A new technique for reducing size of a wpt system using two-loop strongly-resonant inductors

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A New Approach to Peak Threshold Estimation for Impulsive Noise Reduction over Power Line Fading Channels

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Abstract—Impulsive noise (IN) is a major component that degrades signal integrity in power line communication (PLC) systems. PLC systems driven by orthogonal frequency-division multiplexing (OFDM) have Rayleigh distributed amplitudes. Based on the dynamic nature of each OFDM symbol, peak amplitude of the symbol was recently shown to be a suitable threshold for detecting IN and this technique outperforms conventional optimal blanking (COB) scheme. In this study, we improve the dynamic peak-based threshold estimation (DPTE) scheme that relies on the OFDM Rayleigh distributed amplitudes by converting the default Rayleigh distribution to uniform distribution to unveil IN with power levels below that of the conventional peak signal. Then, we perform nonlinear mitigation processing on the received signals whose amplitudes exceed the uniformly distributed amplitude using blanking; a scheme we will refer to as uniformly distributed DPTE (U-DPTE). Our results (based on U-DPTE) significantly outperforms DPTE scheme by up to 4dB gain in terms of output signal-to-noise ratio (SNR). Additionally and unlike earlier DPTE studies, we propose a novel threshold criteria that compensates the Gaussian noise power level amplification (after equalization) for achieving optimal SNR over a log-normal multipath fading channel. The results further reveal the sub-optimality of the DPTE scheme over COB.

Index Terms—Power line communication (PLC), impulsive noise, OFDM, PAPR, dynamic peak threshold estimation (DPTE), uniform distribution, fading channel, equalization, log-normal.

I. INTRODUCTION

Electrical wires, whether indoor or outdoor, are traditionally used to transfer electrical energy. On the other hand, wireless and other wired media such as digital subscriber line, optical fibre, coaxial cables, unshielded twisted pair cables and other similar cables are used to transfer communication data/signal. Now, with the rising dependency on communication technology, the electrical wires usually referred to as power-line volunteer ubiquitous channel for data communications. This method of data communication over power line is referred to as power line communication (PLC) and the electrical wires (power lines) form the power line channel [1], [2].

The scheme subtends the widely known smart grid system today (see [3]). In other words, PLC are overlays of electrical wires (or overlay layer of energy network) for communication data transfer. Outdoor PLC systems involve narrow-band (NB) communication while indoor PLC networks traverse a broadband (BB) communication network [4]. Since the wiring infrastructure already exists, the PLC becomes cost effective for home automation, monitoring and control based on ubiquity of networking points for all rooms.

However, there are many problems that constrain the effective data communication over the channels of PLC systems. Some of these constraints include accurate channel model, impulsive noise (IN), multiple reflections/multipath fading, and frequency-dependent attenuation [4]–[9]. Since some IN have shorter duration, for example, IN generated from electrical/electromagnetic appliances exhibit higher power than background noise [10] and may only last for a fraction of the symbol period [11] while other last for extended periods, the power levels may be higher or lower than some desired OFDM signal power levels. Researchers/engineers are therefore perturbed by the possible accurate noise model and effective mitigation scheme against the IN. Since the received signal integrity depends on the received signal to noise ratio (SNR), high IN level can severely degrade received signal quality. It follows that IN is a major deterring component in data transmission over PLC channels [11].

To improve the received signal integrity, different IN mitigation techniques [2], [9], [12]–[17] were proposed. Recently, reducing the peak-to-average power ratio (PAPR) of OFDM symbol before transmission over PLC channel were considered in [2], [12]–[14], [16], [18]–[20]. This is followed by assuming that the coefficients of IN are present at the receiver of PLC systems, thus blanking, clipping or hybrid clipping-blanking can be used [15] to mitigate the IN. We refer to the method that uses the a priori-knowledge of the IN and the conventional optimal blanking threshold (OBT) scheme [15] in PLC systems as the conventional optimal blanking (COB) in this study. Practically, this is not consistent as the time and probability of IN occurrences can not be predicted precisely. Secondly, OFDM signals are dynamic and exhibit non-constant peaks for each symbol frame which further obloquies the use of perfect knowledge of IN for IN removal in PLC systems. Based on this fact, the peak amplitude of each OFDM symbol is used as the blanking threshold for IN removal; a scheme named as dynamic peak-based threshold estimation (DPTE) [2].

While the IN cannot be obtained precisely, it thus obnoxious to estimate its presence based on a predetermined threshold value. Since each OFDM symbol is dynamic and thus may exhibit both a unique peak at a different time and frequency, the suitable optimal threshold for predicting IN becomes the
maximum amplitude of the OFDM symbol itself. Unfortunately, OFDM signal amplitudes for sufficiently large number of subcarriers is Rayleigh distributed. Thus IN with power level below that of OFDM signal power level may not be detected and thus not mitigated in such PLC system.

To enhance PLC system performance, clipping OFDM signal amplitudes was used to minimize the peak threshold of OFDM signals before transmission in [2]. However, it has been shown that companding achieves better IN reduction in terms of output signal-to-noise ratio (SNR) at the receiver of PLC systems than clipping [19] due to the fact that clipping incurs higher in-band distortion than companding. Later, [21] extended the DPTE study by using look-up table, however, constructing look-up table requires precise knowledge of datasets (received symbols), which is not usually feasible. Other techniques have also been studied such as clipping [22] and adaptive IN mitigation [23] schemes, although these do not operate with DPTE algorithm. Besides, all reported DPTE schemes so far [2], [16], [20], [24] do not adopt realistic fading channels. On the other hand, COB scheme conducts extensive search [12], [19] to achieve optimal SNR performance which expands processing time and depletes system power. Meanwhile, in [9], the IN mitigation was studied in the presence of fading channels, however, without considering optimal output SNR performance. To overcome these aforementioned limitations, we propose to convert the amplitude distribution of the OFDM signals to uniform distribution first before passing the signal over the channel of PLC systems. By this scheme, the probability of missing IN samples with power levels below the peak of desired signal during IN detection is minimized. We refer to this enhanced scheme as uniformly distributed DPTE (U-DPTE) scheme. Secondly, we examine the received signals over log-normal multipath fading channel and then propose a novel criteria for IN detection in this case.

Our contributions include, firstly knowing that clipping incurs higher in-band noise than companding, we establish the general method for converting Rayleigh amplitude distribution to uniform distribution. Secondly, instead of using DPTE in place of COB (as in [2]), we use U-DPTE in place of DPTE to detect IN with power levels below that of desired OFDM signals. Then, we compare the results of OFDM signal transmissions over PLC channels using the U-DPTE scheme with DPTE and COB both over AWGN and log-normal multipath fading channels with IN. We found that the proposed U-DPTE scheme enhanced the output SNR gain by up to 4.2 dB over IN dominated channel with AWGN. Thirdly, we pass the signal, afterwards, through a log-normal fading channel with IN and AWGN; the noise power level is amplified in this case. Thus, we introduced a novel threshold value estimation scheme over fading channel which has never been studied earlier. Based on this, our proposed U-DPTE scheme achieved even better performance of up to 8 dB gain in terms of output SNR in comparison to DPTE technique. Furthermore, we explore four other examples of uniform distribution conversion schemes [25]–[28] to establish the most performing method. From our results, we found that the uniform distribution model whose symbol amplitudes are tightly distributed around the mean amplitude achieves the highest output SNR gain. This can be explained on the premise that uniform distribution transforming function amplifies the amplitudes of low power OFDM signals and simultaneously compresses the amplitudes of high power OFDM signals so that both converge at a uniform amplitude thus exposing all IN of different power levels for detection in the PLC system. Unlike the DPTE scheme in any other existing study in the literature, we also show that over fading channel the COB is more superior in performance than DPTE. Then, on system level performance, comparing COB technique with the proposed U-DPTE scheme, U-DPTE avoids the exhaustive search for optimal power level (unlike [12]) which expends the PLC system power and expands the processing time. Also, comparing with DPTE scheme, the proposed U-DPTE delivers a PLC system that maximizes the signal power level and dissipates high quality output of signal integrity.

Henceforth, we formulate the problem in Section II and the proposed model in Section III. Afterwards, the results are discussed in Section IV with the conclusions following.

II. PROBLEM FORMULATION

Our consideration is given to time-domain OFDM signal frames with frequency domain contents $S_n, \forall n = 0, \cdots, N - 1$ processed using quadrature amplitude modulation (QAM) and transformed by performing inverse fast Fourier transformation (IFFT) on the signal in the form

$$s_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} S_n \exp\left(j2\pi nk/N\right), \forall k = 0, 1, \cdots, N - 1$$

(1)

where $j = \sqrt{-1}$, $k$ and $n$ represent the time and frequency sample indices of the signals respectively and $\frac{1}{\sqrt{N}}$ is a normalization factor. We are concerned with the amplitudes of the OFDM signal, $A_s$, which can be expressed as follows

$$A_s = \sqrt{s_r^2 + s_i^2}, \forall k = 0, 1, \cdots, N - 1$$

(2)

where $s_r$ and $s_i$ are from the fact that the output of Fourier transformed random variable is complex, namely $s = s_r + js_i$. From (2), the symbol peak amplitude can be found as

$$\mathcal{P} = ||A_s||_{\infty}$$

(3)

where $\mathcal{P}$ is the maximum amplitude that exists in every OFDM symbol and $||\cdot||_{\infty}$ represents norm to infinity. Now, from the knowledge of central limit theorem, both $s_r$ and $s_i$ are independently and identically distributed Gaussian random variables, thus implying that (2) can be described using Rayleigh distribution. Analytically, if $s \sim \mathcal{N}(\mu_s, \sigma_s^2)$, then

$$f_{|s|}(s; \mu_s, \sigma_s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{1}{2} \left(\frac{s - \mu_s}{\sigma_s}\right)^2\right)$$

(4)

where $s_0$ is the discrete envelope of $s$, $f_{|s|}(s; \mu_s, \sigma_s)$ is the probability density function (PDF) of $s$ given mean $\mu_s$ and standard deviation $\sigma_s$, $\mu_s = \mathbb{E}\{x(n)\}$, $\mathbb{E}\{\cdot\}$ is the statistical mean operator. We normalized $s_k$ such that the variance $\sigma_s^2 = \frac{1}{N} \mathbb{E}\{|s_k|^2\} = 1$, where $|\cdot|$ computes the absolute value of the input variable.
We model the IN, \( z_k \), as a two-component mixture-Gaussian model with characteristic PDF as follows [12]

\[
f_z(z; \mu_z, \sigma_z^2) = \sum_{l=0}^{L-1} p_l \mathcal{N}(z_0; 0, \sigma_{z,l}^2)
\]

(5)

where \( \mathcal{N}(z_0; 0, \sigma_{z,l}^2) \) is the Gaussian PDF of \( z_k \) with \( z_0 \) discrete envelope, zero-mean \((\mu_z = 0)\), variance \( \sigma_{z,l}^2 \), and \( p_l \) is the mixing probability of the two-component noise model. The characteristic total noise with IN in the PLC system can be expressed explicitly as

\[
z_k = z_{g,k} + z_{i,k}, \quad \forall k = 0, 1, \ldots, N - 1
\]

(6)

where \( z_{g,k} \) is the additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma_{z,0}^2 \). The PDF of \( z_{i,k} \) in (5) is usually described as an approximation from Middleton Class-A noise model [2]. In this study, the IN, \( z_{i,k} \), is assumed to follow the Bernoulli-Gaussian process, in other words, a product of Bernoulli process and Gaussian process [29] such as

\[
z_{i,k} = b_k \cdot n_{w,k}, \quad \forall k = 0, 1, \ldots, N - 1
\]

(7)

where \( n_{w,k} \) is Gaussian noise with zero mean and variance \( \sigma_{n,w}^2 \) and \( b_k \) is the Bernoulli random process with independently and identically distributed sequences of zeros and ones as

\[
Pr\{b_k\} = \begin{cases} p, & b_k = 1 \\ 1 - p, & b_k = 0 \end{cases}, \quad \forall k = 0, 1, \ldots, N - 1
\]

(8)

where \( Pr\{\cdot\} \) is the probability. The mixing probability and variance of \( z_{g,k} \) and \( z_{i,k} \) can be separated into \( p_0 = 1 - p \) and \( p_1 = p \), where \( p \) is the probability of IN occurrence, \( \sigma_{z,0}^2 = \sigma_{w}^2 \) and \( \sigma_{z,1}^2 = \sigma_{w}^2 + \sigma_{I}^2 \). The SNR and signal-to-\( \text{IN} \)-to-\( \text{IN} \) power ratio (SINR) can then be expressed in terms of the variances as

\[
\text{SNR} = 10 \log_{10} \left( \frac{1}{\sigma_w^2} \right)
\]

(9a)

\[
\text{SINR} = 10 \log_{10} \left( \frac{1}{\sigma_I^2} \right)
\]

(9b)

We assume that the receiver is perfectly synchronized to the transmitter, thus the received signal becomes

\[
y_k = \begin{cases} s_k + z_{g,k} & P(H_0) \\ s_k + z_{g,k} + z_{i,k} & P(H_1) \end{cases}
\]

(10)

where \( P(H_0) = 1 - p \) is a null hypothesis that suggests the absence of IN and \( P(H_1) = 1 - P(H_0) = p \) implies the presence of IN. By the perfect synchronization, we imply that the receiver can accurately estimate the OFDM symbol peaks. If this assumption is not met, then there will be some mis-estimation of these peaks which can consequently lead to inefficient noise detection and hence less efficient blanking. In the literature, there are three nonlinear mitigation schemes that can be used to mitigate IN namely blanking, clipping and hybrid clipping-blanking [15]. Although hybrid clipping-blanking achieves better output SNR performance than either clipping or blanking, however, only slightly and it increases receiver complexity. Being light-weight and significantly outperforming clipping scheme, we adopt blanking nonlinear scheme to mitigate the effect of IN at the receiver as follows

\[
y_k = \begin{cases} r_k & A_r \leq T_b \\ 0 & A_r > T_b \end{cases}, \quad \forall k = 0, 1, \ldots, N - 1
\]

(11)

where \( A_r = |r_k| \) is the amplitude of the received signal, \( r_k \), at the receiver and \( T_b \) is the blanking threshold. Clearly, (11) operates on the OFDM signal amplitudes without impacting the phase thus all IN processing will be limited to the amplitude only. We know that if \( T_b \) is too small, most of the desired OFDM signals will be set to zero, also, if \( T_b \) is too large, even the hunted IN will pass through undetected [2]. In [15], the noise parameters are assumed to be known a priori, and then used to determine OBT value. But, a desirable approach does not require knowing the IN a priori as the OFDM signal frames have dynamic amplitudes.

In DPTE scheme [2], the output SNR was measured as

\[
\text{SNR}_{out} = 10 \log_{10} \left( \frac{\mathbb{E}\{|s_k|^2\}}{\mathbb{E}\{|\bar{n}_r|^2\}} \right)
\]

(12)

where \( \bar{n}_r = y_k - s_k \) is the total output noise at the receiver after blanking. This is not exhaustive because in the nonlinear processing of OFDM signal amplitudes, \( \bar{x}_k = \kappa_0 x + n_d, \forall k = 0, 1, \ldots, N - 1 \) according to Bussgang theorem [30], where \( \kappa_0 \) is the amplitude attenuation constant and \( n_d \) is the uncorrelated distortion noise due to blanking. It follows that \( y_k \) can be expressed in terms of Bussgang theorem as

\[
y_k = \kappa_0 r_k + n_d, \quad \forall k = 0, 1, \ldots, N - 1
\]

\[= \kappa_0 s_k + \kappa_0 z_{g,k} + \kappa_0 z_{i,k} + n_d. \]

(13)

From (13), it is observed that in addition to the AWGN \( z_{g,k} \) and IN term \( z_{i,k} \) there is an additional noise term \( n_d \) due to the nonlinear preprocessing at the front-end of the receiver.

To compensate for the nonlinearity effects, a scaling factor is required after the blanking operation [15], so that the output SNR can be measured at the receiver after the IN removal as

\[
\text{SNR}_{out} = 10 \log_{10} \left( \frac{\mathbb{E}\{|\kappa_0 s_k|^2\}}{\mathbb{E}\{|\bar{n}_r|^2\}} \right)
\]

(14a)

\[
= 10 \log_{10} \left( \frac{E_{out}}{2\kappa_0^2} - 1 \right)^{-1}
\]

(14b)

where \( n_r = y_k - \kappa_0 s_k \) is the total output noise at the receiver after blanking, \( E_{out} = \mathbb{E}\{|y_k|^2\} \) is the total power of the nonlinearly mitigated output signal and \( \kappa_0 \) is a scaling factor expressed in [15, Eq. (13)].

In [19], [31], the OFDM signal amplitudes are shown to separate into three, namely low, average and high power signals. Consider an occurrence of IN at periods coinciding with the low amplitude signal sample time, the resultant output PLC data signals at such period exhibits a temporary characteristic amplitude within the neighbourhood of other true OFDM signals. At the receiver, applying nonlinear mitigation techniques will not even detect such INs. As a result, this will
increase the noise power thus diminishing the output SNR at the receiver. Since the BER depends on the received SNR, this phenomenon will consequently degrade the BER performance.

A. IN Mitigation using Conventional DPTE Scheme

In the previous section, we established that conventional OBT requires the knowledge of IN a priori at the receiver. The authors in [2] observed that using $T_b$ to detect IN for all OFDM frames will degrade the system performance, however, setting $T_b$ in (11) as the blanking benchmark may mitigate only a few IN samples. Meanwhile, the occurrence of IN is probabilistic in time and may not be accurate to predict its occurrence precisely. Thus, due to the fact that OFDM signal frame is dynamic and the amplitudes are similarly dynamic, we rewrite the blanking criteria based on (3) as

$$d_k = \begin{cases} r_k & A_r \leq \mathcal{P} \\ 0 & A_r > \mathcal{P} \end{cases}$$

where $d_k$ is the output of IN mitigation using DPTE criteria. In this case, $\mathcal{P}$ assumes the IN detection threshold instead of the conventional $T_b$ which must be known a priori as in (11). Realistically, $T_b$ cannot be obtained however for every OFDM symbol frame, $\mathcal{P}$ can be measured using (3). Also from (3) it must be emphasized that $\mathcal{P}$ does not depend on the phase information of the signal, instead it requires the amplitude only. The output SNR at the receiver after performing DPTE in (15) can proceed from (14) as

$$\text{SNR}_\text{out}^u = 10 \log_{10} \left\{ \left( \frac{E_{\text{out}}^u}{2\kappa_0^2} - 1 \right)^{-1} \right\}$$

where $E_{\text{out}}^u = \mathbb{E} \left\{ |d_k|^2 \right\}$.

The DPTE achieves IN reduction by predetermining the peak amplitude of OFDM signal, $\mathcal{P}$, before transmitting the signal over the PLC channel. Since the amplitudes of OFDM signals are Rayleigh distributed, it is easy to observe that the amplitudes of OFDM signals sometimes exhibit infinitesimally low energy while others exhibit reasonably higher energy. Being time dependent and non-Gaussian randomly distributed, the IN may exist within the OFDM symbol periods coinciding with the low power signals. In such cases, the output may exhibit amplitude sometimes smaller than the normal OFDM signal amplitude and becomes undetected then in turn degrading the received signal integrity. Secondly, the $T_b$ in the COB scheme may be too small that even useful signals are blanked. Thirdly, conducting a holistic search in COB is not only time consuming but also power depleting.

To overcome these, we propose converting the Rayleigh amplitude distribution of OFDM signals to uniform distribution. By this scheme, the uniform distribution of the OFDM signal frame exposes any intrusive amplitude well above the peak of the OFDM signal frame before transmission and used for the nonlinear IN mitigation process at the receiver.

III. PROPOSED SYSTEM MODEL

The algorithm for processing the OFDM signal including the amplitude conversion to uniform distribution is depicted in Fig.1. It shows steps involved in the implementation of the proposed U-DPTE scheme in this study. The amplitude distribution $|s_k|$ of the OFDM signal frame, $s_k$, is converted first to uniform distributed signals namely $u_k$ in the time-domain (the method of converting the amplitude distribution of OFDM signal is described in Section III-A). Afterwards, we compute the maximum amplitude of each OFDM signal (which is always unique for each frame) as

$$\mathcal{T} = \|u_k\|_\infty, \forall k = 0, 1, \ldots, N - 1.$$  \hspace{1cm} (17)

Similar to (3) required in (11), it can be seen that the peak threshold still exhibits amplitude-dependent characteristics and does not require the phase information.

The uniformly distributed amplitude signal is then passed through an IN channel, which gives

$$\bar{u}_k = \begin{cases} u_k + z_{g,k} & P(\mathcal{H}_0) \\ u_k + z_{g,k} + z_{i,k} & P(\mathcal{H}_1) \end{cases}$$

Our interest is to achieve better output SNR at the receiver after blanking when using (14). By the proposed scheme,
one finds that DPTE scheme is more dynamic and adaptive than the conventional COB technique. To achieve optimal performance in COB scheme, an exhaustive search for the optimal threshold is required [12] which expands processing time and depletes system power. As an advancement also, the amplitudes of the uniformly distributed OFDM signals expose the amplitudes of IN component better since the amplitude attains even-distribution.

In the case of (18), the U-DPTE (i.e., enhanced DPTE) criteria becomes

$$y_k^u = \begin{cases} 
\bar{u}_k & A_u \leq \tau, \forall k = 0, 1, \cdots, N - 1 \\
0 & A_u > \tau, \forall k = 0, 1, \cdots, N - 1
\end{cases} \tag{19}$$

where $A_u = |\bar{u}_k|$ is the received signal amplitude, $\bar{u}_k$, and $\tau$ is the U-DPTE blanking threshold. Due to the uniform amplitude conversion, then $\tau = T - P$ where $\tau$ is non-negative. Since $\bar{u}_k$ ensures a reduction in signal amplitude compared to $P$, INs whose power level may be lower than $P$ can be detected and removed. This is absent in DPTE and will enhance the output SNR performance as it will be demonstrated in Section IV.

### A. Uniformly Distributed OFDM signals

In OFDM systems, converting signal amplitudes to uniform distribution can be achieved by [32], [33]

$$u_k(s_k) = F_{|u|}^{-1}\left(F_{|s|}(s_0)\right) \text{sgn}(s_k), \forall k = 0, 1, \cdots, N - 1 \tag{20}$$

where $\text{sgn}(x) = \frac{x}{|x|}$ is the phase of the signal, $F_{|u|}^{-1}(\cdot)$ is the inverse cumulative density function (CDF) of the uniformly distributed signal and $F_{|s|}(\cdot)$ is the CDF of the conventional Rayleigh amplitude distributed signal. From this, it follows that to transform the distribution of an OFDM symbol (e.g. Rayleigh distribution) to another desired distribution (e.g. uniform distribution), one must know the desired PDF and CDF from [32]. Meanwhile, let the CDF of conventional OFDM signal amplitudes be

$$F_{|s|}(s_0) = 1 - \exp\left(-\frac{s_0^2}{\sigma_s^2}\right), s_0 > 0 \tag{21}$$

where $s_0$ is the discrete envelope of $s_k$, $\forall k = 0, 1, \cdots, N - 1$. There are two ways of converting the PDF of (21) to exhibit uniform distribution: i) by imposing a choice constraint on (21) as in [25] or ii) by predetermining a desired PDF and using the CDF in (20) as in [33].

Now, suppose that some signal $|t_k|^m, \forall k = 0, 1, \cdots, N - 1$ with degree $m$, $\forall m \neq 0$ is uniformly distributed within $[0, \alpha]$, then the CDF of $|t_k|^m$ is simply $\Pr\{|t_k| \leq x_0\} = \Pr\{|t_k|^m \leq x_0^m\} = x_0^m/\alpha$ and the inverse CDF $F_{|u|}^{-1} = (\alpha s_0)^m$, where $0 \leq s_0 \leq \sqrt[\alpha]{\alpha}$ [33]. Combining these and substituting accordingly in (20), the uniform distribution criteria is achieved as [33]

$$u_k^1(s_k) = \text{sgn}(s_k) \alpha^{\left\lfloor\frac{1}{\alpha} \left[ 1 - \exp\left(-\frac{s_0^2}{\sigma_s^2}\right)\right]\right\rfloor} \tag{22a}$$

$$\alpha = \left[ \frac{\mathbb{E}\left\{ |s_k|^2 \right\}}{\mathbb{E}\left\{ \left[ 1 - \exp\left(-\frac{s_0^2}{\sigma_s^2}\right)\right]^\frac{1}{\alpha}\right\}} \right]^{\frac{1}{m}} \tag{22b}$$

This approach achieves uniform distribution by expanding the amplitudes of low power signals and also compressing the amplitudes of high power signals. We shall call this exponential converter (EC). A drawback on this (22) model is that when $m$ increases, the expansion of the amplitudes of low power signals reduces, thus limiting the degree of uniformity of transformed signal. Thus, in addition to (22), other models that can achieve reasonable performance also exist [25]–[28], [33]. The first of these uses the airy-function as follows

$$u_k^2(s_k) = \alpha_2 \cdot \text{sgn}(s_k) \cdot \text{airy}(0) - \text{airy}(\beta \cdot |s_k|) \tag{23}$$

where $\text{airy}(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{\left(\frac{x^2}{3} + xv\right)} dv$ is the airy-function of the first kind, $\nu$ is the trailing expansion series and $x$ is the input variable [28], [34]. We call (23) airy-function converter (AC). $\alpha_2$ is a normalization parameter which ensures that the output power of the uniformly distributed signals is equivalent to the power of the input (unmodified) signal as

$$\alpha_2 = \left[ \frac{\mathbb{E}\left\{ |s_k|^2 \right\}}{\mathbb{E}\left\{ \text{sgn}(s_k) \cdot \text{airy}(0) - \text{airy}(\beta \cdot |s_k|) \right\}^2} \right]^{\frac{1}{2}} \tag{24}$$

where $\beta$ is parameter that controls the degree of uniformity of the OFDM signal amplitudes, usually $\beta > 0$. The power normalization parameter in (23) was absent in the foremost uniform distribution amplitude model for PAPR reduction proposed in [26] which can be expressed as

$$u_k^3(s_k) = A_3 \cdot \text{sgn}(s_k) \log_\psi \left(1 + \mu_3 \left|\frac{s_k}{A_3}\right|\right) \tag{25}$$

where $\psi = (1 + \mu)$. $\mu_3 > 0$ and $A_3 = |s_k|$. We call (25) $\mu$-law converter (MC). Thus, the output of (25) requires scaling (multiplying) by $\alpha_3$ to ensure equal power level with the input signal power. In [19], we showed that the power scaling parameter can be derived by comparing the input and output signal powers as $\alpha_3 = \sqrt{\frac{\mathbb{E}\left\{ |s_k|^2 \right\}}{\mathbb{E}\left\{ |u_k^3(s_k)|^2 \right\}}}$.

Unfortunately, the model in (25) does not compress the amplitudes of higher power signals, instead it amplifies the amplitude of weak signals only. Based on this, the authors in [27] imposed the logarithmic constraint to achieve high amplitude compression in addition to the expansion of low amplitude signals as follows

$$u_k^4(s_k) = \text{sgn}(s_k) \left(\alpha_4 \times \ln \left[1 + \mu_3 \left|\frac{s_k}{A_4}\right|\right]\right)^{\frac{1}{\alpha_4}} \tag{26a}$$

$$\alpha_4 = \left(\frac{\mathbb{E}\left\{ |s_k|^2 \right\}}{\mathbb{E}\left\{ \left(1 + \mu_3 \left|\frac{s_k}{A_4}\right|\right)^{\beta_4}\right\}}\right)^{\frac{1}{\alpha}} \tag{26b}$$

where $\alpha_4, \beta_4 > 0$ and $A_4 = |s_k|$. Notice that we have used $\mu_3$ in both (25) and (26) to designate that both are the same. We name (26) log-based MC (LMC). The easier method of deriving the uniform distribution function is simply by imposing a choice constraint on the PDF of the unmodified OFDM signal amplitudes [25] such as

$$f_\nu(s_k) = \varphi f_{|s|}(s_k) = \varphi \frac{s_0}{\sigma_s^2} \exp\left(-\frac{s_0^2}{\sigma_s^2}\right) \tag{27}$$
where $\varphi$ is the choice constraint and can be estimated from the CDF by integrating the PDF in (27) as $\int_{-\infty}^{c} f_{\nu} (s_k) \, ds_k = 1$ or explicitly as $\varphi = 1/\left[1 - \exp \left(-c^2/\sigma_s^2\right)\right]$, $c = \lambda E \{|s_k|\}$ and $\lambda > 0$. Using (20), the uniform distribution function can be realized as

$$u_k^5 (s_k) = \alpha_5 \cdot \text{sgn} (s_k) \left[-\ln \left(1 - \frac{1}{\varphi} \left[1 - \exp \left(-\frac{s_k^2}{\sigma_s^2}\right)\right]\right)\right]^{1/\sigma_s^2}$$  \hspace{1cm} (28a)

$$\alpha_5 = \sqrt{E \{|s_k|^2\}/E \{|u_k^5 (s_k)|^2\}}$$  \hspace{1cm} (28b)

We call (28) Rayleigh distribution constraint-based converter (RCC). Generally, the conversion models (22)-(28) are admissible into the post-modulation PAPR reduction scheme, namely companding.

First, we compare the performance of the conversion transforms (22)-(28) in terms of their respective impacts on the amplitude expansion and/or compression to attain uniform distribution as depicted in Fig. 2. It is observed that AC graciously expands the amplitudes of weaker signals and slightly compresses the high power signals. This will lead to high output SNR due to slight in-band distortion. On the other hand, the RCC model compresses the amplitudes of high power signals but does not impact the amplitude of low power signals. Both LMC and EC simultaneously expand and compress the amplitudes of the signals for both high and weaker signals. Lastly, MC expands the amplitudes of low power signals while not impacting the amplitudes of high power signals. Whether a transform achieves expansion or compression, all schemes follow their own ability to achieving uniform distribution. However, the output SNR performance will demonstrate the degree of uniformity achieved in the conversions.

We can appreciate the PDF distribution of uniformly distributed amplitude signal for all the model transforms (22)-(28) in Fig. 3. In Fig. 3, we depict examples of uniformly distributed amplitude models with reference to the conventional OFDM signal. As the amplitudes approach the mean amplitude tightly, the resulting signal proportionately achieves better uniform distribution. Hence, the presence of IN in the PLC system when passed through the power-line channel is better detected. Notably, due to the poor amplitude expansion of low power signals as identified in Fig. 2, the distribution of (28a) in Fig. 3 is not tightly around the mean - this can be improved by suitably varying $\lambda$.

As the conversion of the amplitude to uniform distribution achieves peak reduction, then our proposal also connote PAPR reduction of OFDM symbols. It follows that in addition to enhancing IN reduction, the amplitude conversion to uniform distribution also achieves PAPR reduction, where

$$\text{PAPR} = \frac{\|u_k\|_\infty^2}{E \{|u_k|^2\}} = \frac{T^2}{E \{|u_k|^2\}}, \quad \forall k = 0, 1, \ldots, N - 1.$$  \hspace{1cm} (29)

Since we scaled the input signal such that $\sigma_s^2 = \frac{1}{2} E \{|u_k|^2\} = 1$, then $E \{|u_k|^2\} = 2\sigma_s^2 = 2$. Thus, the PAPR in (29) relates to the dynamic threshold for mitigating IN in (17) as

$$\mathcal{P} = \|s_k\|_\infty = \sqrt{2\text{PAPR}_s}$$  \hspace{1cm} (30a)

$$\mathcal{T} = \|u_k\|_\infty = \sqrt{2\text{PAPR}_w}$$  \hspace{1cm} (30b)

where $\text{PAPR}_s$ is PAPR of the unmodified OFDM signal and $\text{PAPR}_w$ is PAPR of the uniformly distributed amplitude signals. Since the complementary CDF (CCDF) can be expressed as

$$\text{CCDF} = \Pr \{\text{PAPR} > \text{PAPR}_0\} = \left[1 - (1 - \exp (-\text{PAPR}))^N\right],$$  \hspace{1cm} (31)
both (30a) and (30b) can be rewritten in terms of CCDF as

\[ P = \sqrt{2\text{PAPR}_u} = \sqrt{-2 \ln \left( 1 - \{1 - \text{CCDF}_u \}^{\frac{1}{\beta}} \right)} \]  \hspace{1cm} (32a)

\[ T = \sqrt{2\text{PAPR}_u} = \sqrt{-2 \ln \left( 1 - \{1 - \text{CCDF}_u \}^{\frac{1}{\beta}} \right)} \]  \hspace{1cm} (32b)

Clearly, it follows from (30b) that the threshold for removing IN is directly proportional to the PAPR. That is, reducing the PAPR which is achieved by increasing the degree of uniformity of the signal amplitudes enhances the reduction of IN. In Fig. 4, the CCDF performance is shown in relation to the peaks. As the number of subcarriers increases, the peaks also increase and will in turn impact the IN reduction performance.

Additionally, we exemplify in Fig. 5 the effect of dynamism in the amplitude variation of OFDM signal for the conventional (Rayleigh) distributed amplitude and uniformly distributed amplitudes. While all the models significantly reduce the PAPR, LMC achieves the best PAPR performance corresponding to the PDF behaviour as shown in Fig. 3. One corroborates that the conventional amplitude is well higher compared to the uniformly distributed. Consequently, the PAPR varies significantly than in the case of uniformly distributed amplitudes. By this phenomenon, we infer that the amplitudes of the unmodified signal will mask the low power samples of IN leading to high error floor and output SNR degradation.

B. System Model Processing over PLC Fading Channel

Although PLC involves data transmission over cables, the PLC cable (i.e. PLC channel) differs from the designated data transmission cables such as twisted-pair, fibre-optic or coaxial cables [9]. PLC channels also differ from wireless and other wireline channels in terms of physical properties, topology, propagation and structure. Thus, the fading channel model is modeled differently [6], [7], [35]–[39]. Meanwhile we adopt the log-normal model [35], [38], [39] for its simplicity and wide usage in the literature in which the PDF can be expressed as

\[ f_v(v; \mu_v, \sigma_v) = \frac{\zeta}{\sigma_v \sqrt{2\pi}} \exp \left( -\frac{\left( 10 \log_{10}(v) - \mu_v \right)^2}{2\sigma_v^2} \right) \]  \hspace{1cm} (33)

\[ \sigma_v^2 \] and \( \mu_v \) are the variance and mean of \( 10 \log_{10}(h) \), respectively, \( h \) is the channel impulse response, \( v = h^2 \), and \( \zeta = 10/\ln(10) \) is a scaling constant. Now, after passing the signal (with cyclic prefix added to combat intersymbol interference) over a log-normal multipath fading channel, the received signal becomes

\[ r_k^h = h_k u_k + z_n(n), \forall k = 0, 1, \cdots, N-1 \]

\[ = h_k U(s_k) + z_w(n) + z_i(n), \]  \hspace{1cm} (34)

where \( u_k = U(s_k) \) is the uniform distribution transforming function. The received signal power in this case will be attenuated due to the channel fading and will diminish the output SNR in comparison to the ones described in Section III-A. We can exploit these channel terms as gains in using minimum mean square error (MMSE) equalization scheme since zero-forcing will expand the noise and degrade the SNR. In this case, the received signal after equalization becomes

\[ \tilde{u}_k = \frac{h_k^*}{\left| h_k \right|^2 + SNR_{out}} r_k^h, \forall k = 0, 1, \cdots, N-1 \]  \hspace{1cm} (35)

where in (35), we assumed perfect knowledge of the channel state information at the receiver. Let the MMSE equalizer in (35) be \( h_k^{-1} \) such that

\[ a_k = h_k^{-1} \cdot r_k^h = u_k^* + h_k^{-1} \cdot z_w(n) + h_k^{-1} \cdot z_i(n) \]  \hspace{1cm} (36)

in which we observe that both IN and background noise are modified by \( h_k^{-1} \). Now, the effect of IN can be reduced through blanking the signal samples according to the following criteria

\[ y_k^u = \begin{cases} u_k & A_u \leq T, \forall k = 0, 1, \cdots, N-1 \end{cases} \]  \hspace{1cm} (37)
where $A_u = |\hat{u}_k|$. While IN is non-Gaussian, the background noise is Gaussian distributed. To compensate the signal amplitude due to the $\hat{h}_k^{-1}$ amplification of the noise part, we introduce a pseudo-Gaussian amplitude variable, namely $\epsilon$ to the peak amplitude so that (37) is rewritten as

$$y_k^u = \begin{cases} \hat{u}_k & A_u \leq T + \epsilon, \forall k = 0, 1, \ldots, N - 1 \\ 0 & A_u > T + \epsilon \end{cases}$$

where $\epsilon = \mathbb{E}\{|\mathcal{G}|\}$ and $\mathcal{G}$ is a pseudo-Gaussian random variable generator. By the factor $\epsilon$, the scheme posits the signals beyond the Gaussian noise level but below the IN power level so that correct IN mitigation is achieved - this is pronounced in the results presented in Section IV-B. Explicitly, the output SNR over fading channels after equalization and the nonlinear preprocessor can be calculated as

$$\text{SNR}_u^k = \frac{\mathbb{E}\{\kappa_0 |\hat{u}_k|^2\}}{\mathbb{E}\{|\tilde{y}_k^u - \kappa_0 u_k|^2\}}$$

where $\kappa_0$ is the scaling factor obtained through simulation as

$$k_0 = \frac{\mathbb{E}\{\tilde{y}_k^u \cdot (u_k^u)^*\}}{\mathbb{E}\{|u_k|^2\}}$$

that compensates for the nonlinear processing. These discussions are absent in all the earlier DPTE schemes.

IV. RESULTS AND DISCUSSION

The model system of the foregoing discussions involves $N = 256$ random signals modulated using 16-QAM before passing it through IFFT device to generate time-domain signal as represented in (1). The noise samples are generated as represented in (6) and added to the transmitted signal. In the DPTE scheme, the peak signal amplitude is dynamically estimated as in (3) and used at the receiver to blank received signal amplitudes exceeding $\mathcal{P}$. On the other hand, we convert the amplitude distribution of the time-domain OFDM signals to uniform distribution as described in Section III-A and obtain the peak amplitude $T$ which is used to blank excess amplitude signals. Using (14) and (16), we calculate the output SNR for DPTE and U-DPTE schemes, respectively.

A. Over AWGN Only Channel

In this section, we present the results of the PLC signal processed over AWGN-only channel with IN as shown in Fig. 6. The results are shown for different percentages of IN namely $p = 0.1$, 0.03, 0.01 and 0.003 which represent 10%, 3%, 1% and 0.3% IN in each $N = 256$ subcarrier of OFDM frame, respectively. We average these OFDM symbol frames over $10^4$ samples as depicted in Fig. 1. By these, it is possible to compute the number of OFDM pulses affected by
the IN simply as $p \times N$ which are equivalently 26, 8, 3 and 1, however with the position of OFDM signal pulse affected not static as each symbol frame is unique. Comparing COB to DPTE, it is observed that the DPTE outperforms COB. However, converting the OFDM signal amplitudes towards uniform distribution (using EC scheme) enables the proposed U-DPTE to outperform both COB and DPTE respectively.

Furthermore, we measure the relative gain achieved by the DPTE and proposed U-DPTE (using EC) schemes relative to COB as shown in Fig. 7. Given the output SNR as represented in (14) and the output SNR as represented in (16), the relative gain of the DPTE scheme with respect to that of COB is realized as

$$G_R = \left( \frac{\text{SNR}_{\text{DPTE}}}{\text{SNR}_{\text{COB}}} \right)$$

$$G'_R = \left( \frac{\text{SNR}_{\text{U-DPTE}}}{\text{SNR}_{\text{COB}}} \right)$$

and converted to dB as $10\log_{10}(G_R)$. As remarked in Fig. 6, the output SNR gains become obvious in Fig. 7, with the proposed U-DPTE achieving the most performance in all IN probability measures shown.

Since the U-DPTE system outperforms the DPTE scheme, we extend our investigation to evaluating all the five different uniform distribution models for future design references. In this regard, we investigate first the maximal SNR performances as shown in Figs. 8 and 9. Marginally, the LMC scheme outperforms all other uniform distribution models. This is
related to the tightness of amplitude distribution around the mean amplitude after the conversion to even-distribution. Meanwhile, increasing the percentage of IN in the channel increases error likelihood, further increases the total noise power and diminishes the output SNR as seen in Fig. 8 when compared to Fig. 9.

Also, we investigate the relative gains of the uniform distribution models having established that U-DPTE outperforms DPTE. The results for the models (22)-(28) are depicted in Figs. 10 and 11. We establish that both in terms of output SNR performance and relative gain, the LMC scheme achieves the best performance of 3dB and 2dB for $p = 0.1$, respectively. The proposed U-DPTE scheme also achieves 2 dB relative gain at $p = 0.01$.

Although the proposed U-DPTE scheme achieves excellent output SNR performance compared to the DPTE scheme, we conjecture that the results can still be improved. For example, the U-DPTE scheme is robust over IN power levels larger than that of the uniformly distributed OFDM signals. OFDM signals with power levels below the mean power can be selectively increased to achieve uniform distribution [31]. Thus, by increasing the tightness of OFDM signal amplitude distribution to the mean will further improve mitigation against IN with power level below OFDM signals. In this regard, we point the reader to [10], [12], [31], [40], [41].

B. Over Fading Channels

IN detection and mitigation over fading channel when using DPTE is not popular in the open and available literature as discussed in Section I. This may be due to the fact that it is a more difficult problem to handle IN mitigation when using...
Relative Gain, dB

0
1
2
3
4
5
6
7
8
9
Relative Gain, dB

DPTE after the channel equalization at the receiver. Thus, we approach this problem by considering the fact that after equalization, the Gaussian noise power level is amplified by the equalization filter which now changes the desired signal power level at the receiver. This problem complicates the detection of the desired signal thus increasing the probability of missed detection (and also blanking) which undermines the received signal integrity and diminishes the received SNR. To overcome this problem at the receiver, the power level of detection threshold must be raised above such power level. We achieve this by generating some pseudo-Gaussian distributed variable large enough as the number of signals and compute the mean amplitude. The mean amplitude is then added to the peak amplitude of the OFDM signal to raising the detection threshold power level. Thus, the new amplitude beyond which IN is detected is given by $T' = T + \epsilon$, where $\epsilon$ is the adaptively computed mean amplitude desired to compensate the noise magnification incurred during equalization. To ensure front-end pre-processing, we perform equalization in time domain before IN blanking - the algorithm is represented in Fig. 1.

Comparing the optimal SNR performance over fading channel in Fig. 12 to the AWGN-only channel in Fig. 9, it is observed that the optimal SNR over the fading channel is better than that of the AWGN-only channel for both COB and U-DPTE. However, the DPTE incurs a loss due to the fact that unmodified OFDM signal amplitude masks low power IN. Although the channel coefficients contributed by the log-normal fading channel attenuate the signal power, the IN detection scheme offered by applying $T'$ threshold is therefore robust. These are evident in the results presented in Figs. 12-17.

Next, comparing the DPTE scheme with the COB in Fig. 12, it is seen that there exists only marginal performance of DPTE better than the COB - in fact, during strong IN presence (e.g. when $p = 0.1$), COB outperforms DPTE scheme by about 1.5dB as depicted in Fig. 14. While the optimal search in COB can mitigate some IN with power level below the peak symbol, the DPTE cannot. These INs are better unveiled when the OFDM signal amplitudes are transformed into uniform distribution and these amplitudes appear well-below the conventional peak of the unmodified OFDM signal amplitudes. In return, the optimal SNR is enhanced in using U-DPTE than both COB and DPTE, respectively. Correspondingly, the gains become obvious as depicted in Fig. 13 when $p = 0.01$. Meanwhile, the DPTE scheme shows a loss at about -15 dB SINR implying that the COB outperforms DPTE at this point. Varying the percentage of IN present in the channel does not change performance of the schemes (when using the novel peak threshold estimation over fading channel) as it is found in Figs. 13-17 except that increasing the percentage of IN shows that the DPTE is not robust over fading channels in comparison to COB as shown in Fig. 14. Relative to the COB results, Figs. 13, 15 and 17 expose the inferiority performance of DPTE to the COB. Then to the proposed model, it is observable that U-DPTE achieves up to 8dB better than DPTE technique. From these results and when using our proposed U-DPTE model combined with the novel peak threshold estimation over fading channel, we infer that the probability of missing IN samples with power level below the peak amplitude is significantly minimized up to 8dB better than the DPTE.

In general, we described in Section II that blanking is a nonlinear IN mitigation scheme that significantly outperforms clipping technique in terms of received SNR but performs slightly worse than hybrid clipping-blanking which has significantly higher processing cost. We conjecture that these phenomena will suffice when either clipping or hybrid clipping-blanking is used in similar study; hybrid clipping-blanking incurring significantly higher processing costs.

V. CONCLUSION

OFDM signal transmission over PLC channels is prevaricated by IN which can be mitigated by assuming a prior knowledge of the IN by using nonlinear pre-processing such as blanking. In the literature, it has been shown that since each OFDM symbol frame is unique, a better approach to IN mitigation is to determine the peak amplitude of each of the symbol frame as a threshold for IN mitigation - usually referred to as DPTE. We showed that these peaks masquerade IN with low power level and thus degrades the output SNR performance. Before transmission over the PLC channel, we showed that transforming the amplitude distribution to uniform distribution before applying DPTE achieves excellent SNR performance and mitigates the IN more efficiently than both COB and DPTE. Over fading channels, we introduced a novel method of using a pseudo-Gaussian generator to achieve optimal threshold for IN mitigation. In our results, we found that the proposed threshold value estimation further enhanced the output SNR performance of the proposed U-DPTE technique that a COB and DPTE schemes by up to 8 dB gain. It follows that the proposed U-DPTE scheme is both robust over fading channels and AWGN-only channels.