

Switching multiple model filter for boost-phase missile tracking

MENEGAZ, HMT and BATTISTINI, Simone <<http://orcid.org/0000-0002-0491-0226>>

Available from Sheffield Hallam University Research Archive (SHURA) at:

<https://shura.shu.ac.uk/24856/>

This document is the Accepted Version [AM]

Citation:

MENEGAZ, HMT and BATTISTINI, Simone (2018). Switching multiple model filter for boost-phase missile tracking. IEEE Transactions on Aerospace and Electronic Systems, 54 (5), 2547-2553. [Article]

Copyright and re-use policy

See <http://shura.shu.ac.uk/information.html>

Switching Multiple Model Filter for Boost-Phase Missile Tracking

Henrique M. T. Menegaz, Simone Battistini

Abstract—This paper introduces a filter for tracking a ballistic missile during its boost-phase. This filter includes a new switching algorithm and a modified Interacting Multiple Model Unscented Filter (IMMUF) where the Markov Transition Matrix is time-variable. Position, velocity and all unknown parameters of a medium-range ballistic missile model are reconstructed. Simulations demonstrate the new filter is able to consistently estimate a missile's trajectory and all unknown parameters and to outperform previous forms of the IMMUF.

Boost-Phase Tracking, Interacting Multiple Model Unscented Filter (IMMUF), Time-Varying Markov Transition Matrix (MTM)

I. INTRODUCTION

This paper introduces a new filter for estimating the trajectory of a tactical ballistic missile (TBM) during its *boost-phase*. This filter presents two novelties: i) it is based on a novel *switching algorithm* and ii) is composed of a new *Interacting Multiple Model Filter (IMMF)*. The proposed filter assumes *no a priori* knowledge about the system parameters and the missile maneuvers timing, resulting in a robust estimation scheme for boosted missile tracking.

Estimating a TBM's trajectory during its *boost-phase* is an attractive option because, in this phase, rockets are easy to detect and countermeasures are less effective [1]. Boost-phase estimation is challenging due to i) the strict time available, ii) many unknown parameters in the estimation, and iii) the boost-phase trajectory's multi-phases form. The trajectory of a missile during its propelled phase is limited by physical constraints, such as dynamic pressure, thereby limiting the possibility of the missile to perform maneuvers. At the same time, this defines a sequence of well-known flight phases.

The trajectory's multi-phases form can be described by the formalism of multiple model (MM) systems. An MM system is composed of both discrete and continuous variables. Usually, the continuous variables represent the system's internal state, acquired measurements, and

noises; while the discrete variables denote the system's operating *mode*, and define how the continuous state evolves.

Optimal solutions for the MM filtering problem are computationally intractable because they require exponentially-growing computational effort and memory usage [2]. Thus, suboptimal approaches such as IMMFs are required. Compared with other suboptimal filters for MM systems such as the Generalized Pseudo Bayes, IMMFs greatly improve performance without increasing computational load [3]. As a result, IMMFs have been accepted as solutions to TBM tracking [4]–[6].

IMMFs use Kalman Filters (KFs) [7], and the inherent non-linearity of both dynamics and measurements involved in tracking a TBM calls for *nonlinear* KFs—besides, tracking requires fusing measurements provided by a variety of sensors such as space-based infrared sensors [8] or ground-based radars [9]—. The most widely known nonlinear KF is the Extended Kalman Filter (EKF) [10], [11], but the literature has introduced better alternatives to the EKF, such as the *Unscented Kalman Filters* (UKFs; see Section III-A) [12]–[14]. This work uses an IMMf with UKFs; called *Interacting Multiple Model Unscented Filter (IMMUF)*.

Nevertheless, IMMUFs set-up remains a difficult subject; it relies on *a priori* information [15] or dedicated analysis [16]. Besides, most literature's IMMUFs consider the probabilities of the state transitioning between modes constant [17]–[21]. This requires two quite conservative hypotheses: i) that the (non-constant) probability of the TBM transitioning between phases are well approximated by constant values; and ii) that these constant probabilities are *a priori* known [3], [15], [16].

As a result, this paper proposes the following two modifications to the IMMUF:

- 1) *Time-varying* probability of transitioning between models. This modification relaxes hypothesis i) (see Section III-B).
- 2) A switching strategy between models. This modification relaxes hypothesis ii) (see Section III).

A *Modified IMMUF* (MIMMUF) with time-varying transition matrix has been recently presented by the authors of this paper in [22]. The algorithm presented

H. M. T. Menegaz (henriquemenegaz@gmail.com) and Simone Battistini (simone.battistini@aerospace.unb.br) are with the Faculdade Gama at the Universidade de Brasília, Brazil.

in this paper improves this former concept (See Section III). In numerical simulations, the IMMUF of this work outperforms the MIMMUF of [22] and a standard time-invariant IMMUF (see Section IV).

The paper is organized as follows. Section II defines the model of the dynamical system of the measurements, defining the guidance strategy of the missile during its boost phase. Section III introduces the new switching filter. Section IV presents numerical simulations. Conclusions are given in Section V.

II. BOOST PHASE EQUATIONS AND MEASUREMENTS

This section describes the equations of motion of the target missile and the measurements. The dynamical model that will be employed in the filter is reported as well. The sequence of phases of flight of the missile will be exploited when forming the mode transition matrix of the filter in Section III-B.

The trajectory of a medium range missile during its boost phase is composed of a number of arcs which differ by the direction and magnitude of thrust. Generally speaking, three arcs can always be identified, namely the vertical arc, the pitch maneuver and the gravity turn trajectory [22], [23]. The order of these three phases cannot mutate.

The dynamical model of the missile trajectory used in this work is given in [23] and it is reported hereafter. The equations of motion are written in an inertial reference frame centered at the launch station, known as the Local Horizontal Frame (LHF) ($\hat{r}, \hat{E}, \hat{N}$) with the \hat{N} axis along the North direction of the launch station, the \hat{E} axis along the East direction of the launch station and the \hat{r} axis away from the center of Earth:

$$\begin{cases} \dot{s}_k = v_{k-1} \\ \dot{v}_k = T_{k-1} + G_{k-1} + A_{k-1} = g \frac{n_0 T_{k-1} t_b}{1 - (1 - u_0) t_{k-1}} + \\ - \frac{\mu}{\|s_{k-1}\|^3} s_{k-1} - \rho V_{R,k-1} \frac{\beta_0 V_{R,k-1} t_b}{1 - (1 - u_0) t_{k-1}} \end{cases} \quad (1)$$

having defined the position vector as $\vec{s} = [s_1 \ s_2 \ s_3]^T$ after a transformation from the LHF to Cartesian coordinates, the missile velocity vector as $\vec{v} = [v_1 \ v_2 \ v_3]^T$, the relative wind velocity as the difference between the latter and the local winds $\vec{V}_R = \vec{v} - \vec{V}_w$. The accelerations considered in the model are the gravity \vec{G} , aerodynamic action \vec{A} and thrust \vec{T} . ρ is the air density, which is modeled as a negative exponential depending on the altitude.

Table I reports the parameters of the model. The first four are the fundamental parameters of the model and

TABLE I: Missile model parameters

Parameter	Symbol	Parameter	Symbol
Initial thrust to weight ratio	n_0	Relative mass rate	q_0
Reduced ballistic coefficient	β_0	Burn-out time	t_b
Specific impulse	I_{sp}	Thrust over weight ratio	$n(t)$
Structural over total mass ratio	u_0	Ballistic coefficient	B

they are assumed constant. The remaining four parameters are derived from the former. It is important to notice that the sensitivity of the trajectory to the variations of these parameters (unknown to whom is carrying out the reconstruction of the trajectory) is very high [15]. The n_0 , β_0 , u_0 and t_b parameters will be assumed unknown to the estimator.

At the very beginning of its trajectory, the missile passes through the vertical arc, where it can be assumed

$$\hat{T}_{vert.} = [1 \ 0 \ 0]^T. \quad (2)$$

After the vertical arc, the missile performs the pitch maneuver. Thrust direction at the pitch over is defined from the azimuth angle ψ and the kick angle κ :

$$\hat{T}_{pitchov.} = \begin{bmatrix} \cos \kappa & \sin \kappa \sin \psi & \sin \kappa \cos \psi \end{bmatrix}^T. \quad (3)$$

During the gravity turn, the thrust is aligned with the velocity vector in order to null the incidence [24].

$$\hat{T}_{grav.turn} = \hat{V}_R. \quad (4)$$

The sequence of these three phases of flight is fixed for all ballistic missiles. The switching time between the three phases varies from case to case and cannot be assumed constant. In general, the pitch maneuver is very rapid and the gravity turn lasts until the constraint on the dynamic pressure ceases to exist, i.e. until the missile exits from the atmosphere. In this paper it will be assumed that the boost-phase ends at the exit from the atmosphere. Further maneuvers outside of the atmosphere will be therefore not considered. The following features will be assumed unknown to the estimator:

- 1) Switching time between the phases.
- 2) Direction and magnitude of the pitch maneuver.

The measurements employed in this study are given by a ground based radar, which is assumed to be located at the origin of the coordinate system, without loss of generality. The measurements are shown in Fig. 1: they consist of range $\bar{\rho}$, azimuth angle ψ and elevation

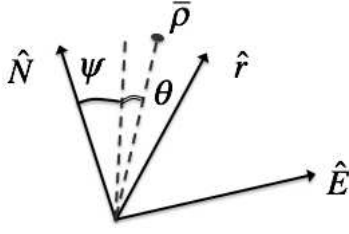


Fig. 1: Radar measurements.

angle θ from the radar to the target [22]. Range-rate measurements were not considered in this case. The azimuth angle in (3) is assumed to be the same returned from the measurements.

III. NEW FILTER FOR TBM TRAJECTORY ESTIMATION

The TBM dynamics considered in this work is composed of the three phases explained in Section II. In order to estimate the state of the TBM during these phases, a filter based on a switching technique is introduced. This filter switches among three (sub)filters: two AdUKFs (see Section III-A) and a new IMMUF (see Section III-B). The *rationale* of the filter for estimating a TBM trajectory during its boost phase is presented here, and in Section III-C an algorithm implementing this rationale is reported as well.

An AdUKF is run in the *first* seconds of the trajectory, the other AdUKF is run in the *last* seconds of the trajectory, and the new IMMUF is run *in the time interval between these first and last seconds*. Fig. 2 sketches the reasoning behind the switching technique. The proposed switching mechanism is based on the following three hypotheses:

- 1) In the first seconds of the trajectory (until $t = t_1$) it is reasonable to say that TBM is following a vertical arc trajectory. Thus, *an AdUKF* is run considering *vertical arc* equations for the time interval $[t_0, t_1]$
- 2) For the last seconds, say from a time instant $t = t_2$ on, one can say the TBM is following a *gravity turn* trajectory. Thus *another AdUKF* is run considering *gravity turn* equations for the time interval $[t_2, t_f]$.
- 3) For the time interval (t_1, t_2) , there is no certainty about which trajectory model the TBM is following. Therefore a multiple model filter is more adequate, and the new IMMUF is run.

The time instants t_1 and t_2 are defined as the following *ad-hoc* functions: $t_1 = \hat{t}_b/2 - 10$ s and $t_2 = \hat{t}_b/2 + 10$ s, where \hat{t}_b is the *estimate* of t_b . These functions of t_1 and t_2 are conservative assumptions in the sense that they result in a *wide* interval (t_1, t_2) of 20 s; this is to ensure that the three hypotheses above hold.

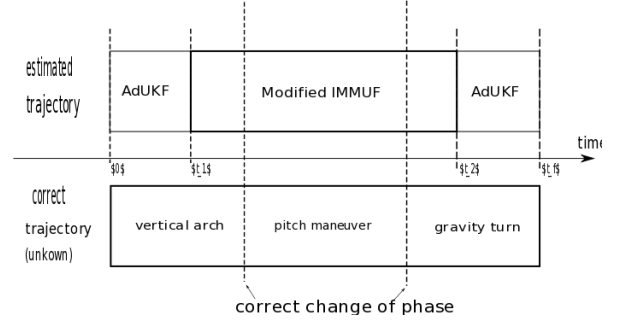


Fig. 2: Time-line of the proposed switching filter.

A. Additive Unscented Kalman Filters

In the time intervals $[t_0, t_1]$ and $[t_2, t_f]$ the trajectory phase of a given TBM can be assumed known. For $[t_0, t_1]$, this trajectory is given by (1) with (2) (vertical arc); and for $[t_2, t_f]$, by (1) with (4) (gravity turn). For both time periods, the measurements are given by the second equation in (5).

In order to write the nonlinear dynamic system for the TBM trajectory, define the internal state vector at the step time k by $x_k := [s_k^T, v_k^T, \beta_{0,k}, n_{0,k}, t_{b,k}, u_{0,k}]^T \in \Phi_{n_x}$. Although the four parameters are considered constant, their values are supposed *unknown* in this work. In order to estimate their correct values, they are therefore included in the state vector x_k . This is a common practice when both the state and the parameters of a given system are estimated [25].

Stochastic filters such as AdUKFs can be used to estimate the internal state x_k of nonlinear dynamic systems, resulting in a good trade-off between computational cost and estimation quality [26]. AdUKFs are based on the concepts of σ -representation (σ R) and Unscented Transformation (UT) [27]. The UT has interesting properties concerning the estimation of \bar{Y} , P_{YY} and P_{XY} :

$$\mu_{\gamma}^{[\mu_x, 2]} = \bar{Y}^{[\bar{X}, 2]}, \Sigma_{\gamma\gamma}^{[\mu_x, 1]} = P_{YY}^{[\bar{X}, 1]}, \Sigma_{X\gamma}^{[\mu_x, 1]} = P_{XY}^{[\bar{X}, 1]},$$

where $Y^{[c, l]}$ stands for the Y 's Taylor Series around c truncated at the l th term, $\Sigma_{\gamma\gamma} := \sum_{i=1}^N w_i^c (\chi_i - \mu_{\chi})(\chi_i - \mu_{\chi})^T$ is the sample covariance, and $\Sigma_{X\gamma} := \sum_{i=1}^N w_i^{cc} (\chi_i - \mu_{\chi})(\gamma_i - \mu_{\gamma})^T$ is the sample cross-covariance.

Properties like these make the UT a good choice to be used in stochastic filters; it can be applied in the Kalman Filter prediction-correction framework to form AdUKFs. There are many definitions of AdUKFs; a systematized presentation of them is given in [27].

AdUKFs are good options for nonlinear problems. However, there are problems that require *more than one* of these systems to properly describe their behavior. This is the case of the TBM trajectory during the time interval (t_1, t_2) .

B. Time-Varying Interacting Multiple Model Unscented Filter

Since in the time interval (t_1, t_2) there is no certainty about which trajectory model the TBM is following, multiple choices have to be considered. In this sense, the TBM trajectory is described by a *MM system* with *M different models*, introduced as

$$\begin{aligned} x_k &= f_{m_k}(x_{k-1}) + \varpi, \\ y_k &= h_{m_k}(x_k) + \vartheta, \quad k \in \mathbb{N}; \end{aligned} \quad (5)$$

where f is the process function; h the measurement function; $y_k := [\hat{\rho}_k \psi_k \theta_k]^T \in \Phi_{n_y}$ the measurement vector; $\varpi \in \Phi_{n_x}$ the process noise; $\vartheta \in \Phi_{n_y}$ the measurement noise; and $m_k \in \mathbb{M} := \{1, \dots, M\}$ is the system's discrete modal state (*mode*). The noises ϖ and ϑ are supposed to i) be uncorrelated, ii) have mean zero, and iii) have covariances Q and R , respectively. The parameter m_k is assumed to follow a *time-varying* Markov Chain with a Markov Transition Matrix (MTM), $\Pi(k)$ defined as:

$$\Pi_{ij}(k) := P\{m_k = j | m_{k-1} = i\}, \quad i, j \in \mathbb{M},$$

where, for a given event e , $P\{e\}$ stands for the probability of e occurring.

The MM system (5) is set with $M = 2$, $h_1 = h_2 = h$, $f_1 = f_{\text{vert.}}$, and $f_2 = f_{\text{pitchov.}}$. In deed, the MM system for the time interval (t_1, t_2) can be written with only *two modes*: one for the vertical arc and another for the pitch maneuver. Even if the TBM is on a gravity turn trajectory, an MM systems with this two-modes formulation can model a gravity turn behavior as a rotation around the missile's transversal axis, just like the pitch maneuver. In this way, the cardinality of \mathbb{M} can be reduced by 1 (instead of 3 modes, there are 2 modes), and the computational cost of the filter is reduced.

In this paper an IMMUF is used to estimate the state of this system. Since optimal solutions for the MM filtering problem are computationally intractable because they require exponentially-growing computational effort and memory usage [2], [25], suboptimal approaches are required. Interacting Multiple Model Filters are computationally cost-efficient suboptimal estimators of MM systems. In comparison with other suboptimal filters for MM systems, such as the Generalized Pseudo Bayes Filters, they greatly improve performance without increasing computational load [3].

However, literature's IMMUFs might fail to estimate a TBM trajectory. In most literature's IMMUFs, the MTM Π is *time-invariant* [$\Pi(k) = \Pi(k+1)$ for every $k \in \mathbb{N}$] [17]–[21]. Nevertheless, with a TBM Π is rarely *well approximated by a constant value*; for instance, when

the TBM system in mode $m_k = 1$, the missile is in the vertical arc phase; and when in mode $m_k = 2$, in the pitch maneuver phase; clearly, $\Pi_{1,2}(k)$ is smaller in the beginning of mode 1 than at its end.

C. Algorithm of the Switching filter

In this paper a modified IMMUF is introduced, where the entries of Π change linearly over time. Being Δt the sampling time, the new matrix Π is given as follows:

$$\pi_{11,k} = \frac{-k\Delta t + 0.5\hat{t}_{b,k-1}}{10} \quad (6)$$

$$\pi_{22,k} = \frac{k\Delta t - 0.5\hat{t}_{b,k-1}}{10} \quad (7)$$

$$\Pi(k) = \begin{bmatrix} \pi_{11,k} & 1 - \pi_{11,k} \\ 1 - \pi_{22,k} & \pi_{22,k} \end{bmatrix}. \quad (8)$$

When time is in the interval (t_1, t_2) , the mode probability vector is initialized with $p_k = [p_{k,1}, p_{k,2}]^T = [1, 0]^T$.

Since $p_{k,1} = 1$, at the beginning of the interval (t_1, t_2) it is sure that the TBM will be following the vertical arc ($m_k = 1$). As time progresses, i) from (6), the probability of the TBM being on the *vertical arc diminishes linearly*; and ii), from (7) the probability of the TBM being in the *pitch maneuver* trajectory ($m_k = 2$) *increases linearly*. At the end of (t_1, t_2) , the probability of the vertical arc is 0 and that of the pitch maneuver is 1. The new filter for estimating the TBM trajectory during its boost phase - called *Switching Modified Interacting Multiple Model Unscented Filter (SMIMMUF)* - is based on i) the switching rationale explained in the beginning of Section III, ii) AdUKFs [27], and iii) the MIMMUF structure described in [22]. Define $k_1 := \text{quo}(t_1, \Delta t)$,

IV. NUMERICAL SIMULATIONS

Numerical simulations have been implemented in order to validate the proposed algorithm against an unknown target. The SMIMMUF will be compared with a classical IMMUF and the MIMMUF of [22]. The AdUKFs used in all cases are Homogeneous Minimum Symmetric AdUKF's ([27], Tab. IV).

A. Simulation parameters

For each algorithm, 300 Monte Carlo simulations have been run, each one differing by the initial guess. The initial guesses belong to a normal distribution with mean value equal to the true value of the state variables. The covariance error matrix $P(0|0)$ is initialized in accordance with the variances of the initial guesses:

$$P_{0|0} = \text{diag} \begin{bmatrix} 2000^2 & 2000^2 & 2000^2 & 100^2 & 100^2 & \dots \\ \dots & 100^2 & (3E-4)^2 & 1^2 & 5^2 & (3E-2)^2 \end{bmatrix}$$

Both the classical IMMUF and the MIMMUF of [22] run *five* models: one for the vertical arc, three for the pitch maneuver with an angle $\kappa \in [3.5^\circ, 6.5^\circ]$ (the true value of κ being 5°), and one for the gravity turn. The reconstruction of the kick angle κ will be therefore left to the filter. In the real model, the transition between the vertical arc and the pitch maneuver occurs after 40 s and the transition between the pitch maneuver and the gravity turn occurs after 46 s. The process noise covariance matrix Q is the same for all the filters and can be found in [22]. The common scenario for all simulations is that of the medium range missile described in [23].

B. Results

Fig. 3 to 5 show some of the results of the three simulations, in particular the estimation errors of the first component of \vec{s} and \vec{v} and of the β_0 parameter. The red line is the mean estimation error over the entire Monte Carlo set; the green line is the estimation error of one sample; the dashed black line is the theoretical σ -bounds of the filter calculated from the error covariance matrix P ; the dashed blue line is the standard deviation of the errors. The other results are not shown for the sake of conciseness, but they are similar to those reported here.

It can be seen that, in general, the algorithms with time-varying MTM - referred to as two-modes (the SMIMMUF) and five-modes (the MIMMUF of [22]) filters in the figures - provide more consistent results than the classical IMMUF with the constant transition matrix - referred to as constant. With the MIMMUF of [22], the mean error diverges at the end in the estimation of \vec{s} and \vec{v} , while the mean error of the SMIMMUF does not. This demonstrates the superiority of the proposed filtering scheme with respect to the other tested algorithms.

V. CONCLUSIONS

This paper introduces a new filter for the tracking of a ballistic missile during the boost phase. The model fed to the filter includes several unknown parameters and dynamics. The new filter endowed with two novelties:

- 1) A modified IMMUF with time-varying transition matrix.
- 2) A switching rationale transitioning between Unscented Kalman Filters and the modified IMMUF.

The algorithm has been tested in a Monte Carlo numerical simulations based on radar measurements. A comparison with other two algorithms, a classic version of the IMMUF and a modified IMMUF with another time-varying MTM, has demonstrated that the proposed solution is a valid alternative to track an unknown ballistic missile.

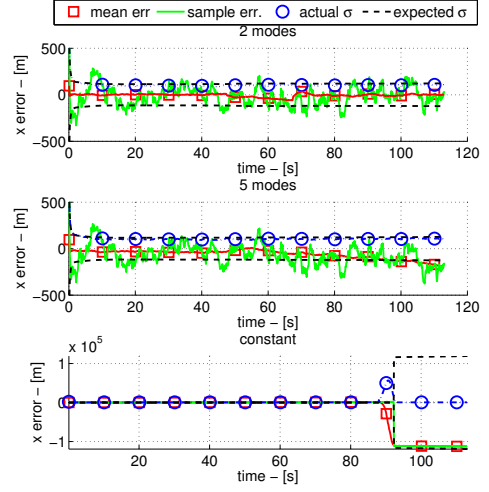


Fig. 3: Estimation error on x.

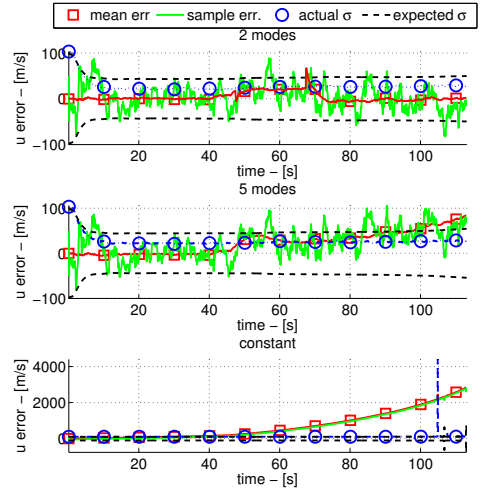


Fig. 4: Estimation error on u.

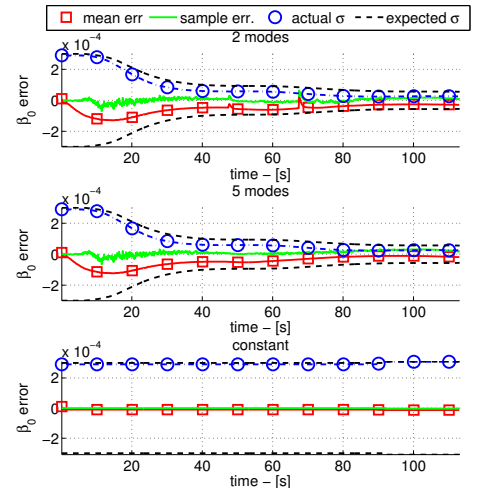


Fig. 5: Estimation error on β_0 .

The proposed filter has shown promising results, but, for completely assessing it, new studies should test its robustness against adverse situations. For instance, when data association due to false alarms or neighboring targets is a significant problem [3], [28]; in this case, it could be compared with filters specifically developed for these situations, such as probabilistic data association filters and joint probabilistic data association filters.

REFERENCES

- [1] D. A. Wilkening, "Airborne boost-phase ballistic missile defense," *Science and Global Security*, vol. 12, pp. 1–67, 2004.
- [2] Y. Bar-Shalom, S. Challa, and H. Blom, "IMM estimator versus optimal estimator for hybrid systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 3, pp. 986–991, 2005.
- [3] E. Mazor, A. Averbuch, Y. Bar-Shalom, and J. Dayan, "Interacting multiple model methods in target tracking: a survey," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 34, 2008.
- [4] W. J. Farrell, "Interacting multiple model filter for tactical ballistic missile tracking," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 44, no. 2, 2008.
- [5] D. Lainiotis, "Optimal adaptive estimation: Structure and parameter adaption," *IEEE Transactions on Automatic Control*, vol. 16, no. 2, pp. 160–170, 1971.
- [6] B. Ristic, S. Arulampalam, and N. Gordon, "Beyond the kalman filter," *IEEE Aerospace and Electronic Systems Magazine*, vol. 19, no. 7, pp. 37–38, 2004.
- [7] H. Blom and Y. Bar-Shalom, "The interacting multiple model algorithm for systems with Markovian switching coefficients," *IEEE Trans. Autom. Control*, vol. 33, no. 8, pp. 780–783, 1988.
- [8] J. Kidd and H. Caldwell, "Defense support program: Support to a changing world," in *Space Progr. and Techn. Conf.*, 1992.
- [9] X. R. Li and V. P. Jilkov, "Survey of maneuvering target tracking: Iii. measurement models," in *International Symposium on Optical Science and Technology*. International Society for Optics and Photonics, 2001, pp. 423–446.
- [10] Y. Bar-Shalom and X.-R. Li, "Estimation and tracking: Principles, techniques, and software," *IEEE Anten. and Propag. Mag.*, vol. 38, no. 1, p. 62, 1996.
- [11] P. Zarchan, "Boost-phase filtering options: is simpler better?" *Journ. of Guid., Cont., and Dyn.*, vol. 33, no. 6, pp. 1724–1731, 2010.
- [12] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte, "A new approach for filtering nonlinear systems," in *Proc. of the American Contr. Conf.*, vol. 3. IEEE, 1995, pp. 1628–1632.
- [13] A. Farina, B. Ristic, and D. Benvenuti, "Tracking a ballistic target: comparison of several nonlinear filters," *IEEE Trans. on Aero. and Elect. Syst.*, vol. 38, no. 3, pp. 854–867, 2002.
- [14] B. Ristic, A. Farina, D. Benvenuti, and M. S. Arulampalam, "Performance bounds and comparison of nonlinear filters for tracking a ballistic object on re-entry," *IEE Proceedings-Radar, Sonar and Navigation*, vol. 150, no. 2, pp. 65–70, 2003.
- [15] A. Benavoli, L. Chisci, and A. Farina, "Tracking of a ballistic missile with a-priori information," *IEEE Trans. on Aero. and Elec. Syst.*, vol. 43, no. 3, pp. 1000–1016, 2007.
- [16] F. Reali, G. B. Palmerini, A. Farina, A. Graziano, and S. Giompapa, "Parametric analysis of ballistic target-tracking problem by multiple model approach," *IET Radar, Sonar & Navigation*, vol. 7, no. 4, pp. 459–469, 2013.
- [17] T. Sathyan, A. Sinha, and T. Kirubarajan, "Passive geolocation and tracking of an unknown number of emitters," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 42, no. 2, pp. 740–750, apr 2006.
- [18] M. R. Morelande and B. Ristic, "Smoothed state estimation for nonlinear Markovian switching systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 4, pp. 1309–1325, oct 2008.
- [19] D. Lee, C. Liu, Y.-W. Liao, and J. K. Hedrick, "Parallel Interacting Multiple Model-Based Human Motion Prediction for Motion Planning of Companion Robots," *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 1, pp. 52–61, jan 2017.
- [20] N. Tudoroiu, M. Zaheeruddin, V. Cretu, and E.-R. Tudoroiu, "IMM-UKF Versus Frequency Analysis," *IEEE Ind. Electron. Mag.*, vol. 4, no. 3, pp. 7–18, sep 2010.
- [21] Y. Tian, Z. Chen, and F. Yin, "Distributed IMM-Unscented Kalman Filter for Speaker Tracking in Microphone Array Networks," *IEEE/ACM Trans. Audio, Speech, Language Process.*, vol. 23, no. 10, pp. 1637–1647, oct 2015.
- [22] S. Battistini and H. M. T. Menegaz, "Interacting multiple model unscented filter for tracking a ballistic missile during its boost phase," in *IEEE Aerospace Conference*. IEEE, 2017, pp. 1–8.
- [23] F. Piergentili and P. Teofilatto, "Algorithm for missile detection from radar data," *Journal of Spacecraft and Rockets*, vol. 44, no. 1, pp. 276–280, 2007.
- [24] G. J. Culler and B. D. Fried, "Universal gravity turn trajectories," *Journal of Appl. Phys.*, vol. 28, no. 6, pp. 672–676, 1957.
- [25] Y. Bar-Shalom, X.-R. Li, and T. Kirubarajan, *Estimation with applications to tracking and navigation*. New York: John Wiley & Sons, 2001.
- [26] S. J. Julier and J. K. Uhlmann, "Unscented Filtering and Nonlinear Estimation," *Proc. IEEE*, vol. 92, no. 3, pp. 401–422, 2004.
- [27] H. M. T. Menegaz, J. Y. Ishihara, G. A. Borges, and A. N. Vargas, "A Systematization of the Unscented Kalman Filter Theory," *IEEE Trans. Autom. Control*, vol. 60, no. 10, pp. 2583–2598, 2015.
- [28] S. S. Blackman, "Multiple hypothesis tracking for multiple target tracking," *IEEE Aero. and Elec. Sys. Mag.*, vol. 19, no. 1, pp. 5–18, 2004.



Henrique M. T. Menegaz received a BS degree in Electrical Engineering from the Universidade de Brasília (UnB), Brazil, in 2007. He received a MS (2011) and a PhD (2016) in Engineering of Electronic Systems and Automation from UnB. He is currently an Assistant Professor with Faculdade Gama, UnB. His major field of study is filtering of nonlinear dynamic systems and their applications.



Simone Battistini received his BSc (2006) and MSc (2009) in Control System Engineering from Sapienza Università di Roma and he holds a PhD in Aerospace Engineering (2013) from the same institution. He is currently an assistant professor at the Faculdade Gama of Universidade de Brasília, Brazil. His main research interests are related with guidance, navigation and control of aerospace vehicles.