Volatility co-movement between Bitcoin and Ether

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Volatility co-movement between Bitcoin and Ether

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Abstract: Using a bivariate Diagonal BEKK model, this paper investigates the volatility dynamics of the two major cryptocurrencies, namely Bitcoin and Ether. We find evidence of interdependencies in the cryptocurrency market, while it is shown that the two cryptocurrencies' conditional volatility and correlation are responsive to major news. In addition, we show that Ether can be an effective hedge against Bitcoin, while the analysis of optimal portfolio weights indicates that Bitcoin should outweigh Ether. Understanding volatility movements and interdependencies in cryptocurrency markets is important for appropriate investment management, and our study can thus assist cryptocurrency users in making more informed decisions.

Keywords: Bitcoin, Ether, Cryptocurrency, Diagonal BEKK, Multivariate GARCH, Conditional volatility

JEL classification: C32, C5, G1
1 Introduction

Cryptocurrency markets have recently received a lot of attention from the media and investors alike. Bitcoin is undoubtedly the most popular cryptocurrency with an estimated market capitalisation currently being worth $167 billion (coinmarketcap.com accessed on 12th March 2018). Since its introduction in 2009, cryptocurrency markets have rapidly grown with a total of more than 1550 existing cryptocurrencies (as of 12th March 2018). Despite its relatively recent launch, Ether constitutes the second largest cryptocurrency in terms of market capitalisation, which is currently estimated at $72 billion (coinmarketcap.com accessed on 12th March 2018). Bitcoin and Ether together represented 60% of the total estimated cryptocurrency market capitalisation at the time of writing. Although the two cryptocurrencies have several fundamental differences in purpose and capability, both of them have recently seen gigantic price fluctuations and are increasingly used for investment and speculation purposes, despite warnings issued by different financial institutions.

Recently the literature on cryptocurrencies has rapidly emerged. For instance, recent studies have examined the hedging capabilities of Bitcoin against other assets (Dyhrberg 2016a, 2016b; Baur et al., 2017; Bouri et al., 2017), the market efficiency of cryptocurrencies (Urquhart, 2016; Nadarajah and Chu, 2017), and the existence of bubbles in cryptocurrencies (Cheah and Fry, 2015; Corbet et al., 2017), while the price volatility of cryptocurrencies has been studied by Katsiampa (2017) and Phillip et al. (2018), among others. More recently, the literature has started examining the connectedness of cryptocurrencies to mainstream assets. For instance, Corbet et al. (2018) and Lee et al. (2018) studied linkages of cryptocurrencies to traditional assets and found that cryptocurrencies are rather isolated from other markets and that correlations between cryptocurrencies and other assets are low. Nevertheless, the literature on interdependencies within cryptocurrency markets is rather limited. To the best of

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2 Due to Ether's fast growth and the fact that several industry giants have backed Ethereum, the network behind Ether, through the formation of the Enterprise Ethereum Alliance, it is believed by some that Ether could possibly overtake Bitcoin in popularity and market value in the future.
the author's knowledge, only Ciaian et al. (2017) and Corbet et al. (2018) have studied interlinkages of cryptocurrencies. More specifically, Ciaian et al. (2017) studied interdependencies between Bitcoin and other cryptocurrencies using an Autoregressive Distributed Lag model and found that the prices of Bitcoin and other cryptocurrencies, such as Ether, are interdependent. However, the authors did not study cryptocurrencies' volatility co-movements. On the other hand, Corbet et al. (2018) studied interlinkages between cryptocurrencies using a Dynamic Conditional Correlation model and similarly found that cryptocurrencies are interconnected with each other. Nevertheless, the authors considered only Bitcoin, Ripple and Litecoin, excluding Ether, though.

As investors in cryptocurrencies are exposed to highly undifferentiated risks (Gkillas and Katsiampa, 2018), examination of cryptocurrency price volatility co-movements is of utmost importance in order for investors and other market participants to better understand interlinkages within the cryptocurrency market and make more informed decisions, and multivariate GARCH models are useful tools for analysing such interdependencies between heteroskedastic time series. Nonetheless, volatility dynamics between Bitcoin and Ether have not been previously explored. Consequently, motivated by the Bitcoin and Ether price fluctuations and the interconnectedness of cryptocurrency markets, by employing a bivariate GARCH model, this study aims to investigate not only the volatility dynamics of Bitcoin and Ether but also their conditional covariance and correlation, examining which important events have led to unprecedented conditional volatility and covariance levels. We also study the optimal portfolio weights and hedging opportunities between the two cryptocurrencies. To the author's best knowledge, this is, therefore, the first study of price volatility dynamics between Bitcoin and Ether and of the hedging opportunities between the two cryptocurrencies.
2 Data and methodology

The dataset consists of daily closing prices for Bitcoin and Ether from 7th August 2015 (as the earliest date available for Ether) to 15th January 2018. The prices are listed in US Dollars and the data are publicly available online at https://coinmarketcap.com/coins/. The returns are defined as

\[ y_{i,t} = \ln p_{i,t} - \ln p_{i,t-1}, \]  

where \( y_{i,t} \) is the logarithmic price change for cryptocurrency \( i, i = 1,2 \), and \( p_{i,t} \) is the corresponding price on day \( t \).

Our empirical analysis begins with producing descriptive statistics for the Bitcoin and Ether price returns. We then perform the Augmented Dickey-Fuller and Phillips-Perron unit-root tests as well as Engle's ARCH-LM test for ARCH effects in order to examine the stationarity of the returns series and whether volatility modelling is required for the price returns of the two cryptocurrencies considered in this study. As shown in section four, the results suggest that the price returns of both cryptocurrencies are stationary but exhibit volatility clustering. Consequently, a bivariate GARCH model can be employed in order to model the conditional variances and covariance of the two cryptocurrencies.

3 Model

The conditional mean equation of the two cryptocurrencies' price returns is given as

\[ y_t = c + \varepsilon_t, \]  

where \( y_t \) is the vector of the price returns as defined in the previous section, \( \varepsilon_t \) is the residual vector with a conditional covariance matrix \( H_t \) given the available information set \( \Omega_{t-1} \), and \( c \) is the vector of parameters that estimates the mean of the return series\(^3\). All the three

\(^3\) It is worth mentioning that in this study a simple specification for the conditional mean equation is employed since our interest lies mainly in the time-varying covariance matrix.
components of the mean equation are 2×1 vectors since here the focus is on the two major cryptocurrencies, namely Bitcoin and Ether.

A popular model of conditional covariances is the BEKK model (Engle and Kroner, 1995), the covariance matrix of which is given as

\[ H_t = W'W + A'\varepsilon_{t-1}'\varepsilon_{t-1}A + B'H_{t-1}B, \]  

(3)

where \( W \), \( A \) and \( B \) are matrices of parameters with appropriate dimensions, with \( W \) being an upper triangular matrix, while the diagonal elements of \( W \), \( A \), and \( B \) are restricted to be positive (Bekiros, 2014). The diagonal elements of \( H_t \), \( h_{ii,t} \), \( i = 1,2 \), represent the conditional variance terms, while the off-diagonal elements of \( H_t \), \( h_{ij,t} \), \( i \neq j \), \( i,j = 1,2 \), represent the conditional covariances. Once the BEKK model parameters are estimated, the conditional correlations can be derived as

\[ \rho_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ii,t}h_{jj,t}}} \]  

(4)

and the BEKK model thus accommodates dynamic conditional correlations as opposed to the Constant Conditional Correlations model. The BEKK model is also viewed as an improvement to the VECH model, as the number of parameters to be estimated is reduced and the positive definiteness of \( H_t \) is ensured provided that \( WW \) is positive definite (Terrell and Fomby, 2006), and to the Dynamic Conditional Correlation model (Boldanov et al., 2016), since consistency and asymptotic normality of the estimated parameters of the latter model have not yet been established (Caporin and McAleer, 2012).

However, the parameters of the BEKK model cannot be easily interpreted, and their net effects on the future variances and covariances cannot be easily observed (Tse and Tsui, 2002). Moreover, the BEKK model is problematic with regards to the existence of its underlying stochastic processes, regularity conditions, and asymptotic properties (Allen and McAleer, 2017). The model most commonly used in practice instead is the first-order Diagonal BEKK model (Ledoit et al., 2003), which addresses the aforementioned issues. In
this model both parameter matrices $A$ and $B$ are diagonal and therefore their off-diagonal elements are all equal to zero. Consequently, under the Diagonal BEKK model, the number of parameters is considerably decreased while maintaining the positive definiteness of $H_t$ (Terrell and Fomby, 2006). Furthermore, the QMLE of the parameters of the Diagonal BEKK model are consistent and asymptotically normal, and hence statistical inference on testing hypotheses is valid (Allen and McAleer, 2017).

For comparison purposes, next the bivariate forms of both models are presented. The unrestricted BEKK model in bivariate form is written as

$$
\begin{pmatrix}
  h_{11,t} & h_{12,t} \\
  h_{21,t} & h_{22,t}
\end{pmatrix}
= W'W + \begin{pmatrix}
  a_{11} & a_{12} \\
  a_{12} & a_{22}
\end{pmatrix}
\begin{pmatrix}
  \varepsilon_{1,t-1}^2 \\
  \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\
  \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\
  \varepsilon_{2,t-1}^2
\end{pmatrix}
\begin{pmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{pmatrix}
+ \begin{pmatrix}
  b_{11} & b_{21} \\
  b_{12} & b_{22}
\end{pmatrix}
\begin{pmatrix}
  h_{11,t-1} \\
  h_{12,t-1} \\
  h_{21,t-1} \\
  h_{22,t-1}
\end{pmatrix}
\begin{pmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22}
\end{pmatrix}
$$

Hence, we have that

$$
h_{11,t} = w_{11}^2 + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 + b_{11}^2h_{11,t-1} + 2b_{11}b_{21}h_{12,t-1}
+ b_{21}^2h_{22,t-1}
$$

$$
h_{22,t} = w_{12}^2 + w_{22}^2 + a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 + b_{12}^2h_{11,t-1}
+ 2b_{12}b_{22}h_{12,t-1} + b_{22}^2h_{22,t-1}
$$

$$
h_{12,t} = w_{12}w_{11} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{12}a_{21} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2
+ b_{11}b_{12}h_{11,t-1} + (b_{12}b_{21} + b_{11}b_{22})h_{12,t-1} + b_{21}b_{22}h_{22,t-1}.
$$

As none of the above single equations solely possesses its own parameters, interpretation of the parameters could be misleading even in the case of only two time series (Terrell and Fomby, 2006). On the other hand, the bivariate form of the Diagonal BEKK model is given by

$$
h_{11,t} = w_{11}^2 + a_{11}^2 \varepsilon_{1,t-1}^2 + b_{11}^2h_{11,t-1},
$$

$$
h_{22,t} = w_{11}^2 + w_{22}^2 + a_{22}^2\varepsilon_{2,t-1}^2 + b_{22}^2h_{22,t-1}.
$$
\[ h_{12,t} = w_{11}w_{22} + a_{11}a_{22}e_{1,t-1}e_{2,t-1} + b_{11}b_{22}h_{12,t-1}. \]

It can be easily noticed that in the case of the Diagonal BEKK model the number of parameters to be estimated is significantly reduced. Therefore, in this study, the Diagonal BEKK model is employed in order to investigate volatility dynamics between Bitcoin and Ether. The model parameters are estimated by the maximum likelihood approach under the multivariate normal and multivariate Student's t error distributions using the BFGS algorithm. The dynamic conditional correlation between Bitcoin and Ether is then calculated as

\[ r_t = \frac{h_{12,t}}{\sqrt{h_{11,t}h_{22,t}}}, \tag{5} \]

where \( h_{11,t} \) is the conditional variance of Bitcoin, \( h_{22,t} \) is the conditional variance of Ether, and \( h_{12,t} \) is their conditional covariance.

The optimal portfolio weights are also constructed, subject to a no-shorting constrain, following Kroner and Ng (1998). The optimal weight of Bitcoin in a one-dollar portfolio consisting only of Bitcoin and Ether is

\[ w_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}}, \text{ if } 0 \leq w_{12,t} \leq 1. \tag{6} \]

Finally, following Dey and Sampath (2018), the dynamic long/short hedge ratio between Bitcoin and Ether is constructed as

\[ \beta_{12,t} = \frac{h_{12,t}}{h_{22,t}}. \tag{7} \]

4 Results

Figure 1 illustrates the prices of Bitcoin and Ether. It can be noticed that although the prices of both cryptocurrencies would increase slowly until the beginning of 2017, there was considerable price appreciation from the second quarter of 2017 onwards, increasing the opportunities for investment and speculation. This indicates that the two cryptocurrencies seem to follow a similar pattern and could be correlated. Indeed, the Pearson correlation
coefficient which measures the linear correlation between Bitcoin and Ether price returns is positive and equal to 0.2507, and significantly different from zero at any conventional level$^4$.

![Graphs of Bitcoin and Ether prices](image)

**Fig. 1** Daily closing prices of Bitcoin and Ether (in US Dollars).

Table 1 (Panel A) presents descriptive statistics for the price returns of the two cryptocurrencies. The average price returns are positive for both Bitcoin and Ether and equal to 0.4373% and 0.6889% with a standard deviation of 3.9092% and 8.5037%, respectively. Furthermore, the price returns of both cryptocurrencies are leptokurtic as a result of significant excess kurtosis - with Bitcoin exhibiting smaller kurtosis than Ether - and negatively skewed suggesting that it is more likely to observe large negative returns. Moreover, the Jarque-Bera test results confirm the departure from normality, while the test results for conditional heteroskedasticity suggest that ARCH effects are present in the price returns of both cryptocurrencies. We can thus proceed with bivariate GARCH modelling to model the conditional variances and covariance of the price returns of Bitcoin and Ether. Furthermore, the results of both unit root tests (Table 1, Panel B) suggest that stationarity is ensured. Consequently, the Bitcoin and Ether price returns are appropriate for further analysis.

$^4$ The Spearman rank-order correlation coefficient, which is a nonparametric measure of correlation, was also found positive and significantly different from zero at all the conventional levels, but equal to 0.1985.
Table 1 Descriptive statistics and unit roots tests for the price returns.

<table>
<thead>
<tr>
<th></th>
<th>Bitcoin</th>
<th>Ether</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Descriptive statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>892</td>
<td>892</td>
</tr>
<tr>
<td>Mean</td>
<td>0.004373</td>
<td>0.006889</td>
</tr>
<tr>
<td>Median</td>
<td>0.003306</td>
<td>0.000000</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.225119</td>
<td>0.412337</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.207530</td>
<td>-1.302106</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.039092</td>
<td>0.085037</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.114590</td>
<td>-3.694999</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.910433</td>
<td>67.17186</td>
</tr>
<tr>
<td>JB</td>
<td>1300.303***</td>
<td>155083.1***</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>48.84901***</td>
<td>40.73491***</td>
</tr>
<tr>
<td>ARCH(5)</td>
<td>61.68204***</td>
<td>90.48128***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Panel B: Unit root test statistics</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-29.35884***</td>
</tr>
<tr>
<td>PP</td>
<td>-29.35593***</td>
</tr>
</tbody>
</table>

Note: *** significant at the 1% level.

The estimation results of the Diagonal BEKK model under the multivariate normal and multivariate Student's t error distributions are reported in Tables 2 and 3, respectively. It can be noticed that in comparison with the results obtained under the multivariate normal distribution, the log-likelihood value is increased and the values of all the three information criteria used in this study (Akaike, Schwarz, and Hannan–Quinn) are decreased under the multivariate Student's t error distribution. The estimated model under the multivariate Student's t error distribution is thus preferred. We notice that the estimated value of the GARCH coefficient, in particular, is equal to 0.8359 and 0.7583 for Bitcoin and Ether, respectively, indicating a relatively high degree of volatility persistence for both cryptocurrencies, with higher volatility persistence displayed in the Bitcoin market, though. Moreover, the ARCH and GARCH coefficients are highly significant for both cryptocurrencies. The significance of the estimated ARCH coefficients suggests that news/shocks in Bitcoin (Ether) are of great importance for Bitcoin's (Ether's) future volatility,
while the significance of the estimated GARCH coefficients indicates that the persistence of shocks also affects the two cryptocurrencies' future volatility. Similar results are obtained for the two cryptocurrencies' conditional covariance which is significantly affected by cross products of previous news/shocks and previous covariance terms.

Table 2 Diagonal BEKK model parameter estimates under multivariate normal error distribution.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>W</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bitcoin</strong></td>
<td><strong>0.002796</strong>*</td>
<td><strong>0.000025</strong>*</td>
<td><strong>0.407807</strong>*</td>
<td><strong>0.920444</strong>*</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td><strong>Ether</strong></td>
<td><strong>0.003900</strong>**</td>
<td><strong>0.000234</strong>*</td>
<td><strong>0.467085</strong>*</td>
<td><strong>0.873196</strong>*</td>
</tr>
<tr>
<td></td>
<td>(0.0279)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th></th>
<th>LL</th>
<th>SIC</th>
<th>$Q^2_{T1}$ (15)</th>
<th>5.2398</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bitcoin</strong></td>
<td>3014.193</td>
<td>6.719916</td>
<td></td>
<td>5.2398</td>
</tr>
<tr>
<td><strong>Ether</strong></td>
<td>6.768452</td>
<td></td>
<td>6.749899</td>
<td></td>
</tr>
</tbody>
</table>

Notes: ** and *** indicate significance at the 5% and 1% levels, respectively. The p-values are presented in brackets. $Q^2_{T1}$ and $Q^2_{T2}$ are the Ljung-Box portmanteau test statistics for serial correlation in the univariate squared standardised residuals of Bitcoin and Ether, respectively.

Conditional variance equations with substituted coefficients:

\[ h_{11,t} = 2.5362e^{-05} + 0.1663\varepsilon^2_{1,t} + 0.8472h_{11,t-1} \]
\[ h_{22,t} = 0.0002 + 0.2182\varepsilon^2_{2,t} + 0.7625h_{22,t-1} \]
\[ h_{12,t} = 1.6115e^{-05} + 0.1905\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.8037h_{12,t-1} \]

Table 3 Diagonal BEKK model parameter estimates under multivariate Student's $t$ error distribution.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>W</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bitcoin</strong></td>
<td><strong>0.002680</strong>*</td>
<td><strong>0.000018</strong>**</td>
<td><strong>0.541649</strong>*</td>
<td><strong>0.914258</strong>*</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0190)</td>
<td>(0.5621)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td><strong>Ether</strong></td>
<td>0.001302</td>
<td>0.000340***</td>
<td>0.622328***</td>
<td>0.870809***</td>
</tr>
<tr>
<td></td>
<td>(0.3314)</td>
<td>(0.0060)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th></th>
<th>t-Distribution</th>
<th>2.686224***</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bitcoin</strong></td>
<td>(Degrees of Freedom)</td>
<td></td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td><strong>Ether</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is also worth mentioning that an asymmetric Diagonal BEKK model under the multivariate Student's $t$ error distribution was also employed but the asymmetric effects between good and bad news were found statistically insignificant for both Bitcoin and Ether and, hence, these results are not reported here as the standard Diagonal BEKK model is preferred.
<table>
<thead>
<tr>
<th>LL</th>
<th>3225.489</th>
<th>SIC</th>
<th>-7.188162</th>
<th>$Q^2_{11}(15)$</th>
<th>$Q^2_{22}(15)$</th>
<th>AIC</th>
<th>-7.242092</th>
<th>HQ</th>
<th>-7.221477</th>
<th>$Q^2_{11}(15)$</th>
<th>$Q^2_{22}(15)$</th>
<th>AIC</th>
<th>-7.242092</th>
<th>HQ</th>
<th>-7.221477</th>
<th>$Q^2_{11}(15)$</th>
<th>$Q^2_{22}(15)$</th>
</tr>
</thead>
</table>

Notes: ** and *** indicate significance at the 5% and 1% levels, respectively. The p-values are presented in brackets. $Q^2_{11}$ and $Q^2_{22}$ are the Ljung-Box portmanteau test statistics for serial correlation in the univariate squared standardised residuals of Bitcoin and Ether, respectively.

Conditional variance equations with substituted coefficients:

$h_{11,t} = 1.7683e^{-05} + 0.2934e^2_{t-1} + 0.8359h_{11,t-1}$
$h_{22,t} = 0.0003 + 0.3873e^2_{t-1} + 0.7583h_{22,t-1}$
$h_{12,t} = 9.0222e^{-06} + 0.3371e_{t-1}e_{2t-1} + 0.7961h_{12t-1}$

The plots of the conditional variances and covariance as well as the plot of the conditional correlations of the price returns of Bitcoin and Ether when using the Diagonal BEKK model under the multivariate Student's t error distribution are depicted in Figures 2 and 3. It can be noticed from Figure 2 that overall Ether exhibits higher conditional volatility than Bitcoin. Moreover, from the evolution of the conditional volatility of Bitcoin, there are few distinct episodes in 2017 that emerge from the plot, where the Bitcoin conditional volatility series has reached unprecedented levels. More specifically, three important spikes which seem to be related to the effects of the Bitcoin hard fork, China banning Bitcoin trading, and the announcement of the CME Group Inc. to launch Bitcoin futures, taking place in July, September, and December 2017, respectively, are observed. On the other hand, for the Ether price volatility, we observe two distinct spikes around June 2016 and February 2017, which seem to be associated with the effects of the Ether hard fork and the formation of the Enterprise Ethereum Alliance, respectively. Furthermore, the conditional covariance between the two cryptocurrencies, which measures the association between Bitcoin and Ether, is time-varying and mostly positive, while the highest peak in the conditional covariance of the two cryptocurrencies is observed in September 2017 and can be associated with China banning Bitcoin trading and initial coin offering. Yet, the conditional correlation plot (Figure 3) confirms time-varying conditional correlations between Bitcoin and Ether, with the dynamic...
correlation between the two cryptocurrencies fluctuating in both positive and negative regions, although positive correlations mostly prevail. More specifically, Figure 3 shows that the conditional correlation between the price returns of Bitcoin and Ether ranges from -0.70 to 0.96, suggesting that checking the unconditional correlation only is not adequate.

Fig. 2 Conditional Variances and Covariance

Fig. 3 Conditional Correlations
Finally, the average hedge ratio and average optimal portfolio weight from the Diagonal BEKK model under the Student's t error distribution are reported in Table 5. The average value of the hedge ratio between Bitcoin and Ether is 0.42, suggesting that a $1 long position in Bitcoin can be hedged for 42 cents with a short position in Ether. In addition, the average optimal weight for the Bitcoin/Ether portfolio is 0.82, suggesting that for a $1 portfolio, 82 cents should be invested in Bitcoin and 18 cents should be invested in Ether on average.  

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Hedge ratio</strong></td>
<td></td>
</tr>
<tr>
<td>Bitcoin/Ether</td>
<td>0.423314</td>
</tr>
<tr>
<td><strong>Panel B: Portfolio weight</strong></td>
<td></td>
</tr>
<tr>
<td>Bitcoin/Ether</td>
<td>0.816894</td>
</tr>
</tbody>
</table>

5 Conclusions

By employing a bivariate Diagonal BEKK model, this study investigated the volatility dynamics of the two largest cryptocurrencies in terms of market capitalisation, namely Bitcoin and Ether. It was found that the price returns of both cryptocurrencies are heteroskedastic, a finding which is consistent with previous studies, and that news/shocks about the two cryptocurrencies as well as their persistence are of great importance for the two cryptocurrencies' future volatility, while the estimated model under the multivariate Student's t error distribution is preferred. It was also found that the two cryptocurrencies' volatility is responsive to major news. Furthermore, the bivariate framework has helped us examine not only the two cryptocurrencies' individual conditional variances but also the movements of their conditional covariance and correlation. More specifically, the two cryptocurrencies' conditional covariance was found to be significantly affected by both cross products of

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6 It should be noticed that the selection of models affects the estimated hedge ratios and optimal portfolio weights (Kroner and Ng, 1998).
previous news/shocks and previous covariance terms, a result that supports the findings of 
previous studies on the interconnectedness of cryptocurrencies. It was also shown that time-
varying conditional correlations between Bitcoin and Ether exist and fluctuate in both 
positive and negative regions, although positive correlations prevail, while the highest 
correlation was observed in September 2017 when China banned digital currency trading. 
Finally, it was shown that Ether can be an effective hedge against Bitcoin, while the analysis 
of optimal portfolio weights suggested that Bitcoin should outweigh Ether.

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