Applications and props: the impact on engagement and understanding

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CASE STUDY

Applications and props: the impact on engagement and understanding

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Abstract

Problems based on applications or objects were added into a first year pure module in gaps where real-life problems were missing. Physical props were incorporated within the teaching sessions where it was possible. The additions to the module were the utilities problem whilst studying planar graphs, data storage when looking at number bases, RSA encryption after modular arithmetic and the Euclidean algorithm, as well as molecules and the mattress problem when looking at group theory. The physical objects used were tori, molecule models and mini mattresses. Evaluation was carried out through a questionnaire to gain the students' opinions of these additions and their general views of applications. Particular attention was paid to the effect on engagement and understanding.

Keywords: Use of applications; physical props; manipulatives; engagement and understanding

1. Introduction and background

The module 'Number and Structure' is situated within a BSc Mathematics programme which focusses on real-world applications and mathematical modelling. The module is one of the purer modules on the course. The content currently consists of sets, graph theory, digraphs, matrices, Boolean algebra, number bases, modular arithmetic, proof, the Euclidean algorithm, irrational numbers, rings, groups and equivalence relations. Some topics were already linked with physical and practical examples. These included Rubik's cubes (discussed by Cornock (2015) for another module) and playing cards when looking at permutations, circuits whilst studying Boolean algebra, the instant insanity problem for graph theory, and symmetries of physical 2D-shapes in group theory.

In a review of the module in 2017, some topics were removed if they could not immediately be linked to any real-life objects / applications and did not feature in subsequent modules. 'Applied mathematical problems' were introduced. Such a problem is where *"the situation and the questions defining it belong to some segment of the real world and allow some mathematical concepts, methods and results to become involved"* (Blum and Niss, 1991). The problems are *"well suited to assist students in acquiring, learning and keeping mathematical concepts, notions, methods and results, by providing motivation for and relevance of mathematical studies"*.

Manipulatives were used where possible. These are physical, pictorial, or virtual representations of abstract concepts, widely used in pre-university mathematics education (Furner and Worrell, 2007), as well as other subjects. Those discussed in this article are often referred to as physical props, though other, virtual and pictorial, manipulatives are used in other parts of the module. Much has been written (Cope, 2015; McNeil and Jarvin, 2007) about the use and effectiveness of manipulatives in teaching mathematics to pre-university students, especially in primary education, but little is written about their use for teaching mathematics in higher education. There has been some discussion as to the use of manipulatives in other areas of undergraduate education such as Chemistry (Saitta, Gittings, and Geiger, 2011), Engineering (Pan, 2013) and particularly in areas related to Biology (Jungck, Gaff, and Weisstein 2010; Krontiris-Litowitz, 2003; Guzman and Bartlett, 2012). The molecular models discussed in this article were themselves borrowed from the university's Biochemistry team.

McNeil and Jarvin (2007) weigh up both sides of the debate on the effective use of mathematical manipulatives, though the counterevidence they provide focuses mainly on the *"cognitive resources"* of children and so is harder to reconcile with the abilities of adults in higher education. We believe that other recommendations, such as principles (c) and (d) described by Laski, Jor'dan, Daoust, and Murray (2015), are in line with the use of manipulatives within the scope of this module, with their other principles again focussing on the cognitive capabilities of children rather than adults. If used in the right way, physical objects can be used to represent abstract ideas in physical form to *"help students deeply understand the math they are learning and needing to apply to our everyday life"* (Furner and Worrell, 2007).

The following problems, topics and activities were brought into the module:

Utilities problem: The utilities problem is a well-known recreational mathematics puzzle (Kullman, 1979). The problem can be described using Figure 1 below in which each house (A, B, C) has to be connected to each of the utilities (Gas, Water, Electricity) in such a way that none of the connections cross one another.

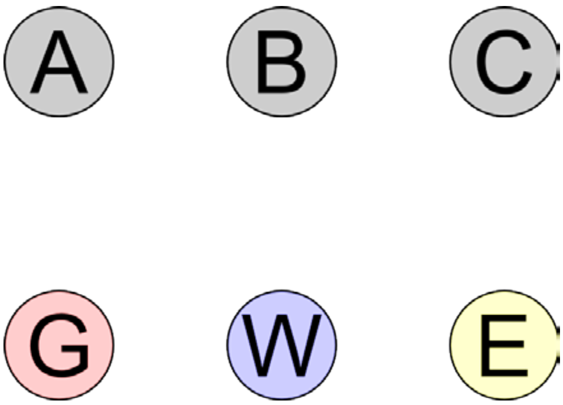

Figure 1. The utilities problem

The students were asked to try and find a solution to this problem, which is not possible, before trying to solve the problem on a labelled polystyrene torus (donut) which was used as a physical prop (see Figure 2). String was attached to represent the connections.

Figure 2. Utility problem on a torus

Data storage: A section on data storage was added after the work on different number bases. This included general information about how data is stored and specific information on the storage of pictures, numerical data, text, and logical data.

RSA encryption: Following a section on the Euclidean algorithm, RSA encryption was used to encrypt and decrypt messages (Katz, Menezes, Van Oorschot, and Vanstone, 1996). An assignment was introduced on RSA encryption which required the students to decrypt an individualised message.

Chemistry: The students were required to look at physical 3D-models of molecules of ammonia (NH_3) (Figure 3) and tetrabromogold ($AuBr_4^-$) (Figure 4) and produce the group tables for their symmetries (Walton, 1998). Such manipulatives are widely used in teaching chemistry, models having been used to demonstrate molecular structures since the middle of the 19th century (Perkins, 2005).

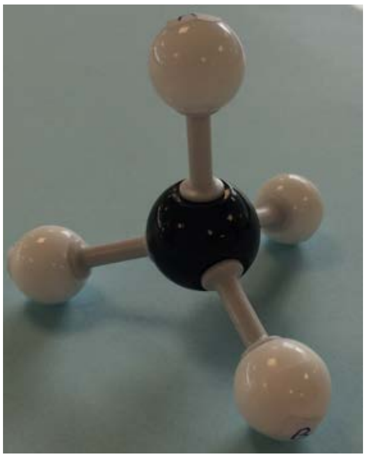

Figure 3. Ammonia molecule model

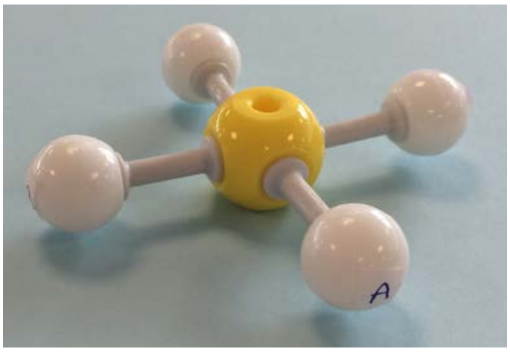

Figure 4. Tetrabromogold molecule model

Mattress problem: The mattress-flipping problem is described by Hayes (2008) as the task of finding *"some set of geometric manoeuvres that you could perform in the same way every time in order to cycle through all the configurations of the mattress"*. The students were given a mini mattress, as shown in Figure 5, and were them to describe the possible movements which would change its orientation. They were then asked whether any combination of such movements would cycle through every possible orientation without as much setup as the tasks with the other physical props.

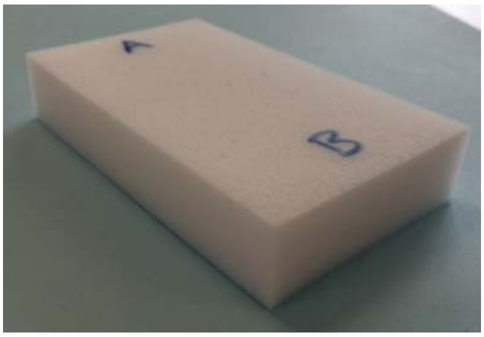

Figure 5. Mini mattress model

Following the introduction of the new topics and practical tasks, evaluation was carried out to gain the students' opinions on the changes, and the impact on their engagement and understanding.

2. Methodology

Evaluation was undertaken through an in-class online questionnaire towards the end of the 2017/18 academic year. Out of the 68 students who attended the classes that week (70% of students taking the module), 51 students filled in the questionnaire.

The questionnaire started with some general questions about applications. The first question was about whether seeing the uses of a topic changes their approach to work. Then the students were asked about their preferences in terms of theory and applications.

The general questions were followed by some statements about specific applications used in the module and whether they helped with understanding topics as well and whether the connections between the applications and the topics were clear. The students were asked to give an indication of how much they agreed or disagreed with the statements by using a 5 point Likert scale. These were followed by statements about physical props used within class, again with a 5 point scale.

Given that more applications were considered when studying group theory than ring theory, a question was included on whether there was a difference learning the topics. This was followed by questions on whether their views of particular topics had changed once the application had been considered.

The students were asked a general question about whether having physical examples in class clarified the concepts that were introduced. The questionnaire concluded with a statement about the module being beneficial to the real-world, which was part of a 5 point scale question and was followed by a comments box for reasons why.

There was a good response rate throughout the questionnaire. A total of 32 students (63% of students who filled in the questionnaire) answered the question about comparing group theory with ring theory, which was the lowest response to a question. All other questions had at least 42 (82%) students answering them. All subsequent percentages will be of the number of students who answered the question, unless otherwise stated.

3. Results

3.1 Use of applications

When asked whether seeing the uses of a topic change how they approach work, 60% of the students gave a positive response, whereas 40% gave a negative response. The most common reason was that it makes them more motivated or interested (36% of 60%). One student said that *"if [they are] able to see the uses of a topic [they are] more likely to want to understand why the procedures work"* and that *"without seeing [an] application [they] tend to just remember the procedure and not ask question[s] about why certain things work".* Other reasons provided by the students included that it helps them, they use different methods for different topics, they may alter the layout, it gives a second angle, it allows the work to be seen differently, it encourages focus, they want to understand it more, they prioritise work that is more useful and their approaches are more analytical.

When asked what they prefer when working on a problem, the majority of students said that they prefer a mixture of theoretical problems and problems based on an application or did not have a preference (82%), some prefer theoretical problems (10%) and some prefer problems being based on an application (8%).

The reasons provided by the students who like a mixture of theoretical and applied questions included that a mixture helps with understanding (27%), it is more appropriate within jobs to do both (4%), it gives an appreciation of the theory and the application (11%), different types are better for different topics (7%), sometimes it is easier out of context first (4%), it enables students to answer a range of questions (7%), and makes it more interesting and enjoyable (9%). One student said that *"the application lets you see the uses of the technique while the theoretical part helps you [to see] how to understand how to perform it"*. Another said that they *"like to learn the theory of something then see its application"* as *"it tends to make [them] think deeper about why processes work and forces [them] to ask knowledge-furthering questions"*. Also, one student said that *"the more theoretical approach…allows [them] to get a base understanding of the concept but [they] also like to be able to apply it to a real world scenario and get some practice with it."*

The students who expressed a preference for problems being based around an application said this was because it would otherwise *"seem pointless",* it is easier to understand, and it makes *"a topic seem more interesting to know how it is used"*. The ones who prefer theoretical problems said they find them interesting, that they can understand the problems better, and they prefer looking at the *"raw maths"*.

3.1.1 The Utilities Problem

The students were asked whether the utilities problem helped them with their understanding of planar graphs. The results in Figure 6 show there was a small number who did not think that the problem helped them. There was a substantial number who agreed with statement (65%) or provided a neutral response (29%).

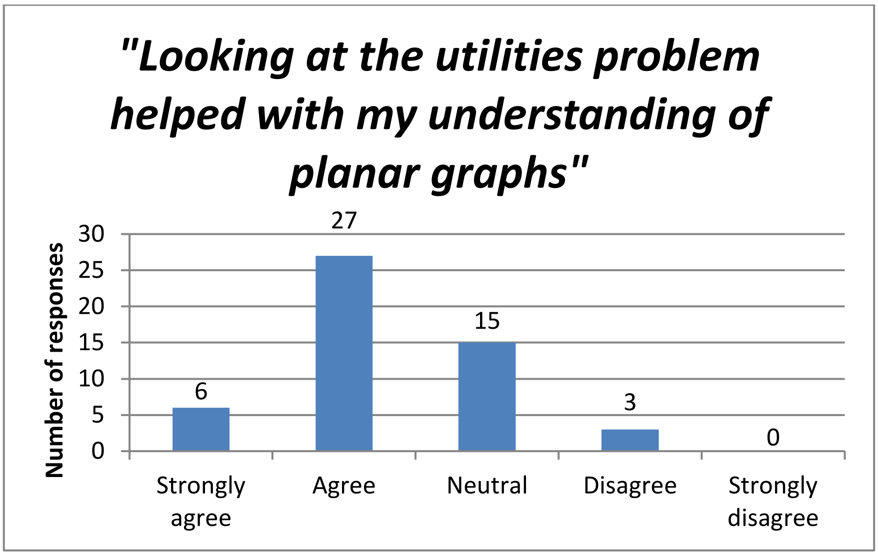

Figure 6

3.1.2 Group Theory

When asked whether learning about ring theory differed from learning about group theory, a large proportion of the students (53%) said it was not different and some said it was (18%). One student in particular was unable to remember ring theory. Another said that it was different *"because [they] used physical objects to help with group theory which helped [them] understand it more"*.

The students were asked whether they agreed with statements about whether the work on molecules and the mattress question helped with their understanding of group theory. A large proportion of the responses were positive (61% for molecules and 73% for the mattress problem) or neutral (29% and 20%) as seen in Figures 7 and 8.

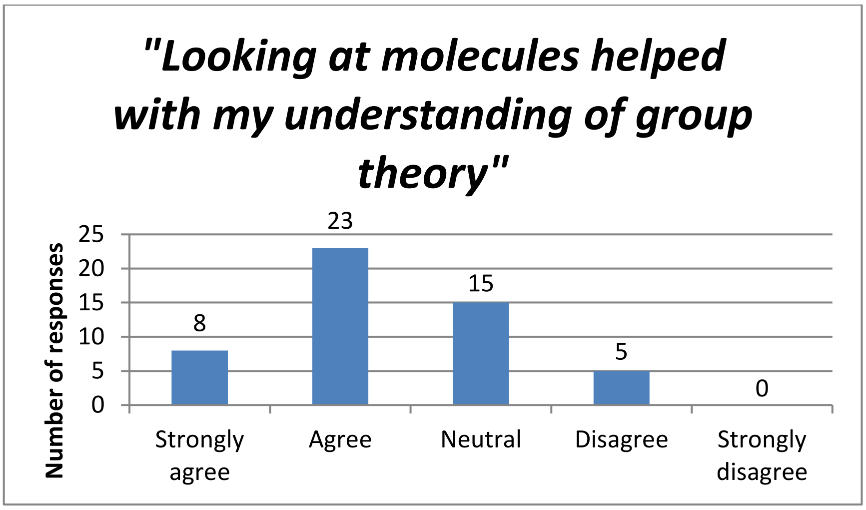

Figure 7

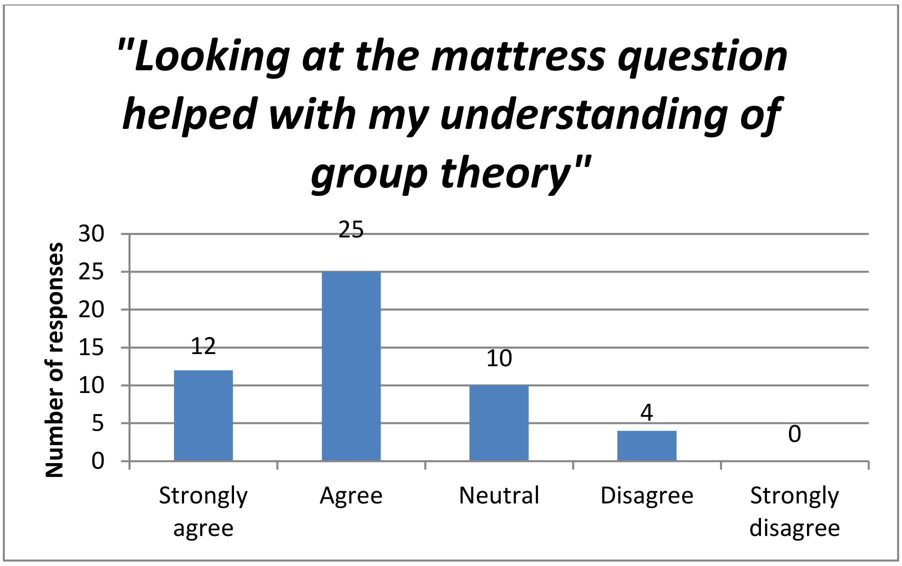

Figure 8

3.1.3 Binary and Hex Numbers

When asked whether their view of hex and binary numbers had changed once they had studied data storage, 40% said it had and 60% said it had not. Comments about why it had not changed included that they had studied it before (19% of 60%), that they had not studied the topic before (12%), data storage was confusing (12%), it did not help (8%), and they did not understand it (12%). Amongst the positive responses, comments about why their view changed included that it was useful to know why it was being studied or how it could be used (53% of 40%), it helped with understanding (35%), and the real-life application helped (12%). One student said that it gave them an *"interesting context"*.

When asked whether they agreed with the statement that looking at data storage helped with the understanding of binary and hex numbers, there was much more of a mixed response as presented in Figure 9. Just over half the students (52%) who answered this question said that it helped, a third provided a neutral response and 15% thought it did not help.

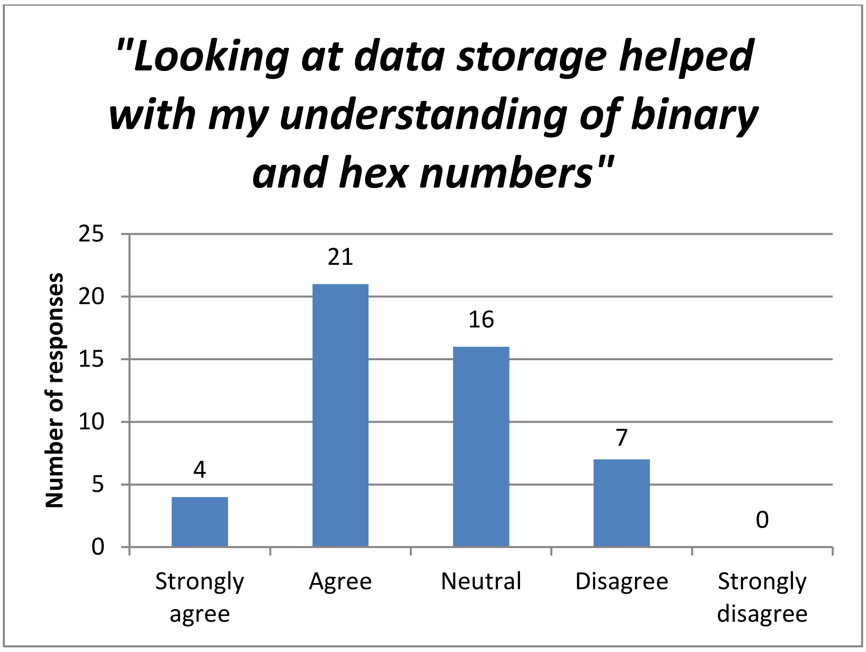

Figure 9

3.1.4 The Euclidean Algorithm and Number Theory

When asked whether their view of number theory and/or the Euclidean algorithm changed once RSA encryption had been studied the response was much more positive than for data storage. A larger proportion of students (69%) found that it had changed their view. The reasons provided included that they could see how it was useful and how it could be applied (52%), it gave them a clearer understanding (23%), it became easier to do (7%) and made it interesting (7%). One student said that *"it was good to see a useful situation where it would be used and [it] motivated [them] to work harder and understand more"*. Other comments included that it allowed them to see a different side of the topic, it *"sparked different ideas"*, they could visualise the ideas and the RSA encryption gave the topics *"purpose"*.

There was one student who was not sure. The students who said it had not changed their view (29%) said that it seemed quite similar (8%), they already had an interest in number theory or had seen the application (23%) and that it made it more complicated (15%).

When asked whether they agreed with a statement that RSA encryption helped them to understand number theory and the Euclidean algorithm, the response was very positive with 82% of the students answering the question either agreeing or strongly agreeing with the statement that was provided (Figure 10).

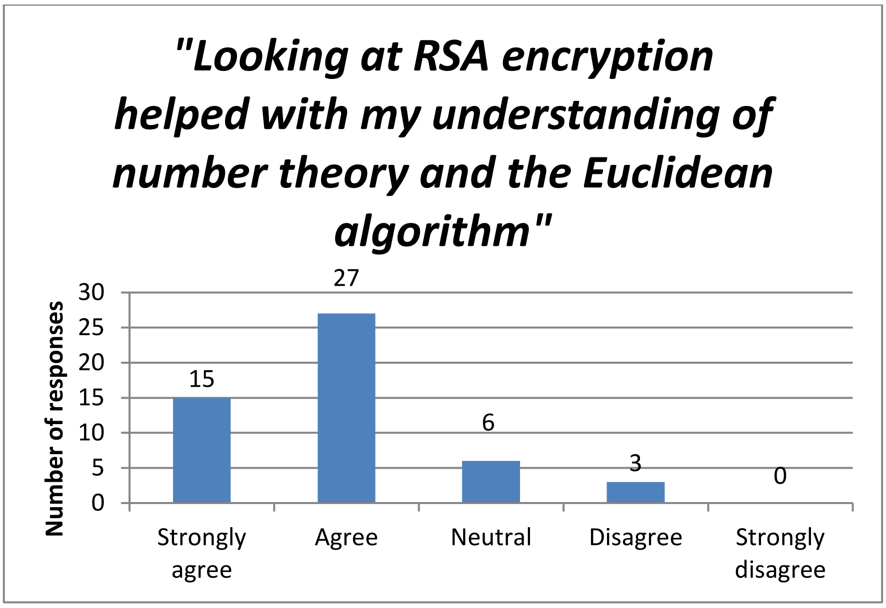

Figure 10

More students were positive about RSA encryption than the other additions. A potential reason is that the students had an assignment on the topic, whereas they did not have any assignment questions on the other additions.

3.2 Manipulatives in Class

Open responses were sought as to whether having physical examples in class clarifies the concepts that are introduced, and why. The majority of these responses were positive (70%), giving varied reasons such as more easily understanding concepts *"viscerally"* or *"physically rather than just reading content about them"*, enabling them to better visualise or *"see the problem"*, and keeping them focused and engaged. Of these responses being able to visualise problems was the most common reason given for the usefulness of the physical props (36% of 70%). Some of the positive comments use the words *"real example"*, *"real life scenario"*, and *"real and understandable context"*, though it is unclear whether the word *real* is being used as a synonym for *physical*, especially as some of the physical props used do not represent an actual real-world problem (for example, the utilities problem on the torus).

About a quarter (26%) of responses to the question were a mixture of positive and negative, generally saying that *"sometimes"* the props were useful in clarifying concepts. These students addressed things like the amount of time spent using the physical props in class (for example, a student said that *"physical examples take up a little too much time in class but may help others"*), that *"sometimes they can help but sometimes they can just be a hassle and seem pointless"*, and *"sometimes as it can help visualise things, other times it makes things more confusing"*.

Of the remaining comments the reasons given for the physical props not helping to clarify concepts were that *"physical examples never really helped [them] in general"* and *"sometimes it just created more confusion"*.

3.2.1 The Utilities Problem on a Torus

When asked whether the torus and string helped with their understanding of planar graphs, 72% agreed that it did (Figure 11). Of the remaining responses 20% were neutral, while 8% disagreed.

Responses to whether they could clearly see the connection between the activity and the topic of planar graphs the results were similar though slightly less strongly positive. About 75% of responses agreed but only 6 of these strongly agreed in contrast to 13 of the responses of the previous question. Of the remaining responses 16% were neutral while 10% disagreed.

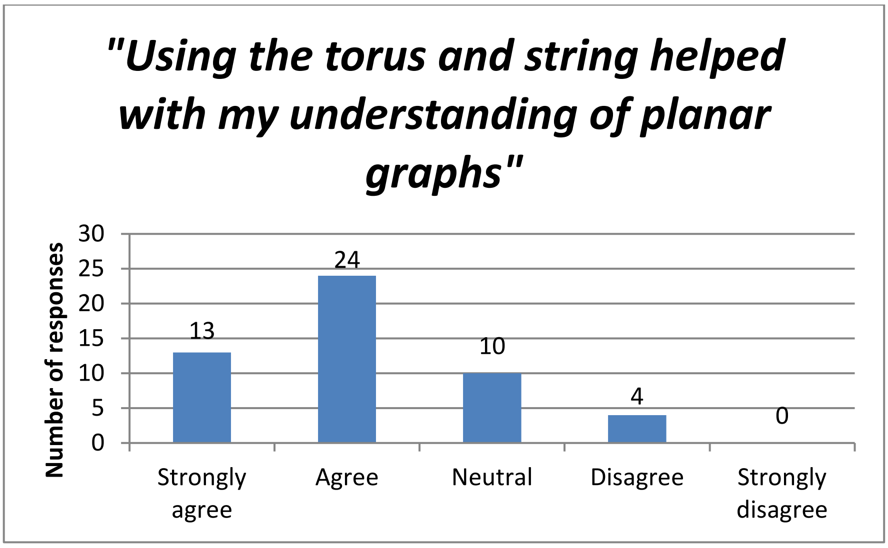

Figure 11

3.2.2 Molecules

The responses in Figure 12 show that 71% of students agreed that looking at physical molecule models helped with their understanding of group theory, while 21% gave neutral responses, and 8% disagreed. Many of the students (79%) agreed that they could clearly see the connection between the molecule models and group theory, 19% gave a neutral response, and 2% disagreed.

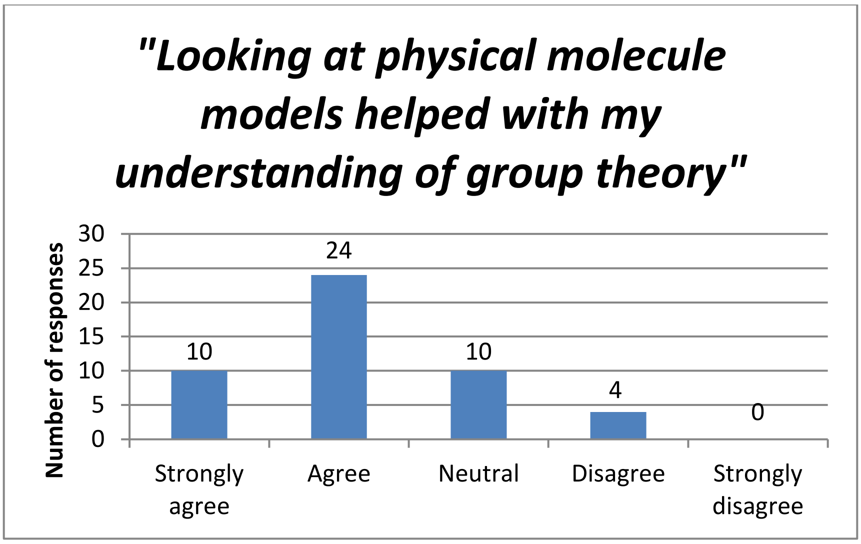

Figure 12

3.2.3 Mattress Task

When asked whether using the mattress models helped their understanding of group theory an 81% majority agreed that it did, 15% were neutral, and 4% disagreed (Figure 13). The distribution of these responses was the same when asked whether they could clearly see the connection between the mattress task and group theory. The responses here were, however, slightly more positive than the other tasks involving physical props.

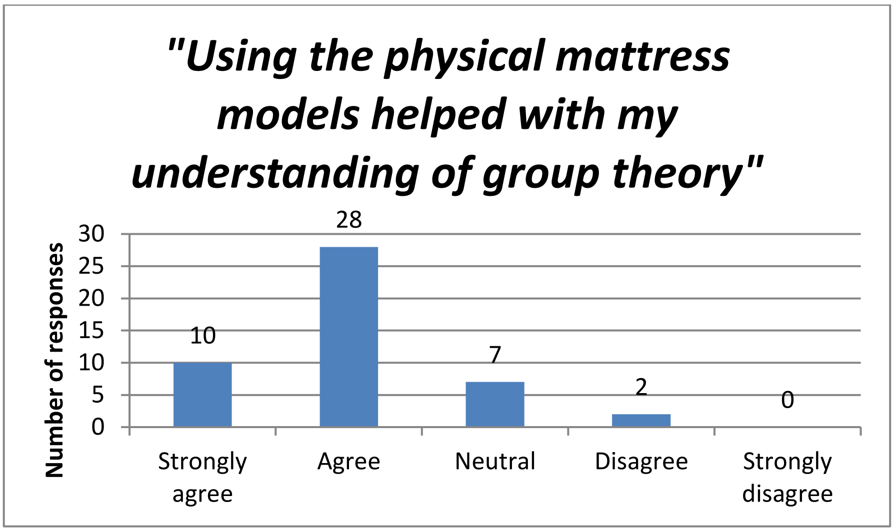

Figure 13

3.3 Benefit of the Content to the Real-World

When asked whether they could see how the content of Number and Structure is beneficial in the real-world, 91% of the students either agreed or strongly agreed that they could as shown in Figure 14.

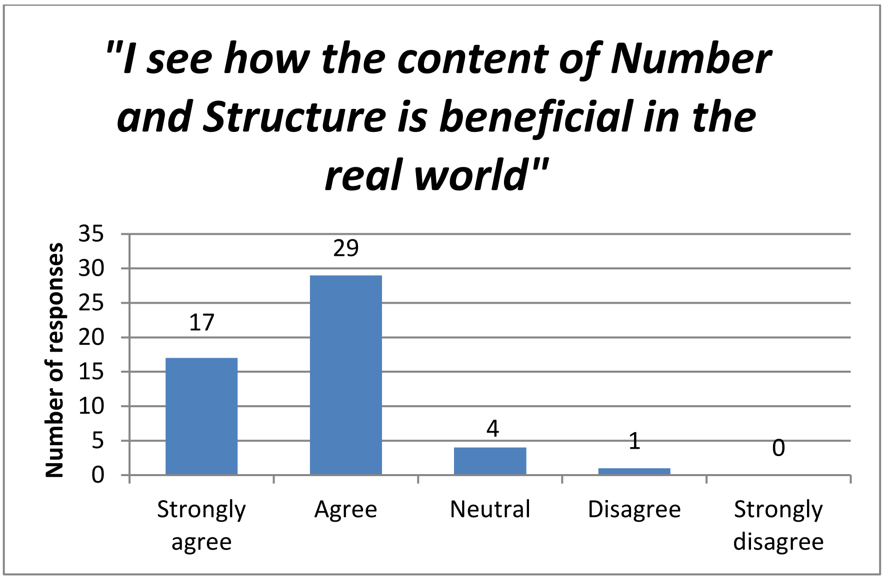

Figure 14

The student who disagreed said that they *"don't see how it could be useful"*. Out of the four students who provided a neutral response one didn't know, one did not always *"understand why [they were] doing it"*, one did not understand the real-world applications, and the other said that they did not know how all of the topics are used in the real-world.

The reasons provided by the students who either agreed or strongly agreed with the statement included comments about how they can see how the ideas are used in everyday life and the real-world (35% of 91%), the content was linked to applications (26%), and there are many applications of the topics (15%). One student mentioned that *"almost every single topic covered this year has had valuable real-world applications"*. Specific mentions of topics were encryption (15%), binary numbers (4%), and graph theory (4%). One student commented about developing problem-solving and logical thinking skills. Other comments included that it *"helped to see how all these ideas are used in everyday lives"*, "*it allows [them] to appreciate and understand more as to how it impacts the real world",* and they *"can see where it is useful"*. In some topics it was easier to see the link than others. One student commented that *"there are times when [they] struggle to see the application of certain theories but in general it has been clearly conveyed in which way some theories are used in the real world"*, whereas another said that *"it was sometimes surprising how some real life situations can be looked at mathematically"*. One student summarised that *"the module uses real world applications to help visualise and solve theoretical questions"*.

4. Conclusions

Following the addition of applied problems and objects, a substantial amount of evidence was collected to suggest that they had an impact on the students. The majority of them can see how the content of the module is beneficial in the real-world as they can see the uses and the links. A fair number of students (60%) said they approach work differently after seeing the uses of a topic, with increased motivation or interest being mentioned as the top reason. The students tend to like a mixture of theoretical and applied problems, which they said increases their understanding and interest / enjoyment. The application of RSA encryption helped the most with understanding (82%), followed by the mattress problem (73%), the utilities problem (65%), molecules (61%) and then data storage (52%). The use of manipulatives was also received well, with 70% of the students being positive about the physical examples. Reasons for this included understanding and engagement. The use of the mattresses helped the most with understanding (81%), followed by the tori (72%) and the molecules (71%).

5. References

Blum, W. and Niss M.,1991. Applied Mathematical Problem Solving, Modelling, Applications, and Links to other Subjects - State, Trends and Issues in Mathematics Instruction. *Educational Studies in Mathematics*, 22(1), pp.37-68. https://doi.org/10.1007/BF00302716

Cope, L., 2015. Math manipulatives: Making the abstract tangible. *Delta Journal of Education*, 5(1), pp.10-19. Available at: http://www.deltastate.edu/PDFFiles/DJE/spring-2015/dje_spring_2015_cope-final.pdf [Accessed 3 July 2018].

Cornock, C., 2015. Teaching group theory using Rubik's cubes. *International Journal of Mathematical Education in Science and Technology*, 46(7), pp.957-967. Available at: https://www.tandfonline.com/doi/pdf/10.1080/0020739X.2015.1070442 [Accessed 5 July 2018].

Furner, J. M., and Worrell, N. L., 2017. The Importance of Using Manipulatives in Teaching Math Today. Transformations, 3(1), p.2. Available at: https://nsuworks.nova.edu/transformations/vol3/iss1/2 [Accessed 3 July 2018].

Guzman, K. and Bartlett, J., 2012. Using simple manipulatives to improve student comprehension of a complex biological process: Protein synthesis. *Biochemistry and Molecular Biology Education*, 40(5), pp.320-327. https://doi.org/10.1002/bmb.20638

Hayes, B., 2005). Group theory in the bedroom. *American Scientist*, 93, pp.395-399. Available at: https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.208.8157&rep=rep1&type=pdf [Accessed 3 July 2018].

Jungck, J.R., Gaff, H. and Weisstein, A.E., 2010. Mathematical manipulative models: In defense of "Beanbag Biology". *CBE—Life Sciences Education*, 9(3), pp.201-211. https://dx.doi.org/10.1187%2Fcbe.10-03-0040

Katz, J., Menezes, A.J., Van Oorschot, P.C. and Vanstone, S.A., 1996. *Handbook of applied cryptography*. Boca Raton, FL: CRC press. Available at: https://cacr.uwaterloo.ca/hac/ [Accessed 3 July 2018].

Krontiris-Litowitz, J., 2003. Using manipulatives to improve learning in the undergraduate neurophysiology curriculum. *Advances in physiology education*, 27(3), pp.109-119. https://doi.org/10.1152/advan.00042.2002

Kullman, D.E., 1979. The utilities problem. *Mathematics Magazine*, 52(5), pp.299-302. https://doi.org/10.2307/2689782

Laski, E.V., Jor'dan, J.R., Daoust, C. and Murray, A.K., 2015. What makes mathematics manipulatives effective? Lessons from cognitive science and Montessori education. *SAGE Open*, 5(2), p.2158244015589588. https://doi.org/10.1177/2158244015589588

McNeil, N. and Jarvin, L., 2007. When theories don't add up: disentangling he manipulatives debate. *Theory into Practice*, 46(4), pp.309-316. https://doi.org/10.1080/00405840701593899

Pan, E.A., 2013. *The use of physical and virtual manipulatives in an undergraduate mechanical engineering (Dynamics) course*. University of Virginia. Available at: https://search.proquest.com/docview/1435605243 [Accessed 3 July 2018].

Perkins, J.A., 2005. A history of molecular representation. Part one: 1800 to the 1960s. *J Biocommun*, 31(1), p.1. Available at: https://jbiocommunication.org/issues/31-1/features3.html [Accessed 3 July 2018].

Saitta, E.K., Gittings, M.J. and Geiger, C., 2011. Learning dimensional analysis through collaboratively working with manipulatives. *Journal of Chemical Education*, 88(7), pp.910-915. https://doi.org/10.1021/ed100019f

Walton, P.H., 1998. *Beginning Group Theory for Chemistry*. London: Oxford University Press.