}$$

We then compute a test statistic as Ttimes the R^2 of the regression: $LM = \ T \ \times \ R^2$, which is asymptotically distributed χ_p^2 where ε_t is the residual in a conditional mean model and T is the total number of observations, with the hypotheses:

$$H_0\!: \upsilon_1 = \upsilon_2 = \cdots = \upsilon_p = 0, \quad \textbf{(9.39)}$$

Versus

H_1: At least one of the parameters is different from zero,

where **(9.39)** corresponds to no persistence/ARCH effect in the conditional variance.

9.2.2 Model selection

Statistical model selection criteria are used to select the orders (p, q) of a GARCH process.

Procedures

1. Fit GARCH(p, q) models with $0 \leq p \leq p_{max}$ and $0 \leq q \leq q_{max}$ for the chosen value of maximal orders.
2. Let $\hat{\sigma}^2\ (p, q)$ be the MLE of $\sigma^2 = Var(\varepsilon_t)$, the variance of GARCH innovations under the Gaussian or Normal assumption for the innovations.
3. Choose (p, q) to maximise one of the following:

- Akaike Information Criterion, **AIC** $(p, q) = \log \tilde{\sigma}^2_{p,q} + 2\left(\frac{p+q}{T}\right)$. **(9.40)**
- Bayesian Information Criterion, **BIC**$(n) = \log \tilde{\sigma}^2_{p,q} + \log\ T\left(\frac{p+q}{T}\right)$. **(9.41)**
- Hannan-Quinn Criterion, **HQ** $(p, q) = \log \tilde{\sigma}^2_{p,q} + 2\log T\left(\frac{p+q}{T}\right)$. **(9.42)**

where T is the total number of observations (or the sample size) and $\tilde{\sigma}^2_{p,q}$ is the maximum likelihood estimate ofσ^2_{ε}, the residual variance. The first term in the equations (**9.40), (9.41)** and **(9.42)** is a measure of the goodness-of-fit of the GARCH (p, q) model to the data, while the second term is the penalty function of the criterion, because each penalizes a candidate model by the number of parameters (e.g. $p + q$) used in the model.

Meanwhile, according to Malmsten and Teräsvirta (2004), a general approach to comparing volatility model is to estimate several models by maximum likelihood and choose the one with the highest log-likelihood (LL) value (see further: Shephard, 1996). However, if the models to be compared have unequal numbers of parameters, one might be willing to favour parsimony by applying a suitable model selection criterion, such as AIC or BIC, for such a purpose. Note that both AIC and BIC are penalized-likelihood criteria, and their only difference in practice is the size of the penalty, but BIC penalizes model complexity more heavily, thereby making it more stringent than AIC. Another method of comparing models is to submit estimated models to misspecification tests (this is beyond the scope of this study) and to see how well they pass the tests.

In this research however, we select the appropriate model by considering the information criterion (IC) with the least value(Zou, 2004) and (Akaike, 1973) especially between **(9.40)** and **(9.41)** having considered other stationarity conditions associated with the respective models.It is important to note that in financial time series, both the AIC and BIC are mostly applied model selection criteria, and in most cases, AIC usually gives the least value, just as it is in the case of this study. For example, going through the results for different models compared using the three criteria, we found that even though the three in each case favoured the same model but the value(s) for AIC was always the least (see Figure 9.2). However, for the purpose of presentation of the models results summary in Table 9.2, we only display AIC.

Tests for Asymmetric Effects

The notes on the news impact curve show that the standard GARCH model has a news impact curve, which is symmetric and centred at ε_{t-1}= 0, indicating that positive and negative returns shocks of the same magnitude generate the same level of volatility. There is a need to explore the presence of asymmetric effects associated with different NICs since the standard GARCH model will not detect such effects of negative (bad) news and positive (good) news on volatility.

Engle and Ng (1993) proposed three diagnostic tests for volatility models: The Sign Bias Test, the Negative Size Bias Test, and the Positive Size Bias Test. The "Sign Bias Test" examines the impact of positive and negative returns shocks on volatility not predicted by the model under consideration. While the negative size bias test focuses on the different effects that large and small negative returns shocks have on volatility, which are not predicted by the volatility model, the positive size bias test deals with the different impacts that large and small positive return shocks may have on volatility that are not explained by the volatility model. In summary, the test is applied to identify possible misspecification of conditional volatility models. This is done by testing whether or not the standardized squared residuals could be predicted using indicator variables.

The joint test statistic for these tests is the Lagrange Multiplier (LM) test statistic, $TR^2 \sim\chi_3^2$ (where T is the total length of the series, R^2 is the coefficient of determination, obtained by estimating the parameters of the regression equation in **9.43** below), under the null hypothesis of no asymmetric effects (Brooks, 2008), which is derived from the multiple regression equation (squared residuals):

$$\hat{z}_t^2 = \ell_0 + \ell_1 S_{t-1}^- + \ell_2 S_{t-1}^- \varepsilon_{t-1} + \ell_3 S_{t-1}^+ \varepsilon_{t-1} + u_t, \qquad \textbf{(9.43)}$$

where z_t is regressed on the constants, $\ell_i, for\ i = 0,1,2,3$, a dummy variable S_{t-1}^-, and an $\mathrm{i.i.d}$ error term, u_t, such that:

$$S_{t-1}^{-} = \begin{cases} 1 & if \quad \hat{\varepsilon}_{t-1} < 0, \\ 0, & otherwise. \end{cases}$$

and $s_{t-1}^{+} = 1 - s_{t-1}^{-}$.

The **sign bias** represented by the dummy variable S_{t-1}^{-} is 1 when $\hat{\varepsilon}_{t-1} < 0$, determines the impact of both negative and positive shocks on volatility not predicted by the model such that when its effect (**sign bias size),** ℓ_1 is present, the following regression is statistically significant:

$$\hat{z}_t^2 = \ell_0 + \ell_1 S_{t-1}^{-} + u_t. \qquad \textbf{(9.44)}$$

Meanwhile, in circumstances when the magnitude or size of the shock impacts on whether the volatility response to shocks is symmetric or not, a negative size bias test is then conducted. A **negative size bias** represented by the term, $S_{t-1}^{-}\varepsilon_{t-1}$ focuses on the effect of large and small negative shocks and it is said to be present if ℓ_2 is statistically significant in the regression:

$$\hat{z}_t^2 = \ell_0 + \ell_2 S_{t-1}^{-}\varepsilon_{t-1} + u_t. \qquad \textbf{(9.45)}$$

Similarly, the **positive sign bias** represented by the dummy, $s_{t-1}^{+} = 1 - s_{t-1}^{-}$, determines the effect of large and small positive shocks and it is said to be present if its effect, $\ell_3$is statistically significant in the equation:

$$\hat{z}_t^2 = \ell_0 + \ell_3 S_{t-1}^{+}\varepsilon_{t-1} + u_t. \qquad \textbf{(9.46)}$$

Goodness-of-fit Test for the Fitted Residuals

Having fitted and chosen the appropriate GARCH candidate model, one of the relevant tests used in assessing the adequacy (or goodness-of-fit) of such a model is to see if the standardized residuals, $\hat{z}_t$, obtained from the GARCH model, are identically and independently Normally distributed ($i.i.d$), and if there is any presence of clustering in the residuals. The common test used for this is a Portmanteau test that examines if several autocorrelations of squared standardized residuals are equal to zero, and the relevant test statistic used is either the Ljung & Box (1978), $Q(p)$ statistic in **equation 4.33** or the Box & Pierce (1970) $Q_*(m)$ statistic in **equation 4.32**.

The underlying assumption is that $\hat{z}_t^2$ are $i.i.d$. The test statistic is compared with $\chi^2_{(p-l)}$(with l as the number of parameters estimated in the model) or the p-value. If the model fits well, then neither the standardized nor squared standardized residuals should exhibit serial correlation.

The Nyblom Test

According to Hansen (1992a, b), the possible challenge with time series models is that the estimated parameters are subject to change over time. When this is left undetected, it may lead to a form of model misspecification, and consequently to false inferences from the model.

To test the stability or constancy of the parameter estimates, we use the Nyblom (1989) test. This test assesses the variance of the errors in the estimated parameters. If the parameter is constant (i.e. there are no errors), then variance of the error term is zero. If it is not constant (and it is related to the past values of the parameter), then the error term has non-zero variance. The test statistic which is derived in Nyblom (1989) and Zivot (2003) as follows:

Consider the linear regression model with k variables

$y_t = x_t^{'}\theta + \eta_t, \ t = 1, 2, \dots, n.$

The time varying parameter (TVP) alternative model assumes that:

$$\theta = \theta_t + \mathcal{E}_t, \qquad \mathcal{E}_{it} \sim N\left(0, \sigma^2_{\mathcal{E}_i}\right), \qquad i = 1, 2, \dots, k$$

The hypotheses of interest are:

H_0: Ψ is constant $\equiv \sigma^2_{\mathcal{E}_t} = 0$, for all i

H_1: $\sigma^2_{\mathcal{E}_t} > 0$, for some i

Nyblom (1989) derives the locally best invariant test as the Lagrange multiplier test. The score assuming Gaussian errors is:

$\sum_{t=1}^{n} x_t \hat{\eta}_t = 0$, where $\hat{\eta}_t = y_t - x_t^{'}\hat{\theta}$, and $\hat{\theta} = (X'X)^{-1}X'Y$

Suppose

$$f_t = x_t \hat{\eta}_t$$

$$S_t = \sum_{j=1}^{t} f_t = cummulative\ sums$$

$$\Lambda = n^{-1}(X'X)$$

Note that

$$\sum_{j=1}^{n} f_t = 0$$

Nyblom derives the Lagrange Multiplier, LM test statistic as:

$L = \frac{1}{n\hat{\sigma}^2} \sum_{t=1}^{n} S_t \Lambda^{-1} S_t,$

$= \frac{1}{n\hat{\sigma}^2} tr\left[\Lambda^{-1} \sum_{t=1}^{n} S_t S_t{}'\right].$ **(9.47)**

Under mild assumptions regarding the behaviour of the covariates, the limiting distribution of L under the null hypothesis follows a Camer-von Mises distribution (Zivot, 2003):

$$L = \int_0^1 \mathbf{H}_k^u(\Psi)\mathbf{H}_k^u(\Psi)' d\Psi$$

$$\mathbf{H}_k^u(\Psi) = \mathbf{W}_k(\Psi) - \Psi\mathbf{W}_k(1)$$

$$\mathbf{W}_k(\Psi) = \ k\ dimensional\ Brownian\ motion$$

Decision: Reject H_0 at 5% if

$L > criticalvalue$ (or p $-$ value $<$ 0.05)

Remarks: •

Distribution of L is non-standard and depends on k.

Critical values are computed by simulation and are given in Nyblom, Hansen (1992) and Hansen (1997)

Test is for constancy of all parameters

Test is not informative about the date or type of structural change

Test is applicable for models estimated by methods other than OLS

Distribution of L is different if x_t is non-stationary (unit root, deterministic trend). See Hansen (1992).

The null hypotheses of stability:

For Individual coefficient tests are set as:

H_0: θ_i $is\ constant, i = 1,2,\ldots,k,$

H_0: $\sigma^2_{\varepsilon_t}$ $is\ constant$, and

For Joint coefficient tests

H_0: θ and σ^2 $are\ constant,$ is accepted if the test statistic does not fall in the rejection region (or $\mathrm{p-value} \geq 0.05$).

The Half-life of a (Shock) Volatility Model

The half-life of a shock, according to Danielson (2011; pp.39-40), is the length of time (n^*) it takes for the conditional variance to revert halfway towards the unconditional variance. It is a measure to determine how long the impact of a shock on the volatility takes to subside.

The general formula for this is:

$$\sigma^2_{t+n^*} - \sigma^2 = \frac{1}{2}(\sigma^2_{t+1} - \sigma^2). \qquad \textbf{(9.48)}$$

For the GARCH (1, 1) process, the half-life is:

$$(\alpha + \beta)^{n^*-1}(\sigma^2_{t+1} - \sigma^2) = \frac{1}{2}(\sigma^2_{t+1} - \sigma^2),$$

and thus

$$n^* = 1 + \frac{\ln(\frac{1}{2})}{ln(\alpha+\beta)}, \qquad \textbf{(9.49)}$$

where $\sigma^2_{t+n^*}$ is the n^* steps ahead conditional volatility, σ^2_{t+1} is the 1-step ahead conditional volatility, σ^2 is the unconditional volatility and $(\alpha + \beta)$ is the measure of persistence in the GARCH (1, 1) model.

9.3 Presentation of Results and Discussions

In this section, the results of the fitted GARCH family models and the relevant tests of model adequacies for the overall and financial crisis periods are presented and discussed.

9.3.1 ARCH Effect Test Results

Table 9.1 below presents the results of the Engle and Breusch-Pagan (B-P) tests across the fifteen banks. From the table, the ARCH effects were significant only in three banks (Diamond, FCMB and Fidelity), using Engle; whereas the effects were significant in eleven banks using Pagan test statistic. Only one bank, UBA, had no detected ARCH effect using either of the test statistics.

Table 9.1: ARCH Effect Test Results

Bank	**Engle LM**	**Breusch-Pagan (B-P) LM**
Access	0.8019(1)	20.7536 (0.000)
Diamond	524.7500 (2.2e-16)	-0.1388 (0.8897)
Eco bank	0.0074 (1)	-67.0785 (0.000)
FCMB	438.4700 (2.2e-16)	-1.6755 (0.094)
Fidelity	443.9500 (2.2e-16)	0.1508 (0.8801)
First	1.8625 (0.9996)	-36.8520 (0.000)
GTB	9.6556 (0.6461)	-30.6169 (0.000)
Skye	0.1261 (1)	28.2494 (0.0000)
STANBIC	0.0140 (1)	-34.4902 (0.000)
Sterling	1.8086 (0.6948)	12.8809 (0.000)
UBA	0.4662 (1)	1.2770 (0.2016)
Union	0.0228 (1)	52.4786 (0.0000)
Unity	0.0178 (1)	83.1671 (0.000)
WEMA	0.0056 (1)	93.7410 (0.000)
Zenith	14.9990 (0.2415)	-21.1845 (0.000)

9. 3.2 Summary of Results for the FITTED Models

In this case two periods were examined- (i) the Overall Period and (ii) the 2007-2009 Financial Crisis Period across the fifteen banks with the sample sizes as presented in **Table 4.1.** Here, column 2 of the table contains the range of periods covered by each bank, while columns 3 and 4 have the sample sizes for the respective bank for the overall and financial crisis periods respectively. The detailed summary statistics tables for the two periods can be found in Tables **5.1** and **5.5** of Chapter five of this thesis.

Summarised Fitted Model Results for the Overall Periods

While fitting the models, all candidate models right from ARCH (1) through to APARCH (2, 2) were with a Normal random error distribution and the student-t error distribution, taking into consideration the skewed forms of the distributions where necessary. **Table 9.2** presents the results of the fitted models with a normal error distribution was considered and **Table 9.3** contains the summary of the final fitted models with a t-error distribution. We see from the three information criteria values that fitting GARCH family models with the assumption of a non-normal error distribution for the Nigerian bank's volatility description for the periods under consideration is more appropriate. This is the general behaviour of the volatility of financial data across different markets (see Tsay, 2012, Taylor, 2011, Danielson, 2011 and Zivot, 2009).

Table 9.2: ARCH/GARCH Models with Student-T-Error Distribution for all the Banks

Bank	Model\Parameters	mu (μ)	omega (ω)	Alpha (α_i)	Beta (β_i)	γ_i ; δ	Shape parameter	AIC	BIC	HQ
Access	APARCH (1,1)	-0.0000 (0.9976)	0.0000 (0.43222)	α_1=0.1862(0.00)	β_1=0.8473(0.00)	γ_1= -0.0998(0.9592) δ=0.3630 (0.00)	3.9261 (0.00)	-8.1900	-8.1787	-8.1860
Diamond	TGARCH (1,1)	- 0.0000(0.9867)	0.0000 (0.9734)	α_1= 0.5826(0.00)	β_1=0.5672(0.00)	γ_1=0.2097(0.000)	3.6845 (0.00)	-7.3100	-7.2954	-7.3047
Eco	APARCH (1,1)	-0.0000 (0.9995)	0.0000 (0.0542)	α_1= 0.2530 (0.00)	β_1=0.7861(0.00)	γ_1=0.1499(0.00); **δ=0.4564(0.000)**	2.6921 (0.00)	-7.7411	-7.7219	-7.7341
FCMB	APARCH (1,1,1)	0.0000 (0.9999)	0.0000 (0.5438)	α_1=0.1440 (0.00)	β_1=0.8616(0.00)	γ_1= 0.0672(0.0018) δ=0.3438(0.00)	3.9696 (0.00)	-8.5594	-8.5429	-8.5534
Fidelity	TGARCH(1,1)	0.0000 (0.976)	0.0000 (0.978)	α_1=0.4354(0.00)	β_1=0.6248(0.00)	γ_1 =0.0990 (0.0007)	3.8707 (0.00)	-8.1600	-8.1454	-8.1547
GTB	APARCH(1, 1)-skew	0.0000 (1.00)	0.0000 (0.2075)	α_1=0.1729(0.00)	β_1=0.8635(0.00)	γ_1 = -0.3440 (0.0); **δ=0.4632**	3.6922 Skew=0.9295	-6.5470	-6.5340	-6.5424
First	APARCH(1,1)-skew	0.0000 (0.999)	0.0000 (0.4176)	α_1= 0.2391(0.00)	β_1=0.7906(0.00)	γ_1 =-0.0895(0.0); **δ=0.5288 (0.00)**	4.1718 (0.00);Skew = 0.9325 (0.00)	-6.4190	-6.4060	-6.4144
Skye	APARCH(2,2, 2)	0.0000 (1.00)	0.0000 (0.07538)	α_1=0.0975(0.0); α_2= 0.0798(0.0)	β_1=0.5259(0.00); β_2=0.3309(0.00)	γ_1 =0.2429 (0.0009); γ_2 =0.2774 (0.0018) **δ=0.4149 (0.0)**	5.3454(0.00)	-8.2640	-8.2385	-8.2547
STANBIC	APARCH (1,2)	0.0000 (0.9998)	0.0000 (0.5323)	α_1=0.1744(0.0)	β_1=0.51805(0.0); β_2=0.3499(0.0)	γ_1 =-0.1874 (0.0009); **δ=0.5109 (0.0)**	3.7918(0.00)	-9.6410	-9.6217	-9.6340
STERLING	TGARCH(1,1)	0.00000 (0.946)	0.0000 (0.9449)	α_1=0.7150 (0.0)	β_1=0.4253(0.0)	γ_1 =0.0652(0.0007)	3.0317(0.00); skew=0.8631 (0.00)	-11.059	- 11.0470	- 11.0550

UBA	APARCH(1,1)	0.0000 (1.00)	0.0000 (0.9929)	α_1=0.1697(0.0)	β_1=0.8557(0.0)	γ_1 =-0.1596(0.00); **δ = 0.3515(0.00)**	4.0555 (0.00)	-5.5093	-5.4980	-5.5053
UNION	TGARCH (1,1)	0.0000 (0.999)	0.0(0.9883)	α_1=0.6101(0.0)	β_1=0.5402(0.0)	γ_1 =-0.1423 (0.00)	2.9917 (0.00)	-7.5056	-7.4958	-7.5021
UNITY	APARCH(1,2)	0.0000 (1.00)	0.0000 (0.0)	α_1=0.2239(0.0);	β_1=0.4846(0.0); β_2=0.3142(0.0)	γ_1 =-0.1101(0.0001); **δ=0.2955(0.00)**	4.0890 (0.00)	-12.7820	-12.7620	-12.7750
WEMA	APARCH(1,2)	0.0000 (0.999)	0.0000 (0.0)	α_1=0.3211(0.0)	β_1=0.6332(0.0); β_2=0.0872(0.0153)	γ_1 =-0.1131(0.0001); **δ=0.4043(0.00)**	3.9827 (0.00)	-13.3180	-13.3050	-12.3130
ZENITH	APARCH(1,2)	0.0000 (0.9988)	0.0000 (0.564)	α_1=0.6493(0.0)	β_1=0.3794(0.0); β_2=0.1438(0.0037)	γ_1 =-0.2713 (0.0420); **δ=0.6908 (0.00)**	2.9003 (0.00)	-6.4477	-6.4292	-6.4410

Table 9.3 gives the summaries of the final fitted models, wherein it can be confirmed that only APARCH and TGARCH models are best models, with the former dominating with eleven banks as against the latter, which is favoured for only four banks. For APARCH, six of them (Access, Eco, FCMB, GTB, First and UBA) are fitted at (1, 1) lag levels; four (STANBIC, Unity, WEMA and Zenith) are fitted at (1, 2) lag levels; and one (Skye bank) is fitted at (2, 2).

Generally, it is not surprising that APARCH, which allows for long-memory, is mostly favoured, because the results on the ACF tests for the absolute returns tests and the plots for most of these banks, obtained in chapter five, had already indicated such behaviour. This implies that the persistence rates of shocks to the future volatilities of the 11 banks with APARCH are longer compared to the other 4 with TGARCH.

Moreover, the suitability of the student-t distribution error model is linked to very high kurtosis observed with the banks' stock returns in chapter five. Another interesting point to note is that while the asymmetric effects of negative shocks only affect seven banks volatility, namely Eco, Skye, Diamond, Fidelity, FCMB, Sterling and Unity banks, the remaining eight with negative values of the leverage parameters are influenced by positive shocks. Further, the effects attributed to all possible sources are different for eleven banks, with Skye bank impacted the most.

Finally, the reasonably high values of the shape parameters across the banks, ranging from approximately 3 to 5 (**see column 8 of Table 9.2**), but with Skye bank having the highest value, indicates that their standardized residuals are fat tailed. And for the three banks, GTB, First and Sterling banks, with the skewed form of the models, the estimated skew parameters are positive and significant.

Table 9.3: Summarised Results on the Fitted Model at Overall level

Bank	**APARCH**		**TGARCH**		**Skewed Form**
	Lag levels	**Leverage Signs**	**Lag levels**	**Signs**	
Access	(1, 1)	Negative	N/A	N/A	No
Diamond	N/A	N/A	(1,1)	Positive	No
Ecobank	(1, 1)	Positive	N/A	N/A	No
First	(1, 1)	Negative	N/A	N/A	Yes
FCMB	(1, 1)	Positive	N/A	N/A	No
Fidelity	N/A	N/A	(1,1)	Positive	No
STANBIC	(1, 2)	Negative	N/A	N/A	No
GTB	(1, 1)	Negative	N/A	N/A	Yes
Skye	(2, 2)	Positive	N/A	N/A	No

Sterling	N/A	N/A	(1,1)	Positive	Yes
UBA	(1, 1)	Negative	N/A	N/A	No
Union	N/A	N/A	(1,1)	Negative	No
Unity	(1, 2)	Positive	N/A	N/A	No
WEMA	(1, 2)	Negative	N/A	N/A	No
Zenith	(1, 2)	Negative	N/A	N/A	No

Figure 9.1below presents the full fitted model for the **Access bank** at the overall level, but the negative and un-significant valueof the leverage parameter (γ_1=-0.09984), at 5% indicates that the bank's volatility responds more to positive shocks than negative shocks overall.

```
> Access.aparch11.t.fit

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model       : apARCH(1,1)
Mean Model        : ARFIMA(0,0,0)
Distribution      : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.000000    0.000000  -0.003033 0.997580
omega   0.000001    0.000001   0.785399 0.432220
alpha1  0.186166    0.003783  49.208178 0.000000
beta1   0.847295    0.003249 260.766421 0.000000
gamma1 -0.099843    0.025401  -3.930690 0.000085
delta   0.363057    0.005725  63.410466 0.000000
shape   3.926067    0.084575  46.421164 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.000000    0.000438 -0.000001 0.999999
omega   0.000001    0.000132  0.005497 0.995614
alpha1  0.186166    0.050288  3.702031 0.000214
beta1   0.847295    0.043874 19.311963 0.000000
gamma1 -0.099843    1.953091 -0.051120 0.959230
delta   0.363057    0.575541  0.630809 0.528166
shape   3.926067   17.956699  0.218641 0.826930

LogLikelihood : 15850.56

Information Criteria
------------------------------------

Akaike       -8.1900
Bayes        -8.1787
Shibata      -8.1900
Hannan-Quinn -8.1860
```

Figure 9.1: APARCH (1, 1, 1) Model for Access Bank at Overall level

Summarised Fitted Model Results for the Financial Crises Periods

Table 9.4 below displays the final fitted model for the fifteen banks during the financial crisis period, which in this research we identified as the period ranging from 2ndJuly 2007 to 30thJune 2009.

Table 9.4: Final Fitted Model Summary for Financial Crisis Periods (2nd July 2007-30th June 2009)

Bank	Model\Parameters	mu ($\boldsymbol{\mu}$)	Omega ($\boldsymbol{\omega}$)	Alpha ($\boldsymbol{\alpha_i}$)	Beta ($\boldsymbol{\beta_i}$)	$\boldsymbol{\gamma_i}$; $\boldsymbol{\delta}$	Shape parameter	AIC	BIC	HQ
Access	GJR-GARCH (1,1)-st	-0.0000 (0.992)	0.0000 (1.000)	α_1= 0.5227 (0.000)	β_1= 0.3648 (0.000)	γ_1= 0.1971 (0.0429)	4.2693 (0.000) Skew = 1.0136 (0.000)	-6.8359	-6.7784	-6.8134
Diamond	GARCH(1,1)-sn	-0.0027 (0.0248)	6.7e-05 (0.0497)	α_1= 0.2708 (0.0003)	β_1=0.6804(0.000)	N/A	Skew = 1.076 (0.000)	-4.0876	-4.0465	-4.0715
Eco	APARCH (1,1)-skew st	-0.0000 (0.9998)	0.3424 (0.000)	α_1= 0.3424 (0.000)	β_1= 0.6690 (0.000)	γ_1 = 0.1360 (0.00063) **$\boldsymbol{\delta}$ = 0.5089 (0.00)**	2.6921 (0.000) Skew = 1.2376 (0.000)	-18.1440	- 18.0780	-18.1180
FCMB	APARCH (1,1,1)	0.0000 (0.9996)	0.0000 (0.9147)	α_1= 0.2474 (0.000)	β_1= 0.7955 (0.000)	γ_1 = 0.2156 (0.00); **$\boldsymbol{\delta}$ = 0.3178 (0.000)**	3.548 (0.000) Skew =1.0131 (0.000)	-7.2478	-7.1821	-7.2210
Fidelity	APARCH(1,1)	0.0000 (0.999)	0.0000 (0.9497)	α_1 = 0.2263 (0.000)	β_1 = 0.7904 (0.000)	γ_1 = 0.2733 (0.0007); **$\boldsymbol{\delta}$ = 0.2179 (0.000)**	3.6925 (0.000)	-9.4046	-9.3470	-9.3820
GTB	GARCH(1, 1)-st	-0.0038 (0.00694)	6.8E-05 (0.0783)	α_1 = 0.3597 (0.000159)	β_1 = 0.6393 (0.000)	N/A	7.3552 (0.000) Skew = 0.9502 (0.000)	-4.1981	-4.1488	-4.1787
First	TGARCH(1,1)	0.0000 (0.995)	0.0000 (0.991)	α_1= 0.8805 (0.000)	β_1 = 0.3695 (0.000)	γ_1 = 0.1270 (0.0053)	3.4048 (0.000)	-5.5757	-5.5264	-5.5564
Skye	APARCH(1,1)	0.0000 (0.99999)	0.0000 (0.46304)	α_1 = 0.3202 (0.000)	β_1= 0.7516 (0.000)	γ_1 = -0.3548 (0.0036); **$\boldsymbol{\delta}$ = 0.3532 (0.000)**	2.9376 (0.000)	-7.4215	-7.3639	-7.3989
STANBIC	TGARCH (1,1)	0.0000 (0.9896)	0.0000 (0.9976)	α_1= 0.9024 (0.000)	β_1= 0.3407 (0.000)	γ_1 = -0.1661 (0.000)	3.0248 (0.000)	-9.1136	-9.0643	-9.0943

STERLING	TGARCH(1,1)-st	0.0000 (0.978)	0.0000 (0.9816)	α_1=0.7950 (0.000)	β_1 = 0.3090 (0.000)	γ_1 = 0.1647 (0.000)	3.0072 (0.00) Skew = 0.8187	-11.5240	-11.4670	-11.5020
UBA	GARCH(1,1)	-0.0022 (0.0268)	8.8E-05 (0.0121)	α_1= 0.5110 (0.000)	β_1=0.4880 (0.000)	N/A	6.6580 (0.000)	-4.3120	-4.2709	-4.2959
UNION	EGARCH (1, 2)	-0.0002 (0.000)	-0.1819 (0.018)	α_1 = 0.1894 (0.0004)	β_1 = 0.99999 (0.000); β_2 = -0.0244 (0.0019)	γ_1 = 0.7403 (0.000)	3.6892 (0.000); Skew = 0.8951 (0.000)	-4.7409	-4.6751	-4.7151
UNITY	GARCH(1,1)-N	-0.0036 (0.021)	0.0001 (0.08399)	α_1 = 0.2529 (0.0034)	β_1 = 0.6760 (0.000)	N/A	N/A	-3.8150	-3.7821	-3.8021
WEMA	APARCH(2,1)	0.0000 (1.000)	0.0000 (0.3218)	α_1 = 0.0492 (0.000); α_2 = 0.0462 (0.000)	β_1 = 0.8838 (0.000)	γ_1 = 0.1715 (0.0208); γ_2 = 0.1587 (0.000) **δ = 0.291 (0.000)**	3.7695 (0.000)	-28.6940	-28.6200	-28.6650
ZENITH	APARCH(1,1)	0.0000 (0.99998)	0.0000 (0.917)	α_1=0.1825(0.000)	β_1=0.8312(0.000)	γ_1 =0.1749(0.000462); **δ=0.2826(0.000)**	3.8812 (0.000)	-7.3935	-7.3359	-7.3709

Apparently, five candidate models- **GARCH, EGARCH, GJR-GARCH, TGARCH** and **APARCH** are favoured by the banks' volatilities as presented in the summary **Tables 9.6 and 9.7** below. While four of the banks, namely- Diamond, Unity, GTB and UBA are fitted with a **standard GARCH** model, the remaining eleven are fitted with an **asymmetric GARCH** model. Specifically, six -Eco, FCMB, Fidelity, Skye, WEMA and Zenith are fitted by **APARCH**; three (3) - First, STANBIC and Sterling use **TGARCH**; Union and Access are respectively fitted with **EGARCH** and **GJR-GARCH** (**see Table 9.5**).

Table 9.5: Fitted Models with their lags and Error Distribution across the Banks during Crisis

GARCH	**EGARCH**	**GJR-GARCH**	**TGARCH**	**APARCH**
Diamond (1,1)-skew-Normal	Union (1, 2)- skew-T	Access (1,1)-skew-T	First (1, 1)-T	Eco (1, 1)-skew-T
Unity (1,1)-Normal			STANBIC (1, 1)-T	FCMB (1, 1)-skew-T
GTB (1, 1)-skew-T			Sterling (1, 1)-skew-T	Fidelity (1, 1)-T
UBA (1, 1)-Normal				WEMA (2, 1)-T
				Skye (1, 1)-T
				Zenith (1, 1)-T
4	**1**	**1**	**3**	**(6)**

Further, while three of the banks- **Diamond, UBA, and Unity** are fitted with Normal error distribution, the remaining twelve follow a student-t- error distribution. For the leverage parameter, out of the eleven that are fitted with asymmetric GARCH model, it is only in eight (**Access, Eco, First, FCMB, Fidelity, Sterling, WEMA and Zenith)** that the negative shocks have a higher impact on their volatilities than positive shocks of the same magnitude, whereas in the remaining three (**STANBIC, Skye and Union),** the impacts of positive shocks on volatility are more than those of negative shocks of equal magnitude. Meanwhile, seven of the models, namely: Access (1.0136), Diamond (1.076), ECO (1.2376), FCMB (1.0131), GTB (0.9502), Union (0.8955) and Sterling (0.8187)[30] accommodate skewness, which reflects the fact that their returns diverge away from a normal distribution (see **Tables 9.4 and 9.6**).

[30]Note that the figures written in front of each of these banks represent the skewness values for the fitted GARCH family models for their respective volatility (see 8th column of Table 9.4 for details)

Table 9.6: Summary of the Fitted Model with Error Distribution, Leverage and Skewness Status

Banks	**Models with Levels**	**Error Distribution**	**Leverage Signs**	**Skewed Form**
Access	GJR-GARCH (1, 1)	Student-T	Positive	Yes
Diamond	GARCH (1, 1)	Normal	N/A	Yes
Ecobank	APARCH (1, 1)	Student-T	Positive	Yes
First	TGARCH (1, 1)	Student-T	Positive	No
FCMB	APARCH (1, 1)	Student-T	Positive	Yes
Fidelity	APARCH (1, 1)	Student-T	Positive	No
STANBIC	TGARCH (1, 1)	Student-T	Negative	No
GTB	GARCH (1, 1)	Student-T	N/A	Yes
Skye	APARCH (1,1)	Student-T	Negative	No
Sterling	TGARCH (1, 1)	Student-T	Positive	Yes
UBA	GARCH (1, 1)	Normal	N/A	No
Union	EGARCH (1, 2)	Student-T	Positive	Yes
Unity	GARCH (1, 1)	Normal	N/A	No
WEMA	APARCH (2, 1)	Student-T	Positive	No
Zenith	APARCH (1, 1)	Student-T	Positive	No

Figure 9.2 below is the full fitted model of GJR-GARCH (1, 1), for Access bank during the financial crisis period, with the positive value of the leverage parameter, gamma (=0.1971) revealing that the model responds to negative shocks more than positive shocks and that it has some degree of skewness.

```
> accessbk.gjrgarch11.st.fit

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : sstd

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu       0.00000    0.000000 -0.009847 0.992143
omega    0.00000    0.000001  0.000000 1.000000
alpha1   0.52268    0.059590  8.771178 0.000000
beta1    0.36479    0.025380 14.372838 0.000000
gamma1   0.19710    0.097328  2.025142 0.042853
skew     1.01364    0.047503 21.338680 0.000000
shape    4.26931    0.319248 13.373007 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu       0.00000  5.1810e-03 -0.000001  1.00000
omega    0.00000  2.0505e-02  0.000000  1.00000
alpha1   0.52268  5.0669e+02  0.001032  0.99918
beta1    0.36479  4.6324e+02  0.000787  0.99937
gamma1   0.19710  4.3425e+01  0.004539  0.99638
skew     1.01364  2.7726e+02  0.003656  0.99708
shape    4.26931  2.8956e+03  0.001474  0.99882

LogLikelihood : 1774.083

Information Criteria
------------------------------------

Akaike       -6.8359
Bayes        -6.7784
Shibata      -6.8363
Hannan-Quinn -6.8134
```

Figure 9.2: Fitted GJR-GARCH Model for Access Bank Returns during the Financial Crisis

Testing the Adequacy of the Models

In this section, tests for: examining the presence of serial correlations, (G)ARCH effects, presence of "Sign bias" in the residuals of the fitted models and the stability of the estimated parameters are presented and briefly discussed for the overall and financial crisis periods considered in this research. This is done to confirm the suitability of the favoured models at both the overall and financial crisis periods.

Testing for the Presence of Serial Correlation Using (Squared) Standardized Residuals

In this case, the results of tests for any left-over serial correlations in the series of each bank after fitting the best models are hereby presented for both the overall and financial crisis periods.

The relevant hypotheses here are:

H_0:No serial correlation in the series in the residuals after the fitting the model

Versus

H_1: There is serial correlation in the residuals after fitting the model.

Having fitted the model, we obtain the residuals as: $\varepsilon_t = r_t - \mu_t$.

The standardized residuals are: $\hat{z}_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}$

where $\hat{\sigma}_t$is the estimated volatility, while $\hat{z}_t$is the standardized residual.

Test statistic: Lung-Box Tests (in **4.33)** on the Standardized and Squared Standardized Residuals.

Decision Rule: Reject H_0 if p-value < 5%

For Access bank at the Overall Level

Figure 9.3 below presents the output of the tests for Access bank at the overall level, covering periods from 2nd June 1999-31stDecember2014, and the fitted model as presented in **Table 9.2** is APARCH (1, 1).

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  3.062e-05  0.9956
Lag[2*(p+q)+(p+q)-1][2] 4.596e-05  0.9999
Lag[4*(p+q)+(p+q)-1][5] 9.203e-05  1.0000
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                         statistic p-value
Lag[1]                    0.001190  0.9725
Lag[2*(p+q)+(p+q)-1][5]   0.003575  1.0000
Lag[4*(p+q)+(p+q)-1][9]   0.005965  1.0000
d.o.f=2
```

Figure 9.3: Standardized Residuals Test Results

Decision: Since none of the p-values at either level of residuals is less than 0.05, we do not reject, H_0; and conclude that there is no further presence of any serial correlation in the series after fitting the model, meaning that the residual is identically and independently distributed. This points to the fact that fitted model is adequate.

For further confirmation, plot c of Figure 9.4 below displays the ACF plot for the squared standardized residuals, wherein none of the points fall outside the 95% confidence limits indicated in red.

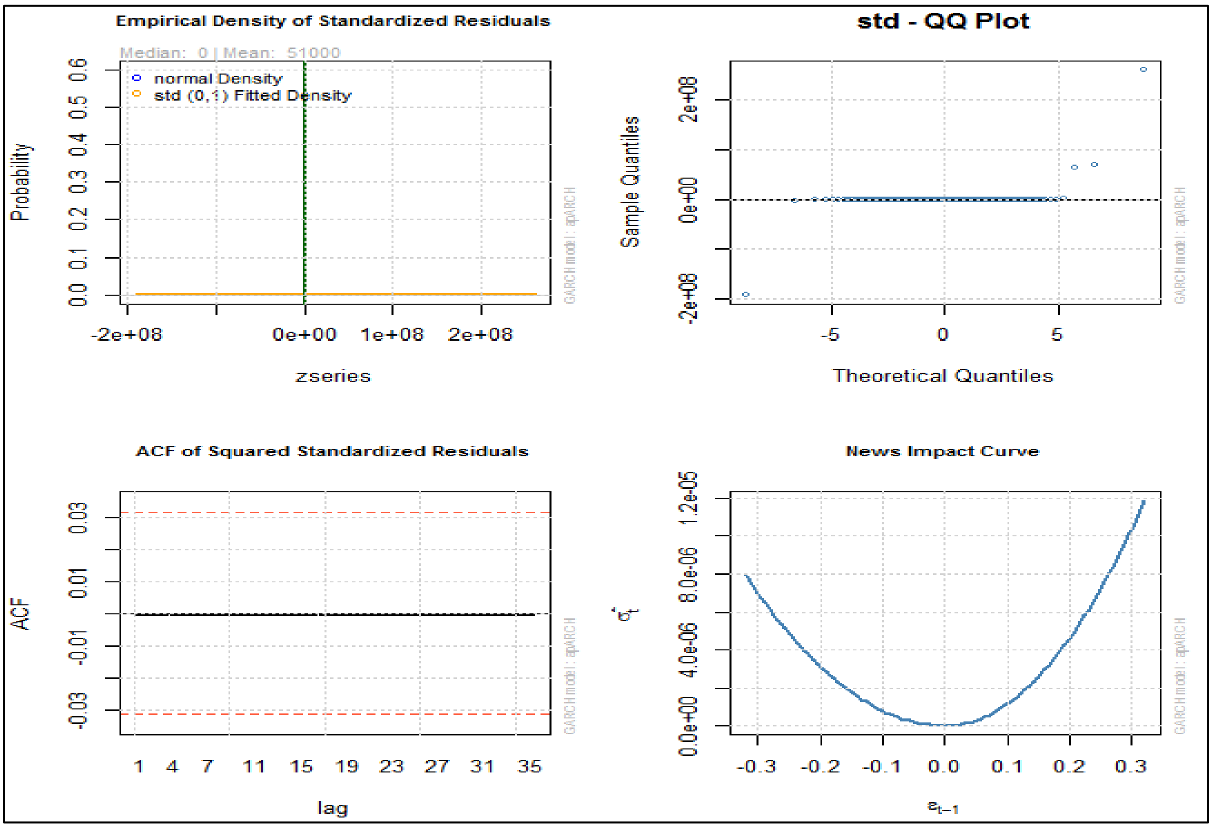

Figure 9.4: Density, ACF's for the Standardized Residuals and NIC for the fitted Model for Access bank

<u>Access bank: Financial Crisis Period</u>

Figure9.5 below displays the adequacy test results for Access bank during the 2007-2009 financial crisis covered in this study, wherein theGJR-GARCH model is favored.

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                  0.0001618  0.9898
Lag[2*(p+q)+(p+q)-1][2] 0.0002433  0.9996
Lag[4*(p+q)+(p+q)-1][5] 0.0004899  1.0000
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.003523  0.9527
Lag[2*(p+q)+(p+q)-1][5]  0.010650  1.0000
Lag[4*(p+q)+(p+q)-1][9]  0.017888  1.0000
d.o.f=2
```

Figure 9.5: Standardized and Standardized Squared Residuals Test Results during the Financial Crisis period for Access bank

The decision in this case is that, since none of the p-values is less than 0.05, we do not reject the null hypothesis; thus we conclude that the model is adequatein this sense at least.

Further, the c portion in **Figure 9.6** below presents the ACF plots for the squared standardized residuals for the model; it can be seen that all the points fall within the 95% confidence limits displayed in red. However, the plot b of both **Figures 9.4 and 9.6**reveals that theresiduals of the fitted models are still fat tailedin the QQ plots. Additionaly, the more points observed on the upper parts of the two plots (Q-Q plots), are further reflection of the fact that the deviation from the conditional normality of the standardized residuals are stronger on the upper parts, this confirms the significance of both the Student-t and the Student-t skew error distribution based models applied. Based on these findings, it suggests that the tail thickness is asymetric, with upper tail thicker than the lower tial for the two cases (see Danielsson, 2011, p. 48-49). Thus, the two plots are consitent with the established fact that the distributions of many high frequency financial time series (such as daily returns) are usually characterised with fatter tails than a normal distribution; meaning, extreme values (or outliers) occur more often than implied by a normal distribution (Zivot, 2009).

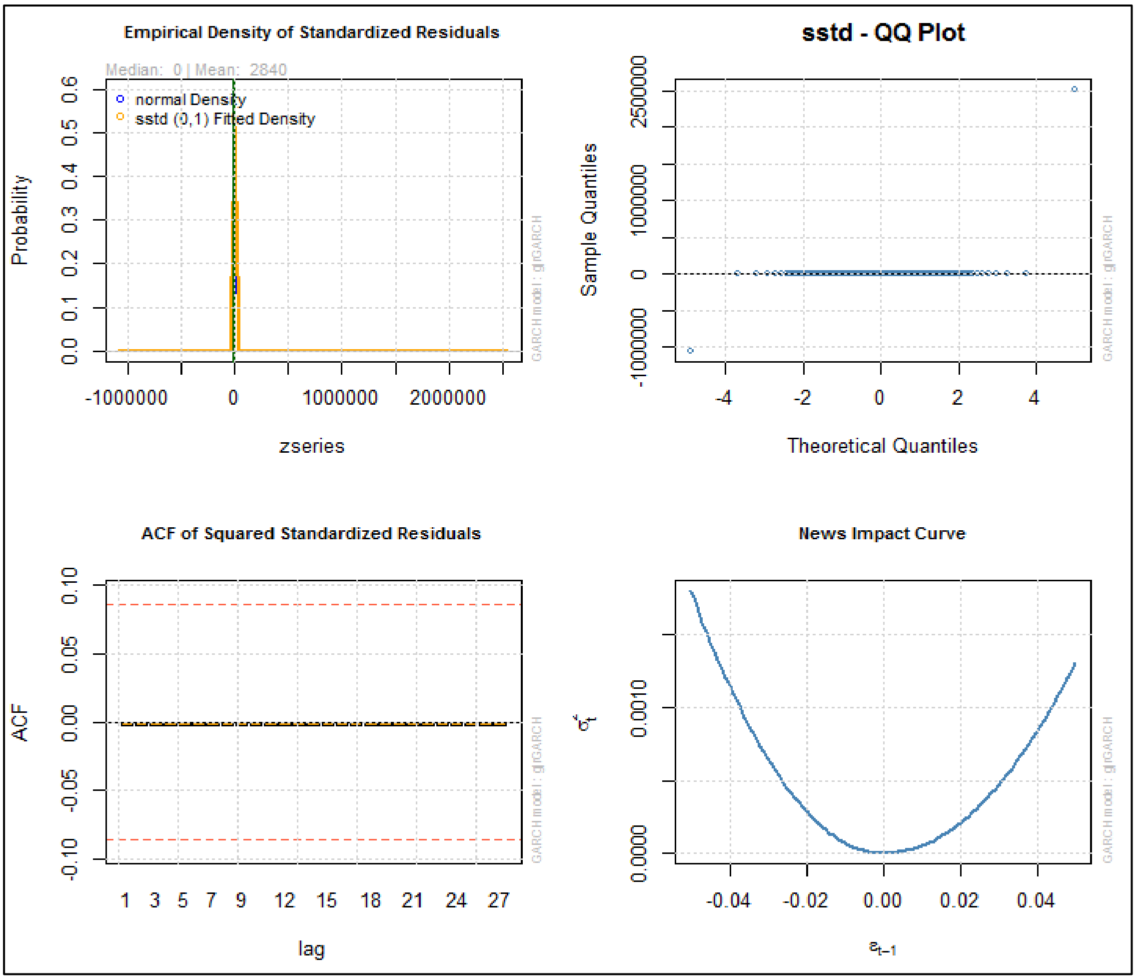

Figure 9.6: Density, ACFs for the Standardized Residuals and NIC for the fitted GJR-GARCH Model during the Crisis for Access bank

Testing for the Presence of Remaining (G) ARCH/ Effects and the Constancy of the Model Parameters

We check to see whether after fitting the model, any (G)ARCH effects are still present.This is achieved using the Engle LM test.Again, the results will be discussed for the two periods examined.

Relevant Hypotheses:

1. For the ARCH test:

   H_0:There is No ARCH Effect in the series after the fitting the model,

   Versus

   H_1: There is an ARCH Effect remaining after fittingthe model

   Decision Rule: Reject the null hypothesis if p-value < 0.05

   Test statistic: Engle-LM.

2. For the Stability of the Estimated Parameters:

H_0:The parameters are constant (jointly and individually)

Versus

H_1: At least one of the parameters is not constant after fitting the model

Decision Rule: Reject the null hypothesis if p-value < 0.05

Test statistic:Nyblom stability statistic.

Access bank's Results at the Overall Level

Figure 9.7 displays the results for Access bank at the overall level, and it can be confirmed that at lags 3, 5 and 7 [31]as presented, none of the p-values (0.9725, 0.9999, 1.000, respectively) is less than 5%. Thus, there is therefore no evidence for the presence of an ARCH effect in the remaining series, indicating that all forms of volatility clustering /heteroscedasticity in the series have been accounted for by the fitted model.

Also for the stability test, as all the p-values are higher than 5% significance level, we do not reject the null hypothesis of constancy of the fitted model parameters either jointly or individually. Implications of not rejecting null hypothesis according to Hansen (1992), Lin and Teräsvirta (1994) and Zivot (2003) means that (1) inference drawn based on the model parameters is relatively accurate and that any decision made based on the model becomes reliable; (2) the out-of-sample predictions should be unbiased, thereby making the forecast errors having approximately zero mean.

[31]Note that the lags: 3, 5 and 7 are arbitrarily and automatically chosen by the R statistical software used for the analysis. These lags are chosen based on the point of convergence of the model.

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[3]   0.001191 0.500 2.000  0.9725
ARCH Lag[5]   0.002859 1.440 1.667  0.9999
ARCH Lag[7]   0.004256 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  511.6708
Individual Statistics:
mu         28.679
omega     143.209
alpha1     44.292
beta1       5.140
gamma1      8.161
delta       3.301
shape      34.897

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:          1.69 1.9 2.35
Individual Statistic:     0.35 0.47 0.75
```

Figure 9.7: ARCH and Stability Test Results for Access Bank at Overall Level

The Financial Crisis Period

In **Figure 9.8**, since none of the p-values (0.9526, 0.9996, and 1.000) for ARCH effect test at lags 3, 5 & 7 is less than 0.05, we do not reject null hypothesis. We conclude that all the ARCH effect in the initial series has been captured by the fitted model.

On the parameters stability, all the p-values, at joint and individual levels are higher than 5% significant level; thus, we reject the null hypothesis of parameter stability, meaning that the model needs to be recalibrated for use in future forecasting of returns.

```
Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[3]   0.003529 0.500 2.000  0.9526
ARCH Lag[5]   0.008497 1.440 1.667  0.9996
ARCH Lag[7]   0.012682 2.315 1.543  1.0000

Nyblom stability test
------------------------------------
Joint Statistic:  82.3708
Individual Statistics:
mu      32.034
omega   21.689
alpha1  13.159
beta1   26.286
gamma1   7.571
skew     4.987
shape    2.258

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75
```
Figure 9.8: ARCH and Stability Test Results for Access Bank during the Financial Crisis

Testing for the Presence of Sign Bias

In this test, we shall be checking to confirm whether the effects of positive and negative shocks are properly captured by the model, such that there is no trace of any difference in the effects due to these two sources of shocks in the remaining series after the model. As we did previously, our presentation shall be based on the two periods of interest in this study across the fifteen banks. Recall that the test procedure as discussed in previous section and presented below was proposed by Engel and Nag (1993).

Relevant Hypothesis:

H_0: There is no Sign Bias Effect (at the Negative, Positive or Joint level),

Versus

H_1: There is evidence of Sign Bias Effect (at the Negative, Positive or Joint level)

Test statistic: Lagrange Multiplier (LM): $TR^2 \sim \chi_3^2$,

where R^2 is obtained from fitting the regression in

$\hat{z}_t^2 = \ell_0 + \ell_1 S_{t-1}^- + \ell_2 S_{t-1}^- \varepsilon_{t-1} + \ell_3 S_{t-1}^+ \varepsilon_{t-1} + u_t$ and T is the series length.

Decision Rule: Reject the null hypothesis if p-value < 0.05.

The Overall Period

Figure 9.9 below presents the output of the fitted model for Access bank at the overall level.

```
Sign Bias Test
------------------------------------
                          t-value   prob sig
Sign Bias               1.004e+00 0.3155
Negative Sign Bias      2.011e-14 1.0000
Positive Sign Bias      1.128e+00 0.2595
Joint Effect            2.390e+00 0.4955


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20      3514            0
2    30      5093            0
3    40      6894            0
4    50      8412            0


Elapsed time : 1.76
```

Figure 9.9: Sign Bias Tests for Access Bank on the fitted APARCH (1, 1) Model at Overall Level

Decision: With the p-values: 0.3155, 1.000, 0.2595 and 0.4955 being all higher than 0.05, we do not reject null hypothesis at the levels of individual or joint tests, indicating that the model has likely accounted for all possible bias effects such that the residuals of the fitted model are identically and independently distributed. This is a sign of a good fit.

Financial Crisis Period

Figure 9.10, which displays the results of the sign bias test for the fitted GJR-GARCH model during the financial crisis period, shows the model is adequate in controlling sign, and indeed negative-positive, effects.

```
Sign Bias Test
------------------------------------
                      t-value   prob sig
Sign Bias           1.197e+00 0.2320
Negative Sign Bias  1.108e-10 1.0000
Positive Sign Bias  1.478e+00 0.1400
Joint Effect        3.238e+00 0.3564


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     359.2    1.297e-64
2    30     489.6    3.941e-85
3    40     555.1    1.773e-92
4    50     663.8   1.102e-108


Elapsed time : 0.8230002
```

Figure 9.10: Sign Bias Tests for Access Bank for the fitted GJR-GARCH (1, 1) Model during the Financial Crisis Period

Distribution of Standardized Residuals

In Figure 9.4, we present empirical distribution and Q-Q plots of the standardized residuals (see **panels 'a' and 'b'** of Figure 9.4 respectively**)** for APARCH (1, 1) model fitted for Access bank at the overall level. The number of points on the straight line reflects that the model's standardized residuals are approximately non-normally distributed (with '4 points' as outliers). This further justifies suitability of a student-t error distribution (which is a non-normal distribution); meaning, it is good for predicting the conditional volatility.

For the periods of the financial crisis, the plots are displayed in panels "**a"& "b" of Figure 9.6,** which also indicates that the standardized residuals from the fitted model are approximately non-normally distributed (with "2 points" as outliers). This also confirms application of a student-t error distribution (which is a non-normal distribution), to capture the clustering in the conditional volatility. It means that the model is good and can likely predict the volatility within the period well.

Discussion on the News Impact Curves

Going by the NIC plot from the APARCH (1, 1) model fitted to the Access bank series at the overall level in panel "d" of **Figure 9.4**, it is obvious that the response to the effects of positive shock on the original series is longer than that of negative shocks. However, in the **panel "d" of Figure 9.6 (that is, the NIC curve),** we can see that the response to the effect of negative shocks dominates that of the positive shocks as indicated by the fitted model (GJR-GARCH(1, 1)) during the financial crisis.

Remarks

It is important to note that our discussions so far, as an example, has been centred on the fitted results for Access bank; for the remaining banks, we see their results in **Appendix 9a and 9b.**

Computation of Persistence Rate, Unconditional Variance and Half-life by Periods

Table 9.7 below presents the computed volatility persistence rates for the fitted models across the 15 banks at both the overall level and during the financial crisis. As the markets experience volatility from time to time, it is expected that the volatility will eventually approach a long run level when the conditional volatility reverts to the unconditional variance. When this happens, the conditional volatility is said to be mean reverted or to have approached stability, and the measure used in controlling the speed of such mean reversion is called persistence.

Table 9.7: Summary Results on the Persistence Rates at the Overall and Financial Crisis Periods

Bank	**OVERALL Level**		**Financial Crisis**	
	Model and the residual distributions	**Persistence**	**Persistence**	**Model and the residual distributions**
Access	APARCH(1, 1)-T	0.9950	0.9867	GJR-GARCH(1,1) skew-T
Diamond	TGARCH(1,1)-T	0.9705	0.9512	GARCH(1, 1)- skew-Normal
Ecobank	APARCH(1, 1)-T	0.9748	0.9238	APARCH (1, 1)-skew-T
First	APARCH(1, 1)-skew T	0.9708	0.9634	APARCH(1, 1)-T
FCMB	APARCH(1, 1)-T	0.9771	**0.9928**	APARCH(1, 1)-skew T
Fidelity	TGARCH(1, 1)-T	0.9303	**0.9818**	APARCH(1, 1)-T
STANBIC	APARCH (1, 2)-T	0.9976	0.9179	TGARCH(1, 1)-T
GTB	APARCH(1, 1)- skew T	0.9928	**0.9953**	GARCH(1, 1)-T
Skye	APARCH (2, 2)-T	0.9975	0.9902	APARCH(1, 1)-T
Sterling	TGARCH(1, 1)-skew T	0.8824	0.8142	TGARCH(1, 1)-skew T

UBA	APARCH(1, 1)- T	0.9913	**0.9990**	GARCH(1, 1)-T
Union	TGARCH(1, 1)- T	0.9280	**0.9755**	GARCH(1, 2)-skew-T
Unity	APARCH(1, 2)- T	0.9829	0.9291	APARCH(1, 1)-Normal
WEMA	APARCH(1, 2)- T	0.9710	0.9616	APARCH(2, 1)-T
Zenith	APARCH(1, 2)- T	0.9428	**0.9812**[32]	APARCH(1, 1)-T

From **Table 9.7**, we see that six banks (**FCMB, Fidelity, GTB, UBA, Union** and **Zenith)** banks' persistent rates have risen higher in the financial crisis compared to what was experienced during the overall period (see **4th columnumn of Table 9.7**). This shows that investment risks were higher for the banks because of the crisis.

Table 9.8 below displays the unconditional variance for the respective banks at the overall level and during the financial crisis. Out of the fifteen banks, ten (**Access, Diamond, Eco, First, FCMB, GTB, Sterling, UBA, Union and Unity bank**), had a higher unconditional variance during the crisis as against the overall period (see **column 4 of Table 9.8**). This is a reflection of the fact that the risks of investments in those banks were higher during the financial crisis.

Table 9.8: Summary Results on the Unconditional Variance at the Overall and Financial Crisis Periods

Bank	**OVERALL Level**		**Financial Crisis**	
	Model	**Unconditional Var.**	**Unconditional Var.**	**Model**
Access	APARCH(1, 1)-T	6.868854e-22	**1.663828e-14**	GJR-GARCH(1,1) skew-T
Diamond	TGARCH(1,1)-T	1.142604e-13	**1.368184e-03**	GARCH(1, 1)- skew-Normal
Ecobank	APARCH(1, 1)-T	3.427144e-19	**5.852198e-17**	APARCH (1, 1)-skew-T
First	APARCH(1, 1)-skew T	3.445528e-18	**7.984797e-14**	APARCH(1, 1)-T
FCMB	APARCH(1, 1)	1.028624e-27	**4.066019e-22**	APARCH(1, 1)-skew T
Fidelity	TGARCH(1, 1)	1.036975e-14	2.982903e-41	APARCH(1, 1)-T
STANBIC	APARCH (1, 2)	1.298849e-15	4.418176e-16	TGARCH(1, 1)-T

[32] The bold face values or estimates (in green colour) indicate the banks where the persistence or where the values increased during financial crisis.

GTB	APARCH(1, 1)-skew T	7.694360e-18	**1.466858e-02**	GARCH(1, 1)-T
Skye	APARCH (2, 2)	2.471256e-17	2.367351e-23	APARCH(1, 1)-T
Sterling	TGARCH(1, 1)-skew T	8.012671e-15	**2.526921e-15**	TGARCH(1, 1)-skew T
UBA	APARCH(1, 1)	3.072200e-23	**8.751997e-02**	GARCH(1, 1)-T
Union	TGARCH(1, 1)	6.093461e-14	**5.934530e-04**	GARCH(1, 2)-skew-T
Unity	APARCH(1, 2)	6.186835e-25	**1.442478e-03**	APARCH(1, 1)-Normal
WEMA	APARCH(1, 2)	6.711079e-20	3.962952e-34	APARCH(2, 1)-T
Zenith	APARCH(1, 2)	1.481989e-15	4.899409e-31	APARCH(1, 1)-T

Table 9.9 below presents half-lives of the fitted volatility models. Recall that the half-life of a volatility shock, which is defined as: $n^* = \frac{\ln(0.5)}{\ln(persistence)}$, and it helps us to measure the average time it takes for the absolute deviation between squared residuals and the long run variance- $|\varepsilon_t^2 - \sigma^2|$ to decrease to half (Zivot, 2009). Then by definition, the closer the persistence to one, the longer the half-life of a volatility shock is.

Table 9.9: Summary Results on the Half-lives at the Overall and Financial Crisis Periods

	OVERALL Level		**Financial Crisis**	
Bank	**Model**	**Half life**	**Half life[33]**	**Model**
Access	APARCH(1, 1)-T	137.2320	51.5917	GJR-GARCH(1,1) skew-T
Diamond	TGARCH(1,1)-T	23.1757	13.8505	GARCH(1, 1)- skew-Normal
Ecobank	APARCH(1, 1)-T	27.1517	8.7428	APARCH (1, 1)-skew-T
First	APARCH(1, 1)-skew T	23.3998	18.5989	APARCH(1, 1)-T
FCMB	APARCH(1, 1)	29.8671	96.4138	APARCH(1, 1)-skew T
Fidelity	TGARCH(1, 1)	9.5930	37.7559	APARCH(1, 1)-T

[33] Cell with yellow color indicates banks where the half-life was the highest during the overall period, green colored cell point to bank was highest half life during the financial crise and red colored cell point to bank with the lowest half lives at both periods.

STANBIC	APARCH (1, 2)	**284.9964**	8.0942	TGARCH(1, 1)-T
GTB	APARCH(1, 1)- skew T	96.4786	148.0095	GARCH(1, 1)-T
Skye	APARCH (2, 2)	273.4625	70.7205	APARCH(1, 1)-T
Sterling	TGARCH(1, 1)-skew T	**5.5420**	**3.3731**	TGARCH(1, 1)-skew T
UBA	APARCH(1, 1)	79.3245	**692.7969**	GARCH(1, 1)-T
Union	TGARCH(1, 1)	9.2715	27.9615	GARCH(1, 2)-skew-T
Unity	APARCH(1, 2)	40.2274	9.4248	APARCH(1, 1)-Normal
WEMA	APARCH(1, 2)	23.5201	17.7170	APARCH(2, 1)-T
Zenith	APARCH(1, 2)	11.7729	36.5677	APARCH(1, 1)-T

Apparently, Sterling bank which accommodates the same candidate model at both levels has the shortest half-lives of approximately six days and three days respectively, both in the overall and financial crisis periods, while STANBIC has the highest half-life of about 285 days at the overall level, and UBA has the highest half-life of about 693 days during the financial crisis.

9.4 Summary and Conclusion

9.4.1 Summary

Recall that this chapter focuses on univariate volatility, and so far, we have succeeded in discussing the need for statistical models, possible causes of time variation in volatility, various types of univariate models and how they are applied and justified our choice for the ARCH/GARCH families adopted in modelling stock volatility of the fifteen Nigerian banks examined. Details on the various GARCH candidate models such as **ARCH, GARCH, EGARCH, GJR-GARCH, TGARCH and APARCH**, applied in this research were explored. Other important concepts touched on include- ARCH effect tests (pre-and-post model fitting), tests of Asymmetry using the sign bias test proposed by Engle and Ng (1993), goodness-of-fit tests by checking for ACFs of the standardized residuals, the Nyblom stability test, News impact curve and possible error distributions.

While presenting the results, we highlighted the procedures that were followed and implemented to generate results on the various tests and models fitted. For the pre-model fitting ARCH effect test, **Engle (1982)-LM and Breusch-Pagan LM** test statistics were applied. Our results revealed that only three banks' stock returns (Diamond, FCMB and Fidelity) were found significant by, Engle-LM, while eleven were identified by the BP statistic and one bank's stock returns (UBA) could not be found to be heteroscedastic by either of the test statistics (see **Table 9.1**).

While fitting the models, two periods, namely the overall and financial periods were focused on, and details on the size of the data for the two periods are presented in **Table 9.2.** For the overall periods, seven banks data run from 2nd June 1999 to 31st December 2014, whereas the remaining eight ran at different periods after the 2004 bank consolidation in Nigeria until 31st December 2014. However, for the financial crisis data across the banks, they all run from 2nd July 2007 to 30th June 2009. **Tables 9.3** and **9.4** provide summary details on the fitted models at the overall level, while **Tables 9.5-9.7** contain detailed summaries on the fitted model for the financial crisis period.

The two GARCH candidate models favoured lags ranging from (1, 1) to (2, 2) at the overall levels are APARCH (11 banks) and TGARCH (4 banks), whereas for the financial crisis periods, five candidate models **GARCH (4 banks), EGARCH (1 bank), GJR-GARCH (1 bank), TGARCH (3 banks) and APARCH (6 banks)**, were favoured; see **Tables 9.4** and **9.6** for details. At the overall level, it was found that the fitted models favoured a student-t error distribution and asymmetric models across the board, with only five banks (Diamond, Eco, Fidelity, Skye and Sterling) having models which responded to negative shocks more than positive shocks (the so-called Leverage Effect). During the financial crisis, however, twelve of the banks were fitted with models favouring a student-t-error distribution, with only three favouring the Gaussian error distribution; 11 of them were fitted with an asymmetric GARCH model and 4 had symmetric GARCH models.

Further, for the financial crisis periods, out of the eleven banks with asymmetric models, eight of them, **Access, Eco, FCMB, Sterling, Union, Fidelity, WEMA** and **Zenith,** responded more to negative shocks than to positive shocks. More specifically, Access bank's volatility, which responded more to positive shocks at the overall level, responded to negative shocks more than the positive shocks during the financial crisis. Also, during the financial crisis, the number of banks whose volatility accommodated skewness increased to seven (Access, Diamond, Eco, FCMB, GTB, Sterling and Union) from three (First, GTB and Sterling), during the overall period (see **Table 9.7**).

All the validation tests conducted show that the fitted models are adequate for both the overall and the financial crisis periods. Persistence rates for the following six banks: FCMB, Fidelity, GTB, UBA, Union and Zenith, were slightly higher during the financial crisis than they were during the overall period (**see column 4 of Table 9.8**). Also, ten banks had their unconditional variances increased during crisis higher than they were at during the overall period (see **Table 9.9** for details).

Lastly, Sterling bank has the least half-life of approximately 6 days and 3 days during the overall and financial crisis periods, respectively, but STANBIC and UBA have the highest half-lives of approximately 285 days and 693 days during the overall and financial crisis periods respectively (see **Table 9.10**).

9.4.2 Conclusion

Given the results and summary made so far, we hereby conclude as follows:

1. Models with a Gaussian error distribution are not appropriate to describe the volatility of the Nigerian banks' stock returns; doing so would certainly lead to underestimating the investment risk in the sector.
2. Asymmetric GARCH family models are largely the appropriate models for modelling Nigerian banks' volatility, at least during the overall period for the periods under investigation.
3. About 67% of the banks responded more to positive shocks than to negative shocks at the overall level. However, during the financial crisis, while about 53% of the banks responded more to negative shocks than positive shocks of equal magnitude, about 20% responded more to positive shocks more than to negative shocks, while about 27% did not respond to either type of the shocks.
4. Five of the banks (Access, Union, FCMB, WEMA and Zenith) that responded to positive shocks during the overall period became exposed to the influence of negative shocks more than positive shocks during the financial crisis.
5. High rates of persistence across the banks indicate predictability, meaning that shocks to any of the banks persist for a reasonably long time.
6. FCMB, Union, Zenith, GTB, Fidelity and STANBIC banks experienced a higher level of shock persistence during the crisis than the overall period.
7. Considering the half-life of each bank in **Table 9.10**, one could conclude that Sterling is the healthiest bank in terms of adjustment to the risk exposure during both periods, whereas

the three riskiest banks at the overall period are respectively: Access, Skye and STANBIC. During the crisis, however, UBA became the most unstable in adjusting to the risk due to the financial crisis.

In summary, the above results show the extent to which the banking sector of the NSM was characterized by the volatility, with implications of the results for comparing the returns and risk profiles of different banks as they are affected differently by shocks in the Nigerian economy during the overall and financial crisis periods. Further, the observation that the crisis in one bank, occasioned by high volatility can easily spread through different means to other banks within the industry, can due to the spill over effects, engulf the NSM and the Nigerian economy, makes the findings in this chapter the core of this research. Our findings reveal how much the 2008 global financial crisis negatively impacted on the stock volatility of the respective Nigerian banks; this we believe is an important observation, especially for risk managers to be able to devise robust trading strategies that would help to either reduce or hedge such likely risks in the future. This is the first study to implement such analysis, which can be extended to other sectors of the NSM in future work. The results also extend those obtained by Omar (2012) at the overall market level. Since volatility is an important parameter in pricing financial derivatives, we believe that the findings of this study could further strengthen the efforts of the management of the NSM to introduce suitable derivative products, aimed at enhancing liquidity in the banking industry, other sectors and entire the stock market. By enhancing liquidity through introductions of other products such as derivatives, options and futures, which are less riskier compared to stocks, (the risk-averse) investors, would have more choices of assets to diversify their investments; enhance they are better able to manage the risks involved in investing in a single and highly risky security asset such as stocks. Further, with the knowledge of how much volatility involved, the impact of news (either negative or positive), the level of volatility persistence and the length of time it takes for the shocks to die off, the investors would be able to make a well-informed investment decisions by timing when to invest, proportion of their funds to invest in an asset and to which product, sector, firm or bank to diversify their investments.

10 CHAPTER TEN: Main Results-Summary, Interpretation and Further Studies

10.1 Introduction

This chapter summarises the main results of the thesis, concludes the thesis and makes suggestions for further work. As stated in chapter one, this study aimed to analyse the behaviour of Nigerian banks' stock returns and related market characteristics in the Nigerian Stock Market (NSM). To achieve this, five concepts were investigated, namely: stylized facts of asset returns, stock market efficiency, anomalies, bubbles and volatility.

The research is an extension of Omar (2012), which focused on the overall market using the All Share Index (ASI), while here we concentrate on the banking sector of the NSM. Specifically, out of about twenty-five banks operating in Nigeria, sixteen were studied, these being among the top leading banks in the NSM. The data used were individual banks' stock returns, generated from the daily closing price of each of the banks within the periods of June 1999 to December 2014 and encompassing various periods of banking consolidation (April 2004-December 2005), global financial crisis (June 2007-June 2009) and post financial crisis banking reform (July 2009-December 2014).

Understanding the key empirical characteristics of these banks, especially considering the impacts of banking reforms and the global financial crisis will be useful for investment decisions by investment managers, market makers and investors; and for enhanced growth of the exchange market and financial policy decisions by the regulatory agencies. We further believe that the research approach could inform the examination of other sectors of the NSM.

As this is the first time such investigation was undertaken in the NSM, which is among the leading markets amongst the African emerging markets, it is hoped that our findings will help to deepen our understanding of NSM and at the same time provide useful insights for development of the NSM and other emerging African markets.

10.2 Stylized Facts of Asset Returns

The analyses of stylised facts of the banks' stock prices and returns in Chapter 5 (Section 5.3) indicate the presence of stylised facts of asset prices commonly found in financial markets across the sixteen banks, though in a few cases some of these characteristics are absent.

In chapter 5, the stylized (or statistical) facts on the empirical behaviour of asset returns of the Nigerian banks, which are common to a large set of assets and different markets, are presented. Amongst the investigated properties are included: the kurtosis and skewness of the empirical distribution of daily returns, the lack of linear autocorrelation of returns, the presence of significant and slowly decaying autocorrelations of absolute and squared returns, and the leverage and Taylor effects. These are some of the fundamental facts or characteristics a true model describing the behaviour of stock prices should possess because they are consistent with the efficient market hypothesis (Thompson, 2011). The lack of autocorrelation between daily returns is in fact what is expected if all publicly available information is fully and instantaneously incorporated into stock prices (Fama, 1965). When this fails through slow response of the prices, then positive autocorrelation of returns is expected or an over-reaction to the information, negative autocorrelation occurs (Thompson, 2011).

The significant autocorrelation of absolute and squared returns indicates violation of the Random Walk hypothesis and the presence of volatility clustering, which according to Mandelbrot (1963), indicates that large (small) absolute returns may tend to follow large (small) absolute returns. Measures of central tendency (arithmetic mean and median) are very close to zero. Thus, corresponding to the standard assumption of the Random Walk model that the expected value of daily returns equals zero is met (see: Taylor, 2005; Miljkovic & Radovic, 2006 and Raheem and Ezepue, 2018).

On the distribution of asset returns, this study only considered the four moments - the mean, variance, skewness and kurtosis, - commonly used to determine the distribution of most series. In doing this, however, we only compare these metrics with that of a normal distribution to conclude that the distribution of the respective banks' returns is non-normal distribution. The limitation is not examining the exact functional form of distribution suitable for describing the returns; this is an area of future study. Empirical distribution functions of most of the banks' returns have a higher peak and longer tails than normal density; the results obtained using both the theoretical tests and appropriate plots. This behaviour commonly characterises daily stock returns of assets across different markets of both the emerging and developed economy (see Tsay, 2014, page 27).

For the leptokurtic behaviour of asset returns, recall that for this to happen, the excess kurtosis (kurtosis-3) of the empirical distribution of an asset return should be higher than that of the normal distribution, which is 0, indicating the distribution is more peaked at the centre than a normal around the mean. Also, such a distribution should have fatter tails than predicted by the normal law. According to Cont (2001), positive values of excess kurtosis indicates a fat tail, meaning, a slow asymptotic decay of the distribution with a sharp peak and heavy tails; this behaviour is known to characterise high frequency financial data such as intraday and daily data. In this study, we confirmed that the majority of the banks possess these characteristics at the overall level and during the financial crisis. This empirical fact was long identified by Mandelbrot (1963). Thereafter, wide ranges of non-normal distributions were suggested but without consensus on actual distribution of the tails for asset returns across different markets (Chakraborti et al., 2011a).

We observe that our findings are in line with those of Tsay (2005, pp. 51-93), regarding the three stylized facts of the daily asset returns: (1.) that the distribution of returns is approximately symmetric with higher kurtosis, fatter tails and more peakedness at the centre than the normal distribution; (2.) That the autocorrelation of the returns is close to zero and (3.) that the autocorrelations of both the absolute and squared returns are positive for many lags except in the case of the skewness where we identified 8 and 5 banks to be positively and negatively skewed respectively for the overall period.

An important observation to be noted is that according to Thompson (2011), techniques used in investigating the stylised facts of stock return series by several studies are apparently quite unsophisticated. Sample autocorrelation and kurtosis estimates in most cases appear to have been satisfactory. Finally, unlike this study where individual returns series of the banks were used, most studies on the developed markets of the US, UK, Japan and many emerging markets utilized market indices to examine the presence of these facts. Studies utilising returns series to investigate these stylised facts that for individual stocks, especially illiquid stocks, listed on such smaller markets are apparently, scant (Thompson, 2011).

In summary, virtually all the stylized facts discussed in Section **3.5** are found in the Nigerian banks' returns. The financial crisis impacted negatively on at least 50% of the banks, and this is confirmed by the negative skewness for 8 out of the 16 banks. There is evidence for presence of a "**Taylor effect**" across the banks; and a "**Leverage effect**" in some banks' returns.

10.3 Market Efficiency

In **Chapter 6**, three concepts of stock returns' properties, focusing on investigating the market efficiency of the banking sector in the NSM, were discussed. The concepts are **stationarity**,

random walks (RW) and the **efficient market hypothesis (EMH)**. Under these concepts, we briefly examined strict and weak stationarity and unit root non-stationarity. Further discussion was focused on random walk theory.

We discussed the Efficient market hypothesis (EMH) and different categories of innovations in a mean returns equation, namely Martingale differences, White noise and independently and identically distributed (iid) innovations. Tests related to the EMH were also discussed. Various forms of market efficiency- strong, semi-strong and weak - were also discussed. Thereafter the relevant test statistics used in examining weak-form market efficiency as found in the literature were surveyed. Finally, tests such as: the variance ratio, BDS, runs, ADF, KPSS and PP tests that were found to be relevant to our objectives were discussed and applied.

From our findings, all the banks are unit root stationary, except for Afribank; and virtually all the banks contradict the RWH based on the test statistics applied. Apparently, there is evidence against randomness and linear dependence across the 16 banks. This behaviour has been found to be common to daily returns (see Tsay, 2005). It is important to state that examining if a market is information efficient especially focusing on weak-form efficiency requires mainly empirical analysis (Muragu, 1990). This test is important because (1.) there is a need to investigate the axiom that emerging markets are generally weak-form inefficient; (2.) there is currently no substantial evidence to support the presence of the weak-form hypothesis across different markets.

Comparing the findings of this study to others cited in chapter 3, we found that our results on the random walk hypothesis are consistent with those of Sharma and Kennedy (1977), Lo and McKinley (1988), across some developed markets and Mobarek and Keasey (2000) for daily data in the Dhaka market. Further, our results on the serial correlation are not significantly different from the results of study by Solnik (1973) on European 277 Stock Exchanges. While Mikailu and Sanda (2007) and Olowe (1999) agree on weak form efficiency of the Nigerian market, Omar (2012), Omar and Ezepue (2012) and this study's findings contradict weak-form market efficiency.

Thus, finding that the Nigerian banks are **weak-form inefficient** conforms to the findings of Omar (2012) which used the All Share Index (ASI) at the overall market level. This confirms that the observed weak-form market inefficiency at the overall NSM market level applies to the banking sector, using the individual return series for the sixteen banks adopted in this research.

It is important to state that of all the studies investigated so far, those that are sectoral as ours is very scant, similar to the fact that virtually all of these studies used a returns index as against the individual returns considered in this study.

10.4 Anomalies

In chapter 7, we defined market anomalies, briefly discussed their attributes and types, with more emphasis on the time series/calendar anomalies as the focus of our research. Implications of the anomalies for stock investment strategies were highlighted. We stated the anomalies considered in this research considered to be important for the NSM and reviewed related literature and methodologies for investigating these anomalies.

Specifically, we looked at the following anomalies: Day-of-the-week/Monday effect, Holiday/January effect, Seasonality effect, with a focus on the Oct-March effect, Turn-of-the-year effect, Month-of-the-year effect and the Yearly average returns obtained.

Our findings reveal that, **56%** of the Nigerian banks were characterised with negative average returns on Monday; holiday and January effects are found in some and absent in others. The October-March effect does not impact on any of the banks returns; while a turn-of-the year effect was present in only one of the banks examined, the turn of the month effects impacts only 50% of the banks. The month of August was when majority of the banks witnessed the lowest average returns.
With these findings, we have identified the key anomalies exhibited by Nigerian banks in the NSM. Investors (either local or foreign), with a good understanding of these anomalies would properly be guided on favourable investment decisions based on their dispositions to risks. However, it is important to note that since each country has its own anomalies, investors aiming at diversification would require better understanding of the anomalies in different markets and market sectors. For example, the above anomalies are exclusive to the Nigerian banking industry and are applicable neither to any other sector nor to any other market/economy.

10.5 Speculative Bubbles

In Chapter 8, the definition of a bubble, its characteristics, and consequences were provided. Reviews of some relevant literature on models for analysing rational speculative bubbles and were presented. We discussed our objectives and motivations for adopting the duration dependent model and unit root tests in this research.

While presenting our results, we first obtained the summary statistics (on the mean, standard deviation, skewness, and kurtosis, including the results of the ACF tests), for the price series of the respective banks, particularly for the suspected "bubble regions" for each bank. We observed that five banks experienced two suspected episodes of bubbles based on the bubble characteristics highlighted (see **Table 8.3**). For the specific summary statistics, we compared their values in the bubble periods to their respective overall values presented in Chapter 5 and obtained percentages of how they compare to the overall values (see **Tables 8.4 and 8.5**).

With the results found, we suspected the presence of bubbles based on the bubble attributes that have been discussed in previous studies. However, since some of these attributes can also be a signal for other stylized facts of stock returns, we performed unit root tests using the **ADF** and **PP** test statistics as the popular tests for identifying presence of unit root against the explosive alternative (which is an indication for bubble presence). The results of the tests are presented in Table **8.6**.

The results revealed the following:

a. Bubbles are present in some of the Nigerian banks.
b. Virtually all the banks found to be characterised with explosiveness under unit root tests were also identified under the Cox proportional hazards model.
c. Duration dependent methods are sensitive to data periods - be it days, weeks or months given that the results from different cases of duration dependence models produced varying results, such that a different number of banks were identified as having bubbles in their returns.

The outcomes of investigating the presence of a bubble is currently and largely dependent on the method being adopted in the data analysis and for the duration dependent models, the units of the "Time" variable largely determines the result.

Finally, it is important to state that, depending on the method used, the number of banks identified in having bubble varies, but largely less than 50% of the investigated banks could be said to be characterised by bubbles considering all the methods.

Also, the use of the Cox proportional hazards model as against the use of the famous log-logistic method proposed by McQueen and Thorley (1994) is premised on the length of the data set (i.e. **daily data**), we have (as against the weekly, monthly and yearly data used in previous studies) and on the suggestion to use a slightly different method from Omar (2012), because of his inability to identify bubbles using the log-logistic method by McQueen and Thorley (1994).

The presence of bubbles in some of the banks could largely be said to be consistent with some of the results obtained in other emerging markets listed in **Table 8.1.** Also, the lack of bubbles in other banks is consistent with some of the results obtained from some African markets such as: Olowe (1999), Okpara (2010) and Omar (2012).

10.6 Volatility

Chapter 9 focused on univariate volatility, and we briefly discussed the following:

The need for statistical models; the importance of Volatility in Financial Markets; the various types of univariate volatility models and how they are applied; the motivation for the choice of the ARCH/GARCH families applied in modelling stock volatility of the Nigerian banks examined; and the details of the various GARCH candidate models such as **ARCH, GARCH, EGARCH, GJR-GARCH, TGARCH and APARCH** applied in this research.

Other important concepts reviewed include ARCH effect tests (pre-and-post model fitting), tests of asymmetry using the sign bias tests proposed by Engle and Ng (1993), goodness-of-fit tests that check the ACFs of the standardized residuals, the Nyblom stability test, the News impact curve, and possible error distributions.
The outcomes of this study show that different ARCH/GARCH models were fitted to the volatility of each of the 16 banks for the overall and financial crisis periods.

The events in the global financial market had significant impacts on the stock returns of these banks; thereby leading to a significant increase in the asymmetric effect of negative news on the models. Fitting of asymmetric GARCH models is consistent with previous research by Poon and Granger (2003), where it was observed that asymmetric models outperform their symmetric counterparts. This simply indicates that the banks' reaction to negative news is apparently different from their reaction to positive news, and that negative shocks tend to hit the banks harder, thereby creating more uncertainty and stronger fluctuations in their returns' volatility. The use of (skewed) Student-t error distribution-based models generally provided an improved fit compared to the Normal error distribution across the 16 banks and the two periods. This could be attributed to excess kurtosis and skewness which are known to characterise financial time series data, which allow for more observations in the tails of their distributions. The high values of persistence (close to 1), and half-lives reveal that the shocks to the volatilities across the banks die off slowly, and show the length of time it took for such shocks to persist with the banks. **Table 10.1** below presents a general summary of the findings made with respect to all the issues investigated.

Table 10.1: General Summary of the Results

OVERALL Period							
Daily Data							
Price: Stationarity	**Returns: Stationarity**	**Leptokurtosis**	**Skewness**	**Non-Normality**	**No Autocorrelation at Lag 1**	**Leverage Effect's sign**	**Long-range Dependence**
None	All	YES-All	8 - "+"; 5 - "-"; 3 - symmetric	All	2-Yes; 14-No	5-Yes; 11-No	All
Monthly Data							
None	N/A	YES-All	6 -"+"; 4: "-"; 6: "symmetric"	15-Yes; 1:No	N/A	N/A	N/A
Financial Crisis Period Data							
None	N/A	9-leptokurtic; 5-platykurtic; 2-mesokurtic	8:"-"; 8:"symmetric"	14-Yes; 2:No	N/A	8:Yes; 8:No	N/A
Efficiency/ Random Walk (RW)Tests							
RUNS Test	**ADF Test**	**BDS Test**		**Variance ratio Test**		**ACF Test of Independence**	
None is Random	None is a RW	All rejected RW		**12**: No to RW; **4**: Yes, to RW (**Eco, STANBIC, Union and WEMA)**		14-No; 2: Yes (**Eco and Unity banks)**	
Anomalies							

Day-of-the-Week Effect	**Turn-of-the-Year Effects**	**Oct-March Seasonality Effect**	**January Effect**	**Holiday Effect**	Month-of-the-year Effects
Monday was lowest and negative for 7; Friday was highest in 5	Yes in **15** banks; No in 1 (**Eco**)	"No" across the banks	Yes in 8 banks	Yes in 9 banks	**7** banks were lowest in **August**;**7** banks were Highest in **May**

Yearly Average
11 banks average returns were lowest in **2008**; **3** (**Afibank, Diamond and Skye**) lowest in **2011** and 2 (**Unity and WEMA)** lowest in **2009**

Stock Bubbles

Bubble Characteristics	**Unit Root Explosiveness**	**Case 1**		**Case 2**	**Case 3**	**Case 4**
7: Afribank, First, Sterling, UBA, Union, WEMA and Zenith	**9-**Access, Eco bank, FCMB, Fidelity, GTB, Skye, UBA, WEMA and Zenith	**7-**Afribank, FCMB, Skye, STANBIC, UBA, WEMA and Zenith		**6-**Access, Skye, Sterling, UBA, WEMA and Zenith	**4**: Afribank, UBA, WEMA and Zenith	**5-** Afribank, FCMB, UBA, WEMA and Zenith

Stock Volatility

Fitted Models: Overall Period

APARCH	**TGARCH**
11 banks: Access, Ecobank, First, FCMB, STANBIC, GTB, Skye, UBA, Unity, WEMA and Zenith	**4 Banks**: Diamond, Fidelity, Sterling and Union

Fitted Models: Financial Crisis Period

GARCH	**EGARCH**	**GJR-GARCH**	**TGARCH**	**APARCH**
4 Banks: Diamond, Unity, GTB and UBA	**1**: Union	1: Access	**3**: First, STANBIC and Sterling	**6**: Eco, FCMB, Fidelity, WEMA, Skye and Zenith

Leverage Effects

Overall		**Financial Crisis**	
Positive Effect	**Negative Effect**	**Positive**	**Negative**
8 banks: Access, First, STANBIC, GTB, UBA, Unity, Union, WEMA and Zenith	**7**: ECO, Skye, Diamond, Sterling, Fidelity, FCMB and Unity	**3**: banks: STANBIC Union and Skye	**8** Banks: Access, ECO, First, FCMB, Fidelity, Sterling, , WEMA and Zenith

				No Leverage-4 banks: Diamond, Unity, GTB and UBA

News Impact Curve-NIC			
Overall		**Financial crisis**	
Positive Domination	Negative Domination	Positive Domination	Negative. Domination
8: Access, GTB, First, STANBIC, UBA, Union, WEMA and Zenith	**7**: Diamond, Eco, FCMB, Fidelity, Skye, Sterling, Unity	**3**: Skye, STANBIC, Union	8: Access, ECO, FCMB, Fidelity, First, Sterling, WEMA and Zenith
Half life			
Overall		**Financial Crisis**	
Highest: **STANBIC** **(**Approximately 285 days**)**	Lowest: **Sterling** **(**Approximately 6 days**)**	Highest: **UBA** **(**Approximately 693 days**)**	Lowest: **Sterling** **(**Approximately 4 days**)**
Error Distribution			
Overall		**Financial Crisis**	
Normal: **None**	Student-T: **All**	Normal: 2**- UBA and Unity**	Student-T: **13 banks**
Skewed Model			
Overall: **3-** First, GTB and Sterling		**Fin. Crisis**:**7-** Access, Diamond, ECO, FCMB, GTB, Sterling and Union	

10.7 Interpretations and Implications of the Research results on the Nigerian Financial System

Many issues related to the systematic characterization of the Nigerian banks' stock prices and returns behaviour are briefly discussed in this section. Specifically, the connections between the fundamental results of the various analyses and the Nigerian financial system are highlighted.

Non-stationarity in price is a general characteristic of financial security prices, and the observed stationarity around a common level in the log returns across the banks is a fundamental stylized fact of asset returns in different global markets. Hence, the bank returns exhibit typical stylised facts of returns in global financial markets.

The observed leptokurtosis across the banks at the overall level for daily returns is in line with the stylized fact of an asset return in different markets. This behaviour implies that small changes occur less frequently because the returns are clustered around the mean, with fat tails showing that large fluctuations in returns are more probable than returns whose distributions have lower kurtosis. Thus, such assets having high kurtosis with fat tails are more prone to crashes than those that are either mesokurtic or platykurtic (thin tailed).

That thirteen of the banks (see **Table 5.14**) are highly leptokurtic with fat tails in the overall period indicates the vulnerability or fragility of the banks returns in the event of market instability. Although the fragility was reflected to some degree in the financial crisis, with nine banks remaining leptokurtic and seven being platykurtic, other factors might have also been responsible for the level of vulnerability of the banks within the periods of our investigation.

Also, having eight banks that have positively skewed returns shows that extreme gains are more probable than losses in their returns; whereas with the five banks that are negatively skewed, extreme losses are more likely to characterise their returns than are gains in the overall period. However, during the financial crisis, half (8) of the banks showed negative skewness, thus, revealing that the crisis negatively impacted on the banks. This might be due to the asymmetric effects of the negative shocks of the financial crisis on the banks' returns.

The fact that 14 of the banks' returns, constituting about 88% of the investigated banks, are auto correlated at lag 1 is a violation of the common stylized fact of no autocorrelation in asset returns, and points to a lack of efficiency in the industry/market. Also, according to Danielson (2001), the presence of serial correlation at lag 1 indicates that the banks were bearish within the periods of investigation.

The presence of the long-range property due to a high rate of persistence observed in the autocorrelation functions of absolute returns of most of the banks is an indication of a prolonged effect of any type of shock on the asset returns of the banks. This behaviour was reflected by the selection of APARCH as the most favoured fitted GARCH volatility model, both for the overall period and during the crisis.

Lack of market efficiency across the banks reflects many challenges, which include: a lack of liquidity, arbitrageurs, and information asymmetry pervading the NSE within the periods of the research (Oteh, 2010; Sanusi, 2011; Omar, 2012; Ezepue and Omar, 2012). These challenges necessitated the introduction of a series of financial and economic reforms into the Nigerian financial system, which were aimed at repositioning both the capital market and the banking industry, both prior to and post the 2008-2009 financial crisis. It is important to state that these kinds challenges are typical of most emerging markets, especially African emerging markets (see Bekaert, Erb, Harvey and Vyskanta (1998); Alagidede and Panagiotidis (2009)), as opposed to developed markets, which are more stable and populated by matured and rational investors.

The presence of Turn-of-the-year, January and Holiday effect anomalies in more than half of the banks could be attributed to the priority that Nigerian banks and Nigerians in general attach to investment in Januarys and days preceding any holiday in the country. Indeed, the choice of January for higher investment could be linked to the fact that, until recently, the month has over the years been the month when Nigeria's yearly budget is read by the federal government. Also, a day preceding any general holiday in the country is often a day with higher volume of trading activities in Nigeria.

Further, that 13 banks (constituting about 82% of the investigated banks) have their lowest average returns between 2008 and 2009 is a confirmation of the negative effects of the 2008-2009 financial crisis, which were made worse by the inadequacies identified with the banks and the NSE outlined in Chapter 2. It is also a justification for the reforms introduced into the Nigerian financial system by both the CBN and SEC post the financial crisis between 2009 and 2011 (see Oteh, 2010 and Sanusi, 2011).

Additionally, the identification of bubbles in the stocks of the banks, especially **UBA**, **WEMA** and **Zenith** may possibly be due to over-subscriptions (or over-valuation) in them, as they are among the most popular banks, with more capital and a wider customer base than other banks in the country. They are among the choice banks by the "top notch" investors and among the leading banks on the gainers table in the NSM. There are few banks with large market capitalization (see the Appendix 2.1 for additional information on the banks). Detection of rational speculative bubbles in the Nigerian banking industry could be attributed to the lack of adequate market information and the information asymmetry charactering the Nigerian market. Thus, relevant regulatory institutions, particularly the SEC should ensure full disclosure of relevant information relating to the value of the proposed traded assets and financial performance of the respective banks to investors, in accordance to the international financial reporting standards (IFRS) procedures. Adherence to capital adequacy regulations and enhanced cooperate governance should be mandated in the NSM.

On the level of volatility, the fact that 11 banks, constituting about 74% of the 15 banks that were modelled, were fitted with a long-memory GARCH model (APARCH), during the overall period reveals persistence in the risks incurred by the investors on their investment within the periods of the research, which further exposed the depth of losses experienced in the NSM within those periods. Clearly, from **Table 10.1** above, all the banks were fitted with asymmetric GARCH models for the overall period, with **8** and **7** banks characterised by positive and negative shocks respectively. During the financial crisis, 11 were fitted with asymmetric GARCH models, while only four banks accommodated standard (symmetric) GARCH models, with 8 and 3 banks characterised by negative and positive shocks respectively. Knowledge of the different volatility characteristics of the banks is useful for investors' understanding of the investment risks facing the banks.

In light of the above, control measures and effective reforms aimed at addressing various challenges such as: lack of depth and sophistication, sharp practices, insider trading, inadequate information, lack of disclosure, and weak regulations oversight (as highlighted in Oteh, 2010 and Sanusi, 2011), that exposed the Nigerian market to different risks investigated in this research should be initiated by the CBN, SEC and NSE management. There is a need for both financial and regulatory policies that will promote the efficient transmission of useful market information within the Nigerian financial system so that development and stability of the economy are maintained.

10.8 Suggestions for Future Studies:

Since this research only concentrates on establishing that the distribution of asset returns is non-normal, the exact non-normal probability distribution for each of the banks' asset returns remains a subject for future research. Further areas of future investigation could be in the determination of the tail index of each of the returns distributions using Hill's method as proposed by Hill (1975) and further applied by Cont. (2010); Hsing, (1991); Resnick and Stărică (1998) given the fact that this research only concentrated on determining if each bank's returns series is either long tailed or not.

Further, trading rules such as: the moving average rule, the channel rule, filter rule and statistical rule proposed by Taylor (2005) could also be explored to identify trends in stock prices of the respective banks to see if these could be exploited by investors to make profitable investment decisions. Further work could also focus on determining the impacts of the calendar anomalies as discussed in this research on the volatility by including the effects of the various anomalies in the volatility equation. Other duration dependent bubble models such as: Log-logistic regression of McQueen and Thorley (1994) with the Weibull distribution could be applied, the results of which could be compared with the Cox-model which is applied in this study.

Further future work may be to apply FIGUREARCH (Fractionalised Integrated Generalized Autoregressive Conditional Heteroscedasticity) models and the Generalized Autoregressive Score (GAS) could be explored to model volatility for the banks and the results compared with the standard GARCH models, since the more than 80% of the banks exhibited long-memory. Replicating this same research in other key sectors could also be of interest.

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