

**A Hybrid Profit and Loss Sharing Model Using Interest Free-Debt and Equity Financing: An Application of Game Theory as a Decision Tool**

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# A Hybrid Profit and Loss Sharing Model Using Interest Free-Debt and Equity Financing: An Application of Game Theory as a Decision Tool

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## Abstract

In this paper two models are contrasted whereby a corporation is seeking to finance the purchase of a merchandise from a supplier through a profit and loss sharing contract. The first mode consist of financing the purchase totally through equity. The second model is a new hybrid model that engages the supplier in the process as a shareholder. Both models are based on the principle of profit and loss sharing which suffers from the issue of moral hazard. This is manifested in the form of the corporation shirking (providing low effort) and/or misreporting profits. It is argued that under equity financing, where the financier is the only shareholder, the corporation can hide part of the merchandise it sold and therefore misreport profits. This is, however, not possible under hybrid financing where, in addition to the financier, the supplier is interested in the financial reporting of the corporation. We apply a game theoretical approach where, under a hybrid financing, the financier and the supplier have mutual interest in true revenue reporting and therefore constitute a coalition (one player) against the corporation. Our game incorporates the effect of sharing markets and corporations' discounts between the game participants under each model. We show theoretically that a non-conditional good Nash equilibrium exists under hybrid financing. This case does not apply to an all equity financing where the existence of a good Nash equilibrium is conditional upon the financier and the supplier sharing ratios. This shows that under the hybrid model the corporation is always induced to provide more effort (not shirk) and truly report profits.

*Keywords:* PLS contracts, asymmetric information, Moral hazard, sharing ratio, Hybrid model. adverse selection

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## 1. Introduction

Profit and Loss Sharing models (PLS) in general represent modes of partnership between two or several parties. whereby profits are shared according to a predetermined ratio and losses are born according the partners' capital contribution (Obaidullah, 2005)

Full capacity of the parties involved coupled with their free consent are indispensable elements for the PLS contract to be valid (Usmani, 2002). All parties contribute a certain capital with some or all acting as agents and managers. One specific type of PLS contracts is the type existing in Islamic finance referred to as Musharakah. In this specific mode of financing, . The Sharing ratio must be based on expected future profits. It cannot be determined as a lump sum or a percentage of the capital investment , otherwise it becomes a fixed income investment. The profit-sharing ratio can eventually be revised upon agreement of all parties. Partners can , if they wish, offer a part of their profit to another partner (Obaidullah, 2005). In terms of liability, the participants in a Musharakah contract normally have unlimited liabilities. The Musharakah contract is terminated if (Usmani, 2002):

- The partnership was limited to a given time frame;
- The purpose of the partnership has been achieved;
- The continuity of the project is compromised by the withdrawal of one or several partners;
- Any of the partners die before the end of the agreement;

**In the case of premature termination, the business shall be liquidated, and the settlement distributed prorata.**

A partner cannot demand that another partner provides security in any form since they have the same rights and obligations towards each other and since they are acting as agents for each other. However, in case of a Musharakah agreement between the bank and the bank's client, the bank can obtain adequate security from the partners against possible misconduct and negligence to ensure the safety of the capital invested.

One specific type of Musharakah is diminishing Musharakah (Musharakah mutanaqisah), where one or several partners progressively increase his/her share in the business or project (Warde, 2010).

An increasing interest is shown by many conventional institutions towards PLS contracts in the form of Musharakah. This is manifested through the opening of Islamic windows to attract investors and depositors. Islamic benchmarks were set to cope with its conventional counterparts. Examples include the Dow Jones Islamic Market Index in New York and the FTSE Global Islamic Index in London (Abidi, 2009).

In 2007, more than 400 billion US Dollars were raised and 600 US Dollars billion were invested in different sectors (Gierath, 2010)

There are many differences between conventional PLS contracts and Islamic Musharakah. The latter is based on ethical considerations governed by the Islamic jurisprudence (Shari'ah). For example, Musharakah is interest-free financing and prohibits the financing of illicit projects (gambling, casinos, wine and pornography). This represents a major limitation to the number of investment opportunities that an Islamic bank can undertake compared to their conventional counterparts.

Islamic Musharakah does not allow the partners to receive fixed compensation like their conventional coun-

terparts. This also represents a drawback to the Islamic bank as it may lose due to the profit and loss sharing principal. In addition, Islamic banks providing Musharakah suffer from higher asymmetric information compared to their conventional counterparts as the entrepreneur may behave in opportunistic ways.

Our paper tries to introduce a new hybrid model to reduce this problem of asymmetric information using a game theoretical approach. In fact, our model proposes giving a partial equity financing to a corporation. The rest of the required fund is given as a debt to the corporation's supplier who enters this game as a shareholder.

This model tries to answer the following research questions:

- Can our model induce the financed corporation to perform better?
- Is there any equilibrium achieved in our model and if so is it better than a total equity financing model?

To answer these research questions, we provide a game theoretical approach comparing the payoffs of the participants under our hybrid model and the total equity financing scenario. Because it is a new model, it is not yet tested in a real case scenario which constitute its main limitation. We can remedy such limitation in future extensions using real case simulations. Also, in the model, there are some of the parameters such as the project's payoffs which are not controllable. We can also think of whether the suppliers can be willing to enter into such contracts. The paper is organized as follows:

Section 2 starts with a literature review of asymmetric information in a financier-entrepreneur environment. Section 3 represents the model. Section 4 presents the methodology. Section 5 represents the results. Finally, section 7 concludes with a summary and possible extensions.

## **2. Literature review**

Despite the big development of Islamic financial literature, especially in the performance and efficiency of banks there is a large gap that is not covered yet. For instance, academic literature does not provide rigorous analysis of the financial capital structure in Islamic private equity (a form of Musharakah) projects, their role under asymmetric information and how to deter opportunistic behaviour of the entrepreneur (Chatti and Yousfi, 2010)

Many researches have tried to minimize the problem of asymmetric information. One such example is the use of dissipative signals. For example, information sharing information is one method to reduce the asymmetry of information. In consistency with this framework, it has been shown that credit bureaus can increase efforts from borrowers (Padilla and Pagano, 1997). At the same time, information sharing may be

used to reduce competition between banks (Gehrig and Stenbacka, 2007). Furthermore, information sharing is more likely if borrower mobility is higher (Pagaon and Jappelli, 1993) and if asymmetric information problems are more important (Brown and Zehnder, 2010). It has also been shown through empirical research that, information sharing is correlated with higher access to credit (Pagaon and Jappelli, 1993), especially in developing countries with inefficient creditor rights (Djankov et al., 2007), but lower lending to low-quality borrowers (Hertzberg et al., 2011). Another way to reduce asymmetric information is the use of collateral. This signalling method is consistent with some research that claim that banks can use collateral in debt contracts to overcome information asymmetries, in particular arising from ex-ante adverse selection (Berger et al., 2011). A confident manager lacking physical collateral can sign for low job protection to prove his confidence in his managerial abilities. This scenario is consistent with the findings of previous research such as the ones in (Subramanian and Sheikh, 2002). Misreporting is one of the biggest problems in Musharakah contracts. Misreporting occurs when the agent under declares the profitability of the project. To reduce this problem, Al-Suwailem (Al-Suwailem, 2006) proposes a higher due diligence from an Islamic institution as compared to conventional banks. Also, it is proposed that the entrepreneur's participation in the capital and the submission of a warranty can resolve the asymmetric information problem (Karim, 2002). We should state however, that one cannot ask for a warranty against performance as this is not permissible under the Shari'ah law. Yet, the recourse to a warranty is allowed when negligence or non-respect of the contract terms by the entrepreneur are proven.

The use of game theory to tackle agency problems in Islamic finance, in general, and particularly, in Musharakah contracts, is very modest in nature. Some implicit research in this area state that the agency problem is based on an unfair distribution of returns if the project fails (Shaikh, 2011). Taking into consideration the risks related to a project, it is proposed that the financial institution may require a higher sharing ratio. In line with the same concept, one research proposes higher incentives for risky projects and lower compensation schemes for less risky projects (Yousfi, 2013). However, giving fewer shares to the entrepreneur may result in less motivation and therefore lower projects returns.

To induce the entrepreneur to exert high effort strategy and therefore reduce moral hazard, a research suggested a minimum capital contribution by the entrepreneur given a minimum sharing ratio (Nabi, 2013).

Another research proposed the usage of two sharing ratios instead of one to reflect the effort of the entrepreneur compared to the financier (Maheran, 2010). This model, however, suffers from the non-treatment of asymmetric information.

Some explicit research tackled the issue of moral hazards and adverse selection in PLS contracts.

For example, in a previous paper an incentive mechanism approach using game theory was introduced to deal with moral hazards. It theoretically allows for higher social value, more freedom to the agent in terms of negotiating the profit sharing ratio, and induced the agents to contribute more with capital (ELFakir and Tkiouat, 2015a).

In another research a new proposed model called ROMCA (EL Fakir and Tkiouat, 2016) was evaluated in relation to other forms of financing like ROSCA and debt contracts. There was a simulation evidence that the ROMCA model can dominate the other forms under adverse random shocks with low market conditions and prevailed in cases of moral hazards (EL Fakir and Tkiouat, 2016).

In order to help financial institution in their agent selection process, we have developed three adverse selection indices in mudaraba financing (a specific form of Musharakah where the financier is the sole provider of funds). These indices should help financial institutions in reducing adverse selection(ELFakir and Tkiouat, 2016a).

In another paper, we tried to test whether the use of a two-contract menu can reduce asymmetric information in an environment of incomplete information. We found a game theoretical evidence that menu contracting is not, always, the optimal option for asymmetric information reduction(ELFakir and Tkiouat, 2016b).

In the same line, we have proposed in another paper the offering of an effort-based Vs output-based contract. In the effort-based contract the remuneration of the agent is assessed ex-post based on effort provided. the project financing can continue, even if it fails in the first stage, if the assessment of profit was positive. The second contract reimburses the agent only based on the project output regardless of the effort provided by the agent. This means that the refinancing can only occur if the output is satisfactory. We found a game theoretical evidence that an effort based contract can give higher compensation to the agent as this contract offers a lower sharing ratio to the financier(ELFakir and Tkiouat, 2015b). This result emphasizes two important Islamic concepts. First it emphasizes the sentiment of altruism which the financier shows by taking a smaller profit-sharing ratio. Second it emphasizes the sentiment of positive reciprocity which the agent exhibits by providing high effort (ELFakir and Tkiouat, 2015b).

### 3. The model

Our model is composed of three agents: A supplier (S), a corporation (C) which is a client of the supplier and a **financier** (F). The normal commercial procedure is that the supplier delivers the merchandise on credit to the corporation making a margin  $M$ . The later pays back after a certain credit period. The drawbacks of this methodology are

- The supplier does not have immediate access to liquidity
- The supplier might not get paid back in case the corporation defaults
- The corporation does not get a discount due to late payment
- The supplier misses the opportunity of investing due to late payments
- The supplier also misses on his payments to his own suppliers. i.e. he cannot pay his suppliers if he does not get paid on time by his client (the corporation)

To overcome these problems, we propose the introduction of a financial institution which act as a partner in this cycle and not just as an intermediary.

To illustrate, the corporation (C) would like to get a merchandise worth  $I$ . The **financier** (F) offers equity financing  $E = W_e \cdot I$  but agrees to deliver the rest of the needed amount  $I - E = (1 - W_e) \cdot I$  as a loan to the supplier(S). The supplier delivers the needed merchandise but on an equity base to the corporation. In this case the supplier as well as the financier are shareholders in the corporation and therefore both have an interest in the corporation financial reporting.

Our model tries to fix two issues. The first relates to the corporation shirking in terms of providing the necessary effort to sell the merchandise. If it shirks, losses will be incurred in the form of revenues  $\underline{R}$  being less than the investment  $I$ . The second issue is that of profit misreporting where there is a possibility that the corporation might hide the actual amount of merchandise it bought from the supplier and therefore declare lower revenues  $\bar{R}$  than normal revenues  $\overline{\bar{R}}$ . This aspect is possible under total equity financing by the financier. This case, **however**, of course, not possible under the hybrid model where the supplier is directly interested in the corporation financial reporting.

We found it **practical** to think of the game as a coalition game of two players against one. The supplier and the **financier** constitute one player (FS) playing against the corporation (C). The intuition behind

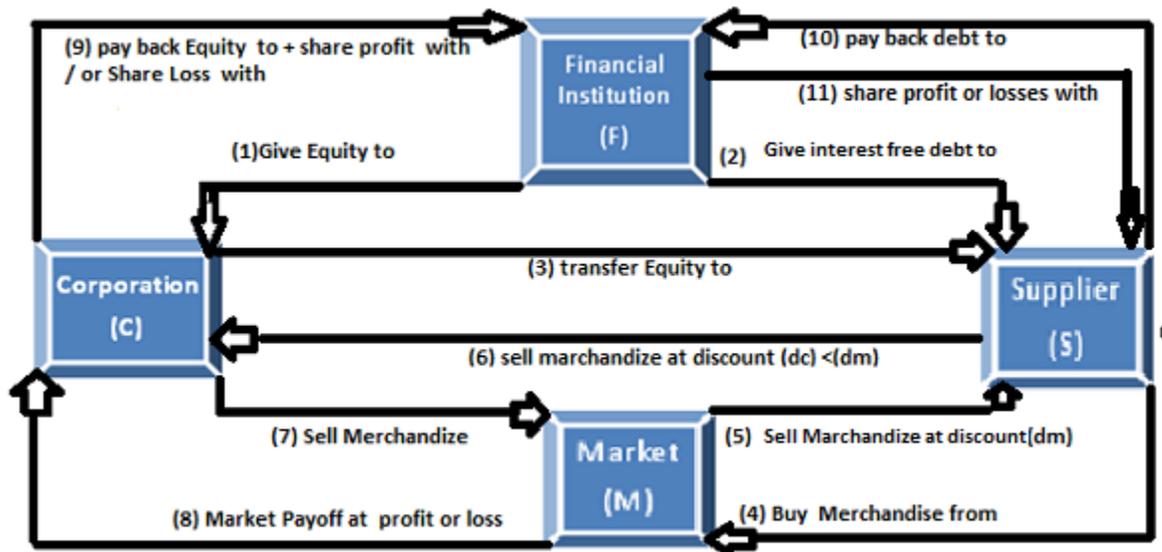
choosing the financial institution and the supplier playing as one coalition is that both have mutual interest in having the corporation not shirking and properly reporting its revenues. The corporation has a high probability not to shirk against the supplier as it values its commercial reputation. To have the supplier participate in the game, we propose that the financier provides him with interest free debt. This allows him to get faster access to liquidity at no interest charges. This also allows the supplier to get his merchandise from the market at a discount ( $d_m$ ), that he eventually will transfer (sell) to the corporation, at a discount ( $d_c$ ). given the later discount ( $d_c$ ), the corporation can sell the merchandise at a competitive price. If the market payoffs to the corporation are favourable, the corporation will share its profit with the financier at an agreed ratio  $\alpha$ . The corporation will also payback its debt to the supplier  $W_d.I$ . The later will transfer the same amount to the financier to redeem his debt.

If the market payoffs are not favorable, the corporation is not obliged to pay the financier equity  $W_e.I$ . It still however obliged to pay its debt  $(1 - W_e).I$  vis-a vis the supplier.

For the financier, this is better than total equity financing. Losing  $W_e.I < I$  is better than losing all equity  $I$ . Also, since the financier has entered into a debt contract with the supplier, it is guaranteed to get part of its investment  $W_d.I$ .

The following diagram illustrates our model:

Figure 1: The Hybrid PLS model using debt and equity



## 4. Methodology

To assess the feasibility of the hybrid model, we compare its payoffs to the participants with those under a total equity financing. We need to take into consideration the discounts  $d_c$  that the corporation is offered when it pays immediately for the merchandise as well as the discount  $d_m$  that the supplier gets from the wholesalesmarket. The supplier makes a net margin  $N_m$  which is equivalent to the total of his margin on sales  $M$  and market discounts  $d_m$  less the discount  $d_c$  he provides to the corporation. i.e.  $N_m = M + d_m - d_c$

We try under the two methods of financing to consider the payoffs to each participant under the shirking and the non-shirking of the corporation. The payoffs are divided to equity share, debt share and discount share. The profit share of the financier and the supplier are given as  $\alpha_f$  and  $\alpha_s$ . The combinations of the financier's and supplier's profit shares is given as  $\alpha_{fs} = \alpha_f + \alpha_s$ .

### 4.1. Payoffs under equity financing

Since this is an equity financing, there is no payoffs from debt. The participants, however, have a payoff from equity and discount share. The following two tables details the payoffs to each participant under the shirking and the non-shirking of the corporation. Since the supplier is not involved in the equity financing, his/her share from the profits of the project is  $\alpha_s = 0$  and therefore  $\alpha_{fs} = \alpha_f$ . Therefore his/her payoff is composed of his/her sales' net margin  $N_m$ .

Table 1: Payoffs to the game participants when the corporation shirks under equity financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	0	0	0	$(1 - \alpha_f)d_c I$	$(1 - \alpha_f)d_c I$
S	0	0	0	$I.N_m$	$I.N_m$
F	$[\underline{R} - I]$	0	0	$\alpha_f I d_c$	$[\underline{R} - I(1 - \alpha_f d_c)]$

Table 2: Payoffs to the game participants when the corporation does not shirk in terms of effort but misreport revenues under equity financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	$[\bar{R} - I](1 - \alpha_f)$	0	$(1 - P)(\bar{\bar{R}} - \bar{R})$	$(1 - \alpha_f)d_c I$	$[\bar{R} - I(1 - d_c)](1 - \alpha_{fs}) + (1 - P)(\bar{\bar{R}} - \bar{R})$
S	0	0	0	$I.N_m$	$I.N_m$
F	$[\bar{R} - I]\alpha_f$	0	$P(\bar{\bar{R}} - \bar{R})$	$\alpha_f I d_c$	$[\bar{R} - I(1 - d_c)]\alpha_f + P(\bar{\bar{R}} - \bar{R})$

Table 3: Payoffs to the game participants when the corporation does not shirk in terms of effort and does not misreport revenues under equity financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	$[\bar{\bar{R}} - I](1 - \alpha_f)$	0	0	$(1 - \alpha_f)d_c I$	$[\bar{\bar{R}} - I(1 - d_c)](1 - \alpha_f)$
S	0	0	0	$I.N_m$	$I.N_m$
F	$[\bar{\bar{R}} - I]\alpha_f$	0	0	$\alpha_f I d_c$	$[\bar{\bar{R}} - I(1 - d_c)]\alpha_f$

#### 4.2. Payoffs under hybrid financing

Since this is a hybrid financing, there is a payoff from debt, equity and discount share. The **financier**, however, has no income from debt since the model assumes an interest free debt. **Unlike the equity case, there is no payoffs from misreporting to the corporation as this strategy is dominated by true-reporting. In other words, the monitoring advantage provided by the hybrid model, allows the financier and the supplier to reclaim back the misreported profit and share it according to their stake in the business. This is given in table 5 where the financier and the supplier claim back  $W_e(\bar{\bar{R}} - \bar{R})$  and  $(1 - W_e)(\bar{\bar{R}} - \bar{R})$  respectively in the case of misreporting. This, therefore, gives no benefits to the corporation from misreporting. The following tables details the payoffs to each participant under the shirking and the non-shirking of the corporation.**

Table 4: Payoffs to the game participants when the corporation shirks in terms of effort under Hybrid financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	0	0	0	$(1 - \alpha_{fs})(d_c + N_m)I$	$(1 - \alpha_{fs})(d_c + N_m)I$
S	$[\underline{R} - I]W_e$	$-(1 - W_e)I$	0	$I.N_m$	$\underline{R} - I[1 - .N_m - \alpha_{fs}d_c]$
F	$[\underline{R} - I]W_e$	$+(1 - W_e)$	0	$\alpha_{fs}Id_c$	$[\underline{R} - I]W_e + (1 - W_e)I + \alpha_{fs}d_cI]$

Table 5: Payoffs to the game participants when the corporation does not shirks in terms of effort but misreport profits under hybrid financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	$[\bar{R} - I](1 - \alpha_{fs})$	0	0	$(1 - \alpha_{fs})(d_c + N_m)I$	$[\bar{R} - I(1 - d_c - N_m)](1 - \alpha_{fs})$
S	$[\bar{R} - I]\alpha_s$	$+(1 - W_e)I$	$(1 - W_e)(\bar{\bar{R}} - \bar{R})$	$\alpha_s I(d_c + N_m)$	$\bar{R} - I(1 - d_c - N_m) + (1 - W_e)(\bar{\bar{R}} - \bar{R})$
F	$[\bar{R} - I]\alpha_f$	$-(1 - W_e)I$	$W_e(\bar{\bar{R}} - \bar{R})$	$\alpha_f I(d_c + N_m)$	$[\bar{R} - I(1 - d_c - N_m)]\alpha_f + W_e(\bar{\bar{R}} - \bar{R})$

Table 6: Payoffs to the game participants when the corporation does not shirks in terms of effort and does not misreport profits under hybrid financing

	Equity Share	Debt Share	Misreporting	Discount Share	Total
C	$[\bar{\bar{R}} - I](1 - \alpha_{fs})$	0	0	$(1 - \alpha_{fs})(d_c + N_m)I$	$[\bar{\bar{R}} - I(1 - d_c - N_m)](1 - \alpha_{fs})$
S	$[\bar{\bar{R}} - I]\alpha_s$	$+(1 - W_e)I$	0	$\alpha_s I(d_c + N_m)$	$[\bar{\bar{R}} - I(1 - d_c - N_m)]\alpha_s$
F	$[\bar{\bar{R}} - I]\alpha_f$	$-(1 - W_e)I$	0	$\alpha_f I(d_c + N_m)$	$[\bar{\bar{R}} - I(1 - d_c - N_m)]\alpha_f$

## 5. Main Results

We now provide a game theoretical approach with the supplier and the **financier** forming a coalition against the corporation shirking. The payoffs of each game participants are simply extracted from the

tables above by adding the supplier's payoff to the **financier** payoff who now constitute one player. The strategy of **the financier/** supplier (FS) are to offer Equity financing or Hybrid Financing. The strategies of the corporation are three: "Shirking (S)" (providing less effort), "Not Shirk + Misreporting (NSM)", "Not Shirk + True-Reporting (NST)".

We have then the following normal form game. In each of the payoff cells, two payoffs are presented. The first payoff is for the financier/supplier (FS). The second is the corporation payoff.

		Corporation		
		Shirk	Not Shirk+True Reporting	Not Shirk+Misreporting
Financier	Hybrid	$U_{fs}(H, S) ; U_C(S, H)$	$U_{fs}(H, NST); U_C(NST, H)$	$U_{fs}(H, NSM); U_C(NSM, H)$
	Pure Equity	$U_{fs}(E, S) ; U_C(S, E)$	$U_{fs}(E, NST); U_C(NST, E)$	$U_{fs}(E, NSM); U_C(NSM, E)$

Where under the hybrid model, the payoffs to the participants under the corporation three strate-

gies are:  $U_{FS}(H, NST) = [\bar{R} - I.(1 - d_c - N_m)]\alpha_{fs}$

$$U_C(NST, H) = [\bar{R} - I.(1 - d_c - N_m)](1 - \alpha_{fs})$$

$$U_{FS}(H, NSM) = [\bar{R} - I.(1 - d_c - N_m)]\alpha_{fs} + \bar{R} - \bar{R}$$

$$U_C(NSM, H) = [\bar{R} - I.(1 - d_c - N_m)](1 - \alpha_{fs})$$

$$U_{FS}(H, S) = [\underline{R} - I[1 - N_m - \alpha_{fs}d_c]]$$

$$U_C(S, H) = d_c.I.(1 - \alpha_{fs})$$

And under the Pure Equity financing model, the payoffs to the participants under the corporation three strategies are:

$$U_{FS}(E, NST) = \alpha_f[\bar{R} - I.(1 - d_c)] + IN_m$$

$$U_C(NST, E) = (1 - \alpha_f)[(\bar{R} - I.(1 - d_c))]$$

$$U_{FS}(E, NSM) = \alpha_f[\bar{R} - I.(1 - d_c)] + IN_m + P(\bar{R} - \bar{R})$$

$$U_C(NSM, E) = (1 - \alpha_f)[(\bar{R} - I.(1 - d_c))] + (1 - P)(\bar{R} - \bar{R})$$

$$U_{FS}(E, S) = [\underline{R} - I.(1 - d_c)] + IN_m$$

$$U_C(S, E) = (1 - \alpha_{fs}).d_c.I$$

## 5.1. Shirking as a dominated strategy

### 5.1.1. Under hybrid financing

Should the coalition (FS) opt for a hybrid financing, the difference in payoffs to the corporation under true-reporting, and misreporting, given the shirking and non shirking cases, would be:

$$U_C(NST, H) - U_C(S, H) = [\bar{R} - I.(1 - N_m)](1 - \alpha_{fs}) > 0 \quad (1)$$

$$U_C(NSM, H) - U_C(S, H) = [\bar{R} - I.(1 - N_m)](1 - \alpha_{fs}) > 0 \quad (2)$$

It is clear from (1) and (2) that, under hybrid financing, "Not shirking" (whether with misreporting or not) is better for the company than "Shirking".

### 5.1.2. Under Pure Equity Financing

If the coalition (FS) opt for an equity financing the corporation gets the following:

$$U_C(NST, E) = (1 - \alpha_f)[(\bar{R} - I.(1 - d_c))]$$

$$U_C(NSM, E) = (1 - \alpha_f)[(\bar{R} - I.(1 - d_c))] + (1 - P)(\bar{R} - \bar{R})$$

$$U_C(S, E) = (1 - \alpha_{fs}).d_c.I$$

Taking the difference of payoffs between the strategies we get:

$$U_C(NST, E) - U_C(S, E) = [\bar{R} - I](1 - \alpha_{fs}) > 0 \quad (3)$$

$$U_C(NSM, E) - U_C(S, E) = [\bar{R} - I](1 - \alpha_{fs}) + (1 - P)(\bar{\bar{R}} - \bar{R}) > 0 \quad (4)$$

It is clear from (3) and (4) that, under Equity financing, "Not shirking" (whether with misreporting or not) is better for the company than "shirking".

### 5.1.3. New normal form of the game

We conclude then from (1) (2) (3) (4) that Not shirking (whether with misreporting or not) dominates shirking. This strategy, then, can safely be excluded from the analysis. The following table presents the new normal form of the game after eliminating shirking as a dominated strategy

Table 7: The new normal form of the game after eliminating shirking as a dominated strategy

		Corporation	
		Not Shirk+True Reporting	Not Shirk+Misreporting
Financier	Hybrid	$U_{fs}(H, NST); U_C(NST, H)$	$U_{fs}(H, NSM); U_C(NSM, H)$
	Pure Equity	$U_{fs}(E, NST); U_C(NST, E)$	$U_{fs}(E, NSM); U_C(NSM, E)$

## 5.2. Nash Equilibrium

To look for a Nash equilibrium over which the participants will settle, we start by comparing the payoff to the corporation under true-reporting and misreporting while the (FS) coalition has chosen the Hybrid model and the equity model respectively.

### 5.2.1. Result 1: Good Nash equilibrium

Under the (FS) hybrid strategy, the difference between the corporation's payoffs from true and misreporting is:

$$U_C(NST, H) - U_C(NSM, H) = \bar{\bar{R}} - \bar{R} = \Delta_R > 0 \quad (5)$$

This means that, under the hybrid strategy, true profit reporting is better in terms of payoff to the corporation than misreporting.

We look then for the best strategy of the financier if the corporation truly report its revenues. To do so we compare the payoff to the (FS) coalition under hybrid and equity financing:

$$U_{FS}(H, NST) = [\bar{R} - I.(1 - d_c - N_m)]\alpha_{fs}$$

$$U_{FS}(E, NST) = \alpha_f[\bar{R} - I.(1 - d_c)] + IN_m$$

Taking the difference we get:

$$U_{FS}(H, NST) - U_{FS}(E, NST) = [\bar{R} - I.(1 - d_c)]\alpha_s - IN_m\alpha_c$$

So, for hybrid financing to prevail against equity financing, the share of the supplier needs to be greater than a threshold value  $\alpha_{s1}$ . Namely we must have:

$$\alpha_s > \alpha_{s1} = \frac{I.N_m}{\bar{R} - I.(1 - d_c)}\alpha_c \quad (6)$$

Based on (6) hybrid financing and revenue true reporting (H , NST) constitute a Nash equilibrium. This is a good Nash equilibrium as it induces the corporation to truly report its revenue.

### 5.2.2. Result 2:Bad Nash equilibrium

Similarly, we test for the best response of the corporation under pure equity financing.

Under the (FS) equity strategy, the difference **between** the corporation's payoffs from true-reporting and misreporting is:

$$U_C(NST, E) - U_C(NSM, E) = \Delta_R[P - \alpha_f] \quad (7)$$

This is more likely to be negative for the following reasons:

- Equity financing (with single monitoring) is riskier compared to hybrid financing (with double monitoring). There is therefore lower chance  $P$  of recovering any misreported profits.
- Equity financing, therefore, suffer from higher agency problem and,hence, the financier requires a higher sharing ratio  $\alpha_f$

The above arguments therefore, states that the corporation would most likely engage in misreporting under total equity financing. From the above discussion, the relevant comparison to the financier/Supplier is between the case of true reporting under hybrid financing and misreporting under equity

financing. In other words the relevant difference in the payoffs to be considered is:  $U_{FS}(H, NST) - U_{FS}(E, NSM)$

$$U_{FS}(H, NST) - U_{FS}(E, NSM) = [\bar{R} - I.(1 - d_c - N_m)]\alpha_{fs} - \alpha_f[\bar{R} - I.(1 - d_c)] - IN_m - P(\bar{R} - \bar{R})$$

Recalling that  $\alpha_{fs} = \alpha_f + \alpha_s$ ,  $\alpha_{fs} + \alpha_c = 1$  and simplifying we get:

So, for Equity financing to prevail against Hybrid financing, the share of the supplier needs to be less than a threshold value  $\alpha_{s2}$ . Namely we must have:

$$\alpha_s < \alpha_{s2} = \frac{I.N_m\alpha_c - \Delta_R[\alpha_f - P]}{\bar{R} - I.(1 - d_c)} \quad (8)$$

The financier preference for equity financing when  $\alpha_s < \alpha_{s2}$  induces the corporation to engage into misreporting. This is considered as a bad Nash Equilibrium

## 6. Avoiding the bad Nash Equilibrium

From (6) the corporation engages in true reporting if  $\alpha_s > \alpha_{s1}$ . Equally likely, from (8), the financier would prefer hybrid financing if  $\alpha_s > \alpha_{s2}$

This means that for Hybrid financing with true reporting to prevail against equity financing with misreporting we must have:

$$\alpha_s > \text{Max}\{\alpha_{s1}, \alpha_{s2}\} \quad (9)$$

## Conclusion

In this research, we have tried to reduce the moral hazard problem in financial contracting using a hybrid financing model. The moral hazard in this research manifests itself in two aspects: Shirking (less effort) and revenue misreporting. We found game theoretical evidence of three results. First, under both equity and hybrid financing, the corporation's shirking strategy is dominated by non-shirking (whether under true-reporting or misreporting). Second, a good Nash equilibrium exists under a hybrid financing model

where the corporation is induced to truly report revenues. A bad Nash equilibrium where the corporation is induced to misreport revenues under equity financing. The supplier's share in hybrid financing should exceed a given threshold to ensure, from one side, his her participation in the contract and, from the other side, the corporation's engagement in the provision of high effort and true reporting.

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