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A Proposal to Modify the Moment Coefficient in Eurocode 2 for Predicting the Residual Strength of Corroded Reinforced Concrete Beams

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Abstract

Ultimate limit state (ULS) criteria are used to design reinforced concrete beams which give~~s~~ a ductile behaviour at failure. This means the resisting moment, M_t, is less than the resisting moment in compression, M_c. Since steel reinforcement is susceptible to corrosion, the ultimate capacity can be seriously affected as the degree of corrosion increases. The impact of corrosion to the main steel reinforcement on the flexural performance of reinforced concrete beams is investigated. Beams measuring 100 mm wide x 150 mm deep with differing levels of under-reinforcement (M_t/M_c ratios) were tested under four-point bending. Although the design code for reinforced concrete beam design has gone through various changes over the years, the fundamentals for design has broadly remained the same in that the beam is designed with an ultimate moment-coefficient ($K = M/f_c bd^2$) with sufficient capacity to be able to easily carry the service loads it is exposed to. However, the long term influence of corrosion on the steel reinforcement is not considered at the design stage although a manufacturing factor of safety is applied. The analysis in this paper uses a modified-moment coefficient ($K_{corr} = M_{corr}/f_{ck}bd^2$) based on EC 2 ultimate limit state design guidelines

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to predict the residual flexural strength of reinforced concrete beams suffering from main steel corrosion. Two grades of concrete (>C35/45 and <C35/45) are considered as used in EC 2 [1]. The results obtained in this investigation show that there is a relationship between the reducing moment-coefficient of corrosion-affected concrete beams and both the degree of under-reinforcement (M_t/M_c) and degree of corrosion to the main steel reinforcement for beams with concrete grades >C35/45. The analysis is then extended to include test data from other researchers to develop a similar simplified empirical analytical expression for beams with concrete grades <C35/45, thereby enabling a prediction of residual strength due to corrosion to be made for any beam size or concrete strength grade.

Keywords: Modified-moment coefficient; ultimate load; degree of corrosion; degree of under-reinforcement, residual strength in bending, EC 2

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Notation

A	Mass of iron per unit volume
A_s	Area of tensile reinforcement
$A_s{}'$	Area of compression steel (hanger bars)
A_r	Rebar surface area before corrosion
b	Width of beam section
β	Degree of under-reinforcement of the beam section
C	Degree of corrosion in the main steel reinforcement
c	Intercept for corrosion slope equation
c'	Intercept for lever arm equation
c_v	Cover to main steel reinforcement
d	Effective depth to main reinforcement
d'	effective depth to compression steel (hanger bars)
δ	Material loss due to corrosion
$\Delta\omega$	Weight loss due to corrosion

F	Faraday's constant
F_{cc}	Force in the concrete in compression
F_{st}	Force in the main steel reinforcement in tension
f_{ce}	Compressive strength of concrete cores
f_{ck}	Compressive strength of concrete cylinders
f_{cu}	Compressive strength of concrete cubes
f_y	Yield strength of main steel reinforcement
γ	Density of steel
γ_c	Partial safety factor for the strength of concrete
γ_s	Partial safety factor for the strength of steel reinforcement
I	Electrical current
i	Corrosion current density
K	Moment coefficient for a reinforced concrete beam
K_{corr}	Moment coefficient for a corroded reinforced concrete beam
l	Lever arm coefficient
M_c	Maximum resisting moment of the concrete in the compression zone
M_{corr}	Resisting moment of the corroded beam
$M_{c(f)}$	Resisting moment in compression at tensile failure
M_{sc}	Resisting moment of the steel hanger bars in compression
M_t	Resisting moment of the control beam in tension (uncorroded)
m	Corrosion slope coefficient
θ	Diameter of steel reinforcement
P_{ult}	Load at ultimate limit state
R_m	Material loss per year due to corrosion
s	Depth of idealised compressive stress block
T'	Time in years
t	Time in seconds
x	Depth to neutral axis
Z	Valence of iron
z	Lever arm
z_{corr}	Lever arm of corroded beam

1 Introduction

Reinforced concrete beams are designed as under-reinforced meaning failure occurs by yielding of the main steel reinforcement. Therefore, the resisting moment in tension, M_t, is less than the resisting moment in compression, M_c. With regards to design to

EC 2 [1], the ultimate allowable resisting moment in compression for singly reinforced concrete beams is limited to $M_c = 0.167 f_{ck} b d^2$ (factored, concrete grade >C35/45). However, the long term influence of reinforcement corrosion is not taken into account at the design stage. Members with corroded main steel reinforcement will exhibit a lower flexural capacity due to the reinforcement corrosion and it is the responsibility of the asset managers to ensure that their structures maintain sufficient residual strength to remain safely in service.

Since corrosion of steel in reinforced concrete is an issue facing many structure owners worldwide, there has been a lot of research conducted on the residual strength of deteriorated concrete. Since reinforced concrete beams are designed as under-reinforced to ensure a ductile failure, EC 2 [1] stipulates that the quantity of main reinforcing steel should be between a minimum of 0.13% bd and maximum of 4% bd respectively. The impact of corrosion on the different degrees of under-reinforcement was first investigated by the authors [2, 3] and Du et al [4]. Hristova [2] and O'Flaherty et al [3] found that lower loss of strength due to corrosion occurred with a higher degree of under-reinforcement (lower M_t/M_c). The recommendation was to keep the quantities of reinforcement as close as possible to 0.13% with preference given to smaller bar diameters. Du et al [4] found that a less brittle failure occurred in over-reinforced beams but under-reinforced beams failed with less ductility. El Maaddawy et al [5] found that the reduction in beam flexural strength was almost proportional to reduced cross sectional area of the steel reinforcement due to corrosion and that for corrosion levels greater than 15% mass loss, the deflection capacity of the beam decreased mainly as a result of pits forming at the higher corrosion levels. Torres-Acosta et al [6] also established that it was the maximum pit size, not the average pit size that was the most important parameter affecting flexural load capacity. In some cases, the pit depth was 73% of the depth of the rebar diameter. Yang et al [7] found that under repeated loading of corrosion damaged reinforced beams, the failure modes changed from plastic to brittle type as the degree of corrosion increased. Yu et al [8] also found that corrosion of the main steel reinforcement also modified the failure mode of the RC beam. The control beam suffered crushing of the concrete in the zone of compression whereas the deteriorated beam failed due to rupture of one main tensile bar. It also suggested that 1% loss in cross sectional area corresponded to a 1% loss in yield and ultimate capacity. Corrosion also led to more brittle behaviour of the steel bars in tension (reduction of ultimate strain), the ductility of the reinforced concrete beam exhibiting main steel corrosion was influenced by the initial ductility of the steel reinforcement. Brittle collapse of corroded reinforced concrete beams could be avoided if highly ductile steel bars are specified in accordance with Class C of EC 2 [1]. However, Malumbela et al [9] reported that 1% loss in steel mass reduced the flexural capacity of deteriorated RC beams by 0.7%, less than that found by Yu et al [8], and that the ultimate moment capacity reduced linearly with the degree of corrosion. Zhu et al [10] reported that corrosion of the main steel significantly influenced the flexural strength of short span beams as a result of reducing the cross sectional area and loss of bond, the flexural capacity being more sensitive to corrosion than the shear capacity. The corroded steel bar exhibited less ductile failure due to corrosion pitting, the degrees of corrosion being between 20 and 30% loss of cross section, similar to that found by El Maaddawy et al [5]. Du et al [11] stated that under simultaneous loading and corrosion of the main steel reinforcement, failure may occur without significant signs of warning, save for the rust stains and surface cracking. There was also a more rapid decrease in ultimate strength as a result of the simultaneous loading and corrosion. Ye et al [12] found that for the same longitudinal crack width, a lower level of corrosion generated via artificial climate exposure was required compared to the galvanic method. However, it was also concluded that the average mass loss ratio of corrosion affected reinforcement or the longitudinal crack widths can be reliably used to predict the flexural capacity of the deteriorated beams. Ballim et al [13] used simultaneous loading and corrosion and determined that a degree of corrosion to the main steel of 6% led to an increase in deflection between

40-70% compared to the control specimen. Dang et al [14] determined that corrosion of the main steel reinforcement led to a change in failure mode from concrete crushing to brittle tensile bar failure, the reason being a decrease in ultimate elongation as a result of the corrosion.

Zhu et al [15] and Dang et al [16] reported that their experimental results not only confirmed that the loss in structural capacity was not only linked to the loss of rebar cross section at the failure location, the post yield hardening properties of the steel reinforcement were modified leading to a difference between the yield capacity and the ultimate capacity.

Both O'Flaherty et al [3], Ahmad [17] and Wang [18] found that cover thickness did not have a significant effect on structural performance of corroded beams although Cabrera [19] found that the crack intensity increased with an increasing depth of cover.

In terms of modelling the influence of main steel corrosion, a number of researchers have attempted to predict the residual strength of deteriorated reinforced concrete beams. Azad et al [20] proposes a model which predicts a lower moment capacity based on reducing the cross sectional area of tension reinforcement. Ahmad's model [17] requires input data such as the steel reinforcement diameter and corrosion rate and corrosion time after initiation. The model was validated based on test data from Rodriguez et al [21]. A time varying model was developed by Zhong et al [22] which predicts the residual flexural capacity of a reinforced concrete member and assess for structural safety at any time in the future. The model was based on experimental data and predicted that by imposing a 10% limit in reduction of flexural capacity, maintenance and strengthening would be required after 24.5 years of service. The work of Lee and Cho [23] concentrated on establishing the reduction in the residual yield strength of the steel reinforcement bar and used this to determine the reduced moment capacity. The model by Han et al [24] considered the corrosion rate and flexural cracks and gave good agreement with others' test data.

This paper aims to develop empirical equations to predict the impact of main steel corrosion on the ultimate strength of reinforced concrete beams using simplified analysis based on EC 2 [1]. Parameters such as the reduced moment coefficient ($K_{corr} = M_{corr}/f_{ck}bd^2$) and lever arm ($l = z_{corr}/d$) are related to the reduction in flexural strength due to the loss in cross sectional area of the main steel reinforcement. The findings are strengthened by combining with similar data from other researchers to cover the full range of beams in-service in accordance with EC 2 [1], those with concrete grades both greater and less than C35/45. These are parameters which the design engineer will be very familiar with and as a result, will help them to simply predict the residual strength of their deteriorating reinforced simply supported concrete beams in-service.

2 Research significance

Over 75 % of the UKs concrete bridge stock was built after the 1960s with the corrosion problem becoming more prominent in the 1980s when these bridges were in service for only a couple of decades [25]. The 1989 survey of two hundred randomly selected concrete bridges found that 72 % of the selection were corrosion affected [26]. Over three decades later, corrosion to steel reinforcement still remains a major problem. The direct cost of reinforcement corrosion to the UK economy is estimated at over £616m per year [27]. In the US, there are 235,000 conventionally reinforced concrete bridges. Approximately 30 % of the bridges are structurally deficient or functionally obsolete. The annual direct cost of corrosion for all highway bridges is estimated to be US$ 13.6 billion [28]. The global cost of corrosion is estimated for all industries to be US$ 2.5 trillion [29]. It has been reported that serviceability failure occurs when a 10-25 % loss in bar diameter occurs due to corrosion [30). However, whilst these figures undoubtedly suggest serviceability issues are likely to occur, it does not provide the engineer with the information to be able to predict the reduced flexural capacity of the deteriorated concrete beams.

Design of under-reinforced concrete beams is based on an ultimate limit state (ULS) approach, limit state being a condition of a structure beyond which it no longer fulfils the relevant design criteria. It assumes the ULS will never be reached as the in-service loads are less than the factored loads used to design to ULS. Irrespective of whether the design engineer used BS 8110 [31] in the past or the current EC 2 [1] design standard, design is based on similar principles. Both standards provide sufficient ultimate moment capacity via a moment coefficient, $K = M/f_c bd^2$, which takes into account the magnitude of the applied moment (M), compressive strength of concrete (f_c) and the width and effective depth of the beam section (bd). However, the influence of long term corrosion to the steel reinforcement is not taken into account at the design stage and this can greatly reduce the ultimate design capacity. Various researchers over the years have put forward different proposals for predicting the residual strength of deteriorated reinforced concrete. The model proposed in this paper differs in that it is based on existing design criteria that engineers will be familiar with by using the moment coefficient approach but accounting for the effects of corrosion by reducing K. The findings in this paper will, therefore, assist engineers to make a more reliable prediction of residual flexural strength of deteriorating reinforced concrete beams since the proposed model is more user friendly as it is based on familiar design principles.

3 Design, manufacture and testing of beams

Reinforced concrete beams, measuring 910 mm long and with a cross-sectional area of 100 mm x 150 mm deep, were made in the laboratory and an accelerated corrosion technique was used to replicate in-situ deterioration. Table 1 and Figure 2 provide details of the test specimens. The main steel reinforcement only was subjected to corrosion. Beams were reinforced with either 2 x 8 mm, 2 x 10mm or 2 x 12 mm high yield main steel and 6 mm mild steel shear reinforcement, Table 1. A corrosion target of 0%-15% weight loss of the main steel reinforcement was decided upon to replicate typical levels of corrosion that may be encountered in-situ.

Main steel reinforcement was specified in accordance with BS 4449 [32] and consisted of high yield (ribbed) bars. An actual stress/strain test was not conducted on the steel reinforcement at the time due to a malfunction with the test rig so the yield stress was taken as 460 N/mm^2 as specified by the supplier. However, it must be noted that the permissible stresses for the design of reinforced concrete building structures were amended in August 2009 [33]. Characteristic yield strengths increased to 500 N/mm^2 for high yield steel which was used for both shear and main reinforcement design. The shear reinforcement had a nominal characteristic strength of 250 N/mm^2 with spacings of either 85, 80 or 60 mm c/c for 2 x 8, 2 x 10 or 2 x 12 respectively. Cover was either 26 mm, 36 mm or 56 mm to the main reinforcement and was specified, along with the variation in area of main steel, to vary the resisting moment of the beams. Beams are identified based on the configuration 2T8/26 (T meaning high yield, Table 1, col. 1). Two longitudinal hanger bars (6 mm diameter, nominal characteristic yield strength of 240 N/mm^2), were provided in the compression zone to support the shear links. The weight of the main steel reinforcement was obtained before casting so an actual percentage corrosion could be calculated after testing. Shrink wrap was used at the intersection between the main and shear reinforcement to eliminate electrical continuity and thereby prevent corrosion to the shear reinforcement during the accelerated corrosion process (see Section 4).

Concrete with target cylinder strength of 40 N/mm^2 was used to cast the beams. Portland cement: fine aggregate: coarse aggregate were used at a ratio of 1:1.5:2.9. All aggregates were oven dried for 24 hours at 100°C. Anhydrous calcium chloride ($CaCl_2$, 1% by weight of cement) was added to the concrete mixture to accelerate corrosion of the main steel reinforcement. The concrete beams were manufactured in steel moulds. Concrete was applied in three layers, each layer being compacted on a vibrating table. The moulds were transferred to a mist curing room (20 °C and 95 % ± 5 % Relative Humidity) for 24 hours. Demoulding occurred after one day followed by curing in water at 20°C for a further 27 days (28 days in total). Electrical connections were made to the steel making the main steel reinforcement anodic and the remaining steel cathodic. Beams were then immersed in a tank filled with an appropriate level of saline solution (to cover the main steel) for accelerated corrosion at 28 days age.

The control specimens with 0% corrosion were tested at 28 days. Due to the time taken to reach the target corrosion in the main reinforcement, corroded beams were tested in bending at either 42, 48 or 45 days age (Table 1, col. 2). After inducing the required degree of corrosion, the beams were tested to failure at the given ages under four point bending in a calibrated ESH600 compression/tension test rig (Figure 1 (a)). The deflection was measured by means of linear variable displacement transducers (LVDT, accurately calibrated using slip gauges) which were positioned under the beams at mid span and protected from damage at beam failure.

In order to confirm the actual percentages of corrosion, at the completion of testing, the steel reinforcement was carefully removed from the concrete and cleaned with 10 % diammonium hydrogen citrate solution. All steel samples were then reweighed and the mass loss determined from which the actual percentages of corrosion were determined (Figure 1 (b)).

4 Accelerated corrosion process

A direct current multi-channel power supply was used to apply a pre-defined current to the longitudinal tensile steel reinforcement. The current was monitored using an ammeter. The corrosion of the steel reinforcement occurred in a plastic tank. Water dosed with 3.5 % $CaCl_2$ was used as the electrolyte. A current density, held constant at 1 mA/cm^2 and based on pilot tests to provide desired levels of corrosion in a reasonable time, was applied to the steel reinforcement. Faraday's Law was used to determine the relationship between the weight of metal lost due to corrosion and current density as shown in Equation 1 [34]:

$$\Delta\omega = \frac{AIt}{ZF}$$

Equation 1

where $\Delta\omega$ = metal weight loss due to corrosion (g); A= atomic weight of iron (56 g); I = electrical current (amps); t = time (sec.); Z = valence of iron (which is 2); and F = Faraday's constant (96 500 coulombs).

Metal weight loss due to corrosion can also be written as:

$$\Delta\omega = A_r \delta \gamma$$

Equation 2

where A_r = rebar surface area before corrosion (cm^2); δ = metal loss (cm); and γ= density of metal (7.86 g/cm^3).

Corrosion current, I, can be expressed as:

$$I = (i)(A_r)$$

Equation 3

where i = corrosion current density (amp/cm^2).

Therefore, Equation 2 and Equation 3 can be combined to give:

$$A_r \delta y = \frac{AIt}{ZF} = \frac{AiA_r t}{ZF}$$

Equation 4

Inserting known values into Equation 4 and simplifying gives:

$$\delta = \frac{(56)(i)(365)(24)(60)(60)}{(7.86)(2)(96500)} = 1165\ (i) \text{ cm/yr}$$

Equation 5

Equation 5 can be written in terms of R_m, which is defined as the metal loss per year (cm/year), giving:

$$R_m = 1165\ (i)\ \text{cm/year}$$

Equation 6

For a corrosion rate, i, of say 1 (mA/cm^2), R_m equals 1.165 (cm/year) (from Equation 6).

If the period of corrosion after initiation is $\boldsymbol{T'}$ years, then:

$$\text{Metal loss after } T' \text{ years} = R_m T' \text{ (cm)}$$

Equation 7

Therefore:

$$\text{Reduction (\%) in bar diameter in } T' \text{ years} = \frac{2R_m T'}{\theta}(100)$$

Equation 8

where $\boldsymbol{\theta}$ is the bar diameter. The reduction in bar diameter due to corrosion in $\boldsymbol{T'}$ years is therefore $\boldsymbol{\frac{2R_m T'}{\theta}(100)}$ and is also identified as the degree of reinforcement corrosion (Table 1, col. 2).

5 Analysis of beam section in flexure

Over the years, there have been a number of changes to the codes for the design of reinforced concrete. BS 8110 [31] is a British Standard for the design and construction of reinforced concrete structures, first introduced in the mid-1980s. In 2010, BS 8110 [31] was superseded by EN 1992 (Eurocode 2, [1]). In terms of design, both of the aforementioned standards are based on similar limit state design principles, limit state being a condition of a structure beyond which it no longer fulfils the relevant design criteria. The analysis presented in this paper concentrates on the design principles from EC 2 [1] since this is the standard currently in use for reinforced concrete beam design and will be familiar to design engineers.

5.1 Compression resisting moment, M_c

The span of the beam was 750mm with symmetrical loads applied at shear spans of 250mm (Figure 2). An idealised stress block was used to determine the maximum compressive resisting moment, M_c, of the section as shown in Figure 3. The contribution of the hanger bars in the compression zone, M_{sc} is omitted during design of under-reinforced concrete beams since the concrete is assumed to carry the compressive forces. Taking moments about the centroid of the tension steel A_s:

$$M_c = (F_{cc})(z)$$

Equation 9

where F_{cc} is the compressive force in the concrete. For beams design in accordance with EC 2 [1], different design criteria exist for concrete cylinder strengths above and below 35 N/mm^2. For concrete cylinder strengths, f_{ck}, < 35 N/mm^2, z is limited to $0.82d$ and when f_{ck} > 35 N/mm^2, z is limited to $0.86d$ in EC 2 [1]. Zero moment distribution is assumed hence the depth of the neutral axis, x, is not greater than $0.45d$ when f_{ck} is less or equal to C35/45. For concrete grades >C35/45, x is limited to $0.35d$.

The ultimate concrete design stress is $0.85f_{ck}/\gamma_c$ where the factor 0.85 relates the cylinder crushing strength to the compressive strength of concrete in the beam element and γ_c is the partial factor of safety (normally 1.5). Referring to Figure 3:

$$F_{cc} = \left(\frac{0.85f_{ck}}{\gamma_c}\right) bs$$

Equation 10

Since

$$z = d - s/2$$

Equation 11

substituting Equation 10 and Equation 11 into Equation 9 gives

$$M_c = \left[\left(\frac{1.7f_{ck}}{\gamma_c}\right) b\right] (d - z)z$$

Equation 12

Taking the partial factor of safety, γ_c equal to 1.0 for laboratory based analysis and taking a concrete grade (f_{ck}) not greater than C35/45 gives $z = 0.82d$. Substituting this into Equation 12 gives

$$M_c = [(0.251 f_{ck} b d^2)] 10^{-6}$$

Equation 13

Similarly, when the concrete grade (f_{ck}) is greater than C35/45, $z = 0.86d$ and substituting into Equation 12 gives

$$M_c = [(0.204 f_{ck} b d^2)] 10^{-6}$$

Equation 14

Equation 13 and Equation 14 were used to calculate the maximum compressive resisting moment of the beams, M_c and were dependent on the cylinder strength of the concrete used in the beam design.

5.2 Tension resisting moment, M_t

The tensile resisting moment, $M_{t(0)}$, was determined for the control beams in each test series (0% corrosion on main reinforcement) corresponding to the ultimate loads in Table 1 (col. 3) using the expression

$$M_t = (P_{ult}/2)(0.25)$$

Equation 15

where P_{ult} is the ultimate load at failure, see Figure 2. Since the beams are designed as under-reinforced, the tension resisting moment (which is also the moment of resistance of the beam) is equal to

$$M_t = \gamma_s f_y A_s z$$

Equation 16

where f_y and A_s is the steel yield stress and steel cross sectional area respectively and z is the lever arm. γ_s is the factor of safety for steel. It was originally 1.15 for design purposes based on EC 2 [1] partial factors of safety but this value has varied between 1.15 and 1.05 in recent decades [35, 36]. It is taken as 1.0 in this analysis since it is laboratory based. For analytical purposes, the tensile resisting moment is taken from Equation 15 in this paper but for prediction purposes (i.e. application of the findings to in-situ deteriorated beams, Section 6), it is taken from Equation 16.

For reinforced concrete beams exhibiting main steel corrosion

$$M_{corr} = (P_{ult}/2)(0.25)$$

Equation 17

where M_{corr} is the resisting moment of the deteriorated beam at failure under the failure load, P_{ult}, applied as shown in Figure 2.

5.3 Degree of under-reinforcement, β

The compressive resisting moment, M_c, was calculated for the beam sections using Equation 14 since f_{ck} >C35/45. The tensile resisting moment, M_t, was calculated using Equation 15. The ratio $M_{t(0)}/M_c$, labelled β, representing the degree of under-reinforcement of the control (uncorroded) beams, was obtained for all beam series. The compressive resisting moment, M_c, remains constant for each beam series as it is not affected by the degree of corrosion to the main steel reinforcement but is based on the properties of the concrete in the compression zone.

5.4 Modified-moment coefficient, K_{corr}

The moment coefficient of the deteriorated beam section is given as

$$K_{corr} = M_{corr}/f_{ck}bd^2$$

Equation 18

where K_{corr} is the modified ultimate moment coefficient at failure due to increasing levels of main reinforcement corrosion in each beam. M_{corr} is the resisting moment at failure due to corrosion of the main steel reinforcement, Equation 17. The compressive cylinder strength of the concrete is f_{ck}. In this analysis, it is taken from crushing cube samples in the laboratory (f_{cu}) but is approximated to the cylinder strength (f_{ck}) via [1]

$$f_{ck} = 0.8f_{cu}$$

Equation 19

For prediction purposes, the compressive strength is either obtained from in-situ measurements such as a (i) Rebound Hammer [37]; (ii) core extraction [38] or (iii) estimated from design data. With regards to (i), the Rebound Hammer is a fairly reliable method of determining the in-situ strength of concrete in a non-destructive manner [39].

However, for (ii), if the actual strength is obtained from crushing extracted core samples, a modification to the compression strength is required. The strength of cores is generally lower than that of standard cylinders, partly as a consequence of the drilling operation [40]. ACI 318-02 [41] suggests an 85% core/cylinder ratio. For simplicity, the cylinder/core ratio will be taken as

$$f_{ck} = (f_{ce}/0.85)$$

Equation 20

where f_{ce} is the core strength. The benefit of using methods (i) or (ii) above is that the estimation of concrete compressive strength takes into account all site conditions during casting and curing, exposure to environmental circumstances whilst in-service and includes possible weaknesses as a result of adverse chemical reactions e.g. alkali-silica reaction (ASR). If f_{ck} is estimated from design strengths (option (iii)), then the strength may be subject to considerable errors. The age of the concrete upon striking the formwork after casting is generally no more than 28 days, hence sufficient strength development is required so that the beam can carry its own and superimposed weight thereafter. However, by the time steel corrosion has occurred, the concrete will be much more mature, likely to be decades old so the compressive strength will have increased from the 28 day value. However, the long term strength gain is influenced by many parameters such as mix quantities, type of constituents, environmental considerations etc. For an accurate estimation of compressive strength, then options (i) or (ii) should be employed for better representation since the compressive strength influences the depth of stress block in the compressive zone. Section 7.2 discusses the implications of f_{ck} on the z_{corr}/d ratio for corrosion affected reinforced concrete beams.

5.5 Lever arm

The lever arm is the distance between the force in the steel in tension, F_{st} and the compressive force in the centroid of the compression zone, F_{cc}. The lever arm, z will vary as the ultimate force in the tension zone reduces due to corrosion reducing the cross-sectional area of the steel reinforcement. According to design guidelines (EC 2 [1]), the lever arm is determined from

$$z = d[0.5 + \sqrt{(0.25 - K/1.134)}$$

Equation 21

Due to the effects of corrosion, z and K are replaced by z_{corr} and K_{corr} respectively in Equation 21 and with the factor of safety, γ_c = 1.0, z_{corr} (unfactored) can be calculated as follows:

$$z_{corr} = d[0.5 + \sqrt{(0.25 - K_{corr}/1.7)}$$

Equation 22

with K_{corr} calculated as shown in Equation 18.

6 Test results and discussion

After testing, the reinforcing bars were removed from the concrete, cleaned using 10% diammonium solution and re-weighed. The weight loss of metal due to corrosion (as a percentage) was subsequently calculated. Corrosion was generally spread along the length of the main steel reinforcement. The following Section 6.1 to Section 8 provides an insight into the influence of corrosion on the flexural capacity along with providing simplified design equations for predicting the residual strength of corrosion affected reinforced concrete beams.

6.1 Influence of steel reinforcement corrosion on load/deflection

Load versus deflection diagrams for all five beam series are given in Figure 4 to Figure 8. Referring to one of these figures, say Figure 4, 2T8/26, the series is identified by the number and type of main steel reinforcement e.g. 2T8 followed by the cover to the main steel (26 mm). Each individual beam within the series again is identified by the number and type of main steel reinforcement followed by the percent of corrosion applied to the main steel e.g. 2T8/1.4% for the beam subjected to 1.4% loss of steel due to corrosion. Therefore, referring to Figure 4, seven different beams with varying degrees of main steel corrosion were tested under beam series 2T8/26, whereas nine, nine, seven and eight were tested for the other beam series, Figure 5 to Figure 8. There is a clear trend across all beam series (Figure 4 to Figure 8) in that the ultimate limit state decreases as the degree of corrosion to the main steel increases as would be expected. Therefore, the engineer needs to have an understanding of the influence of corrosion on beam performance to ensure their structures remain safely in-service.
The following sections will analyse this performance in more detail with the aim of developing simplified analytical equations to assist asset managers in predicting the residual strength of their simply supported beams suffering from main steel corrosion.

6.2 Influence of steel reinforcement corrosion on moment coefficient

Test results are given in Table 1 for all beam series. Referring to col. 1, the effective depth (d) for each beam series and the degree of under-reinforcement ($\beta = M_t/M_c$), calculated as described in Section 5.3, is given. The characteristic yield strengths of the main steel is also given. The degree of corrosion, C, occurring in the main steel of each beam is given as a percentage in Table 1, col. 2. The failure load (P_{ult}) is given in col. 3. Since the data refers to the performance of corroded reinforced concrete, symbols used in EC 2 [1] will be suffixed as appropriate to indicate the data refers to deteriorated concrete beams. Therefore, the resulting modified-moment coefficient, K_{corr}, is given in col. 4, calculated as shown in Equation 18. z_{corr}, from Equation 22, is shown in col. 5 and z_{corr}/d is given in col. 6, Table 1.

The data from columns 2, 4 and 6 in Table 1 are plotted in Figure 9 to Figure 13. Referring to Figure 9 to Figure 13, two relationships are given, namely (i) the lever arm versus moment coefficient (z_{corr}/d vs. K_{corr}, the 'lever-arm curve') and (ii) degree of corrosion versus moment coefficient (C vs. K_{corr}, the 'corrosion gradient'). Despite the level of corrosion being in the 0 % - 20 % range for all beams, the corrosion gradient varies from series to series. The corrosion gradient is linear in the form $Y = mX + c$ and shows that an increase in the degree of corrosion leads to a reduction in the moment coefficient, K_{corr} as would be expected. However, the gradient (slope m of the equation) varies across the beam series from a highest slope of -243 for beam series 2T8/26 (Figure 9) to -61 for beam series 2T10/26 (Figure 12). This relationship will be further analysed in Sections 7 and 8.

The lever arm curve from EC 2 [1] is shown (dashed line) which gives the relationship between the lever arm and the moment coefficient for the design of under-reinforced concrete beams [values taken from (42), unfactored, γ_c taken as 1.0]. Superimposed onto this curve is the modified lever arm (z_{corr}/d) and moment coefficient relationships for the beams exhibiting main steel corrosion (shown as diamond points on the curve). The control beam (0% corrosion) is identified in each figure (larger circular point) and generally yields the highest moment coefficient as would be expected whereas the modified-moment coefficient (K_{corr}) reduces as the lever arm coefficient (z_{corr}/d) increases due to corrosion, in these cases, from a value not less than 0.86 which is the limit for beams designed with concrete grades greater than C35/45. This means that the neutral axis moves upwards as less concrete is required for the concrete compressive resisting moment to balance the reducing tensile resisting moment of the beam.

The magnitude of K_{corr} reduction for each beam series is related to the slope of the corrosion gradient. The more negative the slope m in the corrosion gradient, the less widely spaced are the corresponding points on the lever arm curve for similar degrees of corrosion. Compare 2T8/26, Figure 9 with say 2T8/56, Figure 11, for a similar degree of corrosion of say 10%. K_{corr} decreases by about 0.040 for 2T8/26 ($m = -243$) but by about 0.056 for 2T8/56 ($m = -125$). This indicates a less rapid loss in moment coefficient (reducing K_{corr}) as corrosion increases for beam series 2T8/26. The opposite, therefore, is true when the corrosion slope is less negative, the corresponding values for similar degrees of corrosion on the lever arm curve are more spaced meaning the beam section is less resilient to main steel corrosion and a faster reduction in K_{corr} occurs. The influence of main steel corrosion on the properties of the beam section is further analysed in the following sections, in particular, the relationship between loss in strength and degree of under-reinforcement in the beam section.

7 *Conceptual analysis of corrosion affected singly reinforced concrete beam*

For simplicity, the following analysis concentrates on the performance of one beam series (2T8/26, Figure 9) to develop analytical equations to predict the performance of a corroded reinforced concrete beam. This concept will form the basis for more generic equations to be developed to enable the impact of corrosion on the ultimate limit state of under-reinforced concrete beams to be determined.

7.1 *Corrosion gradient*

The line of best fit for the corrosion equation for beam series 2T8/26 (Figure 9) is

$$C = -243K_{corr} + 26.2$$

Equation 23

where C is the degree of corrosion in the main steel and K_{corr} is the reduced moment coefficient due to main steel corrosion. Rewriting in terms of K_{corr} gives

$$K_{corr} = (26.2 - C)/243$$

Equation 24

As an example, assuming the degree of corrosion from a site inspection is 10% ($C = 10$), by substituting this into Equation 24 gives K_{corr} = 0.067 (actual is 0.068 in Figure 9). At $C = 0$ i.e. control beam, K_{corr} is 0.108 from Equation 24 meaning in this instance, the moment coefficient has decreased by 0.041 for 10% corrosion to the main steel. For comparison, the as-tested control beam also had K_{corr} = 0.108. The actual and predicted K_{corr} values are very similar as would be expected from the regression equation (Equation 23) with a high degree of fit ($R^2 = 0.93$, Figure 9). Clearly, Equation 24 is very specific to the beam under consideration i.e. 2T8/26. This concept will be used to establish more generic equations so residual strengths can be determined for a wide range of beams as described in Section 8.

7.2 Lever arm curve

The lever arm curve in accordance with EC 2 [1] is polynomial (i.e. quadratic) in the form

$$y = -a'X^2 - b'X + c'$$

Equation 25

Referring to Figure 9 to Figure 13, the equation of this polynomial is

$$\frac{z_{corr}}{d} = -0.53\left(K_{corr}{}^2\right) - 0.57K_{corr} + 1$$

Equation 26

which naturally applies to all beam series ($R^2 = 1$). The z_{corr}/d versus K_{corr} values change as a result of corrosion to the main steel reinforcement. However, z_{corr}/d will be minimum for the uncorroded beam when $C = 0$ (no corrosion), but not less than 0.86 d from design specifications.

K_{corr} for this particular case (2T8/26) was estimated from Equation 24 (K_{corr} = 0.108 at $C = 0$). Substituting K_{corr} = 0.108 into Equation 26 gives a value for z_{corr}/d as follows

$$z_{corr}/d = -0.53(0.108^2) - 0.57(0.108) + 1$$

Equation 27

or $z/d = 0.93$ (actual value also 0.93 in Figure 9). Taking the degree of corrosion, for example, as 10% ($C = 10$ and K_{corr} = 0.067 from Equation 24), z/d is 0.96 from Equation 26 (also 0.96 from Figure 9). This means that z_{corr}/d increases due to the lever arm moving upwards as the degree of corrosion increases. The consequence of this is that the cross-sectional area of concrete will decrease to a depth which will no longer be able to carry the forces and compression failure occurs. From a design point of view, z/d is limited to 0.95 [42]. For safety reasons, z_{corr}/d should also be limited to 0.95 but since 10% of main steel corrosion in this example has increased z/d from 0.93 to z_{corr}/d to 0.96, an explosive type compressive failure of the concrete is possible, especially if the level of corrosion increases further.

Referring to Figure 9 to Figure 13, it is clear that an increase in corrosion leads to an increase in z_{corr}/d. This effect should be considered by designing beams with a lever arm ratio closer to the minimum value (0.86 or 0.82) to better accommodate the effects of corrosion throughout its in-service life. However, the data presented in Figure 9 to Figure 13 excludes the factor of safety γ_c of 1.5 for the concrete. This essentially means that the concrete compressive strength used in the design equations is theoretically less than the actual strength. Since $f_{ck}/1.5$ is the strength used in the calculations, the assumed lower strength means that a greater depth of concrete stress block is required to carry the compressive forces. As a result, the neutral axis is lower meaning z/d is lower and theoretically, well below the 0.95 limit. Eliminating the factor of safety leads to a higher compressive strength concrete and as a result, a stress block with a lesser depth (higher neutral axis) is required to balance the compressive/tensile forces. Therefore, in reality, this gives a higher z/d ratio than is predicted via design calculations. The impact of corrosion, therefore, could be more severe as a combination of actual higher compressive strength and increasing z_{corr}/d could lead to premature compressive failure of the concrete. Substituting a maximum value of z_{corr}/d equal to 0.95 into Equation 26 and solving quadratically gives K_{corr} = 0.075. Hence, this shows that for beam 2T8/26, K_{corr} should not decrease to 0.075 (unfactored), otherwise, crushing of the concrete in the compression zone is possible. Substituting K_{corr} = 0.075 into Equation 23 gives C = 8 %, the degree of corrosion required to reduce the moment-coefficient of beam 2T8 to 0.075.

The recommendation, therefore, is to use f_{ck} and not f_{ck}/γ_C when checking z_{corr}/d for corrosion affected, under-reinforced concrete beams in addition to having an accurate estimate of f_{ck} from the options presented in Section 5.4.

7.3 Determination of modified-moment coefficient

To determine the residual moment of resistance of the corroded beam (2T8/26) with say 10% corrosion, the calculated value for K_{corr} from Equation 24 (0.067) is used in Equation 18 where $M_{corr} = K_{corr} f_{ck} b d^2$. Substituting K_{corr} and completing with f_{ck}, b and d gives the reduced moment coefficient due to main steel corrosion

$$M_{corr} = (0.067)(44.8)(100)(120^2)$$

Equation 28

or M_{corr} = 4.3 kNm. The control beam had a K_{corr} value of 0.108 meaning the original resisting moment was 6.9 kNm. In a previous publication by O'Flaherty *et al* [43] based on the same data, the service load was estimated as a percentage of the ultimate (control) load and was assumed as 40% ULS to account for factors of safety included in the design for dead (1.4) and live (1.6) loads and materials (1.5 for concrete and 1.05/1.15 for steel). It was stated that '*beams exhibiting main steel corrosion greater than 10 % generally failed in flexure before reaching the service load. Therefore, beams in practice with main steel corrosion approaching 10% should be considered as reaching their serviceability limit state and repair and maintenance is required to extend their service life*'. Du et al [4] also concluded that beam ductility became an issue when corrosion exceeded about 10%, albeit for very under-reinforced beams (in their case, 0.87% bh). Cabrera [19] found that when the corrosion level reached 9%, the beam deflection increased by 150% compared to the control, uncorroded beams. Almusallam [44] studied the effect of corrosion on the properties of steel reinforcement bars and found that it was possible to have a sudden failure of reinforced concrete slabs in flexure when the degree of corrosion was greater than 13% since the brittle behaviour was evident. In this current example (2T8/26), the resisting moment has reduced to about 60 % ULS for 10% loss of cross sectional area of the main steel due to corrosion so this gives an indication of residual service strengths for simply supported under-reinforced beams exhibiting main steel corrosion. Although it has not quite decreased to 40 % ULS as stated above, only one type of beam section was analysed i.e. 2T8/26 with an overall cross sectional area of 100 mm x 150 mm (bh). This beam section has a degree of under-reinforcement ($\beta = M_t/M_c$) of 53 %. It was stated in Section 6.2, a lower β ratio (higher degree of under-reinforcement) is beneficial when corrosion is present. Lower β leads to a less rapid decline in residual strength.

Concrete compressive strengths can vary as can the quantity of steel reinforcement, which has a minimum quantity of 0.13 % bh to a maximum of 4 % bh. This can lead to a variation in the degree of under-reinforcement of the beam section. It will be shown in Section 8 that the residual strength due to main steel corrosion is influenced by the degree of under-reinforcement so this has to be taken into account in determining the loss in flexural strength due to corrosion.

Regardless of the rapid reduction in ultimate strength as discussed in Section 6.1 and shown in Figure 4 to Figure 8, degrees of corrosion reaching double figures should be a cause for concern irrespective of the degree of under-reinforcement.

Furthermore, it was mentioned in Section 5.2 that the factor of safety of steel reinforcement was reduced from 1.15 to 1.05 [36]. However, in 2005, the standards for reinforcement were changed with the specified characteristic yield strength being increased from 460 N/mm^2 to 500 N/mm^2. As a temporary measure, the partial safety factor was changed back to 1.15 where 500 N/mm^2 reinforcement was used (BS 8110

Amendment 16016, November 2005). Further research was done which argues that the factor of safety could be reduced back to 1.05. What is clear is that the future influence of corrosion on the structural performance of reinforced concrete beams is not taken into account at the design stage, the factor of safety currently used applies to the manufacturing process. The findings in this paper show the severe impact reinforcement corrosion can have on the ultimate strength suggests that a higher factor of safety would be beneficial, hence γ_s = 1.15 would be preferable. However, a detailed analysis of the factor of safety of the steel reinforcement is outside the scope of this paper.

8 *Development of analytical equations for predicting residual strength of corrosion affected singly reinforced concrete beams - Concrete grade >C35/45*

The resisting moment of the control beam in the previous analysis was obtained from flexural testing in the laboratory and the results showed that the beams failed in a ductile manner confirming they were under-reinforced. All beams had concrete cylinder strengths, f_{ck}, greater than C35/45 hence these beams have a lever arm limit $z = 0.86d$ as described in Section 5.1. The equations developed for the corrosion gradients were very specific to the beam series in question (i.e. 2T8/26). In order to be able to apply the laboratory findings to universal cases, the corrosion slope equations need to be generalised to cover a much broader range of beams as would be encountered in-situ. For example, beam properties such as the design moment coefficient will be required from design data as will design z/d before the findings in this paper can be applied.

Referring to Equation 23, the corrosion slope equation was in the form $Y = mX + c$, or rewriting in terms of the degree of corrosion, C

$$C = (m)(K_{corr}) + c$$

Equation 29

Referring to Equation 29, C is the degree of corrosion obtained from a site inspection. The slope m and the intercept c vary from beam to beam (see Figure 9-Figure 13) and these need to be identified for each generic case. The reduced moment coefficient K_{corr} is directly related to the degree of corrosion in the main steel. The following analysis determines K_{corr}, m and c for generic cases based on the findings in Figure 9 to Figure 13 to enable residual strengths to be determined.

8.1 Determination of moment coefficient, K_{corr}

The design moment of resistance of the under-reinforced beam section is $M_t = f_y A_s z$ (unfactored) from Equation 16. At ductile failure, the resisting moment (M_t) will be balanced by a compressive resisting moment, $M_{c(f)}$ so M_t = $M_{c(f)}$. At 0% corrosion, $K_{corr} = K$ so Equation 18 can be written as

$$K = M_t / f_{ck} b d^2$$

Equation 30

Substituting $M_t = f_y A_s z$ into Equation 30 gives

$$K = \frac{f_y A_s z}{f_{ck} b d^2}$$

Equation 31

However,

$$z = d - s/2$$

Equation 32

and

$$s = \frac{f_y A_s}{0.85 f_{ck} b}$$

Equation 33

Therefore,

$$K = \frac{f_y A_s (d - \frac{s}{2})}{0.85 f_{ck} b d^2}$$

Equation 34

Geometric design details such as f_y, A_s, b and d are known or calculated. The concrete compressive strength is obtained as described in Section 5.4. Equation 34 can, therefore, be used to calculate K for an uncorroded section where $C = 0$ which is the moment coefficient for the as-designed (uncorroded) beam section.

8.2 Determination of corrosion slope, m

Referring to Table 1, the beam characteristics for the five series are given in col. 1. The degree of under-reinforcement ranges from 53 % to 96 % (ratios 0.53-0.96), calculated as given in Section 5.3. The analysis of beam Series 2T8/26 in the previous section is applicable only to beams exhibiting similar characteristics, for example a degree of under-reinforcement of 53 % which gives a unique K value and z/d ratio. This section investigates the performance of other beams with different degrees of under-reinforcement so generic equations can be established to predict the residual strength of corroded reinforced concrete beams.

The relationship between the degree of under-reinforcement, β and the corrosion slope coefficient is plotted in Figure 14 for the five beam series in this investigation (concrete grade >C35/45). It shows that a lower β (higher degree of under-reinforcement) leads to a more negative slope, m. The relationship is given by the equation

$$m = 458\beta - 485$$

Equation 35

In the previous example (2T8/26), β was 53% (or 0.53) so substituting this into Equation 35 gives m = -242. m was -243 from Figure 9 so a very good correlation exists.

8.3 Determination of slope intercept, c

Referring to Equation 29, two components of the corrosion slope equation are now known, those being K from Equation 34 and m from Equation 35. Taking $C = 0$ in

Equation 29 and rearranging gives an expression for the intercept for a generic case which involves using the design value for K i.e. the uncorroded moment coefficient which will be a maximum K value

$$0 = (m)(K) + c$$

Equation 36

Simplifying Equation 36 gives

$$c = -(m)(K)$$

Equation 37

Assuming K has been calculated from Equation 34 as 0.108 and m = -242 from Equation 35 as in the above example then

$$c = -(-242)(0.108)$$

Equation 38

or c = 26.1 Therefore, for beam series 2T8/26, line of best fit for the corrosion equation was $C = -243 + 26.2$ from Equation 23 based on actual test data. Using the generic findings from this section gives

$$C = -242K_{corr} + 26.1$$

Equation 39

For the same beam section, assuming a degree of corrosion of say 10%, substituting $C = 10$ into Equation 39 gives K_{corr} = 0.067 which is the same value as determined from Figure 9. Therefore, the generic equations closely matches the actual performance for predicting the residual strength of deteriorated reinforced concrete beams.

9 *Development of analytical equations for predicting residual strength of corrosion affected singly reinforced concrete beams - Concrete grade <C35/45*

Since the findings from this research applied only to beams with concrete grades, f_{ck}, greater than C35/45 (Section 8), this section investigates the relationship between the degree of under-reinforcement and corrosion slope coefficient for beams with the concrete grades, f_{ck}, less than C35/45 since different design criteria apply for concrete strengths above and below this threshold in EC 2 [1]. This was achieved via an extensive literature review. However, despite the availability of an abundance of research publications on the performance of corrosion damaged reinforced concrete beams, only a limited number could be used due to a mismatch between the level of information available to calculate the degree of under-reinforcement and corrosion slope coefficient. However, a sufficient number of publications was obtained and analysed in a similar manner to those presented in Section 8 (it should also be noted that the aim was also to integrate others' findings into the data set in Section 8 but no test data with f_{ck} > C35/45 was found). The findings are shown in Figure 15. Referring to Figure 15, the relationship between degree of under-reinforcement, β, and corrosion slope coefficient, m is

$$m = 257\beta - 385$$

Equation 40

for concrete grades less than C35/45. Also shown in Figure 15 is a snapshot of the assumptions made whilst using the data. The level of detail required was such that it was extremely difficult to have every piece of design data available so a number of assumptions, within reason, had to be made to ensure the data was usable. The regression, R^2, as a result is still quite high (0.70) but it perhaps too much to expect a better linear fit considering the number of variables involved in calculating the relationships. Nevertheless, the data does show that there is a relationship between degree of under-reinforcement and the corrosion slope coefficient for beams with concrete grades <C35/45.

The steps outlined in Sections 8.2 and 8.3 for concrete with grades greater than C35/45 can be followed here to get an estimation for K_{corr} for beams with concrete cylinder strengths less than C35/45.

10 Applications of findings to actual beams

The analytical findings from Section 8 and Section 9 are applied in a step-by-step manner to two real scenarios where simply supported, in-situ under-reinforced beams of dimensions 200 x 550 mm with concrete grades >C35/45 and <C35/45 are

assumed to each suffer from 10% degree of corrosion to the main steel reinforcement. The analysis predicts the loss in moment-coefficient for each beam. It results in a reduction in the ultimate moment of resistance from 294.5 kNm to 171.9 kNm for Scenario (i), 58% residual capacity; for Scenario (ii), the ultimate moment capacity reduces from 150.1 kNm to 101.0 kNm, 67% residual capacity (both unfactored). In addition, z_{corr}/d becomes 0.95 for both which indicates a reduced depth of the stress block compared to the uncorroded sections. The degree of under-reinforcement, β, is 68 % and 45 % respectively. This analysis also shows that a higher degree of under-reinforcement, lower β ratio for a similar degree of main steel corrosion (10%) results in a lower reduction of moment capacity (although Scenario (i) was for concrete grades >C35/45 and Scenario (ii) for concrete grades <C35/45).

It was mentioned in Section 1 that a number of researchers linked the loss in flexural strength to degree of corrosion and generally speaking, approximately 1% degree of corrosion led to a beam flexural strength loss also of about 1%. El Maaddawy et al [5] found that the reduction in beam flexural strength was almost proportional to reduced cross sectional area of the steel reinforcement due to corrosion, Yu et al [8] suggested that 1% loss in cross sectional area corresponded to a 1% loss in yield and ultimate capacity and Malumbela et al [9] reported that 1% loss in steel mass reduced the flexural capacity of deteriorated RC beams by 0.7% and that the ultimate moment capacity reduced linearly with the degree of corrosion. The findings in Table 2 show that this is very conservative and that for 10% degree of corrosion, the loss in moment capacity is much greater than 10%. The findings in this research are in better agreement with findings by Du et al [4], Cabrera [19] and Almusallam [44] as discussed in Section 7.3. In addition, the likelihood is that as the degree of corrosion increases, a larger number of deeper pits will form which will give rise to a much higher loss in cross-sectional area at certain locations along the beam, similar to findings by El Maaddawy et al [5] and Torres Acosta et al [6].

The analysis presented, however, does not take into account characteristics such as different span/depth ratios which may have an influence of overall performance since further categorising the limited data in Figure 14 and Figure 15 would lead to insufficient trends emerging from the data. Likewise, the difference between artificial and naturally generated corrosion could also be investigated i.e. Ye et al [12] but currently there is insufficient data for clear conclusions to emerge.

11 Conclusions

The following are the conclusions emanating from the laboratory study and associated analysis of the influence of main steel corrosion on the performance of singly reinforced concrete beams:

a. The load/deflection characteristics for all under-reinforced beams show increasing reductions in ultimate strength as corrosion increases. This is especially so when degrees of corrosion reach double figures

b. A relationship exists between the increasing degree of corrosion and reducing moment-coefficient, K_{corr}. There is a less rapid loss in moment-coefficient (reducing K_{corr}) as corrosion increases for beams exhibiting higher degrees of under-reinforcement (lower β where $\beta = \frac{M_t}{M_c}$). The opposite is true when β is higher.

c. It is preferable that z/d ratios closer to the lower limit (0.86 and 0.82 for concrete grades greater and less than C35/45 respectively) are specified at the design stage, since corrosion to the main steel during the in-service life leads to a higher z_{corr}/d ratio and possible premature crushing of the concrete in compression due to insufficient depth of the stress block to carry the compressive forces

d. A lower concrete design strength, f_{ck}, is preferable in design as this helps to increase the depth of compressive stress block, thereby keeping the z/d ratio lower. In addition, specifying lower M_t/M_c ratios and keeping the quantity of main reinforcement as close as possible to 0.13% with preference given to smaller bar diameters also provides a more durable beam section.

e. An indication of the residual strength of a corroded, singly reinforced simply supported beam can be determined by knowing geometric details of the beam in addition to having knowledge of the degree of corrosion to the main steel and compressive strength of the concrete.
    - For beams with concrete grades greater than C35/45, the corrosion slope coefficient, m, can be obtained from $m = 458\beta - 485$
    - For beams with concrete grades less than C35/45, the corrosion slope coefficient, m, can be obtained from $m = 257\beta - 385$

Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflict of interest.

Table 1 Reinforced concrete beam design characteristics and test results

Beam Series/ Characteristics	**Corrosion** *(C %)*	P_{ult} *(kN)*	K_{corr} $\left(\frac{M_{corr}}{f_{ck}bd^2}\right)$	z_{corr}	l $\frac{z_{corr}}{d}$
(1)	(2)	(3)	(4)	(5)	(6)
2T8/26	0.0	55.8	0.108	111.8	0.93
	1.4	54.7	0.080	114.1	0.95
A_s = 78.5 mm^2	2.3	43.8	0.106	112.0	0.93
d = 120 mm	8.5	41.1	0.037	117.3	0.98
f_{ck} = 44.8 N/mm^2	10.0	35.0	0.068	115.0	0.96
$\beta = M_t/M_c$ = 0.53	15.5	19.2	0.040	117.1	0.98
f_y = 460 N/mm^2	18.5	20.6	0.085	113.7	0.95
2T8/36	0.0	55.2	0.136	100.4	0.91
	0.8	56.8	0.140	100.1	0.91
A_s = 78.5 mm^2	0.9	52.1	0.128	101.0	0.92
d = 110 mm	1.4	45.0	0.111	102.3	0.93
f_{ck} = 42.0 N/mm^2	4.6	44.6	0.110	102.4	0.93
$\beta = M_t/M_c$ = 0.67	8.5	34.7	0.085	104.2	0.95
f_y = 460 N/mm^2	9.6	34.2	0.084	104.3	0.95
	15.0	24.6	0.060	105.9	0.96
	17.8	20.9	0.051	106.6	0.97
2T8/56	0.0	41.4	0.153	81.0	0.90
	7.7	34.5	0.124	82.9	0.92
A_s = 78.5 mm^2	5.3	33.3	0.092	84.8	0.94
d = 90 mm	2.7	42.9	0.159	80.6	0.90
f_{ck} = 41.8 N/mm^2	3.5	39.2	0.145	81.5	0.91
$\beta = M_t/M_c$ = 0.75	6.9	34.6	0.128	82.6	0.92
f_y = 460 N/mm^2	15.1	17.1	0.063	86.5	0.96
	8.9	26.3	0.097	84.5	0.94
	16.4	10.1	0.037	88.0	0.98
2T10/26	0.0	87.1	0.196	103.2	0.87
A_s = 100.5 mm^2	4.2	87.0	0.195	103.2	0.87
d = 119 mm	4.5	82.5	0.185	104.2	0.88
f_{ck} = 39.3 N/mm^2	2.6	68.5	0.154	107.0	0.90
$\beta = M_t/M_c$ = 0.96	4.0	65.3	0.147	107.6	0.90
f_y = 460 N/mm^2	6.5	64.5	0.145	107.8	0.91
	7.5	62.7	0.141	108.1	0.91
2T10/36	0.0	76.0	0.167	97.0	0.89
A_s = 100.5 mm^2	2.5	69.4	0.199	94.3	0.86
d = 109 mm	3.3	70.7	0.153	98.1	0.90
f_{ck} = 47.8 N/mm^2	6.7	57.7	0.155	97.9	0.90
$\beta = M_t/M_c$ = 0.82	8.3	55.2	0.127	100.2	0.92
f_y = 460 N/mm^2	9.2	54.0	0.121	100.6	0.92
	2.3	70.5	0.119	100.8	0.92

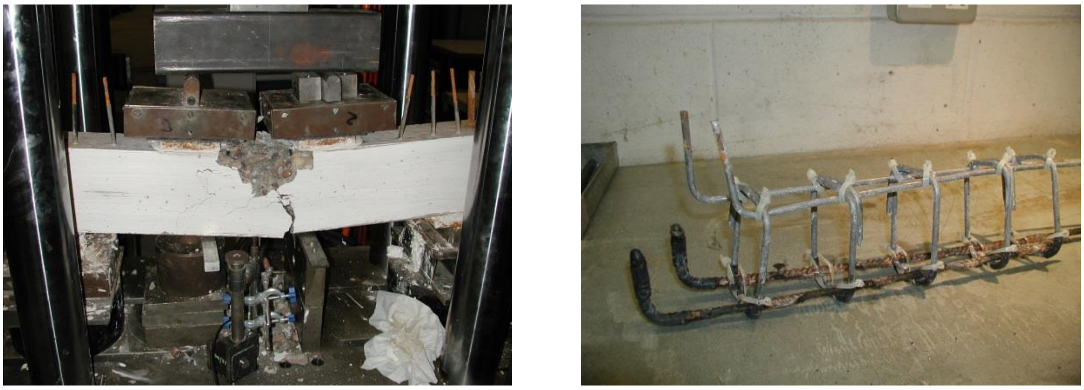
Table 2 Determination of residual strength - worked examples

No	Details	Equation/ Section	EC 2 [1] Concrete > C35/45 ($\beta = 0.68$)	EC 2 [1] Concrete < C35/45 ($\beta = 0.45$)
1	Obtain geometric details of the beam: Width (b) Depth (h). Main steel reinforcement details (A_s), either from drawings or site inspections. Cover (c_v) to the main steel Calculate effective depth (d)	-	*b = 200mm* *h = 550mm* *Main steel = 4 x 20 mm* *(A_s = 1256 mm^2, 1.1% bh)* *c_v = 25 mm* *d = 515 mm*	*b = 200mm* *h = 550mm* *Main steel = 2 x 20 mm* *(A_s = 628 mm^2, 0.57%)* *c_v = 25 mm* *d = 515 mm*
2	Obtain compressive strength of the concrete, either from design data or an on-site rebound hammer test or core extraction	Section 5.4	*f_{ck} = 40 N/mm^2* (design data)	*f_{ck} = 25 N/mm^2* (design data)
3	Estimate the degree of corrosion of the main steel by exposing via break-out. Guidance can be obtained elsewhere [45]	-	*Say 10%*	*Say 10%*
4	Calculate the maximum compressive resisting moment (M_c) of the concrete section	Equation 13 Equation 14	$M_c = 0.204 f_{ck} b d^2$ *(unfactored)* $M_c = (0.204)(40)(200)(515^2)$ $M_c = 432.8\ kNm$	$M_c = 0.251 f_{ck} b d^2$ *(unfactored)* $M_c = (0.251)(25)(200)(515^2)$ $M_c = 332.9\ kNm$
5	Calculate the depth of the stress block, s from the equilibrium of forces in the concrete and steel	Equation 33	$s = (f_y A_s)/(0.85 f_{ck} b)$ $s = (500)(1256)/(0.85)(40)(200)$ $s = 92\ mm$	$s = (f_y A_s)/(0.85 f_{ck} b)$ $s = (500)(628)/(0.85)(25)(200)$ $s = 74 mm$
6	Calculate the lever arm, z	Equation 32	$z = d - s/2$ $z = 515 - 92/2$ $z = 469\ mm$	$z = d - s/2$ $z = 515 - 74/2$ $z = 478 mm$
7	Calculate z/d	No. 6 / No. 1	$z/d = 469/515$ $z/d = 0.91$	$z/d = 478/515$ $z/d = 0.93$
8	Calculate the depth of the neutral axis, x.	No. 5	$x = s/0.8$ $x = 92/0.8$	$x = s/0.8$ $x = 74/0.8$

	If $x < 0.636d$, the section is under-reinforced		$x = 115\ mm$ $(0.35)(515) = 180\ mm > 115\ mm$	$x = 93\ mm$ $(0.45)(515) = 232\ mm > 93\ mm$
9	Calculate the design resisting moment, M_t for the singly reinforced concrete beam	Equation 16	$M_t = f_y A_S z$ $M_t = (500)(1256)(469)$ $M_t = 294.5\ kNm$	$M_t = f_y A_S z$ $M_t = (500)(628)(478)$ $M_t = 150.1\ kNm$
10	Calculate the degree of under-reinforcement of the section, β	No. 9 / No. 4	$\beta = \frac{M_t}{M_c} = \frac{294.5}{432.8} = 0.68$	$\beta = \frac{M_t}{M_c} = \frac{150.1}{332.9} = 0.45$
11	Estimate the corrosion slope factor, m or obtain from the equation	Equation 35 Equation 40	$m = 458\beta - 485$ $m = 458(0.68) - 485$ $m = -173.6$	$m = 257\beta - 385$ $m = 257(0.45) - 385$ $m = -269.4$
12	Calculate K for the as-designed section	No. 9 Equation 30	$K = M_t / f_{ck} b d^2$ $K = 294.5\ x\ 10^6 / (40)(200)(515^2)$ $K = 0.139$	$K = M_t / f_{ck} b d^2$ $K = 150.1\ x\ 10^6 / (25)(200)(515^2)$ $K = 0.113$
13	Determine the corrosion intercept, c	Equation 37	$c = -(m)(K)$ $c = -(-173.6)(0.139)$ $c = 24.1$	$c = -(m)(K)$ $c = -(-269.4)(0.113)$ $c = 30.4$
14	Estimate K_{corr} from degree of corrosion on the main steel (5%), the corrosion slope coefficient m and corrosion intercept, c	Equation 29 No. 13 No. 11	$K_{corr} = C - c/m$ $K_{corr} = 10 - 24.1/-173.6$ $K_{corr} = 0.081$	$K_{corr} = C - c/m$ $K_{corr} = 10 - 30.4/-269.4$ $K_{corr} = 0.076$
15	Calculate change in K as a result of corrosion	No. 14 No. 12	$\Delta K = K - K_{corr}$ $\Delta K = 0.139 - 0.081$ $\Delta K = 0.058$ *(unfactored)*	$\Delta K = K - K_{corr}$ $\Delta K = 0.113 - 0.076$ $\Delta K = 0.037$ *(unfactored)*
16	Calculate loss in moment capacity	No. 15 No. 1	$\Delta M_{corr} = (\Delta K) f_{ck} b d^2$ $\Delta M_{corr} = (0.058)(40)(200)(515^2)$ $\Delta M_{corr} = 122.6\ kNm$ *(unfactored)*	$\Delta M_{corr} = (\Delta K) f_{ck} b d^2$ $AM_{corr} = (0.037)(25)(200)(515^2)$ $AM_{corr} = 49.1\ kNm$ *(unfactored)*
17	Compare as-designed resisting moment to the in-service (corroded) resisting moment	No. 9 No. 16	$M_{corr} = M_t - \Delta M_{corr}$ $M_{corr} = 294.5 - 122.6$ $M_{corr} = 171.9\ kNm$ *(unfactored)* *(58% residual capacity)*	$M_{corr} = M_t - \Delta M_{corr}$ $M_{corr} = 150.1 - 49.1$ $M_{corr} = 101.0\ kNm$ *(unfactored)* *(67% residual capacity)*

18	Calculate z_{corr}/d	Equation 27	$z_{corr}/d = -0.53(0.081^2) - 0.57(0.081) + 1$ $z_{corr}/d = 0.95$	$z_{corr}/d = -0.53(0.076^2) - 0.57(0.076) + 1$ $z_{corr}/d = 0.95$

List of Figures



(a) (b)

Figure 1 (a) Beam 2T10/56 exhibiting classical flexural failure (vertical cracks in the flexural span and crushing of the compressed concrete); (b) removed steel cage showing corrosion to main steel

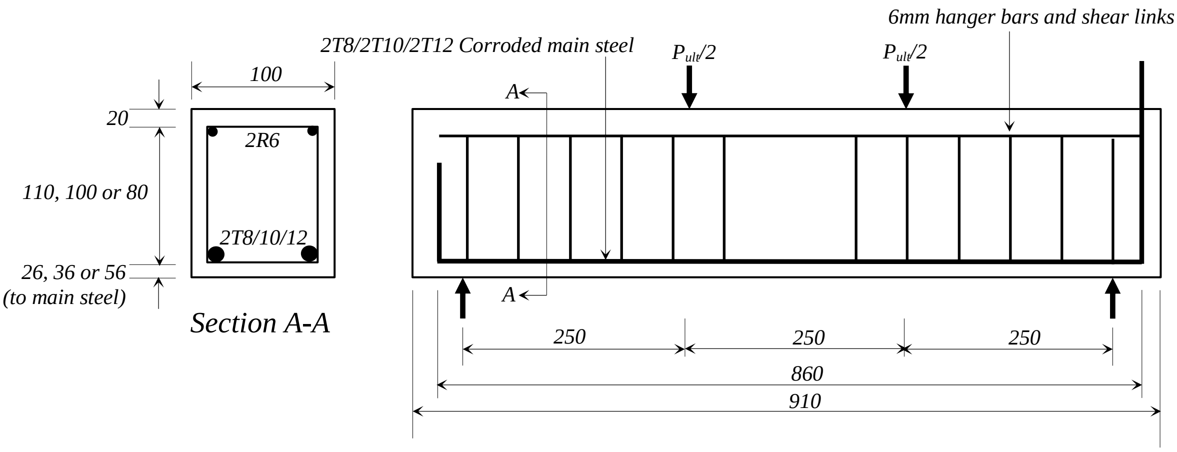


Figure 2 Beam specimen details

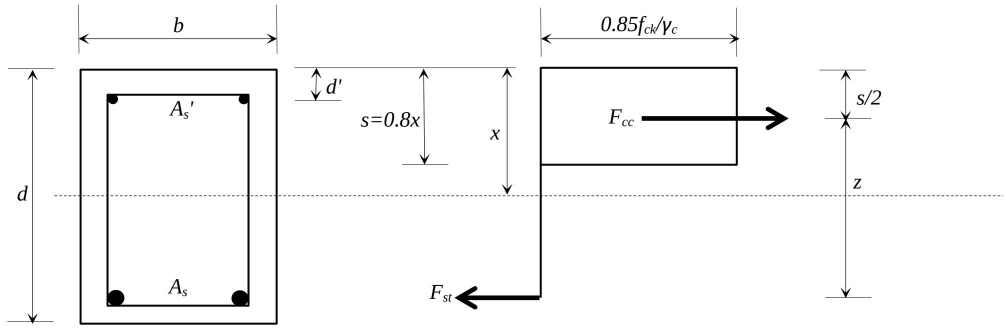


Figure 3 Singly reinforced section

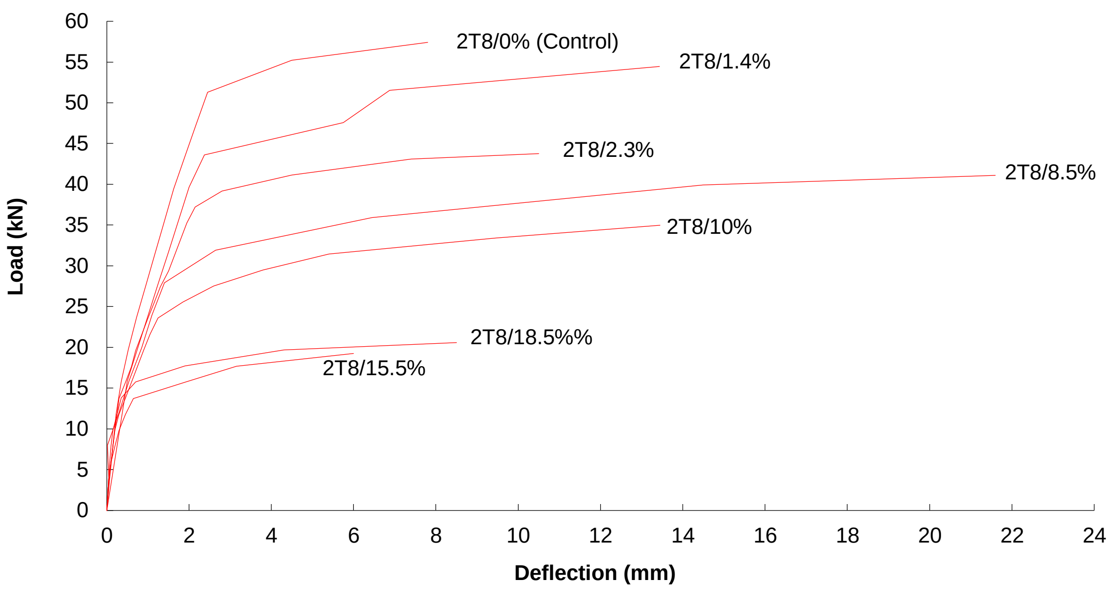


Figure 4 Load vs. deflection graph for Beam Series 2T8/26

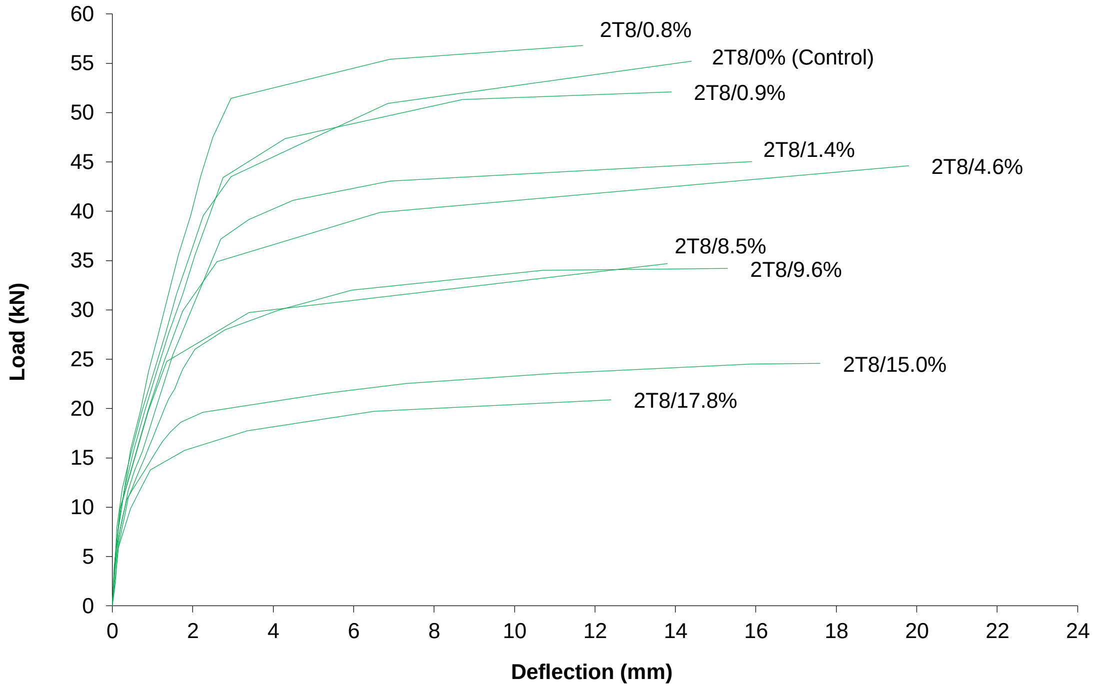


Figure 5 Load vs. deflection graph for Beam Series 2T8/36

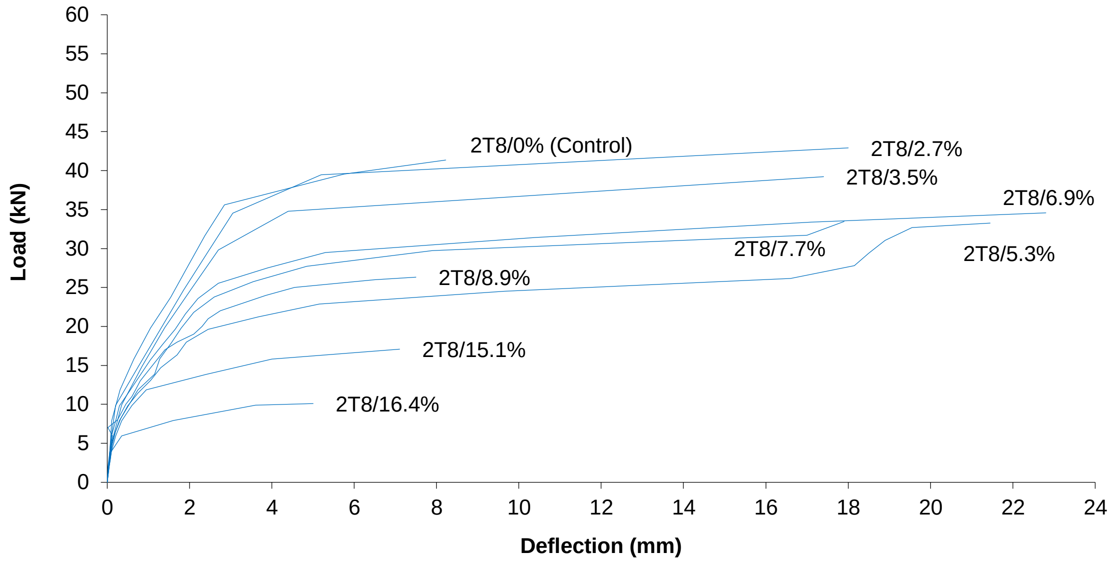


Figure 6 Load vs. deflection graph for Beam Series 2T8/56

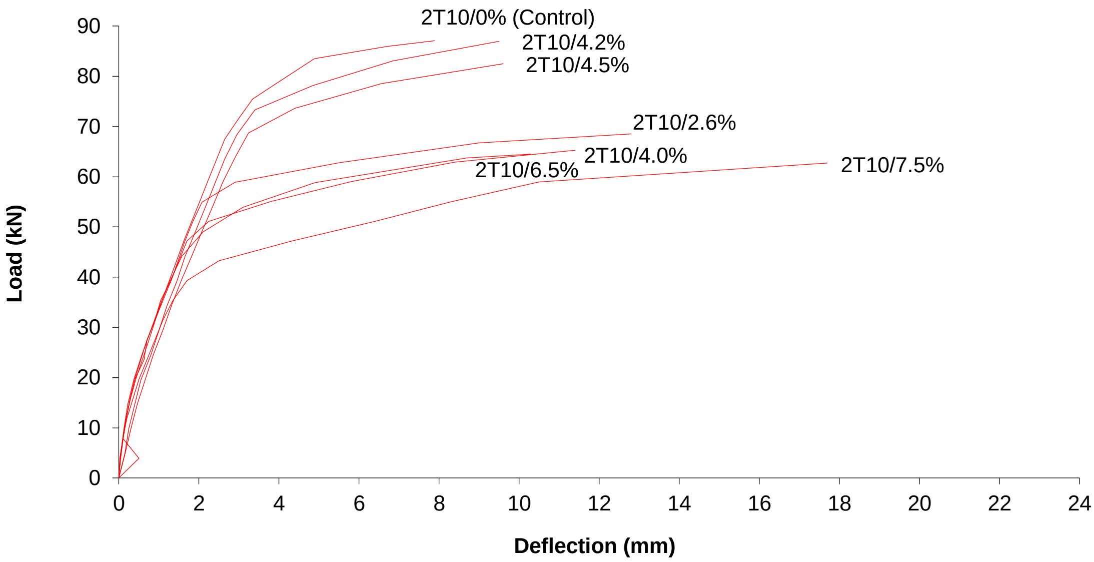


Figure 7 Load vs. deflection graph for Beam Series 2T10/26

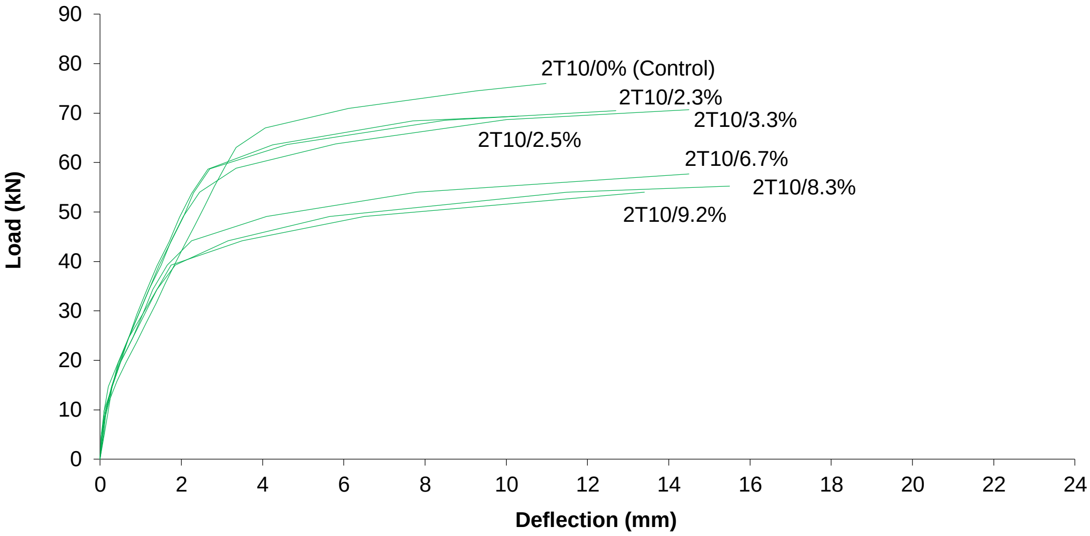


Figure 8 Load vs. deflection graph for Beam Series 2T10/36

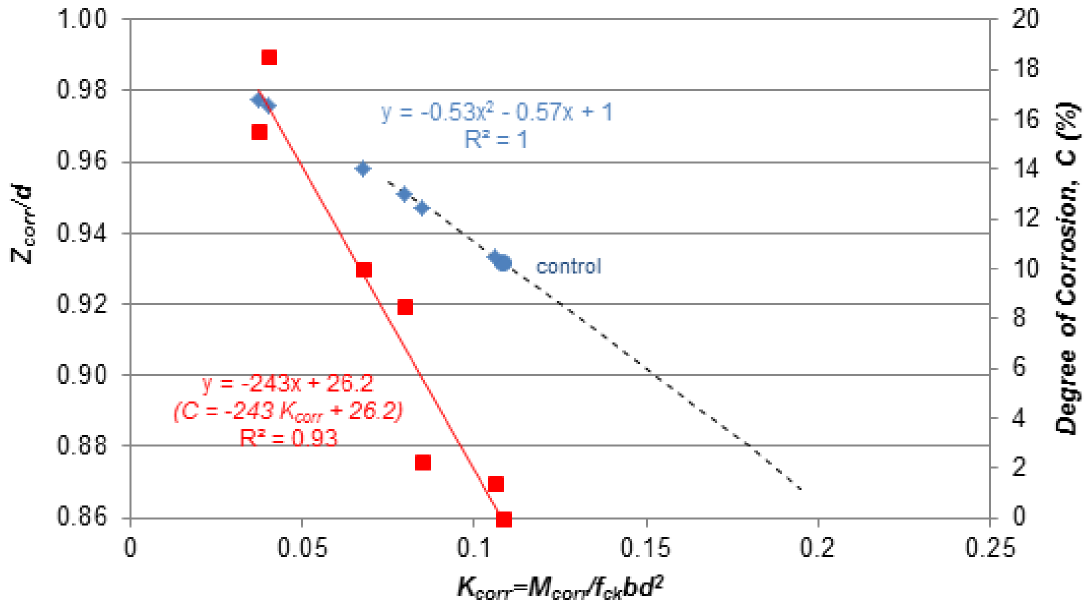


Figure 9 Influence of corrosion on moment coefficient for Beam Series 2T8/26

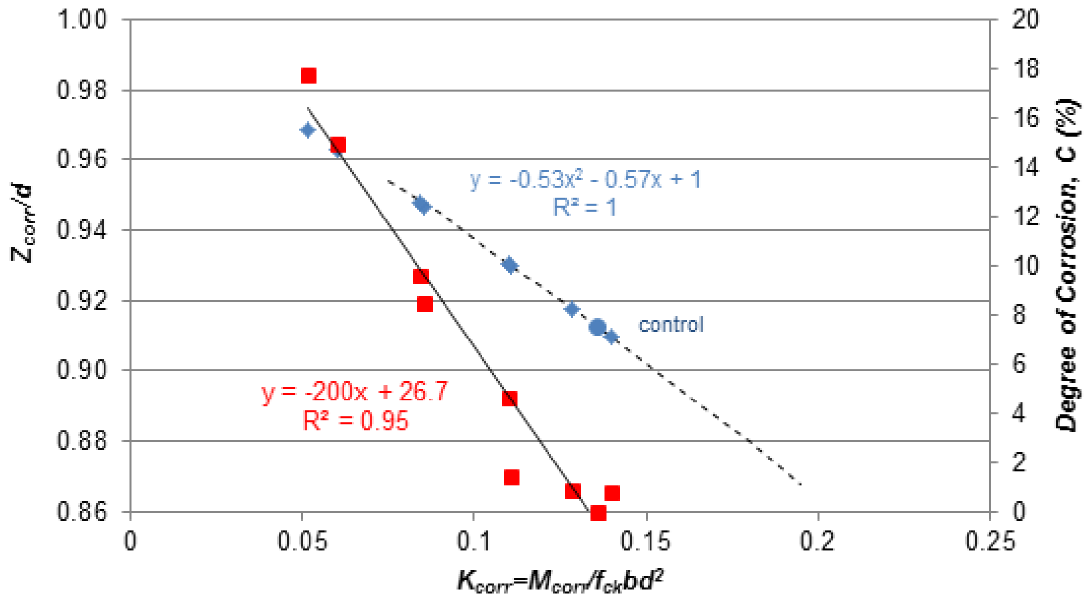


Figure 10 Influence of corrosion on moment coefficient for Beam Series 2T8/36

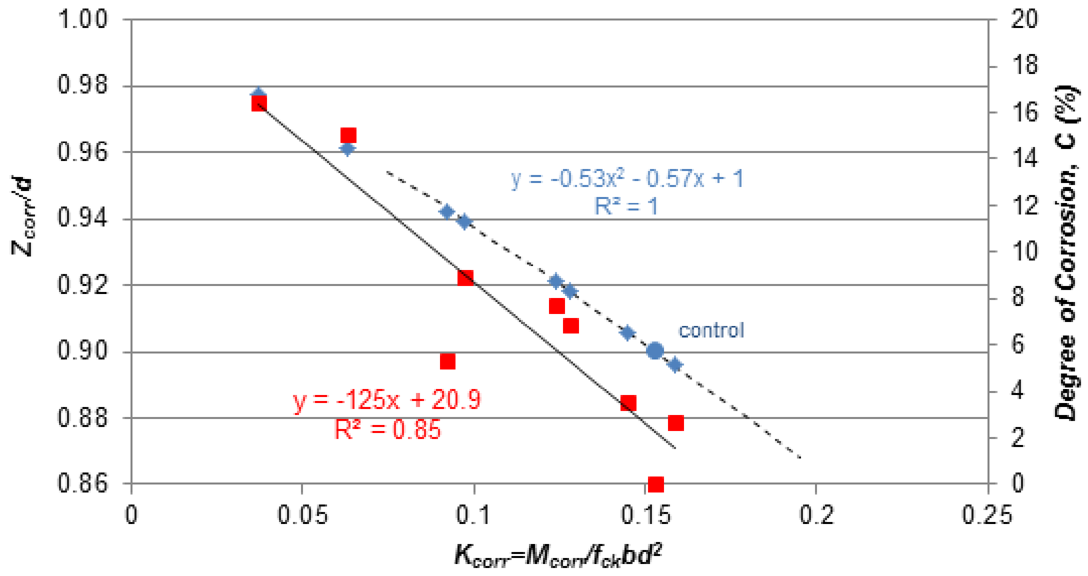


Figure 11 Influence of corrosion on moment coefficient for Beam Series 2T8/56

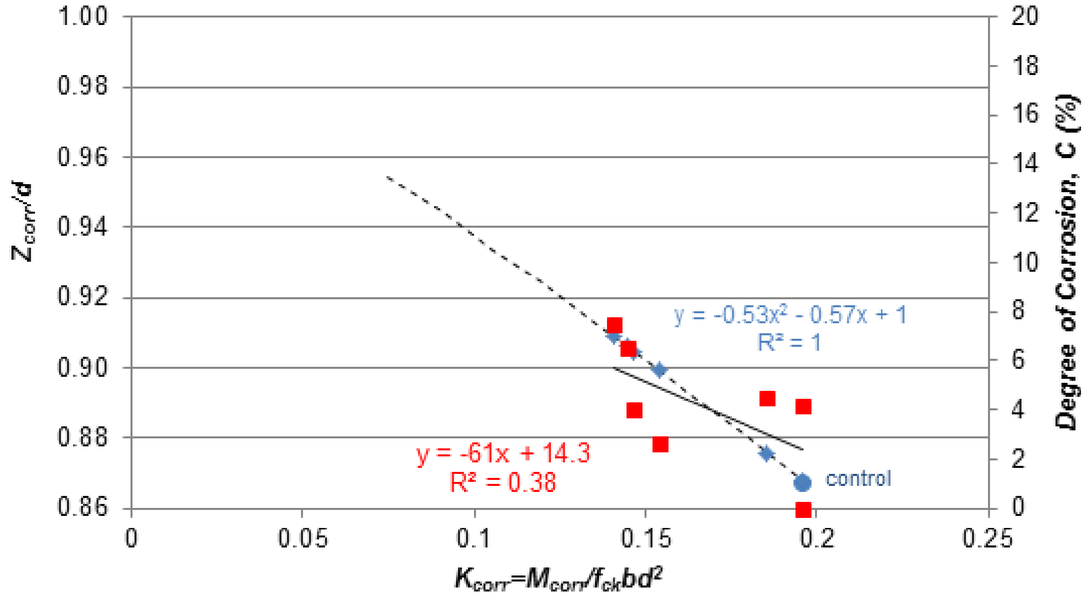


Figure 12 Influence of corrosion on moment coefficient for Beam Series 2T10/26

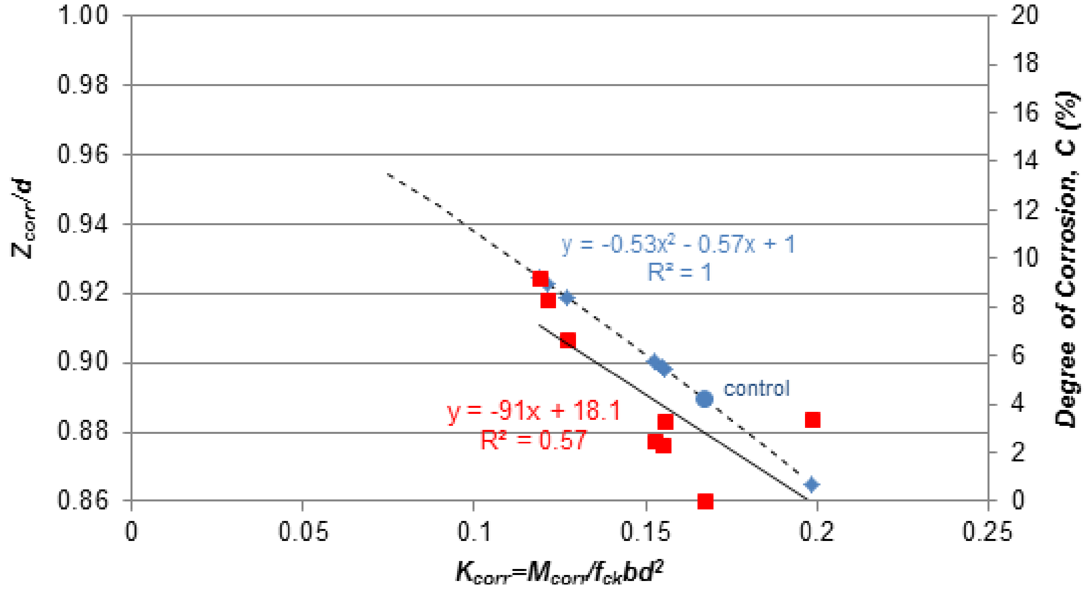


Figure 13 Influence of corrosion on moment coefficient for Beam Series 2T10/36

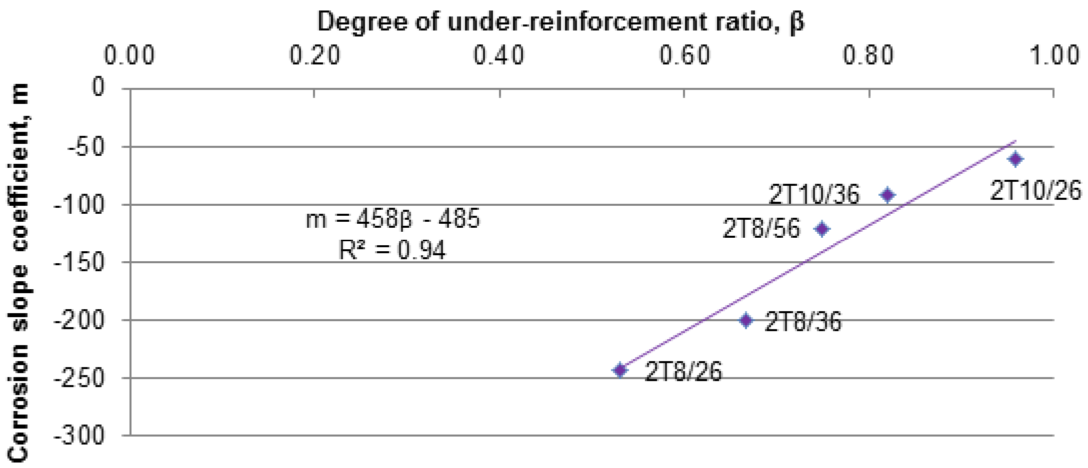
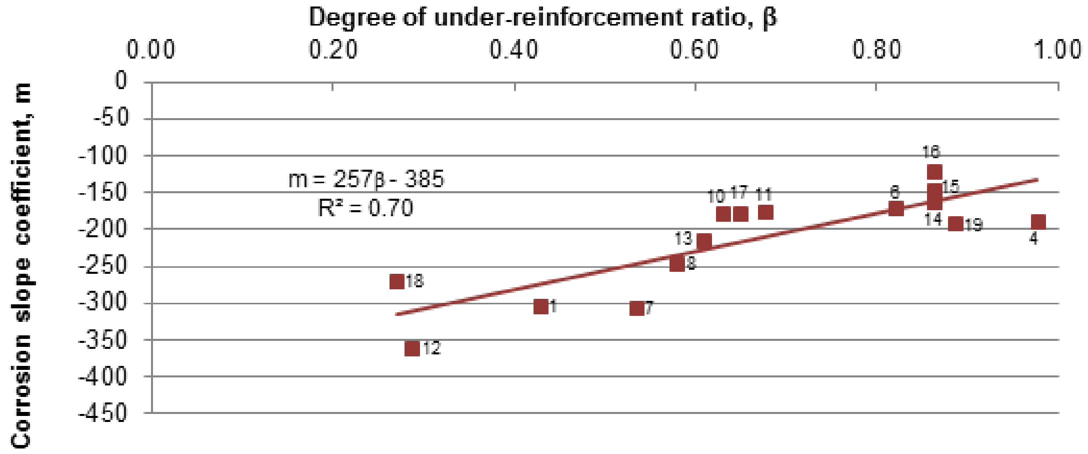


Figure 14 Relationship between degree of under-reinforcement and corrosion slope coefficient (Concrete Grade >C35/45)



No.	Author	Comments
1	Ahmad [17] (a)	Concrete cylinder strength given as 33-47 MPa but assumed as 33 MPa as a worst case scenario
2	Ahmad [17] (b)	Ditto. Excluded, β > 1
3	Ahmad [17] (c)	Ditto. Outlier so excluded from data
4	Ahmad [17] (d)	Ditto.
5	Ahmad & Kamran [46]	Excluded, β > 1
6	Azad et al [47] (a)	-
7	Azad et al [47] (b)	-
8	Azad et al [47] (c)	-
9	Azad et al [47] (d)	Excluded, β > 1
10	Azad et al [47] (e)	-
11	Azad et al [47] (f)	-
12	Naga & Vamsi [48]	Data based on cantilever beam. f_y assumed as 415 MPa from type of steel used (Fe 415)
13	Du et al [4]	Concrete cover assumed as 25 mm. Concrete compressive strengths assumed as cube strengths (UK based testing)
14	Mangat et al [34] (a)	-
15	Mangat et al [34] (b)	-
16	Mangat et al [34] (c)	-
17	Mangat et al [34] (d)	-
18	Torres-Acosta et al [49]	Degrees of corrosion estimated from weight of metal lost
19	Xia et al [50] (a)	-
20	Xia et al [50] (b)	Excluded, β > 1

Figure 15 Relationship between degree of under-reinforcement and corrosion slope coefficient (Concrete Grade <C35/45)

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