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Profit and loss Sharing Negotiations involving a VC and an entrepreneur: A Game Theoretic Approach with Agent Based Simulation

Adil ELFAKIR*, Mohamed TKIOUAT**

Abstract

Profit and Loss Sharing contracts (PLS) are forms of financing where profits are shared according to a predetermined ratio and losses are shared according to each participant’s ratio in the project’s capital. We try to reduce moral hazards by solving for an optimal profit sharing ratio that inhibits the entrepreneur from exerting a lower managerial effort. We follow a game theoretical approach under observable and unobservable entrepreneurial effort. We found theoretical evidence, on one hand, that a specific profit sharing ratio can be developed under observable effort. On the other hand, due to asymmetric information under the unobservable efforts case, a profit sharing span of negotiation was developed. This span of negotiation satisfies the participation and the incentive constraints of the game participants. Within this span of negotiation, we propose a model that helps in identifying an optimum profit sharing ratio based on the participants’ bargaining power. Due to the stochastic nature of the model parameters, we develop a simulation of the game in an agent based platform using Netlogo. Besides serving as a quick tool for numerical calculations and analysis, this platform serves as a decision tool for the VC to decide whether or not to extend the funding contract to the entrepreneur.

Keywords: Finance, Optimal contracts, Moral hazards, Profit and loss sharing contracts, Span of Negotiation.

1. Introduction

VCs are increasingly becoming a vehicle of financing start-up enterprises. The latters however, and due to their innovative nature, carry a high probability of failure risk (Bergemann and Hege, 1998).

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Preprint submitted to British Accounting and Finance Association

February 19, 2019
For this reason, many forms of studies have explored the risks facing the VCs financing. The first form of studies are empirical in nature (Sahlman 1990), (Amit et al. 1990), (Cochrane 2005), (Baierl et al. 2002), (Hall and Lerner 2010), (MacIntosh and Cumming 1997), (Gompers and Lerner 1999) Gompers and Lerner (1999), (Jain 2001), and (Kaplan and Strömberg 2003) and (Tykvová 2007). The second stream of studies are analytical in nature (Casamatta 2003), (Elitzur and Gavious 2003), (Keuschnigg and Nielsen 2003b), (Keuschnigg and Nielsen 2003a) and (Neher 1999).

Despite their different approaches, empirical or analytical, no one research disagree that failure risks of the VCs are mainly due to agency problems between the VC and the financed entrepreneur. One of the sources of the VC’s financed companies’ failure risks is moral hazard, where one party acts in a selfish manner regardless of the actions of their partners (Elitzur and Gavious 2003). Moral hazard is manifested in different forms. One such form is that the entrepreneur knows more about his or her quality/ability than the VC. A second form, which is the focus of this paper, is the shirking of the entrepreneur in terms of performance. Such shirking behavior is unobservable to the VC representing asymmetries of information in the form of moral hazard case. In both cases, it is proposed that the higher are information asymmetries the more the compensation should be tied to the entrepreneur’s performance (Lazear 1986), (Holmstrom 1979). It is this question of compensation and its relation to entrepreneurial performance that is the subject of this paper. We formulate a negotiation environment between a VC and an entrepreneur using game theory and agent based simulation. The purpose is to decide, under uncertainty, whether or not to extend a PLS funding contract? And if we do so, what will the optimal profit sharing ratio be?

In establishing our model, many points have to be taken into consideration. For example, disagreement between the VC and the entrepreneur will occur after the investment is being made. In line with this, control theories (such as (Aghion and Bolton 1992), (Dewatripont and Tirole 1994), and (Dessein 2002)) propose a balanced decision making process where entrepreneurs are given decision making power at some stages while the VCs are given decision power at other stages. In line with these findings, our model proposes a balanced decision power based on each participant’s capital contribution and expertise level. These parameters provide a bargaining tool to decide on the project’s optimal profit sharing ratio.

The second point relates to the opportunity cost of the VC. For example, many studies argue that the VC should require a certain rate of return (Mason and Harrison 2002), (Manigart et al. 2002). This concept is captured in our model by referring to the fact that any future investment by the VC will not occur if the VC does not expect the project to yield an expected profit greater than zero. This concept of at least
breaking even reflects a competition of multiple VCs over the funding of the project.

A third point argues that VC’s face uncertain circumstances, called external risks as cited by Kaplan and Strömberg (2004). These include, demand for new products as well as competitors’ response to new product. For the first issue, our model allows for an interval of expected revenues ranging from lower to upper values. While we believe that future demand can’t be estimated accurately, we establish least best estimates that can be integrated in the model. For the second issue, competition effect should endogenously be reflected in demand estimates.

A Fourth point values the VCs’ expertise and advisory roles Casamatta (2003) compared to other forms of financing such as debt or angel financing. We make use of this expertise feature as a bargaining tool in profit sharing negotiation.

A final point relates to the particularities of our model as opposed to standard VCs contracts. Our model is based on the sharing of profits and losses. The particularity of our model, as opposed to standard VC contracts is that profit is determined based on expected future profits and not as a fixed amount or as a percentage of investment. I.e. there should be no guaranteed returns to the VC as in the case of debt or standard VC contracts and there should be no guaranteed return to the entrepreneur as in the case of fixed wages. Another particularity of the model is that losses are shared according to each participant’s share in the project capital. This is again different from standard VC contracts where in case of losses the VC might totally get the totality of the project’s residual value.

Finally, it is worth mentioning that no model exists to eliminate moral hazard. It can only be reduced but not, entirely, eliminated Elitzur and Gavious (2003). In our model we try to reduce moral hazard, effort shirking, by having the entrepreneur contribute financially in the project.

The rest of the paper is organized as follows: Section 2 proposes our model. Section 3 presents the methodology. Section 4 represents the results and discussion. Finally, section 5 concludes with a summary and possible extensions.
2. The model

Our model is an extension of the original model of ?. The model strives to reduce the moral hazard problem in a sharing contract between risk neutral financier and an entrepreneur. The later is willing to undertake a project which requires funding $F$. He is endowed with an initial fund $A$ but requires an additional funding $I - A$. The success of the project depends on the effort of the entrepreneur.

The project is estimated to result in a stochastic verifiable output $R$ conditional dependent on a high or low managerial effort $e_i; i \in \{l, h\}$:

$$E(R|e_i) = \int_0^R Rf(R|e_i)dR$$

Where the share of the entrepreneur is $R_e$ and the share of the financier is $R_f$ such that $R = R_e + R_f$. This output can take upper and lower values depending on the effort being taken. In fact the output can be $\bar{R} (R)$ with probability $\theta_h (1 - \theta_h)$ in case of high effort and $\theta_l (1 - \theta_l)$ in case of low efforts.

The manager has a dis-utility of effort $D(e_i) = d(e_i).(1 - \beta).I$. This dis-utility is manifested as a percentage of his investment in the project when exercising effort $e_i; i \in \{l, h\}$.

A higher disutility is manifested through exercising higher effort such that $D(e_h) > D(e_l)$

The manager also has a reservation utility $U = u.(1 - \beta).I$ as a percentage of his investment in the project.

The expected NPV under the high effort and low effort case are given respectively as:

$$\overline{NPV} = \theta_h R - F > 0$$

$$NPV = \theta_l R - F + S < 0$$

**Assumption1:** We assume that under equation 2 the NPV is negative even if the entrepreneur enjoys some private benefits $S$ when he performs a low effort.
3. Methodology

We consider a one period contract. The entrepreneur and the financier agree on a partnership contract \((x; F, \alpha, \beta=x)\) whereby the entrepreneur commits to undertake a high effort and invest \(f= (1-x) F\). Two sharing ratio \(\alpha\) and \(\beta\) such that \(0 \leq \beta \leq 1\) and \(0 \leq \alpha \leq 1\), are given to the financier in case of success and Loss of the project respectively as in Nabi [20].

Taking \(r\) as the project rate of return, if the project is successful, yielding \(\bar{R} = (1 + \bar{r}) I\) the share of the financier and Entrepreneur respectively are:

\[
\bar{R}_f = \alpha \bar{R} = \alpha (1 + \bar{r}) I \quad \text{and} \quad \bar{R}_e = (1 - \alpha) \bar{R} = (1 - \alpha)(1 + \bar{r}) I \quad (4)
\]

If the project is unsuccessful, yielding \(\bar{R} = (1 + r) I\) the share of the financier and Entrepreneur respectively are:

\[
\bar{R}_f = \beta \bar{R} = \beta r I \quad \text{and} \quad \bar{R}_e = (1 - \beta) \bar{R} = (1 - \beta) r I \quad (5)
\]

We should note the distinguishing characteristic of the model where each participant cannot loose more than his/her capital contribution. This a distinguishing feature from the conventional setting where the financier might demand guarantees against losses of more than his/her capital contribution.

We start by developing a sharing ratio under managerial observable effort. We then develop a span of profit sharing ratio in an incomplete information setting where managerial effort is unoservable. We then provide the rational of our results using a game theory approach. Finally we test our results using agent based simulation tool (Netlogo).

4. Results and Discussion

4.1. The model under managerial observable effort

Under this scenario, the manager can’t deviate from providing his commitments of high effort and therefore the financier is in a comparative advantage in terms of profit sharing ratio negotiations. In
other words, the objective of the financer is to minimize the remuneration $R_m$ of the manager subject to the manager breaking even. Formally:

$$\min_{R_m(R)} \theta_h \int_I^R R_m f(R|e_h) dR + (1 - \theta_h) \int_0^I R_m f(R|e_h) dR$$

S.t

$$\theta_h \int_I^R R_m f(R|e_h) dR + (1 - \theta_h) \int_0^I R_m f(R|e_h) dR - (1 - \beta) I - D(e_h) \geq U$$

Since notations inside integrals represent expectations of returns we can re-express the problem in a shorthand form:

$$\min_{R_m(R)} \theta_h E(R_m|e_h) + (1 - \theta_h) E(R_m|e_h)$$

S.t

$$\theta_h E(R_m|e_h) + (1 - \theta_h) E(R_m|e_h) - (1 - \beta_h) I - d(e_h)(1 - \beta). I \geq u(1 - \beta). I$$

Using the rate of returns from (4) and (5) we get:

$$\theta_h (1 - \alpha)(1 + \tau). I + (1 - \theta_h)(1 - \beta)(1 + Er). I - (1 - \beta_h) I - d(e_h)(1 - \beta). I \geq u(1 - \beta). I$$

Solving for the share of the manager $(1 - \alpha)$ we get:

$$1 - \alpha \geq (1 - \beta) \frac{1 + d_h + u - (1 - \theta_h)(1 + \bar{r})}{\theta_h(1 + \bar{r})}$$

(6)

or

$$1 - \alpha \geq (1 - \beta) M_{MPC}$$

(7)

Where we can name the manager participation constraint multiplier $M_{MPC}$:

$$M_{MPC} = \frac{1 + d_h + u - (1 - \theta_h)(1 + \bar{r})}{\theta_h(1 + \bar{r})}$$

(8)

This multiplier which is clearly greater than 1, can represent what the manager demands as profit sharing ratio in excess of his contribution in the project.
Now solving for the financier share $\alpha$ that will allow the manager to participate in the contract we get:

$$\alpha \leq \alpha_{MPC} = 1 - (1 - \beta)M_{MPC} \quad (9)$$

4.2. **The model under managerial unobservable effort**

In this case the financier is facing a situation with regards to the type of the manager. In other words the financier is questioning whether the manager is going to exercise a high effort or not while undertaking the project.

so in addition to fulfilling the participation constraints using the sharing ratio at (6), the financier must also give an incentive so that the entrepreneur is at least indifferent between exercising low effort (Not Shirking) or exercising low effort (Shirking).

4.2.1. **Problem preliminaries**

We can establish a condition for which the entrepreneur is to perform a high effort. i.e we must have the Expected profit to the entrepreneur under no shirking $U_e(NS)$ to be higher than his profit under shirking $U_e(S)$ . i.e

$$U_e(NS) \geq U_e(S)$$

which means:

The financier then works out his payoff taking into consideration two probabilities:

- type probabilities $p_h$: regarding the probability that a manager is going to perform a high effort.
- performance conditional probabilities: regarding the probability that the project will be successful conditional on the manager’s effort. in our case, this is $\theta_h$ under high effort and $\theta_l$ under low effort.

From these two probabilities we can easily infer the joint probability of success $P(S)$ of the
project:

\[ P(S) = p_h \cdot \theta_h + (1 - p_h) \theta_l \] \quad (10)

This situation gives rise to private benefits \( S \) drawn by the manager if he performs a lower effort. Taking this into consideration, the financier is in a competitive disadvantage and therefore his objective will be to at least break even.

The contract being assigned needs to take into consideration three main constraints:

- **Participation constraints PCF and PCM**: where both participants (Financier Manager) are at least breaking even.
- **Incentive compatibility constraints ICM**: where only the manager is offered a profit sharing ratio that will encourage him to exert high effort rather than shirking.

So the objective of the financier is to maximize his return subject to the above mentioned constraints. Formally:

\[
\max_{R_f} P(S) \int_I^R R_f f(R|e_i) dR + (1 - P(S)) \int_0^I R_f f(R|e_i) dR \]

subject to constraints:

\[
PCF : P(S) \int_I^R R_f f(R|e_i) dR + (1 - P(S)) \int_0^I R_f f(R|e_i) dR - \beta I \geq 0 \] \quad (12)

\[
PCM : \theta_h \int_I^R R_m f(R|e_h) dR + (1 - \theta_h) \int_0^I R_m f(R|e_h) dR - D(e_h) \geq U \] \quad (13)

\[
ICM : \theta_l \int_I^R R_m f(R|e_l) dR + (1 - \theta_h) \int_0^I R_m f(R|e_l) dR - D(e_l) \geq S
\]
Using the expectations, rate of returns, percentage dis-utility percentage utilities and percentage private benefits, we can figure out a shorthand of the formula for the constrained problem:

$$\max_{R_f} P(S)\alpha(1 + E(\tau))I + (1 - P(S))\beta(1 + E(\tau))I$$

subject to constraints:

$$PCF : P(S)\alpha(1 + E(\tau))I + (1 - P(S))\beta(1 + E(\tau))I - \beta I \geq 0$$

$$PCM : \theta h(1 - \alpha)(1 + E(\tau)) + (1 - \beta)[(1 - \theta h)(1 + E(\tau)) - 1 - d(e_h) - u] \geq 0$$

$$ICM : \theta l(1 - \alpha)(1 + E(\tau)) + (1 - \beta)[(1 - \theta h)(1 + E(\tau)) - 1 - d(e_l) - u + s]$$

4.3. Satisfying the Manager’s participation and incentive compatibility constraints

We have already solved for the participation constraints of the manager (PCM) in equations (6) and (7). We need now to solve for the Incentive constraint of the manager (INC).

Solving for the share of the entrepreneur \((1 - \alpha)\) we get:

$$(1 - \alpha) = (1 - \beta)\frac{\Delta \theta(1 + E(\tau)) + \Delta d + s}{\Delta \theta(1 + E(\tau))}$$
\[ 1 - \alpha \geq (1 - \beta)M_{ICM} \] (20)

where we can name the manager incentive compatible constraint multiplier \( M_{ICM} \):

\[ M_{ICM} = \frac{1 + d_h + u - (1 - \theta_h)(1 + \bar{r})}{\theta_h(1 + \bar{r})} \] (21)

This multiplier can represent what the manager demands as profit sharing ratio in excess of his contribution in the project to induce him to exercise a high effort.

Now solving for \( \alpha \) we get:

\[ \alpha \leq \alpha_{ICM} = 1 - (1 - \beta)M_{ICM} \] (22)

We can infer then from (9)(22) that For \( \alpha \) to be fulfill both the incentive and the participation constraints, \( \alpha \) has to fulfill the following condition:

\[ \alpha \leq \min\{\alpha_{ICM}; \alpha_{PCM}\} \] (23)

4.4. Satisfying the Financier’s participation constraints

Now, we turn to the less competitive participant in this game, the financier. He needs a sharing ratio \( \alpha_{pcf} \) that enables him to at least break even. We give shorthand formula of the integrals of the financier participation constraints (12) by introducing expectations forms as follows:

\[ P(S)\alpha(1 + E(\bar{r}).I) + (1 - P(S))\beta(1 + E(\underline{r}).I) - \beta I \] (24)
Solving for $\alpha$ we get:

$$\alpha \geq \alpha_{PCF} = \beta \frac{1 - (1 - P(S))(1 + E(R))}{(1 + E(\bar{r}))P(S)}$$

(25)

from (25) and (23) we can figure out an interval of the sharing ratio $\alpha$ that the financier should get an which should be satisfying for both parties:

$$\alpha_{PCF} \leq \alpha \leq \min\{\alpha_{ICM}; \alpha_{PCM}\}$$

(26)

4.5. Span of Negotiation

We can notice that there is a span of negotiation $SN$ in terms of the profit sharing ratio such that:

$$SN = \min\{\alpha_{ICM}; \alpha_{PCM}\} - \alpha_{PCF}.$$ 

(27)

The larger is this span of negotiation the more likelihood that a contract can be materialized. A negative span of negotiation suggests the non-concluding of the contract.

4.6. Bargaining power

From the earlier discussion, we have identified the span of negotiation over the profit sharing ratio that the VC can get from the contract. We propose finding an optimum value within that interval taking into consideration the bargaining power of each participant. We propose that the bargaining power depend on three parameters: The VC capital contribution ratio $\beta$ and her expertise level $\mu$ and the weight ($W_i$ where $i \in (\beta, \mu)$ attached to those two parameters. For example if $W_\beta = 50\%$ then the financier gives equal importance to the capital provided and expertise provided in running the project.

We start from a basic point where both participants equally share the project capital and have the same level of expertise (50% each). In this case the span of negotiation is then shared equally:

$$\alpha_{average} = \frac{\min\{\alpha_{icm}; \alpha_{pcm}\} + \alpha_{pcf}}{2}$$

(28)
This constitutes our starting point above which the entrepreneur has less bargaining power and vice-versa. A change in the capital contribution is \( \% \Delta B \). A change in the VC’s expertise level is noted as \( \Delta \mu \in [-0.5;0.5] \). For example if \( \Delta \mu = -0.5 \) then \( \mu = 50\% - 0.5 = 0 \). i.e the VC has no expertise and therefore it is the entrepreneur who has full expertise.

We propose the following contractual agreement.

\[
\alpha_{opt} = \begin{cases} 
\alpha_{average} + [W_\beta(\% \Delta \beta) + W_\mu \Delta \mu][\min\{\alpha_{icm}; \alpha_{pcm}\} - \alpha_{pcf}] , & \text{if } \min\{\alpha_{icm}; \alpha_{pcm}\} > \alpha_{pcf} , \\
0 , & \text{otherwise}. 
\end{cases}
\]

(29)

5. Numerical Simulation

We do run our model on an agent based simulation model, Netlogo. We use the following data for the sake of exposition:

\[
I = 100000 ; \beta = 70\% \quad U = 5\% ; \quad S = 5\% ; \quad D(e_h) = 10\% ; \quad D(e_l) = 5\% ; \quad R = 150000 \\
R = 50000 ; \quad \theta_h = 80\% \quad \text{with } \sigma_h = 10\% ; \quad \theta_l = 40\% \quad \text{with } \sigma_l = 10\% ; \quad p_h = 50\% ; \quad \mu = 0\% ; \quad W_\beta = 50\% .
\]

Of course these data can be changed according to the users specific parameters.

The following figure shows the result of our model for 1000 simulation.
We can see that out of 1000 simulations, 109 contracts were void. This represents 12% which is below the rejection threshold of 30%. We can see that the minimum acceptable sharing ratio by the VC is \( \alpha_{pcf} = 63.03\% \) While the maximum sharing ratio for the VC tolerated by the entrepreneur : \( \alpha_{max} = \min\{\alpha_{icm}; \alpha_{pcm}\} = 71.02\% \)

Now, consider the bargaining power model proposed. If we start by the initial scenario where both participants have the same capital contribution and expertise level, then the agreed VC profit sharing ratio is:

\[ \alpha_{average} = 67.11\% \]

Now since \( \beta = 70\% \) then \( \%\Delta = 20\% \). Also since \( \mu = 0 \) this means that the VC has no expertise in the project. Therefore \( \Delta_{\mu} = 0 - 0.5 = -0.5 \). Also capital contribution is given equal importance as the level of expertise in deciding on the optimum profit ratio. So the optimum profit ratio for the financier is given as: So \( \alpha_{opt} = 67.11\% + [50\%10\% + 50\%(-0.5)][71.08\% - 63.03\%] \)

\[ \alpha_{opt} = 68.89\% \]

We should note that the optimum sharing ratio is different from the average resulting from
the span of negotiation. In this case we can see that the optimum sharing ratio is closer to the minimal acceptable ratio by the financier. This indicates that the entrepreneur possesses more bargaining power in deciding on the level of profit sharing.

6. Conclusion

In this Model we tried reducing moral hazards in a profit and loss sharing contract involving a VC and an entrepreneur. Moral hazards in our context manifest itself in the entrepreneur exerting of a low effort and thereby leading to a higher probability of the project’s failure. We applied game theory techniques under both observable effort (symmetric information) and unobservable effort (asymmetric case). We found theoretical evidence that under observable efforts a specific profit sharing ratio can be developed. Under unobservable effort however a span of negotiation is developed which is both incentive and participation constraints satisfying to both participants. Unlike the case of observable effort, an optimum profit sharing ratio can only be developed if the bargaining power of each participant is determined. The theoretical modeling of the game was then simulated in an agent based platform. The simulation allows for faster numerical calculation and enables the VC to decide whether or not a funding contract should be extended to the entrepreneur.

References


