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Profit Ratio Negotiability Model in Entrepreneurial Financing
Using Game Theory and Agent based Simulation as an aid to
Decision Making:
An application to Profit and Loss Sharing Contracts

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Abstract
Profit and Loss (PLS) sharing contracts in Islamic finance are considered to be
fair economic practices as they focus on sharing profits and losses between the
project’s participants. This mode, however, suffers from asymmetric information
in the form of moral hazards and adverse selection. The purpose of this paper is
to reduce moral hazards by developing an equilibrium profit sharing ratio’s span
of negotiation in a PLS contract involving a financier and an entrepreneur. We aim
to establish an agent based model that will help the financier decide whether to
accept financing a contract. We make use of game theory techniques and we test
our results using an agent based simulation tool (Netlogo). We found theoretical
evidence that a Nash equilibrium span of negotiation, for both profit sharing
ratios, can be developed which is both rational and incentive compatible to both
participants. However, the simulation tool suggests that despite the existence of an
average positive span of negotiation, financial contracts might not be extendes if
the number of void contracts in a simulation exceeds a specified threshold. The
usefulness of the agent based simulation tool has added value to our theoretical
finding by suggesting when PLS contracts can or can’t be signed.

Keywords: Entrepreneurial Financing, sharing ratio, Profit ratio negotiability,

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1. Introduction

Profit and Loss contracts are partnerships agreement in which profits are shared according to a pre-specified ratio\cite{Obaidullah2005} while losses are born subject to each party capital contribution. Free consent of the partners and their full capacity are required before the signing of a contract\cite{UsmaniAnsari2010}.

In case of a partnership of more than two, one or more of the parties may act as agent or managers. It can also be agreed that management can be undertaken by one rather than all parties\cite{Warde2010}.

Profit determination should be based on expected future profits and not as a fixed amount or as a percentage of investment. In case of multi-period PLS contracts, profit ratios can be renegotiated. In such type of contracts one or more partners can increase their share in the partnership progressively\cite{Warde2010}.

From a cooperation point of view parties can sacrifice part of their parts as an incentive to other partners\cite{Obaidullah2005}. Subject to agreements between partners, provisional profits can be distributed before maturity of the contract. Also profits can be retained for future investments. In contrast to a conventional setting security against losses is not allowed in a PLS contract. However the financier might require security against misconduct of the partnership manager.

An increasing interest has been shown towards Islamic PLS contracts. In fact many Islamic windows were opened to match their conventional parts. Such examples of windows include the FTSE Global Islamic Index in London and Dow Jones Islamic Market Index in New York\cite{Abidi2009}. In addition to this, fund raising amounted to more than 400 billion US Dollars while investment funds amounted to 600 US Dollars billion\cite{Gierath2010}.

Conventional and Islamic PLS are different in many settings. Islamic PLS contracts are based on ethical considerations. For example Islamic PLS is non-interest-based financing and do not engage into illicit projects (gambling, casinos, and pornography). This restriction, however limits investment opportunities of Islamic PLS compared to their conventional counterpart.
Islamic PLS are more riskier in a sense that the financier is neither allowed to receive fixed compensation nor it is allowed to receive guarantees against losses. This may lead to more moral hazards issues manifested in the opportunistic behavior of the PLS managers.

Due to their complexity, this research tries to establish a model that will help in deciding whether to extend financing to an entrepreneur. The research methodology applied include two parts. the first part is theoretical in nature and uses game theory technique. The purpose of this strategy is to decide on the best strategy of each participant (financier and the entrepreneur) in response to the strategy of the other participant. The second part, takes the results from the game theory analysis and feed it in a built in simulation model using netlogo. the purpose of using this simulation tool is to make the decision faster and accurate based on the data entered by the financier.

Our Paper proposes a PLS contract model that tries to reduce the moral hazard problem in the form of the entrepreneur undertaking a low effort (Shirking). This paper tries to develop a model that is both participative and incentive compatible for both parties using game theory and agent based simulation. to do so we try to answer the following two questions:

1. Given a specific financier capital contribution can we develop a rational and incentive compatible equilibrium span of negotiation for the profit sharing ratio in a PLS contract?
2. Can our model help in deciding whether to accept a financial contract or reject it?

2. Literature review

Asymmetric information between an entrepreneur and a financier manifest itself in two ways: Adverse Selection and Moral Hazards. Adverse selection, which refers to lack of information about the participants before the contract is established. Moral Hazards, refers to the risk that one, some, or all participants deviate from their commitments after the contract. In our paper we are interested in reducing moral hazards.

To reduce asymmetric information, that an agent holds against a financier, many dissipative signals can be used.

One such signal is collateral that efficient agents need to use to signal their efficient type. This is consistent with works of Berger et al. (2011) and Karim (2002). While this method is allowed in a conventional system, it is prohibited in Islamic jurisprudence (Shari’ah law). The recourse to a warranty is allowed only if
there is a proof of negligence or violation of the terms of the PLS contracts.

Benchmarking on collateral, efficient managers might sign for low job protection (Subramanian and Sheikh, 2002). This is in agreement with other research as in (Subramanian and Sheikh, 2002). Yet, signing for low job protection is considered unfair to the entrepreneur since the project failures can be due to some uncontrollable factors (ELFakir and Tkiouat, 2015a, ELFakir and Tkiouat, 2015b).

Due diligence can be used to reduce moral hazards, in the form of misreporting, in Musharakah. However, the extent of it is more in Musharaka than in conventional PLS (Al-Suwailem, 2006).

The Entrepreneur’s participation in the project’s capital might be used to reduce information asymmetries in musharakah contracts (Karim, 2002). In agreement with this argument, we have allowed the agent to participate in the project capital in our model.

In dealing with moral hazard, in the form of low effort, Nabi (2013) suggested that the entrepreneur should participate with a minimum capital contribution coupled with a minimum profit sharing ratio. In our model we propose we allowed for more flexibility in this parameter. The model allows the entrepreneur to fix his capital contribution based on his financial capacity and, based on that, allows him to negotiate on the profit sharing ratio.

The same research (Nabi, 2013) suggested that the entrepreneur suffers a disutility when exercising effort. We have allowed for such parameter, in our model, recognizing the fact that higher effort commands a greater disutility than low effort.

One research suggested that moral hazards can be reduced under Mudaraba but cannot be solved under musharakah (YOUSFI, 2013). We can criticize this argument in the following way. First under Mudaraba, the financier is the sole provider of capital and subsequently supports all types of risks. On the other hand, under PLS losses are shared. Our reasoning is consistent with the findings of Nabi (2013) and Innes (1990). Furthermore, allows for both forms, i.e., the entrepreneur may or may not participate in the capital of the project.

In a previous paper (ELFakir and Tkiouat, 2015a) of ours we have proposed an incentive mechanism to reduce asymmetric information. This mechanism results in higher social value and more entrepreneurial negotiation power in terms of the profit sharing ratio. The model however does not provide an agent based simulation tool to illustrate it.

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3 Adoption of AAOIFI Shariah Standard No. 12, Clause 3/1/4/1
4 A form of business where the financier provides capital while the entrepreneur provides work
In dealing with adverse selection and moral hazards, we have provided some series of publications using two contracts. The purpose is to allow for agents type separation. In the first paper, we suggested using two types of contracts: one is effort based and the second is output based. A theoretical evidence showed that an effort based contract can give higher compensation to the agent as this contract offers a lower sharing ratio to the financier [ELFakir and Tkiouat (2015b)]. This result emphasizes two important Islamic concepts. First it emphasizes the sentiment of altruism which the financier shows by taking a smaller profit sharing ratio. Second it emphasizes the sentiment of positive reciprocity which the agent exhibits by providing high effort.

Our new model goes in line with our previous research in promoting and inciting entrepreneurs to perform higher efforts.

In the second paper, we tried to reduce the adverse selection with respect to Mudaraba using a model of two contracts combined with adverse selection index for each contract. We have managed to develop three types of indices that can help financial institutions in their agent selection process (ELFakir and Tkiouat, 2016a).

In the third paper, we tried to use a two contract concept in a game theoretical approach under incomplete information. Menu contracting was found not to be always the optimal option for moral hazard reduction (ELFakir and Tkiouat, 2016b).

Our main difference from prior literature is to develop an agent based simulation that gives the opportunity for profit sharing negotiability and hence the development of a span of negotiation. The agent based simulation model allows for faster decision making about whether to extend financing for a PLS contract. This tool as well is very flexible as all the model parameters can be modified depending on each user preferences and scenario circumstances. This tool is useful as well in that it allows for the minimum acceptable profit sharing ratio of each participant. Therefore an interval, or span, of negotiation can be established.

3. The model

Our model strives to reduce the moral hazard problem in a sharing contract between risk neutral financier and an entrepreneur. The later is willing to undertake a project which requires funding F. He is endowed with an initial fund A but requires an additional funding I−A. The success of the project depends on the effort of the entrepreneur.

The project is estimated to result in a stochastic verifiable output R conditional
dependent on a high or low managerial effort $e_i: i \in \{l, h\}$:

$$E(R|e_i) = \int_0^R R f(R|e_i)dR$$  \hspace{1cm} (1)

Where the share of the entrepreneur is $R_e$ and the share of the financier is $R_f$ such that $R = R_e + R_f$. This output can take upper and lower values depending on the effort being taken. In fact the output can be $\bar{R}(R)$ with probability $\theta_h(1-\theta_h)$ in case of high effort and $\theta_l(1 - \theta_l)$ in case of low efforts.

The manager has a disutility of effort $D(e_i) = d(e_i).(1 - \beta).I$. This disutility is manifested as a percentage of his investment in the project when exercising effort $e_i: i \in \{l, h\}$. A higher disutility is manifested through exercising higher effort such that $D(e_h) > D(e_l)$.

The manager also has a reservation utility $U = u.(1 - \beta).I$ as a percentage of his investment in the project.

The expected NPV under the high effort and low effort case are given respectively as:

$$\overline{NPV} = \theta_h R - F > 0$$  \hspace{1cm} (2)

$$\overline{NPV} = \theta_l R - F + S < 0$$  \hspace{1cm} (3)

**Assumption1:** We assume that under equation 2 the NPV is negative even if the entrepreneur enjoys some private benefits $S$ when he performs a low effort.

### 4. Methodology

We consider a one period contract. The entrepreneur and the financier agree on a partnership contract $(x; F, \alpha, \beta=x)$ whereby the entrepreneur commits to undertake a high effort and invest $f = (1 - x) F$. Two sharing ratio $\alpha$ and $\beta$ such that $0 \leq \beta \leq 1$ and $0 \leq \alpha \leq 1$, are given to the financier in case of success and Loss of the project respectively as in Nabi [20].

Taking $r$ as the project rate of return, if the project is successful, yielding $\bar{R} = (1 + \bar{r}).I$ the share of the financier and Entrepreneur respectively are:

$$\bar{R}_f = \alpha \bar{R} = \alpha(1 + \bar{r}).I \quad \text{and} \quad \bar{R}_e = (1 - \alpha) \bar{R} = (1 - \alpha)(1 + \bar{r}).I$$  \hspace{1cm} (4)
If the project is unsuccessful, yielding $R = (1 + r)I$ the share of the financier and Entrepreneur respectively are:

$$R_f = \beta R = \beta rI \quad \text{and} \quad R_e = (1 - \beta)R = (1 - \beta)rI$$

(5)

We should note the distinguishing characteristic of the model where each participant cannot loose more than his/her capital contribution. This a distinguishing feature from the conventional setting where the financier might demand guarantees against losses of more than his/her capital contribution.

We start by developing a sharing ratio under managerial observable effort. We then develop a span of profit sharing ratio in an incomplete information setting where managerial effort is unoservable. We then provide the rational of our results using a game theory approach. Finally we test our results using agent based simulation tool (Netlogo).

5. Results and Discussion

5.1. The model under managerial observable effort

Under this scenario, the manager can’t deviate from providing his commitments of high effort and therefore the financier is in a comparative advantage in terms of profit sharing ratio negotiations. In other words, the objective of the financier is to minimize the remuneration $R_m$ of the manager subject to the manager breaking even. We can, then, formally express the maximum share share of the financier ($\alpha$) as:

$$\alpha \leq \alpha_{PCM} = 1 - (1 - \beta)\frac{1 + d_h + u - (1 - \theta_h)(1 + r)}{\theta_h(1 + \bar{r})}$$

(6)

5.2. The model under managerial unobservable effort

In this case the financier is facing a situation with regards to the type of the manager. In other words the financier is questioning whether the manager is going to exercise a high effort or not while undertaking the project.

So in addition to fulfilling the participation constraints using the sharing ratio at (6), the financier must also give an incentive so that the entrepreneur is at least indifferent between exercising low effort (Not Shirking) or exercising low effort (Shirking).
5.2.1. Problem preliminaries

We can establish a condition for which the entrepreneur is to perform a high effort, i.e. we must have the Expected profit to the entrepreneur under no shirking $U_e(NS)$ to be higher than his profit under shirking $U_e(S)$, i.e

$$U_e(NS) \geq U_e(S)$$

which means:

The financier then works out his payoff taking into consideration two probabilities:

- type probabilities $W_h$: regarding the probability that a manager is going to perform a high effort.
- performance conditional probabilities: regarding the probability that the project will be successful conditional on the manager’s effort, in our case, this is $\theta_h$ under high effort and $\theta_l$ under low effort.

From these two probabilities we can easily infer the joint probability of success $P(S)$ of the project:

$$P(S) = W_h \cdot \theta_h + (1 - W_h) \theta_l$$

This situation give rise to private benefits $S$ drawn by the manager if he performs a lower effort. Taking this into consideration, the financier is in a competitive disadvantage and therefore his objective will be to at least break even.

The contract being assigned need to take into consideration three main constraints:

- Participation constraints PCF and PCM: where both participants (Financier Manager) are at least breaking even.
- Incentive compatibility constraints ICM: where only the manager is offered a profit sharing ratio that will encourage him to exert high effort rather than shirking.

So the objective of the financier, using the expectations, rate of returns, percentage disutility percentage utilities and percentage private benefits, is to maximize his return subject to the above mentioned constraints. Formally:

$$\max_{R_f} P(S)\alpha(1 + E(\bar{r}))I + (1 - P(S))\beta(1 + E(\bar{r}))I$$
subject to constraints:

\[ PCF : P(S)\alpha(1 + E(\bar{r}))I + (1 - P(S))\beta(1 + E(\bar{r}))I - \beta I \geq 0 \]  \hspace{1cm} (9)

\[ PCM : \theta_h(1 - \alpha)(1 + E(\bar{r})) + (1 - \beta)((1 - \theta_h)(1 + E(\bar{r})) - 1 - d(e_h) - u) \geq 0 \]  \hspace{1cm} (10)

\[ ICM : \theta_l(1 - \alpha)(1 + E(\bar{r})) + (1 - \beta)((1 - \theta_h)(1 + E(\bar{r})) - 1 - d(e_l) - u + s) \]

\hspace{1cm} (11)

5.3. Satisfying the Manager’s participative and incentive compatibility constraints

We have already solved for the participation constraints of the manager (PCM) in equations (6) and (7). We need now to solve for the Incentive constraint of the manager (INC)

So, the maximum financier share (\(\alpha\)) to give an incentive for the entrepreneur to exert a high effort is:

\[ \alpha \leq \alpha_{ICM} = 1 - (1 - \beta)\frac{\Delta \theta.(1 + E(\bar{r})) + \Delta d + s}{\Delta \theta(1 + E(\bar{r}))} \]  \hspace{1cm} (12)

We can infer then from (9)(22) that For \(\alpha\) to be fulfill both the incentive and the participation constraints, \(\alpha\) has to fulfill the following condition:

\[ \alpha \leq \min\{\alpha_{ICM}; \alpha_{PCM}\} \]  \hspace{1cm} (13)

5.4. Satisfying the Financier’s participative constraints

Now, we turn to the less competitive participant in this game, the financer. He needs a sharing ratio \(\alpha_{pcf}\) that enables him to at least break even. We give shorthand formula of the integrals of the financier participation constraints (12) by introducing expectations forms as follows:

\[ P(S)\alpha(1 + E(\bar{r}).I) + (1 - P(S))\beta(1 + E(\bar{r}).I) - \beta I \]  \hspace{1cm} (14)
Solving for $\alpha$ we get:

$$\alpha \geq \alpha_{PCF} = \beta \frac{1 - (1 - P(S))(1 + E(R))}{(1 + E(\bar{r}))P(S)} \quad (15)$$

from (25) and (23) we can figure out an interval of the sharing ratio $\alpha$ that the financier should get an which should be satisfying for both parties:

$$\alpha_{PCF} \leq \alpha \leq \min\{\alpha_{ICM}; \alpha_{PCM}\} \quad (16)$$

5.5. Span of Negotiation

We can notice that there is a span of negotiation $SN$ in terms of the profit sharing ratio such that:

$$SN = \min\{\alpha_{ICM}; \alpha_{PCM}\} - \alpha_{PCF}. \quad (17)$$

The larger is this span of negotiation the more likelihood that a contract can be materialized. A negative span of negotiation suggests the non-concluding of the contract.

5.6. a word about the invested capital

Something which emerged against our odds is the non-existence of the investment parameter $I$ in the equations (6)(20) (25) leading to the decision on the profit sharing ratio and to the span of negotiation. This shows the irrelevance of the investment parameter in the analysis we provided.

6. Agent based simulation

6.1. preliminaries

In this section we will provide a case based scenario to illustrate our findings using an agent based simulation. The usefulness of the simulation tool is that its decision parameters can be changed to reflect different scenarios settings. We run the scenarios based o the decision parameters for 1000 simulation to give us robustness in our decision whether to refuse or accept a financial contract. The decision to reject or accept a contract depends on the number of void contracts during our simulation. If the number of void contracts exceed a certain threshold decided by the simulation’s user then the contract is rejected. the simulation are parameters are as follows:
Table 1: The simulation parameters initial values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment</strong></td>
<td>100,000£</td>
</tr>
<tr>
<td>Bank – Contribution</td>
<td>80%</td>
</tr>
<tr>
<td>Return – upper – value</td>
<td>50%</td>
</tr>
<tr>
<td>Mean – Probability – success/high – effort</td>
<td>80%</td>
</tr>
<tr>
<td>Standard – Deviation/High</td>
<td>10%</td>
</tr>
<tr>
<td>Return – lower – value</td>
<td>-50%</td>
</tr>
<tr>
<td>Mean – Probability – success/Low – effort</td>
<td>70%</td>
</tr>
<tr>
<td>Standard – Deviation/Low</td>
<td>10%</td>
</tr>
<tr>
<td>Mean – probability of being high – manager</td>
<td>10%</td>
</tr>
<tr>
<td>Standard – deviation/high – manager</td>
<td>10%</td>
</tr>
<tr>
<td>Manager reservation utility</td>
<td>10%</td>
</tr>
<tr>
<td>disutility from high effort</td>
<td>10%</td>
</tr>
<tr>
<td>disutility from low effort</td>
<td>10%</td>
</tr>
</tbody>
</table>

The following figure shows the initial outlook of the simulation interface with the assigned simulation parameters on the left hand side of the figure. The simulation is run 1000 times and the rejection threshold is 30% of void contracts from the whole simulation.
The average bank ratio monitors and average manager ratio monitors, represents the minimum profit ratio the financier and the entrepreneur would accept to participate in the project.
A bank decision monitor is provided to decide whether to accept or refuse the contract.
A graphical monitor and numerical monitor are shown to display the span of negotiation between the financier and the entrepreneur. The numerical labels under the agents represent their share of the profit if the project is successful under the high effort.
6.2. Running the model under initial simulation values

Launching the model under the initial values gives the following results as shown on the interface:

Figure 2: The game under initial values of the simulation parameters

The figure shows that despite the span of negotiation being positive, this contract should be refused as the number of void contracts (426) in the simulation is beyond the threshold of 30%.

6.3. Running the model under a decrease in banks capital contribution

What can we suggest to make the contract acceptable? One of the answers is to decrease the contribution of the financier. The usefulness of the model is that
it allows for a desired decrease in the contribution until an acceptable contract is reached. The following figure shows the results if the contribution is changed from 80% to 70%.

Figure 3: The game under decrease in banks capital contribution

We can notice that the decrease in the level of bank contribution in our case results in the contract being marginally acceptable as the percentage of void contracts matched the rejection threshold of 30%.

6.4. The importance of probabilities assessments

One important point to highlight is the sensitivity of the model to probabilities assessments. In fact proper assessment needs to be done in terms of how likely the project is to succeed depending on managerial type and performance type. The
following figure shows the change a slight change of the probability that the agent is of high type from 50% to 60%

Figure 4: The game under an increase in the likelihood of high managerial effort

We can notice that a slight increase in the manager type probability assessment resulted in the acceptability of the contract while before at 50% it was refused.

6.5. The importance of volatilities of probabilities assessments

In this case we would like to assess if a high optimism about the success of project results in a contract acceptance. To do this we run the model under a high probability of success at 90% but with a higher volatility (from 10% to 30% the figure bellow shows our results:
In this case we notice that even under high optimism about the success of project, a higher volatility of such optimism might result in contract refusal.

7. Conclusion

In this research we have tried to reduce the moral hazard between a financier and an entrepreneur problem in a profit and loss sharing contract. We tried to develop a rational and incentive compatible equilibrium span of negotiation for the profit sharing ratio. The whole mechanism of the PLS contract was developed using an agent based simulation tool (Netlogo). The purpose of the simulation was to test whether a PLS contract should be accepted or not in a desired setting.
We found simulation evidence that an initial refusal or acceptance of a financial contract can be reversed if a slight adjustment of the model parameters can be made. The model also suggests that a positive span (see figure 2) of negotiation does not necessarily guarantee the acceptance of a contract. Even if the whole parametric values were hypothetical in nature, they can be re-adjusted to new scenarios allowing for more flexibility in the module. From a management practice point of view, we believe that this simulation model can be a useful tool in profit and loss financial contract decision making. In fact the model we are proposing can help in making faster decisions regarding whether or not to extent financing. The model is also flexible as it allows for adjustment of the parameters depending on the managerial beliefs and scenarios circumstances. From an academic practice point of view, this research fills a gap in the literature about PLS contracts and can open doors for other extensions. For example, more participants (entrepreneurs and financiers) in the model game can be added. This will test how competition over financing can affect the results of the model.

References


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