Volume 1

Analysing Quizzes using a Census

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Abstract.

This is a theoretical study of the marks that could be obtained from quizzes, when a census is done. The quizzes are limited to 'special' quizzes and we look at how many obtain a particular mark, and what happens to the distribution of marks when the guessing problem is tackled.

Without building any tools we can do some calculations, but then we are driven to writing computer programs that calculate and display the distribution of marks. When computer programs are written they involve such large numbers that computer programs cannot normally handle them. Thus we are driven to languages, like C, where integers of an arbitrary size can be handled in libraries like LEDA.

Quizzes can either have all their stems answered or have some of their stems missed. The philosophy of this study is to start off simply and gradually get more complicated, and so 'special' quizzes where all stems are answered are looked at first followed by 'special' quizzes where stems can be missed.

The guessing problem is tackled, in this study, by:

a) using negative marking,
b) raising the pass mark,
c) using a more complex quiz.

What we find is that when the guessing problem of 'special' quizzes is exactly tackled the distribution of marks moves to the left (when the mean mark is 0), and that the tackling of the guessing problem leads to more problems. All 'special' stems include the answer, which can be guessed. The likelihood of guessing the correct answer to a stem decreases as the complexity of the quiz increases, but then the likelihood of passing also decreases.

As the complexity of the quiz increases the distribution is 'bunched' at the lower end which means that half the marks of distribution are between the lowest mark and the mark where the mean occurs.

The main conclusion is not quantitative but qualitative and is "Do not use 'special' quizzes for *measuring* **understanding**" because 'special' quizzes do not provide a space that allows one to *demonstrate* one's understanding and are therefore **in-valid** when understanding is claimed to be measured.

Acknowledgements.

I would to thank my Director of Studies (Dr. Keith Burley) who in my previous life was my student. My Director of Studies met me most weeks and suggested the direction I should go in for the next week. I would also like to thank my Supervisor (Dr. Paul Crowther) both he and Dr. Burley took time to read my thesis and give me feedback.

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Glossary.

Criterion Reference Marking	You get a mark independent of how others did.
EMSQ	Extended Matching Set Question.
Epistemology	The study of knowledge.
False positive	When a sample says that something is present when it is not.
False negative	When a sample says that something is not present when it is.
Grade Point Average	The mean mark of each module, scaled to 4.
IMRAD	Introduction, Method, Results and Discussion.
KISS	Keep It Simple Stupid.
Kurtosis	A descriptive statistic that measures the 'peakiness' of a distribution.
LEDA	A set of routines to handle integers of an arbitrary size. These routines are written/maintained by Algorithmic Solutions of Germany. LEDA does not stand for anything.
Methodology	The study of methods. It is not synonymous with method. It is not a family of methods. It might be a name given to a family of methods, as long as it is not Methodology.
MAQ	Multiple **Answer** Question.
MCQ	Multiple **Choice** Question.
Median	The 'middle' x value of a distribution. This depends on whether the population/number in the sample is odd or even
Mode	The most frequently occurring value of a distribution (as a 1^{st} approximation).
Norm Reference Marking	You get a mark dependent on how others did.
Ontology	The study of existence.
Option	A choice that the learner has.
Pivot	e.g. How many times does a mark occur?
Precise	How close to the actual are you?
RAM	Random Access Memory.
Reliable	Are the measurements the same every time?
Standard Deviation	The positive square root of the Variance.
Stem	The question that the setter is asking.
Ticks	The number of answers to a stem that are ticked as correct.
Skewness	A descriptive statistic of whether a distribution has a long 'tail' to the left or right.

Symmetric	A distribution which has equal 'tails'. When a distribution is symmetric, its mean, mode and median are co-incident.
Valid	Does it measure what it claims to measure?
Validation	Are we building the product right?
Variance	A descriptive statistic of the spread of a distribution.
Verification	Are we building the right product?
VS	Visual Studio.

Papers that were published during the thesis.

1. LAFFERTY, H.H. and BURLEY, K. (2009). Do preferred learning styles exist? *In 2nd Institutional Research Conference* 8/9/2009
2. LAFFERTY, H.H. and BURLEY, K. (2011). Do learning styles exist? *Education horizons. Journal of excellence in teaching.* 11 (4), 17.
3. LAFFERTY, H.H. and BURLEY, K. (2012a). Analysing some quizzes. *In Annual HEIR conference at Liverpool University* 12 July 2012.
4. LAFFERTY, H.H. and BURLEY, K. (2012b). Think before using quizzes. *In Learning and Teaching conference at Sheffield Hallam University* 11 July 2012.

Chapter 1. Introduction.

Abstract.

This is a theoretical study where the Research Question (**RQ**) is "What is the distribution of marks of special quizzes when a census is done?" We define what we mean by a special quiz, which is exemplified by a quiz made up of only Multiple-choice questions (**MCQ**s).

The distribution is drawn, described and measured. Limits are placed on the types of analysis and the types of quizzes special. The measurements are mainly restricted to the production of 'descriptive statistics'.

We ask a subsidiary question "What is the effect of tackling guessing the correct answer to stems/questions of special quizzes?" and that is refined to "What does the distribution look like when the guessing problem is tackled?"

The structure of the thesis is **ILMRADRA**. Each Chapter has a different emphasis, and this Chapter concentrates on the **I**ntroduction, where the title of the research, the Research Question, the philosophy, the motivation, the contribution, the journey, what we did and what we did not do, are all explained.

1.1 Introduction.

Quizzes are one form of assessment, which we deal with much more fully later on in this chapter. We took quizzes for Flash and Dreamweaver, both packages marketed by Macromedia (later part of Adobe) and we were intrigued by whether the quizzes did measure how well you **understood** the packages and why they seemed to "not want you to pass" (Cooper (2011)) and did not allow you to explain *why* you gave an answer. Another motivation for looking at quizzes was that for certain tasks quizzes seemed both inappropriate and unfair.

McCoubrie (2004) thinks that using quizzes to assess doctors is questionable and Naeem (2010) thinks that quizzes are sometimes unfair.

A personal motivation for undertaking this study was "To stay alive". It is believed that a healthy body and a healthy mind lead to longevity, and as I had an un-healthy body, it was left to maintaining a healthy mind. A belief in "Learning for Life" has a resonance here.

1.2 Special quizzes.

Having explained the motivation of looking at quizzes, we now show that our study of quizzes was limited to looking at what we call 'special' quizzes, which all have the property of having one answer that might be guessed. We will say, first of all, what a special quiz is *not*, and then say what it *is* and on the way define a few terms.

A 'special' quiz is NOT like an **expert system,** where the next question depends on the previous answer. The questions/stems must be as independent as possible. We know that true independence of stems is impossible, because they are all about the same subject e.g. relational databases.

> *Two events A and B, with nonzero probabilities, are independent when knowing that A occurs has no effect on the probability that B occurs.*
>
> Ian (2013).

We assume that stems are as independent as possible.

Quizzes can either have all their stems answered or some stems can be missed.

We have used the word 'stem' twice now, and so we had better define it.

> A **stem** is the question that the setter is asking.

For example,

"Is a record a row of a table?"

Figure 1-1. An example of a stem.

Figure 1-1 is an example of a stem and has 2 answers "Yes" or "No" but also can be not answered, or missed. Thus, this is an example of a Yes/No stem, and we are told that the stem can be missed.

We do not argue whether this is a good stem. It might be worded badly; it might not make sense; it might be ambiguous. We, just agree, that "Is a record a row of a table?" is a stem.

We will say that a quiz has **n** stems.

A 'pool' is where a group of stems is formed e.g. there are 100 of them, and a random number between 1 and 100 is generated and a stem corresponding to the random number is chosen. We do NOT examine quizzes that have **pools** in them (which complicate the issue), on the grounds that "The guy next to you does not sit the same exam as you". We only examine quizzes where the guy next to you *does* take the same test as you.

A 'special' quiz is NOT a **matching** quiz, where for example, there are pictures of many birds, and there are many texts describing those birds. One picture is dragged to the text that is judged, by the learner, to the best text that describes that bird, and then the next bird is dragged etc. The odds of dragging a bird to the best describing text depend on the order in which the birds are dragged. The odds also depend on how many birds there are and how many texts there are. This all 'hurts our head', because there might be more texts than pictures, or more pictures than texts, or an equal number of them both. To stop 'our heads hurting' we limit quizzes to special quizzes, where the odds of answering are always the *same* and the likelihood of choosing an option, is always the same.

A 'special' quiz, is NOT like a pub quiz, where the learner can say anything. We call that an '**open**' quiz. 'Open' quizzes also include quizzes where the learner '**fills in the blanks**" and hence can fill in with anything. We are limiting special quizzes to '**closed**' quizzes where one (or more) of the options is/are correct, and hence given. The reason for only looking at 'closed' quizzes is because then the user is limited in the answers.

That is the second time we have used the word 'option' and so we had better define that.

An **option** is one of the choices that the learner has.

For example,

When designing a database system, which of the following do you have to consider? This is the **stem** (we say there are **n** stems)

- ☐ A database consists, solely, of tables.
- ☑ Relationships are between tables.
- ☑ Columns are within a row.
- ☐ Indexes, are always used for sorting

There are 4 **options** to the stem and 2 **ticks**
Here, **P**=4 and **T**=2

Score of [**c**, **a**], because the answer is deemed to be correct, and here **a** = 0, **c** = 1.
Here, the stem cannot be missed i.e. an answer must be given.

Figure 1-2. An example of a Multiple Answer stem.

Figure 1-2 is an example of a **M**ultiple **A**nswer **Q**uestion (**MAQ**) stem with 4 (**P**) options, two of which are ticked, correct. The number of ticks is generalised to **T**, and in this thesis we do not ask "What should **P** and **T** be?" that is left to other papers. Rodriguez (2005) says that for MCQ quizzes the optimum **P** is 3

"Three options are optimal for Multiple Choice items".

The arguments revolve around the setter being able to set more questions than with **P** = 4 while still covering the subject matter. We argue that (for a 3-option MCQ-only quiz) if you get 30 (out of 100) stems right and ***guess*** the rest you will get a mean mark of 30 +70/3 = 53.33 (for **n** = 100, **a** = 0, **c** = 1) and so pass by guessing most of the quiz. For a 4-option MCA-only quiz where all stems are answered and you get 30 (out of 100) stems right and guess the rest you will get a mean mark of 30 + 70/4 =47.5 Thus you pass (at a 40% institute) by guessing most of the quiz.

We think that a **M**ultiple **C**hoices **Q**uestion (**MCQ**) is a specialisation of an **MAQ** where **T** = 1.

We cannot find any papers on what the optimal value of **T** is.

We assume that options are independent as possible, but know they cannot be truly independent, because they all refer to same stem and so, we assume it away.

We do not argue that each option and stem is of equal 'hardness'. We do not argue that this stem is 'harder' to answer than the previous stem. We just assume that all the stems are equally 'hard' and each option is equally hard. The notion of hardness cannot be defined, and so we just assume it away. What is 'hard' to 1 person might be 'easy' to another. For example "Is Sheffield (in England) north of London?" is easy for a Sheffielder but hard for President Obama. We could say "This stem is twice as hard to answer as that stem" and so, for example, this stem gets a score of 2 and that stem gets a score of 1. Some systems allow this, for example

> *"...allows the Instructor to enter any string as the score for a Grade Center item...".*
>
> BlackBoard (2012).

But, who decides that one stem is twice as 'hard' as another. This introduces subjectivity (which complicates the issue) and in an attempt to reduce this subjectivity we will assume that all stems are scored the same, assuring that, from a scoring point of view, all stems are equally 'hard'.

That brings in scoring. The usual score is 1 for a correct answer to a stem and 0 for an in-correct answer, as is shown in Figure 1-3 below

> When designing a database system, which of the following do you have to consider?
>
> ☑ A database consists, solely, of tables.
> ☐ Relationships are between tables.
> ☐ Columns are within a row.
> ☐ Indexes, are always used for sorting
>
> The **score** here is 0, because the answer to the stem is in-correct.

Figure 1-3. An example of score.

A raw mark is the summation of scores for 1 person doing a quiz i.e. Raw mark = $\sum$score.

We generalise the scoring to:

c for a correct answer to the stem
a for an in-correct answer to the stem
b for a missing the stem.

It can be argued that **c** can always be made into 1 by a subtraction or division (unless we divide by 0) and so for special quizzes the score fo*r every* stem is

[**b**], [1, **a**].

The assumption that we make is **every** stem is scored:

b if the stem is missed, or
1 if the answer is correct, or
a if the answer is in-correct.

It can be, further, argued, that **b** should *always* be 0 on the grounds that the learner by missing a stem did not answer it correctly or in-correctly and so should not be punished or rewarded and so should receive 0. If you believe this argument the general score is

[**0**], [1, **a**].

When **a** is negative we are said to be *negative* marking, and when **a** ≥ 0 we are said to be using *positive* marking. We should say that we are using negative **scoring** and not say negative *marking*, because a mark = ∑scores.

We do not argue what the size of **T** should be. But, note that

P and **T** are integers
T cannot be greater than **P** (you cannot have more Ticks than Options)

if **T** = 1 we have an MCQ stem
This shows that an MCQ stem is a specialisation of an MAQ stem.

We have questions now. "What is the minimum **P** and **T**?" "Does it make sense to have no options?" Figure 1-3 had no options or did it have 2? i.e. "Yes" and "No" Or did it have 1? Are we muddling up the "The total number of answers" with "The number of options"? To clarify this we draw a picture, and see what happens.

A Yes/No question looks like this

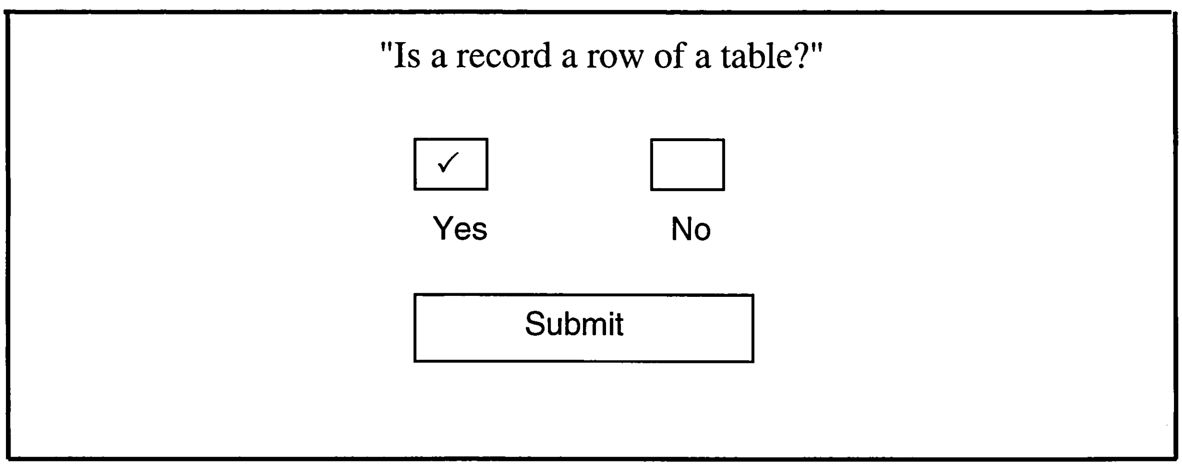


Figure 1-4. What does a Yes/No question look like?

Behind the scenes this gets a score of 1, because the answer to the stem is deemed to be correct.

What is happening is that, in the limit of a Multiple Yes/No, when there is 1 option, the stem and the option coalesce, and the total number of answers is 2. So, here, **P** = 1, **T** = 1 and the total number of answers is 2. So, here, **P** = **T** = 1 and the total number of answers is 2^P (or 2^T). So, the minimum **P** is 1. If **T** = 0 then the stem has been missed, and so the minimum **T** is 0. A Yes/No stem is a specialisation of a Multiple Yes/No stem with **P** = 1, and when there is only 1 option, the option disappears and only the stem survives. But, is a Yes/No stem a specialisation of an MCQ stem? The answer to this question is, when the number of options of an MCQ is 1, the options and the stem coalesce, but now the total number of answers to an MCQ stem is 1 whereas the total number of answers to a Yes/No stem is 2. The odds of answering an MCQ stem correctly (when there is 1 option) are 1/1 (when stems cannot be missed) whereas the odds of answering a Yes/No stem correctly (when there is 1 option) are ½. So, a Yes/No stem is NOT a specialisation of an MCQ stem with **P** = 1, it is a specialisation of a Multiple Yes/No stem with **P** = 1.

The hardness of *stems* problem is assumed away, because, for example, the score is 1 for *every* stem that is answered correctly.

The hardness of *options* problem is also, assumed away when the odds of choosing an option to be ticked are assumed to the same as the odds for any option. Thus, we assume that what happens in the TV programme "Eggheads" is not possible, where the contestants can eliminate an option from their considerations, and thus a 3-option **M**ultiple **C**hoice **Q**uestion (**MCQ**) stem becomes a 2-option MCQ stem.

A 'special' quiz is one where the types of stems are NOT **mixed**. For example, in the same quiz we do not have Yes/No stems and MCQ stems, because then the odds of guessing the correct answer depend on the stem type. But we have to be a bit careful, because you can mix 10-option MCQ stems with 5-option MAQ stems with 2 ticks, because they both have the same odds of the correct answer being guessed. To make life easy we assume no mixing occurs but we do assume that 10-option MCQ quizzes and 5-option MAQ quizzes with 2 ticks are the same.

A 'special' quiz is where the type of quiz does NOT **change**. For example a quiz does not start off as a MCQ quiz and then *changes* to an 'open' quiz as can happen in the TV programme "Eggheads" when there is a draw between the Eggheads and the contestants, or in the TV programme "In It To Win it" where one (or more) of the contestants goes into the red zone when a 3-option Multiple Choice Stem is answered wrongly, and then an 'open' stem is asked.

We do not use **C**ertainty-**B**ased **M**arking (**CBM**) in a 'special' quiz. We should say when Certainty Based ***Scoring*** is not used, but we will use the word *marking* because Garner-Medwin, the inventor of CBM does (Gardner-Medwin 2008, Gardner-Medwin and Cutin 2007) do. A variation, due to Davies (2001a, 2001b) is also not considered. The variation is where the Certainty is input first and then the options are shown, one (or none) of which is then chosen as the answer. We consider CBM and its variation to be a type of scoring where the user inputs 2 things.

Gardner-Medwin has changed his mind about what the **C**, in **C**BM, should be. It was **Confidence** before but now is **Certainty**, (Gardner-Medwin 2005, University College London 2010). What the C in CBM is, we do not care as we do not use CBM. CBM has the following scoring system.

Table 1-1. How a stem is scored in Certainty-Based Marking.

Certainty	1		2		3	
Answer	Correct	In-correct	Correct	In-correct	Correct	In-correct
Score	1	0	2	-2	3	-6

Also a miss is scored 0

We do not consider CBM because, that requires a fast computer for the calculation of ALL the marks, that is not available to us, and sampling requires that the sample represents the population (about which there is some doubt).

The learner can 'cheat' in many ways. For example, the "**authenticity**" problem is exemplified by "Did Daddy write the essay?" another example is in the 'knowledge' test of London taxi drivers and "Who is doing the knowledge test?" is asked, and yet another example is the citizens' test where it is asked "Who is doing the citizens test?" A, partial, solution to the authentication problem is provided by TurnitIn (2012) which searches its own database to see if a similar essay is there, but that does not cater for a student paying for a unique essay. We think that every type of assessment is open to cheating, with some more open than others. For example, group work is open to cheating because it has to be ascertained who did the most, relevant, work.

So, a 'special' quiz is NOT:

- like an expert system, where the answer to a stem determines the next stem,
- a matching quiz,
- 'open' like a 'pub' quiz,
- one that changes from 'closed' to 'open'
- one where there is mixing,
- scored by CBM (or its variation),
- one where 'hardness' of stems or options is taken into account,
- one where learners 'cheat' in any way.

A 'special' quiz IS:

- 'closed',
- where ,*every* one of the **n** *stems* is *scored the same* for:
    - a correct answer — we argue that it is always scored 1,
    - an-incorrect answer — we say that it is scored **a,**
    - a miss — we say that it is scored **b**, but an argument is that **b** should be 0,
- there, is still only 1 correct answer when many options are ticked,
- there is only 1 way that a stem can be missed,
- where every option within a stem has the same odds of being chosen to be ticked,
- where the learner/user does not 'cheat',
- where $\mu_n = n^*\mu_1$, where μ_n=the mean mark when **n** stems are used in a quiz

It is intuitive that for 'special' quizzes the mean mark for **n** stems is **n** * (the mean mark for 1 stem). For example, for a Yes/No stem the mean mark is (**c** + **a** + **b**)/3 for quizzes where every stem is NOT answered, and when **c** = 1, **a** = **b** = 0 we have the mean mark for 1 stem μ_1 = 1/3 and for the second stem the mean = 1/3 and so for **n** stems the mean $\mu_n = n^*\mu_1$. If you do not like this intuitive approach, then Straker (2012), with input from us, tries to prove it (see Appendix 4).

This makes a 'special' quiz, very special indeed.

1.3 What 'special' quizzes do we consider?

We have mentioned Yes/No quizzes, MCQ quizzes and MAQ quizzes, but what about other quizzes?

A multiple Yes/No quiz is one that has many options and to each option the learner/user answers "Yes" or "No". The total number of answers is 2^P. We immediately see that a Yes/No quiz is a specialisation of a Multiple Yes/No quiz where the number of options (**P**) is 1.

The format of **Multiple Yes/No**-only quizzes is given by Frisbie and Sweeney (1982). Frisbie and Druva (1986) showed that Multiple True-False questions are reliable. The ineffectiveness of Multiple Yes/No-only quizzes is shown in a paper by Ebel (1978) but even their name is not consistent as Alonso et al (2006) talk about *Complex* Multiple Choice questions/quizzes, but we will stick to the term *Multiple* Yes/No-only quizzes. Haladyana and Downing (2002) are ambivalent about them and are not sure about using them. Bender (2003) talks about Multiple Yes/Nos or **M**ultiple **T**rue/**F**alse **Q**uestions (MTFQs) and says that the quizzes are MAQs.

> *"Neither Wood nor Harper discusses a third type of computer-marked test, the multiple true/false question (MTFQ), in which the student is presented with a brief lead in, followed by four or five statements, each of which must be marked true (or correct) or false (or wrong). Any number of the possible answers may be correct or incorrect."*

This type of quiz is mentioned by Gerberich in 1956, so it is not new.

We *do* analyse quizzes that consist of only MTFQs which we call **Multiple** Yes/No-only questions, and where the user is asked how many options, **P**, there are (4 or 5 in the above example) and how many ticks, **T**, there are (not specified in the above example).

EMSQs (**E**xtended **M**atching **S**et **Q**uestions) are proposed, and mentioned by Wood (2003), which have a long stem, sometimes called a **vignette**, that describes a scenario, or case study, and has up to 26 options. A challenge by Harper (2003) suggests that MCQs will do anyway.

> *" Contrary to some suggestions, ... MCQ's can be used to assess a variety of outcomes including all of the competences in Bloom's taxonomy..."*

We ask if this is true and whether special quizzes (that include MCQs) can assess all of Bloom's competences?

It is unclear whether EMSQ quizzes are MCQs or MAQs. Alcolalado and Mir (2002) suggest that the quizzes are MCQs although MAQs sometimes.

> learners *".. are offered a list of up to about 20 possible responses from which to choose the (usually) one best answer."*

We regard EMSQs as either MCQs or MAQs that simply have a large number of options, **P**. If the quizzes are MAQs then the user is asked how many ticks, **T**, there are.

An EMSQ quiz can either be an MCQ quiz or an MAQ quiz and we have seen that MCQ quizzes are a specialisation of MAQ quizzes where **T** = 1.

This thesis covers quizzes that are 'special' quizzes:

- Yes/No questions only.
- Multiple Choice Questions (MCQs) only.
- Multiple Answer Questions (MAQs) only.
- Multiple Yes/No question only.
- Extended Matching Set Questions (EMSQ) (MAQs or MCQs) only.

We have seen that Yes/No-only quizzes are a specialisation of Multiple Yes/No-only quizzes, and that MCQ-only quizzes are a specialisation of MAQ-only quizzes and that EMSQ-only quizzes are either MCQ or MAQ quizzes. So we are only dealing with:

- MAQ-only quizzes, or
- Multiple Yes/No-only quizzes.

1.4 The structure of the thesis.

The structure of this thesis is **I**ntroduction **L**iterature review **M**ethods **R**esults **a**nd **D**iscussion, **R**eferences and **A**ppendices (**ILMRADRA**). We changed the **D** into a **C** for **C**onclusions. The IMRAD bit comes from comes from a book by Day and Gastel (2011 p10) to which we have added **L**iterature Review, **R**eferences and **A**ppendices. The additions are because most people have a Literature Review and References in their thesis and Appendices are there to back up what was done but not 'get in the way'. We also got the structure of this thesis by looking at the structure of other theses at this University and others (c.f. Russell 2010), seeing how we would be judged at Sheffield Hallam University (2011), the feedback from of the RF2, reading and attending lectures and conferences. The Conclusions chapter has been split into two sections:

a) one discussing the pros and cons of *this* research,
b) one discussing what could be done in the future.

The Introduction chapter 'sets the scene' which includes explaining the title, the Research Question (RQ), the philosophy, the motivation for doing the study and the contribution to knowledge that this study makes. After setting the scene, the journey undertaken is explained and involves finding a subject and placing limits on the analysis and types of quizzes. After this, comes what we did do and what we did not do. During the whole of the journey reading was done, a summary of part of the reading appears in Chapter 2 (i.e. The **L** of **L**iterature Review).

The first two chapters of an **I**ntroduction and **L**iterature review explain why this study was done and that a) we know what we are talking about b) a gap was spotted.

The next chapter contain the **M**ethod and the tools that we used and their explanation (phase 1). The next three chapters contain the analysis of special quizzes (phase 2) and the next chapter contains the **R**esults that we obtained from the tools. Chapter 3 shows what tools we built and why.

The analysis that could be done without any tools being built is shown in Chapters 4, 5 and 6 (phase 1).

The Results of the tools (*tool1* and *tool2*) is shown in Chapter 7.

These results are talked about in the Summary chapter i.e. Chapter 8..

There is a list of references in the **R**eferences section, and that is followed by 18 **A**ppendices, which are there to back up our results and discussion/conclusion, but not 'get in the way'.

This way we have the **ILMRADRA** structure of this thesis (even though the D was changed into an S).

1.5 Explanation of the title.

The title says "Analysis of Quizzes using a Census". It does not say ALL analyses of ALL quizzes. **Some** analysis of **some** quizzes will be done, where a limitation occurs on analysis and quizzes. The limitation on quizzes is where we only consider '*special*' quizzes, and the limitation of analysis is mainly producing 'descriptive statistics' of the marks of special quizzes of a census. A sample is where *part* of the population is looked at and a census is where *all* the population is looked at. The only difficulty is deciding what the population is. It is not everybody in a country. Here, the population is ∑((a mark) x (all the ways of getting that mark of a 'special' quiz)).

1.6 The Research Question.

The **R**esearch **Q**uestion (**RQ**) is "What is the distribution of marks of **special** quizzes when a census is done?" A subsidiary question is "What happens to the distribution when the answer to a stem is guessed?" This we call the guessing problem and is tackled (in this thesis) by a) negative marking b) raising the pass mark c) using a 'more complex' quiz.

We are looking at marks of special quizzes and produce mainly descriptive statistics by building tools (i.e. by writing programs) but we ask "What can be done without building any tools?", and that is what Chapters 4, 5 and 6 are about. Much of the rest of the thesis is about when tools are built. The tools were built because the distributions had such large numbers that 'ordinary' computer programs could not deal with.

1.7 The Philosophy.

The philosophy of doing the study was **simplicity** and to have as few uncontrolled variables as possible. We did not want to open the 'can of worms' that contained 'good' classes and 'bad' teachers. That includes at least 4 variables:

a) class,
b) teacher,
c) good,
d) bad.

and that is not simple. We see that 'real' people were not be involved, which leads to the data being *generated* and *not collected*. In line with our philosophy is to try and start with a simple problem and then move on to a more complicated problem, rather than start with a complicated problem. This way you have got a chance to solve a simple problem rather than fail to solve a complex problem. This is in stark contrast to The General Problem Solver of Artificial Intelligence (Newell, Shaw and Simon 1959) that tried to solve complex general problems, until it was realised that particular problems like Draughts were more amenable to solution than general ones. Nelson (2012) said

> *"An invincible checkers-playing program named Chinook has solved a game…".*

Einstein (1950) is quoted as saying

> *"Make everything as simple as possible, but not simpler"*

and that is in line with our philosophy of simplicity. Thus, we do not collect data; we do not do sampling; we do not have a hypothesis; we make lots of simplifying assumptions; we make simplifying limitations.

1.8 The contribution to knowledge.

This study shows that the distribution of marks of a 'special' quiz is bell shaped and that when the guessing problem is tackled (by negative marking or allowing learners to miss answering a stem with no penalty or using a more 'complex'

1.10 What is a mark?

Marks come in many varieties raw and modified in some way e.g. percentage, adjusted, scaled, aligned, standardised.

A **RAW** mark is the sum of scores for an individual who takes a quiz.

For example, a learner might get a raw mark of 69 in a 100 stem quiz, by answering 69 stems correctly, all of which are scored 1 and 31 incorrectly each of which is scored 0.

A **PERCENTAGE** mark is 100* (the raw mark)/(mark that could have been obtained)

For example, if the learner got a raw mark of 69 in a quiz where **n** = 80, **a** = 0, **c** = 1 and all stems are answered, then the percentage mark is 100* 69/80 = 85.19.

Percentage Mark = 100 *(Raw Mark)/(**n***Max(**a**, **b**, **c**)).

Students tend not be given raw marks but percentage marks.

An **ADJUSTED** mark is where if the raw mark that is less than 0 it is returned as 0.

For example, a raw mark of -69 is retuned as 0.

A **SCALED** mark is scale (or grade) to which a raw or transformed mark is allocated. The scales/grades at British Universities are Fail, Pass/3rd 2:2 2:1 or 1st.

For example a raw mark of -69 is adjusted to 0 and then the percentage mark of 0 is calculated and then the scale or grade of Fail is assigned. Scaling does not just apply to Universities, it also applies to GCSE and A-levels, where a scaling of A* might be applied to a raw mark.

In America, the mean of all the module marks is formed and the Grade Point average is calculated, and they say that their degrees are not scaled. But, they are. The Americans do not say "I got a 2.2" but say "My Grade point average was x". But, the grade point average is scaled with the maximum being 4, and even further they say "I averaged between an A and a B+ on my under-graduate degree", which further indicates that a scaling of their undergraduate degrees goes on. Their postgraduate degrees are scaled where, for example, the Carnegie-Mellon MBA is scaled to "Pass" or "Fail".

An **ALIGNED** mark depends on how others did, in that year, and so the alignment process, involves Norm marking. The alignment process first of all decides what the cut-off points should be. The process might decide that Band

A should be marks above 90 and Band B should be the marks between 80 and 89, Band C should be 70 to 88, Band D should be the marks between 65 and 69. If you got 67 for your raw mark, then your aligned mark is ((69-67)/(69-65)) * 5 + 65 = 65 +10/4 = 67.5

An example of **STANDARDISED** mark is what is done in the Management Department of Canterbury University (NZ) where the marks are standardised as follows

Standardised mark = 65 + 18* (your percentage raw mark -class mean percentage mark)/(class percentage standard deviation).

> *"If the mark is negative it is set to 0*
> *If the mark is over 100 it is set to 100"*
>
> James (2012).

This is Norm marking and this thesis assumes Criterion marking is used.

z-SCORE

Another form of standardisation is

$$z = (x-\mu)/\sigma$$

and is often called **Normalisation**, but this term has many meanings e.g. in statistics it might refer to the Normal distribution, and in relational databases it might refer to the process of converting flat files.

where z = z-score
μ = mean mark of population
σ = standard deviation marks of population
x = raw mark

When $x > \mu$ then z is positive
When $x < \mu$ then z is negative

The z-score is just a generalisation of standardisation

We will say that the transformation of raw marks e.g. standardisation, is not straightforward.

A raw mark is the sum of scores. We immediately see that there is a relationship between a raw mark and:

a) how many stems are (**n**),
b) how many stems are answered correctly (**w**),
c) how many stems are answered in-correctly (**x**),
d) how many stems are missed (**y**), given that all stems are scored the same,
e) how we score i.e. what are **a, b** and **c**
f) the type of quiz.

We will look at this relationship in the thesis. For now we will summarise and say

$$\mathbf{n} = \mathbf{w} + \mathbf{x} + \mathbf{y}$$

$$\text{Mark} = \mathbf{yb} + \mathbf{wc} + \mathbf{xa}$$

but there might be 2 or more ways of obtaining the same mark and so a Pivot is necessary. After a Pivot, **M** is the mark that can be obtained in a number of ways.

1.11 What is a distribution?

Graphs have an independent variable that is measured along the x-axis, and a variable, called the dependent variable, that is shown on the y-axis. A distribution shows how the dependent variable varies with changes in the independent variable. On the x-axis we show a mark (**M**), and on the y-axis we show the number of ways of getting that mark ($\mathbf{Y_M}$) i.e. the frequency of getting a mark.

There are two types of independent variables:

- continuous,
- discrete.

As the marks are not continuous, we are dealing with discrete distributions in this thesis.

For example, we are considering marks after a Pivot is done. A Pivot is where a mark and how many got that mark is calculated.

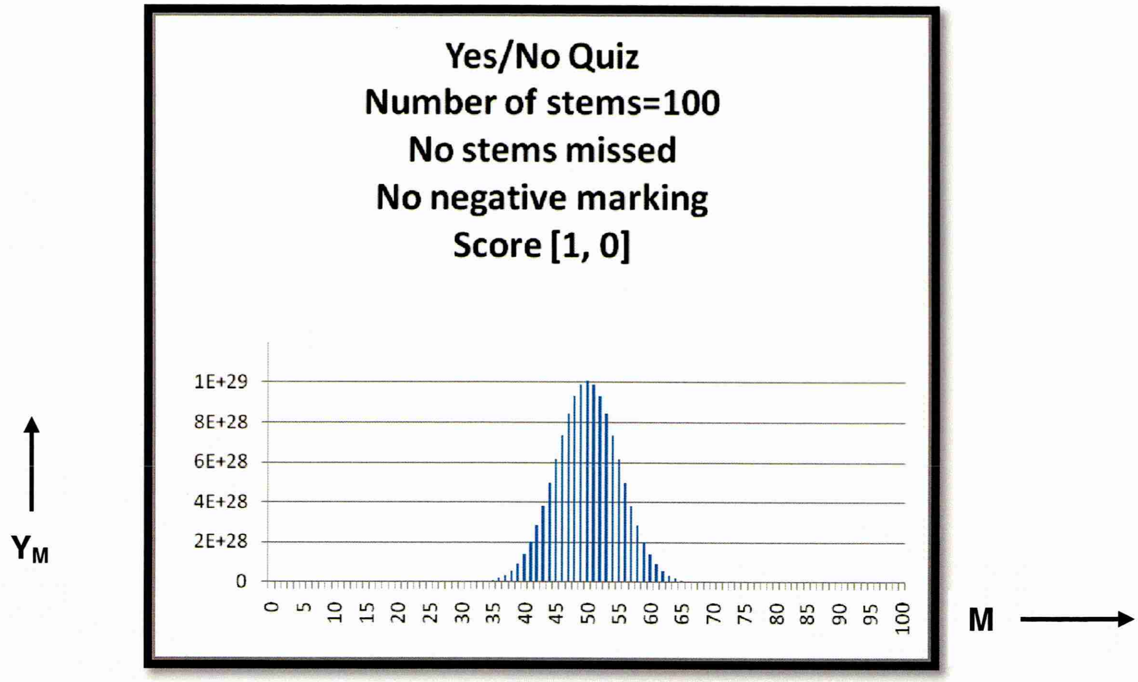


Figure 1-5. The distribution of marks.

Figure 1-5 is an example of a distribution once a Pivot is done and shows that very 'large' numbers are involved.

1.12 What is a census and what is a sample?

A **census** concerns data about the *whole* population that get marks from a quiz. For example it is $\sum Y_M M$ in the picture shown above i.e. the 'area' (later on we will call this a pseudo area) of the distribution. The above picture shows the marks that can be obtained in a 100 stem Yes/No quiz, where every stem is answered and scored [1, 0], and shows how many ways there are of obtaining a particular mark. For example, 1E+29 means $1*10^{29}$, so, there are about 10^{29} ways of getting the mark 50.

A **sample** concerns data from *part* of the population. A sample must represent the population. If it does not, there are problems, like the possibility of introducing bias and extrapolation/generalisation from the sample to the population. If the sample does represent the population there are no problems.

To eliminate any possibility of bias and being able to generalise, this thesis looks at the whole population.

1.13 What are descriptive statistics?

Descriptive statistics try to describe a distribution, by mentioning a few measures, and as this thesis deals with censi it deals with measures of censi and not estimates of these measures. Thus, for example, this thesis calculates **μ** and not $\bar{x}$, and talks about **range** (that gives min(x) *to* max(x)), **mean**, **mode**, **median, variance, standard deviation, skewness, kurtosis**.

1.14 The journey.

The journey consisted of finding a subject and refining that subject. A subject had to be found because a subject was not given to me by my supervisor.

The whole journey was not 'clean'. Finding a subject involved going down many 'blind' alleys and coming back e.g. dropping "how the brain works". Refining the subject until it provided a topic that was 'doable' required going down more blind alleys e.g. dropping CBM (Gardner-Merwin (2005) and its variation, due to Davies (2001a). But, going down 'blind alleys' can prove useful, by telling future workers "Do not go down that alley".

The journey started with wondering what the Flash software gave us and that was soon dropped, after models and their types were looked at. The dropping occurred because it was felt that the subject could not sustain a PhD.

Because I had been a lecturer at Universities, for a long time, lecturing was looked at, and as lecturing involves teaching, learning and assessment, they were all looked at.

Teaching, Learning and assessment involve at least 3 types of people viz. teachers, learners and support staff, and they need to form a Community of Inquiry (**CoI**) in order for the learner to have an enhanced Educational Experience, according to Garrison and Anderson (2003).

Lecturers/teachers are told that they should be *reflexive,* in order to become better teachers e.g. Lewis (1987) who says

"...centres in the UK appear to make significant attempts to encourage teachers to engage in critical reflection..".

Assessment of whom, leads to questions such as "who should do the assessment of lecturers?" and "how should this assessment be done?", "what happens if a lecturer fails an assessment?" We do not explore lecturers/teachers being reflexive, nor these questions.

Lecturers/teachers are told that *embedding* assessment into the teaching leads to 'better' learning e.g. Black , Harrison and Lee (2003, p3) who say

"..test scores can be raised .." .

Embedding is where assessment is inside the teaching, and leads to questions like "is the learning process improved by embedding the assessment?" Which, in turn, leads to further questions like "what does the learning process consist of?", "who learns?", "how do we learn?" "Do we retain knowledge longer?" "Can we get knowledge out faster?" "Do we have more knowledge?" We do not explore the embedding of assessment, or these questions, but the question of "how do we learn?" was looked at and later dropped, because we did not find out how we learn.

Looking at assessment leads to questions like "why bother with assessment?" which is asked by Conner (1991) and that leads to answers like "to measure what learners have learnt" (this is called **summative** assessment), "to obtain an opinion of the learner" (this is called **formative** assessment), "to **improve** the **learning** process" (e.g. by **embedding**), which in turn lead to more questions. The terms summative and formative are used by Black, Harrison and Lee (2003)

Another type of assessment is **ipsative** assessment which marathon runners use to see if they have improved. Ipsative assessment is mentioned by Freeman and Lewis (1998), who also mention that teachers can get better. We note that with ipsative assessment, if negative marks are all given 0 then improvement cannot be measured.

We do not explore why assessment is done, but note that our results apply to summative, formative, ipsative and embedded assessment (Wilson and Sloane 2000), but are most forceful when applied to summative assessment by special quizzes.

1.14.1 Deep or Surface learning and Assessment

Teaching, has so much written about it, that providing something new looked unlikely. But, looking at teaching proved useful as it was thought by Biggs and Tang (2009) that Universities sometimes impart non-procedural knowledge and sometimes procedural (or shallow) knowledge. Procedural knowledge is exemplified by "do this then that" (i.e. a procedure is followed) and non-procedural knowledge is exemplified by "*why* do this then that?" Biggs and Tang (2009) further thought that Teaching, **Assessment** and Learning should all be '*aligned*'. One way that this alignment can be done is if we are teaching procedural knowledge then we should assess this procedural knowledge. Biggs backs this up with

> *"Then they are told about creative problem solving in packed lecture halls and tested with multiple-choice tests. It's all out of kilter."*
>
> Biggs (2004).

We e-mailed Biggs with some ideas c.f. Biggs (2009) about the measurement of 'deep' because we were concerned about 'how' deep. He maintains that Multiple-choice tests encourage 'surface' learning (Biggs 2004) and if you want to encourage 'deep' learning (where the learner **understands** a concept) then you have to assess in a different way to testing by MCQ. Heywood (2000) has a section in Chapter 9 about 'deep' and 'surface' (on p221 seq.) but notices that assessment drives learning and students want a 'good' degree and if a 'good' degree can be obtained by 'surface' learning then so be it. A 'special' quiz does not have room where an answer is explained and so a learner cannot explain how the answer was obtained. The learner cannot *demonstrate* that he/she *understands* a concept by using a 'special' quiz and so alignment is impossible when a 'special' quiz is used and understanding is claimed to be measured. But we do not care about how the students learn, be it deeply or sufacely, or whether the assessment/teaching/learning are aligned, we only want to see how the marks are distributed.

We left Teaching out, but carried on looking at Learning and Assessment, but it did not restrict us from wondering "can Yes/No quizzes ask deep questions?"

The next part of the journey involved looking at Learning and wondering "what does Learning mean?" and "how does the brain work?" This led to wondering whether Learning Styles exist, and two papers were produced (one a near copy of the other) Lafferty and Burley (2009) and Lafferty and Burley (2011). Three books written by Greenfield (2000a, 2000b, 2008) on how the brain works were read, but did not enlighten us on what Learning meant. Despite that Greenfield (2007) 'panned' the idea that learning styles exist.

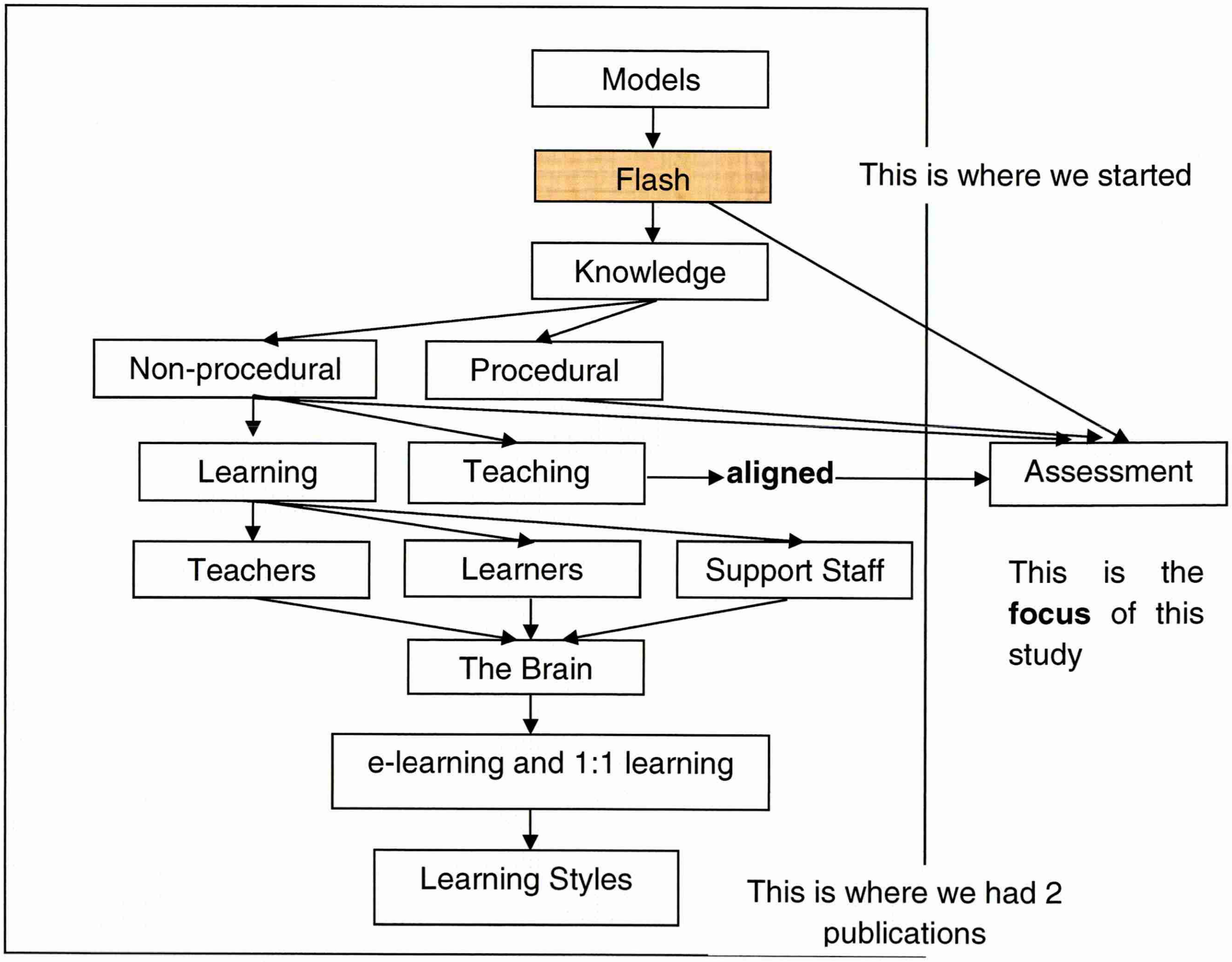
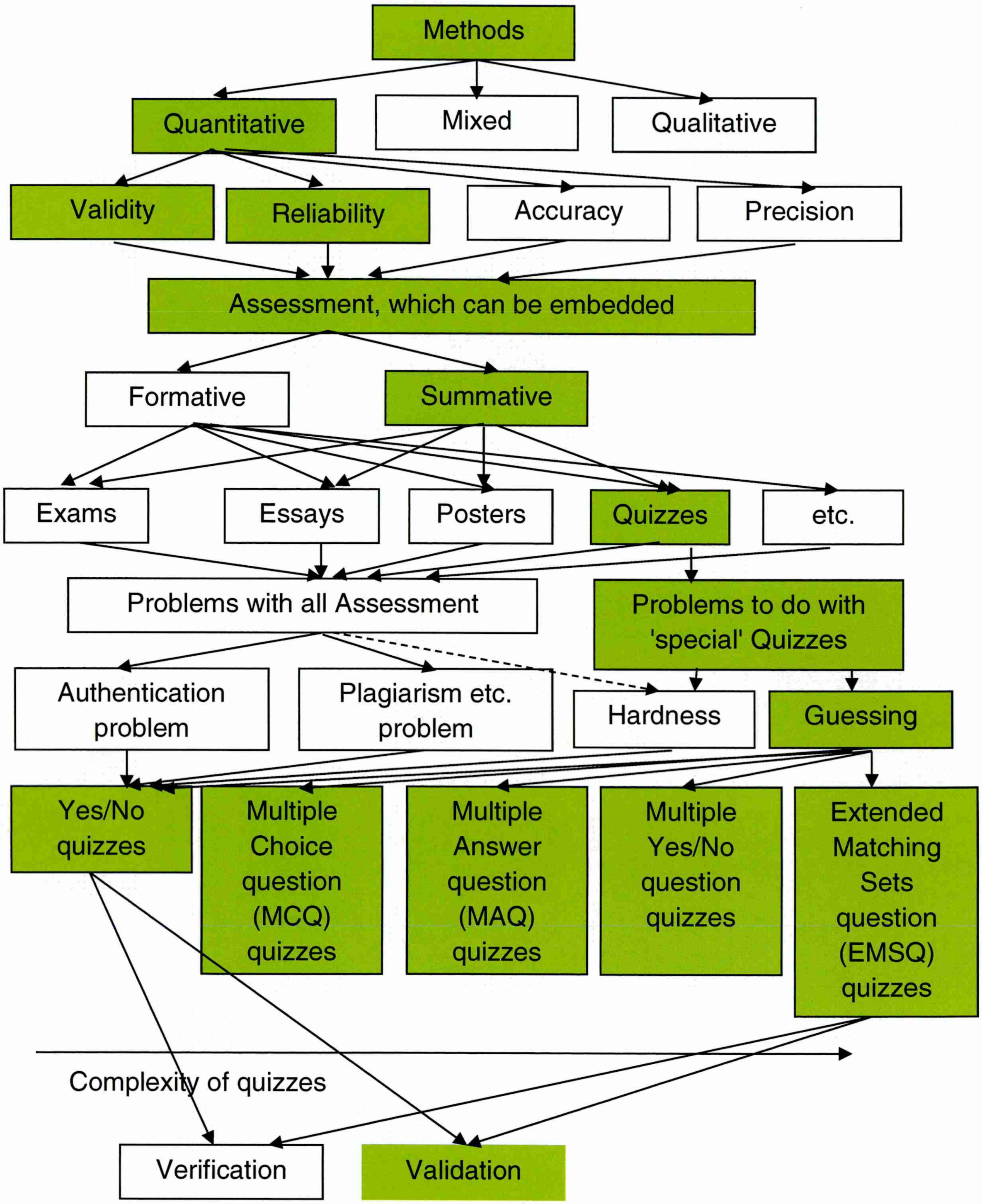


Figure 1-6. Finding a subject.

There are many arrows left out, to make the diagram readable. For example, Teaching can be of Procedural knowledge.

After much soul searching it was decided that Assessment was going to be looked at, and the production of Assessment by Flash (a computer system) was going to be dropped, leaving only Assessment, by other methods, to be looked at.

Figure 1-6 shows how a subject was found.



The solutions to the guessing problem include

Using more complex quizzes (our idea)
Using negative marking (see Burton 2010)
Raising the pass mark (see Gov.uk. 2013)

We do not deal with quizzes that are scored by CBM or its variant by Davies.

Figure 1-7. Finding a subset of quizzes to look at.

Figure 1-7 shows that a subset of quizzes was chose and again, many arrows are missed out in order to make the diagram readable, for example, all quizzes *do* have the authentication, plagiarism, hardness and guessing problems.

This thesis is about looking at the marks produced by a census, of a subset of quizzes that are used summatively, and tackle the guessing problem. We write programs that help, and then validate that they produce right results.

We try using mores complex quizzes as a way of tackling guessing the correct answer to a stem. For example, we try 5-option Multiple Answer Quizzes where 2 answers are chosen as correct, to see what happens, when guessing is tackled.

Burton (2010) considers using negative marking for tackling the problem of guessing the correct answer, but goes further than we do and considers taking off some marks for guessing. We only consider 'exact' negative marking in this thesis. For example, for Yes/No-only quizzes we use a value of -1 for the negative mark, and for 4-option MCQ-only quizzes we only consider -1/3 as the value of the negative mark.

In the British government driving test:

a) the pass mark for multiple choice questions for motorcycles and cars is 43 out of 50 and the test lasts for 57 minutes,
b) for the hazard perception test the pass mark is 44 out of 75 (Gov.uk. 2013).

We look at the effect of raising the pass mark

We had publications that asked "Can this subset of quizzes be valid?" Lafferty and Burley (2012a) and "What happens when you try and solve the guessing problem of this subset of quizzes?" Lafferty and Burley (2012b).

1.15 Some of the assumptions that were made.

Some of the assumptions that were made were:

- quizzes are 'closed' and do not change (during the quiz),
- the likelihood of choosing an option was equally likely,
- the marking of the quiz is done by a computer, and then there is 100% reliability because there is no variation in the marks between markers or within the same marker,
- the stems are as independent as possible,
- the options are as independent as possible, and equally likely,
- the learner cannot work out the right answer from the options,
- pools are not used,
- the authentication problem existed but was ignored,
- the plagiarism problem existed but was ignored,
- the hardness problem existed but was ignored,
- the quiz consisted of questions that had been taught,
- criterion marking and not Norm marking is used .

We do not try and solve the hardness problem which for Yes/No questions can stated as "stems for Yes/No questions should be equally hard or reflect their hardness in their score" A similar statement can be made for MCQs MAQs and Multiple Yes/Nos, but then options also have to be mentioned. A solution to the hardness problem is so problematic/subjective/argumentative that it is avoided.

1.16 What we did NOT do.

We do not do:

- a longitudinal study, where many students are studied over many years This involves matching students to remove bias, and how does one deals with 'good' classes and 'bad' teachers and vice versa,
- a micro study, in that an individual is not studied,
- a comparative study, in that the results of quizzes are not compared to the results of exams for example,
- we did not try to solve the 'nearly right' problem, which is exemplified by a 6-option-only MAQ quiz where 3 options should be ticked as correct, and only 2 options are ticked,
- a gender study e.g. where males and females are compared,
- a feminist study e.g. where only females are studied.

We did not use

- action research, where the researcher engages in trying to change an organization.

"Action research involves the process of actively participating in an organization change situation whilst conducting research."

Anon (2012).

- grounded research, where a theory emerges from the data. We were not looking for a theory to explain our data.

  "..generating theory is the researcher's main aim .."

  Strauss and Corbin (1997).

1.17 What we did do.

We did do a series of:

- case studies,
- explanatory studies, where we were trying to explain the results,
- **theoretical** studies, all using the same method whose limit was shown.

We did do a series of *case studies,* where for example a case is "a quiz consisting entirely of Yes/No questions, all of which are answered and scored [1, 0] "

We did not have to gather any data, we *produced* the data. So, we do not have to say what are our sources were, or explain whether they are primary or secondary, nor do we have to explain the tools used for gathering the data.

We do try to explain the results that we obtained, so we could be said to doing an explanatory study in part.

We did not try to use 'real' people to see what marks they got, but looked at what marks could be obtained, and thus could be said to be doing a theoretical study.

1.18 Conclusions.

We conclude that this study tries by limitations and assumptions to **simplify**. We limit ourselves to looking at 'special' quizzes, and the assumptions include that every stem is equally 'hard'.

A Literature Review, that shows what others have said and that a 'gap' was spotted, is coming next.

Chapter 2. Literature Review.

Abstract.

Reading round the subject of assessment, we came to the conclusion that previous research had been complicating the issue. The complications arose from a) writing 'good' MCQs b) good/bad classes c) god/bad teachers d) sampling e) confidence/certainty f) learning styles g) sorts of assessment e.g. quizzes, examinations h) types of assessment (formative, summative, ipsative) i) reasons for assessment (e.g. to aid learning) j) effectiveness of assessment k) who does the assessment (e.g. by students of teachers).

We were *not* concerned with writing good MCQs or with good/bad teachers/classes or with the optimal number of options of MCQs or the optimal number of ticks of MAQs or with sampling (see next chapter) or with confidence or with learning styles or with the sorts of assessment or with the reasons for assessment or with its effectiveness or with who does the assessment. We, simply, wanted to know the distribution of marks of special quizzes.

From our reading we concluded that a learning styles does not exist and so a) academics should not accept papers that *assume* they do b) salesmen should not promote products that purport to measure a learning style c) learners should not be encouraged to find out their learning style d) teachers should not split their classes by learning style e) teachers do not need to match their teaching style to the learners' learning style

It is claimed that to have an enhanced educational experience that the people involved with the learning should form a **C**ommunity **of** **I**nquiry (**CoI**).

There was the gap in the previous research which did not ask "What is the distribution of marks of quizzes?" By limiting quizzes to 'special' quizzes and by only considering a census of all the marks we could make our question simpler.

Out **R**esearch **Q**uestion (**RQ**) "What is the distribution of marks that could be obtained from a 'special' quiz when a census is done?" and what happens to the distribution when the guessing problem is tackled in 3 ways. The 3 ways are a) using negative marking b) raising the pass mark c) using a more complex quiz. We show in this chapter, why negative marking is used, and the opinions held on it and in later chapters show the effect of using negative marking. Raising the pass mark is examined in later chapters, as is the use of more complex quizzes (but brief mention is made in this chapter).

The central issue is that we wanted to make was that our **RQ** was simple as possible.

2.1 Introduction.

A purpose of a **L**iterature Review is

> *"All research needs to be informed by existing knowledge in a subject area"*
>
> Rowley and Slack (2004, p31).

They also say that the literature should be evaluated and we will show what previous research has been done on MCQs which has been concentrated on writing 'good' MCQs. Our simplification is to assume that 'good' MCQs have been written.

By reading the literature, a *gap* is found that establishes that a question has not been asked by anyone else which must be 'doable' in the time allowed. In our case the gap is "What is the distribution of marks of a special quiz when a census is done?" A supplementary question is "What happens to the distribution when the guessing problem is tackled?" The whole thing lasts about 6 years, because the PhD was being done *part-time*. The guessing problem is tackled, in this thesis, in 3 ways a) by using negative marking: an old idea and we will show (in later chapters) what its effect is and in this chapter show what the opinions held on its use are b) raising the pass mark: which is done by lots of organisations and we will show in later chapters what its effect is c) using a more complex quiz, an idea of ours, and we will show in later chapters what its effect is.

A Literature Review does not contain *everything* we have read. For example, a Literature Review does not contain what has been read about Methods; that is put in the Methods chapter.

A Literature Review must not be 'too long' or repetitious and must say how one came to the chosen method. In our case the chosen method was to write computer programs that could handle integers of an arbitrary length. The programs produced a drawing of the distribution and some descriptive statistics. The next chapter on Methods will go into more detail of the method chosen for this study and the measures used in the method.

Special quizzes are in assessment and so it is asked "Is assessment effective?" Assessment occurs for many reasons and is done on many different types of people (e.g. students) for many different types of people (e.g. teachers).

2.2 Research must be new and Deductive and Inductive reasoning.

Dawson (2005) says the contribution of the research must add to the body of knowledge. He even draws a diagram to illustrate this point. This is in contrast to undergraduate degrees, which, in the main, do not have to demonstrate any originality.

Rudestam & Newton (1992) say that research is like a wheel and goes on for ever, with iterations of Deductive reasoning and Inductive reasoning. Deductive reasoning goes from the general to the particular and Inductive reasoning goes from the particular to the general. In our case, looking at the particular case of Yes/No-only quizzes where every stem is answered, we do *not* try to generalise and induce what would happen to 4-option MCQ-only quizzes, but we did use Inductive reasoning and we did use Deductive reasoning (see next section).

Many studies start off with an hypothesis, which forms the RQ, but what is unusual about this study is that it does not start with an hypothesis, it wants to know the distribution of marks of a 'special' quiz.

2.3 The funnel idea.

It is not known who came up with the funnel idea, but following on from the cycle of Deductive/Inductive reasoning and doing research and then asking further questions, it seems that research first of all is wide and then gets narrower and then broadens out. Research is rather like a funnel attached to an inverted funnel.

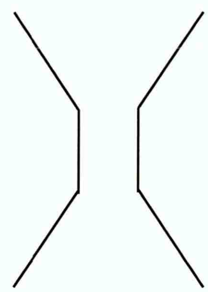


Figure 2-1. The Funnel idea.

The narrowing of our study to quizzes and limit them to 'special' quizzes shows a form of Deductive reasoning and the broad parts are where Inductive reasoning is used.

Thus, the Research Wheel and the funnel idea say the same thing viz. research goes on for ever with bursts of deductive reasoning followed by inductive reasoning.

2.4 Prior relevant research on Multiple-choice questions.

'Special' quizzes are exemplified by Multiple-Choice Questions (MCQs) and the research on MCQs has spawned many papers on the writing of 'good' stems and options of MCQs. Gerberich (1956) gives examples (at least 150) of 'good' MCQs and on many pages says where the examples come from. Other papers on writing 'good' MCQs are by Kehoe (1995), Haladyna and Downing (1989), Cheung and Bucat (2002), Boland, Lester and Williams (2010). Haladyna and

Downing (1989) include a 43-long list of do's-and-don't's to do with writing 'good' options of MCQs that include

"Avoid, or use sparingly, the phrase 'all of the above'".

and this is repeated by Cheung and Bucat (2002). Boland, Lester and Williams (2010) concentrate on the medical world and they think that MCQs are useful in this domain but we and McCoubrie (2004) question that.

Kehoe (1995) urges us (amongst other things) to avoid irrelevant material when writing stems.

Berk (1998) shows some of the flaws that can occur with MCQs and contains a short history of MCQs. He gives an example of a flaw on page 98 which contains an option of "Who cares" which can be eliminated and thus increase the odds of choosing the correct option.

We simplify the situation by *assuming* that 'good' stems and options have been written.

Other research has focussed on how many options in MCQs there should be (Rodriguez 2005), Vyas and Supe 2008). Little research, if any, has been focussed on how many ticks there should be and we cannot find any papers on what the *optimal* number of ticks is. Because we do not know the optimal number of ticks we simplify the situation by asking the user how many options/ticks there are.

Another area of research is how good a quiz is at discriminating between 'good' and 'bad' students (Macintosh and Morison 1969), Hoffman 1962). Our view is that this only complicates the issue and so we avoid it.

It is claimed that MCQs are capable of tackling all of Bloom's taxonomy which is shown in Dzuiban (2011) but we say that as you cannot show how you came to the correct answer, you might have guessed, you might have got the correct answer by an in-correct method. So, this does not show that MCQs can always tackle all of Bloom's taxonomy.

2.5 The effectiveness of assessment

Assessment comes in many forms e.g. exams (c.f. Chapter 1) and is of 3 types a) formative b) summative c) ipsative (again c.f. Chapter 1). Assessment is *of* many different types of people (e.g. students) *by* many different types of people (e.g. by students of teachers) (c.f. Chen and Hoshower 2003) and is for *many reasons*:

1. to find out what the learners know,
2. to help the learning,
3. to accredit and give grades,
4. to rank the learners,
5. to help the teaching. (cf. Freeman and Lewis (1998, p10))

More reasons are given in Brown & Bull & Pendlebury (1997, p11) and we add

6. to find out whether the learners *understand*.

The effectiveness of assessment depends on the purpose of assessment and what we mean by effective. If by effective we mean cost-effective then we will *not* look at that, although Hornby and Laing (2003) did.

This thesis does look at the marks that are given to learners and they can be used in items 1, 3 and 4 above. The embedding of assessment in the learning materials is claimed by Clark and Mayer (2008) to assist the learning.

The University of Texas, Stuart (2013) think that assessment should help students to learn, but do not say whether the assessment is effective. The University of the State of Dakota, DSU (2013) imply that assessment should be done, as do the Austin Community College (2001) but nowhere say that the assessment is effective.

I strongly believe that quizzes cannot measure *understanding* as there is nowhere to explain this understanding and if they claim to measure understanding they are **in-valid** (see later where we define validity). Thus, 'special' quizzes are in-effective and in-valid because they do not measure what they claim to measure.

The conclusion from this is that assessment should be done, but we do not know if it is effective. This thesis only considers assessment by special quizzes and limits itself to the marks produced by special quizzes, and does not assume that special quizzes are effective.

2.6 Does a Preferred Learning Style exist?

Whilst engaged in divergent thinking we thought a preferred learning style might not exist, and trainee teachers at Sheffield Hallam University are taught not to use learning styles (Griffiths 2013) and Kara (2009) says:

> *"Learning styles are, simply put, various approaches or ways of learning"*

A **"Preferred"** learning style means that the learner prefers to learn in a chosen way.

From our reading we came to the conclusion that a preferred learning style might not exist c.f. Lafferty & Burley (2009) and were asked by Wright (2010) for a copy. The subject was started by Kolb (1984) and since then a whole 'army' of practitioners has sprung up c.f. Honey (2009), Honey and Mumford (1992), Fleming (2001), Gardner (Memletics, 2008). Honey even went so far as to send Clark (2007) 'abusive' e-mails, when he reported Coffield's (2004) findings, that there were at least 70 ways of measuring learning styles, and more are suggested by Martinez (2009).

People learn in different ways (Biggs and Tang, 2009), Biggs (2009) and at different rates (Christensen 2008) and to different 'depths' (Reynold, Caley and Mason, 2008). But do learners have *one preferred* way of learning?

The arguments for a preferred learning style being a myth are supported by Willingham (2009a, 2009b), Curwin and Mendler (1999), Greenfield (2000a, 2000b, 2007a, 2007b,2009) , Rowbotham (1999) , Demos Think Tank (2004), Stahl (1999), Siegfried and Fels (1979), Occam's donkey (2008), Nicholl (1988), Musa and Wood (2003), Reid (1987), Shih and Gamon (2001), Cambiano , De Vore and Harvey (2001), Van Wagner (2005), Martinez (2009), Anon (2009), Denzine (2009), Western Nevada (2009) and that people learn in the same ways has been shown by Draper (2012). We were not allowed to borrow a non-invasive device c.f. Emsense (2009). There is a paper that says that neuro-scientists cannot teach educationalists anything (Snook 2007) despite what Baroness Greenfield thinks.

The arguments for a preferred learning style being NOT a myth are mainly a refutation of the arguments that a preferred learning style is a myth. These arguments are based on the fact that they do not prove that a preferred learning style does not exist. But see Lu and Liu (2013) and there is anecdotal evidence from Walus (2009) that might show that a preferred learning style does exist. A web-site claims that learning styles are static (SeeChange 2005) and there is a paper that learning styles affect how people learn to read (Snider 1992).

Learning styles and a preferred learning style are not to be confused. We all learn in different ways and so learning styles do exist, but do we prefer to learn in one way? We think not, and in the lack of evidence of a preferred learning existing we hypothesize/conjecture that a preferred learning style does not exist.

The Association of Psychological Science said in a report in 2008,

> "*We conclude therefore, that at present, there is no adequate evidence base to justify incorporating learning-styles assessments into general educational practice.*" Pashler, McDaniel, Rohrer, & Bjork, R. (2008)

Despite this report McTighe and O'Connor (2004) say that assessment can throw light on learning styles.

Based on the lack of evidence of learning styles existing then it follows that:

- papers should not be accepted that *assume* that learning styles exist, (see Akdemir and Kozalka (2008), but see Brown (2006)),
- salesmen should not promote products that support learning styles,

(see Honey (1992), Honey and Mumford (2009), Fleming's VARK (2001), Gardner's Memletics (2009))

- learners should not be told to find out what their learning style is, (see Southampton University (2009))
- teachers should not split their classes by learning styles, (but see Bostrom and Lassen (2006))
- teachers should not modify their teaching to *match* learners' learning styles.
  (see Rowbotham (1999), Siegfried and Fels (1979), Occam's donkey (2008))

2.7 Community of Inquiry.

Garrison & Anderson's (2003) thought that what was needed to learn was for the people involved to form a Community of Inquiry (CoI) that required from each a:

- Social Presence.
- Cognitive Presence.
- Teaching Presence.

Where they all overlapped was called an **Educational Experience** for the learner, and was exemplified by a diagram on p28.

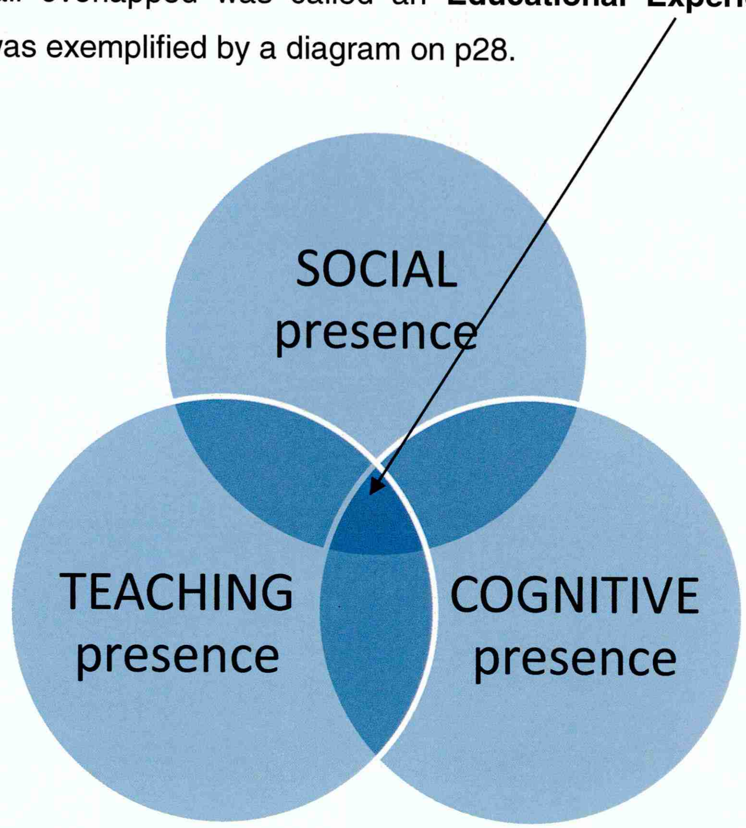


Figure 2-2. Garrison & Anderson's idea of a Community of Inquiry (CoI).

For example, a distance learner's *Cognitive* presence includes an element of critical thinking, whereby what he/she is taught is analysed and put in place, and to that end has an enhanced Educational Experience. The teachers have a cognitive presence when preparing their lectures, and when they become learners. The support staff and the teachers must know the needs of the learners, and hence have a *Cognitive* presence.

A distance learner must be able to ask the right questions of the right people, e.g. teachers, support staff, other students. So, must 'get on' with other people, or is said to have a *Social* presence, so that he/she learns more/faster, understands more, retains more, and hence has an enhanced Educational Experience. The support staff of distance learners must know who is on what course and where they live; tell that to other learners. They must know what the teachers are teaching and know when hand-ins are due and deal with them. The support-staff liaise with academics and students. Thus, teachers, support-staff and the students have a *Social* Presence.

The learners learn when they teach their 'mates' (Peer supported learning). The teachers of a distance learning module should teach differently because the learners are not face-to-face. Teachers learn from the learners, and hence become learners themselves. The support staff treat distance learners differently (maybe), and so the teachers, learners and support stall, all, have a distinctive *Teaching* presence, whether a distance learning module is being given or not.

The assumption, or conjecture, is that once all 3 presences are in harmony an enhanced *Educational Experience* will result.

The idea of a **CoI** does not just apply to distance learning and it leads to many questions like:

- "Can critical thinking be taught?" Others, think it can e.g. Browne and Keeley (2007), Bullen (1998), Kelley-Riley et al (2001).
- "Who should do critical thinking teaching?", "Should critical thinking be taught to everybody?"
- "How is Social Presence taught?", "Should it be taught to everybody?"

- "How should teachers be taught to teach on Distance Learning modules?"

CoI maps the 'who' and the 'how' of learning, in that, how we learn involves who learns and the who, includes teachers, learners and support staff. We even, looked at the processes that each type of person does, and used nVivo's modelling capability, but that lead nowhere.

Another facet of learning is exemplified by "How does the brain work?" but reading 3 books by Baroness Greenfield (2000a, 2000b, 2008) did not enlighten us about how we learn.

2.8 e-learning.

Informal discussions with staff led to the notion that Sheffield Hallam University should provide better Distance Learning modules, using e-learning.

Clark & Mayer's (2008) book about the science of e-learning contained much sense about how e-teaching was to be conducted. This book is about teaching based on how we learn. Some of their models e.g. on p33, were rudimentary, and they did not consider:

- making the assessment elective,
- making the learning material elective,
- doing different assessment when the learners got something wrong,
- using other material for swotting for material that was for learning.

Horton (2006) also wrote a book about e-learning that also contained much sense, e.g. how big a class ought to be, but again, was about e-teaching

Michael Allen's (2007) book on e-learning which leads to a question "should it be e-*learning* or e-*teaching*?" The books by Clark & Mayer, Allen, Garrison & Anderson and Horton (2006) all have "e-learning" in their titles but only Garrison & Anderson's book is about learning and the others are about teaching. So, maybe, the subject should be called e-teaching (based on how we learn).

2.9 1:1 Teaching/Learning.

It was thought that e-teaching could be one-to-one. If preferred learning styles existed then the learners only had to have their preferred learning style measured once, and then they could choose the appropriate lesson that addressed their own preferred learning style. We even thought that with the advent of modern computers it might be possible to do 1:1 teaching Kruse (2002). When it was realised that, perhaps, a preferred learning style did not exist, learning, e-teaching, and 1:1 teaching/learning were all dropped.

2.10 Validity, Reliability, Accuracy and Precision.

Special quizzes must be valid, reliable, accurate and precise.

A **valid** measure is, when the instrument that does the measurement, claims to measure what it actually measures

> *"In plain language, it's valid if it measures what it's supposed to."*
>
> Hopkins (2001).

We e-mailed Hopkins in 2010 about being accurate but got back a scathing reply.

For example, if a quiz claims to measure somebody's IQ but actually measures their short-term memory, then the quiz is said to be invalid. More importantly, from this thesis's point of view, if a quiz passes you when it should have failed you it is invalid. If the mark from a quiz claims to represent your knowledge of a subject, and passes you when you know less than 30% of the subject, then the quiz is invalid. We know that this statement will make us very un-popular, especially as we think that using CBM (or its variation due to Davies) to score a quiz is also invalid (at a 40% institute). When you get 30 stems right (using CBM) with C = 3 and guess the rest with C = 1 (where **n**=100) you get a percentage mark of 100*(30*3 + 70*(1 +0)/2)300 = 125/300 = 41.67, and you pass (at a 40% institute) when you have guessed 70% of the questions. C has a value of 1, 2 or 3 and represents the Certainty of an answer to a stem. Using this argument it shows that the variation of CBM used by Davies (2001a and 2001b) is also invalid.

Cohen, Manion and Morrison (2007, p133) say that there are at least 18 measures of validity and at its root validity boils down to an *opinion* about the evidence that an instrument is valid. Hopkins (2001), on the other hand, only deals with 1 type of validity

> *".. concurrent validity, and it's the only one I will deal with here."*

A **reliable** measure is when the measure is *repeatable.*

> *"It is generally understood to be the extent to which a measure is stable or consistent and produces similar results when administered repeatedly."*
>
> Stroke Engine (2012).

Also, see Colossi (1997) who tries to explain the difference between Reliability and Validity.

There are many types of reliability including, internal consistency, inter-rater agreement, intra-rater agreement and test-retest. We consider that quizzes marked by a computer are 100% reliable. You will get the same mark time and time again, because there are no inter-rater or intra-rater variations. As our marks are about a census, there is no need to do a test-retest. We assume that our quizzes are internally consistent.

An **accurate** measure is how far away the measure is from the true measure. We think that if you are playing darts and consistently miss double 20 when you are aiming at double 20 then you reliably miss double 20 but are not very accurate. If you always put your darts in a small circle, when you are aiming at double 20, then you are precise in your misses

> " Accuracy is how close a measured value is to the **actual (true) value**."
>
> *"Precision is how close the measured values are **to each other**."*
>
> Pierce (2014).

We asked lots of questions (not about precision) like:

- "Is Reliability inside Validity, and if so by how much?"
- "Can something be unreliable but still valid?"
- "Where does verification, fit in the picture?"
- "Where does validation, fit in the picture?"

The University of Georgetown (2012) think that reliability is the precursor to validity and do think that something can be reliable but not valid

> *"A test can be both reliable and valid, one or the other, or neither. Reliability is a prerequisite for measurement validity".*

Our feeling is that that something can be in-valid even if it is reliable, and we think that University of Georgetown agree with us. For example, if the bathroom scales always gives your weight as 5lbs more than it should, it reliably/consistently gives your weight, but not validly/accurately. Hopkins (2010) agrees with us when he says

> *"For example, a little thought will satisfy you that measurements can be reliable but not valid".*

Trochim (2006) also thinks that something can be reliable but not valid when he says

> *"This measure is reliable, but no valid (that is, it's consistent but wrong)"*

2.11 Verification and Validation.

Verification can be said to be

> *"Am I building the right product?"*

and **validation** can be said to be

> *"Am I building the product right?"*

Ants (2012).

For example, we can be said to *validate* Yes/No-only quizzes by seeing if they give the correct descriptive statistics for 'small' sets.

This thesis does little verification but lots of validation. We validate our analysis and validate our programs.

2.12 The problems with all assessment.

The problems that are inherent in all assessments, including quizzes, are the authentication problem and the plagiarism problem, which are described in Chapter 1.

The authentication problem is "Who is doing the test?" and the plagiarism problem is "Can the learners 'cheat' in any other ways?"

We assume that the authentication and the plagiarism problems exist, but do not apply here, and cheating does not occur.

2.13 What quizzes are out there?

A paper by Haladyna, Downing and Rodriguez (2002) indicate that there are the following types of stems:

- Alternate choice, which is essentially a 2-option MCQ.
- Yes/No, which they call T/F, and recommend.
- P-option MCQs. What P should be, we look at later.
- R-option MAQs, which they call Complex MCQs or type K stems, and do not recommend them.
- S-option Multiple Yes/No, which they call Multiple T/F.
- Matching. We regard that matching requires a change in the odds. as each match is done, and as such we do not analyse them.
- Context-dependent Item and Item set. We think this is an **EMSQ** i.e. an Extended Matching set Question.

There are also many other types other of quizzes, 'open', 'closed', those that change, 'fill in the blanks', where all stems are answered, where all stems are not answered, those that are scored by CBM (or its variation), those that are scored depending on 'how hard' the stems are, and a mixture of stem types.

It is our job to simplify these, and see what they have in common. We think that a Yes/No stem is a limiting case of a Multiple Yes/No stem that has 1 option. We think that the number of options that are ticked correct (**T**) distinguishes an MAQ stem from a MCQ stem. So, and MCQ stem is just an MAQ stem with **T** = 1. An MCQ, MAQ and an EMSQ each have many options (**P**). An EMSQ is either an MCQ or an MAQ with a high **P**. We do not consider matching stems, 'open' quizzes, quizzes that change, those that are scored by CBM (or its variation), or how hard their stems are. So our simplification is to only consider MAQ quizzes (where MCQ are a limiting case) and Multiple Yes/No quizzes (where Yes/No is a limiting case). EMSQs quizzes are MAQ or MCQ quizzes that have many options.

Another simplification that we make is that each stem is scored the same.

Whether tools are built or not we analyse quizzes where all stems are:

a) answered,
b) NOT answered.

2.14 'Open' v 'Closed'.

'Open' and 'closed' are *our* terms and 'open' quizzes are exemplified by 'pub' quizzes where the user can put anything. 'Closed' quizzes are exemplified by **M**ultiple **C**hoice **Q**uestion (MCQ) quizzes, where the correct answer is given. The correct answer is given in 'special' quizzes, it can be guessed, and is known as the guessing problem.

We regard 'fill in the blanks' quizzes as 'open' quizzes where the user can fill in the blanks with anything.

We will restrict our quizzes to 'closed' quizzes, and ones that do not change from e.g. from 'closed' to 'open'.

2.15 The Hardness Problem.

The hardness problem of quizzes has 2 parts:

a) the hardness of *stem*s problem,
b) the hardness of *options* problem.

We simply assume that all stems are scored the same, and so from a scoring point of view, are equally 'hard'. If we do not assume that stems are equally hard then we can say that our quiz contains stems that are not equally hard where their hardness is reflected in the scoring. This scoring is then very subjective. Who decides that this stem is twice as hard as that stem? Should it be 1.9 or 1.2? All options are also assumed to be equally hard. If they are not, then their analysis becomes difficult. We do not understand the argument that says "that because one of the options is correct it follows that it is easier or harder to answer".

The hardness problem applies to quizzes, in particular, but, maybe, applies to exam questions as well. If the rubric of exams says that you can get full marks by answering 4 out of 7, then all the questions must be equally hard or the hardness problem can be ignored (to say nothing of the "the guy next you not sitting the same exam as you").

BlackBoard (2012) and Authoring Manager (2009) allow stems of different 'hardness' to exist which this thesis does not.

2.16 How many options should there be in MCQs?

Vyas and Supe (2008) did a literature review of options of an MCQ stem and came to the view that 3-option MCQs are to be preferred to 4-option and 5-option MCQs, because they are more reliable. This is the same conclusion that Rodriguez (2005) came to.

We do not worry about what is the optimum number of options for MCQs; we simply ask the user how many options there are.

2.17 The effect of negative marking and the opinions held

Negative marking is one way of tackling guessing.

Negative marking means the taking away some marks for a wrong answer. In this thesis we give 1 for a correct answer to a stem and take away 1/X for an in-correct answer (where X=the number of in-correct answers to a stem) when negative marking is used. For example, for a Yes/No stem we will give 1 for a correct answer and -1 for an incorrect answer; for a 4-option MCQ we will give 1 for a correct answer and -1 for an incorrect answer (but there are 3 ways to get a wrong answer).

The opinions held about negative marking revolve around fairness. We will consider the simple case of a quiz made up of only Yes/No stems. A similar argument can be made for more complex stems. Let us consider a quiz made up of 100 Yes/No stems, and where we use negative marking of -1 for an incorrect answer to a stem. Is it fair to receive a mark of 38 (i.e. a fail) when 69 stems are answered correctly (31 are answered in-correctly)? Is it fair for one institution to use negative marking and another not. How do you now compare institutions? Naeem (2010) has put this unfairness issue eloquently.

We then ask "What is the pass mark?" For an essay it is 40% (at a 40% institution) and for a Yes/No-only quiz that uses negative marking it is 70%., say. Now, the pass mark depends on the type of assessment, and comparison of marks becomes difficult. An even more sensitive question is "How do you compare marks (and grades) between a 40% institution and a 50% institution?"

2.18 Raising the pass mark.

Another way of tackling the guessing problem is to raise the pass mark e.g. to 70% and similar to using negative marking we ask "What is the pass mark?" It depends on the type of assessment.

Another problem is "What grade should we give to 80% (say)?" Let us say it is a 2.2. Now the grade that somebody gets is dependent on the type of assessment. A 2.2 for an essay (at a 40% institution) might get a mark of 50% and for a 'special' quiz they have to have a mark of 80%.

Adobe, in their quizzes (for both Flash and Dreamweaver), raise the pass mark, but in this study we do not consider *why* they do it, because that makes the conclusions complicated and arguable.

This study tries 5 different pass marks to see what effect it has.

2.19 Using a more complex quiz.

Using a more complex quiz is our idea, and so there are no references in this section.

Yet another way of tackling the guessing problem is to use a more complex quiz. The idea is that when using a more complex quiz the likelihood of guessing the correct answer to a stem is reduced.

We will use the value of X to mean complexity where X is the number of incorrect answers to a stem. For example a) for a Yes/No stem X=1 b) for a 4-option MCQ X=3, but for a 2-option Multiple Yes/No X is also 3 i.e. the same as 4-option MCQ. So, a number of different types of stem might have the same X, and from that point of view are equally complex.

We say that a more complex quiz has a higher X than a less complex quiz.

2.20 The rationale for the philosophy and the Research Question (RQ).

It was felt that the emphasis of the previous research was to make the subject more complicated. This research tries to make the subject as simple as possible.

Because we had a philosophy of *simplicity* we did not want to do a sample, with all its attendant difficulties (like size of the sample, the estimated error of the sample, whether the sample represents the population). Thus, we wanted to do a census. This immediately causes us to ask "Can a census be done?" and later chapters show that a census can be done, and that causes the word census to be included in our **RQ** i.e. "Can special quizzes be analysed when a *census* is done?" The answer to this is "Only if large integers can be handled". Further chapters show that large integers can be handled.

Again, because we had a philosophy of simplicity we did not want to complicate the matter and wanted as few variables as possible. Thus we did not want questions of the kind "What happens if you have a *good* class?" or "What happens if you have a *bad* teacher?" or "Can you generalise from the sample you have?" The *good* class question leads to a longitudinal study like Russel (2010) did and the *bad* teacher question leads to other **RQs** like "How do you measure the effectiveness of a teacher?" The generalising question leads to "Does this sample represent the population?" If you consider the whole of the population this question does not need to be asked, and hence a sample of the population need not be taken and that makes the **RQ** simpler.

The *What* and *Why* questions are interesting but lead to disagreement and so another modification of the RQ is "What is the distribution of marks of special quizzes when a census is done?" If you can ignore good classes and bad teachers and do not have to use a sample then you are simplifying a problem and not complicating it by for example asking "Do the students suffer stress by being assessed?" as Miller and Parlett (1974) did.

Further simplification can occur when certain assumptions are made of special quizzes:

- the stems are all equally 'hard',
- the options of a stem are all equally 'hard'.

If the stems are all equally hard then it follows that stems are not 'mixed'. If stems are all equally hard then they can all be scored the same. If options of a stem are all equally hard it follows that stems can be all scored the same.

Other assumptions are made (in the interests of simplicity) like:

- the learners do not 'cheat', for example, letting others sit the test,
- CBM (or Davies's (2001a, 2001b) modification) is not used for scoring, where the learner has to input 2 things.

The rationale for the RQ is simplicity.

2.21 Conclusions.

From our reading we have shown what others think and how their research complicates things. We wanted to consider as **simple** a scenario as possible.

We also spotted a gap where nobody had asked a question about the distribution of marks of quizzes.

By restricting quizzes to 'specia'l quizzes and making assumptions we could simplify the situation.

2.22 What is coming next?

The Methods we have chosen and their limitations are shown next.

Chapter 3. Methodology, Methods, tools and measures.

Abstract.

We describe where Methodology fits in and then describe quantitative, qualitative and mixed methods. We used mixed methods but mainly quantitative. The categories of methods that we used were mathematics, statistics, programming and arguments. The method, MyMethod, we have chosen is quantitative and what it means and how to use the results is qualitative.

The mathematical tools' analysis (*phase 2*) is shown in Chapters 4, 5, 6 and the other quantitative tools that we used are shown here are in *phase 1*:

- Visual Studio,
- the LEDA library to calculate 'descriptive' statistics of the marks
- the programming language C to build *tool1* and *tool2* which allowed the calculation of the descriptive statistics.
- Excel™ was used to draw the distribution of marks,

The method, MyMethod, we used was to form a distribution by using *tools* 1 and 2 and we used Excel™ to draw the distribution, and all 3 tools used intermediate tab-delimited text files, which themselves are tools.

The measures we used in the tools were:

- the minimum mark,
- the maximum mark,
- the range of marks,
- the mean mark,
- the variance of the marks, (c.f. Straker (2011)
- the standard deviation of the marks,
- the skewness of the marks,
- the kurtosis of the marks,
- the proportion of passes,
- the number of marks (**M_g**) in each passing grade at a British University.

Descriptive statistics

We used statistics to aid the calculation of the descriptive statistics.

3.1 Introduction.

The structure of this chapter follows Grix (2010 p68) who says the order is ontology, epistemology, methodology, methods and sources. The tools that we used in our method, MyMethod, were a) *tool1* to calculate how many got a mark in a particular way b) Excel™ to do a Pivot and draw the resulting distribution c) *tool2* to calculate descriptive statistics (in the main) d) intermediate tab-delimited text files that allowed *tool1*, Excel™ and *tool2* to communicate.

Others think that phenomenology also comes in somewhere (Smith 2008) and is separate from but related to ontology, epistemology, logic and ethics. We want to simplify as much as possible and so leave out phenomenology, ethics and logic.

We also, will leave out *sources* because we *generate* the data, and add tools and measures, to get ontology, epistemology, methodology, methods, tools and measures, as the structure of this chapter. Methodology consists of a number of methods, which form a spectrum, with Quantitative methods at one end and Qualitative methods at the other. We, briefly, mention the spectrum here and will expand on it later, in this chapter. We will discuss quantitative methods, qualitative methods and mixed methods (which are a combination of quantitative and qualitative methods). The type of quantitative methods, that we used were, mathematics, statistics, and programming. The qualitative methods that we used were arguments about the meaning of the results and how they were to be used. So, we used mixed methods, but mainly quantitative methods.

By leaving out sources, we do not have data collection and so do not need to discuss a) how the data was gathered b) whether our sources are primary or secondary. This leads to the concept of simplicity. From simplicity we see that we use quantitative methods and not qualitative ones, otherwise the conclusions are arguable. Further, we used a reduced subset of quantitative methods. We used a census, and hence do not need to use sampling with all its inherent problems (e.g. does the sample represent the population?). We do not have a hypothesis, and so do not have to show whether it is confirmed or not. We do not have to bother with 'good' classes or 'bad' teachers. We find that we cannot use just quantitative methods; we still have to use qualitative methods to explain what our results mean and how to use them; this is done in the Conclusions chapter (Chapter 8).

We will also leave out ethics, because we are not going to hurt anybody (either mentally or physically) or going to use animals (who cannot give their consent). So, we do not need to consult the ethics committee. This is another reason that this thesis follows the path of simplicity.

We will quickly deal with ontology, epistemology (which both can have a positivist or interpretivist stance) and methodology; to leave the main thrust of this chapter viz. our method, MyMethod, tools and measures.

Before programming is done we perform an analysis (in the A of SSADM sense) and this is described in Chapters 4, 5 and 6 which constitute *phase 2*. We discuss *phase 1* (the creation of the tools) in this chapter.

The programs (produced in *phase 1*) were validated on small sets which could be handled 'by hand'. The 'large' sets relied on our proofs for validation (see Chapters 4, 5, 6 that constitute *phase 2*). The validation was done using mathematic and statistics and 'hand' calculations. Once the programs were validated, it was assumed that the programs would work on *all* special quizzes.

We had to write our own programs because 'normal' languages could not handle very large integers and so we used a programming language and a library of routines that could handle arbitrarily large integers. We used the programming language C that used the LEDA library (that could handle arbitrarily large integers). We wrote two programs (*tool1* and *tool2*) in the programming language C that ran in the Visual Studio (VS) environment:

a) the calculation of marks and how many get a mark in a particular way was done by *tool1*,
b) the calculation of descriptive statistics and the proportion of passes was done by *tool2*.

The Pivoting and the drawing of the distribution of marks were done using the Excel™ tool.

The results of these 3 tools and their intermediate files (which allowed them to communicate) are shown in Chapter 7.

This chapter is accompanied by Appendices 9, 10, 11 that describe:

a) how Excel™ *imports* a tab-delimited text file
b) how Excel™ does a *Pivot* of the data in the file
c) how Excel™ *exports* its results to a tab-delimited text file.

This chapter is also accompanied by Appendices 12, 13, 14 that describe:

a) the tool, *tool1*,
b) the validation of *tool1* where ALL stems are answered,
c) the validation of *tool1* where stems can be missed.

This chapter is further accompanied by Appendices 15, 16 that describe:

a) the tool, *tool2*,
b) the validation of *tool2*.

We do not use an Appendix to show that *tool1*, Excel™ and *tool2* all work together; we do it here.

This chapter describes the programming that was done and where the tools (*tool1* and *tool2*) were built and that constitutes *phase 1*

The measures that we used were mainly based on descriptive statistics including mean mark, variance of marks, standard deviation of marks.

3.2 What is ontology?

Grix (2010, p68) says that **ontology** can be summarised by

> *"What's out there to know?"*

Gruber (2013) on the other hand says that ontology is defined by

> *"An* ***ontology*** *is an explicit specification of a conceptualization. The term is borrowed from philosophy, where an ontology is a systematic account of Existence…."*

We say that entities are out there, and ontology is a listing of these entities, one of the entities is quizzes. So, we are going to use Grix's definition.

Johnson and Duberley (2000) hardly mention ontology at all, and Cohen, Manion and Morrison (2007) have nearly the same structure as Grix but introduce 'human nature' between epistemology and methodology.

3.3 What is epistemology?

Epistemology is the study of knowledge and how it is acquired.

> *"Epistemological considerations depend on beliefs about the nature of knowledge…ways of acquiring and gathering knowledge.."*
>
> Grix (2010, p166)

The epistemology of quizzes has its limits and what we know about quizzes is small. This thesis is about how we can increase this knowledge by limiting the quizzes to 'special' quizzes and limit their analysis and limit their results to mainly descriptive statistics.

Grix (2010, p 68) says that **epistemology** can be summarised by

> *"What and how do we know about it?"*

Gruber (1993) does not mention epistemology at all, and it is my opinion that Gruber conflates ontology and epistemology in that what he recommends is rather like a dictionary which gives many definitions of a concept from which we can arrive at the meaning of the concept. Epistemology is about the meaning of a concept; ontology is about what concepts there are.

Ontology and Epistemology can have at least two stances:

1. positivism
2. interpretivism

3.3.1 What is positivism?

Cohen, Manion and Morrison (2007, p9) say that

> *"…**positivism** which held that all genuine knowledge is based on sense experience and can be advanced only by observation and experiment"*

Creswell (2013) agrees but calls positivism post-positivism.

We take a positivistic viewpoint (or stance) by saying that special quizzes *can* have their descriptive statistics and other statistics calculated.

The ontological/epistemological stance and the structure of this thesis can all be said to be influenced by our experience. My experience was a lecturer in Computer Science and so from that point of view I 'knew' what an entity was, if entities are what are 'out there'. Thus, my stance, or point of view, was mainly quantitative.

Johnson & Duberley (2000) agree that our stance is influenced by what precedes it, but think that epistemology is the driving force, although they do mention ontology on at least 3 pages (2, 10, 78).

Cohen, Mannion and Morrison (2007) have many chapters and they, in Chapter 1, deal with ontology and epistemology (p9), and come to more or less the same conclusions as Grix (2010).

3.3.2 What is interpretivism?

Interpretivism can be called anti-positivism (Abbot 2010) who maintains that human beings are not like plants and so positivism is not appropriate.

> "..*we have to delve into the reasons and meanings which that action has for people.*"

Thus people are constructing meaning from the world around them. This leads to Creswell (2016, p8) calling interpretivism constructivism.

We did *not* take an interpretivist stance to *produce* the results, but took an interprevist view to *explain* the results.

3.4 Qualitative and Quantitative data.

If the data that is being used is numeric then we use quantitative methods, but if the data is non-numeric then we are using qualitative methods (c.f. McNeil

1990). Bryman (2012) and Creswell (2013) maintain that the two types of method require different *strategies*, and that one can combine them to obtain the advantages of both types. To his credit, Bryman (2008) shows an argument that says the two strategies are incompatible and cannot be combined. Nevertheless, Bryman (2012/) and Creswell (2013) maintain that the *combination* of types of method *is possible*, which results in mixed methods.

3.5 Methodology.

Methodology, according to Grix, comes after ontology and epistemology and before Methods.

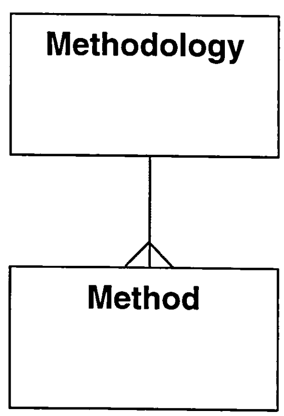
The word Methodology can either mean the study of methods or a collection of methods. By Methodology we are going to mean "the study of methods" and it is not to be confused with a collection of methods.

> Methodology is *"...the study of methods..."*.
>
> Grix (2010, p169).

But, how often do you hear "the methodology is flawed" (Swaminathan 2010) (ERM Strategies 2013) (King 2011)? We do not use methodology in that way here. If you want to mean Methodology is a collection of methods then do not call that collection Methodology.

Methodology has the following Entity Diagram of

Attributes of a record of the entity Methodology.
<u>Key.</u> Name. MyMethod ....



Thus, Methodology consists of a number of Methods.

Figure 3-1. The Entity Relationship diagram for Methodology.

Each record of Methodology consists of a number of Methods and if you give a record a name it must not be Methodology. Calling a "collection of methods"

Methodology leads to an inconsistency (you will have an entity and a record with the same name or you will have two records with same name).

A thing to do in a science of methods is to say what methods, in general, consist of, and what influences them. Ontology, Epistemology, Methodology and Methods are all influenced by our experience which gives us a world view.

3.6 Interviews and questionnaires.

Interviewing and questionnaires can be a qualitative or a quantitative method. If the data of our interviews or questionnaires is numeric then we can be said to be using a quantitative method.

The data from un-structured interviews is words and hence is qualitative. The words might be translated to categories and left as words e.g. sensitive, uncaring. The data is still qualitative and can have Factor Analysis applied to it. If the categories are turned into numbers then the data is quantitative and we can use quantitative methods to analyse it. For example, the quantitative data can be counted and have regression analysis applied to it. In this scenario, we first of all, gather our qualitative data and gradually turn it into quantitative data and analyse it using statistics and then we are said to be using a mixed method. But, in this thesis, we do not call the mixture of methods a Methodology.
Early in the study it was thought that gathering the data might involve interviewing people or getting them to fill in questionnaires. To that end we read a book by Oppenheim (1992) and chapters in Cohen, Manion and Morrison (2007) and McNeil (1990:p34) and found that doing a *pilot study* was the most important process. There are other processes that depend on the type of survey that is done, partial or complete. The complete survey is called a *census*, and these are where we got the idea of a census from. We also got the same idea of a census from Ruane (2005), where a study of part of the population is called a sample study, and a study that is of the whole population is called a *census* study.

We dropped the idea of gathering data when it was realised that we could do a census study and generate the data. But, pilot studies of quizzes, still ought to be done. For example, if everybody answers stem 15 correctly, then it is perhaps too easy; and if nobody answered stem 17 correctly, then it is, maybe, badly worded or too hard. In both cases the stems might be dropped after a pilot study is done.

3.7 Population.

The *population* is not everybody in the world or a country. In our case the population is ∑(a mark) x (how many obtained that mark) of a special quiz. For example, if the quiz is made up of only 50 Yes/No stems and negative marking is not used then the marks that can be obtained are 0, 1, 2…50. Thus, the population is the ∑(marks 0…50 in steps of 1) x (how many got a particular mark).

3.8 Quantitative methods.

Quantitative methods are said by Bryman (2008) and Creswell (2013) to use a research strategy and qualitative methods use a different strategy. It can be said that the two strategies are incompatible but both Bryman and Creswell say that if both quantitative and qualitative methods are used you can obtain the advantages of both.

The quantitative view can be said to be scientific. That is we can sense things and experiment on them (the positivist approach), and that the results of these experiments are repeatable (reliable).

Quantitative methods are said by Bryman (2008) to have a typical sequence. We did not follow the typical sequence. For example, a) we did not have an hypothesis b) we did not gather data c) we did not use sampling d) we do not have to select research sites e) we did not have to select research subjects.

3.9 Qualitative methods.

Qualitative methods can be said to be *non-scientific*, and include, grounded research and action research as well as many others (e.g. gender studies, feminist studies), generally called Social Science. They are non-scientific in that the results are not *repeatable* (c.f. ethnography in Bryman (2008) and O'Reilly (2005)). We will get different results with different groups of people. Bryman (2008) and McNeil (1990) also say that the *sample* that is used might not represent the population. The sample might be 'too small' and the characteristics of the sample might not reflect those of the population. McNeil (1990), Bryman (2008), Creswell (2013), Cohen, Manion and Morrison (2007) all say that a sample must *represent* the population for any generalisation from the sample to carry any weight.

We only used the qualitative method of argument of interpreting the results of our quantitative methods.

The non-repeatability, smallness of sample (maybe), sampling (although quantitative methods also use sampling) and possibility of non-generalisation, drove us to use quantitative methods to obtain our results, and our philosophy of simplicity meant that we also used a restricted form of quantitative methods, that did not use sampling.

The qualitative point of view has many adherents, and asks interesting, but arguable, questions, like "Can special quizzes assess anybody's 'deep' learning?" This question is asked in the Conclusions section and basically is asking "Are we building the right product?" i.e. verification. In Social Science one is often driven to a qualitative point of view when asking questions like "Why do people say that capital punishment is good?" In the Conclusions chapter we argue that in constructing special quizzes we might not be building the correct product, because the learners are not allowed to explain how they came to an answer.

A book by Ruane (2005) shows which methods have the qualitative view, but also, says on p34

> *"When we claim **measurement validity** we claim that we have been successful at measuring what we say we have measured"*

This is discussed in Chapter 2.

On p105 she also says

> *"Samples that do a good job at conveying accurate information about the whole are referred to as **representative samples**"*

and that is 1 reason why CBM was dropped because it might involve using a sample that did not represent the population.

3.9.1 What is Grounded research?

Grounded research is a qualitative method, where a theory to explain the data comes from the data. This is the reverse of the traditional research method where we start with a theory and see whether the data backs it up. Papers by Bringer, Johnston and Brackenridge (2004, 2006) were basically about using nVivoth which is based on grounded research. A book by Bazeley (2007) was about how to use nVivo, and a course on nVivo proved useful in organising our note taking.

3.9.2 What is Action research?

Action research is where the user is actively engaged in the study and changes an organisation (c.f. Anon (212)).

As we used quantitative methods, in the main, qualitative methods will not be discussed further in this thesis. We did not use Grounded research or Action research.

3.10 Mixed methods.

One of the first things to do in a science is to form a taxonomy. Creswell (2013) and Bryman (2008) say that methods form a spectrum with Qualitative methods at one end and Quantitative methods at the other. According to both Bryman (2008) and Creswell (2013), mixed methods use both Quantitative and Qualitative methods, one of which (grounded research) is described by Richards (2005) and Richards and Morse (2007). Both Bryman (2008) and

Creswell (2013) imply that using 2 or more quantitative methods is *not* using mixed methods. In order to be said to be using mixed methods you must be using some qualitative and some quantitative methods.

> *"The term mixed method research is used as a simple shorthand to stand for research that integrates quantitative and qualitative research within a single project"*
>
> Bryman (2008, p603)

We used mixed methods, but mainly quantitative ones. The type of quantitative methods that we used are mathematics, statistics and computer programming. The qualitative methods are how to interpret the results and use them. It might even be argued that we used quantitative methods to obtain our results and then had used qualitative methods to explain them and how to use them. In creating the results we did not use mixed methods, but used many quantitative ones.

With mixed methods you try to use the advantages of both types of method, for example you try to triangulate the data. You try to see whether other data supports you.

3.11 Contrasting quantitative, qualitative and mixed methods.

Quantitative methods are said to be *scientific* in that they can have experiments done using them whose results are repeatable. The viewpoint (or world-view) of quantitative methods is positivist c.f. Cohen Manion & Morrison 2007 (sometimes called post-positivist). This viewpoint is coloured by our experience and mine was as a lecturer in Computer Science which had its basis in Mathematics and physics where the world could be appreciated by our use of our senses and so experiments could be done (the positivist approach). Thus the strengths of quantitative methods lie in their use of the scientific method. But they also have their weaknesses c.f. Gödel's theorem (Denton 2014, Bragg 2008 below) which shows that there are some things that cannot be proven by using quantitative methods.

A weakness of quantitative methods is that they often use a sample, because the whole population cannot be measured, and so we have to worry about a) the size of the sample b) whether it represents the population.

Another weakness of quantitative methods can be summed up by saying that *causality* cannot be *proven*. There is a saying in statistics that a relationship between A and B does not mean that A causes B even if A happened before B For example, does smoking cause lung cancer (Cancer Research UK 2012) and does HIV cause Aids (NIH 2012)? Science is trying to show causality much of the time, but Science has its limits and the best that we can say "It is probable that A causes B, but there might be exceptions" If A is related to B it does not *prove* that A *caused* B. B might have caused A. C might have caused A and then A causes B. C might have caused both A and B, and so C is the primary cause. An argument is that if A came *before* B in time then A caused B. This is known as the Past Hoc fallacy (Nizcor 2012) and can lead to difficulties. Let us try an example. The front of a house heats up and then the back of the house heats up. We could conclude that the front of the house heating up caused the back of the house to heat up. But, the sun caused them both due to the sun moving relative to the earth. This is simply C causing both A and B even though A occurred before B.

Yet another weakness of quantitative methods is illustrated by the following argument. If the world is made up of only physical entities then the scientific method rules, but if the world contains non-physical elements as well then there are things that quantitative methods cannot address because maybe, they cannot be sensed. Even if the world is composed of only physical entities Gödel's incompleteness theorems show that there are some questions that quantitative methods cannot answer:

> "*Gödel proved that there were some problems in maths that were impossible to solve, that the bright clear plain of mathematics was in fact a labyrinth filled with potential paradox.*"
>
> Bragg (2008).

Thus, mathematics is flawed and any method based on mathematics (e.g. statistics) is also flawed. Thus, quantitative methods are flawed.

Another weakness of quantitative methods is that the world also contains human beings that can say "No" and behave irrationally and so Social Science and qualitative methods were born to answer questions that quantitative methods could not.

Qualitative methods are said to be *non-scientific* in that the experimental results, most probably, are not repeatable, as with ethnographic research and that their sample might not represent the population. They have a number of world-views (e.g. constructivist, transformative, pragmatic c.f. Creswell 2013) all of which are non-positivist. With qualitative methods you most probably cannot perform experiments and sometimes (e.g. in Grounded research c.f. Bazeley 2007) do not start with a hypothesis although even a theoretical quantitative study (like ours) did not have an hypothesis. Qualitative methods come into their own when quantitative methods fail or are in-appropriate e.g. when asking "Why do people support capital punishment?" Quite often a questionnaire is used for data collection and Oppenheim (1992) stresses that in that case a pilot study should be done. Thus, one is sometimes driven to use qualitative methods but they, also, have their weaknesses e.g. sometimes not being able to repeat results and their sample might not represent the population.
We could have experimented on 'real' people, by using qualitative methods and done a longitudinal study (Russell 2010), to find out their marks, but that wold leave us open to the arguments of 'good/bad' teachers and 'god/bad/ classes. We avoided these arguments by using a census.

Mixed methods use some quantitative and some qualitative methods and are trying to get the best 'of both worlds' even though their world-views might be incompatible c.f. Brymam 2012, Cresswell 2013.

Having set out the pros and cons of various types of research we need to justify our choice of type. We used quantitative techniques for our method and qualitative techniques to explain our results and how to use them. Thus, we used mixed methods but quantitative in the main. We chose quantitative methods, mainly, in our study so that our results could be repeated by somebody else which is a weakness of qualitative methods. We did not need a sample (as a census was performed) and so we did not need to consider the size of the sample or whether it represented the population. Thus, the pitfalls of qualitative methods were avoided (as far as possible) and the pitfalls of sampling (see next section) were avoided (which apply to both quantitative methods and qualitative methods) by the use of a census.

3.12 Sampling.

In many cases (of both Quantitative and Qualitative methods) a sample of the population is looked at. As long as the sample a) is not 'too small' b) represents the population (Abbott 2010) then we can generalise from the sample to the population.

There are many types of sampling a) simple random b) systematic c) stratified d) cluster e) stage f) multi-phase which are all probability sampling, and non-probability sampling includes a) convenience b) quota c) purposive d) dimensional e) snowball f) volunteer g) theoretical according to Cohen, Manion and Morrison (2007).

There are no answer to the question "How big should the sample be?" but the general advice is "The bigger the sample is the better".

We avoid the problems associated with sampling a) which type of sampling did you use b) how big should the sample be c) does the sample represent that data, by using a *census* and not a sample. Thus we avoid sampling altogether and its attendant problems.

3.13 Method.

A method uses a number of tools.

In the method, MyMethod, we used 5 tools a) *tool1* b) a tab delimited text file c) Excel™ d) a tab delimited text file e) *tool2.*

We wrote one program (*tool1*) in the programming language "C", that asked the user a number of questions, and then:

a) calculated a mark and the number that got that mark in that manner,
b) outputted these to a tab-delimited text file (this is a tool),
c) calculated some descriptive statistics and outputted these to the debug window.

The tool Excel™ then:

a) read the tab-delimited text file,
b) did a Pivot to sort the marks and how many ways there are of getting that mark, into ascending order of marks,
c) drew the resulting distribution of the marks (which was copied to Word™),
d) outputted to a tab-delimited text file (this is a tool).

The 2nd "C" program (*tool2*) calculated the descriptive statistics (including mode and median, which required that a Pivot is done) and other statistics (including the proportion that were at or above various pass marks) of the distribution of the marks e.g. the range of the marks. The distribution drawn by Excel™ was transferred to Word™ and the descriptive statistics were super-imposed on a Word™ document.

The whole method, MyMethod, is shown in the following diagram.

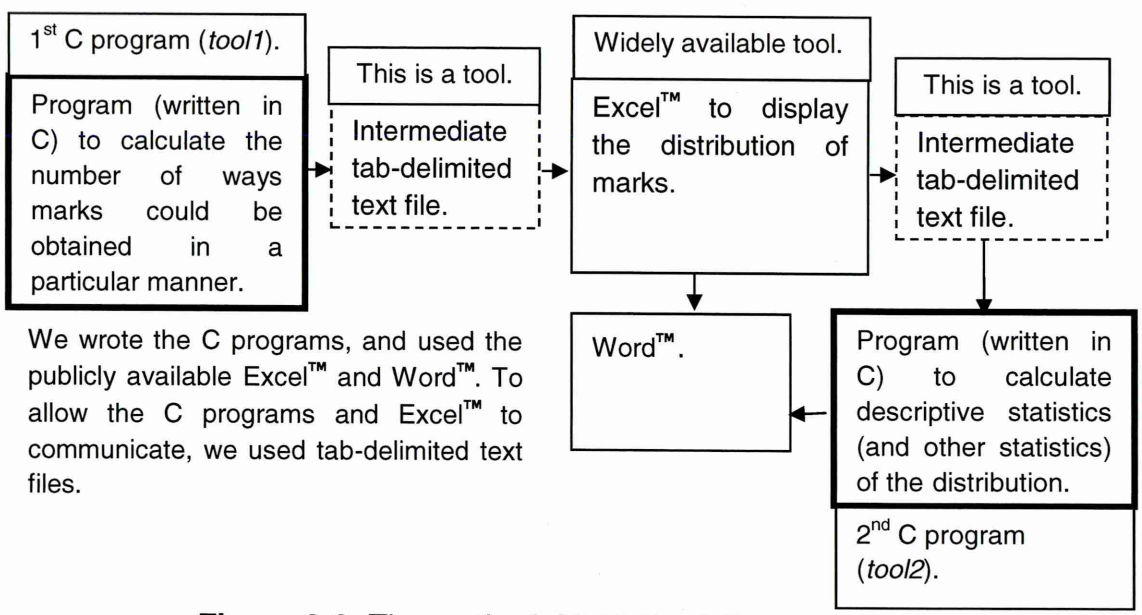


Figure 3-2. The method, MyMethod, that we used.

The marks might not be ordered or added and so a Pivot is needed to sort the marks and how many got a mark into ascending order of marks and there is a possibility of impossible marks. The user is asked for the number of stems **n**, e.g. 100, the values of **a**, **b** and **c** (e.g. 0, 0, 1), and what type of quiz is to be used e.g. 4-option MAQ-only (so the number of options, **P**, and the number of ticks, **T**, have to be obtained from the user). A mark = **w*c** + **x*a** + **y*b** and there is 1 less degree of freedom because **n** = **w** + **x** + **y**. **a** is the value of the negative score when a stem is answered in-correctly, **b** is the score when a stem is missed, **c** is the score given to a stem that is answered correctly. **w** is the number of stems that are answered correctly. **y** is the number of stems that are missed. **x** is the number of stems answered incorrectly. **y** is varied from 0 to **n** and inside that **w** is varied from 0 to **n**. An impossible mark is exemplified by answering 99 stems correctly and missing 2 when **n**=100. An impossible mark occurs when **x** is negative. The total number of impossible marks is 0 + 1 +2 + ...**n** which is an Arithmetic Progression of **w**. If the mark is not impossible, then the mark and the number of ways that this mark could be obtained (with this combination of **w** and **y**) was output to a file. Excel™ read this file, did a Pivot to sort a mark, **M,** and the number of ways that **M** could be obtained, i.e. $\mathbf{Y_M}$. Excel™ drew the distribution of $\mathbf{Y_M}$ v **M** and both $\mathbf{Y_M}$ and **M** were output to tab-delimited text file for its descriptive statistics and other statistics to be calculated.

The method is now shown, in more detail, in a second diagram

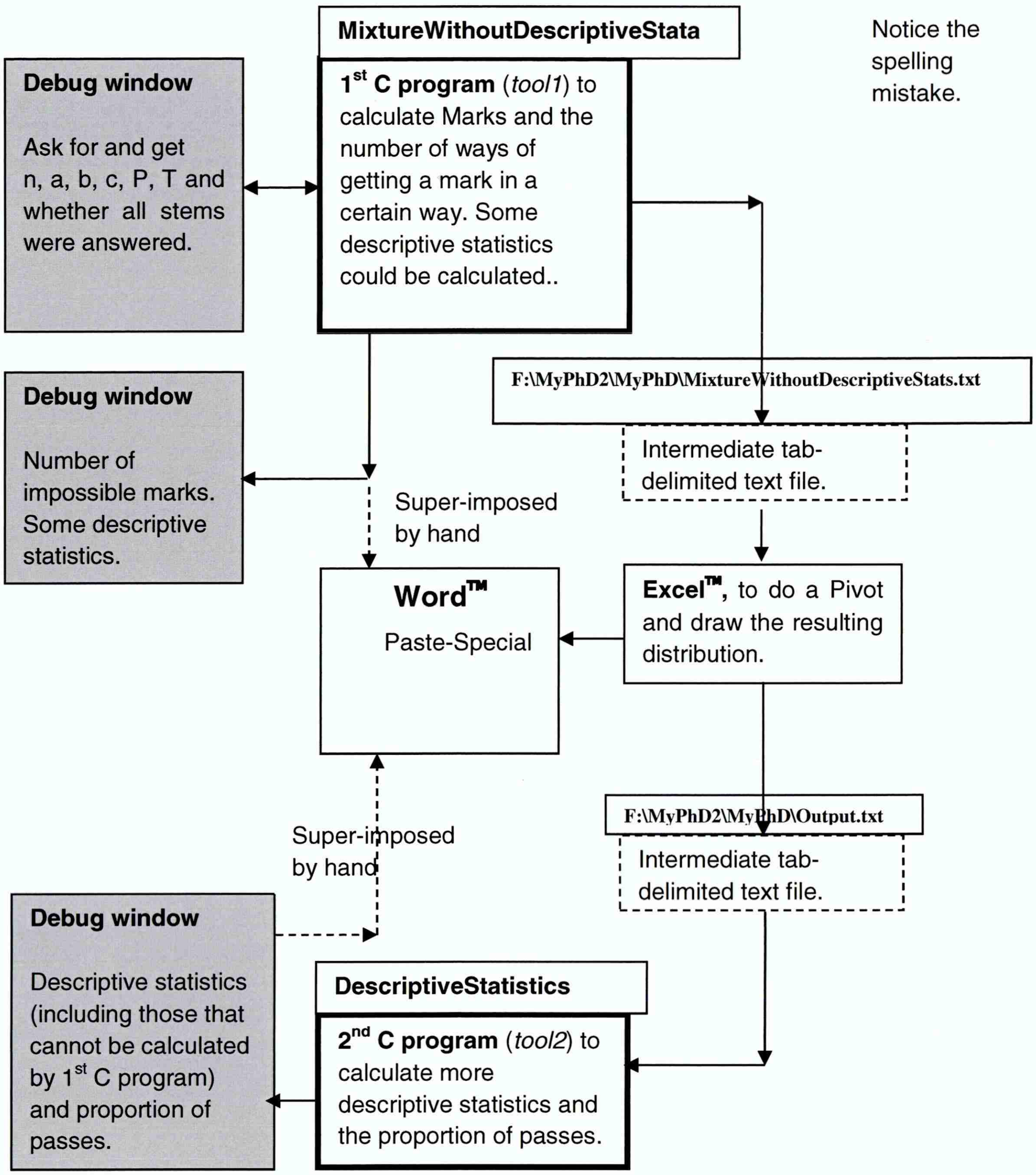


Figure 3-3. More details of the method, MyMethod, that we used.

The method, MyMethod, introduced an accuracy error, because Excel™, even when it could handle the numbers we used, it could not do it accurately.

3.13.1 How did we analyse our programs?

Before the writing of programs comes the analysis but we show. The use of the word "analysis" does not mean the analysis of the data and its interpretation as Judd, McLelland and Ryan (2009) did, but is meant in the A of SSADM sense. The two uses of the word "analysis" is more confusing in that the *results* of our method could be said to be analysed in the Judd, McLelland and Ryan (2009) sense. To avoid confusion we will always use the word "analysis" in the SSADM sense in this thesis.

We asked "What can be done without any tools being built?" and in that sense we analysed our programs. The analysis consisted of a) some mathematics b) some proofs c) showing some calculations done 'by hand' on small sets.

After the analysis comes the writing of programs, which have to be shown to be correct

The method, thus, is divided into two:

- phase 1, where the analysis is done (shown in Chapters, 4, 5 and 6),
- phase 2, where the programs were written (shown in this Chapter).

3.13.2 How did we validate our programs?

We validated our programs by:

- hand calculations on small quizzes to validate *tool1* (see Appendices 13 and 14),
- using our proofs for large quizzes,
- showing that *tool2* gave the same results as a well-known tool, SPSS, until SPSS reaches its limits(see Appendix 16),
- showing that *tool1*, *tool2* and Excel™ worked together.

3.14 Measures of the method, MyMethod.

The **measures** that we used in the method, MyMethod, were:

a) descriptive statistics of the minimum mark, the maximum mark, the range of marks, the variance of the distribution marks, the standard deviation of the distribution of marks, the skewness of the distribution of marks, the kurtosis of the distribution of marks,

b) the proportion of passes
c) how many marks there are in each grade, for which we used Excel™ to help us.

We used statistics to form these measures in *tool1* and *tool2*.

3.14.1 Statistics.

We kept in mind that the use of statistics is the last resort.

> *"Statistical tests are too often treated as if science were a religion and a statistical test a required ritual in its practice. That's nonsense."*
> Rothchild (2010).

It took us a long time to find out what formulae SPSS (a well-known statistics package) used for calculating the estimates of skewness and kurtosis. When these formulae were found out, we could do them ourselves, and check (validate) that SPSS used the same formulae. But, SPSS uses estimates of statistics, and we wanted to do a census study that used statistics and not their estimates, and so we had to find out the statistics and not their estimates. When we then moved on to doing a census using LEDA and C (a programming language that we had to learn), SPSS became redundant.

Cohen, Manion and Morrisonl (2007, pp501-592) have many chapters on statistics taking qualitative and quantitative views, and Field (2005) takes a mainly quantitative view. Kurtosis is a measure of how 'peaky' a distribution is.

In order to learn how kurtosis is defined we had to learn about:

- continuous,
- discrete,
- probability density functions,
- distributions,
    - the normal distribution,
- expectations, E(x),
- moments,
- census, the whole population,
- samples, part of the population,
- estimates of parts of the population,
- outliers,
- descriptive statistics,
    - measures of 'the middle' of a distribution

        - arithmetic mean of the population is μ, arithmetic mean of the sample is $\overline{x}$. μ = (∑x)/n, $\overline{x}$ = (∑x)/m where n = size of the population and m = the size of the sample,
        - geometric mean i.e.$\sqrt[p]{x_1 + x_2 + .. x_p}$,
        - harmonic mean, the reciprocal of the arithmetic mean of the reciprocals,
            - there is even a subject called 'central tendency',
        - mode, the most frequently occurring value,
        - median, the 'middle' value,
        - mean, ignoring outliers. SPSS has an estimate of the mean that ignores outliers c.f. trimmed mean,
    - variance, which is the 2nd moment about the mean,

"The population variance is calculated using the population mean

$$\sigma^2 = \frac{1}{n}\sum_i (x_i - \mu)^2$$

The sample variance is calculated using the sample mean

$$s^2 = \frac{1}{n-1}\sum_i (x_i - \overline{x})^2$$
"

Stone (2006)

        - standard deviation, which is the positive square root of variance,
    - skewness, which is the 3rd moment about the mean. See Weisstein (2014) which says

"Skewness is a measure of the degree of asymmetry of a distribution."

    - kurtosis, which is the 4th moment about the mean.

The variance and the standard deviation are measures of the spread of a distribution is

"The variance and standard deviation are related indicators of the spread of data within a population or sample"

Stone (2006)

The skewness is a measure of whether the distribution has long 'tails' to the left or right. A distribution that has long tails on the right is positively skewed and one that has long tails on the left is negatively skewed.

3.15 An outline description of tool1.

The tool, *tool1*, calculates a mark and how many ways there are of getting that mark in a particular way.

The tool has a number of parts:

- Definitions, declarations and initialisation.
- Getting information from the user.
- Calculation of a mark m : and how many get that mark in a particular way y_m.
- Output m and y_m to a tab-delimited text file.
- Calculation of some descriptive statistics that do not depend on the marks being sorted, or how they could be obtained being added. This calculation uses a method validated in *tool2*.
- Output the descriptive statistics to the debug window.

3.15.1 A more detailed description of tool1

The **definitions** and any functions that we are going to use come *before* the `main()` function. These definitions include

- `# includes` which say (amongst other things where the LEDA library is),
- `using namespaces` which allow us to refer to things without repetition (unless that results in ambiguity).

The **declarations** (which are *inside* the `main()` function) include the LEDA `integer` and `bigfloat` as well as the normal `int` and `float`.

The **initialisation** is mostly setting variables to 0, but some are set to something very 'big'.

The **information we get from the user** is from the debug window and is **a**, **b**, **c**, **n**, **P**,**T** and whether stems can be missed.

We can do limited checking of these:

- **n** is an integer between 1 and 100,
- **T** must not equal **P,**
- For an MAQ quiz the minimum **P** is 2.

The **calculation of a mark and how many ways there are of getting that mark in a particular manner** is done inside two loops:

- one varying **y**,
- and inside that, one varying **w**.

Outputting to a tab-delimited text file is done in one statement viz.

```
cout   << "Mark=" << mark[way] << "\t" << " number getting that mark that way="
<< numberOfWaysOfGettingThatMarkThatWay[way]  << " n=" << n << " w=" << w << "
x=" << x << " y=" << y << endl;
```

(See Appendix 12 for full code)

Calclation of some descriptive statistics is done by calculating:

- the mean, **μ**,
- then in a loop calculating the
  - variance,
  - standard deviation,
  - skewness,
  - kurtosis.

all of whom do not require that the marks are sorted and how they are obtained are added.

The **output of these descriptive statistics** is to the debug window via `cout`.

3.15.2 The validation of tool1.

Validation of *tool1* involves showing it works i.e. gives the same results on as many examples as we can. Appendices 13 and 14 contain the evidence that *tool1* works, as far as we can tell.

3.16 An outline description of tool2.

The tool, *tool2:*

- inputs data from a tab-delimited text file,
- calculates descriptive statistics and other statistics of a distribution.

Before *tool2* is Excel™ which:

- inputs data from a tab-delimited text file,
- does a Pivot,
- outputs results of the Pivot to a tab-delimited text file.

The tool, *tool2* makes 3 assumptions:

- the data concerns a *census* and not a sample. SPSS assumes that the data is a *sample*, and thus, the descriptive statistics of SPSS use estimates,
- what is measured on the x-axis is sorted from lowest to highest. In our case a **mark** is measured on the x-axis, and if **a** = 0 the lowest mark is 0,
- what is measured on the y-axis is added. In our case if the number getting a mark is measured on the y-axis. What is presented to Excel™ is how many get a mark in a **particular way**. So, for example, if the mark 0 can be got in 3 ways when **y** = 1 and **w** = 2 and the mark 0 can be got in 6 ways when **y** = 3 and **w** = 5, then the mark 0 can be got in 9 ways i.e. an addition is necessary, and thus a Pivot is necessary.

i.e. *tool2* assumes that the data concerns a census and that a Pivot is done. The tool (*tool2*) has a number of parts:

- Definitions, declarations and initialisation,
- inputs data from a tab-delimited text file:
  - if we do an experiment to see if we get the same results as SPSS we get the input from `F:\MyPhD2\MyPhD\Output.txt` which is in two columns but the data that corresponds to the mark is ignored,
  - if we do NOT do an experiment to see if we get the same results as SPSS we get the input from `F:\MyPhD2\MyPhD\Output.txt` which is in 2 columns

3.16.1 A more detailed description of tool2.

The **definitions** and any functions that we are going to use come *before* the main() function. These definitions include

- # includes which say (amongst other things where the LEDA library is),
- using namespaces which allow us to refer to things without repetition (unless that results in ambiguity).

The **declarations** (which are *inside* the main() function) include the LEDA integer and bigfloat as well as the normal int and float.

The **initialisation** is mostly setting variables to 0, but some are set to something very 'big'. There is a limitation where a LEDA variable cannot be initialised to ten nines.

Then comes a section where the descriptive statistics (and other statistics) are calculated. The descriptive statistics are range, mean, mode, median, variance, standard deviation, skewness, kurtosis. The other statistics include the proportion that pass at various pass marks.

The descriptive statistics (and other statistics) are then output to the debug window via "cout".

The section on descriptive statistics (and other statistics) is divided into two:

a) one where we try and show that *tool2* gives the same results as a well-known package e.g. SPSS,
b) one that calculates and outputs the descriptive statistics (and other statistics).

The trouble with SPSS is that the descriptive statistics are calculated for a *single* column and we want the descriptive statistics of 2 columns that form a distribution. The data for validating *tool2* and using *tool2* are both in the same file called **Output.txt**, but when we are validating we ignore the column containing the mark, because we do not know how to make Excel™ calculate the descriptive statistics of 2 columns.

3.16.2 The validation of tool2.

We need to show that *tool2* gives the same results as a well-known tool like SPSS, and the evidence of the validation is contained in Appendix 16.

The difficulty of using SPSS is that SPSS treats everything as a *sample*, and we want to do the analysis of a *census*. SPSS uses *estimates* of its statistics and these estimates are in-accurate for small populations, but quickly become accurate as larger and larger samples of the population are used. We had to find out what formulae SPSS uses for its descriptive statistics and then see what the formulae should be used for a census. The following table shows the estimate that SPSS uses and the formulae that should be used for a census.

Table 3-1. Formulae used for estimates and censi.

	Estimates. **Formulae that SPSS uses.** **n=size of sample**	**Census** **Formulae** **N=size of population**
Mean	$\bar{x}=\frac{\sum_{i=1}^{n} x_i}{n}$	$\mu=\frac{\sum_{i=1}^{N} x_i}{N}=\sum x_i P(x_i)$
Variance	VarSam= $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$	VarPop= $\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$
Standard Deviation	$s=\sqrt{VarSam}$	$\sigma=\sqrt{VarPop}$
Skewness	$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^3}{N\sigma^3}$
Kurtosis	$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^4} - \frac{3(n-1)^2}{(n-1)(n-2)}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^4}{N\sigma^4}$ -3

We had n=N and used the same x_i, thus $\overline{x}= \mu$ because

$$\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}=\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$$

n is the number of cases i.e. the number of different x_i there are (it is not the number of stems) N = the size of the population.

The Variance of the *Sample* (VarSam) is divided by (n-1) and the Variance of the (VarPop) is divided by N and so s is not the same as σ and the skewness of the sample is not the same as skewness of the population nor is the kurtosis the same in both cases.

We used Stone (2006) for the variance equations and Weisstein (2014) for the skewness equations. For the kurtosis equation we used

$$\text{Kurtosis}=\sum(x_i-\mu)^4/(n\sigma^4).$$

Both Field (2005, p11) and the SPSS manual (PASW 2010: p11-14) get kurtosis wrong

" in a normal distribution the value of skew and kurtosis are 0..",
"...normal curve (for which the kurtosis is zero."

The kurtosis for a Normal distribution is 3.

*"The kurtosis for a standard normal distribution is **three**."*
Engineering Statistics Handbook (2010).

What Field and an SPSS manual mean, is that for a Normal distribution the ***excess*** kurtosis is 0 (by the subtraction of 3), which they both continue to call kurtosis, and as a consequence, so will we.

3.17 What are the limitations of validation of tool2?

We want to see if SPSS gives the same answers as *tool2*. We could *not* get SPSS to work out the descriptive statistics of 2 columns e.g. the variance of Y v

X. Thus we had to present SPPS with 1 column of data, and so we presented SPSS with 2 columns of data but ignored 1 of them.

Because SPSS treats its data as coming from a sample it uses estimates, and these estimates are wrong when n is small, where n is the number of cases e.g. the variance is divided by (n-1) for a sample and n for a census. We know that the formulae are wrong for 'small' n but SPSS itself has limitations at the 'upper' end as well. The 'upper' end is when SPSS is dealing with 'large' numbers and is almost impossible to find out the limits of SPS.

If a variable is *said to be* Numeric or Scientific Notation, then it has a maximum width of 40. The maximum number that SPSS can handle is about 1.79769 E+308, using Scientific Notation. If SPSS is presented with a file that contains numbers that are bigger than about 1.79769 E+308 SPSS will try and represent it in Scientific Notation and in the process most probably gets an overflow, so gets a truncated number, **but does not tell you so**. If you then ask for Descriptive Statistics of the truncated contents of this variable, you will get the wrong answers.

So, SPSS has its limits at the 'bottom' end and at the 'top' end.

3.18 What are the limits of Excel™?

At least the limits of Excel™ are easy to find out. We are using Excel™ 2007 and its limits are published (Microsoft 2012) and from these we find

- The maximum length of a cell is 255 characters.
- The maximum number is 9.999999999999 E+307 i.e. about 1 E+308

The only difficulty is when these limits are exceeded. If Excel™ is presented with a file that has a line where a number is bigger that about 1 E+308 then Excel™ treats this number as text, and Excel™ puts the rest of the number into succeeding cells. Even if a cell is made wider than its maximum width then Excel™ still puts the rest of the number into succeeding cells; so that the number in the 1st cell is truncated. If the number represented is less than about

E+308 then Excel™ uses Scientific Notation to represent it, with a subsequent loss of accuracy. If the number represented is greater than about 1 E+308 Excel™ **does not stop and tell you** it cannot cope but tries to do a Pivot (using the truncated data) and makes such a mess of it that is obviously wrong.
Thus Excel™ has its limits.

3.19 What are the limitations of the intermediate files?

The limitations of the size of the intermediate files are dependent on the **O**perating **S**ystem (**OS**) used. We stated off using the XP™ OS and then moved to Windows 7™ but never came across the OS's limits.

3.20 What are the limits of this method?

The method, MyMethod, uses *tool1* to calculate, and outputs the results of the calculation to a tab-delimited text file. The programming language, C, has an upper limit on the index of an array of about 86,000. If this upper limit is not exceeded, then the there is no limit to *tool1*. If the upper limit is exceeded then the calculation does not need to store its intermediate results in arrays but calculate them again, with a subsequent increase of time that *tool1* runs. We use two loops, one inside the other, each of which goes from 0 to 100 in steps of 1. Thus the loops are gone through 101 * 101 times = 10, 201 times, which is much less than 86,000 (and takes no account of impossible situations). Thus, *tool1* is only limited by the upper limit on the index of arrays, and even this can be got around.

Excel™ is limited when it uses 'big' numbers i.e. above about E+308. Unfortunately, we use numbers that are bigger than E+308 e.g. when we consider 20-option EMSQ quizzes with 10 ticks. Thus, in this situation the method fails, to draw the distribution of marks, but we are able to calculate the mean mark and the range of marks, by other means.

SPSS, also, is limited when it uses 'big' numbers and so the validation of *tool2* that tries to see if *tool2* and SPSS use the same formulae, fails when the limits of SPSS are reached. SPSS also produces the wrong results when a 'small' number of cases is used.

3.21 Do the tools work together?

The tools are:

- the tool, *tool1* which:

  a) inputs from the debug widow
  b) calculates a mark and how many ways there are of getting that mark in a particular fashion,
  c) calculates some descriptive statistics and outputs them to the debug window
  d) outputs to an intermediate tab-delimited text file.

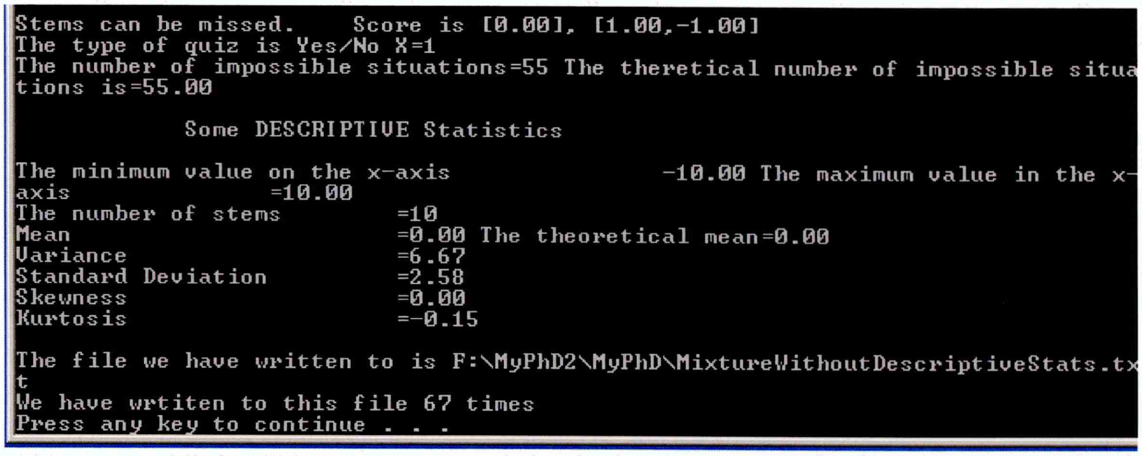
- **Excel™** which:

  a) inputs this text file,
  b) does a Pivot ,
  c) draws the result of the Pivot
  d) outputs the results of the Pivot to a tab-delimited text file

- the tool, *tool2* which:
  a) inputs this text file,
  b) calculates some descriptive and other statistics,
  c) outputs the statistics to the debug window.

We will take an example and show that, for this example, the tools work together. Let us take an example of a Yes/No quiz that has 10 stems, where stems can be MISSED with **b** = 0 and the guessing problem is exactly tackled. In this example X = 1, **n** = 10, **a** = -1, **b** = 0 and stems can be missed.

The tool, *tool1* produces in the debug window



```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the x-
axis            =10.00
The number of stems          =10
Mean                         =0.00 The theoretical mean=0.00
Variance                     =6.67
Standard Deviation           =2.58
Skewness                     =0.00
Kurtosis                     =-0.15

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
Press any key to continue . . .
```

Figure 3-4. Some descriptive statistics, from *tool1*.

Notice that **the mean is 0** and a tab-delimited text file is produced.

Excel™ does a Pivot of this data and produces, in Excel™

Row Labels	Sum of Number getting that mark
-10	1
-9	10
-8	55
-7	210
-6	615
-5	1452
-4	2850
-3	4740
-2	6765
-1	8350
0	8953
1	8350
2	6765
3	4740
4	2850
5	1452
6	615
7	210
8	55
9	10
10	1
Grand Total	**59049**

This line is got rid of, when the results of the Pivot are exported.

These 2 lines are are got rid of.

We then export this data to **Ouput.txt** and get in that file

RowLabels	Sum of Number getting that mark
-10	1
-9	10
-8	55
-7	210
-6	615
-5	1452
-4	2850
-3	4740
-2	6765
-1	8350
0	8953
1	8350
2	6765
3	4740
4	2850
5	1452
6	615
7	210
8	55
9	10
10	1
Grand Total	59049

The file is then 'cleaned up', by taking out the 3 lines. *tool2* then works on the 'cleaned up' version of **Output.txt** and produces in the debug window.

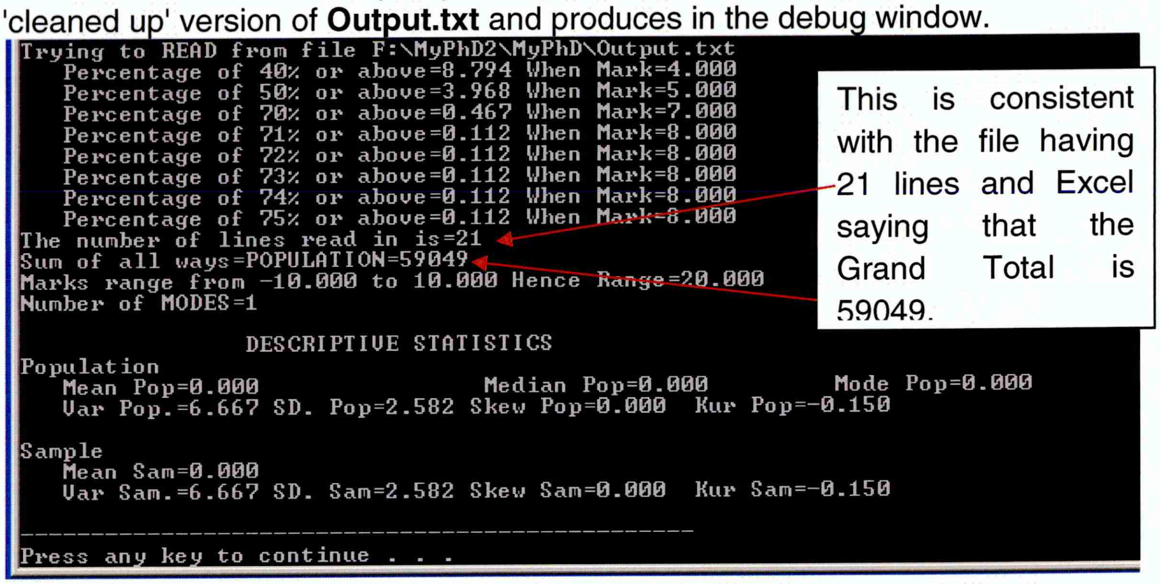


Figure 3-5. Some descriptive statistics produced by *tool2.*

The two tools that we have written give the **same** result in their debug windows.

However, the calculation of mean in *tool2* involves a multiplication of:

- a mark,
- how many ways there are of getting that mark.

and if either of these involve an approximation then the calculation of mean might be in error.

As an example, consider a Yes/No quiz with 100 stems where stems can be missed and **a** = -1 and **b** = 0. The two debug windows (of *tool1* and *tool2*) contain

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -100.00 The maximum value in the x
-axis                =100.00
The number of stems           =100
Mean                          =-0.00 The theoretical mean=0.00
Variance                      =66.67
Standard Deviation            =8.16
Skewness                      =-0.00
Kurtosis                      =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

Figure 3-6. The de-bug window of *tool1*.

```
Sum of all ways=POPULATION=515377520732010594027850537408483178751413190656
Marks range from -100.000 to 100.000 Hence Range=200.000
Number of MODES=1

                DESCRIPTIVE STATISTICS
First Mode occurs at mark 0.000 Last  Mode occurs at Mark 0.000Number of modes=1
 Mode=0.000
Population
   Mean Pop=-0.000               Median Pop=0.000          Mode Pop=0.000
   Var Pop.=66.667 SD. Pop=8.165         Skew Pop=0.000  Kur Pop=-0.015

Sample
   Mean Sam=-0.000
   Var Sam.=66.667 SD. Sam=8.165         Skew Sam=0.000  Kur Sam=-0.015
```

Figure 3-7. The de-bug window of *tool2*.

The two debug windows contain the **same** information. But, if the **marks** contain an approximation and even if there is no scientific notation involved the 2 debug windows do not contain the same information, but nearly the same, as the following example shows. We take an example where X = 3 (i.e. a 4-option MCQ-only quiz) where **a** = -0.33333 (it should be 1/3 but we approximate that), **b** = 0 and there are 40 stems which can be missed.

The tool, *tool1* outputs to its debug window

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -13.33 The maximum value in the x-
axis                =40.00
The number of stems         =40
Mean                        =0.00 The theoretical mean=0.00
Variance                    =10.67
Standard Deviation          =3.27
Skewness                    =0.20
Kurtosis                    =-0.00
```

Figure 3-8. The debug window of *tool1*.

The tool, *tool2* outputs to its debug window

```
Sum of all ways=POPULATION=909494701772926396374450l760
Marks range from -13.333 to 40.000 Hence Range=53.333
Number of MODES=1

                DESCRIPTIVE STATISTICS
First Mode occurs at mark -0.667 Last  Mode occurs at Mark 0.000Number of modes=
1 Mode=-0.667
Population
   Mean Pop=0.160                 Median Pop=0.000         Mode Pop=-0.667
   Var Pop.=10.692 SD. Pop=3.270          Skew Pop=0.057  Kur Pop=-0.042
```

Figure 3-9. The debug window of *tool2*.

Now, the 2 debug windows do not contain the same information. For example, the mean mark in one is 0 and in the other is 0.16, the standard deviation is the same in both windows but the skewness and kurtosis are not.

The 2 tools do not agree and so which one do we believe? In *tool1* the mean mark should be 0 and it agrees with the theoretical: so we should believe that.

So, why use *tool2* at all? Excel™ does a Pivot and draws the resultant distribution. We cannot easily do the Pivot or draw the distribution and so we use a widely available tool to do these. *tool2* assumes that a Pivot has been done in order to calculate mode, median and the proportion that pass. If you think that the calculation of mode and median are not very important and that you should believe *tool1* rather than *tool2* then *tool2* is only useful for obtaining an approximation to the proportion of passes.

3.22 What other questions are there?

3.22.1 Why did any programs in C have to be written?

The number of ways a mark could be obtained was so large that 'ordinary' programs could not cope. A library was used, that contained routines for handling arbitrarily large integers. The library chosen was called LEDA, written/maintained by Algorithmic Solutions, of Germany, in C. This library could run in Visual Studio, but it meant that:

a) The programming language, C, had to be learnt.
b) LEDA had to be learnt.
c) Visual Studio had to be learnt.
d) The incorporation of C and LEDA into Visual Studio had to be learnt. This learning was not easy and occupied about 9 months.

3.22.2 Why did intermediate files have to be used?

Excel™ could easily do:

a) a Pivot,
b) the drawing of the distribution of the Pivot. In order for Excel™ to do a Pivot it needed to import some data.

We decided that its data was going to be in a tab-delimited text file. Excel™ could not easily calculate the descriptive statistics of the distribution and so we decided to do that ourselves by a C program that worked on data in a text file.

3.22.3 Could everything be done in Excel™?

Inside Excel™ there are many functions like, MAX, MIN, MEDIAN, MODE but MODE and MEDIAN, for example, require that each range of cells they work on be specified for each distribution. Thus we were driven to writing our own programs that calculated descriptive statistics and other statistics, like MODE MEDIAN and the proportion of passes, once Excel™ had done its work.

3.22.4 What other tools could have been used?

It is claimed that in the .NET framework there is support for arbitrarily large integers

> *"Support for arbitrarily large integers"*
>
> Patten (2012).

Weinstein (2011) also talks about IronPythton supporting arbitrary length integer but these systems would have involved just as much learning as we did in learning about C, LEDA, Visual Studio and their integration. So, it is possible that other tools could have been used, but we think that using them would not have saved much, if any, time.

3.23 Conclusion.

The method we used was quantitative. The meaning and use of the results was qualitative. Thus, it could be argued that we were using mixed methods.

The method that we used was to write programs in C that a) calculated a mark and how many received that mark (*tool1*) b) calculated the descriptive statistics of the distribution of marks (*tool2*). The distribution was drawn by Excel™. The writing of the programs was necessary because 'normal' programs could not handle the large integers needed for the calculations. The programs ran in VS and used the LEDA library to handle integers of an arbitrary size.

The measures we used in *tools* 1 and 2 were mainly descriptive statistics like, mean, variance, standard deviation, skewness and kurtosis.

3.24 What is coming next?

Chapters 4, 5 and 6 describe the analysis that was done when no tools are built *(phase 2)*. Chapter 7 describes the *results* of building tools.

Chapter 4 (the next chapter) describes the analysis that applies to all special quizzes.

Chapter 4. Analysis that applies to all 'special' quizzes.

Abstract.

For 'special' quizzes we calculate:

- the range of marks,
- the mean mark,
- the number of marks in each grade.

Using

- the number of stems,
- the number of correct answers to stems,
- the number of in-correct answers to stems,
- the number of stems missed.

4.1 Introduction.

'Special' quizzes are where a restriction is put on quiz types and their scoring. The quizzes are called mixed when they do not have stems *all* of the same type. For example, a quiz is mixed when some stems are Yes/No stems and some stems are 4-option MCQs. 'Special' quizzes are not mixed when the odds of guessing the correct answer to a stem depends on the stem type. But, the odds of guessing the correct answer to a stem are the *same* for a 10-option MCQ stem and a 5-option MAQ stem with 2 ticks. Thus, in this case the quiz is not mixed.

After Analysis comes design which is about specifying and writing programs which we do in the programming language, C, and which are described in a further chapter. The validation of the programs is contained in Appendices 13, 14 and 16, and sees whether the output from the programs agrees with the Analysis described in this and other chapters.

Further information can be found in Appendices 1, 2, 3, 4, with Appendix 4 containing the details of Straker's (2012) proof. The validation that applies to 'special' quizzes is contained in Appendices 7 and 8.

4.2 Methods/Measures.

The **methods** that we are going to use in this Chapter are, in the main, the *quantitative* methods of mathematics and statistics. We shall also use the *qualitative* method of arguing.

The **measures** we are going to use in this Chapter are:

- The **Range** of marks.
- The **Mean** mark.
- The number of marks in each grade at University (in Britain), $\mathbf{M_g}$.

These are all measures of what is shown on the x-axis.

4.3 Definitions.

n = the number of stems, an integer starting at 1 and goes up in steps of 1
Having a negative number of stems or no stems or having a non-integer number of stems does not make sense.

w = the number of correct answers to stems of a quiz
w is an integer and goes from 0 to **n** is steps of 1
x = the number of in-correct answers to stems of a quiz
x is an integer and goes from 0 to **n** in steps of 1
y = the number of stems that are missed of a quiz
y is an integer and goes from 0 to **n** in steps of 1
So, **n** = **w** + **x** + **y**

a = the *score* when a stem is answered in-correctly
b = the *score* when a stem is missed
c = the *score* when a stem is answered correctly
a, **b** and **c** can be any number.
When **b** does not exist and **a** is negative, we are said to be using **negative *marking***. This is in-accurate as we should say we are using negative *scoring*. A mark is the sum of the scores.

The general *score* is shown as [**b**], [**c**, **a**]

We argue that one of **a**, **b** and **c** can be made into 1 by a subtraction e.g. subtract (**c** -1) (or division e.g. divide by **c**, unless we divide by 0). We arbitrarily decide that **c** should be 1. We make one variable into the constant 1 and then we only have to vary **a** and **b**.

So, the *score* is [**b**], [1, **a**]

A mark, **m** = **w*c** + **x*a**+ **y*b**, and when **c** = 1
m = **w** + **x*a** + **y*b**

Another argument is that **b** should *always* be 0 because by missing a stem the learners did not answer it correctly or in-correctly, and so should not be rewarded or punished, and so **b** should always be 0.

If you believe these two arguments then

m = **w** + **x*a**

and the score is [**0**], [1, **a**]. This way, we only have to decide what **a** should be.

Part 1, is about when **y** does not exist and hence **b** does not exist (**b** does not have the value 0: it does not exist).

Part 2, is about when **y** does exist (Here **b** exists and can have the value 0).

P = the number of *options* of each stem

- **P** is an integer
  - The least **P** is 1.
  - **P** being negative or 0 does not make sense
    - What the optimal **P** is, is left to other papers
  - Assumption.
    - Every stem has the same number of options, **P**

T = the number of *ticks* that each stem has.

- **T** is an integer
  - The least **T** is 0.
    - When **T** = 0 it is assumed that the stem has been missed
    - When a stem is answered the least **T** is 1
  - What the optimal **T** is we do not know.
    - **T** cannot be greater than **P**
      - You cannot have more ticks than options.
  - Assumptions
    - a) Every stem has the same number of ticks, **T,**
    - b) The learner uses *exactly* **T** ticks on every stem (except where the stem is missed),
    - c) **P≠T.**

Mg = the number of marks there are in each passing grade, and is written as something like [10, 9, 8, 1]. 10 means the number of marks in the 3rd grade, 9 is the number of mark in the 2.2 grade, 8 is the number of marks in the 2.1 grade and 1 is the number of marks in the 1st grade.

A 40% institution is where the pass mark is 40%. A 50% institution is where the pass mark is 50%.

4.4 Proof that applies to all 'special' quizzes.

Whether stems are answered or not, Straker's (2012) proof (see Appendix 4) with some input from us shows that $\mu_n = n*\mu_1$ where μ_1 = Mean mark of 1 stem.

When every stem is scored alike the mean mark μ_1 for a quiz that has 1 stem is ((the number of right answers to 1 stem)*(1 + (number of wrong answers to 1 stem) ***a** +(number of ways of missing 1 stem)***b**))/(The total number of ways for 1 stem). The (number of right answers to 1 stem) = 1 for 'special' quizzes. The mean mark for a quiz that has 2 stems is $\mu_1 + \mu_2$ when both stems have the same **a** and **b**. The mean mark for **n** stems is **n** times the mean mark for 1 stem. Appendix 4 gives a full proof.

4.5 Analysis that applies to all 'special' quizzes.

4.5.1 The Range of marks.

The **Range** of raw marks goes from the minimum raw mark to the maximum raw mark. To make life easier we will only consider quizzes that have 100 stems, and then we do not have to bother about percentage marks.

The minimum raw mark of 'special' quizzes is **n*** Min (**a**, **b**, **c**)
For example in a 100 stem quiz where **a** = -1, **b** = 0, **c** = 1 the minimum raw mark is 100*(-1) = -100

The maximum raw mark of 'special' quizzes is **n*** Max (**a**, **b**, **c**)
For example in a 100 stem quiz where **a** = -1, **b** = 0, **c** = 1 the maximum raw mark is 100*(1) = 100

Thus, **Range** of raw marks is **n*** Min (**a**, **b**, **c**) to **n*** Max (**a**, **b**, **c**).
When **b** does not exist i.e. when all stems are answered and **c** = 1 the
Range of raw marks is **n*** Min (**a**, **1**) to **n*** Max (**a**, **1**).

We can use the range of raw marks as part of the validation of our programs. If our programs do not show that the minimum mark for a 100 stem quiz where **a** = -1, **b** = 0 and **c** = 1 is -100 then the program is wrong. If our programs do not show that the maximum mark for a 100 stem quiz where **a** = -1, **b** = 0 and **c** = 1 is 100 then the programs are wrong.

4.5.2 The number of answers to a stem.

For a **Yes/No** stem the number of answers is 2 i.e. "Yes" or "No"

For **4-option MCQ** stem the number of answers to 1 stem = 4

Hence, for a **P**-option MCQ stem, the number of in-correct answers to 1 stem = **P-1.**

For **5-option MAQ** stem, with 2 ticks, the number of answers to 1 stem = 10, when 2 ticks are used for each answered stem. We do not consider when **T** is greater than 2 or when **T** is less than 2. We assume that the learner always uses *exactly* 2 ticks to answer *every* stem. If the stem is missed **T** is 0.

Hence, for a **P**-option MAQ stem, with **T** ticks, the number of answers to 1 stem = ${}_PC_T$, when **T** ticks are used for each answered stem.

An **EMSQ** quiz is either an MAQ quiz or MCQ quiz and we can deal with either of them.

For a **3-option-only Multiple Yes/No** quiz the number of answers to 1 stem = 8

Hence, for a **P**-option-only Multiple Yes/No quiz, the number of answers to 1 stem = 2^P

An MCQ is a specialisation of an MAQ where **T** = 1
A Yes/No is a specialisation of a Multiple Yes/No where **P** = 1
We are only dealing with MAQs or Multiple Yes/Nos, where (the number of answers to 1 stem) = ${}_PC_T$ (when exactly **T** ticks are used for each answered stem) or 2^P. For example, the number of answers to a 4-option MCQ stem is ${}_4C_1$ = 4. Another example is the number of answers to a 5-option Multiple Yes/No stem is $2^5 = 32$.

4.5.3 The number of in-correct answers to a stem.

The number of answers to a stem is ${}_pC_T$. or 2^P
The number of in-correct answers to a stem is 1 less than the number of answers to a stem, is ${}_pC_T$ -1 or 2^P -1. Let X= ${}_pC_T$ -1 or 2^P -1.

For example, the number of in-correct answers to a 5-option MAQ that has 2 ticks = ${}_5C_2$ - 1= 10-1 = 9. This assumes that the number of correct answers to a stem is 1. This not the case with CBM (where the number of correct answers to a stem is 3 depending on the value of C) and so this is one reason why scoring by CBM is not allowed for 'special' quizzes.

4.5.4 The mean mark.

The **mean mark** for 1 stem is given by

μ_1 = ((number of correct answers to 1 stem)***c** +
(number of in-correct answers to 1 stem)***a**
(number of ways of missing 1 stem)***b**)/
(the number of possibilities of 1 stem)

For 'special' quizzes,

(number of correct answers to 1 stem) = 1
(number of ways of missing 1 stem) = 1
(the number of possibilities of 1 stem)= (number of correct answers to 1 stem) +
(number of in-correct answers to 1 stem) +
(number of ways of missing 1 stem)
=1 + (number of in-correct answers to 1 stem) + 1

So, μ_1 = (**c*1** + (number of in-correct answers to 1 stem)***a** + **b*1**)/
((number of in-correct answers to 1 stem) +2)

As **c** = 1 and we shall say that

X=**w**= (the number of in-correct answers to 1 stem, when exactly T ticks are used in an answer)

Then μ_1 = (1 +X***a** + **b**)/(X +2)
and using Straker's (2012) proof we have μ_n = **n***(1 +X***a** + **b**)/(X +2)

For example, the mean mark for a 5-option-only MAQ quiz with 2 ticks that has 100 stems (where stems can be missed)) and **a** = **b** = 0 is 100*(1 + 0 + 0)/(9+2) = 9.09

When **y** does not exist (so, neither does **b**) we have
μ_1 = (1 +X***a**)/(X + 1)
and μ_n = **n***(1 +X***a**)/(X + 1)

For example, the mean mark for a 4-option-only MCQ quiz that has 100 stems (where all stems are answered) and **a** = 0 is 100*(1 + 0)/(3+1) = 25

4.5.5 The number of marks in a grade.

If you are at a **40%** institution University in Britain then the pass marks might be

40, 41, 42, 43, 44, 45, 46, 47, 48, 49	**10** marks in the 3rd range
50, 51, 52, 53, 54, 55, 56, 57, 58, 59	**10** marks is the 2.2 range
60, 61, 62, 63, 64, 65, 66, 67, 68, 69	**10** marks in the 2.1 range
70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**31** marks in the 1st range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	
100	

A measure of the pass marks at a 40% British University is

$\mathbf{M_g}$ = [10, 10, 10, 31]

If you are at a **50%** institution University in Britain then the pass marks might be

50, 51, 52, 53, 54, 55, 56, 57, 58, 59	**10** marks is the 3rd range
60, 61, 62, 63, 64, 65, 66, 67, 68, 69	**10** marks in the 2.2 range
70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**10** marks in the 2.1 range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**21** marks in the 1st range
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	
100	

A measure of the pass marks at a 50% British University is

$\mathbf{M_g}$ = [10, 10, 10, 21]

4.6 Conclusion.

We calculate measures that apply to *all* 'special' quizzes viz.

- the *range* of marks **n*** Min (**a**, **b**, **c**) to **n*** Max (**a**, **b**, **c**).
- the *mean* mark (using Straker's proof) $\mu_n = n^* \mu_1$

where μ_1 = the mean mark of a quiz that has 1 stem and this depends on whether stems can be missed

and **n** = the number of stems in the quiz.

Thus $\mu_n = n^*(1 + X^*a)/(X + 1)$ for stems that are ALL answered

$\mu_n = n^*(1 + X^*a + b)/(X + 2)$ for stems that are ALL answered

- the *number of marks in each grade* M_g.

Using

- the number of stems, **n**,
- the number of correct answers to stems, **w**,
- the number of in-correct answers to stems, **x**,
- the number of stems missed, **y**.
- the *score* when a stem is answered in-correctly,**a**
- the *score* when a stem is missed, **b,**
- the *score* when a stem is answered correctly, **c**
- X=either $({}_PC_T - 1)$ or $(2^P - 1)$

4.7 What is coming next.

The next Chapter will look at the Analysis that applies to all 'special' quizzes when:

a) all stems are answered,
b) stems are not 'mixed'

Chapter 5. Analysis where ALL stems are answered.

Abstract.

When 'special' quizzes have ALL their stems answered, we have 3 proofs. We use these proofs to help to build tools and validate them.

The guessing problem can be tackled by:

- using a more complex quiz,
- using negative marking,
- raising the pass mark.

or a combination of these. Each way of tackling the guessing problem leads to more problems.

Using a more complex quiz leads to the distribution of marks being concentrated in the lower region. For example, a 100 stem 5-option-only Multiple Yes/No quiz (with **a** = 0) has its mean mark at 3.125. That means that half of the area of the distribution of marks is between the marks 0 and 3.125. The distribution is thus positively skewed.

Using negative marking leads to difficulties of 'fairness' and raising the pass mark leads to difficulties of meaning.

Using a combination of ways to tackle the guessing problem just compounds difficulties.

All the ways of tackling the guessing problem lead to problems of comparison of marks obtained by different types of assessment because the distribution of marks and grades might be different for different types of assessment.

5.1 Introduction.

This Chapter is about Analysis of 'special' quizzes that have ALL their stems answered and the guessing problem is tackled.

If you force stems to be answered then *guessing is forced*, when the correct answer is not known. This is a general problem and applies to all 'special' quizzes where all stems are answered. However, if the learner *chooses* to answer all stems (perhaps he/she sees an advantage) then no forced guessing goes on.

Further information can be found in Appendices 1 to 5. Appendix 7 shows the validation of our analysis of 'special' quizzes where ALL stems are answered.

A resumé of this Chapter, the previous one and the next one was given as a conference paper, Lafferty & Burley (2012a).

5.2 Methods/Tool.

The **methods** that we are going to use in this Chapter are, in the main, *quantitative*.

The **tool** that we use to help the analysis is the widely available Excel™ using files **MeanMark.xlsx** and **MG.xlsx**.

5.3 The number of ways of getting a mark.

The number of ways of getting a mark **M** depends on

- quiz type e.g. Yes/No-only,
- the number of stems, **n**,
- the number of stems that are answered correctly, **x,**
- the number of stems that are answered in-correctly, **w,**
- .

Thus

For **Yes/No-only** quizzes

the number of ways of getting a mark **M** = ${}_nC_w$

For **P-option MCQ**-only quizzes

the number of ways of getting a mark **M** = $(\mathbf{P}\text{-}1)^x \; {}^*{}_nC_w$

For **P-option MAQ**-only quizzes with **T** ticks

the number of ways of getting a mark **M** $=({}_PC_T\text{-}1)^x \; {}^*{}_nC_w$

(when *exactly* **T** ticks are used to answer each of the **n** stems)

EMSQ quizzes are either MCQs or MAQs

For **Multiple Yes/No-only** quizzes

the number of ways of getting a mark **M** = $(2^P\text{-}1)^x \; {}^*{}_nC_w$.

Thus, there are only 2 types of quizzes where the number of ways of getting a mark **M** is either:

- $({}_PC_T\text{-}1)^x \; {}^*{}_nC_w$ for MAQs with MCQs are a specialisation where **T** = 1.
- $(2^P\text{-}1)^x \; {}^*{}_nC_w$ for Multiple Yes/Nos where Yes/Nos are a specialisation where **P** = 1.

We can generalise even further and only have 1 type of quiz, by saying X represents quiz type (i.e. X is either $({}_PC_T\text{-}1)$ or $(2^P\text{-}1)$) and then the number of ways of getting a mark **M** is $X^x \; {}^*{}_nC_w$.

An immediate question is "Where does this come from?" and another question is "What has it got to do with **x**?" (as **x** is the number of stems that are *in-correctly* answered). The philosophy of this study was to start off simply and gradually get more complicated, and so, we started off with Yes/No-only quizzes where all stems were answered and **a** = 0, **n** = 100. In this case, we had a 'hunch' (the posh word is conjecture) that the number of ways of getting a mark was dependent on **n** and **w**. It took about 3 months to realise that the formula was ${}_nC_w$, (Appendices 1, 2 and 3 justify where it came from). To obtain the mark 0 you answer all the stems in-correctly, and there is only 1 way of doing this, and so $\mathbf{Y_0}$ = 1. To obtain the mark 100 you answer all the stems correctly, and there is only 1 way of doing this, and $\mathbf{Y_{100}}$ = 1. We get the impression that $\mathbf{Y_M}$ is symmetrical (Investopedia 2011). This is bolstered by considering the marks 1 and 99. There are 100 ways to obtain the mark 1 i.e. answer the 1st stem correctly and the rest in-correctly, or answer the 2nd stem correctly and the rest in-correctly etc. Similarly, there are 100 ways of getting the mark 99 i.e. answer the 1st stem in-correctly and the rest correctly etc, so $\mathbf{Y_1} = \mathbf{Y_{99}} = 100$. To get the mark 2 we can answer the fist 2 stems correctly and the rest in-correctly. We immediately ask "How many ways of doing that are there?" and by making **n** = 100 we go against the KISS principle (Keep it **simple** stupid). So, we make **n** 'small' e.g. 3. When **n** = 3, the number of ways of getting the mark 2 is to answer the 1st two stems correctly and the 3rd stem in-correctly or answer the 1st and 3rd stems correctly and the 2nd stem in-correctly or answer the 2nd and 3rd stems correctly and the 1st stem in-correctly. There are 3 ways of getting the mark 2 when **n** = 3 i.e. ${}_3C_2 = 3!/(2!*(3-2)!)) = 3$. When **n** = 4, there is only 1 way of getting the marks 0 or 4; there are 3 ways of getting the marks 1 or 3. This only leaves the mark 2. How many ways are there of getting that? Answer the 1st two stems correctly and the last 2 in-correctly or answer the 1st and 3rd stems correctly and the other two in-correctly, or answer the 1st and 4th stems correctly and the other two in-correctly, or answer the 2nd and 3rd stems correctly and the other two in-correctly, or answer the 2nd and 4th stem correctly and the other two in-correctly, or answer the 3rd and 4th stem correctly and the other two in-correctly. Thus there are ${}_4C_2$ ways of getting the mark 2 i.e. $4!/(2!2!) = 4.3.2.1/(2*2) = 6$.

The symmetry comes about because ${}_nC_r = {}_nC_{n-r}$ (see Appendices 1, 2 and 3)

because $\quad {}_nC_r = \mathbf{n!/(r!*(n-r)!)}$

$${}_nC_{n-r} = n!/((n-r)!*(n-n+r)!) = \mathbf{n!/(r!*(n-r)!)}.$$

Thus, the number of ways of getting the mark 0 $= {}_nC_0 = 1$

and when **n**=100

the number of ways of getting the mark 100 $= {}_{100}C_{100} = 1$

and

the number of ways of getting the mark 1 $= {}_{100}C_1 = 100$

the number of ways of getting the mark 99 $= {}_{100}C_{99} = 100$.

When the quiz type is 4-option MCQ there are 3 ways of getting an answer to a stem in-correct. Similarly, for a **P**-option MCQ there are (**P** -1) ways of getting an answer to a stem in-correctly and so, the number of ways of getting a mark $= (\mathbf{P}\text{-}1)^{\mathbf{x}} * {}_{\mathbf{n}}C_{w}$. Similarly, for a **P**-option MAQ with **T** ticks there are $({}_{P}C_{T}\text{-}1)$ ways of getting an answer to a stem in-correctly (when exactly **T** ticks are used for each answered stems) and so, the number of ways of getting a mark $= ({}_{P}C_{T}\text{-}1)^{\mathbf{x}} * {}_{\mathbf{n}}C_{\mathbf{w}}$. Similarly, for **P**-option Multiple Yes/No-only quizzes the number of ways of getting a mark $= (2^{\mathbf{P}}\text{-}1)^{\mathbf{x}} * {}_{\mathbf{n}}C_{\mathbf{w}}$.

This explains where ${}_{\mathbf{n}}C_{\mathbf{w}}$ comes from, and answers what it has to do with answering stems in-correctly. The expressions are validated in Appendix 7.

The method now becomes:

a) find out the quiz type and find out **P** and **T**, **n** and **a** (so that **M** can be calculated),
b) vary **w** from 0 to **n** in steps of 1 in a loop and calculate **M** and $\mathbf{Y_M}$ inside this loop,
c) draw $\mathbf{Y_M}$ v **M**,
d) calculate descriptive and other statistics.

5.4 Proofs that apply to 'special' quizzes where all stems are answered.

We have 3 proofs here:

- Proof 1. The gap between marks is |1 - **a**|. (See Appendix 5).
- Proof 2. The number of impossible marks is 0. (See Appendix 5).
- Proof 3. The number of marks is (**n** + 1) (except where **a** = 1). (See Appendix 5).

An 'impossible' mark is where the conditions for calculating the mark are impossible. We cannot give an example here, but where stems can be missed it is impossible to answer 99 stems correctly and miss 2 stems when the number of stems is 100.

These proofs can be used to help construct the programs that form our tools. For example, one tool should have a loop in it that represents **w**, and goes from 0 to **n** in steps of 1. This implies that before the loop **n** must be known. If we knew the value of **a** then inside the loop we could calculate **M** and $\mathbf{Y_M}$ inside the loop, if the quiz type is known. So, a partial specification of 1 tool is:

Ask for **n**, **a** and quiz type. Hence we can calculate
For **w** going from 0 to n in steps of 1
    Calculate **M** and $\mathbf{Y_M}$

This is a precursor to Chapter 6.

The results of these proofs can, also, be used as validation of our tools. For example:

a) if our programs (tools) do not show that the gap between successive marks is |1 -**a**| then they are wrong,
b) if our programs do not produce 0 for the number of impossible marks, the programs are wrong.

The conclusions from these proofs is that when all stems of 'special' quizzes are answered

- The distribution of marks is 'stretched', as **a** varies. The mean mark is changed, and anything that depends on the mean is also changed (variance, standard deviation, skewness and kurtosis).
- There is no need to do an experiment to see if the rank-order of people depends on the value of **a**.

To illustrate the first conclusion we draw 2 distributions

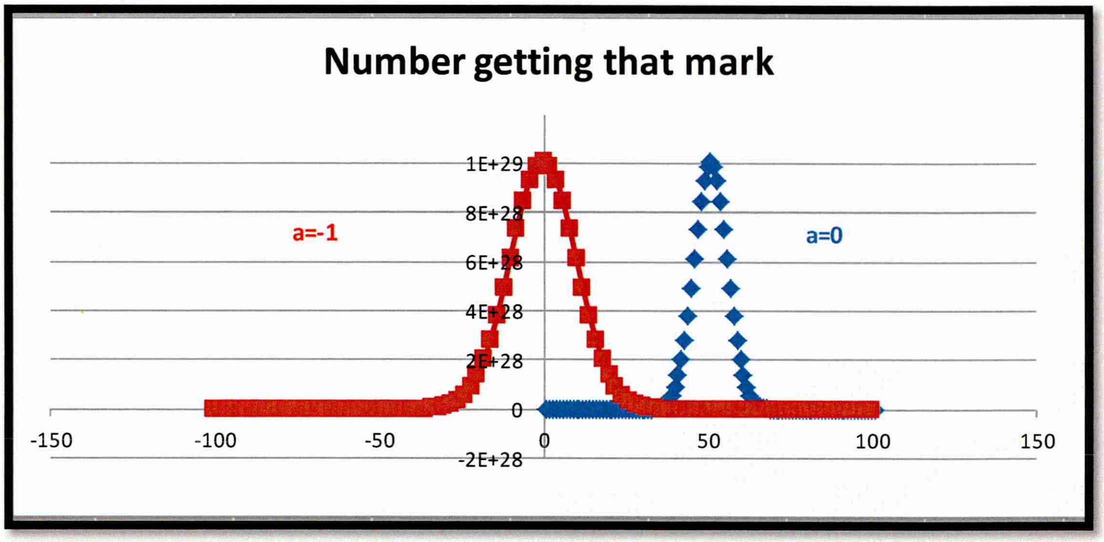


Figure 5-1. The distribution of marks when **a** = 0 and when **a** = -1, when **n** = 100, for Yes/No-only quizzes.

(This was drawn using the Excel™ file **MeanMark.xlsx**)

This becomes a topological problem, where we draw the distribution on a piece of elastic. If we are using negative marking, then we hold the elastic at the mark 100 with our right hand and stretch the elastic along the x-axis leftwards with our left hand. If we are not using negative marking, then we hold the elastic in our left hand at the lowest mark, and stretch the elastic rightwards along the x-axis, with our right hand. The lowest mark is shown by examples. Let us say **a**= 0.5, **n** = 100 then, the lowest mark is 50 (viz. 100 * Min (1, 0.5)) i.e. get the answers to all the stems wrong. When the answers to all the stems are right, the highest mark =100 (viz. 100 * Max (1, 0.5). The marks go from 50 to 100 in steps of 0.5 (viz. |1 - 0.5|). When **a** = 1 there is only 1 mark viz. 100 (when **n** = 100) and it does not matter how the stems are answered. When **a** = 2, say, the minimum mark is 100 (i.e. get the answers to all the stems right) and the maximum mark is 200 (i.e. get the answers to all the stems wrong). The marks, now, range from 100 to 200 in steps of 1 (viz. |1 - 2|). When **a** = 3, the minimum mark is 100 (i.e. get the answers to all stems right) and the maximum mark is 300 (i.e. get the answers to all the stems wrong). The range is 100 to 300 in steps of 2 (viz. |1 - 3|).

This causes problems when we use positive marking (i.e. **a** >1), because we give more marks when answers are wrong.

The second conclusion comes from argument, and also involves topology. Imagine two people D and E where D gets a better mark than E. If we are using negative marking then D will always beat E no matter what **a** is. This follows when we draw the distribution and stretch it leftwards in 2 dimensions. If we are using non-negative marking, then the argument is that D will beat E, no matter what **a** is, still holds. If **a** = 1 then D cannot get a better mark than E. The conclusion is that D will always beat E no matter what **a** is, unless **a** = 1 and then everybody gets 100% and D cannot beat E.

5.5 The mean mark.

Using Straker' (2011) proof (see Appendix 4), we have $\mu_n = n*(1 + X*a)/(X + 1)$ and then we have

For **Yes/No-onl**y quizzes $X = 1$
so $\mu_{100} = 100*(1 + a)/(2)$
when **a** = 0 $\mu_{100} = 50$

For **P-option-only MCQ** quizzes $X = (P-1)$
so $\mu_{100} = 100*(1 + (P - 1)*a)/(P)$
when **a** = 0, **P** = 4, $\mu_{100} = 25$

For **P-option MAQ** -only quizzes with **T** ticks $X = ({}_PC_T - 1)$
so $\mu_{100} = 100*(1 + ({}_PC_T - 1)*a)/({}_PC_T)$
when **a** = 0, **P** = 5, **T** = 2 $\mu_{100} = 10$

For **P**-option-only **Multiple Yes/No** quizzes $X = (2^P - 1)$
so $\mu_{100} = 100*(1 + (2^P - 1)*a)/(2^P)$
when **a** = 0, **P** = 5, $\mu_{100} = 3.125$

We can use this analysis to help us with the validation of the programs we write. For example, if our program does not produce 25 as the mean for a 4-option MCQ-only quiz (where all stem are answered) then it is wrong.

5.5.1 By guessing we can get the mean mark.

The mean mark is the mark that can be obtained by guessing. We show this as follows. The calculation of $\boldsymbol{\mu_1}$ and hence $\boldsymbol{\mu_n}$ is the same calculation that would be done if guessing had taken place. For example,

μ_1 = ((number of correct answers to 1 stem)***c** +
(number of in-correct answers to 1 stem)***a** +
(number of ways of missing 1 stem)***b**)/
(the number of possibilities of 1 stem)

This is only true if *every* stem has the *same* **a**, **b** and **c**.

5.6 Tackling the guessing problem.

Guessing the correct answer to a stem is called the guessing problem, and the solutions to the guessing problem include:

- Using a more complex quiz.
- Using negative marking.
- Raising the pass mark.

and combinations of these.

We will try them individually and then in tandem, and show that each of these solutions brings with it more problems.

Appendix 6 shows tables of how to tackle the guessing problem and the way to read these tables is to ask "What way do we want to tackle the guessing problem?" If the answer is "Use a more complex quiz" and then we ask "What complex quiz do you want to use?" and if the answer is "4-option MCQ" then we go to the relevant table and then ask "What value of **a** should we use and what should we raise the pass mark to?" If the answers are to choose **a** = -1/3 and raise the **P**ass **M**ark (**PM**) to 72, we see that the mean mark is 0 and the number of pass marks is given by [6, 8, 7, 1] i.e. there are 6 marks in the Pass grade (72, 73 +1/3, 74 + 2/3, 76, 77 +1/3, 78+ 2/3) for **n** = 100, 8 marks in the 2.2 grade, 7 marks in the 2.1 grade and 1 mark in the 1st grade

We, now, have to make a decision:

- we have to compare marks obtained by this quiz where the pass mark is 72 to marks obtained by other types of assessment where the pass mark is 40 (at a 40% institution),

- we have to compare marks obtained by this quiz that go from -100/3 to +100 in steps of |1+1/3| with marks obtained from essays, say, that go from 0 to 100 in unknown steps,

- we have to compare grades obtained by this quiz where there is only 1 mark in the 1st grade to grades obtained by other types of assessment (e.g. exams) where there are 31 marks in the 1st grade (viz. 70, 71…99, 100) (at a 40% institution).

If we do what the Americans do to form the Grade Point Average, then we are asking "Can the Grade Point Average be computed when an addition and a division (and a scaling) is done?" Can we add marks that mean different things in different contexts? For example, can we add 71 that in one context is a fail and in another is a 1st.

This is not just 'getting at' Americans: it is 'getting at' any institution that adds marks obtained from sources where the marks mean different things. For example, can we add marks obtained from essays to those obtained from electronic tests?

5.6.1 *Using a more complex quiz.*

If we look at **using a more complex quiz** alone and draw a table using the tables of Appendix 7 we get

Table 5-1. Mean mark of various quizzes when **a** = 0, **n** = 100 where all stems are answered.

Type of quiz	Mean Mark when n = 100, a = 0 and c = 1	Comment
Yes/No-only	**50**	
MCQ 3-option -only	**33.33**	
4-option-only	**25**	Same as a 2-option Multiple Yes/ No
5-option-only	**20**	
6-option-only	**16.67**	
7-option-only	**14.29**	
8-option-only	**12.5**	Same as a 3-option Multiple Yes/ No
MAQ 5-option with 2 ticks-only	**10**	Same as a 10-option MCQ
6-option with 3 ticks-only	**5**	Same as a 20-option MCQ
7-option with 4 ticks-only	**2.86**	
8-option with 5 ticks -only	**1.79**	
Multiple Yes/No 5-option-only	**3.125**	
6-option-only	**1.5625**	
7-option-only	**0.78125**	
8-option-only	**0.390625**	
EMSQ 20-option (MCQ)	**5**	
20-option (MAQ with 10 ticks)	**0.00054**	About 0

Minimum mark for them all = 0. Maximum mark for them all = 100

$\mathbf{M_g}$ at a 40% institution is [10, 10, 10, 31] for them all, and

at a 50% institution is [10, 10, 10, 21] for them all

(This table, and subsequent tables, used the Excel™ file **MG.xlsx** see Appendix 6).

It looks as though as we go down this table we can conclude that "using a more complex quiz results in the mean mark going down". But, we have to be a bit careful about complexity. Complexity depends on how many options there are and how many ticks. For example a 5-option-only MAQ quiz with each of 100 stems having 5 ticks results in a mark of 100 (as long as the learner is told the number of ticks and sticks to it) which is worse than a 4-option MCQ-only quiz and worse than a Yes/No-only quiz. It gets even more complicated when a 5-option-only MAQ quiz with 2 ticks has the *same* mean mark as a 10-option-only

MCQ. A 3-option Multiple Yes/No quiz has the same mean mark as an 8-option MCQ. With the proviso that a more complex quiz means that the odds of guessing the correct answer to a stem goes down then we can say that the mean mark goes down as we use a more complex quiz.

5.6.1.1 Difficulties of using a more complex quiz.

Again, as we go down this table, it looks as though you could use a 5 or more *Multiple* Yes/No quiz (because it is doubtful if one can mark to an accuracy of 3.125 or less), but then:

a) you have to ensure that each option is equally likely to be chosen (if that is possible),
b) you have got to write and *test* 5 or more options,
c) taking a 5-option Multiple Yes/No-only quiz as an example, the mean mark is 3.125 and the minimum mark is 0.

Thus, half the area of the distribution of marks is between the marks 0 and 3.125, and is 'bunched' at the bottom end. The distribution of marks is said to be positively skewed, and the comparison of marks obtained by essays, say, and the marks obtained by quizzes, is 'hard' (as they may have different distributions). Using a 'more complex' quiz on its own, to tackle the guessing problem, leads to problems.

5.6.2 *Using negative marking.*

For **Yes/No-only** quizzes with **a** = -1 is it 'fair' to get a mark of 38 when 69 stems (out of 100) are answered correctly? Is it 'fair' to *assume* that guessing is going on? The learner might genuinely have thought that answer to a stem was correct. The same argument applies for **4-option-only MCQ** quizzes where **a** = -1/3. Perhaps, it is 'fair' when the learners know how they are going to be marked. We do not answer these questions, because 'fair' cannot be defined. To cut this argument short, using negative marking brings with it, its own problems. The comparison of marks obtained by quizzes, with those obtained by other types of assessment is 'hard' because the marks obtained by quizzes might have a different distribution to those obtained by other types of assessment. For a start, when negative marking is used then the minimum mark is negative, but for exams, say, the minimum mark is 0. We can modify the marks to make all negative marks into 0, but then the distribution of marks obtained by quizzes and those obtained by exams might be different and comparing the marks obtained by different types of assessment becomes 'hard'.

If we ask "What should **a** be?" The answer is $-1/({}_PC_T -1)$ for non-multiple Yes/No quizzes and $-1/(2^P -1)$ for Multiple Yes/No quizzes, to make μ_n into 0. For example, for a Yes/No quiz we make **a** = $-1/(2^1-1) = -1$, and for 4-option MCQ quiz we make **a** = $-1/({}_4C_1 -1) = -1/3$. (**Note**. It is NOT -1/4). We see that we can reduce the guessing problem by making **a** more and more negative. At some point we will make the mean mark 0, where we say the guessing problem is *exactly* tackled, and beyond that the mean mark is negative and we say that the guessing problem is *over* tackled. For example, for a Yes/No-only quiz where **n** = 100

a = 0	μ_{100} = 100*(1+**a**)/2 = 100/2 =	50
a = -0.25	μ_{100} = 100*(1+**a**)/2 = 100*(3/4)/2 =	37.5
a = -0.5	μ_{100} = 100*(1+**a**)/100*(1/2)/2 =	25
a = -0.75	μ_{100} = 100*(1+**a**)/2=100*(1/4)/2 =	12.5
a = -1	μ_{100} = 100*(1+**a**)/2 = 100*(0)/2 =	0
	Guessing is **exactly** tackled	
a = -1.25	μ_{100} = 100*(1+**a**)/2 = 100*(-1/4)/2 =	-12.5
	Guessing is **over** tackled	

If we use *exact* negative marking at a 40% institution and **c**=1 then

For **Yes/No-only** quizzes (where **a**=-1) some of the marks are

40, 42, 44, 46, 48	**5** Marks in the 3rd range	
50, 52, 54, 56, 58	**5** Marks in the 2.2 range	
60, 62, 64, 66, 68	**5** Marks in the 2.1 range	
70, 72, 74, 76, 78	**16** Marks in the 1st range	
80, 82, 84, 86, 88		
90, 92, 94, 96, 98	A measure is	[5, 5, 5, 16]
100	compared to	[10, 10, 10, 31]

Note. It is not possible to get the mark 41. The mark 40 comes from 70 answers being correct and 30 in-correct. The mark 42 comes from 71 answers being correct and 29 in-correct which results in the mark 71-29=42.

Thus, there are only even marks for a **Yes/No** quiz when **a**=-1 and all 100 stems are answered. There are thus, about half the marks in each range.

For **4-option-only MCQ** quizzes (where **a**=-1/3) some of the marks are

40, 41 +1/3, 42 + 2/3, 44, 45 +1/3, 46 + 2/3, 48, 49 +1/3
8 Marks in the 3rd range
50+2/3, 52, 53 +1/3, 54 +2/3, 56, 57 + 1/3, 58 + 2/3
7 Marks in the 2.2 range
60, 61 +1/3, 62 +2/3, 64, 65 +1.3, 66 +2/3, 68, 69 +1/3
8 Marks in the 2.1 range
70 +2/3, 72, 73 +1/3, 74 +2/3, 76, 77 + 1/3, 78 +2/3
23 Marks in the 1st range
80, 81 +1/3, 82+ 2/3, 84, 85 + 1/3, 86 +2/3, 88, 89 +1/3
90 +2/3, 92, 93 +1/3, 94 +2/3, 96, 97 + 1/3, 98 + 2/3

100	A measure is	[8, 7, 8, 23]
	compared to	[10, 10, 10, 31]

The mark 40 comes from 55 stems being answered correctly and 45 in-correctly (and then the mark is 55*1 -45/3=40).

We see that pass marks are affected by the *type* of quiz and the conclusion is that if you tackle the guessing problem *exactly* by negative marking then the marks you get do depend on the quiz type, but that results in the number of marks in grades being altered.

5.6.2.1 Difficulties of using negative marking.

There are at least 3 problems associated with negative marking:

- Is it 'fair' to fail when 69% of the stems of a 'special' quiz are answered correctly and 31% answered in-correctly
- Is it 'fair' to *assume* that guessing has gone on?
- How do you compare grades, when the number of marks in each grade is not the same?

Negative marking gets a 'bad' press (Naeem 2010) and many of her comments are about fairness, the tests taking longer, demoralising students, comparing marks obtained from establishments that use negative marking with those that do not use negative marking.

In quizzes where there are multiple correct answers then a 'partially' correct answer rears its ugly head. We do not consider answers that are 'nearly right'. That consideration is left to future work. Here, we only consider answers that are correct or not.

Another concept is that every option is right and the learner is asked for "the most right" option. We simply consider one option to be right and the others wrong, and so the concept of "the most right" option does not apply.

5.6.3 Raising the pass mark.

If we **only raise the pass mark** then it does not matter whether stems are missed or not. The pass mark is raised to 70, say, and anybody that gets a mark below 70 fails, whether the stems can be missed or not. Two problems associated with raising the pass mark are:

a) it is 'hard' to compare marks obtained by quizzes to those obtained by other assessments. For example, what does a mark of 69 mean when you raise the pass mark to 70? At a 40% institution it is almost a 1st (and many times it will be given as a 1st) when it is obtained by an exam, say, but it is a fail when obtained by a quiz,
b) there are less marks associated with a 1st.

But, what should we raise the pass mark to?

If we raise the pass mark to **69**, some of the marks are

69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**11** Marks in the 3rd range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 2.2 range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**10** Marks in the 2.1 range
100	**1** Mark in the 1st range

	A measure is	[11, 10, 10, 1]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

But, it could be argued that there is only 1 mark in the 3rd range viz. 69, and then the marks are

69	**1** Mark in the 3rd range
70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**10** Marks in the 2.2 range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 2.1 range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**11** Marks in the 1st range
100	

	A measure is	[1, 10, 10, 11]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

If we raise the pass mark to **70**, some of the marks are

70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**10** Marks in the 3rd range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 2.2 range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**10** Marks in the 2.1 range
100	**1** Mark in the 1st range

	A measure is	[10, 10, 10, 1]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

If we raise the pass mark to **71**, some of the marks are

71, 72, 73, 74, 75, 76, 77, 78, 79	**9** Marks in the 3rd range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 2.2 range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**10** Marks in the 2.1 range
100	**1** Mark in the 1st range

	A measure is	[9, 10, 10, 1]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

If we raise the pass mark to **72**, some of the marks are

72, 73, 74, 75, 76, 77, 78, 79	**8** Marks in the 3rd range
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 2.2 range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**10** Marks in the 2.1 range
100	**1** Mark in the 1st range

	A measure is	[8, 10, 10, 1]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

If we raise the pass mark to **80** some of the marks are

80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Marks in the 3rd range
90, 91, 92, 93, 94, 95, 96, 97, 88, 99	**10** Marks in the 2.2 range
100	**1** Mark in the 2.1 range

	A measure is	[10, 10, 1, 0]
compared to	(when the pass mark is 40)	[10, 10, 10, 31]
and	(when the pass mark is 50)	[10, 10, 10, 21]

We do not know what is the optimal Pass Mark and thus cannot answer "What should we raise the Pass Mark to?"

5.6.3.1 Difficulties of raising the pass mark.

We can see that raising the pass mark on its own has its difficulties.

We simply ask "What is the **pass mark** for this piece of work?" Is it 70% or 40% (at a 40% institute)? How do you compare a piece of work which has a pass mark of 70% to a piece of work that has a pass mark of 40%?

There are 'less marks to play with' and this results in comparing marks and grades obtained by quizzes and those obtained by other assessments which is 'hard' because they might have different distributions and mean different things.

5.6.4 Using a more complex quiz + negative marking.

We want to see the effect of using more complex quizzes and using negative marking. If we vary **a** for some 'special' quizzes we end up with the tables below garnered from **MG.xlsx** and/or Appendix 6. For this table we will keep the **P**ass **M**ark (**PM**) constant at 40% (i.e. a 40% institution) and for the next table we shall keep the **PM** constant at 50%.

Table 5-2. Values of μ_{100} and M_g for various values of a, for various quiz types, for quizzes where every stem is answered at a 40% institution.

Quiz type	**Value of μ_{100} and M_g for various values of negative marking and various quizzes at a 40% institution**					
Yes/No	a=0	a=-1/4	a=-2/4	a=-3/4	a=-4/4	a=-5/4
μ_{100}	50	37.5	25	12.5	0	-12.5
M_g	[10, 10, 10, 31]	[8, 8,8, 25]	[7, 7,6, 21]	[6, 6, 5,18]	[5, 5, 5,16]	[4, 5, 4,14]
MCQ	a=0	a=-1/8	a=-2/8	a=-3/8	a=-4/8	a=-1
3-options μ_{100}	33.33	25	16.67	8.33	0	-33.33
M_g	[10, 10, 10, 31]	[9, 9,9, 27]	[8, 8,8, 25]	[7, 7, 8, 22]	[7, 7, 6,21]	[5, 5, 5,16]
4-options	a=0	a=-1/12	a=-2/12	a=-3/12	a=-4/12	a=-1
μ_{100}	25	18.75	12.5	6.25	0	-50
M_g	[10, 10, 10, 31]	[9, 10,9, 28]	[9, 8,9, 26]	[8, 8, 8,25]	[8, 7, 8, 23]	[5, 5, 5,16]
5 options	a=0	a=-1/16	a=-2/16	a=-3/16	a=-4/16	a=-1
μ_{100}	20	15	10	5	0	-60
M_g	[10, 10, 10, 31]	[9, 10,9, 29]	[9, 8,9, 27]	[8, 9, 8,26]	[8, 8, 8, 25]	[5, 5, 5,16]
6 options	a=0	a=-1/20	a=-2/20	a=-3/20	a=-4/20	a=-1
μ_{100}	16.67	12.5	8.33	4.17	0	-66.67
M_g	[10, 10, 10, 31]	[10, 9,10, 29]	[9, 9, 9, 28]	[9, 9, 8,27]	[8, 8, 8, 26]	[5, 5, 5,16]
7-options	a=0	a=-1/24	a=-2/24	a=-3/24	a=-4/24	a=-1
μ_{100}	14.29	10.71	7.14	3.57	0	-71.43
M_g	[10, 10, 10, 31]	[10, 9,10, 29]	[9, 10,9, 28]	[9, 9, 8,27]	[9, 8, 9, 26]	[5, 5, 5,16]
8-options	a=0	a=-1/28	a=-2/28	a=-3/28	a=-4/28	a=-1

μ_{100}	12.5	9.375	6.25	3.125	0	-75
M_g	[10, 10, 10, 31]	[9, 10,10, 29]	[10, 9,9, 28]	[9, 9, 9,28]	[9, 8, 9, 27]	[5, 5, 5,16]
MAQ	**a**=0	**a**=-1/36	**a**=-2/36	**a**=-3/36	**a**=-4/36	**a**=-1
5/2 μ_{100}	10	7.5	5	2.5	0	-80
M_g	[10, 10, 10, 31]	[10,10,9, 30]	[9, 10,9, 29]	[9, 10, 9,28]	[9, 8, 9, 28]	[5, 5, 5,16]
6/3	**a**=0	**a**=-1/76	**a**=-2/76	**a**=-3/76	**a**=-4/76	**a**=-1
μ_{100}	5	3.75	2.5	1.25	0	-90
M_g	[10, 10, 10, 31]	[10,10,10, 30]	[10, 10,9, 30]	[9, 10, 10,29]	[10,9,10, 29]	[5, 5, 5,16]
7/4	**a**=0	**a**=-1/136	**a**=-2/136	**a**=-3/136	**a**=-4/136	**a**=-1
μ_{100}	2.86	2.14	1.43	0.714	0	-94.29
M_g	[10, 10, 10, 31]	[10,10,10, 30]	[10,10,10, 30]	[10, 9, 10,30]	[10,10,9, 30]	[5, 5, 5,16]
8/5	**a**=0	**a**=-1/220	**a**=-2/220	**a**=-3/220	**a**=-4/220	**a**=-1
μ_{100}	1.79	1.34	0.893	0.446	0	-96.43
M_g	[10, 10, 10, 31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[9,10,10,30]	[5, 5, 5,16]
Multiple Yes/No	**a**=0	**a**=-1/124	**a**=-2/124	**a**=-3/124	**a**=-4/124	**a**=-1
5-options μ_{100}	3.125	2.34	1.56	0.781	0	-93.75
M_g	[10, 10, 10, 31]	[10,10,10,30]	[10,10,10,30]	[10,9,10,30]	[10, 10,9,30]	[5, 5, 5,16]
6-options	**a**=0	**a**=-1/252	**a**=-2/252	**a**=-3/252	**a**=-4/252	**a**=-1
μ_{100}	1.56	1.172	0.781	0.39	0	-96.88
M_g	[10, 10, 10, 31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[10, 10,10,30]	[5, 5, 5,16]
7-options	**a**=0	**a**=-1/508	**a**=-2/508	**a**=-3/508	**a**=-4/508	**a**=-1
μ_{100}	0.78	0.59	0.39	0.195	0	-98.44
M_g	[10, 10, 10, 31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[10, 10,10,30]	[5, 5, 5,16]
8-options	**a**=0	**a**=-1/1020	**a**=-2/1020	**a**=-3/1020	**a**=-4/1020	**a=-1**
μ_{100}	0.39	0.293	0.195	0.098	0	-99.2
M_g	[10, 10, 10, 31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[10, 10,10,30]	[5, 5, 5,16]

Table 5-3. Values of μ_{100} and M_g for various values of a, for various quiz types, for quizzes where every stem is answered at a 50% institution.

Quiz type	**Value of μ_{100} and M_g for various values of negative marking and various quizzes at a 50% institution**					
Yes/No	**a**=0	**a**=-1/4	**a**=-2/4	**a**=-3/4	**a**=-4/4	**a**=-5/4
μ_{100}	50	37.5	25	12.5	0	-12.5
M_g	[10, 10, 10, 21]	[8, 8,8, 17]	[7, 6,7, 14]	[6, 5, 6,12]	[5, 5, 5,11]	[5, 4, 5,9]
MCQ	**a**=0	**a**=-1/8	**a**=-2/8	**a**=-3/8	a=-4/8	**a**=-1
3-options μ_{100}	33.33	25	16.67	8.33	0	-33.33
M_g	[10, 10, 10, 21]	[9, 9,9, 18]	[8, 8,8, 17]	[7, 8, 7, 15]	[7, 6, 7,14]	[5, 5, 5,11]
4-options	**a**=0	**a**=-1/12	**a**=-2/12	**a**=-3/12	**a**=-4/12	**a**=-1
μ_{100}	25	18.75	12.5	6.25	0	-50
M_g	[10, 10, 10, 21]	[10, 9,9,19]	[8, 9,8, 18]	[8, 8, 8,17]	[7, 8, 7, 16]	[5, 5, 5,11]
5 options	**a**=0	**a**=-1/16	**a**=-2/16	**a**=-3/16	**a**=-4/16	**a**=-1
μ_{100}	20	15	10	5	0	-60
M_g	[10, 10, 10, 21]	[10, 9,10, 19]	[9, 9,9, 18]	[9, 8, 9,17]	[8, 8, 8, 17]	[5, 5, 5,11]
6 options	**a**=0	**a**=-1/20	**a**=-2/20	**a**=-3/20	**a**=-4/20	**a**=-1
μ_{100}	16.67	12.5	8.33	4.17	0	-66.67
M_g	[10, 10, 10, 21]	[9, 10,9, 20]	[9, 9, 9, 19]	[9, 8, 9,18]	[8, 8, 9, 17]	[5, 5, 5,11]
7-options	**a**=0	**a**=-1/24	**a**=-2/24	**a**=-3/24	**a**=-4/24	**a**=-1
μ_{100}	14.29	10.71	7.14	3.57	0	-71.43
M_g	[10, 10, 10, 21]	[10, 10,9, 20]	[10, 9,9, 19]	[9, 9, 9,18]	[8, 9, 8, 18]	[5, 5, 5,11]

8-options	a=0	a=-1/28	a=-2/28	a=-3/28	a=-4/28	a=-1
μ_{100}	12.5	9.375	6.25	3.125	0	-75
M_g	[10, 10, 10, 21]	[10, 10,9, 20]	[9, 9,10, 19]	[9, 9, 9,19]	[8, 9, 9, 18]	[5, 5, 5,11]
MAQ 5/2	a=0	a=-1/36	a=-2/36	a=-3/36	a=-4/36	a=-1
μ_{100}	10	7.5	5	2.5	0	-80
M_g	[10, 10, 10, 21]	[10,9,10, 20]	[10, 9,10, 19]	[10, 9, 9,19]	[9, 9, 9, 19]	[5, 5, 5,11]
6/3	a=0	a=-1/76	a=-2/76	a=-3/76	a=-4/76	a=-1
μ_{100}	5	3.75	2.5	1.25	0	-90
M_g	[10, 10, 10, 21]	[10,10,10, 20]	[10, 9,10, 20]	[10, 10, 9,20]	[9,10,9, 20]	[5, 5, 5,11]
7/4	a=0	a=-1/136	a=-2/136	a=-3/136	a=-4/136	a=-1
μ_{100}	2.86	2.14	1.43	0.714	0	-94.29
M_g	[10, 10, 10, 21]	[10,10,10, 20]	[10,10,10, 20]	[9, 10, 10,20]	[10,9,10, 20]	[5, 5, 5,11]
8/5	a=0	a=-1/220	a=-2/220	a=-3/220	a=-4/220	a=-1
μ_{100}	1.79	1.34	0.893	0.446	0	-96.43
M_g	[10, 10, 10, 21]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[5, 5, 5,11]
Multiple Yes/No 5-options	a=0	a=-1/124	a=-2/124	a=-3/124	a=-4/124	a=-1
μ_{100}	3.125	2.34	1.56	0.781	0	-93.75
M_g	[10, 10, 10, 21]	[10,10,10,20]	[10,10,10,20]	[9,10,10,20]	[10, 9,10,20]	[5, 5, 5,11]
6-options	a=0	a=-1/252	a=-2/252	a=-3/252	a=-4/252	a=-1
μ_{100}	1.56	1.172	0.781	0.39	0	-96.88
M_g	[10, 10, 10, 21]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[10, 10,10,20]	[5, 5, 5,11]
7-options	a=0	a=-1/508	a=-2/508	a=-3/508	a=-4/508	a=-1
μ_{100}	0.78	0.59	0.39	0.195	0	-98.44
M_g	[10, 10, 10, 21]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[10, 10,10,20]	[5, 5, 5,11]
8-options	a=0	a=-1/1020	a=-2/1020	a=-3/1020	a=-4/1020	a=-1
μ_{100}	0.39	0.293	0.195	0.098	0	-99.2
M_g	[10, 10, 10, 21]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[10, 10,10,20]	[5, 5, 5,11]

As we go *across* these tables **a** increases in negativity, and as we go *down* the tables we try to increase the complexity of the quiz type. We say 'try', because for example, a 5-option Multiple Yes/No-only quiz is not as complex as a 7-option MAQ-only quiz with 4 ticks.

We could conclude that using 5 or more option Multiple Yes/No-only quiz with **a** = 0 will do (as we cannot mark to that accuracy) but we have already seen that using a Multiple Yes/No quiz the marks are 'bunched' and the distribution of marks is skewed, and then using negative marking as well (as long as we do not overdo it) reduces the 'small' mean mark to zero. Thus, using a 5 or more option Multiple Yes/No-only quiz and using negative marking is not much different to using a 5 or more option Multiple Yes/No-only quiz with **a** = 0.

Using 2 ways of combating the guessing problem compounds the difficulties associated with just using 1 way of combating the guessing problem.

5.6.5 Using a more complex quiz + raising the pass mark.

We want to see what the effect of using more complex quizzes and raising the pass mark is, whilst keeping **a** = 0. Thus, we will draw a table shown below:

Table 5-4. The effect of a) using a more complex quiz and b) raising the pass mark.

Type of quiz			Raising the pass mark to				
			70	**71**	**72**	**73**	**80**
Yes/No -only		μ_{100}	50	50	50	50	50
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
MCQ	4	μ_{100}	25	25	25	25	25
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	5-options	μ_{100}	20	20	20	20	20
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	6-options	μ_{100}	16.67	16.67	16.67	16.67	16.67
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
MAQ	**5**-options-**2** ticks	μ_{100}	10	10	10	10	10
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	6-options-**3** ticks	μ_{100}	5	5	5	5	5
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	7-options-4 ticks	μ_{100}	2.86	2.86	2.86	2.86	2.86
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
Multiple Yes/No	**6**	μ_{100}	1.56	1.56	1.56	1.56	1.56
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	7-options	μ_{100}	0.78	0.78	0.78	0.78	0.78
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]
	8-options	μ_{100}	0.39	0.39	0.39	0.39	0.39
		M_g	[10, 10, 10, 1]	[9, 10, 10, 1]	[8, 10, 10, 1]	[7, 10, 10, 1]	[10, 10, 1, 0]

We see, that:

- the mean mark is not affected by raising the pass mark, but the mean mark does depend on the type of quiz.
- $\mathbf{M_g}$ does not depend on the quiz type, but does depend on what mark the pass mark is

Raising the pass mark has the effect that that there are 'less marks to play with' and hence the number of marks in grades is reduced

Using a 6-option Multiple Yes/No-only quiz (for example) with **a** = 0 and the pass mark raised to 72 results in the mean mark being 1.56 and there being 29 pass marks and thus 101-29 = 72 failure marks (72/101 = 71.29%), which are distributed [8, 10, 10, 1]. Whereas, for an essay for example, the mean mark is unknown, but is most probably higher than 1.56, and number of pass marks is 61 (at a 40% institution) and distributed [10, 10, 10, 31]. This results in the comparison of marks obtained by a 6-option Multiple Yes/No-only quiz with raised pass mark, to marks obtained from other sources difficult.

Using a more complex quiz and raising the pass mark results in the mean mark going down and there being less marks 'to play with'.

5.6.6 Negative marking + raising the pass mark.

We want to see the effect of using negative marking and raising the pass mark, whilst keeping the quiz type constant. We will keep the quiz type constant at **Yes/No-only** quiz. The measures we have are μ_{100}, M_g. Thus, we will draw the table shown below

Table 5-5. Tackling the guessing problem by a) negative and b) raising the pass mark.

Pass mark (PM)		a=0	a=-0.25	a=-0.5	a=-0.75	a=-1
40	μ_{100}	50	37.5	25	12.5	0
	M_g	[10, 10, 10, 31]	[8, 8, 8, 25]	[7, 7, 6, 21]	[6, 6, 5, 18]	[5, 5, 5, 16]
50	μ_{100}	50	37.5	25	12.5	0
	M_g	[10, 10, 10, 21]	[8, 8, 8, 17]	[7, 6, 7, 14]	[6, 5, 6, 12]	[5, 5, 5, 11]
70	μ_{100}	50	37.5	25	12.5	0
	M_g	[10, 10, 10, 1]	[8, 8, 8, 1]	[7, 7, 6, 1]	[6, 6, 5, 1]	[5, 5, 5, 1]
71	μ_{100}	50	37.5	25	12.5	0
	M_g	[9, 10, 10, 1]	[7, 8, 8, 1]	[6, 7, 6, 1]	[5, 6, 5, 1]	[4, 5, 5, 1]
72	μ_{100}	50	37.5	25	12.5	0
	M_g	[8, 10, 10, 1]	[6, 8, 8, 1]	[5, 7, 6, 1]	[5, 6, 5, 1]	[4, 5, 5, 1]
73	μ_{100}	50	37.5	25	12.5	0
	M_g	[7, 10, 10, 1]	[5, 8, 8, 1]	[5, 7, 6, 1]	[4, 6, 5, 1]	[3, 5, 5, 1]
74	μ_{100}	50	37.5	25	12.5	0
	M_g	[6, 10, 10, 1]	[4, 8, 8, 1]	[4, 7, 6, 1]	[3, 6, 5, 1]	[3, 5, 5, 1]
80	μ_{100}	50	37.5	25	12.5	0
	M_g	[10, 10, 1, 0]	[8, 8, 1, 0]	[7, 6, 1, 0]	[6, 5, 1, 0]	[5, 5, 1, 0]

As before, the mean mark is not affected by raising the pass mark, but it is affected by the value of **a**. The mean is not changed when we go down the table but it does change when we go across the table.
These results were generated by the Excel™ program, held in the file **MG.xlsx,** which is described in Appendix 6.

Using the two methods of tackling the guessing problem viz. negative marking and raising the pass mark, simply results in the mean mark going down and there being less marks to 'play with'.

5.6.7 Using a more complex quiz + using negative marking + raising the pass mark.

If one tries all three ways of tackling the guessing problem at the same time, one arrives at the tables shown below (again, garnered from Appendix 6)

Table 5-6. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a Yes/No-only quiz.

Quiz type			**Value of *a***					
Yes/No-only	**Pass Mark**	**Measures**	**0**	**-0.25**	**-0.5**	**-0.75**	**-1**	**-1.25**
	40	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[10,10,10,31]	[8,8,8,25,]	[7,7,7,21]	[6,6,6,18]	[5,5,5,16]	[4,5,4,14]
	50	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[10,10,10,21]	[8,8,8,17]	[7,6,7,14]	[6,5,6,12]	[5,5,5,11]	[5,5,4,9]
	70	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[10,10,10,1]	[8,8,8,1]	[7,7 6,1]	[6,6,5,1]	[5,5,5,1]	[5,4,4,1]
	71	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[9,10,10,1]	[7,7,8,1]	[6,7,6,1]	[5,6,5,1]	[4,5,5,1]	[4,4,4,1]
	72	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[8,10,10,1]	[6,8,8,1]	[5,7,6,1]	[5,6,5,1]	[4,5,5,1]	[4,4,4,1]
	73	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[7,10,10,1]	[5,8,8,1]	[5,7,6,1]	[4,6,5,1]	[3,5,5,1]	[4,4,4,1]
	74	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[6,10,10,1]	[4,8,8,1]	[4,7,6,1]	[3,6,5,1]	[3,5,5,1]	[3,4,4,1]
	80	μ_{100}	50	37.5	25	12.5	0	-12.5
		M_g	[10,10,1,0]	[8,8,1,0]	[7,6,1,0]	[6,5,1,0]	[5,5,1,0]	[4,4,1,0]

Table 5-7. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 5-option-only MCQ quiz.

Quiz type			Value of *a*					
5-option MCQ	**Pass Mark**		**0**	**-1/16**	**-2/16**	**-3/16**	**-1/4**	**-1**
	40	μ_{100}	20	15	10		0	-60
		M_g	[10,10,10,31]	[9,10,9,29]	[9,9, 8,27]	[8,9, 8,26]	[8,8, 8,25]	[5,5, 5,16]
	50	μ_{100}	20	15	10	5	0	-60
		M_g	[10,10,10,21]	[10,9, 10,19]	[9,9, 8,18]	[9,8, 9,17]	[8,8, 8,17]	[5,5, 5,11]
	70	μ_{100}	20	15	10	5	0	-60
		M_g	[10,10,10,1]	[9,9, 10,1]	[9,9, 8,1]	[9,8, 9,1]	[8,8, 8,1]	[5,5, 5,1]
	71	μ_{100}	20	15	10	5	0	-60
		M_g	[9,10,10,1]	[9,9, 10,1]	[8,9, 8,1]	[8,8, 8,1]	[7,8, 8,1]	[4,5, 5,1]
	72	μ_{100}	20	15	10	5	0	-60
		M_g	[8,10,10,1]	[8,9, 9,1]	[7,9, 8,1]	[7,8, 8,1]	[6,8, 8,1]	[4,5, 5,1]
	73	μ_{100}	20	15	10	5	0	-60
		M_g	[7,10,10,1]	[7,9, 9,1]	[7,9, 8,1]	[6,8, 8,1]	[5,8, 8,1]	[3,5, 5,1]
	74	μ_{100}	20	15	10	5	0	-60
		M_g	[6,10,10,1]	[6,9, 9,1]	[6,9, 8,1]	[5,8, 8,1]	[4,8, 8,1]	[3,5, 5,1]
	80	μ_{100}	20	15	10	5	0	-60
		M_g	[10,10,1,0]	[9,9, 1,0]	[9,8, 1,0]	[8,8, 1,0]	[8,8,1,0]	[5,5, 1,0]

These are only two of many tables taken from Appendix 6 and show that using all three methods of tackling the guessing problem:

a) reduces the mean mark, as **a** is reduced (but it must not be over-done)
b) the mean mark does depend on the quiz type
c) raising the pass mark does reduce the number of marks to 'play with'.

What we have shown is that the ways of tackling the guessing problem just compound the difficulties associated with only 1 way of tackling the guessing problem.

5.7 Discussion.

The problems of just using a more complex quiz are to do with:

a) the distribution of marks being positively skewed,
b) simply writing (and testing) quizzes that obey all the rules e.g. stems and options are as independent as they can be.

The problems of just using negative marking are numerous:

a) Is it 'fair' to only get 40 when 70 stems have been answered correctly?
b) Is it 'fair' to assume that guessing is going on?
c) Is it 'fair' to put off able students, because they do not like guessing?
d) Is it 'fair' that completing a quiz that has negative marking takes longer to complete, than one that does not (again, because of the dislike of guessing)?
e) Is it 'fair' that one institute uses negative marking and another does not?
f) The number of marks in a grade is different from when negative marking is not used

The problems of just raising the pass mark are:

a) what does the mark 69 now mean? (it is a fail when the Pass Mark is raised to 70)
b) the number of marks in a grade is different from when the pass mark is not raised.

When one uses a more complex quiz with negative marking one has all the problems associated with a complex quiz PLUS all the problems associated with negative marking. For example, the marks are positively skewed AND the number of marks in the pass grade are reduced.

When one uses a more complex quiz with raising the pass mark one has all the problems associated with a complex quiz PLUS all the problems associated with raising the pass mark. For example, the marks are positively skewed AND

69 now means a fail, whereas before it most probably was a 1st e.g. for an essay (at a 40% institution).

When one uses negative marking with raising the pass mark one has all the problems associated with a negative marking PLUS all the problems associated with raising the pass mark. For example, is it 'fair' to get a mark of 40 when 70 stems were answered correctly AND 69 now means a fail, whereas before it most probably was a 1st (at a 40% institution).

If one uses all 3 ways of tackling the guessing problem at the same time one has problems associated with a complex quiz PLUS all the problems associated with a negative marking PLUS all the problems associated with raising the pass mark.

Thus one augments the problems when one tries a combination of ways of tackling the guessing problem.

All the ways of tackling the guessing problem have one difficulty in common viz. they make comparison of marks obtained by quizzes 'hard' to compare to marks obtained by other forms of assessment.

When we ask "Does using negative marking result in the same problems that raising the pass mark does?" we now see that they result in different problems, but they have a problem in common viz. the number of marks in a grade. We modify the question to "Does using negative marking or raising the pass mark result in the same number of marks in each grade?" and the answer is "No", we simply have to look at Table 5-5 to see this where the number of marks in each passing grade is not the same when **a** = -1 and PM = 40 to where **a** = 0 and PM = 72

The proofs help us to write programs for quizzes where every stem is answered. For example, a tool (i.e. a program) has in it a loop for **w** going from 0 to **n** in steps of 1. Before the loop we ask for the values of **a** and **n**. Inside the loop we calculate a mark that corresponds to **w**, **a** and **n**. Appendices 12 and 15 show the coding for the two tools from which the following fragment is taken

```
//Input a
                correctInput=false;
                while (!correctInput)
                {
                        cout << "NEGATIVE marking (a). Input either 0 for no
negative marking, or a fraction something like 1.0, or 0.5. This fraction is
usually negative"  << endl;
                        cin >> a;
                        correctInput=true;
                }
```

Because the marks are unique i.e. each mark correspond to only 1 value of **w**, it does not require an addition to be done (if a mark can be obtained in 2 or more different ways, then an addition of marks for that way is necessary). The number obtaining a mark marks is symmetric (e.g. the number obtaining the mark 1 = the number obtaining the mark 99) and so this means that the number obtaining a mark starts low, goes up and ends up low.

The proofs also help us with validation of our programs. For example, if the gap between successive marks is not |1 - **a**| then the programs are wrong.

5.7.1 Is M_g a good measure?

$\mathbf{M_g}$ is a measure of how many marks there are in a passing grade, but it does not measure the proportion of passes there are. This requires that tools are built and that is what most of the rest of the thesis is about. $\mathbf{M_g}$ is what we can do *without* tools being built, but it does not measure the pass rate e.g. the ratio of the number of passes to the total number taking a test. To measure the pass rate (or the failure rate), tools have to be built.

The calculation of $\mathbf{M_g}$ meant that Excel™ had to be learnt and that involved reading Walkenbatch (2010) and knowing Excel's™ limitations from Microsoft (2012).

5.8 Conclusion.

This chapter is about the Analysis that can be done on 'special' quizzes where ALL stems are answered.

The *mean* mark μ_n = **n***(1 +X***a**)/(X + 1). For example when **a**=0 and **n**=100 and X=1, μ_n = 50. We notice that under these conditions as X gets bigger μ_n gets smaller until it is about 0 (c.f. Table 5-1).

There are 3 proofs which apply, here, and they and with the above can be used to validate the programs that we write. Sometimes, what follows from the proofs can be used also.

- Proof 1. The gap between successive marks is |1 - **a**|. (See Appendix 5).
- Proof 2. The number of impossible marks is 0. (See Appendix 5).
- Proof 3. The number of marks is (**n** + 1) (except where **a** = 1). (See Appendix 5).

From these proofs we see that:

- The distribution of marks is 'stretched', as **a** varies and X is kept constant. The mean mark is changed, and anything that depends on the mean is also changed (variance, standard deviation, skewness and kurtosis).
- There is no need to do an experiment to see if the rank-order of people depends on the value of **a**.

Using a more complex quiz, on its own (to tackle the guessing problem) leads to extra problems e.g. how you compare the marks obtained by a quiz to those obtained by essays.

Using negative marking, on its own (to tackle the guessing problem) leads to extra problems e.g. a) how you compare the marks obtained by a quiz to those obtained by essays b) is it fair?

Raising the pass mark, on its own (to tackle the guessing problem) leads to extra problems e.g. a) how you compare the marks obtained by a quiz to those obtained by essays.

Using these measures in tandem only compounds the problems.

So, no matter what we do to tackle the guessing problem leads to more problems.

Notice that **y** (the number of stems that are missed) does not exist here. It does not have the value 0, otherwise we would use $\mu_n = \mathbf{n}*(1 + X*\mathbf{a})/(X + 2)$ for the mean mark.

5.9 What is coming next?

Chapter 6 contains that Analysis when stems can be missed

i.e. $\mu_n = \mathbf{n}*(1 + X*\mathbf{a} + \mathbf{b})/(X + 2)$.

Chapter 6. Analysis when all stems are NOT answered.

Abstract.

The distribution of marks is not always symmetric when stems can be missed.

X is the number of in-correct answers that each stem has and we assume that $X \neq 0$ and to calculate the mean mark we use $\mu_n = \mathbf{n}*(1 + X*\mathbf{a} + \mathbf{b})/(X + 2)$.

The number getting a mark in a particular way is $X^{X}*{}_{n}C_{y}*{}_{n-y}C_{w}$ but there might be 2 or more ways of getting the same mark, and so a Pivot is necessary, before *all* the descriptive statistics can be calculated. This leads to a method where one C program calculates the marks and how many ways there are of getting that mark in that manner. Excel™ does the Pivoting and another C program calculates *all* the descriptive statistics and some other statistics like the proportion of pass marks.

Each way of tackling the guessing problem has its own problems, and using the ways in combination, compounds the problems.

Using a more complex quiz leads to the distribution of marks being positively skewed.

Using a modification of negative marking where **b** = 0, leads to unfairness.

Raising the pass mark leads to:

a) questioning what a mark means,
b) the number of marks in a grade not being the same as when the pass mark is not raised.

6.1 Introduction.

This Chapter is about analysing quizzes, without any tools being built, where NOT every stem is answered. When stems can be missed with **a** = **b** = 0 and **c** = 1 there are many ways of getting the mark 0, but only 1 way of getting the mark 100 (for **n** = 100) and so the distribution of marks cannot be symmetric (where [the *mean* mark] = [*mode* mark] = [*median* mark]). But, if the number of ways of getting the mark 50, for example, is 10^6 and the number of ways of getting the mark 0 is 50 and the number of ways of getting the mark 100 is 1, then the number of ways of getting the mark 50 'swamp' the ways of getting the marks 0 and 100, the distribution of marks might *appear* to be symmetric.

Accompanying this Chapter are 2 Appendices:

- Appendix 6: Three proofs of quizzes where stems can be missed.
- Appendix 8: Validation of the analysis that we do.

The marks are not sorted and the number that get a mark, in a particular way, requires an addition and so a Pivot is needed when stems can be missed. This leads to a change of method. Even if a Pivot is not needed (in the case of all stems being answered) we can do a Pivot without any harm being done.

The number getting a mark in a particular way is generalised to $X^x * {}_nC_y * {}_{n-y}C_w$ where X is the number of in-correct answers to a stem and represents the quiz type, and the other variables have been defined in earlier Chapters. For example, when

X = 1 we have a Yes/No-only quiz

X = 2 we have an MCQ-only quiz with 3 options.

If X = 0 we have no in-correct answers to a stem and that leads to inconsistencies and so we assume that there is at least 1 wrong answer to a stem. When X = 0 then **P** = **T**, because ${}_PC_P = 1$, and then $({}_PC_P - 1) = 0$. So, we assume that for 'special' quizzes the number of options does not equal the number of ticks. The inconsistencies include the fact that when **P** = **T** there is only 1 mark viz. 100%.

We ask the user for **n** and vary **y** and **w**. Then **x** is *given* because

$$\mathbf{n} = \mathbf{w} + \mathbf{x} + \mathbf{y}$$

hence $$\mathbf{x} = \mathbf{n} - \mathbf{w} - \mathbf{y}$$

and **x** is impossible when **x** is negative, because **x** (the number of stems answered in-correctly) is a strictly positive integer (see later).

We prove in Appendix 6 that the gap between successive marks is NOT the constant |1 -**a**| where **a** = the score when a stem is answered in-correctly. In the same Appendix we also prove that the number of impossible marks is the sum of **w** (where **w** = the number of stems answered correctly, and an impossible mark is where the conditions are such that it is impossible to calculate a mark). For example, for **n** = 100 it is impossible to answer 98 stems correctly and miss 3 stems. In the same Appendix we also prove, by exception, that the number of marks on the x-axis is NOT (**n** + 1). We look at **n** = 3, **a** = -1/2, **b** = 0 and conclude that the number of marks is at least 9, which is NOT 4 viz. (**n** + 1).

In Appendix 8 we try to answer "How do you know that your analysis is right?" This is called validation of our analysis, which we do by doing 'hand' calculations for 'small' X and 'small' **n**. We do this validation for X = 1 and **n** = 1 to 5, and X = 3 and **n** = 1 to 4 and partially for **n** = 5. After that we assume that our validation is correct.

Because we are dealing with 'special' quizzes we can calculate their mean mark, μ_{100}, but we cannot use Excel™ to help us calculate how many marks there are because there are not a fixed number of marks for a given **n**. For example, for a given **n**, there are about 10 marks between 40 and 50 when **a** = -1, and about 20 marks when **a** = -1/2 and about 30 when **a** = -1/3.

The guessing problem can be tackled by a combination of:

- using a more complex quiz,
- using a modification of negative marking where **b** = 0,
- raising the pass mark.

but each combination leads to more problems.

6.2 Methods/Tool.

The **methods** that we are going to use in this Chapter are, in the main *quantitative*.

The **tool** that we use to help the analysis is the widely available Excel™ using file **Mean Mark when stems are missed.xlsx**.

6.3 Mark.

When stems can be missed the mark obtained is

m = **w*c** + **x*a** + **y*b** when each stem is scored **a**, **b**, **c**.

If we assume that **c** = 1 and **b** = 0, then

m = **w** + **x*a**

As **n** = **w** + **x** + **y** and if we assume that **n** is given by a user, then **n** is a constant and there is 1 less degree of freedom, and for example **x** = **n** - **w** - **y** and so **m** = **w** + **(n** - **w** - **y)*a**.

The quiz type is either $_PC_T$ or 2^P. **X** *represents* the quiz type, because it is either ($_PC_T$-1) or (2^P -1) and ($_PC_T$-1) and as (2^P -1) are both strictly positive and so X is strictly positive.

An *impossible* situation occurs when **x** is negative, and **x** is an integer going from 0 to **n** in steps of 1.

When **a** = **b** = 0, **m** = **M** = **w**, i.e. the marks are the same as **w**, and **w** goes from 0 to **n** in steps of 1, and so the marks include 40, 41 and 42. It does *not* mean that the mean mark is 50. The mean mark depends on *how many* obtain a mark i.e. the mean mark depends on the distribution of marks. If the distribution of marks is *symmetric* (Investopedia 2011), (Symmetrical distribution 2013) then the mean, mode and median are coincident i.e. they occur at the same mark. With **a** = **b** = 0 and **c** = 1 the distribution of marks cannot be symmetric, as there are at least 2 ways of getting the mark 0 (i.e. miss all the stems, answer all the stems in-correctly), but only 1 way of getting the mark 100 (when **n** = 100) i.e. answer all the stems correctly.

Thus, the distribution of marks is not always symmetric when stems can be missed.

Another example, is when **a** = -1 and with **b** = 0, **c** = 1 and **n** = 100 and for any 'special' quiz there are many ways of getting the mark 0 (e.g. answer 2 stems correctly, 2 stems in-correctly and miss 96), but still only 1 way of getting the mark 100.

6.4 The number of ways of getting a mark.

The number of ways of getting a mark in a particular way depends on:

a) the quiz type,
b) the number of stems, **n**,
c) the number of stems that are answered correctly, **w**,
d) the number of stems that are missed , **y**.

Thus, the number of ways of getting a mark m in a particular way

For **Yes/No-only** quizzes = ${}_{n}C_{y} * {}_{n-y}C_{w}$.

For **P-option MCQ-only** quizzes = $(\mathbf{P}-1)^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$.

For **P-option MAQ-only** with **T** ticks quizzes = $({}_{P}C_{T}-1)^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$.

EMSQ quizzes are either MCQs or MAQs.

For **Multiple Yes/No-only** quizzes = $(2^{P}-1)^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$.

A particular way means "for a particular combination of **y** and **w**".

Again, we see that we can generalise and see that quizzes fall into 2 camps:

- MAQs with MCQs as a special case where **T** = 1, and EMSQs are either MCQs or MAQs
- Multiple Yes/Nos with Yes/Nos being a special case where **P** = 1.

We can generalise still further by saying that the number of in-correct answers to a stem (if the user always uses exactly **T** ticks for MAQ quizzes, and **P ≠ T**) is X and then the number of ways of getting a mark in a particular way is

$X^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$.

For example

for **Yes/No-only** quizzes $X = 1$

for **P-option MCQ-only** quizzes $X = \mathbf{P}-1$

for **P-option MAQ-only** quizzes, with T ticks $X = {}_{P}C_{T}-1$

if $\mathbf{T} = 1$ ${}_{P}C_{T} = \mathbf{P}$ and so MCQs are a special case of MAQs

for **Multiple Yes/No-only** quizzes $X = 2^{P}-1$

if $\mathbf{P} = 1$, $2^{P} = 2$ and so Yes/Nos are a specialisation of Multiple Yes/Nos where **P**=1)

EMSQ quizzes (MCQ) are MCQs where **P** is 'large' e.g. 20

EMSQ quizzes (MAQ) are MAQs where **P** is 'large'. What **T** is, is not defined.

We have explained most of it. When **y** stems are allowed to be missed, the number of ways of missing stems is ${}_{n}C_{y}$. Because **y** stems are missed there are (**n** - **y**) stems left and that is where ${}_{n-y}C_{w}$ comes from. If you do not like this explanation then Appendix 8 covers the validation of these formulae.

6.4.1 Why is a Pivot necessary?

A Pivot is necessary because there are many ways of getting a mark and the marks are not sorted, there is a need for a Pivot to be done that sorts the marks and adds together the number that get that mark in a particular manner. The necessity of a Pivot is exemplified in the table below.

Table 6-1. Why a Pivot is necessary.

Number of Stems (n)	Mark (m)	Number of stems missed (y)	Number of stems answered correctly (w)	Number of stems answered in-correctly (x)	Number of ways of getting that mark, in that manner (Y_m)
MCQ with 3 options	0	0	0	3	All wrong **27**
	1	0	1	2	1 right, 2 wrong **27**
	2	0	2	1	2 right 1 wrong **9**
	3	0	3	0	All right **1**
	0	1	0	2	1 missed, 2 wrong **27**
	1	1	1	1	1 missed, 1 right, 1 wrong. For each stem right there are 6 possibilities , and there are 3 ways of getting a stem right **18**
	2	1	2	0	1 missed, 2 right **3**
		1	3	-1	**Impossible**
	0	2	0	1	2 missed, 1 wrong **9**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

We see, for example, that the mark 0 appears many times, and each time it appears (except the first) requires that an addition is performed. Excel™ and can do a Pivot.

Because the **ms** are not sorted and their associated $\mathbf{Y_m}$s are not added (when their **ms** are equal), there is a necessity to do a Pivot, which sorts the **ms** and adds their associated $\mathbf{Y_m}$s. For example, if there are 3 ways to get the mark 0 and their associated numbers are 100, 200 and 300, then after a Pivot we should get 600 for the total number of ways that the mark 0 can be obtained.

The 'grey' column is for the *given* **x** (**n = w + x + y**). When **n**, **w** and **y** are known then **x** is given. An impossible situation is when **x** is negative.

6.4.2 What method does that lead to?

We ask the user for quiz type (that reflects the value of X), **n** (the number of stems), **a** (how each of the **n** stems is scored if the stem is answered incorrectly), **b** (how each of the **n** stems is scored if the stem is missed), **P** (the number of *options* that each stem has), **T** (the number of *ticks* that each of the **n** stems has). Then we vary **w** and **y**.

The method now becomes:

a) find out the quiz type, **n**, **P**, **T**, **a** and **b** (so that **m** can be calculated),
b) vary **w** from 0 to **n** in steps of 1 in a loop,
c) vary **y** from 0 to **n** in steps of 1 in a loop inside the varying of **w**, and calculate **m** and $\mathbf{Y_m}$ inside this loop,
d) do a Pivot which sorts the **ms** and adds the associated $\mathbf{Y_m}$s because the **ms** are not in any order (this produces **M** and $\mathbf{Y_M}$),
e) draw $\mathbf{Y_M}$ v **M**,
f) calculate descriptive and other statistics of the distribution of $\mathbf{Y_M}$ v **M**.

Our calculation of the mode assumes that the $\mathbf{Y_M}$s are added, because we go from the lowest **M** to the highest **M** an ask "is the associated $\mathbf{Y_M}$ bigger than the largest found so far?" Our calculation of the median works from the lowest **M** to the highest and adds their associated $\mathbf{Y_M}$s and then asks "Have we passed the halfway point?" if we have then the previous mark is the median.

6.5 Proofs that apply when stems can be missed.

We have 3 proofs here:

- Proof 1: The gap between successive marks is NOT |1 - **a**|. (See Appendix 6).
- Proof 2: The number of impossible marks is an AP of **w**. (See Appendices 1, 2, 3 and 6).
- Proof 3: The number of marks is NOT (**n** + 1). (See Appendix 6).

The results of these proofs can be used as validation of our programs. For example:

a) if our programs do not produce the sum of an AP of **w** for the number of impossible marks, the programs are wrong ,

b) if our programs do show that the gap between successive marks is |1 - **a**| then the programs are wrong.

6.6 The mean mark.

Using Straker's (2012) proof (see Appendix 4) we have $\mu_n = n*(1 + X*a + b)/(X + 2)$ and then we have

For **Yes/No-only** quizzes X = 1
so $\mu_{100} = 100*(1 + a + b)/(3)$
when **a** = 0 and **b** = 0 $\mu_{100} = 33.33$

For **P-option-only MCQ** quizzes X = (**P**-1)
so $\mu_{100} = 100*(1 + (P - 1)*a + b)/(P+1)$
when **a** = **b** = 0, **P** = 4, $\mu_{100} = 20$

For **P-option MAQ quizzes with T ticks** only X = $({}_PC_T - 1)$
so $\mu_{100} = 100*(1 + ({}_PC_T - 1)*a + b)/({}_PC_T + 1)$
when **a** = **b** = 0, **P** = 5, **T** = 2 $\mu_{100} = 9.09$

For **P-option-only Multiple Yes/No** quizzes X = $(2^P - 1)$
so $\mu_{100} = 100*(1 + (2^P - 1)*a + b)/(2^P + 1)$
when **a** = **b** = 0, **P** = 5, $\mu_{100} = 3.03$.

We can use this analysis to help us with the validation of the programs we write. For example, if our program does not produce 20 as the mean for a 4-option MCQ-only quiz (where stems can be missed) then it is wrong.

"False positive" and "false negative" errors are called Type I and Type II errors (Shuttleworth 2011). A Type I error is where we believe something to be true when it is not. A Type II error is when we fail to accept something is true when it is true. They are both to do with hypothesis testing and the probability of an hypothesis being accepted when a *sample* is done and as we are NOT doing sampling there is no possibility of Type I and Type II errors.

6.7 Tackling the guessing problem.

The ways of tackling the guessing problem, include:

- Using a more complex quiz,
- Using a modification of negative marking where **b** = 0,

- Raising the pass mark.

As before, each of these ways of tackling the guessing problem brings with it more problems.

6.7.1 Using a more complex quiz.

When a **'more complex' quiz** is used, we draw a table of more complex quizzes and assume that **b** = 0 (in line with one argument).

Table 6-2. Mean mark of various quizzes when **a**=0 and **b**=0, **n**=100 where all stems are NOT answered.

Type of quiz	Mean Mark i.e. μ_{100} when n=100, and a=0 b=0 and c=1	Comment
Yes/No- only	**33.33**	
MCQ 3-option -only	**25**	
4-option-only	**20**	Same as a 2-option Multiple Yes/ No
5-option-only	**16.67**	
6-option-only	**14.29**	
7-option-only	**12.5**	
8-option-only	**11.11**	Same as a 3-option Multiple Yes/ No
MAQ 5-option with 2 ticks -only	**9.09**	Same as a 10-option MCQ
6-option with 3 ticks -only	**4.76**	Same as a 20-option MCQ
7-option with 4 ticks -only	**2.78**	
8-option with 5 ticks -only	**1.75**	
Multiple Yes/No 5-option -only	**3.03**	
6-option-only	**1.54**	
7-option-only	**0.78**	
8-option-only	**0.39**	
EMSQ 20-option (MCQ)	**4.76**	
20-option (MAQ with 10 ticks)	**0.00541251**	About 0

Minimum mark for them all = 0. Maximum mark for them all = 100
$\mathbf{M_g}$ at a 40% institution is [10, 10, 10, 31] for them all, and
at a 50% institution is [10, 10, 10, 21] for them all

6.7.1.1 Difficulties of using a more complex quiz.

As we go down this table, it looks as though, again, you could use a 5 or more option *Multiple* Yes/No quiz (because it is doubtful if one can mark to an accuracy of 3 or less), but you might, now, use 5 or more option MAQ quizzes (as long as you are careful about complexity and use an appropriate number of ticks). But then:

a) you have got to write and *test* 5 or more options,
b) taking a 6-option Multiple Yes/No-only quiz as an example, the mean mark is 1.54 and the minimum mark is 0.

Thus, half the pseudo area of the distribution of marks is between the marks 0 and 1.54 and is 'bunched' at the bottom end. The distribution of marks is said to be positively skewed, and the comparison of marks obtained by essays, say, and the marks obtained by quizzes, is 'hard' (as they may have different distributions), hence using a 'more complex' quiz leads to problems.

6.7.2 Using a modification negative marking.

If we look at **using a modification of negative marking** (where **b** = 0) then we ask "What should **a** be?" The answer is **a** = $-1/({}_PC_T -1)$ for non-multiple Yes/No quizzes and $-1/(2^P -1)$ for Multiple Yes/No quizzes. To make μ_n into 0 we could have **b** = 0 and then **a** is the same as before. M_g is a matrix holding the number of marks in each passing grade but can the number of marks in a grade, now, be calculated (without tools being built)? The marks, now, depend on stems being missed. For example, when **n** = 100, **b** = 0, **c** = 1, **a** = -1 (for a Yes/No-only quiz) some of the marks are

40, **41**, 42, 43, 44, 45, 46, 47, 48, 49	**10** Marks in the 3rd grade (at a 40% institution)

The mark 41 comes from answering 41 stems correctly and missing 59. The mark 42 can come from answering 42 stems correctly and missing 58, but it can also be got by answering 71 stems correctly and 29 stems in-correctly. In addition it can also be got by answering 62 stems correctly 20 in-correctly and missing 18. There are other ways. It is not clear whether the number of marks in a grade can be calculated by knowing the values of **n**, **a**, **b** and **c**. Let us take another example, where **n** = 100, **b** = 0, **c** = 1, and **a** = -1/2 (for a Yes/No-only quiz). **a** does not have be integer/integer: it could be 2.7/3.4, for example. Some of the marks when **a** = -1/2 are

40, 40+0.5, 41, 41 +0.5, 42, 42 +0.5, 43, 43 +0.5, 44, 44 +0.5, 45,
45 +0.5, 46, 46 +0.5, 47, 47 +0.5, 48, 47 +0.5, 49, 49+0.5

20 Marks in the Pass grade (at a 40% institution).

The mark 40 can be obtained by answering 40 stems correctly, and missing 60 (the mark 40 can also be obtained in numerous other ways). The mark 40+0.5 can be obtained by 41 stems correctly, 1 stem in-correctly and missing 58.

When **a** = -1/10 the marks include

40, 40 +1/10, 40 +2/10.

Now there are about 100 marks in the Pass grade (at a 40% institution) and can we use Excel™ to help us to count how many marks there are in the Pass Grade when the number of marks depends on the value of **a**? We certainly cannot use Excel™ in the same manner as before, where we proved that there were 101 marks (for **n** = 100) no matter what **a** is (except when **a** = 1), and for a given Pass Mark count how many marks are in Pass, 2.2. 2.1 and 1st grades.

Now, if the Pass Mark is 40 **a** = -1 then there are 10 marks in the 3rd grade
there are 10 marks in the 2.2 grade
there are 10 marks in the 2.1 grade
there are 31 marks in the 2.1 grade
The same number of marks as when **a** = 0.

Now, if the Pass Mark is 40 **a** = -1/2 then there are 20 marks in the 3rd grade
there are 20 marks in the 2.2 grade
there are 20 marks in the 2.1 grade
there are about 61 marks in the 1st grade.

So, comparing marks between various types of assessment is 'hard' when they have different numbers of marks in each grade.

Table 6-3. Values of μ_{100} for various values of **a**, with **b**=0, for various quiz types, for quizzes where every stem is NOT answered.

Quiz type	**μ_{100} while varying *a* and keeping *b*=0**					
Yes/No	a=0	a=-0.25	a=-0.5	a=-0.75	a=-1	a=-1.25
μ_{100}	33.33 ✓	25	6.67	8.33	0	-8.33
MCQ	a=0	a=-1/8	a=-2/8	a=-3/8	a=-1/2	a=-1
3-option μ_{100}	25 ✓	18.75	12.5	6.25	0	-25
4-option-only	a=0	a=-1/12	a=-2/12	a=-3/12	a=-1/3	a=-1
μ_{100}	20✓	15	10	5	0	-40
5-option-only	a=0	a=-1/16	a=-2/16	a=-3/16	a=-1/4	a=-1
μ_{100}	16.67✓	12.5	8.33	4.17	0	-50
6-option-only	a=0	a=-1/20	a=-2/20	a=-3/20	a=-1/5	a=-1
μ_{100}	14.29✓	10.71	7.14	3.57	0	-57.14
7-option-only	a=0	a=-1/24	a=-2/24	a=-3/24	a=-1/6	a=-1
μ_{100}	12.5✓	9.375	6.25	3.125	0	-62.5
8-option-only	a=0	a=-1/28	a=-2/28	a=-3/28	a=-1/7	a=-1
μ_{100}	11.11✓	8.33	5.56	2.78	0	-66.67
MAQ 5-option -only with 2 ticks	a=0	a=-1/36	a=-2/36	a=-3/36	a=-1/9	a=-1
μ_{100}	9.09✓	6.82	4.56	2.27	0	-72.73
6-option-only with 3 ticks	a=0	a=-1/76	a=-2/76	a=-3/76	a=-1/19	a=-1
μ_{100}	4.76✓	3.57	2.38	1.19	0	-85.71
7-option-only with 4 ticks	a=0	a=-1/136	a=-2/136	a=-3/136	a=-1/34	a=-1
μ_{100}	2.78✓	2.08	1.39	0.69	0	-91.67
8-option-only with 5 ticks	a=0	a=-1/220	a=-2/220	a=-3/220	a=-1/55	a=-1
μ_{100}	1.75✓	1.32	0.88	0.44	0	-94.74
Multiple Yes/No-only	a=0	a=-1/124	a=-2/124	a=-3/124	a=-1/31	a=-1
5-option μ_{100}	3.03✓	2.27	1.52	0.76	0	-90.90
6-option	a=0	a=-1/252	a=-2/252	a=-3/252	a=-1/63	a=-1
μ_{100}	1.54✓	1.16	0.77	0.38	0	-95.39
7-option	a=0	a=-1/508	a=-2/508	a=-3/508	a=-1/127	a=-1
μ_{100}	0.78✓	0.74	0.39	0.19	0	-97.67
8-option	a=0	a=-1/1020	a=-2/1020	a=-3/1020	a=-1/255	a=-1
μ_{100}	0.39✓	0.29	0.19	0.097	0	-98.83
EMSQ 20-optiom	a=0	a=-1/76	a=-2/76	a=-3/76	a=-1/19	a=-1
(MCQ) μ_{100}	4.76✓	3.57	2.38	1.19	0	-85.71
20-optiom	a=0	a=-1/4A	a=-2/4A	a=-3/4A	a=-4/4A	a=-1
(MAQ) with 10 ticks μ_{100}	About 0 0.00541✓	About 0 0.0004	About 0 0.00003	About 0 0.00014	0	About -100 -99.99

4A=184755

(This was helped by the Excel™ program in file **Mean Mark when stems are missed.xlsx**).

The **a** = 0 column is the same as Table 5-2 (That's what the tick marks are, and form one of the validations).

The 'grey' column is where the guessing problem is *exactly* tackled (this is another form of validation) and shows yet another reason why stems should not be mixed in the same quiz. They have different **a**'s in order to make μ_{100} into 0 (as long as **b** = 0).

As we go across this table **a** increases in negativity and as we go down the table we increase the complexity of the quiz type. Yet again it looks as though Multiple Yes/No quizzes could be used, and some MAQ quizzes, but, yet again, we can argue that the distribution of marks of these quizzes is positively skewed. Hence, comparison of marks obtained by 'special' quizzes is 'hard' to compare with marks obtained by other forms of assessment.

6.7.2.1 Difficulties of a modification of negative marking.

All of Naeem's (2010) comments about fairness, tests taking longer, demoralising students and comparing marks obtained from establishments that use this modification of negative marking to marks obtained from establishments that do not use this modification, maybe still apply (maybe not), when stems can be missed.

The following remarks are when **a** = -1, **b** = 0, **c** = 1 and **n** = 100

> Is it 'fair' to obtain a mark of 40 (at a 40% institution) when 70 stems are answered correctly and 30 stems answered in-correctly, for any special quiz? If the learner *chooses* to answer all stems we are back to the previous Chapter where all stems are answered.
>
> The mark 40 could also be obtained by answering 40 stems correctly and missing 60 stems. The mark 40 could also be obtained by answering 41 stems correctly, 1 in-correctly and missing 58. In fact there are many ways of getting the mark 40, each way with a different 'fairness'.
>
> As 'fairness' is hard to define, if not impossible, we will cut the argument short.

Are the learners now more likely to miss a stem than answer it in-correctly?

Do the tests take longer?

If tests take 1 hr, say, then they do not take any longer. They last for 1 hr and any stems not answered are missed.

Are good students demoralised by this modification of negative marking?

If they can miss a question with no penalty are they demoralised?

Is it 'fair' to *assume* that guessing is going on? Some learners might genuinely believe they have answered a stem correctly. But, how do you deal with elevated marks when guessing might have gone on?

Is it 'better' to not guess and just answer those stems that you are confident about? This leads to CBM scoring and we assume that special quizzes are not scored by CBM.

6.7.3 Raising the pass mark.

As was said in the last Chapter if we ***only* raise the pass mark** then it does not matter whether stems are missed or not. The pass mark, for example, is raised to 70 and whether stems are missed or not the pass mark is 70.

Table 6-4. M_g when the pass mark is raised with **a**=0.

Pass Mark (PM)	**M_g**	**Without raising the pass mark at a 40% institution**	**Without raising the pass mark at a 50% institution**
70	[10, 10, 10, 1]	[10, 10, 10, 31]	[10, 10, 10, 21]
71	[9, 10, 10, 1]	[10, 10, 10, 31]	[10, 10, 10, 21]
72	[8, 10, 10, 1]	[10, 10, 10, 31]	[10, 10, 10, 21]
73	[7, 10, 10, 1]	[10, 10, 10, 31]	[10, 10, 10, 21]
74	[6, 10, 10, 1]	[10, 10, 10, 31]	[10, 10, 10, 21]
80	[10, 10, 1, 0]	[10, 10, 10, 31]	[10, 10, 10, 21]

The same argument can be used as in the last Chapter that if the pass mark is raised to 70, for example, then how can we compare grades with other modules?

If the pass mark is raised to 70, for example, a mark of 69 obtained by a quiz is a fail, but the same mark of 69 might be given a 1st (at a 40% institution). Thus, the *same* mark obtained by a quiz is a fail and when it is obtained by an essay, say, it a 1st. So, marks *mean* different things depending on how they were obtained, and so how can marks be combined to form the final grade when a module has several types of assessment? The comparison of marks obtained from different types of assessment is problematic, to say the least.

6.7.3.1 Difficulties of raising the pass mark.

As in the last Chapter the difficulties of raising the pass mark are to do with comparing marks and grades between two types of assessment, because:

- the marks and grades might have different distributions,
- the marks mean different things.

6.7.4 Using a more complex quiz + using a modification of negative marking.

We want to see the effect of using more complex quizzes and using a modification of negative marking, where **b** = 0.

We think that does Table 6-3 does this, but it does not show the values of $\mathbf{M_g}$ which require tools to be built.

6.7.5 Using a more complex quiz + raising the pass mark.

We want to see what the effect of using more complex quizzes and raising the pass mark is, whilst keeping **a** = **b** = 0. Raising the pass mark does not affect the mean mark, but using a more complex quiz does. We cannot calculate $\mathbf{M_g}$ and so end up with a very simple table like Table 6-2.

6.7.6 Using a modification of negative marking + raising the pass mark.

We want to see the effect of using a modification of negative marking where **b** = 0 and raising the pass mark, whilst keeping the quiz type constant. We will keep the quiz type constant at **Yes/No-only** quiz. The measures we have are $\boldsymbol{\mu}_{100}$,

M_g, and it needs tools to be built to calculate M_g (because the number of marks in a grade is not constant and so Excel™ cannot be used). As the mean mark (μ_{100}) is not affected by raising the pass mark, again we think Table 6-3 does this.

6.7.7 Using a more complex quiz + using negative marking + raising the pass mark.

The measures we are using are μ_{100} and M_g. M_g requires tools to be built for its calculation when stems can be missed and a modification of negative marking is used. μ_{100} is not affected by raising the pass mark. If one tries all 3 ways of tackling the guessing problem at the same time we think that Table 6-3 will do.

6.8 Conclusion and Discussion.

This Chapter has shown that the Analysis of special quizzes where every stem is NOT answered is limited and requires that tools are built.

Pulling together the Analysis of this Chapter and the last we obtain the following table.

Table 6-5. Pulling together the Analysis of Chapters 5 and 6.

Type of quiz	**All stems answered** **Mark M=w + x*a**	**All stems NOT answered** **Mark m=w + x*A + y*b**
P-option **MAQ**-only with T ticks (when **T**=1 we have MCQ)	Number of ways of getting a mark $y_m = Y_M = ({}_PC_T-1)^x * {}_nC_w$	Number of ways of getting a mark in a particular way $y_m = ({}_PC_T-1)^x * {}_nC_y * {}_{n-y}C_w$
	Mean mark $\mu_n = n*(1+({}_PC_T-1) * a)/({}_PC_T)$ To make $\mu_n = 0$ we make $a = -1/({}_PC_T-1)$	Mean mark $\mu_n = n*(1+({}_PC_T-1)*a + b)/({}_PC_T + 1)$ To make $\mu_n = 0$ we could make $a = -1/({}_PC_T-1)$ and $b = 0$
Multiple Yes/No-only (when **P**=1 we have Yes/No)	Number of ways getting a mark $y_m = Y_M = (2^P-1)^x * {}_nC_w$	Number of ways of getting a mark in a particular way $y_m = (2^P-1)^x * {}_nC_y * {}_{n-y}C_w$
	Mean mark $\mu_n = n*(1+(2^P-1)*a)/(2^P)$ To make $\mu_n = 0$ we make $a = -1/(2^P-1)$	Mean mark $\mu_n = n*(1+(2^P-1)*a + b)/(2^P + 1)$ To make $\mu_n = 0$ we could make $a = -1/(2^P-1)$ and $b = 0$

When **a** = **b**= 0, the minimum mark is 0.

This table leads to a method of:

- ask the user for
  - whether stems can be missed,,
  - the type of quiz e.g. 4-option MCQ-only,
  - the number of stems, **n**,
  - the score when a stem is answered in-correctly, represented by **a**,
  - the score when a stem is missed , **b** (an argument is that **b** should always be 0),
  - the number of options a stem has, **P**,
  - the number of ticks a correct answer to a stem has, **T** (this only applies to MAQ quizzes),
- vary **w** and inside this variation vary **y**,
- inside these loops calculate Mark and (the number that get that mark in a particular way).

This means that a program has to be written by us that calculates **m** and the number that get the mark **m** that way. Because the number that get the mark **m** that way can be very large, 'normal' programs cannot deal with it and we need to resort to using something like the LEDA routines to deal with $\mathbf{y_m}$.

- Do a Pivot of **m** and the number that get the mark **m** that way to form $\mathbf{Y_M}$ v **M.** Draw the distribution of $\mathbf{Y_M}$ v **M** using the widely available tool Excel™. The Pivot becomes necessary when :

  a) the marks are not ordered,
  b) there is more than one way to get a mark.

- Calculate the descriptive statistics and other statistics of $\mathbf{Y_M}$ v **M**.

This means we need to write another program to calculate the descriptive statistics and other statistics. The descriptive statistics include mean, variance and standard deviation. The other statistics include median, mode and proportion of passes, which all require a sort **m** and an addition of the associated the number that get the mark **m** to be done (hence the Pivot).

- Copy the drawing to Word™ from Excel™ and super-impose the statistics and other statistics.

The programs we write and Excel™ have to communicate, and we do that by intermediate tab-delimited text files. This means that:

- Our first program has to be able to export to a tab-delimited text file.
- Excel™ has to import from a tab-delimited text file (see Appendix 9).
- Excel™ has to do a Pivot (see Appendix 10).
- Excel™ has to export to a tab-delimited text file (see Appendix 11).
- Our second program has to import from a tab-delimited text file.

The copying of the distribution to Word™ from Excel™ involves a **Paste Special**, because we do not want Excel™ to change anything.

There are other ways of making μ_n 0, in the case of quizzes where stems can be missed: we make $(1+({}_{P}C_{T}-1)*\mathbf{a}+\mathbf{b}) = 0$ or $(1+(2^{P}-1)*\mathbf{a}+\mathbf{b}) = 0$. That is left to future work, but if you believe that **b** should *always* be 0 this future work is unnecessary.

A message is DO NOT **MIX**. Quizzes that have a mixture of stem types, where the odds of guessing the correct answer depends on the quiz type, and so have different **a**'s.. Another way of saying it is, stems with different **a**s (and **b**s) have different means (unless they have the same ${}_{P}C_{T}$ or 2^{P}). An example of stems having the same ${}_{P}C_{T}$ or 2^{P} are, an 8-option MCQ stem which has a ${}_{P}C_{T}$ of 8 and a 3-option Multiple Yes/No stem which has a 2^{P} of 8.

Mean Mark when ALL stems are answered $\mu_n = \mathbf{n}*(1 + X*\mathbf{a})/(X + 1)$
Mean Mark when all stems are NOT answered $\mu_n = \mathbf{n}*(1 + X*\mathbf{a} + \mathbf{b})/(X + 2)$

Putting these into a table, where just the complexity of the quiz is altered, we have:

Table 6-6. Comparing means obtained when a) ALL stems are answered b) stems can be missed.

Type of quiz	Mean Mark when ALL are answered when n=100, a=0 and c=1	Mean Mark when all stems are NOT answered n=100, and a=0 b=0 and c=1
Yes/No- only	50	33.33
MCQ 3-option -only	33.33	25
4-option-only	25	20
5-option-only	20	16.67
6-option-only	16.67	14.29
7-option-only	14.29	12.5
8-option-only	12.5	11.11
MAQ 5-option with 2 ticks -only	10	9.09
6-option with 3 ticks -only	5	4.76
7-option with 4 ticks -only	2.86	2.78
8-option with 5 ticks -only	1.79	1.75
Multiple Yes/No 5-option -only	3.125	3.0
6-option-only	1.5625	1.54
7-option-only	0.78125	0.7752
8-option-only	0.390625	0.3891
EMSQ 20-option (MCQ)	5	4.76
20-option (MAQ with 10 ticks)	0.000541254	0.0054125

We see that the mean mark goes down when stems can be missed, and we suspect that the distribution of the marks moves to the left, but this suspicion will have to wait for tools to be built.

The guessing problem is where the correct answer to a stem is guessed, and that occurs because the correct answer is *given*, to special quizzes. The ways of tackling the guessing problem are shown in the table below and the big message is each of the ways of tackling the guessing problem have problems themselves.

Table 6-7. Problems associated with tackling the guessing problem.

All stems answered	All stems NOT answered
Using more complex quizzes, with a=0	**Using more complex quizzes with a=b=0**
The distribution of marks is positively skewed	The distribution of marks is positively skewed
Using negative marking	**Using a modification negative marking with b=0**
Is it 'fair' to get a mark of 40 when 70 stems are answered correctly (when **a**=-1)?	Is it 'fair' to get a mark of 40 when 60 stems are answered correctly, 20 stems are answered incorrectly and 20 stems missed (when **a**=-1, **b**=0)?
Raising the pass mark	**Raising the pass mark**
What does a mark of 69 mean? Fail of a quiz and nearly a 1st for an essay (at a 40% institution).	
How do you compare grades when there are less marks in a grade?	How do you compare grades when there are more marks in a grade?

The above table is when steps to tackle the guessing problem are taken individually. When steps are taken in tandem e.g. for quizzes where stems can be missed **and** a more complex quiz is used **and** a modification of negative marking is used, there are *more* marks than when **a** = **b** = 0. For example, when **a** = -1/2, **b** = 0, **c** = 1, **n** = 100 some of the marks are

40, 40 + 1/2, 41, 41 + ½, 42, 42 + ½, 43, 43 +1/2, 44, 44+ ½, 45, 45 +1/2, 46, 46 +1/2, 47,47 +1/2,48, 48 +1/2, 49, 49 +1/2 **20** Marks in the 3rd range

and when **a** = **b** = 0 some of the marks are

40, 41, 42, 43, 44, 45, 46, 47, 48, 49, **10** Marks in the 3rd range

How do you compare grades when there are different numbers of marks in each grade?

It is argued that:

a) MCQs are a special case of MAQs where **T** = 1,
b) Yes/Nos are a special case of Multiple Yes/Nos where **P** = 1,
c) EMSQ quizzes are either MCQs or MAQs and we can deal with either ,
d) the score for a correct answer to a stem, **c**, can be made into 1,
e) the score for a missed stem, **b,** is 0.

We are going to show that we need to build tools in order calculate the Pass Rate. For 100 stems where $\mathbf{M_g}$ = [10, 9, 8, 1] there are 10, pass marks and 9, 2.2 marks 8 2.1 marks and 1 1st mark. The total number of marks is 101. The Pass *Rate* is not (10 + 9 + 8 +1)/101: it depends on *how many* pass. The pass rate is the (pseudo area of passes)/(total pseudo area). We say *pseudo* because we are dealing with a discrete distribution, and thus we are dealing with a summation and not an integration. Thus we need tools to calculate the total pseudo area, and the pseudo area of passes and as the calculations involve large numbers we need to use tools that can handle large numbers (see Chapter 3).

6.9 What is coming next?

Next, we are going to present the results of building tools.

Chapter 7. Results.

Abstract.

What we learn from drawing the distribution of marks from 100 stem 'special' quizzes that work on a census and super-imposing some statistics, on the distribution, is:

- the distribution is bell shaped,
- the likelihood of passing, for all but the simplest quiz, is very low.

If we tackle the guessing problem by using *exact* negative marking, or its equivalent, then the distribution moves to the left, making the likelihood of passing even lower. There are also problems to do with fairness e.g. for a Yes/No quiz where stems can be missed, if score when negative marking is used is -1 and the score when a stem is missed is 0, and **n**=100, is it fair to :

- **Fail** when 65 stems are answered correctly, 26 in-correctly and 9 missed?
- Assume that guessing is going on?
- Compare the marks gained from institutions that use negative marking with those that do not?

If we tackle the guessing problem by raising the pass mark, we have the problem that comparing marks from parts of modules that raise the mark with those that do not raise the pass mark is 'hard'.

If we tackle the guessing problem by using a more complex quiz, we have the problem that half the area of the distribution is between the lowest mark and the mean mark, and the mean mark goes down the more complex the quiz is. The more complex quiz has to be written, checked, piloted and marked: this all takes time, and so, using a more complex quiz is longer process.

So, no matter how we tackle the guessing problem there are difficulties.

7.1 Introduction.

We built tools (*tool1* and *tool2*) to see what the distribution of marks was for special quizzes that use a *census*.. Excel™ was used to draw the distribution produced by *tool1* and *tool2* was used to calculate the descriptive statistics and other statistics of the distribution. The other statistics include mode, median and the likelihood of passing when the pass mark is raised.

This Chapter is about what we can learn from drawing the distribution and superimposing our statistics on them.

We use *census* data and do not want estimates and so *tool2's* statistics for census data are bound to disagree with the statistics for *sample* data, for 'small' samples. The question really is "When do the statistics from *tool2* settle down?" and that is answered in the first part of this Chapter.

The second part of this Chapter is about seeing the distribution of marks (until the method we use 'runs out of steam'), and see what happens if we use negative marking, or raise the pass mark or use a more 'complex' quiz.

The third part of this Chapter is about explaining the results we obtain from the second part of the Chapter.

7.2 Part 1. When do the descriptive statistics 'settle down'?

The *population* (Pop.) descriptive statistics and the *sample* descriptive statistics (Sam.) are both calculated by *tool2*.

Table 7-1. Yes/No quiz. ALL stems answered. Negative marking is not used i.e. a=0.

Number Stems (n)	Pop Sam	Mean/Mode/ Median (n/2)	Variance	Standard Deviation	Skewness	Kurtosis
1	Pop. Sam.	0.5/0.5/0.5 0.5	**0.25** **0.5**	**0.5** **0.707**	0 N.A	-2.0 N.A.
2	Pop. Sam.	1.0/1.0/1.0 1.0	**0.5** **0.667**	**0.707** **0.816**	0 0	**-1.0** **-1.5**
3	Pop. Sam.	1.5/1.5/1.5 1.5	**0.75** **0.857**	**0.866** **0.926**	0 0	**-0.667** **0.000**
4	Pop. Sam.	2.0/2.0/2.0 2.0	**1.0** **1.067**	**1.0** **1.033**	0 0	**-0.5** **-0.206**
5	Pop. Sam.	2.5/2.5/2.5 2.5	**1.25** **1.29**	**1.118** **1.136**	0 0	**-0.4** **-0.257**
6	Pop. Sam.	3.0/3.0/3.0 3.0	**1.5** **1.524**	**1.225** **1.234**	0 0	**-0.333** **-0.261**
7	Pop. Sam.	3.5/3.5/3.5 3.5	**1.750** **1.764**	**1.323** **1.328**	0 0	**-0.286** **-0.249**
8	Pop. Sam.	4.0/4.0/4.0 4.0	**2.000** **2.008**	**1.414** **1.417**	0 0	**-0.250** **-0.231**
9	Pop. Sam.	4.5/4.5/4.5 4.5	2.250 2.254	1.500 1.501	0 0	-0.222 -0.213
10	Pop. Sam.	5.0/5.0/5.0 5.0	2.500 2.502	1.581 1.582	0 0	-0.200 -0.195
11	Pop. Sam.	5.5/5.5/5.5 5.5	2.750 2.751	1.658 1.659	0 0	-0.182 -0.179
20	Pop. Sam.	10/10/10 10 This is by implication 10	5 5	2.236 2.236	0 0	-0.1 -0.1
40	Pop. Sam	20/20/20 This is by implication 20	10 10	3.162 3.162	0 0	-0.050 -0.050
80	Pop. Sam.	40/40/40 This is by implication 40	20 20	4.472 4.472	0 0	-0.025 -0.025
100	Pop. Sam.	50/50/50 This is by implication 50	25 25	5 5	0 0	-0.02 -0.02

For a symmetric distribution, the Mean mark, the Mode mark and the Median mark should always occur at the same mark. So, our calculations, by program, seem to work for symmetric distributions, up to **n** = 20, and after **n** = 20 the median is inferred for a symmetric distribution.

As we go down this table, we notice:

- the mean of the population and the mean of the sample are always the same (which we would expect as the means work on the same data using equivalent formulae),
- the skewness (when it can be calculated) is always 0 because the distribution is symmetric and then we would expect the mean to equal the mode and the median,
- the distribution 'settles down' at about **n** = 9 i.e. the descriptive statistics are about the same for the population and the sample,
- the mean mark = **n**/2 (which we can show),
- the variance of the distribution = **n**/4, which we cannot show and this left as a conjecture.

Table 7-2. Yes/No quiz. ALL stems answered Negative marking IS **a**=-1.

Number of Stems	Pop Sam	**Mean/Mode/ Median**	**Variance**	**Standard Deviation**	**Skew.**	**Kurtosis**
1	Pop.	0/0/0	**1**	**1**	0	-2
	Sam	0	**2**	**1.414**	N.A.	N.A.
2	Pop.	0//0	**2**	**1.414**	0	**-1**
	Sam	0	**2.667**	**1.633**	0	**+1.5**
3	Pop.	0/0/0	**3**	**1.732**	0	**-0.667**
	Sam	0	**3.429**	**1.852**	0	**0**
4	Pop.	0/0/0	**4**	**2**	0	**-0.5**
	Sam	0	**4.267**	**2.066**	0	**-0.206**
5	Pop.	0/0/0	**5**	**2.236**	0	**-0.400**
	Sam	0	**5.161**	**2.272**	0	**-0.257**
6	Pop.	0/0/0	**6**	**2.440**	0	**-0.333**
	Sam	0	**6.095**	**2.469**	0	**-0.261**
7	Pop.	0/0/0	**7**	**2.646**	0	**-0.286**
	Sam	0	**7.055**	**2.656**	0	**-0.249**
8	Pop.	0/0/0	**8**	**2.828**	0	**-0.250**
	Sam	0	**8.31**	**2.834**	0	**-0.231**
9	Pop.	0/0/0	9	3	0	-0.222
	Sam	0	9.018	3.003	0	-0.213
10	Pop.	0/0/0	10	3.162	0	-0.200
	Sam	0	10.10	3.164	0	-0.195
11	Pop.	0/0/0	11	3.317	0	-0.182
	Sam	0	11.005	3.317	0	-0.179
20	Pop.	0/0/0	20	4.472	0	-0.100
	Sam	0	20	4.472	0	-0.100
40	Pop.	0/0/0	40	6.325	0	-0.050
	Sam	0	40	6.325	0	-0.050
80	Pop.	0/0/0	80	8.944	0	-0.025
	Sam	0	80	8.944	0	-0.025
100	Pop.	0/0/0	100	10	0	-0.020
	Sam	0	100	10	0	-0.020

As we go down this table, we notice:

- the mean of the distribution = 0
  (because we tackle the guessing problem *exactly*),
- the skewness is always 0 and hence the mean mode and median are co-incident,
- the descriptive statistics of the distribution 'settle down' at about **n** = 9.

Table 7-3. Yes/No quiz. Stems can be MISSED. Negative marking is NOT used a=b=0.

Number of Stems	Pop Sam	Mean/Mode/ Median	Variance	Standard Deviation	Skew.	Kurtosis
1	Pop.	0.33/0/0	**0.222**	**0.471**	**0.707**	-1.500
	Sam	0.33	**0.333**	**0.577**	**1.732**	N.A.
2	Pop.	0.667/0.5/1.0	**0.444**	**0.667**	**0.500**	**-0.750**
	Sam	0.667	**0.500**	**0.707**	**0.606**	**-0.286**
3	Pop.	1.0/1.0/1.0	**0.667**	**0.816**	**0.400**	**-0.500**
	Sam	1.0	**0.692**	**0.832**	**0.433**	**-0.347**
4	Pop.	1.333/1.00/1.00	**0.889**	**0.943**	**0.354**	**-0.375**
	Sam	1.333	**0.900**	**0.949**	**0.360**	**-0.321**
5	Pop.	1.667/1.5/2.0	**1.111**	**1.054**	**0.316**	**-0.300**
	Sam	1.667	**1.116**	**1.056**	**0.318**	**-0.282**
6	Pop.	2.00/2.0/2.0	1.333	1.155	0.289	-0.250
	Sam	2.00	1.335	1.155	0.289	-0.243
7	Pop.	2.33/2.0/2.0	1.556	1.247	0.267	-0.214
	Sam	2.333	1.556	1.248	0.267	-0.212
8	Pop.	2.667/2.5/3.0	1.778	1.333	0.250	-0.188
	Sam	2.667	1.778	1.333	0.250	-0.187
9	Pop.	3.0/3.0/3.0	2	1.414	0.236	-0.167
	Sam	3.0	2	1.414	0.236	-0.166
10	Pop.	3.33/3.00/3.00	2.222	1.491	0.224	-0.150
	Sam	3.333	2.222	1.491	0.224	-0.150
11	Pop.	3.667/3.5/4.0	2.444	1.563	0.213	-0.136
	Sam	3.667	2.444	1.563	0.213	-0.136
20	Pop.	6.667/6.5/7.0	4.444	2.100	0.158	-0.075
	Sam	6.667	4.444	2.100	0.158	-0.075
40	Pop.	13.33/13./13	8.889	2.981	0.112	-0.030
	Sam	13.33	8.889	2.981	0.112	-0.030
80	Pop.	26.667/26.5/27	17.778	4.216	0.079	-0.019
	Sam	26.667	17.778	4.216	0.079	-0.019
100	Pop.	33.333/33/33	22.222	4.714	0.071	-0.015
	Sam	33.333	22.222	4.714	0.071	-0.015

As we go down this table the variance of the distribution is conjectured to be $2\mu/3$.

What we notice is that, here, a Pivot is necessary because, for example, the mark 0 can be obtained in many ways. Scientific notation is used in Excel™ when the numbers are above 11 digits. *tool2* is expecting a number that get a mark to be an integer, and so column B (in Excel™) has to be changed to integer format.

The descriptive statistics 'settle down' a bit earlier than before, at **n** = 6. Because **the number of cases** is a bit bigger and so, dividing by the number of cases does not matter much, when calculating variance.

Table 7-4. Yes/No quiz. Stems can be MISSED. A modification of negative marking IS used where **a**=-1 and **b**=0.

Number of Stems	Pop Sam	**Mean/Mode/ Median**	**Variance**	**Standard Deviation**	**Skew.**	**Kurtosis**
1 This distn. is very unusual in that it has **3** modes	Pop. Sam	0/0/0 0	**0.667** **1.000**	**0.816** **1.000**	0 0	-1.5 N.A.
2	Pop. Sam	0/0/0 0	**1.333** **1.500**	**1.155** **1.225**	0 0	**-0.750** **-0.286**
3	Pop. Sam	0/0/0 0	**2** **2.077**	**1.414** **1.441**	0 0	**-0.5** **-0.347**
4	Pop. Sam	0/0/0 0	**2.667** **2.700**	**1.633** **1.643**	0 0	**-0.375** **-0.321**
5	Pop. Sam	0/0/0 0	**3.333** **3.347**	**1.826** **1.830**	0 0	**-0.3** **-0.281**
6	Pop. Sam	0/0/0 0	4 4.005	2 2.001	0 0	-0.250 -0.243
7	Pop. Sam	0/0/0 0	4.667 4.669	2.160 2.162	0 0	-0.214 -0.212
8	Pop. Sam	0/0/0 0	5.333 5.334	2.309 2.310	0 0	-0.187 -0.187
9	Pop. Sam	0/0/0 0	6 6	2.449 2.50	0 0	-0.167 -0.166
10	Pop. Sam	0/0/0 0	6.667 6.667	2.582 2.582	0 0	-0.15 -0.15
11	Pop. Sam	0/0/0 0	7.333 7.333	2.708 2.708	0 0	-0.136 -0.136
20	Pop. Sam	0/0/0 0	13.333 13.333	3.651 3.651	0 0	-0.075 -0.075
40	Pop. Sam	0/0/0 0	26.667 26.667	5.164 5.164	0 0	-0.038 -0.038
80	Pop. Sam	0/0/0 0	53.333 53.333	7.303 7.303	0 0	-0.19 -0.19
100	Pop. Sam	0/0/0 0	66.667 66.667	8.165 8.165	0 0	-0.015 -0.015

The number of cases when **n** = 9 and all stems are answered for Yes/No quizzes is 10 (i.e. **n** +1), and the number of cases when stems can be missed for Yes/No quizzes where **a** = -1 and **b** = 0 and **n** = 6 is 13.

7.3 Part 2.

This part of the Chapter considers the results from:

- Yes/No-only quizzes, X = 1
- 4-option MCQ-only quizzes, X = 3
- 5-option MAQ quizzes with 2 ticks -only, X = 9
- 20-option EMSQ (MCQ)-only quizzes, X = 19
- 20-option EMSQ (MAQ) quizzes with 10 ticks-only. X = 184755

All 'special' quizzes cannot be examined, and so, a sample of them is examined.

7.3.1 Results for Yes/No-only quizzes (X=1).

Yes/No-only quizzes, can either:

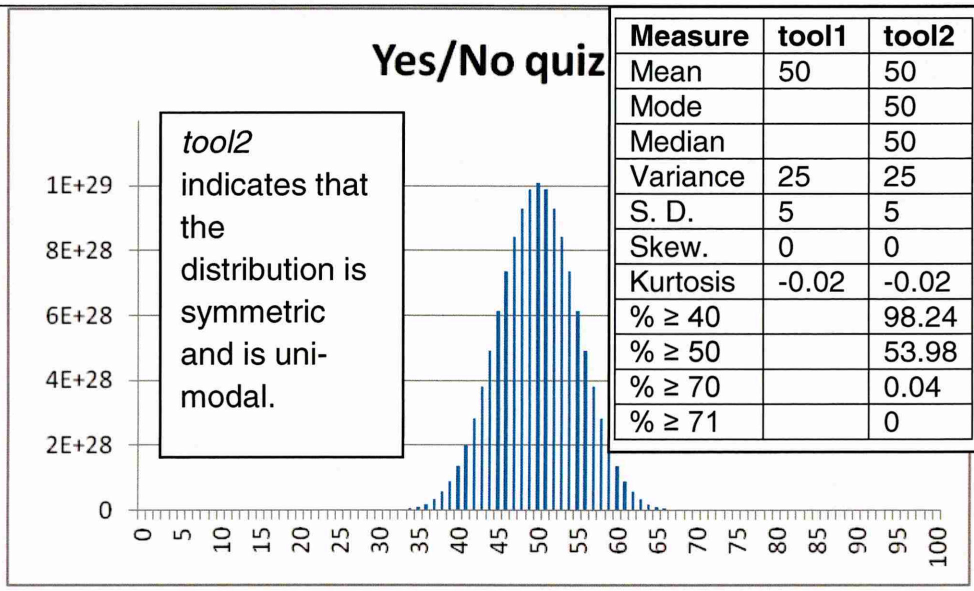
- have ALL their stems answered, or,
- have their stems MISSED.

For each of these we can use negative marking (or the modification where **b** = 0) and/or raise the pass mark to tackle the guessing problem.

The tools, *tool1*, *tool2* and Excel™ (and their intermediate files), produce the following results, for quizzes that have 100 stems.

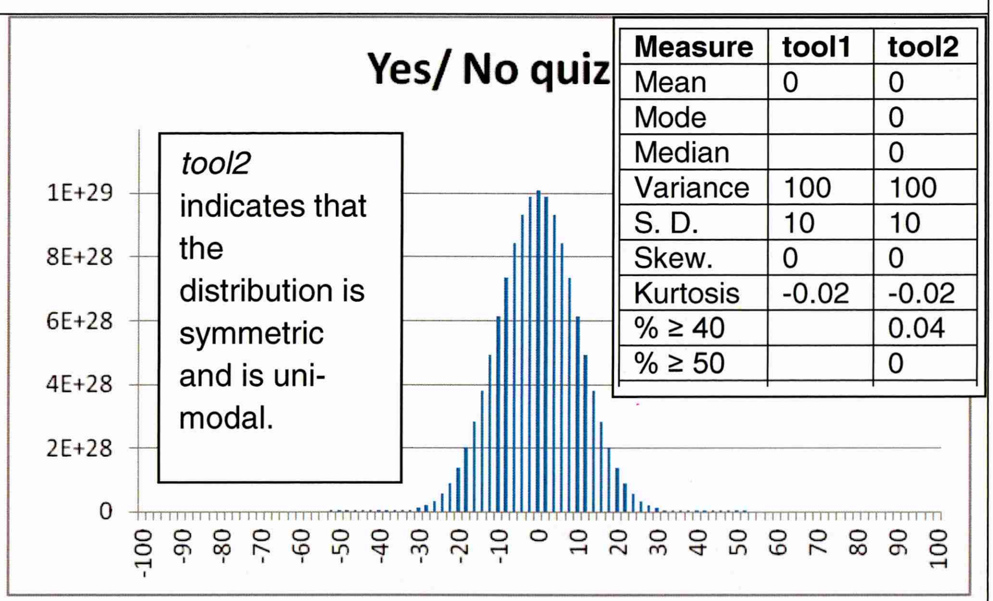
Table 7-5. Comparison of distribution of marks for a 100 stem Yes/No quiz when ALL stems are answered.

ALL stems are answered, and negative marking is NOT used (i.e. **a**=0)



Measure	tool1	tool2
Mean	50	50
Mode		50
Median		50
Variance	25	25
S. D.	5	5
Skew.	0	0
Kurtosis	-0.02	-0.02
% ≥ 40		98.24
% ≥ 50		53.98
% ≥ 70		0.04
% ≥ 71		0

ALL stems are answered, and negative marking IS used (i.e. **a**=-1)

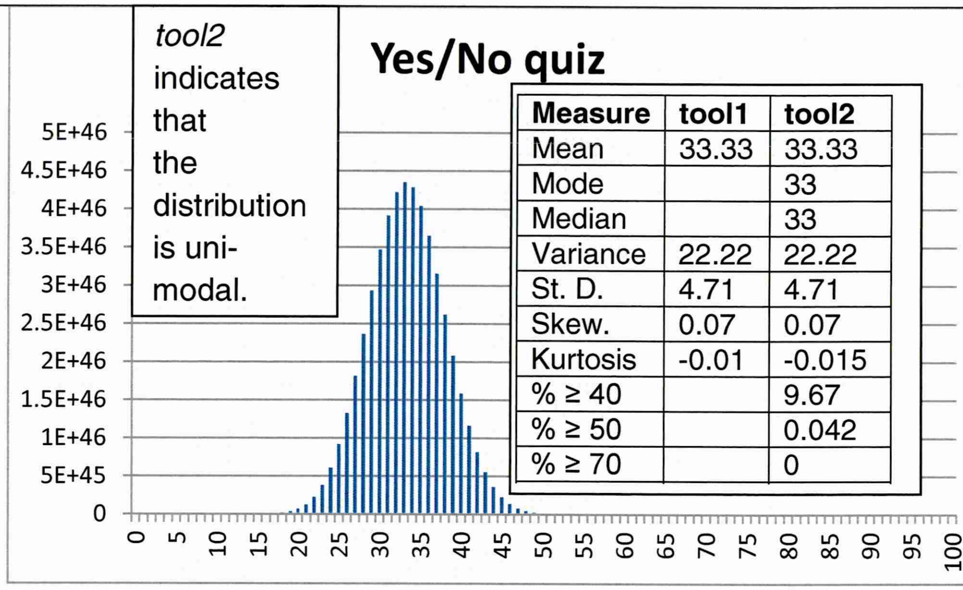


Measure	tool1	tool2
Mean	0	0
Mode		0
Median		0
Variance	100	100
S. D.	10	10
Skew.	0	0
Kurtosis	-0.02	-0.02
% ≥ 40		0.04
% ≥ 50		0

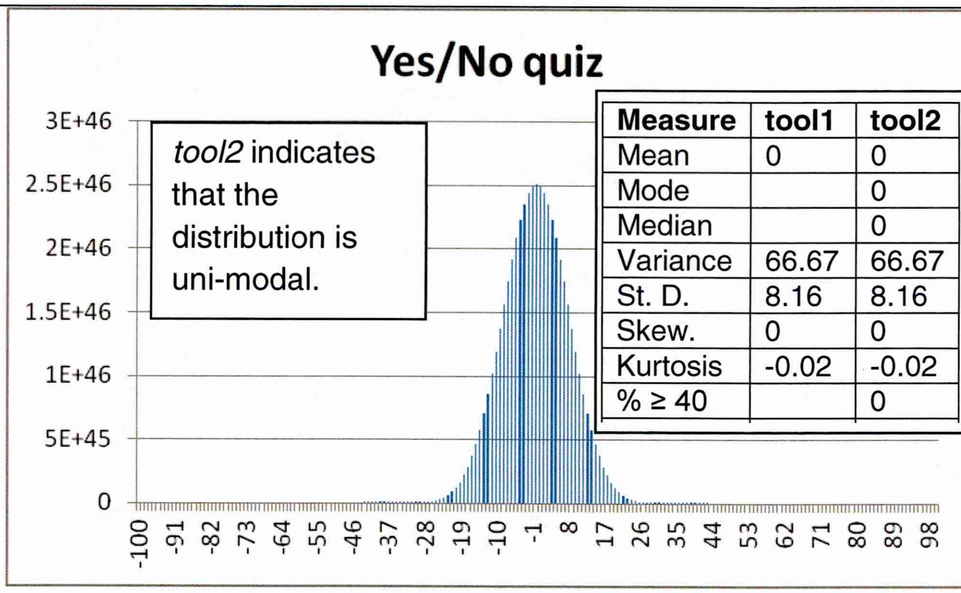
Thus, for Yes/No quizzes where all stems are answered, the distribution moves to the left when exact negative marking is used. The likelihood of passing when the pass mark is raised to 70. say, is about 0.

Table 7-6. Comparison of distribution of marks for a 100 stem Yes/No quiz when stems can be MISSED.

Stems can be MISSED and a modification of negative marking is NOT used (i.e. **a**=0, **b**=0).



Stems can be MISSED and a modification of negative marking IS used (i.e. **a**=-1) **b**=0).



For Yes/No quizzes where stems can be missed, the distribution moves to the left when the equivalent of exact negative marking is used. The likelihood of passing when the pass mark is raised to 70, say, is about 0 in both cases.

7.3.2 Results for 4-option MCQ-only quizzes (X=3).

4-option MCQ-only quizzes can either:

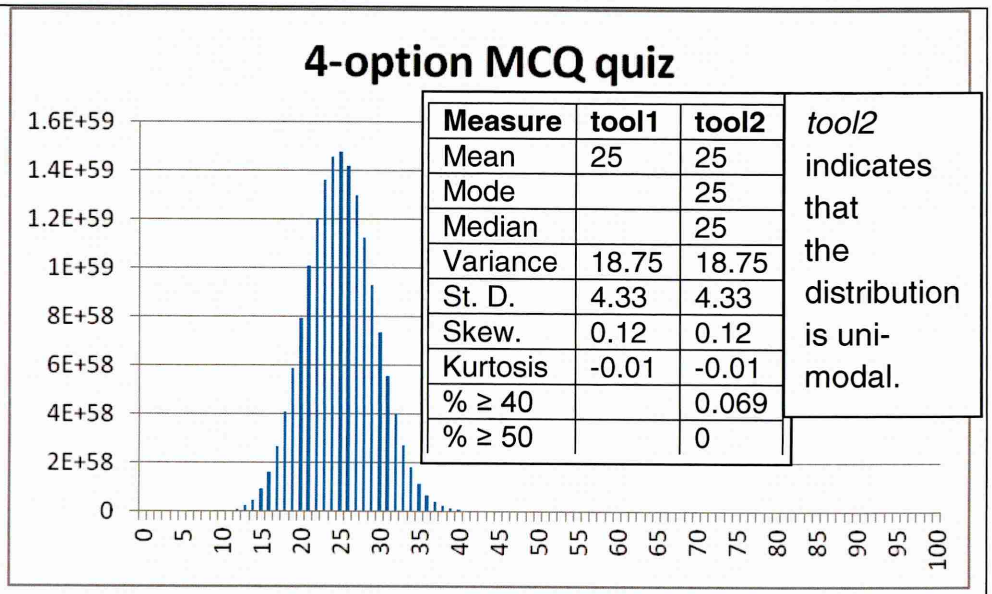
- have ALL their stems answered, or,
- have their stems MISSED.

For each of these we can use negative marking (or the modification where **b** = 0) and/or raising the pass mark to tackle the guessing problem.

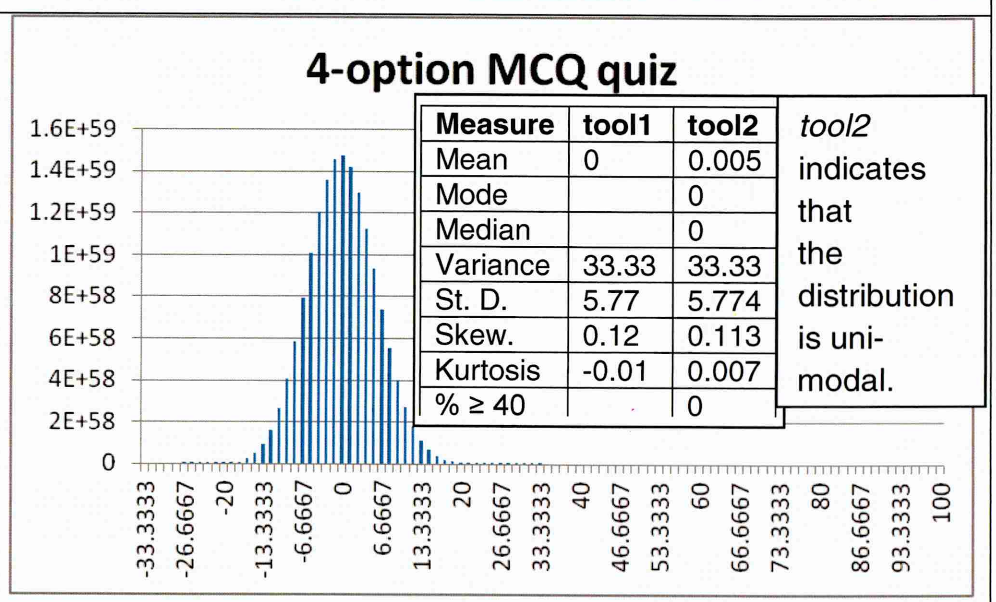
The tools, *tool1*, *tool2* and Excel™ (and their intermediate files), produce the following results, for quizzes that have 100 stems,

Table 7-7. Comparison of distribution of marks for a 4-option MCQ quiz when ALL stems are answered.

ALL stems are answered, and negative marking is NOT used (i.e. **a**=0)

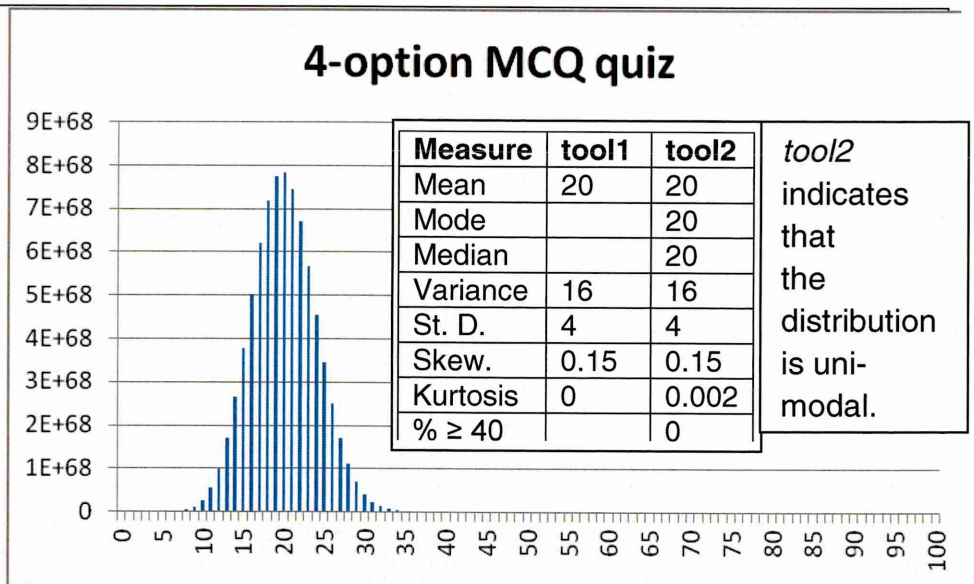
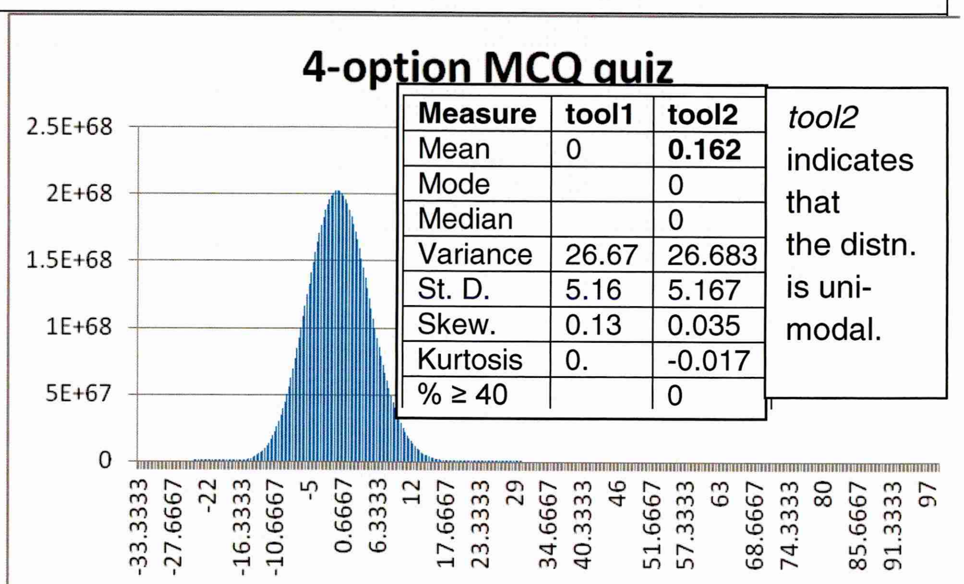


ALL stems are answered, and a negative marking IS used (with **a**=-0.33333)



Thus, for 4-option MCQ quizzes where all stems are answered, the distribution moves to the left when exact negative marking is used. The likelihood of passing when the pass mark is raised to 50, say, is about 0 in both cases.

Table 7-8. Comparison of distribution of marks for a 4-option MCQ quiz when stems can be MISSED.

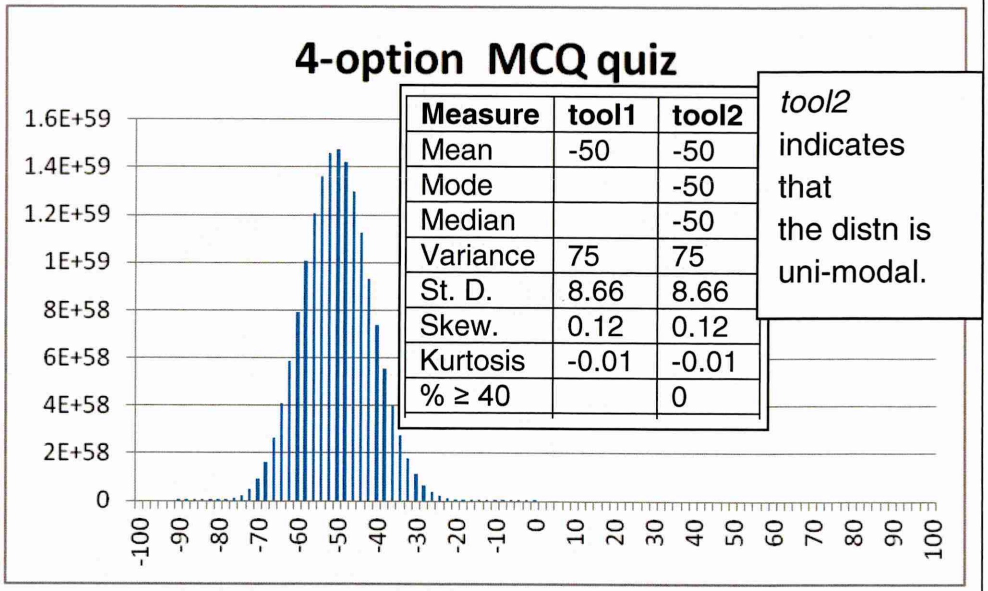
Stems can be MISSED and modification of negative marking is NOT used (i.e. **a**=**b**=0)	
Stems can be MISSED, and a modification of negative marking IS used (with **a**=-0.33333) **b**=0	

For 4-option MCQ quizzes where stems can be missed the distribution moves to the left when the equivalent of exact negative marking is used. The likelihood of passing when the pass mark is raised to 40, say, is about 0 in both cases.

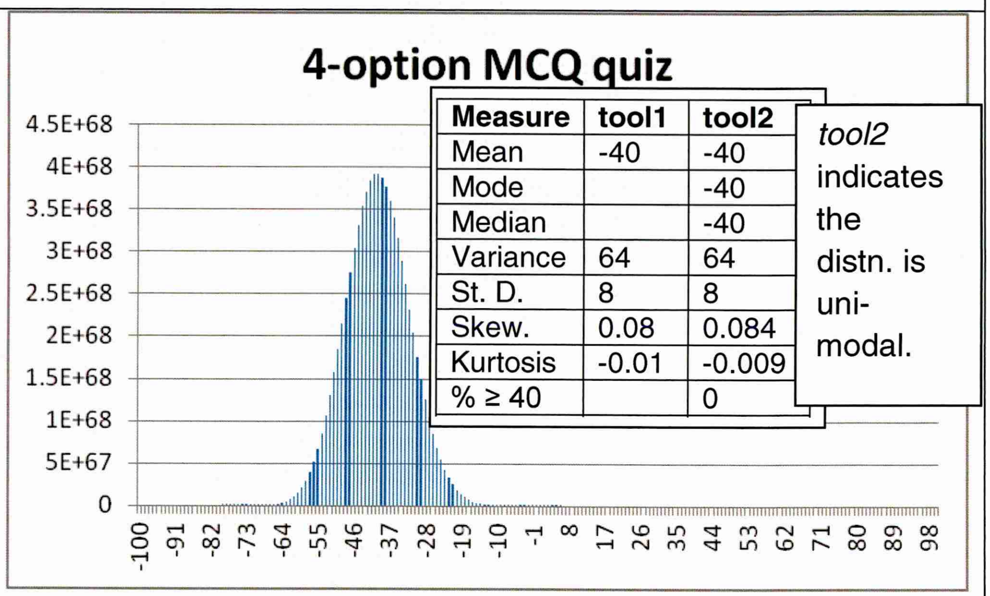
We also ask the question "What happens if we score **-1** for a stem that is answered in-correctly?"

Table 7-9. Comparison of distribution of marks for a 4-option MCQ quiz when **a**=-1 and when **b**=0.

ALL Stems are answered and negative marking IS used (i.e. **a**=-1)



Stems can be MISSED and a modification of negative marking used with **a**=-1) **b**=0.



Now, the distribution moves to the **right** when stems can be missed, because the score for answering a stem in-correctly is negative and the score for missing a stem is 0. But, this is 'overkill' in that the likelihood of getting a mark above **40** is about 0.

7.3.3 Results for 5-option MAQ-only quizzes with 2 ticks (X=9).

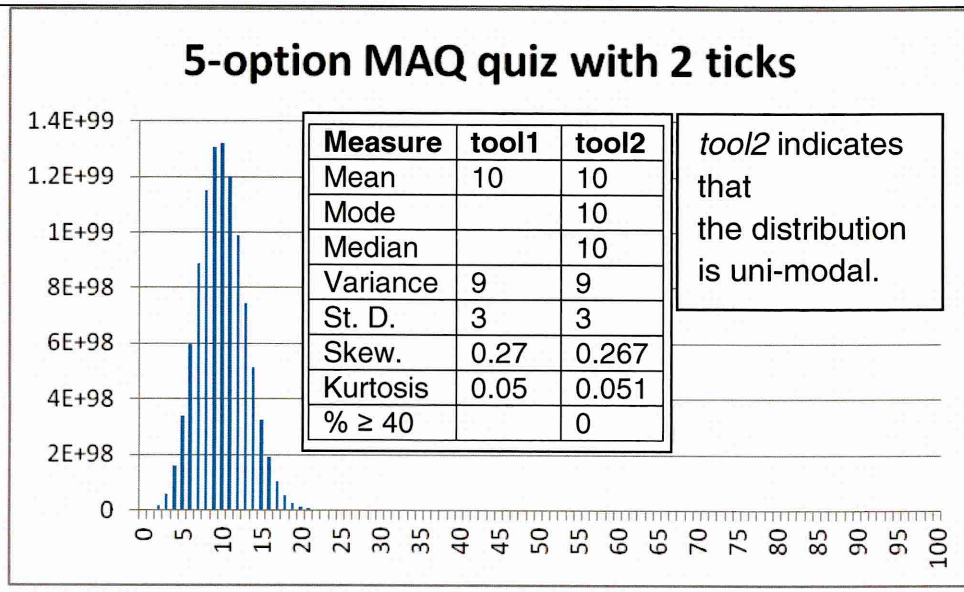
5-option MAQ-only quizzes with 2 ticks can either:

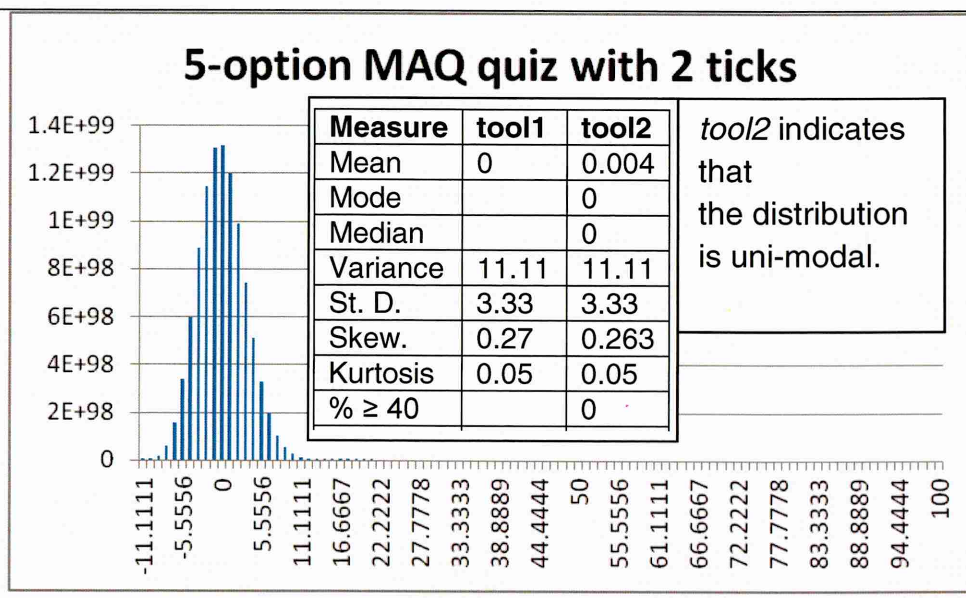
- have ALL their stems answered, or,
- have their stems MISSED.

For each of these we can use negative marking (or the modification where **b** = 0) and/or raising the pass mark to tackle the guessing problem.

The tools, *tool1*, *tool2* and Excel™ (and their intermediate files), produce the following results, for 100 stem quizzes.

Table 7-10. Comparison of distribution of marks for a 5-option MAQ quiz with 2 ticks, when ALL stems are answered.

<table>
<tr><td>ALL stems are answered, and negative marking is NOT used (i.e. a=0) X=9</td><td>


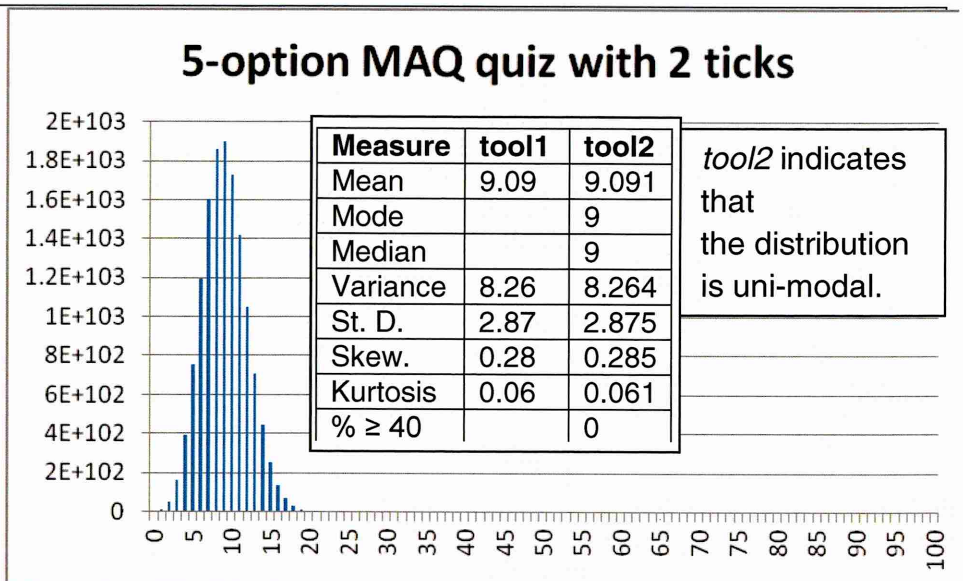
</td></tr>
<tr><td>ALL stems are answered, and negative marking IS used (with a=-0.111111) a=-1/X</td><td>


</td></tr>
</table>

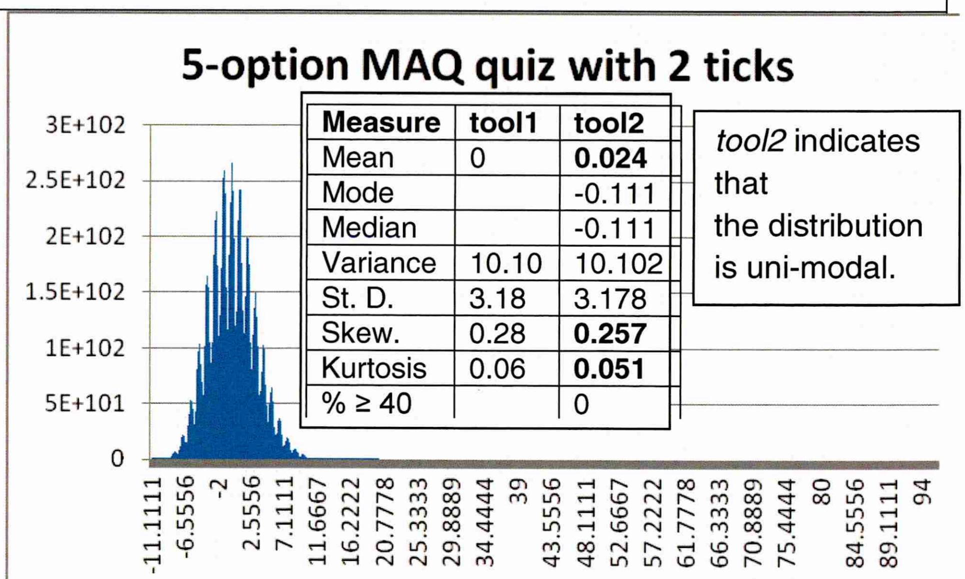
Thus, for 5-option MCQ quizzes with 2 ticks where all stems are answered, the distribution moves to the left when exact negative marking is used. The likelihood of passing when the pass mark is raised to 40, say, is about 0 in both cases.

Table 7-11. Comparison of distribution of marks for a 5-option MAQ quiz with 2 ticks, when stems can be MISSED.

Stems can be MISSED and a modification of negative marking is NOT used (i.e. **a**=0, **b**=0) X=9



Stems can be MISSED, and a modification of negative marking IS used (with **a**=-0.11111) **a**=-1/X **b**=0.



For 5-option MCQ quizzes with 2 ticks where stems can be missed the distribution moves to the left when the equivalent of exact negative marking is used. The likelihood of passing when the pass mark is raised to 40, say, is about 0 in both cases.

7.3.4 Results for 20-option EMSQ (MCQ)-only quizzes (X=19).

20-option EMSQ (MCQ)-only quizzes can either:

- have ALL their stems answered, or,
- have their stems MISSED.

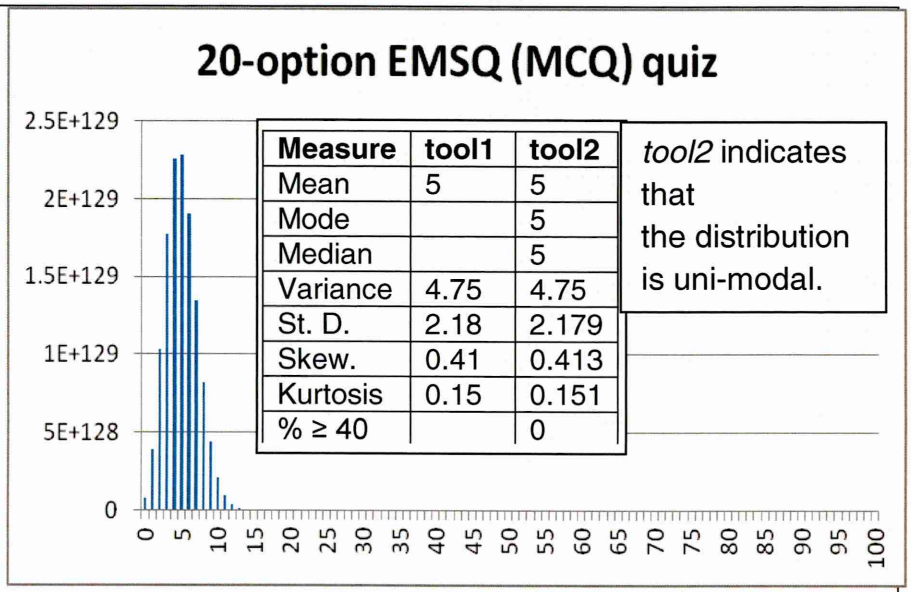
For each of these we can use negative marking (or the modification where **b** = 0) and/or raising the pass mark to tackle the guessing problem.

The tools, *tool1*, *tool2* and Excel™ (and their intermediate files), produce the following results, for quizzes that have 100 stems

Table 7-12. Comparison of distribution of marks for a 20-option EMSQ (MCQ) quiz when ALL stems are answered.

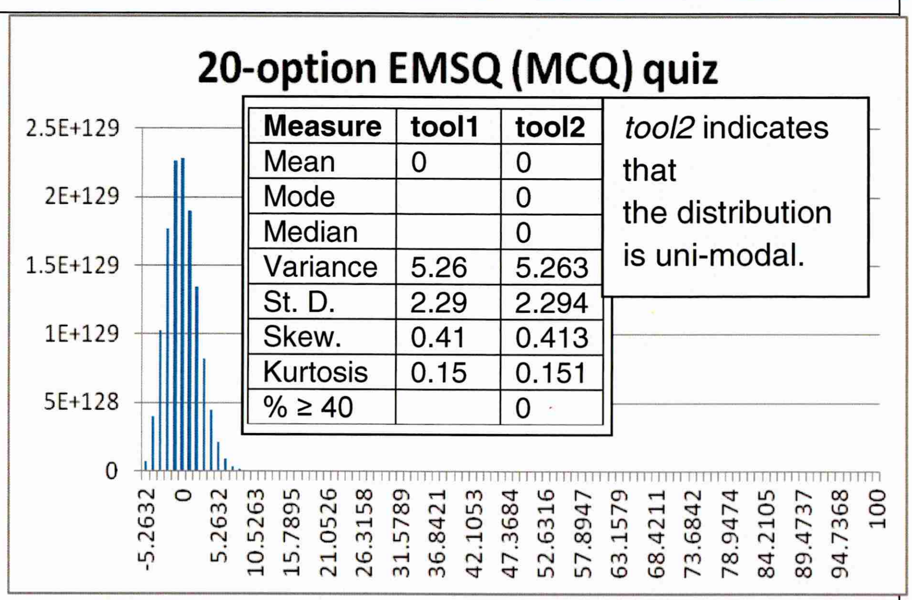
ALL stems are answered, and negative marking is NOT used (i.e. **a**=0)

X=19



Measure	tool1	tool2
Mean	5	5
Mode		5
Median		5
Variance	4.75	4.75
St. D.	2.18	2.179
Skew.	0.41	0.413
Kurtosis	0.15	0.151
% ≥ 40		0

ALL stems are answered, and negative marking IS used (with **a**=-0.052631578947) **a**=-1/X

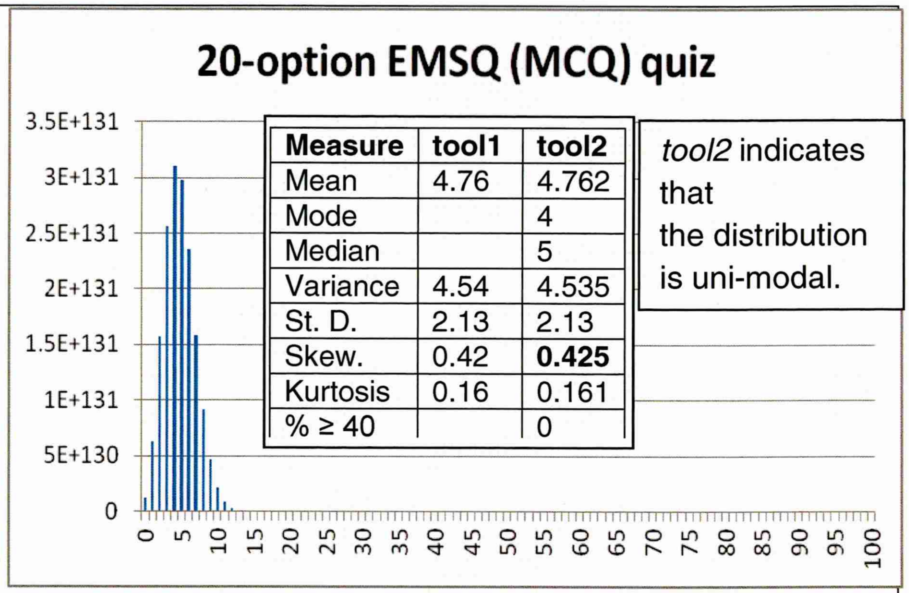


Measure	tool1	tool2
Mean	0	0
Mode		0
Median		0
Variance	5.26	5.263
St. D.	2.29	2.294
Skew.	0.41	0.413
Kurtosis	0.15	0.151
% ≥ 40		0

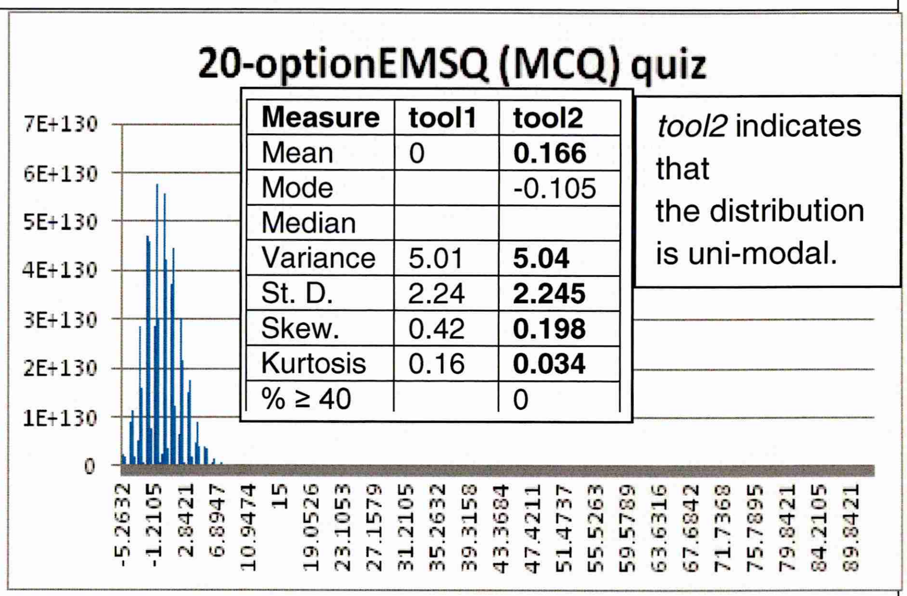
Thus, for 20-option EMCQ (MCQ) quizzes where all stems are answered, the distribution moves to the left when exact negative marking is used. The likelihood of passing when the pass mark is raised to 40, say, is about 0 in both cases.

Table 7-13. Comparison of distribution of marks for a 20-option EMSQ (MCQ) quiz stems can be MISSED.

Stems can be MISSED and a modification of negative marking is NOT used (i.e. **a**=**b**=0) X=19



Stems can be MISSED, and a modification of negative marking IS used (with **a**=- 0.052631578947) **a**=-1/19 **b**=0.



For 20-option EMCQ (MCQ) quizzes where stems can be missed, the distribution moves to the left when the equivalent of exact negative marking is used. The likelihood of passing when the pass mark is raised to 40, say, is about 0 in both cases.

7.3.5 Results for20-option EMSQ (MAQ with 10 ticks)-only quizzes (X=184755).

We cannot draw the results for 20-option EMSQ (MAQ with 10 ticks)-only quizzes for **100** stem quizzes for reasons that are explained below (basically, Excel™ 'runs out of steam').

We can, however, calculate the mean, which is about 0. The range is as follows

for quizzes where every stem is answered and **a** = 0

The range of marks is 100*Min(1,0) to 100*Max(1,0) i.e. 0 to 100.

for quizzes where every stem is answered and **a** = -about 0

The range of marks is 100*Min(1,-about 0) to 100*Max(1,-about 0) i.e. -about 0 to 100.

for quizzes where stems is can be MISSED and **a** = 0, **b** = 0

The range of marks is 100*Min(1,0,0) to 100*Max(1,0,0) i.e. 0 to 100.

for quizzes where every stem is answered and **a** = -about 0, **b** = 0

The range of marks is 100*Min(1,-about 0, 0) to 100*Max(1,-about 0,0) i.e. -about 0 to 100.

When we try and compute the results for 20-option EMSQ (MAQ with 10 ticks)-only quizzes, we come across the limitations of Excel™, where we find that

"a cell cannot be wider than 255 characters"
"the maximum number is 9.99999999999999E+307"

Microsoft (2012).

For example, if either of these conditions applies then Excel™ cannot cope. When we import into Excel™ (from *tool1*) the 1st line of data (for **n** = 100, **a** = 0 and stems cannot be missed) is

0.0000
4566884708903691125661469296950879378305716027010065006361060244
835183030694884976996240347064746874399231037262948171959633492453419
591834556562384422071214252502868166939122389600145405873068666859039
326591965787655037452149880736324258114317043931735317705099760505469
297465077148872541294163673495783128015682768149944039109157966115164
134035282757246561554670030362173244174676227903269412256539449145129
651814202027407540515519741445215348782777528430827380297830000430980
0556368849344668348066988983191549777984619140625

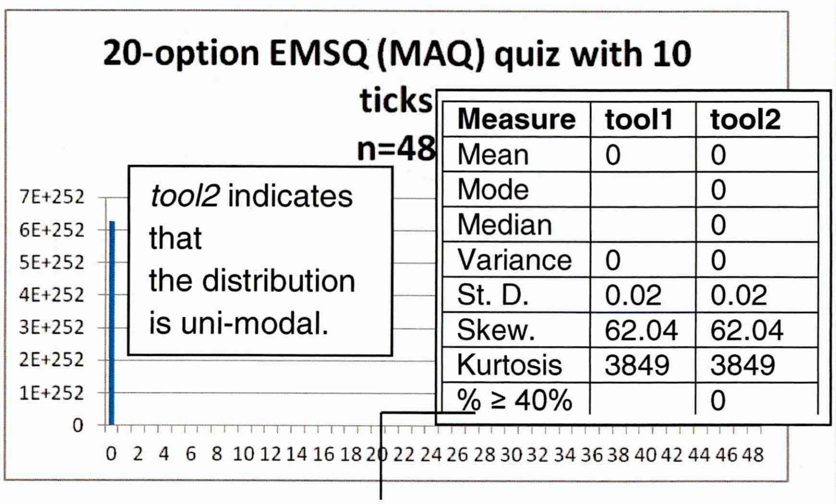
which is more than 9.99999999999999999E+307 and occupies more than 255 characters. So, Excel™ cannot cope for 2 reasons. Excel™ tries to read in the characters into column B and fails, and then uses other columns for the data. Other rows can cope with the data with the help of Scientific Notation (e.g. 1.237E121).

Because Excel™ cannot cope with this data, it cannot do a Pivot correctly, **but it does not tell you**, and goes ahead with the Pivot, but is so obviously wrong that the results of the Pivot can be ignored.

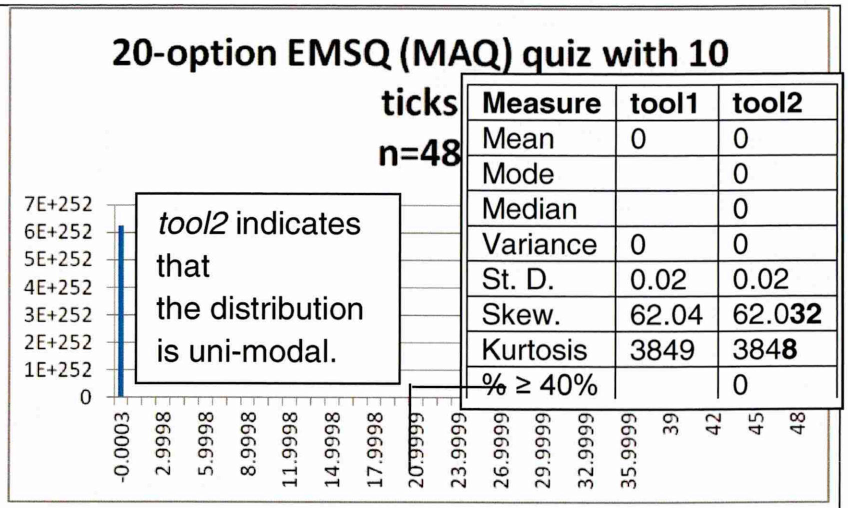
Thus, there are limits on the method i.e. when Excel™ cannot cope.

We can get *tool1* and *tool2* (and their intermediate files) to work together up to **n** = 48 for quizzes where stems are ALL answered.

Table 7-14. Both tools (*tool1* and *tool2*) work together up to **n**=48.

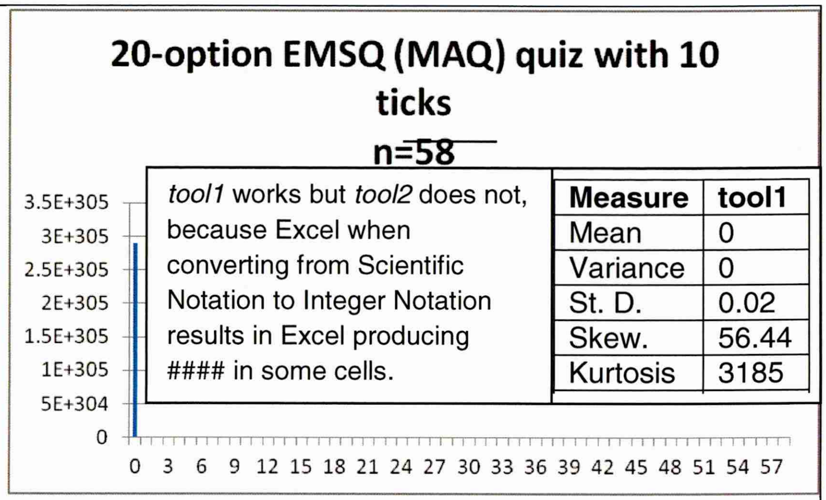
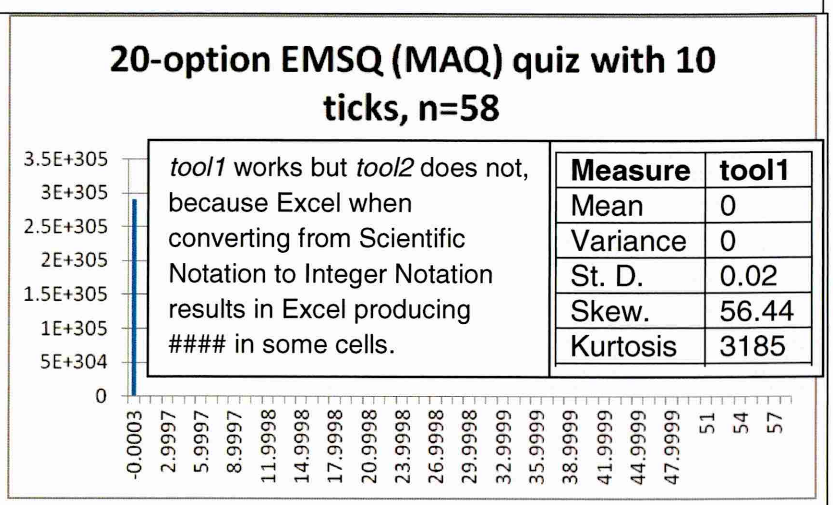
ALL stems answered and negative marking is NOT used (i.e. **a**=0)	

Measure	tool1	tool2
Mean	0	0
Mode		0
Median		0
Variance	0	0
St. D.	0.02	0.02
Skew.	62.04	62.04
Kurtosis	3849	3849
% ≥ 40%		0

ALL stems answered and negative marking IS used (i.e. **a**= -about 0)	

Then *tool1* works *alone* up to **n** = 58

Table 7-15. From **n**=49 to **n**=58 we can get *tool1* to work alone.

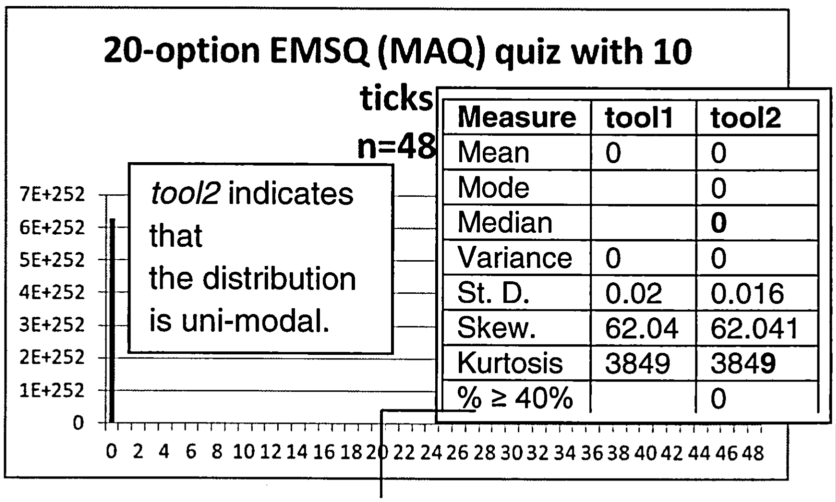
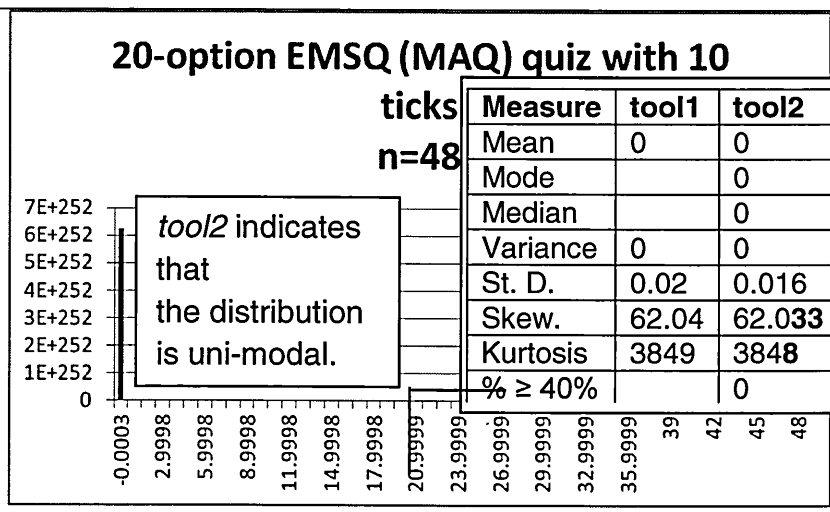
ALL stems answered and negative marking is NOT used (i.e. **a**=0) Before, the conversion to Integer Notation is done in Excel™.	
ALL stems answered and negative marking IS used with **a**=-about 0 Before, the conversion to Integer Notation is done in Excel™.	

Excel™, does its best up to **n**=58 (by using Scientific Notation), but then, when we try to get *tool2* to work by converting the numbers from Scientific Notation to Integer Notation we find that Excel™ produces #### in some cells, indicating that the conversion cannot be done (because the numbers would now occupy more than 255 characters).

So, where stems are ALL answered, the method we use, works up to **n** = 48, and then *tool2* stops working. From **n** = 49 to **n** = 58 *tool1* works alone and after that the method packs up altogether.

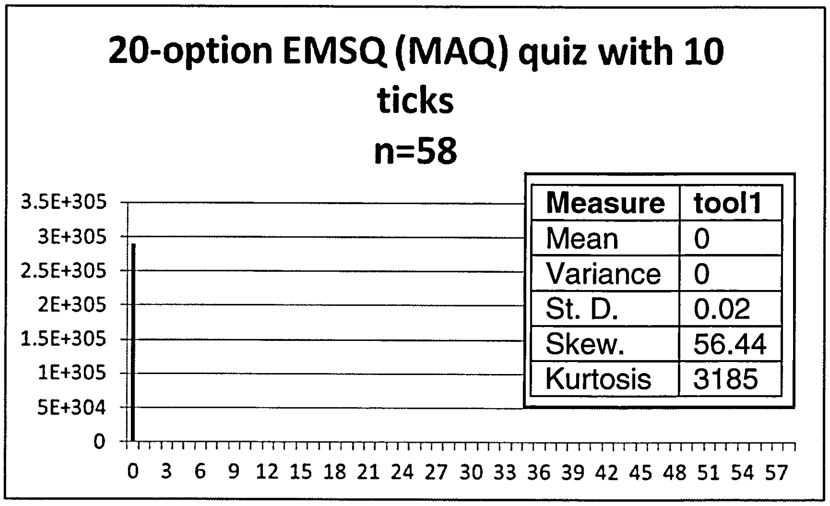
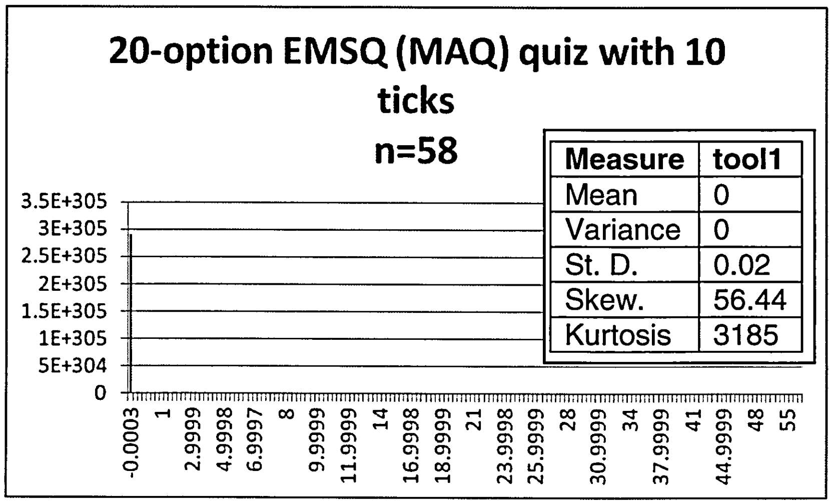
Now, looking at where stems can be MISSED.

Table 7-16. Both tools work together up to **n**=48 for quizzes where stems can be missed.

<table>
<tr>
<td>Stems can be MISSED and the modification of negative marking is NOT used (i.e. a=0, b=0)</td>
<td>
</td>
</tr>
<tr>
<td>Stems can be MISSED and the modification of negative marking IS used (i.e. a=- about 0, b=0)</td>
<td>
</td>
</tr>
</table>

We can get *tool1* and *tool2* to work together up **n** = 48.

Table 7-17. From **n**=49 to **n**=58 *tool1* works by itself.

Stems can be MISSED and the modification of negative marking is NOT used (i.e. **a**=0, **b**=0)	
Stems can be MISSED and the modification of negative marking IS used (i.e. **a**=- about 0, **b**=0)	

Then *tool1* works alone from **n** = 49 to **n** = 58. Then the method packs up altogether.

We can conclude for 20-option EMSQ (MAQ) quizzes-only with 10 ticks, that the method we use:

- works completely up to **n** = 48,
- partially works up to **n** = 58,
- packs up when **n** > 58.

7.4 Part 3. Discussion.

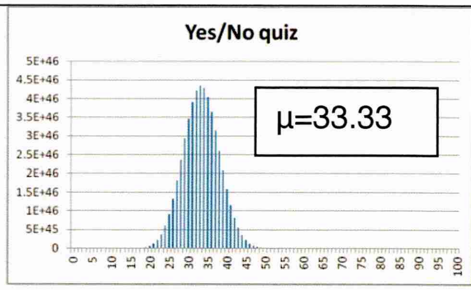
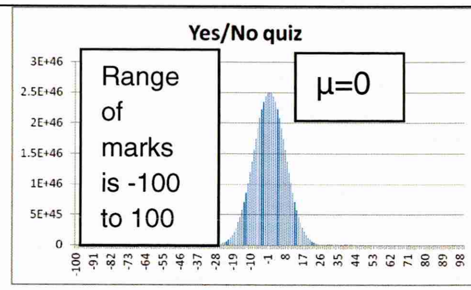
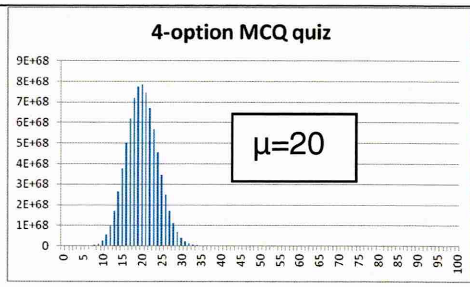
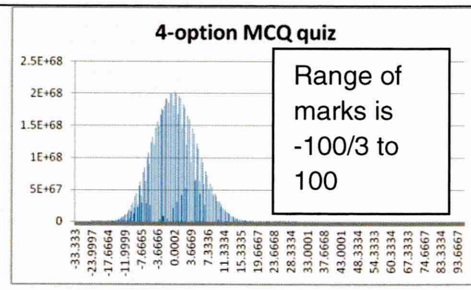
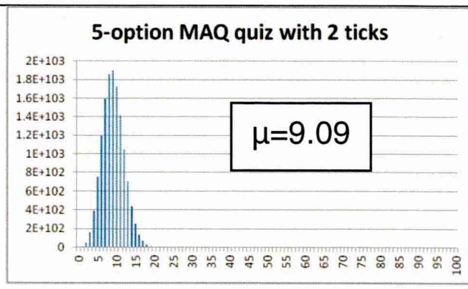
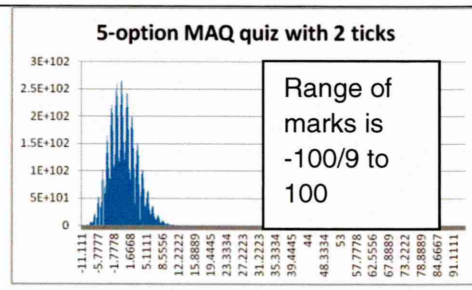
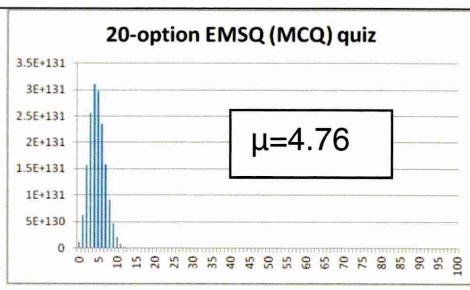
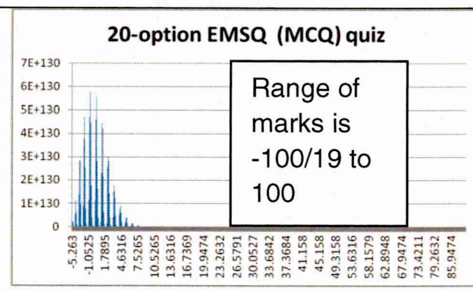
Part 1 of this Chapter shows that the descriptive statistics of a population 'settle down' to the same as those for a sample when the sample size is 'big' enough, and that occurs:

a) when **n** = 9 for Yes/No-only quizzes where ALL stems are answered and negative marking is not used (i.e. **a** = 0),
b) when **n** = 6 for Yes/No-only quizzes where stems can be missed and the modification of negative marking is not used (i.e. **a** = **b** = 0).

As we are concerned, here, about populations and not samples of populations, the formulae that we use for populations will *always* work no matter how small the population is. That is one reason for not using SPSS.

Part 2 of this Chapter shows what results we got and this part of the Chapter discusses the results. If we conclude that quizzes where all stems are answered, **forces** guessing the correct answer to stems where the correct answer is not known, then we can drop the quizzes where all stems are answered and draw the distribution of marks for *only quizzes* where stems can be *missed*. This results in:

Table 7-18. Comparison of distribution of marks, for various quiz types, when stems can be missed.

Type of quiz	**Stems MISSED** **a=b=0 Range of marks is 0 to 100** **Not symmetric**	**Stems can be MISSED** **a=-1/X, b=0, μ=0** **Mostly not symmetric**
Yes/No (X=1)		
4-option MCQ (X=3)		
5-option MAQ with 2 ticks (X=9)		
20--option EMSQ (MCQ) (X=19)		
20-option EMSQ (MAQ with 10 ticks) (X=184755)	Cannot be shown, because of the restrictions of Excel™. μ= 0	Cannot be shown, because of the restrictions of Excel™. μ=0 Range of marks is -100/184755 to 100 .

For the last row we can show that can use the method works completely up to **n** = 48, and then only *tool1* works up to **n** = 49, and then the method we use, stops working.

But:

- How many 20-option EMSQ (MAQ) quizzes with 10 ticks are there?
- How many 20-option EMSQ (MAQ) quizzes with 10 ticks have more than 48 stems?
- *Can* all the stems of a 20-option EMSQ (MAQ) quiz with 10 ticks have the *same* probability of being guessed correctly?
- How is the 'nearly' right problem dealt with, exemplified by answering 9 stems being correctly answered in the above. Then what score should be given a) 9/10 b) 0 c) something else?

If we look at **Yes/No quizzes,4-option MCQ quizzes, 5-option MAQ quizzes with 2 ticks, 20-option EMSQ (MCQ) quizzes**, (where **a** = **b** = 0), and try raising the pass mark to 70 then the proportion of pass marks is very low (0.0).

If we look at **Yes/No quizzes,4-option MCQ quizzes, 5-option MAQ quizzes with 2 ticks, 20-option EMSQ (MCQ) quizzes**, (where **a** = -1/X, **b** = 0), and try raising the pass mark to 70 then the proportion of pass marks is very low (0.0).

So, raising, the pass mark to 70 for special quizzes does not work, for quizzes where stems can be missed, and negative marking (or its equivalent) is used or not.

This is in conjunction with all the difficulties that have been found, so far:

- If the pass mark for this module has been raised to 70 then how do you *compare* this module to others where the pass mark is 40? The pass marks are different and the number of marks in the 2.1 grade, say, is different (i.e. $\mathbf{M_g}$ is different). How do you compare marks of two *different* distributions?

- Fairness raises its ugly head e.g. is it 'fair' to answer 65 stems correctly and *fail* (in the case of a **Yes/No** quiz where stems can be missed where **a** = -1, **b** = 0? If 65 stems are answered correctly, 26 are answered incorrectly and 9 stems are missed then a mark of 39 is obtained, and a fail results even though 65 stems have been answered correctly. The same argument applies to other special quizzes, where the equivalent *exact* negative marking is used.

- Is it fair to *assume* guessing is going on? The learner might have genuinely got an answer to a stem in-correct, without any guessing going on.

- Is it fair for one institution to use negative marking and another not?

If we try using a more 'complex' quiz (without using the modification of negative marking being used where stems can be missed) e.g. 4-option MCQ-only quiz instead of a Yes/No-only quiz we find that mean mark is 20. The mean mark can be obtained by guessing. If this is deemed to be 'too high' then an even more complex quiz can be tried e.g. a 5-option MAQ quiz with 2 ticks. This has a mean mark of 9.09. This might be deemed to be 'too high' and so an even more complex quiz can be tried e.g. a 20-option EMSQ (MCQ)-only quiz with a mean of 4.76. This might be deemed to be acceptable, but then we run into the difficulties of:

The setting, checking, piloting and marking of a more complex quiz all take time and so using more complex quiz is harder.

- the probability of passing to be very low,
- the possibility of an option not being as 'plausible' as another option,
- the lowest mark being 0, the mean mark = 4.76, so half of the area of the distribution is between the marks of 0 and 4.67.

So, no matter what we do to tackle the guessing problem of special quizzes we run into difficulties.

7.5 What is coming next?

The next chapter, the last, is the Summary chapter, and is split into 2:

- what we have concluded about *this* research,
- what could be done next.

Chapter 8. Summary.

Abstract.

This chapter is split into 2:

- Part 1, where we summarise what we have done in *this* study.
- Part 2, where we discuss some of what could be done in *the future*.

In **Part 1**:
We show that the **R**esearch **Q**uestion (**RQ**) (aim) of ascertaining the distribution of marks of 'special' quizzes when a census is done, and the objectives of the study of seeing how the distribution changed when the guessing problem was tackled were **met**, but the method, MyMethod, used, has its limitations.

The method was to build two tools, *tool1* and *tool2* (in the programming language C). The tool *tool1* fed into the widely available Excel™, and Excel™ fed into *tool2* and between the tools were tab-delimited text files. The limitations of the method were down to limitations of Excel™.

All the ways of tackling the guessing problem, of 'special' quizzes, result in difficulties, and so a conclusion is not to use 'special' quizzes for summative assessment of 'deep' learning, especially where a comparison of marks (with a different type of assessment) is going to be done. This conclusion is asking "Are we building the right product?" i.e. verification.

We question whether the assumptions that we make can ever be satisfied. For example, can we ever ensure that stems are equally 'hard'? Can we ever ensure that options within a stem options are equally 'hard'?

We end Part 1 with some recommendations for practice.

In **Part 2**:
We show how *proposed* research projects are judged worthy of progressing to *actuals* by going through a process that is based on a triage that is performed in an hospital. The process starts off with a broad test and then a detailed test and then a ranking.

We end Part 2 with some recommendations for further work.

8.1 Introduction.

This chapter is split into 2:

- Conclusions of this research. Part 1.
- What could be done in the future? Part 2.

The **R**esearch **Q**uestion (**RQ**) was to ascertain the distribution of marks of a 'special' quiz when a census was performed. We further asked "How this distribution changed when the guessing problem was tackled?" In order to answer the **RQ** and its sub-aims, tools (*tool1* and *tool2*) were built, and Excel™ (a widely available tool) was used. The tools communicated by using tab-delimited text files, which, themselves, are tools.

We wrote programs for *tool1* and *tool2* (c.f. Chapter 3) in the programming language C (which we had to learn) and analysed the tools in Chapters 4, 5 and 6, which contained 6 proofs. The proofs and 'hand' calculations were used to validate tools 1 and 2. We question, in Part1 whether, in fact, we are building the 'right product' i.e. do some *verification.*

The tools have their limitations due to limitations of Excel™ that restricted how big a number could be and how much space it occupied.

Assumptions are made in order to do the analysis that caused *tool1* and *tool2* to be built and we see if these assumptions can ever be met.

We ask what, in retrospect, we would do differently if we did the research again.

Part 1 ends up with some recommendations, concerning *this* study.

Part 2 is about what could be done in the *future.*

8.2 Part 1.

This part of this chapter contains

- A summary of what we did,
- A summary of the contribution *this study* makes to the body of knowledge.
- Recommendations for practice.

8.3 A summary of what we did.

8.3.1 Chapter 1. Introduction.

Chapter 1 'sets the scene' and defines the **R**esearch **Q**uestion (**RQ**) (aim) of our study which was to ascertain the distribution of marks of 'special' quizzes when a census is done. The objectives of this research were to see what happened to the distribution of marks when 3 ways of tackling the guessing problem were used viz. negative marking (an old idea), raising the pass mark (an old idea), using a more 'complex' quiz (our idea). For negative marking we used 'exact' negative marking where the mean mark is 0. For raising the pass mark we tried eight values of the pass mark. For 'complexity' we used the value of X (the number of in-correct answers to a stem (see Ch. 4). We defined 'special' quizzes which are exemplified by 4-option **M**ultiple-**C**hoice-**Q**uestions (**MCQ**)-only. All 'special' quizzes are all a) closed b) have only one correct answer to a stem (which is given and hence can be guessed, which gives rise to the guessing problem) c) the stem can only be missed in one way. 'Special' quizzes can either a) have ALL their stems answered, or b) have some stems missed.

Because we were not following the Director of Studies ideas (which concentrated on student retention) we had to find our own subject/topic and that took a long time and involved going down many 'blind alleys' (e.g. how do we learn) and dropping topics e.g. teaching and scoring by CBM.

8.3.2 Chapter 2. Literature Review

The Literature Review shows that we 'know what we are talking about' and that a gap was spotted which led to the **RQ** mentioned in Chapter 1. The Literature relating to methods will be reviewed in the methods chapter.

8.3.3 Chapter 3. Methods.

We first of all, show where Methods fit in, and we show what kind of methods we used. For where methods fit in we used a modified suggestion by Grix (2010) so that the order we used was Ontology, Epistemology, Methodology, Methods, Tools and Measures. We quickly dealt with Ontology and Epistemology and Methodology to leave the main thrust of this chapter viz. Methods, Tools and Measures.

Ontology is the study of existence or "What entities are out there?" (Grix 2010). Epistemology is the study of knowledge and can be summarised as "What do we know about these entities?" (Grix 2010). Methodology is the study of methods (in this thesis).

Methods form a spectrum going from Qualitative methods (like action research and grounded research) to Quantitative methods (like mathematics, statistics and computer programming). If you use methods that are a mixture of Quantitative and Qualitative then you are using Mixed methods (Bryman 2008, Cresswell 2013). A particular method, MyMethod, was introduced, that was mixed but mainly Quantitative and had its own tools and measures. The qualitative part came when the results were explained and whether we were building the 'right product' (i.e. verification) which is discussed in this chapter.

Quantitative methods can be said to be 'scientific' in that they often contain an experiment that can be repeated by somebody else. Qualitative methods (e.g. ethnography) are said to be non-scientific in that they cannot be repeated (and get the same results) and sometimes cannot be generalised, because their sample does not represent the population. Mixed methods try to get 'the best of both worlds' even though their underlying world-views might be incompatible.

The strengths and weaknesses of each type of method were shown and a justification of choosing quantitative methods was given.

We chose to use quantitative methods because we wanted the results to be repeatable by somebody else and we chose to use a census so that we did not use a sample with its attendant problems. But, we were not completely successful in just using quantitative methods, because we used qualitative methods to explain the results and somebody else might have a different explanation.

We used a subset of quantitative tools in that we did not:

- have an hypothesis
- do sampling, with all its attendant difficulties (e.g. how big is the sample, does it represent the population, what type of sampling did we use (e.g. random sampling). So, we did not use a sample but did a census of our population, which was ∑(a mark) x (how many got that mark).

The five tools used in MyMethod (constituting phase 1) were:

- *tool1*, which calculated the points of the distribution of marks of a special quiz,
- *tool1* outputs the points to a tab-delimited text file..
- Excel ™ imported the text file, did a Pivot and drew the resulting distribution.
- Excel ™ outputted the points of the distribution to another tab-delimited text file
- *tool2* read this text file and calculated descriptive statistics (in the main) of the distribution.

We did a census by using the LEDA library of routines in tools 1 and 2 which could handle arbitrary large integers.

The measures we used in tools 1 and 2 were a) descriptive statistics e.g. mean, standard deviation b) the proportion of passes c) the number in each grade ($\mathbf{M_g}$)

The limitations of a) MyMethod and b) quantitative methods, in general, were both discussed.

8.3.4 Chapters 4, 5 and 6. Analysis

This group of chapters is known as phase 2 and contains a) snapshots of 'hand' calculations (that were shown in more detail in the relevant Appendices) b) proofs that applied to b1) stems which were all answered b2) stems which could be missed. The proofs were used in the validation of tools 1 and 2.

The most interesting proof concerned the gap between marks of a quiz where all stems were answered which was $|1-\mathbf{a}|$. This led to a) the distribution being 'stretched' along the x-axis b) the rank order of people not being affected by the value of **a** (except when **a**=1).

In phase 2 we tried to be as general as possible. For example, we said that MCQs were a subset of Multiple Answer Questions (**MAQ**s) and that Yes/No questions were a subset of Multiple Yes/No questions. **E**xtended **M**atching **S**et **Q**uestions (**EMSQ**s) could either be MCQs or M**A**Qs and so we were only dealing with 2 types of quiz a) MAQs or b) Multiple Yes/No. In MAQs the number of in-correct answers to a stem is ${}_{P}C_{T}$ -1 and the number of in-correct answers to Multiple Yes/No stems is 2^{P} -1. Where **P**=the number of options and **T**=the number of ticks (so an MCQ is an M**AQ** where **T**=1). The generalisation of this is to let X= ${}_{P}C_{T}$ - 1 for MCQs, M**A**Qs and EMSQs or X=2^{P} - 1 for multiple Yes/Nos.

A conclusion of these chapters is do NOT MIX i.e. do not use stems of two different types in the same quiz. For example, do not mix Yes/No stems and 4-option MCQ stems in the same quiz, because they have a different likelihood of being answered correctly. The likelihood of answering a Yes/No stem correctly is ½ (where all stems are answered and negative marking is not done) and the likelihood of answering a 4-option MCQ stem correctly is ¼. A Yes/No stem, here, is twice as likely to be answered correctly as a 4-option MCQ stem. So, from this point of view a Yes/No stem is twice as *easy* as a 4-option MCQ stem.

8.3.5 Chapter 7. Results

We, first of all, found out that the descriptive statistics "settle" down quite quickly, but if we are using low values of **n** then SPSS is not to be trusted to give the correct results, because a) SPSS *assumes* that the data refers to a *sample*, and we were using a *census* b) SPSS is wrong when **n** is very small c) SPSS is wrong when very large numbers are involved. Those are the reasons for not using SPSS.

Much work was done (c.f. Appendices 1..17) to show that a) the tools that we used in MyMethod were correct (validated) b) worked together, within their limitations c) gave correct results for quizzes where every stem is answered d) gave correct results for quizzes where stems could be missed.
The difficulties that we faced were shown in Appendix 18 and future work ideas are contained in Appendix 19.

We tried quizzes increasing values of X where a) all stems were answered b) some stems could be missed. For each quiz we showed what happens when the guessing problem is tackled by a) negative marking b) raising the pass mark. The 3rd way of tackling the guessing problem was to use a more complex quiz which is given by the value of X. For example, a) when X=1 we have a Yes/No quiz b) when X=3 we have either of b1) a 4-option MCQ quiz, or b2) a 2-option Multiple Yes/No quiz.

The results showed that, for a 100 stem quiz the distribution is 'bell' shaped (which we did not know) for all but the most extreme values of X (which we did not know). We superimposed on this distribution 8 pass marks and the proportion that passed, from *tool2*. For each value of X that we used, the results showed:

- using a Yes/No quiz (where X=1) (where all stems are answered and no negative marking is used) the mean mark is 50, which we already knew from our analysis (and thus helped with the validation of the programs that produced 50),
- using a Yes/No quiz where all stems are answered and *exact* negative marking is used (**a**=-1/X) the mean mark is 0 (as we expected).
    - A conclusion is that the distribution moves to the *left*.
- using a Yes/No quiz, where stems can be missed and negative marking is not used, the mean mark is 33,
- using a Yes/No quiz, where stems can be missed and *exact* negative marking **and** used **b**=0, the mean mark is 0.
    - A conclusion is that the distribution moves to the *left*.

Thus, we can conclude that when X=1 the distribution moves to the *left* no matter whether stems can be missed or not and whether exact negative marking (its modification) is used or not.

The same conclusion applies for the various values of X that we tried (viz. 1, 3, 9, 19, 184755) (except for the highest value of X, where the distribution did not move). If we try to tackle the guessing problem by using negative marking (or its equivalent) there are 'fairness' problems

Now, we look at raising the pass mark (which we superimposed by hand onto the distribution) and find that for all but where X=1 (and negative marking is not used), the proportion that pass is low.

We, further, find that as X increases, the mean mark decreases, and that the marks are 'bunched' at the 'bottom' end and the likelihood of passing is low (except for when X=1).

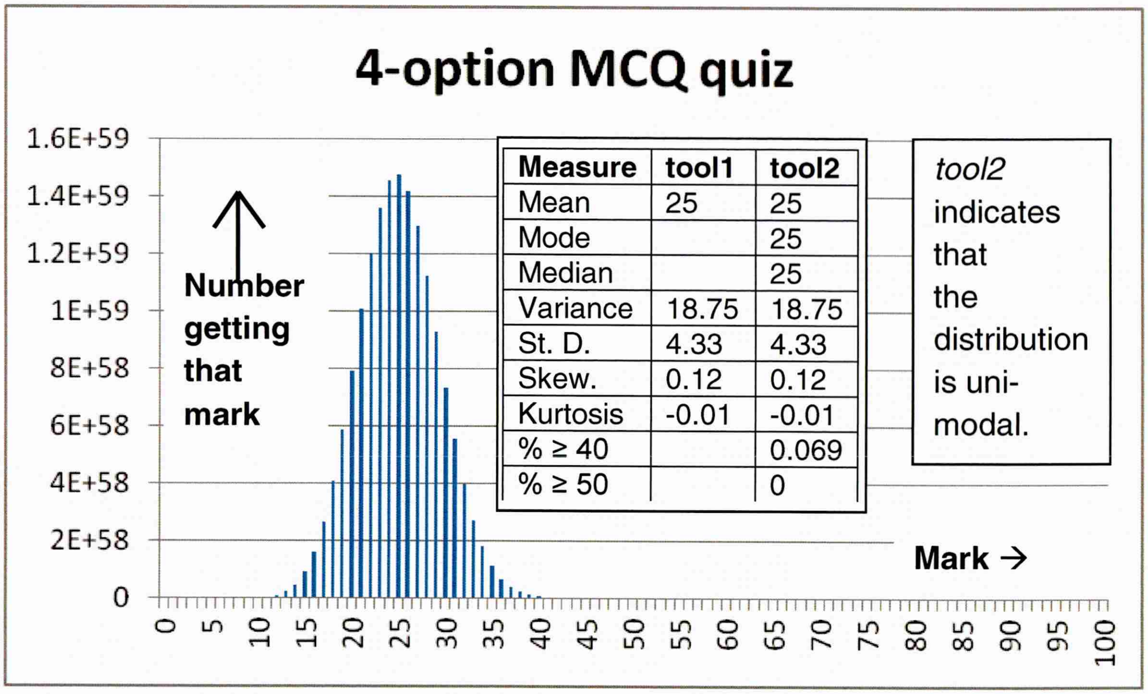
That means that:

- if we try to tackle the guessing problem by using *negative marking* (or its equivalent) alone there are 'fairness' problems,
- if we try to solve the guessing problem by *raising the pass mark* alone there are problems of comparison of marks,
- if we try to solve the guessing problem by *using a more complex* quiz there are again problems of the likelihood of passing being low,
- if you try to tackle the guessing problem by any *combination* of the above it only compounds the problems.

So, tackling the guessing problem brings with it more problems.

8.4 Did we achieve our RQ?

Yes we did achieve our RQ (aim), which was "What is distribution of marks of 'special' quizzes when a census is done?" 'Special' quizzes, which are exemplified by 4-option Multiple Choice Question (MCQ)-only quizzes. 4-option Multiple Choice Question (MCQ)-only quizzes have a distribution of marks as follows, for a quiz having 100 stems and every stem is answered and the score for an incorrect answer to a stem is 0 i.e. negative marking is not used, and the score for a correct answer to a stem is 1.



Measure	tool1	tool2
Mean	25	25
Mode		25
Median		25
Variance	18.75	18.75
St. D.	4.33	4.33
Skew.	0.12	0.12
Kurtosis	-0.01	-0.01
% ≥ 40		0.069
% ≥ 50		0

Figure 8-1. Distribution of marks of a 4-option MCQ-only quiz consisting of 100 stems, where ALL stems are answered and **a**=0.

The distribution of marks was drawn by Excel™, which was fed into by *tool1*, and Excel™ fed into *tool2*. We built *tool1* and *tool2* in the programming language C which, as you can see, use very large *integers* e.g. 1.6E+59 which is $1.6*10^{59}$ and is much larger than C can normally deal with. The maximum size of an integer in C depends on the number of bits the Operating System uses and the number of bits required to store an integer. Whatever the largest integer is, we are dealing with much bigger integers than this. Hence, *tool1* and *tool2* had to use a library of routines that could handle arbitrary large integers. The library we used is known as LEDA, which does not seem to stand for anything. The LEDA routines are written/maintained by **A**lgorithmic **S**olutions (**AS**) of Germany and run in **V**isual **S**tudio (**VS**). There were difficulties associated with:

- loading the latest version of VS,
- loading the latest version of LEDA,
- loading LEDA into a space that would not be 'wiped',
- learning C,
- getting C and LEDA and VS to work together,
- writing and re-writing programs in C, when later versions of VS and the LEDA routines became available.

See Appendix 18 on difficulties.

This section prompts three questions:

- "What is a stem?"
- "What are 'special' quizzes?"
- "What is the guessing problem?"

A *stem* is the question that is asked. For example, a Yes/No stem is "Does a key uniquely identify a record?"

'*Special*' quizzes are exemplified by 4-option **M**ultiple **C**hoice **Q**uestion (**MCQ**)-only stems. When we talk about 4-option **M**ultiple **C**hoice **Q**uestion (**MCQ**)-only quizzes we mean that *every* stem is a 4-option **M**ultiple **C**hoice **Q**uestion (**MCQ**) stem.

The *guessing problem* is where the correct answer to a stem is guessed.

8.4.1 What were the objectives of the study?

The objectives of the study were to see how the distribution changes when the guessing problem was tackled by:

- using negative marking,
- increasing the pass mark,
- using a more 'complex' quiz.

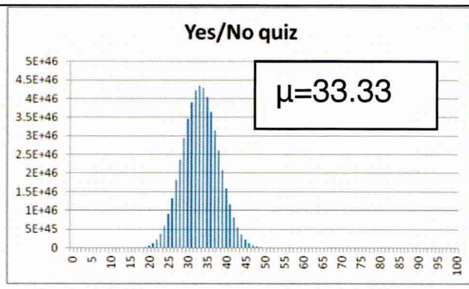
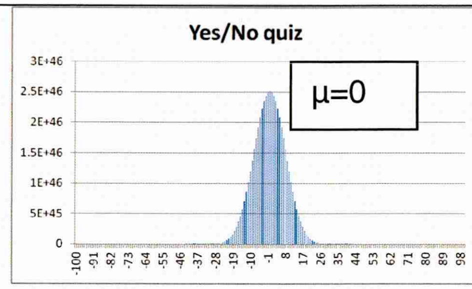
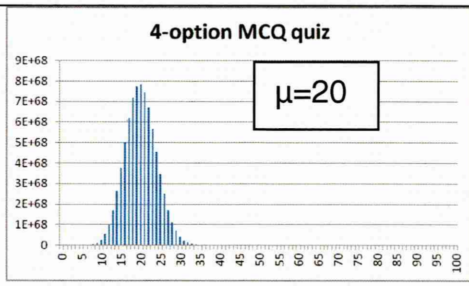
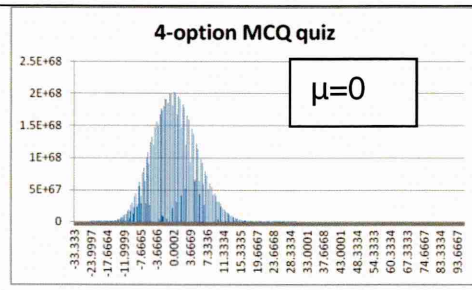
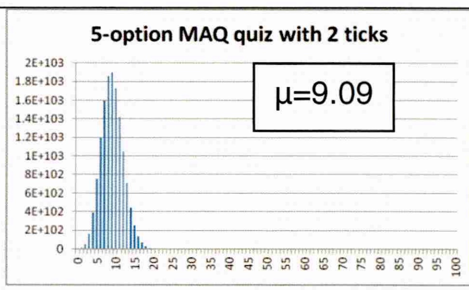
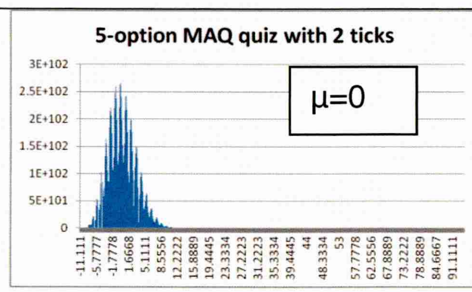
8.4.2 Did we achieve our objectives?

Yes, we achieved our objectives.

If the guessing-problem is tackled by *exact* **negative** marking (or its equivalent where **b** = 0), then the distribution moves to the left.

For example:

Table 8-1. The distribution moves to the left when stems can be MIISSED and the equivalent of exact negative marking is used (i.e. where **a**=-1/X and **b**=0).

Type of quiz	**Stems MISSED** **a=b=0 Range of marks is 0 to 100.** **Not symmetric**	**Stems can be MISSED** **a=-1/X, b=0 (equivalent of exact negative marking)** **Mostly not symmetric**
Yes/No-only (X=1)		
4-option MCQ-only (X=3)		
5-option MAQ-only with 2 ticks (X=9)		

Exact negative marking (or its equivalent) is where an incorrect answer to a stem is given a negative score that makes the mean mark 0. For example, when a quiz is a Yes/No-only quiz the score for an incorrect answer is -1. The generalisation is **a**=-1/X. The equivalent is when stems that are missed are given a score of 0. The conclusion we came to were:

- If the guessing-problem is tackled by **increasing** the **pass mark** then the proportion of passes goes down as the pass mark is increased. If we look at the left hand pictures of the set of diagrams above, we see that, it is pointless raising the pass mark above 40, for 4-option MCQ-only quizzes.

- If the guessing problem is tackled by **using a more complex quiz** then the distribution moves to the left and mean mark goes down and half the area of the distribution of marks is between the lowest mark and this mean. When the mean is low then the likelihood of passing is low. We, again, can see this by looking at the pictures above.
- If we combine the ways of tackling the guessing problem e.g. using exact negative marking and raising the pass mark, then we combine the difficulties. For example, if we use a Yes/No-only quiz, and use the equivalent of exact negative marking and raise the pass mark to 70 (i.e. the 'top' row of the above set of diagrams), then we tackle the guessing problem, but then the likelihood of passing is about 0, and there are problems to do with 'fairness' and comparison.

8.5 What are the limitations of the method that was used?

The method that was used, MyMethod, was to build two tools, *tool1* and *tool2*, and use the widely available Excel™ and use tab-delimited text files in between. *tool1* and Excel™ that communicated via a tab-delimited text file. Excel™ and *tool2* communicated through another tab-delimited text file. The limitations of the method were down to Excel™ having limitations of:

a) the maximum width of a cell,
b) the maximum number that Excel™ could deal with.

For the numbers that we dealt with there were no limitations on the sizes of the tab-delimited text files, but when mixing is considered there might be.

The tools, *tool1* and *tool2* have a limitation on the maximum size of an index of an array. For the numbers that we dealt with, this maximum size was not encountered.

8.6 What assumptions are made, and can they be achieved?

The assumptions that we made in the study, included:

- all stems are equally 'hard',
- within a stem all options are equally 'hard',
- all stems are scored the same, and for this thesis the score for a stem is [0], [1, **a**], where 0 is the score of a MISSED stem (i.e. **b**=0), 1 is score when a stem is answered correctly and **a** is the score when a stem is answered in-correctly. If negative marking is used we make **a**=-1/X and if the equivalent of negative is used then (**a**=-1/X **and b**=0).

We cannot define 'hard' (because what might be 'hard' to 1 person might be 'easy' to another) and so we assume that the correct answer to a stem has the same probability of being guessed as any other stem. This means that every stem is scored the same. Thus, in this study, every stem has the same likelihood of being answered as other stems and stems have an equal likelihood of being missed. A further assumption is that all stems in a particular quiz the same number of options which are equally likely to be chose. Yet another assumption is that we do not mix in the same quiz stems that have a different X. The 'hardness' problem disappears with these assumptions. We further assume that the learner cannot work out the correct answer to a stem, and that the user cannot calculate better odds of an answer being correct.

In 'real' life the likelihood of every stem being equally hard is not measured and the likelihood of every option within a stem being equally 'hard' is also not measured. It is postulated that this likelihood gets lower the more when there are and the more stems. For example, if a quiz has 60 stems, it is very unlikely that the correct answer to each of the 60 stems can be guessed with the same likelihood. But it is easier to achieve with a quiz that only has 5 stems. Can the aim of making each stem equally hard ever be achieved? It is unlikely, but is assumed to be the case, in this study. If each stem has 20 options then *can* each *option* be equally hard? Again this is unlikely, but is assumed to be the case, in this study, in order for the analysis to be done.

For this study we simply assume that every stem is equally 'hard' and within a stem every option is equally 'hard'. We simply ignore the 'hardness' problem in this study, and then the analysis can be done. These assumptions might not be ever possible to be achieved. The more stems there are and the more options there are, the less likely the assumptions are going to be to be achieved.

Another assumption is that CBM is *not* used for scoring. The main reason is that, that for the method we use cannot easily calculate the number of ways that a mark can be obtained. Because, there are 7 marks when a quiz consisting of 1 stem is used, and there are 7^2 marks when 2 stems are used, and 7^{100} marks (some of the marks might be repeated) when a quiz of 100 stems is used. This simply is too big, and a sample does not represent the population.

Another assumption is that 'guy' next to you sits the same test as you. One way to do this is to have a different quiz for every cohort, but then setting the quiz is not quick. Then, neither is an 'ordinary' exam, but the quiz is at least reliable (but if might be invalid) and quick to mark by a computer. Another way is to use a different pool for each cohort. But, comparing cohorts might prove problematic. This argument applies to all type of assessment, and so comparing cohorts might never be possible.

We assume that no cheating goes on and so even though the authentication problem exists it is assumed that it does not apply and nor does any other form of cheating.

Another assumption is that *norm* reference marking is *not* used. We assume that the mark you obtain does not depend on how well others do. One form of Norm reference marking makes the assumption that the sample of marks is so large that there is a fixed proportion of Passes/3rds, 2.2s, 2.1s and Firsts. Another form of Norm reference marking calculates every time how many obtain a Pass etc. An objection to this form of marking is that it does not take into account the 'good' class.

8.7 Can these assumptions ever be met.

Here, we are only considering a) that stems are equally 'hard' b) options within a stem are equally 'hard'. If we say that stems are equally 'hard' then they can all be scored the same e.g. 1. That is a justification of using 1 for the correct answer to a stem. If we use 1 for every stem that is answered correctly then all stems are equally hard and **c**=1.

If stems are equally 'hard' then all options within a stem must be equally 'hard' and then if we assume that stems are all of the same type then all in-correct options are scored the same. The independence problem rears its 'ugly' head in that we assume that all stems are independent and we know they cannot be truly independent, because every stem is about the same subject e.g. data mining. We simply say that the stems must be **as independent as they can be and** that options of one stem must be **as independent as possible**.

So, some of the assumptions cannot ever be met

8.8 Closed quizzes are easier to answer correctly than open quizzes.

An 'open' quiz is where the user can say *anything* and 'closed' quizzes include the correct answer to a stem, as one (or more) of the options. We only analyse quizzes that are 'closed', but that means that:

a) the correct answer to a stem is told to the learner as one (or more) of the options
b) the correct answer to a stem can be guessed.

In an 'open' quiz, like a 'pub' quiz, the likelihood of guessing the correct answer exists, but is very low and cannot be calculated.

'Closed' quizzes are 'easier' to answer than open quizzes because there are an infinite number of answers to an 'open' quiz and a small number in a 'closed' quiz. Furthermore, with 'closed' quizzes it is not possible to *explain* how the correct answer was arrived at. A way of summarising these arguments is to say "'Closed' quizzes take away the element of inventiveness". This suggests that another form of assessment, like a proctored examination, is required. But the explanation of an answer, or the use of an 'open' quiz, requires the use of judgement (by the marker(s)) to see whether a stem was answered correctly and as a consequence to arrive at a mark.

8.9 What have we leant?

Biggs and Tang (2009) says that *learning* can be compartmentalised into two:

a) 'deep',
b) 'surface'.

and that *knowledge* can also be compartmentalised into two:

a) non-procedural
b) procedural

They also say that *knowledge*, *assessment* and *learning* should be all *aligned*.

Biggs and Tang (2009) maintain that MCQs (which are a type of assessment) encourage 'surface' learning.

We go one step further and say that 'special' quizzes *cannot be aligned* when **understanding** of a concept is tried to be measured, because there is no space to allow the learner to *demonstrate* his/her understanding. This makes 'special' quizzes **in-valid** when understanding is claimed to be measured.

Another form of in-validity is when 30% of the stems are answered correctly and the rest guessed, at a 40% institution (where the pass mark is 40%). The same form of invalidity occurs at a 50% institution when 40% of the stems are answered correctly and the rest guessed. Here, 'special' quizzes are **in-valid** because you pass when you should have failed.

Quizzes are a type of assessment. There are many types of assessment e.g. examinations, group work, posters, quizzes, screencasts. These are not mutually exclusive. An assessment can be of two (or more) types e.g. group work of a poster. If Lecturers are urged to include many types of assessment then it leads to difficulties of comparison of marks gained from quizzes and examinations, say. The basic difficulty is because the marks might have come from different distributions.

Quizzes can have all their stems answered or their stems can be missed. For both of these we can tackle the guessing problem by:

a) using exact negative marking e.g. **a** = -1/3 or its modification with **a** = -1/3 and **b** = 0
b) raising the pass mark e.g. to 70
c) using a more 'complex quiz e.g. a 5-option MCQ-only quiz.

Quizzes where we insist that their stems are all answered either last for ever or **guessing** the correct answer to stems is **forced** (when the correct answer is not known). A much more realistic situation is to allow stems to be missed, with a consequent reduction in the maximum mark that can be obtained.

Why is assessment done at all? Assessment is done for at least three reasons:

a) to see whether learners have met the standards of the learning outcomes,
b) to form an opinion of learners,
c) to assist the learning process, c.f. Black (1999).

Assessment comes in three types:

a) formative,
b) summative,
c) ipsative. (c.f. Freeman and Lewis (1989)

Formative assessment might be to see whether learners know the syllabus and to form an opinion of learners and to assist the learning process or to find out any mis-conceptions. Summative assessment is used to see whether learners know the syllabus, and is where the learner is given a mark (sometimes a learner is given a mark in formative assessment).

Ipsative assessment (Freeman and Lewis 1998) is often applied to oneself, and measures if there has been an improvement. It is often done by marathon runners. We note that when it is applied to marks of a quiz where negative marking has been used and all negative marks are made into 0, then ipsative assessment cannot be used.

An instrument is **valid** if it *claims* to measure what it *actually* measures (Cohen, Manion, Morrision (2007)), Hopkins (2010), and an instrument is **reliable** if the measurement it gives is *repeatable* (Trochim (2006)). An example of an in-valid instrument is one that claims to measure IQ but in fact measures memory. An example of an un-reliable instrument is one that gives a reading of 100 on one day and 98 on another, ceteris paribus.

For a 100 stem quiz where every stem is answered and **a** = 0 where 30 stems are answered correctly and 70 guessed, the mark obtained, when the correct answer to a stem is scored 1, is:

- for Yes/No-only quiz 30 + 70/2 = 65
- for a 4-option MCQ-only quiz 30 + 70/4 = 47.5

- for a 5-option MCQ-only quiz   30 + 70/5 = 44
- for a 6-option MCQ-only quiz   30 + 70/6 = 41.67

Thus, at a 40% institution, using a more 'complex' quiz for some quizzes, results in a pass when 70% of stems are guessed and 30% answered correctly. Thus, for some quizzes, they are *in-valid* (they do not measure how much of the syllabus is known), even though they are reliable.

So, validity adds to reliability. We might be measuring the wrong thing even though we can repeat the measurement.

If we ask "Why use quizzes at all for assessment?" Some answers are, because:

a) if they are marked by computer they are 100% reliable and the marking is quick and there is no inter-rater variation or intra-rater variation (Trochim (2008)),
b) they are objective (Macintosh and Morrison (1969)).

There are many objections/questions to the quickness argument. Oppenheim (1992) and McNeil (1990) say that a *pilot* study ought to be done. That takes time. Some stems might be taken out, some stems revised and some stems added. That also takes time; stems have to be written. Stems have options, which have to be written. Correct answers have to be checked. That, all takes time. If 'pools' are going to be used, then many stems each with many options have to be created and tested. This takes time. If pools are used then "the guy next to you might not take the same assessment as you" and then how can you compare your mark to that of the guy sat next to you? Pools can be used, if the same quiz is given to a cohort. This gets round the problem of "the guy sitting next to you does not sit the same test". But what happens if you want to compare cohorts? If pools are not used, then very quickly, cohorts will know what the test consists of. If pools are not used then a different quiz for every cohort has to be created. Then this takes time and teachers prepared for this burden? Other objections to setting a different quiz for each cohort are a) how do you compare cohorts that have different assessments? b) how do you cope with a 'good' cohort who have received a 'hard' assessment? Another way of putting this is how do you assure that all the quizzes are equally 'hard'?

The use of pools has a knock on effect to examinations. Should the rubric of a 10 question examination say "full marks can be obtained by answering 7 questions"? The guy next to you might answer different questions to you, and yet the examiners might say he got a 2.1 and you got a 2.2. A fairer rubric might say "Full marks can be obtained by answering ALL questions".

Then we come to the objective argument. Are stems objective or subjective? Why was this stem set by the setter and not that one? Do stems reflect the bias of the setter? The same questions apply to options. For example, "is each option plausible?" or can some options be ignored, because they are 'obviously' wrong. An example of this is a **stem** which says:

What is official currency of Malaysia?

The **options** are

Dong

Ringitt

Rupee

A learner might *not* know the correct answer to the stem, but knows that the Rupee is to do with India. This means that either Dong or Ringitt is the correct answer. This increases the likelihood of guessing the correct answer from 1/3 to 1/2. If on further reflection the competitor says that the Dong is to do with China or Japan, and so says that the Ringitt is official currency of Malaysia. The competitor gets the answer to the stem correct even though the reasoning about the Dong was in-correct. The conclusions from this example are a) that the options are not equally plausible b) the correct answer was obtained by the wrong method because the correct answer was given. But, using the wrong method is arguable . What happens if the competitor did not know that the Rupee was to do with India? This way the options are equally plausible and, if the correct answer is not known, a guess is called for, with the likelihood of guessing the correct answer to be 1/3. Another argument is that 'special' quizzes, do not have space to show *how* the correct answer was arrived at and so the correct answer to a stem might be arrived at by the wrong process (as shown in the above example).

Thus, the questions to ask about quizzes include:

- Do 'special' quizzes show that the learner **understands** a concept?
- What type of knowledge is being imparted?
- Is the assessment *aligned* to the type of knowledge being imparted and the type of learning?
- Why bother with assessment?
- Why bother with quizzes?
- Is formative, summative or ipsative assessment being done?
- Are quizzes valid?
- Do quizzes show any bias?
- Are stems equally 'hard'?
- Are options within a stem equally 'hard'?
- How are pools used?
- Are comparisons of marks and grades done, this includes within a cohort and between cohorts and between institutions?
- Is negative marking (or its equivalent) going to be used?
- Is the pass mark going to be raised?

Some of these questions were asked in a conference paper by Lafferty & Burley (2012b).

8.10 Why is the pass mark raised?

The pass mark is raised for quizzes for, at least two possible reasons, which might overlap:

- To tackle the guessing problem,
- To make passing the quizzes 'harder' and thus make the quizzes 'more exclusive'.

The tackling of the guessing problem by raising the pass mark brings in questions of 'fairness' and comparison with other methods of assessment which do not raise the pass mark. If comparison is NOT done, then raising the pass mark is allowable to tackle the guessing problem, and then all that has to be asked is "What should the pass mark be raised to?" We attempt to answer that for different pass marks and different types of quizzes. For example, where stems can be missed and when **a** = **b** = 0

For a 100 stem Yes/No-only quiz then raise the mark to **71**.
For a 100 stem 4-option MCQ-only quiz then raise the pass mark to **50**.
For a 100 stem quiz:

- for a 5-option MAQ-only quiz with 2 ticks
- or for a 20-option EMSQ (MCQ)-only quiz

- or for a 20-option ENMSQ (MAQ)-only quiz with 10 ticks

there is no point in raising the pass marks above **40.**

This does not answer "What is the **best** pass mark to the raise to?" because it depends on the type of quiz that is used. It also does not deal with the unlikeliness of using a 100 stem quiz. So, let us see what a 60 stem quiz yields

For a 60 stem Yes/No-only quiz then raise the pass mark to **42** (i.e. **70**%).

For a 60 stem 4-option MCQ-only quiz then raise the pass mark to **25** (i.e. **40**%).

A measure of the number of marks in a grade is $\mathbf{M_g}$ and the way to use it is as follows. When a quiz consists of 100 Yes/No stems only where all stems are answered with **a** = 0 then the number of marks in a grade is given by [10, 10, 10, 31] i.e. there are 10 marks in the grades Pass/3rd , 2.2 and 2.1 and 31 marks in the First grade (at a 40% British University). If now, a quiz consisting of 100 Yes/No stems only, where all stems are answered and **a** = -1 then the number of marks in a grade is [5, 5, 5, 16]. Thus, by using *exact* negative marking to counteract the guessing problem, where all stems are answered, results in about half the number of marks in a grade when a quiz consisting of 100 Yes/No stems is used. If we now look at a 100 stem 4-option MCQ-only quiz, the number of marks in a grade is given by [10, 10, 10, 31] when **a** = 0 and all stems are answered. Counteracting the guessing problem *exactly* with **a** = -1/3 results in [27, 27, 27, 91] where all stems are answered, and so how can you compare [10, 10, 10, 31] to [27, 27, 27, 91]? When you try to counteract the guessing problem you end up, this way, with more marks in a grade. Similarly, for 100 stem 5-option-only quiz with 2 ticks where stems are all answered, with **a** = 0, the number of marks in grades is [10, 10, 10, 31]. When the guessing problem is tackled by **a** = -1/9 then the number of marks in grades is [90, 90, 90, 271]. Again, how can you compare [10, 10, 10, 31] to [90, 90, 90, 271]? What you are doing is comparing two different distributions and all that can be said is "They are different" and comparing them is 'hard'".

8.11 What we have not done.

We do not tackle the 'hardness' problem: we simply assumed it away. We do not consider whether one stem is 'harder' than another, and we do not allow one stem to have more options than another. A stem that is scored more than another implies that one stem is 'harder' than another. This becomes 'too subjective' and all stems have the same score in our system.

We have not tried to tackle the 'nearly right' problem which is exemplified by 6-option MAQ quizzes that have 4 ticks and a learner only has 3 correct ticks. Do we give that a score of ¾ or 0 or something else? What *we* do is to say that this stem has been answered in-correctly and hence is scored 0.

We have not tried to decide:

- how many stems there should be in a quiz,
- how many options a stem should have,
- how long, in time, a quiz should be.

We know that all of these problems are related in that it takes time to read a stem and if a stem has many options that takes more time to read. If the quiz lasts for 1 hour then the more options stems have the fewer stems the quiz can have.

8.12 What would I do differently if I could do the study again?

The things that I might change, include:

- It is debatable whether I would find the subject of the thesis more quickly.
- Notes would be taken more carefully.
- Notes would be put in a place that could be searched easily.
- Does going on an nVivo courses help?
- Going on a longer statistics course.

Using my supervisor to give me a subject to study would be faster, but is it any challenge? Am I interested enough to spend 6 years studying something that does not interest me much? Am I better off reading around and gradually finding a subject that interests me? So, maybe I could find a subject more quickly but would that be sustainable?

Notes would be taken more carefully. Despite all the warnings, understanding what I read is the most important and being able to find what I have read (and the page number) is also important, without wasting time. Being able to quote what I have read is also important, and being able to find the quotes quickly of what others said (without recourse to re-reading the article, and thus wasting time) is important. Thus searching easily what I had read is of supreme importance.

A statistics course provides what tests there are and why. These tests might prove to be un-necessary in a particular thesis, but at least they are in the armoury, and might be applied. For example, "Can parametric tests be applied to see whether the data is Normal?" Another example is "Should the Kolmogorov-Smirnov be used or Shapiro-Wilks for tests of normality?"

A recommendation is **all** quantitative PhD students take a statistics course at Sheffield Hallam University.

8.13 A critical evaluation of the thesis.

The time that was spent on quizzes where ALL stems are answered has not been wasted but these quizzes can be ignored if we posit that each quiz can either have ALL its stems answered or have some of its stems MISSED. If one insists on ALL stems being answered then **forcing** of *guessing* the correct answer to a stem goes on, if the correct answer to a stem is not known. You cannot force anybody and if they do not want to answer a stem then they will not, and obtain a reduced maximum mark. You cannot force anybody and if they do not want to answer a stem they will not, but they will obtain a reduced mark. Thus, we can ignore those parts of the thesis where ALL the stems are insisted on being answered, and we can ignore all those (interesting?) proofs where ALL stems are answered. But, if formative assessment is being done it might be argued that forcing of guessing does not matter much and in formative assessment any of the assumptions that we make, for the analysis to be done, might be broken. So, we assume that summative assessment is being done. We simply ignore ipsative assessment.

This means that stems can be MISSED, and so we ask "What score should we give to a MISSED stem?" An argument is that the learner did not answer a MISSED stem in-correctly or correctly and so the learner should not be punished or rewarded and thus the score should be 0. Throughout this thesis we have assumed that the score for a MISSED stem is 0 (i.e. **b** = 0). Future work will look at what score should be allocated to **A** and **b** (i.e. **A** is the score given when a stem is answered in-correctly, and **b** is the score when a stem is MISSED).

We cannot force a learner to answer a stem. There is always a chance of some stems being missed. Thus, we must allow stems to be missed and half of the thesis can be ignored. When stems can be MISSED, without negative marking being used, the distribution moves to the left. But, "how far does the distribution move when he/she is taught?" and "what measure shall we use?"

This brings us to the question of "Should we use mean mode or median and does it matter"? They all move to the left for 'special' quizzes and if we are only bothered about the distribution moving to the left we can use any one of them. If we are bothered about *how far* the distribution moves we can use one of mean, mode and median. But, if they are all about the same, then it does not matter which we use and we might as well use only one of them, say mean. The mean is accurate (when *tool1* is used) and mode and median (when *tool2* is used) might be inaccurate, but if they are all about the same, we might as well use mean.

This brings us to the questions:

- Do we need *tool1*?
- Do we need *Excel*™?
- Do we need *tool2*?

The tool, *tool1* calculates:

- a mark
- how many ways there are of obtaining that mark for a particular combination of **w** and **y**.

w is the *number* of stems that are answered correctly and **y** is the *number* of stems that are missed. The numbers involved in (how many ways there are of obtaining a mark) are so huge that normal programming languages cannot cope.

Thus, something must be done to allow the calculations to be done. A solution is the adoption of a library of routines that cater for arbitrary length integers. We used the LEDA routines that are in the programming language C and run in VS. SPSS a) assumes a sample b) estimates the statistics of the sample. We wanted to use a census and not estimates. So, something like *tool1* is necessary. The tool *tool1* is limited by the size of the index of an array in the programming language C, but this limitation can be avoided if we are willing to calculate a mark **again** when calculating variance, standard deviation, skewness and kurtosis. So, *tool1* has limitations that can be avoided and so *tool1* has no limitations until we reach the limitations of the operating system. For example, when an integer is so large that it will not fit the memory of the computer. This then complicates the issue and we ask "What memory?" Are we just talking about RAM or do we included disc(s) (what about networked discs)? Do we include sticks? To not complicate the issue we will just talk about RAM.

The mark obtained by *tool1* might not be unique and so the **same** mark might be obtained by a different combination of **w** and **y**.

So, a tool is needed to:

a) *do a Pivot*,
b) draw the resultant distribution.

Excel™ is one of the tools that can do a Pivot and draw the resultant distribution. *tool1* and Excel™ need to communicate and they do that via a tab-delimited text file. Excel™ can do a Pivot and draw the resultant distribution until it reaches its limits, but in doing a Pivot, Excel™ introduces an approximation, that is passed on to *tool2*. Excel™ does 4 things for us:

a) *imports* the data that is produced by *tool1* that resides in a tab-delimited text file,
b) performs a *Pivot* on this data,
c) *draws* the distribution of marks when a Pivot is done,
d) *exports* the results of the Pivot to another tab-delimited text file.

The tool, *tool2* calculates some descriptive statistics, from this tab-delimited text file. Some statistics can be calculated before a Pivot is done, which *tool1* also does. But, *tool2* also calculates the **mode**(s) and **median**, and also calculates the 'area' of the distribution that covers 40% (say) or more of the maximum mark. All of these require a Pivot to be done. It could be argued that the mode and median are close to the mean and hence do not need to be calculated by *tool2*, because the mean will do. Anyway the data that *tool2* works on is an approximation, and so the mode and median are approximations. For these reasons parts of *tool2* can be ignored, and we are only left with dealing with raising the pass mark to be dealt with by *tool2*.

The conclusions of all this are:

- *tool1* has no discernible limitations,
- Excel™ has limitations which it passes on *tool2,*
- *tool2* has the limitations that are passed to it, but it is only necessary to calculate the proportion.

All the tools (or something like them) are necessary, and they work together.

8.14 Verification.

In this section we are asking "Are we building the right tool?" An argument goes that 'special' quizzes do not allow learners to show that they *understand* a concept. A plausible argument is that learners go through the process of understanding to obtain the correct answer, but nevertheless, they might have guessed the correct answer and not gone through this process.

Another form of assessment that allows the demonstration of *understanding* a concept is needed, for summative assessment, where a comparison of marks/grades is performed.

8.15 The contribution that this research makes to the body of knowledge.

This research makes sure that quizzes are based on a sound footing, so that the distribution of marks of a 'special' is known. Nobody has asked the question before "What is the distribution of marks of a quiz?" but they have asked other questions like "What rules should we follow to create 'good' stems and options?" (c.f. Kehoe (1995), Haladyna and Downing (1989), Cheung and Bucat (2002), Boland, Lester and Williams (2010)). Another question is "How well do these quizzes *discriminate* students?" (c.f. Macintosh and Morison 1969), Hoffman 1962)). Yet another question is "What is the optimal number of options?" (c.f. Rodriguez 2005), Vyas and Supe 2008).

Quizzes (in this thesis) are limited to 'special' quizzes exemplified by 4-option MCQ-only quizzes. These 'special' quizzes are further restricted by assumptions made about them, so that their analysis can be performed.

'Special' quizzes all have the property that they contain the correct answer, which can be guessed. This 'guessing problem' is tackled in 3 ways (in this thesis) and each of them (or a combination of them) results in more problems. The 3 ways and their associated problems are

- using negative marking
    - fairness e.g. is it 'fair' to answer 69% of stems correctly and fail,
- using a more complex quiz,
    - fewer passes the more complex the quiz is,
- raising the pass mark
    - "what is the pass mark for other parts of this module?".

All 'special' quizzes are **in-valid** when **understanding** is **claimed** to be **measured**, and they should not be used for summative assessment where a comparison of marks obtained by other assessment types is done.

The contribution of this research to the body of knowledge is that 'special' quizzes should not be used for assessing anybody *summatively* when a claim of understanding is made and when a comparison of marks/grades is done. Some research has been done on using quizzes to assess formatively and summatively concentrating on what stress the students are under (c.f.Cassady and Gridley, 2005, Miller and Parlett, 1974).

8.16 Recommendations for practice.

- Do not use 'special' quizzes for measuring understanding.
- Under some conditions 'special' quizzes are *in-valid*, so do not use them under these conditions to assess somebody summatively. For example, when 100 Yes/No stems are used in the quiz and 30 are correct and 70 guessed (with **a**=0) the mark obtained is 65. Under these conditions marking by CBM can also be in-valid (when 30 stems are answered correctly with C=3 and 70 are guessed with C=1),
- Do not use 'special' quizzes to assess someone summatively, where the mark that the quiz produces is compared to the marks produced by other forms of assessment.
- Do not use 'special' quizzes for summative assessment, because they include the answer, which can be guessed. Tackling this guessing problem produces more problems. You can use 'special' quizzes for *formative* assessment even when they are in-valid.
- Do not 'mix' questions in the same quiz e.g. do not use Yes/No stems and 4-option MCQ stems in the same quiz, because the chance of guessing the correct answer to a Yes/No stem is 1/3 and the chance of guessing the correct answer to 4-option MCQ stem is 1/5 (in a quiz where stems can be missed with **b**=0).
- Do not use pools of stems, to assess somebody summatively, because 'the guy sitting next to you' might not sit the same exam/assessment.
- Do not use negative marking, to assess somebody summatively, because that leads to unfairness when people or modules or institutions are compared where negative marking is not used.
- If you are still going to use a 'special' quiz, summatively, ask lots of questions first. For example, a) what type of knowledge is being imparted, b) are the type of knowledge and the type of learning and the assessment all aligned?
- All quantitative PhD students go on a statistics course.

8.17 Part 2. Future work.

This Part contains work that might be done in the future, by me or somebody else. The future work does *not* include writing up *this* research, even though, some of the write up, is to be done in the future.

It is suggested that, in order for the *proposed* research project to become an *actual* project it should pass 2 tests:

- the broad test,
- the detailed test.

Then if many proposed projects pass both tests, they should be *ranked*, and work can start on the proposed research project which becomes the top ranked actual project.

We will show some proposed research that passes:

- both tests,
- the 'broad' test and fails the detailed test.

8.17.1 The 'broad' test

The new research should be 'doable' and the short report about performing the new research should contain

- what question is being asked? i.e. what is the **RQ**.
- what **experiment(s)** should be done, if any,
- an indication **how long** it will take e.g. if it indicated that it will take longer than 6 years then it is not suitable for a PhD,
- what **data** is needed, if any,
- have we got the necessary expertise (can we obtain the necessary expertise).

If the proposed project passes the 'doable' ('broad') test then a more detailed test should be done. Only when a proposed project passes these detailed criteria should the project be considered, but even then it might still be rejected.

8.17.2 The detailed test.

The detailed test includes:

- What type of research is being proposed? e.g. longitudinal, 'snapshot'.
- What type of method should be used? Qualitative, Quantitative, Mixed?
- If Quantitative, what hypothesis, if any, is being used?
    - How do you show that the hypothesis stands or falls
- What type of sampling should be used, if any? Probabilistic or non-probabilistic? Within this, what method of sampling should be used? e.g. random, snowball.
- What is scale that the data is measured on? e.g. Nominal, ordinal, interval (e.g.Likert), ratio.
- What tools should be used to gather the data? e.g. open-ended questions via interviews.
- Is a pilot study going to be used and how long does that take? Give details.
- Who is going to gather the data? Do they need training? How long will that take? Is the training material extant?
- Where is the data going to be gathered? Why?

- Do the providers of the data have to be trained? How long will that take? Is the training material extant?
- How long will the data gathering take?
- Do we have to go on courses to gain the necessary expertise, to gather data and how long will that take?
- How is the data going to be analysed?
- Do any other tools have to be built? Have we got the expertise to build them? How can the expertise be acquired? How long will it take?
- Do we have to obtain the Ethics Committee's permission?
- Do we have to do a Risk Analysis?
- Can any of these be done be done in parallel? Maybe a Gantt chart could help with the planning.

There is no doubt that more elements could be added to this list, but the thrust of these lists is rather like doing a triage and then doing a detailed examination.

If many projects pass both tests then they can be ranked. We know that this requires a judgement and this judgement will not be discussed.

8.17.3 The ranking.

The idea of ranking *proposed* research projects into *actual* research projects is to start on the 'most important' first. 'Most important' is a judgement that involves, 'easiest' first, does not take very long, amongst others.

8.18 Idea 1. Using words in other contexts.

The English language changes all the time and some words are borrowed from one context and mean something subtly different in another context. Some words even mean something completely different. For example, the word 'wicked' can now mean the exact opposite of what it did and can also mean 'with a wick' and for you crossword buffs can indicate the presence of an anagram. The word 'decimate' originally mean to cut **by** a tenth and is now used regularly to mean 'cut **to** a tenth'.

I think a subtle shift of the word 'ontology' is now thought of in Knowledge Engineering to mean that everybody in a field must understand one another. Knowledge Engineers in the Information world talk about 'portable ontologies' and these ontologies are like dictionaries which give many examples and from these examples one gathers a meaning. It is a laudable aim but I think that an ontology is about what concepts and ideas are 'out there' and epistemology is about what they mean. The word ontology has been borrowed from philosophy and now embraces epistemology as well.

There is even a subject in Computing that uses the term Profile Ontologies (Sutterer, Droeghorn, David, 2008) and one wonders do they mean the same thing as we mean when we use the word Ontology? A solution to this problem is to put a word in the Glossary of a publication, to define how that word is used in this publication.

The word 'vector' is used in mathematics, botany and medicine and in each occurrence involves a subtle shift in meaning. In mathematics we can talk about a vector having a magnitude and a direction, whereas in medicine we can talk about the vector of a disease, and in botany we can talk about a vector of a plant and not mean what mathematicians mean by the word vector. The word vector is even used for the path taken by a bee. It is used to mean that when a bee flies over a field that has been treated by a type of fertilizer it gets confused and cannot get back to its hive.

8.19 Idea 2. Sampling.

In order to be able to make generalisations from a sample of the population, the sample must *represent* the population. If the sample does not represent the population then one cannot generalise from it to the population. One reason for using a sample is that the whole population is not available.

This looks like a statistics paper/thesis. Some textbooks say that the sample size should be at least 30.

If the population is homogeneous then a sample of size 1 will do, because the sample does represent the population. So, we have to assume that the population is heterogeneous. Then, at least three questions raise their ugly heads:

- How *big* should the sample be? We are told that the bigger the sample size the better.
- Do we introduce any *bias* into our data? For example,
  - if we only sample women, then we cannot generalise to men and women,
  - if we only sample educated male Chinese between the ages of 18 and 21, then we cannot generalise to the whole population (unless the population is educated male Chinese between the ages of 18 and 21). We have introduced at least 4 biases
    - a nationality bias,
    - a sex bias,
    - an education bias
    - an age bias
- What is the *population*? For example, if we assume that the population is the queries that are issued to Google then do not just collect data from Oman, because the data does not represent **all** queries that are sent to Google.

What statistics are there to answer this question?

Straker (2013) says that the probability of answering the question "Does the sample represent the population?" depends on 3 things:

- the method that was chosen to do the sampling,
- the population,
- the question that you are trying to answer.

We, further, ask "What assumptions can be made in order that the problem is tractable?"

For example, do you assume that the data that is in the sample is Normally distributed?

Table 8-2. Can we answer "Does a sample represent the data?"

Question(s)	**RQ**	**Experiment**	**Time**	**Data**	**Have we got the necessary expertise?**
What statistics are there to see whether a sample represents a population?	?	?	?	?	Most probably not

It is not clear that this proposed project passes the 'doable' test and so is rejected.

8.20 Idea 3. Widening the discussion of validity, reliability, precision and accuracy.

The argument about the validity can be widened when we can ask "What is the relationship between validity and reliability?" Can an instrument be reliable and not valid? If we take an example of dart throwing, where a competitor consistently throws the darts on the floor when going for double 20, the dart-throwing by the competitor can be said to be reliable but not valid. The competitor reliably misses double twenty; in fact he throws the darts on the floor every time he is going for double 20. So, something can be reliable but not valid, but in order for an instrument to be valid its measurements must be repeatable.

So, we think that reliability comes before validity.

> *"Reliability is a prerequisite for measurement validity"*
>
> University of Georgetown (2012).

Similarly we have to ask "Can an instrument be valid and yet unreliable?" and it can be. If the claim is that a dart thrower can hit the board but not hit the correct segment, then the dart thrower validly hits the board but does not hit the correct segment repeatedly. Thus the dart thrower has valid throw but not a reliable one.

Thus an instrument might be both valid and reliable, might be valid but not reliable, might be reliable but not valid and might be both un-reliable and invalid.

For example, we are questioning whether scoring by CBM is a) *always* valid b) always reliable c) *always* precise d) *always* accurate e) the relationship between these. We also need to ask "Whether it is valid, reliable accurate and precise to use a quiz to assess somebody's knowledge and understanding, if comparison between marks obtained by a quiz and other forms of assessment is done?"

This turns out to be a number of *proposed* research projects, simply by asking a question (simply by being *curious*).

8.21 Idea 4. Can sampling be ever used?

It might be argued that a sample can never represent the population and so sampling should never be used. This is not quite true. If the population is homogeneous then a sample of 1 would do to represent the population. Another argument is that if the sample has most of the characteristics of the population then the sample nearly represents the population. The argument boils down to when the sample displays most of the important characteristics of the population, can it be said to represent the population? The only difficulty with this argument is "What are the important characteristics?" This can argued about for ever, and except for homogeneous populations, whether a sample can be used is best judged on a case to case footing.

The trouble with this is that sampling *is* used, because measuring the whole population is not feasible. We do not know any statistics that can be used. But, we must be careful that the sample represents the population if we want to extrapolate from the sample to the population.

Table 8-3. Can sampling be ever used?

RQ	Experiment	Time	Data	Have we got the necessary expertise?
Can sampling be *ever* used?	?	?	?	Yes.

This fails the broad test.

8.22 Idea 5. Should pools/banks of questions, be used?

An argument that has been touched on is whether pools of questions should be used, and this expands into whether the 'guy next to you sits the same test as you', and expands still further into what the rubric of an exam should say.

An argument for quizzes using pools (and even pools of pools) is that the quiz can be generated automatically with the use of a random number generator and can be automatically marked by a computer. But, an objection to the use of pools is that the guy next to you might not sit the same test as you and he may fortuitously pass and you may fail. It could be argued that both of you should sit the same test then you both can be compared. To this end, the rubric for an exam could say something like "Full marks can be obtained when all questions are answered" rather than saying something like "Full marks can be obtained from answering 7 out of 10 questions" where the guy next to you might not sit the same test as you. This sort of rubric does not appear on quizzes, but does appear on other tests and we are asking "Should it?"

We do not know what experiments to do when the guy next to you does not sit the same test as you. We have to assume that the guy next to you does sit the same test as you for comparison of marks to be done. Hence, we assume that pools are not used.

Table 8-4. Should pools be used?

RQ	**Experiment**	**Time**	**Data**	**Have we got the necessary expertise?**
Should pools be used?	?	?	?	Yes.

This fails the broad test and counts as a publication associated with *this* study.

Table 8-5. What happens when the 'guy next to you does not take the same assessment as you'?

RQ	Experiment	Time	Data	Have we got the necessary expertise?
Does the guy next to you take the same assessment as you?	?	?	?	?

This fails the broad test comprehensively, because it counts as a publication associated with *this* study, so do not proceed with it.

8.23 Idea 6. What should a and b be?

When stems can be missed in a quiz, we can ask what score is given to stem that is missed and we can ask what score should we give to a stem that is answered in-correctly? We know that an in-correct answer to a 'special' quiz is scored **a**X where X is the number of in-correct answers a stem has. . If we assume that a correct answer to a stem is scored 1 then what should **a** and **b** be, where **b** is the score given to a MISSED stem? We are only considering stems of a quiz that can be missed, because we are looking at when **b** exists. The mean mark for a quiz that has **n** stems is

μ_n = **n***(1 + **a***X + **b**)/(X +2) where X represents a quiz type

So, we ask "What value of **a** and **b** should be used to make μ_n into 0?"

In *this* study we make **b** = 0 but it does not have to be. For a particular quiz type we can ask "What values of **a** and **b** make (1 + **a**X + **b**) into 0?" There is an infinity of the variations of **a** and **b** for a particular quiz. For example, for a Yes/No quiz (where X = 1) we could make (**a** = 2 and **b** = -3) or (**a** = 1.5 and **b** = -2.5), in order to make (1 + **a**X + **b**) into 0. But, then we use a positive value for **a** which encourages in-correct answers, because the tactic, then, should be to answer *all* stems in-correctly. Is this because **a** > **b**. Should we never use a positive value for **a**? Even if it is not known whether a stem is answered correctly or nor, an in-correct answer, it could be argued, that guessing should never be encouraged. Missing is encouraged when **b** > **a**X and so limits can be put on **a** and **b**, where guessing is not encouraged and missing is not encouraged.

Table 8-6. What should **a** and **b** be?

RQ	Experiment	Time	Data	Have we got the necessary expertise?
What should **a** and **b** be?	1. Argue that **a** and **b** should be restricted 2. Within these restrictions vary **a** and **b**.	This will take a long time and is suitable for doing a PhD that follows on from this.	We do not have to collect data, but generate it. The data is quantitative i.e. values of **a** and **b**	Yes

This passes the broad test and if the detailed test is applied we obtain.

1. What type of research is being proposed?	**Theoretical.**
2. What type of method should be used?	**Quantitative.**
3. If Quantitative what hypothesis, if any, is being used?	**None.**
4. What type of sampling should be used, if any?	**None**
5. What is *scale* that the data is measured on?	**Ratio.**
6. What tools should be used to *gather* the data?	**None**
7. Is a *pilot study* going to be used and how long does that take?	**No**
8. *Who* is going to gather the data? Do they need training? How long will that take? Is the training material extant?	**Data is generated.**
9. *Where* is the data going to be gathered? Why?	**Data is generated.**
10. Do the providers of the data have to be *trained*? How long will that take? Is the training material extant?	**Data is generated.**
11. *How long* will the data gathering take?	**Data is generated.**
12. Do we have to go on courses to gain the necessary expertise, to gather data and how long will that take?	**Data is generated.**
13. How is the data going to be *analysed*?	**Don't know.**
14. Do any other tools have to be built? Have we got the expertise to build them? How can the expertise be acquired? How long will it take?	**We do have the expertise.**
15. Do we have to obtain the *Ethics Committee's* permission?	**No.**
16. Do we have to do a *Risk Analysis*?	**Don't know.**
17. Can any of these tasks be done be done in parallel? Maybe a Gantt chart could help with the planning.	**Doubt it.**

This *proposed* project passes the detailed test and can be done (when the **don't knows** are answered) and then a PhD can be started, that follows on from this study.

8.24 Idea 7. Mixing stems.

If we have, in the *same* quiz, stems of *different* types we are said to be 'mixing' stems. For example, if we have Yes/No stems and 4-option MCQ stems in the same quiz we are said to be mixing stems. The difficulty is that that we have to decide how many of each type of stem there should be in the quiz. Consider that the quiz has 100 stems, are there 20 Yes/No stems or 50 or 70? In order to fix on a number we could say that there are 20% Yes/No stems and 80% 4-option MCQs. If we do that can we generalise about mixing stems? We might come to a conclusion about mixing stems that only applies to a 20% mixing. What happens when 50% mixing is done? The whole thing becomes complicated when 3, or more, types of stem are 'mixed' in the same quiz. For example, Yes/No stems, 4-option MCQ stems and 5-option MCQ stems in the same quiz. Then what proportion should be Yes/No stems, what proportion should be 4-option MCQ stems and what proportion 5-option MCQ stems?

About the only thing we know, a priori, is that when 2 types of stems are mixed in the same quiz, the likelihood of guessing the correct answer to a Yes/No stem is 1/3 and the likelihood of guessing the correct answer to a 4-option MCQ stem is 1/5 (when stems can be missed).

A principle of starting off simply and getting more complicated suggests that we choose to mix 2 types of stems and then vary their proportion and see what happens. We might be able to draw some conclusions. But, starting off with mix of many types of stem mixed might lead to no conclusions.

So, a research idea is to see what happens when 2 types of stem are mixed in the same quiz.

Table 8-7. What happens when stems are mixed?

RQ	Experiment	Time	Data	Have we got the necessary expertise?
What happens when stems are mixed?	Only consider 2 types of stems mixed e.g. Yes/No and 4-option MCQs. Vary the no. of stems of the 1st type. The no. of stems of the 2nd type is given. Then vary the type of 2nd quiz. Then vary the type of 1st quiz. The 1st and 2nd type should be limited to about 5.	This will take a long time and is suitable for doing a PhD that follows on from this.	We do not have to collect data, but generate it. The data is quantitative i.e. marks are frequency that that mark appears.	Yes

This *proposed* project passes the broad test. That answers to the detailed test are the same as above. So, this proposed project passes the two tests, and can now be ranked.

8.25 The Ranking.

Proposed project	Comment
The use of words in different contexts	Paper. Can be started now by me.
What should **a** and **b** be?	PhD for somebody else.
What happens when stems are mixed?	PhD for somebody else, it involves some assumptions being made
Can teachers be assessed by special quizzes.	Maybe a PhD for somebody else

8.26 Recommendations for Further work.

- Write a *paper* about the use of words in different contexts. This can be started quite quickly.
- What should **a** and **b** be if we make some assumptions. This might take quite a long time and is most probably suitable for a **PhD**.
- What happens when stems of special quizzes are mixed? This might take quite a long time and is most probably suitable for a **PhD**.
- Can the effectiveness of teaching be assessed? This might take quite a long time and is most probably suitable for a **PhD**. If the quizzes are in-valid it cannot be done by assessing by quizzes.

Even if 'special' quizzes are in-valid we should find as much as we can about them.

8.27 Conclusion.

We can ask lots of questions but the projects suggested should pass a broad test that asks "Is the proposed project 'doable'?" If the proposed project passes this test then if it passes a more detailed test it is a contender for a project to be done in the future. Ranking these projects tells us the order of doing the projects. The whole process is like a triage in an hospital.

We could write letters to journals, write papers in journals and supervise PhDs (from which there could be conference papers, letters and papers). This then brings us to "How does one decide to supervise a research project? and "What are the advantages to be gained by supervising? These are further questions and are not discussed in this thesis.

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Appendix 1. Sum of Progressions.

1.1 Sum of an Arithmetic Progression (AP).

1.1.1 Why should we want the Sum of an Arithmetic Progression (AP)?

The number of impossible situations for non-mixture quizzes where stems can be missed, is the sum of an AP.

1.1.2 Sum of an Arithmetic Progression to n terms.

Let $\mathbf{AP_n}$ be the sum of an **A**rithmetic **P**rogression, which starts at $\mathbf{a_0}$, adds **d** to each term thereafter and has **n** terms

$AP_n=a_0+(a_0+d)+(a_0+2d)\ldots+(a_0+(n-2)d)+(a_0+(n-1)d)$

Reversing this we have
$AP_n=(a_0+(n-1)d)+(a_0+(n-2)d)+\ldots(a_0+2d)+(a_0+d)+a_0$

Adding them together we have
$2.AP_n=(2a_0+(n-1)d)+(2a_0+(n-1).d)\ldots(2a_0+(n-1))d)$
$2.AP_n=n.(a_0+(n-1)d)$

$\mathbf{AP_n=n(2a_0+(n-1)d)/2}$

Check. When n=2, a=1, d=1 we should get 3. $AP_n=2.(2+1.1)/2=3$✓
When n=3, a=1, d=1 we should get 6. $AP_n=3.(2+2.1)/2=6$✓

When n=101, a=1, d=0
$AP_n=n.(2a_0+(n-1)d)/2=101.(2.0+100.1)/2=101.100/2=101.50=5050$

When we ask 100 Yes/No questions, that are NOT all answered, there are 5050 impossible situations. There are 101*101 situations and 101*101 -5050 =5151 possible situations

Another example is in Killer Sudoku where a small square has the numbers 1 to 9 in it and their sum is where n=9. $a_0=1$ and d=1. So, $\mathbf{AP_n}=9*(2*1+8*1)/2=90/2=45$. Thus the numbers in a small square add up to 45.

Here, n is the number of terms and is NOT the number of stems.

1.2 Sum of an Geometric Progression (GP)

1.2.1 Why should we want the Sum of an Geometric Progression (GP)?

The number of marks for a Yes/No questions that all marked by CBM is the sum of a GP. There are 7 to begin with, then 49, then 333 etc.

1.2.2 Sum of an Geometric Progression to n terms

Let GP_n be the sum of a **G**eometric **P**rogression, which starts at a and each term is r times the previous term.

$GP_n = a+a.r^1+ a.r^2+... a.r^{(n-2)}+a.r^{(n-1)}$...1

Multiplying both sides by r we have
$r. GP_n = a.r^1+a .r^2+ ....+a.r^{(n-1)}+a.r^n$..................................2

Subtracting 1 from 2 i.e. 2-1 we have
$(r-1). GP_n = a.r^n -a$
$GP_n = a.(r^n-1)/(r-1)$..........................r>1..............................3

Subtracting 2 from 1 i.e. 1-2 we have
$(1-r) . GP_n = a.(1-r^n)/(1-r)$
$GP_n = a.(1-r^n)/(1-r)$..........................r<1 and r≠ 0......................4

In the case where r = 7, r is greater than 1, so we use equation 3.

Check

At level 1 we should have 7 marks.
When n = 1, a = 7 GPn = $7(7^1-1)/6 = 7$✓
At level 2 we should have 49 marks, but the sum should be 7+49 = 56
When n = 2, a = 7 GPn = $7(7^2-1)/6 = 56$✓
At level 3 we should have 343marks, but the sum should be 7+49 +343=399
When n = 3, a = 7 GPn = $7(7^3-1)/6 = 399$✓

Here, n is the number of terms and is NOT the number of stems.

Appendix 2. Factorials.

2.1 Why should we want Factorials?

Permutations and Combinations are defined in terms of factorials, see next Appendix.

2.2 What does factorial mean, and what is its notation?

Factorial only applies to integers that are ≥0 i.e. positive integers

The notation for factorial n is n!

The definition of factorial is recursive, with n! = n(n-1)!

We need some stopping condition, and we use 0! = 1

Example. 6! = 6.5.4.3.2.1.1 = 720

We cannot work out:

- 2.7! because n must be an *integer*
- -3! because n must be a *positive* integer

2.3 What is the effect of factorials growing very quickly?

Factorials grow very quickly e.g. 0! = 1,1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720, 7! = 5040 with 100! having of the order of 158 digits. This soon outgrows a modern digital computer running 'ordinary' programs.

We are driven to handling integers of an arbitrary size, and that is where LEDA fits in.

LEDA routines include a function factorial (n) that calculates factorial for us.

Appendix 3. Permutations and Combinations.

3.1 Why should we want Permutations and Combinations?

The number getting a mark, in a particular way, is either a Permutation or a Combination. Permutations and Combinations are related, in that permutations are bigger than combinations where in

- permutations
  - Choice1 and Choice2 is different from Choice2 and Choice1
- combinations
  - Choice1 and Choice2 are the same as Choice2 and Choice1

In our case Choice1 and Choice2 are the same as Choice2 and Choice1so we use combinations, and thus, the number getting a mark in a particular way is a Combination.

3.2 We do not allow repetitions.

Something like 3, 5, 5, 5, 6, 10 we do not allow since 5 is repeated. The following is simplified by not allowing repetition.

3.3 What do we want to do?

We want to choose r things from n, where repetition is disallowed and Choice1 and Choice 2 is the same as Choice 2 and Choice 1, and $n \geq r$

3.4 What is the notation of Permutations and Combinations?

For Permutations the notation is ${}_nP_r$
For Combinations the notation is ${}_nC_r$ and so, we will use this.
where there are r permutations/combinations from n, with $n \geq r$

3.5 What is the relationship between Permutations and Combinations?

There are (n-r)! combinations in ${}_nP_r$

$${}_nC_r = {}_nP_r / (n-r)! \quad \ldots\ldots\ldots\ldots 1.$$

Once we have calculated ${}_nP_r$ we only have to divide by (n-r)! to calculate ${}_nC_r$

3.6 What is $_nP_r$?

$_nP_r$ is the number of permutations of r from n where $r \neq 0$, and $n \geq r$

$_nP_r = n!/r!$..2.

3.7 What is $_nC_r$?

$_nC_r$ is the number of combinations of r from n

From 1. we have

$_nC_r = n!/(r!(n-r!)$................................3.

3.8 How can we use this result?

3.8.1 Yes/ No quizzes.

3.8.1.1 All stems answered

Where all **n** stems are ALL answered, the number of ways of answering **w** stems correctly = **$_nC_w$**

3.8.1.2 Some stems missed

Where **y** stems can be missed, the number answering **w** stems correctly

$= {}_{n-y}C_w * {}_nC_y$

w ,**n** and **y** correspond to a mark m.

n is how many stems/questions there are

w is how many stems are answered correctly

x is how many stems are answered in-correctly

y is how many stems are missed

We can say "the mark m was obtained by particular values of **w**, **y** and **n**". For example, when **n** = 3 and the score is [0], [1, 0],

the mark 1 is obtained when 1 question is answered correctly and 0 stems are missed (i.e. **y** = 0 and **w** = 1). There are other ways of getting the mark 1 e.g. answer 1 stem correctly and miss 1 stem.

In both these cases **x** (how many questions are answered in-correctly) is given because **n** = **w** + **x** + **y.**

The marks are *not* now unique. Pivot Table/Charts allow the calculation of how many there are of obtaining marks 0, 1 2 etc. We can plot the distribution of marks, and calculate the descriptive statistics of the distribution. So, the Pivot Tables/Charts do a summation for us and arrive at how many ways there are of getting mark 0, how may ways here are of getting mark 1, etc.

The generalisation here is by letting **y** = 0, because then $_nC_0 = 1$

Appendix 4. Straker's proof.

Abstract.

The mean is a familiar concept, but here, we use the concept that if mean mark of a special quiz consisting of 1 stem is μ_1 then the mean mark of **n** stems

$$\mu_n = n^* \mu_1$$

4.1 Introduction.

The following is due to Christine Straker (2012), a statistician from Sheffield Hallam University.

For **n** stems of a Yes/No-only quiz where there are three possible marks; right, wrong and missing

The number of times each separate "value" appears = **n.** (3^{n-1})

1 appears n. (3^{n-1}) times

a appears n. (3^{n-1}) times

b appears n. (3^{n-1}) times

For example:

For 1 stem, each value appears $1 \times 3^0 = 1$ time

For 2 stems, each value appears $2 \times 3^1 = 6$ times

For 3 stems, each value appears $3 \times 3^2 = 27$ times

For 4 stems, each value appears $4 \times 3^3 = 108$ times

For 5 stems, each value appears $5 \times 3^4 = 405$ times and so on...

The expected value for 1 stem is

$$E(\text{Stem } 1) = \frac{(1+a+b)}{3} = \mu_1$$

Expected value for Stem1 and Stem 2

$$E(\text{Stem } 1 + \text{Stem } 2) = (2.\ 3^1)\frac{(1+a+b)}{3^2} = \frac{2.3^1}{3}\frac{(1+a+b)}{3} = \mu_2 = 2E(\text{Stem } 1) = 2\,\mu_1$$

and so on...

So in general

$$E(\text{Stem 1 to Stem } \mathbf{n} \text{ inclusive}) = (\mathbf{n}3^{n-1})\frac{(1+a+b)}{3^n} = \mu_n$$

$$= \frac{n3^{n-1}}{3^{n-1}}\left[\frac{1+A+b}{3}\right] = \mathbf{n}\{\text{Expected value of Stem 1}\}$$

So, $\mu_n = n\,\mu_1$

This proof works for Yes/No-only quizzes, and we extend this proof to cater for more general quizzes.

For some, more general 'special' quizzes, that do not include Multiple Yes/No-only quizzes

$$E(\text{Stem 1}) = \frac{(1+({}_PC_T-1)a+b)}{(1+\ {}_PC_T)} = \mu_1$$

For **n** stems

1 and **b** appear n. $(1 + {}_PC_T)^{n-1})$ times

and **a** appears $({}_PC_T - 1)$ n. $(1 + {}_PC_T)^{n-1})$ times

$$E(\text{Stem 1 to Stem } \mathbf{n} \text{ inclusive}) = (n(1 + {}_PC_T)^{n-1})\frac{(1+({}_PC_T-1)a+b)}{(1+\ {}_PC_T)^n} = \mu_n = \mathbf{n}\mu_1$$

For other special quizzes i.e. Multiple Yes/No-only quizzes

$$E(\text{Stem 1}) = \frac{(1+(2^P-1)a+b)}{(1+2^P)} = \boldsymbol{\mu_1}$$

For **n** stems

1 and **b** appear n. $(1 + 2^P)^{n-1}$ times

and **a** appears (2^P-1) **n**. $(1 + 2^P)^{n-1})$ times

$$E(\text{Stem 1 to Stem n inclusive}) = (\mathbf{n}(1 + 2^P)^{n-1})\frac{(1+(2^P-1)a+b)}{(1+2^P)^n} = \boldsymbol{\mu_n = n\mu_1}$$

4.2 Results of this analysis

4.2.1 For 2 stem of a Yes/No-only quiz we have the marks

Mark for 1st stem	Mark for 2nd stem
1	1
1	a
1	b
a	1
a	a
a	b
b	1
b	a
b	b

App. 4- 1. Marks for a 2-stem Yes/No-only quiz.

The mean mark μ_2 is

1+1 + 1 +**a** + 1 + **b** + **a** +1 + **a** + **a** + **a** + **b** + **b** +1 +**b** + **a** + **b** + **b**)/9

= (3.1 +(1 + **a** + b) +3.**a** +(1 +**a** + **b**) +3**b** +(1 + **a** + **b**))/9
= 6(1 + **a** +**b**)/9
= 2(1 + **a** +**b**)/3 So $\mu_2 = 2\mu_1$

So, the theorem is true for Yes/No quizzes when **n** = 2

4.3 For 3 stem of a Yes/No-only quiz we have the marks

Mark for 1st stem	Mark for 2nd stem	Mark for 3rd stem
1	1	1
1	1	a
1	1	b
1	a	1
1	a	a
1	a	b
1	b	1
1	b	a
1	b	b
a	1	1
a	1	a
a	1	b
a	a	1
a	a	a
a	a	b
a	b	1
a	b	a
a	b	b
b	1	1
b	1	A
b	1	b
b	a	1
b	a	a
b	a	b
b	b	1
b	b	a
b	b	b

App. 4- 2. Marks for a 3-stem Yes/No-only quiz.

The mean mark is

$= (3 \quad +2 \quad +\mathbf{a} \quad +2 \quad +\mathbf{b}$
$+2 \quad +\mathbf{a} \quad +1 \quad +2\mathbf{a} \quad +1 \quad +\mathbf{a} \quad +\mathbf{b}$
$+2 \quad +\mathbf{b} \quad +1 \quad +\mathbf{b} \quad +\mathbf{a} \quad +1 \quad +2\mathbf{b}$
$+\mathbf{a} \quad +2 \quad +2\mathbf{a} \quad +1 \quad +1 \quad +\mathbf{a} \quad +\mathbf{b}$
$+2\mathbf{a} \quad +1 \quad +3\mathbf{a} \quad +2\mathbf{a} \quad +\mathbf{b}$
$+\mathbf{a} \quad +\mathbf{b} \quad +1 \quad +2\mathbf{a} \quad +\mathbf{b} \quad +\mathbf{a} \quad +2\mathbf{b}$
$+\mathbf{b} \quad +2 \quad +\mathbf{a} \quad +\mathbf{b} \quad +1 \quad +2\mathbf{b} \quad +1$
$+\mathbf{a} \quad +\mathbf{b} \quad +1 \quad +2\mathbf{a} \quad +\mathbf{b} \quad +2\mathbf{b} \quad +\mathbf{a}$
$+2\mathbf{b} \quad +1 \quad +2\mathbf{b} \quad +\mathbf{a} \quad +3\mathbf{b}$
$)/27$

$= (3 \quad + 3\mathbf{a} \quad + \quad 3\mathbf{b}$
$+6 \quad + 6\mathbf{a} \quad + \quad 6\mathbf{b}$
$+3(\mathbf{a}+2)$
$+3(\mathbf{b}+2)$
$+3(1 +2\mathbf{a})$
$+3(1+2\mathbf{b})$
$+3(2\mathbf{a}+\mathbf{b})$
$+3(\mathbf{a}+2\mathbf{b})$
$)/27$

$= (9(1+ \quad \mathbf{a} \quad + \quad \mathbf{b}) \quad +$
$3\mathbf{a} \quad + \quad 6 \quad + \quad 3 \quad + \quad 6\mathbf{a} \quad +$
$3\mathbf{b} \quad + \quad 6 \quad + \quad 3 \quad + \quad 6\mathbf{b} \quad +$
$6\mathbf{a} \quad + \quad 3\mathbf{b} \quad + \quad 3\mathbf{a} \quad + \quad 6\mathbf{b}$
$)/27$

$= (9(1 +\mathbf{a} +\mathbf{b}) + (9 +9\mathbf{a}) +(9+9\mathbf{b})$
$+ 9\mathbf{a} +9\mathbf{b})/27$
$= 27\ (1 +\mathbf{a} +\mathbf{b})/27$ So, $3(1 +\mathbf{a} +\mathbf{b})/3$ and so $\boldsymbol{\mu_3 = 3\mu_1}$

So, the theorem is true for **n** = 3

4.3.1 For a 2 stem 3-option MCQ-only quiz we have the marks

Mark for 1st stem	Mark for 2nd stem
1	1
1	a
1	a
1	b
a	1
a	a
a	a
a	b
a	1
a	a
a	a
a	b
b	1
b	a
b	a
b	b

App. 10- 3. Marks for a 2-stem MCQ-only quiz.

$\mu_1 = (1 + 2a + b)/4$

$\mu_2 = (4 + (1 + 2a + b) + 4a + (1 + 2a + b) + 4a + (1 + 2a + b) + 4b + (1 + 2a + b))/16$
$= (8(1 + 2a + b)/16 = 2(1 + 2a + b)/4 = 2\mu_1$. So, the theorem is true, here.

4.4 Conclusions.

We show that the mean mark of a special quiz, that consists of **n** stems (all of the same type) is **n*** (the mean of a quiz consisting of 1 stem), and we try this result out on 3 quizzes.

The mean mark or 1 stem is

$$\mu_1 = (1 + (1 - {}_PC_T)\,\mathbf{a} + \mathbf{b})/(1 + {}_PC_T) \text{ or}$$
$$\mu_1 = (1 + (1 - 2^P)\,\mathbf{a} + \mathbf{b})/(1 + 2^P)$$

and the mean mark for **n** stems is

$$\mu_n = \mathbf{n}(1 + (1 - {}_PC_T)\,\mathbf{a} + \mathbf{b})/(1 + {}_PC_T) \text{ or}$$
$$\mu_n = \mathbf{n}(1 + (1 - 2^P)\,\mathbf{a} + \mathbf{b})/(1 + 2^P)$$

Hence $\mu_n = n\mu_1$ for 'special' quizzes.

Appendix 5. Proofs where ALL stems are answered.

5.1 Proof 1. The gap between successive marks is |1 - a|.

The step-size between successive **w**s depends on

- how you define successive
- whether stems are all answered or not.

This Appendix deals with when stems are all answered.

Successive depends on whether you are working up the **w**s or down the **w**s i.e. it depends on the direction of the subtraction of the **w**s. We will do the subtraction in both directions and come to a conclusion and a proof.

Let us give 1 example and then a proof.

For example

For **n** = 100, and **a** = -1/2.

App. 5. Table 1. An example of the marks on the x-axis.

Mark	Stems answered correctly, w	Stems answered in-correctly x
-50	0	100
-48.5	1	99
-47	2	98
-45.5	3	97
….		
98.5	99	1
100	100	0

Here, the range of marks is -50 to 100

the gap between marks of successive **w**s is 1.5, when we work *down* the marks, but is -1.5 if we work *up* the marks.

Proof 1, of step-size

Let us take successive **ws** so $w_1 = w_2 + 1$ and $x_1 = x_2 - 1$

If w goes up by 1, then x goes down by one, and if w goes down by 1 then x goes up by 1.

The 2 marks concerned are

$Mark_1 = w_1*1 + x_1*\mathbf{a}$.

$Mark_2 = w_2*1 + x_2*\mathbf{a}$.

As long as the same **a** is used for both marks, and we assume that both stems have the same **c** (which in this case is 1).

$$\begin{aligned} Mark_1 - Mark_2 &= w_1 + x_1{*}\mathbf{a} - (w_1 - 1) - (x_1 - 1){*}\mathbf{a} \\ &= w_1 - w_1 + x_1{*}\mathbf{a} - x_1{*}\mathbf{a} + 1 - \mathbf{a} \\ &= 1 - \mathbf{a} \\ &= +(1 - \mathbf{a}) \end{aligned}$$

If we do the subtraction the other way round we have

$$\begin{aligned} Mark_2 - Mark_1 &= w_2 + x_2{*}\mathbf{a} - w_1 - x_1{*}\mathbf{a} \\ &= w_2 + x_2{*}\mathbf{a} - (w_2 + 1) - (x_2 - 1){*}\mathbf{a} \\ &= -1 + \mathbf{a} \\ &= -(1 - \mathbf{a}) \end{aligned}$$

The gap between marks for successive **ws** is **|1-a|.**

Q.E.D

This proof applies to all special quizzes where ALL stems are answered, and even applies when **a** = 1, where there is only 1 mark.

Here, the step size is a constant, given that **a** does not change.

We will prove, (see proof 3), that the marks correspond in a 1:1 way with **w** and because the **ws** are unique, so are the marks. Thus, successive marks are unique and are separated by |1-**a**|.

5.2 Proof 2. The number of impossible marks is zero.

Marks are impossible when the conditions that lead to the marks being calculated are impossible.

Proof 2. There are no impossible marks when all stems are answered.

1. **w** takes on the values 0, 1, 2….**n** i.e. **w** ranges from 0 to **n** in steps of 1.
2. As **w** is varied, we calculate a mark that corresponds to **w**.
3. An impossible mark occurs when **w** takes on an impossible value, in this range.
4. It is not possible for there to be a mark where **w** does not exist, in this range.
5.There are no impossible **ws** in the range 0, 1, 2 ..n and so there are no impossible marks in this range of **w**.

Q.E.D.

For example, for a score of [1, **a**] and **n** = 100, in a Yes/No-only quiz, it is not possible to obtain a mark that corresponds to **w** = 99.5 no matter what **a** is, because **w** = 99.5 is impossible (**w** must be an integer between 0 and **n** inclusive).That is an indication that there are no impossible marks, here.

If the algorithm is vary the integer **w** in steps of 1 between 0 and **n**, then the number of impossible marks that correspond to **w** should be 0. If our program prints out 0 in this situation, it is a check that our program works.

5.3 Proof 3. The number of marks is (n +1).

When we ask "How many marks there are on the x-axis, for distributions that we are dealing with?" the answer depends on:

a) how many stems there are, let us say there are **n** stems,
b) how we score stems, the scoring is [**b**], [1, **a**] if **b** exists and [1, **a**] if **b** does not exist,
c) whether stems can be missed or not.

This Appendix deals with all stems being answered, where **b** does not exist, and so stems cannot be missed.

When all stems are answered and **a** = 0, the score is [1, 0], there are (**n**+1) marks on the x-axis i.e. 0, 1, 2....**n** where **b** does not exist

A conjecture is

"When all stems are answered and **a**≠ 0 there are still (**n**+1) marks on the x-axis, unless **a** = 1". The proof of this conjecture relies on an earlier proof (see line 4 below).

Proof 3, of the number of marks on the x-axis

1. There are (**n**+1) unique **w**s	because	**w** is 0, 1, 2, 3 ...n
2. A mark **M**, corresponds to one **w**	because	**M** = w +x**a**, but **n**=w + x
	therefore	**M** = w + (**n**-w)**a**

3. But, the same mark might correspond to 2 (or more) **w**s
4. The difference between marks for successive **w**s is |1 - **a**|. (see proof 1)
5. If **a** does not change then |1-**a**| is a constant.
6. Adding a constant to a unique **w** results in a unique mark.
7. Subtracting a constant from a unique **w** results in a unique mark.
8. So, marks are unique, and correspond to a unique **w** and 3 is impossible
9. As there are (**n**+1) **w**s then there are (**n**+1) marks.

Q.E.D.

In quizzes where all stems are answered, and each stem is scored [1, **a**], we have a proof that, there are (**n** + 1) marks on the x-axis, no matter what **a** is, (unless **a** = 1 in which case there is only 1 mark).

Appendix 6. Proofs where stems can be missed.

This Appendix concerns 3 proofs about quizzes where stems can be missed.

6.1 Proof 1. The gap between successive marks is NOT |1 - a|.

This proof depends on finding just one occasion where the gap between successive marks is not |1 - a|.

Proof 1, of the step-size
For **n** = 100, **b** = 0, **a** = -0.25

The mark **100** can only be obtained by answering all the stems correctly

The next lowest mark **99** can only be obtained by answering 99 stems correctly and missing 1 stem

The step-size = **1**

The next lowest mark **98.75**, can only be obtained by answering 99 stems correctly, and answering 1 stem in-correctly.

The step-size = **0.25**

Which illustrates that, here, the step-size is not |1 -**a**|. Hence, we *cannot* say, here, that the marks go from minimum to maximum in the same steps.

Q.E.D.

This is called a *proof by exception*. We have shown that in one case, at least, the step-size between marks is not a constant.

Other examples show that where **a** is an integer e.g. **n** = 100, **a** = -1, **b** = 0, the marks on the x-axis are -100, -98, -96 etc. and that in some instances the step-size between marks for quizzes, where every stem is NOT answered, is a constant.

6.2 Proof 2. The number of impossible marks is the sum of an AP.

We extend the idea that a mark corresponding to an impossible **w** is itself impossible, to a situation where an impossible *combination* of **w** and **y** results in an impossible corresponding mark. For example, for 100 stems of any quiz it is impossible to answer 99 stems correctly and miss 2 and so the combination of **w** = 99 and **y** = 2 is an impossible situation, and thus the mark that corresponds to **w** = 99, an **y** = 2 is impossible. The impossible situation occurs when **x** (the number of incorrect answers to stems) is negative, and **x** is a strictly positive integer in the range 0, 1, 2..**n**.

This result does not depend on the values of **a** and **b**, as the following argument shows. The way we are arguing is that an impossible mark comes from a combination of **w** and **y** being impossible, and so any mark that corresponds to an impossible combination of **w** and **y** cannot be calculated. We are not arguing about the marks at all, and so **z** and **b** can be anything. So, this result does not depend on **a** or **b**.

We have a conjecture that the number of impossible situations is equal to **w**.

The conjecture arises when we consider the marks for a 'small' population. For example when the number of stems = **3** and the score is [0], [1, -1]

App 6. Table 1. Conjecture that the number of impossible situations is w.

Mark	**w**	**y**	**x (given)**	**Comment**	**Number of impossible situations**
-3	0	0	3	All wrong	0
-2		1	2	1 missed 2 wrong	
-1		2	1	2 missed 1 wrong	
0		3	0	All missed	
-1	1	0	2	1 right 2 wrong	1
0		1	1	1 right 1 missed 1 wrong	
1		2	0	1 right 2 missed	
		3	**-1**	**Impossible.** 1 right and 3 missed	
1	2	0	1	2 right 1 wrong	2
2		1	0	2 right 1 missed	
		2	**-1**	**Impossible.** 2 right and 2 missed	
		3	**-2**	**Impossible.** 2 right and 3 missed	
3	3	0	0	All right	3
		1	**-1**	**Impossible.** 3 right and 1 missed	
		2	**-2**	**Impossible.** 3 right and 2 missed	
		3	**-3**	**Impossible.** 3 right and 3 missed	

Proof 2 of the conjecture that the number of impossible situations is w

1. **w** goes from 0 to **n** in steps of 1 i.e. **w** takes on the values 0, 1, 2, 3....**n**
2. **n** = **w** + **y** +**x** . We ask the user for **n**, and so **n** is a constant. We vary **w** and **y** from 0 to **n** in steps of 1
3. When **w** = 0 and **y** is varied from 0 to **n** then **x** takes on the values 0 to **n,** and **x** 'takes up the slack' and in each case is not impossible. Here, there are 0 impossible marks i.e. when **w** = 0 the number of impossible situations = 0
4. When **w** = 1 and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y** = **n** i.e. when **w** = 1 the number of impossible situations = 1
5. When **w** = 2 and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y** = **n**-1 and **y** = **n** i.e. when **w** = 2 the number of impossible situations = 2
6. When **w** = **m** and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y** = **n-m**, **n-m**-1,..**n**-1 and **y** = **n** i.e. when **w** = **m** the number of impossible situations = **m**
7. As the conjecture is true when **w** = 0 and **w** = 1 and **w** = 2 and when **w** = **m** it is always true, as long as **w** ≤ **n**

This is a proof by *induction*.

So, if the number of impossible situations = **w** and **w** ranges from 0 to **n** in steps of 1, then the sum of **w** = 0 +1 +2 +....**n** = total number of impossible situations
Sum of an AP_m = (**n**+1)*(0 + **n***d)/2 because a_0=0
= (**n**+1)*(**n**)/2 because d = 1
and when **n** = 100
Sum of the AP_{100} = 101*50 = 5050
So, the total number of impossible situations when **n** = 100, for 'special' quizzes where some stems can be missed, and there are no mixtures, is 5050.

Q.E.D.

6.3 Proof 3. The number of marks is NOT (n +1).

Again, we only have to find 1 occasion where the number of marks on the x-axis is not (**n** + 1).

Proof 3, of the number of marks.

Consider, **n** = 3, **a** = -1/2, **b** = 0.

The marks include

-3/2	Answer all 3 stems in-correctly
-2/2	Answer 2 stems in-correctly and miss 1
-1/2	Answer 1 stem in-correctly and miss 2
0	Miss all stems
1/2	Answer 1 stem correctly, 1 stem in-correctly and miss 1
2/2	Answer 1 stem correctly and miss 2
3/2	Answer 2 stems correctly and 1 in-correctly
4/2	Answer 2 stems correctly and miss 1
5/2	Answer 3 stems correctly and 1 in-correctly **Impossible**
6/2	Answer all 3 stems correctly

There are 9 possible marks on the x-axis for **n** = 3 and as that is not (**n** +1)

Q.E.D.

Again, this is a *proof by exception*.

Appendix 7. Validation of analysis of quizzes where ALL stems are answered.

7.1 Introduction.

This Appendix is in 2 parts.

Part 1 is where we validate the expression $X^{x*}{}_nC_w$
for quizzes where all stems are answered. The expression says how many obtain a mark for a combination of X, **x**, **n** and **w**.

X	is the number of answers to 1 stem that are in-correct
n	is the number of stems that a quiz has
x	is the number of stems answered in-correctly
w	is the number of stems answered correctly.

n = w + x

We ask the user for **n**, and so for any 1 quiz **n** is constant (e.g. 100) and so **x** (say) is given.

A mark = **w***1 + **x*****a** assuming that a correct answer to a stem is scored 1 and an in-correct answer is scored **a.** We ask the use for **a.** When **a** is negative we are said to use negative marking.

Part 2 is where we validate an Excel™ program that helps us to calculate **Mg**.

7.2 Part 1.

The 'grey' column in the tables below is for **x** which is given.

7.3 Yes/No-only quizzes where X=1 and negative marking is NOT used i.e. a = 0.

One thing that we know about the distribution of marks is that it is symmetric and hence the skewness is 0.

App 7. Table 1. Yes/No questions. Stems can NOT be missed. Score [1, 0]. n=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
1 The mean Mark=0.5	**0**	**1**	0	1	Get them all wrong **1** $1^{1}*{}_{1}C_{0}=1*1=1$
	1	**1**	1	0	Get them all right **1** $1^{0}*{}_{1}C_{1}=1*1=1$

We would expect our tool (when it is built) to output (0 1) (1 1) and to say that the skewness = 0. The gap between marks should be |1 -**a**| (see Appendix 5) and as **a** = 0 the gap should be 1, and by inspection it is.

This distribution has 2 modes and hence is said to be *bi-modal.*

App7. Table 1. Yes/No questions. Stems can NOT be missed. Score [1, 0]. n=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
2 The mean Mark=1=(0*1 +1*2 + 2*1)/4	**0**	**1**	0	2	Get them all wrong **1** $1^{2}*{}_{2}C_{0}=1*1=1$
	1	**2**	1	1	Get 1 right **2** $1^{1}*{}_{2}C_{1}=1*2=2$
	2	**1**	2	0	Get them all right **1** $1^{0}*{}_{2}C_{2}=1*1=1$

This distribution has 1 mode and hence is said to be *uni-modal.*

App7. Table 2. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean Mark=(0*1 +1*3 + 2*3 +3*1)/8=1.5	**0**	**1**	0	3	Get them all wrong **1** $1^{1}3_{3}C_{0}=1*1=1$
	1	**3**	1	2	Get 1 right **3** $1^{2*}{}_{3}C_{1}=1*3=3$
	2	**3**	2	1	Get 1 wrong **3** $1^{1*}{}_{3}C_{2}=1*3=3$
	3	**1**	3	0	Get them all right **1** $1^{0*}{}_{3}C_{3}=1*1=1$

This distribution has 2 modes and hence is said to be *bi-modal*.

App7. Table 3. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The mean Mark=(0*1 +1*4 +2*6 +3*4 +4*1)/16=2	**0**	**1**	0	4	Get them all wrong **1** $1^{4*}{}_{4}C_{0}=1*1=1$
	1	**4**	1	3	Get 1 right **4** $1^{3*}{}_{4}C_{1}=1*4=4$
	2	**6**	2	2	Get 2 right **6** $1^{2*}{}_{4}C_{2}=1*6=6$
	3	**4**	3	1	Get 1 wrong $1^{1*}{}_{4}C_{3}=1*4=4$ **4**
	4	**1**	4	0	Get them all right **1** $1^{0*}{}_{4}C_{4}=1*1=1$

7.4 Part 2.

We have written an Excel™ program to help with our analysis of special quizzes where all stems are answered. This is held in the file **MG.xlsx** and is described here. The main part of the file looks like this

	No. Right	No.wrong	Mark	Grade	
D 10	0	100	0	Fail	G 10
	1	99	1	Fail	
	2	98	2	Fail	
	3	97	3	Fail	
	4	96	4	Fail	
	5	95	5	Fail	
	6	94	6	Fail	
	7	93	7	Fail	
	8	92	8	Fail	
	9	91	9	Fail	
	97	3	97	First	
	98	2	98	First	
	99	1	99	First	
D 110	100	0	100	First	G 110
		Mean	5		
		No. of Fails	40		
		No.of Passes	10	F 113	
		No. of 2.2s	10		
		No. of 2.1s	10		
		No. of Firsts	31		
		Total	101	F 117	

App 7. Figure 1. Part of the Excel™ program.

The number of stems that are answered correctly is varied from 0 to 100. This is achieved by putting 0 in D10 and then pressing Ctrl and dragging down to D 110.

The number of stems that are answered in-correctly is calculated in E10 to be 100-D10. This formula i.e. =100-D10 is copied down to E110.

The mark that corresponds to this is calculated in F10 as = D10*1 + E10*$C$7 and this formula is copied down to F110.

This demands that the *constant* in C7 is looked at. Constants in Excel™ are where dollar signs are put round the column letter of a cell.

Pass Mark		C6	40
a		C7	0
X		C8	19

We see that this Excel™ program needs 3 things to be changed viz. the contents of C6, C7 and C8. C6 represents the **P**ass **M**ark (**PM**) and in this case is 40. C7 represents the value of negative marking, **a**, and in this case is 0. C8 represents X and in this case is 19 (e.g. a 6-option MAQ with 3 ticks)

We assume that **c** 1 and so

Mark = (number of stems answered correctly)*1 + (number of stems answered in-correctly) * **a**

and so D10*1 + E10*$C$7

The **Mean** in F111 is n*(1+X*a)/(X +1) which in Excel™'s terms is 100*(1 + $C$8*$C$7)/($C$8 +1) when **n** = 100.

(A better way of doing this is by using names and Names Manager in Formulas).

The Total number of Fails is how many times the word Fail appears and is calculated in F112 by = COUNTIF(G10:G110, "Fail").
The Total number of Passes is how many times the word Pass appears and is calculated in F113 by = COUNTIF(G10:G110, "Pass").
The Total number of 2.2s is how many times the string 2.2 appears and is calculated in F114 by = COUNTIF(G10:G110, "2.2").
The Total number of 2.1s is how many times the string 2.1 appears and is calculated in F115 by = COUNTIF(G10:G110, "2.1").
The Total number of 1^{st}s is how many times the 1^{st} appears and is calculated in F116 by = COUNTIF(G10:G110, "First").

A check is that everything should add up to 101 when n = 100. We do this check in F117 by = SUM(F112:F116).

All we have to do is calculate when a Fail, 2.2, 2.1 and 1^{st} occurs. We do this in G10 and copy the formula down to G110. The calculation depends on the highest pass mark being calculated. This depends on the Pass Mark and the accuracy of Excel™. The highest pass mark is calculated in G6 and uses the formula

= IF($C$6 <50,49.99999999, IF($C$6 <60,59.9999999999, IF($C$6 <70, 69.9999999999, IF($C$6 <80, 79.9999999999, IF($C$6 <90, 89.9999, 89.9999)))))

This says that the highest pass mark is 49.9999999 if the Pass Mark is less than 50

otherwise it is 59.99999999999
otherwise it is 69.99999999999
otherwise it is 79.99999999999
otherwise it is 89.99999999999

We assume that the pass mark is between 50 and 100, and that the IF statement stops at the 1st condition it finds.

Fail, Pass, 2.2, 2.1 and 1st can be calculated in G10 and this formula can be copied down to G110

= IF(F10<$C$6,"Fail",IF(AND(F10>=$C$6,F10<$G$6+0),"Pass",IF(AND(F10>=$C$6,F10<$G$6+10), "2.2", IF(AND(F10>=$C$6,F10<$G$6+20),"2.1", IF(AND(F10>=$C$6,F10<$G$6+30), "First", "First")))))

We have to learn:

- How to handle constants.
- How to copy an increment a cell.
- How IF works.
- How COUNTIF works.

This Excel™ program can now be used to produce Tables (see later) which can be checked by 'hand'.

One of the difficulties of raising the pass mark is whether one should go from a fail to a 2.2 if there are no passes. For example, if one raises the pass mark to 69 and **a** = -1/2, the gap between marks is |1.5| and so the marks include

78 stems are answered correctly and 22 incorrectly, the mark is 78-22/2 = 66
79 stems are answered correctly and 21 incorrectly, the mark is 78-21/2 = 67.5
80 stems are answered correctly and 20 incorrectly, the mark is 80-20/2 = 70

The top mark for a pass grade is 69.99999999 and so there are *no* passes.

The results of this Excel™ program are shown in the Tables below.

App 7 Table 4. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using Yes/No-only quiz

Quiz type			**Value of *a***					
Yes/No-only	**Pass Mark**	**Measures**	**0**	**-0.25**	**-0.5**	**-0.75**	**-1**	**-1.25**
	40	μ_{100} M_g	50 [10,10,10,31]	37.5 [8,8,8,25,]	25 [7,7,7,21]	12.5 [6,6,6,18]	0 [5,5,5,16]	-12.5 [4,5,4,14]
	50	μ_{100} M_g	50 [10,10,10,21]	37.5 [8,8,8,17]	25 [7,6,7,14]	12.5 [6,5,6,12]	0 [5,5,5,11]	-12.5 [5,5,4,9]
	70	μ_{100} M_g	50 [10,10,10,1]	37.5 [8,8,8,1]	25 [7,7 6,1]	12.5 [6,6,5,1]	0 [5,5,5,1]	-12.5 [5,4,4,1]
	71	μ_{100} M_g	50 [9,10,10,1]	37.5 [7,7,8,1]	25 [6,7,6,1]	12.5 [5,6,5,1]	0 [4,5,5,1]	-12.5 [4,4,4,1]
	72	μ_{100} M_g	50 [8,10,10,1]	37.5 [6,8,8,1]	25 [5,7,6,1]	12.5 [5,6,5,1]	0 [4,5,5,1]	-12.5 [4,4,4,1]
	73	μ_{100} M_g	50 [7,10,10,1]	37.5 [5,8,8,1]	25 [5,7,6,1]	12.5 [4,6,5,1]	0 [3,5,5,1]	-12.5 [4,4,4,1]
	74	μ_{100} M_g	50 [6,10,10,1]	37.5 [4,8,8,1]	25 [4,7,6,1]	12.5 [3,6,5,1]	0 [3,5,5,1]	-12.5 [3,4,4,1]
	80	μ_{100} M_g	50 [10,10,1,0]	37.5 [8,8,1,0]	25 [7,6,1,0]	12.5 [6,5,1,0]	0 [5,5,1,0]	-12.5 [4,4,1,0]

We, partially, validate this table by choosing a square at random, **a** = -0.5 and the Pass Mark is raised to 73, say (shown in grey above)

μ_{100} = 100*(1+**a**)/2 and when **a** = -0.5 we have μ_{100} = 25 ✓

The marks should include

73, 74.5, 76, 77.5, 79 — **5** 3rd marks

80.5, 82, 83.5, 85, 86.5, 88, 89.5 — **7** 2.2 marks

91, 92.5, 94, 95.5, 97, 98.5 — **6** 2.1 marks

100 — **1** 1st mark

So, M_g should be [5, 7, 6, 1] ✓

The mark 71.5 would come from 81 stems being answered correctly and 19 answered in-correctly. The mark therefore is 81*1 -19*1/2 = 81- 9.5 = 71.5.

The mark 73 comes from 82 stems being answered correctly and 18 answered in-correctly. The mark therefore is 82*1 -18*1/2 = 82-9 = 73.

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a** = - 0.5 the gap between successive marks |1.5|. ✓

There are 101 marks, -50 to +100 in steps of 1.5 (see Appendix 5). Thus, this table is partially validated.

7.5 Discussion.

The expression $X^{x*}{}_nC_w$ has been validated for many values of X but each for 'small' values of **n**.

The Excel™ program that helps us calculate $\mathbf{M_g}$ has been validated for many quiz types, and for many values of **a** and for many values of the Pass Mark.

Appendix 8. Validation of analysis of quizzes where stems can be MISSED.

8.1 Introduction.

We only consider, in this Appendix, special quizzes where stems can be missed.

One of the purposes of validation of our Analysis is to show that the Analysis is correct 'as far as we can tell'. We check on a *subset* and then believe our Analysis is correct for the *whole set*. The sort of quiz that we validate is where stems can be missed.

We are validating that the number getting a mark in a particular way is $X^x * {}_nC_y * {}_{n-y}C_w$ where X is number of answers to *a stem* that are wrong and (X +1) reflects the quiz type. For example, when

X = 7 we have a Multiple Yes/No with 3 options or
we have an 8-option MCQ.

We assume that:

a) there is only 1 right answer, and that X cannot be 0,
b) the number of ticks, **T,** is not 0, and does not equal **P.**

The validation, by hand calculation, can only be done for 'small' sets. We validate for 'small' **n**. After that we assume that the expression is true for 'large' sets. We will validate for:

X = 1 and **n** = 1 to 5	Yes/No-only quizzes
X = 3 and **n** = 1 to 5	4-option MCQ-only quizzes
X = 7 and **n** = 1 to 5	3-option Multiple Yes/No-only quizzes
X = 9 and **n** = 1 to 5	5-option MAQ with 2 ticks-only quizzes
X = 19 and **n** = 1 to 5	20-option EMSQ (MCQ)-only quizzes or a 6-option MAQ with 3 ticks-only
X = 184755 and **n** = 1 to 5	20-option EMSQ (MAQ with 10 ticks)-only quizzes

For each of these we try to validate our expressions for:

a) **a** = **b** = 0.
b) **b** = 0 and **a** has a value that exactly tackles the guessing problem. For example, **a** =-1/X= -1 for Yes/No quizzes.

This means that there are 12 possibilities to check.

In line with the philosophy of this thesis we will start off simply and gradually get more complicated. Thus, we will start off with Yes/No quizzes with 1 stem and then 2 stems, then look at 4-optiion MCQs, then look at 3-option Multiple Yes/Nos and then 5-option MAQs with 2 ticks etc.

The 'grey' column in the tables below is for the *given* **x** (the number of stems that are answered in-correctly)

We, first of all, show how the expression was arrived at, and then validate it on 12 sets.

8.2 Yes/No-only quizzes where X=1 and negative marking is NOT used i.e. a = b = 0.

App. 8 Table 1. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Comment**
1 The mean Mark=(0*1 +0*1 +1*1)/3=**0.33**	0	1	0	0	1	Get them all wrong
	0	1		1	0	Miss them all
	1	1	1	0	0	Get them all right
				1	-1	**Impossible.** You cannot miss them all and get 1 right.

App. 8 Table 2. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Comment**
2 The mean Mark=(0*3 +1*4 +2*1)/9=**0.67**	0	1	0	0	2	Get them all wrong
		2		1	1	Miss 1 and get the other wrong
		1		2	0	Miss them all
	1	2	1	0	1	Get 1 right and 1 wrong
		2		1	0	Get 1 right and miss 1
				2	-1	**Impossible**
	2	1	2	0	0	Get them all right
				1	-1	**Impossible**
				2	-2	**Impossible**

App. 8 Table 3. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=3.

Number of Stems n	Mark M	Number of ways to get a mark in that manner	w	y	x	Comment
3	0	1	0	0	3	Get all wrong
The mean Mark=(0*8 + +1*12 +2*6 +3*1)/27=1		3		1	2	Miss 1 of the 3 and get 0 right of 2
		3		2	1	Get 1 wrong and miss the remainder
		1		3	0	Miss them all
	1	3	1	0	2	1 right and 2 wrong =3
		6		1	1	
		3		2	0	
				3	-1	**Impossible**
	2	3	2	0	1	
		3		1	0	
				2	-1	**Impossible**
				3	-2	**Impossible**
	3	1	3	0	0	Get them all right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**

App 8. Table 4. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=4.

Number of Stems n	Mark M	Number of ways to get that mark in that manner	w	y	x	Comment
4	0	1	0	0	4	Get all wrong
The mean Mark=(0*16 + 1*32 + 2*24 +3*8 +4*1)/(16+32 +24 +8+1)=108/81=**1.33**		4		1	3	0 right 1 missed
		6		2	2	0 right 2 missed i.e. missed 1 2, 1 3, 1 4, 2 3, 2 4, 3,4
		4		3	1	0 right 3 missed
		1		4	0	Miss them all
	1	4	1	0	3	1 right 0 missed
		12		1	2	1 right 1 missed i.e. q right q missed 1 2 1 3 1 4 2 1 2 3 2 4 3 1 3 2 3 4 4 1

						4 2 4 3
		12		2	1	1 right 2 missed i.e. q right qs missed 1 2,3 1 2,4 1 3,4 2 1,3 2 1,4 2 3,4 3 1,2 3 1,4 3 2,4 4 1,2 4 1,3 4 2,3
		4		3	0	1 right 3 missed
				4	-1	**Impossible**
	2	6	2	0	2	2 right 0 missed i.e. right 1 2, 1 3, 1 4, 2 3, 2 4, 3,4
		12		1	1	2 right 1 missed i.e. qs right q missed 1,2 3 1,2 4 1,3 2 1,3 4 1,4 2 1,4 3 2,3 1 2,3 4 2,4 1 2,4 3 3,4 1 3,4 2
		6		2	0	2 right 2 missed i.e. qs right qs missed 1,2 3,4 1,3 2,4 1,4 2,3 2,3 1,4 2,4 1,3 3,4 1, 2
				3	-1	**Impossible**
				4	-2	**Impossible**

	3	4	3	0	1	3 right 0 missed i.e 1 2 3, 1 2 4, 1 3 4, 2 3 4.
		4		1	0	3 right 1 missed
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
	4	1	4	0	0	All right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**
				4	-4	**Impossible**

8.3 Discussion.

Appendix 7 showed that the expression $X^x * {}_nC_y * {}_{n-y}C_w$ worked for quizzes where stems could not be missed (The expression that we have used works for **y** = 0).

This Appendix shows that the expression $X^x * {}_nC_y * {}_{n-y}C_w$ works for quizzes where stems can be missed (i.e. **y** = 0 and *can be* ≠0).

Thus, the expression $X^x * {}_nC_y * {}_{n-y}C_w$ has been validated, for 'small' values of **n**, for both sorts of quizzes and we believe this expression is correct for 'large' values of **n**.

The expression tells us the number of ways of getting a mark in a particular manner. The particular manner is given by the values of X, **x**, **n**, **y**, and **w**.

Appendix 9. Import a tab-delimited text file into Excel™.

The tool, ***tool1*** outputs to a tab-delimited file called
F:\MyPhD2\MyPhD\MixtureWirhoutDescriptiveStats.txt

The importation of a tab-delimited text file into Excel™ has the following steps

1. Open Excel™.
    a. Choose the cell A1 to work in.
    b. We assume that this is obvious, and so it will be missed out of the following.
2. Say we are going to import a text file.
3. Navigate to the text file.
4. Follow the 'wizard' instructions
    a. Excel™ will work out for itself that the file is delimited, or you have to tell it.

Appendix 10. Excel's™ Pivot Chart.

- The *usual* process where 1 file is imported;
  - The 1st file is imported into Sheet 1 and then a Pivot is performed on Sheet 1's data and put into Sheet 4.
  - Titles are added to Sheet 4
  - The results of Sheet 4 are copied to Word™, so that the results are not changed by , by using Paste Special in Word™.
  - In Word™ some 'fiddling about ' is done. The 'fiddling' includes a) adding titles to the axes b) formatting the picture of the Pivot
  - In Excel™, Sheet 4 is changed and exported to **Output.txt** and this file is 'cleaned up' before its descriptive statistics are calculated. (See Appendix 12)

10.1 The usual process.

The **usual** process is to only have 1 input file and this part of the Appendix shows how Pivot Charts is Excel™ are used on 1 input file.

The **input** is **F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.txt**
that is a tab-delimited text file
This file contains Many lines, which consist of

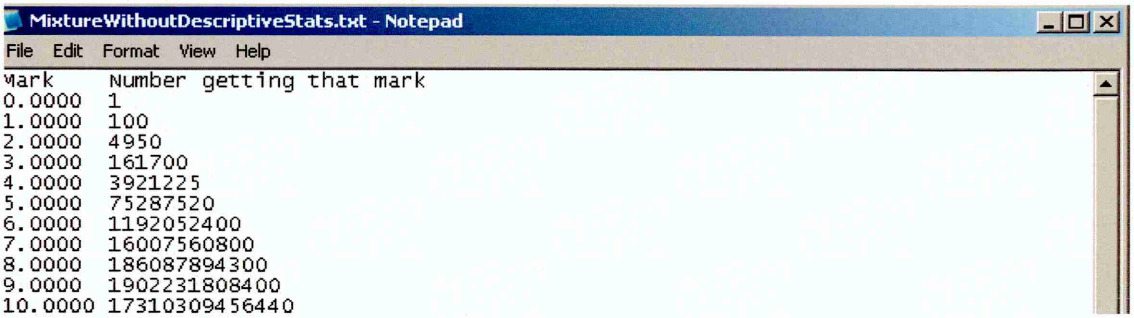
On the 1st line are headers, which are strings
Mark
tab
Number getting that mark
On each line thereafter are floats that represent
Mark
tab
Number of ways of getting that mark in that manner

The **outputs** are

1. A plot of the distribution of the results of the Pivot Chart
2. Some titles are added to these results
3. The results plus titles are copied to Word™ such that they are not altered
4. The changed results are exported to **Output.txt**
   (See Appendix 11)
5. **Output.txt** is 'cleaned up'
6. A calculation tool uses **Output.txt** to form the descriptive statistics of the distribution.(See Appendix ??)

This Appendix only describes steps 1, 2., 3 and 4

The **input** file starts off something like this,



MixtureWithoutDescriptiveStats.txt - Notepad

File Edit Format View Help

```
Mark     Number getting that mark
0.0000   1
1.0000   100
2.0000   4950
3.0000   161700
4.0000   3921225
5.0000   75287520
6.0000   1192052400
7.0000   16007560800
8.0000   186087894300
9.0000   1902231808400
10.0000 17310309456440
```

This is a file that represents Yes/No quiz where stems cannot be missed
Negative marking is NOT used
The score is [1, 0].
There is 1 header and 101 data lines.

The marks might not be unique and are not ordered, (but they are in this case i.e. the marks are unique and ordered). We want to perform a Pivot chart on this data to make the marks unique and order them.

Appendix 11. Export a tab-delimited text file from Excel™ and clean up this file.

1. A calculation tool (*tool1*) exports its results to a file.
2. The display tool takes in these results uses a Pivot Chart (see Appendix 10) and displays the result of the Pivot Chart
    a. You then take this display into Word™, and make sure that the results of this display are not changed, by using Paste Special and not Paste.
3. You then have to export this display this display into **Output.txt** and clean up **Output.txt**
4. Another calculation tool works out the descriptive statistics associated with **Output.txt**

This Appendix is concerned with the *exportation* and *cleaning up* i.e. step 3.

Appendix 12. Tool1.

```
//Includes
      #include <LEDA/numbers/integer.h>  //Include LEDA
      #include <LEDA/numbers/bigfloat.h>
      #include <LEDA\core\array.h>           //This was helped by VS 10 system
      #include <LEDA\numbers\real.h>

      #include <iostream>                    //Include ordinary C++
      #include <fstream>
      #include <string>
      #include <math.h>
  //Namespaces
      using namespace std;
      using namespace leda;

  //Definition of a function isDivisibleBy () that we are going to use

         bool isDivisibleBy(int firstNumber, int secondNumber)
         {
                 if (firstNumber % secondNumber ==0)
                 {
                        return(true);
                 }
                 else
                 {
                     return (false);
                 }
         }


  int main()
    {

      //Declarations
         //LEDA declarations
          integer X,XToThex, startingValue, maximumY, minimumY,twoToTheP,
twoTothePMinus1oThex,twoTothePMinus1,P,T,pCT,pCTMinus1,pCTMinus1ToThex,
twoTothePMinus1ToThex,maximum, pow[101], nCy,nMinusyCw,
numberOfWaysOfGettingThatMarkThatWay[6000],numberGettingThatMarkThatWayByMultiple
,numberGettingThatMarkThatWayByMAQ,sumOfNumberOfWaysOfGettingThatMarkThatWay;
          bigfloat
temp1,sumOfProbabilities,momentAboutMean,pseudoArea,mu,variance,standardDeviation
,skewness, kurtosis,varianceSum,skewnessSum,kurtosisSum;
      //Ordinary C++ declarations
         int numberOfModes,noOfTicks,n,w,x, y, noOfOptions, noOfOptionMinus1, way,
impossible, missLimit;
         float maximumMark, minimumMark,
scoreForMAQright,scoreForMAQwrong,scoreForMAQmissed,scoreForMultipleright,scoreFo
rMultiplewrong,scoreForMultiplemissed,a,b,c;
         double mark[6000], a0, d;
         bool correctInput;
//Strings
                std::string fileName,typeOfQuiz,canStemsBeMissed;
//Files
                ofstream myfile;              //I think the o means output



//Input a
                correctInput=false;
```

```
                while (!correctInput)
                {
                        cout << "NEGATIVE marking (a). Input either 0 for no
negative marking, or a fraction something like 1.0, or 0.5. This fraction is
usually negative"  << endl;
                        cin >> a;
                        correctInput=true;
                }
        scoreForMAQwrong=scoreForMultiplewrong=a;

//Input c.
                correctInput=false;
                while (!correctInput)
                {
                        cout << "How do you score a CORRECT answer to a stem.
Quite often this is 1.0"  << endl;
                        cin >> c;
                        correctInput=true;
                }
                scoreForMAQright=scoreForMultipleright=c;

//Input whether stems can be missed
                correctInput=false;
                while (!correctInput)
                {
                 cout << "Can Stems be missed. Input either Yes or No"  << endl;
                 cin >> canStemsBeMissed;                               if
(canStemsBeMissed=="Yes" || canStemsBeMissed=="No")
                        {
                         correctInput=true;
                        }
                        else
                        {
                          cout << "Can Stems be missed is NOT Yes or No.  You
inputted "<< canStemsBeMissed << endl;
                        }
                }

//Input the number of stems, n
                correctInput=false;
                while (!correctInput)
                {
                        cout << "Input the TOTAl number of stems(n). An integer
between 1 and 100 inclusive" << endl;
                        cin >> n;
                        if (n>=1 && n <=100)
                        {
                         correctInput=true;
                        }
                        else
                        {
                         cout << "The total number of stems(n) is NOT an integer
between 1 and 100 inclusive" << endl;
                        }
                }

//Input b, only when stems can be missed
                scoreForMAQmissed=scoreForMultiplemissed=0;

                if (canStemsBeMissed=="No")
                {
                 missLimit=0;
                }
                else
```

```
                {
                   missLimit=n;
                   correctInput=false;
                   while (!correctInput)
                    {
                         cout << "How do you score a MISSED stem (b) ? Input
either 0 , or a  number"  <<endl;
                         cin >> b;
                            correctInput=true;
                      }
                      scoreForMAQmissed=scoreForMultiplemissed=b;
//Keith argues that b should always be 0.
                }

                 correctInput=false;
                  while (!correctInput)
                  {
                       cout << "Input a string (into typeOfQuiz) of MAQ   or
Multiple" << endl;
                       cin >> typeOfQuiz;
                       if (typeOfQuiz == "MAQ" || typeOfQuiz =="Multiple"  )
                       {
                            correctInput=true;
                       }
                       else
                       {
                            cout << "The type of quiz is NOT   MAQ     or
Multiple" << endl;
                            cout << "You input " << typeOfQuiz << endl;
                       }
                  }

       if (typeOfQuiz == "MAQ")
          {
                 //The minimimum number of options is 2

                 //How many options have we.

                             correctInput=false;
                       while (!correctInput)
                        {
                                 cout << "How many options (P) do MAQs have? Input
a number" << endl;
                                 cin >> noOfOptions ;

                                    if (noOfOptions >=2 && noOfOptions <=26)
//MAQS have a minimum of 2. EMSQs have a maximum of 26 (say)
                                    {
                       correctInput=true;
                                    }
                                    else
                                    {
                                           cout << "The number of options is NOT 2
to 26 inclusive. You input " << noOfOptions << endl;
                                    }
                            }
                          P=noOfOptions ;

                         //How many ticks have we.

                              correctInput=false;
                          while (!correctInput)
                           {
```

```
                                    cout << "How many ticks (T) do MAQs have? Input
an integer that is greater than or equal to 1 and less than or equal to " <<
noOfOptions -1 << endl;
                                    cin >> noOfTicks ;
                                      //cout << " For the number of ticks you
typed " << noOfTicks << endl;
                                    if (noOfTicks >= 1 && noOfTicks < noOfOptions)
                                     {
                                         correctInput=true;
                                     }
                                     else
                                     {
                                         cout << "The  number of ticks cannot be
less than 1 or greater than " << noOfOptions - 1  << endl;
                                           if ( noOfTicks == noOfOptions )
                                           {
                              cout << "When the number of ticks=the number of options
it leads to inconsistencies, like all the marks are the same" << endl;
                                             correctInput=false;
                                           }
                                     }
                                }
                                  /
                               T=noOfTicks;
            }

          if (typeOfQuiz =="Multiple"  )
          {
                //The minimimum number of options is 1 (for Yes/Nos)
                //How many options have we.

                                correctInput=false;
                           while (!correctInput)  /
                            {
                                  cout << "How many options (P) do Mutiples have?
Input a number" << endl;
                                  cin >> noOfOptions ;

                                    if (noOfOptions >=1 && noOfOptions <=26)
//Multiples have a minimum of 1. EMSQs have a maximum of 26 (say)
                                    {
                           correctInput=true;
                                    }
                                    else
                                    {
                                           cout << "The number of options is NOT 1
to 26 inclusive. You input " << noOfOptions << endl;
                                    }
                              }
                            P=noOfOptions ;

          }

 //Initialisation
             way=0;

          sumOfNumberOfWaysOfGettingThatMarkThatWay=0.0;
          pseudoArea=0.0;

                maximumMark=-9999999999.0;
            minimumMark=999999999.0;

                minimumY=99999999.9;
            maximumY=-999999999.9;
```

```
          numberOfModes=0;
          varianceSum=0.0;
          skewnessSum=0.0;
          kurtosisSum=0.0;

          sumOfProbabilities=0.0;
                impossible=0;

         fileName= "F:\\MyPhD2\\MyPhD\\MixtureWithoutDescriptiveStats.txt";

         myfile.open (fileName);
         myfile << fixed;
         myfile << setprecision(4);

//Output Headers to  file
          myfile << "Mark"                  << "\t" << "Number getting that mark
" <<   endl;
          //cout << "Mark"                  << "\t" << "Number getting that mark
" <<   endl;
          way=way+1;

               cout   << fixed; cout  << setprecision(4);

          for (y=0;  y < missLimit + 1; y++ )   // y is how many stems are missed
           {
                      for (w =0; w < n + 1; w++)      //w is how many stems are
answered correctly
              {

                              x=n-w-y;                //x is how many stems are
answered in-correctly
                                                       if (x < 0)
                              {
                   impossible=impossible + 1;
                                                        }

                             if (x >=0)
                             {

                                    if (typeOfQuiz=="MAQ")
                                    {

pCT=factorial(P)/(factorial(T)*factorial(P - T));
                                           pCTMinus1=pCT -1;

                                           X=pCTMinus1=pCT -1;
                                    }
                                    if (typeOfQuiz=="Multiple")
                        {
                                           twoToTheP=1;


                                           for (int j =0; j < P; j++)
                                           {
                           twoToTheP=twoToTheP*2;
                                           }

                                           twoTothePMinus1=twoToTheP -1;

                                           X=twoTothePMinus1;
                                    }

                                     startingValue=1;
                                    XToThex=1;
```

```
                                        for (int j=0; j < x; j++)
                                        {

                                               XToThex = XToThex * X;

                                        }


                                        nCy=factorial(n)/(factorial(y)*factorial(n-
y));
                                        nMinusyCw=factorial(n-
y)/(factorial(w)*factorial(n-y-w));
                                        way=way+1;
                                        mark[way]= scoreForMAQright * w +
scoreForMAQwrong *(n-w-y) +scoreForMAQmissed * y;
                                        if(mark[way] > maximumMark)
maximumMark=mark[way];
                                        if(mark[way] < minimumMark)
minimumMark=mark[way];



numberOfWaysOfGettingThatMarkThatWay[way]=XToThex * nCy * nMinusyCw;

                                        if (canStemsBeMissed=="No")
                                         {

if(numberOfWaysOfGettingThatMarkThatWay[way ]> maximumY)
maximumY=numberOfWaysOfGettingThatMarkThatWay[way];

if(numberOfWaysOfGettingThatMarkThatWay[way] < minimumY)
minimumY=numberOfWaysOfGettingThatMarkThatWay[way];
                                         }

sumOfNumberOfWaysOfGettingThatMarkThatWay=sumOfNumberOfWaysOfGettingThatMarkThatW
ay+numberOfWaysOfGettingThatMarkThatWay[way];

pseudoArea=pseudoArea+(bigfloat)mark[way]*numberOfWaysOfGettingThatMarkThatWay[wa
y];
//Write them to a file
                                         myfile << mark[way] << "\t" <<
numberOfWaysOfGettingThatMarkThatWay[way]  << endl;

                      cout   << "Mark=" << mark[way] << "\t" << " numbner getting
that mark that way=" << numberOfWaysOfGettingThatMarkThatWay[way]  << " n=" << n
<< " w=" << w << " x=" << x << " y=" << y << endl;

                             }
                       }
               }

//Close file
         myfile.close();

//Work out some descriptive stats
               mu=pseudoArea/sumOfNumberOfWaysOfGettingThatMarkThatWay;
               for (int i=1;i<way +1;i++)    //way is how many times we have
written to the file
            {
              momentAboutMean=mark[i] - mu;
              varianceSum=varianceSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean;
```

```
              skewnessSum=skewnessSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean*momentAbo
utMean;
               kurtosisSum=kurtosisSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean*momentAbo
utMean*momentAboutMean;
                  }

          variance=varianceSum/sumOfNumberOfWaysOfGettingThatMarkThatWay;
          standardDeviation=sqrt(variance);
          skewness=
skewnessSum/(sumOfNumberOfWaysOfGettingThatMarkThatWay*standardDeviation*standard
Deviation*standardDeviation);
          kurtosis=-
3+kurtosisSum/(sumOfNumberOfWaysOfGettingThatMarkThatWay*standardDeviation*standa
rdDeviation*standardDeviation *standardDeviation);


//Output the descriptive statistics

         cout << fixed << endl;
         cout << setprecision(2);

               cout << "   " << endl;

               if (canStemsBeMissed=="Yes")
               {
           cout << "Stems can be missed.    Score is [" << b << "], [" << c <<
"," << a << "]" << endl;
               }
               else
               {
           cout << "Stems CANNOT be missed. Score is [" << c << "," << a << "]"
<< endl;
               }

               cout << "The type of quiz is ";
               if (typeOfQuiz=="Multiple" && P==1 )
               {
           cout << "Yes/No" ;
               }
               if (typeOfQuiz=="Multiple" && P >1 )
               {
           cout << "Multiple Yes/No with " << P << " options" ;
               }
               if (typeOfQuiz=="MAQ" && P >1 && T==1)
               {
           cout << "MCQ with " << P << " options" ;    //This also caters for
EMSQs where P is big

               }
                if (typeOfQuiz=="MAQ" && P >1 && T > 1)
               {
           cout << "MAQ with " << P << " options and " << T << " ticks"; //This
also caters for EMSQ of type MAQ

               }

               cout << " X=" << X << endl;
               cout << "The number of impossible situations=" << impossible;
               if (canStemsBeMissed=="Yes")
               {
                      a0=0;
                      d=1;
```

```
                cout << " The theretical number of impossible situations is="
<<(n + 1)*(2*a0+n*d)/2 << endl; //We have to be careful because this involves a
division
               }
               else
               {
          cout << " The theretical number of impossible situations is=" << 0 <<
endl;
               }
               cout << "   " << endl;
               cout << "            Some DESCRIPTIVE Statistics" << endl;
               cout << "   " << endl;
               cout << "The minimum value on the x-axis                 "
<<minimumMark << " The maximum value in the x-axis              =" << maximumMark
<< endl;
               if (canStemsBeMissed=="No")
                {
             cout << "The minimum Y                =" << minimumY   << " The
maximum Y              =" << maximumY    << endl;
                }

               cout << "The number of stems         ="  << n
<< endl;
               cout << "Mean                        ="  << mu.to_double();
                     if (canStemsBeMissed=="No")
                           {
                             cout<<      " The theoretical mean=" << (n *(c +
X.to_double()  * a))    /(X.to_double() +1) << endl; //Be careful here as this
involve a division
                        }
                     else
                        {
                              cout<<     " The theoretical mean=" << (n*(c +
X.to_double()  * a) + b)/(X.to_double()  +2)<< endl; //Be careful here as this
involve a division
                        }

               cout << "Variance                    ="  << variance.to_double()
<< endl;
               cout << "Standard Deviation          ="  <<
standardDeviation.to_double()  << endl;
               cout << "Skewness                    ="  << skewness.to_double()
<< endl;
               cout << "Kurtosis                    ="  << kurtosis.to_double()
<< endl;
               cout << "   " << endl;
               cout << "The file we have written to is "<< fileName << endl;
              cout << "We have wrtiten to this file " << way << " times" << endl;

    } //End of main
```

Appendix 13. Validation of *tool1* where stems are ALL answered.

13.1 Introduction.

This Appendix is about validating *tool1* for quizzes where stems are ALL answered.

The tool, *tool1* is where the calculation of a mark and how the mark was got in a particular way is done (see Appendix 12).

After *tool1* comes the widely available tool Excel™. *tool1* writes to a tab-delimited text file. Thus Excel™ has to import this file (see Appendix 9), Then Excel™ does a Pivot (see Appendix 10) and produces a distribution of marks and how many get that mark. Excel™ then outputs this distribution to tab-delimited text file (see Appendix 11).

The tool, *tool2* (see Appendix 15) is where descriptive statistics (and other statistics) of the distribution of marks and the number getting that mark are calculated. The tool, *tool2* assumes that the marks have been sorted, and the total number of ways a mark can be obtained involves an addition. If the marks are not sorted or the addition has not been done then:

a) the mode,
b) the median cannot be calculated, because the mode is the most frequently occurring value and the median mark is the 'middle' mark when the marks have been sorted.

The tool, *tool2* also calculates the proportion of passing grades that there are.

The tools work together. They communicate via intermediate tab-delimited text files.

This Appendix is accompanied by Appendix 7 where the validation of the expression that *tool1* uses is done. We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 7.
Where stems are ALL answered the validation of *tool1* is done 'by hand'

X = 1 i.e. Yes/No quizzes	for **n** = 1 to 5
X = 3 i.e. 4-option MCQ quizzes	for **n** = 1 to 5
X = 7 i.e. 3-option Multiple Yes/No quizzes	for **n** = 1 to 5
X = 9 i.e. 5-option MAQ quizzes with 2 ticks	for **n** = 1 to 5
X = 19i.e. 2-option EMSQ (MCQ)	for **n** = 1 to 5
X = 187455 i.e.20-option EMSQ (MAQ with 10 ticks)	for **n** = 1 to 5

Beyond **n** = 5 we use our proofs and other means (see below) to validate *tool1*, where stems can be missed.

For each of the above we:

- do not use negative marking, and have **a** = 0,
- do use marking where **b** = 0 and **a** has a value that exactly tackles the guessing problem.

So that means that there are 12 sections, in this Appendix, where our validation is done.

With *tool1* we can do a bit more validation:

- We can calculate the minimum and maximum value of what is measured on the x-axis, and check that they agree with a table in Appendix 7. The range of marks should be from **n***min(1,**a**) to **n***max(1,**a**).

- We can calculate the minimum and maximum value of what is measured on the y-axis, and check that they agree with a table in Appendix 7.

- There should be no impossible situations, here, where stems cannot be missed.

- The mean and the theoretical mean should be the same (and they are).

- The gap between successive marks should be |1-**a**|.

One validation that we can do is that, here, the distribution should not be skewed, and that for every case, in this Appendix, the skewness should be zero.

The validation is an attempt to answer "Does the tool give the right answers?" and the answer is "YES, as far as we can tell". Because we cannot check every situation it requires an assumption to say that *tool1* will **always** work for quizzes where stems can be missed.

13.2 Yes/No quizzes (X = 1) where no negative marking is used i.e. a = 0.

```
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-a
is              =1.00
The minimum Y                =1 The maximum Y              =1
The number of stems          =1
Mean                         =0.50 The theoretical mean=0.50
Variance                     =0.25
Standard Deviation           =0.50
Skewness                     =0.00
Kurtosis                     =-2.00
```

The range of marks should be 0 to 1, and is.

This agrees with a table in Appendix 7, where **n** = 1.

```
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is              =2.00
The minimum Y                =1 The maximum Y              =2
The number of stems          =2
Mean                         =1.00 The theoretical mean=1.00
Variance                     =0.50
Standard Deviation           =0.71
Skewness                     =0.00
Kurtosis                     =-1.00
```

The range of marks should be 0 to 2, and is.

This agrees with a table in Appendix 7, where **n** = 2.

```
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =3.00
The minimum Y                  =1 The maximum Y              =3
The number of stems            =3
Mean                           =1.50 The theoretical mean=1.50
Variance                       =0.75
Standard Deviation             =0.87
Skewness                       =0.00
Kurtosis                       =-0.67
```

The range of marks should be 0 to 3, and is.

This agrees with a table in Appendix 7, where **n** = 3

```
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =4.00
The minimum Y                  =1 The maximum Y              =6
The number of stems            =4
Mean                           =2.00 The theoretical mean=2.00
Variance                       =1.00
Standard Deviation             =1.00
Skewness                       =0.00
Kurtosis                       =-0.50
```

The range of marks should be 0 to 4, and is.

This agrees with a table in Appendix 7, where **n** = 4

```
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =5.00
The minimum Y                  =1 The maximum Y              =10
The number of stems            =5
Mean                           =2.50 The theoretical mean=2.50
Variance                       =1.25
Standard Deviation             =1.12
Skewness                       =0.00
Kurtosis                       =-0.40
```

The range of marks should be 0 to 5, and is.

This agrees with a table in Appendix 7, where **n** = 5

Now, we try and validate by observing that:

a) the range of marks is correct,

b) the skewness is always 0.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                 =1 The maximum Y               =252
The number of stems           =10
Mean                          =5.00 The theoretical mean=5.00
Variance                      =2.50
Standard Deviation            =1.58
Skewness                      =0.00
Kurtosis                      =-0.20
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =20.00
The minimum Y                 =1 The maximum Y               =184756
The number of stems           =20
Mean                          =10.00 The theoretical mean=10.00
Variance                      =5.00
Standard Deviation            =2.24
Skewness                      =0.00
Kurtosis                      =-0.10
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =30.00
The minimum Y                 =1 The maximum Y               =155117520
The number of stems           =30
Mean                          =15.00 The theoretical mean=15.00
Variance                      =7.50
Standard Deviation            =2.74
Skewness                      =0.00
Kurtosis                      =-0.07
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =40.00
The minimum Y                 =1 The maximum Y               =137846528820
The number of stems           =40
Mean                          =20.00 The theoretical mean=20.00
Variance                      =10.00
Standard Deviation            =3.16
Skewness                      =0.00
Kurtosis                      =-0.05
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =50.00
The minimum Y                 =1 The maximum Y             =126410606437752
The number of stems           =50
Mean                          =25.00 The theoretical mean=25.00
Variance                      =12.50
Standard Deviation            =3.54
Skewness                      =-0.00
Kurtosis                      =-0.04
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =60.00
The minimum Y                 =1 The maximum Y             =118264581564861424
The number of stems           =60
Mean                          =30.00 The theoretical mean=30.00
Variance                      =15.00
Standard Deviation            =3.87
Skewness                      =-0.00
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is                =70.00
The minimum Y                 =1 The maximum Y             =112186277816662845432
The number of stems           =70
Mean                          =35.00 The theoretical mean=35.00
Variance                      =17.50
Standard Deviation            =4.18
Skewness                      =0.00
Kurtosis                      =-0.03
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =80.00
The minimum Y                 =1 The maximum Y             =1075072087333361764616
20
The number of stems           =80
Mean                          =40.00 The theoretical mean=40.00
Variance                      =20.00
Standard Deviation            =4.47
Skewness                      =-0.00
Kurtosis                      =-0.02
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =90.00
The minimum Y                 =1 The maximum Y                =10382742128755341136
71120
The number of stems           =90
Mean                          =45.00 The theoretical mean=45.00
Variance                      =22.50
Standard Deviation            =4.74
Skewness                      =-0.00
Kurtosis                      =-0.02
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =100.00
The minimum Y                 =1 The maximum Y                =1008913445455641933
12497256
The number of stems           =100
Mean                          =50.00 The theoretical mean=50.00
Variance                      =25.00
Standard Deviation            =5.00
Skewness                      =-0.00
Kurtosis                      =-0.02
```

Here, **n**=100

Appendix 14. Validation of *tool1* where stems can be missed.

14.1 Introduction.

This Appendix attempts to validate *tool1* for quizzes where stems can be missed. The validation is done 'by hand' for:

X = 1 i.e. Yes/No quizzes	for **n** = 1 to 5
X = 3 i.e. 4-option MCQ quizzes	for **n** = 1 to 5
X = 7 i.e. 3-option Multiple Yes/No quizzes	for **n** = 1 to 5
X = 9 i.e. 5-option MAQ quizzes with 2 ticks	for **n** = 1 to 5
X = 19i.e. 2-option EMSQ (MCQ)	for **n** = 1 to 5
X = 187455 i.e.20-option EMSQ (MAQ with 10 ticks)	for **n** = 1 to 5

Beyond **n** = 5 we use or proofs, and other indications, to validate *tool1*, where stems can be missed.

For each of the above we:

- do not use negative marking, and have **a** = 0,
- do use negative marking where **b** = 0 and **a** has a value that exactly tackles the guessing problem i.e. **a**=-1/X

So that means that there are 12 sections where our validation is done.

This Appendix is accompanied by Appendix 8.

We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 8.

Appendix 8 is where the validation of the **expression** that is used by *tool1* is done, for quizzes where stems can be missed. We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 8.

One validation that we can do is that the distribution should be skewed.

14.2 Yes/No quizzes (X = 1) where no negative marking is used i.e. a = 0.

```
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1

Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =1.00
The number of stems          =1
Mean                         =0.33 The theoretical mean=0.33
Variance                     =0.22
Standard Deviation           =0.47
Skewness                     =0.71
Kurtosis                     =-1.50

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 4 times
Press any key to continue . . .
```

The distribution is skewed as the Skewness is non-zero

The file has a header written to it and three pairs of numbers; each pair on a separate line.

Notice that, here, the marks are not sorted, and the numbers getting mark 0, say, are not added. So a Pivot is necessary.

This agrees with a table in Appendix 8, where **n** = 1. So, *tool1* produces the same results as the expression. It also agrees with the theoretical number of impossible situations and produces a mean that agrees with the theoretical.

```
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000       numbner getting that mark that way=2 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2

Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =2.00
The number of stems          =2
Mean                         =0.67 The theoretical mean=0.67
Variance                     =0.44
Standard Deviation           =0.67
Skewness                     =0.50
Kurtosis                     =-0.75

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 7 times
Press any key to continue . . .
```

This is Multiple Yes/No quiz with 1 option (and hence is a Yes/No quiz). There are 2 stems i.e. **n** = 2 and agrees with a table in Appendix 8, where **n** = 2.

```
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000      numbner getting that mark that way=3 n=3 w=0 x=2 y=1
Mark=1.0000      numbner getting that mark that way=6 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000      numbner getting that mark that way=3 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is              =3.00
The number of stems         =3
Mean                        =1.00 The theoretical mean=1.00
Variance                    =0.67
Standard Deviation          =0.82
Skewness                    =0.41
Kurtosis                    =-0.50

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 11 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8, where **n** = 3

```
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000      numbner getting that mark that way=4 n=4 w=0 x=3 y=1
Mark=1.0000      numbner getting that mark that way=12 n=4 w=1 x=2 y=1
Mark=2.0000      numbner getting that mark that way=12 n=4 w=2 x=1 y=1
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000      numbner getting that mark that way=6 n=4 w=0 x=2 y=2
Mark=1.0000      numbner getting that mark that way=12 n=4 w=1 x=1 y=2
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000      numbner getting that mark that way=4 n=4 w=0 x=1 y=3
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is              =4.00
The number of stems         =4
Mean                        =1.33 The theoretical mean=1.33
Variance                    =0.89
Standard Deviation          =0.94
Skewness                    =0.35
Kurtosis                    =-0.37

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 16 times
Press any key to continue . . . _
```

This agrees with a table in Appendix 8, where **n** = 4

```
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000       numbner getting that mark that way=5 n=5 w=0 x=4 y=1
Mark=1.0000       numbner getting that mark that way=20 n=5 w=1 x=3 y=1
Mark=2.0000       numbner getting that mark that way=30 n=5 w=2 x=2 y=1
Mark=3.0000       numbner getting that mark that way=20 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000       numbner getting that mark that way=10 n=5 w=0 x=3 y=2
Mark=1.0000       numbner getting that mark that way=30 n=5 w=1 x=2 y=2
Mark=2.0000       numbner getting that mark that way=30 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000       numbner getting that mark that way=10 n=5 w=0 x=2 y=3
Mark=1.0000       numbner getting that mark that way=20 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000       numbner getting that mark that way=5 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is           =5.00
The number of stems         =5
Mean                        =1.67 The theoretical mean=1.67
Variance                    =1.11
Standard Deviation          =1.05
Skewness                    =0.32
Kurtosis                    =-0.30

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 22 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8, where **n** = 5

We do not check by 'by hand' beyond **n** = 5. Beyond that we know:

a) the mean mark,
b) the range of marks,
c) the number of impossible situations,
d) the name of the file we have written to.

We can use these as the validation.

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is           =10.00
The number of stems         =10
Mean                        =3.33 The theoretical mean=3.33
Variance                    =2.22
Standard Deviation          =1.49
Skewness                    =0.22
Kurtosis                    =-0.15

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =20.00
The number of stems             =20
Mean                            =6.67 The theoretical mean=6.67
Variance                        =4.44
Standard Deviation              =2.11
Skewness                        =0.16
Kurtosis                        =-0.08

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 232 times
Press any key to continue . . .
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=465 The theretical number of impossible sit
ations is=465.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is                =30.00
The number of stems             =30
Mean                            =10.00 The theoretical mean=10.00
Variance                        =6.67
Standard Deviation              =2.58
Skewness                        =0.13
Kurtosis                        =-0.05

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 497 times
Press any key to continue . . .
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =40.00
The number of stems             =40
Mean                            =13.33 The theoretical mean=13.33
Variance                        =8.89
Standard Deviation              =2.98
Skewness                        =0.11
Kurtosis                        =-0.04

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 862 times
Press any key to continue . . .
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =50.00
The number of stems           =50
Mean                          =16.67 The theoretical mean=16.67
Variance                      =11.11
Standard Deviation            =3.33
Skewness                      =0.10
Kurtosis                      =-0.03

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1327 times
Press any key to continue . . .
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =60.00
The number of stems           =60
Mean                          =20.00 The theoretical mean=20.00
Variance                      =13.33
Standard Deviation            =3.65
Skewness                      =0.09
Kurtosis                      =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1892 times
Press any key to continue . . .
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =70.00
The number of stems           =70
Mean                          =23.33 The theoretical mean=23.33
Variance                      =15.56
Standard Deviation            =3.94
Skewness                      =0.08
Kurtosis                      =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 2557 times
Press any key to continue . . .
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =80.00
The number of stems          =80
Mean                         =26.67 The theoretical mean=26.67
Variance                     =17.78
Standard Deviation           =4.22
Skewness                     =0.08
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 3322 times
Press any key to continue . . .
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=4095 The theretical number of impossible s
uations is=4095.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-
is                =90.00
The number of stems          =90
Mean                         =30.00 The theoretical mean=30.00
Variance                     =20.00
Standard Deviation           =4.47
Skewness                     =0.07
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 4187 times
Press any key to continue . . .
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =100.00
The number of stems          =100
Mean                         =33.33 The theoretical mean=33.33
Variance                     =22.22
Standard Deviation           =4.71
Skewness                     =0.07
Kurtosis                     =-0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

Here, **n**=100

So, for Yes/ No-only stems where stems can be missed and the score is [0], [1, 0] *tool1* works, as far as we can tell.

Appendix 15. Tool2.

```
//Descriptive Stats Source File
//Includes
    //Include LEDA
      #include <LEDA/numbers/integer.h>
      #include <LEDA/numbers/bigfloat.h>
      #include <LEDA\core\array.h>           //This was helped by VS 10 system
    //Include ordinary C++
      #include <iomanip>
      #include <string>
      #include <iostream>
      #include <typeinfo> //To allow typeid to work. Do we ever use typeid?
    //Namespaces
      using namespace std;
         using namespace leda;

       //Definition of a function isDivisibleBy () that we are going to use
         bool isDivisibleBy(integer firstNumber, int secondNumber)
         {
                if (firstNumber % secondNumber ==0)
                {
                       return(true);
                }
                else
                {
                    return (false);
                }
         }

  int main()
   {
     //Declarations
          std::string fileName, fileName2,line,modalType,experiment;
          integer maxForWorkingOutMode,maxForWorkingOutMedian,sum, swap,
howMany[9900], cumulativeFrequency[9900],minimumWay,maximumWay,maximumY,
minimumY,sumToDate[9900];
          int way,sizeOfn, numberOfModes, position;
          double mark[9900],firstMarkForMode,lastMarkForMode,maximumMark,
minimumMark;
          bigfloat
xbar,pseudoArea,sumOfWays,maximumNumberOfWays,mu,median,mode,variance,
sd,skewness,
kurtosis,varianceSum,varianceSumSam,skewnessSum,skewnessSumSam,kurtosisSum,kurtos
isSumSam,momentAboutMean,
momentAboutMeanSam,varianceSam,sdSam,skewnessSam,kurtosisSam;
          bool
cannotWorkOutMode,cannotWorkOutMedian,weHaveFoundMedian,weHaveFoundIt,medianFirst
,medianSecond,doneModeOnce,doneModeTwice,fortyFirst,fiftyFirst,seventyFirst,
seventyOneFirst, seventyTwoFirst, seventyThreeFirst,
seventyFourFirst,seventyFiveFirst, populationIsEven,
populationIsOdd,medianTakesTooLong;
          ifstream myfile;
        //Initialisation
          //experiment="Checking that we are using the same formulae as SPSS";
//Options are "No" or "Checking that we are using the same formulae as SPSS"
          experiment="No";
          if(experiment=="No")
           {
            fileName="F:\\MyPhD2\\MyPhD\\Output.txt";
            myfile.open(fileName);
```

```
            if (!myfile)  //the file might not be open or is in use by another
program
             {
           cout << "Unable to open file " << fileName << endl;   //What is wrong
with this?
           exit(1); // terminate with error
         }
           }
          else
          {
          fileName2="F:\\MyPhD2\\MyPhD\\Output.txt";
             myfile.open(fileName2);
             if (!myfile)  //the file might not be open or is in use by another
program
              {
            cout << "Unable to open file " << fileName2 << endl;
            exit(1); // terminate with error
              }
          }
          maxForWorkingOutMedian=maxForWorkingOutMode=999999999; //There appears
to be an upper limit here!!! I have got 9 nines. But if I use 10 I get an error
                                              //Why? I thought a LEDA integer was
of an arbirary length.!!!!!!! Algorithmic Solutions did not reply to my query.

          for(int i=1;i <5; i++)            //This is part of the solution to the
above problem
          {
                  maxForWorkingOutMedian=maxForWorkingOutMode=
maxForWorkingOutMedian*10+9;
          }

          way=0;
          sum=0;
          sumOfWays=0;
          numberOfModes=0;
          varianceSum=0;
          skewnessSum=0;
          kurtosisSum=0;
          sumToDate[1]=0;

          varianceSumSam=0.0;
          skewnessSumSam=0.0;
          kurtosisSumSam=0.0;

          maximumNumberOfWays=-9999999999.0; //Why is that?
          minimumWay=9999999999.0;//I thought this was an integer
          maximumWay=-999999999;
          maximumMark=-9999999999.0;
          minimumMark=999999999.0;

          minimumY=99999999.9;
          maximumY=-999999999.9;
          medianFirst=false;

          modalType="Uni-modal";
          doneModeOnce=false;
          doneModeTwice=false;
          medianFirst=false;
          medianSecond=false;

          fortyFirst=false;
          fiftyFirst=false;
          seventyFirst=false;
          seventyOneFirst=false;
```

```
            seventyTwoFirst=false;
            seventyThreeFirst=false;
            seventyFourFirst=false;
            seventyFiveFirst=false;

            populationIsEven=false;
            populationIsOdd=false;

            medianTakesTooLong=false;
            weHaveFoundIt=false;
            weHaveFoundMedian=false;
            cannotWorkOutMedian=false;
            cannotWorkOutMode=false;

            if(experiment=="No")
            {
             cout << "Trying to READ from file " << fileName  << endl;
            }
           else
            {
          cout << "Trying to read from file " << fileName2 << endl;
            }
            //cout << "We have read the following " << endl;
            if(experiment=="Checking that we are using the same formulae as SPSS")
            {
                   //Debug
                   cout << "We are in Checking" << endl;
                   while(!myfile.eof()) //We should do lots of checks on this file
                 {
                        way++;
                myfile >> mark[way] >> howMany[way];
                        cout  << howMany[way] << endl;
                           sum=sum+howMany[way];
                           if(howMany[way] > maximumWay)
                        {
                 maximumWay=howMany[way];
                        }
                        if (howMany[way] < minimumWay)
                        {
                            minimumWay=howMany[way];
                        }
                           if(howMany[way]==maximumNumberOfWays) //There is a small
chance that howMany[i]=initial value of maximumNumberOfWays
                        {
                 modalType="Bi-modal";
                            mode=(mode+howMany[way])/2.0;

                        }
                       if(howMany[way] > maximumNumberOfWays )
                       {
                       maximumNumberOfWays=howMany[way] ;
                          mode=howMany[way] ;
                          }
                     } // End while
                   xbar=(bigfloat)sum/(1.0*way);
                   cout << fixed;
                   for (int i=1;i<way+1;i++)
                 {
                        sumOfWays=sumOfWays+howMany[i];
                        momentAboutMeanSam=howMany[i] - xbar;
                        varianceSumSam=varianceSumSam +
momentAboutMeanSam*momentAboutMeanSam;
                        skewnessSumSam=skewnessSumSam +
momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
```

```
                    kurtosisSumSam=kurtosisSumSam +
momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
               }
               sizeOfn=way;
                  varianceSam=varianceSumSam/(sizeOfn-1); //Is sizeOfn right?
Should it be sumOfWays This way variance will be 0
            sdSam=sqrt(varianceSam);

skewnessSam=sizeOfn*skewnessSumSam/((sdSam*sdSam*sdSam)*(sizeOfn-1)*(sizeOfn-2));
                  kurtosisSam=-3.0*(bigfloat)(sizeOfn-1)*(sizeOfn-1)/((sizeOfn-
2)*(sizeOfn-3))+kurtosisSumSam*sizeOfn*(sizeOfn+1)/((sizeOfn-1)*(sizeOfn-
2)*(sizeOfn-3)*varianceSam*varianceSam);
            //Output
                      cout << fixed;
                   cout << setprecision( 3 ) ;  //What is wrong with that? Is it
a LEDA word?
                      cout << "Range goes from "  << minimumWay << " to " <<
maximumWay << endl;
                      cout << "Mean Sam="       << xbar.to_double()     << "
Mode=" << mode.to_double()     << endl;
                   cout << "Var Sam.="       << varianceSam.to_double()
                           << " SD. Sam="       << sdSam.to_double() <<
"\t"
                           << "Skew Sam="       << skewnessSam.to_double() <<
"\t"
                           << "Kur Sam="        << kurtosisSam.to_double() <<
endl;


                      cout << "-----------------------------" << endl;
                      cout << "          Scientific notation" << endl;
                      cout << scientific << "Mean Sam="       << xbar.to_double()
<< " Mode=" << mode.to_double()     << endl;
                   cout << scientific << "Var Sam.="       <<
varianceSam.to_double()
                           << " SD. Sam="       << sdSam.to_double() <<
"\t"
                           << "Skew Sam="       << skewnessSam.to_double() <<
"\t"
                           << "Kur Sam="        << kurtosisSam.to_double() <<
endl;
          }

          else
          {

           while(!myfile.eof()) //We are now reading from the file
F:\\MyPhD2\\MyPhD\\Output.txt
           {
               way++;
         myfile >> mark[way] >> howMany[way];
               if(mark[way] > maximumMark)
               {
           maximumMark=mark[way];
               }
               if (mark[way] < minimumMark)
               {
                 minimumMark=mark[way];
               }

               if(howMany[way] > maximumY)
               {
           maximumY=howMany[way];
               }
```

```
                if (howMany[way] < minimumY)  //This does not work now
                {
                        minimumY=howMany[way];
                }

                pseudoArea=pseudoArea+mark[way]*howMany[way];
                sum=sum+howMany[way];

            }

            mu=pseudoArea/((bigfloat)sum);
            xbar=mu;



               //Calculate other descriptive statistics
           for (int i=1;i<way+1;i++)
             {


            momentAboutMean=mark[i]-mu;   //Moment about mean
                  momentAboutMeanSam=mark[i] - xbar;

                   cout << fixed;
                   cout << setprecision( 3 ) ;

                      //Work out the proportion of passes

                  if (mark[i] >=40.0*maximumMark/100.0 && !fortyFirst)
                  {
                       fortyFirst=true;
                       cout << "   Percentage of 40% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                  }
                  if (mark[i] >=50.0*maximumMark/100.0 && !fiftyFirst)
                  {
                       fiftyFirst=true;
                       cout << "   Percentage of 50% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                  }

                  if (mark[i] >=70.0*maximumMark/100.0 && !seventyFirst)
                  {
                       seventyFirst=true;
                       cout << "   Percentage of 70% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=71.0*maximumMark/100.0 && !seventyOneFirst)
                  {
                       seventyOneFirst=true;
                       cout << "   Percentage of 71% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=72.0*maximumMark/100.0 && !seventyTwoFirst)
                  {
                       seventyTwoFirst=true;
                       cout << "   Percentage of 72% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=73.0*maximumMark/100.0 && !seventyThreeFirst)
```

```
                {
                     seventyThreeFirst=true;
                     cout << "   Percentage of 73% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                }

                 if (mark[i] >=74.0*maximumMark/100.0 && !seventyFourFirst)
                {
                     seventyFourFirst=true;
                     cout << "   Percentage of 74% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                }

                if (mark[i] >=75.0*maximumMark/100.0 && !seventyFiveFirst)
                {
                     seventyFiveFirst=true;
                     cout << "   Percentage of 75% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;

                }

                 sumOfWays=sumOfWays+howMany[i];

                //Work out HOW MANY MODES there are.

          if(howMany[i]==maximumY) numberOfModes=numberOfModes+1;

        //Now work out the MODE. Later on the MODE will be output

                if((howMany[i]==maximumY) && numberOfModes >1)
                       {
                              lastMarkForMode=mark[i];
                    }

                if((howMany[i] == maximumY) && numberOfModes==1 )
                 {
                                 firstMarkForMode=-mark[i] ;
                                 lastMarkForMode=0;
                 }

                //Work out MEDIAN

         if (way==1) //If the census only has 1 value
              {
           median=mark[1];
                 maximumY=minimumY=howMany[1];
                 maximumWay=minimumWay=mark[1];
              }
              //if ((sum >  maxForWorkingOutMedian))
              //{
                //cannotWorkOutMedian=true;
              //}

         //if(!cannotWorkOutMedian)
              //{
                 if(!weHaveFoundMedian)
              {
                 if(i==1) cumulativeFrequency[1] =
howMany[1];
                       else     cumulativeFrequency[i]= cumulativeFrequency[i-1]
+ howMany[i];
                          if(cumulativeFrequency[i] >=sum/2)
                  {
                          position=i;
```

```
                        if ((howMany[i] >  maxForWorkingOutMedian)) //if
Scientific Notation is used anywhere e cannot work out median
                                   {
                         cannotWorkOutMedian=true;
                                   }

                                   if (!isDivisibleBy(sum,2)) //The population
is odd.
                        {
                          median=mark[position];
                                  weHaveFoundMedian=true;
                        }
                                   else                         //population is
even
                                   {

                                     if(cumulativeFrequency[i]==sum/2)
                          {
                                      median=(mark[position]  +
mark[position+1])/2; //What happens if there is no (position +1). Does not happen
in our case

//Be careful there is a devisiion involved

//median is of type bigfloat, mark is of type float. Should median be of type
float?
                                      weHaveFoundMedian=true;
                                 }
                                    else
                                {
                              median=mark[position];
                                      weHaveFoundMedian=true;
                                 }
                                   }
                             }
                  }
             //}

                //Work out the sums for censi and samples
                 varianceSum=varianceSum +
howMany[i]*momentAboutMean*momentAboutMean;
                 skewnessSum=skewnessSum +
howMany[i]*momentAboutMean*momentAboutMean*momentAboutMean;
                 kurtosisSum=kurtosisSum +
howMany[i]*momentAboutMean*momentAboutMean*momentAboutMean*momentAboutMean;

                 varianceSumSam=varianceSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam;
                 skewnessSumSam=skewnessSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
                 kurtosisSumSam=kurtosisSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam*momentAboutMe
anSam;
            }
               //Work out descriptive stats for censi and samples
               variance=varianceSum/sumOfWays;
               sd=sqrt(variance);
         skewness=     skewnessSum/(sumOfWays*sd*sd*sd);
               kurtosis=-3 + kurtosisSum/(sumOfWays*sd*sd*sd*sd);


               varianceSam=varianceSumSam/(sumOfWays-1);
         sdSam=sqrt(varianceSam);
```

```
skewnessSam=sumOfWays*skewnessSumSam/((sdSam*sdSam*sdSam)*(sumOfWays-
1)*(sumOfWays-2));
               kurtosisSam=-3.0*(bigfloat)(sumOfWays-1)*(sumOfWays-
1)/((sumOfWays-2)*(sumOfWays-
3))+kurtosisSumSam*sumOfWays*(sumOfWays+1)/((sumOfWays-1)*(sumOfWays-
2)*(sumOfWays-3)*varianceSam*varianceSam);

          //Output
            cout << fixed;
               cout << setprecision( 3 ) ;

               cout << "The number of lines read in is=" << way << endl;
               cout << "Sum of all ways=POPULATION=" << sumOfWays << endl;
               cout << "Marks range from " << minimumMark << " to " <<
maximumMark << " Hence Range=" <<maximumMark-minimumMark << endl;
               cout << "Number of MODES=" << numberOfModes << endl;
               cout << "  " <<endl;
               cout << "          DESCRIPTIVE STATISTICS" << endl;

               mode =(lastMarkForMode - firstMarkForMode)/numberOfModes;
//Careful, that involves a division.
               //Debug
               cout << "First Mode occurs at mark " << -firstMarkForMode << "
Last  Mode occurs at Mark " << lastMarkForMode << "Number of modes=" <<
numberOfModes << " Mode=" << mode.to_double() << endl;

               //if (!cannotWorkOutMedian)
               //{
                 cout << "Population" << endl;
                 cout << "   Mean Pop="       << mu.to_double()          << "\t\t
Median Pop=" << median.to_double() << "\t " << " Mode Pop=" << mode.to_double()
<< endl;
               //}
               //else
               //{
                       //cout << "Population" << endl;
                       //cout << "  We cannot calculate MEDIAN (there are
approximations due to Scientific Notation)" << endl;
                    //cout << "   Mean Pop="       << mu.to_double() ;
                       //cout << "\t " << " Mode Pop=" << mode.to_double() <<
endl;

               //}


               cout << "   Var Pop.="       << variance.to_double()
                       << " SD. Pop="          << sd.to_double() << "\t"
                       << "Skew Pop="          << skewness.to_double()    <<
"\t"
                      << "Kur Pop="           << kurtosis.to_double()    << endl;
               cout << "  " << endl;
               cout << "Sample" << endl;
               cout << "   Mean Sam="       << mu.to_double()          << endl;
               cout << "   Var Sam.="       << varianceSam.to_double()
                       << " SD. Sam="          << sdSam.to_double() <<
"\t"
                       << "Skew Sam="          << skewnessSam.to_double() <<
"\t"
                      << "Kur Sam="           << kurtosisSam.to_double() << endl;
               cout << "  " << endl;
              cout << "---------------------------------------------------" << endl;
```

Appendix 16. Validation of *tool2*.

16.1 Introduction.

We will validate *tool2* by showing:

a) that *tool2* calculates the descriptive statistics properly by showing that it gets the same results as SPSS in many cases and hence uses the same formulae as SPSS,
b) the tool, *tool2* calculates *median* properly (where it can),
c) the tool, *tool2* calculates *mode* properly,
d) the tool, *tool2* calculates the proportion of the population that obtain a certain mark or above.

The mode and median cannot be calculated, in general, until a Pivot is done, because:

a) the mode (as a 1st approximation) is the most frequently occurring y-value,
b) the median is the 'middle' x-value.

16.2 Does *tool2* use the same formulae as SPSS?

The descriptive statistics include:

- Mean,
- Variance,
- Standard Deviation,
- Skewness,
- Kurtosis.

We do not know how to ask SPSS to calculate the descriptive statistics for a *distribution*, but only for a single column, because that is what a statistician said

> *"... you can only get descriptive stats of one variable"*
>
> Straker (2011).

So, we ask SPSS to give us the descriptive statistics of a single column. This part of the validation of *tool2* is done by inputting from **Output.txt** (which has 2 columns:

a) a mark,
b) how many obtain that mark and ignoring the mark.

This is done when `experiment="Checking that we are using the same formulae as SPSS"`

SPSS also calculates minimum, maximum and range. So *tool2* calculates and outputs to the debug window:

a) the *minimum* number that get a mark,
b) the *maximum* number that get a mark,
c) the range of marks.

When in experimental mode *tool2* produces something like

```
Range goes from 1 to 1182645815648610000
```

When in non-experimental mode *tool2* produces something like

```
larks range from 0.000 to 60.000 Hence Range=60.000
```

The range of marks given by *tool2* can be observed.

There is some confusion between the *range* of marks and the *number* of marks. Consider that there are only **2** marks, 0 and 3. The range of marks is **3** (3-0). Do we now output 2 or 3? To avoid any argument, *tool2* shows:

a) the *minimum* mark,
b) the *maximum* mark,
c) the range of marks which is (*maximum* - *minimum*).

The formulae that are used are:

App. 16 Table 1. Formulae that SPSS and *tool2* use.

	Estimates. **Formulae that *SPSS* uses. n=size of sample**	**Census** **Formulae that *tool2* uses. N=size of population**
Mean	$\bar{x}=\frac{\sum x_i}{n}$	$\mu=\frac{\sum x_i}{N}=\sum x_i P(x_i)$
Variance	VarSam= $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$.	VarPop= $\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$
Standard Deviation	s=$\sqrt{VarSam}$	σ=$\sqrt{VarPop}$
Skewness	$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^3}{N\sigma^3}$
Kurtosis	$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^4}$ - $\frac{3(n-1)^2}{(n-1)(n-2)}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^4}{N\sigma^4}$ -3

Here, n is any integer above 0 and not the number of stems. n is the size of the sample and N is the size of the population.

The data of the following table corresponds to X = 1. Yes/No quizzes where stems are ALL answered with **a** having any value (except 1). All that changes with a change of **a** is the mark, and we ignore that.

We use Scientific Notation (e.g. 428 E76) where necessary, because SPSS does.

App 16 Table 2. *tool2* compared to SPSS for Yes/No quizzes where ALL stems are answered and no negative marking is used.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	93.909	8943.091	94.568	0.624	-1.273
	SPSS	93.909	8943.091	94.568	0.624	-1.273
20	tool2	49932.19	4.274E9.	65379.214	1.071	-0.381
	SPSS	49932.19	4.274E9	65379.214	1.071	-0.381
40	tool2	2.68E10	1.951E21	4.416E10	1.539	0.943
	SPSS	2.68E10	1.951E21	4.4168E10	1.539	0.943
60	tool2	1.89E16	1.247E33	3.531E16	1.832	1.989
	SPSS	1.89E16	1.247E33	3.531E16	1.832	1.989
80	tool2	1.49E22	9.250E44	3.041E22	2.050	2.887
	SPSS	1.49E22	9.250E44	3.041E22	2.050	2.887
100	tool2	1.256E28	7.464 E56	2.732E28	2.228	3.685
	SPSS	1.26E28	7.464E56	2.732E28	2.228	3.685

For Yes/No quizzes where stems are ALL answered, *tool2* and SPSS give the same results (as far as we can tell) for 10 to 100 stems (inclusive).

The data for the following table corresponds to X = 1. Yes/No quizzes. Stems are can be MISSED and **a** = 0.

App. 16 Table 3. *tool2* compared to SPSS for Yes/No quizzes where stems are be MISSED and **a**=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	5368.09	3.368E7	5803.691	0.721	-1.125
	SPSS	5368.09	3.368E7	5803.691	0.721	-1.125
20	tool2	1.66E8	5.202E16	2.28E8	1.164	-0.1462
	SPSS	1.66E8	5.202E16	2.28E8	1.164	-0.146
40	tool2	2.97E17	2.587E35	5.086E17	1.633	1.269
	SPSS	2.97E17	2.587E35	5.086E17	1.633	1.269
60	tool2	6.95E26	1.819E54	1.349E27	1.928	2.383
	SPSS	6.95E26	1.819E54	1.349E27	1.928	2.383
80	tool2	1.825 E36	1.488E73	3.857E36	2.149	3.336
	SPSS	1.82 E36	1.488E73	3.857E36	2.149	3.336
85	tool2	4.176 E38	8.076 E77	8.986 E38	2.198	3.557
	SPSS	4.18 E38	8.076 E77	8.986 E38	2.198	3.557
86	tool2	1.239 E39	7.151 E78	2.674 E39	2.207	3.600
	SPSS	1.24 E39	7.151 E78	2.674 E39	2.207	3.600
87	tool2	3.673 E39	6.333 E79	7.958 E39	2.216	3.643
	SPSS	**8.26 E38**	**2.834 E78**	**1.683 E39**	**2.770**	**8.521**
90	tool2	9.591 E40	4.403 E82	2.098 E41	2.243	3.771
	SPSS	**1.54 E39**	**6.120 E78**	**2.474 E39**	**1.506**	**0.957**
100	tool2	5.103E45	1.325 E92	1.151 E46	2.329	4.185
	SPSS	**2.01 E38**	**6.21 E78**	**2.494 E39**	**1.310**	**1.110**

It goes wrong at **n** = 87 onwards. The size of **n** is not important. *tool2* and SPSS disagree when the numbers being represented are 'too big'.

So, *tool2* does use the same formulae as SPSS, until SPSS reaches its limitations.

The data for the following table corresponds to X = 1. Yes/No quizzes. Stems are can be MISSED and a = -1.

App. 16 Table 4. *tool2* compared to SPSS for Yes/No quizzes where stems are can be MISSED and a=-1, b=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	2811.86	1.057 E7	3250.708	0.832	-0.891
	SPSS	2811.86	1.051 E7	3250.708	0.832	-0.891
20	tool2	8.504 E7	1.596 E16	1.263 E8	1.309	0.224
	SPSS	8.504 E7	1.596 E16	1.263 E8	1.309	0.224
40	tool2	1.50 E17	7.788 E34	2.791 E17	1.813	1.893
	SPSS	1.50 E17	7.788 E34	2.791 E17	1.813	1.893
60	tool2	3.503 E26	5.431 E53	7.370 E26	2.129	3.209
	SPSS	3.503 E26	5.431 E53	7.370 E26	2.129	3.209
80	tool2	9.181 E35	4.420 E72	2.102 E36	2.366	4.333
	SPSS	9.181 E35	4.420 E72	2.102 E36	2.366	4.333
85	tool2	2.10 E38	2.397 E77	4.896 E38	2.417	4.593
	SPSS	2.10 E38	2.397 E77	4.896 E38	2.417	4.593
86	tool2	6.228 E38	2.122 E78	1.457 E39	2.427	4.644
	SPSS	6.23 E38	2.122 E78	1.457 E39	2.427	4.644
87	tool2	1.847 E39	1.870 E79	1.335 E39	2.437	4.695
	SPSS	**7.25 E38**	**3.305 E78**	**1.817 E39**	**3.331**	**11.348**
90	tool2	4.822 E40	1.306 E82	1.143 E41	2.466	4.845
	SPSS	**1.07 E39**	**3.72 E78**	**1.930 E39**	**2.044**	**3.872**
100	tool2	2.564 E45	3.923 E91	6.264 E45	2.558	5.331
	SPSS	**1.58 E39**	**5.145 E79**	**2.268 E39**	**1.605**	**1.828**

tool2 and SPSS do not agree when **n** = 87 or more.

SPSS goes wrong, here, at **n** = 87 and it appears to be when a number occupies more than 40 characters. *tool2* appears to be all right (all the descriptive statistics appear to be going in the right direction at the right speed, whereas:

a) the mean as measured by SPSS goes down and up,
b) the kurtosis as measured by SPSS suddenly jumps.

The data for the following table corresponds to X = 9. 5-option MAQ quizzes with 2 ticks. Stems are ALL answered and **a** = 0.

App. 16 Table 5. *tool2* compared to SPSS for 5-option MAQ quizzes with 2 ticks where stems are ALL answered and *a*=0

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	9.09 E8	2.217 E18	1.489 E9	1.454	0.571
	SPSS	9.09 E8	2.217 E18	1.489 E9	1.454	0.571
20	tool2	4.76 E18	8.339 E37	9.132 E18	1.904	2.471
	SPSS	4.76 E18	8.339 E37	9.132 E18	1.904	2.471
40	tool2	2.44 E38	3.140 E77	5.603 E38	2.398	4.674
	SPSS	2.44 E38	3.140 E77	5.603 E38	2.398	4.674
60	tool2	1.639 E58	1.761 E117	4.197 E58	2.724	6.418
	SPSS	**2.17 E39**	**5.948 E78**	**2.439 E39**	**1.047**	**0.317**
80	tool2	1.235 E78	1.165 E157	3.3414 E78	2.975	7.915
	SPSS	**2.68 E39**	**7.51 E78**	**2.741 E39**	**1.046**	**0.055**
90	tool2	1.099 E88	9.833 E176	3.136 E88	3.082	8.599
	SPSS	**3.33 E39**	**9.424 E78**	**3.070 E39**	**0.684**	**-0.808**
100	tool2	9.901 E97	8.444 E196	2.906 E98	3.181	9.249
	SPSS	**3.20 E39**	**7.263 E78**	**2.695 E39**	**0.656**	**-0.496**

Overall, we can conclude that *tool2* and SPSS use the same formulae, but they differ in their results when the limits of SPSS are reached. It looks as though the limits of *tool2* are not reached, here.

The data for the following table corresponds to X = 9. 5-option MAQ quizzes with 2 ticks. Stems are can be MISSED with **a** = **b** = 0.

App. 16 Table 5. *tool2* compared to SPSS for 5-option MAQ quizzes with 2 ticks wher stems cab be MISSED and *a*=*b*=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	2.36 E9	1.606 E19	4.007 E9	1.544	0.846
	SPSS	2.36 E9	1.606 E19	4.007 E9	1.544	0.846
20	tool2	3.20 E19	4.007 E39	6.330 E19	1.975	2.802
	SPSS	3.20 E19	4.007 E39	6.330 E19	1.975	2.802
40	tool2	1.104 E40	6.780 E80	2.604 E40	2.475	5.092
	SPSS	**1.58 E39**	**9.621 E78**	**3.102 E39**	**1.856**	**2.039**
60	tool2	4.992 E60	1.717 E122	1.310 E61	2.805	6.909
	SPSS	**2.60 E39**	**6.468 E78**	**2.91 E39**	**0.952**	**-0.175**
80	tool2	2.529 E81	5.135 E163	7.166 E81	3.059	8.470
	SPSS	**3.08 E39**	**9.058 E78**	**3.010 E39**	**0.835**	**-0.497**

90	tool2	5.838 E91	2.914 E184	1.707 E92	3.169	9.183
	SPSS	**3.39 E39**	**8.799 E78**	**2.966 E39**	**0.574**	**-0.834**
100	tool2	1.364 E102	1.683 E205	4.102 E102	3.269	9.861
	SPSS	**3.54 E39**	**9.281 E78**	**3.046 E39**	**0.614**	**-0.841**

Here, *tool2* and SPSS disagree at about **n** = 40. The disagreement comes about when SPSS cannot cope with 'large' numbers. But, the main conclusion is that tool2 and SPSS do use the same formulae until SPSS 'runs out of steam'.

The data for the following table corresponds to X = 9. 5-option MAQ quizzes with 2 ticks. Stems are can be MISSED with

App. 16 Table 6. *tool2* compared to SPSS for 5-option MAQ quizzes with 2 ticks where stems cab be MISSED with *a*=-1/9 and *b*=0**.**

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	3.990 E8	9.665 E17	9.831 E8	2.787	6.901
	SPSS	3.990 E8	9.665 E17	9.831 E8	2.787	6.901
20	tool2	3.93 E18	1.186 E38	1.089 E19	3.433	12.082
	SPSS	3.93 E18	1.186 E38	1.089 E19	3.433	12.082
40	tool2	1.096 E39	1.105 E79	3.325 E39	3.879	15.634
	SPSS	**5.78 E38**	**2.562 E78**	**1.601 E39**	**3.604**	**13.189**
60	tool2	4.381 E59	2.013 E120	1.419 E60	4.115	17.783
	SPSS	**2.40 E39**	**7.249 E79**	**2.692 E39**	**1.053**	**0.062**
80	tool2	2.014 E80	4.860 E161	6.972 E80	4.256	19.029
	SPSS	**2.93 E39**	**7.505 E78**	**2.740 E39**	**0.853**	**-0.271**
90	tool2	4.453 E90	2.536 E182	1.592 E91	4.327	19.570
	SPSS	**3.14 E39**	**7.702 E78**	**2.775 E39**	**0.743**	**-0.531**
100	tool2	9.993 E100	1.358 E203	3.685 E101	4.412	39.049
	SPSS	**3.22 E39**	**7.551 E78**	**2.748 E39**	**0.797**	**-0.415**

The same conclusions a last time can be found.

The reader can reasonably ask "Why do you believe *tool2* and not SPSS?" The answer is with *tool2* all the descriptive statistics keep on going up but they do not with SPSS.

The main conclusion is that *tool2* and SPSS use the same formulae, until SPSS 'runs out of steam'.

16.3 Does *tool2* calculate the MEDIAN calculated properly?

The median is defined as the 'middle' value. So, for marks, the median mark is the middle mark. It all depends on what 'middle' means. Let us try a small example. If we have

Mark	**How Many Get That Mark**
0	5
1	4

Then there are 5 0s and 4 1s here, i.e. 000001111

This is the middle value → and so the **median** mark is **0**.

However, if the data is

Mark	**How Many Get That Mark**
0	6
1	4

There is no 'middle' value of 00000 01111 (because the population is even). The 'middle' is between the two noughts and so the mean is taken, thus the **median** mark is **0**. In this case the mean is 0, but it does not have to always be, as the following data shows.

Mark	**How Many Get That Mark**
0	4
1	4

Then the data means 000 01 111. The middle mark is the mean of 0 and 1 i.e. 1/2, a mark that does not occur in the data (but it might if the two marks are further apart).

Thus, if the population size is odd then the median mark is the mark that corresponds to **n**/2 (in C) and if the population is even the median mark is the mean of the marks that correspond to the mark we are looking at and the next mark. So, if we are looking at the mark 0 and the next mark is 1, then the mean of the two marks is (0 +1)/2 = 0.5 and thus the **median** mark is **0.5.**

If Scientific Notation is used anywhere in Excel™, then we cannot work out if the population is even or odd, because Scientific Notation uses 10 to the something which is always even.

We work out the cumulative frequency, and our data now looks like

Mark	How Many Get That Mark	Cumulative Frequency
0	5	5
1	4	9

or

Mark	How Many Get That Mark	Cumulative Frequency
0	6	6
1	4	10

or

Mark	How Many Get That Mark	Cumulative Frequency
0	4	4
1	4	8

If this looks a 'small' amount of data we only have to consider population sizes that are:

- even, or
- odd

Of these, the only difficulty is with even populations, where there is no 'middle' mark, and then we take the mean of the values on either side of the 'middle'. If the values the marks on either side of the 'middle' are the same then the median is one of them. If the values the marks on either side of the 'middle' are NOT the same then the median mark is the mean of the marks on either side of the 'middle'. Our data reflects these conditions.

So, *tool2* appears to calculate the Median properly.

16.4 Does *tool2* calculate the MODE properly?

The first approximation of mode is the mark that occurs most frequently. Our data shows the frequency that a mark occurs, so as a 1st approximation we need to find the mark where howMany[i] is greatest. As a 2nd approximation when there are 2 equal most frequently occurring values (then we have a bi-modal distribution) and the mode is the mean of where the 2 values occur. Our data is such that we often have uni-modal or bi-modal distributions. We rarely have tri or more modal distributions.

tool2 only works out the mode for any distribution, and tells you how many modes there are. SPSS does that for a single column, but not for a distribution (that we can find out) when **Frequencies** are used.

In a loop *tool2* goes through all the data and reads in:

a) a mark (**M**),
b) the number obtaining that mark ($\mathbf{Y_M}$), and puts them both into arrays.

If $\mathbf{Y_M}$ is greater than the maximum then it becomes the maximum. In another loop *tool2* goes through the data again and if $\mathbf{Y_M}$ equals the maximum then the number of modes is incremented and remembered:

a) what mark the first mode occurred at,
b) what mark the last mode occurred at.

At the end:

a) the number of modes,
b) the mode mark = (mark where last mode occurred - mark where 1st mode occurred)/(number of modes), are output to the debug window.

If the number of modes is 1 and $\mathbf{Y_M}$ is equal to the maximum then the mode mark is the mark we have just read in.

Before that, if the number of modes is 2 and $\mathbf{Y_M}$ is equal to the maximum then the mode mark is the mean of the mode and the mark we have just read in.

For symmetric distributions the mean mark, the mode mark and the median mark should all be co-incident.

This symmetry occurs when all stems are answered (and X = 1), and so we can do a check that mode and median are calculated properly by *tool2* for quizzes where all stems are answered (and X = 1), where Scientific Notation is not used by Excel™.

This check is positive, where it can be done. For example, for a quiz where X = 1 (i.e. a Yes/No quiz) where all stems are answered, where **a** = 0 and **n** = 40. The mean mark = 20.0 (which agrees with the theoretical) and so are the Median and the Mode.

```
                    DESCRIPTIVE STATISTICS
Population
   Mean Pop=20.000                  Median Pop=20.000         Mode Pop=20.000
   Var Pop.=10.000 SD. Pop=3.162           Skew Pop=0.000   Kur Pop=-0.050
```

When X = 7 (i.e. a 3-option Multiple Yes/No quiz) where all stems are answered with **a** = 0 i.e. the score is [1, 0], then the distribution of marks is *not* symmetric. There is only 1 way of getting the answers to all stems right and there are many ways of getting the answer to all stems wrong. For example, for 40 stems we have

```
The number of lines read in is=41
Sum of all ways=POPULATION=1329227995784911150537324190635130880
Marks range from 0.000 to 40.000 Hence Range=40.000
Number of MODES=1

                    DESCRIPTIVE STATISTICS
First Mode occurs at mark 5.000 Last  Mode occurs at Mark 0.000Number of modes=1
 Mode=5.000
Population
   Mean Pop=5.000                   Median Pop=5.000          Mode Pop=5.000
   Var Pop.=4.375 SD. Pop=2.092 Skew Pop=0.359  Kur Pop=0.079
```

Here, the skewness is not 0 and so we do not have a symmetric distribution with a uni-modal distribution.

We now, try *tool2* for a bi-modal distribution with the maximum marks next to each other (and hence the mode is not a mark that occurs)

```
The number of lines read in is=11
Sum of all ways=POPULATION=1167464
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=2

                     DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 3.000Number of modes=
Mode=2.500
Population
   Mean Pop=2.693                 Median Pop=3.000          Mode Pop=2.500
   Var Pop.=2.085 SD. Pop=1.444 Skew Pop=0.215  Kur Pop=-0.555
```

You will notice that, here, we have some debugging information.

We now, try *tool2* for a bi-modal distribution with the maximum marks, far apart, and hence the mode might be a mark that occurs.

```
The number of lines read in is=11
Sum of all ways=POPULATION=1404395
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=2

                     DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 6.000Number of modes=2
Mode=4.000
Population
   Mean Pop=3.352                 Median Pop=3.000          Mode Pop=4.000
   Var Pop.=3.507 SD. Pop=1.873 Skew Pop=0.096  Kur Pop=-1.209
```

We now, try *tool2* for a tri-modal distribution

```
The number of lines read in is=11
Sum of all ways=POPULATION=1451470
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=3

                     DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 6.000Number of modes=3
Mode=2.667
Population
   Mean Pop=3.340                 Median Pop=3.000          Mode Pop=2.667
   Var Pop.=3.398 SD. Pop=1.843 Skew Pop=0.116  Kur Pop=-1.151
```

So, *tool2* does better than SPSS which tells you the mode for a uni-modal distribution and then says "Multiple modes exist. The smallest value is shown".

The tool, *tool2* gives:

a) how many modes there are,
b) the value of the mode.

16.5 Does *tool2* calculate the proportion of passes properly?

The initialisation that we do includes

```
fortyFirst=false;
```

and our coding includes

```
if (mark[i] >=40.0*maximumMark/100.0 && !fortyFirst)
                {
                        fortyFirst=true;
                        cout << "    Percentage of 40% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                }
```

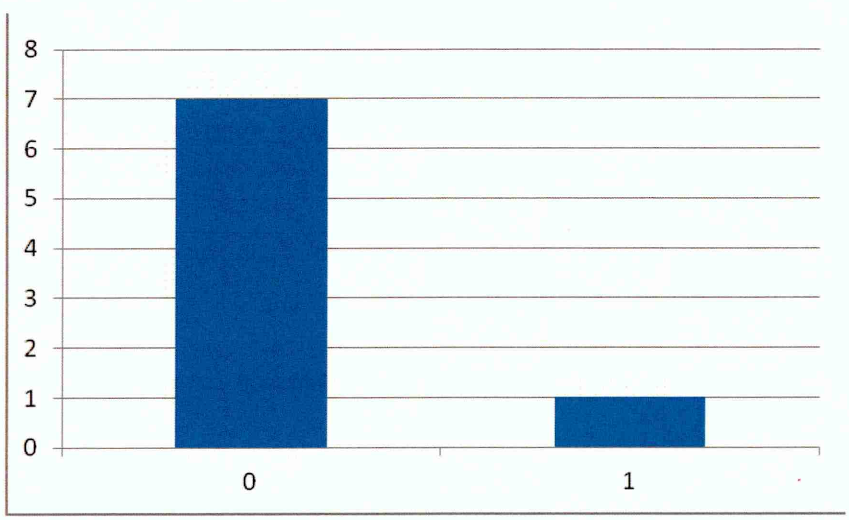
This coding assumes that 40% is the raised pass mark.

When the mark that we are considering is equal to or greater than 40% of the maximum mark then the proportion of marks that are equal to or greater than 40% is (all of them -the proportion to date).

We do this calculation for 40%, 50%, 70%, 71%, 72%, 73%, 74% and 75%.

To check our calculations we do the checks for a few cases.

For X = 7, **a** = 0, **n** = 1 where ALL stems are answered we have



The data in file **Output.txt** is

0 7

1 1

The output from *tool2* is

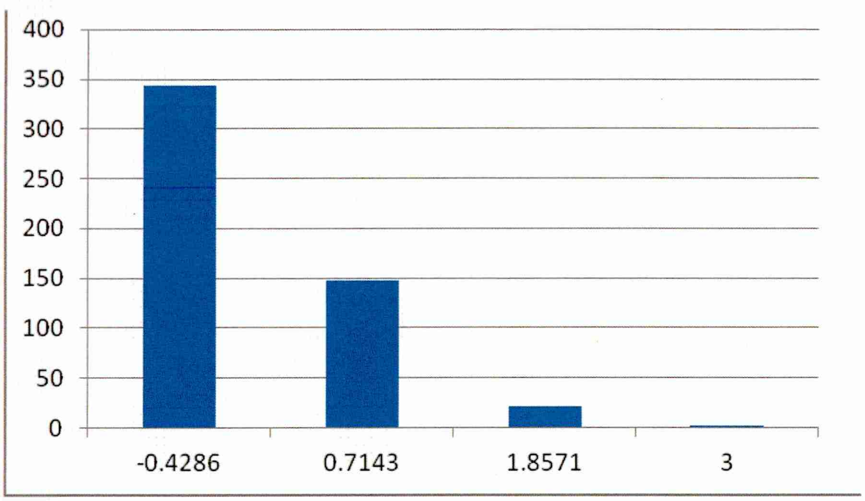
```
Trying to READ from file F:\MyPhD2\MyPhD\Output.txt
     Percentage of 40% or above=12.500 When Mark=1.000
     Percentage of 50% or above=12.500 When Mark=1.000
     Percentage of 70% or above=12.500 When Mark=1.000
     Percentage of 71% or above=12.500 When Mark=1.000
     Percentage of 72% or above=12.500 When Mark=1.000
     Percentage of 73% or above=12.500 When Mark=1.000
     Percentage of 74% or above=12.500 When Mark=1.000
     Percentage of 75% or above=12.500 When Mark=1.000
The number of lines read in is=2
Sum of all ways=POPULATION=8
Marks range from 0.000 to 1.000 Hence Range=1.000
Number of MODES=1

                          DESCRIPTIVE STATISTICS
Population
     Mean Pop=0.125                      Median Pop=0.000          Mode Pop=0.000
     Var Pop.=0.109 SD. Pop=0.331 Skew Pop=2.268   Kur Pop=3.143
```

With the mean mark being 1/8 = 0.125

The marks go from 0 to 1 (as shown above).

At mark 0, we are not above 40%. At mark 1, we are above (or equal to) 40% and there is 12.5% of the area left. At mark 1, we are also above (or equal to) 50% and there is 12.5% of the area left.

Let us try another, example of X = 7 **n** = 3, with all stems answered, **a** = -1/X



The data in **Output.txt** is

-0.4286	343
0.7143	147
1.8571	21
3	1

This is what *tool2* gives

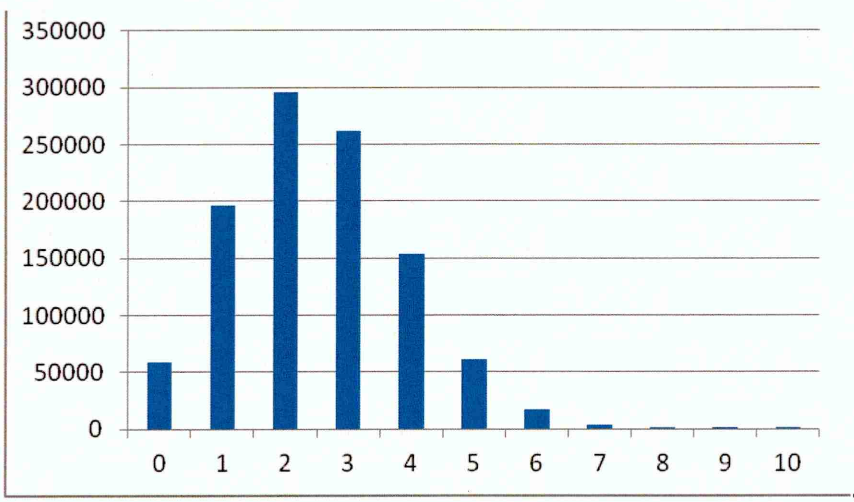
```
Trying to READ from file F:\MyPhD2\MyPhD\Output.txt
   Percentage of 40% or above=4.297 When Mark=1.857
   Percentage of 50% or above=4.297 When Mark=1.857
   Percentage of 70% or above=0.195 When Mark=3.000
   Percentage of 71% or above=0.195 When Mark=3.000
   Percentage of 72% or above=0.195 When Mark=3.000
   Percentage of 73% or above=0.195 When Mark=3.000
   Percentage of 74% or above=0.195 When Mark=3.000
   Percentage of 75% or above=0.195 When Mark=3.000
The number of lines read in is=5
Sum of all ways=POPULATION=512
Marks range from -92559631349317830000000000000000000000000000000000000000000000
.000 to 3.000 Hence Range=9255963134931783000000000000000000000000000000000000000
00000000.000
Number of MODES=1

                DESCRIPTIVE STATISTICS
Population
   Mean Pop=0.047                Median Pop=-0.429          Mode Pop=-0.429
   Var Pop.=0.431 SD. Pop=0.656 Skew Pop=1.086   Kur Pop=0.665
```

Population = 343 +147 +21 +1 = 512.

When the mark 1.8571 is considered, the mark is above or equal 40*3/100 = 1.2 and then there are 22 of 512 marks left i.e. 100*22/512 = 4.297

When mark 1.8571 is considered we are equal to or above 40% of the maximum mark (3) and then there is 4.297% of the 'area' of marks left. So, the proportion of the marks that are above (or equal to) 40% of the maximum mark is 4.297%. There is no point in raising the pass mark beyond 70%.

Let us try another example. X = 3 i.e. a 4-option Multiple Choice quiz, with **n** = 10 and **a** = 0 where ALL stems are answered.

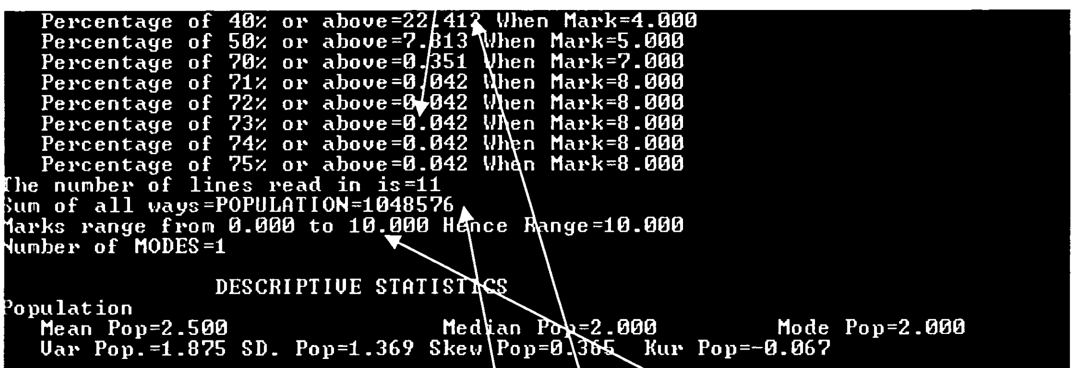


The data in **Output.txt** is

0	59049
1	196830
2	295245
3	262440
4	153090
5	61236
6	17010
7	3240
8	405
9	30
10	1

There are 11 lines here

The output from *tool2* is



```
   Percentage of 40% or above=22.412 When Mark=4.000
   Percentage of 50% or above=7.813 When Mark=5.000
   Percentage of 70% or above=0.351 When Mark=7.000
   Percentage of 71% or above=0.042 When Mark=8.000
   Percentage of 72% or above=0.042 When Mark=8.000
   Percentage of 73% or above=0.042 When Mark=8.000
   Percentage of 74% or above=0.042 When Mark=8.000
   Percentage of 75% or above=0.042 When Mark=8.000
The number of lines read in is=11
Sum of all ways=POPULATION=1048576
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=1

                 DESCRIPTIVE STATISTICS
Population
   Mean Pop=2.500                 Median Pop=2.000          Mode Pop=2.000
   Var Pop.=1.875 SD. Pop=1.369 Skew Pop=0.365  Kur Pop=-0.067
```

The population = 59049 + 196830 + 295245 + 262440 + 153090 + 61236 + 17010 + 3240 + 405 + 30 +1 = 1048576

When the mark 4 is considered, it equal to 40% of the maximum mark (10) and then the percentage of marks left is 100*(153090 + 61236 + 17010 + 3240 + 405 + 30 + 1)/1048576 = 235012/1048576 = 22.412.

Similarly, when the mark 8 is considered the percentage of marks left = 100*436/1048576 = 0.042

So, for the few cases we have tried, *tool2* does calculate correctly the proportion that pass.

16.6 Conclusions.

The tool, *tool2* does:

- use the correct formulae,
- calculate MEDIAN properly,
- calculate MODE properly,
- calculate correctly the proportion that pass for various pass marks.

Appendix 17. Loading the LEDA routines and integrating with Visual Studio 10.

17.1 Introduction.

The abbreviation VS will be used throughout this appendix. VS stands for **V**isual **S**tudio 10. VS can run programs written in the programming language C, but C normally has an upper limit for integers and we go past this upper limit. The LEDA routines (written by **A**lgorithmic **S**olutions(AS) of Germany) allow integers of any size to be used, but this involves a 4 stage process

1. Installing VS.
2. Installing the LEDA routine.
3. Configure VS to work with LEDA.
4. Setting up system variables.

We shall describe, here, steps 2, 3 and 4 and assume 1 has been done.

Step 2 is relatively pain free, but the step 3 is very long and hence error prone.

The numbers and instructions used in step 3 are an amalgam of those provided by AS and those provided by us.

Appendix 18. Difficulties we had.

18.1 Introduction.

We describe the difficulties we encountered, which can be classed into:

- Difficulties with installing **V**isual **S**tudio (VS).
- Difficulties with the LEDA routines.
- Difficulties with making VS work with the LEDA routines.
- Difficulties with understanding how the programming language C works.
- Difficulties with understanding some Statistical concepts.
- Difficulties with SPSS.
- Difficulties with the limits of Excel™.
- Difficulties with the tools we wrote.

Some of the solutions involved more than 1 difficulty. For example, a new version of VS was loaded and that meant that a new version of the LEDA routines had to waited for (and then downloaded to a 'safe' place) and the programs that were written for the older versions of VS and LEDA had to be re-written to work with the newer versions.

18.2 Difficulties with installing Visual Studio (VS).

We wanted to use the latest version of VS and so the technicians were asked to install VS 8. They could not

> *"Still failed, I'm afraid, so I'll have to approach it from another angle..."*
>
> Flowers (2010).

Because I had administrator rights I could install VS 8, which I did onto **C:** drive and wrote programs in the programming language C that used the LEDA routines inside VS **8**.

Without me being told, a version of VS viz. VS **10**, was later installed and then the programs that had worked in VS 8 had to be re-written.

So, this was an installation problem.

18.3 Difficulties with the LEDA routines.

We wanted to use the latest version of the LEDA routines, which, at the time was a version that worked inside VS **8.** We also did not want to pay anything for the privilege of using the LEDA routines that would run on our machine. The free version, which has no support, depended on what Operating System (OS) was used, a fact that we did not know. The LEDA free edition talked about i386, FC, Sparc Solaris and msc 8 and 9 (32 bit and 64 bit) and multithread safe. The newer version only talks about Linux OS and Windows OS (and not about Sparc Solaris OS). The technicians were asked whether we used 32 bits or 64, and after a long conversation it was thought that 32 bits was relevant (whether multithread was safe was unclear). When **A**lgorithmic **S**olutions (AS) of Germany were asked what version we should download we got a very 'frosty' reply,

> *"Are you writing a book or a blog about what you think are bad product descriptions or cryptic websites and you would like to engage us into some funny conversation that you can publish there?"*
>
> Support Desk (2010).

But, they eventually replied that:

- **FC** stood for **F**edora **C**ore
- **i386** implies 32 bits
- **msc** stood for **M**icro**s**oft **C**orporation

and so, we downloaded **LEDA-6.6, i386, msc 8 (.NET 2005), 32 bit** to **C:\Algorithmic Solutions**.

The people around me complained that the system was running slowly and the technicians thought that was due to us using an old image. So, a new image was put onto **C:** that used VS 10 (despite the technicians saying they could not download VS 8), and that had 3 effects. Everything that was on **C:** was 'wiped'. A new version of the LEDA routines that worked in VS10 (when they had been written by AS) had to be downloaded into a 'safe' place, and it was decided that

D: drive was 'safe' i.e. would not be 'wiped'. The programs that I had written in the programming language C for VS 8 had to be re-written to work in VS 10.

So, the problems break down into:

- *What* version of LEDA should be downloaded, despite problems with AS?
- *Where* should the LEDA routines be downloaded to?
- Anew version of the LEDA routines, might cause the programs that use them to be re-written.

A good feature of the LEDA routines was that they included the function factorial(m)

18.4 Difficulties with making VS work with the LEDA routines.

The difficulties of making VS work with the LEDA routines can be broken down into:

- configuring VS and telling VS where the LEDA routines are,
- telling the system what the PATH, INCLUDE and LIB environment variables are.

We attempted to do this in Appendix 17.

Configuring VS means telling it things like

1. the name of a project,
2. where is the solution,
3. what sort of application we are writing,
4. what sort of source file we are going to write (anything ending in .cpp is a C++ file),
5. what the source file is called,
6. how line numbers are included in the source file. This way we can understand, easily, the messages VS outputs. We only have to add line numbers *once* per project,
7. the source code,
8. what the compiler flag is,
9. where to find the LEDA dll,
10. where to find the LEDA library,
11. what directory to use for the #includes (this involves telling VS where the LEDA includes are),
12. what directory to use for the LEDA libraries.

This is a very error-prone process, because it is so long, and in order to reduce the number of errors, we wrote Appendix 17. But, there is still the difficulty of knowing whether to take out the _DEBUG statement and if there is an error we take it out

> *"The compiler defines _DEBUG when you specify the /MTd or /Mdd option. These options specify debug versions of the C runtime library."*
> Masur (2011).

Telling the system what 3 environment variables are is not a hard process and only has to be done *once*.

In order to build and run the source file we press Ctrl +F5.

So, the difficulties associated with VS are in configuring it to work with LEDA.

18.5 Difficulties with understanding how the programming language C works.

I had extensive knowledge of computer languages, but did not know C. The trouble with learning C which runs inside VS using the LEDA routines, was knowing whether an error:

a) was due to a lack of understanding how C works,
b) or due to a poor understanding of how VS works,
c) or due to a lack of understanding of how the LEDA routines work.

Words like namespace and define are new to me, although I am used to the concept of a constant (that define uses), and the concept of avoidance of name clashes (that namespace uses). include is not new to me but what is in what libraries sometimes baffles me, and so what to include is of concern. The idea of libraries is not new to me but getting C to work with the included libraries caused me great problems. So much so, that asking Algorithmic Solutions for help often implied that it was a C problem, not a LEDA problem

"The problems you have are not related to any problems with the LEDA package but to the correct use of the developer environment from Microsoft, the correct use of LEDA and correct C++ programming."
Support Desk Algorithmic Solutions (2010).

In the end they got completely fed up and refused any help at all

"We would like to kindly inform you, that the LEDA free edition comes without support."
Support Desk Algorithmic Solutions (2010).

18.5.1 C did not work as I expected it to.

The programming language C did not do what was expected of it in a number of ways

- in a statement like c = a/b; if a and b are both integers then this statement might give the wrong answer, especially if b is bigger than a, and thus c is given the value 0 (no matter what the type of c is). In the expression a/b if both sides of the operator / are integers then the operation yields an integer,
- this means that all divisions have to be looked at,
- in a loop like

```
for (i=1; i< k; i++)
        {
        }
```

  Is this repeat loop or a while loop? The basic question is "What is the minimum number of times that this loop can be executed?" The loop starts off with i=1 and then tests whether i is less than k, and then increments i. An even more basic question "Is the test done at the beginning of the loop or at the end?" So, if k is less than 1, how many times is this loop executed? Is it 0 or 1? I thought that this loop would be executed a minimum of 1 time. I was wrong.

  In Algol 60 a for-loop is executed at least once and in FORTRAN a do-loop is executed at least once.

- in an initialisation like maxForWorkingOutMedian=9999999999; (this has ten 9s). If maxForWorkingOutMedian is declared as of type integer (a LEDA type) then there is an error. A way round this is to initialise maxForWorkingOutMedian to nine 9s and then add to it e.g.

```
for (int i =1; i<5; i++)
  maxForWorkingOutMedian =10* maxForWorkingOutMedian +9;
```

- the main program is an integer function called **main** e.g.

```
int main()
{
```

}

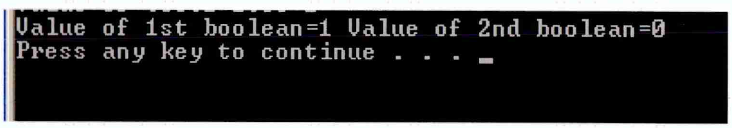
- we can declare variables to be of type bool (i.e. boolean) but booleans are held as integers, with 0 representing false and 1 representing true.

"When boolean values are evaluated, they actually don't evaluate to true or false. They evaluate to the numbers 0 (false) or 1 (true). Consequently, when we print their values with cout, it prints 0 for false, and 1 for true:"
LearnCpp.com (2012).

For example

```
int main ()
  {
    //Declarations

         bool firstBoolean,  secondBoolean;

    //Initialisation

         firstBoolean=true;
         secondBoolean=false;

    //Output
         cout <<  "Value of 1st boolean=" << firstBoolean;
         cout << " Value of 2nd boolean=" << secondBoolean << endl;
  }
```

and then in the debug window we should see



18.6 Difficulties of learning some statistical concepts.

I had to learn many statistical concepts that include

- What is a probability density function?
- What do *discrete* and *continuous* mean?
- What is a *distribution*?
- What does our data mean?
- What does *population* mean?
- What is a *census*?
- What is a *sample*?
- What does a sample have to be?
- What is *expectation*?
- What is *mean*?
- What is *mode*?
- What is *median*?
- What is *variance*?

- What is *standard deviation*?
- What is *skewness*?
- What is *kurtosis*?
- What does *symmetric* mean?
- What is the *moment* about the mean?
- What are *descriptive statistics*?
- What are *parametric* and *non-parametric* statistics?

18.6.1 What is a probability density function?

A probability density function of a *continuous* variable is the function that gives the probability of a value occurring.

18.6.2 What do discrete and continuous mean?

Variables come in two types discrete and continuous.

A discrete variable (like Mark) takes on a finite number of values.

A continuous variable takes on an infinite number of values.

18.7 What is a distribution?

A distribution has an x-value and a y-value.

In our case the x-value is a Mark, and the y-value is number of times that mark occurs.

18.7.1 What does our data mean?

The data, *before* a Pivot, might look like this

Mark	**How many get that mark that way**
0.111	278
0.222	3010
0.111	56

So, the data is *discrete*.

The 1st column is a mark and the 2nd column is the number obtaining that mark *that way*. So, 278 obtained the mark 0.111 *that way* and 56 also obtained the same mark in another way: explaining why a Pivot is necessary. How the mark was obtained is not recorded.

So, the 2nd column is the frequency of obtaining a mark, *that way*.

In order to explain the words *that way*, we will look at another example

0	265
1	304
0	27

and then *after* a Pivot, if we see

0	292
1	304

Then we can say there were 292 occurrences of the mark 0, and 304 occurrences of the mark 1. We can only say this after a Pivot is done.

So, the probability of the mark 0 occurring is 292/596 = 0.4899 and the probability of the mark 1 occurring is 304/596 = 0.5101. Their sum is 1.

The probability density function of the marks 0 and 1 occurring is (0.4899, 0.5101)

Excel™ can draw this distribution.

18.7.2 What does population mean?

A *population* is the whole set. In our case the *population* is all the marks that can be obtained in the quiz.

18.7.3 What is a census?

In our case a census looks at *all* the marks.

18.7.4 What is a sample?

A *sample* is a subset of the population. There are many types of sample e.g. random, systematic, convenience and snowball. For example, a systematic sample might be every 5th one. In our case a *sample* is some of the marks that can be obtained in the quiz.

18.7.5 What does a sample have to be?

In order to generalise about the *sample*, it must **represent the population**. For example, if the population has a characteristic that the number of marks possible with a quiz consisting of m stems is 7 times the number of marks with a quiz consisting of (m - 1) stems, and a sample does not have this characteristic, then the sample does not represent the population. So, a sample of the marks of a quiz that is marked by CBM might not represent all the marks.

18.7.6 What is expectation?

The *expected* mark for a discrete distribution is defined as

$\sum$ Mark *(Probability of that mark occurring)

Generalising, where Mark = x

$$E(x) = \sum x.\ p(x)$$

18.7.7 What is mean?

The *mean* mark for a discrete distribution is defined as

$\sum$ (Mark)/(How many marks there are)

It can be shown that the mean mark and the expected mark are the same.

The mean for a sample is denoted by $\overline{x}$ and the mean for a census is denoted by μ.

18.7.8 What is mode?

The *mode* mark can be defined as the most frequently occurring mark, and in SPSS the mode is obtained under **Frequencies**.

But then we have to ask "What happens if there are 2 equal most frequently occurring marks?" In which case we have a bi-modal distribution. But if there is only one most frequently occurring mark we have a uni-modal distribution. In the case of a bi-modal distribution of marks obtained by a quiz then the mode mark is the mean of the two mode marks.

We then ask "What happens when there are more than 2 modes?" and then in our case, the mode is (the highest mode mark - the lowest mode mark)/(the number of modes). The tossing of an un-biased die is an example of a 6-mode distribution.

Finally we ask "What happens when mode does not exist?" as with the function tan(x). In that case, mode should not be calculated. In our case we mostly have uni and bi-modal distributions and very occasionally a tri-modal distribution, and so *tool2* always calculates mode. After a Pivot is done our data is

Mark **The number of times that mark occurs in the data**

So, the most frequently occurring mark is the *maximum* of the number of times that mark occurs in the data.

18.7.9 What is the median?

The *median* mark is the 'middle' mark. This has the difficulty of "what does **middle** mean?" It is best demonstrated by an example. Say, that our data, after a Pivot is

Mark	**Number of times that mark occurs**
0	5
1	4

Then there are 5 marks of 0 and 4 marks of 1

So, the marks are
0000 0 1111
and the 'middle' mark is 0.

If, however, the data is

Mark	**Number of times that mark occurs**
0	5
1	5

Then there are 5 marks of 0 and 5 marks of 1

So, the marks are

0000 01 1111

and the 'middle' mark is between the 0 and the 1, and so we take the mean and get 0.5 (a mark that is not possible, in this data).

So, the definition of the median mark is

If the number of data points (n) is **odd** then take the mark that corresponds to n/2 (in C), as long as $n \geq 2$ (after a Pivot is done).

If the number of data points (n) is **even** then the median mark = (mean of the marks that correspond to most frequently occurring values) (after a Pivot is done).

(Here, n is NOT the number of stems, but the number of data points)

This means that the evenness or oddness of the number of data points has to be calculated. When Excel™ uses Scientific Notation the evenness or oddness cannot be calculated, because the number that get a mark is always even (e.g. 1.3 E^{27}) which is a multiple of 10 and thus even.

A natural question is "What are the advantages and disadvantages of mean, mode and median?" We can generalise this question a bit further by noting that mean, mode and median are all measures of the 'middle' of the data. There are many measures of the 'middle' of the data that include, arithmetic mean (the one we use), geometric mean, harmonic mean, weighted mean, truncated mean, midrange, midhinge, trimean, Winsorized mean, mode, median and geometric median. These are some of the measures of Central Tendency and so the question becomes "What are pros and cons of the measures of Central Tendency?" The answer is left to the reader. But, one of the answers might be "Mean is distorted by high/low values". For example, the annual salaries of a sample of people might be £12,000, £26,000, £1,000,000. The mean salary of this sample is (12,000 + 26,000 +1,000,000)/3 = £346,000 but in most of this sample people receive an annual salary much less than this. So, mean, here, is not a very good measure. Perhaps the median salary of £26,000 is a better measure. Perhaps, the *outlier* of £1,000,000 should be ignored, and that is where truncated mean or Winsorized mean come in. This opens a whole

Pandora's box of what measures of Central Tendency are most appropriate to us and should outliers be ignored. We will not open this Pandora's box.

18.7.10 What is variance?

Variance measures the spread of a distribution about its mean $= \sum x\ p(x)$

18.7.11 What is standard deviation?

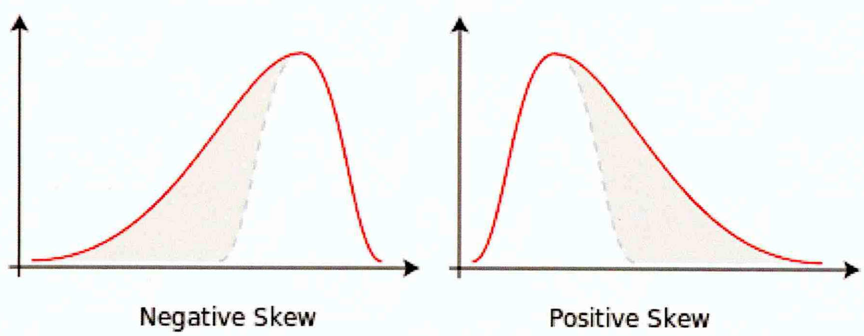
***S**tandard **D**eviation* (S.D.) of a distribution is the positive square root of the variance of the distribution.

The symbol for the S.D. of a census is **σ** and the symbol for the S.D. of a sample is **s**.

A natural question is "Why do you need both variance and S.D.?" and an answer is that you do *not* need both.

18.7.12 What is skewness?

Skewness measures whether a distribution has a long tail to the left or right.



18.7.13 What is kurtosis?

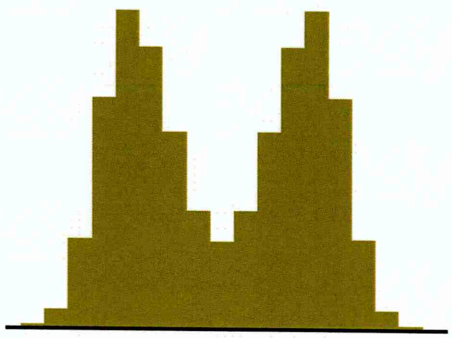
Kurtosis measures how 'peaky' the data is.

Distributions with a negative value of kurtosis are called *platykurtic*, and those with a positive value of kurtosis are called *leptokurtic.*

18.7.14 What does symmetric mean?

The distribution is not skewed (skewness = 0) when a distribution is symmetric. This is a necessary condition but not sufficient.

> *"A situation in which the values of variables occur at regular frequencies, and the mean, median and mode occur at the same point"*
>
> Investopedia (2011).



This is an image of bi-modal, symmetric, discrete distribution.

18.7.15 What is moment about the mean?

The p^{th} moment about the mean for a census of a discrete distribution is

$$\sum (x_i - \mu)^p p(x)$$

Thus, *variance* is the 2^{nd} moment about the mean i.e. where p = 2.
and *skewness* is the 3^{rd} (moment about the mean)/σ^3.
and *kurtosis* is the 4^{th} (moment about the mean)/σ^4.
and **excess** *kurtosis* is kurtosis -3

18.7.16 What are descriptive statistics?

Descriptive statistics are measures used to describe a distribution, without drawing it. For example, we talk about *range* of the x-axis, and in our case it means what is the range of marks that we are considering e.g. from 0 to 100.

In this thesis we both draw a distribution and superimpose the descriptive statistics of the distribution by hand.

The descriptive statistics include, range, minimum x-value, maximum x-value, mean, mode, median, variance, standard deviation, skewness and kurtosis.

18.7.17 What are parametric and non-parametric statistics?

Parametric statistics assume that the data has parameters and non-parametric statistics do not make any assumptions about the data. Typically, parametric statistics assume that the data is Normal.

We do not assume that our data has any parameters and so we can be said to be using non-parametric statistics.

18.8 Difficulties of SPSS

SPSS treats the data as a *sample*, and not as a census, and hence uses **estimates** of its statistics which are inaccurate for small populations. For example,

App. 18 Table 4. Samples and estimates.

	Estimates. **Formulae that SPSS uses.** **n=size of sample**	**Census** **Formulae** **N=size of population**
Variance	VarSam= $\frac{\sum (x_i - \bar{x})^2}{n-1}$	VarPop=∑X P(x) = $\frac{\sum (x_i - \mu)^2}{N}$
Standard Deviation	s=$\sqrt{VarSam}$	σ=$\sqrt{VarPop}$

where n is the size of the sample and N is the size of the population. So, the variance of the sample is divided by (n -1) and the variance of the population is divided by N. This leads to inaccuracies when n is 'small'. For example, the formulae 'settle down' when the number of stems is about 9 for Yes/No quizzes where all stems are answered (see Appendix 16).

We had to find out what formulae SPSS uses for its descriptive statistics and then see what the formulae should be used for a census. The following table shows the estimates that SPSS uses and the formulae that should be used for a census.

App. 18 Table 2. Table of descriptive statistics, for estimates and measures.

	Estimates. **Formulae that SPSS uses.** **n=size of sample**	**Census** **Formulae** **N=size of population**
Mean	$\bar{x}=\frac{\sum x_i}{n}$	$\mu=\frac{\sum x_i}{N}=\sum x_i\, p(x_i)$
Variance	VarSam= $\frac{\sum (x_i-\bar{x})^2}{n-1}$	VarPop= $\frac{\sum (x_i-\mu)^2}{N}$
Standard Deviation	$s=\sqrt{VarSam}$	$\sigma=\sqrt{VarPop}$
Skewness	$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^3}{N\sigma^3}$
Kurtosis	$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^4}-\frac{3(n-1)^2}{(n-2)(n-3)}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^4}{N\sigma^4}-3$

Another difficulty we had was that we could not find out how SPSS calculates its descriptive statistics for a distribution (i.e. 2 columns). We could only find out how SPSS calculates its descriptive statistics for1 column. Our data consisted of 2 columns, a mark and the number that got that mark, and we ignored the mark, so that our data, finally, was in 1 column and we could compare estimate formulae with census formulae. We found that the results were different when n was 'small' and when SPSS could not handle very large numbers.

Another difficulty we had was that the textbooks appeared to be wrong about Kurtosis. Field (2005, p11) and an SPSS manual (Introduction to PASW 2010, p14-11) say

> *" in a normal distribution the value of skew and kurtosis are 0..",*
> *"...normal curve (for which the kurtosis is zero)."*

But, the kurtosis for a Normal distribution is 3.

> *"The kurtosis for a standard normal distribution is* ***three.****"*
> Engineering Statistics Handbook (2010).

What Field and an SPSS manual mean is that for a normal distribution the ***excess*** kurtosis is 0, which they both continue to call kurtosis, and so will we.

18.8.1 The limits of SPSS

The data for SPSS is coming from the file **Output.txt** and when the numbers that it uses are 'too big' SPSS calculates the wrong answers for its descriptive statistics (see Appendix 16). The reason for this is that SPSS can only handle numbers in a file that occupy up to 40 characters (even if the variable is of type Scientific notation). That is why you see E39 such a lot where mean is concerned. The square of this is about E78, and so the variance is about E78 (see Appendix 16).

So, we cannot use SPSS because:

- it calculates the descriptive statistics wrongly, for a situation where a division by n results in an answer that is significantly different from the answer when a division is done by (n - 1). This is because SPSS treats the data as a sample, and thus uses estimates for its descriptive statistics.

- It calculates the descriptive statistics wrongly, for a situation where the data comes from a file and the numbers that the data represent occupy more than 40 characters.

- We do not know how to receive the descriptive statistics of a distribution. We do not know how to obtain the descriptive statistics of 2 columns (like the variance)

- The data that SPSS uses does not mean the same thing as we mean. For example, the data might be

Mark	
0	292
1	304
0	3

We mean that the mark 0 appears 295 times (after a Pivot), and if we see in SPSS, how many times the mark 0 appears SPSS says 2 (when we look at the column called **Mark**)

18.9 Difficulty with the limits of Excel™

The limits of Excel™ are easy to find as they are published (http://office.microsoft.com/en-001/excel-help/excel-specifications-and-limits-HP010073849.aspx), and the ones that interest us include

- the number of rows that Excel™ could deal with is (about 1 million)

  We never came across this restriction, but when we loosen the restriction on the types of stems in a quiz (e.g. have Yes/No stems and 4-option MCQ stems in the same quiz), we might.

- the maximum length of a cell (255 characters)

Excel™, like SPSS, uses Scientific Notation (when it has to), and so the evenness or oddness of a number cannot be calculated when Scientific Notation is used. Further, if we have a number that occupies more than 255 characters then a Pivot cannot be done. If we have a number that occupies, say, 300 characters, then the first 255 characters are put in cell A26 (say) and the characters of the number from 256 to 300 are put in cell A27. The number that corresponds to characters 1 to 255 in A26 is not the number that corresponds to the number in A26 and A27. If we ask for a Pivot to be done on the number in A26 we get the wrong answer because the maximum number that can be dealt with in a cell (about 1 E+308)

18.9.1 Getting Excel™ to import a tab-delimited text file.

We wrote Appendix 9 to deal with getting Excel™ to *import* a tab-delimited text file.

18.9.2 Getting Excel™ to do a Pivot

The necessity for a Pivot is that the data before a Pivot is

Mark **Number getting that mark that way**

and the *same* mark might be obtained in 2, or more, ways.

After a Pivot, the data is now

Mark **Number getting that mark**

and so after the Pivot:

- the marks increase, and there is only 1 mark 0 (say)
- the number getting that mark might involve numerous additions

Appendix 10 shows how Excel™ does a *Pivot*.

18.9.3 Getting Excel™ to export a tab-delimited text file

We wrote Appendix 11 to deal with getting Excel™ to *export* a tab-delimited text file.

18.10 Difficulties with the tools we wrote

The tool, *tool1* calculated a mark and how many got that mark that way, and then outputted to a tab-delimited text file (see Appendices 12, 13 and 14) for Excel™ to do a Pivot on this data.

Excel™ imported this text file (see Appendix 9) did a Pivot (see Appendix 10) drew a distribution and then exported to another tab-delimited text file (see Appendix 11). The drawing of the distribution was copied to Word™ by hand.

The tool, *tool2* then imported this text file and calculates some descriptive and other statistics (see Appendices 15 and 16). These statistics were then superimposed on the distribution, by hand.

Both, a mark and how many get that mark that way are held in arrays, and there is a limit of about 86,000 on the index of an array in the programming language C. But arrays do not need to be used, if we are willing to calculate a mark again. Thus there does not seem to be a limit on *tool1*.

The limits of Excel™ impose limits on *tool2*. Because Excel™ can only handle about 1 million rows, if we want to use more than 1 million rows we have to use several intermediate files, between *tool1* and Excel™. This limitation was never encountered in our case, but it might be if the quiz can contain a mixture of stem types (e.g. Yes/No stems and 4-option MCQs). Excel™ cannot handle numbers that occupy more 255 characters, and in a few cases (e.g. a 20-option EMSQ quiz where 10 stems are correct and we use a quiz that has100 stems) we meet this restriction of Excel™. There is also a restriction that Excel™ cannot handle numbers that are greater than about E308, and again this restriction was met for 'large' EMSQ quizzes. But, at least the mean mark of 'large' EMSQ quizzes could be calculated (which is about 0).

Thus, Excel™ has a few restrictions, which are rarely met, but they impinge upon *tool2*.

The tool, *tool2* comes after Excel™ has done its work and is therefore is subject to Excel's™ restrictions. *tool2* calculates descriptive and other statistics of a distribution, and it does its calculations on any distribution, and is only limited by what it receives.

The tool, t*ool2:*

a) tells us how many modes are,
b) works on census data and not samples,
c) does not use estimates for its descriptive statistics,
d) does more than SPSS e.g. tells us the proportion of passes,
e) works for 'small' samples,
f) uses the same formulae as SPSS until SPSS 'runs out of steam'.

So, we think that *tool2* is only restricted by the limitations of Excel™.

Thus, the tools that we have built are only restricted by Excel™.

quiz), the distribution moves to the left (when the guessing problem is tackled 'exactly' for negative marking. If the guessing problem is tackled by raising the pass mark to 70%, say, then for some modules the pass mark is 40% (at a 40% institution) and for others it is 70%. So, what is the pass mark?

Also, this study should make academics ask qualitative questions before using quizzes for assessment. The type of questions is illustrated by:

- "What is the type of knowledge that is being taught?"
- "Is the assessment *aligned* to the type of knowledge being taught?"
- "Can special quizzes assess people's deep learning?"

The main contribution to knowledge is DO NOT USE 'SPECIAL' QUIZZES TO MEASURE UNDERSTANDING, because quizzes do not provide *space* for understanding to be *demonstrated.* This conclusion could have been come to without the study being done at all, but we could not have come to conclusions about the distribution of marks or what happens to the distribution when the guessing problem is tackled, without this study being done.

1.9 What types of marking are there?

There are at least two types of marking:

a) **Norm** Reference marking.
b) **Criterion** Reference marking.

Both types of marking are mentioned by Freeman and Lewis (1998).

Criterion marking is where the learner gets a mark regardless of what others do. For example if the learner gets a mark of 40% he/she passes (at a 40% institute). Norm marking is where the learner gets a mark depending on what others do. For example, every year 10% of the learners get a 1st, 20% get a 2:1, 30% get a 2:2, 30% get a 3rd and 10% fail. This assumes that the sample size is so big that in norm marking the proportions remain static for every cohort.

This thesis assumes **Criterion** marking is done whereby if you get 40% you pass (at a 40% institute).

Volume 2

Full Appendices

Table of Contents.

Appendix 1. Sum of Progressions

1.1 Sum of an Arithmetic Progression (AP)

1.1.1 Why should we want the Sum of an Arithmetic Progression (AP)?

The number of possible situations for non-mixture quizzes where stems can be missed, is the sum of an AP

1.1.2 Sum of an Arithmetic Progression to n terms

Let **AP_n** be the sum of an **A**rithmetic **P**rogression, which starts at **a_0**, adds **d** to each term thereafter and has **n** terms

$AP_n=a_0+(a_0+d)+(a_0+2d)\ldots+(a_0+(n-2)d)+(a_0+(n-1)d)$

Reversing this we have
$AP_n=(a_0+(n-1)d)+(a_0+(n-2)d)+\ldots(a_0+2d)+(a_0+d)+a_0$

Adding them together we have
$2.AP_n=(2a_0+(n-1)d)+(2a_0+(n-1).d)\ldots(2a_0+(n-1))d)$
$2.AP_n=n.(a_0+(n-1)d)$

$AP_n=n(2a_0+(n-1)d)/2$

Check. When n=2, a=1, d=1 we should get 3. $AP_n=2.(2+1.1)/2=3$✓
When n=3, a=1, d=1 we should get 6. $AP_n=3.(2+2.1)/2=6$✓

When n=101, a=1, d=0
$AP_n=n.(2a_0+(n-1)d)/2=101.(2.0+100.1)/2=101.100/2=101.50=5050$

When we ask 100 Yes/No questions, that are NOT all answered, there are 5050 impossible situations. There are 101*101 situations and 101*101 -5050 =5151 possible situations

Another example is in Killer Sudoku where a small square has the numbers 1 to 9 in it and their sum is where n=9. a_0=1 and d=1. So, **AP_n**=9*(2*1 +8*1)/2=90/2=45. Thus the numbers in a small square add up to 45.

Here, n is the number of terms and is NOT the number of stems.

1.2 Sum of an Geometric Progression (GP)

1.2.1 Why should we want the Sum of an Geometric Progression (GP)?

The number of marks for a Yes/No questions that all marked by CBM is the sum of a GP. There are 7 to begin with, then 49, then 333 etc.

1.2.2 Sum of an Geometric Progression to n terms

Let GP_n be the sum of a **G**eometric **P**rogression, which starts at a and each term is r times the previous term.

$GP_n=a+a.r^1+ a.r^2+... a.r^{(n-2)}+a.r^{(n-1)}$..1

Multiplying both sides by r we have
$r. GP_n = a.r^1+a .r^2+$ ....$+a.r^{(n-1)}+a.r^n$..................................2

Subtracting 1 from 2 i.e. 2-1 we have
$(r-1). GP_n =a.r^n -a$
$GP_n= a.(r^n-1)/(r-1)$..........................r>1...............................3

Subtracting 2 from 1 i.e. 1-2 we have
$(1-r) . GP_n =a.(1-r^n)/(1-r)$
$GP_n= a.(1-r^n)/(1-r)$..........................r<1 and $r\neq 0$...................4

In the case where r=7, r is greater than 1, so we use equation 3.

Check

At level 1 we should have 7 marks.

When n=1, a=7 $GPn=7(7^1-1)/6=7$✓

At level 2 we should have 49 marks, but the sum should be 7+49=56

When n=2, a=7 $GPn=7(7^2-1)/6=56$✓

At level 3 we should have 343marks, but the sum should be 7+49 +343=399

When n=3, a=7 $GPn=7(7^3-1)/6=399$✓

Here, n is the number of terms and is NOT the number of stems.

Appendix 2. Factorials.

2.1 Why should we want Factorials?

Permutations and Combinations are defined in terms of factorials, see next Appendix.

2.2 What does factorial mean, and what is its notation?

Factorial only applies to integers that are ≥0 i.e. positive integers

The notation for factorial n is n!

The definition of factorial is recursive, with n!=n(n-1)!

We need some stopping condition, and we use 0!=1

Example. 6!=6.5.4.3.2.1.1=720

We cannot work out :

- 2.7! because n must be an *integer*
- -3! because n must be a *positive* integer

2.3 What is the effect of factorials growing very quickly?

Factorials grow very quickly e.g. 0!=1,1!=1, 2!=2, 3!=6, 4!=24, 5!=120, 6!=720, 7!=5040 with 100! having of the order of 158 digits. This soon outgrows a modern digital computer running 'ordinary' programs.

We are driven to handling integers of an arbitrary size, and that is where LEDA fits in.

LEDA routines include a function factorial (n) that calculates factorial for us.

Appendix 3. Permutations and Combinations.

3.1 Why should we want Permutations and Combinations?

The number getting a mark, in a particular way, is either a Permutation or a Combination. Permutations and Combinations are related, in that permutations are bigger than combinations where in

- permutations
    - Choice1 and Choice2 is different from Choice2 and Choice1
- combinations
    - Choice1 and Choice2 are the same as Choice2 and Choice1

In our case Choice1 and Choice2 are the same as Choice2 and Choice1so we use combinations, and thus, the number getting a mark in a particular way is a Combination.

3.2 We do not allow repetitions

Something like 3, 5, 5, 5, 6, 10 we do not allow since 5 is repeated. The following is simplified by not allowing repetition.

3.3 What do we want to do?

We want to choose r things from n, where repetition is disallowed and Choice1 and Choice 2 is the same as Choice 2 and Choice 1, and n≥r.

3.4 What is the notation of Permutations and Combinations?

For Permutations the notation is $_nP_r$
For Combinations the notation is $_nC_r$ and so, we will use this.
where there are r permutations/combinations from n, with n≥r.

3.5 What is the relationship between Permutations and Combinations?

There are (n-r)! combinations in $_nP_r$

$_nC_r = {}_nP_r /(n-r)!$ ....1.

Once we have calculated $_nP_r$ we only have to divide by (n-r)! to calculate $_nC_r$

3.6 What is $_nP_r$?

$_nP_r$ is the number of permutations of r from n where r≠0, and n≥r.

$_nP_r$=n!/r!..2.

3.7 What is nCr?

$_nC_r$ is the number of combinations of r from n
From 1. we have

$_nC_r$=n!/(r!(n-r!)...............................3.

3.8 How can we use this result?

3.8.1 Yes/ No quizzes

3.8.1.1 All stems answered

Where all **n** stems are ALL answered, the number of ways of answering **w** stems correctly = $_n\mathbf{C}_w$

3.8.1.2 Some stems missed

Where **y** stems can be missed, the number answering **w** stems correctly

= $_{n-y}\mathbf{C}_w{}^*{}_n\mathbf{C}_y$

w ,**n** and **y** correspond to a mark m.

n is how many stems/questions there are
w is how many stems are answered correctly
x is how many stems are answered in-correctly
y is how many stems are missed

We can say "the mark m was obtained by particular values of **w**, **y** and **n**". For example, when **n**=3 and the score is [0], [1, 0],

the mark 1 is obtained when 1 question is answered correctly and 0 stems are missed (i.e. **y**=0 and **w**=1). There are other ways of getting the mark 1 e.g. answer 1 stem correctly and miss 1 stem.

In both these cases **x** (how many questions are answered in-correctly) is given because **n=w + x + y.**

The marks are *not* now unique. Pivot Table/Charts allow the calculation of how many there are of obtaining marks 0, 1 2 etc. We can plot the distribution of marks, and calculate the descriptive statistics of the distribution. So, the Pivot Tables/Charts do a summation for us and arrive at how many ways there are of getting mark 0, how may ways here are of getting mark 1, etc.

The generalisation here is by letting **y**=0, because then $_nC_0$=1 and **n-y=n**

Appendix 4. Straker's proof

Abstract

The mean is a familiar concept, but here, we use the concept that if mean mark of a special quiz consisting of 1 stem is **μ_1** then the mean mark of **n** stems

$$\mu_n = n^* \mu_1$$

4.1 Introduction

The following is due to Christine Straker (2012), a statistician from Sheffield Hallam University.

For **n** stems of a Yes/No-only quiz where there are three possible marks; right, wrong and missing

The number of times each separate "value" appears = **n**. (3^{n-1})

1 appears n. (3^{n-1}) times

a appears n. (3^{n-1}) times

b appears n. (3^{n-1}) times

For example:

For 1 stem, each value appears $1 \times 3^0 = 1$ time

For 2 stems, each value appears $2 \times 3^1 = 6$ times

For 3 stems, each value appears $3 \times 3^2 = 27$ times

For 4 stems, each value appears $4 \times 3^3 = 108$ times

For 5 stems, each value appears $5 \times 3^4 = 405$ times and so on…

The expected value for 1 stem is

$$E(\text{Stem } 1) = \frac{(1+a+b)}{3} = \mu_1$$

Expected value for Stem1 and Stem 2

$$E(\text{Stem } 1 + \text{Stem } 2) = (2.\ 3^1)\frac{(1+a+b)}{3^2} = \frac{2.3^1}{3}\frac{(1+a+b)}{3} = \mu_2 = 2E(\text{Stem } 1) = 2\,\mu_1$$

and so on…

So in general

$$E(\text{Stem 1 to Stem } n \text{ inclusive}) = (n3^{n-1})\frac{(1+a+b)}{3^n} = \mu_n$$

$$= \frac{n3^{n-1}}{3^{n-1}}\left[\frac{1+a+b}{3}\right] = n\{\text{Expected value of Stem 1}\}$$

So, $\mu_n = n\,\mu_1$

This proof works for Yes/No-only quizzes, and we extend this proof to cater for more general quizzes.

For some, more general 'special' quizzes, that do not include Multiple Yes/No-only quizzes

$$E(\text{Stem } 1) = \frac{(1+({}_{P}C_{T}-1)a+b)}{(1+\ {}_{P}C_{T})} = \mu_1$$

For **n** stems

1 and **b** appear n. $(1 + {}_{P}C_{T})^{n-1}$) times

and **a** appears $({}_{P}C_{T}-1)$ n. $(1 + {}_{P}C_{T})^{n-1}$) times

$$E(\text{Stem 1 to Stem } \mathbf{n} \text{ inclusive}) = (n(1 + {}_{P}C_{T})^{n-1})\frac{(1+({}_{P}C_{T}-1)a+b)}{(1+\ {}_{P}C_{T})^{n}} = \mu_n = \mathbf{n}\mu_1$$

For other special quizzes i.e. Multiple Yes/No-only quizzes

$$E(\text{Stem } 1) = \frac{(1+(2^{P}-1)a+b)}{(1+2^{P})} = \boldsymbol{\mu_1}$$

For **n** stems

1 and **b** appear n. $(1 + 2^{P})^{n-1}$ times

and **a** appears $(2^{P}-1)$ **n.** $(1 + 2^{P})^{n-1}$) times

$$E(\text{Stem 1 to Stem n inclusive}) = (\mathbf{n}(1 + 2^{P})^{n-1})\frac{(1+(2^{P}-1)a+b)}{(1+2^{P})^{n}} = \boldsymbol{\mu_n} = \mathbf{n}\boldsymbol{\mu_1}$$

4.2 Results of this analysis

4.2.1 For 2 stem of a Yes/No-only quiz we have the marks

Mark for 1st stem	**Mark for 2nd stem**
1	1
1	**a**
1	**b**
a	1
a	**a**
a	**b**
b	1
b	**a**
b	**b**

App. 4- 1 .Marks for a 2-stem Yes/No-only quiz.

The mean mark μ_2 is

(1 + 1 + 1 + **a** + 1 + **b** +
a + 1 + **a** + **a** + **a** + **b** +
b + 1 + **b** + **a** + **b** + **b**)/9

=(3.1 +(1 + **a** + b) +3.**a** +(1 +**a** + **b**) +3**b** +(1 + **a** + **b**))/9
=6(1 + **a** +**b**)/9
=2(1 + **a** +**b**)/3 So $\mu_2 = 2\mu$

So, the theorem is true for **n**=2

4.3 For 3 stem of a Yes/No-only quiz we have the marks

Mark for 1st stem	Mark for 2nd stem	Mark for 3rd stem
1	1	1
1	1	a
1	1	b
1	a	1
1	a	a
1	a	b
1	b	1
1	b	a
1	b	b
a	1	1
a	1	a
a	1	b
a	a	1
a	a	a
a	a	b
a	b	1
a	b	a
a	b	b
b	1	1
b	1	a
b	1	b
b	a	1
b	a	a
b	a	b
b	b	1
b	b	a
b	b	b

App. 4- 2. Marks for a 3-stem Yes/No-only quiz.

The mean mark is

=(3 + 2 + **a** + 2 + **b**
+2 + **a** + 1 + 2**a** + 1 + **a** + **b**
+2 + **b** + 1 + **b** + **a** + 1 + 2**b**
+**a** + 2 + 2**a** + 1 + 1 + **a** + **b**
+2**a** + 1 + 3**a** + 2**a** + **b**
+**a** + **b** + 1 + 2**a** + **b** + **a** + 2**b**
+**b** + 2 + **a** + **b** + 1 + 2**b** + 1
+**a** + **b** + 1 + 2**a** + **b** + 2**b** + **a**
+2**b** + 1 + 2**b** + **a** + 3**b**
)/27
=(3 + 3**a** + 3**b**
+6 + 6**a** + 6**b**
+3(**a**+2)
+3(**b**+2)
+3(1 +2**a**)
+3(1+2**b**)
+3(2**a**+**b**)
+3(**a**+2**b**)
)/27
=(9(1 + **a** + **b**) +
3**a** + 6 + 3 + 6**a** +
3**b** + 6 + 3 + 6**b** +
6**a** + 3**b** + 3**a** + 6**b**
)/27
=(9(1 +**a** +**b**) + (9 +9**a**) +(9+9**b**)
+ 9**a** +9**b**)/27
=27 (1 +**a** +**b**)/27

So, the theorem is true for **n**=3 and, here **n**= $(1+ {}_{P}C_{T})$

4.3.1 For a 2 stem 3-option MCQ-only quiz we have the marks

Mark for 1st stem	Mark for 2nd stem
1	1
1	**a**
1	**a**
1	**b**
a	1
a	**a**
a	**a**
a	**b**
a	1
a	**a**
a	**a**

a	b
b	1
b	a
b	a
b	b

App. 4-3. Marks for a 2-stem MCQ-only quiz.

$\mu_1=(1 + 2a + b)/4$

$\mu_2=(4 +(1 + 2a +b) + 4a + (1 + 2a +b) + 4a + (1 +2a +b) + 4b + (1 +2a +b))/16$
$=(8(1 +2a +b)/16=2\mu_1$. So, the theorem is true, here.

4.4 Conclusions

We show that the mean mark of a special quiz, that consists of **n** stems (all of the same type) is **n*** (the mean of a quiz consisting of 1 stem), and we try this result out on 3 quizzes.

The mean mark or 1 stem is

$$\boldsymbol{\mu_1} =(1 +(1- {}_{P}C_{T})\ \mathbf{a} + \mathbf{b})/(1 + {}_{P}C_{T}) \text{ or}$$
$$\boldsymbol{\mu_1} =(1 + (1- 2^{P})\ \ \mathbf{a} + \mathbf{b})/(1 + 2^{P})$$

and the mean mark for **n** stems is

$$\boldsymbol{\mu_n} =\mathbf{n}(1 +(1- {}_{P}C_{T})\ \mathbf{a} + \mathbf{b})/(1 + {}_{P}C_{T}) \text{ or}$$
$$\boldsymbol{\mu_n}= \mathbf{n}(1 + (1- 2^{P})\ \ \mathbf{a} + \mathbf{b})/(1 + 2^{P})$$

Hence $\boldsymbol{\mu_n}= \mathbf{n}\boldsymbol{\mu_1}$ for 'special' quizzes.

Appendix 5. Proofs where ALL stems are answered

5.1 Proof 1. The gap between successive marks is |1 - a|

The step-size between successive **w**s depends on:

- how you define successive
- whether stems are all answered or not.

This Appendix deals with when stems are all answered.

Successive depends on whether you are working up the **w**s or down the **w**s i.e. it depends on the direction of the subtraction of the **w**s. We will do the subtraction in both directions and come to a conclusion and a proof.

Let us give 1 example and then a proof.

For example

For **n**=100, and **a**=-1/2.

App. 5 Table 1. An example of the marks on the x-axis.

Mark	Stems answered correctly, w	Stems answered in-correctly x
-50	0	100
-48.5	1	99
-47	2	98
-45.5	3	97
....		
98.5	99	1
100	100	0

Here, the range of marks is -50 to 100
the gap between marks of successive **w**s is 1.5, when we work *down* the marks, but is -1.5 if we work *up* the marks.

Proof 1, of step-size

Let us take successive **w**s so $w_1=w_2 + 1$ and $x_1=x_2 - 1$

If w goes up by 1, then x goes down by one, and if w goes down by 1 then x goes up by 1.

The 2 marks concerned are

$Mark_1= w_1*1 + x_1*\mathbf{a}$.

$Mark_2= w_2*1 + x_2*\mathbf{a}$.

As long as the same **a** is used for both marks, and we assume that both stems have the same **c** (which in this case is 1).

$$\begin{aligned} Mark_1 - Mark_2 &= w_1 + x_1*\mathbf{a} - (w_1 - 1) - (x_1 - 1)*\mathbf{a} \\ &= w_1 - w_1 + x_1*\mathbf{a} - x_1*\mathbf{a} + 1 - \mathbf{a} \\ &= 1 - a \\ &= +(1 - \mathbf{a}) \end{aligned}$$

If we do the subtraction the other way round we have

$$\begin{aligned} Mark_2 - Mark_1 &= w_2 + x_2*\mathbf{a} - w_1 \quad - x_1*\mathbf{a} \\ &= w_2 + x_2*\mathbf{a} - (w_2 + 1) - (x_2 - 1)*\mathbf{a} \\ &= -1 + \mathbf{a} \\ &= -(1 - \mathbf{a}) \end{aligned}$$

The gap between marks for successive **w**s is $|1-\mathbf{a}|$.

Q.E.D

This proof applies to all special quizzes where ALL stems are answered, and even applies when **a**=1, where there is only 1 mark.

Here, the step size is a constant, given that **a** does not change.

We will prove, (see proof 3), that the marks correspond in a 1:1 way with **w** and because the **w**s are unique, so are the marks. Thus, successive marks are unique and are separated by $|1-\mathbf{a}|$.

5.2 Proof 2. The number of impossible marks is zero

Marks are impossible when the conditions that lead to the marks being calculated are impossible.

Proof 2. There are no impossible marks when all stems are answered.

1. **w** takes on the values 0, 1, 2....**n** i.e. **w** ranges from 0 to **n** in steps of 1.
2. As **w** is varied, we calculate a mark that corresponds to **w**.
3. An impossible mark occurs when **w** takes on an impossible value, in this range.
4. It is not possible for there to be a mark where **w** does not exist, in this range.
5.There are no impossible **w**s in the range 0, 1, 2 ..n and so there are no impossible marks in this range of **w**.

Q.E.D.

For example, for a score of [1, **a**] and **n**=100, in a Yes/No-only quiz, it is not possible to obtain a mark that corresponds to **w**=99.5 no matter what **a** is, because **w**=99.5 is impossible (**w** must be an integer between 0 and **n** inclusive).That is an indication that there are no impossible marks, here.

If the algorithm is vary the integer **w** in steps of 1 between 0 and **n**, then the number of impossible marks that correspond to **w** should be 0. If our program prints out 0 in this situation, it is a check that our program works.

5.3 Proof 3. The number of marks is (n +1)

When we ask "How many marks there are on the x-axis, for distributions that we are dealing with?" the answer depends on a) how many stems there are b) how we score stems c) whether stems can be missed or not.

a) let us say there are **n** stems
b) the scoring is [**b**], [1, **a**] if **b** exists and [1, **a**] if **b** does not exist

This Appendix deals with all stems being answered, where **b** does not exist, and so stems cannot be missed.

When all stems are answered and **a**=0, the score is [1, 0], there are (**n**+1) marks on the x-axis i.e. 0, 1, 2....**n** where **b** does not exist

A conjecture is

"When all stems are answered and **a**≠ 0 there are still (**n**+1) marks on the x-axis, unless **a**=1". The proof of this conjecture relies on an earlier proof (see line 4 below).

Proof 3, of the number of marks on the x-axis

1. There are (**n**+1) unique **w**s	because	**w** is 0, 1, 2, 3 ...n
2. A mark **M**, corresponds to one **w**	because	M=w +x**a**, but **n**=w + x
	therefore	M=w + (**n**-w)**a**

3. But, the same mark might correspond to 2 (or more) **w**s
4. The difference between marks for successive **w**s is |1 - **a**|. (see proof 1)
5. If **a** does not change then |1-**a**| is a constant.
6. Adding a constant to a unique **w** results in a unique mark.
7. Subtracting a constant from a unique **w** results in a unique mark.
8. So, marks are unique, and correspond to a unique **w** and 3 is impossible
9. As there are (**n**+1) **w**s then there are (**n**+1) marks.

Q.E.D.

In quizzes where all stems are answered, and each stem is scored [1, **a**], we have a proof that, there are (**n** + 1) marks on the x-axis, no matter what **a** is, (unless **a**=1 in which case there is only 1 mark).

Appendix 6. Proofs where stems can be missed

This Appendix concerns 3 proofs about quizzes where stems can be missed.

6.1 Proof 1. The gap between successive marks is NOT |1 - a|

This proof depends on finding just one occasion where the gap between successive marks is not |1 - a|.

Proof 1, of the step-size
For **n**=100, **b**=0, **a**=-0.25

The mark **100** can only be obtained by answering all the stems correctly

The next lowest mark **99** can only be obtained by answering 99 stems correctly and missing 1 stem

The step-size=**1**

The next lowest mark **98.75**, can only be obtained by answering 99 stems correctly, and answering 1 stem in-correctly.

The step-size=**0.25**

Which illustrates that, here, the step-size is not |1 -**a**|. Hence, we *cannot* say, here, that the marks go from minimum to maximum in the same steps.

Q.E.D.

This is called a *proof by exception*. We have shown that in one case, at least, the step-size between marks is not a constant.

Other examples show that where **a** is an integer e.g. **n**=100, **a**=-1, **b**=0, the marks on the x-axis are -100, -98, -96 etc. and that in some instances the step-size between marks for quizzes, where every stem is NOT answered, is a constant.

6.2 Proof 2. The number of impossible marks is the sum of an AP

We extend the idea that a mark corresponding to an impossible **w** is itself impossible, to a situation where an impossible *combination* of **w** and **y** results in an impossible corresponding mark. For example, for 100 stems of any quiz it is impossible to answer 99 stems correctly and miss 2 and so the combination of **w**=99 and **y**=2 is an impossible situation, and thus the mark that corresponds to **w**=99, an **y**=2 is impossible. The impossible situation occurs when **x** (the number of incorrect answers to stems) is negative, and **x** is a strictly positive integer in the range 0, 1, 2..**n**.

This result does not depend on the values of **a** and **b**, as the following argument shows. The way we are arguing is that an impossible mark comes from a combination of **w** and **y** being impossible, and so any mark that corresponds to an impossible combination of **w** and **y** cannot be calculated. We are not arguing about the marks at all, and so **a** and **b** can be anything. So, this result does not depend on **a** or **b**.

We have a conjecture that the number of impossible situations is equal to **w**.

The conjecture arises when we consider the marks for a 'small' population. For example when the number of stems =**3** and the score is [0], [1, -1]

App 6. Table 1. Conjecture that the number of impossible situations is w.

Mark	**w**	**y**	**x (given)**	**Comment**	**Number of impossible situations**
-3	0	0	3	All wrong	0
-2		1	2	1 missed 2 wrong	
-1		2	1	2 missed 1 wrong	
0		3	0	All missed	
-1	1	0	2	1 right 2 wrong	1
0		1	1	1 right 1 missed 1 wrong	
1		2	0	1 right 2 missed	
		3	**-1**	**Impossible.** 1 right and 3 missed	
1	2	0	1	2 right 1 wrong	2
2		1	0	2 right 1 missed	
		2	**-1**	**Impossible.** 2 right and 2 missed	
		3	**-2**	**Impossible.** 2 right and 3 missed	
3	3	0	0	All right	3
		1	**-1**	**Impossible.** 3 right and 1 missed	
		2	**-2**	**Impossible.** 3 right and 2 missed	
		3	**-3**	**Impossible.** 3 right and 3 missed	

Proof 2 of the conjecture that the number of impossible situations is w

1. **w** goes from 0 to **n** in steps of 1 i.e. **w** takes on the values 0, 1, 2, 3....**n**
2. n=w + **y** +**x** . We ask the user for **n**, and so **n** is a constant. We vary **w** and **y** from 0 to **n** in steps of 1
3. When **w**=0 and **y** is varied from 0 to **n** then **x** takes on the values 0 to **n,** and **x** 'takes up the slack' and in each case is valid. So, here, there are 0 impossible marks .i.e. when **w**=0 the number of impossible situations=0
4. When **w**=1 and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y**=**n** i.e. when **w**=1 the number of impossible situations=1
5. When **w**=2 and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y**=**n**-1 and **y**=**n** i.e. when **w**=2 the number of impossible situations=2
6. When **w**=**m** and **y** is varied from 0 to **n** then **x** cannot 'take up the slack' when **y**=**n**-**m**, **n**-**m**-1,..**n**-1 and **y**=**n** i.e. when **w**=**m** the number of impossible situations=**m**
7. As the conjecture is true when **w**=0 and **w**=1 and **w**=2 and when **w**=**m** it is always true, as long as **w** ≤ **n**

This is a proof by *induction*.

So, if the number of impossible situations=**w** and **w** ranges from 0 to **n** in steps of 1, then the sum of **w** =0 +1 +2 +....**n**=total number of impossible situations = Sum of an AP_m =(**n**+1)*(0 + **n***d)/2 because a_0=0

=(**n**+1)*(**n**)/2 because d=1

and when **n**=100

Sum of the AP_{100}=101*50=5050

So, the total number of impossible situations when **n**=100, for 'special' quizzes where some stems can be missed, and there are no mixtures, is 5050.

Q.E.D.

6.3 Proof 3. The number of marks is NOT (n +1)

Again, we only have to find 1 occasion where the number of marks on the x-axis is not (**n** + 1).

Proof 3, of the number of marks.

Consider, **n**=3, **a**=-1/2, **b**=0.

The marks include

-3/2	Answer all 3 stems in-correctly
-2/2	Answer 2 stems in-correctly and miss 1
-1/2	Answer 1 stem in-correctly and miss 2
0	Miss all stems
1/2	Answer 1 stem correctly, 1 stem in-correctly and miss 1
2/2	Answer 1 stem correctly and miss 2
3/2	Answer 2 stems correctly and 1 in-correctly
4/2	Answer 2 stems correctly and miss 1
5/2	Answer 3 stems correctly and 1 in-correctly
	Impossible
6/2	Answer all 3 stems correctly

There are 9 possible marks on the x-axis for **n**=3 and as that is not (**n** +1)

Q.E.D.

Again, this is a *proof by exception*.

Appendix 7. Validation of analysis of quizzes where ALL stems are answered.

7.1 Introduction

This Appendix is in 2 parts.

Part 1 is where we validate the expression $X^{x*}{}_nC_w$

for quizzes where all stems are answered. The expression says how many obtain a mark for a combination of X, **x**, **n** and **w**.

X	is the number of answers to 1 stem that are in-correct
n	is the number of stems that a quiz has
x	is the number of stems answered in-correctly
w	is the number of stems answered correctly.

n=w + x

We ask the user for **n**, and so for any 1 quiz **n** is constant (e.g. 100) and so **x** (say) is given.

A mark =**w***1 + **x*a** assuming that a correct answer to a stem is scored 1 and an in-correct answer is scored **a.** We ask the use for the value of **a**. When **a** is negative we are said to use negative marking.

Part 2 is where we validate an Excel™ program that helps us to calculate **Mg**.

7.2 Part 1

The 'grey' column in the tables below is where **x** is given.

7.2.1 Yes/No-only quizzes where X=1 and negative marking is NOT used i.e. a=0.

One thing that we know about the distribution of marks is that it is symmetric and hence the skewness is 0.

App 7. Table 1. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The mean Mark=0.5	**0**	**1**	0	1	Get them all wrong **1** $1^{1}*{}_1C_0=1*1=1$
	1	**1**	1	0	Get them all right **1** $1^{0}*{}_1C_1=1*1=1$

We would expect our tool (when it is built) to output (0 1) (1 1) and to say that the skewness=0. The gap between marks should be |1 -**a**| (see Appendix 5) and as **a**=0 the gap should be 1, and by inspection it is.

This distribution has 2 modes and hence is said to be *bi-modal.*

App7. Table 1. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The mean Mark=1=(0*1 +1*2 + 2*1)/4	**0**	**1**	0	2	Get them all wrong **1** $1^{2}*{}_2C_0=1*1=1$
	1	**2**	1	1	Get 1 right **2** $1^{1}*{}_2C_1=1*2=2$
	2	**1**	2	0	Get them all right **1** $1^{0}*{}_2C_2=1*1=1$

This distribution has 1 mode and hence is said to be *uni-modal.*

App7. Table 2. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean Mark=(0*1 +1*3 + 2*3 +3*1)/8=1.5	**0**	**1**	0	3	Get them all wrong **1** $1^{1}3_{3}C_{0}=1*1=1$
	1	**3**	1	2	Get 1 right **3** $1^{2*}{}_3C_1=1*3=3$
	2	**3**	2	1	Get 1 wrong **3** $1^{1*}{}_3C_2=1*3=3$
	3	**1**	3	0	Get them all right **1** $1^{0*}{}_3C_3=1*1=1$

This distribution has 2 modes and hence is said to be *bi-modal.*

App7. Table 3. Yes/No questions. Stems can NOT be missed. Score [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The mean Mark=(0*1 +1*4 +2*6 +3*4 +4*1)/16=2	**0**	**1**	0	4	Get them all wrong **1** $1^{4*}{}_4C_0=1*1=1$
	1	**4**	1	3	Get 1 right **4** $1^{3*}{}_4C_1=1*4=4$
	2	**6**	2	2	Get 2 right **6** $1^{2*}{}_4C_2=1*6=6$
	3	**4**	3	1	Get 1 wrong $1^{1*}{}_4C_3=1*4=4$ **4**
	4	**1**	4	0	Get them all right **1** $1^{0*}{}_4C_4=1*1=1$

App 7 Table 4. Yes/No questions. Stems NOT can be missed. Score [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5 The mean Mark=(0*1 + 1*5 + 2*10 + 3*10 + 4*5 +5*1)/32=2.5	**0**	**1**	0	5	Get them all wrong **1** $1^{5*}{}_5C_0=1*1=1$
	1	**5**	1	4	Get 1 right **5** $1^{4*}{}_5C_1=1*5=5$
	2	**10**	2	3	Get 2 right **10** 1,2 1,3, 1,4 1,5 2,3 2,4 2,5 3,4 3, 5 4, 5 $1^{3*}{}_5C_2=1*10$
	3	**10**	3	2	Get 2 wrong **10** 1,2 1,3, 1,4 1,5 2,3 2,4 2,5 3,4 3, 5 4, 5 $1^{2*}{}_5C_3=1*10$
	4	**5**	4	1	Get 1 wrong **5** $1^{1*}{}_5C_4=1*5$
	5	**1**	5	0	Get them all right **1** $1^{0*}{}_5C_5=1*1=1$

7.2.2 Yes/No-only quizzes where X=1 and negative marking IS used i.e. a=-1.

We will use a value of **a** that exactly tackles the guessing problem, where the **mean** mark is 0. This is a validation that we can do when *tool1* is built.

App 7 Table 5. Yes/No questions. Stems can NOT be missed. Negative marking is used. Score [1, -1]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**-1**	**1**	0	1	Get them all wrong **1** $1^{1*}{}_1C_0=1*1=1$
	1	**1**	1	0	Get them all right **1** $1^{0*}{}_1C_1=1*1=1$

App 7 Table 6. Yes/No questions. Stems can NOT be missed. Negative marking is used. Score [1, -1]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**-2**	**1**	0	2	Get them all wrong **1** $1^{2*}{}_2C_0=1*1=1$
	0	**2**	1	1	Get 1 right **2** $1^{1*}{}_2C_1=1*2=1$
	2	**1**	2	0	Get them all right **1** $1^{0*}{}_2C_2=1*1=1$

App 7 Table 7. Yes/No questions. Stems can NOT be missed. Negative marking is used. Score [1, -1]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3	**-3**	**1**	0	3	Get them all wrong **1** $1^{3*}{}_3C_0=1*1=1$
	-1	**3**	1	2	Get 1 right **3** $1^{2*}{}_3C_1=1*3=3$
	1	**3**	2	1	Get 1 wrong **3** $1^{1*}{}_3C_2=1*3=3$
	3	**1**	3	0	Get them all right **1** $1^{0*}{}_3C_3=1*1=1$

App 7 Table 8. Yes/No questions. Stems can NOT be missed. Negative marking is used. Score [1, -1]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4	**-4**	**1**	0	4	Get them all wrong **1** $1^{4*}{}_4C_0=1*1=1$
	-2	**4**	1	3	Get 1 right **4** $1^{3*}{}_4C_1=1*4=4$
	0	**6**	2	2	Get 2 right **6** $1^{2*}{}_4C_2=1*6=6$
	2	**4**	3	1	Get 3 right $1^{1*}{}_4C_3=1*4=4$ **4**
	4	**1**	4	0	Get them all right **1** $1^{0*}{}_4C_4=1*1=1$

App 7 Table 9. Yes/No questions. Stems can NOT be missed. Negative marking is used. Score [1, -1]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5	**-5**	**1**	0	5	Get them all wrong **1** $1^{5*}{}_5C_0=1*1=1$
	-3	**5**	1	4	Get 1 right **5** $1^{4*}{}_5C_1=1*5=5$
	-1	**10**	2	3	Get 2 right **10** $1^{3*}{}_5C_2=1*10$
	1	**10**	3	2	Get 3 right **10** $1^{2*}{}_5C_3=1*\ 10$
	3	**5**	4	1	Get 4 right **5** $1^{1*}{}_5C_4=1*5$
	5	**1**	5	0	Get them all right **1** $1^{0*}{}_5C_5=1*1=1$

From this, we conclude that the expression $X^{x*}{}_nC_w$ works for X=1.

7.2.3 4-option MCQ-only quizzes where X=3 and negative marking is NOT used i.e. a=0.

App 7 Table 10. 4-option MCQ. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The mean Mark=(0*3 + 1*1)/4=0.25	**0**	**3**	0	1	Get them all wrong **3** $3^{1*}{}_1C_0=3*1=3$
	1	**1**	1	0	Get them all right **1** $3^{0*}{}_1C_1=1*1=1$

App 7 Table 10. 4-option MCQ. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The mean Mark=(0*9 +1*6 +2*1)/16=0.5	**0**	**9**	0	2	Get them all wrong 3^2= **9** $3^{2*}{}_2C_0$=9*1=9
	1	**6**	1	1	Get 1 right 1 wrong **6** $3^{1*}{}_2C_1$=3*2=6
	2	**1**	2	0	Get them all right **1** $3^{0*}{}_2C_2$=1*1=1

App 7 Table 11. 4-option MCQ. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean Mark=(0*27 +1*27 +2*9 +3*1)/64=0.75	**0**	**27**	0	3	Get them all wrong 3^3= **27** $3^{3*}{}_3C_0$=1*1=1
	1	**27**	1	2	Get 1 right **27** $3^{2*}{}_3C_1$=9*3=27
	2	**9**	2	1	Get 1 wrong **9** $3^{1*}{}_3C_2$=3*3=9
	3	**1**	3	0	Get them all right **1** $3^{0*}{}_3C_3$=1*1=1

App 7 Table 12. 4-option MCQ. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. n=**4**.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The mean Mark=(0*81 + 1*108 + 2*54 +3*12 +4*1)/256=1	**0**	**81**	0	4	Get them all wrong 3^4= **81** $3^{4*}{}_4C_0$=81*1=81
	1	**108**	1	3	Get 1 right $3^{3*}{}_4C_1$=27*4= **108**
	2	**54**	2	2	Get 2 right $3^{2*}{}_4C_2$=9*6= **54**
	3	**12**	3	1	Get 1 wrong $3^{1*}{}_4C_3$=3*4= **12**
	4	**1**	4	0	Get them all right **1** $3^{0*}{}_4C_4$=1*1=1

App 7 Table 13. 4-option MCQ. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5 The mean Mark=(0*243 +1*405 +2*270+3*90 +4*15 +5*1)/1024=1.25	**0**	**243**	0	5	Get them all wrong **243** $3^{5*}{}_5C_0$=243*1=243
	1	**405**	1	4	Get 1 right **405** $3^{4*}{}_5C_1$=81*5=405
	2	**270**	2	3	Get 2 right 10*27=**270** $3^{3*}{}_5C_2$=27*10=270
	3	**90**	3	2	Get 2 wrong 10*9= **90** $3^{2*}{}_5C_3$=9* 10 =90
	4	**15**	4	1	Get 1 wrong 5*3= **15** $3^{1*}{}_5C_4$=3*5
	5	**1**	5	0	Get them all right **1** $3^{0*}{}_5C_5$=1*1=1

7.2.4 4-option MCQ-only quizzes where X=3 and negative marking IS used

These all show that the number of marks =**n** + 1

App 7 Table 14. 4-option MCQ. Stems can NOT be missed. Negative marking IS used. Score [1, -1/3]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**-0.3333**	**3**	0	1	Get them all wrong **3** $3^{1*}{}_1C_0$=3*1=1
	1	**1**	1	0	Get them all right **1** $3^{0*}{}_1C_1$=1*1=1

App 7 Table 15. 4-option MCQ. Stems can NOT be missed. Negative marking IS used. Score [1, -1/3]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**-0.6667**	**9**	0	2	Get them all wrong 3^2= **9** $3^{2*}{}_2C_0$=9*1=1
	0.6667	**6**	1	1	Get 1 right 1 wrong **6** $3^{1*}{}_2C_1$=3*2=6
	2	**1**	2	0	Get them all right **1** $3^{0*}{}_2C_2$=1*1=1

App 7 Table 18. 4-option MCQ. Stems can NOT be missed. Negative marking IS used. Score [1, -1/3]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x}*{}_{n}C_{w}$
3	**-1**	**27**	0	3	Get them all wrong 3^{3}= **27** $3^{3}*{}_{3}C_{0}$=1*1=1
	0.3333	**27**	1	2	Get 1 right **27** $3^{2}*{}_{3}C_{1}$=9*3=27
	1.6667	**9**	2	1	Get 1 wrong **9** $3^{1}*{}_{3}C_{2}$=3*3=9
	3	**1**	3	0	Get them all right **1** $3^{0}*{}_{3}C_{3}$=1*1=1

App 7 Table 16. 4-option MCQ. Stems can NOT be missed. Negative marking IS used. Score [1, -1/3]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x}*{}_{n}C_{w}$
4	**-1.3333**	**81**	0	4	Get them all wrong 3^{4}= **81** $3^{4}*{}_{4}C_{0}$=81*1=81
	0	**108**	1	3	Get 1 right $3^{3}*{}_{4}C_{1}$=27*4= **108**
	1.3333	**54**	2	2	Get 2 right $3^{2}*{}_{4}C_{2}$=9*6= **54**
	2.6667	**12**	3	1	Get 1 wrong $3^{1}*{}_{4}C_{3}$=4=3*4= **12**
	4	**1**	4	0	Get them all right **1** $3^{0}*{}_{4}C_{4}$=1*1=1

App 7 Table 20. 4-option MCQ. Stems can NOT be missed. Negative marking IS used. Score [1, -1/3]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x}*{}_{n}C_{w}$
5	**-1.6667**	**243**	0	5	Get them all wrong 3^{5}=**243** $3^{5}*{}_{5}C_{0}$=243*1=243
	-0.3333	**405**	1	4	Get 1 right **405** $3^{4}*{}_{5}C_{1}$=81*5=405
	1	**270**	2	3	Get 2 right 10*27=**270** $3^{3}*{}_{5}C_{2}$=27*10=270
	2.3333	**90**	3	2	Get 2 wrong 10*9= **90** $3^{2}*{}_{5}C_{3}$=9* 10 =90
	3.6667	**15**	4	1	Get 1 wrong 5*3= **15** $3^{1}*{}_{5}C_{4}$=3*5
	5	**1**	5	0	Get them all right **1** $3^{0}*{}_{5}C_{5}$=1*1=1

From this, we conclude that the expression $X^{x*}{}_nC_w$ works for X=1 and X=3, for 'small' values of **n**.

7.2.5 3-option Multiple Yes/No-only quizzes where X=7 and negative marking is NOT used.

App 7 Table 17. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The Mean mark =(0*7 +1*1)/8=0.125	**0**	**7**	0	1	Get them all wrong **7** $7^{1*}{}_1C_0=7*1=7$
	1	**1**	1	0	Get them all right **1** $7^{0*}{}_1C_1=1*1=1$

App 7 Table 18. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The Mean mark =(0*49 + 1*14 +2*1)/64=0.25	**0**	**49**	0	2	Get them all wrong 7^2= **49** $7^{2*}{}_2C_0=49*1=1$
	1	**14**	1	1	Get 1 right 1 wrong **14** $7^{1*}{}_2C_1=7*2=14$
	2	**1**	2	0	Get them all right **1** $7^{0*}{}_2C_2=1*1=1$

App 7 Table 19. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The Mean mark =(0*343 +1*147+2*21 +3*1)/512=192/512=**0.375**	**0**	**343**	0	3	Get them all wrong 7^3= **343** $7^{3*}{}_3C_0=343*1=343$
	1	**147**	1	2	Get 1 right $7^{2*}{}_3C_1=49*3=$ **147**
	2	**21**	2	1	Get 1 wrong $7^{1*}{}_3C_2=7*3=$ **21**
	3	**1**	3	0	Get them all right **1** $7^{0*}{}_3C_3=1*1=1$

App 7 Table 20. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The Mean mark =(0*2401 + 1*1372 + 2*294 + 3*28 +4*1)/4096=2048/4096=**0.5**	**0**	**2401**	0	4	Get them all wrong 7^4= **2401** $7^{4*}{}_4C_0$=2401*1=2401
	1	**1372**	1	3	Get 1 right $7^{3*}{}_4C_1$=343*4= **1372**
	2	**294**	2	2	Get 2 right $7^{2*}{}_4C_2$=49*6= **294**
	3	**28**	3	1	Get 1 wrong $7^{1*}{}_4C_3$=7*4= **28**
	4	**1**	4	0	Get them all right **1** $7^{0*}{}_4C_4$=1*1=1

App 7 Table 21. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5 The Mean mark =(0*16807 + 1*12005 + 2*3430 + 3*490 + 4*35 +5*1)=20480/32768=**0.625**	**0**	**16807**	0	5	Get them all wrong 7^5= **16807** $7^{5*}{}_5C_0$=16807*1=16807
	1	**12005**	1	4	Get 1 right $7^{4*}{}_5C_1$=2401*5=**12005**
	2	**3430**	2	3	Get 2 right 10*7^3= **3430** $7^{3*}{}_5C_2$=342*10=3430
	3	**490**	3	2	Get 2 wrong 10*49= **490** $7^{2*}{}_5C_3$=49* 10 =490
	4	**35**	4	1	Get 1 wrong 5*7= **35** $7^{1*}{}_5C_4$=7*5
	5	**1**	5	0	Get them all right **1** $7^{0*}{}_5C_5$=1*1=1

7.2.6 3-option Multiple Yes/No-only quizzes where X=7 and negative marking IS used.

The mean Mark=0, because the guessing problem is exactly tackled.

App 7 Table 22. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking IS used. Score [1, -1/7]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**-0.1429**	**7**	0	1	Get them all wrong **7** $7^{1*}{}_1C_0=7*1=7$
	1	**1**	1	0	Get them all right **1** $7^{0*}{}_1C_1=1*1=1$

App 7 Table 23. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking IS used. Score [1, -1/7]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**-0.2857**	**49**	0	2	Get them all wrong $7^2=$ **49** $7^{2*}{}_2C_0=49*1=49$
	0.8571	**14**	1	1	Get 1 right 1 wrong **14** $7^{1*}{}_2C_1=7*2=14$
	2	**1**	2	0	Get them all right **1** $7^{0*}{}_2C_2=1*1=1$

App 7 Table 24. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking IS used. Score [1, -1/7]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3	**-0.4286**	**343**	0	3	Get them all wrong $7^3=$ **343** $7^{3*}{}_3C_0=343*1=343$
	0.7143	**147**	1	2	Get 1 right $7^{2*}{}_3C_1=49*3=$ **147**
	1.8571	**21**	2	1	Get 1 wrong $7^{1*}{}_3C_2=7*3=$ **21**
	3	**1**	3	0	Get them all right **1** $7^{0*}{}_3C_3=1*1=1$

App 7 Table 25. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking IS used. Score [1, -1/7]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
4	**-0.5714**	**2401**	0	4	Get them all wrong 7^{4}= **2401** $7^{4*}{}_{4}C_{0}$=2401*1=2401
	0.5714	**1372**	1	3	Get 1 right $7^{3*}{}_{4}C_{1}$=343*4= **1372**
	1.7143	**294**	2	2	Get 2 right $7^{2*}{}_{4}C_{2}$=49*6= **294**
	2.8571	**28**	3	1	Get 1 wrong $7^{1*}{}_{4}C_{3}$=7*4= **28**
	4	**1**	4	0	Get them all right **1** $7^{0*}{}_{4}C_{4}$=1*1=1

App 7 Table 26. 3-option Multiple Yes/No. Stems can NOT be missed. Negative marking IS used. Score [1, -1/7]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
5	**-0.7143**	**16807**	0	5	Get them all wrong 7^{5}= **16807** $7^{5*}{}_{5}C_{0}$=*1=
	0.4286	**12005**	1	4	Get 1 right $7^{4*}{}_{5}C_{1}$=2401*5=**12005**
	1.5714	**3430**	2	3	Get 2 right 10*7^{3}= **3430** $7^{3*}{}_{5}C_{2}$=342*10=3430
	2.7143	**490**	3	2	Get 2 wrong 10*49= **490** $7^{2*}{}_{5}C_{3}$=49* 10 =490
	3.8571	**35**	4	1	Get 1 wrong 5*7= **35** $7^{1*}{}_{5}C_{4}$=7*5
	5	**1**	5	0	Get them all right **1** $7^{0*}{}_{5}C_{5}$=1*1=1

From this, we conclude that the expression $X^{x*}{}_{n}C_{w}$ works for X=1, X=3 and X=7, for 'small' values of **n**.

7.2.7 5-option MAQ with 2 ticks-only quizzes where X=9 and negative marking is NOT used i.e. a=0.

App 7 Table 27. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The mean Mark=(0*9 +1*1)/10=**0.1**	**0**	**9**	0	1	Get them all wrong **9** $9^{1*}{}_1C_0=9*1=9$
	1	**1**	1	0	Get them all right **1** $9^{0*}{}_1C_1=1*1=1$

App 7 Table 28. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The mean Mark=(0*81 +1*18 +2*1)/100=**0.2**	**0**	**81**	0	2	Get them all wrong $9^2=$ **81** $9^{2*}{}_2C_0=49*1=1$
	1	**18**	1	1	Get 1 right 1 wrong **18** $9^{1*}{}_2C_1=9*2=18$
	2	**1**	2	0	Get them all right **1** $9^{0*}{}_2C_2=1*1=1$

App 7 Table 29. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean Mark=(0*729 + 1*243 + 2*27 +3*1)/1000=**0.3**	**0**	**729**	0	3	Get them all wrong $9^3=$ **729** $9^{3*}{}_3C_0=1*1=1$
	1	**243**	1	2	Get 1 right $9^{2*}{}_3C_1=81*3=$ **243**
	2	**27**	2	1	Get 1 wrong $9^{1*}{}_3C_2=9*3=$ **27**
	3	**1**	3	0	Get them all right **1** $9^{0*}{}_3C_3=1*1=1$

App 7 Table 30. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The mean Mark=(0*6561 +1* 2916 + 2*486 + 3*36 +4*1)/10000=**0.4**	**0**	**6561**	0	4	Get them all wrong 9^4= **6561** $9^{4}*{}_4C_0$=6561*1=6561
	1	**2916**	1	3	Get 1 right $9^{3}*{}_4C_1$=729*4= **2916**
	2	**486**	2	2	Get 2 right $9^{2}*{}_4C_2$=81*6= **486**
	3	**36**	3	1	Get 1 wrong $9^{1}*{}_4C_3$=9*4= **36**
	4	**1**	4	0	Get them all right **1** $9^{0}*{}_4C_4$=1*1=1

App 7 Table 31. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5 The mean Mark=(0*59049 +1*32805 + 2*7290 + 3*810 + 4*45 + 5*1)/100000=**0.5**	**0**	**59049**	0	5	Get them all wrong 9^5=**59049** $9^{5}*{}_5C_0$=59049*1=59049
	1	**32805**	1	4	Get 1 right $9^{4}*{}_5C_1$=6561*5=**32805**
	2	**7290**	2	3	Get 2 right 10*9^3= **7290** $9^{3}*{}_5C_2$=729*10=7290
	3	**810**	3	2	Get 2 wrong 10*81= **810** $9^{2}*{}_5C_3$=81* 10 =490
	4	**45**	4	1	Get 1 wrong 5*9= **45** $9^{1}*{}_5C_4$=7*5
	5	**1**	5	0	Get them all right **1** $9^{0}*{}_5C_5$=1*1=1

7.2.8 3-option Multiple Yes/No-only quizzes where X=9 and negative marking IS used i.e. a=-1/9.

App 7 Table 32. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking IS used. Score [1, -1/9]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**-0.11111**	**9**	0	1	Get them all wrong **9** $9^{1*}{}_1C_0$=9*1=9
	1	**1**	1	0	Get them all right **1** $9^{0*}{}_1C_1$=1*1=1

App 7 Table 33. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking IS used. Score [1, -1/9]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**-0.22222**	**81**	0	2	Get them all wrong 9^2= **81** $9^{2*}{}_2C_0$=49*1=1
	0.8888	**18**	1	1	Get 1 right 1 wrong **18** $9^{1*}{}_2C_1$=9*2=18
	2	**1**	2	0	Get them all right **1** $9^{0*}{}_2C_2$=1*1=1

App 7 Table 34. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking IS used. Score [1, -1/9]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3	**-0.33333**	**729**	0	3	Get them all wrong 9^3= **729** $9^{3*}{}_3C_0$=729*1=729
	0.7778	**243**	1	2	Get 1 right $9^{2*}{}_3C_1$=81*3= **243**
	1.8889	**27**	2	1	Get 1 wrong $9^{1*}{}_3C_2$=9*3= **27**
	3	**1**	3	0	Get them all right **1** $9^{0*}{}_3C_3$=1*1=1

App 7 Table 35. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking IS used. Score [1, -1/9]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
4	**-0.44444**	**6561**	0	4	Get them all wrong 9^4= **6561** $9^{4*}{}_4C_0$=6561*1=6561
	0.6667	**2916**	1	3	Get 1 right $9^{3*}{}_4C_1$=729*4= **2916**
	1.7778	**486**	2	2	Get 2 right $9^{2*}{}_4C_2$=81*6= **486**
	2.8889	**36**	3	1	Get 1 wrong $9^{1*}{}_4C_3$=9*4= **36**
	4	**1**	4	0	Get them all right **1** $9^{0*}{}_4C_4$=1*1=1

App 7 Table 36. 5-option MAQ with 2 ticks. Stems can NOT be missed. Negative marking IS used. Score [1, -1/9]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
5	**-0.5556**	**59049**	0	5	Get them all wrong 9^5=**59049** $9^{5*}{}_5C_0$=59049*1=59049
	0.5556	**32805**	1	4	Get 1 right $9^{4*}{}_5C_1$=6561*5=**32805**
	1.6667	**7290**	2	3	Get 2 right 10*9^3= **7290** $9^{3*}{}_5C_2$=729*10=7290
	2.7778	**810**	3	2	Get 2 wrong 10*81= **810** $9^{2*}{}_5C_3$=81* 10 =810
	3.8889	**45**	4	1	Get 1 wrong 5*9= **45** $9^{1*}{}_5C_4$=9*5
	5	**1**	5	0	Get them all right **1** $9^{0*}{}_5C_5$=1*1=1

From this, we conclude that the expression $X^{x*}{}_{n}C_{w}$ works for X=1, X=3, X=7 and X=9, for 'small' values of **n**.

7.2.9 20-option EMSQ (MCQ)-only quizzes where X=19 and negative marking is NOT used i.e. a=0.

App 7 Table 37. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The mean	**0**	**19**	0	1	Get them all wrong **19** $19^{1}*{}_1C_0=19*1=19$
Mark=(0*19 +1*1)/20=**0.05**	**1**	**1**	1	0	Get them all right **1** $19^{0}*{}_1C_1=1*1=1$

App 7Table 38. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The mean	**0**	**361**	0	2	Get them all wrong 19^2= **361** $19^{2}*{}_2C_0=361*1=361$
Mark=(0*361 +1*38	**1**	**38**	1	1	Get 1 right 1 wrong $19^{1}*{}_2C_1=19*2=$ **38**
+2*1)400=**0.1**	**2**	**1**	2	0	Get them all right **1** $19^{0}*{}_2C_2=1*1=1$

App 7 Table 39. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean	**0**	**6859**	0	3	Get them all wrong 19^3= **6859** $19^{3}*{}_3C_0=6859*1$
Mark=(0*6859 +1*1083 +2*57	**1**	**1083**	1	2	Get 1 right $19^{2}*{}_3C_1=$ 361*3= **1083**
+3*1)/8000=**0.15**	**2**	**57**	2	1	Get 1 wrong $19^{1}*{}_3C_2=19*3=$ **57**
	3	**1**	3	0	Get them all right **1** $19^{0}*{}_3C_3=1*1=1$

App 7 Table 40. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=4.

Number of Stems **n**	**Mark** **M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
4 The mean Mark=(0*130321 +1*27436 + 2*2166 +3*76 +4*1)/160000=**0.2**	**0**	**130321**	0	4	Get them all wrong 19^4= **130321** $19^{4}*{}_{4}C_{0}$=130321*1=130321
	1	**27436**	1	3	Get 1 right 4*19^3 $19^{3}*{}_{4}C_{1}$=6859*4= **27436**
	2	**2166**	2	2	Get 2 right 6*19^2 $19^{2}*{}_{4}C_{2}$=361*6= **2166**
	3	**76**	3	1	Get 1 wrong 4*19 $19^{1}*{}_{4}C_{3}$=19*4= **76**
	4	**1**	4	0	Get them all right **1** $19^{0}*{}_{4}C_{4}$=1*1=1

App 7 Table 41. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=5.

Number of Stems n	**Mark** **M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_{n}C_{w}$
5 The mean Mark=(0*2476099 +1*651605 + 2*68590 +3*3610 +4*95 +5*1)/3200000=**0.25**	**0**	**2476099**	0	5	Get them all wrong 19^5= **2476099** $19^{5}*{}_{5}C_{0}$=59049*1=59049
	1	**651605**	1	4	Get 1 right $19^{4}*{}_{5}C_{1}$=130321*5= **651605**
	2	**68590**	2	3	Get 2 right 10*19^3= **68590** $19^{3}*{}_{5}C_{2}$=6859*10=68590
	3	**3610**	3	2	Get 2 wrong 10*19^2= **3610** $19^{2}*{}_{5}C_{3}$=81* 10 =490
	4	**95**	4	1	Get 1 wrong 5*19= **95** $19^{1}*{}_{5}C_{4}$=7*5
	5	**1**	5	0	Get them all right **1** $19^{0}*{}_{5}C_{5}$=1*1=1

7.2.10 20-option EMSQ (MCQ)-only quizzes where X=19 and negative marking IS used i.e. a=-1/19.

Here, the mean Mark is 0, because the guessing problem is exactly tackled.

App 7 Table 42. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking IS used. Score [1, -1/19]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**-0.0527**	**19**	0	1	Get them all wrong **19** $19^{1*}{}_1C_0$=19*1=19
	1	**1**	1	0	Get them all right **1** $19^{0*}{}_1C_1$=1*1=1

App 7 Table 43. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking IS used. Score [1, -1/19]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**-0.1053**	**361**	0	2	Get them all wrong 19^2= **361** $19^{2*}{}_2C_0$=361*1=361
	1.0526	**38**	1	1	Get 1 right 1 wrong $19^{1*}{}_2C_1$=19*2= **38**
	2	**1**	2	0	Get them all right **1** $19^{0*}{}_2C_2$=1*1=1

App 7 Table 44. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking IS used. Score [1, -1/19]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3	**-0.1579**	**6859**	0	3	Get them all wrong 19^3= **6859** $19^{3*}{}_3C_0$=6859*1
	0.8947	**1083**	1	2	Get 1 right $19^{2*}{}_3C_1$= 361*3= **1083**
	1.9474	**57**	2	1	Get 1 wrong $19^{1*}{}_3C_2$=19*3= **57**
	3	**1**	3	0	Get them all right **1** $19^{0*}{}_3C_3$=1*1=1

App 7 Table 45. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking IS used. Score [1, -1/19]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4	**-0.2105**	**130321**	0	4	Get them all wrong 19^4= **130321** $19^{4*}{}_4C_0$=130321*1=130321
	0.8421	**27436**	1	3	Get 1 right 4*19^3 $19^{3*}{}_4C_1$=6859*4= **27436**
	1.8947	**2166**	2	2	Get 2 right 6*19^2 $19^{2*}{}_4C_2$=361*6= **2166**
	2.9474	**76**	3	1	Get 1 wrong 4*19 $19^{1*}{}_4C_3$=19*4= **76**
	4	**1**	4	0	Get them all right **1** $19^{0*}{}_4C_4$=1*1=1

App 7 Table 46. 20-option EMSQ (MCQ). Stems can NOT be missed. Negative marking IS used. Score [1, -1/19]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5	**-0.2632**	**2476099**	0	5	Get them all wrong 19^5= **2476099** $19^{5*}{}_5C_0$=59049*1=59049
	0.7895	**651605**	1	4	Get 1 right $19^{4*}{}_5C_1$=130321*5= **651605**
	1.8421	**68590**	2	3	Get 2 right 10*19^3= **68590** $19^{3*}{}_5C_2$=6859*10=68590
	2.8947	**3610**	3	2	Get 2 wrong 10*19^2= **3610** $19^{2*}{}_5C_3$=81* 10 =490
	3.9474	**95**	4	1	Get 1 wrong 5*19= **95** $19^{1*}{}_5C_4$=7*5
	5	**1**	5	0	Get them all right **1** $19^{0*}{}_5C_5$=1*1=1

From this, we conclude that the expression $X^{x*}{}_nC_w$ works for X=1, X=3, X=7, X=9 and X=19, for 'small' values of **n**.

7.2.11 20-option EMSQ (MAQ with 10 ticks)-only quizzes where X=184755 and negative marking is NOT used i.e. a=0.

App 7 Table 47. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1 The mean mark is about **0**.	**0**	**184755**	0	1	Get them all wrong **184755** $187455^{1*}{}_1C_0$=187455*1=187455
	1	**1**	1	0	Get them all right **1** $187455^{0*}{}_1C_1$=1*1=1

App 7 Table 48. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2 The mean mark is about **0**.	**0**	**34134410025**	0	2	Get them all wrong 184755^2=**34134410025** $184755^{2*}{}_2C_0$=
	1	**369510**	1	1	Get 1 right 1 wrong $184755^{1*}{}_2C_1$=184755*2 = **369510**
	2	**1**	2	0	Get them all right **1** $184755^{0*}{}_2C_2$=1*1=1

App 7 Table 49. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3 The mean mark is about **0**.	**0**	**6306502924168875**	0	3	Get them all wrong 184755^3=**6306502924168875** $184755^{3*}{}_3C_0$=
	1	**102403230075**	1	2	Get 1 right $184755^{2*}{}_3C_1$= **102403230075**
	2	**554265**	2	1	Get 1 wrong $184755^{1*}{}_3C_2$=184755*3 = **554265**
	3	**1**	3	0	Get them all right **1** $184755^{0*}{}_3C_3$=1*1=1

App 7 Table 50. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4 The mean mark is about **0**.	**0**	**116515794775482005006255**	0	4	Get them all wrong 184755^4= **116515794775482005006255** $184755^{4*}{}_4C_0$=
	1	**25226011696675500**	1	3	Get 1 right $4*184755^3$ $184755^{3*}{}_4C_1$= **25226011696675500**
	2	**204806460150**	2	2	Get 2 right $6*184755^2$ $184755^{2*}{}_4C_2$= **204806460150**
	3	**739020**	3	1	Get 1 wrong 4*184755 $184755^{1*}{}_4C_3$= **739020**
	4	**1**	4	0	Get them all right **1** $184755^{0*}{}_4C_4$=1*1=1

App 7 Table 51. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking is NOT used. Score [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5 The mean mark is about **0**.	**0**		0	5	Get them all wrong 184755^5=**215268756637441861592971875** $184755^{5*}{}_5C_0$=
	1		1	4	Get 1 right $184755^{4*}{}_5C_1$=**5825789738774102503125**
	2	**63065029241688750**	2	3	Get 2 right $10*184755^3$ = **63065029241688750** $184755^{3*}{}_5C_2$
	3	**341341100250**	3	2	Get 2 wrong $10*184755^2$= $184755^{2*}{}_5C_3$= **341341100250**
	4	**923775**	4	1	Get 1 wrong 5*184755= **923775** $184755^{1*}{}_5C_4$
	5	**1**	5	0	Get them all right **1** $184755^{0*}{}_5C_5$=1*1=1

7.2.12 20-option EMSQ (MAQ with 10 ticks)-only quizzes where X=184755 and negative marking IS used i.e. a=-1/184755.

Here, the mean Mark is 0, because the guessing problem is exactly tackled, but the mean Mark was about 0 before the guessing problem was tackled.

App 7 Table 52. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking IS used. Score [1, -1/184755]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
1	**0**	**184755**	0	1	Get them all wrong **184755** $187455^{1*}{}_1C_0$=187455*1=187455
	1	**1**	1	0	Get them all right **1** $187455^{0*}{}_1C_1$=1*1=1

App 7 Table 53. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking IS used. Score [1, -1/184755]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
2	**0**	**34134410025**	0	2	Get them all wrong 184755^2=**34134410025** $184755^{2*}{}_2C_0$=
	1	**369510**	1	1	Get 1 right 1 wrong $184755^{1*}{}_2C_1$=184755*2 = **369510**
	2	**1**	2	0	Get them all right **1** $184755^{0*}{}_2C_2$=1*1=1

App 7 Table 54. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking IS used. Score [1, -1/184755]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
3	**0**	**6306502924168875**	0	3	Get them all wrong 184755^3=**6306502924168875** $184755^{3}*{}_3C_0$=
	1	**102403230075**	1	2	Get 1 right $184755^{2}*{}_3C_1$= **102403230075**
	2	**554265**	2	1	Get 1 wrong $184755^{1}*{}_3C_2$=184755*3 = **554265**
	3	**1**	3	0	Get them all right **1** $184755^{0}*{}_3C_3$=1*1=1

App 7 Table 55. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking IS used. Score [1, -1/184755]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
4	**0**	**1165157947754820050 0625**	0	4	Get them all wrong 184755^4= **11651579477548200500625** $184755^{4}*{}_4C_0$=
	1	**25226011696675500**	1	3	Get 1 right 4*184755^3 $184755^{3}*{}_4C_1$= **25226011696675500**
	2	**204806460150**	2	2	Get 2 right 6*184755^2 $184755^{2}*{}_4C_2$= **204806460150**
	3	**739020**	3	1	Get 1 wrong 4*184755 $184755^{1}*{}_4C_3$= **739020**
	4	**1**	4	0	Get them all right **1** $184755^{0}*{}_4C_4$=1*1=1

App 7 Table 56. 20-option EMSQ (MAQ with 10 ticks). Stems can NOT be missed. Negative marking IS used. Score [1, -1/184755]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get a mark**	**w**	**x**	**Comment** $X^{x*}{}_nC_w$
5	**0**		0	5	Get them all wrong 184755^5=**215268756637441861592971875** $184755^{5*}{}_5C_0$=
	1		1	4	Get 1 right $184755^{4*}{}_5C_1$=**5825789738774102503125**
	2	**63065029241688750**	2	3	Get 2 right 10*184755^3 = **63065029241688750** $184755^{3*}{}_5C_2$
	3	**341341100250**	3	2	Get 2 wrong 10*184755^2= $184755^{2*}{}_5C_3$= **341341100250**
	4	**923775**	4	1	Get 1 wrong 5*184755= **923775** $184755^{1*}{}_5C_4$
	5	**1**	5	0	Get them all right **1** $184755^{0*}{}_5C_5$=1*1=1

From this, we conclude that the expression $X^{x*}{}_nC_w$ works for X=1, X=3, X=7, X=9 , X=19 and X=184755, for 'small' values of **n**.

7.3 Part 2

We have written an Excel™ program to help with our analysis of special quizzes where all stems are answered. This is held in the file **MG.xlsx** and is described here. The main part of the file looks like this

	No. Right	No.wrong	Mark	Grade	
D 10	0	100	0	Fail	G 10
	1	99	1	Fail	
	2	98	2	Fail	
	3	97	3	Fail	
	4	96	4	Fail	
	5	95	5	Fail	
	6	94	6	Fail	
	7	93	7	Fail	
	8	92	8	Fail	
	9	91	9	Fail	
	97	3	97	First	
	98	2	98	First	
	99	1	99	First	
D 110	100	0	100	First	D 110
		Mean	5		
		No. of Fails	40		
		No.of Passes	10	F 113	
		No. of 2.2s	10		
		No. of 2.1s	10		
		No. of Firsts	31		
		Total	101	F 117	

App 7 Figure 1. Part of the Excel™ program.

The number of stems that are answered correctly is varied from 0 to 100. This is achieved by putting 0 in D10 and then pressing Ctrl and dragging down to D 110.

The number of stems that are answered in-correctly is calculated in E10 to be 100-D10. This formula i.e. =100-D10 is copied down to E110.

The mark that corresponds to this is calculated in F10 as =D10*1 + E10*$C$7 and this formula is copied down to F110.

This demands that the *constant* in C7 is looked at. Constants in Excel™ are where dollar signs are put round the column letter of a cell.

Pass Mark		C6	40
a		C7	0
X		C8	19

We see that this Excel™ program needs 3 things to be changed viz. the contents of C6, C7 and C8. C6 represents the **P**ass **M**ark (**PM**) and in this case is 40. C7 represents the value of negative marking, **a**, and in this case is 0. C8 represents X and in this case is 19 (e.g..a 6-option MAQ with 3 ticks)

We assume that **c**=1 and so

Mark=(number of stems answered correctly)*1 + (number of stems answered in-correctly) * **a**

and so D10*1 + E10*$C$7

The **Mean** in F111 is n*(1+X*a)/(X +1) which in Excel™'s terms is 100*(1 + $C$8*$C$7)/($C$8 +1) when **n**=100.

(A better way of doing this is by using names and Names Manager in Formulas).

The Total number of Fails is how many times the word Fail appears and is calculated in F112 by =COUNTIF(G10:G110, "Fail")..
The Total number of Passes is how many times the word Pass appears and is calculated in F113 by =COUNTIF(G10:G110, "Pass").
The Total number of 2.2s is how many times the string 2.2 appears and is calculated in F114 by =COUNTIF(G10:G110, "2.2").
The Total number of 2.1s is how many times the string 2.1 appears and is calculated in F115 by =COUNTIF(G10:G110, "2.1").
The Total number of 1^{st}s is how many times the 1^{st} appears and is calculated in F116 by =COUNTIF(G10:G110, "First").

A check is that everything should add up to 101 when n=100. We do this check in F117 by =SUM(F112:F116).

All we have to do is calculate when a Fail, 2.2, 2.1 and 1^{st} occurs. We do this in G10 and copy the formula down to G110. The calculation depends on the highest pass mark being calculated. This depends on the Pass Mark and the accuracy of Excel™. The highest pass mark is calculated in G6 and uses the formula

=IF($C$6 <50,49.99999999, IF($C$6 <60,59.9999999999, IF($C$6 <70, 69.99999999999, IF($C$6 <80, 79.9999999999, IF($C$6 <90, 89.9999, 89.9999)))))

This says that the highest pass mark is 49.9999999 if the Pass Mark is less than 50

otherwise it is 59.99999999999
otherwise it is 69.99999999999
otherwise it is 79.99999999999
otherwise it is 89.99999999999

We assume that the pass mark is between 50 and 100, and that the IF statement stops at the 1st condition it finds.

Fail, Pass, 2.2, 2.1 and 1st can be calculated in G10 and this formula can be copied down to G110

=IF(F10<$C$6,"Fail",IF(AND(F10>=$C$6,F10<$G$6+0),"Pass",IF(AND(F10>= $C$6,F10<$G$6+10), "2.2", IF(AND(F10>=$C$6,F10<$G$6+20),"2.1", IF(AND(F10>=$C$6,F10<$G$6+30), "First", "First")))))

We have to learn

- How to handle constants
- How to copy an increment a cell
- How IF works
- How COUNTIF works

This Excel™ program can now be used to produce Tables (see later) which can be checked by 'hand'.

One of the difficulties of raising the pass mark is whether one should go from a fail to a 2.2 if there are no passes. For example, if one raises the pass mark to 69 and **a**=-1/2, the gap between marks is |1.5| and so the marks include

78 stems are answered correctly and 22 incorrectly, the mark is 78-22/2=66
79 stems are answered correctly and 21 incorrectly, the mark is 78-21/2=67.5
80 stems are answered correctly and 20 incorrectly, the mark is 80-20/2=70

The top mark for a pass grade is 69.99999999 and so there are *no* passes.

The results of this Excel™ program are shown in the Tables below

App 7 Table 60. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using Yes/No-only quiz.

Quiz type			**Value of *a***					
Yes/No-only	**Pass Mark**	**Measures**	**0**	**-0.25**	**-0.5**	**-0.75**	**-1**	**-1.25**
	40	μ_{100} M_g	50 [10,10,10,31]	37.5 [8,8,8,25,]	25 [7,7,7,21]	12.5 [6,6,6,18]	0 [5,5,5,16]	-12.5 [4,5,4,14]
	50	μ_{100} M_g	50 [10,10,10,21]	37.5 [8,8,8,17]	25 [7,6,7,14]	12.5 [6,5,6,12]	0 [5,5,5,11]	-12.5 [5,5,4,9]
	70	μ_{100} M_g	50 [10,10,10,1]	37.5 [8,8,8,1]	25 [7,7 6,1]	12.5 [6,6,5,1]	0 [5,5,5,1]	-12.5 [5,4,4,1]
	71	μ_{100} M_g	50 [9,10,10,1]	37.5 [7,7,8,1]	25 [6,7,6,1]	12.5 [5,6,5,1]	0 [4,5,5,1]	-12.5 [4,4,4,1]
	72	μ_{100} M_g	50 [8,10,10,1]	37.5 [6,8,8,1]	25 [5,7,6,1]	12.5 [5,6,5,1]	0 [4,5,5,1]	-12.5 [4,4,4,1]
	73	μ_{100} M_g	50 [7,10,10,1]	37.5 [5,8,8,1]	25 [5,7,6,1]	12.5 [4,6,5,1]	0 [3,5,5,1]	-12.5 [4,4,4,1]
	74	μ_{100} M_g	50 [6,10,10,1]	37.5 [4,8,8,1]	25 [4,7,6,1]	12.5 [3,6,5,1]	0 [3,5,5,1]	-12.5 [3,4,4,1]
	80	μ_{100} M_g	50 [10,10,1,0]	37.5 [8,8,1,0]	25 [7,6,1,0]	12.5 [6,5,1,0]	0 [5,5,1,0]	-12.5 [4,4,1,0]

We, partially, validate this table by choosing a square at random, **a**=-0.5 and the Pass Mark is raised to 73, say (shown in grey above)

μ_{100}=100*(1+**a**)/2 and when **a**=-0.5 we have μ_{100}=25 ✓

The marks should include

73, 74.5, 76, 77.5, 79	**5** 3rd marks
80.5, 82, 83.5, 85, 86.5, 88, 89.5	**7** 2.2 marks
91, 92.5, 94, 95.5, 97, 98.5	**6** 2.1 marks
100	**1** 1st mark

So, M_g should be [5, 7, 6, 1] ✓

The mark 71.5 would come from 81 stems being answered correctly and 19 answered in-correctly. The mark therefore is 81*1 -19*1/2=81-9.5=71.5.

The mark 73 comes from 82 stems being answered correctly and 18 answered in-correctly. The mark therefore is 82*1 -18*1/2=82-9=73.

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-0.5 the gap between successive marks=|1.5|. ✓

There are 101 marks, -50 to +100 in steps of 1.5 (see Appendix 5).

Thus, this table is partially validated.

App 7 Table 61. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 4-option MCQ-only quiz.

Quiz type			Value of *a*					
4 -opt. MCQ	**Pass Mark**	**Measures**	**0**	**-1/12**	**-2/12**	**-3/12**	**-1/3**	**-1**
	40	μ_{100} M_g	25 [10,10, 10,31]	18.75 [9,10, 9,28]	12.5 [9,8,9,26]	6.25 [8,8,8,25]	0 [8,7,8,23]	-50 [5,5,5 ,16]
	50	μ_{100} M_g	25 [10,10, 10,21]	18.75 [9,10, 9,19]	12.5 [9,8,9,18]	6.25 [8,8,8,17]	0 [7,8,7,16]	-50 [5,5,5 ,11]
	70	μ_{100} M_g	25 [10,10, 10,1]	18.75 [9,9,9,1]	12.5 [8,9,8,1]	6.25 [8,8,8,1]	0 [7,8,7,1]	-50 [5,5,5 ,1]
	71	μ_{100} M_g	25 [9,10, 10,1]	18.75 [8,9,9,1]	12.5 [7,9,8,1]	6.25 [7,8,8,1]	0 [6,8,7,1]	-50 [4,5,5 ,1]
	72	μ_{100} M_g	25 [8,10, 10,1]	18.75 [7,9,9,1]	12.5 [7,9,8,1]	6.25 [6,8,8,1]	0 [6,8,7,1]	-50 [4,5,5 ,1]
	73	μ_{100} M_g	25 [7,10, 10,1]	18.75 [6,9,9,1]	12.5 [6,9,8,1]	6.25 [5,8,8,1]	0 [5,8,7,1]	-50 [3,5,5 ,1]
	74	μ_{100} M_g	25 [6,10, 10,1]	18.75 [6,9,9,1]	12.5 [5,9,8,1]	6.25 [4,8,8,1]	0 [4,8,7,1]	-50 [3,5,5 ,1]
	80	μ_{100} M_g	25 [10,10, 1,0]	18.75 [9,9,1,0]	12.5 [9,8 ,1,0]	6.25 [8,8,1,0]	0 [8,7,1,0]	-50 [5,5,1 ,0]

Going down this table for a fixed **a** we would not expect the *mean* mark to change even though the Pass Mark changes (and it does not change). Going across the table for a fixed Pass Mark, we would expect the mean to change (and it does). For a 4-option MCQ-only quiz we would expect the mean mark to be 0 when **a**=-1/3 (and it is). Further validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=3 and **a**=-1

we have μ_{100}=100(1-3)/4=-50 ✓

The marks should include

80, 82, 84, 86, 88, 89.5	**5** Pass marks
90, 92, 94, 96, 98,	**5** 2.2 marks
100	**1** 2.1 mark

So, M_g should be [5, 5, 1, 0]

✓

The mark 78 would come from 89 stems being answered correctly and 11 answered in-correctly. The mark therefore is 89*1 -11*1=89-11=78.

The mark 80 comes from 90 stems being answered correctly and 10 answered in-correctly. The mark therefore is 90*1 -10*1=90-10=80.

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1.0 the gap between successive marks=|2| ✓

There are 101 marks, -100 to +100 in steps of 2 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 62. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 5-option MCQ-only quiz.

Quiz type			**Value of *a***					
5-option MCQ	**Pass Mark**		**0**	**-1/16**	**-2/16**	**-3/16**	**-1/4**	**-1**
	40	μ_{100} M_g	20 [10,10,10,31]	15 [9,10,9,29]	10 [9,9, 8,27]	5 [8,9, 8,26]	0 [8,8, 8,25]	-60 [5,5, 5,16]
	50	μ_{100} M_g	20 [10,10,10,21]	15 [10,9, 10,19]	10 [9,9, 8,18]	5 [9,8, 9,17]	0 [8,8, 8,17]	-60 [5,5, 5,11]
	70	μ_{100} M_g	20 [10,10,10,1]	15 [9,9, 10,1]	10 [9,9, 8,1]	5 [9,8, 9,1]	0 [8,8, 8,1]	-60 [5,5, 5,1]
	71	μ_{100} M_g	20 [9,10,10,1]	15 [9,9, 10,1]	10 [8,9, 8,1]	5 [8,8, 8,1]	0 [7,8, 8,1]	-60 [4,5, 5,1]
	72	μ_{100} M_g	20 [8,10,10,1]	15 [8,9, 9,1]	10 [7,9, 8,1]	5 [7,8, 8,1]	0 [6,8, 8,1]	-60 [4,5, 5,1]
	73	μ_{100} M_g	20 [7,10,10,1]	15 [7,9, 9,1]	10 [7,9, 8,1]	5 [6,8, 8,1]	0 [5,8, 8,1]	-60 [3,5, 5,1]
	74	μ_{100} M_g	20 [6,10,10,1]	15 [6,9, 9,1]	10 [6,9, 8,1]	5 [5,8, 8,1]	0 [4,8, 8,1]	-60 [3,5, 5,1]
	80	μ_{100} M_g	20 [10,10,1,0]	15 [9,9, 1,0]	10 [9,8, 1,0]	5 [8,8, 1,0]	0 [8,8,1,0]	-60 [5,5, 1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=4 and **a**=-3/16

we have μ_{100}=100(1-4.3/16)/5=100.4/16.5=5 ✓

The marks should include

73.875, 75.0625, 76.25, 77.4375, 78.625, 79.8125 **6** Pass marks

81, 82.175, 83.375, 84.5625, 85.75, 86.9375, 88.125, 89.3125

8 2.2 marks

90.5, 91.6875, 92.875, 94.0625, 95.25, 96.4375, 97.625, 98.8125,

8 2.1 mark

100 **1** 1st mark

So, $\mathbf{M_g}$ should be [6, 8, 8, 1] ✓

The mark 73.875 comes from 78 stems being answered correctly and 22 answered in-correctly. The mark therefore is 78*1 -22*3/16= 78-4.125=73.875

The mark 75.0625 comes from 79 stems being answered correctly and 21 answered in-correctly. The mark therefore is 79*1 -21*3/16= 79-3.9375=75.0625

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-3/16 the gap between successive marks=|1.1875| ✓

There are 101 marks, -18.75 to +100 in steps of 1.1875 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 63. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 6-option MCQ-only quiz.

Quiz type			**Value of *a***					
6-option MCQ	**PM**	**Mean**	**0**	**-1/20**	**-2/20**	**-3/20**	**-1/5**	**-1**
	40	μ_{100} $\mathbf{M_g}$	16.67 [10,10, 10, 31]	12.5 [10,9,10, 29]	8.33 [9,9,9,28]	4.167 [9,9,8,27]	0 [9,8,8,26]	-66.67 [5,5,5,16]
	50	μ_{100} $\mathbf{M_g}$	16.67 [10,10,10,21]	12.5 [9,10,9,20]	8.33 [9,9,9,19]	4.167 [9,8,9,18]	0 [8,8,9,17]	-66.67 [5,5,5,11]
	70	μ_{100} $\mathbf{M_g}$	16.67 [10,10,10,1]	12.5 [9,10,9,1]	8.33 [9,9,9,1]	4.167 [9,9,8,1]	0 [9,8,8,1]	-66.67 [5,5,5,1]
	71	μ_{100} $\mathbf{M_g}$	16.67 [9,10,10,1]	12.5 [8,10,9,1]	8.33 [8,9,9,1]	4.167 [8,9,8,1]	0 [8,8,8,1]	-66.67 [4,5,5,1]
	72	μ_{100} $\mathbf{M_g}$	16.67 [8,10,10,1]	12.5 [7,10,9,1]	8.33 [7,9,9,1]	4.167 [7,9,8,1]	0 [7,8,8,1]	-66.67 [4,5,5,1]
	73	μ_{100} $\mathbf{M_g}$	16.67 [7,10,10,1]	12.5 [6,10,9,1]	8.33 [6,9,9,1]	4.167 [6,9,8,1]	0 [6,8,8,1]	-66.67 [3,5,5,1]
	74	μ_{100} $\mathbf{M_g}$	16.67 [6,10,10,1]	12.5 [5,10,9,1]	8.33 [5,9,9,1]	4.167 [5,9,8,1]	0 [5,8,8,1]	-66.67 [3,5,5,1]
	80	μ_{100} $\mathbf{M_g}$	16.67 [10,10,1,0]	12.5 [10,9,1,0]	8.33 [9,9,1,0]	4.167 [9,8,1,0]	0 [8,8,1,0]	-66.67 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=5 and **a**=-2/20

we have μ_{100}=100(1-5*0.1)6=50/6=8.33 ✓

The marks should include

72.5, 73.6, 74.7, 75.8, 76.9, 78, 79.1	**7** Pass marks
80.2, 81.3, 82.4, 83.5, 84.6. 85.7, 86.8, 87.9, 89	**9** 2.2 marks
90.1, 91.2, 92.3. 93.4, 94.5, 95.6, 96.7, 97.8, 98.9	**9** 2.1 mark
100	**1** 1st mark

So, $\mathbf{M_g}$ should be [7, 9, 9, 1] ✓

The mark 72.5 comes from 75 stems being answered correctly and 25 answered in-correctly. The mark therefore is 75*1 -25*2/20=75-2.5=72.5

The mark 73.6 comes from 76 stems being answered correctly and 24 answered in-correctly. The mark therefore is 76*1 -24*2/20=76-2.4=73.6

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-2/20 the gap between successive marks=|1.1| ✓

There are 101 marks, -10 to +100 in steps of 1.1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 64. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 7-option MCQ-only quiz.

Quiz type			Value of *a*					
7-option MCQ	PM	Mean	0	-1/24	-2/24	-3/24	-1/6	-1
	40	μ_{100} M_g	14.29 [10,10, 10, 31]	10.71 [9,10,10, 29]	7.14 [9,10,9,28]	3.57 [9,9,8,27]	0 [9,8,8,26]	-71.43 [5,5,5,16]
	50	μ_{100} M_g	14.29 [10,10,10,21]	10.71 [10,10,9,20]	7.14 [10,9,9,19]	3.57 [9,9,9,18]	0 [8,9,8,18]	-71.43 [5,5,5,11]
	70	μ_{100} M_g	14.29 [10,10,10,1]	10.71 [9,10,9,1]	7.14 [9,9,9,1]	3.57 [9,9,9,1]	0 [8,9,8,1]	-71.43 [5,5,5,1]
	71	μ_{100} M_g	14.29 [9,10,10,1]	10.71 [8,10,9,1]	7.14 [8,9,9,1]	3.57 [8,9,8,1]	0 [7,9,8,1]	-71.43 [4,5,5,1]
	72	μ_{100} M_g	14.29 [8,10,10,1]	10.71 [7,10,9,1]	7.14 [7,9,9,1]	3.57 [7,9,8,1]	0 [7,9,8,1]	-71.43 [4,5,5,1]
	73	μ_{100} M_g	14.29 [7,10,10,1]	10.71 [6,10,9,1]	7.14 [6,9,9,1]	3.57 [7,9,8,1]	0 [6,9,8,1]	-71.43 [3,5,5,1]
	74	μ_{100} M_g	14.29 [7,10,10,1]	10.71 [5,10,9,1]	7.14 [6,9,9,1]	3.57 [6,9,8,1]	0 [5,9,8,1]	-71.43 [3,5,5,1]
	80	μ_{100} M_g	14.29 [10,10,1,0]	10.71 [10,9,1,0]	7.14 [9,9,1,0]	3.57 [9,8,1,0	0 [9,8,1,0]	-71.43 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=6 and **a**=-1/24

we have μ_{100}=100(1-6/24)/7=100*18/24*7=300/28=10.71 ✓

The marks should include

75, 76+1/24, 77 + 2/24, 78 + 3/24, 79 + 4/25 **5** Pass marks

80 + 5/24, 81 +6/24, 82 +7/24, 83 +8/24, 84 + 9/24, 85 +10/24, 86 +11/24, 87 +12/24, 88 + 13/24, 89 +14/24 **10** 2.2 marks

90 + 15/24, 91 + 16/24, 92 + 17/24, 93 + 18/24, 94 + 19/24, 95 +20/24, 96 + 21/24, 97 + 22/24, 98 + 23/24 **9** 2.1 marks

100 **1** 1st mark

So, M_g should be [5, 10, 9, 1] ✓

The previous mark 73.95833 comes from 75 stems being answered correctly and 25 answered in-correctly. The mark therefore is 75*1 - 25*1/24=73.95833 Fail

The mark 75 comes from 76 stems being answered correctly and 24 answered in-correctly. The mark therefore is 76*1 -24*1/24=76-1=75

The mark 76 +1/24 comes from 77 stems being answered correctly and 23 answered in-correctly. The mark therefore is 77*1 -23*1/24=76 +1/24

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1/24 the gap between successive marks=|1 + 1/24| ✓

There are 101 marks, -100/24= -4.167 to +100 in steps of 1+1/24 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 65. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 8-option MCQ-only quiz.

Quiz type			Value of *a*					
8-option MCQ	PM	Mean	0	-1/28	-2/28	-3/28	-1/7	-1
	40	μ_{100} M_g	12.5 [10,10,10,31]	9.375 [9,10,10, 29]	6.25 [10,9,9,29]	3.125 [9,9,9,28]	0 [9,8,9,27]	-75 [5,5,5,16]
	50	μ_{100} M_g	12.5 [10,10,10,21]	9.375 [10,10,9, 20]	6.25 [9,9,10,19]	3.125 [9,9,9,19]	0 [8,9,9,18]	-75 [5,5,5,11]
	70	μ_{100} M_g	12.5 [10,10,10,1]	9.375 [9,10,9, 1]	6.25 [10,9,9,1]	3.125 [9,9,9,1]	0 [9,9,8,1]	-75 [5,5,5,1]
	71	μ_{100} M_g	12.5 [9,10,10,1]	9.375 [9,10,9, 1]	6.25 [9,9,9,1]	3.125 [8,9,9,1]	0 [8,9,8,1]	-75 [4,5,5,1]
	72	μ_{100} M_g	12.5 [8,10,10,1]	9.375 [8,10,9, 1]	6.25 [8,9,9,1]	3.125 [7,9,9,1]	0 [7,9,8,1]	-75 [4,5,5,1]
	73	μ_{100} M_g	12.5 [7,10,10,1]	9.375 [7,10,9, 1]	6.25 [7,9,9,1]	3.125 [6,9,9,1]	0 [6,9,8,1]	-75 [3,5,5,1]
	74	μ_{100} M_g	12.5 [6,10,10,1]	9.375 [6,10,9, 1]	6.25 [6,9,9,1]	3.125 [5,9,9,1]	0 [5,9,8,1]	-75 [3,5,5,1]
	80	μ_{100} M_g	12.5 [10,10,1,0]	9.375 [10,9, 1,0]	6.25 [9,9,1,0]	3.125 [9,9,1,0]	0 [9,8,1,0]	-75 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=7 and **a**=0

we have μ_{100}=100(1-7*0)/8=100/8=12.5 ✓

The marks should include

71, 72, 73, 74, 75, 76, 77, 78, 79	**9** Pass marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** 2.2 marks
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	**10** 2.1 marks
100	**1** 1st mark

So, M_g should be [9, 10, 10, 1] ✓

The previous mark 70 comes from 70 stems being answered correctly and 30 answered in-correctly. The mark therefore is 70*1 - 30*0=70. Fail

The mark 71 comes from 71 stems being answered correctly and 29 answered in-correctly. The mark therefore is 71*1 -29*0=71

The mark 72 comes from 72 stems being answered correctly and 28 answered in-correctly. The mark therefore is 72*1 -28*0=72

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 66. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 5-option MAQ-only quiz with 2 ticks.

Quiz type			**Value of *a***					
5-option MAQ with 2 ticks	**PM**	**Mean**	**0**	**-1/36**	**-2/36**	**-3/36**	**-1/9**	**-1**
	40	μ_{100} M_g	10 [10,10,10,31]	7.5 [10,10,9,30]	5 [10,10,9,29]	2.5 [9,10,9,28]	0 [9,8,9,28]	-80 [5,5,5,16]
	50	μ_{100} M_g	10 [10,10,10,21]	7.5 [10,9,10,20]	5 [10,10,9,19]	2.5 [10,9,9,19]	0 [9,9,9,19]	-80 [5,5,5,11]
	70	μ_{100} M_g	10 [10,10,10,1]	7.5 [10,10,9,1]	5 [10,10,9,1]	2.5 [9,9,9,1]	0 [9,9,9,1	-80 [5,5,5,1]
	71	μ_{100} M_g	10 [9,10,10,1]	7.5 [9,10,9,1]	5 [9,9,9,1]	2.5 [8,9,9,1]	0 [8,9,9,1	-80 [4,5,5,1]
	72	μ_{100} M_g	10 [8,10,10,1]	7.5 [8,10,9,1]	5 [8,9,9,1]	2.5 [7,9,9,1]	0 [7,9,9,1	-80 [4,5,5,1]
	73	μ_{100} M_g	10 [7,10,10,1]	7.5 [7,10,9,1]	5 [7,9,9,1]	2.5 [6,9,9,1]	0 [6,9,9,1	-80 [3,5,5,1]
	74	μ_{100} M_g	10 [6,10,10,1]	7.5 [6,10,9,1]	5 [6,9,9,1]	2.5 [6,9,9,1]	0 [5,9,9,1	-80 [3,5,5,1]
	80	μ_{100} M_g	10 [10,10,1,0]	7.5 [10,9,1,0]	5 [9,9,1,0]	2.5 [9,9,1,0]	0 [9,9,1,0]	-80 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=9 and **a**=0

we have μ_{100}=100(1-9*0)/10=100/10=10 ✓

The marks should include

40, 41, 42, 43, 44, 45, 46, 47, 48, 49 — **10** Pass marks
50, 51, 52, 53, 54, 55, 56, 57, 58, 59 — **10** 2.2 marks
60, 61, 62, 63, 64, 65, 66, 67, 68, 69 — **10** 2.1 marks
70, 71, 72, 73, 74, 75, 76, 77, 78, 79 — **31** 1st marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89
90, 91, 92, 93, 94, 95, 96, 97, 98, 99
100

So, M_g should be [10, 10, 10, 31] ✓

The previous mark 39 comes from 39 stems being answered correctly and 61 answered in-correctly. The mark therefore is 39*1 - 61*0=39. Fail

The mark 40 comes from 40 stems being answered correctly and 60 answered in-correctly. The mark therefore is 40*1 -60*0=40

The mark 41 comes from 41 stems being answered correctly and 59 answered in-correctly. The mark therefore is 41*1 -59*0=41

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 67. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 6-option MAQ-only quiz with 3 ticks.

Quiz type			**Value of *a***					
6-option MAQ with 3 ticks	**PM**	**Mean**	**0**	**-1/76**	**-2/76**	**-3/76**	**-1/19**	**-1**
	40	μ_{100} M_g	5 [10,10,10,31]	3.75 [10,10,10,30]	2.5 [10,10,9,30]	1.25 [9,10,10,29]	0 [10,9,10,29]	-90 [5,5,5,16]
	50	μ_{100} M_g	5 [10,10,10,21]	3.75 [10,10,10,20]	2.5 [10,9,10,20]	1.25 [10,10,9,20]	0 [9,10,9,20]	-90 [5,5,5,11]
	70	μ_{100} M_g	5 [10,10,10,1]	3.75 [10,10,9,1]	2.5 [10,10,9,1]	1.25 [9,10,9,1]	0 [9,10,9,1]	-90 [5,5,5,1]
	71	μ_{100} M_g	5 [9,10,10,1]	3.75 [9,10,9,1]	2.5 [9,10,9,1]	1.25 [8,10,9,1]	0 [8,10,9,1]	-90 [4,5,5,1]
	72	μ_{100} M_g	5 [8,10,10,1]	3.75 [8,10,9,1]	2.5 [8,10,9,1]	1.25 [7,10,9,1]	0 [7,10,9,1]	-90 [4,5,5,1]
	73	μ_{100} M_g	5 [7,10,10,1]	3.75 [7,10,9,1]	2.5 [7,10,9,1]	1.25 [6,10,9,1]	0 [6,10,9,1]	-90 [3,5,5,1]
	74	μ_{100} M_g	5 [6,10,10,1]	3.75 [6,10,9,1]	2.5 [6,10,9,1]	1.25 [6,10,9,1]	0 [5,10,9,1]	-90 [3,5,5,1]
	80	μ_{100} M_g	5 [10,10,1,0]	3.75 [10,9,1,0]	2.5 [10,9,1,0]	1.25 [10,9,1,0]	0 [10,9,1,0]	-90 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=19 and **a**=0

we have μ_{100}=100(1-19*0)/20=100/20=5 ✓

The marks should include

50, 51, 52, 53, 54, 55, 56, 57, 58, 59 — **10** Pass marks
60, 61, 62, 63, 64, 65, 66, 67, 68, 69 — **10** 2.2 marks
70, 71, 72, 73, 74, 75, 76, 77, 78, 79 — **10** 2.1 marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89 — **12** 1st marks

90, 91, 92, 93, 94, 95, 96, 97, 98, 99
100

So, $\mathbf{M_g}$ should be [10, 10, 10, 21] ✓

The previous mark 49 comes from 49 stems being answered correctly and 51 answered in-correctly. The mark therefore is 49*1 - 51*0=49. Fail

The mark 50 comes from 50 stems being answered correctly and 50 answered in-correctly. The mark therefore is 50*1 -50*0=50

The mark 51 comes from 51 stems being answered correctly and 49 answered in-correctly. The mark therefore is 51*1 -49*0=51

The gap between successive marks is $|1 - \mathbf{a}|$ (see Appendix 5) and as $\mathbf{a}$=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 68. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 7-option MAQ-only quiz with 4 ticks.

Quiz type			Value of *a*					
	PM		0	-1/136	-2/136	-3/136	-1/34	-1
7-option MAQ with 4 ticks	40	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[10,10,10,31]	[10,10,10,30]	[10,10,10,30]	[10,9,10,30]	[10,10,9,30]	[5,5,5,16]
	50	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[10,10,10,21]	[10,10,10,20]	[10,10,10,20]	[9,10,10,20]	[10,9,10,20]	[5,5,5,11]
	70	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[10,10,10,1]	[10,10,9,1]	[10,10,9,1]	[10,9,9,1]	[10,10,9,1]	[5,5,5,1]
	71	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[9,10,10,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[4,5,5,1]
	72	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[8,10,10,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[4,5,5,1]
	73	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[7,10,10,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[3,5,5,1]
	74	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[6,10,10,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[3,5,5,1]
	80	μ_{100}	2.86	2.14	1.43	0.71	0	-94.29
		M_g	[10,10,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=34 and **a**=0

we have μ_{100}=100(1-34*0)/35=100/35=2.86 ✓

The marks should include

70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**10** Pass marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** 2.2 marks
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	**10** 2.1 marks
100	**1** 1st mark

So, M_g should be [10, 10, 10, 1] ✓

The previous mark 69 comes from 69 stems being answered correctly and 31 answered in-correctly. The mark therefore is 69*1 - 31*0=69. Fail

The mark 70 comes from 70 stems being answered correctly and 30 answered in-correctly. The mark therefore is 70*1 -30*0=70

The mark 71 comes from 71 stems being answered correctly and 29 answered in-correctly. The mark therefore is 71*1 -29*0=71

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 69. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 8-option MAQ-only quiz with 5 ticks.

Quiz type			**Value of *a***					
8-option MAQ with 5 ticks	**PM**		**0**	**-1/220**	**-2/220**	**-3/220**	**-1/55**	**-1**
	40	μ_{100} M_g	1.79 [10,10,10,31]	1.34 [10,10,10,30]	0.89 [10,10,10,30]	0.446 [10,10,10,30]	0 [9,10,10,30]	-96.43 [5,5,5,16]
	50	μ_{100} M_g	1.79 [10,10,10,21]	1.34 [10,10,10,20]	0.89 [10,10,10,20]	0.446 [10,10,10,20]	0 [10,10,10,20]	-96.43 [5,5,5,11]
	70	μ_{100} M_g	1.79 [10,10,10,1]	1.34 [10,10,9,1]	0.89 [10,10,9,1]	0.446 [10,10,9,1]	0 [10,10,9,1]	-96.43 [5,5,5,1]
	71	μ_{100} M_g	1.79 [9,10,10,1]	1.34 [9,10,9,1]	0.89 [9,10,9,1]	0.446 [9,10,9,1]	0 [9,10,9,1]	-96.43 [4,5,5,1]
	72	μ_{100} M_g	1.79 [8,10,10,1]	1.34 [8,10,9,1]	0.89 [8,10,9,1]	0.446 [8,10,9,1]	0 [8,10,9,1]	-96.43 [4,5,5,1]
	73	μ_{100} M_g	1.79 [7,10,10,1]	1.34 [7,10,9,1]	0.89 [7,10,9,1]	0.446 [7,10,9,1]	0 [7,10,9,1]	-96.43 [3,5,5,1]
	74	μ_{100} M_g	1.79 [6,10,10,1]	1.34 [6,10,9,1]	0.89 [6,10,9,1]	0.446 [6,10,9,1]	0 [6,10,9,1]	-96.43 [3,5,5,1]
	80	μ_{100} M_g	1.79 [10,10,1,0]	1.34 [10,9,1,0]	0.89 [10,9,1,0]	0.446 [10,9,1,0]	0 [10,9,1,0]	-96.43 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=55 and **a**=0

we have μ_{100}=100(1-55*0)/56=100/356= 1.79 ✓

The marks should include

72, 73, 74, 75, 76, 77, 78, 79	**8** Pass marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** 2.2 marks
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	**10** 2.1 marks
100	**1** 1st mark

So, M_g should be [8, 10, 10, 1] ✓

The previous mark 71 comes from 71 stems being answered correctly and 29 answered in-correctly. The mark therefore is 71*1 - 29*0=71. Fail

The mark 72 comes from 72 stems being answered correctly and 28 answered in-correctly. The mark therefore is 72*1 -28*0=72

The mark 73 comes from 73 stems being answered correctly and 27 answered in-correctly. The mark therefore is 73*1 -27*0=73

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 70. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 4-option Multiple Yes/No-only quiz.

Quiz type			**Value of *a***					
4-option Multiple	**PM**		**0**	**-1/60**	**-2/60**	**-3/60**	**-1/15**	**-1**
	40	μ_{100} M_g	6.25 [10,10,10,31]	4.69 [10,10,10,30]	3.125 [10,10,9,30]	1.5625 [10,9,9,29]	0 [10,9,9,29]	-87.5 [5,5,5,16]
	50	μ_{100} M_g	6.25 [10,10,10,21]	4.69 [10,10,10,20]	3.125 [10,9,10,20]	1.5625 [9,10,9,20]	0 [9,9,10,19]	-87.5 [5,5,5,11]
	70	μ_{100} M_g	6.25 [10,10,10,1]	4.69 [10,10,9,1]	3.125 [10,10,9,1]	1.5625 [9,10,9,1]	0 [10,9,9,1]	-87.5 [5,5,5,1]
	71	μ_{100} M_g	6.25 [9,10,10,1]	4.69 [9,10,9,1]	3.125 [9,10,9,1]	1.5625 [8,10,9,1]	0 [9,9,9,1]	-87.5 [4,5,5,1]
	72	μ_{100} M_g	6.25 [8,10,10,1]	4.69 [8,10,9,1]	3.125 [8,10,9,1]	1.5625 [7,10,9,1]	0 [8,9,9,1]	-87.5 [4,5,5,1]
	73	μ_{100} M_g	6.25 [7,10,10,1]	4.69 [7,10,9,1]	3.125 [7,10,9,1]	1.5625 [6,10,9,1]	0 [7,9,9,1]	-87.5 [3,5,5,1]
	74	μ_{100} M_g	6.25 [6,10,10,1]	4.69 [6,10,9,1]	3.125 [6,10,9,1]	1.5625 [5,10,9,1]	0 [6,9,9,1]	-87.5 [3,5,5,1]
	80	μ_{100} M_g	6.25 [10,10,1,0]	4.69 [10,9,1,0]	3.125 [10,9,1,0]	1.5625 [10,9,1,0]	0 [9,9,1,0]	-87.5 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=15 and **a**=0

we have μ_{100}=100(1-15*0)/36=100/36=6.25 ✓

The marks should include

40, 41, 42, 43, 44, 45, 46, 47, 48, 49	**10** Pass marks
50, 51, 52, 53, 54, 55, 56, 57, 58, 59	**10** 2.2 marks
60, 61, 62, 63, 64, 65, 66, 67, 68, 69	**10** 2.1 marks
70, 71, 72, 73, 74, 75, 76, 77, 78, 79	**31** 1st marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	
100	

So, $\mathbf{M_g}$ should be [7, 10, 10, 1] ✓

The previous mark 72 comes from 72 stems being answered correctly and 28 answered in-correctly. The mark therefore is 72*1 - 28*0=72. Fail

The mark 73 comes from 73 stems being answered correctly and 27 answered in-correctly. The mark therefore is 73*1 -27*0=73

The mark 74 comes from 74 stems being answered correctly and 26 answered in-correctly. The mark therefore is 74*1 -26*0=74

The gap between successive marks is $|1 - \mathbf{a}|$ (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 71. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 5-option Multiple Yes/No-only quiz.

Quiz type			Value of *a*					
	PM		0	-1/124	-2/124	-3/124	-1/31	-1
5-option Multiple	40	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[10,10,10,31]	[10,10,10,30]	[10,10,10,30]	[10,9,10,30]	[10,10,9,30]	[5,5,5,16]
	50	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[10,10,10,21]	[10,10,10,20]	[10,10,10,20]	[9,10,10,20]	[10,9,19,20]	[5,5,5,11]
	70	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[10,10,10,1]	[10,10,9,1]	[10,10,9,1]	[10,10,9,1]	[10,10,9,1]	[5,5,5,1]
	71	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[9,10,10,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[4,5,5,1]
	72	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[8,10,10,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[4,5,5,1]
	73	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[7,10,10,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[3,5,5,1]
	74	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[6,10,10,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[3,5,5,1]
	80	μ_{100}	3.125	2.34	1.5625	0.78125	0	-93.75
		M_g	[10,10,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=31 and **a**=0

we have $\boldsymbol{\mu_{100}}$=100(1-31*0)/32=100/32=2.125 ✓

The marks should include

74, 75, 76, 77, 78, 79	**6** Pass marks
80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** 2.2 marks
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	**10** 2.1 marks
100	**1** 1st mark

So, $\mathbf{M_g}$ should be [6, 10, 10, 1] ✓

The previous mark 73 comes from 73 stems being answered correctly and 27 answered in-correctly. The mark therefore is 73*1 - 27*0=73. Fail

The mark 74 comes from 74 stems being answered correctly and 26 answered in-correctly. The mark therefore is 74*1 -26*0=74

The mark 75 comes from 75 stems being answered correctly and 25 answered in-correctly. The mark therefore is 75*1 -25*0=75

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 72. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 6-option Multiple Yes/No-only quiz.

Quiz type			Value of *a*					
6-option Multiple	**PM**		**0**	**-1/252**	**-2/252**	**-3/252**	**-1/63**	**-1**
	40	μ_{100} $\mathbf{M_g}$	1.5625 [10,10,10,31]	1.17 [10,10,10,30]	0.78125 [10,10,10,30]	0.39 [10,10,10,30]	0 [10,10,10,30]	-98.88 [5,5,5,16]
	50	μ_{100} $\mathbf{M_g}$	1.5625 [10,10,10,21]	1.17 [10,10,10,20]	0.78125 [10,10,10,20]	0.39 [10,10,10,20]	0 [10,10,10,20]	-98.88 [5,5,5,11]
	70	μ_{100} $\mathbf{M_g}$	1.5625 [10,10,10,1]	1.17 [10,10,9,1]	0.78125 [10,10,9,1]	0.39 [10,10,9,1]	0 [10,10,9,1]	-98.88 [5,5,5,1]
	71	μ_{100} $\mathbf{M_g}$	1.5625 [9,10,10,1]	1.17 [9,10,9,1]	0.78125 [9,10,9,1]	0.39 [9,10,9,1]	0 [9,10,9,1]	-98.88 [4,5,5,1]
	72	μ_{100} $\mathbf{M_g}$	1.5625 [8,10,10,1]	1.17 [8,10,9,1]	0.78125 [8,10,9,1]	0.39 [8,10,9,1]	0 [8,10,9,1]	-98.88 [4,5,5,1]
	73	μ_{100} $\mathbf{M_g}$	1.5625 [7,10,10,1]	1.17 [7,10,9,1]	0.78125 [7,10,9,1]	0.39 [7,10,9,1]	0 [7,10,9,1]	-98.88 [3,5,5,1]
	74	μ_{100} $\mathbf{M_g}$	1.5625 [6,10,10,1]	1.17 [6,10,9,1]	0.78125 [6,10,9,1]	0.39 [6,10,9,1]	0 [6,10,9,1]	-98.88 [3,5,5,1]
	80	μ_{100} $\mathbf{M_g}$	1.5625 [10,10,1,0]	1.17 [10,9,1,0]	0.78125 [10,9,1,0]	0.39 [10,9,1,0]	0 [10,9,1,0]	-98.88 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=63 and **a**=0

we have μ_{100}=100(1-63*0)/64=100/64=1.5625 ✓

The marks should include

80, 81, 82, 83, 84, 85, 86, 87, 88, 89	**10** Pass marks
90, 91, 92, 93, 94, 95, 96, 97, 98, 99	**10** 2.2 marks
100	**1** 2.1 mark

So, $\mathbf{M_g}$ should be [10, 10, 1, 0] ✓

The previous mark 79 comes from 79 stems being answered correctly and 21 answered in-correctly. The mark therefore is 79*1 - 21*0=79. Fail

The mark 80 comes from 80 stems being answered correctly and 20 answered in-correctly. The mark therefore is 80*1 -20*0=80

The mark 81 comes from 81 stems being answered correctly and 19 answered in-correctly. The mark therefore is 81*1 -19*0=81

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=0 the gap between successive marks=|1| ✓

There are 101 marks, 0 to +100 in steps of 1 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 73. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 7-option Multiple Yes/No-only quiz.

Quiz type			Value of *a*					
7-option Multiple	PM		0	-1/508	-2/508	-3/508	-1/127	-1
	40	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[10,10,10,31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[5,5,5,16]
	50	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[10,10,10,21]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[10,10,10,20]	[5,5,5,11]
	70	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[10,10,10,1]	[10,10,9,1]	[10,10,9,1]	[10,10,9,1]	[10,10,9,1]	[5,5,5,1]
	71	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[9,10,10,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9.1]	[9,10,9,1]	[4,5,5,1]
	72	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[8,10,10,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9.1]	[8,10,9,1]	[4,5,5,1]
	73	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[7,10,10,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[3,5,5,1]
	74	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[6,10,10,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[3,5,5,1]
	80	μ_{100}	0.78125	0.586	0.39	0.195	0	-98.44
		M_g	[10,10,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[10,9,1,0]	[5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=127 and **a**=-1

we have μ_{100}=100(1-127*1)/128=-100*126/128=-98.44 ✓

The marks should include

40, 42, 44, 46, 48 — **5** Pass marks

50, 52, 54, 56, 58 — **5** 2.2 marks

60, 62, 64, 66, 68 — **5** 2.1 marks

70, 72, 74, 76, 78 — **16** 1st marks

80, 82, 84, 86, 88

90, 92, 94, 96, 98

100

So, M_g should be [5, 5, 5, 16] ✓

The previous mark 39 comes from 69 stems being answered correctly and 31 answered in-correctly. The mark therefore is 69*1 - 31*1=38. Fail

The mark 40 comes from 70 stems being answered correctly and 30 answered in-correctly. The mark therefore is 70*1 -30*1=40

The mark 42 comes from 71 stems being answered correctly and 29 answered in-correctly. The mark therefore is 71*1 -29*1=42

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1 the gap between successive marks=|2| ✓

There are 101 marks, -100 to +100 in steps of 2 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 74. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 8-option Multiple Yes/No-only quiz.

Quiz type			Value of *a*					
8-option Multiple	**PM**		**0**	**-1/1020**	**-2/1020**	**-3/1020**	**-1/255**	**-1**
	40	μ_{100} M_g	0.39 [10,10,10,31]	0.29 [10,10,10,30]	0.195 [10,10,10,30]	0.098 [10,10,10,30]	0 [10,10,10,30]	-99.22 [5,5,5,16]
	50	μ_{100} M_g	0.39 [10,10,10,21]	0.29 [10,10,10,20]	0.195 [10,10,10,20]	0.098 [10,10,10,20]	0 [10,10,10,20]	-99.22 [5,5,5,11]
	70	μ_{100} M_g	0.39 [10,10,10,1]	0.29 [10,10,9,1]	0.195 [10,10,9,1]	0.098 [10,10,9,1]	0 [10,10,9,1]	-99.22 [5,5,5,1]
	71	μ_{100} M_g	0.39 [9,10,10,1]	0.29 [9,10,9,1]	0.195 [9,10,9,1]	0.098 [9,10,9,1]	0 [9,10,9,1]	-99.22 [4,5,5,1]
	72	μ_{100} M_g	0.39 [8,10,10,1]	0.29 [8,10,9,1]	0.195 [8,10,9,1]	0.098 [8,10,9,1]	0 [8,10,9,1]	-99.22 [4,5,5,1]
	73	μ_{100} M_g	0.39 [7,10,10,1]	0.29 [7,10,9,1]	0.195 [7,10,9,1]	0.098 [7,10,9,1]	0 [7,10,9,1]	-99.22 [3,5,5,1]
	74	μ_{100} M_g	0.39 [6,10,10,1]	0.29 [6,10,9,1]	0.195 [6,10,9,1]	0.098 [6,10,9,1]	0 [6,10,9,1]	-99.22 [3,5,5,1]
	80	μ_{100} M_g	0.39 [10,10,1,0]	0.29 [10,9,1,0]	0.195 [10,9,1,0]	0.098 [10,9,1,0]	0 [10,9,1,0]	-99.22 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=255 and **a**=-1

we have μ_{100}=100(1-255*1)/256=-100*254/256=-99.21875 ✓

The marks should include

50, 52, 54, 56, 58	**5** Pass marks
60, 62, 64, 66, 68	**5** 2.2 marks
70, 72, 74, 76, 78	**5** 2.1 marks
80, 82, 84, 86, 88	**11** 1^{st} marks
90, 92, 94, 96, 98	
100	

So, $\mathbf{M_g}$ should be [5, 5, 5, 11] ✓

The previous mark 48 comes from 74 stems being answered correctly and 26 answered in-correctly. The mark therefore is 74*1 -26*1=48. Fail

The mark 50 comes from 75 stems being answered correctly and 25 answered in-correctly. The mark therefore is 75*1 -25*1=50

The mark 52 comes from 76 stems being answered correctly and 24 answered in-correctly. The mark therefore is 76*1 -24*1=52

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1 the gap between successive marks=|2| ✓

There are 101 marks, -100 to +100 in steps of 2 (see Appendix 5)

Thus, this table is partially validated.

App 7 Table 75. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 20-option EMSQ-only quiz (MCQ).

Quiz type			**Value of *a***					
20-option EMSQ (MCQ)	**PM**		**0**	**-1/76**	**-2/76**	**-3/76**	**-1/19**	**-1**
	40	μ_{100} M_g	5 [10,10,10,31]	3.75 [10,10,10,30]	2.5 [10,10,9,30]	1.25 [9,10,10,29]	0 [10,9,10,29]	-90 [5,5,5,16]
	50	μ_{100} M_g	5 [10,10,10,21]	3.75 [10,10,10,20]	2.5 [10,9,10,20]	1.25 [10,10,9,20]	0 [9,10,9,20]	-90 [5,5,5,11]
	70	μ_{100} M_g	5 [10,10,10,1]	3.75 [10,10,9,1]	2.5 [10,10,9,1]	1.25 [9,10,9,1]	0 [9,10,9,1]	-90 [5,5,5,1]
	71	μ_{100} M_g	5 [9,10,10,1]	3.75 [9,10,9,1]	2.5 [9,10,9,1]	1.25 [8,10,9,1]	0 [8,10,9,1]	-90 [4,5,5,1]
	72	μ_{100} M_g	5 [8,10,10,1]	3.75 [8,10,9,1]	2.5 [8,10,9,1]	1.25 [7,10,9,1]	0 [7,10,9,1]	-90 [4,5,5,1]
	73	μ_{100} M_g	5 [7,10,10,1]	3.75 [7,10,9,1]	2.5 [7,10,9,1]	1.25 [6,10,9,1]	0 [6,10,9,1]	-90 [3,5,5,1]
	74	μ_{100} M_g	5 [6,10,10,1]	3.75 [6,10,9,1]	2.5 [6,10,9,1]	1.25 [6,10,9,1]	0 [5,10,9,1]	-90 [3,5,5,1]
	80	μ_{100} M_g	5 [10,10,1,0]	3.75 [10,9,1,0]	2.5 [10,9,1,0]	1.25 [10,9,1,0]	0 [10,9,1,0]	-90 [5,5,1,0]

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=19 and **a**=-1

we have μ_{100}=100(1-19*1)/20=-100*19/20=-90 ✓

The marks should include

70, 72, 74, 76, 78	**5** Pass marks
80, 82, 84, 86, 88	**5** 2.2marks
90, 92, 94, 96, 98	**5** 2.1 marks
100	**1** 1st marks

So, M_g should be [5, 5, 5, 1] ✓

The previous mark 68 comes from 84 stems being answered correctly and 16 answered in-correctly. The mark therefore is 84*1 - 16*1=68. Fail

The mark 70 comes from 85 stems being answered correctly and 15 answered in-correctly. The mark therefore is 85*1 -15*1=70

The mark 72 comes from 86 stems being answered correctly and 14 answered in-correctly. The mark therefore is 86*1 -14*1=72

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1 the gap between successive marks=|2| ✓

There are 101 marks, -100 to +100 in steps of 2 (see Appendix 5)

Thus, this table is partially validated, and it should be the same as a 6-option MAQ quiz with 3 ticks (see App 7 Table)

App 7 Table 76. Tackling the guessing problem by a) using negative marking and b) raising the pass mark and c) using a 20-option EMSQ-only quiz (MAQ with 10 ticks).

Quiz type			**Value of *a***					
	PM		**0**	**-1/A**	**-2/A**	**-3/A**	**-1/184755**	**-1**
20-option EMSQ (MAQ with 10 ticks)	40	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[10,10,10,31]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[10,10,10,30]	[5,5,5,16]
	50	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[10,10,10,21]	[10,10,10,20]	[10,10,10,20]	[10,10,9,20]	[10,10,10,20]	[5,5,5,11]
	70	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[10,10,10,1]	[10,10,9,1]	[10,10,9,1]	[9,10,9,1]	[10,10,9,1]	[5,5,5,1]
	71	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[9,10,10,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[9,10,9,1]	[4,5,5,1]
	72	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[8,10,10,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[8,10,9,1]	[4,5,5,1]
	73	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[7,10,10,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[7,10,9,1]	[3,5,5,1]
	74	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[6,10,10,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[6,10,9,1]	[3,5,5,1]
	80	μ_{100}	about 0	about 0	about 0	about 0	0	-100
		M_g	[10,10,1,0]	[10,9,1,0]	[9,10,1,0]	[10,9,1,0]	[10,9,1,0]	[5,5,1,0]

where **A**=4*184755=4*($_{20}C_{10}$ -1)

Validation of this table is done when we take a cell (marked above) and check its contents by hand.

μ_{100}=100*(1+X**a**)/(X +1) when X=$_{20}C_{10}$-1 and **a**=-1

we have μ_{100}=100(1-($_{20}C_{10}$-1)*1)/20=-100*19/20=-about 100 ✓

The marks should include

72, 74, 76, 78	**4** Pass marks
80, 82, 84, 86, 88	**5** 2.2marks
90, 92, 94, 96, 98	**5** 2.1 marks
100	**1** 1^{st} marks

So, $\mathbf{M_g}$ should be [4, 5, 5, 1] ✓

The previous mark 70 comes from 85 stems being answered correctly and 15 answered in-correctly. The mark therefore is 85*1 - 15*1=70. Fail

The mark 72 comes from 86 stems being answered correctly and 14 answered in-correctly. The mark therefore is 86*1 -14*1=72

The mark 74 comes from 87 stems being answered correctly and 13 answered in-correctly. The mark therefore is 87*1 -13*1=74

The gap between successive marks is |1 -**a**| (see Appendix 5) and as **a**=-1 the gap between successive marks=|2| ✓

There are 101 marks, -100 to +100 in steps of 2 (see Appendix 5)

Thus, this table is partially validated.

7.4 Discussion

The expression $X^{x*}{}_{n}C_{w}$ has been validated for many values of X but each for 'small' values of **n**.

The Excel™ program that helps us calculate $\mathbf{M_g}$ has been validated for many quiz types, and for many values of **a** and for many values of the Pass Mark.

Appendix 8. Validation of analysis of quizzes where stems can be MISSED.

8.1 Introduction

We only consider, in this Appendix, special quizzes where stems can be missed.

One of the purposes of validation of our Analysis is to show that the Analysis is correct 'as far as we can tell'. We check on a *subset* and then believe our Analysis is correct for the *whole set*. The sort of quiz that we validate is where stems can be missed.

We are validating that the number getting a mark in a particular way is $X^x * {}_nC_y * {}_{n-y}C_w$ where X is number of answers to *a stem* that are wrong and (X +1) reflects the quiz type. For example, when

X=7 we have a Multiple Yes/No with 3 options or
we have an 8-option MCQ.

We assume that a) there is only 1 right answer, and that X cannot be 0 b) the number of ticks, **T,** is not 0, and does not equal **P**

The validation, by hand calculation, can only be done for 'small' sets. We validate for 'small' **n**. After that we assume that the expression is true for 'large' sets. We will validate for

X=1 and **n**=1 to 5	Yes/No-only quizzes
X=3 and **n**=1 to 5	4-option MCQ-only quizzes
X=7 and **n**=1 to 5	3-option Multiple Yes/No-only quizzes
X=9 and **n**=1 to 5	5-option MAQ with 2 ticks-only quizzes
X=19 and **n**=1 to 5	20-option EMSQ (MCQ)-only quizzes or a 6-option MAQ with 3 ticks-only
X=184755 and **n**=1 to 5	20-option EMSQ (MAQ with 10 ticks)-only quizzes

For each of these we try to validate our expressions for a) **a**=**b**=0 b) **b**=0 and **a** has a value that exactly tackles the guessing problem. For example, **a**=-1 for Yes/No quizzes. This means that there are 12 possibilities to check.

In line with the philosophy of this thesis we will start off simply and gradually get more complicated. Thus, we will start off with Yes/No quizzes with 1 stem and then 2 stems, then look at 4-optiion MCQs, then look at 3-option Multiple Yes/Nos and then 5-option MAQs with 2 ticks etc.

The 'grey' column in the tables below is for the *given* **x** (the number of stems that are answered in-correctly)

We, first of all, show how the expression was arrived at, and then validate it on 12 sets.

8.2 Yes/No-only quizzes where X=1 and negative marking is NOT used i.e. a=b=0.

App 8 Table 1. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Comment**
1	0	1	0	0	1	Get them all wrong
	0	1		1	0	Miss them all
The mean Mark=(0*1 +0*1 +1*1)/3=**0.33**	1	1	1	0	0	Get them all right
				1	-1	**Impossible.** You cannot miss them all and get 1 right.

App 8 Table 2. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Comment**
2	0	1	0	0	2	Get them all wrong
The mean Mark=(0*3 +1*4 +2*1)/9=**0.67**		2		1	1	Miss 1 and get the other wrong
		1		2	0	Miss them all
	1	2	1	0	1	Get 1 right and 1 wrong
		2		1	0	Get 1 right and miss 1
				2	-1	**Impossible**
	2	1	2	0	0	Get them all right
				1	-1	**Impossible**
				2	-2	**Impossible**

App 8 Table 3. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Comment**
3 The mean Mark=(0*8 + +1*12 +2*6 +3*1)/27=1	0	1	0	0	3	Get all wrong
		3		1	2	Miss 1 of the 3 and get 0 right of 2
		3		2	1	Get 1 wrong and miss the remainder
		1		3	0	Miss them all
	1	3	1	0	2	1 right and 2 wrong =3
		6		1	1	
		3		2	0	
				3	-1	**Impossible**
	2	3	2	0	1	
		3		1	0	
				2	-1	**Impossible**
				3	-2	**Impossible**
	3	1	3	0	0	Get them all right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**

App 8 Table 4. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get that mark in that manner**	**w**	**y**	**x**	**Comment**
4 The mean Mark=(0*16 + 1*32 + 2*24 +3*8 +4*1)/(16+32 +24 +8+1)=108/81=**1.33**	0	1	0	0	4	Get all wrong
		4		1	3	0 right 1 missed
		6		2	2	0 right 2 missed i.e. missed 1 2, 1 3, 1 4, 2 3, 2 4, 3,4
		4		3	1	0 right 3 missed
		1		4	0	Miss them all
	1	4	1	0	3	1 right 0 missed
		12		1	2	1 right 1 missed i.e. q right q missed 1 2 1 3 1 4 2 1 2 3 2 4 3 1 3 2 3 4 4 1

						4 2 4 3
		12		2	1	1 right 2 missed i.e. q right qs missed 1 2,3 1 2,4 1 3,4 2 1,3 2 1,4 2 3,4 3 1,2 3 1,4 3 2,4 4 1,2 4 1,3 4 2,3
		4		3	0	1 right 3 missed
				4	-1	**Impossible**
	2	6	2	0	2	2 right 0 missed i.e. right 1 2, 1 3, 1 4, 2 3, 2 4, 3,4
		12		1	1	2 right 1 missed i.e. qs right q missed 1,2 3 1,2 4 1,3 2 1,3 4 1,4 2 1,4 3 2,3 1 2,3 4 2,4 1 2,4 3 3,4 1 3,4 2
		6		2	0	2 right 2 missed i.e. qs right qs missed 1,2 3,4 1,3 2,4 1,4 2,3 2,3 1,4 2,4 1,3 3,4 1, 2
				3	-1	**Impossible**
				4	-2	**Impossible**
	3	4	3	0	1	3 right 0 missed i.e

						1 2 3, 1 2 4, 1 3 4, 2 3 4.
		4		1	0	3 right 1 missed
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
	4	1	4	0	0	All right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**
				4	-4	**Impossible**

App 8 Table 5. Yes/No questions. Stems can be missed. Score [0], [1, 0]. **n**=5.

Number of Stems n	**Mark M**	**Number of ways to get that mark in that manner**	**w**	**y**	**x**	**Comment**
5 The mean Mark=(0*32 + 1*80 + 2* 80 +3*40 + 4*10 +5*1)/277=(80+160 +120+40+5)/(32+80 80 + 40 +10 +1)=405/243=**1.67**	0	1	0	0	5	Get all wrong
		5		1	4	0 right 1 missed
		10		2	3	0 right 2 missed i.e. missed 1 2, 1 3, 1 4, 1, 5. 2 3, 2 4, 2,5 3 4 , 3 5, 4 5
		10		3	2	0 right 3 missed i.e. missed 1 2 3, 1 2 4, 1 2 5, 1 3 4, 1 3 5 2 3 1, 2 3 4, 2 3 5 2 4 5 3 4 5
		5		4	1	0 right 4 missed. 1 answered wrongly.
		1		5	0	Miss them all
	1	5	1	0	4	1 right 0 missed
		20		1	3	1 right 1 missed i.e. q r qm qr qm 1 2 1 3 1 4 1 5 2 1 2 3 2 4 2 5 3 1 3 2 3 4 3 5 4 1 4 2 4 3 4 5 5 1 5 2 5 3 5 4
		30		2	2	1 right 2 missed i.e.

						q right qs missed 1 2, 3 1 2, 4 1 2, 5 1 3, 4 1 3, 5 1 4, 5 2 1, 3 2 1, 4 2 1, 5 2 3, 1 2 3, 4 2 3, 5 3 1, 2 3 1, 4 3 1, 5 3 2, 4 3 2, 5 3 4, 5 4 1, 2 4 1, 3 4 1, 5 4 2, 3 4 2, 5 4 3, 5
		20		3	1	1 right 3 missed i.e qr qsm qr qsm 1 2,3,4 1 2,3,5 1 2,4,5 1 3,4,5 2 1,3,4 2 1,3,5 2 1,4 5 2 3,4,5 3 1,2,4 3 1,2,5 3 1,4,5 3 2,4,5 4 1,2,3 4 1,2,5 4 1,3,5 4 2,3,5 5 1,2,3 5 1,2,4 5 1,3,4 5 2,3,4
		5		4	0	1 right 4 missed
				5	-1	**Impossible**
	2	10		0	3	2 right 0 missed i.e. right 1 2, 1 3, 1 4, 1,5 2 3, 2 4, 2 5 3 4, 3,5 4 5.
		30		1	2	2 right 1 missed i.e. qs right q missed 1,2 3 1,2 4 1,2 5

						1,3 2 1,3 4 1,3 5 1,4 2 1,4 3 1,4 5 1,5 2 1,5 3 1,5 4 2,3 1 2,3 4 2,3 5 2,4 1 2,4 3 2,4 5 2,5 1 2,5 3 2,5 4 3,4 1 3,4 2 3,4 5 3,5 1 3,5 2 3,5 4 4,5 1 4,5 2 4,5 3
		30		2	1	2 right 2 missed i.e. qs right qs missed 1,2 3,4 1,2 3,5 1,2 4,5 1,3 2,4 1,3 2,5 1,3 4,5 1,4 2,3 1,4 2,5 1,4 3,5 1,5 2,3 1,5 2,4 1,5 3,4 2,3 1,4 2,3 1,5 2,3 4,5

						2,4 1,3 2,4 1,5 2,4 3,5 2,5 1,3 2,5 1,4 2,5 3,4 3,4 1,2 3,4 1,5 3,4 2,5 3,5 1,2 3,5 1,4 3,5 2,4 4,5 1,2 4,5 1,3 4,5 3,2
		10		3	0	2 right 3 missed. By symmetry
				4	-1	**Impossible**
				5	-2	**Impossible**
	3	10	3	0	2	3 right 0 missed i.e 1 2 3, 1 2 4, 1 2 5, 1 3 4, 1 3 5 ,1 4 5 2 3 4, 2 3 5 3 4 5 **Missed 1** **Another way of looking at this is there are 2 wrong.** **1 2, 1 3, 1,4 1, 5** **2 3 , 2 4, 2,5** **3,4 3, 5** **4,5**
		20		1	1	3 right 1 missed i.e qm qsr qm qsr 1 2,3,4 1 2,3.5 1 3,4,5 2 1,3,4 2 1,3,5 2 3,4,5 3 1,2,4 3 1,2,5 3 2,4,5 4 1,2,3 4 1,2,5 4 2,3,5 5 1,2,3 5 1,2,4 5 2,3,4
		10		2	0	3 right 2 missed
				3	-1	**Impossible**
				4	-2	**Impossible**
				5	-3	**Impossible**
	4	5	4	0	1	4 right 0 missed i.e. 1 2 3 4, 1 2 3 5, 1 2 4 5, 1 3 4 5,

						2, 3, 4, 5
		5		1	0	4 right 1 missed i.e. miss 1 question in turn q missed qs right 1 2, 3, 4, 5 2 1, 3, 4, 5 3 1, 2, 4, 5 4 1, 2, 3, 5 5 1 ,2, 3, 4
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
				5	-4	**Impossible**
	5	1	5	0	0	All right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**
				4	-4	**Impossible**
				5	-5	**Impossible**

Checking becomes a nightmare above **n**=5. There are just too many combinations, that checking any further is out of the question.

At last, we see the pattern to be the number of scores at a particular possible mark, got in a particular way and it is $X^{x*}{}_{n}C_{y}*{}_{n-y}C_{w}$

If x is -ve then we have an impossible situation.

We arrived at that pattern by first of all considering how many stems are missed **($_{n}C_{y}$)** and then considering how many answers are right from those that are left **($_{n-y}C_{w}$)**

We have to show that this pattern works

App 8 Table 6. Our pattern works for **n**=1 for Yes/No-only quizzes. Score [0], [1, 0].

Number of Stems n	**Mark M**	**Number of ways to get that mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything is 1
1	**0**	**1**	0	0	1	${}_1C_0{}^*{}_1C_0=1^*1=\mathbf{1}$
		1		1	0	${}_1C_1{}^*{}_1C_0=1^*1=\mathbf{1}$
	1	**1**	1	0	0	${}_1C_0{}^*{}_1C_1=1^*1=\mathbf{1}$
				1	-1	**Impossible.**

Here the number of marks is (**n** + 1) i.e. 0 and 1. The gap between successive marks is |1-**a**| and as **a**=0, the gap is 1. But, the proofs of the gap being |1-**a**| and the number of marks only apply when ALL stems are answered. It is true here, but not *always* true. The number of impossible marks is 0 +1 =1. The pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ for X=1, works for **n**=1 for Yes/No-only quizzes where stems can be missed and gives the number that get a mark in a particular way.

App 8 Table 7. Our pattern works for **n**=2 for Yes/No-only quizzes. Score [0], [1, 0**]**.

Number of Stems n	**Mark M**	**Number of ways to get that mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything strictly positive to the 1 is 1
2	0	**1**	0	0	2	${}_2C_0{}^*{}_2C_0=1^*1=\mathbf{1}$
		2		1	1	${}_2C_1{}^*{}_1C_0=2^*1=\mathbf{2}$
		1		2	0	${}_2C_2{}^*{}_0C_0=1^*1=\mathbf{1}$
	1	**2**	1	0	1	${}_2C_0{}^*{}_2C_1=1^*2=\mathbf{2}$
		2		1	0	${}_2C_1{}^*{}_1C_1=2^*1=\mathbf{2}$
				2	-1	**Impossible**
	2	**1**	2	0	0	${}_2C_0{}^*{}_2C_2=1^*1=\mathbf{1}$
				1	-1	**Impossible**
				2	-2	**Impossible**

The number of impossible marks is 0 +1 +2 =3. The pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ for X=1, works for **n**=2 and gives the number that get a mark in a particular way.

App 8 Table 8. Our pattern works for n=3 for Yes/No-only quizzes. Score [0], [1,0].

Number of Stems n	Mark M	Number of ways to get that mark in that manner	w	y	x	Pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything strictly positive to the 1 is 1
3	0	**1**	0	0	3	${}_3C_0{}^*{}_3C_0=1*1=$**1**
		3		1	2	${}_3C_1{}^*{}_2C_0=3*1=$**3**
		3		2	1	${}_3C_2{}^*{}_1C_0=3*1=$**3**
		1		3	0	${}_3C_3{}^*{}_0C_0=1*1=$**1**
	1	**3**	1	0	2	${}_3C_0{}^*{}_3C_1=1*3=$**3**
		6		1	1	${}_3C_1{}^*{}_2C_1=3*2=$**6**
		3		2	0	${}_3C_2{}^*{}_1C_1=3*1=$**3**
				3	-1	**Impossible**
	2	**3**	2	0	1	${}_3C_0{}^*{}_3C_2=1*3=$**3**
		3		1	0	${}_3C_1{}^*{}_2C_2=3*1=$**3**
				2	-1	**Impossible**
				3	-2	**Impossible**
	3	**1**	3	0	0	${}_3C_0{}^*{}_3C_3=1*1=$**1**
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**

The number of impossible marks is 0 +1 +2 + 3 =6. The pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ for X=1, works for **n**=3 and gives the number that get a mark in a particular way.

App 8 Table 9. Our pattern works for **n**=4 for Yes/No-only quizzes. Score [0], [1, 0].

Number of Stems n	Mark M	Number of ways to get that mark in that manner	w	y	x	Pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything strictly positive to the 1 is 1
4	0	**1**	0	0	4	${}_4C_0{}^*{}_4C_0=1*1=$**1**
		4		1	3	${}_4C_1{}^*{}_3C_0=4*1=$**4**
		6		2	2	${}_4C_2{}^*{}_2C_0=6*1=$**6**
		4		3	1	${}_4C_3{}^*{}_1C_0=4*1=$**4**
		1		4	0	${}_4C_4{}^*{}_0C_9=1*1=$**1**
	1	**4**	1	0	3	${}_4C_0{}^*{}_4C_1=1*4=$**4**
		12		1	2	${}_4C_1{}^*{}_3C_1=4*3=$**12**
		12		2	1	${}_4C_2{}^*{}_2C_1=6*2=$**12**
		4		3	0	${}_4C_3{}^*{}_1C_1=4*1=$**4**
				4	-1	**Impossible**
	2	**6**	2	0	2	${}_4C_0{}^*{}_4C_2=1*6=$**6**
		12		1	1	${}_4C_1{}^*{}_3C_2=4*3=$**12**
		6		2	0	${}_4C_2{}^*{}_2C_2=6*1=$**6**
				3	-1	**Impossible**
				4	-2	**Impossible**
	3	**4**	3	0	1	${}_4C_0{}^*{}_4C_3=1*4=$**4**

		4		1	0	$_4C_1{}^*{}_3C_3=4*1=$**4**
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
	4	**1**	4	0	0	$_4C_0{}^*{}_4C_4=1*1=$**1**
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**
				4	-4	**Impossible**

The number of impossible marks is 0 +1 +2 + 3 + 4=10. The pattern $X^{X*}{}_nC_y{}^*{}_{n-y}C_w$ for X=1, works for **n**=4 and gives the number that get a mark in a particular way.

We would expect the programs (tools) that we are going to write, to output that there are 10 impossible situations.

App 8 Table 10. Our pattern works for n=5 for Yes/No-only quizzes. Score [0], [1, 0].

Number of Stems n	Mark M	Number of ways to get that mark in that manner	w	y	x	Pattern $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything strictly positive to the 1 is 1
5	0	**1**	0	0	5	$_5C_0{}^*{}_5C_0$=1*1=**1**
		5	0	1	4	$_5C_1{}^*{}_4C_0$=5*1=**5**
Symmetric		**10**	0	2	3	$_5C_2{}^*{}_3C_0$=10*1=**10**
		10	0	3	2	$_5C_3{}^*{}_2C_0$=10*1=**10**
Sum=**32**		**5**	0	4	1	$_5C_4{}^*{}_1C_0$=5*1=**5**
		1	0	5	0	$_5C_5{}^*{}_0C_0$=1*1=**1**
	1	**5**	1	0	4	$_5C_0{}^*{}_5C_1$=1*5=**5**
Symmetric		**20**	1	1	3	$_5C_1{}^*{}_4C_1$=5*4=**20**
		30	1	2	2	$_5C_2{}^*{}_3C_1$=10*3=**30**
Sum=**80**		**20**	1	3	1	$_5C_3{}^*{}_2C_1$=10*2=**20**
		5	1	4	0	$_5C_4{}^*{}_1C_0$=5*1=**5**
			1	5	-1	**Impossible**
	2	**10**	2	0	3	$_5C_0{}^*{}_5C_2$=1*10=**10**
Symmetric		**30**	2	1	2	$_5C_1{}^*{}_4C_2$=5*6=**30**
		30	2	2	1	$_5C_2{}^*{}_3C_2$=10*3=**30**
Sum=**80**		**10**	2	3	0	$_5C_3{}^*{}_2C_2$=10*1=**10**
			2	4	-1	**Impossible**
			2	5	-2	**Impossible**
	3	**10**	3	0	2	$_5C_0{}^*{}_5C_3$=1*10=**10**
Symmetric		**20**	3	1	1	$_5C_1{}^*{}_4C_3$=5*4=**20**
		10	3	2	0	$_5C_2{}^*{}_3C_3$=10*1=**10**
Sum=**40**			3	3	-1	**Impossible**
			3	4	-2	**Impossible**
			3	5	-3	**Impossible**
	4	**5**	4	0	1	$_5C_0{}^*{}_5C_4$=1*5=**5**
Symmetric		**5**	4	1	0	$_5C_1{}^*{}_4C_4$=5*1=**5**
			4	2	-1	**Impossible**
Sum=**10**			4	3	-2	**Impossible**
			4	4	-3	**Impossible**
			4	5	-4	**Impossible**
	5	**1**	5	0	0	$_5C_0{}^*{}_5C_5$=1*1=**1**
Symmetric			5	1	-1	**Impossible**
			5	2	-2	**Impossible**
Sum=**1**			5	3	-3	**Impossible**
			5	4	-4	**Impossible**
			5	5	-5	**Impossible**

We would expect the programs that we are going to write, to output that there are 15 impossible situations.

We would expect the Pivot Table/Chart for **n**=5 to have 32, 80, 80, 40, 10 and 1 in it, *after* the Pivot is done

Thus, for Yes/No-only quizzes, where stems can be missed, the number getting a mark in a particular way is $X^{X}*{}_{n}C_{y}*{}_{n-y}C_{w}$ and for Yes/No-only quizzes the marks are sorted, but the number getting a particular mark is not added to the number already getting that mark. That is 1 reason that a Pivot should be done.

We have shown that the number getting a mark in a particular manner is $X^{X}*{}_{n}C_{y}*{}_{n-y}C_{w}$ a) for quizzes where stems can be missed b) when X=1, because when X to the anything strictly positive =1 c) **a**=**b**=0

8.3 Yes/No quizzes (X=1)where a modification of negative marking is used where a=-1 and b=0

When **a**=-1 i.e. guessing is exactly tackled, here, and the mean mark is 0.

App 8 Table 11. Yes/No questions. Stems can be missed. Score [0], [1, -1]. **n**=1.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything is 1
1 The mean Mark=(-1*1 +0*1 + 1*1)/3=**0**	**-1**	**1**	0	0	1	Get them all wrong $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{1*}{}_1C_0{}^*{}_1C_0=$ **1**
	0	**1**		1	0	Miss them all $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{0*}{}_1C_1{}^*{}_0C_0=$ **1**
	1	**1**	1	0	0	Get them all right $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{0*}{}_1C_0{}^*{}_1C_1=$ **1**
				1	-1	**Impossible.** You cannot miss them all and get 1 right.

App 8 Table 12. Yes/No questions. Stems can be missed. Score [0], [1, -1]. **n**=2.

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything is 1
2	**-2**	**1**	0	0	2	Get them all wrong $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{2*}{}_2C_0{}^*{}_2C_0=$ **1**
	-1	**2**		1	1	Miss 1 and get the other wrong $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{1*}{}_2C_1{}^*{}_1C_0=1*2*1=$ **2**
	0	**1**		2	0	Miss them all $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{0*}{}_2C_2{}^*{}_0C_0=1*1*1=$ **1**
	0	**2**	1	0	1	Get 1 right and 1 wrong $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1_1{}^*{}_2C_0{}^*{}_2C_1=1*2*1=$ **2**
	1	**2**		1	0	Get 1 right and miss 1 $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{0*}{}_2C_1{}^*{}_1C_0=1*2*1=$**2**
				2	-1	**Impossible**
	2	**1**	2	0	0	Get them all right $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=$ $1^{0*}{}_2C_0{}^*{}_2C_2=1*1*1=$**1**
				1	-1	**Impossible**
				2	-2	**Impossible**

So, we would expect *tool1* (when it is built) to produce

Mark	Number getting that mark that way
-2	1
-1	2
0	1
0	2
1	2
2	1

We then expect Excel™ to a) input the above b) do a Pivot c) output the result of doing a Pivot. We would then 'clean up' the result and run *tool2*.

The result of doing the Pivot should be

-2	1	
-1	2	
0	3	The number of ways of getting mark 0 are added
1	2	
2	1	

tool2 should then calculate the descriptive statistics (and other statistics) of the results of the Pivot.

App 8 Table 13 . Yes/No questions. Stems can be missed. Score [0], [1, -1]. **n=3.**

Number of Stems n	**Mark M**	**Number of ways to get a mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ Here, X=1 and 1 to the anything is 1
3	**-3**	**1**	0	0	3	Get all wrong $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$= $1^{3*}{}_{3}C_{0}{}^{*}{}_{3}C_{0}$= **1**
	-2	**3**		1	2	Miss 1 of the 3 and get 0 right of 2 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$= $1^{2*}{}_{3}C_{1}{}^{*}{}_{2}C_{0}$=1*2*1=**2**
	-1	**3**		2	1	Miss 2 of the 3 and get the remaining 1 wrong $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$= $1^{1*}{}_{3}C_{2}{}^{*}{}_{1}C_{0}$=1*3*1=**3**
	0	**1**		3	0	Miss them all $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$= $1^{0*}{}_{3}C_{3}{}^{*}{}_{0}C_{0}$=1*1*1=**1**
	-1	**3**	1	0	2	Get 1 right and 2 wrong right wrong 1 2, 3 2 1,3 3 1,2

						$X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{2*}{}_{3}C_{0}{}^{*}{}_{3}C_{1}=1*1*3=$**3**
	0	**6**		1	1	1 right, 1 wrong, 1 missed qr qw qm 1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{1*}{}_{3}C_{1}{}^{*}{}_{2}C_{1}=1*3*2=$**6**
	1	**3**		2	0	1 right, 2 missed $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{3}C_{2}{}^{*}{}_{2}C_{2}=1*3*1=$**3**
				3		**Impossible**
	1	**3**	2	0	1	Get 2 right and 1 wrong $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{1*}{}_{3}C_{0}{}^{*}{}_{3}C_{1}=1*1*3=$**3**
	2	**3**		1	0	Get 2 right and 1 missed $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{3}C_{1*2}C_{2}=1*3*1=$ **3**
				2	-1	**Impossible**
				3	-2	**Impossible**
	3	**1**	3	0	0	Get them all right $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{3}C_{0}{}^{*}{}_{3}C_{0}=$ **1**
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**

App 8 Table 14. Yes/No questions. Stems can be missed. Score [0], [1, -1]. **n**=4.

Number of Stems n	**Mark M**	**Number of ways to get that mark in that manner**	**w**	**y**	**x**	**Pattern** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ Here, X=1 and 1 to the anything is 1
4	**-4**	**1**	0	0	4	Get all wrong $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{4*}{}_{4}C_{0}{}^{*}{}_{4}C_{0}=1*1*1=$ **1**
	-3	**4**		1	3	0 right 1 missed 3 wrong $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{3*}{}_{4}C_{1}{}^{*}{}_{3}C_{0}=1*4*1=$ **4**
	-2	**6**		2	2	0 right 2 missed i.e. missed 1 2, 1 3, 1 4, 2 3, 2 4, 3,4 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{2*}{}_{4}C_{2}{}^{*}{}_{2}C_{0}=1*6*1=$ **6**
	-1	**4**		3	1	0 right 3 missed 1 wrong

						$X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^1*{}_4C_3*{}_1C_0=1*4*1=$ **4**
	0	**1**		4	0	Miss them all $X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^0*{}_4C_4*{}_0C_0=1*1*1=$ **1**
	-2	**4**	1	0	3	1 right 0 missed 3 wrong $X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^3*{}_4C_0*{}_4C_1=1*1*4=$ **4**
	-1	**12**		1	2	1 right 1 missed 2 wrong i.e. qr qm qw 1 2 3, 4 1 3 2, 4 1 4 2, 3 2 1 2 3 2 4 3 1 3 2 3 4 4 1 4 2 4 3 $X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^2*{}_4C_1*{}_3C_1=1*4*3=$ **12**
	0	**12**		2	1	1 right 2 missed 1 wrong i.e. q right qs missed 1 2,3 1 2,4 1 3,4 2 1,3 2 1,4 2 3,4 3 1,2 3 1,4 3 2,4 4 1,2 4 1,3 4 2,3 $X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^1*{}_4C_2*{}_1C_1=$ 1*6*1=**6** here n=4, y=3, w=1 so n-y=1
	1	**4**		3	0	1 right 3 missed $X^{x}*{}_nC_y*{}_{n-y}C_w=$ $1^0*{}_4C_3*{}_1C_0=1*4*1=$ **4**
				4	-1	**Impossible**
	0	**6**	2	0	2	2 right 0 missed 2 wrong i.e. right 1 2, 1 3, 1 4, 2 3, 2 4,

						3,4 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{2*}{}_{4}C_{0}{}^{*}$
	1	**12**		1	1	2 right 1 missed 1 wrong i.e. qs right q missed 1,2 3 1,2 4 1,3 2 1,3 4 1,4 2 1,4 3 2,3 1 2,3 4 2,4 1 2,4 3 3,4 1 3,4 2 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{1*}{}_{4}C_{1}{}^{*}$
	2	**6**		2	0	2 right 2 missed 0 wrong i.e. qs right qs missed 1,2 3,4 1,3 2,4 1,4 2,3 2,3 1,4 2,4 1,3 3,4 1, 2 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{4}C_{2}{}^{*}$
				3	-1	**Impossible**
				4	-2	**Impossible**
	2	**4**	3	0	1	3 right 0 missed 1 wrong i.e 1 2 3, 1 2 4, 1 3 4, 2 3 4. $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{1*}{}_{4}C_{0}{}^{*}$
	3	**4**		1	0	3 right 1 missed $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{4}C_{1}{}^{*}$
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
	4	**1**	4	0	0	All right $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}=$ $1^{0*}{}_{4}C_{0}{}^{*}$
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**

				4	-4	**Impossible**

App 8Table 15. Yes/No questions. Stems can be missed. Score [0], [1, -1]. =5.

Number of Stems n	**Mark M**	**Number of ways to get that mark** in that manner	**w**	**y**	**x**	**Pattern** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ Here, X=1 and 1 to the anything is 1
5	-5	1	0	0	5	Get all wrong
	-4	5		1	4	0 right 1 missed, 4 wrong
	-3	10		2	3	0 right 2 missed 3 wrong i.e. missed 1 2, 1 3, 1 4, 1, 5. 2 3, 2 4, 2,5 3 4 , 3 5, 4 5
	-2	10		3	2	0 right 3 missed 2 wrong i.e. missed 1 2 3, 1 2 4, 1 2 5, 1 3 4, 1 3 5 2 3 1, 2 3 4, 2 3 5 2 4 5 3 4 5
	-1	5		4	1	0 right 4 missed. 1 answered wrongly.
	0	1		5	0	Miss them all
	-3	5	1	0	4	1 right 0 missed 4 wrong
	-2	20		1	3	1 right 1 missed 3 wrong i.e. q r qm qr qm 1 2 1 3 1 4 1 5 2 1 2 3 2 4 2 5 3 1 3 2 3 4 3 5 4 1 4 2 4 3 4 5 5 1 5 2 5 3 5 4
	-1	30		2	2	1 right 2 missed 2 wrong i.e. q right qs missed 1 2, 3 1 2, 4 1 2, 5 1 3, 4 1 3, 5 1 4, 5

						2 1, 3 2 1, 4 2 1, 5 2 3, 1 2 3, 4 2 3, 5 3 1, 2 3 1, 4 3 1, 5 3 2, 4 3 2, 5 3 4, 5 4 1, 2 4 1, 3 4 1, 5 4 2, 3 4 2, 5 4 3, 5
	0	20		3	1	1 right 3 missed 1 wrong i.e qr qsm qr qsm 1 2,3,4 1 2,3,5 1 2,4,5 1 3,4,5 2 1,3,4 2 1,3,5 2 1,4 5 2 3,4,5 3 1,2,4 3 1,2,5 3 1,4,5 3 2,4,5 4 1,2,3 4 1,2,5 4 1,3,5 4 2,3,5 5 1,2,3 5 1,2,4 5 1,3,4 5 2,3,4
	1	5		4	0	1 right 4 missed
				5	-1	**Impossible**
	-1	10	2	0	3	2 right 0 missed 3 wrong i.e. right 1 2, 1 3, 1 4, 1,5 2 3, 2 4, 2 5 3 4, 3,5 4 5.
	0	30		1	2	2 right 1 missed 2 wrong i.e. qs right q missed 1,2 3 1,2 4 1,2 5 1,3 2 1,3 4 1,3 5

						1,4 2 1,4 3 1,4 5 1,5 2 1,5 3 1,5 4 2,3 1 2,3 4 2,3 5 2,4 1 2,4 3 2,4 5 2,5 1 2,5 3 2,5 4 3,4 1 3,4 2 3,4 5 3,5 1 3,5 2 3,5 4 4,5 1 4,5 2 4,5 3
	1	30		2	1	2 right 2 missed 1 wrong i.e. qs right qs missed 1,2 3,4 1,2 3,5 1,2 4,5 1,3 2,4 1,3 2,5 1,3 4,5 1,4 2,3 1,4 2,5 1,4 3,5 1,5 2,3 1,5 2,4 1,5 3,4 2,3 1,4 2,3 1,5 2,3 4,5 2,4 1,3 2,4 1,5 2,4 3,5

						2,5 1,3 2,5 1,4 2,5 3,4 3,4 1,2 3,4 1,5 3,4 2,5 3,5 1,2 3,5 1,4 3,5 2,4 4,5 1,2 4,5 1,3 4,5 3,2
	2	10		3	0	2 right 3 missed 0 wrong. By symmetry
				4	-1	**Impossible**
				5	-2	**Impossible**
	1	10	3	0	2	3 right 0 missed 2 wrong i.e 1 2 3, 1 2 4, 1 2 5, 1 3 4, 1 3 5 ,1 4 5 2 3 4, 2 3 5 3 4 5 **Missed 1** **Another way of looking at this is there are 2 wrong.** **1 2, 1 3, 1,4 1, 5** **2 3 , 2 4, 2,5** **3,4 3, 5** **4,5**
	2	20		1	1	3 right 1 missed 1 wrong i.e qm qsr qm qsr 1 2,3,4 1 2,3.5 1 3,4,5 2 1,3,4 2 1,3,5 2 3,4,5 3 1,2,4 3 1,2,5 3 2,4,5 4 1,2,3 4 1,2,5 4 2,3,5 5 1,2,3 5 1,2,4 5 2,3,4
	3	10		2	0	3 right 2 missed
				3	-1	**Impossible**
				4	-2	**Impossible**
				5	-3	**Impossible**
	3	5	4	0	1	4 right 0 missed 1 wrong i.e. 1 2 3 4, 1 2 3 5, 1 2 4 5, 1 3 4 5, 2, 3, 4, 5

	4	5		1	0	4 right 1 missed 0 wrong i.e. miss 1 question in turn q missed qs right 1 2, 3, 4, 5 2 1, 3, 4, 5 3 1, 2, 4, 5 4 1, 2, 3, 5 5 1 ,2, 3, 4
				2	-1	**Impossible**
				3	-2	**Impossible**
				4	-3	**Impossible**
				5	-4	**Impossible**
	5	1	5	0	0	All right
				1	-1	**Impossible**
				2	-2	**Impossible**
				3	-3	**Impossible**
				4	-4	**Impossible**
				5	-5	**Impossible**

We have shown that the number getting a mark in a particular manner is $X^x * {}_nC_y * {}_{n-y}C_w$ a) for quizzes where stems can be missed b) when X=1 because X to the anything =1 c) **a**=**b**=0 d) **b**=0 and **a** has a value that exactly tackles the guessing problem i.e.-1 e) **n**=1,2,3,4,5.

8.4 4-option Multiple-Choice Questions (MCQ) where X=3 and a modification of negative marking is NOT used i.e. a=b=0.

The number of ways of getting a mark in a particular way is given by $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations. So, in the following the 3 is obtained by hand and should be equal to $X^{x*}\,{}_nC_y{}^*{}_{n-y}C_w$ when X=3, **y**=**w**=**x**=0 and **n**=1 i.e. $3^{1*}{}_1C_0{}^*{}_1C_0=3*1*1=3$, and so it is. Thus, the expression has been validated for this 1 example.

App 8 Table 16. 4-option MCQ-only. Stems can be missed. Score is [0], [1 0]. **n**=1.

Number of Stems n	Mark M	Number of ways of getting that mark in that way	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
1 The mean Mark=(0*4 + 1*1)/5=**0.2**	0	**3**	0	0	1	All wrong **3=3^1**
	1	**1**	0	1	0	All right **1**
	0	**1**	1	0	0	All missed **1**
	1		1	1	-1	**Impossible**

App 8 Table 17. 4-option MCQ-only. Stems can be missed. Score is [0], [1 0]. **n**=2

Number of Stems n	Mark M	Number of ways of getting that mark in that way	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
2 The mean Mark=(0*16 +1*8 +2*1)/25=10/25=**0.4**	0	**9**	0	0	2	All wrong **9=3^2**
	1	**6**	0	1	1	1 right out of 2 and the other wrong **6**
	2	**1**	0	2	0	All right **1**

	0	**6**	1	0	1	1 missed and 1 wrong **6**
	1	**2**	1	1	0	1 right out of 2 and the other missed **2**
			1	2	-1	**Impossible**
	0	**1**	2	0	0	All missed **1**
			2	1	-1	**Impossible**
			2	2	-2	**Impossible**

App 8 Table 18. 4-option MCQ-only. Stems can be missed. Score is [0], [1 0]. **n**=3.

Number of Stems n	**Mark M**	**Number of ways of getting that mark in that way**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
3 The mean Mark=(0*64+1*48 + 2*12 + 3*1)/125=75125= **0.6**	0	**27**	0	0	3	All wrong **27=3³**
	1	**27**	0	1	2	1 right, 2 wrong **27**
	2	**9**	0	2	1	2 right 1 wrong **9**
	3	**1**	0	3	0	All right **1**
	0	**27**	1	0	2	1 missed, 2 wrong **27**
	1	**18**	1	1	1	1 missed, 1 right, 1 wrong. For each question right there are 6 possibilities , and there are 3 ways of getting a question right **18**
	2	**3**	1	2	0	1 missed, 2 right **3**
			1	3	-1	**Impossible**
	0	**9**	2	0	1	2 missed, 1 wrong **9**

	1	**3**	2	1	0	2 missed, 1 right **3**
			2	2	-1	**Impossible**
			2	3	-2	**Impossible**
	0	**1**	3	0	0	All missed **1**
			3	1	-1	**Impossible**
			3	2	-2	**Impossible**
			3	3	-3	**Impossible**

App 8Table 19. 4-option MCQ-only. Stems can be missed. Score is [0], [1 0]. **n**=4.

Number of Stems **n**	**Mark** **M**	**Number of ways of getting that mark in that way**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way**
4	0	**81**	0	0	4	All wrong **81** $=3^4$
The mean Mark=(0*255 + 1*256 + 2*96 + 3*16 +4*1)/576=500/625= **0.8**	1	**108**	0	1	3	1 right 3 wrong **108** There are 4 ways of choosing the right answer and in each of these there are 3^3 ways. So, 4*27=108
	2	**54**	0	2	2	2 right 2 wrong **54** There are 6 ways of choosing 2 right and in each of these there are 3^2 ways. So 6*9=54
	3	**12**	0	3	1	1 wrong, 3 right **12**
	4	**1**	0	4	0	All right **1**
	0	**108**	1	0	3	1 miss, 3 wrong **108**

						There are 4 ways of choosing the miss and in each of these there are 3^3 ways. So, 4*27=108
	1	**108**	1	1	2	1 miss, 1 right, 2 wrong **108** There are 4 ways of choosing the miss. In each of these there are 3 ways of choosing the right answer, and in each of these there are 3^2 ways of choosing the wrong answer. So, $4*3*3^2=108$
	2	**36**	1	2	1	1 missed, 2 right, 1 wrong **36** There are 4 ways of choosing which question to miss. In this there are 3 ways of getting 2 questions right. This only leaves 1 question and there are 3 ways to get that wrong, So, 4*3*3=36
	3	**4**	1	3	0	3 right, miss 1 **4**

			1	4	-1	**Impossible**
	0	**54**	2	0	2	2 missed, 2 wrong **54** There are 6 ways of choosing 2 missed and in each of these there are 3^2 ways. So 6*9=54
	1	**36**	2	1	1	2 missed, 1 right, 1 wrong **36** There are 4 ways to choose the right question. In that there are 3 ways of missing 2 questions. That only leaves 1 question and there are 3 ways of getting that wrong. So, 4*3*3=36
	2	**6**	2	2	0	2 right miss 2 **6**
			2	3	-1	**Impossible**
			2	4	-2	**Impossible**
	0	**12**	3	0	1	3 miss 1 wrong **12**
	1	**4**	3	1	0	1 right, miss 3 **4**
			3	2	-1	**Impossible**
			3	3	-2	**Impossible**
			3	4	-3	**Impossible**
	0	**1**	4	0	0	All missed **1**
			4	1	-1	**Impossible**
			4	2	-2	**Impossible**
			4	3	-3	**Impossible**
			4	4	-4	**Impossible**

App8 Table 20. Some of the simpler situations for **n**=5 (i.e. not all filled in)

Number of Stems **n**	**Mark** **M**	**Number of ways of getting that mark in that way**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way**
5	0	**243**	0	0	5	All wrong **243=3^5**
	1	**405**	0	1	4	1 right 4 wrong **405** There are 5 ways of choosing the right answer and in each of these are 3^4 ways of getting a wrong answer. So, $5*3^4=5*81=405$
	2	**270**	0	2	3	2 right 3 wrong. **270** There are 10 ways of getting 2 right and in each of these there are 3^3 ways of getting wrong answers. So, $10*3^3=10*27=270$
	3	**90**	0	3	2	3 right 2 wrong **90** There are 10 ways of getting 3 answers right, and in each of these there are 3^2 ways of getting wrong answers. So, $10*3^2=10*9=90$
	4	**15**	0	4	1	4 right 1 wrong **15**
	5	**1**	0	5	0	All right **1**
	0		1	0	4	
	1		1	1	3	
	2		1	2	2	
	3		1	3	1	
	4	**5**	1	4	0	4 right missed 1 **5**
			1	5	-1	**Impossible**
	0		2	0	3	
	1		2	1	2	
	2		2	2	1	**1 wrong 2 right**
	3	**10**	2	3	0	2 missed 3 right **10**

			2	4	-1	**Impossible**
			2	5	-2	**Impossible**
	0		3	0	2	
	1		3	1	1	
	2	**10**	3	2	0	3 missed 2 right **10** Got by writing out all the members of the set 1 2 3, 1 2 4, 1 2 5 1 3 4, 1 3 5 1 4 5 2 3 4, 2 3 5 2 4 5 3 4 5
			3	3	-1	**Impossible**
			3	4	-2	**Impossible**
			3	5	-3	**Impossible**
	0	**15**	4	0	1	4 missed 1 wrong **15**
	1	**5**	4	1	0	4 out of 5 missed, 1 right **5**
			4	2	-1	**Impossible**
			4	3	-2	**Impossible**
			4	4	-3	**Impossible**
			4	5	-4	**Impossible**
	0	**1**	5	0	0	All missed **1**
			5	1	-1	**Impossible**
			5	2	-2	**Impossible**
			5	3	-3	**Impossible**
			5	4	-4	**Impossible**
			5	5	-5	**Impossible**

We see that, for 4-option-only MCQ quizzes the number getting a particular mark in a particular manner is $3^{x*}{}_{n}C_{y}*{}_{n-y}C_{w}$ (as far as we can tell) and for these MCQ quizzes the marks are NOT sorted.

That is another reason for a Pivot to be done viz. the marks should be sorted.

8.5 4-option Multiple-Choice Questions (MCQ) where X=3 and a modification of negative marking IS used. i.e. a=-1/3 and b=0

Let us assume that guessing is *exactly* tackled and then **a**=-1/3.

The number of ways of getting a mark in a particular way is given by $X^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$ and this is the expression that is used to check our 'hand' calculations.

The mean Mark should be 0 because the guessing problem is *exactly* tackled.

App 8Table 21. 4-option MCQ-only. Stems can be missed.
Score is [0], [1, -1/3]. **n**=1.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
1	-1/3	0	0	1	All wrong **$3=3^1$**
The mean	1	0	1	0	All right **1**
Mark=	0	1	0	0	All missed **1**
(-1/3*3 + 1*1 +0*1)/3=**0**		1	1	-1	**Impossible**

App 8 Table 22. 4-option MCQ-only. Stems can be missed.
Score is [0], [1, -1/3]. **n**=2

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
2	-2/3	0	0	2	All wrong **$9=3^2$**
The mean	+2/3	0	1	1	1 right out of 2 and the other wrong **6**
Mark=	2	0	2	0	All right **1**
(-2*9/3 +2*6/3 +	-1/3	1	0	1	1 missed and 1 wrong **6**
2*1 -1*6/3 + 1*2	1	1	1	0	1 right out of 2 and the other missed **2**
+0*1)/24=		1	2	-1	**Impossible**
(-6 +4 +2 -	0	2	0	0	All missed **1**
2 + 2+		2	1	-1	**Impossible**
0)/25=**0**		2	2	-2	**Impossible**

App 8 Table 23. 4-option MCQ-only. Stems can be missed. Score is [0], [1, -1/3]. **n**=3

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
3	-3/3	0	0	3	All wrong **27=3^3**
	1/3	0	1	2	1 right, 2 wrong **27**
	1 +2/3	0	2	1	2 right 1 wrong **9**
	3	0	3	0	All right **1**
	-2/3	1	0	2	1 missed, 2 wrong **27**
	0	1	1	1	1 missed, 1 right, 1 wrong. For each question right there are 6 possibilities , and there are 3 ways of getting a question right **18**
	2	1	2	0	1 missed, 2 right **3**
		1	3	-1	**Impossible**
	-1/3	2	0	1	2 missed, 1 wrong **9**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8Table 24. 4-option MCQ-only. Stems can be missed. Score is [0], [1, -1/3]. n=4.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
4	- 4*0.3333 =1.3332	0	0	4	All wrong **81 =3^4**
	1- 3*0.333 about 0	0	1	3	1 right 3 wrong **108** There are 4 ways of choosing the right answer and in each of these there are 3^3 ways. So, 4*27=108
	2- 2*0.3333 =1.3334	0	2	2	2 right 2 wrong **54** There are 6 ways of choosing 2 right and in each of these there are 3^2 ways. So 6*9=54
	3-0.3333 =2.6667	0	3	1	1 wrong, 3 right **12**
	4	0	4	0	All right **1**
	-	1	0	3	1 miss, 3 wrong **108**

	3*0.3333 =-0.9999				There are 4 ways of choosing the miss and in each of these there are 3^3 ways. So, 4*27=108
	1- 2*0.3333 =0.3334	1	1	2	1 miss, 1 right, 2 wrong **108** There are 4 ways of choosing the miss. In each of these there are 3 ways of choosing the right answer, and in each of these there are 3^2 ways of choosing the wrong answer. So, $4*3*3^2$=108
	2-0.3333 =1.6667	1	2	1	1 missed, 2 right, 1 wrong **36** There are 4 ways of choosing which question to miss. In this there are 3 ways of getting 2 questions right. This only leaves 1 question and there are 3 ways to get that wrong, So, 4*3*3=36
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	- 2*0.3333 =-0.6666	2	0	2	2 missed, 2 wrong **54** There are 6 ways of choosing 2 missed and in each of these there are 3^2 ways. So 6*9=54
	1-0.3333 =0.6667	2	1	1	2 missed, 1 right, 1 wrong **36** There are 4 ways to choose the right question. In that there are 3 ways of missing 2 questions. That only leaves 1 question and there are 3 ways of getting that wrong. So, 4*3*3=36
	2	2	2	0	2 right miss 2 **6**
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	-0.333	3	0	1	3 miss 1 wrong **12**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 25. 4-option MCQ-only. Stems can be missed.
Score is [0], [1, -1/3]. n=5 (not all filled in).

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
5	-5*0.33333 =**-1.6667**	0	0	5	All wrong **243=3^5**
	1-4*0.3333 =**-0.3333**	0	1	4	1 right 4 wrong **405** There are 5 ways of choosing the right answer and in each of these are 3^4 ways of getting a wrong answer. So, $5*3^4$=5*81=405
	2-3*0.33333 =**1**	0	2	3	2 right 3 wrong. **270** There are 10 ways of getting 2 right and in each of these there are 3^3 ways of getting wrong answers. So, $10*3^3$=10*27=270
	3-2*0.33333 =**2.3333**	0	3	2	3 right 2 wrong **90** There are 10 ways of getting 3 answers right, and in each of these there are 3^2 ways of getting wrong answers. So, $10*3^2$=10*9=90
	4-0.3333 =**3.6667**	0	4	1	4 right 1 wrong **15**
	5	0	5	0	All right **1**
	-4*0.33333 =**-1.3333**	1	0	4	1 missed, 0 right. 4 wrong,
	1-3*0.33333 = **0**	1	1	3	1 missed, 1 right, 3 wrong
	2-2*0.33333 =**1.3333**	1	2	2	1 missed, 2 right, 2 wrong
	3-0.33333 =**2.6667**	1	3	1	1 missed, 3 right, 1 wrong
	4	1	4	0	4 right missed 1 **5**
		1	5	-1	**Impossible**
	-1	2	0	3	3 wrong
	1-2*0.33333 =**0.3333**	2	1	2	1 right 2 wrong
	2-0.3333 =1.6667	2	2	1	2 right 1 wrong
	3	2	3	0	2 missed 3 right **10**
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**

	-0.6667	3	0	2	2 wrong
	1-0.33333 =**0.6667**	3	1	1	1 right, 1 wrong
	2	3	2	0	3 missed 2 right **10** Got by writing out all the members of the set 1 2 3, 1 2 4, 1 2 5 1 3 4, 1 3 5 1 4 5 2 3 4, 2 3 5 2 4 5 3 4 5
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	-0.3333	4	0	1	4 missed 1 wrong **15**
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

The conclusion from all this that the expression $X^{X*}\ {}_nC_y{}^*{}_{n-y}C_w$ works for X=1 and X=3 (as far as we can tell)

8.6 3-option Multiple Yes/No (where X=7) and stems can be missed and a modification of negative marking is NOT used i.e. a=b=0.

The number of ways of getting a mark in a particular way is given by $X^{X*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations.

App 8Table 26. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1 0]. **n=1.**

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
1	**0**	0	0	1	All wrong **7**=7^1
The mean	**1**	0	1	0	All right **1**
Mark=(0*7	**0**	1	0	0	All missed **1**
+1*1 +0*1)/9=**0.11**	1	1	1	-1	**Impossible**

App 8Table 27. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1 0]. **n=2.**

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
2	**0**	0	0	2	All wrong 7^2=**49**
The mean Mark=(0*49 + 1*14 +2*1 +0*14 + 1 *2	**1**	0	1	1	1 right out of 2 and the other wrong 2*7=**14**
+0*1)/81=(18)/81=**0.22**	**2**	0	2	0	All right **1**
	0	1	0	1	1 missed and 1 wrong 2*7=**14**
	1	1	1	0	1 right out of 2 and the other missed **2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1**
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8 Table 28. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1 0]. **n=3.**

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
3	0	0	0	3	All wrong **343**=7^3
The mean Mark=(0*343 + 1* 147 + 2* 21 +3*1+ 0*147 + 1*42 +2*3 +0*21 +1*3 +0*1)/729=243/729=**0.33**	1	0	1	2	1 right, 2 wrong =3*7^2=3*49= **147**
	2	0	2	1	2 right 1 wrong 3*7=**21**
Check 9^3=729✓	3	0	3	0	All right **1**
	0	1	0	2	1 missed, 2 wrong 3*7^2=3*49= **147**
	1	1	1	1	1 missed, 1 right, 1 wrong. 3*2*7= **42**
	2	1	2	0	1 missed, 2 right **3** i.e. right 1, 2 1, 3 and 2, 3
		1	3	-1	**Impossible**
	0	2	0	1	2 missed, 1 wrong 3*7= **21**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 29. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1 0]. **n=4**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
4	**0**	0	0	4	All wrong **2401=7^4**
	1	0	1	3	1 right 3 wrong $4*7^3$= **1372**
	2	0	2	2	2 right 2 wrong $6*7^2$= **294**
	3	0	3	1	1 wrong, 3 right 4*7= **28**
	4	0	4	0	All right **1**
	0	1	0	3	1 miss, 3 wrong $4*7^3$=4*343= **1372**
	1	1	1	2	1 miss, 1 right, 2 wrong $4*3*7^2$=12*49= **588**
	2	1	2	1	1 missed, 2 right, 1 wrong 4*3*7= **84**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	0	2	0	2	2 missed, 2 wrong $6*7^2$= **294**
	1	2	1	1	2 missed, 1 right, 1 wrong There are 4 ways of getting 1 right. That leaves 3. There are 3 *7 ways of getting 1 wrong. The misses are given 4*3*7= **84**
	2	2	2	0	2 right miss 2 **6** i.e. miss 1,2, 1, 3, 1, 4 2, 3, 2, 4 3, 4
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	0	3	0	1	3 miss 1 wrong 4*7= **28**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 30. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1, 0]. **n=5**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
5	0	0	0	5	All wrong **16807**=7^5
	1	0	1	4	1 right 4 wrong $5*7^4$=5*2401= **12005**
	2	0	2	3	2 right 3 wrong. $10*7^3$=10*343=**3430**
	3	0	3	2	3 right 2 wrong $10*7^2$= **490**
	4	0	4	1	4 right 1 wrong 5*7= **35**
	5	0	5	0	All right **1**
	0	1	0	4	1 missed 4 wrong $5*7^4$=5*2401= **12005**
	1	1	1	3	1 missed 1 right 3 wrong $5*4*7^3$=20*343=**6860**
	2	1	2	2	1 missed 2 right 2 wrong =$5*6*7^2$= 30*49=**1470**
	3	1	3	1	1 missed 3 right 1 wrong =5*4*7= **140**
	4	1	4	0	4 right missed 1 **5**
		1	5	-1	**Impossible**
	0	2	0	3	2 missed 3 wrong $10*7^3$= **3430**
	1	2	1	2	2 missed 1 right 2 wrong =${}_5C_2*3*7^2$=10*3*49=**1470**
	2	2	2	1	2 missed 2 right 1 wrong =${}_5C_2*{}_3C_2$ = 10*3*7=**210**
	3	2	3	0	2 missed 3 right **10** =${}_5C_2$
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	0	3	0	2	3 missed 2 wrong =$10*7^2$=**490**
	1	3	1	1	3 missed 1 right 1 wrong =10*2*7=**140**
	2	3	2	0	3 missed 2 right **10**
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	0	4	0	1	4 missed 1 wrong **35**
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**

		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

The number of ways of getting a mark in a particular way is checked against $X^{X*}{}_nC_y{}^*{}_{n-y}C_w$. where stems can be missed (i.e. **y** can be ≠0), X=7 and **a**=b=**0**.

8.7 3-option Multiple Yes/No (X=7) ad stems can be missed and a modification of negative marking is used, where a=-1/7 and b=0

The number of ways of getting a mark in a particular way is given by $X^{X*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations.

The mean Mark should be 0, because the guessing problem is *exactly* tackled.

App 8 Table 31. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1, -1/7]. n=1.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
1	**-0.1429**	0	0	1	All wrong **$7=7^1$**
The mean	**1**	0	1	0	All right **1**
Mark=	**0**	1	0	0	All missed **1**
(-7/7 +1*1 +0*1)/9=**0**	1	1	1	-1	**Impossible**

App 8Table 32. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1, -1/7]. **n=2.**

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
2	-0.2857	0	0	2	All wrong **$49=7^2$**
	0.6571	0	1	1	1 right out of 2 and the other wrong **14**
	2	0	2	0	All right **1**
	-0.1329	1	0	1	1 missed and 1 wrong **14**
	1	1	1	0	1 right out of 2 and the other missed **2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1**
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8 Table 33. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1, -1/7]. **n=3.**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
3	- 3*0.142857= **-0.4286**	0	0	3	All wrong 7^3= **343**
	1- 2*0.142857= **0.7143**	0	1	2	1 right, 2 wrong $3*7^2$= 3*49= **147**
	2-0.142857= **1.8571**	0	2	1	2 right 1 wrong 3*7= **21**
	3	0	3	0	All right **1**
	- 2*0.142857= **-0.2857**	1	0	2	1 missed, 2 wrong $3*7^2$= **147**
	1-0.142857= **0.8571**	1	1	1	1 missed, 1 right, 1 wrong. For each question right there are 6 possibilities , and there are 7 ways of getting a question wrong 3*2*7= **42**
	2	1	2	0	1 missed, 2 right **3**
		1	3	-1	**Impossible**
	-0.1429	2	0	1	2 missed, 1 wrong 3*7 **21**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 34. 3-option Multiple Yes/No-only. Stems can be missed. Score is [0], [1, -1/7]. **n=4**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
4	4*0.142857= **-0.5714**	0	0	4	All wrong 7^4= **2401**
	1- 3*0.142857= **0.5714**	0	1	3	1 right 3 wrong $4*7^3$= **1372**
	2- 2*0.142857= **1.7143**	0	2	2	2 right 2 wrong $6*7^2$= **294**

	3-0.142857= **2.8571**	0	3	1	1 wrong, 3 right 4*7= **28**
	4	0	4	0	All right **1**
	- 3*0.142857= **-0.4286**	1	0	3	1 miss, 3 wrong 4*73= **1372**
	1- 2*0.142857= **0.7143**	1	1	2	1 miss, 1 right, 2 wrong $4*3*7^2$= **588**
	2-0.142857= **1.8571**	1	2	1	1 missed, 2 right, 1 wrong 4*3*7= **84**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	-0.2857	2	0	2	2 missed, 2 wrong $6*7^2$=6*49= **294**
	1-0.142857= **0.8571**	2	1	1	2 missed, 1 right, 1 wrong 6*2*7= **84**
	2	2	2	0	2 right 2 miss **6**
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	-0.1429	3	0	1	3 miss 1 wrong 4*7= **28**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

The conclusion from all this that the expression $X^{X*}\ {}_nC_y*{}_{n-y}C_w$ works for X=1 and X=3 and X=7 (as far as we can tell) for quizzes where stems can be missed.

8.8 5-option Multiple Answer Questions (MAQ) with 2 ticks (where X=9) and stems can be missed and a modification of negative marking is NOT used (i.e. a=b=0)

The number of ways of getting a mark in a particular way is given by $X^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$ and this is the expression that is used to check our 'hand' calculations.

App 8 Table 35. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1 0]. **n**=1.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
1	**0**	0	0	1	All wrong 9^1=**9**
The mean Mark=(0*9 +1 *1 + 0*1)/11=**0.0909**	**1**	0	1	0	All right **1**
	0	1	0	0	All missed **1**
	1	1	1	-1	**Impossible**

App 8 Table 36. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1 0]. **n**=2.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
2	**0**	0	0	2	All wrong 9^2=**81**
The mean Mark=(0*81 + 1*18 +2*1 +0*18 +1*2 +0*1)/121=22/121=**0.1818**	**1**	0	1	1	1 right out of 2 and the other wrong 2*9=**18**
Check 11^2=121✓	**2**	0	2	0	All right **1**
	0	1	0	1	1 missed and 1 wrong 2*9=**18**
	1	1	1	0	1 right out of 2 and the other missed **2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1**
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8 Table 37. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1 0]. **n**=3.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
3	0	0	0	3	All wrong **729**=9^3
The mean Mark=(0*729 + 1*243+ 2*27 +3*1 + 0*243 + 1*54 +2*3 + 0*27 + 1*3 +0*1)/1331=363/1331= **0.27**	1	0	1	2	1 right, 2 wrong **243**=3*9^2=3*81
	2	0	2	1	2 right 1 wrong 3*9=**27**
	3	0	3	0	All right **1**
Check 11^3=1331	0	1	0	2	1 missed, 2 wrong **243**=3*9^2=3*81
	1	1	1	1	1 missed, 1 right, 1 wrong. 3*2*9=**54**
	2	1	2	0	1 missed, 2 right **3** i.e. right 1, 2 1, 3 and 2, 3
		1	3	-1	**Impossible**
	0	2	0	1	2 missed, 1 wrong 3*9=**27**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 38. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1 0]. **n**=4.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
4	**0**	0	0	4	All wrong **6561**=9^4
	1	0	1	3	1 right 3 wrong 4*9^3= **2916**
	2	0	2	2	2 right 2 wrong 6*9^2= **486**
	3	0	3	1	1 wrong, 3 right 4*9= **36**
	4	0	4	0	All right **1**
	0	1	0	3	1 miss, 3 wrong 4*9^3= **2916**
	1	1	1	2	1 miss, 1 right, 2 wrong 4*3*9^2=12*81=**972**

	2	1	2	1	1 missed, 2 right, 1 wrong 4*3*9=**108**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	0	2	0	2	2 missed, 2 wrong $6*9^2$= **486**
	1	2	1	1	2 missed, 1 right, 1 wrong There are 4 ways of getting 1 right. That leaves 3. There are 3 *9 ways of getting 1 wrong. The misses are given 4*3*9=**108**
	2	2	2	0	2 right miss 2 **6** i.e. miss 1,2, 1, 3, 1, 4 2, 3, 2, 4 3, 4
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	0	3	0	1	3 miss 1 wrong 4*9=**36**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 39. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1 0]. **n**=5.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
5	0	0	0	5	All wrong **59049**=9^5
	1	0	1	4	1 right 4 wrong $5*9^4$=5*6561= **32805**
	2	0	2	3	2 right 3 wrong. $10*9^3$=10*729=**7290**
	3	0	3	2	3 right 2 wrong $10*9^2$= **810**
	4	0	4	1	4 right 1 wrong 5*9= **45**
	5	0	5	0	All right **1**
	0	1	0	4	1 missed 4 wrong $5*9^4$= **32805**
	1	1	1	3	1 missed 1 right 3 wrong $5*4*9^3$=20*729=**14580**
	2	1	2	2	1 missed 2 right 2 wrong =$5*6*9^2$= **2430**
	3	1	3	1	1 missed 3 right 1 wrong

					=5*4*9= **180**
	4	1	4	0	4 right missed 1 **5**
		1	5	-1	**Impossible**
	0	2	0	3	2 missed 3 wrong $10*9^3$= **7290**
	1	2	1	2	2 missed 1 right 2 wrong $={}_5C_2*3*9^2$=10*3*81=**2430**
	2	2	2	1	2 missed 1 wrong 2 right $_5C_2*{}_3C_1*9$=10*3*9=**270**
	3	2	3	0	2 missed 3 right **10** $={}_5C_2$
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	0	3	0	2	3 missed 2 wrong $=10*9^2$=**810**
	1	3	1	1	3 missed 1 right 1 wrong =10*2*9=**180**
	2	3	2	0	3 missed 2 right **10**
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	0	4	0	1	4 missed 1 wrong **45**
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

8.9 5-option Multiple Answer Questions (MAQ) with 2 ticks (where X=9) and stems can be missed and a modification of negative marking IS used with a=-1/9 and b=0

Let us assume that guessing is *exactly* tackled and then **a**=-1/9. By exactly tackling the guessing problem we make the mean zero, and so the tools that calculate the mean should yield zero.

The number of ways of getting a mark in a particular way is given by $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations.

App 8 Table 40. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1, -1/9]. **n**=1.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
1	-0.1111	0	0	1	All wrong **9**=**9**^{1}
The mean Mark=	1	0	1	0	All right **1**
(-9/9 +1*1 +0*1)/11=**0**	0	1	0	0	All missed **1**
		1	1	-1	**Impossible**

App 8 Table 41. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1, -1/9]. **n**=2/

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
2	**-0.2222**	0	0	2	All wrong **81** =9^2
The mean Mark=	1-0.1111 =**0.8889**	0	1	1	1 right out of 2 and the other wrong 2*9= **18**
(-2*81/9 + 8*18/9	**2**	0	2	0	All right **1**
+2*1 - 1*18/9	**-0.1111**	1	0	1	1 missed and 1 wrong 2*9= **18**
+1*2 + 0*1)/121=	**1**	1	1	0	1 right out of 2 and the other missed **2**
(-18 + 16 +		1	2	-1	**Impossible**
2 -2 +2	**0**	2	0	0	All missed **1**
)/121=**0**		2	1	-1	**Impossible**
Check 11^2=121✓		2	2	-2	**Impossible**

App 8 Table 42. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1, -1/9]. **n**=3.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
3	**-0.3333**	0	0	3	All wrong 9^3=**729**
	1-0.2222 =**.7778**	0	1	2	1 right, 2 wrong $3*9^2$=**243**
	2-0.1111 =**1.8889**	0	2	1	2 right 1 wrong =3*9=**27**
	3	0	3	0	All right **1**
	-0.2222	1	0	2	1 missed, 2 wrong $3*9^2$=**243**
	1-0.1111 =**0.8889**	1	1	1	1 missed, 1 right, 1 wrong. 3*2*9=**54**
	2	1	2	0	1 missed, 2 right **3**
		1	3	-1	**Impossible**
	-0.1111	2	0	1	2 missed, 1 wrong **27**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 43. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1, -1/9]. =4.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
4	**-0.4444**	0	0	4	All wrong 9^4=**6561**
	1-0.3333 =**0.6667**	0	1	3	1 right 3 wrong $4*9^3$= 4*729= **2916**
	2-0.2222 =**1.7778**	0	2	2	2 right 2 wrong 6 $_4C_2*9^2$=6*81= **486**
	3-0.1111 =**2.8889**	0	3	1	1 wrong, 3 right 9*4=**36**
	4	0	4	0	All right **1**
	-0.3333	1	0	3	1 miss, 3 wrong $4*9^3$= 4*729= **2916**
	1-0.2222 =**0.7778**	1	1	2	1 miss, 1 right, 2 wrong $4*3*9^2$=12*81=**972**
	2-0.111 =**1.8889**	1	2	1	1 missed, 2 right, 1 wrong 4*3*9= **108**
	3	1	3	0	3 right, miss 1 **4**

		1	4	-1	**Impossible**
	-0.2222	2	0	2	2 missed, 2 wrong $6*9^2=6*81=$**486**
	1-0.1111=**0.8889**	2	1	1	2 missed, 1 right, 1 wrong 6*2*9=**108**
	2	2	2	0	2 right miss 2 $_4C_2=$**6**
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	-0.1111	3	0	1	3 miss 1 wrong **36**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 44. 5-option MAQ-only with 2 ticks. Stems can be missed. Score is [0], [1, -1/9]. **n**=5.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
5	**-0.5555**	0	0	5	All wrong $9^5=$**59049**
	1-0.4444=**0.5556**	0	1	4	1 right 4 wrong $5*9^4=5*6561=$ **32805**
	2-0.3333=**1.6667**	0	2	3	2 right 3 wrong. $_5C_2*9^3=10*729=$ **7290**
	3-0.2222=**2.7778**	0	3	2	3 right 2 wrong $_5C_3*9^2=10*81=$ **810**
	4-0.1111=3.8889	0	4	1	4 right 1 wrong $_5C_4*9=5*9=$ **45**
	5	0	5	0	All right **1**
	-0.4444	1	0	4	1 missed, 0 right. 4 wrong, $5*9^4=5*6561=$ **32805**
	1-0.3333=**0.6667**	1	1	3	1 missed, 1 right, 3 wrong $5*4*9^3=20*729=$**14580**
	2-0.2222=**1.7778**	1	2	2	1 missed, 2 right, 2 wrong $=5*{}_4C_2*9^2=5*6*81=$**2430**
	3-0.1111=**2.8889**	1	3	1	1 missed, 3 right, 1 wrong 5 *4*9= **180**
	4	1	4	0	4 right 1 missed **5**
		1	5	-1	**Impossible**
	-0.333	2	0	3	3 wrong

					${}_5C_2*9^3$=10*729=**7290**
	1- 0.2222= **0.6667**	2	1	2	2 missed 1 right 2 wrong ${}_5C_1*{}_4C_2*9^2$=5*6*81=**2430**
	2- 0.1111= **1.8889**	2	2	1	2 missed 2 right 1 wrong ${}_5C_2*{}_3C_2$*9= 10*3*9=**270**
	3	2	3	0	2 missed 3 right **10**
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	-0.2222	3	0	2	3 missed 2 wrong ${}_5C_3*9^2$=10*81= **810**
	1- 0.1111= **0.8889**	3	1	1	1 right, 1 wrong 5*4*9= **180**
	2	3	2	0	3 missed 2 right ${}_5C_2$= **10**
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	-0.1111	4	0	1	4 missed 1 wrong 5*9= **45**
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

The conclusion from all this that the expression $X^{x}* {}_nC_y*{}_{n-y}C_w$ works for X=1, X=3, X=7 and X=9 (as far as we can tell)

8.10 20-option EMSQ (MCQ) (where X=19) and stems can be MISSED and a modification of negative marking is NOT used i.e. a=b=0.

The number of ways of getting a mark in a particular way is given by $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations.

App 8 Table 45. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1 0]. **n=**1.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
1	**0**	0	0	1	All wrong **19**=19^1
The mean	**1**	0	1	0	All right **1**
Mark=(0*19	**0**	1	0	0	All missed **1**
+1*1 +0*1)/21=**0.0476**	1	1	1	-1	**Impossible**

App 8 Table 46. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1 0]. **n=**2.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
2	**0**	0	0	2	All wrong 19^2=**361**
The mean Mark=(0*361 + 1*38 +2*1 +0*38 + 1*2 +0*1)/42=42/441=**0.095**	**1**	0	1	1	1 right out of 2 and the other wrong 2*19=**38**
	2	0	2	0	All right **1**
	0	1	0	1	1 missed and 1 wrong 2*19=**38**
	1	1	1	0	1 right out of 2 and the other missed **2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1**
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8 Table 47. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1 0]. **n**=3.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
3	**0**	0	0	3	All wrong **6859** $=19^3$
	1	0	1	2	1 right, 2 wrong $=3*19^2=3*361=$ **1083**
	2	0	2	1	2 right 1 wrong 3*19=**57**
	3	0	3	0	All right **1**
	0	1	0	2	1 missed, 2 wrong $3*19^2=3*361=$ **1083**
	1	1	1	1	1 missed, 1 right, 1 wrong. 3*2*19= **114**
	2	1	2	0	1 missed, 2 right **3** i.e. right 1, 2 1, 3 and 2, 3
		1	3	-1	**Impossible**
	0	2	0	1	2 missed, 1 wrong 3*19=**57**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 48. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1 0]. **n**=4.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
4	**0**	0	0	4	All wrong **13021** $= 19^4$
	1	0	1	3	1 right 3 wrong $4*19^3= 4*6859 =$**27436**
	2	0	2	2	2 right 2 wrong $6*19^2=6*361=$ **2166**
	3	0	3	1	1 wrong, 3 right 4*19= **76**
	4	0	4	0	All right **1**
	0	1	0	3	1 miss, 3 wrong $4*19^3=4*6859=$**27436**
	1	1	1	2	1 miss, 1 right, 2 wrong $4*3*19^2=12*361=$**4332**
	2	1	2	1	1 missed, 2 right, 1 wrong 4*3*19=**228**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	0	2	0	2	2 missed, 2 wrong $6*19^2=6*361=$**2166**
	1	2	1	1	2 missed, 1 right, 1 wrong

					There are 4 ways of getting 1 right. That leaves 3. There are 3 *19 ways of getting 1 wrong. The misses are given 4*3*19=**228**
	2	2	2	0	2 right miss 2 **6** i.e. miss 1,2, 1, 3, 1, 4 2, 3, 2, 4 3, 4
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	0	3	0	1	3 miss 1 wrong 4*19= **76**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 49. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1 0]. **n=**5.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
5	**0**	0	0	5	All wrong **2476099** =19^5
	1	0	1	4	1 right 4 wrong 5*19^4=5*130321= **651605**
	2	0	2	3	2 right 3 wrong. 10*19^3=10*6859=**68590**
	3	0	3	2	3 right 2 wrong 10*19^2= **3610**
	4	0	4	1	4 right 1 wrong 5*19= **95**
	5	0	5	0	All right **1**
	0	1	0	4	1 missed 4 wrong 5*19^4=5*13021= **651605**
	1	1	1	3	1 missed 1 right 3 wrong 5*4*19^3=20*6859= **137180**
	2	1	2	2	1 missed 2 right 2 wrong =5*6*19^2= 30*361=**10830**
	3	1	3	1	1 missed 3 right 1 wrong =5*4*19= **380**
	4	1	4	0	4 right missed 1 **5**
		1	5	-1	**Impossible**
	0	2	0	3	2 missed 3 wrong 10*19^3= **68590**
	1	2	1	2	2 missed 1 right 2 wrong =${}_5C_2$*3*19^2=10*3*361=

					10830
	2	2	2	1	2 missed 2 right 1 wrong $={}_5C_2{*}\ {}_3C_2 =$ 10*3*19= **570**
	3	2	3	0	2 missed 3 right **10** $={}_5C_2$
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	0	3	0	2	3 missed 2 wrong $=10{*}19^2=$**3610**
	1	3	1	1	3 missed 1 right 1 wrong =10*2*19=**380**
	2	3	2	0	3 missed 2 right **10**
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	0	4	0	1	4 missed 1 wrong 5*19= **95**
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

8.11 20-option EMSQ (MCQ) (where X=19) and stems can be MISSED and a modification of negative marking is used i.e. a= -1/19 and b=0.

The number of ways of getting a mark in a particular way is given by $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ and this is the expression that is used to check our 'hand' calculations.

The mean Mark should be 0, because the guessing problem is *exactly* tackled.

App 8 Table 50. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1, -1/19]. **n=**1.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
1	**-0.0526**	0	0	1	All wrong **19**=$19^{1}*{}_1C_0*{}_1C_0$
The mean	**1**	0	1	0	All right **1**=$19^{0}*{}_1C_0*{}_1C_1$
Mark=(-19/19 +1*1	**0**	1	0	0	All missed **1**=$19^{0}*{}_1C_0*{}_0C_0$
+0*1)/21=**0**		1	1	-1	**Impossible**

There is 1 impossible situation.
We know (from our proofs) that mean mark should be 1*(1 -19/19)/(19 +2)=0
The gap between successive marks is not |1 **-a**| (in line with 1 of our proofs)

These can be checked when our tools are validated.

App 8 Table 51. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1, -1/19]. **n=**2.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
2	**-0.1053**	0	0	2	All wrong **361** =$19^{2}*{}_2C_0*{}_2C_0$
The mean Mark=(-2*19²/19 +18*38/19	**0.9474**	0	1	1	1 right out of 2 and the other wrong **38** =$19^{1}*{}_2C_0*{}_2C_1$
+2*1 - 1*38/19 +	**2**	0	2	0	All right =$19^{0}*{}_2C_0*{}_2C_0$ **1**
1*2+0*1)/441 =(-38 +36 + 2 -2	**-0.0526**	1	0	1	1 missed and 1 wrong **38** =$19^{1}*{}_2C_1*{}_1C_1$
+2)/441= **0**	**1**	1	1	0	1 right out of 2 and the other missed **2** =$19^{0}*{}_2C_1*{}_1C_0$
		1	2	-1	**Impossible**
	0	2	0	0	All missed=$19^{0}*{}_0C_0*{}_0C_0$ **1**

		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

The gap between successive marks is not |1 **-a**| (in line with 1 of our proofs)
There is 3 impossible situations.
We know that mean mark should be 2*(1 -19/19)/(19 +2)=0

The last two, can be checked when our tools are validated

App 8 Table 52. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1, -1/19]. **n**=3.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way** $X^{x*}{}_nC_y*{}_{n-y}C_w$
3	**-0.1579**	0	0	3	All wrong $19^3=19^{3}*{}_3C_0*{}_3C_0=$ **6859**
	0.8947	0	1	2	1 right, 2 wrong $3*19^2=3*361=$**1083**$=19^{2}*{}_3C_0*{}_3C_2$
	1.9474	0	2	1	2 right 1 wrong
	3	0	3	0	All right **1** $19^{0}*{}_3C_0*{}_3C_0$
	0.1053	1	0	2	1 missed, 2 wrong $3*19^2=19^{2}*{}_3C_1*{}_2C_0=$ **1083**
	0.9474	1	1	1	1 missed, 1 right, 1 wrong. $=19^{1}*{}_3C_1*{}_2C_1$ = 19*3*2= **114** =3*2* 19
	2	1	2	0	1 missed, 2 right **3** $=19^{0}*{}_3C_1*{}_0C_0$ =3
		1	3	-1	**Impossible**
	-0.0526	2	0	1	2 missed, 1 wrong $19^{1}*{}_3C_2*{}_1C_0=$ **57** =3*19
	1	2	1	0	2 missed, 1 right =3*1= **3**= $19^{0}*{}_3C_2*{}_1C_1$
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

The gap between successive marks is not |1 **-a**| (in line with 1 of our proofs)

There is 6 impossible situations.
We know that mean mark should be 3*(1 -19/19)/(19 +2)=0

The last two can be checked when our tools are validated

App 8 Table 53. 20-option EMSQ (MCQ). Stems can be missed.
Score is [0], [1, -1/19]. **n=4.**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
4	-4**a**= **-0.2105**	0	0	4	All wrong 19^4 = **130321** $= 19^{4*}{}_4C_0{}^*{}_4C_0$
	1-3**a**= **0.8421**	0	1	3	1 right 3 wrong 4^*19^3= **27436**
		0	2	2	2 right 2 wrong 6 ${}_4C_2{}^*19^2$=6*361= **2166**
	3- **a**=**2.9474**	0	3	1	1 wrong, 3 right 19*4=**76** $=19^{1*}{}_4C_0{}^*{}_4C_1$
	4	0	4	0	All right **1**
	-3**a**= **-0.1579**	1	0	3	1 miss, 3 wrong 4^*19^3= 4*= **27436** $=19^{3*}{}_4C_1{}^*{}_3C_3$
	1-2**a**= **0.8947**	1	1	2	1 miss, 1 right, 2 wrong $4^*3^*19^2$=12* = **4332** $19^{2*}{}_4C_1{}^*{}_3C_1$
	2-**a**= **1.9474**	1	2	1	1 missed, 2 right, 1 wrong 4*3*19= **228**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	-2**a**=	2	0	2	2 missed, 2 wrong 6^*19^2=6*361=**2166** $= 19^{2*}{}_4C_2{}^*{}_2C_2$
		2	1	1	2 missed, 1 right, 1 wrong 6*2*19=**228** $= 19^{1*}{}_4C_2{}^*{}_2C_1$
	2	2	2	0	2 right miss 2 ${}_4C_2$=**6** $=19^{0*}{}_4C_2{}^*{}_2C_2$
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
		3	0	1	3 miss 1 wrong 4*19=**76** $= 19^{1*}{}_4C_3{}^*{}_1C_1$
	1	3	1	0	1 right, miss 3 **4** $19^{0*}{}_4C_1{}^*{}_1C_0$
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1** $19^{0*}{}_4C_4{}^*{}_0C_0$
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

The gap between successive marks is not |1 **-a**| (in line with 1 of our proofs)

There is 10 impossible situations.
We know that mean mark should be 4*(1 -19/19)/(19 +2)=0

The last two can be checked when our tools are validated

App 8 Table 54. 20-option EMSQ (MCQ). Stems can be missed. Score is [0], [1, -1/19]. **n**=5.

Number of Stems **n**	**Mark** **M**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
5	-5*0.052632 =**-0.2632**	0	0	5	All wrong 19^5=**2476099** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{5*}{}_5C_0{}^*{}_5C_0$
	1-4*0.052632 =**0.7895**	0	1	4	1 right 4 wrong $5*19^4$=**651605** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{4*}{}_5C_0{}^*{}_5C_1$
	2-3*0.052632= **1.8421**	0	2	3	2 right 3 wrong. ${}_5C_2*19^3$=10*6859= **68590** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{3*}{}_5C_0{}^*{}_5C_2$
	3-2*0.056232= **2.8947**	0	3	2	3 right 2 wrong ${}_5C_3*19^2$=10*361=**3610** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{x*}{}_5C_0{}^*{}_5C_3$
	4-0.052632= **3.9474**	0	4	1	4 right 1 wrong ${}_5C_4*19$= 5*19= **95** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{1*}{}_5C_0{}^*{}_5C_4$
	5	0	5	0	All right **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{0*}{}_5C_0{}^*{}_5C_5$
	-4*0.052632= **-0.2105**	1	0	4	1 missed, 0 right. 4 wrong, $5*19^4$=5*130321=**651605** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{4*}{}_5C_1{}^*{}_4C_4$
	1-3*0.056232= **0.8421**	1	1	3	1 missed, 1 right, 3 wrong $5*4*19^3$=20*6859= **137180** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{3*}{}_5C_1{}^*{}_4C_3$
	2-2*0.052632= **1.8947**	1	2	2	1 missed, 2 right, 2 wrong =$5*{}_4C_2*19^2$= 5*6*361= **10830** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{2*}{}_5C_1{}^*{}_4C_2$
	3-0.052632= **-2.9474**	1	3	1	1 missed, 3 right, 1 wrong 5 *4*19= **380** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{1*}{}_5C_1{}^*{}_4C_1$=19*5*4
	4	1	4	0	4 right 1 missed **5** $19^{0*}{}_5C_1{}^*{}_4C_0$
		1	5	-1	**Impossible**
	-3*0.052632=	2	0	3	2 missed 3 wrong ${}_5C_2*19^3$=10*6859=**68590**

	-0.1579				$19^{3*}{}_5C_2{}^*{}_3C_3$
	1- 2*0.052632= **0.8947**	2 y	1 w	2 x	2 missed 1 right 2 wrong $_5C_1{}^*{}_4C_2{}^*19^2$=5*6*361=**10830** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{2*}{}_5C_2$*3C1= 361*10*3
	2-0.052632= **1.9474**	2	2	1	2 missed 2 right 1 wrong $_5C_2{}^*{}_3C_2$*19= 10*3*19=**570** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $19^{1*}{}_5C_2{}^*{}_3C_1$=19*10*3
	3	2	3	0	2 missed 3 right **10**
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	- 2*0.052632= **-0.1053**	3	0	2	3 missed 2 wrong $_5C_3{}^*19^2$=10*361= **3610** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{2*}{}_5C_3{}^*{}_2C_0$
	1-0.052632= **0.9474**	3	1	1	1 right, 1 wrong 5*4*19= **380** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{1*}{}_5C_3{}^*{}_2C_1$
	2	3	2	0	3 missed 2 right $_5C_2$= **10** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{0*}{}_5C_3{}^*{}_2C_0$
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	-0.0526	4	0	1	4 missed 1 wrong 5*19=**95** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{1*}{}_5C_4{}^*{}_1C_1$
	1	4	1	0	4 out of 5 missed, 1 right **5** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{0*}{}_5C_4{}^*{}_1C^0$
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w=19^{0*}{}_5C_5{}^*{}_0C_0$
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

The gap between successive marks is not |1 **-a**| (in line with 1 of our proofs).

There are 15 impossible situations.
We know that mean mark should be 3*(1 -19/19)/(19 +2)=0.

The last two can be checked when our tools are validated.

The conclusion from all this that the expression $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ works for X=1, X=3, X=7, X=9 and X=19 (as far as we can tell). .

8.12 20-option EMSQ (MAQ with 10 ticks)-only (where X=184755) and stems can be MISSED and a modification of negative marking is NOT used i.e. a=b=0.

The number of ways of getting a mark in a particular way is calculated using $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$.

App 8 Table 55. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, 0]. **n**=1.

Number of Stems **n**	**Mark** **M**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
1 The mean Mark=(0*184755 +1*1 +0*1)184755= 1/184755= about **0**	**0**	0	0	1	All wrong **184755** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $184755^{1*}{}_1C_0{}^*{}_1C_0$=184755*1*1
	1	0	1	0	All right **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $184755^{0*}{}_1C_0{}^*{}_1C_1$=1*1*1
	0	1	0	0	All missed **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$= $184755^{0*}{}_1C_1{}^*{}_0C_0$=1*1*1
	1	1	1	-1	**Impossible**

App 8 Table 56. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, 0]. **n**=2.

Number of Stems **n**	**Mark** **M**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way**
2	**0**	0	0	2	All wrong 184755^2= $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_2C_0{}^*{}_2C_0$= **34134410025***1*1
	1	0	1	1	1 right out of 2 and the other wrong 2*184755= $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_2C_0{}^*{}_2C_1$= 184755*1*2=**369510**
	2	0	2	0	All right **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_2C_0{}^*{}_2C_2$=1*1**1
	0	1	0	1	1 missed and 1 wrong 2*184755= $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_2C_1{}^*{}_1C_0$= 184755*2*1=**369510**
	1	1	1	0	1 right out of 2 and the other missed **2**

					$X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{0*}{}_{2}C_{1}{}^{*}{}_{1}C_{1}=$ 1*2*1=**2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{0*}{}_{2}C_{2}{}^{*}{}_{0}C_{0}$=1*1*1
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8Table 57. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, 0]. **n**=3.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$
3	**0**	0	0	3	All wrong =184755^{3} **6306502924168875** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{3*}\,{}_{3}C_{0}{}^{*}{}_{3}C_{0}$
	1	0	1	2	1 right, 2 wrong =3*184755^{2}= **1024032230075** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{2*}\,{}_{3}C_{0}{}^{*}{}_{3}C_{1}$
	2	0	2	1	2 right 1 wrong 3*184755=**554265** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{1*}\,{}_{3}C_{0}{}^{*}{}_{3}C_{2}$
	3	0	3	0	All right **1** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{0*}\,{}_{3}C_{0}{}^{*}{}_{3}C_{0}$=1*1*1
	0	1	0	2	1 missed, 2 wrong 3*184755^{2}=**1024032230075** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{2*}\,{}_{3}C_{1}{}^{*}{}_{2}C_{0}$
	1	1	1	1	1 missed, 1 right, 1 wrong. 3*2*184755= **1108530** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{1*}\,{}_{3}1_{y}{}^{*}{}_{2}C_{1}$
	2	1	2	0	1 missed, 2 right **3** i.e. right 1, 2 1, 3 and 2, 3 $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $184755^{0*}\,{}_{3}C_{1}{}^{*}{}_{2}C_{2}$
		1	3	-1	**Impossible**
	0	2	0	1	2 missed 1 wrong 3*184755=**554265** $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$ $X^{x*}\,{}_{n}C_{y}{}^{*}\,{}_{n-y}C_{w}$
	1	2	1	0	2 missed, 1 right **3** $184755^{0*}\,{}_{3}C_{2}{}^{*}{}_{1}C_{0}$
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**

					$X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_3C_3{}^*{}_3C_0$
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

App 8 Table 58. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, 0]. **n=**4.

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$
4	**0**	0	0	4	All wrong = 184755^4 **1165157947754820500626** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{4*}{}_4C_0{}^*{}_4C_0$
	1	0	1	3	1 right 3 wrong $4*184755^3$=**25226011696675500** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^*{}_4C_0{}^*{}_4C_1$
	2	0	2	2	2 right 2 wrong $6*184755^2$=**204806460150** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_4C_9{}^*{}_4C_2$
	3	0	3	1	1 wrong, 3 right 4*184755= **739020** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_4C_0{}^*{}_4C_3$
	4	0	4	0	All right **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_4C_0{}^*{}_4C_0$
	0	1	0	3	1 miss, 3 wrong $4*184755^3$=**25226011696675500** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{3*}{}_4C_1{}^*{}_3C_3$
	1	1	1	2	1 miss, 1 right, 2 wrong $4*3*184755^2$=**409612920300** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_4C_1{}^*{}_3C_1$
	2	1	2	1	1 missed, 2 right, 1 wrong 4*3*184755=**2217060** $184755^{2*}{}_nC_y{}^*{}_{n-y}C_w$
	3	1	3	0	3 right, miss 1 **4** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_4C_1{}^*{}_3C_0$=1*4*1
		1	4	-1	**Impossible**
	0	2	0	2	2 missed, 2 wrong $6*184755^2$=**204806460150** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_4C_2{}^*{}_2C_2$
	1	2	1	1	2 missed, 1 right, 1 wrong There are 4 ways of getting 1 right. That leaves 3. There are 3 *184755 ways of getting 1 wrong. The misses are given

					4*3*184755=**2217060** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{1*}{}_{4}C_{2}{}^{*}{}_{2}C_{1}$
	2	2	2	0	2 right miss 2 **6** i.e. miss 1,2, 1, 3, 1, 4 2, 3, 2, 4 3, 4 $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{0*}{}_{4}C_{2}{}^{*}{}_{2}C_{0}$
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	0	3	0	1	3 miss 1 wrong 4*184755= **739020** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{1*}{}_{4}C_{3}{}^{*}{}_{1}C_{1}$
	1	3	1	0	1 right, miss 3 **4** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{0*}{}_{4}C_{3}{}^{*}{}_{1}C_{1}$
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{0*}{}_{4}C_{4}{}^{*}{}_{0}C_{0}$
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 59. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, 0]. **n=5.**

Number of Stems **n**	**Mark** **M**	**Number missed** **y**	**Number right** **w**	**Number wrong** **x**	**Number of ways of getting that mark, that way** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$
5	**0**	0	0	5	All wrong =184755^{5} **215268756637441861592971875** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{5*}{}_{5}C_{0}{}^{*}{}_{5}C_{0}$
	1	0	1	4	1 right 4 wrong $5*184755^{4}$= **5825789738773102503125** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{4*}{}_{5}C_{0}{}^{*}{}_{5}C_{1}$
	2	0	2	3	2 right 3 wrong. $10*184755^{3}$=**63065029241688750** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{3*}{}_{5}C_{0}{}^{*}{}_{5}C_{2}$
	3	0	3	2	3 right 2 wrong $10*184755^{2}$=**341344100250** $X^{x*}{}_{n}C_{y}{}^{*}{}_{n-y}C_{w}$ $184755^{2*}{}_{5}C_{0}{}^{*}{}_{5}C_{3}$
	4	0	4	1	4 right 1 wrong

					5*184755=**923775** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_5C_0{}^*{}_5C_4$
	5	0	5	0	All right **1** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_5C_0{}^*{}_5C_5$
	0	1	0	4	1 missed 4 wrong $5*184755^4$= **5825789738773102503125** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{5*}{}_5C_1{}^*{}_4C_4$
	1	1	1	3	1 missed 1 right 3 wrong $5*4*184755^3$= **126130058483377500** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_5C_1{}^*{}_4C_1$
	2	1	2	2	1 missed 2 right 2 wrong $=5*6*184755^2$=**1024032300750** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_5C_1{}^*{}_4C_2$
	3	1	3	1	1 missed 3 right 1 wrong =5*4*184755= **3695100** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_5C_0{}^*{}_4C_1$
	4	1	4	0	4 right missed 1 **5** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_5C_0{}^*{}_4C_0$
		1	5	-1	**Impossible**
	0	2	0	3	2 missed 3 wrong $10*184755^3$= **63065029241688750** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{3*}{}_5C_2{}^*{}_3C_0$
	1	2	1	2	2 missed 1 right 2 wrong $=10*3*184755^2$= **1024032300750** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_5C_2{}^*{}_3C_2$
	2	2	2	1	2 missed 2 right 1 wrong $={}_5C_2{}^*{}_3C_2$*184755=**5542650** 184755*10*3 $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{1*}{}_5C_2{}^*{}_3C_2$
	3	2	3	0	2 missed 3 right **10** $={}_5C_2$ $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{0*}{}_5C_2{}^*{}_3C_0$
		2	4	-1	**Impossible**
		2	5	-2	**Impossible**
	0	3	0	2	3 missed 2 wrong $=10*184755^2$= **341344100250** $X^{x*}{}_nC_y{}^*{}_{n-y}C_w$ $184755^{2*}{}_5C_3{}^*{}_2C_2$
	1	3	1	1	3 missed 1 right 1 wrong =10*2*184755=**3695100**

					$X^{x*}\ {}_nC_y{}^{*}{}_{n-y}C_w$ $184755^{1*}\ {}_5C_2{}^{*}{}_2C_1$
	2	3	2	0	3 missed 2 right **10** $X^{x*}\ {}_nC_y{}^{*}{}_{n-y}C_w$ $184755^{0*}\ {}_5C_3{}^{*}{}_2C_0$
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	0	4	0	1	4 missed 1 wrong 5*184755= **923775** $X^{x*}\ {}_nC_y{}^{*}{}_{n-y}C_w$ $184755^{1*}\ {}_5C_0{}^{*}{}_5C_9$
	1	4	1	0	4 out of 5 missed, 1 right **5** $X^{x*}\ {}_nC_y{}^{*}{}_{n-y}C_w$ $184755^{0*}\ {}_5C_4{}^{*}{}_3C_0$
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1** $X^{x*}\ {}_nC_y{}^{*}{}_{n-y}C_w$ $184755^{0*}\ {}_5C_5{}^{*}{}_0C_0$
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

8.13 20-option EMSQ (MAQ with 10 ticks)-only (where X=184755) and stems can be MISSED and a modification of negative marking is used i.e. a=-1/184755, b=0.

Thus **a** is -0.0000054125734 i.e. about 0

Let us assume that guessing is *exactly* tackled and then **a**=-1/184755. By exactly tackling the guessing problem we make the mean zero, and so the tools (when they are built) that calculate the mean should yield zero.

The number of ways of getting a mark in a particular way is given by $X^{x} * {}_{n}C_{y} * {}_{n-y}C_{w}$ and this is the expression that is used to check our 'hand' calculations.

The mean mark should be 0, because the guessing problem is *exactly* tackled.

App 8 Table 60. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, -1/184755]. **n**=1.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
1 The mean Mark=(0*184755 + 1*1 +0*1)/184759=about **0**	**0** (to 4 decimal places)	0	0	1	All wrong **184755** =184755^1
	1	0	1	0	All right **1**
	0	1	0	0	All missed **1**
		1	1	-1	**Impossible**

App 8 Table 61. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, -1/184755]. **n**=2.

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
2	**0** (to 4 decimal places)	0	0	2	All wrong 34134410025=184755^2
	1 (to 4 decimal places)	0	1	1	1 right out of 2 and the other wrong 2*184755= **369510**
	2	0	2	0	All right **1**
	0 (to 4 decimal places)	1	0	1	1 missed and 1 wrong 2*184755= **369510**

	1	1	1	0	1 right out of 2 and the other missed **2**
		1	2	-1	**Impossible**
	0	2	0	0	All missed **1**
		2	1	-1	**Impossible**
		2	2	-2	**Impossible**

App 8 Table 62. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, -1/184755]. **n=3.**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
3	**0**	0	0	3	All wrong 184755^3= **63065029168875**
	1	0	1	2	1 right, 2 wrong 3*184755^2=**102403230075**
	2	0	2	1	2 right 1 wrong =3*184755=**554265**
	3	0	3	0	All right **1**
	0	1	0	2	1 missed, 2 wrong 3*184755^2=**102403230075**
	1	1	1	1	1 missed, 1 right, 1 wrong. 3*2*184755= **1108530**
	2	1	2	0	1 missed, 2 right **3**
		1	3	-1	**Impossible**
	0	2	0	1	2 missed, 1 wrong 3*184755=**554265**
	1	2	1	0	2 missed, 1 right **3**
		2	2	-1	**Impossible**
		2	3	-2	**Impossible**
	0	3	0	0	All missed **1**
		3	1	-1	**Impossible**
		3	2	-2	**Impossible**
		3	3	-3	**Impossible**

The number of ways of getting a mark in a particular way is getting 'very big'.

App 8 Table 63. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, -1/184755]. **n=4.**

Number of Stems n	**Mark M**	**Number missed y**	**Number right w**	**Number wrong x**	**Number of ways of getting that mark, that way**
4	**0**	0	0	4	All wrong 1847455^4= **1165157947754820500625**
	1	0	1	3	1 right 3 wrong 4*184755^3= **252260116966675500**
	2	0	2	2	2 right 2 wrong $_4C_2$*184755^2 = **204806460150**
	3	0	3	1	3 right,1 wrong,

					$_4C_3$* 184755=**739020**
	4	0	4	0	All right **1**
	0	1	0	3	1 miss, 3 wrong $4*184755^3$= **252226011696675500**
	1	1	1	2	1 miss, 1 right, 2 wrong $4*3*184755^2$=**409612920300**
	2	1	2	1	1 missed, 2 right, 1 wrong 4*3*184755=**2217060**
	3	1	3	0	3 right, miss 1 **4**
		1	4	-1	**Impossible**
	0	2	0	2	2 missed, 2 wrong $6*184755^2$=**204806460150**
	1	2	1	1	2 missed, 1 right, 1 wrong 6*2*184755=**2217060**
	2	2	2	0	2 right miss 2 $_4C_2$=**6**
		2	3	-1	**Impossible**
		2	4	-2	**Impossible**
	0	3	0	1	3 miss 1 wrong $_4C_3$*184755=**739020**
	1	3	1	0	1 right, miss 3 **4**
		3	2	-1	**Impossible**
		3	3	-2	**Impossible**
		3	4	-3	**Impossible**
	0	4	0	0	All missed **1**
		4	1	-1	**Impossible**
		4	2	-2	**Impossible**
		4	3	-3	**Impossible**
		4	4	-4	**Impossible**

App 8 Table 64. 20-option EMSQ (MAQ with 10 ticks)-only. Stems can be missed. Score is [0], [1, -1/184755]. **n**=5 (Partial).

Number of Stems n	Mark M	Number missed y	Number right w	Number wrong x	Number of ways of getting that mark, that way
5	**0**	0	0	5	All wrong 1847455^5
	1	0	1	4	1 right 4 wrong
	2	0	2	3	2 right 3 wrong.
	3	0	3	2	3 right 2 wrong
	4	0	4	1	4 right 1 wrong
	5	0	5	0	All right **1**
	0	1	0	4	1 missed, 0 right. 4 wrong,
	1	1	1	3	1 missed, 1 right, 3 wrong
	2	1	2	2	1 missed, 2 right, 2 wrong
	3	1	3	1	1 missed,3 right, 1 wrong
	4	1	4	0	4 right 1 missed **5**
		1	5	-1	**Impossible**
	0	2	0	3	3 wrong
	1	2	1	2	2 missed 1 right 2 wrong
	2	2	2	1	2 missed 2 right 1 wrong
	3	2	3	0	2 missed 3 right **10**
		2	4	-1	**Impossible**

		2	5	-2	**Impossible**
	0	3	0	2	3 missed 2 wrong
	1	3	1	1	1 right, 1 wrong
	2	3	2	0	3 missed 2 right ${}_5C_2$= **10**
		3	3	-1	**Impossible**
		3	4	-2	**Impossible**
		3	5	-3	**Impossible**
	0	4	0	1	4 missed 1 wrong
	1	4	1	0	4 out of 5 missed, 1 right **5**
		4	2	-1	**Impossible**
		4	3	-2	**Impossible**
		4	4	-3	**Impossible**
		4	5	-4	**Impossible**
	0	5	0	0	All missed **1**
		5	1	-1	**Impossible**
		5	2	-2	**Impossible**
		5	3	-3	**Impossible**
		5	4	-4	**Impossible**
		5	5	-5	**Impossible**

The conclusion from all this that the expression $X^{X*}\,{}_nC_y{}^*{}_{n-y}C_w$ works for X=1, X=3, X=7, X=9 , X=19 and X=184755 (as far as we can tell), for quizzes where stems can be missed.

8.14 Discussion

Appendix 7 showed that the expression $X^{x}\,{}^*{}_nC_y{}^*{}_{n-y}C_w$ worked for quizzes where stems could not be missed (i.e. **y**=0).

This Appendix shows that the expression $X^{x}\,{}^*{}_nC_y{}^*{}_{n-y}C_w$ works for quizzes where stems can be missed (i.e. **y** =0 and *can be* ≠0).

Thus, the expression $X^{x}\,{}^*{}_nC_y{}^*{}_{n-y}C_w$ has been validated, for 'small' values of **n**, for both sorts of quizzes and we believe this expression is correct for 'large' values of **n**.

The expression tells us the number of ways of getting a mark in a particular manner. The particular manner is given by the values of X, **x**, **n**, **y**, and **w**.

Appendix 9. Import a tab-delimited text file into Excel™

The tool, ***tool1*** outputs to a tab-delimited file called

F:\MyPhD2\MyPhD\MixtureWirhoutDescriptiveStats.txt

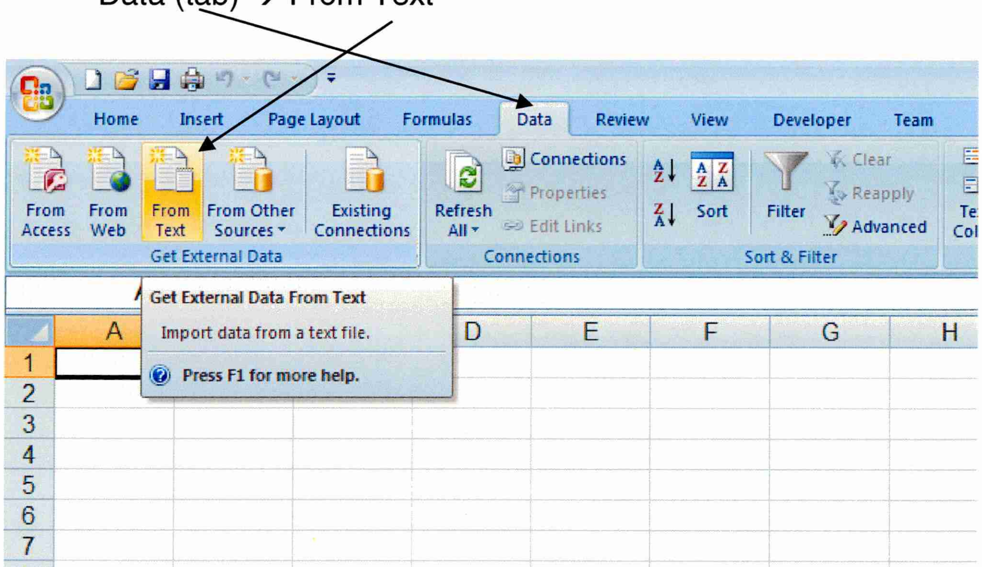
The importation of a tab-delimited text file into Excel™ has the following steps

1. Open Excel™.
    a. Choose the cell A1 to work in
    b. We assume that this is obvious, and so will be missed out of the following
2. Say we are going to import a text file
3. Navigate to the text file
4. Follow the 'wizard' instructions
    a. Excel™ will work out for itself that the file is delimited, or you have to tell it.

We will expand on steps 2, 3, 4

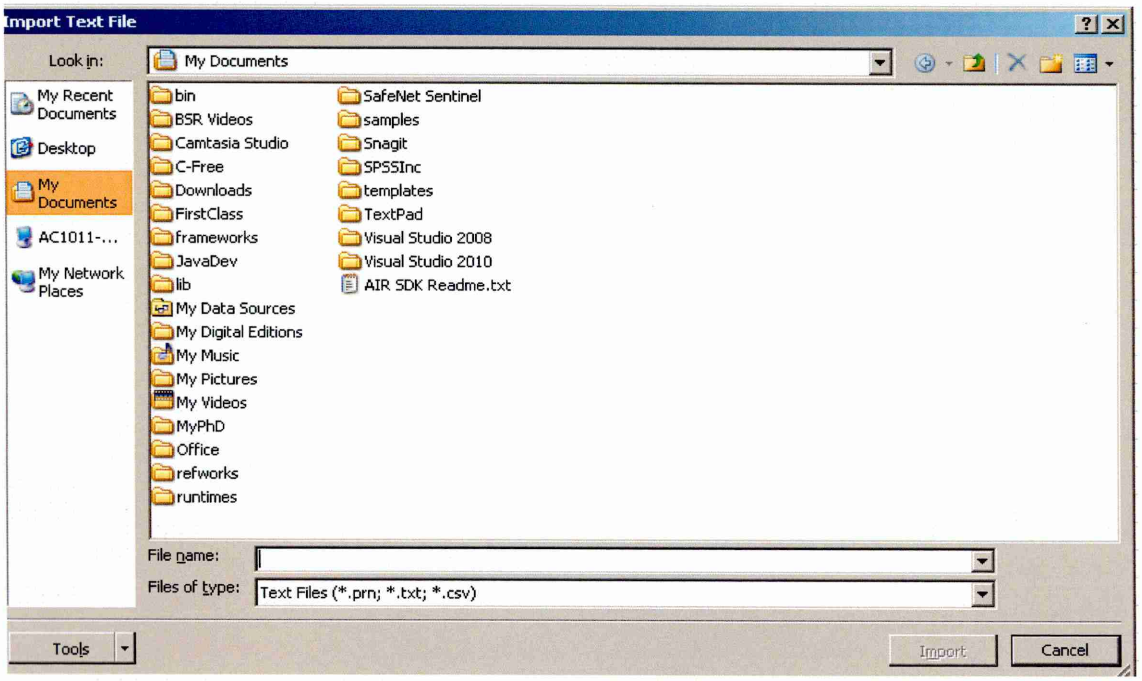
Step 2. Say that we are going to import a text file

Data (tab) → From Text

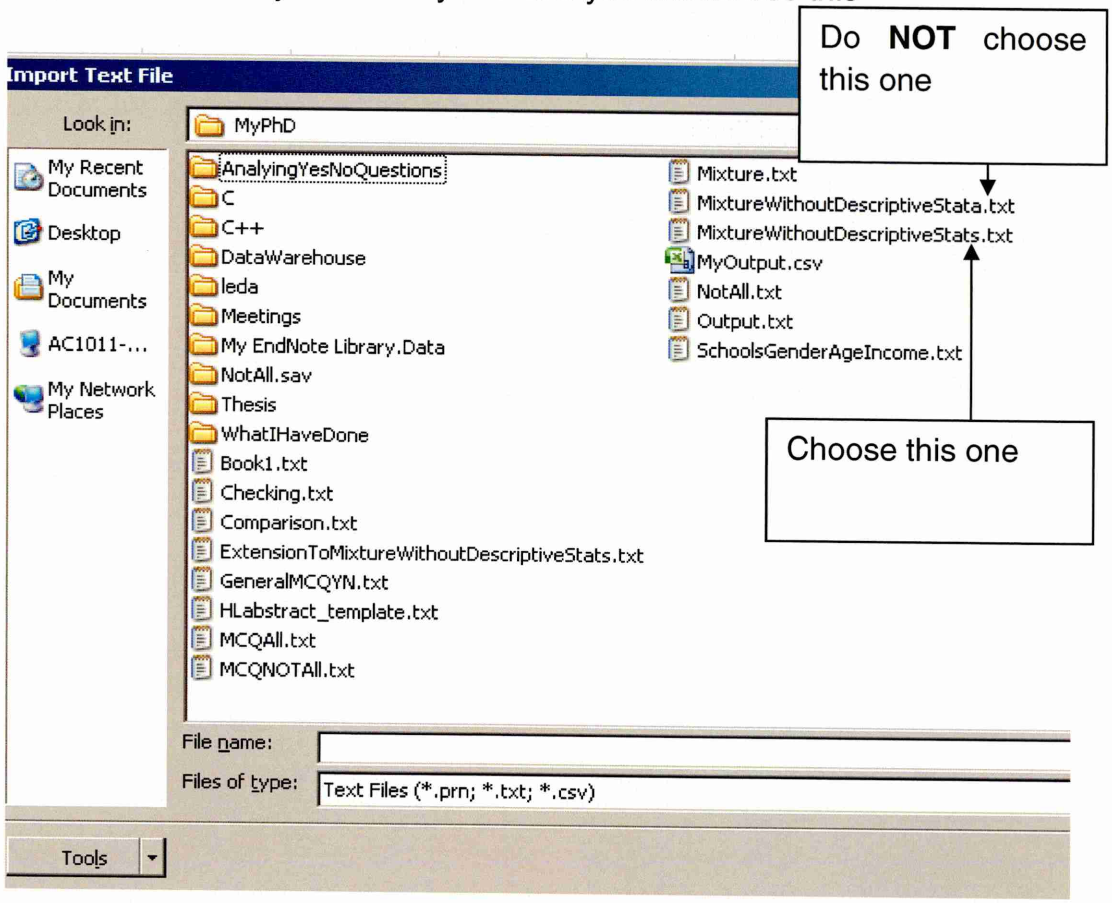


Step 3. Navigate to the text file

This is the 1st screen that you will see.



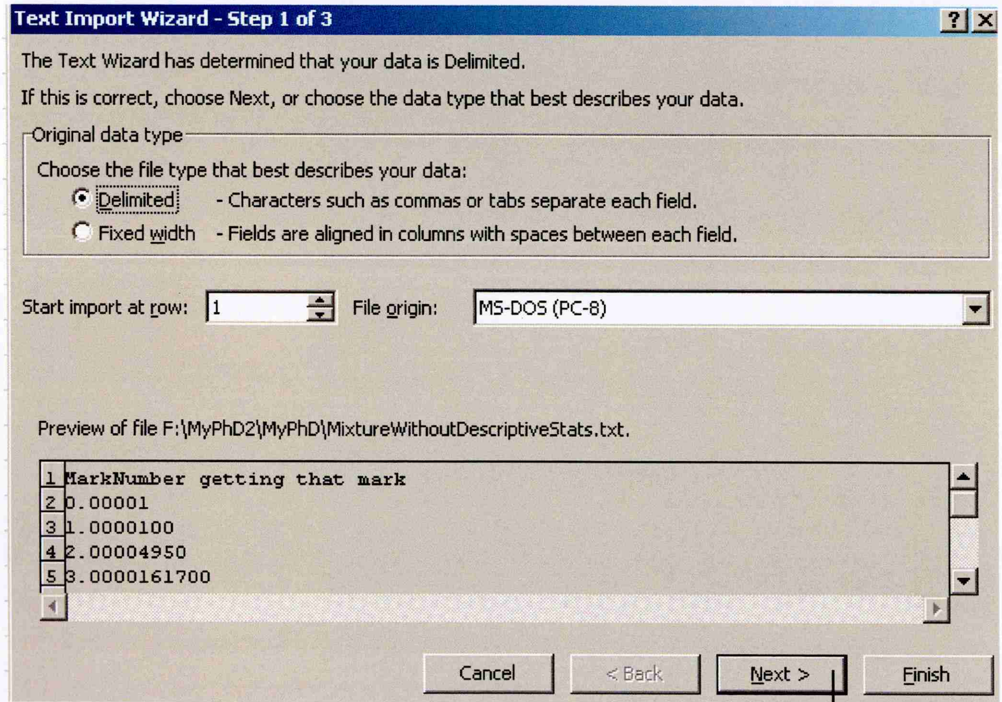
Go to F: drive → MyPhD2→ MyPhD and you should see this



Choose **MixtureWithoutDescriptiveStats.txt** NOT MixtureWithoutDescriptive Stata.txt

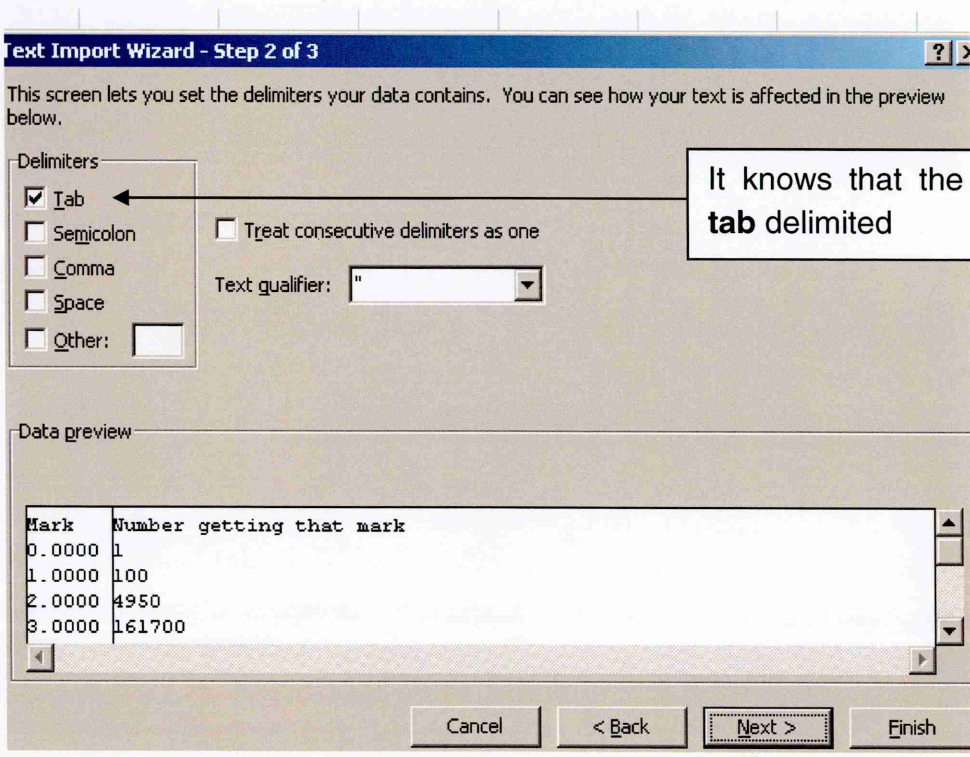
Step 4. Follow the wizard

The first screen you will see is



ALWAYS choose **Delimited** and then choose **Next** (Excel™ can usually work this out for itself) and if **Delimited** is already chosen then just choose **Next**

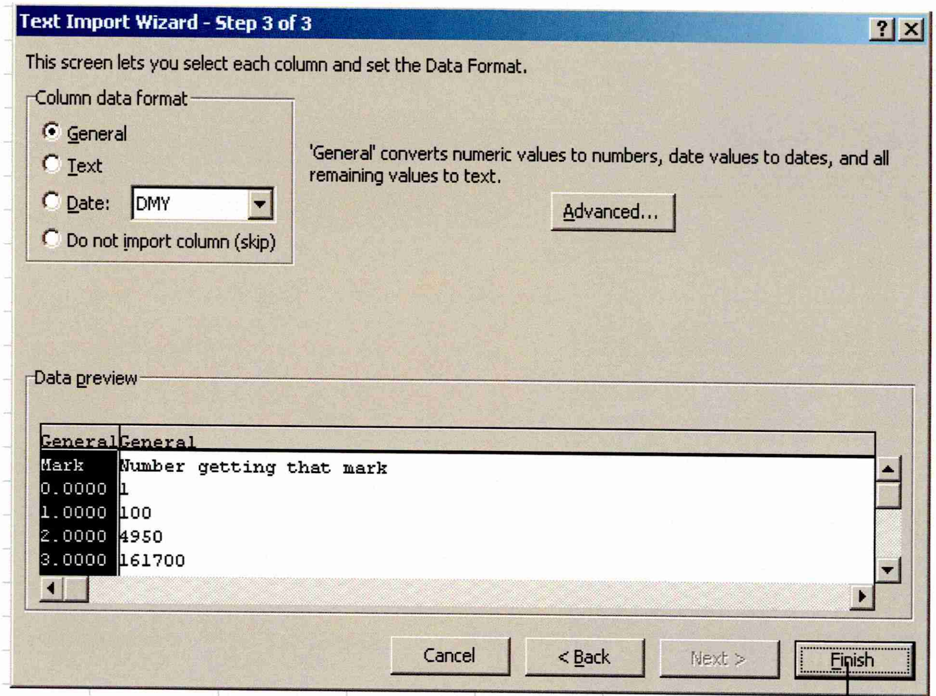
So, now you should see something like



It knows that the file is **tab** delimited

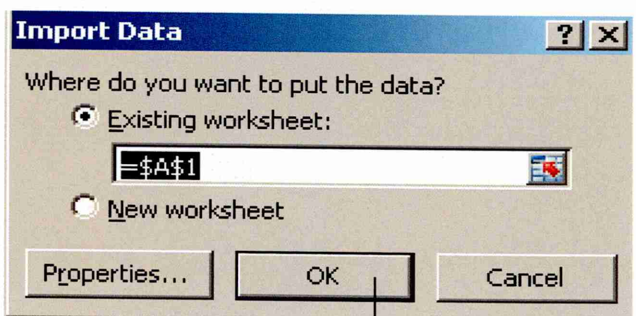
Just choose **Next**

You should see something like this



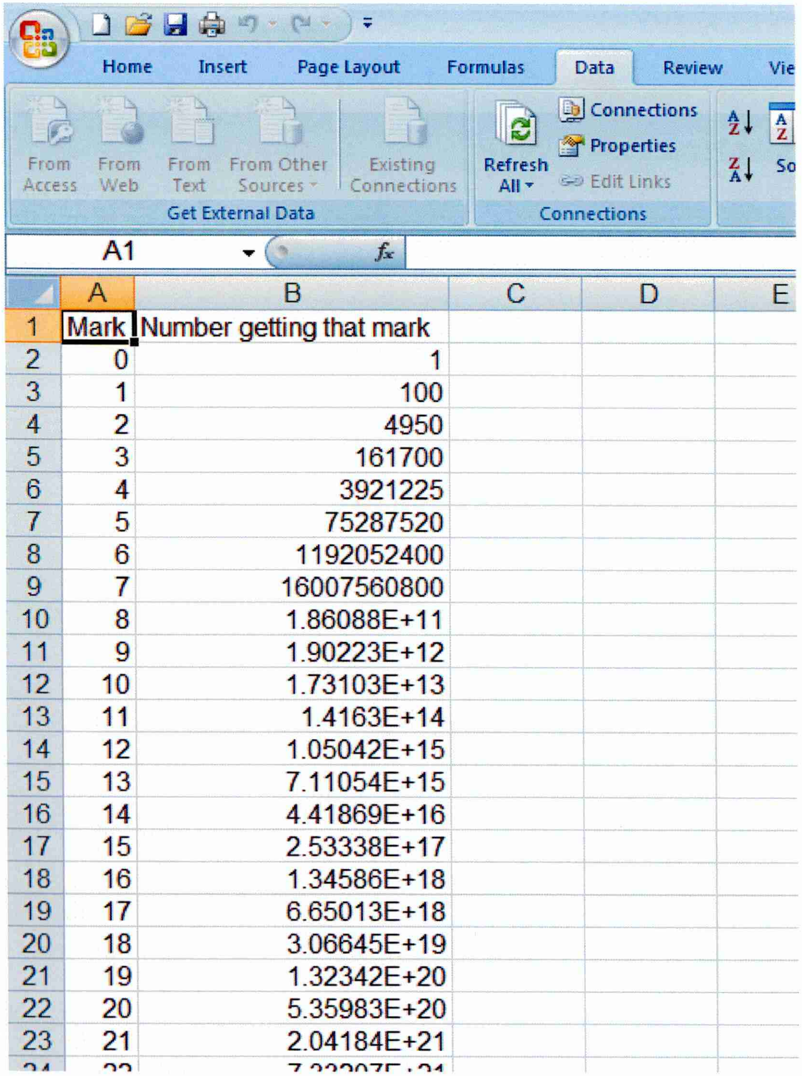
Just choose **Finish**

and then you will see



Just choose **OK**

and then you will see something like



	A	B
1	Mark	Number getting that mark
2	0	1
3	1	100
4	2	4950
5	3	161700
6	4	3921225
7	5	75287520
8	6	1192052400
9	7	16007560800
10	8	1.86088E+11
11	9	1.90223E+12
12	10	1.73103E+13
13	11	1.4163E+14
14	12	1.05042E+15
15	13	7.11054E+15
16	14	4.41869E+16
17	15	2.53338E+17
18	16	1.34586E+18
19	17	6.65013E+18
20	18	3.06645E+19
21	19	1.32342E+20
22	20	5.35983E+20
23	21	2.04184E+21

BUT, notice the E format and even if we Format the B column to be integers with 0 decimal place WE WILL GET A LOSS OF ACCURACY.

This loss of accuracy sometimes results in a slight disagreement between the tools

Appendix 10. Excel's™ Pivot Chart

The *usual* process (where 1 file is imported) is:

- The 1st file is imported into Sheet 1 and then a Pivot is performed on Sheet 1's data and put into Sheet 4.
- Titles are added to Sheet 4
- The results of Sheet 4 are copied to Word, so that the results are not changed by Excel™, by using Paste Special in Word.
- In Word some 'fiddling about ' is done. The 'fiddling' includes a) adding titles to the axes b) formatting the picture of the Pivot
- In Excel™, Sheet 4 is changed and exported to **Output.txt** and this file is 'cleaned up' before its descriptive statistics are calculated. (See Appendix 12)

10.1 The usual process

The **usual** process is to only have 1 input file and this part of the Appendix shows how Pivot Charts is Excel™ are used on 1 input file.

The **input** is **F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.txt**
that is a tab-delimited text file

This file contains Many lines, which consist of

On the 1st line are headers, which are strings
Mark
tab
Number getting that mark

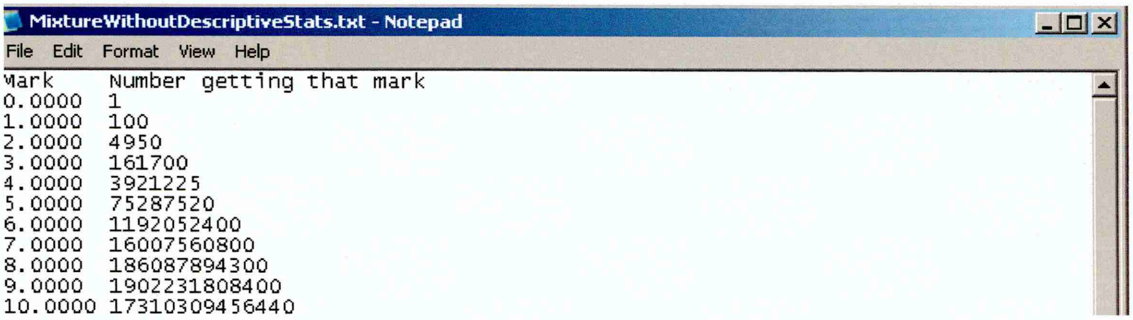
On each line thereafter are floats that represent
Mark
tab
Number of ways of getting that mark in that manner

The **outputs** are

1. A plot of the distribution of the results of the Pivot Chart
2. Some titles are added to these results
3. The results plus titles are copied to Word such that they are not altered
4. The changed results are exported to **Output.txt**
   (See Appendix 11)
5. **Output.txt** is 'cleaned up'
6. A calculation tool uses **Output.txt** to form the descriptive statistics of the distribution.(See Appendix ??)

This Appendix only describes steps 1, 2., 3 and 4

The **input** file starts off something like this,



MixtureWithoutDescriptiveStats.txt - Notepad

File Edit Format View Help

```
Mark    Number getting that mark
0.0000  1
1.0000  100
2.0000  4950
3.0000  161700
4.0000  3921225
5.0000  75287520
6.0000  1192052400
7.0000  16007560800
8.0000  186087894300
9.0000  1902231808400
10.0000 17310309456440
```

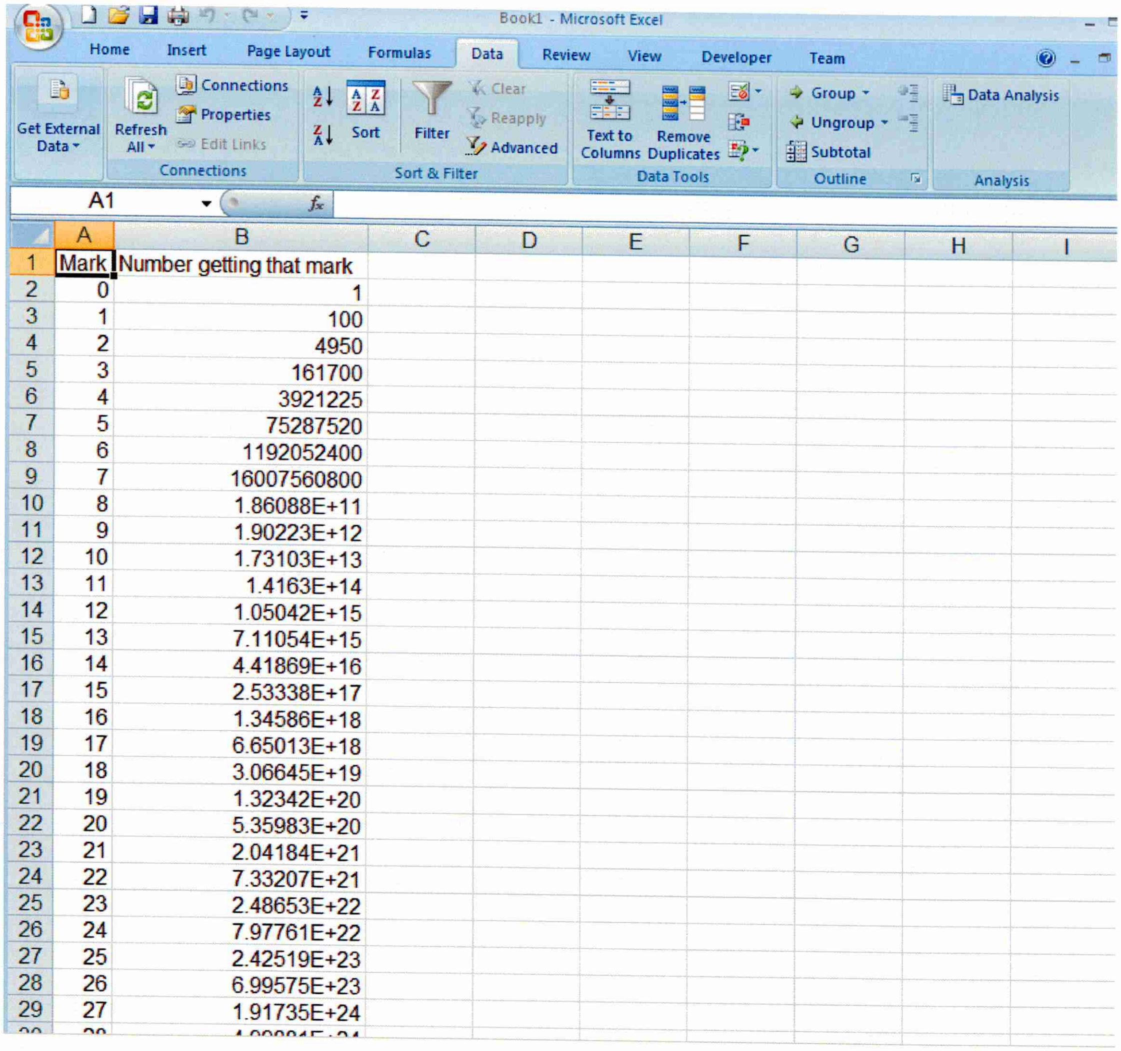
This is a file that represents Yes/No quiz where stems cannot be missed
Negative marking is NOT used
The score is [1, 0].
There is 1 header and 101 data lines.

The marks might not be unique and are not ordered, (but they are in this case i.e. the marks are unique and ordered). We want to perform a Pivot chart on this data to make the marks unique and order them.

The first thing we have to do is get this data into Excel™.

Open Excel™
Data (tab) → From Text → Navigate to
F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.txt and Import this data in Excel™ (see Appendix 9).

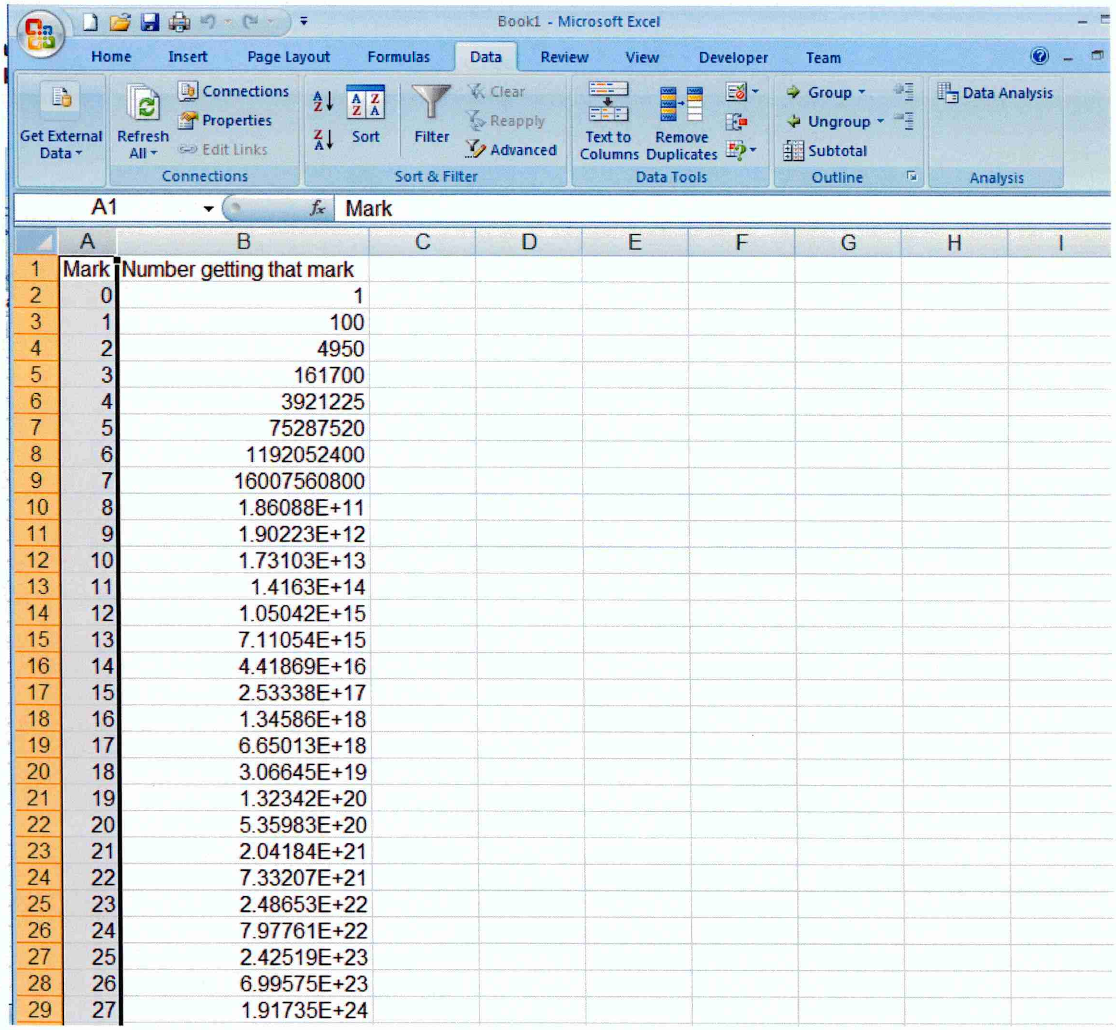
You should see something like this



Mark	Number getting that mark
0	1
1	100
2	4950
3	161700
4	3921225
5	75287520
6	1192052400
7	16007560800
8	1.86088E+11
9	1.90223E+12
10	1.73103E+13
11	1.4163E+14
12	1.05042E+15
13	7.11054E+15
14	4.41869E+16
15	2.53338E+17
16	1.34586E+18
17	6.65013E+18
18	3.06645E+19
19	1.32342E+20
20	5.35983E+20
21	2.04184E+21
22	7.33207E+21
23	2.48653E+22
24	7.97761E+22
25	2.42519E+23
26	6.99575E+23
27	1.91735E+24

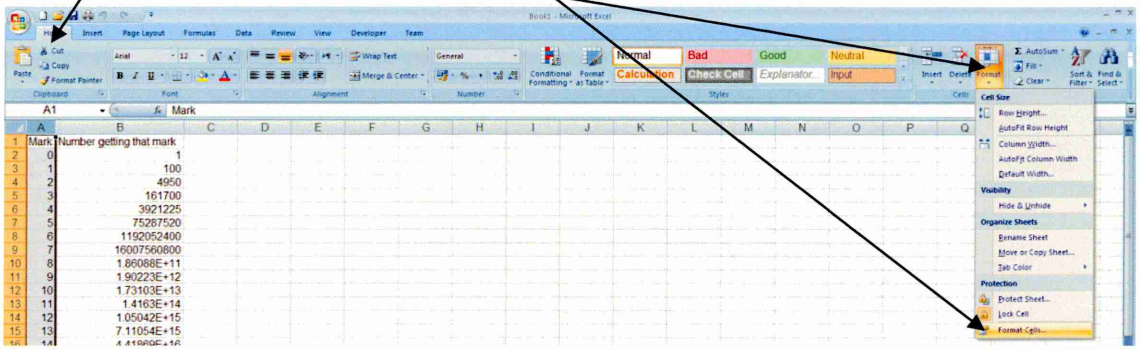
Highlight the first column of the data i.e. the A column

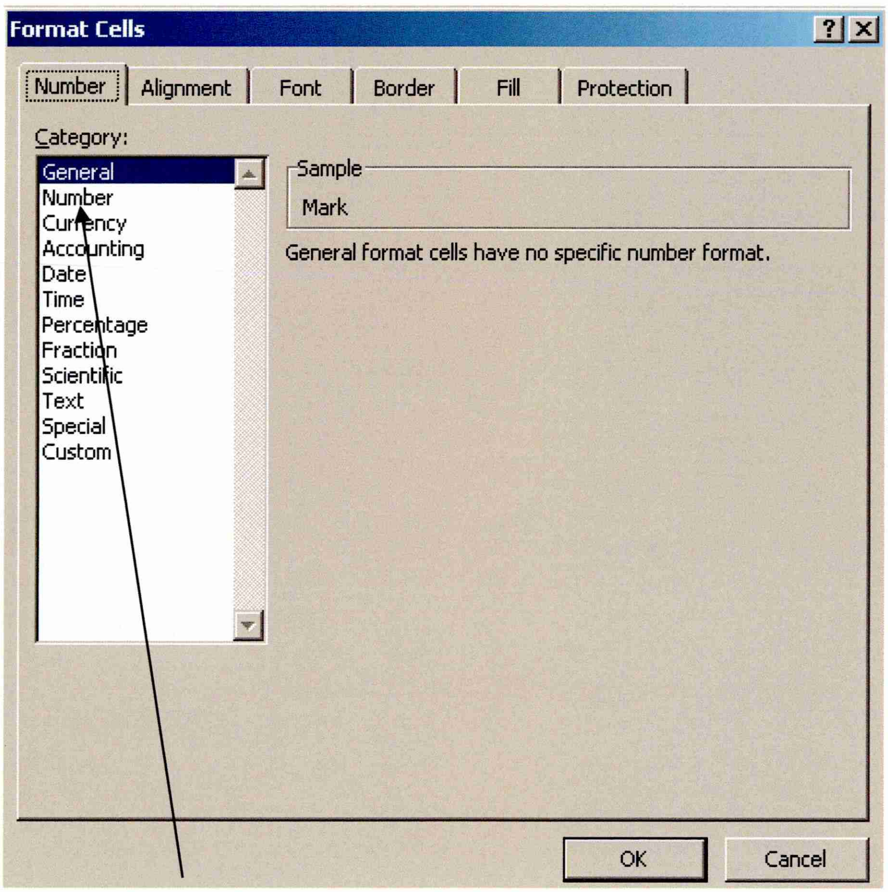
You should see something like this



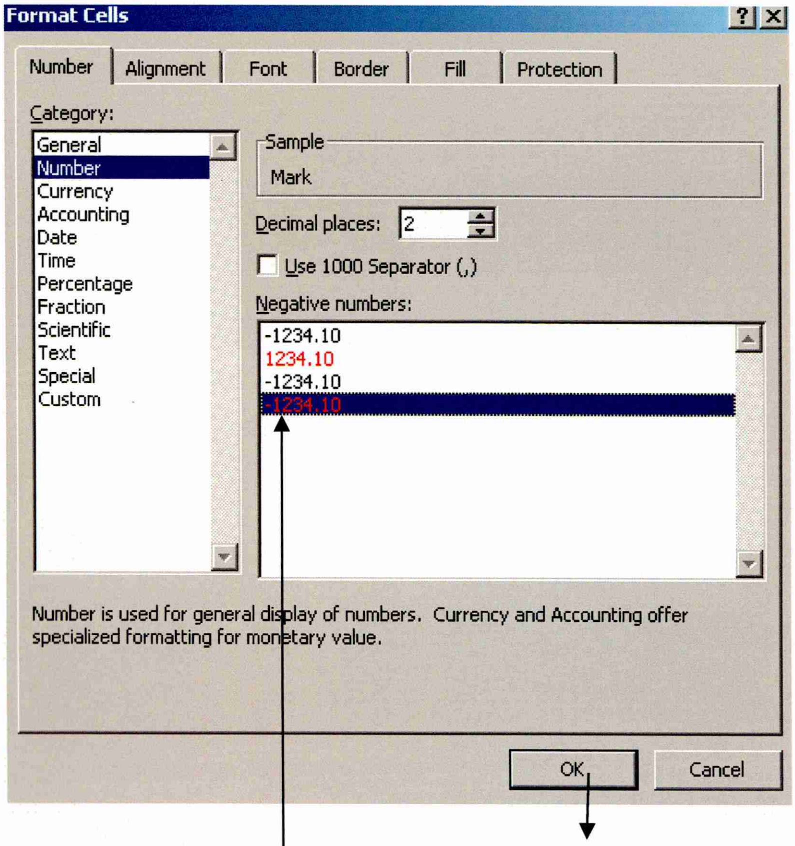
Convert column A to number format with negative numbers shown in red

Home →Format→Format Cells

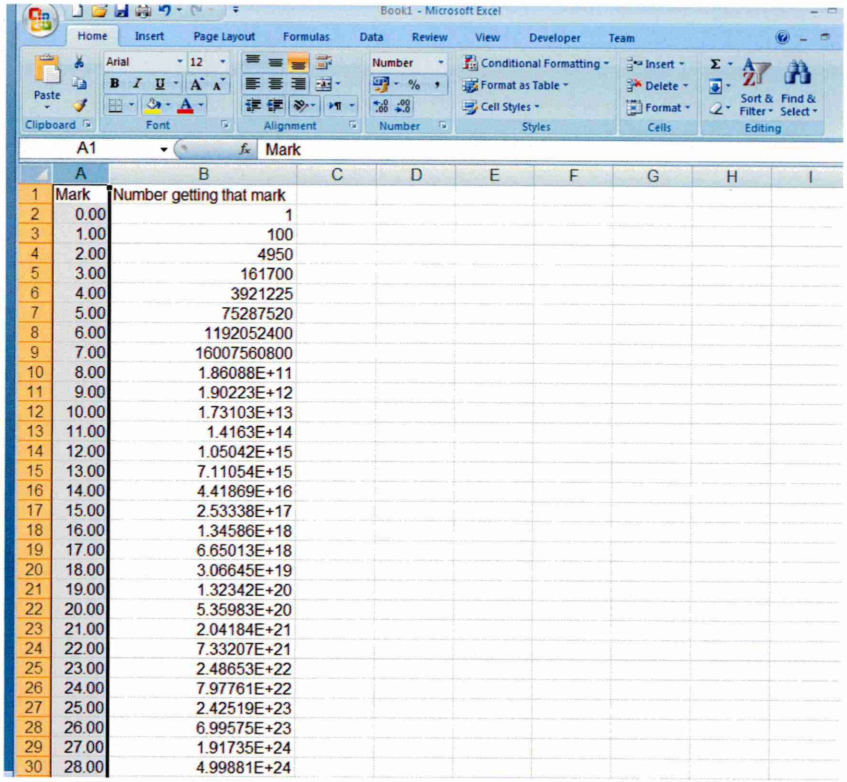




Choose **Number**



Choose negative number to be in red, and then choose **OK** and you should see something like this



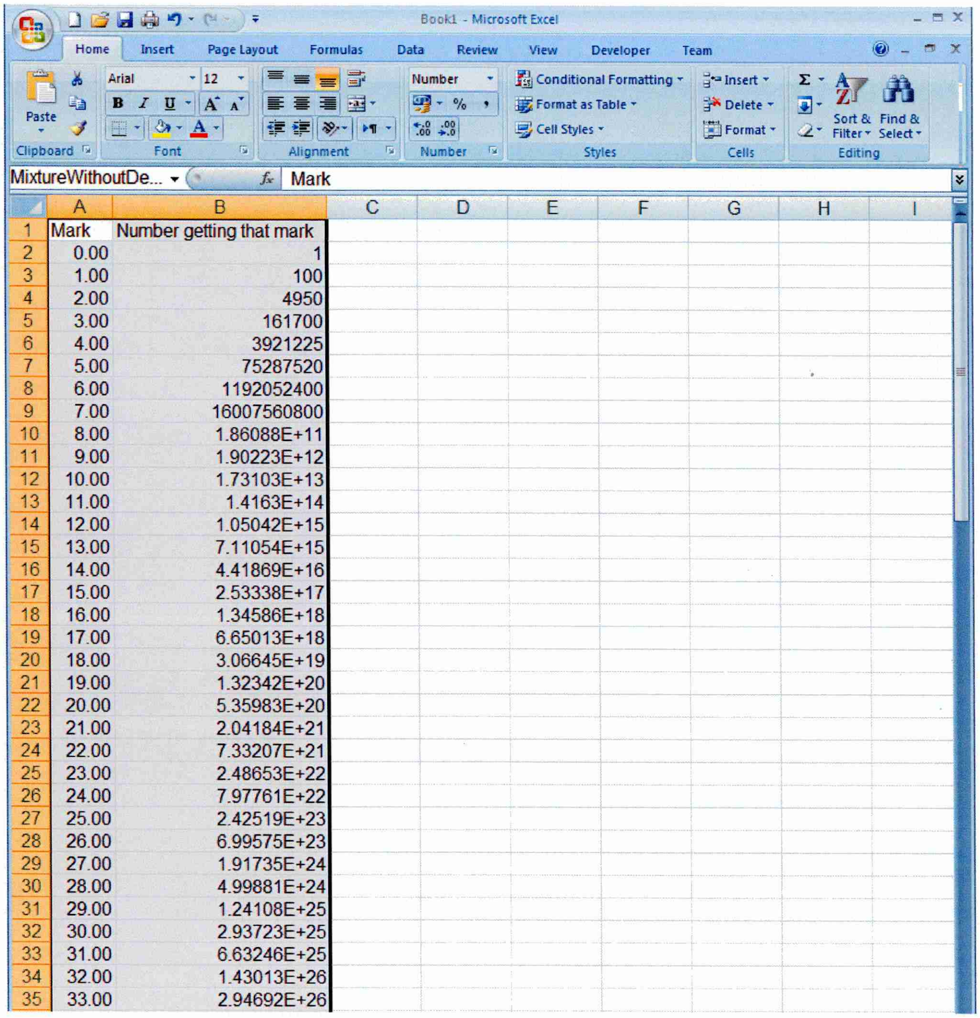
	A	B
1	Mark	Number getting that mark
2	0.00	1
3	1.00	100
4	2.00	4950
5	3.00	161700
6	4.00	3921225
7	5.00	75287520
8	6.00	1192052400
9	7.00	16007560800
10	8.00	1.86088E+11
11	9.00	1.90223E+12
12	10.00	1.73103E+13
13	11.00	1.4163E+14
14	12.00	1.05042E+15
15	13.00	7.11054E+15
16	14.00	4.41869E+16
17	15.00	2.53338E+17
18	16.00	1.34586E+18
19	17.00	6.65013E+18
20	18.00	3.06645E+19
21	19.00	1.32342E+20
22	20.00	5.35983E+20
23	21.00	2.04184E+21
24	22.00	7.33207E+21
25	23.00	2.48653E+22
26	24.00	7.97761E+22
27	25.00	2.42519E+23
28	26.00	6.99575E+23
29	27.00	1.91735E+24
30	28.00	4.99881E+24

Notice that the marks are now to 2 decimal places.

When marks are negative (i.e. when negative marking is used) the negative marks are shown in red.

Highlight ALL the data (including the headers)

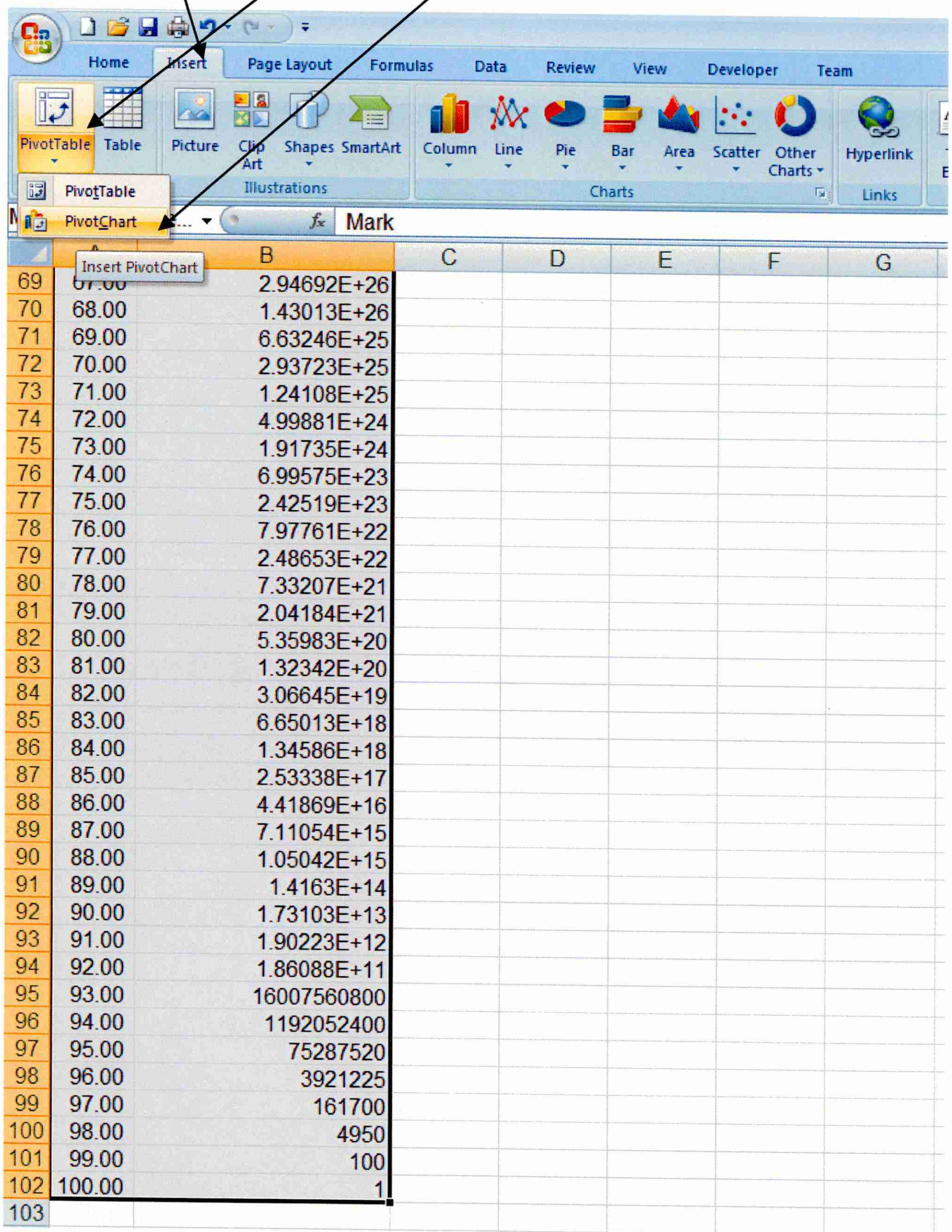
Go to the bottom of the data
Choose the first 2 items of the bottom line
Hold the Shift arrow
Go to the top of the data
Choose Mark
You should see something like this

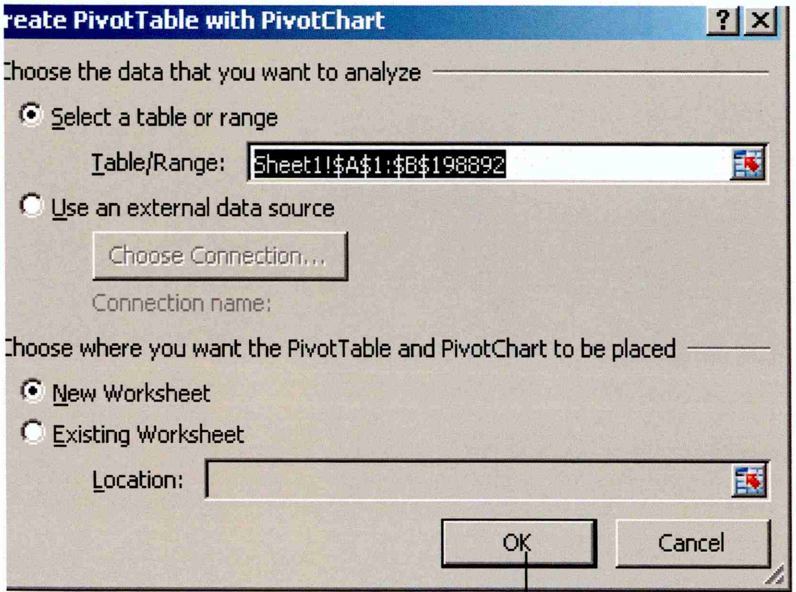


	A	B
1	Mark	Number getting that mark
2	0.00	1
3	1.00	100
4	2.00	4950
5	3.00	161700
6	4.00	3921225
7	5.00	75287520
8	6.00	1192052400
9	7.00	16007560800
10	8.00	1.86088E+11
11	9.00	1.90223E+12
12	10.00	1.73103E+13
13	11.00	1.4163E+14
14	12.00	1.05042E+15
15	13.00	7.11054E+15
16	14.00	4.41869E+16
17	15.00	2.53338E+17
18	16.00	1.34586E+18
19	17.00	6.65013E+18
20	18.00	3.06645E+19
21	19.00	1.32342E+20
22	20.00	5.35983E+20
23	21.00	2.04184E+21
24	22.00	7.33207E+21
25	23.00	2.48653E+22
26	24.00	7.97761E+22
27	25.00	2.42519E+23
28	26.00	6.99575E+23
29	27.00	1.91735E+24
30	28.00	4.99881E+24
31	29.00	1.24108E+25
32	30.00	2.93723E+25
33	31.00	6.63246E+25
34	32.00	1.43013E+26
35	33.00	2.94692E+26

Now you can do a Pivot Chart on the data.

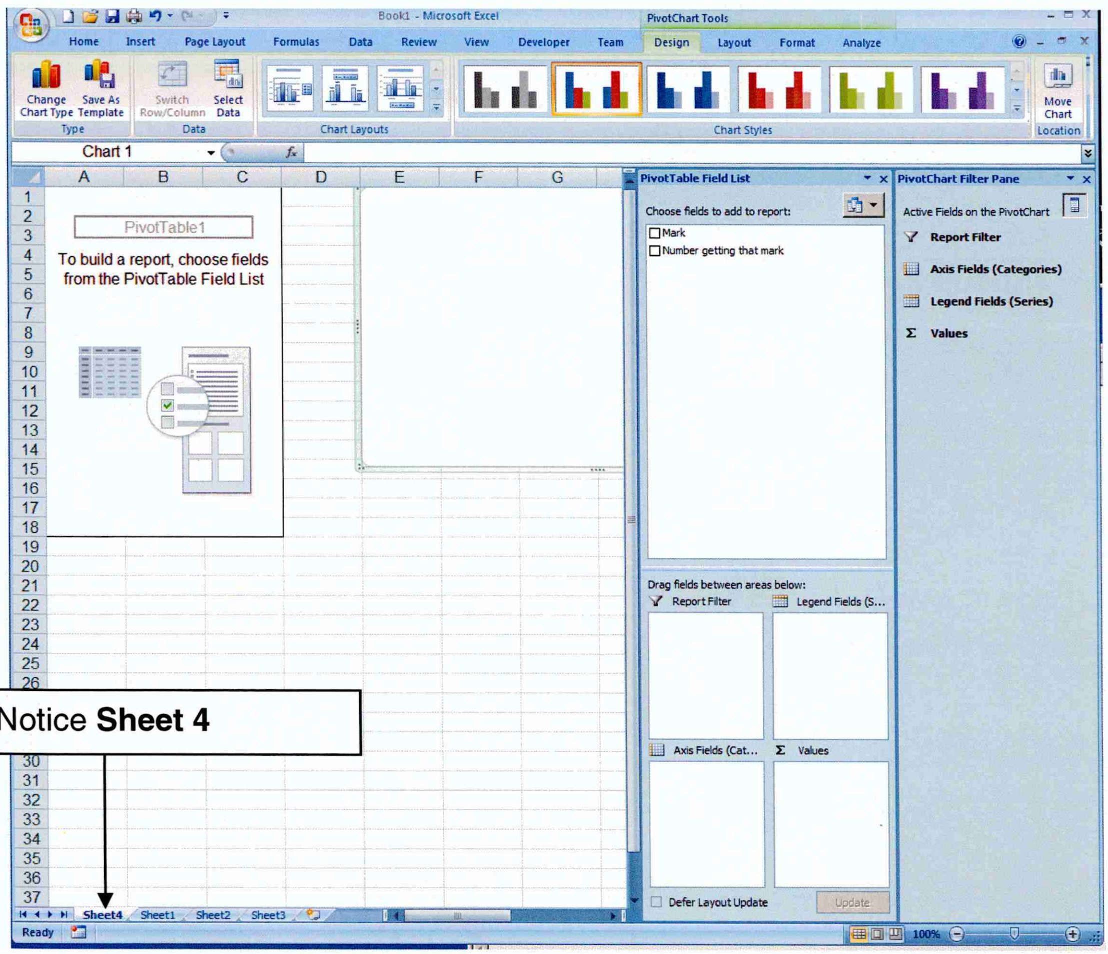
Insert →Pivot Table→Pivot Chart
and you should see something like this





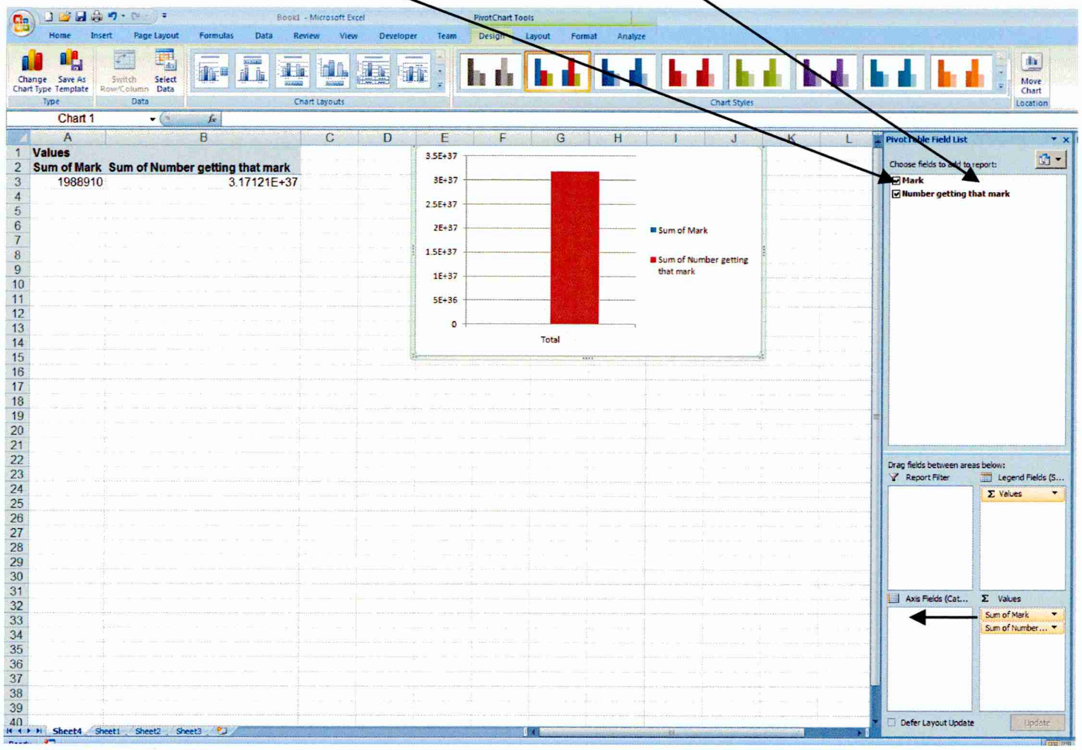
Choose **OK**

The results of doing a Pivot Chart are put in Sheet 4

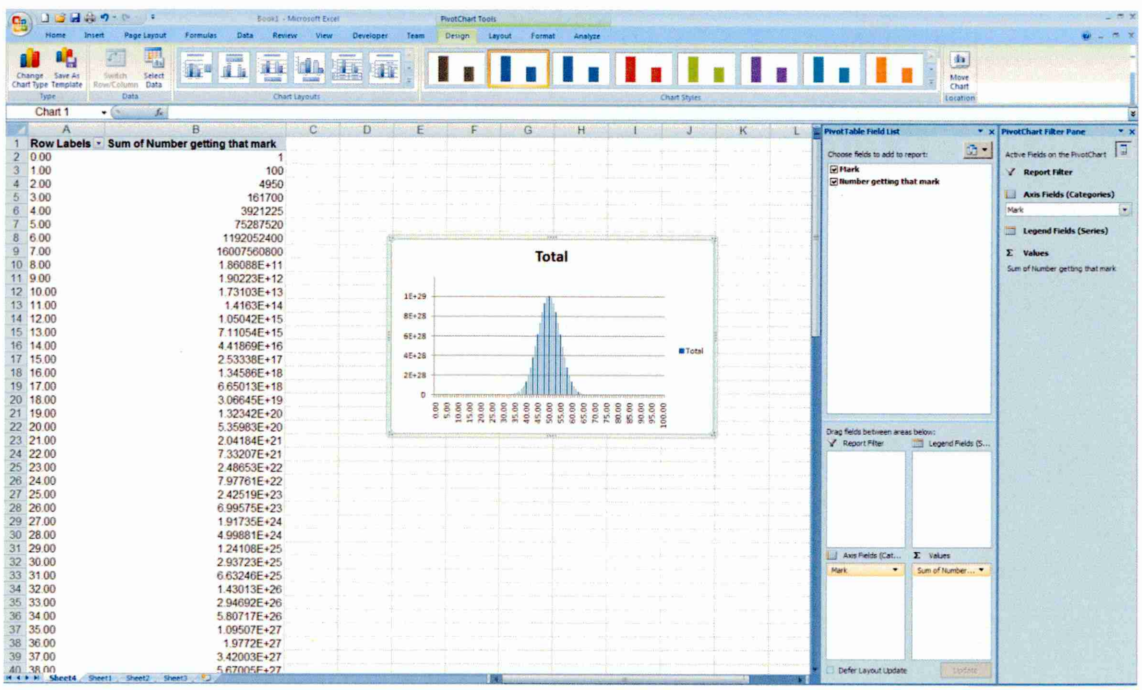


Notice **Sheet 4**

Highlight **Mark** and **Number getting that mark**



Transfer ∑ **Values** Sum of Mark to Axis Field Mark



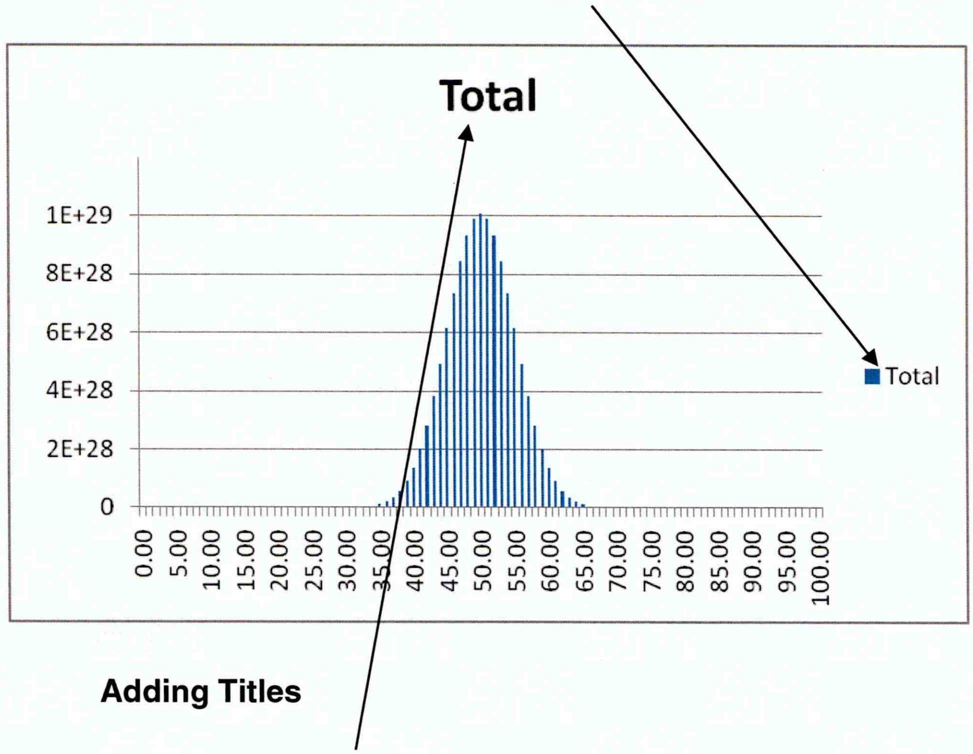
So, now, we have plotted the distribution of the product of Pivot Chart and we have to add titles to this display, and then copy that to Word such that it is not linked, and so will not be changed (i.e. we use Paste Special and not Paste)

Notice that the data is in **Sheet 1** and the Pivot Chart is in **Sheet 4.**

Adding titles (and removing a bit)

Removing a bit

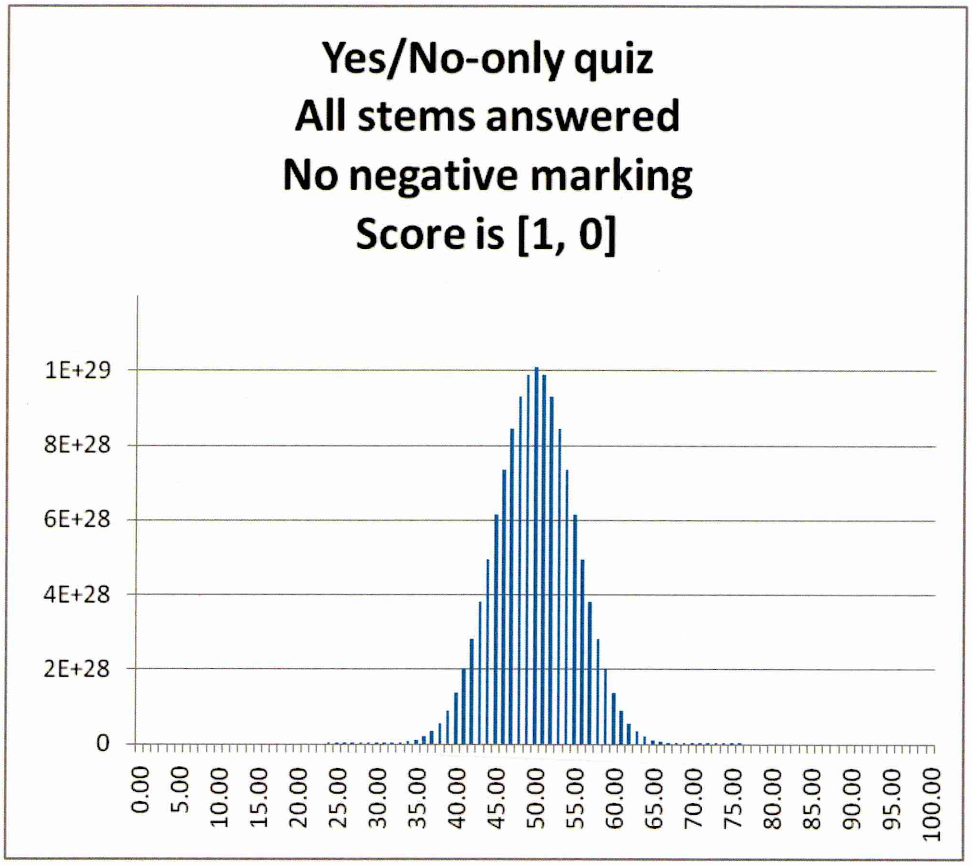
We want to remove this, which we do by choosing it and removing it.



Adding Titles

Go into **Total** and add titles which reflect the distribution.
The display is changed
Expand the display
Copy the display

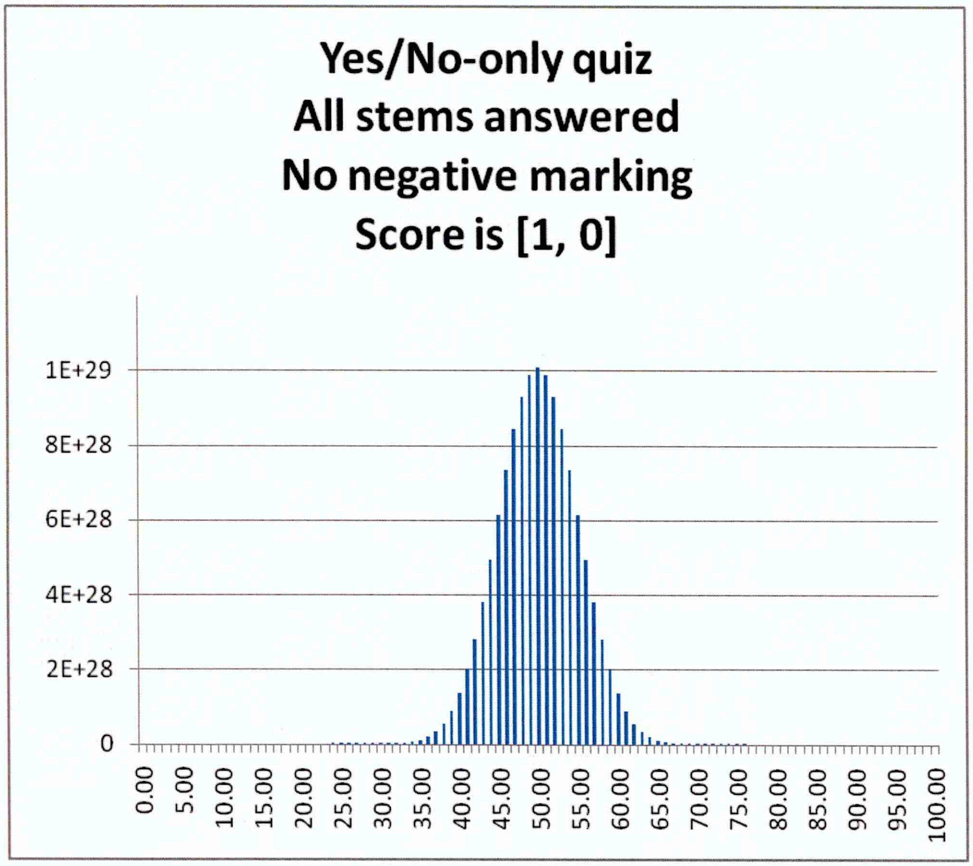
You should see something like this

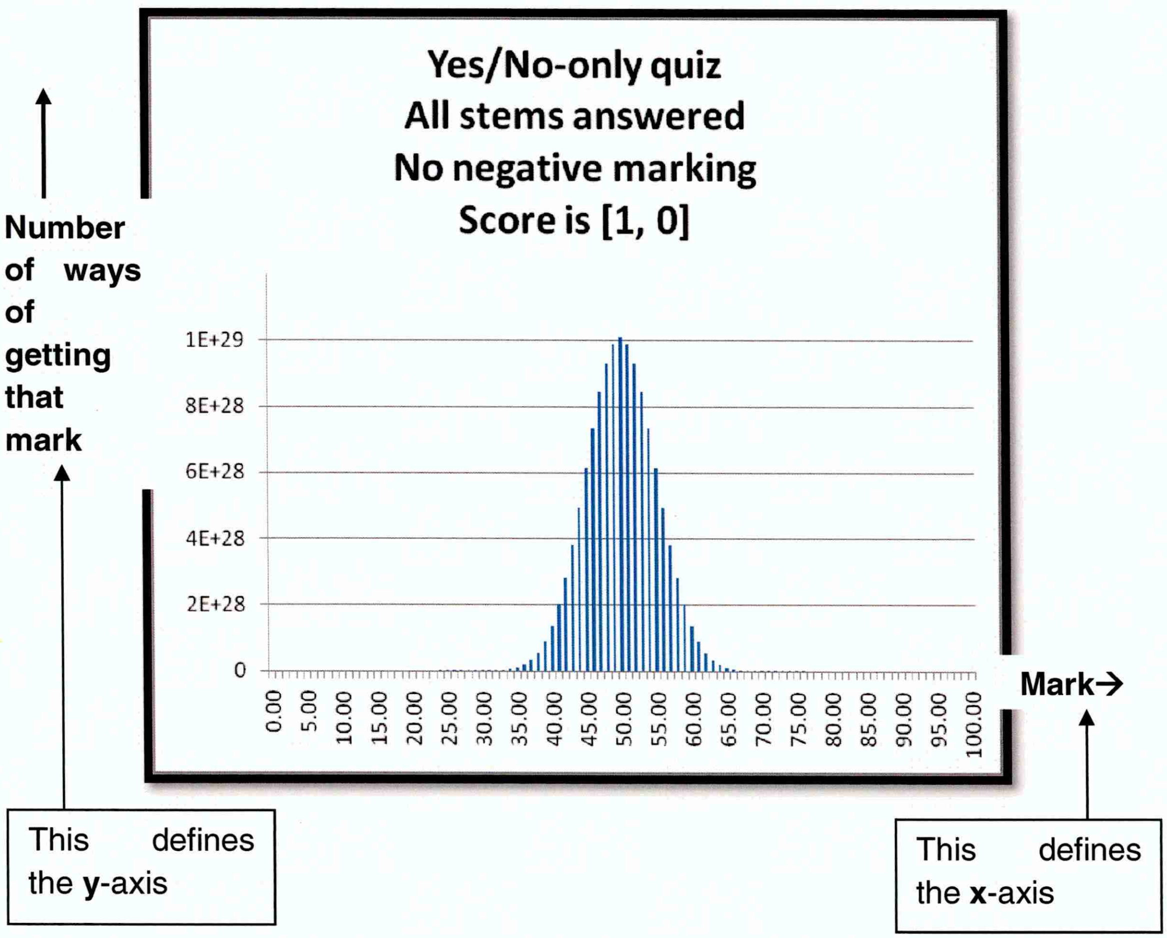


Paste Special into Word to prevent the display being changed.

Go to Word
In Word Paste Special and save it as a PNG picture
If you use Paste the picture is linked to Excel™ and can be changed in Excel™
Use Text Wrapping→ Square
Move the picture
Add text boxes that define the x and y axes.
The text boxes have no outline
Format the picture
The picture and the text boxes fit into the margins
Add a caption to the picture

You should see something like this





Appendix 11. Export a tab-delimited text file from Excel™ and clean up this file.

1. A calculation tool exports its results to 1 or 2 files.
2. The display tool takes in these results uses a Pivot Chart (see Appendix 10) and displays the result of the Pivot Chart
   a. You then take this display into Word, and make sure that the results of this display are not changed, by using Paste Special and not Paste.
3. You then have to export this display this display into **Output.txt** and clean up **Output.txt**
4. Another calculation tool works out the descriptive statistics associated with **Output.txt**

This Appendix is concerned with the *exportation* and *cleaning up* i.e. step 3.

You do the display in a worksheet and export from the worksheet. But, the exportation adds 2 lines (the first and last lines) and so the cleaning up consists of removing these 2 lines.

So, step 4 consists of stages:

a. Make sure that integers in column B, representing How Many are exported OK
b. Export from Excel™ to a tab-delimited file called **Output.txt**
c. Clean up **Output.txt**

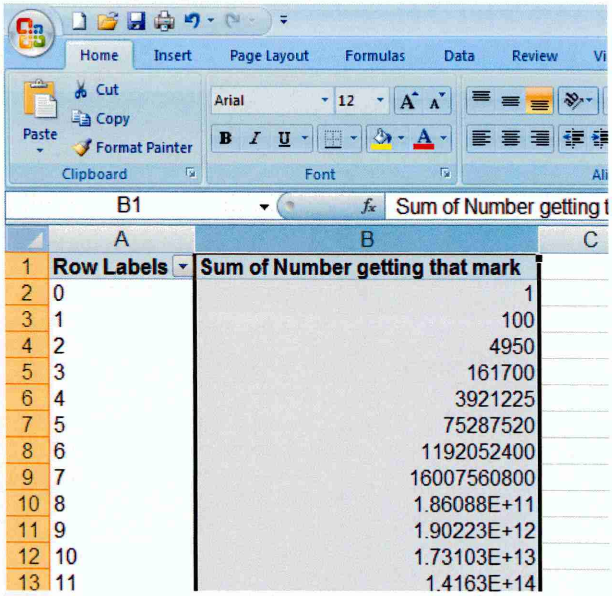
Step 3a. Make sure that column B are integers.

If you miss this step out and then the numbers read out of **Output.txt** are all 'out of kilter. For example if
0 456.00
1 789.00 is exported
Then what is read is
0 456 and it stops at the full stop (as it is not an integer) then it reads 00 as the mark 0 and then 1 as howMany and then the reading of the file is 'out of kilter'. So, what you need to do is:

- highlight column B
- and make them all integer numbers with 0 decimal places

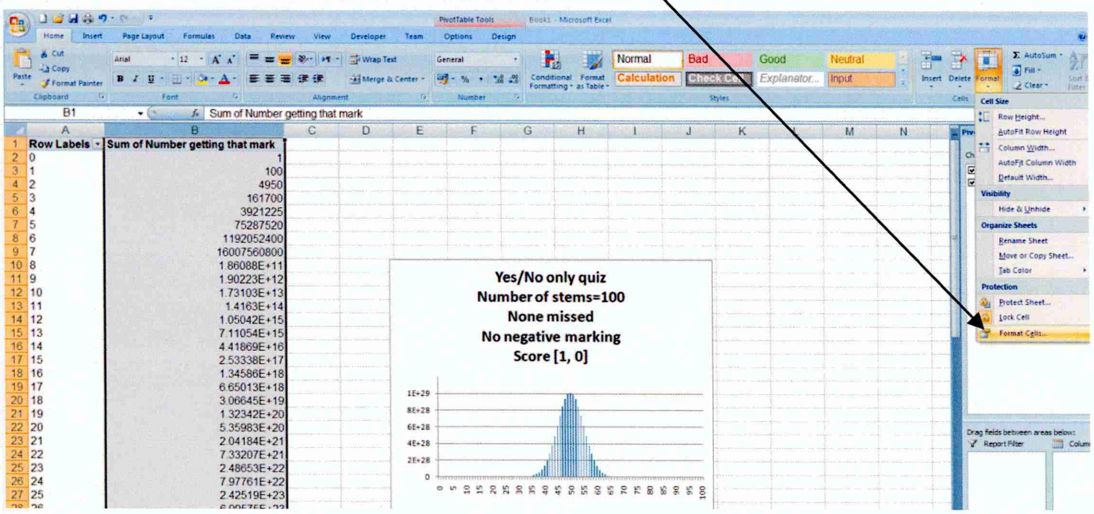
You should see something like this

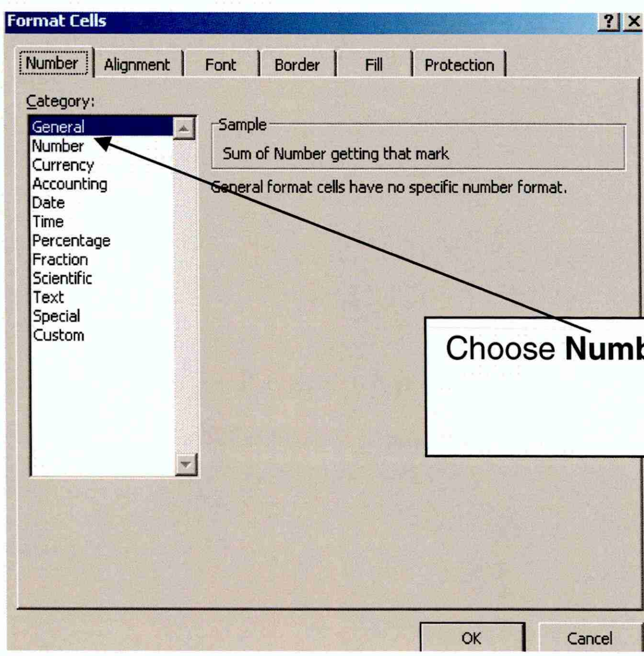
Highlight column B



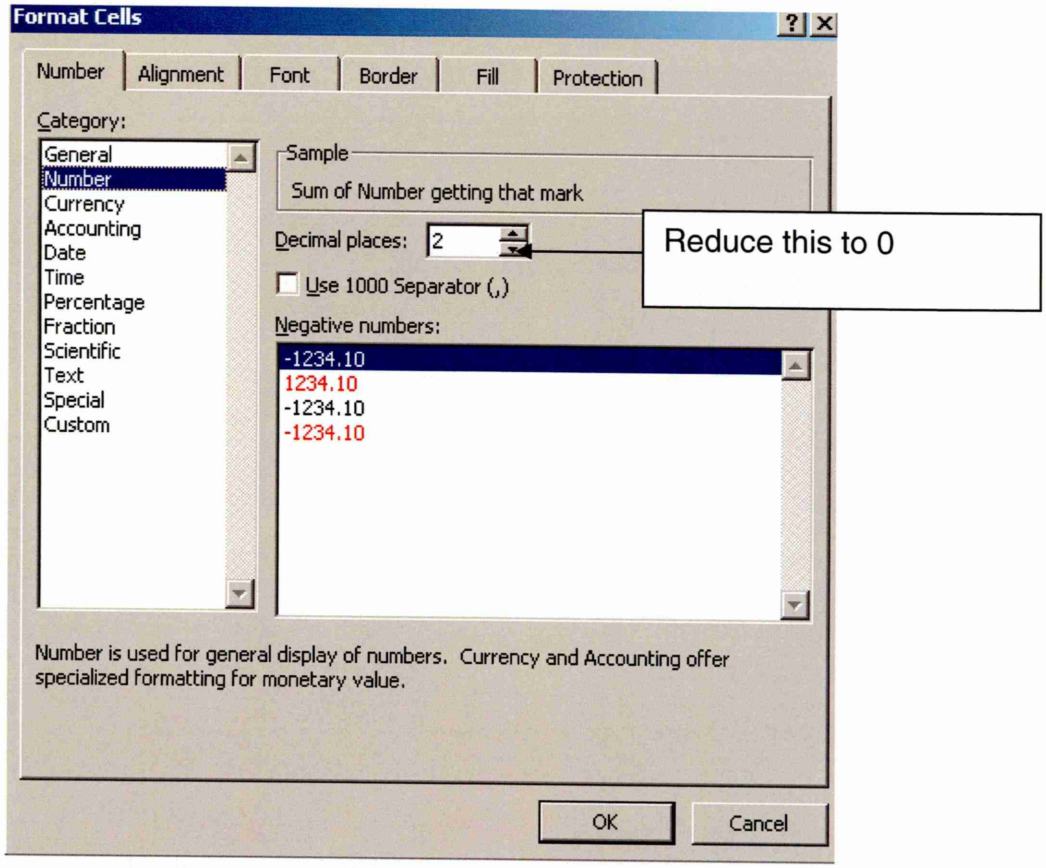
Notice that these have an E associated with them

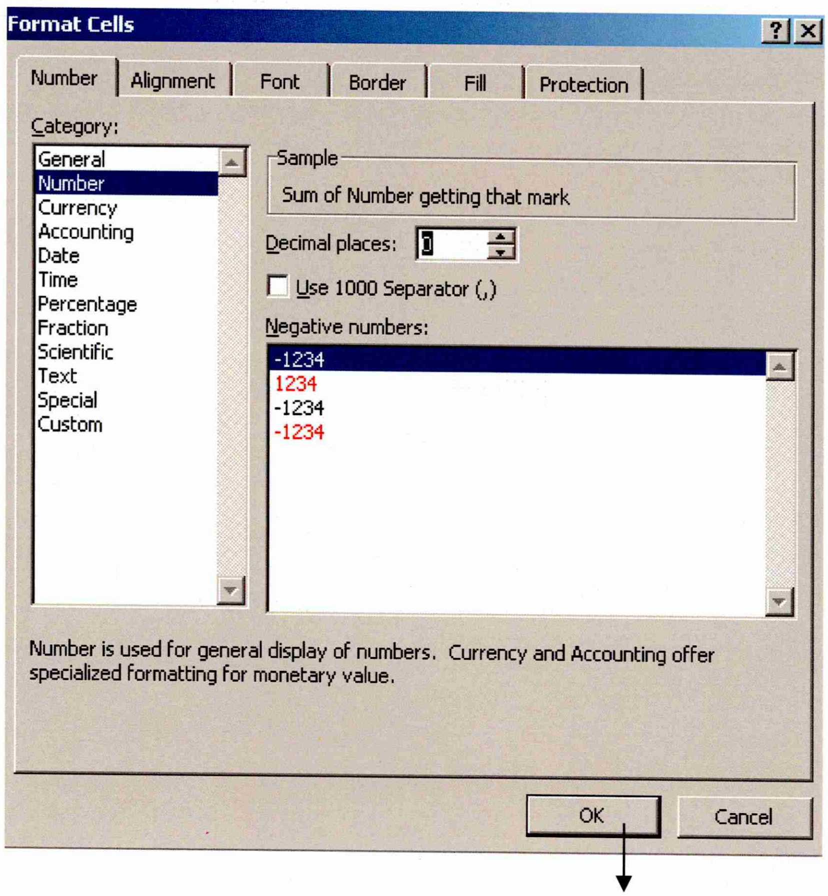
From **Home → Format→Format Cells**





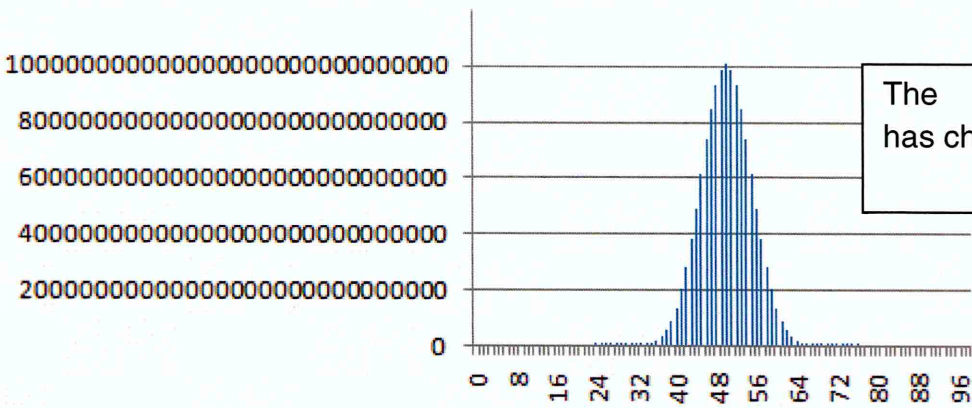
Choose **Number**



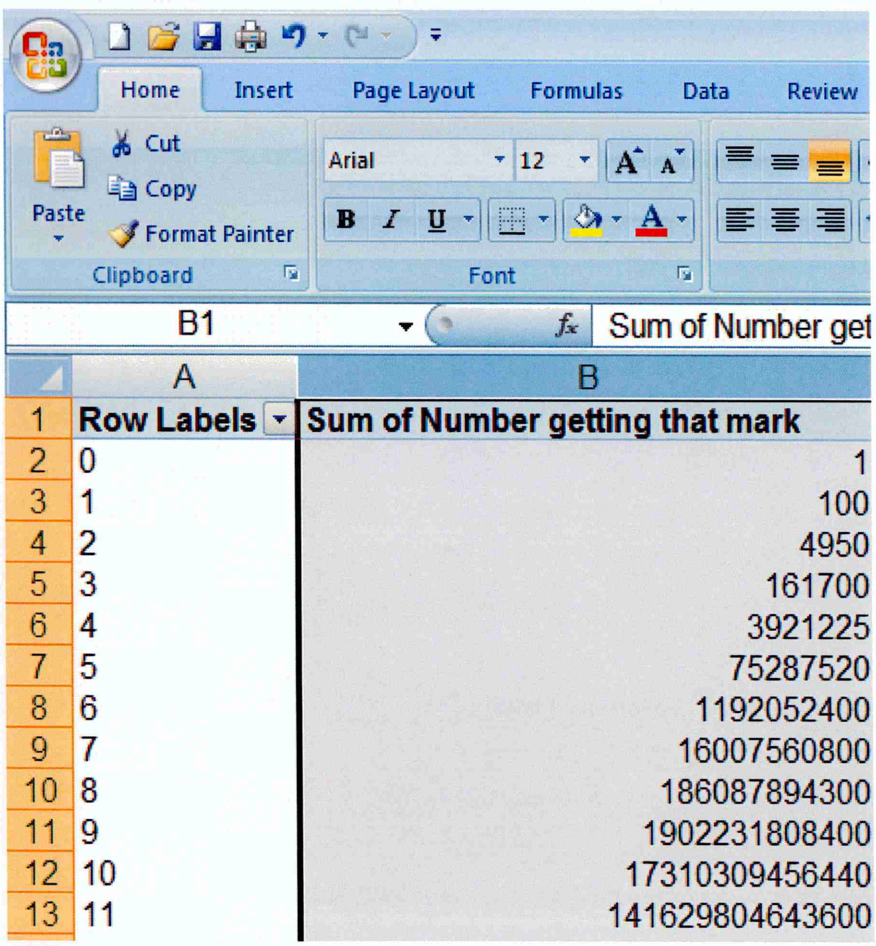


Then choose OK and the numbers on the y-axis should change, and the numbers now do not have an E associated with them.

Yes/No only quiz
Number of stems=100
None missed
No negative marking
Score [1, 0]



The display has changed



	A	B
1	Row Labels	Sum of Number getting that mark
2	0	1
3	1	100
4	2	4950
5	3	161700
6	4	3921225
7	5	75287520
8	6	1192052400
9	7	16007560800
10	8	186087894300
11	9	1902231808400
12	10	17310309456440
13	11	141629804643600

The numbers no longer have an E associated with them

Step 3b. Export a tab-delimited file

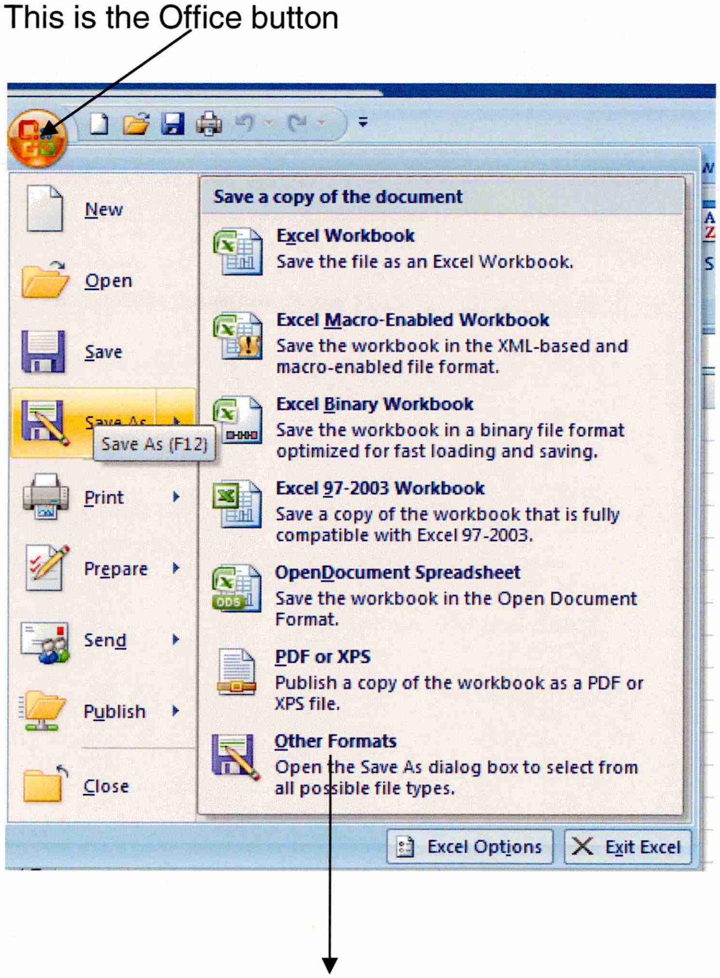
This step consists of the stages:

- Use the Office button → Save As → Other Formats
- Navigate to where **Output.txt** should be or already is and choose tab-delimited txt file
- Save **Output.txt** as tab-delimited txt file

You should see something like

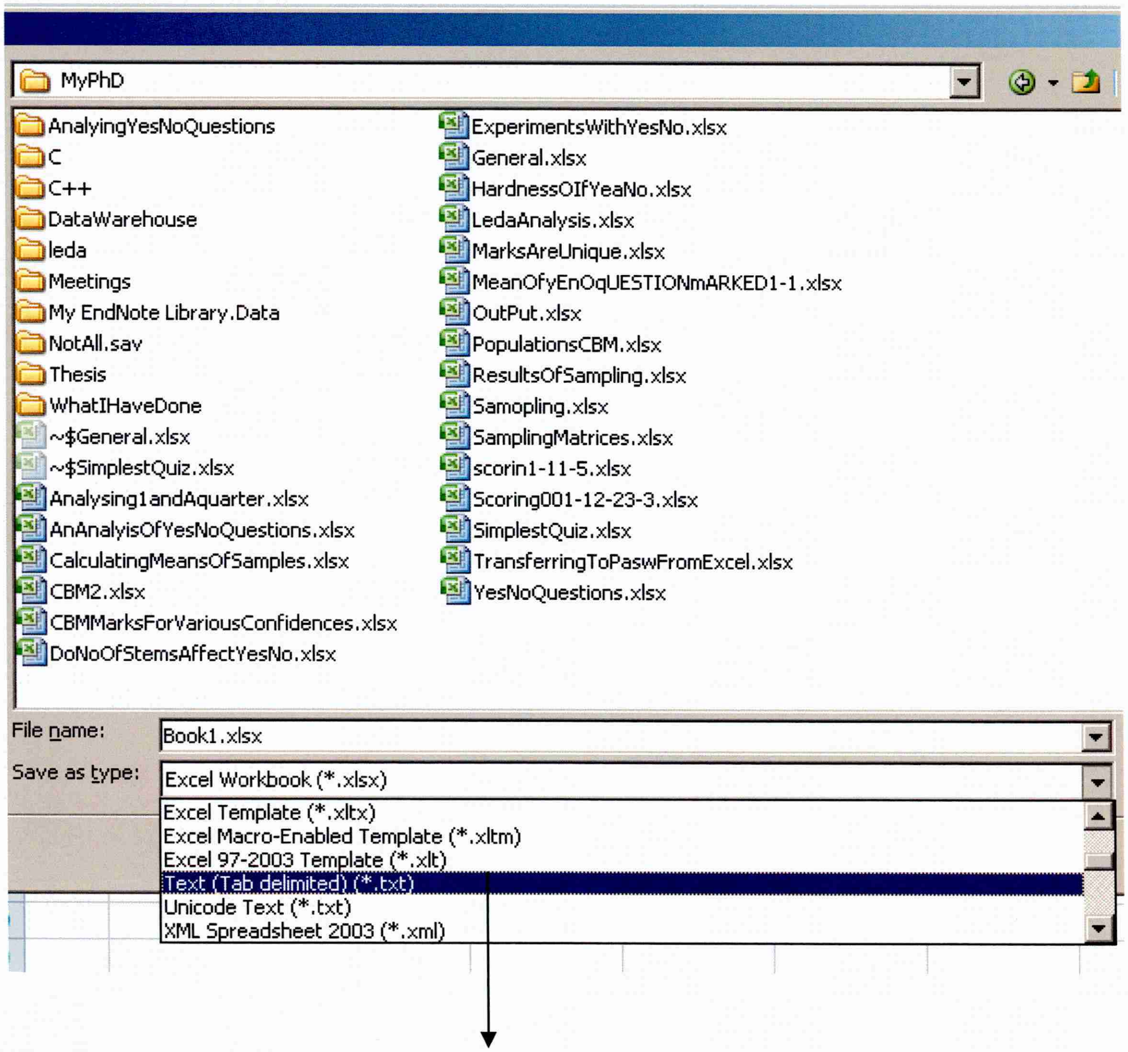
Use the Office button → **Save As→ Other Formats** (or Excel™ Workbook)

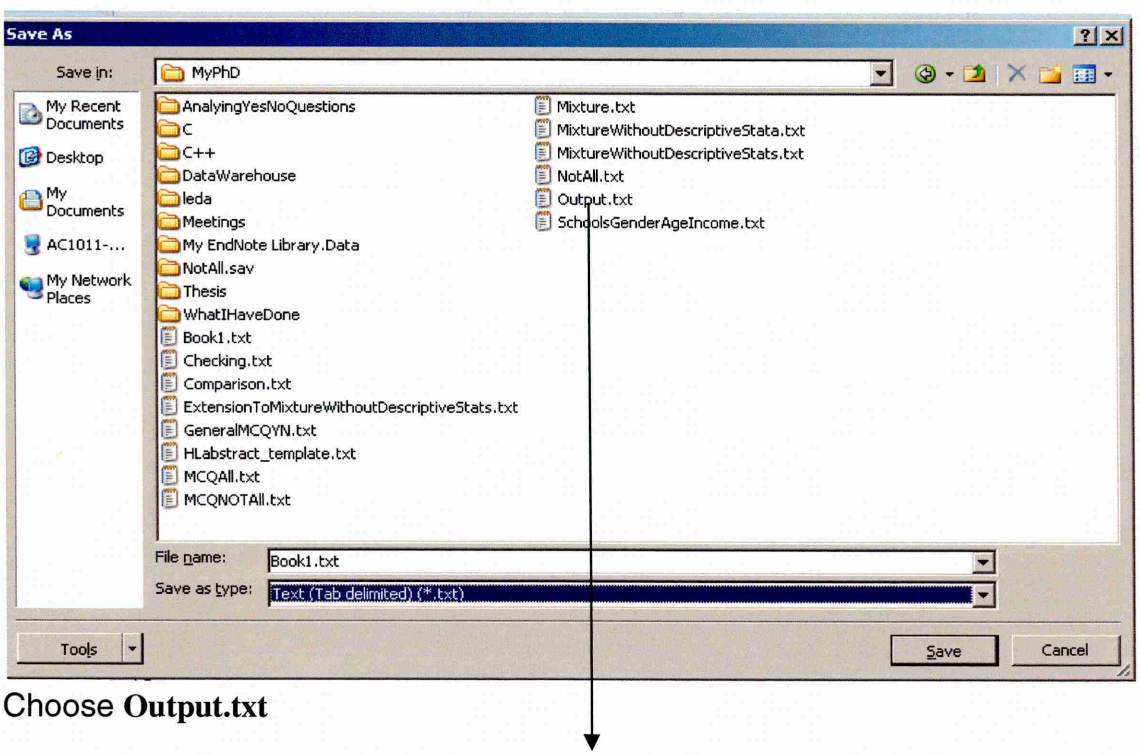
This is the Office button



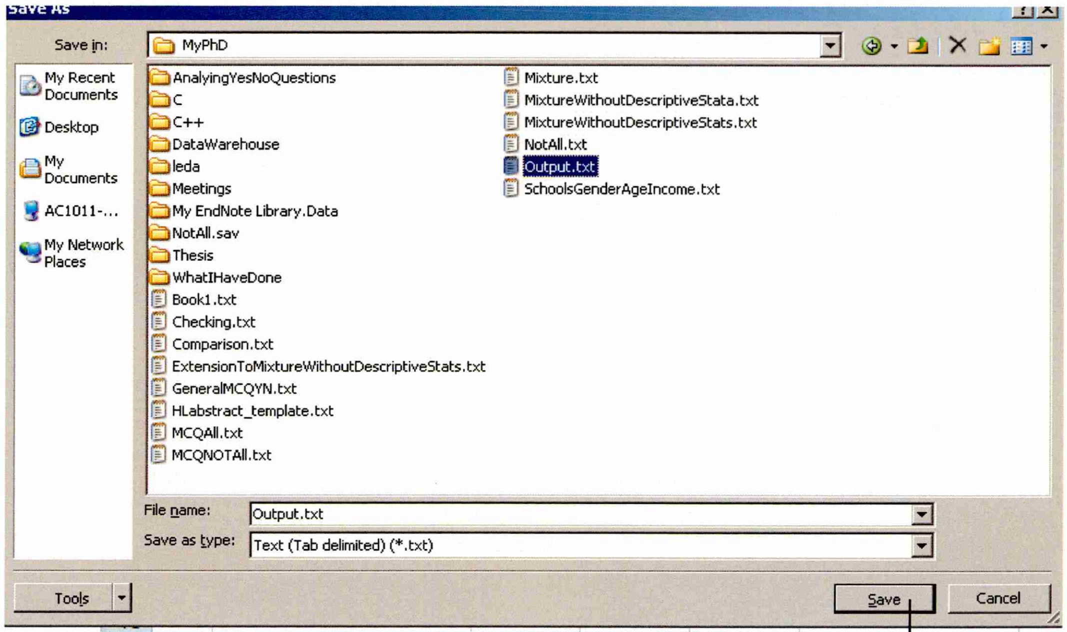
Choose **Other Formats** (you could choose **Excel™ Workbook**)

Navigate to where **Output.txt** should be or already is and choose tab-delimited txt file

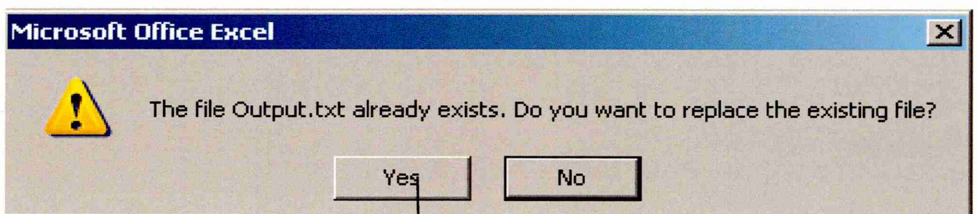




Choose **Output.txt**

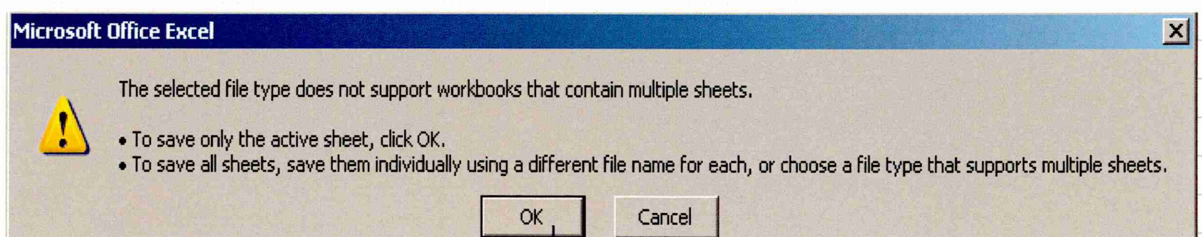


If the file already exists you will see this



Choose **Yes**

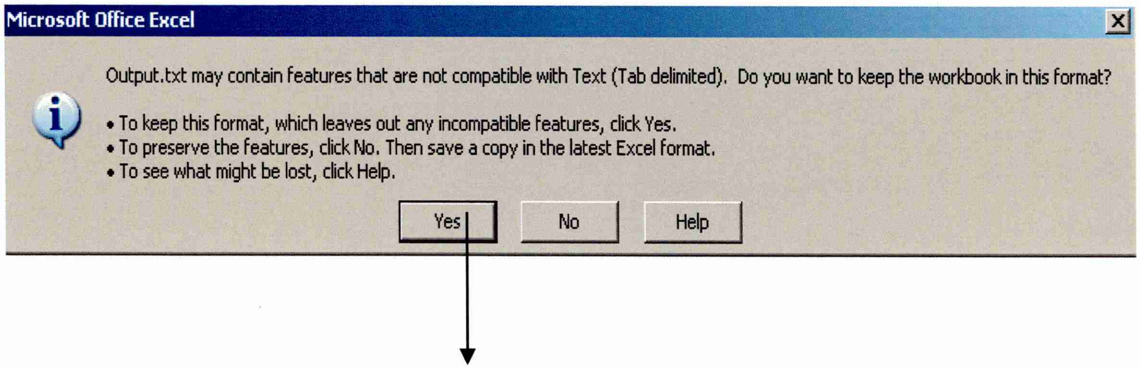
Then you will see this



Choose **OK**

This saves the WHOLE active part of the Worksheet and you cannot change the results of a Pivot Chart in Excel™. It will not allow you to.

Then you will see this



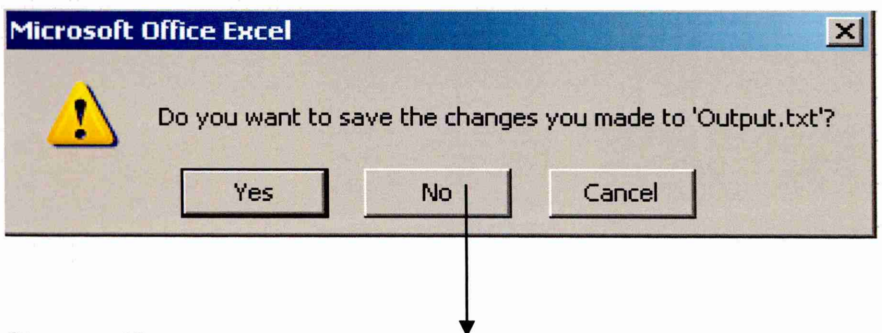
Choose **Yes**

That's it. You have saved tab-delimited text fie **Output.txt**

Now you have to clean up this file by taking out the first and last lines

Close Excel™, because it is attached to **Output.txt** and you want to work on it

You will see this



Choose **No**

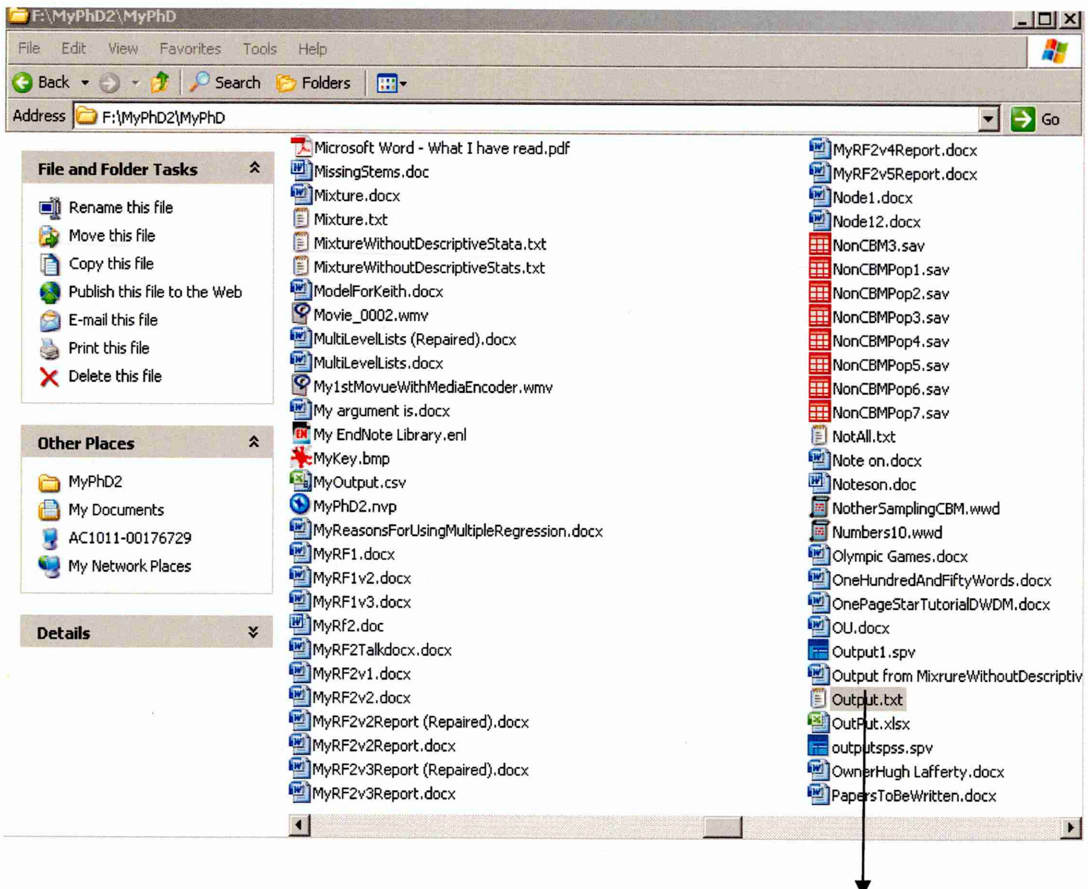
Step 3b. Clean up Output.txt

This step consists of the stages:

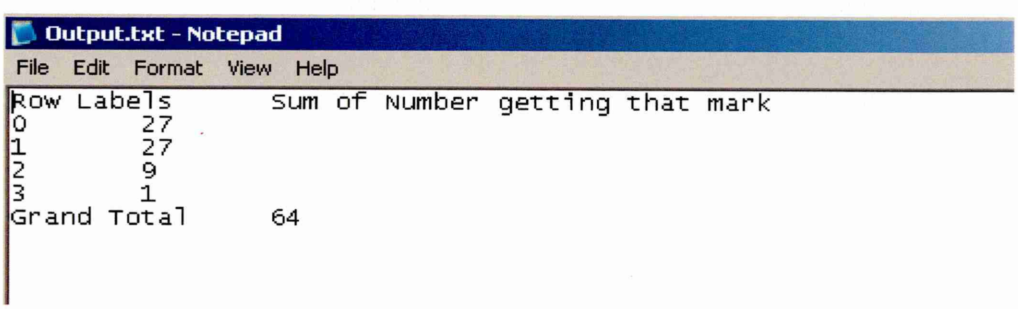
- Open **Output.txt**
- Take out the 1st and last line
- Save what you have done

It is best to keep open the file utility open that allows you to navigate to a file. So that all you have to do is choose Output.txt to achieve the first part, when you do it more than once.

Open **Output.txt**



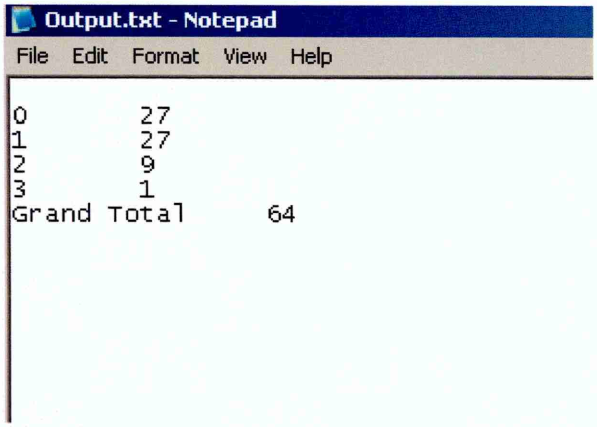
You will see something like this



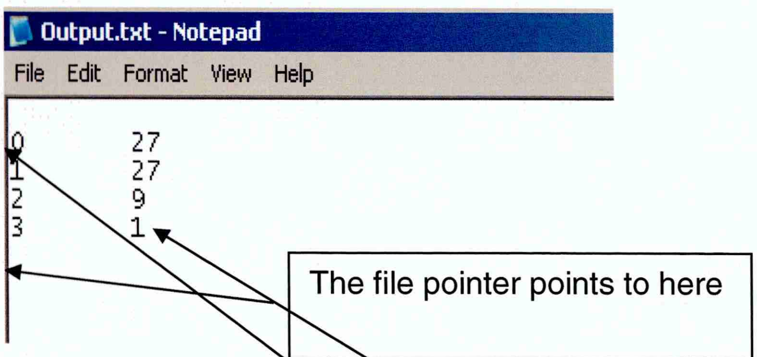
Take out the 1st and last line

So, what you have to do is take out the first and last lines and leave the file pointer after the last character

So, you should see something like this when you take out the first line



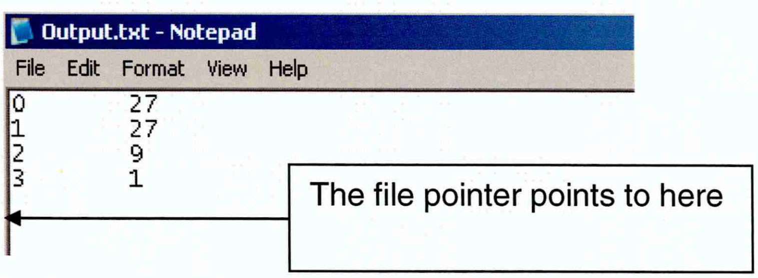
and something like this when you take out the last line



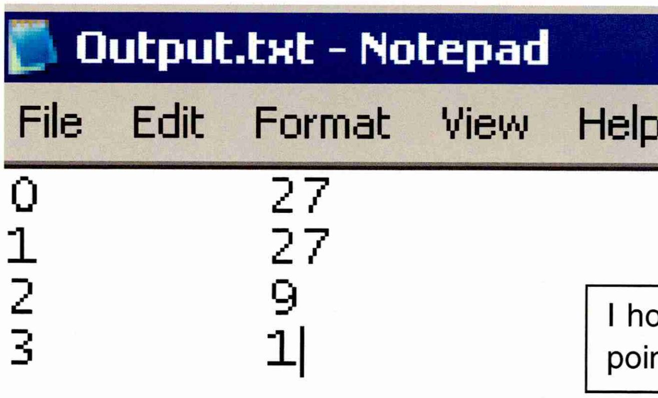
You want the file pointer to point to

To do that point to just before the 0 and take out the first line completely

You should then see something like this

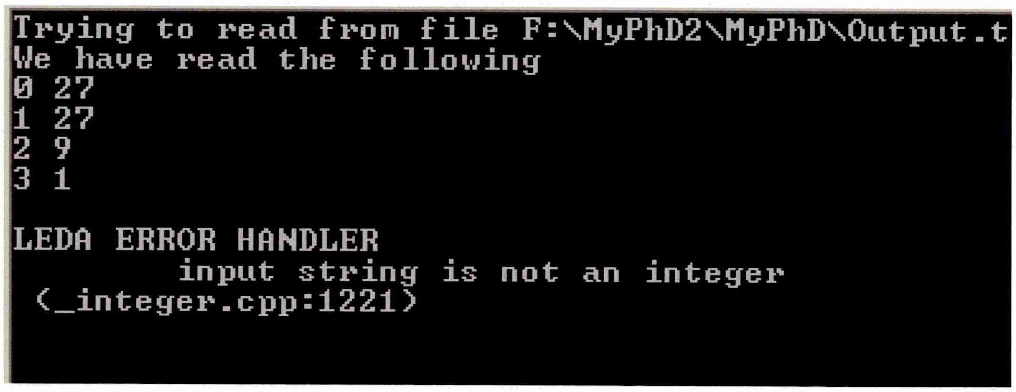


To make the file pointer point to just after the 1 take out two lines and any spaces after the 1. You should see something like this

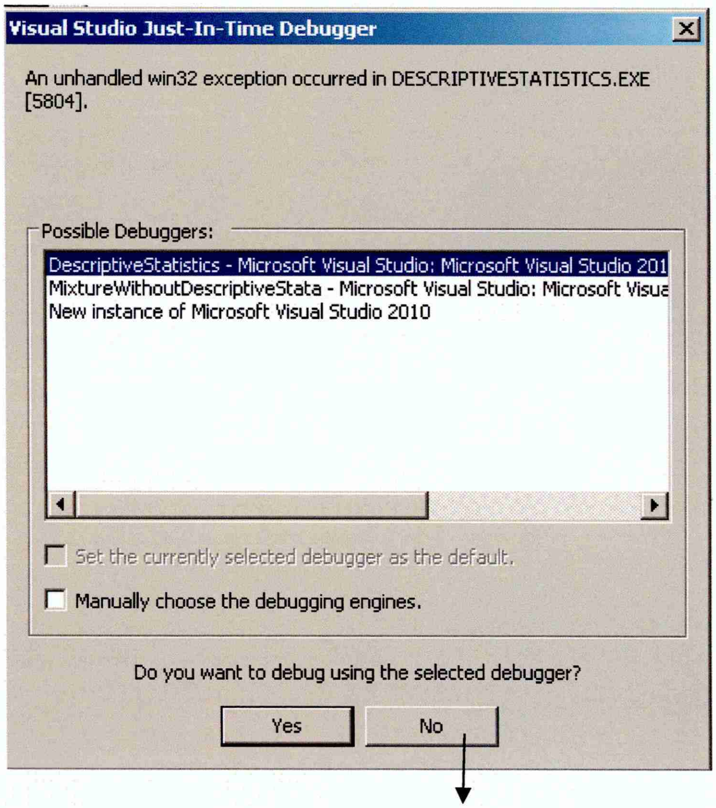


I hope you can see the file pointer pointing to just after the 1

If you make any mistakes in this process you will get an error message from C++ when you run DescriptiveStatistics you will see something like

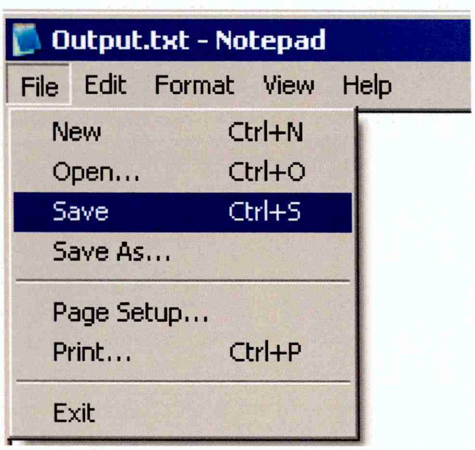


and then you will be asked to use the de-bugger



Which is very confusing.

Save what you have done



Use the File menu to do this or use the shorthand Ctrl+S

In conclusion

1. Make sure you export integers
    a. highlight column B
    b. Format this column to be Number
    c. Reduce the number of decimal places to 0
2. Export from Excel™ to a tab-delimited file called **Output.txt**
    a. Use the Office button → Save As → Other Formats
    b. Navigate to where **Output.txt** should be or already is and choose tab-delimited txt file
    c. Save **Output.txt** as tab-delimited txt file
3. Clean up **Output.txt**
    a. Open **Output.txt**
    b. Take out the 1st and last lines
    c. Save what you have done (Ctrl+S)

Appendix 12. Tool1.

```
//Includes
      #include <LEDA/numbers/integer.h>  //Include LEDA
      #include <LEDA/numbers/bigfloat.h>
      #include <LEDA\core\array.h>          //This was helped by VS 10 system
      #include <LEDA\numbers\real.h>

      #include <iostream>                   //Include ordinary C++
      #include <fstream>
      #include <string>
      #include <math.h>
  //Namespaces
      using namespace std;
      using namespace leda;

  //Definition of a function isDivisibleBy () that we are going to use

         bool isDivisibleBy(int firstNumber, int secondNumber)
         {
                if (firstNumber % secondNumber ==0)
                {
                       return(true);
                }
                else
                {
                    return (false);
                }
         }


  int main()
    {

      //Declarations
         //LEDA declarations
          integer X,XToThex, startingValue, maximumY, minimumY,twoToTheP,
twoTothePMinus1oThex,twoTothePMinus1,P,T,pCT,pCTMinus1,pCTMinus1ToThex,
twoTothePMinus1ToThex,maximum, pow[101], nCy,nMinusyCw,
numberOfWaysOfGettingThatMarkThatWay[6000],numberGettingThatMarkThatWayByMultiple
,numberGettingThatMarkThatWayByMAQ,sumOfNumberOfWaysOfGettingThatMarkThatWay;
          bigfloat
temp1,sumOfProbabilities,momentAboutMean,pseudoArea,mu,variance,standardDeviation
,skewness, kurtosis,varianceSum,skewnessSum,kurtosisSum;
      //Ordinary C++ declarations
         int numberOfModes,noOfTicks,n,w,x, y, noOfOptions, noOfOptionMinus1, way,
impossible, missLimit;
         float maximumMark, minimumMark,
scoreForMAQright,scoreForMAQwrong,scoreForMAQmissed,scoreForMultipleright,scoreFo
rMultiplewrong,scoreForMultiplemissed,a,b,c;
         double mark[6000], a0, d;
         bool correctInput;
//Strings
                std::string fileName,typeOfQuiz,canStemsBeMissed;
//Files
                ofstream myfile;              //I think the o means output



//Input a
                correctInput=false;
                while (!correctInput)
```

```
                {
                        cout << "NEGATIVE marking (a). Input either 0 for no
negative marking, or a fraction something like 1.0, or 0.5. This fraction is
usually negative"  << endl;
                        cin >> a;
                        correctInput=true;
                }
        scoreForMAQwrong=scoreForMultiplewrong=a;

//Input c.
                correctInput=false;
                while (!correctInput)
                {
                        cout << "How do you score a CORRECT answer to a stem.
Quite often this is 1.0"  << endl;
                        cin >> c;
                        correctInput=true;
                }
                scoreForMAQright=scoreForMultipleright=c;

//Input whether stems can be missed
                correctInput=false;
                while (!correctInput)
                {
                 cout << "Can Stems be missed. Input either Yes or No"  << endl;
                 cin >> canStemsBeMissed;                               if
(canStemsBeMissed=="Yes" || canStemsBeMissed=="No")
                        {
                         correctInput=true;
                        }
                        else
                        {
                          cout << "Can Stems be missed is NOT Yes or No.  You
inputted "<< canStemsBeMissed << endl;
                        }
                }

//Input the number of stems, n
                correctInput=false;
                while (!correctInput)
                {
                        cout << "Input the TOTAl number of stems(n). An integer
between 1 and 100 inclusive" << endl;
                        cin >> n;
                        if (n>=1 && n <=100)
                        {
                         correctInput=true;
                        }
                        else
                        {
                         cout << "The total number of stems(n) is NOT an integer
between 1 and 100 inclusive" << endl;
                        }
                }

//Input b, only when stems can be missed
                scoreForMAQmissed=scoreForMultiplemissed=0;

                if (canStemsBeMissed=="No")
                {
                 missLimit=0;
                }
                else
                {
                   missLimit=n;
```

```
                correctInput=false;
                while (!correctInput)
                 {
                       cout << "How do you score a MISSED stem (b) ? Input
either 0 , or a  number"  <<endl;
                       cin >> b;
                          correctInput=true;
                   }
                  scoreForMAQmissed=scoreForMultiplemissed=b;
//Keith argues that b should always be 0.
              }

               correctInput=false;
                while (!correctInput)
                {
                     cout << "Input a string (into typeOfQuiz) of MAQ   or
Multiple" << endl;
                     cin >> typeOfQuiz;
                     if (typeOfQuiz == "MAQ" || typeOfQuiz =="Multiple"  )
                     {
                          correctInput=true;
                     }
                     else
                     {
                          cout << "The type of quiz is NOT   MAQ     or
Multiple" << endl;
                          cout << "You input " << typeOfQuiz << endl;
                     }
                }

        if (typeOfQuiz == "MAQ")
           {
                 //The minimimum number of options is 2

                 //How many options have we.

                             correctInput=false;
                       while (!correctInput)
                        {
                               cout << "How many options (P) do MAQs have? Input
a number" << endl;
                               cin >> noOfOptions ;

                                  if (noOfOptions >=2 && noOfOptions <=26)
//MAQS have a minimum of 2. EMSQs have a maximum of 26 (say)
                                  {
                        correctInput=true;
                                  }
                                  else
                                  {
                                         cout << "The number of options is NOT 2
to 26 inclusive. You input " << noOfOptions << endl;
                                  }
                          }
                         P=noOfOptions ;

                        //How many ticks have we.

                              correctInput=false;
                         while (!correctInput)
                          {
                                 cout << "How many ticks (T) do MAQs have? Input
an integer that is greater than or equal to 1 and less than or equal to " <<
noOfOptions -1 << endl;
                                 cin >> noOfTicks ;
```

```
                                    //cout << " For the number of ticks you
typed " << noOfTicks << endl;
                                 if (noOfTicks >= 1 && noOfTicks < noOfOptions)
                                 {
                                     correctInput=true;
                                 }
                                 else
                                 {
                                     cout << "The  number of ticks cannot be
less than 1 or greater than " << noOfOptions - 1  << endl;
                                       if ( noOfTicks == noOfOptions )
                                       {
                           cout << "When the number of ticks=the number of options
it leads to inconsistencies, like all the marks are the same" << endl;
                                         correctInput=false;
                                       }
                                 }
                           }
                             /
                            T=noOfTicks;
          }

        if (typeOfQuiz =="Multiple"  )
        {
               //The minimimum number of options is 1 (for Yes/Nos)
               //How many options have we.

                           correctInput=false;
                     while (!correctInput)  /
                      {
                             cout << "How many options (P) do Mutiples have?
Input a number" << endl;
                             cin >> noOfOptions ;

                                if (noOfOptions >=1 && noOfOptions <=26)
//Multiples have a minimum of 1. EMSQs have a maximum of 26 (say)
                                {
                     correctInput=true;
                                }
                                else
                                {
                                       cout << "The number of options is NOT 1
to 26 inclusive. You input " << noOfOptions << endl;
                                }
                         }
                       P=noOfOptions ;

        }

 //Initialisation
          way=0;

        sumOfNumberOfWaysOfGettingThatMarkThatWay=0.0;
        pseudoArea=0.0;

             maximumMark=-9999999999.0;
          minimumMark=999999999.0;

             minimumY=99999999.9;
          maximumY=-999999999.9;

             numberOfModes=0;
        varianceSum=0.0;
        skewnessSum=0.0;
        kurtosisSum=0.0;
```

```
          sumOfProbabilities=0.0;
                impossible=0;

         fileName= "F:\\MyPhD2\\MyPhD\\MixtureWithoutDescriptiveStats.txt";

         myfile.open (fileName);
         myfile << fixed;
         myfile << setprecision(4);

//Output Headers to  file
          myfile << "Mark"                    << "\t" << "Number getting that mark
" <<   endl;
          //cout << "Mark"                    << "\t" << "Number getting that mark
" <<   endl;
          way=way+1;

               cout   << fixed; cout  << setprecision(4);

          for (y=0;  y < missLimit + 1; y++ )   // y is how many stems are missed
           {
                      for (w =0; w < n + 1; w++)      //w is how many stems are
answered correctly
              {

                              x=n-w-y;                    //x is how many stems are
answered in-correctly
                                                            if (x < 0)
                              {
                    impossible=impossible + 1;
                                                             }
                             if (x >=0)
                             {

                                    if (typeOfQuiz=="MAQ")
                                    {

pCT=factorial(P)/(factorial(T)*factorial(P - T));
                                           pCTMinus1=pCT -1;

                                           X=pCTMinus1=pCT -1;
                                    }
                                    if (typeOfQuiz=="Multiple")
                      {
                                           twoToTheP=1;


                                           for (int j =0; j < P; j++)
                                           {
                         twoToTheP=twoToTheP*2;
                                           }

                                           twoTothePMinus1=twoToTheP -1;

                                           X=twoTothePMinus1;
                                    }

                                     startingValue=1;
                                    XToThex=1;

                                    for (int j=0; j < x; j++)
                                    {

                                           XToThex = XToThex * X;
```

```
                                    }


                                    nCy=factorial(n)/(factorial(y)*factorial(n-
y));
                                    nMinusyCw=factorial(n-
y)/(factorial(w)*factorial(n-y-w));
                                    way=way+1;
                                    mark[way]= scoreForMAQright * w +
scoreForMAQwrong *(n-w-y) +scoreForMAQmissed * y;
                                    if(mark[way] > maximumMark)
maximumMark=mark[way];
                                    if(mark[way] < minimumMark)
minimumMark=mark[way];



numberOfWaysOfGettingThatMarkThatWay[way]=XToThex * nCy * nMinusyCw;

                                    if (canStemsBeMissed=="No")
                                     {

if(numberOfWaysOfGettingThatMarkThatWay[way ]> maximumY)
maximumY=numberOfWaysOfGettingThatMarkThatWay[way];

if(numberOfWaysOfGettingThatMarkThatWay[way] < minimumY)
minimumY=numberOfWaysOfGettingThatMarkThatWay[way];
                                     }

sumOfNumberOfWaysOfGettingThatMarkThatWay=sumOfNumberOfWaysOfGettingThatMarkThatW
ay+numberOfWaysOfGettingThatMarkThatWay[way];

pseudoArea=pseudoArea+(bigfloat)mark[way]*numberOfWaysOfGettingThatMarkThatWay[wa
y];
//Write them to a file
                                     myfile << mark[way] << "\t" <<
numberOfWaysOfGettingThatMarkThatWay[way]  << endl;

                          cout   << "Mark=" << mark[way] << "\t" << " numbner getting
that mark that way=" << numberOfWaysOfGettingThatMarkThatWay[way]  << " n=" << n
<< " w=" << w << " x=" << x << " y=" << y << endl;

                              }
                          }
                  }

//Close file
         myfile.close();

//Work out some descriptive stats
              mu=pseudoArea/sumOfNumberOfWaysOfGettingThatMarkThatWay;
              for (int i=1;i<way +1;i++)    //way is how many times we have
written to the file
            {
             momentAboutMean=mark[i] - mu;
             varianceSum=varianceSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean;
             skewnessSum=skewnessSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean*momentAbo
utMean;
              kurtosisSum=kurtosisSum +
numberOfWaysOfGettingThatMarkThatWay[i]*momentAboutMean*momentAboutMean*momentAbo
utMean*momentAboutMean;
               }
```

```
          variance=varianceSum/sumOfNumberOfWaysOfGettingThatMarkThatWay;
          standardDeviation=sqrt(variance);
          skewness=
skewnessSum/(sumOfNumberOfWaysOfGettingThatMarkThatWay*standardDeviation*standard
Deviation*standardDeviation);
          kurtosis=-
3+kurtosisSum/(sumOfNumberOfWaysOfGettingThatMarkThatWay*standardDeviation*standa
rdDeviation*standardDeviation *standardDeviation);


//Output the descriptive statistics

         cout << fixed << endl;
         cout << setprecision(2);

               cout << "   " << endl;

               if (canStemsBeMissed=="Yes")
               {
           cout << "Stems can be missed.    Score is [" << b << "], [" << c <<
"," << a << "]" << endl;
               }
               else
               {
           cout << "Stems CANNOT be missed. Score is [" << c << "," << a << "]"
<< endl;
               }

               cout << "The type of quiz is ";
               if (typeOfQuiz=="Multiple" && P==1 )
               {
           cout << "Yes/No" ;
               }
               if (typeOfQuiz=="Multiple" && P >1 )
               {
           cout << "Multiple Yes/No with " << P << " options" ;
               }
               if (typeOfQuiz=="MAQ" && P >1 && T==1)
               {
           cout << "MCQ with " << P << " options" ;    //This also caters for
EMSQs where P is big

               }
                if (typeOfQuiz=="MAQ" && P >1 && T > 1)
               {
           cout << "MAQ with " << P << " options and " << T << " ticks"; //This
also caters for EMSQ of type MAQ

               }

               cout << " X=" << X << endl;
               cout << "The number of impossible situations=" << impossible;
               if (canStemsBeMissed=="Yes")
               {
                       a0=0;
                       d=1;
                cout << " The theretical number of impossible situations is="
<<(n + 1)*(2*a0+n*d)/2 << endl; //We have to be careful because this involves a
division
               }
               else
               {
          cout << " The theretical number of impossible situations is=" << 0 <<
endl;
               }
```

```
              cout << "   " << endl;
              cout << "              Some DESCRIPTIVE Statistics" << endl;
              cout << "   " << endl;
              cout << "The minimum value on the x-axis                "
<<minimumMark << " The maximum value in the x-axis            =" << maximumMark
<< endl;
              if (canStemsBeMissed=="No")
               {
             cout << "The minimum Y                 ="  << minimumY   << " The
maximum Y              =" << maximumY    << endl;
               }

              cout << "The number of stems         ="  << n
<< endl;
              cout << "Mean                        ="  << mu.to_double();
                    if (canStemsBeMissed=="No")
                          {
                           cout<<      " The theoretical mean=" << (n *(c +
X.to_double()  * a))    /(X.to_double() +1) << endl; //Be careful here as this
involve a division
                        }
                    else
                        {
                            cout<<     " The theoretical mean=" << (n*(c +
X.to_double()  * a) + b)/(X.to_double()  +2)<< endl; //Be careful here as this
involve a division
                        }

              cout << "Variance                    ="  << variance.to_double()
<< endl;
              cout << "Standard Deviation          ="  <<
standardDeviation.to_double()  << endl;
              cout << "Skewness                    ="  << skewness.to_double()
<< endl;
              cout << "Kurtosis                    ="  << kurtosis.to_double()
<< endl;
              cout << "   " << endl;
              cout << "The file we have written to is "<< fileName << endl;
             cout << "We have wrtiten to this file " << way << " times" << endl;

    } //End of main
```

Appendix 13. Validation of *tool1* where stems are ALL answered

13.1 Introduction

This Appendix is about validating *tool1* for quizzes where stems are ALL answered.

The tool, *tool1* the calculation of a mark and how the mark was got in a particular way is done (see Appendix 12).

After *tool1* comes the widely available tool Excel™. *tool1* writes to a tab-delimited text file. Thus Excel™ has to import this file (see Appendix 9), Then Excel™ does a Pivot (see Appendix 10) and produces a distribution of marks and how many get that mark. Excel™ then outputs this distribution to tab-delimited text file (see Appendix 11).

The tool, *tool2* (see Appendix 15) is where descriptive statistics (and other statistics) of the distribution of marks and the number getting that mark are calculated. *tool2* assumes that the marks have been sorted, and the total number of ways a mark can be obtained involves an addition. If the marks are not sorted or the addition has not been done then a) the mode b) the median cannot be calculated, because the mode is the most frequently occurring value and the median mark is the 'middle' mark when the marks have been sorted. *tool2* also calculates the proportion of passing grades that there are.

The tools work together. They communicate via intermediate tab-delimited text files.

This Appendix is accompanied by Appendix 7 where the validation of the expression that *tool1* uses is done. We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 7.

Where stems are ALL answered the validation of *tool1* is done 'by hand'.

X=1	i.e. Yes/No quizzes	for **n**=1 to 5
X=3	i.e. 4-option MCQ quizzes	for **n**=1 to 5
X=7	i.e. 3-option Multiple Yes/No quizzes	for **n**=1 to 5
X=9	i.e. 5-option MAQ quizzes with 2 ticks	for **n**=1 to 5
X=19	i.e. 2-option EMSQ (MCQ)	for **n**=1 to 5
X=187455	i.e.20-option EMSQ (MAQ with 10 ticks)	for **n**=1 to 5

Beyond **n**=5 we use our proofs and other means (see below) to validate *tool1*, where stems can be missed.

For each of the above we:

- do not use negative marking, and have **a**=0
- do use marking where **b**=0 and **a** has a value that exactly tackles the guessing problem.

So that means that there are 12 sections, in this Appendix, where our validation is done.

With *tool1* we can do a bit more validation:

- We can calculate the minimum and maximum value of what is measured on the x-axis, and check that they agree with a table in Appendix 7. The range of marks should be from **n***min(1,**a**) to **n***max(1,**a**).
- We can calculate the minimum and maximum value of what is measured on the y-axis, and check that they agree with a table in Appendix 7.
- There should be no impossible situations, here, where stems cannot be missed.
- The mean and the theoretical mean should be the same (and they are).
- The gap between successive marks should be |1-**a**|.

One validation that we can do is that, here, the distribution should not be skewed, and that for every case, in this Appendix, the skewness should be zero.

The validation is an attempt to answer "Does the tool give the right answers?" and the answer is "YES, as far as we can tell". Because we cannot check every situation it requires an assumption to say that *tool1* will **always** work for quizzes where stems can be missed.

13.2 Yes/No quizzes (X=1) where no negative marking is used i.e. a=0

```
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =1.00
The minimum Y                  =1 The maximum Y              =1
The number of stems            =1
Mean                           =0.50 The theoretical mean=0.50
Variance                       =0.25
Standard Deviation             =0.50
Skewness                       =0.00
Kurtosis                       =-2.00
```

The range of marks should be 0 to 1, and is.

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =2.00
The minimum Y                  =1 The maximum Y              =2
The number of stems            =2
Mean                           =1.00 The theoretical mean=1.00
Variance                       =0.50
Standard Deviation             =0.71
Skewness                       =0.00
Kurtosis                       =-1.00
```

The range of marks should be 0 to 2, and is.

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =3.00
The minimum Y                  =1 The maximum Y              =3
The number of stems            =3
Mean                           =1.50 The theoretical mean=1.50
Variance                       =0.75
Standard Deviation             =0.87
Skewness                       =0.00
Kurtosis                       =-0.67
```

The range of marks should be 0 to 3, and is.

This agrees with a table in Appendix 7, where **n**=3

```
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =4.00
The minimum Y                 =1 The maximum Y              =6
The number of stems           =4
Mean                          =2.00 The theoretical mean=2.00
Variance                      =1.00
Standard Deviation            =1.00
Skewness                      =0.00
Kurtosis                      =-0.50
```

The range of marks should be 0 to 4, and is.

This agrees with a table in Appendix 7, where **n**=4

```
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =5.00
The minimum Y                 =1 The maximum Y              =10
The number of stems           =5
Mean                          =2.50 The theoretical mean=2.50
Variance                      =1.25
Standard Deviation            =1.12
Skewness                      =0.00
Kurtosis                      =-0.40
```

The range of marks should be 0 to 5, and is.

This agrees with a table in Appendix 7, where **n**=5

Now, we try and validate by observing that a) the range of marks is correct b) the skewness is always 0

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                 =1 The maximum Y              =252
The number of stems           =10
Mean                          =5.00 The theoretical mean=5.00
Variance                      =2.50
Standard Deviation            =1.58
Skewness                      =0.00
Kurtosis                      =-0.20
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =20.00
The minimum Y                  =1 The maximum Y                =184756
The number of stems            =20
Mean                           =10.00 The theoretical mean=10.00
Variance                       =5.00
Standard Deviation             =2.24
Skewness                       =0.00
Kurtosis                       =-0.10
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =30.00
The minimum Y                  =1 The maximum Y                =155117520
The number of stems            =30
Mean                           =15.00 The theoretical mean=15.00
Variance                       =7.50
Standard Deviation             =2.74
Skewness                       =0.00
Kurtosis                       =-0.07
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =40.00
The minimum Y                  =1 The maximum Y                =137846528820
The number of stems            =40
Mean                           =20.00 The theoretical mean=20.00
Variance                       =10.00
Standard Deviation             =3.16
Skewness                       =0.00
Kurtosis                       =-0.05
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =50.00
The minimum Y                  =1 The maximum Y                =126410606437752
The number of stems            =50
Mean                           =25.00 The theoretical mean=25.00
Variance                       =12.50
Standard Deviation             =3.54
Skewness                       =-0.00
Kurtosis                       =-0.04
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   0.00 The maximum value in the x-ax
is                =60.00
The minimum Y                     =1 The maximum Y                =1182645815648614 24
The number of stems               =60
Mean                              =30.00 The theoretical mean=30.00
Variance                          =15.00
Standard Deviation                =3.87
Skewness                          =-0.00
Kurtosis                          =-0.02
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   0.00 The maximum value in the x-a
is                =70.00
The minimum Y                     =1 The maximum Y                =112186277816662845432
The number of stems               =70
Mean                              =35.00 The theoretical mean=35.00
Variance                          =17.50
Standard Deviation                =4.18
Skewness                          =0.00
Kurtosis                          =-0.03
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   0.00 The maximum value in the x-ax
is                =80.00
The minimum Y                     =1 The maximum Y                =107507208733336176461
20
The number of stems               =80
Mean                              =40.00 The theoretical mean=40.00
Variance                          =20.00
Standard Deviation                =4.47
Skewness                          =-0.00
Kurtosis                          =-0.02
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   0.00 The maximum value in the x-ax
is                =90.00
The minimum Y                     =1 The maximum Y                =1038274212875534113696
71120
The number of stems               =90
Mean                              =45.00 The theoretical mean=45.00
Variance                          =22.50
Standard Deviation                =4.74
Skewness                          =-0.00
Kurtosis                          =-0.02
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-a
is              =100.00
The minimum Y               =1 The maximum Y              =100891344545564193334
12497256
The number of stems         =100
Mean                        =50.00 The theoretical mean=50.00
Variance                    =25.00
Standard Deviation          =5.00
Skewness                    =-0.00
Kurtosis                    =-0.02
```

Here, **n**=100

13.3 Yes/No quizzes (X=1) where negative marking IS used i.e. a=-1.

Here, a validation that can be done is that the step-size between marks should be 2 when **a**=-1 i.e. |1-**a**| (see Appendix 5)

Another validation is that the range of marks should be **n***min (1, **a**) to **n***max(1, **a**)

Another validation we can do is that the mean mark should be 0, because we exactly tackle the guessing problem.

```
Mark=-1.0000     numbner getting that mark that way=1 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0

Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.00 The maximum value in the x-a
xis              =1.00
The minimum Y              =1 The maximum Y              =1
The number of stems        =1
Mean                       =0.00 The theoretical mean=0.00
Variance                   =1.00
Standard Deviation         =1.00
Skewness                   =0.00
Kurtosis                   =-2.00
```

The range of marks should be -1 to 1, and is.

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=-2.0000     numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=0.0000      numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0

Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -2.00 The maximum value in the x-a
xis              =2.00
The minimum Y              =1 The maximum Y              =2
The number of stems        =2
Mean                       =0.00 The theoretical mean=0.00
Variance                   =2.00
Standard Deviation         =1.41
Skewness                   =0.00
Kurtosis                   =-1.00
```

The range of marks should be -2 to 2, and is.

This agrees with a table in Appendix 7, where **n**=2

```
Mark=-3.0000      numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=-1.0000      numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=1.0000       numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -3.00 The maximum value in the x-a
xis               =3.00
The minimum Y                  =1 The maximum Y              =3
The number of stems            =3
Mean                           =0.00 The theoretical mean=0.00
Variance                       =3.00
Standard Deviation             =1.73
Skewness                       =0.00
Kurtosis                       =-0.67
```

The range of marks should be -3 to 3, and is.

This agrees with a table in Appendix 7, where **n**=3

```
1
Mark=-4.0000      numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=-2.0000      numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=0.0000       numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=2.0000       numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.00 The maximum value in the x-a
xis               =4.00
The minimum Y                  =1 The maximum Y              =6
The number of stems            =4
Mean                           =0.00 The theoretical mean=0.00
Variance                       =4.00
Standard Deviation             =2.00
Skewness                       =0.00
Kurtosis                       =-0.50
```

The range of marks should be -4 to 4, and is.

This agrees with a table in Appendix 7, where **n**=4

```
Mark=-5.0000      numbner getting that mark that way=1 n=5 w=0 x=5 y=0
Mark=-3.0000      numbner getting that mark that way=5 n=5 w=1 x=4 y=0
Mark=-1.0000      numbner getting that mark that way=10 n=5 w=2 x=3 y=0
Mark=1.0000       numbner getting that mark that way=10 n=5 w=3 x=2 y=0
Mark=3.0000       numbner getting that mark that way=5 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -5.00 The maximum value in the x-a
xis               =5.00
The minimum Y                  =1 The maximum Y              =10
The number of stems            =5
Mean                           =0.00 The theoretical mean=0.00
Variance                       =5.00
Standard Deviation             =2.24
Skewness                       =0.00
Kurtosis                       =-0.40
```

The range of marks should be -5 to 5, and is.

This agrees with a table in Appendix 7, where **n**=5

Now, we try and validate by observing that a) the range of marks are correct b) the skewness is always 0

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the x-
axis                 =10.00
The minimum Y                 =1 The maximum Y                =252
The number of stems           =10
Mean                          =0.00 The theoretical mean=0.00
Variance                      =10.00
Standard Deviation            =3.16
Skewness                      =0.00
Kurtosis                      =-0.20
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -20.00 The maximum value in the x-
axis                 =20.00
The minimum Y                 =1 The maximum Y                =184756
The number of stems           =20
Mean                          =0.00 The theoretical mean=0.00
Variance                      =20.00
Standard Deviation            =4.47
Skewness                      =0.00
Kurtosis                      =-0.10
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -30.00 The maximum value in the x-
axis                 =30.00
The minimum Y                 =1 The maximum Y                =155117520
The number of stems           =30
Mean                          =0.00 The theoretical mean=0.00
Variance                      =30.00
Standard Deviation            =5.48
Skewness                      =0.00
Kurtosis                      =-0.07
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -40.00 The maximum value in the x-
axis                 =40.00
The minimum Y                 =1 The maximum Y                =137846528820
The number of stems           =40
Mean                          =0.00 The theoretical mean=0.00
Variance                      =40.00
Standard Deviation            =6.32
Skewness                      =0.00
Kurtosis                      =-0.05
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -50.00 The maximum value in the x-
axis                  =50.00
The minimum Y                   =1 The maximum Y                  =126410606437752
The number of stems             =50
Mean                            =0.00 The theoretical mean=0.00
Variance                        =50.00
Standard Deviation              =7.07
Skewness                        =-0.00
Kurtosis                        =-0.04
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -60.00 The maximum value in the x-
axis                  =60.00
The minimum Y                   =1 The maximum Y                  =118264581564861424
The number of stems             =60
Mean                            =0.00 The theoretical mean=0.00
Variance                        =60.00
Standard Deviation              =7.75
Skewness                        =-0.00
Kurtosis                        =-0.03
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -70.00 The maximum value in the x-
axis                  =70.00
The minimum Y                   =1 The maximum Y                  =112186277816662845432
The number of stems             =70
Mean                            =0.00 The theoretical mean=0.00
Variance                        =70.00
Standard Deviation              =8.37
Skewness                        =-0.00
Kurtosis                        =-0.03
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -80.00 The maximum value in the x-
axis                  =80.00
The minimum Y                   =1 The maximum Y                  =1075072087333361764616
20
The number of stems             =80
Mean                            =0.00 The theoretical mean=0.00
Variance                        =80.00
Standard Deviation              =8.94
Skewness                        =-0.00
Kurtosis                        =-0.02
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -90.00 The maximum value in the x-
axis                 =90.00
The minimum Y                 =1 The maximum Y                =103827421287553411369
671120
The number of stems           =90
Mean                          =0.00 The theoretical mean=0.00
Variance                      =90.00
Standard Deviation            =9.49
Skewness                      =0.00
Kurtosis                      =-0.02
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -100.00 The maximum value in the x
-axis                 =100.00
The minimum Y                 =1 The maximum Y               =1008913445455641933348
12497256
The number of stems           =100
Mean                          =0.00 The theoretical mean=0.00
Variance                      =100.00
Standard Deviation            =10.00
Skewness                      =0.00
Kurtosis                      =-0.02
```

Here, **n**=100

13.4 4-option MCQ quizzes (X=3) where negative marking NOT used i.e. a=0.

```
Mark=0.0000       numbner getting that mark that way=3 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =1.00
The minimum Y                 =1 The maximum Y              =3
The number of stems           =1
Mean                          =0.25 The theoretical mean=0.25
Variance                      =0.19
Standard Deviation            =0.43
Skewness                      =1.15
Kurtosis                      =-0.67
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=0.0000       numbner getting that mark that way=9 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=6 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =2.00
The minimum Y                 =1 The maximum Y              =9
The number of stems           =2
Mean                          =0.50 The theoretical mean=0.50
Variance                      =0.38
Standard Deviation            =0.61
Skewness                      =0.82
Kurtosis                      =-0.33
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000       numbner getting that mark that way=27 n=3 w=0 x=3 y=0
Mark=1.0000       numbner getting that mark that way=27 n=3 w=1 x=2 y=0
Mark=2.0000       numbner getting that mark that way=9 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =3.00
The minimum Y                 =1 The maximum Y              =27
The number of stems           =3
Mean                          =0.75 The theoretical mean=0.75
Variance                      =0.56
Standard Deviation            =0.75
Skewness                      =0.67
Kurtosis                      =-0.22
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=0.0000       numbner getting that mark that way=81 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=108 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=54 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=12 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =4.00
The minimum Y                =1 The maximum Y               =108
The number of stems          =4
Mean                         =1.00 The theoretical mean=1.00
Variance                     =0.75
Standard Deviation           =0.87
Skewness                     =0.58
Kurtosis                     =-0.17
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=0.0000       numbner getting that mark that way=243 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=405 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=270 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=90 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=15 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =5.00
The minimum Y                =1 The maximum Y               =405
The number of stems          =5
Mean                         =1.25 The theoretical mean=1.25
Variance                     =0.94
Standard Deviation           =0.97
Skewness                     =0.52
Kurtosis                     =-0.13
```

This agrees with a table in Appendix 7, where **n**=5.

Now, we try and validate by observing that a) the range of marks is correct

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                =1 The maximum Y               =295245
The number of stems          =10
Mean                         =2.50 The theoretical mean=2.50
Variance                     =1.88
Standard Deviation           =1.37
Skewness                     =0.37
Kurtosis                     =-0.07
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is               =20.00
The minimum Y                   =1 The maximum Y                =222465454128
The number of stems             =20
Mean                            =5.00 The theoretical mean=5.00
Variance                        =3.75
Standard Deviation              =1.94
Skewness                        =0.26
Kurtosis                        =-0.03
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is               =30.00
The minimum Y                   =1 The maximum Y                =191656683456006600
The number of stems             =30
Mean                            =7.50 The theoretical mean=7.50
Variance                        =5.63
Standard Deviation              =2.37
Skewness                        =0.21
Kurtosis                        =-0.02
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is               =40.00
The minimum Y                   =1 The maximum Y                =17452578574186791731466
72
The number of stems             =40
Mean                            =10.00 The theoretical mean=10.00
Variance                        =7.50
Standard Deviation              =2.74
Skewness                        =0.18
Kurtosis                        =-0.02
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is               =50.00
The minimum Y                   =1 The maximum Y                =16399292721333694349776
60147900
The number of stems             =50
Mean                            =12.50 The theoretical mean=12.50
Variance                        =9.37
Standard Deviation              =3.06
Skewness                        =0.16
Kurtosis                        =-0.01
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =60.00
The minimum Y                =1 The maximum Y               =1571519736154510755962
91591629478960
The number of stems          =60
Mean                         =15.00 The theoretical mean=15.00
Variance                     =11.25
Standard Deviation           =3.35
Skewness                     =0.15
Kurtosis                     =-0.01
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =70.00
The minimum Y                =1 The maximum Y               =1526999021844733973370
58560146236153005720
The number of stems          =70
Mean                         =17.50 The theoretical mean=17.50
Variance                     =13.12
Standard Deviation           =3.62
Skewness                     =0.14
Kurtosis                     =-0.01
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =80.00
The minimum Y                =1 The maximum Y               =1498661461374430380050
98608269863062186207598320
The number of stems          =80
Mean                         =20.00 The theoretical mean=20.00
Variance                     =15.00
Standard Deviation           =3.87
Skewness                     =0.13
Kurtosis                     =-0.01
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =90.00
The minimum Y                =1 The maximum Y               =1482369987636075301322
16600454730839885207974988533150
The number of stems          =90
Mean                         =22.50 The theoretical mean=22.50
Variance                     =16.87
Standard Deviation           =4.11
Skewness                     =0.12
Kurtosis                     =-0.01
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =100.00
The minimum Y                =1 The maximum Y            =1475164171513788398220
09229108064149381071967804239972703728
The number of stems          =100
Mean                         =25.00 The theoretical mean=25.00
Variance                     =18.75
Standard Deviation           =4.33
Skewness                     =0.12
Kurtosis                     =-0.01
```

Here, **n**=100

13.5 4-option MCQ quizzes (X=3) where negative marking IS used i.e. a=-1/3.

Here, the mean should be 0.

```
Mark=-0.3333     numbner getting that mark that way=3 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.33 The maximum value in the x-a
xis             =1.00
The minimum Y                =1 The maximum Y            =3
The number of stems          =1
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.33
Standard Deviation           =0.58
Skewness                     =1.15
Kurtosis                     =-0.67
```

This agrees with a table in Appendix 7, where **n**=1.

```
ark=-0.6667     numbner getting that mark that way=9 n=2 w=0 x=2 y=0
ark=0.6667      numbner getting that mark that way=6 n=2 w=1 x=1 y=0
ark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0


tems CANNOT be missed. Score is [1.00,-0.33]
he type of quiz is MCQ with 4 options X=3
he number of impossible situations=0 The theretical number of impossible situat
ons is=0

            Some DESCRIPTIVE Statistics

he minimum value on the x-axis                -0.67 The maximum value in the x-a
is              =2.00
he minimum Y                =1 The maximum Y            =9
he number of stems          =2
ean                         =0.00 The theoretical mean=0.00
ariance                     =0.67
tandard Deviation           =0.82
kewness                     =0.82
urtosis                     =-0.33
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=-1.0000      numbner getting that mark that way=27 n=3 w=0 x=3 y=0
Mark=0.3333       numbner getting that mark that way=27 n=3 w=1 x=2 y=0
Mark=1.6667       numbner getting that mark that way=9 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.00 The maximum value in the x-a
xis              =3.00
The minimum Y                    =1 The maximum Y               =27
The number of stems              =3
Mean                             =0.00 The theoretical mean=0.00
Variance                         =1.00
Standard Deviation               =1.00
Skewness                         =0.67
Kurtosis                         =-0.22
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=-1.3333      numbner getting that mark that way=81 n=4 w=0 x=4 y=0
Mark=0.0000       numbner getting that mark that way=108 n=4 w=1 x=3 y=0
Mark=1.3333       numbner getting that mark that way=54 n=4 w=2 x=2 y=0
Mark=2.6667       numbner getting that mark that way=12 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.33 The maximum value in the x-a
xis              =4.00
The minimum Y                    =1 The maximum Y               =108
The number of stems              =4
Mean                             =0.00 The theoretical mean=0.00
Variance                         =1.33
Standard Deviation               =1.15
Skewness                         =0.58
Kurtosis                         =-0.17
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=-1.6667      numbner getting that mark that way=243 n=5 w=0 x=5 y=0
Mark=-0.3333      numbner getting that mark that way=405 n=5 w=1 x=4 y=0
Mark=1.0000       numbner getting that mark that way=270 n=5 w=2 x=3 y=0
Mark=2.3333       numbner getting that mark that way=90 n=5 w=3 x=2 y=0
Mark=3.6667       numbner getting that mark that way=15 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.67 The maximum value in the x-a
xis              =5.00
The minimum Y                    =1 The maximum Y               =405
The number of stems              =5
Mean                             =0.00 The theoretical mean=0.00
Variance                         =1.67
Standard Deviation               =1.29
Skewness                         =0.52
Kurtosis                         =-0.13
```

This agrees with a table in Appendix 7, where **n**=5.

Here, we observe whether a) the range of marks is correct b) the mean is 0.

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -3.33 The maximum value in the x-
xis               =10.00
The minimum Y                =1 The maximum Y              =295245
The number of stems          =10
Mean                         =0.00 The theoretical mean=0.00
Variance                     =3.33
Standard Deviation           =1.83
Skewness                     =0.37
Kurtosis                     =-0.07
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -6.67 The maximum value in the x-a
xis               =20.00
The minimum Y                =1 The maximum Y              =222465454128
The number of stems          =20
Mean                         =0.00 The theoretical mean=0.00
Variance                     =6.67
Standard Deviation           =2.58
Skewness                     =0.26
Kurtosis                     =-0.03
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the x-
axis               =30.00
The minimum Y                =1 The maximum Y              =191656683456006600
The number of stems          =30
Mean                         =0.00 The theoretical mean=0.00
Variance                     =10.00
Standard Deviation           =3.16
Skewness                     =0.21
Kurtosis                     =-0.02
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -13.33 The maximum value in the x-
axis               =40.00
The minimum Y                =1 The maximum Y              =174525785741867917314
72
The number of stems          =40
Mean                         =0.00 The theoretical mean=0.00
Variance                     =13.33
Standard Deviation           =3.65
Skewness                     =0.18
Kurtosis                     =-0.02
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -16.67 The maximum value in the x-
axis                =50.00
The minimum Y                 =1 The maximum Y                 =16399292721333694349760
60147900
The number of stems           =50
Mean                          =0.00 The theoretical mean=0.00
Variance                      =16.67
Standard Deviation            =4.08
Skewness                      =0.16
Kurtosis                      =-0.01
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -20.00 The maximum value in the x-
axis                =60.00
The minimum Y                 =1 The maximum Y                 =15715197361545107559622
91591629478960
The number of stems           =60
Mean                          =0.00 The theoretical mean=0.00
Variance                      =20.00
Standard Deviation            =4.47
Skewness                      =0.15
Kurtosis                      =-0.01
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -23.33 The maximum value in the x-
axis                =70.00
The minimum Y                 =1 The maximum Y                 =15269990218447339733370
58560146236153005720
The number of stems           =70
Mean                          =0.00 The theoretical mean=0.00
Variance                      =23.33
Standard Deviation            =4.83
Skewness                      =0.14
Kurtosis                      =-0.01
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -26.67 The maximum value in the x-
axis                =80.00
The minimum Y                 =1 The maximum Y                 =14986614613744303800500
98608269863062186207598320
The number of stems           =80
Mean                          =0.00 The theoretical mean=0.00
Variance                      =26.67
Standard Deviation            =5.16
Skewness                      =0.13
Kurtosis                      =-0.01
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -30.00 The maximum value in the x-
axis              =90.00
The minimum Y                 =1 The maximum Y              =1482369987636075301322
16600454730839885207974988533150
The number of stems           =90
Mean                          =0.00 The theoretical mean=0.00
Variance                      =30.00
Standard Deviation            =5.48
Skewness                      =0.12
Kurtosis                      =-0.01
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -33.33 The maximum value in the x-
axis              =100.00
The minimum Y                 =1 The maximum Y              =1475164171513788398220
09229108064149381071967804239972703728
The number of stems           =100
Mean                          =0.00 The theoretical mean=0.00
Variance                      =33.33
Standard Deviation            =5.77
Skewness                      =0.12
Kurtosis                      =-0.01
```

Here, **n**=100

13.6 3-option Multiple Yes/No quizzes (X=7) where negative marking is NOT used i.e. a=0.

```
Mark=0.0000      numbner getting that mark that way=7 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =1.00
The minimum Y                =1 The maximum Y              =7
The number of stems          =1
Mean                         =0.13 The theoretical mean=0.13
Variance                     =0.11
Standard Deviation           =0.33
Skewness                     =2.27
Kurtosis                     =3.14
```

This agrees with a table in Appendix 7, where **n**=1 (to 2 decimal figures)

```
Mark=0.0000      numbner getting that mark that way=49 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=14 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =2.00
The minimum Y                =1 The maximum Y              =49
The number of stems          =2
Mean                         =0.25 The theoretical mean=0.25
Variance                     =0.22
Standard Deviation           =0.47
Skewness                     =1.60
Kurtosis                     =1.57
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000      numbner getting that mark that way=343 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=147 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=21 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =3.00
The minimum Y                =1 The maximum Y              =343
The number of stems          =3
Mean                         =0.38 The theoretical mean=0.38
Variance                     =0.33
Standard Deviation           =0.57
Skewness                     =1.31
Kurtosis                     =1.05
```

This agrees with a table in Appendix 7, where **n**=3 (to 2 decimal places) .

```
Mark=0.0000       numbner getting that mark that way=2401 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=1372 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=294 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=28 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =4.00
The minimum Y                 =1 The maximum Y               =2401
The number of stems           =4
Mean                          =0.50 The theoretical mean=0.50
Variance                      =0.44
Standard Deviation            =0.66
Skewness                      =1.13
Kurtosis                      =0.79
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=0.0000       numbner getting that mark that way=16807 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=12005 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=3430 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=490 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=35 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =5.00
The minimum Y                 =1 The maximum Y               =16807
The number of stems           =5
Mean                          =0.63 The theoretical mean=0.63
Variance                      =0.55
Standard Deviation            =0.74
Skewness                      =1.01
Kurtosis                      =0.63
```

This agrees with a table in Appendix 7, where **n**=5 (to 2 decimal places).

Here, we can observe that the range of marks is correct.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                 =1 The maximum Y               =403536070
The number of stems           =10
Mean                          =1.25 The theoretical mean=1.25
Variance                      =1.09
Standard Deviation            =1.05
Skewness                      =0.72
Kurtosis                      =0.31
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =20.00
The minimum Y                  =1 The maximum Y                =309398583602985310
The number of stems            =20
Mean                           =2.50 The theoretical mean=2.50
Variance                       =2.19
Standard Deviation             =1.48
Skewness                       =0.51
Kurtosis                       =0.16
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-a
is                =30.00
The minimum Y                  =1 The maximum Y                =266792191195949177366
44580
The number of stems            =30
Mean                           =3.75 The theoretical mean=3.75
Variance                       =3.28
Standard Deviation             =1.81
Skewness                       =0.41
Kurtosis                       =0.10
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =40.00
The minimum Y                  =1 The maximum Y                =2492657300603455516654
81659611501544
The number of stems            =40
Mean                           =5.00 The theoretical mean=5.00
Variance                       =4.38
Standard Deviation             =2.09
Skewness                       =0.36
Kurtosis                       =0.08
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =50.00
The minimum Y                  =1 The maximum Y                =2429163737320098955222
644375038472652903707 00
The number of stems            =50
Mean                           =6.25 The theoretical mean=6.25
Variance                       =5.47
Standard Deviation             =2.34
Skewness                       =0.32
Kurtosis                       =0.06
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =60.00
The minimum Y                  =1 The maximum Y              =23824081818304431205306
4759794708674984434909577976440
The number of stems            =60
Mean                           =7.50 The theoretical mean=7.50
Variance                       =6.56
Standard Deviation             =2.56
Skewness                       =0.29
Kurtosis                       =0.05
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =70.00
The minimum Y                  =1 The maximum Y              =23499932738025735054636
7956957842950734782610961589939584075080
The number of stems            =70
Mean                           =8.75 The theoretical mean=8.75
Variance                       =7.66
Standard Deviation             =2.77
Skewness                       =0.27
Kurtosis                       =0.04
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =80.00
The minimum Y                  =1 The maximum Y              =23627754782445210738248
316392725883464808191172492023211876722282209988 0
The number of stems            =80
Mean                           =10.00 The theoretical mean=10.00
Variance                       =8.75
Standard Deviation             =2.96
Skewness                       =0.25
Kurtosis                       =0.04
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =90.00
The minimum Y                  =1 The maximum Y              =24092812323038956438399
468484182105499124154852707870156022163856318655220781912 0
The number of stems            =90
Mean                           =11.25 The theoretical mean=11.25
Variance                       =9.84
Standard Deviation             =3.14
Skewness                       =0.24
Kurtosis                       =0.03
```

Here, **n**=90

The maximum Y is 'large' and we see the need for something like the LEDA routines.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is                =100.00
The minimum Y                 =1 The maximum Y               =245465769597967938911б
27204123609383643731873809300797538837388495577425708171282236866700
The number of stems           =100
Mean                          =12.50 The theoretical mean=12.50
Variance                      =10.94
Standard Deviation            =3.31
Skewness                      =0.23
Kurtosis                      =0.03
```

Here, **n**=100

13.7 3-option Multiple Yes/No quizzes (X=7) where negative marking IS used i.e. a=-1/7.

The validation that can be done is a) the mean should be 0 b) the range of marks can be observed.

```
Mark=-0.1429      numbner getting that mark that way=7 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.14 The maximum value in the x-a
xis                 =1.00
The minimum Y                  =1 The maximum Y                =7
The number of stems            =1
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.14
Standard Deviation             =0.38
Skewness                       =2.27
Kurtosis                       =3.14
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=-0.2857      numbner getting that mark that way=49 n=2 w=0 x=2 y=0
Mark=0.8571       numbner getting that mark that way=14 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.29 The maximum value in the x-a
xis                 =2.00
The minimum Y                  =1 The maximum Y                =49
The number of stems            =2
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.29
Standard Deviation             =0.53
Skewness                       =1.60
Kurtosis                       =1.57
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=-0.4286      numbner getting that mark that way=343 n=3 w=0 x=3 y=0
Mark=0.7143       numbner getting that mark that way=147 n=3 w=1 x=2 y=0
Mark=1.8571       numbner getting that mark that way=21 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.43 The maximum value in the x-a
xis                 =3.00
The minimum Y                  =1 The maximum Y                =343
The number of stems            =3
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.43
Standard Deviation             =0.65
Skewness                       =1.31
Kurtosis                       =1.05
```

This agrees with a table in Appendix 7, where **n**=3.

```
3
Mark=-0.5714      numbner getting that mark that way=2401 n=4 w=0 x=4 y=0
Mark=0.5714       numbner getting that mark that way=1372 n=4 w=1 x=3 y=0
Mark=1.7143       numbner getting that mark that way=294 n=4 w=2 x=2 y=0
Mark=2.8571       numbner getting that mark that way=28 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -0.57 The maximum value in the x-a
xis              =4.00
The minimum Y                    =1 The maximum Y               =2401
The number of stems              =4
Mean                             =0.00 The theoretical mean=0.00
Variance                         =0.57
Standard Deviation               =0.76
Skewness                         =1.13
Kurtosis                         =0.79
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=-0.7143      numbner getting that mark that way=16807 n=5 w=0 x=5 y=0
Mark=0.4286       numbner getting that mark that way=12005 n=5 w=1 x=4 y=0
Mark=1.5714       numbner getting that mark that way=3430 n=5 w=2 x=3 y=0
Mark=2.7143       numbner getting that mark that way=490 n=5 w=3 x=2 y=0
Mark=3.8571       numbner getting that mark that way=35 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -0.71 The maximum value in the x-a
xis              =5.00
The minimum Y                    =1 The maximum Y               =16807
The number of stems              =5
Mean                             =0.00 The theoretical mean=0.00
Variance                         =0.71
Standard Deviation               =0.85
Skewness                         =1.01
Kurtosis                         =0.63
```

This agrees with a table in Appendix 7, where **n**=5.

Here, we can observe that range of marks is correct.

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -1.43 The maximum value in the x-a
xis              =10.00
The minimum Y                    =1 The maximum Y               =403536070
The number of stems              =10
Mean                             =0.00 The theoretical mean=0.00
Variance                         =1.43
Standard Deviation               =1.20
Skewness                         =0.72
Kurtosis                         =0.31
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -2.86 The maximum value in the x-a
xis              =20.00
The minimum Y                  =1 The maximum Y             =309398583602985310
The number of stems            =20
Mean                           =0.00 The theoretical mean=0.00
Variance                       =2.86
Standard Deviation             =1.69
Skewness                       =0.51
Kurtosis                       =0.16
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.29 The maximum value in the x-a
xis              =30.00
The minimum Y                  =1 The maximum Y             =266792191195949177366
44580
The number of stems            =30
Mean                           =0.00 The theoretical mean=0.00
Variance                       =4.29
Standard Deviation             =2.07
Skewness                       =0.41
Kurtosis                       =0.10
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -5.71 The maximum value in the x-a
xis              =40.00
The minimum Y                  =1 The maximum Y             =249265730060345551665
81659611501544
The number of stems            =40
Mean                           =0.00 The theoretical mean=0.00
Variance                       =5.71
Standard Deviation             =2.39
Skewness                       =0.36
Kurtosis                       =0.08
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -7.14 The maximum value in the x-a
xis              =50.00
The minimum Y                  =1 The maximum Y             =242916373732009895522
64437503847265290370700
The number of stems            =50
Mean                           =0.00 The theoretical mean=0.00
Variance                       =7.14
Standard Deviation             =2.67
Skewness                       =0.32
Kurtosis                       =0.06
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -8.57 The maximum value in the x-a
xis              =60.00
The minimum Y              =1 The maximum Y                =23824081818304431205306
6475979470867498443490957797644
The number of stems        =60
Mean                       =-0.00 The theoretical mean=-0.00
Variance                   =8.57
Standard Deviation         =2.93
Skewness                   =0.29
Kurtosis                   =0.05
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -10.00 The maximum value in the x-
axis              =70.00
The minimum Y              =1 The maximum Y                =23499932738025735054636
7956957842950734782610961589939584075080
The number of stems        =70
Mean                       =0.00 The theoretical mean=0.00
Variance                   =10.00
Standard Deviation         =3.16
Skewness                   =0.27
Kurtosis                   =0.04
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -11.43 The maximum value in the x-
axis              =80.00
The minimum Y              =1 The maximum Y                =23627754782445210738248
83163927258834648081911724920232118767222822099880
The number of stems        =80
Mean                       =0.00 The theoretical mean=0.00
Variance                   =11.43
Standard Deviation         =3.38
Skewness                   =0.25
Kurtosis                   =0.04
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -12.86 The maximum value in the x-
axis              =90.00
The minimum Y              =1 The maximum Y                =24092812323038956438339
94684841821054991241548527078701560221638563186552207819120
The number of stems        =90
Mean                       =0.00 The theoretical mean=0.00
Variance                   =12.86
Standard Deviation         =3.59
Skewness                   =0.24
Kurtosis                   =0.03
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -14.29 The maximum value in the x-
axis              =100.00
The minimum Y                  =1 The maximum Y                 =24546576959796793891116
272041236093836437318738093007975388373884955774257081712822368667OO
The number of stems            =100
Mean                           =0.00 The theoretical mean=0.00
Variance                       =14.29
Standard Deviation             =3.78
Skewness                       =0.23
Kurtosis                       =0.03
```

Here, **n**=100

13.8 5-option MAQ with 2 ticks quizzes (X=9) where negative marking is NOT used i.e. a=0.

```
Mark=0.0000      numbner getting that mark that way=9 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =1.00
The minimum Y                  =1 The maximum Y                 =9
The number of stems            =1
Mean                           =0.10 The theoretical mean=0.10
Variance                       =0.09
Standard Deviation             =0.30
Skewness                       =2.67
Kurtosis                       =5.11
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=0.0000      numbner getting that mark that way=81 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=18 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =2.00
The minimum Y                  =1 The maximum Y                 =81
The number of stems            =2
Mean                           =0.20 The theoretical mean=0.20
Variance                       =0.18
Standard Deviation             =0.42
Skewness                       =1.89
Kurtosis                       =2.56
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000        numbner getting that mark that way=729 n=3 w=0 x=3 y=0
Mark=1.0000        numbner getting that mark that way=243 n=3 w=1 x=2 y=0
Mark=2.0000        numbner getting that mark that way=27 n=3 w=2 x=1 y=0
Mark=3.0000        numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =3.00
The minimum Y                   =1 The maximum Y               =729
The number of stems             =3
Mean                            =0.30 The theoretical mean=0.30
Variance                        =0.27
Standard Deviation              =0.52
Skewness                        =1.54
Kurtosis                        =1.70
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=0.0000        numbner getting that mark that way=6561 n=4 w=0 x=4 y=0
Mark=1.0000        numbner getting that mark that way=2916 n=4 w=1 x=3 y=0
Mark=2.0000        numbner getting that mark that way=486 n=4 w=2 x=2 y=0
Mark=3.0000        numbner getting that mark that way=36 n=4 w=3 x=1 y=0
Mark=4.0000        numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =4.00
The minimum Y                   =1 The maximum Y               =6561
The number of stems             =4
Mean                            =0.40 The theoretical mean=0.40
Variance                        =0.36
Standard Deviation              =0.60
Skewness                        =1.33
Kurtosis                        =1.28
```

This agrees with a table in Appendix 7, where **n**=4.

```
lark=0.0000        numbner getting that mark that way=59049 n=5 w=0 x=5 y=0
lark=1.0000        numbner getting that mark that way=32805 n=5 w=1 x=4 y=0
lark=2.0000        numbner getting that mark that way=7290 n=5 w=2 x=3 y=0
lark=3.0000        numbner getting that mark that way=810 n=5 w=3 x=2 y=0
lark=4.0000        numbner getting that mark that way=45 n=5 w=4 x=1 y=0
lark=5.0000        numbner getting that mark that way=1 n=5 w=5 x=0 y=0


tems CANNOT be missed. Score is [1.00,0.00]
he type of quiz is MAQ with 5 options and 2 ticks X=9
he number of impossible situations=0 The theretical number of impossible situat
ons is=0

             Some DESCRIPTIVE Statistics

he minimum value on the x-axis                0.00 The maximum value in the x-ax
s               =5.00
he minimum Y                    =1 The maximum Y               =59049
he number of stems              =5
lean                            =0.50 The theoretical mean=0.50
ariance                         =0.45
tandard Deviation               =0.67
kewness                         =1.19
urtosis                         =1.02
```

This agrees with a table in Appendix 7, where **n**=5.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                 =1 The maximum Y                   =3874204890
The number of stems           =10
Mean                          =1.00 The theoretical mean=1.00
Variance                      =0.90
Standard Deviation            =0.95
Skewness                      =0.84
Kurtosis                      =0.51
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =20.00
The minimum Y                 =1 The maximum Y                   =28517980706429832990
The number of stems           =20
Mean                          =2.00 The theoretical mean=2.00
Variance                      =1.80
Standard Deviation            =1.34
Skewness                      =0.60
Kurtosis                      =0.26
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =30.00
The minimum Y                 =1 The maximum Y                   =2360879322323426423429
84086140
The number of stems           =30
Mean                          =3.00 The theoretical mean=3.00
Variance                      =2.70
Standard Deviation            =1.64
Skewness                      =0.49
Kurtosis                      =0.17
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =40.00
The minimum Y                 =1 The maximum Y                   =2058870434411991149498
071114275471660990
The number of stems           =40
Mean                          =4.00 The theoretical mean=4.00
Variance                      =3.60
Standard Deviation            =1.90
Skewness                      =0.42
Kurtosis                      =0.13
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =50.00
The minimum Y                 =1 The maximum Y                =1849246008952152157948
1657323616110818986220483240
The number of stems           =50
Mean                          =5.00 The theoretical mean=5.00
Variance                      =4.50
Standard Deviation            =2.12
Skewness                      =0.38
Kurtosis                      =0.10
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =60.00
The minimum Y                 =1 The maximum Y                =1692855313637338784507
70597562839027031055250555503099105460
The number of stems           =60
Mean                          =6.00 The theoretical mean=6.00
Variance                      =5.40
Standard Deviation            =2.32
Skewness                      =0.34
Kurtosis                      =0.09
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =70.00
The minimum Y                 =1 The maximum Y                =1570419468436320919404
071815399487874215907967959025065369133208402880
The number of stems           =70
Mean                          =7.00 The theoretical mean=7.00
Variance                      =6.30
Standard Deviation            =2.51
Skewness                      =0.32
Kurtosis                      =0.07
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =80.00
The minimum Y                 =1 The maximum Y                =1471200954050474922170
3440444379457549911825137205451552367812322064587555329150
The number of stems           =80
Mean                          =8.00 The theoretical mean=8.00
Variance                      =7.20
Standard Deviation            =2.68
Skewness                      =0.30
Kurtosis                      =0.06
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is               =90.00
The minimum Y                =1 The maximum Y                =138868351595417889240
30072472416260530726409173230840212202607641267955550702128237141670
The number of stems          =90
Mean                         =9.00 The theoretical mean=9.00
Variance                     =8.10
Standard Deviation           =2.85
Skewness                     =0.28
Kurtosis                     =0.06
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-a
is               =100.00
The minimum Y                =1 The maximum Y                =1318653468244882166165
94146715124364906201198902990547517518412232297907791576654182041240375680440
The number of stems          =100
Mean                         =10.00 The theoretical mean=10.00
Variance                     =9.00
Standard Deviation           =3.00
Skewness                     =0.27
Kurtosis                     =0.05
```

Here, **n**=100

13.9 5-option MAQ with 2 ticks quizzes (X=9) where negative marking IS used i.e. a=-1/9.

Here, the mean Mark should be 0, because the guessing problem is exactly tackled.

```
Mark=-0.1111      numbner getting that mark that way=9 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.11 The maximum value in the x-a
xis              =1.00
The minimum Y              =1 The maximum Y              =9
The number of stems        =1
Mean                       =0.00 The theoretical mean=0.00
Variance                   =0.11
Standard Deviation         =0.33
Skewness                   =2.67
Kurtosis                   =5.11
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=-0.2222      numbner getting that mark that way=81 n=2 w=0 x=2 y=0
Mark=0.8889       numbner getting that mark that way=18 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.22 The maximum value in the x-a
xis              =2.00
The minimum Y              =1 The maximum Y              =81
The number of stems        =2
Mean                       =0.00 The theoretical mean=0.00
Variance                   =0.22
Standard Deviation         =0.47
Skewness                   =1.89
Kurtosis                   =2.56
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=-0.3333      numbner getting that mark that way=729 n=3 w=0 x=3 y=0
Mark=0.7778       numbner getting that mark that way=243 n=3 w=1 x=2 y=0
Mark=1.8889       numbner getting that mark that way=27 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.33 The maximum value in the x-a
xis              =3.00
The minimum Y              =1 The maximum Y              =729
The number of stems        =3
Mean                       =0.00 The theoretical mean=0.00
Variance                   =0.33
Standard Deviation         =0.58
Skewness                   =1.54
Kurtosis                   =1.70
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=-0.4444      numbner getting that mark that way=6561 n=4 w=0 x=4 y=0
Mark=0.6667       numbner getting that mark that way=2916 n=4 w=1 x=3 y=0
Mark=1.7778       numbner getting that mark that way=486 n=4 w=2 x=2 y=0
Mark=2.8889       numbner getting that mark that way=36 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.44 The maximum value in the x-a
xis              =4.00
The minimum Y                  =1 The maximum Y              =6561
The number of stems            =4
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.44
Standard Deviation             =0.67
Skewness                       =1.33
Kurtosis                       =1.28
```

This agrees with a table in Appendix 7, where **n**=4.

```
2
Mark=-0.5556      numbner getting that mark that way=59049 n=5 w=0 x=5 y=0
Mark=0.5556       numbner getting that mark that way=32805 n=5 w=1 x=4 y=0
Mark=1.6667       numbner getting that mark that way=7290 n=5 w=2 x=3 y=0
Mark=2.7778       numbner getting that mark that way=810 n=5 w=3 x=2 y=0
Mark=3.8889       numbner getting that mark that way=45 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.56 The maximum value in the x-a
xis              =5.00
The minimum Y                  =1 The maximum Y              =59049
The number of stems            =5
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.56
Standard Deviation             =0.75
Skewness                       =1.19
Kurtosis                       =1.02
```

This agrees with a table in Appendix 7, where **n**=5.

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -1.11 The maximum value in the x-a
xis              =10.00
The minimum Y                  =1 The maximum Y              =3874204890
The number of stems            =10
Mean                           =0.00 The theoretical mean=0.00
Variance                       =1.11
Standard Deviation             =1.05
Skewness                       =0.84
Kurtosis                       =0.51
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -2.22 The maximum value in the x-a
xis              =20.00
The minimum Y                =1 The maximum Y               =28517980706429832990
The number of stems          =20
Mean                         =0.00 The theoretical mean=0.00
Variance                     =2.22
Standard Deviation           =1.49
Skewness                     =0.60
Kurtosis                     =0.26
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -3.33 The maximum value in the x-a
xis              =30.00
The minimum Y                =1 The maximum Y               =2360879322323426423429
84086140
The number of stems          =30
Mean                         =0.00 The theoretical mean=0.00
Variance                     =3.33
Standard Deviation           =1.83
Skewness                     =0.49
Kurtosis                     =0.17
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -4.44 The maximum value in the x-a
xis              =40.00
The minimum Y                =1 The maximum Y               =2058870434411991149498
0711142754716609 90
The number of stems          =40
Mean                         =0.00 The theoretical mean=0.00
Variance                     =4.44
Standard Deviation           =2.11
Skewness                     =0.42
Kurtosis                     =0.13
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -5.56 The maximum value in the x-
xis              =50.00
The minimum Y                =1 The maximum Y               =184924600895215215794
1657323616110818986220483240
The number of stems          =50
Mean                         =0.00 The theoretical mean=0.00
Variance                     =5.56
Standard Deviation           =2.36
Skewness                     =0.38
Kurtosis                     =0.10
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -6.67 The maximum value in the x-a
xis              =60.00
The minimum Y                    =1 The maximum Y                    =16928553136373387845070
70597562839027031055250555503099105460
The number of stems              =60
Mean                             =0.00 The theoretical mean=0.00
Variance                         =6.67
Standard Deviation               =2.58
Skewness                         =0.34
Kurtosis                         =0.09
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -7.78 The maximum value in the x-a
xis              =70.00
The minimum Y                    =1 The maximum Y                    =1570419468436320919404
071815399487874215907967959025065369133208402880
The number of stems              =70
Mean                             =0.00 The theoretical mean=0.00
Variance                         =7.78
Standard Deviation               =2.79
Skewness                         =0.32
Kurtosis                         =0.07
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -8.89 The maximum value in the x-a
xis              =80.00
The minimum Y                    =1 The maximum Y                    =1471200954050474922170
3440444379457549911825137205451552367812322064587555329150
The number of stems              =80
Mean                             =0.00 The theoretical mean=0.00
Variance                         =8.89
Standard Deviation               =2.98
Skewness                         =0.30
Kurtosis                         =0.06
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -10.00 The maximum value in the x-
axis              =90.00
The minimum Y                    =1 The maximum Y                    =1388683515954178892401
30072472416260530726409173230840212202607641267955550702128237141670
The number of stems              =90
Mean                             =0.00 The theoretical mean=0.00
Variance                         =10.00
Standard Deviation               =3.16
Skewness                         =0.28
Kurtosis                         =0.06
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -11.11 The maximum value in the x-
axis                 =100.00
The minimum Y                 =1 The maximum Y               =1318653468244882166163
5941467151243649062011989029905475175184122322979077915766541820412403756804 40
The number of stems           =100
Mean                          =0.00 The theoretical mean=0.00
Variance                      =11.11
Standard Deviation            =3.33
Skewness                      =0.27
Kurtosis                      =0.05
```

Here, **n**=100

13.10 20-option EMSQ (MCQ) quizzes (X=19) where negative marking is NOT used i.e. a=0.

```
1
Mark=0.0000      numbner getting that mark that way=19 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =1.00
The minimum Y                 =1 The maximum Y               =19
The number of stems           =1
Mean                          =0.05 The theoretical mean=0.05
Variance                      =0.05
Standard Deviation            =0.22
Skewness                      =4.13
Kurtosis                      =15.05
```

This agrees with a table in Appendix 7, where **n**=1.

```
1
Mark=0.0000      numbner getting that mark that way=361 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=38 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =2.00
The minimum Y                 =1 The maximum Y               =361
The number of stems           =2
Mean                          =0.10 The theoretical mean=0.10
Variance                      =0.10
Standard Deviation            =0.31
Skewness                      =2.92
Kurtosis                      =7.53
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000      numbner getting that mark that way=6859 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=1083 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=57 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =3.00
The minimum Y                 =1 The maximum Y                 =6859
The number of stems           =3
Mean                          =0.15 The theoretical mean=0.15
Variance                      =0.14
Standard Deviation            =0.38
Skewness                      =2.38
Kurtosis                      =5.02
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=0.0000      numbner getting that mark that way=130321 n=4 w=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=27436 n=4 w=1 x=3 y=0
Mark=2.0000      numbner getting that mark that way=2166 n=4 w=2 x=2 y=0
Mark=3.0000      numbner getting that mark that way=76 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =4.00
The minimum Y                 =1 The maximum Y                 =130321
The number of stems           =4
Mean                          =0.20 The theoretical mean=0.20
Variance                      =0.19
Standard Deviation            =0.44
Skewness                      =2.06
Kurtosis                      =3.76
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=0.0000      numbner getting that mark that way=2476099 n=5 w=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=651605 n=5 w=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=68590 n=5 w=2 x=3 y=0
Mark=3.0000      numbner getting that mark that way=3610 n=5 w=3 x=2 y=0
Mark=4.0000      numbner getting that mark that way=95 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0

Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =5.00
The minimum Y                 =1 The maximum Y                 =2476099
The number of stems           =5
Mean                          =0.25 The theoretical mean=0.25
Variance                      =0.24
Standard Deviation            =0.49
Skewness                      =1.85
Kurtosis                      =3.01
```

This agrees with a table in Appendix 7, where **n**=5.

Here, we can observe that the range of marks is correct.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =10.00
The minimum Y                =1 The maximum Y               =6131066257801
The number of stems          =10
Mean                         =0.50 The theoretical mean=0.50
Variance                     =0.47
Standard Deviation           =0.69
Skewness                     =1.31
Kurtosis                     =1.51
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =20.00
The minimum Y                =1 The maximum Y               =39568393113206271782 47
9580
The number of stems          =20
Mean                         =1.00 The theoretical mean=1.00
Variance                     =0.95
Standard Deviation           =0.97
Skewness                     =0.92
Kurtosis                     =0.75
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =30.00
The minimum Y                =1 The maximum Y               =3638946598376766553350
15030640947305370
The number of stems          =30
Mean                         =1.50 The theoretical mean=1.50
Variance                     =1.42
Standard Deviation           =1.19
Skewness                     =0.75
Kurtosis                     =0.50
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =40.00
The minimum Y                =1 The maximum Y               =3053032580444397557610
99083330109914717497357810398 0
The number of stems          =40
Mean                         =2.00 The theoretical mean=2.00
Variance                     =1.90
Standard Deviation           =1.38
Skewness                     =0.65
Kurtosis                     =0.38
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    0.00 The maximum value in the x-ax
is                =50.00
The minimum Y                       =1 The maximum Y                  =29397400860851230395l2
1736276529459725217203590605255221850745225
The number of stems                 =50
Mean                                =2.50 The theoretical mean=2.50
Variance                            =2.38
Standard Deviation                  =1.54
Skewness                            =0.58
Kurtosis                            =0.30
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    0.00 The maximum value in the x-ax
is                =60.00
The minimum Y                       =1 The maximum Y                  =26499352331846251251 99
496402135718153492339885174139750835599080220825225285 80
The number of stems                 =60
Mean                                =3.00 The theoretical mean=3.00
Variance                            =2.85
Standard Deviation                  =1.69
Skewness                            =0.53
Kurtosis                            =0.25
```

Here, **n**=60

The maximum Y is now 'very big' and the need for something like LEDA becomes necessary.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    0.00 The maximum value in the x-ax
is                =70.00
The minimum Y                       =1 The maximum Y                  =2598938824477428289089
4889669407040635582987320728834095685222733803013162939120756908408 60
The number of stems                 =70
Mean                                =3.50 The theoretical mean=3.50
Variance                            =3.32
Standard Deviation                  =1.82
Skewness                            =0.49
Kurtosis                            =0.22
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    0.00 The maximum value in the x-ax
is                =80.00
The minimum Y                       =1 The maximum Y                  =2423063730013761715450
225141387532247018419671274378747524356925844685129163138820844029136395784677 99
80
The number of stems                 =80
Mean                                =4.00 The theoretical mean=4.00
Variance                            =3.80
Standard Deviation                  =1.95
Skewness                            =0.46
Kurtosis                            =0.19
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =90.00
The minimum Y                =1 The maximum Y              =24001195865742967700083
55309441053122796097517319468528364049646383401895520932796894211299948743056459
696162914608390
The number of stems          =90
Mean                         =4.50 The theoretical mean=4.50
Variance                     =4.27
Standard Deviation           =2.07
Skewness                     =0.44
Kurtosis                     =0.17
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =100.00
The minimum Y                =1 The maximum Y              =2281997067911412364298
219494738374443220404907511590015254940215625927689091081720989676701132300278 79
6059987867803115059178680480
The number of stems          =100
Mean                         =5.00 The theoretical mean=5.00
Variance                     =4.75
Standard Deviation           =2.18
Skewness                     =0.41
Kurtosis                     =0.15
```

Here, **n**=100

13.11 20-option EMSQ (MCQ) quizzes (X=19) where negative marking IS used i.e. a=-1/19.

Here, we can observe that the mean=0, because the guessing problem is exactly tackled,

```
Mark=-0.0526      numbner getting that mark that way=19 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.05 The maximum value in the x-a
xis             =1.00
The minimum Y                =1 The maximum Y              =19
The number of stems          =1
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.05
Standard Deviation           =0.23
Skewness                     =4.13
Kurtosis                     =15.05
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=-0.1053      numbner getting that mark that way=361 n=2 w=0 x=2 y=0
Mark=0.9474       numbner getting that mark that way=38 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.11 The maximum value in the x-a
xis              =2.00
The minimum Y                  =1 The maximum Y               =361
The number of stems            =2
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.11
Standard Deviation             =0.32
Skewness                       =2.92
Kurtosis                       =7.53
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=-0.1579      numbner getting that mark that way=6859 n=3 w=0 x=3 y=0
Mark=0.8947       numbner getting that mark that way=1083 n=3 w=1 x=2 y=0
Mark=1.9474       numbner getting that mark that way=57 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.16 The maximum value in the x-a
xis              =3.00
The minimum Y                  =1 The maximum Y               =6859
The number of stems            =3
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.16
Standard Deviation             =0.40
Skewness                       =2.38
Kurtosis                       =5.02
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=-0.2105      numbner getting that mark that way=130321 n=4 w=0 x=4 y=0
Mark=0.8421       numbner getting that mark that way=27436 n=4 w=1 x=3 y=0
Mark=1.8947       numbner getting that mark that way=2166 n=4 w=2 x=2 y=0
Mark=2.9474       numbner getting that mark that way=76 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.21 The maximum value in the x-a
xis              =4.00
The minimum Y                  =1 The maximum Y               =130321
The number of stems            =4
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.21
Standard Deviation             =0.46
Skewness                       =2.06
Kurtosis                       =3.76
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=-0.2632      numbner getting that mark that way=2476099 n=5 w=0 x=5 y=0
Mark=0.7895       numbner getting that mark that way=651605 n=5 w=1 x=4 y=0
Mark=1.8421       numbner getting that mark that way=68590 n=5 w=2 x=3 y=0
Mark=2.8947       numbner getting that mark that way=3610 n=5 w=3 x=2 y=0
Mark=3.9474       numbner getting that mark that way=95 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.26 The maximum value in the x-a
xis              =5.00
The minimum Y                  =1 The maximum Y               =2476099
The number of stems            =5
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.26
Standard Deviation             =0.51
Skewness                       =1.85
Kurtosis                       =3.01
```

This agrees with a table in Appendix 7, where **n**=5.

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.53 The maximum value in the x-a
xis              =10.00
The minimum Y                  =1 The maximum Y               =6131066257801
The number of stems            =10
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.53
Standard Deviation             =0.73
Skewness                       =1.31
Kurtosis                       =1.51
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.05 The maximum value in the x-a
xis              =20.00
The minimum Y                  =1 The maximum Y               =3956839311320627178247
9580
The number of stems            =20
Mean                           =0.00 The theoretical mean=0.00
Variance                       =1.05
Standard Deviation             =1.03
Skewness                       =0.92
Kurtosis                       =0.75
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.58 The maximum value in the x-a
xis              =30.00
The minimum Y                  =1 The maximum Y               =3638946598376766553350
15030640947305370
The number of stems            =30
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =1.58
Standard Deviation             =1.26
Skewness                       =0.75
Kurtosis                       =0.50
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -2.11 The maximum value in the x-a
xis                 =40.00
The minimum Y                  =1 The maximum Y               =3053032580444397557610
990833301099147174973578103980
The number of stems            =40
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =2.11
Standard Deviation             =1.45
Skewness                       =0.65
Kurtosis                       =0.38
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -2.63 The maximum value in the x-a
xis                 =50.00
The minimum Y                  =1 The maximum Y               =2939740086085123039512
17362765294597252172035906052555221850745225
The number of stems            =50
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =2.63
Standard Deviation             =1.62
Skewness                       =0.58
Kurtosis                       =0.30
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -3.16 The maximum value in the x-a
xis                 =60.00
The minimum Y                  =1 The maximum Y               =2649935233184625125199
49640213571815349233988517413975083559908022082522528580
The number of stems            =60
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =3.16
Standard Deviation             =1.78
Skewness                       =0.53
Kurtosis                       =0.25
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -3.68 The maximum value in the x-
xis                 =70.00
The minimum Y                  =1 The maximum Y               =259893882447742828908
488966940704063558298732072883409568522273380301316293912075690840860
The number of stems            =70
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =3.68
Standard Deviation             =1.92
Skewness                       =0.49
Kurtosis                       =0.22
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.21 The maximum value in the x-a
xis                 =80.00
The minimum Y                   =1 The maximum Y                =2423063730013761715450
22514138753224701841967127437874752435692584468512916313882084402913639578467799
80
The number of stems             =80
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =4.21
Standard Deviation              =2.05
Skewness                        =0.46
Kurtosis                        =0.19
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.74 The maximum value in the x-a
xis                 =90.00
The minimum Y                   =1 The maximum Y                =2400119586574296770083
55309441053122796097517319468528364049646383401895520932796894211299948743056459
696162914608390
The number of stems             =90
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =4.74
Standard Deviation              =2.18
Skewness                        =0.44
Kurtosis                        =0.17
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -5.26 The maximum value in the x-a
xis                 =100.00
The minimum Y                   =1 The maximum Y                =2281997067911412364298
21949473837444322040490751159001525494021562592768909108172098967670113230027879
6059987867803115059178680480
The number of stems             =100
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =5.26
Standard Deviation              =2.29
Skewness                        =0.41
Kurtosis                        =0.15
```

Here, **n**=100

13.12 20-option EMSQ (MAQ with 10 ticks) quizzes (X=184755) where negative marking is NOT used i.e. a=0.

```
Mark=0.0000       numbner getting that mark that way=184755 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =1.00
The minimum Y                  =1 The maximum Y                 =184755
The number of stems            =1
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.00
Skewness                       =429.83
Kurtosis                       =184751.00
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=0.0000       numbner getting that mark that way=34134410025 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=369510 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =2.00
The minimum Y                  =1 The maximum Y                 =34134410025
The number of stems            =2
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.00
Skewness                       =303.94
Kurtosis                       =92375.50
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=0.0000       numbner getting that mark that way=6306502924168875 n=3 w=0 x=3
 y=0
Mark=1.0000       numbner getting that mark that way=102403230075 n=3 w=1 x=2 y=0

Mark=2.0000       numbner getting that mark that way=554265 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =3.00
The minimum Y                  =1 The maximum Y                 =6306502924168875
The number of stems            =3
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.00
Skewness                       =248.16
Kurtosis                       =61583.67
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=0.0000      numbner getting that mark that way=1165157947754820500625 n=4 v
=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=25226011696675500 n=4 v=1 x=
3 y=0
Mark=2.0000      numbner getting that mark that way=204806460150 n=4 v=2 x=2 y=0

Mark=3.0000      numbner getting that mark that way=739020 n=4 v=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 v=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is                =4.00
The minimum Y                 =1 The maximum Y              =1165157947754820500625

The number of stems           =4
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =214.91
Kurtosis                      =46187.75
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=0.0000      numbner getting that mark that way=2152687566374418615929971875
n=5 v=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=582578973877410250З125 n=5 v
=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=63065029241688750 n=5 v=2 x=
3 y=0
Mark=3.0000      numbner getting that mark that way=341344100250 n=5 v=3 x=2 y=0

Mark=4.0000      numbner getting that mark that way=923775 n=5 v=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 v=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is                =5.00
The minimum Y                 =1 The maximum Y              =215268756637441861592
71875
The number of stems           =5
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =192.23
Kurtosis                      =36950.20
```

This agrees with a table in Appendix 7, where **n**=5.

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is                =10.00
The minimum Y                 =1 The maximum Y              =463406375842301695412
859465958851239293439454101562 5
The number of stems           =10
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =135.92
Kurtosis                      =18475.10
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-
is               =20.00
The minimum Y                   =1 The maximum Y                =214745469171296576364
025867701282985087303621345255862894784826964923161318776538314529800510406494
0625
The number of stems             =20
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =96.11
Kurtosis                        =9237.55
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =30.00
The minimum Y                   =1 The maximum Y                =9951441959722527326786
7875969997004972814319394269737958357596890370525232787260330146259957198753803З
02389429636694843090422436670815573073923587799072265625
The number of stems             =30
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =78.48
Kurtosis                        =6158.37
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =40.00
The minimum Y                   =1 The maximum Y                =4611561652960028828814
9566594191622292361051345438717192951554719440263812750202425670601089049971098
0789435944426170834197261617704814558569972366932335339474901662704732790097550
1451869551092386245727539062
The number of stems             =40
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =67.96
Kurtosis                        =4618.78
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =50.00
The minimum Y                   =1 The maximum Y                =2939740086085123039512
17362765294597252172035906052555221850745225
The number of stems             =50
Mean                            =2.50 The theoretical mean=2.50
Variance                        =2.38
Standard Deviation              =1.54
Skewness                        =0.58
Kurtosis                        =0.30
```

Here, **n**=50

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =60.00
The minimum Y                  =1 The maximum Y               =99031197077726135194142
3312171608654319585676362284765927888134085862713075566106306249371857881574960
6193207061481552114155696232291873228261746453710277997173632115119189754641016
1190649310973346774315791015309232299535411432583305506187933941836019767671677681
105173278461664687633003723021829500794410705566406 25
The number of stems            =60
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =55.49
Kurtosis                       =3079.18
```

Here, **n**=60

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =70.00
The minimum Y                  =1 The maximum Y               =45891688133113806750 34
445707322529380460631151227422290583787236932646879428674231527001893614590362710
50687458279262802155254245190836724434403969887772054406925858821083410512724362
44096992449407603043700292188316285015001046320120157794556347839415499751765059
94666949552303754248905793127748865141386169282302111926863695057568492963184780
819574370980262756347656 25
The number of stems            =70
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =51.37
Kurtosis                       =2639.30
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =80.00
The minimum Y                  =1 The maximum Y               =21266500879051433368476
7088213946836185142126926880897408297595497640336089108349365848256005548116414
8984552766774996240320706212507996059897466565541638435511380300176251389098143
0184593078306579096617083514523565178972448968757411116487052466652875631014189
6468603629686167830972554620435373597946420783980930549520028275000098617096721
3193382712069821475381324101061569917996228046286333324133011046797037124633789
0625

The number of stems            =80
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =48.06
Kurtosis                       =2309.39
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =90.00
The minimum Y                 =1 The maximum Y                 =98550320992083479217 49
59602091456234570835259838120726619246309706658513426166956112813596773929730336
66897014990394950602305873640225953018244396202232056681022218182836986848968601
52277824726792336034045830367878582375797015904912052980741634808462449773829117
84405374298654905277451897319544148850795179336421900221153325815216511605765642
57895894550588715644206862511409858413699223253579290403921804794558462428290158
07494129558855022743557583453366532921791076660156 25
The number of stems           =90
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.02
Skewness                      =45.31
Kurtosis                      =2052.79
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =100.00
The minimum Y                 =1 The maximum Y                 =45668847089036911256 61
46929695087937830571602701006500636106024483518303069488497699624034706474687439
92310372629481719596334924534195918345565623844220712142525028681669391223896001
45405873068666859039326591965787655037452149880736324258114317043931735317705099
76050546929746507714887254129416367349578312801568276814994403910915796611516413
40352827572465615546700303621732441746762279032694122565394491451296518142020274
07540515519741445215348782777528430827380297830000430980055636884934466834806698
89831915497779846191 40625
The number of stems           =100
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.02
Skewness                      =42.98
Kurtosis                      =1847.51
```

Here, **n**=100

13.13 20-option EMSQ (MAQ with 10 ticks) quizzes (X=184755) where negative marking IS used i.e. a=-1/184755.

```
Mark=-0.0000      numbner getting that mark that way=184755 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis              =1.00
The minimum Y                 =1 The maximum Y              =184755
The number of stems           =1
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =429.83
Kurtosis                      =184751.00
```

This agrees with a table in Appendix 7, where **n**=1.

```
Mark=-0.0000      numbner getting that mark that way=34134410025 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=369510 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis              =2.00
The minimum Y                 =1 The maximum Y              =34134410025
The number of stems           =2
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =303.94
Kurtosis                      =92375.50
```

This agrees with a table in Appendix 7, where **n**=2.

```
Mark=-0.0000      numbner getting that mark that way=6306502924168875 n=3 w=0 x=3
 y=0
Mark=1.0000       numbner getting that mark that way=102403230075 n=3 w=1 x=2 y=0

Mark=2.0000       numbner getting that mark that way=554265 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis              =3.00
The minimum Y                 =1 The maximum Y              =6306502924168875
The number of stems           =3
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =248.16
Kurtosis                      =61583.67
```

This agrees with a table in Appendix 7, where **n**=3.

```
Mark=-0.0000      numbner getting that mark that way=116515794775482050062 5 n=4 w
=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=25226011696675500 n=4 w=1 x=
3 y=0
Mark=2.0000       numbner getting that mark that way=20480646015 0 n=4 w=2 x=2 y=0

Mark=3.0000       numbner getting that mark that way=739020 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis               =4.00
The minimum Y                 =1 The maximum Y               =116515794775482050062 5

The number of stems           =4
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =214.91
Kurtosis                      =46187.75
```

This agrees with a table in Appendix 7, where **n**=4.

```
Mark=-0.0000      numbner getting that mark that way=215268756637441861592971875
n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=582578973877410250312 5 n=5 w
=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=63065029241688750 n=5 w=2 x=
3 y=0
Mark=3.0000       numbner getting that mark that way=341344100250 n=5 w=3 x=2 y=0

Mark=4.0000       numbner getting that mark that way=923775 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0


Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis               =5.00
The minimum Y                 =1 The maximum Y               =21526875663744186159 2 9
71875
The number of stems           =5
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =192.23
Kurtosis                      =36950.20
```

This agrees with a table in Appendix 7, where **n**=5.

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis               =10.00
The minimum Y                 =1 The maximum Y               =4634063750423016954121
85946595885123929343945410 15625
The number of stems           =10
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =135.92
Kurtosis                      =18475.10
```

Here, **n**=10

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.00 The maximum value in the x-a
xis              =20.00
The minimum Y                =1 The maximum Y               =21474546917129657636411
0258677012829850873036213452558628947848269649231613187765383145298005104064941
0625
The number of stems          =20
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =96.11
Kurtosis                     =9237.55
```

Here, **n**=20

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situa
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.00 The maximum value in the x-
xis              =30.00
The minimum Y                =1 The maximum Y               =99514419597225273267
78759699970049728143193942697379583575968903705252327872603301462599571987538
0238942963669484309042243667081557307392358779907226 5625
The number of stems          =30
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =78.48
Kurtosis                     =6158.37
```

Here, **n**=30

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.00 The maximum value in the x-a
xis              =40.00
The minimum Y                =1 The maximum Y               =4611561652960028828814
956659419162229236105134543871719295155471944026381275020242567060108904997109 85
078943594442617083419726161770481455856997236693233533947490166270473279009755 06
1451869551092386245727539 0625
The number of stems          =40
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =67.96
Kurtosis                     =4618.78
```

Here, **n**=40

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.00 The maximum value in the x-a
xis              =50.00
The minimum Y                =1 The maximum Y               =2137027072571541178242
689566293860579841782524101101322457409520289863076870097074757906761440558215 68
391016439189759148681241918412841277227779739536791802450654610913570023866561 78
261278981919357913902889018601513177028493917228857412737852428108453750610351 56
25
The number of stems          =50
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.02
Skewness                     =60.79
Kurtosis                     =3695.02
```

Here, **n**=50

```
C:\WINDOWS\system32\cmd.exe
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-
xis                 =60.00
The minimum Y                  =1 The maximum Y                 =990311970777261351941
2331217160865431958567636228476592788813408586271307556610630624937185788157496
6193207061481552114155696232291873228261746453710277997173632115119189754641016
1906493109733467743157910153092322995354114325833055061879339418360197676716776
1105173278461664687633003723021829500794410705566406 25
The number of stems            =60
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =55.49
Kurtosis                       =3079.18
```

Here, **n**=60

```
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis                 =70.00
The minimum Y                  =1 The maximum Y                 =45891688133113806750 34
4457073225293804606311512274222905837872369326468794286742315270018936145903627 1
0506874582792628021552542451908367244344039698877720544069258588210834105127243 6
2440969924494076030437002921883162850150010463201201577945563478394154997517650 5
9946669495523037542489057931277488651413861692823021119268636950575684929631847 8
08195743709802627563476 5625
The number of stems            =70
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =51.37
Kurtosis                       =2639.30
```

Here, **n**=70

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis                 =80.00
The minimum Y                  =1 The maximum Y                 =2126650087905143336847
6708821394683618514212692688089740829759549764033608910834936584825600554811641 4
8984552766774996240320706212507996059897466565541638435511380300176251389098143 0
1845930783065790966170835145235651789724489687574111164870524666528756310141896 4
6860362968616783097255462043537359794642078398093054952002827500009861709672131 9
3382712069821475381324101061569917996228046286333324133011046797037124633789062 5

The number of stems            =80
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =48.06
Kurtosis                       =2309.39
```

Here, **n**=80

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis              =90.00
The minimum Y                =1 The maximum Y               =985503209920834792174959602091456234570835259838120726619246309706658513426166956112813596773929730336668970149903949506023058736402259530182443962022320566810222181828369868489686015227782472679233603404583036787858237579701590491205298074163480846244977382911784405374298654905277451897319544148850795179336421900221153325815216511605765642578958945505887156442068625114098584136992232535792904039218047945584624282901580749412955885502274355758345336653292179107666015625
The number of stems          =90
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.02
Skewness                     =45.31
Kurtosis                     =2052.79
```

Here, **n**=90

```
Stems CANNOT be missed. Score is [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=0 The theretical number of impossible situat
ions is=0

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis              =100.00
The minimum Y                =1 The maximum Y               =456688470890369112566146929695087937830571602701006500636106024483518303069488497699624034706474687439923103726294817195963349245341959183455656238442207121425250286816693912238960014540587306866685903932659196578765503745214988073632425811431704393173531770509976050546929746507714887254129416367349578312801568276814994403910915796611516413403528275724656155467003036217324417467622790326941225653944914512965181420202740754051551974144521534878277752843082738029783000043098005563688493446683480669889831915497779846191406 25
The number of stems          =100
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.02
Skewness                     =42.98
Kurtosis                     =1847.51
```

Here, **n**=100

Appendix 14. Validation of *tool1* where stems can be missed.

14.1 Introduction

This Appendix attempts to validate *tool1* for quizzes where stems can be missed. The validation is done 'by hand' for:

X=1	i.e. Yes/No quizzes	for **n**=1 to 5
X=3	i.e. 4-option MCQ quizzes	for **n**=1 to 5
X=7	i.e. 3-option Multiple Yes/No quizzes	for **n**=1 to 5
X=9	i.e. 5-option MAQ quizzes with 2 ticks	for **n**=1 to 5
X=19	i.e. 2-option EMSQ (MCQ)	for **n**=1 to 5
X=187455	i.e.20-option EMSQ (MAQ with 10 ticks)	for **n**=1 to 5

Beyond **n**=5 we use or proofs, and other indications, to validate *tool1*, where stems can be missed.

For each of the above we:

- do not use negative marking, and have **a**=0
- do use negative marking where **b**=0 and **a** has a value that exactly tackles the guessing problem.

So that means that there are 12 sections where our validation is done.

This Appendix is accompanied by Appendix 8.

We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 8.

Appendix 8 is where the validation of the **expression** that is used by *tool1* is done, for quizzes where stems can be missed. We use this validation (and our proofs) to show that *tool1* produces the same results as Appendix 8.

One validation that we can do is that the distribution should be skewed.

14.2 Yes/No quizzes (X=1) where no negative marking is used i.e. a=0

```
Mark=0.0000        numbner getting that mark that way=1 n=1 w=0 x=1 y=0
Mark=1.0000        numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000        numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =1.00
The number of stems          =1
Mean                         =0.33 The theoretical mean=0.33
Variance                     =0.22
Standard Deviation           =0.47
Skewness                     =0.71
Kurtosis                     =-1.50

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 4 times
Press any key to continue . . .
```

The distribution is skewed as the Skewness is non-zero

The file has a header written to it and three pairs of numbers; each pair on a separate line.

Notice that, here, the marks are not sorted, and the numbers getting mark 0, say, are not added. So a Pivot is necessary.

This agrees with a table in Appendix 8, where **n**=1. So, *tool1* produces the same results as the expression. It also agrees with the theoretical number of impossible situations and produces a mean that agrees with the theoretical.

```
Mark=0.0000        numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=1.0000        numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000        numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000        numbner getting that mark that way=2 n=2 w=0 x=1 y=1
Mark=1.0000        numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000        numbner getting that mark that way=1 n=2 w=0 x=0 y=2

Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is               =2.00
The number of stems          =2
Mean                         =0.67 The theoretical mean=0.67
Variance                     =0.44
Standard Deviation           =0.67
Skewness                     =0.50
Kurtosis                     =-0.75

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 7 times
Press any key to continue . . .
```

This is Multiple Yes/No quiz with 1 option (and hence is a Yes/No quiz). There are 2 stems i.e. **n**=2 and agrees with a table in Appendix 8, where **n**=2.

```
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000      numbner getting that mark that way=3 n=3 w=0 x=2 y=1
Mark=1.0000      numbner getting that mark that way=6 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000      numbner getting that mark that way=3 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =3.00
The number of stems          =3
Mean                         =1.00 The theoretical mean=1.00
Variance                     =0.67
Standard Deviation           =0.82
Skewness                     =0.41
Kurtosis                     =-0.50

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 11 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8, where **n**=3

```
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000      numbner getting that mark that way=4 n=4 w=0 x=3 y=1
Mark=1.0000      numbner getting that mark that way=12 n=4 w=1 x=2 y=1
Mark=2.0000      numbner getting that mark that way=12 n=4 w=2 x=1 y=1
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000      numbner getting that mark that way=6 n=4 w=0 x=2 y=2
Mark=1.0000      numbner getting that mark that way=12 n=4 w=1 x=1 y=2
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000      numbner getting that mark that way=4 n=4 w=0 x=1 y=3
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =4.00
The number of stems          =4
Mean                         =1.33 The theoretical mean=1.33
Variance                     =0.89
Standard Deviation           =0.94
Skewness                     =0.35
Kurtosis                     =-0.37

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 16 times
Press any key to continue . . . _
```

This agrees with a table in Appendix 8, where **n**=4

```
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=3 y=0
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=2 y=0
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000      numbner getting that mark that way=5 n=5 w=0 x=4 y=1
Mark=1.0000      numbner getting that mark that way=20 n=5 w=1 x=3 y=1
Mark=2.0000      numbner getting that mark that way=30 n=5 w=2 x=2 y=1
Mark=3.0000      numbner getting that mark that way=20 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000      numbner getting that mark that way=10 n=5 w=0 x=3 y=2
Mark=1.0000      numbner getting that mark that way=30 n=5 w=1 x=2 y=2
Mark=2.0000      numbner getting that mark that way=30 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000      numbner getting that mark that way=10 n=5 w=0 x=2 y=3
Mark=1.0000      numbner getting that mark that way=20 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000      numbner getting that mark that way=5 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is             =5.00
The number of stems          =5
Mean                         =1.67 The theoretical mean=1.67
Variance                     =1.11
Standard Deviation           =1.05
Skewness                     =0.32
Kurtosis                     =-0.30

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 22 times
Press any key to continue . . . _
```

This agrees with a table in Appendix 8, where **n**=5

We do not check by 'by hand' beyond **n**=5. Beyond that we know a) the mean mark b) the range of marks c) the number of impossible situations d) the name of the file we have written to. We can use these as the validation.

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is            =10.00
The number of stems          =10
Mean                         =3.33 The theoretical mean=3.33
Variance                     =2.22
Standard Deviation           =1.49
Skewness                     =0.22
Kurtosis                     =-0.15

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =20.00
The number of stems           =20
Mean                          =6.67 The theoretical mean=6.67
Variance                      =4.44
Standard Deviation            =2.11
Skewness                      =0.16
Kurtosis                      =-0.08

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 232 times
Press any key to continue . . .
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=465 The theretical number of impossible sit
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is                =30.00
The number of stems           =30
Mean                          =10.00 The theoretical mean=10.00
Variance                      =6.67
Standard Deviation            =2.58
Skewness                      =0.13
Kurtosis                      =-0.05

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 497 times
Press any key to continue . . .
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =40.00
The number of stems           =40
Mean                          =13.33 The theoretical mean=13.33
Variance                      =8.89
Standard Deviation            =2.98
Skewness                      =0.11
Kurtosis                      =-0.04

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 862 times
Press any key to continue . . .
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =50.00
The number of stems          =50
Mean                         =16.67 The theoretical mean=16.67
Variance                     =11.11
Standard Deviation           =3.33
Skewness                     =0.10
Kurtosis                     =-0.03

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1327 times
Press any key to continue . . .
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =60.00
The number of stems          =60
Mean                         =20.00 The theoretical mean=20.00
Variance                     =13.33
Standard Deviation           =3.65
Skewness                     =0.09
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1892 times
Press any key to continue . . .
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is               =70.00
The number of stems          =70
Mean                         =23.33 The theoretical mean=23.33
Variance                     =15.56
Standard Deviation           =3.94
Skewness                     =0.08
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 2557 times
Press any key to continue . . .
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =80.00
The number of stems           =80
Mean                          =26.67 The theoretical mean=26.67
Variance                      =17.78
Standard Deviation            =4.22
Skewness                      =0.08
Kurtosis                      =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 3322 times
Press any key to continue . . .
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=4095 The theretical number of impossible s
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-
is                =90.00
The number of stems           =90
Mean                          =30.00 The theoretical mean=30.00
Variance                      =20.00
Standard Deviation            =4.47
Skewness                      =0.07
Kurtosis                      =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 4187 times
Press any key to continue . . .
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Yes/No X=1
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =100.00
The number of stems           =100
Mean                          =33.33 The theoretical mean=33.33
Variance                      =22.22
Standard Deviation            =4.71
Skewness                      =0.07
Kurtosis                      =-0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

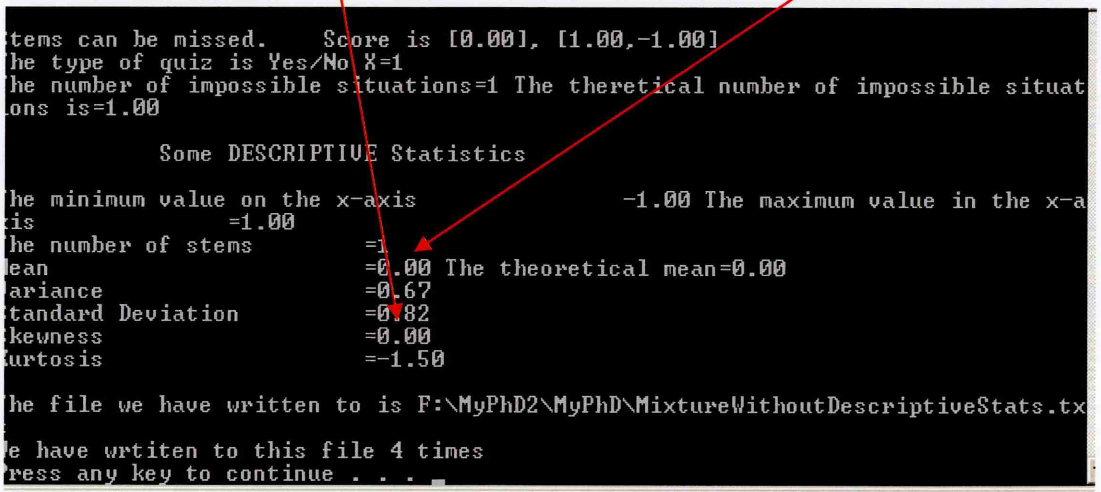
Here, **n**=100

So, for Yes/ No-only stems where stems can be missed and the score is [0], [1, 0] *tool1* works, as far as we can tell.

14.3 Yes/No quizzes (X=1) where a modification of negative marking is used i.e. a=-1 and b=0

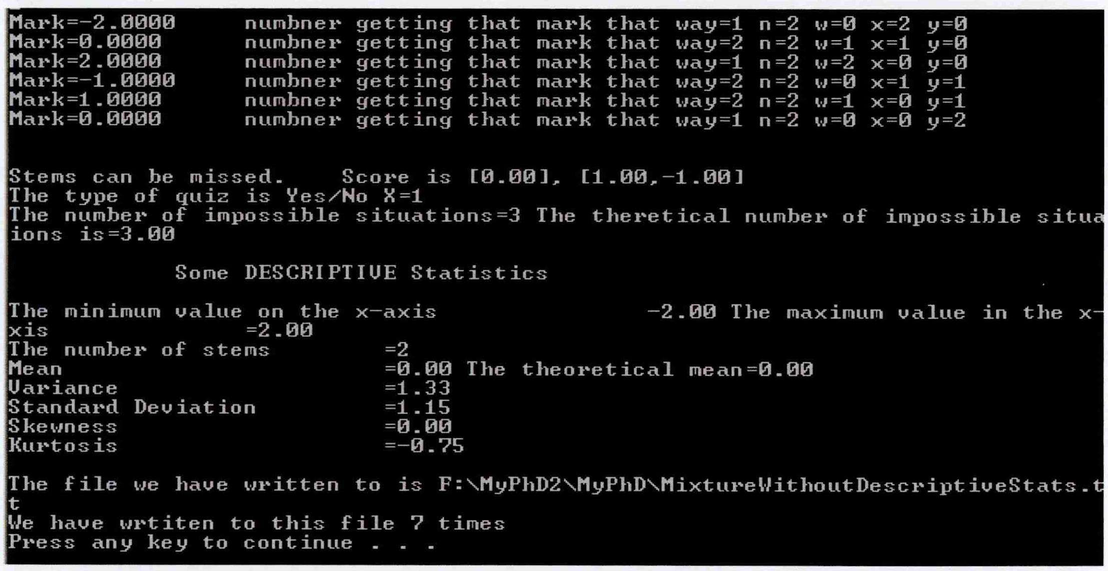
Because we are tackling the guessing problem *exactly*, the mean mark is 0, and that can be used as part of the validation.

Here, the skewness is 0, but that is not always the case (see Appendix 6).



```
tems can be missed.    Score is [0.00], [1.00,-1.00]
he type of quiz is Yes/No X=1
he number of impossible situations=1 The theretical number of impossible situat
ons is=1.00

           Some DESCRIPTIVE Statistics

he minimum value on the x-axis               -1.00 The maximum value in the x-a
is              =1.00
he number of stems          =1
ean                         =0.00 The theoretical mean=0.00
ariance                     =0.67
tandard Deviation           =0.82
kewness                     =0.00
urtosis                     =-1.50

he file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx

e have wrtiten to this file 4 times
ress any key to continue . . .
```

This agrees with a table in Appendix 8, where **n** (the number of stems) =1



```
Mark=-2.0000      numbner getting that mark that way=1 n=2 w=0 x=2 y=0
Mark=0.0000       numbner getting that mark that way=2 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-1.0000      numbner getting that mark that way=2 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3 The theretical number of impossible situa
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -2.00 The maximum value in the x-
xis               =2.00
The number of stems          =2
Mean                         =0.00 The theoretical mean=0.00
Variance                     =1.33
Standard Deviation           =1.15
Skewness                     =0.00
Kurtosis                     =-0.75

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 7 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8, where **n** (the number of stems) =2

```
Mark=-3.0000      numbner getting that mark that way=1 n=3 w=0 x=3 y=0
Mark=-1.0000      numbner getting that mark that way=3 n=3 w=1 x=2 y=0
Mark=1.0000       numbner getting that mark that way=3 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-2.0000      numbner getting that mark that way=3 n=3 w=0 x=2 y=1
Mark=0.0000       numbner getting that mark that way=6 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-1.0000      numbner getting that mark that way=3 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -3.00 The maximum value in the x-a
xis              =3.00
The number of stems          =3
Mean                         =0.00 The theoretical mean=0.00
Variance                     =2.00
Standard Deviation           =1.41
Skewness                     =0.00
Kurtosis                     =-0.50

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 11 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=-4.0000      numbner getting that mark that way=1 n=4 w=0 x=4 y=0
Mark=-2.0000      numbner getting that mark that way=4 n=4 w=1 x=3 y=0
Mark=0.0000       numbner getting that mark that way=6 n=4 w=2 x=2 y=0
Mark=2.0000       numbner getting that mark that way=4 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-3.0000      numbner getting that mark that way=4 n=4 w=0 x=3 y=1
Mark=-1.0000      numbner getting that mark that way=12 n=4 w=1 x=2 y=1
Mark=1.0000       numbner getting that mark that way=12 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-2.0000      numbner getting that mark that way=6 n=4 w=0 x=2 y=2
Mark=0.0000       numbner getting that mark that way=12 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-1.0000      numbner getting that mark that way=4 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=10 The theretical number of impossible situ
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -4.00 The maximum value in the x-
xis              =4.00
The number of stems          =4
Mean                         =0.00 The theoretical mean=0.00
Variance                     =2.67
Standard Deviation           =1.63
Skewness                     =0.00
Kurtosis                     =-0.38

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 16 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=4

```
1
Mark=-5.0000      numbner getting that mark that way=1 n=5 v=0 x=5 y=0
Mark=-3.0000      numbner getting that mark that way=5 n=5 v=1 x=4 y=0
Mark=-1.0000      numbner getting that mark that way=10 n=5 v=2 x=3 y=0
Mark=1.0000       numbner getting that mark that way=10 n=5 v=3 x=2 y=0
Mark=3.0000       numbner getting that mark that way=5 n=5 v=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 v=5 x=0 y=0
Mark=-4.0000      numbner getting that mark that way=5 n=5 v=0 x=4 y=1
Mark=-2.0000      numbner getting that mark that way=20 n=5 v=1 x=3 y=1
Mark=0.0000       numbner getting that mark that way=30 n=5 v=2 x=2 y=1
Mark=2.0000       numbner getting that mark that way=20 n=5 v=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 v=4 x=0 y=1
Mark=-3.0000      numbner getting that mark that way=10 n=5 v=0 x=3 y=2
Mark=-1.0000      numbner getting that mark that way=30 n=5 v=1 x=2 y=2
Mark=1.0000       numbner getting that mark that way=30 n=5 v=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 v=3 x=0 y=2
Mark=-2.0000      numbner getting that mark that way=10 n=5 v=0 x=2 y=3
Mark=0.0000       numbner getting that mark that way=20 n=5 v=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 v=2 x=0 y=3
Mark=-1.0000      numbner getting that mark that way=5 n=5 v=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 v=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 v=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -5.00 The maximum value in the x-a
xis           =5.00
The number of stems         =5
Mean                        =0.00 The theoretical mean=0.00
Variance                    =3.33
Standard Deviation          =1.83
Skewness                    =0.00
Kurtosis                    =-0.30

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 22 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=5

We do not check by 'by hand' beyond **n**=5. Beyond that we know a) the mean mark b) the range of marks c) the number of impossible situations d) the name of the file we have written to. We can use these as the validation.

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -10.00 The maximum value in the x-
axis          =10.00
The number of stems         =10
Mean                        =0.00 The theoretical mean=0.00
Variance                    =6.67
Standard Deviation          =2.58
Skewness                    =0.00
Kurtosis                    =-0.15

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
Press any key to continue . . .
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=210 The theretical number of impossible sit
ations is=210.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -20.00 The maximum value in the x
axis                =20.00
The number of stems           =20
Mean                          =0.00 The theoretical mean=0.00
Variance                      =13.33
Standard Deviation            =3.65
Skewness                      =0.00
Kurtosis                      =-0.08

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 232 times
Press any key to continue . . .
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=465 The theretical number of impossible s
ations is=465.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -30.00 The maximum value in the
axis                =30.00
The number of stems           =30
Mean                          =0.00 The theoretical mean=0.00
Variance                      =20.00
Standard Deviation            =4.47
Skewness                      =0.00
Kurtosis                      =-0.05

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats
t
We have wrtiten to this file 497 times
Press any key to continue . . .
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -40.00 The maximum value in the x-
axis                =40.00
The number of stems           =40
Mean                          =0.00 The theoretical mean=0.00
Variance                      =26.67
Standard Deviation            =5.16
Skewness                      =-0.00
Kurtosis                      =-0.04

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 862 times
Press any key to continue . . .
```

Here, **n**=40

```
Stems can be missed.     Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -50.00 The maximum value in the x-
axis                 =50.00
The number of stems          =50
Mean                         =0.00 The theoretical mean=0.00
Variance                     =33.33
Standard Deviation           =5.77
Skewness                     =-0.00
Kurtosis                     =-0.03

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1327 times
Press any key to continue . . .
```

Here, **n**=50

```
Stems can be missed.     Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -60.00 The maximum value in the x-
axis                 =60.00
The number of stems          =60
Mean                         =-0.00 The theoretical mean=0.00
Variance                     =40.00
Standard Deviation           =6.32
Skewness                     =-0.00
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1892 times
Press any key to continue . . .
```

Here, **n**=60

```
Stems can be missed.     Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=2485 The theretical number of impossible s
uations is=2485.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -70.00 The maximum value in the
axis                 =70.00
The number of stems          =70
Mean                         =0.00 The theoretical mean=0.00
Variance                     =46.67
Standard Deviation           =6.83
Skewness                     =-0.00
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 2557 times
Press any key to continue . . .
```

Here, **n**=70

```
Stems can be missed.     Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=3240 The theretical number of impossible s
uations is=3240.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -80.00 The maximum value in the
axis                 =80.00
The number of stems          =80
Mean                         =-0.00 The theoretical mean=0.00
Variance                     =53.33
Standard Deviation           =7.30
Skewness                     =-0.00
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 3322 times
Press any key to continue . . .
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -90.00 The maximum value in the x-
axis               =90.00
The number of stems         =90
Mean                        =-0.00 The theoretical mean=0.00
Variance                    =60.00
Standard Deviation          =7.75
Skewness                    =0.00
Kurtosis                    =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 4187 times
Press any key to continue . . .
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-1.00]
The type of quiz is Yes/No X=1
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -100.00 The maximum value in the x
-axis               =100.00
The number of stems         =100
Mean                        =-0.00 The theoretical mean=0.00
Variance                    =66.67
Standard Deviation          =8.16
Skewness                    =-0.00
Kurtosis                    =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

Here, **n**=100

So, for Yes/ No-only stems where stems can be missed and the score is [0], [1, 0] or when stems can be missed, and a modification of negative marking is used with **b**=0, when the score is [0], [1, -1] *tool1* works, as far as we can tell.

14.4 4-option MCQ (X=3) without a modification of negative marking i.e. a=b=0

We now validate 4-option MCQ quizzes. MCQs are a specialisation of MAQs where **T**=1.

```
Mark=0.0000      numbner getting that mark that way=3 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1 The theretical number of impossible situa
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-a
is                =1.00
The number of stems          =1
Mean                         =0.20 The theoretical mean=0.20
Variance                     =0.16
Standard Deviation           =0.40
Skewness                     =1.50
Kurtosis                     =0.25

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=0.0000      numbner getting that mark that way=9 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=6 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000      numbner getting that mark that way=6 n=2 w=0 x=1 y=1
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=3 The theretical number of impossible situ
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-
is                =2.00
The number of stems          =2
Mean                         =0.40 The theoretical mean=0.40
Variance                     =0.32
Standard Deviation           =0.57
Skewness                     =1.06
Kurtosis                     =0.13

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 7 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=2

```
1
Mark=0.0000       numbner getting that mark that way=27 n=3 w=0 x=3 y=0
Mark=1.0000       numbner getting that mark that way=27 n=3 w=1 x=2 y=0
Mark=2.0000       numbner getting that mark that way=9 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000       numbner getting that mark that way=27 n=3 w=0 x=2 y=1
Mark=1.0000       numbner getting that mark that way=18 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000       numbner getting that mark that way=9 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is               =3.00
The number of stems          =3
Mean                         =0.60 The theoretical mean=0.60
Variance                     =0.48
Standard Deviation           =0.69
Skewness                     =0.87
Kurtosis                     =0.08

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 11 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=0.0000       numbner getting that mark that way=81 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=108 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=54 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=12 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000       numbner getting that mark that way=108 n=4 w=0 x=3 y=1
Mark=1.0000       numbner getting that mark that way=108 n=4 w=1 x=2 y=1
Mark=2.0000       numbner getting that mark that way=36 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000       numbner getting that mark that way=54 n=4 w=0 x=2 y=2
Mark=1.0000       numbner getting that mark that way=36 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000       numbner getting that mark that way=12 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=10 The theretical number of impossible situ
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-a
is               =4.00
The number of stems          =4
Mean                         =0.80 The theoretical mean=0.80
Variance                     =0.64
Standard Deviation           =0.80
Skewness                     =0.75
Kurtosis                     =0.06

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 16 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=4

```
Mark=0.0000      numbner getting that mark that way=243 n=5 w=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=405 n=5 w=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=270 n=5 w=2 x=3 y=0
Mark=3.0000      numbner getting that mark that way=90 n=5 w=3 x=2 y=0
Mark=4.0000      numbner getting that mark that way=15 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000      numbner getting that mark that way=405 n=5 w=0 x=4 y=1
Mark=1.0000      numbner getting that mark that way=540 n=5 w=1 x=3 y=1
Mark=2.0000      numbner getting that mark that way=270 n=5 w=2 x=2 y=1
Mark=3.0000      numbner getting that mark that way=60 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000      numbner getting that mark that way=270 n=5 w=0 x=3 y=2
Mark=1.0000      numbner getting that mark that way=270 n=5 w=1 x=2 y=2
Mark=2.0000      numbner getting that mark that way=90 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000      numbner getting that mark that way=90 n=5 w=0 x=2 y=3
Mark=1.0000      numbner getting that mark that way=60 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000      numbner getting that mark that way=15 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=15 The theretical number of impossible si
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-
is              =5.00
The number of stems           =5
Mean                          =1.00 The theoretical mean=1.00
Variance                      =0.80
Standard Deviation            =0.89
Skewness                      =0.67
Kurtosis                      =0.05

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 22 times
Press any key to continue . . . _
```

This (partly) agrees with a table in Appendix 8 where **n**=5.

We do not check by 'by hand' beyond **n**=5. Beyond that we know a) the mean mark b) the range of marks c) the number of impossible situations d) the name of the file we have written to. We can use these as the validation.

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is              =10.00
The number of stems           =10
Mean                          =2.00 The theoretical mean=2.00
Variance                      =1.60
Standard Deviation            =1.26
Skewness                      =0.47
Kurtosis                      =0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
Press any key to continue . . .
```

The *theoretical* and *actual* are equal

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =20.00
The number of stems          =20
Mean                         =4.00 The theoretical mean=4.00
Variance                     =3.20
Standard Deviation           =1.79
Skewness                     =0.34
Kurtosis                     =0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 232 times
Press any key to continue . . .
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =30.00
The number of stems          =30
Mean                         =6.00 The theoretical mean=6.00
Variance                     =4.80
Standard Deviation           =2.19
Skewness                     =0.27
Kurtosis                     =0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 497 times
Press any key to continue . . .
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=820 The theretical number of impossible sit
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-a
is              =40.00
The number of stems          =40
Mean                         =8.00 The theoretical mean=8.00
Variance                     =6.40
Standard Deviation           =2.53
Skewness                     =0.24
Kurtosis                     =0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 862 times
Press any key to continue . . .
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =50.00
The number of stems           =50
Mean                          =10.00 The theoretical mean=10.00
Variance                      =8.00
Standard Deviation            =2.83
Skewness                      =0.21
Kurtosis                      =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1327 times
Press any key to continue . . .
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =60.00
The number of stems           =60
Mean                          =12.00 The theoretical mean=12.00
Variance                      =9.60
Standard Deviation            =3.10
Skewness                      =0.19
Kurtosis                      =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1892 times
Press any key to continue . . .
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =70.00
The number of stems           =70
Mean                          =14.00 The theoretical mean=14.00
Variance                      =11.20
Standard Deviation            =3.35
Skewness                      =0.18
Kurtosis                      =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 2557 times
Press any key to continue . . .
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is               =80.00
The number of stems          =80
Mean                         =16.00 The theoretical mean=16.00
Variance                     =12.80
Standard Deviation           =3.58
Skewness                     =0.17
Kurtosis                     =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 3322 times
Press any key to continue . . .
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=4095 The theretical number of impossible s
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-
is               =90.00
The number of stems          =90
Mean                         =18.00 The theoretical mean=18.00
Variance                     =14.40
Standard Deviation           =3.79
Skewness                     =0.16
Kurtosis                     =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 4187 times
Press any key to continue . . .
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is               =100.00
The number of stems          =100
Mean                         =20.00 The theoretical mean=20.00
Variance                     =16.00
Standard Deviation           =4.00
Skewness                     =0.15
Kurtosis                     =0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

Here, **n**=100

So, for 4-option MCQ-only stems where stems can be missed and the score is [0], [1, 0] *tool1* works, as far as we can tell.

14.5 4-option MCQ (X=3) with a modification of negative marking where b=0 and a =-1/3.

If the guessing is *exactly* tackled with **a**=-1/3 then the mean mark is 0. The distribution is skewed.

```
1
Mark=-0.3333     numbner getting that mark that way=3 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1 The theretical number of impossible situa
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.33 The maximum value in the x-
xis              =1.00
The number of stems          =1
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.27
Standard Deviation           =0.52
Skewness                     =1.29
Kurtosis                     =-0.08

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 4 times
Press any key to continue . . .
```

The mean is 0

The skewness is not 0

This agrees with a table in Appendix 8 where **n**=1

```
1
Mark=-0.6667     numbner getting that mark that way=9 n=2 w=0 x=2 y=0
Mark=0.6667      numbner getting that mark that way=6 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-0.3333     numbner getting that mark that way=6 n=2 w=0 x=1 y=1
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=3 The theretical number of impossible situa
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.67 The maximum value in the x-
xis              =2.00
The number of stems          =2
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.53
Standard Deviation           =0.73
Skewness                     =0.91
Kurtosis                     =-0.04

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 7 times
Press any key to continue . . .
```

The *theoretical* and the *actual* are the same

This agrees with a table in Appendix 8 where **n**=2

```
Mark=-1.0000      numbner getting that mark that way=27 n=3 w=0 x=3 y=0
Mark=0.3333       numbner getting that mark that way=27 n=3 w=1 x=2 y=0
Mark=1.6667       numbner getting that mark that way=9 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-0.6667      numbner getting that mark that way=27 n=3 w=0 x=2 y=1
Mark=0.6667       numbner getting that mark that way=18 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-0.3333      numbner getting that mark that way=9 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00, 0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=6 The theretical number of impossible situ
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                1.00 The maximum value in the x
xis            =3.00
The number of stems          =3
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.80
Standard Deviation           =0.89
Skewness                     =0.75
Kurtosis                     = 0.03

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 11 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=-1.3333      numbner getting that mark that way=81 n=4 w=0 x=4 y=0
Mark=0.0000       numbner getting that mark that way=108 n=4 w=1 x=3 y=0
Mark=1.3333       numbner getting that mark that way=54 n=4 w=2 x=2 y=0
Mark=2.6667       numbner getting that mark that way=12 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-1.0000      numbner getting that mark that way=108 n=4 w=0 x=3 y=1
Mark=0.3333       numbner getting that mark that way=108 n=4 w=1 x=2 y=1
Mark=1.6667       numbner getting that mark that way=36 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-0.6667      numbner getting that mark that way=54 n=4 w=0 x=2 y=2
Mark=0.6667       numbner getting that mark that way=36 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-0.3333      numbner getting that mark that way=12 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=10 The theretical number of impossible situ
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -1.33 The maximum value in the x-
xis            =4.00
The number of stems          =4
Mean                         =0.00 The theoretical mean=0.00
Variance                     =1.07
Standard Deviation           =1.03
Skewness                     =0.65
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
```

This agrees with a table in Appendix 8 where **n**=4

```
I
Mark=-1.6667     numbner getting that mark that way=243 n=5 w=0 x=5 y=0
Mark=-0.3333     numbner getting that mark that way=405 n=5 w=1 x=4 y=0
Mark=1.0000      numbner getting that mark that way=270 n=5 w=2 x=3 y=0
Mark=2.3333      numbner getting that mark that way=90 n=5 w=3 x=2 y=0
Mark=3.6667      numbner getting that mark that way=15 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=-1.3333     numbner getting that mark that way=405 n=5 w=0 x=4 y=1
Mark=0.0000      numbner getting that mark that way=540 n=5 w=1 x=3 y=1
Mark=1.3333      numbner getting that mark that way=270 n=5 w=2 x=2 y=1
Mark=2.6667      numbner getting that mark that way=60 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=-1.0000     numbner getting that mark that way=270 n=5 w=0 x=3 y=2
Mark=0.3333      numbner getting that mark that way=270 n=5 w=1 x=2 y=2
Mark=1.6667      numbner getting that mark that way=90 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=-0.6667     numbner getting that mark that way=90 n=5 w=0 x=2 y=3
Mark=0.6667      numbner getting that mark that way=60 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=-0.3333     numbner getting that mark that way=15 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -1.67 The maximum value in the x-a
xis                 =5.00
The number of stems          =5
Mean                         =0.00 The theoretical mean=0.00
Variance                     =1.33
Standard Deviation           =1.15
Skewness                     =0.58
Kurtosis                     =-0.02

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 22 times
Press any key to continue . . . _
```

This agrees (where it can) with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -3.33 The maximum value in the x-a
xis                 =10.00
The number of stems          =10
Mean                         =0.00 The theoretical mean=0.00
Variance                     =2.67
Standard Deviation           =1.63
Skewness                     =0.41
Kurtosis                     =-0.01

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 67 times
Press any key to continue . . .
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=210 The theretical number of impossible si
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -6.67 The maximum value in the x
xis              =20.00
The number of stems         =20
Mean                        =0.00 The theoretical mean=0.00
Variance                    =5.33
Standard Deviation          =2.31
Skewness                    =0.29
Kurtosis                    =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 232 times
Press any key to continue . . .
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=465 The theretical number of impossible si
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the
axis             =30.00
The number of stems         =30
Mean                        =0.00 The theoretical mean=0.00
Variance                    =8.00
Standard Deviation          =2.83
Skewness                    =0.24
Kurtosis                    =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
t
We have wrtiten to this file 497 times
Press any key to continue . . .
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=820 The theretical number of impossible sit
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -13.33 The maximum value in the x
axis             =40.00
The number of stems         =40
Mean                        =0.00 The theoretical mean=0.00
Variance                    =10.67
Standard Deviation          =3.27
Skewness                    =0.20
Kurtosis                    =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 862 times
Press any key to continue . . .
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -16.67 The maximum value in the x-
axis                 =50.00
The number of stems           =50
Mean                          =0.00 The theoretical mean=0.00
Variance                      =13.33
Standard Deviation            =3.65
Skewness                      =0.18
Kurtosis                      =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1327 times
Press any key to continue . . .
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -20.00 The maximum value in the x-
axis                 =60.00
The number of stems           =60
Mean                          =0.00 The theoretical mean=0.00
Variance                      =16.00
Standard Deviation            =4.00
Skewness                      =0.17
Kurtosis                      =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 1892 times
Press any key to continue . . .
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -23.33 The maximum value in the x-
axis                 =70.00
The number of stems           =70
Mean                          =0.00 The theoretical mean=0.00
Variance                      =18.67
Standard Deviation            =4.32
Skewness                      =0.15
Kurtosis                      =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.tx
t
We have wrtiten to this file 2557 times
Press any key to continue . . .
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=3240 The theretical number of impossible si
uations is=3240.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -26.67 The maximum value in the x
axis                =80.00
The number of stems          =80
Mean                         =0.00 The theoretical mean=0.00
Variance                     =21.33
Standard Deviation           =4.62
Skewness                     =0.14
Kurtosis                     =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 3322 times
Press any key to continue . . .
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=4095 The theretical number of impossible si
uations is=4095.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -30.00 The maximum value in the x
axis                =90.00
The number of stems          =90
Mean                         =0.00 The theoretical mean=0.00
Variance                     =24.00
Standard Deviation           =4.90
Skewness                     =0.14
Kurtosis                     =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 4187 times
Press any key to continue . . .
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-0.33]
The type of quiz is MCQ with 4 options X=3
The number of impossible situations=5050 The theretical number of impossible si
uations is=5050.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -33.33 The maximum value in the x
axis                =100.00
The number of stems          =100
Mean                         =0.00 The theoretical mean=0.00
Variance                     =26.67
Standard Deviation           =5.16
Skewness                     =0.13
Kurtosis                     =-0.00

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 5152 times
Press any key to continue . . .
```

Here, **n**=100

The sort of validation we can do is:

- The range of marks, here, should be, here, -33.33 to +100.
- The number of impossible situations should be, here, 5050 i.e. the sum of 0 + 1 + 2 +....99, 100.
- The mean mark should be 0.
- The theoretical should be the same as the actual for both the number of impossible situations and the mean.
- The name of the intermediate file should be **MixtureWithoutDecriptiveStats.txt**
- We should write to the intermediate file, here, (101 * 101 -5050) + 1 (for the header) times i.e. 5152.
- For 'small' **n** we calculate 'by hand'

14.6 3-option Multiple Yes/No quiz (X=7) and a=b=0

```
Mark=0.0000      numbner getting that mark that way=7 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1 The theretical number of impossible situa
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =1.00
The number of stems           =1
Mean                          =0.11 The theoretical mean=0.11
Variance                      =0.10
Standard Deviation            =0.31
Skewness                      =2.47
Kurtosis                      =4.13

The file we have written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
t
We have wrtiten to this file 4 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=0.0000      numbner getting that mark that way=49 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=14 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000      numbner getting that mark that way=14 n=2 w=0 x=1 y=1
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=3 The theretical number of impossible situa
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =2.00
The number of stems           =2
Mean                          =0.22 The theoretical mean=0.22
Variance                      =0.20
Standard Deviation            =0.44
Skewness                      =1.75
Kurtosis                      =2.06

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
xt
We have wrtiten to this file 7 times
Press any key to continue . . .
```

This agrees with a table in Appendix 8 where **n**=2

```
Mark=0.0000      numbner getting that mark that way=343 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=147 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=21 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000      numbner getting that mark that way=147 n=3 w=0 x=2 y=1
Mark=1.0000      numbner getting that mark that way=42 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000      numbner getting that mark that way=21 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is              =3.00
The number of stems          =3
Mean                         =0.33 The theoretical mean=0.33
Variance                     =0.30
Standard Deviation           =0.54
Skewness                     =1.43
Kurtosis                     =1.37
```

This agrees with a table in Appendix 8 where **n**=3

```
3
Mark=0.0000      numbner getting that mark that way=2401 n=4 w=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=1372 n=4 w=1 x=3 y=0
Mark=2.0000      numbner getting that mark that way=294 n=4 w=2 x=2 y=0
Mark=3.0000      numbner getting that mark that way=28 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1372 n=4 w=0 x=3 y=1
Mark=1.0000      numbner getting that mark that way=588 n=4 w=1 x=2 y=1
Mark=2.0000      numbner getting that mark that way=84 n=4 w=2 x=1 y=1
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000      numbner getting that mark that way=294 n=4 w=0 x=2 y=2
Mark=1.0000      numbner getting that mark that way=84 n=4 w=1 x=1 y=2
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000      numbner getting that mark that way=28 n=4 w=0 x=1 y=3
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=10 The theretical number of impossible situ
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-a
is              =4.00
The number of stems          =4
Mean                         =0.44 The theoretical mean=0.44
Variance                     =0.40
Standard Deviation           =0.63
Skewness                     =1.24
Kurtosis                     =1.03

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.
xt
We have wrtiten to this file 16 times
```

This (partly) agrees with a table in Appendix 8 where **n**=4

```
Mark=0.0000      numbner getting that mark that way=16807 n=5 w=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=12005 n=5 w=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=3430 n=5 w=2 x=3 y=0
Mark=3.0000      numbner getting that mark that way=490 n=5 w=3 x=2 y=0
Mark=4.0000      numbner getting that mark that way=35 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000      numbner getting that mark that way=12005 n=5 w=0 x=4 y=1
Mark=1.0000      numbner getting that mark that way=6860 n=5 w=1 x=3 y=1
Mark=2.0000      numbner getting that mark that way=1470 n=5 w=2 x=2 y=1
Mark=3.0000      numbner getting that mark that way=140 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000      numbner getting that mark that way=3430 n=5 w=0 x=3 y=2
Mark=1.0000      numbner getting that mark that way=1470 n=5 w=1 x=2 y=2
Mark=2.0000      numbner getting that mark that way=210 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000      numbner getting that mark that way=490 n=5 w=0 x=2 y=3
Mark=1.0000      numbner getting that mark that way=140 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000      numbner getting that mark that way=35 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =5.00
The number of stems            =5
Mean                           =0.56 The theoretical mean=0.56
Variance                       =0.49
Standard Deviation             =0.70
Skewness                       =1.11
Kurtosis                       =0.83

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
```

This (partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =10.00
The number of stems            =10
Mean                           =1.11 The theoretical mean=1.11
Variance                       =0.99
Standard Deviation             =0.99
Skewness                       =0.78
Kurtosis                       =0.41
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=210 The theretical number of impossible sit
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is              =20.00
The number of stems            =20
Mean                           =2.22 The theoretical mean=2.22
Variance                       =1.98
Standard Deviation             =1.41
Skewness                       =0.55
Kurtosis                       =0.21
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =30.00
The number of stems          =30
Mean                         =3.33 The theoretical mean=3.33
Variance                     =2.96
Standard Deviation           =1.72
Skewness                     =0.45
Kurtosis                     =0.14
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =40.00
The number of stems          =40
Mean                         =4.44 The theoretical mean=4.44
Variance                     =3.95
Standard Deviation           =1.99
Skewness                     =0.39
Kurtosis                     =0.10
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =50.00
The number of stems          =50
Mean                         =5.56 The theoretical mean=5.56
Variance                     =4.94
Standard Deviation           =2.22
Skewness                     =0.35
Kurtosis                     =0.08
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =60.00
The number of stems          =60
Mean                         =6.67 The theoretical mean=6.67
Variance                     =5.93
Standard Deviation           =2.43
Skewness                     =0.32
Kurtosis                     =0.07
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 0.00 The maximum value in the x-ax
is                =70.00
The number of stems          =70
Mean                         =7.78 The theoretical mean=7.78
Variance                     =6.91
Standard Deviation           =2.63
Skewness                     =0.30
Kurtosis                     =0.06
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =80.00
The number of stems           =80
Mean                          =8.89 The theoretical mean=8.89
Variance                      =7.90
Standard Deviation            =2.81
Skewness                      =0.28
Kurtosis                      =0.05
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =90.00
The number of stems           =90
Mean                          =10.00 The theoretical mean=10.00
Variance                      =8.89
Standard Deviation            =2.98
Skewness                      =0.26
Kurtosis                      =0.05
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =100.00
The number of stems           =100
Mean                          =11.11 The theoretical mean=11.11
Variance                      =9.88
Standard Deviation            =3.14
Skewness                      =0.25
Kurtosis                      =0.04
```

Here, **n**=100

14.7 3-option Multiple Yes/No (X=7) and a=-1/7 and b=0 i.e. a=-0.142857

The mean Mark should be 0, because the guessing problem is *exactly* tackled.

```
3
Mark=-0.1429      numbner getting that mark that way=7 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.     Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.14 The maximum value in the x-a
xis              =1.00
The number of stems          =1
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.13
Standard Deviation           =0.36
Skewness                     =2.41
Kurtosis                     =3.91
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=-0.2857      numbner getting that mark that way=49 n=2 w=0 x=2 y=0
Mark=0.8571       numbner getting that mark that way=14 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-0.1429      numbner getting that mark that way=14 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.     Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=3 The theretical number of impossible situa
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  -0.29 The maximum value in the x-
xis             =2.00
The number of stems          =2
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.25
Standard Deviation           =0.50
Skewness                     =1.70
Kurtosis                     =1.96
```

This agrees with a table in Appendix 8 where **n**=2

```
Mark=-0.4286      numbner getting that mark that way=343 n=3 w=0 x=3 y=0
Mark=0.7143       numbner getting that mark that way=147 n=3 w=1 x=2 y=0
Mark=1.8571       numbner getting that mark that way=21 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-0.2857      numbner getting that mark that way=147 n=3 w=0 x=2 y=1
Mark=0.8571       numbner getting that mark that way=42 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-0.1429      numbner getting that mark that way=21 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.43 The maximum value in the x-a
xis              =3.00
The number of stems           =3
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.38
Standard Deviation            =0.62
Skewness                      =1.39
Kurtosis                      =1.30
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=-0.5714      numbner getting that mark that way=2401 n=4 w=0 x=4 y=0
Mark=0.5714       numbner getting that mark that way=1372 n=4 w=1 x=3 y=0
Mark=1.7143       numbner getting that mark that way=294 n=4 w=2 x=2 y=0
Mark=2.8571       numbner getting that mark that way=28 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-0.4286      numbner getting that mark that way=1372 n=4 w=0 x=3 y=1
Mark=0.7143       numbner getting that mark that way=588 n=4 w=1 x=2 y=1
Mark=1.8571       numbner getting that mark that way=84 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-0.2857      numbner getting that mark that way=294 n=4 w=0 x=2 y=2
Mark=0.8571       numbner getting that mark that way=84 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-0.1429      numbner getting that mark that way=28 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.57 The maximum value in the x-a
xis              =4.00
The number of stems           =4
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.51
Standard Deviation            =0.71
Skewness                      =1.20
Kurtosis                      =0.98
```

This agrees with a table in Appendix 8 where **n**=4

```
Mark=-0.7143     numbner getting that mark that way=16807 n=5 w=0 x=5 y=0
Mark=0.4286      numbner getting that mark that way=12005 n=5 w=1 x=4 y=0
Mark=1.5714      numbner getting that mark that way=3430 n=5 w=2 x=3 y=0
Mark=2.7143      numbner getting that mark that way=490 n=5 w=3 x=2 y=0
Mark=3.8571      numbner getting that mark that way=35 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=-0.5714     numbner getting that mark that way=12005 n=5 w=0 x=4 y=1
Mark=0.5714      numbner getting that mark that way=6860 n=5 w=1 x=3 y=1
Mark=1.7143      numbner getting that mark that way=1470 n=5 w=2 x=2 y=1
Mark=2.8571      numbner getting that mark that way=140 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=-0.4286     numbner getting that mark that way=3430 n=5 w=0 x=3 y=2
Mark=0.7143      numbner getting that mark that way=1470 n=5 w=1 x=2 y=2
Mark=1.8571      numbner getting that mark that way=210 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=-0.2857     numbner getting that mark that way=490 n=5 w=0 x=2 y=3
Mark=0.8571      numbner getting that mark that way=140 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=-0.1429     numbner getting that mark that way=35 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=15 The theretical number of impossible situ
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.71 The maximum value in the x-
xis             =5.00
The number of stems             =5
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.63
Standard Deviation              =0.80
Skewness                        =1.08
Kurtosis                        =0.78
```

This agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -1.43 The maximum value in the x-a
xis             =10.00
The number of stems             =10
Mean                            =0.00 The theoretical mean=0.00
Variance                        =1.27
Standard Deviation              =1.13
Skewness                        =0.76
Kurtosis                        =0.39
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -2.86 The maximum value in the x-a
xis             =20.00
The number of stems             =20
Mean                            =0.00 The theoretical mean=0.00
Variance                        =2.54
Standard Deviation              =1.59
Skewness                        =0.54
Kurtosis                        =0.20
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.29 The maximum value in the x-a
xis                 =30.00
The number of stems           =30
Mean                          =0.00 The theoretical mean=0.00
Variance                      =3.81
Standard Deviation            =1.95
Skewness                      =0.44
Kurtosis                      =0.13
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -5.71 The maximum value in the x-a
xis                 =40.00
The number of stems           =40
Mean                          =0.00 The theoretical mean=0.00
Variance                      =5.08
Standard Deviation            =2.25
Skewness                      =0.38
Kurtosis                      =0.10
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1275 The theretical number of impossible si
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -7.14 The maximum value in the x-
xis                 =50.00
The number of stems           =50
Mean                          =0.00 The theoretical mean=0.00
Variance                      =6.35
Standard Deviation            =2.52
Skewness                      =0.34
Kurtosis                      =0.08
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=1830 The theretical number of impossible si
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -8.57 The maximum value in the x-
xis                 =60.00
The number of stems           =60
Mean                          =0.00 The theoretical mean=0.00
Variance                      =7.62
Standard Deviation            =2.76
Skewness                      =0.31
Kurtosis                      =0.07
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the x-
axis                =70.00
The number of stems           =70
Mean                          =0.00 The theoretical mean=0.00
Variance                      =8.89
Standard Deviation            =2.98
Skewness                      =0.29
Kurtosis                      =0.06
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -11.43 The maximum value in the x-
axis                =80.00
The number of stems              =80
Mean                             =0.00 The theoretical mean=0.00
Variance                         =10.16
Standard Deviation               =3.19
Skewness                         =0.27
Kurtosis                         =0.05
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -12.86 The maximum value in the x-
axis                =90.00
The number of stems              =90
Mean                             =0.00 The theoretical mean=0.00
Variance                         =11.43
Standard Deviation               =3.38
Skewness                         =0.25
Kurtosis                         =0.04
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-0.14]
The type of quiz is Multiple Yes/No with 3 options X=7
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -14.29 The maximum value in the x-
axis                =100.00
The number of stems              =100
Mean                             =0.00 The theoretical mean=0.00
Variance                         =12.70
Standard Deviation               =3.56
Skewness                         =0.24
Kurtosis                         =0.04
```

Here, **n**=100

14.8 5-option MAQ with 2 ticks (X=9) and a=b=0

```
Mark=0.0000      numbner getting that mark that way=9 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    0.00 The maximum value in the x-ax
is              =1.00
The number of stems              =1
Mean                             =0.09 The theoretical mean=0.09
Variance                         =0.08
Standard Deviation               =0.29
Skewness                         =2.85
Kurtosis                         =6.10

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 4 times
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=0.0000      numbner getting that mark that way=81 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=18 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000      numbner getting that mark that way=18 n=2 w=0 x=1 y=1
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is             =2.00
The number of stems          =2
Mean                         =0.18 The theoretical mean=0.18
Variance                     =0.17
Standard Deviation           =0.41
Skewness                     =2.01
Kurtosis                     =3.05
```

This agrees with a table in Appendix 8 where **n**=2

```
Mark=0.0000      numbner getting that mark that way=729 n=3 w=0 x=3 y=0
Mark=1.0000      numbner getting that mark that way=243 n=3 w=1 x=2 y=0
Mark=2.0000      numbner getting that mark that way=27 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000      numbner getting that mark that way=243 n=3 w=0 x=2 y=1
Mark=1.0000      numbner getting that mark that way=54 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000      numbner getting that mark that way=27 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is             =3.00
The number of stems          =3
Mean                         =0.27 The theoretical mean=0.27
Variance                     =0.25
Standard Deviation           =0.50
Skewness                     =1.64
Kurtosis                     =2.03

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 11 times
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=0.0000      numbner getting that mark that way=6561 n=4 w=0 x=4 y=0
Mark=1.0000      numbner getting that mark that way=2916 n=4 w=1 x=3 y=0
Mark=2.0000      numbner getting that mark that way=486 n=4 w=2 x=2 y=0
Mark=3.0000      numbner getting that mark that way=36 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000      numbner getting that mark that way=2916 n=4 w=0 x=3 y=1
Mark=1.0000      numbner getting that mark that way=972 n=4 w=1 x=2 y=1
Mark=2.0000      numbner getting that mark that way=108 n=4 w=2 x=1 y=1
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000      numbner getting that mark that way=486 n=4 w=0 x=2 y=2
Mark=1.0000      numbner getting that mark that way=108 n=4 w=1 x=1 y=2
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000      numbner getting that mark that way=36 n=4 w=0 x=1 y=3
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =4.00
The number of stems          =4
Mean                         =0.36 The theoretical mean=0.36
Variance                     =0.33
Standard Deviation           =0.57
Skewness                     =1.42
Kurtosis                     =1.53

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 16 times
```

This (partly) agrees with a table in Appendix 8 where **n**=4

```
Mark=0.0000      numbner getting that mark that way=59049 n=5 w=0 x=5 y=0
Mark=1.0000      numbner getting that mark that way=32805 n=5 w=1 x=4 y=0
Mark=2.0000      numbner getting that mark that way=7290 n=5 w=2 x=3 y=0
Mark=3.0000      numbner getting that mark that way=810 n=5 w=3 x=2 y=0
Mark=4.0000      numbner getting that mark that way=45 n=5 w=4 x=1 y=0
Mark=5.0000      numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000      numbner getting that mark that way=32805 n=5 w=0 x=4 y=1
Mark=1.0000      numbner getting that mark that way=14580 n=5 w=1 x=3 y=1
Mark=2.0000      numbner getting that mark that way=2430 n=5 w=2 x=2 y=1
Mark=3.0000      numbner getting that mark that way=180 n=5 w=3 x=1 y=1
Mark=4.0000      numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000      numbner getting that mark that way=7290 n=5 w=0 x=3 y=2
Mark=1.0000      numbner getting that mark that way=2430 n=5 w=1 x=2 y=2
Mark=2.0000      numbner getting that mark that way=270 n=5 w=2 x=1 y=2
Mark=3.0000      numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000      numbner getting that mark that way=810 n=5 w=0 x=2 y=3
Mark=1.0000      numbner getting that mark that way=180 n=5 w=1 x=1 y=3
Mark=2.0000      numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000      numbner getting that mark that way=45 n=5 w=0 x=1 y=4
Mark=1.0000      numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000      numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =5.00
The number of stems          =5
Mean                         =0.45 The theoretical mean=0.45
Variance                     =0.41
Standard Deviation           =0.64
Skewness                     =1.27
Kurtosis                     =1.22

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 22 times
```

This (partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =10.00
The number of stems            =10
Mean                           =0.91 The theoretical mean=0.91
Variance                       =0.83
Standard Deviation             =0.91
Skewness                       =0.90
Kurtosis                       =0.61
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =20.00
The number of stems            =20
Mean                           =1.82 The theoretical mean=1.82
Variance                       =1.65
Standard Deviation             =1.29
Skewness                       =0.64
Kurtosis                       =0.30
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =30.00
The number of stems            =30
Mean                           =2.73 The theoretical mean=2.73
Variance                       =2.48
Standard Deviation             =1.57
Skewness                       =0.52
Kurtosis                       =0.20
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=820 The theretical number of impossible sit
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is                =40.00
The number of stems            =40
Mean                           =3.64 The theoretical mean=3.64
Variance                       =3.31
Standard Deviation             =1.82
Skewness                       =0.45
Kurtosis                       =0.15
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =50.00
The number of stems            =50
Mean                           =4.55 The theoretical mean=4.55
Variance                       =4.13
Standard Deviation             =2.03
Skewness                       =0.40
Kurtosis                       =0.12
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =60.00
The number of stems          =60
Mean                         =5.45 The theoretical mean=5.45
Variance                     =4.96
Standard Deviation           =2.23
Skewness                     =0.37
Kurtosis                     =0.10
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =70.00
The number of stems          =70
Mean                         =6.36 The theoretical mean=6.36
Variance                     =5.79
Standard Deviation           =2.41
Skewness                     =0.34
Kurtosis                     =0.09
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =80.00
The number of stems          =80
Mean                         =7.27 The theoretical mean=7.27
Variance                     =6.61
Standard Deviation           =2.57
Skewness                     =0.32
Kurtosis                     =0.08
```

Here, **n**=80

```
tems can be missed.    Score is [0.00], [1.00,0.00]
he type of quiz is MAQ with 5 options and 2 ticks X=9
he number of impossible situations=4095 The theretical number of impossible sit
ations is=4095.00

              Some DESCRIPTIVE Statistics

he minimum value on the x-axis                0.00 The maximum value in the x-ax
s              =90.00
he number of stems          =90
ean                         =8.18 The theoretical mean=8.18
ariance                     =7.44
tandard Deviation           =2.73
kewness                     =0.30
urtosis                     =0.07
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00
              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =100.00
The number of stems          =100
Mean                         =9.09 The theoretical mean=9.09
Variance                     =8.26
Standard Deviation           =2.87
Skewness                     =0.28
Kurtosis                     =0.06
```

Here, **n**=100

So, for **a**=**b**=0 for 5-option MAQs with 2 ticks tool1 works

14.9 5-option MAQ with 2 ticks (X=9) and a=-1/9, b=0

The mean Mark should be 0, because the guessing problem is *exactly* tackled.

```
Mark=-0.1111      numbner getting that mark that way=9 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.11 The maximum value in the x-a
xis              =1.00
The number of stems           =1
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.10
Standard Deviation            =0.32
Skewness                      =2.80
Kurtosis                      =5.92

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 4 times
```

This agrees with a table in Appendix 8 where **n**=1.

```
2
Mark=-0.2222      numbner getting that mark that way=81 n=2 w=0 x=2 y=0
Mark=0.8889       numbner getting that mark that way=18 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-0.1111      numbner getting that mark that way=18 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.22 The maximum value in the x-a
xis              =2.00
The number of stems           =2
Mean                          =0.00 The theoretical mean=0.00
Variance                      =0.20
Standard Deviation            =0.45
Skewness                      =1.98
Kurtosis                      =2.96

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 7 times
```

This agrees with a table in Appendix 8 where **n**=2.

```
Mark=-0.3333     numbner getting that mark that way=729 n=3 w=0 x=3 y=0
Mark=0.7778      numbner getting that mark that way=243 n=3 w=1 x=2 y=0
Mark=1.8889      numbner getting that mark that way=27 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-0.2222     numbner getting that mark that way=243 n=3 w=0 x=2 y=1
Mark=0.8889      numbner getting that mark that way=54 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-0.1111     numbner getting that mark that way=27 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.33 The maximum value in the x-a
xis             =3.00
The number of stems         =3
Mean                        =0.00 The theoretical mean=0.00
Variance                    =0.30
Standard Deviation          =0.55
Skewness                    =1.61
Kurtosis                    =1.97
```

This (partly) agrees with a table in Appendix 8 where **n**=3.

```
Mark=-0.4444     numbner getting that mark that way=6561 n=4 w=0 x=4 y=0
Mark=0.6667      numbner getting that mark that way=2916 n=4 w=1 x=3 y=0
Mark=1.7778      numbner getting that mark that way=486 n=4 w=2 x=2 y=0
Mark=2.8889      numbner getting that mark that way=36 n=4 w=3 x=1 y=0
Mark=4.0000      numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-0.3333     numbner getting that mark that way=2916 n=4 w=0 x=3 y=1
Mark=0.7778      numbner getting that mark that way=972 n=4 w=1 x=2 y=1
Mark=1.8889      numbner getting that mark that way=108 n=4 w=2 x=1 y=1
Mark=3.0000      numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-0.2222     numbner getting that mark that way=486 n=4 w=0 x=2 y=2
Mark=0.8889      numbner getting that mark that way=108 n=4 w=1 x=1 y=2
Mark=2.0000      numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-0.1111     numbner getting that mark that way=36 n=4 w=0 x=1 y=3
Mark=1.0000      numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000      numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.44 The maximum value in the x-a
xis             =4.00
The number of stems         =4
Mean                        =0.00 The theoretical mean=0.00
Variance                    =0.40
Standard Deviation          =0.64
Skewness                    =1.40
Kurtosis                    =1.48
```

This (partly) agrees with a table in Appendix 8 where **n**=4

```
z
Mark=-0.5555      numbner getting that mark that way=59049 n=5 w=0 x=5 y=0
Mark=0.5556       numbner getting that mark that way=32805 n=5 w=1 x=4 y=0
Mark=1.6667       numbner getting that mark that way=7290 n=5 w=2 x=3 y=0
Mark=2.7778       numbner getting that mark that way=810 n=5 w=3 x=2 y=0
Mark=3.8889       numbner getting that mark that way=45 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=-0.4444      numbner getting that mark that way=32805 n=5 w=0 x=4 y=1
Mark=0.6667       numbner getting that mark that way=14580 n=5 w=1 x=3 y=1
Mark=1.7778       numbner getting that mark that way=2430 n=5 w=2 x=2 y=1
Mark=2.8889       numbner getting that mark that way=180 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=-0.3333      numbner getting that mark that way=7290 n=5 w=0 x=3 y=2
Mark=0.7778       numbner getting that mark that way=2430 n=5 w=1 x=2 y=2
Mark=1.8889       numbner getting that mark that way=270 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=-0.2222      numbner getting that mark that way=810 n=5 w=0 x=2 y=3
Mark=0.8889       numbner getting that mark that way=180 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=-0.1111      numbner getting that mark that way=45 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.56 The maximum value in the x-a
xis             =5.00
The number of stems             =5
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.51
Standard Deviation              =0.71
Skewness                        =1.25
Kurtosis                        =1.18

The file we havec written to is F:\MyPhD2\MyPhD\MixtureWithoutDescriptiveStats.t
xt
We have wrtiten to this file 22 times
```

This(partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -1.11 The maximum value in the x-a
xis             =10.00
The number of stems             =10
Mean                            =0.00 The theoretical mean=0.00
Variance                        =1.01
Standard Deviation              =1.01
Skewness                        =0.88
Kurtosis                        =0.59
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -2.22 The maximum value in the x-a
xis             =20.00
The number of stems             =20
Mean                            =0.00 The theoretical mean=0.00
Variance                        =2.02
Standard Deviation              =1.42
Skewness                        =0.63
Kurtosis                        =0.30
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -3.33 The maximum value in the x-a
xis              =30.00
The number of stems           =30
Mean                          =0.00 The theoretical mean=0.00
Variance                      =3.03
Standard Deviation            =1.74
Skewness                      =0.51
Kurtosis                      =0.20
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -4.44 The maximum value in the x-a
xis              =40.00
The number of stems           =40
Mean                          =0.00 The theoretical mean=0.00
Variance                      =4.04
Standard Deviation            =2.01
Skewness                      =0.44
Kurtosis                      =0.15
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -5.56 The maximum value in the x-
xis              =50.00
The number of stems           =50
Mean                          =0.00 The theoretical mean=0.00
Variance                      =5.05
Standard Deviation            =2.25
Skewness                      =0.40
Kurtosis                      =0.12
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -6.67 The maximum value in the x-a
xis              =60.00
The number of stems           =60
Mean                          =0.00 The theoretical mean=0.00
Variance                      =6.06
Standard Deviation            =2.46
Skewness                      =0.36
Kurtosis                      =0.10
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -7.78 The maximum value in the x-a
xis              =70.00
The number of stems           =70
Mean                          =0.00 The theoretical mean=0.00
Variance                      =7.07
Standard Deviation            =2.66
Skewness                      =0.33
Kurtosis                      =0.08
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -8.89 The maximum value in the x-a
xis              =80.00
The number of stems          =80
Mean                         =0.00 The theoretical mean=0.00
Variance                     =8.08
Standard Deviation           =2.84
Skewness                     =0.31
Kurtosis                     =0.07
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -10.00 The maximum value in the x-
axis              =90.00
The number of stems          =90
Mean                         =0.00 The theoretical mean=0.00
Variance                     =9.09
Standard Deviation           =3.02
Skewness                     =0.29
Kurtosis                     =0.07
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-0.11]
The type of quiz is MAQ with 5 options and 2 ticks X=9
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -11.11 The maximum value in the x-
axis              =100.00
The number of stems          =100
Mean                         =0.00 The theoretical mean=0.00
Variance                     =10.10
Standard Deviation           =3.18
Skewness                     =0.28
Kurtosis                     =0.06
```

Here, **n**=100

So, for 5-option MAQs (with 2 ticks) where X=9, *tool1* works for both when **a**=0 and when **a**=-1/9. In both cases **b**=0.

So, *tool1* works for X=1, X=3, X=7 and X=9 for quizzes where stems can be missed.

14.10 20-option EMSQ (MCQ) (X=19) and a=b=0

```
Mark=0.0000       numbner getting that mark that way=19 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is            =1.00
The number of stems           =1
Mean                          =0.05 The theoretical mean=0.05
Variance                      =0.05
Standard Deviation            =0.21
Skewness                      =4.25
Kurtosis                      =16.05
```

This agrees with a table in Appendix 8 (to 2 decimal places) where **n**=1.

```
Mark=0.0000       numbner getting that mark that way=361 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=38 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000       numbner getting that mark that way=38 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is            =2.00
The number of stems           =2
Mean                          =0.10 The theoretical mean=0.10
Variance                      =0.09
Standard Deviation            =0.30
Skewness                      =3.00
Kurtosis                      =8.03
```

This agrees with a table in Appendix 8 (to 2 decimal places) where **n**=2

```
Mark=0.0000       numbner getting that mark that way=6859 n=3 w=0 x=3 y=0
Mark=1.0000       numbner getting that mark that way=1083 n=3 w=1 x=2 y=0
Mark=2.0000       numbner getting that mark that way=57 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1083 n=3 w=0 x=2 y=1
Mark=1.0000       numbner getting that mark that way=114 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000       numbner getting that mark that way=57 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=6 The theretical number of impossible situa
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is            =3.00
The number of stems           =3
Mean                          =0.14 The theoretical mean=0.14
Variance                      =0.14
Standard Deviation            =0.37
Skewness                      =2.45
Kurtosis                      =5.35
```

This (partly) agrees with a table in Appendix 8 where **n**=3

```
Mark=0.0000       numbner getting that mark that way=130321 n=4 w=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=27436 n=4 w=1 x=3 y=0
Mark=2.0000       numbner getting that mark that way=2166 n=4 w=2 x=2 y=0
Mark=3.0000       numbner getting that mark that way=76 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000       numbner getting that mark that way=27436 n=4 w=0 x=3 y=1
Mark=1.0000       numbner getting that mark that way=4332 n=4 w=1 x=2 y=1
Mark=2.0000       numbner getting that mark that way=228 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000       numbner getting that mark that way=2166 n=4 w=0 x=2 y=2
Mark=1.0000       numbner getting that mark that way=228 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000       numbner getting that mark that way=76 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is               =4.00
The number of stems          =4
Mean                         =0.19 The theoretical mean=0.19
Variance                     =0.18
Standard Deviation           =0.43
Skewness                     =2.12
Kurtosis                     =4.01
```

This (partly) agrees with a table in Appendix 8 where **n**=4

```
Mark=0.0000       numbner getting that mark that way=2476099 n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=651605 n=5 w=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=68590 n=5 w=2 x=3 y=0
Mark=3.0000       numbner getting that mark that way=3610 n=5 w=3 x=2 y=0
Mark=4.0000       numbner getting that mark that way=95 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000       numbner getting that mark that way=651605 n=5 w=0 x=4 y=1
Mark=1.0000       numbner getting that mark that way=137180 n=5 w=1 x=3 y=1
Mark=2.0000       numbner getting that mark that way=10830 n=5 w=2 x=2 y=1
Mark=3.0000       numbner getting that mark that way=380 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000       numbner getting that mark that way=68590 n=5 w=0 x=3 y=2
Mark=1.0000       numbner getting that mark that way=10830 n=5 w=1 x=2 y=2
Mark=2.0000       numbner getting that mark that way=570 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000       numbner getting that mark that way=3610 n=5 w=0 x=2 y=3
Mark=1.0000       numbner getting that mark that way=380 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000       numbner getting that mark that way=95 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               0.00 The maximum value in the x-ax
is               =5.00
The number of stems          =5
Mean                         =0.24 The theoretical mean=0.24
Variance                     =0.23
Standard Deviation           =0.48
Skewness                     =1.90
Kurtosis                     =3.21
```

This (partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =10.00
The number of stems          =10
Mean                         =0.48 The theoretical mean=0.48
Variance                     =0.45
Standard Deviation           =0.67
Skewness                     =1.34
Kurtosis                     =1.60
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =20.00
The number of stems          =20
Mean                         =0.95 The theoretical mean=0.95
Variance                     =0.91
Standard Deviation           =0.95
Skewness                     =0.95
Kurtosis                     =0.80
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =30.00
The number of stems          =30
Mean                         =1.43 The theoretical mean=1.43
Variance                     =1.36
Standard Deviation           =1.17
Skewness                     =0.78
Kurtosis                     =0.54
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =40.00
The number of stems          =40
Mean                         =1.90 The theoretical mean=1.90
Variance                     =1.81
Standard Deviation           =1.35
Skewness                     =0.67
Kurtosis                     =0.40
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1275 The theretical number of impossible si
uations is=1275.00

             Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-a
is                =50.00
The number of stems          =50
Mean                         =2.38 The theoretical mean=2.38
Variance                     =2.27
Standard Deviation           =1.51
Skewness                     =0.60
Kurtosis                     =0.32
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =60.00
The number of stems           =60
Mean                          =2.86 The theoretical mean=2.86
Variance                      =2.72
Standard Deviation            =1.65
Skewness                      =0.55
Kurtosis                      =0.27
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =70.00
The number of stems           =70
Mean                          =3.33 The theoretical mean=3.33
Variance                      =3.17
Standard Deviation            =1.78
Skewness                      =0.51
Kurtosis                      =0.23
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =80.00
The number of stems           =80
Mean                          =3.81 The theoretical mean=3.81
Variance                      =3.63
Standard Deviation            =1.90
Skewness                      =0.48
Kurtosis                      =0.20
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =90.00
The number of stems           =90
Mean                          =4.29 The theoretical mean=4.29
Variance                      =4.08
Standard Deviation            =2.02
Skewness                      =0.45
Kurtosis                      =0.18
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                =100.00
The number of stems           =100
Mean                          =4.76 The theoretical mean=4.76
Variance                      =4.54
Standard Deviation            =2.13
Skewness                      =0.42
Kurtosis                      =0.16
```

Here, **n**=100

14.11 20-option EMSQ (MCQ) (X=19) and a=-1/19, b=0

The mean Mark should be 0, because the guessing problem is *exactly* tackled.

```
Mark=-0.0526      numbner getting that mark that way=19 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.05 The maximum value in the x-a
xis              =1.00
The number of stems           =1
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.05
Standard Deviation            =0.22
Skewness                      =4.23
Kurtosis                      =15.96
```

This agrees with a table in Appendix 8 where **n**=1

```
1
Mark=-0.1053      numbner getting that mark that way=361 n=2 w=0 x=2 y=0
Mark=0.9474       numbner getting that mark that way=38 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-0.0526      numbner getting that mark that way=38 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=3 The theretical number of impossible situa
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.11 The maximum value in the x-
xis              =2.00
The number of stems           =2
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.10
Standard Deviation            =0.32
Skewness                      =2.99
Kurtosis                      =7.98
```

This agrees with a table in Appendix 8 where **n**=2

```
Mark=-0.1579      numbner getting that mark that way=6859 n=3 w=0 x=3 y=0
Mark=0.8947       numbner getting that mark that way=1083 n=3 w=1 x=2 y=0
Mark=1.9474       numbner getting that mark that way=57 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-0.1053      numbner getting that mark that way=1083 n=3 w=0 x=2 y=1
Mark=0.9474       numbner getting that mark that way=114 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-0.0526      numbner getting that mark that way=57 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.16 The maximum value in the x-a
xis              =3.00
The number of stems           =3
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.15
Standard Deviation            =0.39
Skewness                      =2.44
Kurtosis                      =5.32
```

This (partly) agrees with a table in Appendix 8 where **n**=3

```
Mark=-0.2105      numbner getting that mark that way=130321 n=4 w=0 x=4 y=0
Mark=0.8421       numbner getting that mark that way=27436 n=4 w=1 x=3 y=0
Mark=1.8947       numbner getting that mark that way=2166 n=4 w=2 x=2 y=0
Mark=2.9474       numbner getting that mark that way=76 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-0.1579      numbner getting that mark that way=27436 n=4 w=0 x=3 y=1
Mark=0.8947       numbner getting that mark that way=4332 n=4 w=1 x=2 y=1
Mark=1.9474       numbner getting that mark that way=228 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-0.1053      numbner getting that mark that way=2166 n=4 w=0 x=2 y=2
Mark=0.9474       numbner getting that mark that way=228 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-0.0526      numbner getting that mark that way=76 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.21 The maximum value in the x-a
xis              =4.00
The number of stems           =4
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.20
Standard Deviation            =0.45
Skewness                      =2.12
Kurtosis                      =3.99
```

This (partly) agrees with a table in Appendix 8 where **n**=4

```
Mark=-0.2632      numbner getting that mark that way=2476099 n=5 w=0 x=5 y=0
Mark=0.7895       numbner getting that mark that way=651605 n=5 w=1 x=4 y=0
Mark=1.8421       numbner getting that mark that way=68590 n=5 w=2 x=3 y=0
Mark=2.8947       numbner getting that mark that way=3610 n=5 w=3 x=2 y=0
Mark=3.9474       numbner getting that mark that way=95 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=-0.2105      numbner getting that mark that way=651605 n=5 w=0 x=4 y=1
Mark=0.8421       numbner getting that mark that way=137180 n=5 w=1 x=3 y=1
Mark=1.8947       numbner getting that mark that way=10830 n=5 w=2 x=2 y=1
Mark=2.9474       numbner getting that mark that way=380 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=-0.1579      numbner getting that mark that way=68590 n=5 w=0 x=3 y=2
Mark=0.8947       numbner getting that mark that way=10830 n=5 w=1 x=2 y=2
Mark=1.9474       numbner getting that mark that way=570 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=-0.1053      numbner getting that mark that way=3610 n=5 w=0 x=2 y=3
Mark=0.9474       numbner getting that mark that way=380 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=-0.0526      numbner getting that mark that way=95 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.26 The maximum value in the x-a
xis              =5.00
The number of stems           =5
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.25
Standard Deviation            =0.50
Skewness                      =1.89
Kurtosis                      =3.19
```

This (partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.53 The maximum value in the x-a
xis                 =10.00
The number of stems           =10
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.50
Standard Deviation            =0.71
Skewness                      =1.34
Kurtosis                      =1.60
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -1.05 The maximum value in the x-a
xis                 =20.00
The number of stems           =20
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =1.00
Standard Deviation            =1.00
Skewness                      =0.95
Kurtosis                      =0.80
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -1.58 The maximum value in the x-a
xis                 =30.00
The number of stems           =30
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =1.50
Standard Deviation            =1.23
Skewness                      =0.77
Kurtosis                      =0.53
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -2.11 The maximum value in the x-a
xis                 =40.00
The number of stems           =40
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =2.01
Standard Deviation            =1.42
Skewness                      =0.67
Kurtosis                      =0.40
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -2.63 The maximum value in the x-a
xis                 =50.00
The number of stems           =50
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =2.51
Standard Deviation            =1.58
Skewness                      =0.60
Kurtosis                      =0.32
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -3.16 The maximum value in the x-a
xis             =60.00
The number of stems            =60
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =3.01
Standard Deviation             =1.73
Skewness                       =0.55
Kurtosis                       =0.27
```

Here, **n**=60

```
Stems can be missed.    Score is [0.00], [1.00,-0.95]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=2485 The theretical number of impossible sit
uations is=2485.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -66.68 The maximum value in the x-
axis             =70.00
The number of stems            =70
Mean                           =-57.00 The theoretical mean=-57.00
Variance                       =14.39
Standard Deviation             =3.79
Skewness                       =0.39
Kurtosis                       =0.14
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=3240 The theretical number of impossible si
uations is=3240.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -4.21 The maximum value in the x-
xis             =80.00
The number of stems            =80
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =4.01
Standard Deviation             =2.00
Skewness                       =0.47
Kurtosis                       =0.20
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -4.74 The maximum value in the x-a
xis             =90.00
The number of stems            =90
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =4.51
Standard Deviation             =2.12
Skewness                       =0.45
Kurtosis                       =0.18
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-0.05]
The type of quiz is MCQ with 20 options X=19
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                    -5.26 The maximum value in the x-a
xis             =100.00
The number of stems            =100
Mean                           =-0.00 The theoretical mean=-0.00
Variance                       =5.01
Standard Deviation             =2.24
Skewness                       =0.42
Kurtosis                       =0.16
```

Here, **n**=100

14.12 20-option EMSQ (MAQ with 10 ticks) (X=184755) and a=b=0

```
Mark=0.0000      numbner getting that mark that way=184755 n=1 w=0 x=1 y=0
Mark=1.0000      numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000      numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =1.00
The number of stems          =1
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.00
Standard Deviation           =0.00
Skewness                     =429.83
Kurtosis                     =184752.00
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=0.0000      numbner getting that mark that way=361 n=2 w=0 x=2 y=0
Mark=1.0000      numbner getting that mark that way=38 n=2 w=1 x=1 y=0
Mark=2.0000      numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=0.0000      numbner getting that mark that way=38 n=2 w=0 x=1 y=1
Mark=1.0000      numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000      numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 19 ticks X=19
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =2.00
The number of stems          =2
Mean                         =0.10 The theoretical mean=0.10
Variance                     =0.09
Standard Deviation           =0.30
Skewness                     =3.00
Kurtosis                     =8.03
```

This (partly) agrees with a table in Appendix 8 where **n**=2

```
Mark=0.0000      numbner getting that mark that way=6306502924168875 n=3 w=0 x=3
 y=0
Mark=1.0000      numbner getting that mark that way=102403230075 n=3 w=1 x=2 y=0

Mark=2.0000      numbner getting that mark that way=554265 n=3 w=2 x=1 y=0
Mark=3.0000      numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=0.0000      numbner getting that mark that way=102403230075 n=3 w=0 x=2 y=1

Mark=1.0000      numbner getting that mark that way=1108530 n=3 w=1 x=1 y=1
Mark=2.0000      numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=0.0000      numbner getting that mark that way=554265 n=3 w=0 x=1 y=2
Mark=1.0000      numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000      numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is              =3.00
The number of stems          =3
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.00
Standard Deviation           =0.00
Skewness                     =248.16
Kurtosis                     =61584.00
```

This (partly) agrees with a table in Appendix 8 where **n**=3

```
Mark=0.0000       numbner getting that mark that way=1165157947754820500625 n=4 w
=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=25226011696675500 n=4 w=1 x=
3 y=0
Mark=2.0000       numbner getting that mark that way=204806460150 n=4 w=2 x=2 y=0

Mark=3.0000       numbner getting that mark that way=739020 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=0.0000       numbner getting that mark that way=25226011696675500 n=4 w=0 x=
3 y=1
Mark=1.0000       numbner getting that mark that way=409612920300 n=4 w=1 x=2 y=1

Mark=2.0000       numbner getting that mark that way=2217060 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=0.0000       numbner getting that mark that way=204806460150 n=4 w=0 x=2 y=2

Mark=1.0000       numbner getting that mark that way=2217060 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=0.0000       numbner getting that mark that way=739020 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-ax
is              =4.00
The number of stems          =4
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.00
Standard Deviation           =0.00
Skewness                     =214.92
Kurtosis                     =46188.00
```

This(partly) agrees with a table in Appendix 8 where **n**=4

```
Mark=0.0000       numbner getting that mark that way=2152687566374418615929718750
n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=5825789738774102503125 n=5
=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=63065029241688750 n=5 w=2 x
3 y=0
Mark=3.0000       numbner getting that mark that way=341344100250 n=5 w=3 x=2 y=

Mark=4.0000       numbner getting that mark that way=923775 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=0.0000       numbner getting that mark that way=5825789738774102503125 n=5
=0 x=4 y=1
Mark=1.0000       numbner getting that mark that way=126130058483377500 n=5 w=1
=3 y=1
Mark=2.0000       numbner getting that mark that way=1024032300750 n=5 w=2 x=2 y
1
Mark=3.0000       numbner getting that mark that way=3695100 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=0.0000       numbner getting that mark that way=63065029241688750 n=5 w=0 x
3 y=2
Mark=1.0000       numbner getting that mark that way=1024032300750 n=5 w=1 x=2 y
2
Mark=2.0000       numbner getting that mark that way=5542650 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=0.0000       numbner getting that mark that way=341344100250 n=5 w=0 x=2 y=

Mark=1.0000       numbner getting that mark that way=3695100 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=0.0000       numbner getting that mark that way=923775 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=15 The theretical number of impossible situ
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis              0.00 The maximum value in the x-a
is              =5.00
The number of stems          =5
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =192.23
Kurtosis                     =36950.40
```

This (partly) agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=55 The theretical number of impossible sit
tions is=55.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-
is                 =10.00
The number of stems             =10
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =135.92
Kurtosis                        =18475.20
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=210 The theretical number of impossible sit
ations is=210.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-a
is                 =20.00
The number of stems             =20
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =96.11
Kurtosis                        =9237.60
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=465 The theretical number of impossible sit
ations is=465.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-a
is                 =30.00
The number of stems             =30
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =78.48
Kurtosis                        =6158.40
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                 =40.00
The number of stems             =40
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.01
Skewness                        =67.96
Kurtosis                        =4618.80
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                  0.00 The maximum value in the x-ax
is                 =50.00
The number of stems             =50
Mean                            =0.00 The theoretical mean=0.00
Variance                        =0.00
Standard Deviation              =0.02
Skewness                        =60.79
Kurtosis                        =3695.04
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =60.00
The number of stems            =60
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =55.49
Kurtosis                       =3079.20
```

Here, **n**=60

```
tems can be missed.    Score is [0.00], [1.00,0.00]
he type of quiz is MAQ with 20 options and 10 ticks X=184755
he number of impossible situations=2485 The theretical number of impossible sit
ations is=2485.00

            Some DESCRIPTIVE Statistics

he minimum value on the x-axis                0.00 The maximum value in the x-ax
s                =70.00
he number of stems            =70
ean                           =0.00 The theoretical mean=0.00
ariance                       =0.00
tandard Deviation             =0.02
kewness                       =51.37
urtosis                       =2639.31
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =80.00
The number of stems            =80
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =48.06
Kurtosis                       =2309.40
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =90.00
The number of stems            =90
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =45.31
Kurtosis                       =2052.80
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                0.00 The maximum value in the x-ax
is                =100.00
The number of stems            =100
Mean                           =0.00 The theoretical mean=0.00
Variance                       =0.00
Standard Deviation             =0.02
Skewness                       =42.98
Kurtosis                       =1847.52
```

Here, **n**=100

14.13 20-option EMSQ (MAQ with 10 ticks) (X=184755) and a=-1/184755, b=0

The mean mark should be 0, because the guessing problem is *exactly* tackled.

```
Mark=-0.0000      numbner getting that mark that way=184755 n=1 w=0 x=1 y=0
Mark=1.0000       numbner getting that mark that way=1 n=1 w=1 x=0 y=0
Mark=0.0000       numbner getting that mark that way=1 n=1 w=0 x=0 y=1


Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1 The theretical number of impossible situat
ions is=1.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis             =1.00
The number of stems           =1
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =429.83
Kurtosis                      =184752.00
```

This agrees with a table in Appendix 8 where **n**=1

```
Mark=-0.0000      numbner getting that mark that way=34134410025 n=2 w=0 x=2 y=0
Mark=1.0000       numbner getting that mark that way=369510 n=2 w=1 x=1 y=0
Mark=2.0000       numbner getting that mark that way=1 n=2 w=2 x=0 y=0
Mark=-0.0000      numbner getting that mark that way=369510 n=2 w=0 x=1 y=1
Mark=1.0000       numbner getting that mark that way=2 n=2 w=1 x=0 y=1
Mark=0.0000       numbner getting that mark that way=1 n=2 w=0 x=0 y=2


Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=3 The theretical number of impossible situat
ions is=3.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis             =2.00
The number of stems           =2
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =303.94
Kurtosis                      =92376.00
```

This agrees with a table in Appendix 8 where **n**=2

```
Mark=-0.0000      numbner getting that mark that way=6306502924168875 n=3 w=0 x=3
 y=0
Mark=1.0000       numbner getting that mark that way=102403230075 n=3 w=1 x=2 y=0

Mark=2.0000       numbner getting that mark that way=554265 n=3 w=2 x=1 y=0
Mark=3.0000       numbner getting that mark that way=1 n=3 w=3 x=0 y=0
Mark=-0.0000      numbner getting that mark that way=102403230075 n=3 w=0 x=2 y=1

Mark=1.0000       numbner getting that mark that way=1108530 n=3 w=1 x=1 y=1
Mark=2.0000       numbner getting that mark that way=3 n=3 w=2 x=0 y=1
Mark=-0.0000      numbner getting that mark that way=554265 n=3 w=0 x=1 y=2
Mark=1.0000       numbner getting that mark that way=3 n=3 w=1 x=0 y=2
Mark=0.0000       numbner getting that mark that way=1 n=3 w=0 x=0 y=3


Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=6 The theretical number of impossible situat
ions is=6.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.00 The maximum value in the x-a
xis              =3.00
The number of stems           =3
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =248.16
Kurtosis                      =61584.00
```

This agrees with a table in Appendix 8 where **n**=3

```
Mark=-0.0000      numbner getting that mark that way=116515794775482050O625 n=4 w
=0 x=4 y=0
Mark=1.0000       numbner getting that mark that way=2522601169667550O n=4 w=1 x=
3 y=0
Mark=2.0000       numbner getting that mark that way=204806460150 n=4 w=2 x=2 y=0

Mark=3.0000       numbner getting that mark that way=739020 n=4 w=3 x=1 y=0
Mark=4.0000       numbner getting that mark that way=1 n=4 w=4 x=0 y=0
Mark=-0.0000      numbner getting that mark that way=25226011696675500 n=4 w=0 x=
3 y=1
Mark=1.0000       numbner getting that mark that way=409612920300 n=4 w=1 x=2 y=1

Mark=2.0000       numbner getting that mark that way=2217060 n=4 w=2 x=1 y=1
Mark=3.0000       numbner getting that mark that way=4 n=4 w=3 x=0 y=1
Mark=-0.0000      numbner getting that mark that way=204806460150 n=4 w=0 x=2 y=2

Mark=1.0000       numbner getting that mark that way=2217060 n=4 w=1 x=1 y=2
Mark=2.0000       numbner getting that mark that way=6 n=4 w=2 x=0 y=2
Mark=-0.0000      numbner getting that mark that way=739020 n=4 w=0 x=1 y=3
Mark=1.0000       numbner getting that mark that way=4 n=4 w=1 x=0 y=3
Mark=0.0000       numbner getting that mark that way=1 n=4 w=0 x=0 y=4


Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=10 The theretical number of impossible situa
tions is=10.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis               -0.00 The maximum value in the x-a
xis              =4.00
The number of stems           =4
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.00
Skewness                      =214.92
Kurtosis                      =46188.00
```

This agrees with a table in Appendix 8 where **n**=4

```
Mark=-0.0000      numbner getting that mark that way=21526875663744186159297l875
n=5 w=0 x=5 y=0
Mark=1.0000       numbner getting that mark that way=582578973877410250З125 n=5 w
=1 x=4 y=0
Mark=2.0000       numbner getting that mark that way=6306502924168875O n=5 w=2 x=
3 y=0
Mark=3.0000       numbner getting that mark that way=341344100250 n=5 w=3 x=2 y=0

Mark=4.0000       numbner getting that mark that way=923775 n=5 w=4 x=1 y=0
Mark=5.0000       numbner getting that mark that way=1 n=5 w=5 x=0 y=0
Mark=-0.0000      numbner getting that mark that way=5825789738774102503125 n=5 w
=0 x=4 y=1
Mark=1.0000       numbner getting that mark that way=1261300584833775OO n=5 w=1 x
=3 y=1
Mark=2.0000       numbner getting that mark that way=1024032300750 n=5 w=2 x=2 y=
1
Mark=3.0000       numbner getting that mark that way=3695100 n=5 w=3 x=1 y=1
Mark=4.0000       numbner getting that mark that way=5 n=5 w=4 x=0 y=1
Mark=-0.0000      numbner getting that mark that way=63065029241688750 n=5 w=0 x=
3 y=2
Mark=1.0000       numbner getting that mark that way=1024032300750 n=5 w=1 x=2 y=
2
Mark=2.0000       numbner getting that mark that way=5542650 n=5 w=2 x=1 y=2
Mark=3.0000       numbner getting that mark that way=10 n=5 w=3 x=0 y=2
Mark=-0.0000      numbner getting that mark that way=341344100250 n=5 w=0 x=2 y=3

Mark=1.0000       numbner getting that mark that way=3695100 n=5 w=1 x=1 y=3
Mark=2.0000       numbner getting that mark that way=10 n=5 w=2 x=0 y=3
Mark=-0.0000      numbner getting that mark that way=923775 n=5 w=0 x=1 y=4
Mark=1.0000       numbner getting that mark that way=5 n=5 w=1 x=0 y=4
Mark=0.0000       numbner getting that mark that way=1 n=5 w=0 x=0 y=5


Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=15 The theretical number of impossible situa
tions is=15.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis                =5.00
The number of stems          =5
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =192.23
Kurtosis                     =36950.40
```

This agrees with a table in Appendix 8 where **n**=5

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=55 The theretical number of impossible situa
tions is=55.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis                =10.00
The number of stems          =10
Mean                         =-0.00 The theoretical mean=-0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =135.92
Kurtosis                     =18475.20
```

Here, **n**=10

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=210 The theretical number of impossible situ
ations is=210.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                -0.00 The maximum value in the x-a
xis                =20.00
The number of stems          =20
Mean                         =0.00 The theoretical mean=0.00
Variance                     =0.00
Standard Deviation           =0.01
Skewness                     =96.11
Kurtosis                     =9237.60
```

Here, **n**=20

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=465 The theretical number of impossible situ
ations is=465.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -0.00 The maximum value in the x-a
xis                 =30.00
The number of stems           =30
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =78.48
Kurtosis                      =6158.40
```

Here, **n**=30

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=820 The theretical number of impossible situ
ations is=820.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -0.00 The maximum value in the x-a
xis                 =40.00
The number of stems           =40
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.01
Skewness                      =67.96
Kurtosis                      =4618.80
```

Here, **n**=40

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1275 The theretical number of impossible sit
uations is=1275.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -0.00 The maximum value in the x-a
xis                 =50.00
The number of stems           =50
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.02
Skewness                      =60.79
Kurtosis                      =3695.04
```

Here, **n**=50

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=1830 The theretical number of impossible sit
uations is=1830.00

              Some DESCRIPTIVE Statistics

The minimum value on the x-axis                   -0.00 The maximum value in the x-a
xis                 =60.00
The number of stems           =60
Mean                          =-0.00 The theoretical mean=-0.00
Variance                      =0.00
Standard Deviation            =0.02
Skewness                      =55.49
Kurtosis                      =3079.20
```

Here, **n**=60

```
tems can be missed.    Score is [0.00], [1.00,0.00]
he type of quiz is MAQ with 20 options and 10 ticks X=184755
he number of impossible situations=2485 The theretical number of impossible sit
ations is=2485.00

              Some DESCRIPTIVE Statistics

he minimum value on the x-axis                   0.00 The maximum value in the x-ax
s                 =70.00
he number of stems           =70
ean                          =0.00 The theoretical mean=0.00
ariance                      =0.00
tandard Deviation            =0.02
kewness                      =51.37
urtosis                      =2639.31
```

Here, **n**=70

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=3240 The theretical number of impossible sit
uations is=3240.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis                 =80.00
The number of stems             =80
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =0.00
Standard Deviation              =0.02
Skewness                        =48.06
Kurtosis                        =2309.40
```

Here, **n**=80

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=4095 The theretical number of impossible sit
uations is=4095.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis                 =90.00
The number of stems             =90
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =0.00
Standard Deviation              =0.02
Skewness                        =45.31
Kurtosis                        =2052.80
```

Here, **n**=90

```
Stems can be missed.    Score is [0.00], [1.00,-0.00]
The type of quiz is MAQ with 20 options and 10 ticks X=184755
The number of impossible situations=5050 The theretical number of impossible sit
uations is=5050.00

            Some DESCRIPTIVE Statistics

The minimum value on the x-axis                 -0.00 The maximum value in the x-a
xis                 =100.00
The number of stems             =100
Mean                            =-0.00 The theoretical mean=-0.00
Variance                        =0.00
Standard Deviation              =0.02
Skewness                        =42.98
Kurtosis                        =1847.52
```

Here, **n**=100

Appendix 15. Tool2.

```
//Descriptive Stats Source File
//Includes
    //Include LEDA
      #include <LEDA/numbers/integer.h>
      #include <LEDA/numbers/bigfloat.h>
      #include <LEDA\core\array.h>           //This was helped by VS 10 system
    //Include ordinary C++
      #include <iomanip>
      #include <string>
      #include <iostream>
      #include <typeinfo> //To allow typeid to work. Do we ever use typeid?
    //Namespaces
      using namespace std;
         using namespace leda;

       //Definition of a function isDivisibleBy () that we are going to use
         bool isDivisibleBy(integer firstNumber, int secondNumber)
         {
                if (firstNumber % secondNumber ==0)
                {
                       return(true);
                }
                else
                {
                    return (false);
                }
         }

  int main()
   {
     //Declarations
          std::string fileName, fileName2,line,modalType,experiment;
          integer maxForWorkingOutMode,maxForWorkingOutMedian,sum, swap,
howMany[9900], cumulativeFrequency[9900],minimumWay,maximumWay,maximumY,
minimumY,sumToDate[9900];
          int way,sizeOfn, numberOfModes, position;
          double mark[9900],firstMarkForMode,lastMarkForMode,maximumMark,
minimumMark;
          bigfloat
xbar,pseudoArea,sumOfWays,maximumNumberOfWays,mu,median,mode,variance,
sd,skewness,
kurtosis,varianceSum,varianceSumSam,skewnessSum,skewnessSumSam,kurtosisSum,kurtos
isSumSam,momentAboutMean,
momentAboutMeanSam,varianceSam,sdSam,skewnessSam,kurtosisSam;
          bool
cannotWorkOutMode,cannotWorkOutMedian,weHaveFoundMedian,weHaveFoundIt,medianFirst
,medianSecond,doneModeOnce,doneModeTwice,fortyFirst,fiftyFirst,seventyFirst,
seventyOneFirst, seventyTwoFirst, seventyThreeFirst,
seventyFourFirst,seventyFiveFirst, populationIsEven,
populationIsOdd,medianTakesTooLong;
          ifstream myfile;
        //Initialisation
          //experiment="Checking that we are using the same formulae as SPSS";
//Options are "No" or "Checking that we are using the same formulae as SPSS"
          experiment="No";
          if(experiment=="No")
           {
            fileName="F:\\MyPhD2\\MyPhD\\Output.txt";
            myfile.open(fileName);
```

```
            if (!myfile)  //the file might not be open or is in use by another
program
             {
           cout << "Unable to open file " << fileName << endl;   //What is wrong
with this?
           exit(1); // terminate with error
         }
          }
          else
          {
          fileName2="F:\\MyPhD2\\MyPhD\\Output.txt";
             myfile.open(fileName2);
             if (!myfile)  //the file might not be open or is in use by another
program
              {
            cout << "Unable to open file " << fileName2 << endl;
            exit(1); // terminate with error
              }
          }
          maxForWorkingOutMedian=maxForWorkingOutMode=999999999; //There appears
to be an upper limit here!!! I have got 9 nines. But if I use 10 I get an error
                                                   //Why? I thought a LEDA integer was
of an arbirary length.!!!!!!! Algorithmic Solutions did not reply to my query.

          for(int i=1;i <5; i++)            //This is part of the solution to the
above problem
          {
                  maxForWorkingOutMedian=maxForWorkingOutMode=
maxForWorkingOutMedian*10+9;
          }

          way=0;
          sum=0;
          sumOfWays=0;
          numberOfModes=0;
          varianceSum=0;
          skewnessSum=0;
          kurtosisSum=0;
          sumToDate[1]=0;

          varianceSumSam=0.0;
          skewnessSumSam=0.0;
          kurtosisSumSam=0.0;

          maximumNumberOfWays=-9999999999.0; //Why is that?
          minimumWay=9999999999.0;//I thought this was an integer
          maximumWay=-999999999;
          maximumMark=-9999999999.0;
          minimumMark=999999999.0;

          minimumY=99999999.9;
          maximumY=-999999999.9;
          medianFirst=false;

          modalType="Uni-modal";
          doneModeOnce=false;
          doneModeTwice=false;
          medianFirst=false;
          medianSecond=false;

          fortyFirst=false;
          fiftyFirst=false;
          seventyFirst=false;
          seventyOneFirst=false;
          seventyTwoFirst=false;
```

```
         seventyThreeFirst=false;
         seventyFourFirst=false;
         seventyFiveFirst=false;

         populationIsEven=false;
         populationIsOdd=false;

         medianTakesTooLong=false;
         weHaveFoundIt=false;
         weHaveFoundMedian=false;
         cannotWorkOutMedian=false;
         cannotWorkOutMode=false;

         if(experiment=="No")
         {
          cout << "Trying to READ from file " << fileName  << endl;
         }
        else
         {
        cout << "Trying to read from file " << fileName2 << endl;
         }
         //cout << "We have read the following " << endl;
         if(experiment=="Checking that we are using the same formulae as SPSS")
         {
                //Debug
                cout << "We are in Checking" << endl;
                while(!myfile.eof()) //We should do lots of checks on this file
              {
                   way++;
             myfile >> mark[way] >> howMany[way];
                   cout  << howMany[way] << endl;
                      sum=sum+howMany[way];
                      if(howMany[way] > maximumWay)
                   {
              maximumWay=howMany[way];
                   }
                   if (howMany[way] < minimumWay)
                   {
                       minimumWay=howMany[way];
                   }
                      if(howMany[way]==maximumNumberOfWays) //There is a small
chance that howMany[i]=initial value of maximumNumberOfWays
                   {
              modalType="Bi-modal";
                       mode=(mode+howMany[way])/2.0;

                   }
                  if(howMany[way] > maximumNumberOfWays )
                  {
                  maximumNumberOfWays=howMany[way] ;
                     mode=howMany[way] ;
                     }
                 } // End while
                xbar=(bigfloat)sum/(1.0*way);
                cout << fixed;
                for (int i=1;i<way+1;i++)
              {
                    sumOfWays=sumOfWays+howMany[i];
                    momentAboutMeanSam=howMany[i] - xbar;
                    varianceSumSam=varianceSumSam +
momentAboutMeanSam*momentAboutMeanSam;
                    skewnessSumSam=skewnessSumSam +
momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
                    kurtosisSumSam=kurtosisSumSam +
momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
```

```
               }
               sizeOfn=way;
                  varianceSam=varianceSumSam/(sizeOfn-1); //Is sizeOfn right?
Should it be sumOfWays This way variance will be 0
            sdSam=sqrt(varianceSam);

skewnessSam=sizeOfn*skewnessSumSam/((sdSam*sdSam*sdSam)*(sizeOfn-1)*(sizeOfn-2));
                  kurtosisSam=-3.0*(bigfloat)(sizeOfn-1)*(sizeOfn-1)/((sizeOfn-
2)*(sizeOfn-3))+kurtosisSumSam*sizeOfn*(sizeOfn+1)/((sizeOfn-1)*(sizeOfn-
2)*(sizeOfn-3)*varianceSam*varianceSam);
            //Output
                      cout << fixed;
                    cout << setprecision( 3 ) ;  //What is wrong with that? Is it
a LEDA word?
                      cout << "Range goes from "  << minimumWay << " to " <<
maximumWay << endl;
                      cout << "Mean Sam="       << xbar.to_double()     << "
Mode=" << mode.to_double()     << endl;
                    cout << "Var Sam.="       << varianceSam.to_double()
                          << " SD. Sam="       << sdSam.to_double() <<
"\t"
                          << "Skew Sam="       << skewnessSam.to_double() <<
"\t"
                          << "Kur Sam="        << kurtosisSam.to_double() <<
endl;


                      cout << "------------------------------" << endl;
                      cout << "          Scientific notation" << endl;
                      cout << scientific << "Mean Sam="       << xbar.to_double()
<< " Mode=" << mode.to_double()     << endl;
                    cout << scientific << "Var Sam.="       <<
varianceSam.to_double()
                          << " SD. Sam="       << sdSam.to_double() <<
"\t"
                          << "Skew Sam="       << skewnessSam.to_double() <<
"\t"
                          << "Kur Sam="        << kurtosisSam.to_double() <<
endl;
          }

          else
          {

           while(!myfile.eof()) //We are now reading from the file
F:\\MyPhD2\\MyPhD\\Output.txt
           {
               way++;
         myfile >> mark[way] >> howMany[way];
               if(mark[way] > maximumMark)
               {
           maximumMark=mark[way];
               }
               if (mark[way] < minimumMark)
               {
                 minimumMark=mark[way];
               }

               if(howMany[way] > maximumY)
               {
           maximumY=howMany[way];
               }

               if (howMany[way] < minimumY)  //This does not work now
               {
```

```
                        minimumY=howMany[way];
                }

                pseudoArea=pseudoArea+mark[way]*howMany[way];
                sum=sum+howMany[way];

            }

            mu=pseudoArea/((bigfloat)sum);
            xbar=mu;



              //Calculate other descriptive statistics
           for (int i=1;i<way+1;i++)
             {


            momentAboutMean=mark[i]-mu;    //Moment about mean
                  momentAboutMeanSam=mark[i] - xbar;

                   cout << fixed;
                   cout << setprecision( 3 ) ;

                       //Work out the proportion of passes

                  if (mark[i] >=40.0*maximumMark/100.0 && !fortyFirst)
                  {
                       fortyFirst=true;
                       cout << "   Percentage of 40% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                  }
                  if (mark[i] >=50.0*maximumMark/100.0 && !fiftyFirst)
                  {
                       fiftyFirst=true;
                       cout << "   Percentage of 50% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                  }

                  if (mark[i] >=70.0*maximumMark/100.0 && !seventyFirst)
                  {
                       seventyFirst=true;
                       cout << "   Percentage of 70% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=71.0*maximumMark/100.0 && !seventyOneFirst)
                  {
                       seventyOneFirst=true;
                       cout << "   Percentage of 71% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=72.0*maximumMark/100.0 && !seventyTwoFirst)
                  {
                       seventyTwoFirst=true;
                       cout << "   Percentage of 72% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                  }

                  if (mark[i] >=73.0*maximumMark/100.0 && !seventyThreeFirst)
                  {
                       seventyThreeFirst=true;
                       cout << "   Percentage of 73% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
```

```
                }

                 if (mark[i] >=74.0*maximumMark/100.0 && !seventyFourFirst)
                {
                     seventyFourFirst=true;
                     cout << "   Percentage of 74% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;
                }

                if (mark[i] >=75.0*maximumMark/100.0 && !seventyFiveFirst)
                {
                     seventyFiveFirst=true;
                     cout << "   Percentage of 75% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i] << endl;

                }

                 sumOfWays=sumOfWays+howMany[i];

                //Work out HOW MANY MODES there are.

        if(howMany[i]==maximumY) numberOfModes=numberOfModes+1;

      //Now work out the MODE. Later on the MODE will be output

                if((howMany[i]==maximumY) && numberOfModes >1)
                       {
                              lastMarkForMode=mark[i];
                     }

                if((howMany[i] == maximumY) && numberOfModes==1 )
                 {
                                  firstMarkForMode=-mark[i] ;
                                  lastMarkForMode=0;
                 }

                //Work out MEDIAN

       if (way==1) //If the census only has 1 value
             {
         median=mark[1];
               maximumY=minimumY=howMany[1];
               maximumWay=minimumWay=mark[1];
             }
             //if ((sum >  maxForWorkingOutMedian))
             //{
               //cannotWorkOutMedian=true;
             //}

       //if(!cannotWorkOutMedian)
             //{
                if(!weHaveFoundMedian)
             {
                 if(i==1) cumulativeFrequency[1] =
howMany[1];
                       else     cumulativeFrequency[i]= cumulativeFrequency[i-1]
+ howMany[i];
                          if(cumulativeFrequency[i] >=sum/2)
                  {
                          position=i;

                       if ((howMany[i] >  maxForWorkingOutMedian)) //if
Scientific Notation is used anywhere e cannot work out median
                                     {
                        cannotWorkOutMedian=true;
```

```
                                }

                                if (!isDivisibleBy(sum,2)) //The population
is odd.
                        {
                          median=mark[position];
                                weHaveFoundMedian=true;
                        }
                                else                    //population is
even
                                {

                                  if(cumulativeFrequency[i]==sum/2)
                          {
                                   median=(mark[position]  +
mark[position+1])/2; //What happens if there is no (position +1). Does not happen
in our case

//Be careful there is a devisiion involved

//median is of type bigfloat, mark is of type float. Should median be of type
float?
                                   weHaveFoundMedian=true;
                               }
                                 else
                              {
                            median=mark[position];
                                   weHaveFoundMedian=true;
                               }
                                 }
                            }

                    }
                //}

                  //Work out the sums for censi and samples
                   varianceSum=varianceSum +
howMany[i]*momentAboutMean*momentAboutMean;
                   skewnessSum=skewnessSum +
howMany[i]*momentAboutMean*momentAboutMean*momentAboutMean;
                   kurtosisSum=kurtosisSum +
howMany[i]*momentAboutMean*momentAboutMean*momentAboutMean*momentAboutMean;

                   varianceSumSam=varianceSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam;
                   skewnessSumSam=skewnessSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam;
                   kurtosisSumSam=kurtosisSumSam +
howMany[i]*momentAboutMeanSam*momentAboutMeanSam*momentAboutMeanSam*momentAboutMe
anSam;
              }
                  //Work out descriptive stats for censi and samples
                  variance=varianceSum/sumOfWays;
                  sd=sqrt(variance);
             skewness=     skewnessSum/(sumOfWays*sd*sd*sd);
                  kurtosis=-3 + kurtosisSum/(sumOfWays*sd*sd*sd*sd);


                  varianceSam=varianceSumSam/(sumOfWays-1);
             sdSam=sqrt(varianceSam);

skewnessSam=sumOfWays*skewnessSumSam/((sdSam*sdSam*sdSam)*(sumOfWays-
1)*(sumOfWays-2));
                  kurtosisSam=-3.0*(bigfloat)(sumOfWays-1)*(sumOfWays-
1)/((sumOfWays-2)*(sumOfWays-
```

```
3))+kurtosisSumSam*sumOfWays*(sumOfWays+1)/((sumOfWays-1)*(sumOfWays-
2)*(sumOfWays-3)*varianceSam*varianceSam);

          //Output
            cout << fixed;
               cout << setprecision( 3 ) ;

               cout << "The number of lines read in is=" << way << endl;
               cout << "Sum of all ways=POPULATION=" << sumOfWays << endl;
               cout << "Marks range from " << minimumMark << " to " <<
maximumMark << " Hence Range=" <<maximumMark-minimumMark << endl;
               cout << "Number of MODES=" << numberOfModes << endl;
               cout << "  " <<endl;
               cout << "           DESCRIPTIVE STATISTICS" << endl;

               mode =(lastMarkForMode - firstMarkForMode)/numberOfModes;
//Careful, that involves a division.
               //Debug
               cout << "First Mode occurs at mark " << -firstMarkForMode << "
Last  Mode occurs at Mark " << lastMarkForMode << "Number of modes=" <<
numberOfModes << " Mode=" << mode.to_double() << endl;

               //if (!cannotWorkOutMedian)
               //{
                 cout << "Population" << endl;
                 cout << "   Mean Pop="        << mu.to_double()          << "\t\t
Median Pop=" << median.to_double() << "\t " << " Mode Pop=" << mode.to_double()
<< endl;
               //}
               //else
               //{
                       //cout << "Population" << endl;
                       //cout << "  We cannot calculate MEDIAN (there are
approximations due to Scientific Notation)" << endl;
                    //cout << "   Mean Pop="       << mu.to_double() ;
                       //cout << "\t " << " Mode Pop=" << mode.to_double() <<
endl;

               //}


               cout << "   Var Pop.="       << variance.to_double()
                       << " SD. Pop="          << sd.to_double() << "\t"
                       << "Skew Pop="          << skewness.to_double()    <<
"\t"
                     << "Kur Pop="            << kurtosis.to_double()    << endl;
               cout << "  " << endl;
               cout << "Sample" << endl;
               cout << "   Mean Sam="       << mu.to_double()          << endl;
               cout << "   Var Sam.="       << varianceSam.to_double()
                       << " SD. Sam="          << sdSam.to_double() <<
"\t"
                       << "Skew Sam="          << skewnessSam.to_double() <<
"\t"
                     << "Kur Sam="            << kurtosisSam.to_double() << endl;
               cout << "  " << endl;
              cout << "-----------------------------------------------" << endl;


        } //End of else
      //Close file
           myfile.close();

    } //End of main
```

Appendix 16. Validation of tool2.

16.1 Introduction

We will validate *tool2* by showing that:

a) The tool, *tool2* calculates the descriptive statistics properly by showing that it gets the same results as SPSS in many cases and hence uses the same formulae as SPSS.
b) The tool, *tool2* calculates *median* properly (where it can).
c) The tool, *tool2* calculates *mode* properly.
d) The tool, *tool2* calculates the proportion of the population that obtain a certain mark or above.

The mode and median cannot be calculated, in general, until a Pivot is done, because:

a) the mode (as a 1st approximation) is the most frequently occurring y-value,
b) the median is the 'middle' x-value.

16.2 Does tool2 use the same formulae as SPSS?

The descriptive statistics include:

- Mean,
- Variance,
- Standard Deviation,
- Skewness,
- Kurtosis.

We do not know how to ask SPSS to calculate the descriptive statistics for a *distribution*, but only for a single column, because that is what a statistician said

> *"... you can only get descriptive stats of one variable" (Straker, 2011).*

So, we ask SPSS to give us the descriptive statistics of a single column. This part of the validation of *tool2* is done by inputting from **Output.txt** (which has 2 columns a) a mark b) how many obtain that mark) and ignoring the mark. This is done when `experiment="Checking that we are using the same formulae as SPSS"`

SPSS also calculates minimum, maximum and range. So *tool2* calculates and outputs to the debug window a) the *minimum* number that get a mark b) the *maximum* number that get a mark c) the range of marks.

When in experimental mode *tool2* produces something like

```
Range goes from 1 to 1182645815564861000
```

When in non-experimental mode *tool2* produces something like

```
Marks range from 0.000 to 60.000 Hence Range=60.000
```

The range of marks given by *tool2* can be observed..

There is some confusion between the *range* of marks and the *number* of marks. Consider that there are only **2** marks, 0 and 3. The range of marks is **3** (3-0). Do we now output 2 or 3? To avoid any argument, *tool2* shows:

a) the *minimum* mark
b) the *maximum* mark,
c) the range which is (maximum - minimum).

The formulae that are used are,

App 16 Table 57. Formulae that SPSS and *tool2* use.

	Estimates. **Formulae that *SPSS* uses.** **n=size of sample**	**Census** **Formulae that *tool2* uses.** **N=size of population**
Mean	$\bar{x}=\frac{\sum x_i}{n}$	$\mu=\frac{\sum x_i}{N}=\sum x_i P(x_i)$
Variance	VarSam= $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$	VarPop= $\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$
Standard Deviation	s= $\sqrt{VarSam}$	σ= $\sqrt{VarPop}$
Skewness	$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^3}{N\sigma^3}$
Kurtosis	$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^4} - \frac{3(n-1)^2}{(n-1)(n-2)}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^4}{N\sigma^4}$ -3

Here, n is any integer above 0 and not the number of stems. n is the size of the sample and N is the size of the population.

The data of the following table corresponds to X=1. Yes/No quizzes where stems are ALL answered with **a** having any value (except 1). All that changes with a change of **a** is the mark, and we ignore that.

We use Scientific Notation (e.g. 428 E76) where necessary, because SPSS does.

App 16 Table 58. Tool2 compared to SPSS for a) Yes/No quizzes b) ALL stems are answered c) no negative marking is used d) a has any value (except 1).

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	93.909	8943.091	94.568	0.624	-1.273
	SPSS	93.909	8943.091	94.568	0.624	-1.273
20	tool2	49932.19	4.274E9.	65379.214	1.071	-0.381
	SPSS	49932.19	4.274E9	65379.214	1.071	-0.381
40	tool2	2.68E10	1.951E21	4.416E10	1.539	0.943
	SPSS	2.68E10	1.951E21	4.4168E10	1.539	0.943
60	tool2	1.89E16	1.247E33	3.531E16	1.832	1.989
	SPSS	1.89E16	1.247E33	3.531E16	1.832	1.989
80	tool2	1.49E22	9.250E44	3.041E22	2.050	2.887
	SPSS	1.49E22	9.250E44	3.041E22	2.050	2.887
100	tool2	1.256E28	7.464 E56	2.732E28	2.228	3.685
	SPSS	1.26E28	7.464E56	2.732E28	2.228	3.685

For Yes/No quizzes where stems are ALL answered, *tool2* and SPSS give the same results (as far as we can tell) for 10 to 100 stems (inclusive).

The data for the following table corresponds to X=1. Yes/No quizzes. Stems are can be MISSED and **a**=0.

App 16 Table 59. Tool2 compared to SPSS for a) Yes/No quizzes b) stems are can be MISSED c) a=0.

Number of stems	**Tool**	**Mean**	**Variance**	**Standard Deviation**	**Skewness**	**Kurtosis**
10	tool2	5368.09	3.368E7	5803.691	0.721	-1.125
	SPSS	5368.09	3.368E7	5803.691	0.721	-1.125
20	tool2	1.66E8	5.202E16	2.28E8	1.164	-0.1462
	SPSS	1.66E8	5.202E16	2.28E8	1.164	-0.146
40	tool2	2.97E17	2.587E35	5.086E17	1.633	1.269
	SPSS	2.97E17	2.587E35	5.086E17	1.633	1.269
60	tool2	6.95E26	1.819E54	1.349E27	1.928	2.383
	SPSS	6.95E26	1.819E54	1.349E27	1.928	2.383
80	tool2	1.825 E36	1.488E73	3.857E36	2.149	3.336
	SPSS	1.82 E36	1.488E73	3.857E36	2.149	3.336
85	tool2	4.176 E38	8.076 E77	8.986 E38	2.198	3.557
	SPSS	4.18 E38	8.076 E77	8.986 E38	2.198	3.557
86	tool2	1.239 E39	7.151 E78	2.674 E39	2.207	3.600
	SPSS	1.24 E39	7.151 E78	2.674 E39	2.207	3.600
87	tool2	3.673 E39	6.333 E79	7.958 E39	2.216	3.643
	SPSS	**8.26 E38**	**2.834 E78**	**1.683 E39**	**2.770**	**8.521**
90	tool2	9.591 E40	4.403 E82	2.098 E41	2.243	3.771
	SPSS	**1.54 E39**	**6.120 E78**	**2.474 E39**	**1.506**	**0.957**
100	tool2	5.103E45	1.325 E92	1.151 E46	2.329	4.185
	SPSS	**2.01 E38**	**6.21 E78**	**2.494 E39**	**1.310**	**1.110**

It goes wrong at **n**=87 onwards. The size of **n** is not important. *tool2* and SPSS disagree when the numbers being represented are 'too big'.

So, *tool2* does use the same formulae as SPSS, until SPSS reaches its limitations.

The data for the following table corresponds to X=1. Yes/No quizzes. Stems are can be MISSED and a=-1.

App 16 Table 60. Tool2 compared to SPSS for a) Yes/No quizzes b) stems are can be MISSED c) a=-1, b=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	2811.86	1.057 E7	3250.708	0.832	-0.891
	SPSS	2811.86	1.051 E7	3250.708	0.832	-0.891
20	tool2	8.504 E7	1.596 E16	1.263 E8	1.309	0.224
	SPSS	8.504 E7	1.596 E16	1.263 E8	1.309	0.224
40	tool2	1.50 E17	7.788 E34	2.791 E17	1.813	1.893
	SPSS	1.50 E17	7.788 E34	2.791 E17	1.813	1.893
60	tool2	3.503 E26	5.431 E53	7.370 E26	2.129	3.209
	SPSS	3.503 E26	5.431 E53	7.370 E26	2.129	3.209
80	tool2	9.181 E35	4.420 E72	2.102 E36	2.366	4.333
	SPSS	9.181 E35	4.420 E72	2.102 E36	2.366	4.333
85	tool2	2.10 E38	2.397 E77	4.896 E38	2.417	4.593
	SPSS	2.10 E38	2.397 E77	4.896 E38	2.417	4.593
86	tool2	6.228 E38	2.122 E78	1.457 E39	2.427	4.644
	SPSS	6.23 E38	2.122 E78	1.457 E39	2.427	4.644
87	tool2	1.847 E39	1.870 E79	1.335 E39	2.437	4.695
	SPSS	**7.25 E38**	**3.305 E78**	**1.817 E39**	**3.331**	**11.348**
90	tool2	4.822 E40	1.306 E82	1.143 E41	2.466	4.845
	SPSS	**1.07 E39**	**3.72 E78**	**1.930 E39**	**2.044**	**3.872**
100	tool2	2.564 E45	3.923 E91	6.264 E45	2.558	5.331
	SPSS	**1.58 E39**	**5.145 E79**	**2.268 E39**	**1.605**	**1.828**

tool2 and SPSS do not agree when **n**=87 or more.

SPSS goes wrong, here, at **n**=87 and it appears to be when a number occupies more than 40 characters. *tool2* appears to be all right (all the descriptive statistics appear to be going in the right direction at the right speed, whereas a) the mean as measured by SPSS goes down and up b) the kurtosis as measured by SPSS suddenly jumps).

The data for the following table corresponds to X=9. 5-option MAQ quizzes with 2 ticks. Stems are ALL answered and a=0. But **a** can have any value (except 1).

App 16 Table 61. Tool2 compared to SPSS for a) 5-option MAQ quizzes with 2 ticks b) stems are ALL answered c) *a*=0 (but *a* can be any value (except 1).

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	9.09 E8	2.217 E18	1.489 E9	1.454	0.571
	SPSS	9.09 E8	2.217 E18	1.489 E9	1.454	0.571
20	tool2	4.76 E18	8.339 E37	9.132 E18	1.904	2.471
	SPSS	4.76 E18	8.339 E37	9.132 E18	1.904	2.471
40	tool2	2.44 E38	3.140 E77	5.603 E38	2.398	4.674
	SPSS	2.44 E38	3.140 E77	5.603 E38	2.398	4.674
60	tool2	1.639 E58	1.761 E117	4.197 E58	2.724	6.418
	SPSS	**2.17 E39**	**5.948 E78**	**2.439 E39**	**1.047**	**0.317**
80	tool2	1.235 E78	1.165 E157	3.3414 E78	2.975	7.915
	SPSS	**2.68 E39**	**7.51 E78**	**2.741 E39**	**1.046**	**0.055**
90	tool2	1.099 E88	9.833 E176	3.136 E88	3.082	8.599
	SPSS	**3.33 E39**	**9.424 E78**	**3.070 E39**	**0.684**	**-0.808**
100	tool2	9.901 E97	8.444 E196	2.906 E98	3.181	9.249
	SPSS	**3.20 E39**	**7.263 E78**	**2.695 E39**	**0.656**	**-0.496**

Overall, we can conclude that *tool2* and SPSS use the same formulae, but they differ in their results when the limits of SPSS are reached. It looks as though the limits of *tool2* are not reached, here.

The data for the following table corresponds to X=9. 5-option MAQ quizzes with 2 ticks. Stems are can be MISSED with **a=b**=0.

App 16 Table 62. Tool2 compared to SPSS for a) 5-option MAQ quizzes with 2 ticks b) stems cab be MISSED c) *a*=*b*=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2	2.36 E9	1.606 E19	4.007 E9	1.544	0.846
	SPSS	2.36 E9	1.606 E19	4.007 E9	1.544	0.846
20	tool2	3.20 E19	4.007 E39	6.330 E19	1.975	2.802
	SPSS	3.20 E19	4.007 E39	6.330 E19	1.975	2.802
40	tool2	1.104 E40	6.780 E80	2.604 E40	2.475	5.092
	SPSS	**1.58 E39**	**9.621 E78**	**3.102 E39**	**1.856**	**2.039**
60	tool2	4.992 E60	1.717 E122	1.310 E61	2.805	6.909
	SPSS	**2.60 E39**	**6.468 E78**	**2.91 E39**	**0.952**	**-0.175**

80	tool2 SPSS	2.529 E81 **3.08 E39**	5.135 E163 **9.058 E78**	7.166 E81 **3.010 E39**	3.059 **0.835**	8.470 **-0.497**
90	tool2 SPSS	5.838 E91 **3.39 E39**	2.914 E184 **8.799 E78**	1.707 E92 **2.966 E39**	3.169 **0.574**	9.183 **-0.834**
100	tool2 SPSS	1.364 E102 **3.54 E39**	1.683 E205 **9.281 E78**	4.102 E102 **3.046 E39**	3.269 **0.614**	9.861 **-0.841**

Here, *tool2* and SPSS disagree at about **n**=40. The disagreement comes about when SPSS cannot cope with 'large' numbers. But, the main conclusion is that tool2 and SPSS do use the same formulae until SPSS 'runs out of steam'.

The data for the following table corresponds to X=9. 5-option MAQ quizzes with 2 ticks. Stems are can be MISSED with **a=-1/9** and **b**=0

App 16 Table 63. Tool2 compared to SPSS for a) 5-option MAQ quizzes with 2 ticks b) stems cab be MISSED c) *a*=-1/9 and *b*=0.

Number of stems	Tool	Mean	Variance	Standard Deviation	Skewness	Kurtosis
10	tool2 SPSS	3.990 E8 3.990 E8	9.665 E17 9.665 E17	9.831 E8 9.831 E8	2.787 2.787	6.901 6.901
20	tool2 SPSS	3.93 E18 3.93 E18	1.186 E38 1.186 E38	1.089 E19 1.089 E19	3.433 3.433	12.082 12.082
40	tool2 SPSS	1.096 E39 **5.78 E38**	1.105 E79 **2.562 E78**	3.325 E39 **1.601 E39**	3.879 **3.604**	15.634 **13.189**
60	tool2 SPSS	4.381 E59 **2.40 E39**	2.013 E120 **7.249 E79**	1.419 E60 **2.692 E39**	4.115 **1.053**	17.783 **0.062**
80	tool2 SPSS	2.014 E80 **2.93 E39**	4.860 E161 **7.505 E78**	6.972 E80 **2.740 E39**	4.256 **0.853**	19.029 **-0.271**
90	tool2 SPSS	4.453 E90 **3.14 E39**	2.536 E182 **7.702 E78**	1.592 E91 **2.775 E39**	4.327 **0.743**	19.570 **-0.531**
100	tool2 SPSS	9.993 E100 **3.22 E39**	1.358 E203 **7.551 E78**	3.685 E101 **2.748 E39**	4.412 **0.797**	39.049 **-0.415**

The same conclusions a last time can be found.

The reader can reasonably ask "Why do you believe *tool2* and not SPSS?" The answer is with *tool2* all the descriptive statistics keep on going up but they do not with SPSS.

The main conclusion is that *tool2* and SPSS use the same formulae, until SPSS 'runs out of steam'.

16.3 Does tool2 calculate the MEDIAN calculated properly?

The median is defined as the 'middle' value. So, for marks, the median mark is the middle mark. It all depends on what 'middle' means. Let us try a small example. If we have

Mark	**How Many Get That Mark**
0	5
1	4

Then there are 5 0s and 4 1s here, i.e. 000001111

This is the middle value → and so the **median** mark is **0**.

However, if the data is

Mark	**How Many Get That Mark**
0	6
1	4

There is no 'middle' value of 00000 01111 (because the population is even). The 'middle' is between the two noughts and so the mean is taken, thus the **median** mark is **0**. In this case the mean is 0, but it does not have to always be, as the following data shows.

Mark	**How Many Get That Mark**
0	4
1	4

Then the data means 000 01 111. The middle mark is the mean of 0 and 1 i.e. 1/2, a mark that does not occur in the data (but it might if the two marks are further apart).

Thus, if the population size is odd then the median mark is the mark that corresponds to **n**/2 (in C) and if the population is even the median mark is the mean of the marks that correspond to the mark we are looking at and the next mark. So, if we are looking at the mark 0 and the next mark is 1, then the mean of the two marks is (0 +1)/2=0.5 and thus the **median** mark is **0.5.**

If Scientific Notation is used anywhere in Excel™, then we cannot work out if the population is even or odd, because Scientific Notation uses 10 to the something which is always even.

We work out the cumulative frequency, and our data now looks like

Mark	**How Many Get That Mark**	**Cumulative Frequency**
0	5	5
1	4	9

Mark	**How Many Get That Mark**	**Cumulative Frequency**
0	6	6
1	4	10

Mark	**How Many Get That Mark**	**Cumulative Frequency**
0	4	4
1	4	8

For our data, we get

Median
0
0
0.5

This is what *tool2* produces. If this looks a 'small' amount of data we only have to consider population sizes that are:

- even, or

- odd

Of these, the only difficulty is with even populations, where there is no 'middle' mark, and then we take the mean of the values on either side of the 'middle'. If the values the marks on either side of the 'middle' are the same then the median is one of them. If the values the marks on either side of the 'middle' are NOT the same then the median mark is the mean of the marks on either side of the 'middle'. Our data reflects these conditions.

So, *tool2* appears to calculate the Median properly.

16.4 Does *tool2* calculate the MODE properly?

The first approximation of mode is the mark that occurs most frequently. Our data shows the frequency that a mark occurs, so as a 1st approximation we need to find the mark where howMany[i] is greatest. As a 2nd approximation when there are 2 equal most frequently occurring values (then we have a bi-modal distribution) and the mode is the mean of where the 2 values occur. Our data is such that we often have uni-modal or bi-modal distributions. We rarely have tri or more modal distributions.

tool2 only works out the mode for any distribution, and tells you how many modes there are. SPSS does that for a single column, but not for a distribution (that we can find out) when **Frequencies** are used.

In a loop *tool2* goes through all the data and reads in a) a mark (**M**) and b) the number obtaining that mark ($\mathbf{Y_M}$), and puts them both into arrays. If $\mathbf{Y_M}$ is greater than the maximum then it becomes the maximum. In another loop *tool2* goes through the data again and if $\mathbf{Y_M}$ equals the maximum then the number of modes is incremented and remembered a) what mark the first mode occurred at and b) what mark the last mode occurred at. At the end, a) the number of modes and b) the mode mark=(mark where last mode occurred - mark where 1st mode occurred)/(number of modes), are output to the debug window.

If the number of modes is 1 and $\mathbf{Y_M}$ is equal to the maximum then the mode mark is the mark we have just read in.

Before that, if the number of modes is 2 and $\mathbf{Y_M}$ is equal to the maximum then the mode mark is the mean of the mode and the mark we have just read in.

For symmetric distributions the mean mark, the mode mark and the median mark should all be co-incident.

This symmetry occurs when all stems are answered (and X=1), and so we can do a check that mode and median are calculated properly by *tool2* for quizzes where all stems are answered (and X=1), where Scientific Notation is not used by Excel™.

This check is positive, where it can be done. For example, for a quiz where X=1 (i.e. a Yes/No quiz) where all stems are answered, where **a**=0 and **n**=40. The mean mark=20.0 (which agrees with the theoretical) and so are the Median and the Mode.

```
                    DESCRIPTIVE STATISTICS
Population
   Mean Pop=20.000                  Median Pop=20.000        Mode Pop=20.000
   Var Pop.=10.000 SD. Pop=3.162           Skew Pop=0.000   Kur Pop=-0.050
```

When X=7 (i.e. a 3-option Multiple Yes/No quiz) where all stems are answered with **a**=0 i.e. the score is [1, 0], then the distribution of marks is *not* symmetric. There is only 1 way of getting the answers to all stems right and there are many ways of getting the answer to all stems wrong. For example, for 40 stems we have

```
The number of lines read in is=41
Sum of all ways=POPULATION=1329227995784911150537324190635130880
Marks range from 0.000 to 40.000 Hence Range=40.000
Number of MODES=1

                DESCRIPTIVE STATISTICS
First Mode occurs at mark 5.000 Last  Mode occurs at Mark 0.000Number of modes=1
 Mode=5.000
Population
   Mean Pop=5.000                Median Pop=5.000         Mode Pop=5.000
   Var Pop.=4.375 SD. Pop=2.092 Skew Pop=0.359  Kur Pop=0.079
```

Here, the skewness is not 0 and so we do not have a symmetric distribution with a uni-modal distribution.

We now, try *tool2* for a bi-modal distribution with the maximum marks next to each other (and hence the mode is not a mark that occurs)

```
The number of lines read in is=11
Sum of all ways=POPULATION=1167464
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=2

                DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 3.000Number of modes=2
Mode=2.500
Population
   Mean Pop=2.693                Median Pop=3.000         Mode Pop=2.500
   Var Pop.=2.085 SD. Pop=1.444 Skew Pop=0.215  Kur Pop=-0.555
```

You will notice that, here, we have some debugging information.

We now, try *tool2* for a bi-modal distribution with the maximum marks, far apart, and hence the mode might be a mark that occurs.

```
The number of lines read in is=11
Sum of all ways=POPULATION=1404395
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=2

                DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 6.000Number of modes=2
Mode=4.000
Population
   Mean Pop=3.352                Median Pop=3.000         Mode Pop=4.000
   Var Pop.=3.507 SD. Pop=1.873 Skew Pop=0.096  Kur Pop=-1.209
```

We now, try *tool2* for a tri-modal distribution

```
The number of lines read in is=11
Sum of all ways=POPULATION=1451470
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=3

                DESCRIPTIVE STATISTICS
Firt Mode occurs at mark 2.000 Last  Mode occurs at Mark 6.000Number of modes=3
Mode=2.667
Population
   Mean Pop=3.340                Median Pop=3.000         Mode Pop=2.667
   Var Pop.=3.398 SD. Pop=1.843 Skew Pop=0.116  Kur Pop=-1.151
```

So, *tool2* does better than SPSS which tells you the mode for a uni-modal distribution and then says "Multiple modes exist. The smallest value is shown". *tool2* gives a) how many modes there are and b) the value of the mode.

16.5 Does tool2 calculate the proportion of passes properly?

The initialisation that we do includes

```
fortyFirst=false;
```

and our coding includes

```
if (mark[i] >=40.0*maximumMark/100.0 && !fortyFirst)
                {
                        fortyFirst=true;
                        cout << "   Percentage of 40% or above=" << (100 -
100*sumOfWays/sum).to_double() << " When Mark=" << mark[i]<< endl;
                }
```

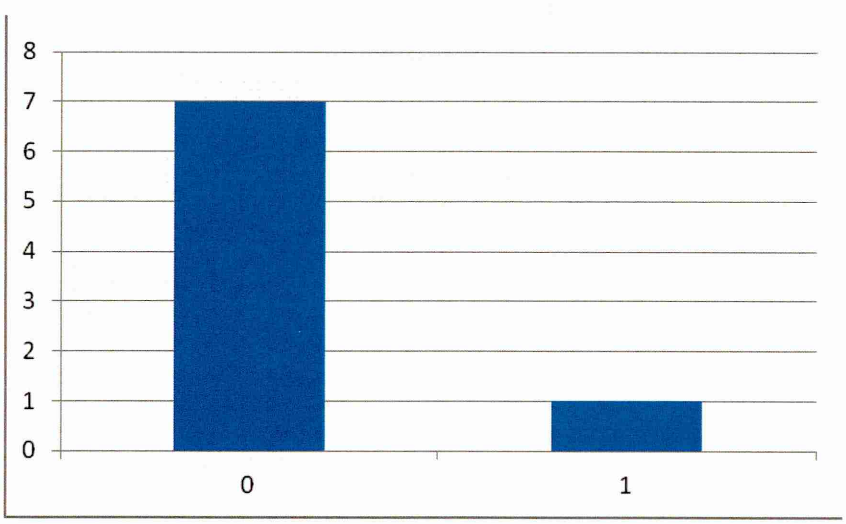
This coding assumes that 40% is the raised pass mark.

When the mark that we are considering is equal to or greater than 40% of the maximum mark then the proportion of marks that are equal to or greater than 40% is (all of them -the proportion to date).

We do this calculation for 40%, 50%, 70%, 71%, 72%, 73%, 74% and 75%.

To check our calculations we do the checks for a few cases.

For X=7, **a**=0, **n**=1 where ALL stems are answered we have



The data in file **Output.txt** is

0 7

1 1

The output from *tool2* is

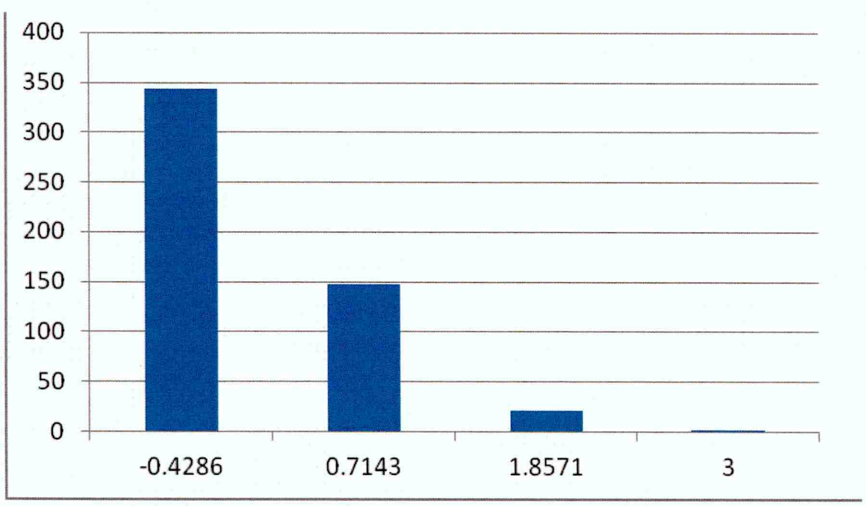
```
Trying to READ from file F:\MyPhD2\MyPhD\Output.txt
   Percentage of 40% or above=12.500 When Mark=1.000
   Percentage of 50% or above=12.500 When Mark=1.000
   Percentage of 70% or above=12.500 When Mark=1.000
   Percentage of 71% or above=12.500 When Mark=1.000
   Percentage of 72% or above=12.500 When Mark=1.000
   Percentage of 73% or above=12.500 When Mark=1.000
   Percentage of 74% or above=12.500 When Mark=1.000
   Percentage of 75% or above=12.500 When Mark=1.000
The number of lines read in is=2
Sum of all ways=POPULATION=8
Marks range from 0.000 to 1.000 Hence Range=1.000
Number of MODES=1

                DESCRIPTIVE STATISTICS
Population
   Mean Pop=0.125                  Median Pop=0.000             Mode Pop=0.000
   Var Pop.=0.109 SD. Pop=0.331 Skew Pop=2.268   Kur Pop=3.143
```

With the mean mark being 1/8=0.125

The marks go from 0 to 1 (as shown above).

At mark 0, we are not above 40%. At mark 1, we are above (or equal to) 40% and there is 12.5% of the area left. At mark 1, we are also above (or equal to) 50% and there is 12.5% of the area left.

Let us try another, example of X=7 **n**=3, with all stems answered, **a**=-1/7



The data in **Output.txt** is

-0.4286	343
0.7143	147
1.8571	21
3	1

This is what *tool2* gives

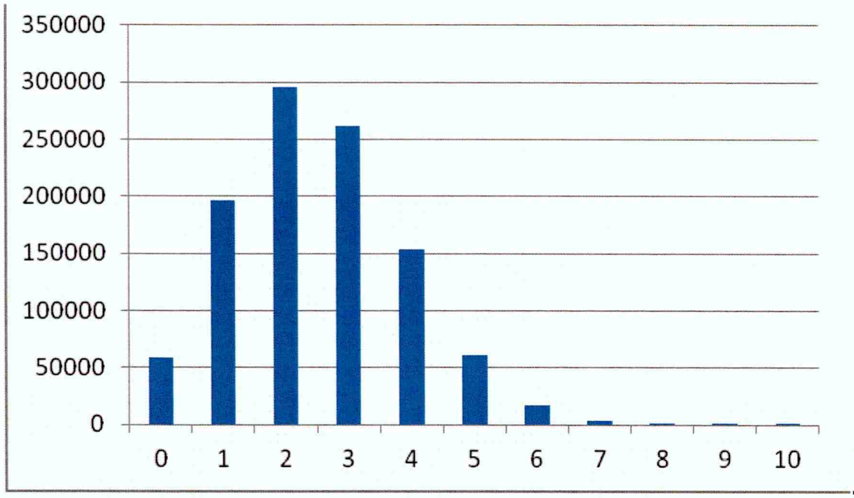
```
Trying to READ from file F:\MyPhD2\MyPhD\Output.txt
   Percentage of 40% or above=4.297 When Mark=1.857
   Percentage of 50% or above=4.297 When Mark=1.857
   Percentage of 70% or above=0.195 When Mark=3.000
   Percentage of 71% or above=0.195 When Mark=3.000
   Percentage of 72% or above=0.195 When Mark=3.000
   Percentage of 73% or above=0.195 When Mark=3.000
   Percentage of 74% or above=0.195 When Mark=3.000
   Percentage of 75% or above=0.195 When Mark=3.000
The number of lines read in is=5
Sum of all ways=POPULATION=512
Marks range from -925596313493178300000000000000000000000000000000000000000000000
.000 to 3.000 Hence Range=9255963134931783000000000000000000000000000000000000
00000000.000
Number of MODES=1

                DESCRIPTIVE STATISTICS
Population
   Mean Pop=0.047              Median Pop=-0.429        Mode Pop=-0.429
   Var Pop.=0.431 SD. Pop=0.656 Skew Pop=1.086  Kur Pop=0.665
```

Population=343 +147 +21 +1=512.

When the mark 1.8571 is considered, the mark is above or equal 40*3/100=1.2 and then there are 22 of 512 marks left i.e. 100*22/512=4.297

When mark 1.8571 is considered we are equal to or above 40% of the maximum mark (3) and then there is 4.297% of the 'area' of marks left. So, the proportion of the marks that are above (or equal to) 40% of the maximum mark is 4.297%. There is no point in raising the pass mark beyond 70%.

Let us try another example. X=3 i.e. a 4-option Multiple Choice quiz, with **n**=10 and **a**=0 where ALL stems are answered.



The data in **Output.txt** is

0	59049
1	196830
2	295245
3	262440
4	153090
5	61236
6	17010
7	3240
8	405
9	30
10	1

There are 11 lines here

The output from *tool2* is

```
   Percentage of 40% or above=22.412 When Mark=4.000
   Percentage of 50% or above=7.813 When Mark=5.000
   Percentage of 70% or above=0.351 When Mark=7.000
   Percentage of 71% or above=0.042 When Mark=8.000
   Percentage of 72% or above=0.042 When Mark=8.000
   Percentage of 73% or above=0.042 When Mark=8.000
   Percentage of 74% or above=0.042 When Mark=8.000
   Percentage of 75% or above=0.042 When Mark=8.000
The number of lines read in is=11
Sum of all ways=POPULATION=1048576
Marks range from 0.000 to 10.000 Hence Range=10.000
Number of MODES=1

                 DESCRIPTIVE STATISTICS
Population
   Mean Pop=2.500                  Median Pop=2.000          Mode Pop=2.000
   Var Pop.=1.875 SD. Pop=1.369 Skew Pop=0.365  Kur Pop=-0.067
```

The population=59049 + 196830 + 295245 + 262440 + 153090 + 61236 + 17010 + 3240 + 405 + 30 +1 =1048576

When the mark 4 is considered, it equal to 40% of the maximum mark (10) and then the percentage of marks left is 100*(153090 + 61236 + 17010 + 3240 + 405 + 30 + 1)/1048576=235012/1048576=22.412.

Similarly, when the mark 8 is considered the percentage of marks left=100*436/1048576 =0.042

So, for the few cases we have tried, *tool2* does calculate correctly the proportion that pass.

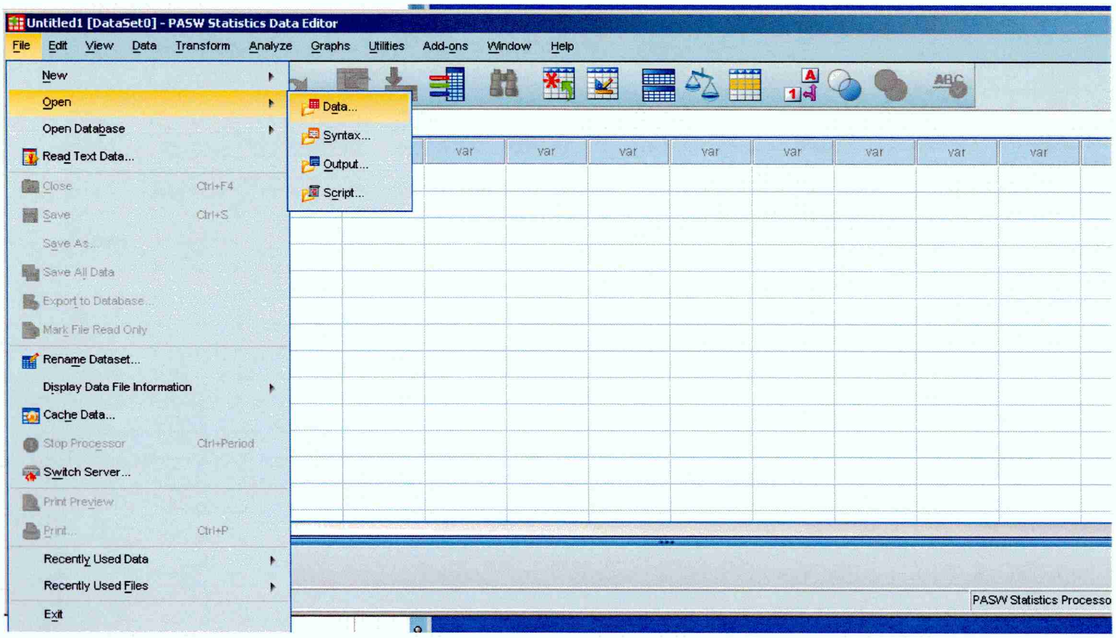
16.6 Conclusions

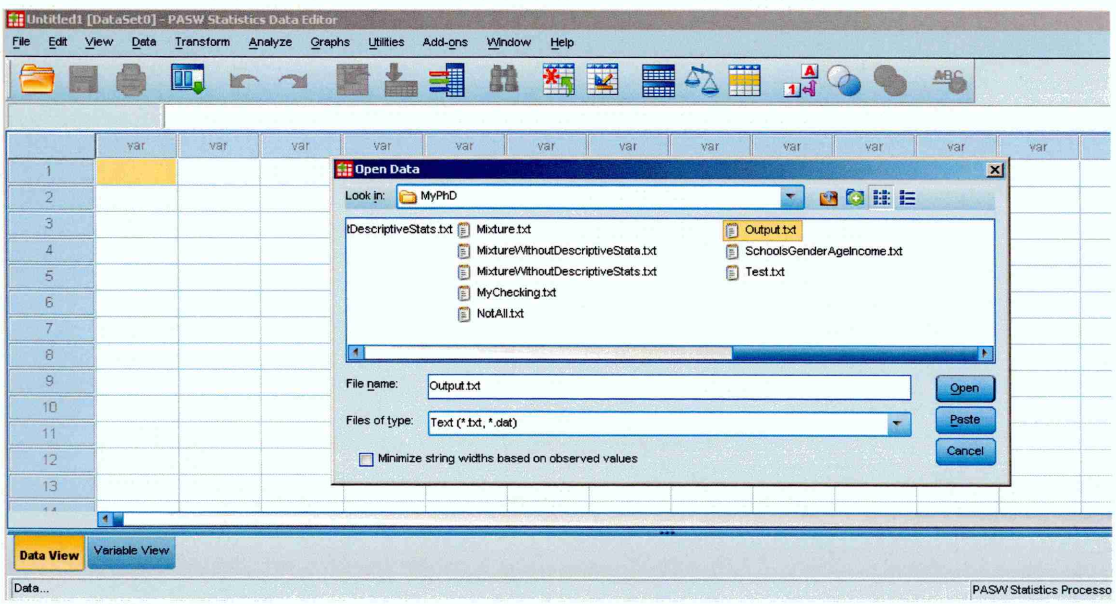
The tool, *tool2* does:

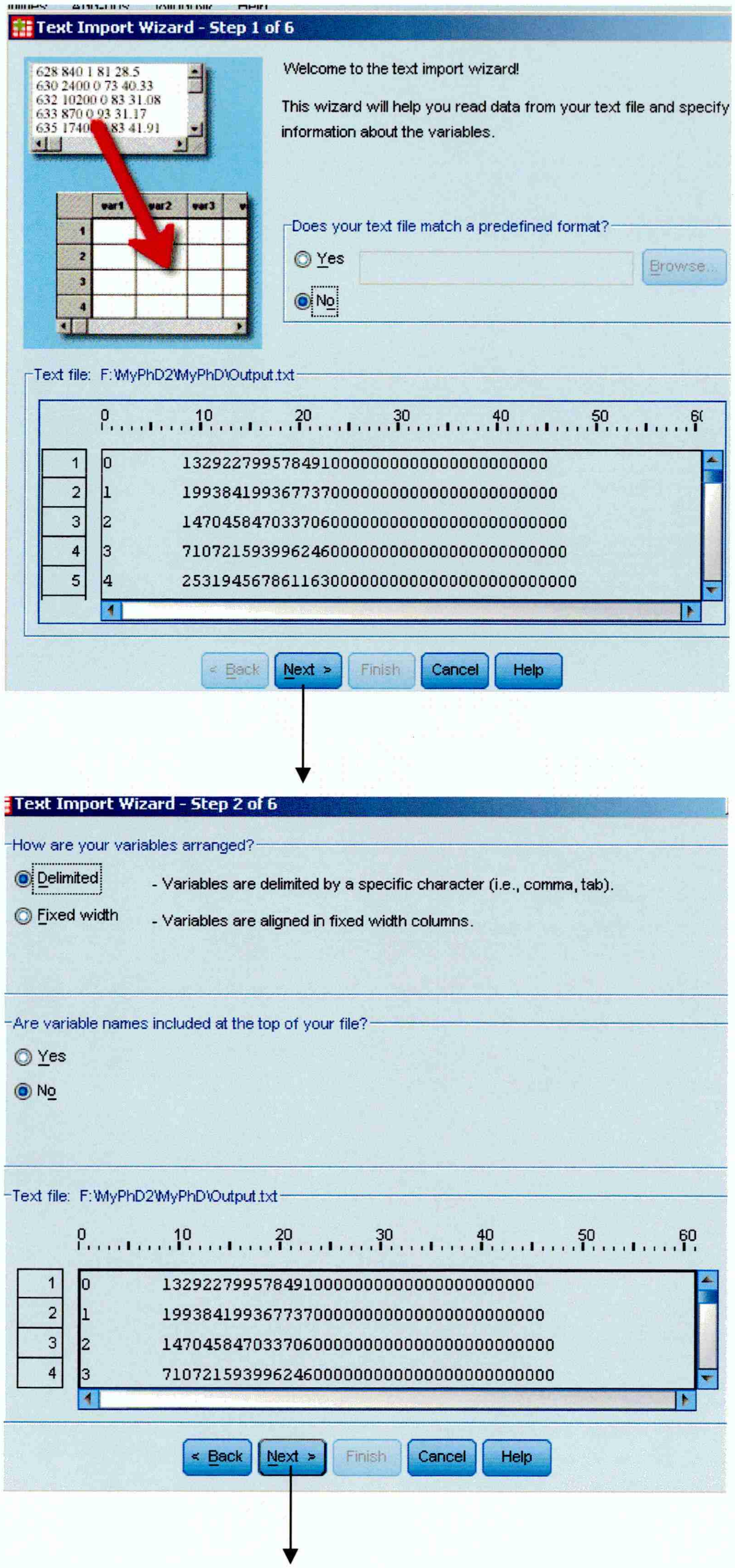
- use the correct formulae,
- calculate MEDIAN properly,
- calculate MODE properly,
- calculate correctly the proportion that pass for various pass marks.

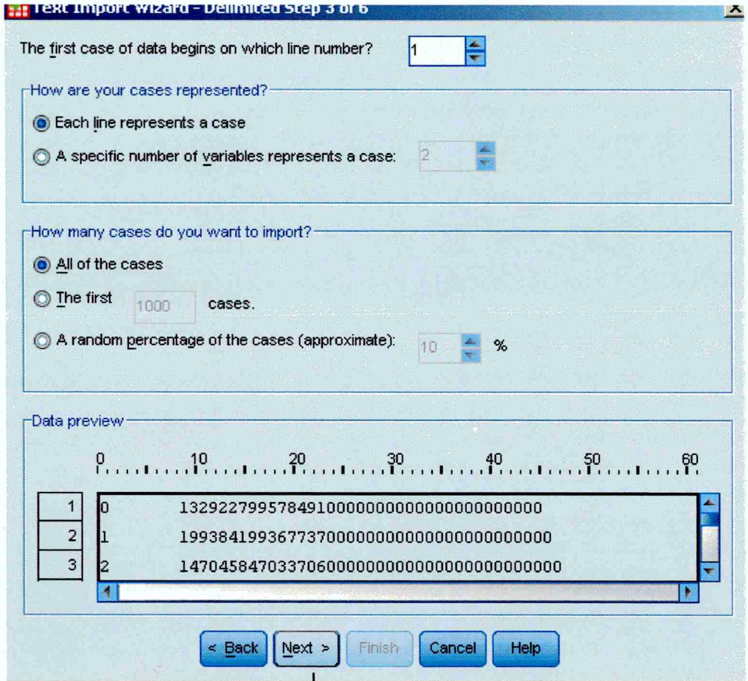
This Part of the Appendix shows how use SPSS to calculate descriptive statistics of 1 column.

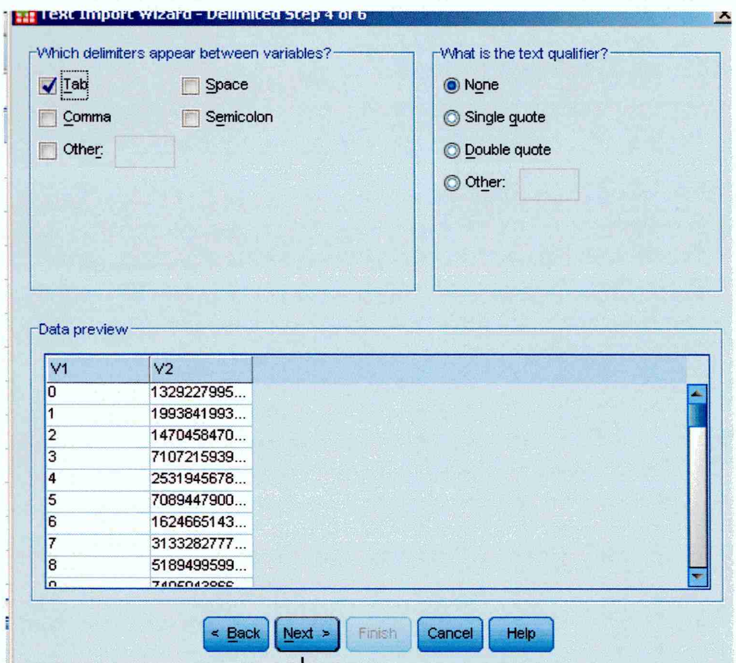
Open a file that contains tab-delimited text data.



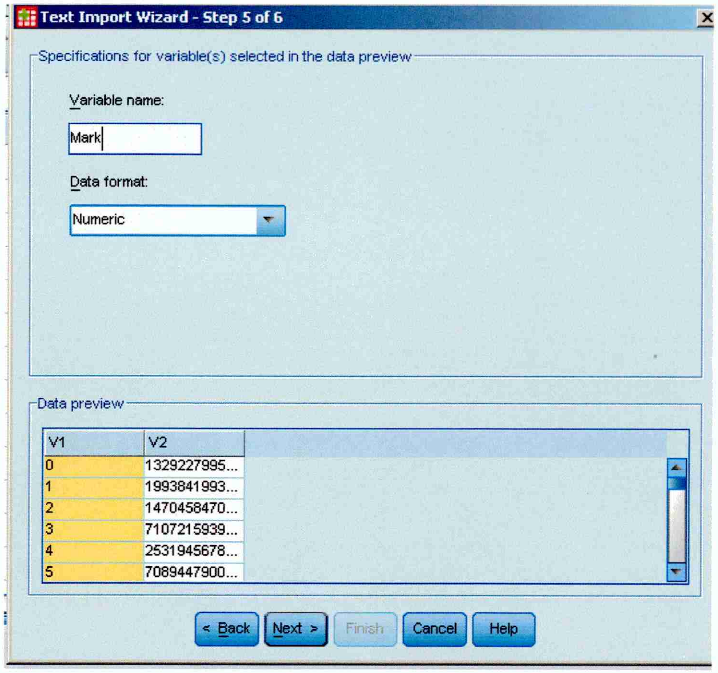



Text Import Wizard - Step 1 of 6
Welcome to the text import wizard!
This wizard will help you read data from your text file and specify information about the variables.
Does your text file match a predefined format?
Yes
No
Browse...
Text file: F:\MyPhD2\MyPhD\Output.txt
0 1329227995784910000000000000000000000
1 1993841993677370000000000000000000000
2 1470458470337060000000000000000000000
3 7107215939962460000000000000000000000
4 2531945678611630000000000000000000000
< Back
Next >
Finish
Cancel
Help
Text Import Wizard - Step 2 of 6
How are your variables arranged?
Delimited - Variables are delimited by a specific character (i.e., comma, tab).
Fixed width - Variables are aligned in fixed width columns.
Are variable names included at the top of your file?
Yes
No
Text file: F:\MyPhD2\MyPhD\Output.txt
0 1329227995784910000000000000000000000
1 1993841993677370000000000000000000000
2 1470458470337060000000000000000000000
3 7107215939962460000000000000000000000
< Back
Next >
Finish
Cancel
Help

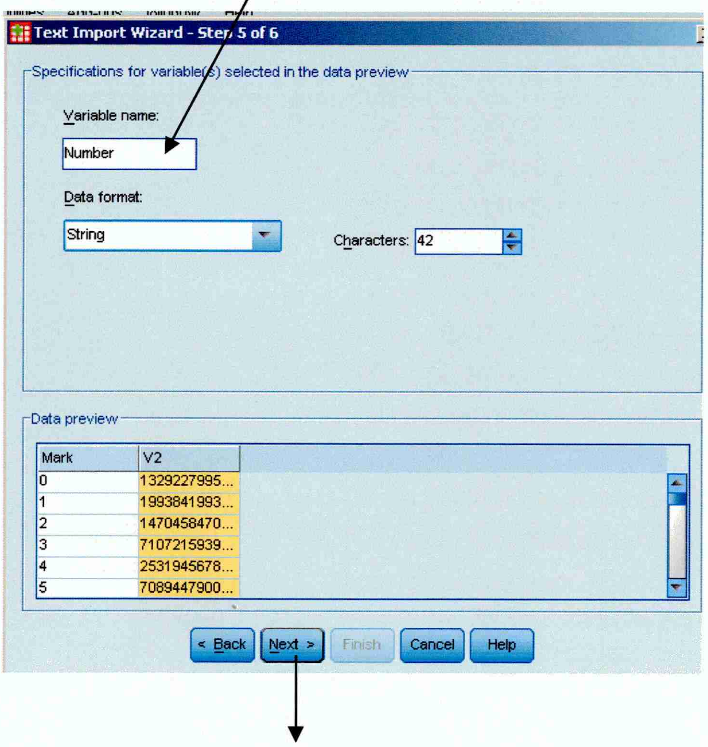

Text Import Wizard - Delimited Step 3 of 6
The first case of data begins on which line number?
How are your cases represented?
Each line represents a case
A specific number of variables represents a case:
How many cases do you want to import?
All of the cases
The first
1000
cases.
A random percentage of the cases (approximate):
10
%
Data preview
0 13292279957849100000000000000000000000
1 19938419936773700000000000000000000000
2 14704584703370600000000000000000000000
< Back
Next >
Finish
Cancel
Help

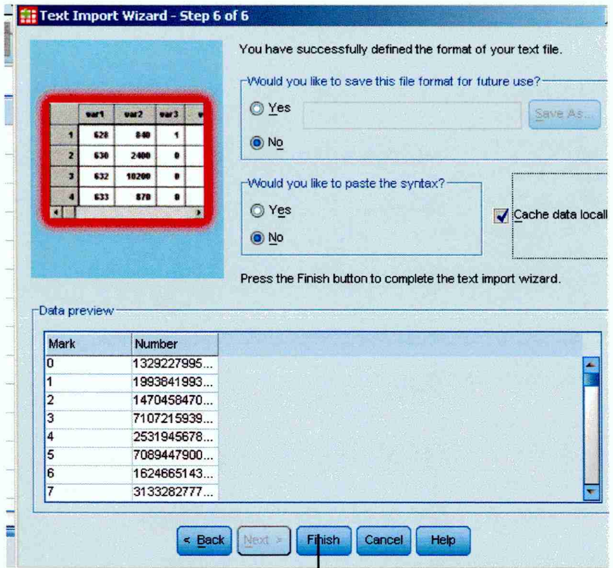

Text Import Wizard - Delimited Step 4 of 6
Which delimiters appear between variables?
Tab
Space
Comma
Semicolon
Other:
What is the text qualifier?
None
Single quote
Double quote
Other:
Data preview
V1 V2
0 1329227995...
1 1993841993...
2 1470458470...
3 7107215939...
4 2531945678...
5 7089447900...
6 1624665143...
7 3133282777...
8 5189499599...
< Back
Next >
Finish
Cancel
Help

Change **V1** into **Mark** (it does not matter as **Mark** will be ignored)

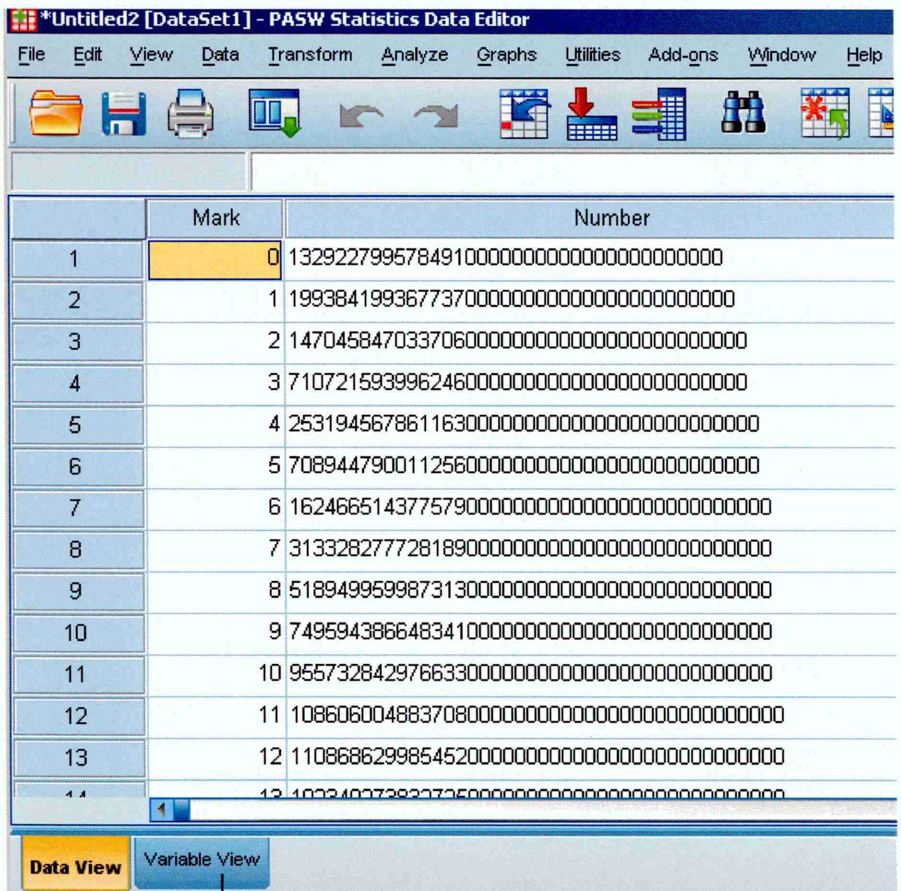


Change the variable **V2** into the variable **Number** (you will see that **Number** is of type String, which we will have to change back into an integer)



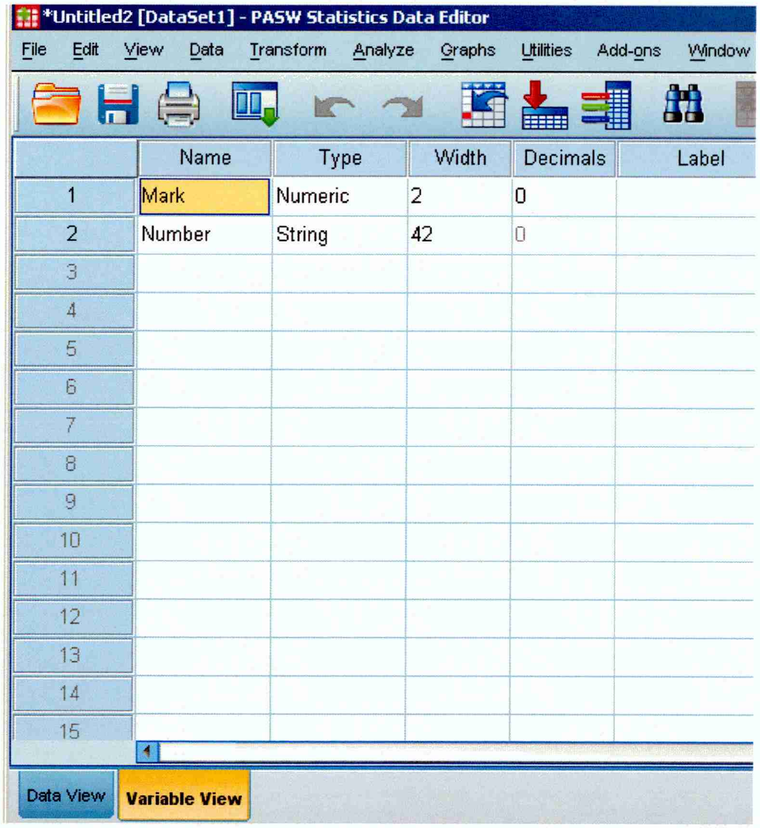


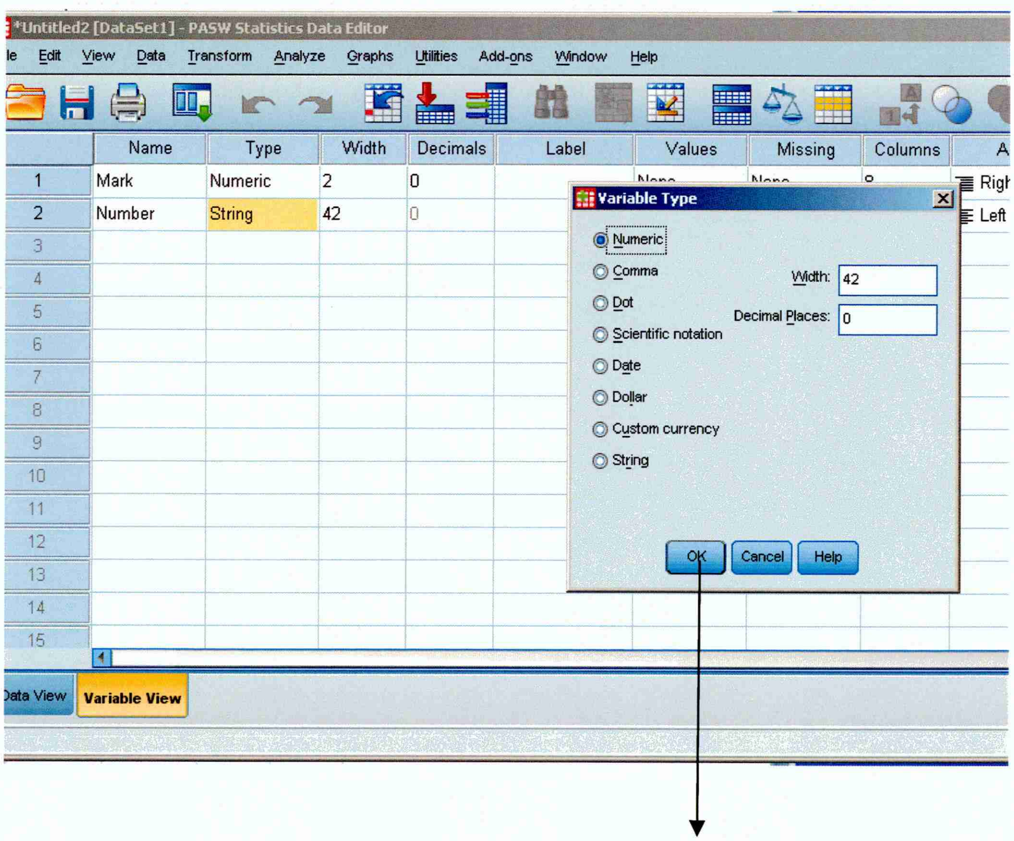
Text Import Wizard - Step 6 of 6
You have successfully defined the format of your text file.
Would you like to save this file format for future use?
Yes
Save As...
No
Would you like to paste the syntax?
Yes
No
Cache data locall
Press the Finish button to complete the text import wizard.
Data preview
Mark
Number
0 1329227995...
1 1993841993...
2 1470458470...
3 7107215939...
4 2531945678...
5 7089447900...
6 1624665143...
7 3133282777...
< Back
Next >
Finish
Cancel
Help

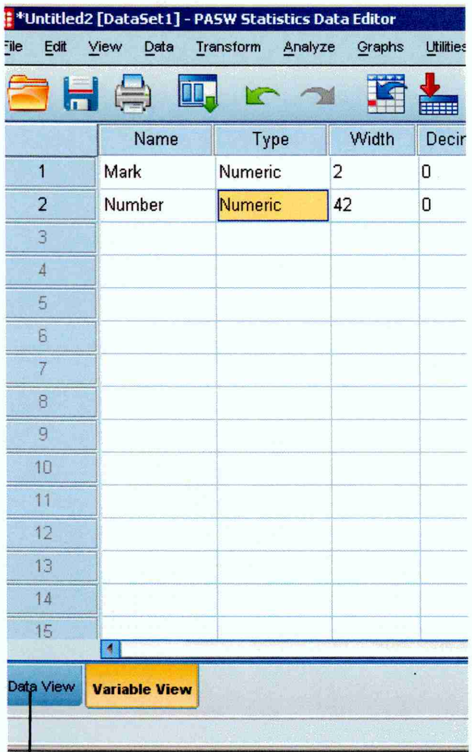


*Untitled2 [DataSet1] - PASW Statistics Data Editor
File Edit View Data Transform Analyze Graphs Utilities Add-ons Window Help
Mark
Number
1 0 1329227995784910000000000000000000000
2 1 1993841993677370000000000000000000000
3 2 1470458470337060000000000000000000000
4 3 7107215939962460000000000000000000000
5 4 2531945678611630000000000000000000000
6 5 7089447900112560000000000000000000000
7 6 1624665143775790000000000000000000000
8 7 3133282777281890000000000000000000000
9 8 5189499599873130000000000000000000000
10 9 7495943866483410000000000000000000000
11 10 9557328429766330000000000000000000000
12 11 10860600488370800000000000000000000000
13 12 11086862998545200000000000000000000000
Data View
Variable View

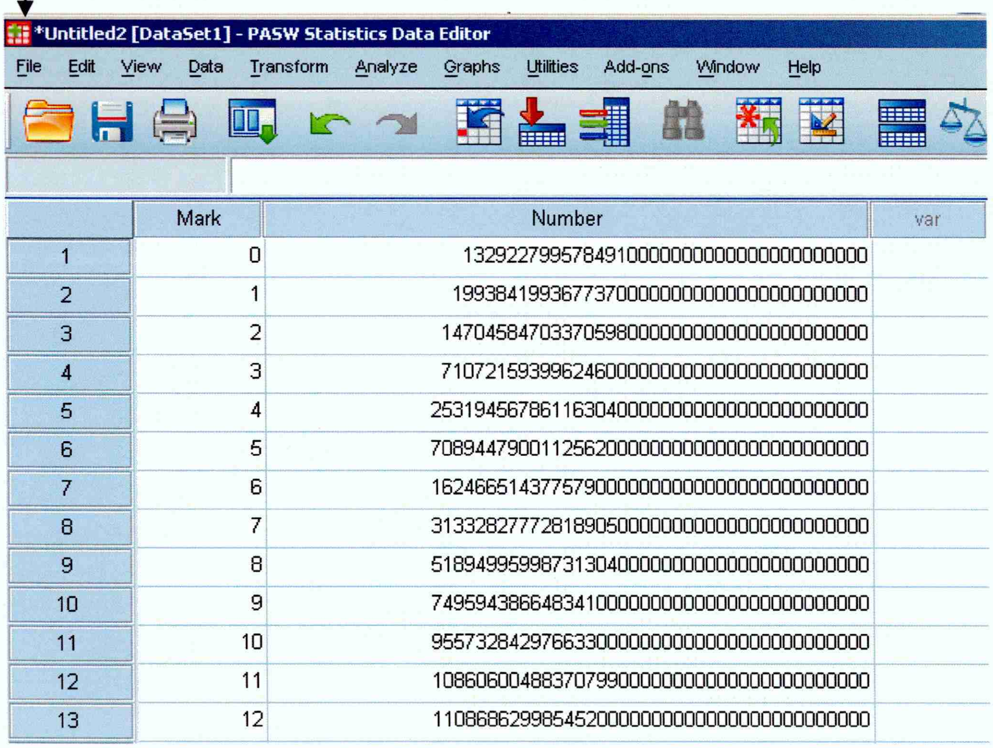
Change the **String** back into an **integer** (i.e. a **numeric** with 0 decimal places)





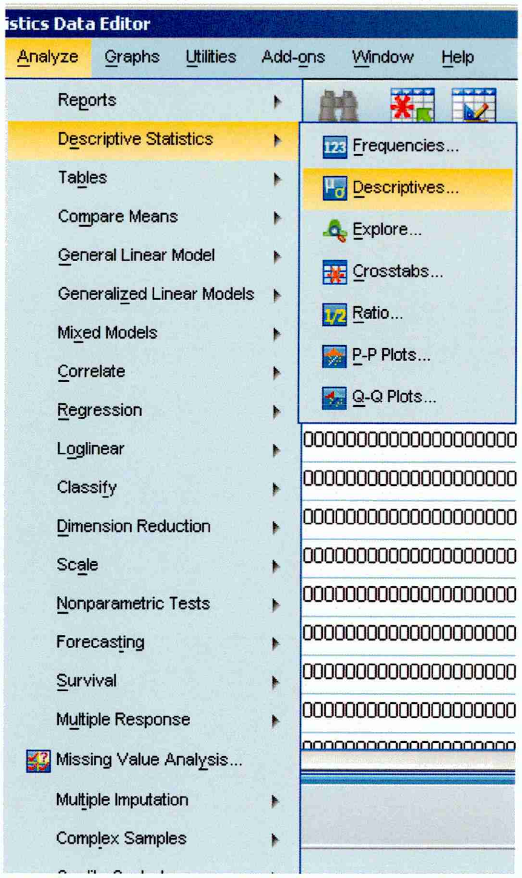


	Name	Type	Width	Decir
1	Mark	Numeric	2	0
2	Number	Numeric	42	0
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

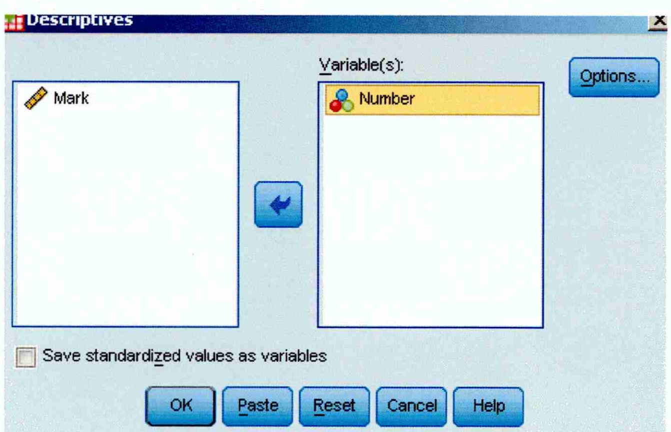


	Mark	Number	var
1	0	132922799578491000000000000000000000000000	
2	1	1993841993677370000000000000000000000000000	
3	2	14704584703370598000000000000000000000000000	
4	3	71072159399624600000000000000000000000000000	
5	4	253194567861163040000000000000000000000000000	
6	5	708944790011256200000000000000000000000000000	
7	6	162466514377579000000000000000000000000000000	
8	7	313328277728189050000000000000000000000000000	
9	8	518949959987313040000000000000000000000000000	
10	9	749594386648341000000000000000000000000000000	
11	10	955732842976633000000000000000000000000000000	
12	11	108606004883707990000000000000000000000000000	
13	12	110868629985452000000000000000000000000000000	

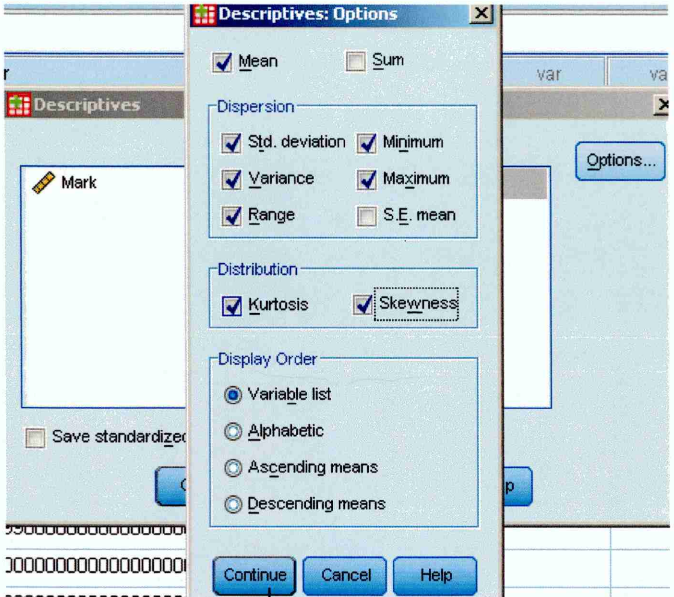
If you **Analyze Number** now and ask for its descriptive statistics, you get

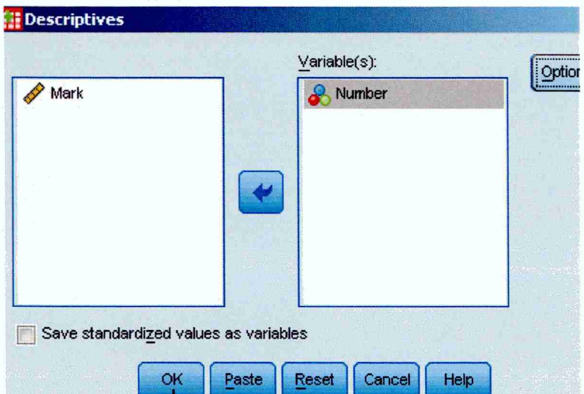


Put **Number** in the pane where analysis is done



and then ask for the Options that you want.





Descriptives

[DataSet1]

Descriptive Statistics

	N	Range	Minimum	Maximum	Mean	Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Number	61	1.E40	1	1.E40	1.38E39	2.621E39	6.868E78	2.045	.306	3.186	.604
Valid N (listwise)	61										

So, now, we have the descriptive statistics produced by SPSS for 1 column.

Appendix 17. Loading the LEDA routines and integrating with Visual Studio 10.

17.1 Introduction

The abbreviation VS will be used throughout this appendix. VS stands for **V**isual **S**tudio 10. VS can run programs written in the programming language C, but C normally has an upper limit for integers and we go past this upper limit. The LEDA routines (written by **A**lgorithmic **S**olutions(AS) of Germany) allow integers of any size to be used, but this involves a 4 stage process

1. Installing VS
2. Installing the LEDA routine.
3. Configure VS to work with LEDA.
4. Setting up system variables

We shall describe, here, steps 2, 3 and 4 and assume 1 has been done.

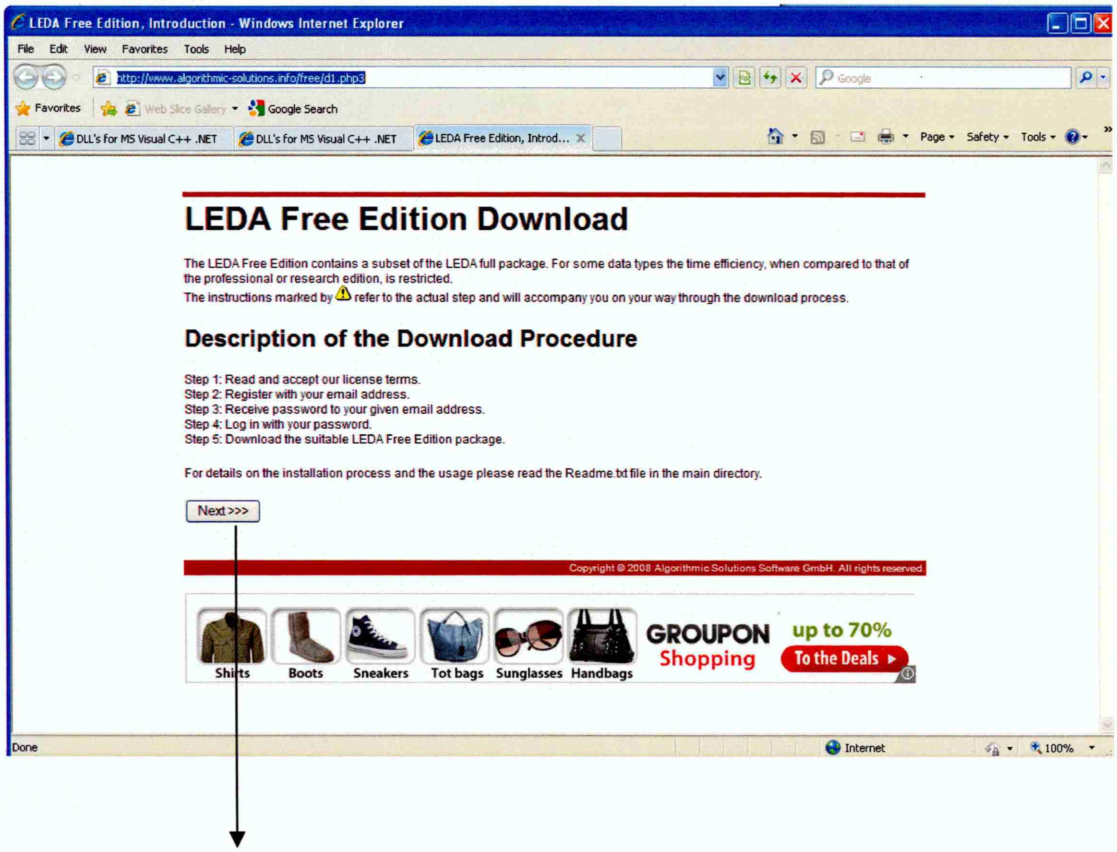
Step 2 is relatively pain free, but the step 3 is very long and hence error prone.

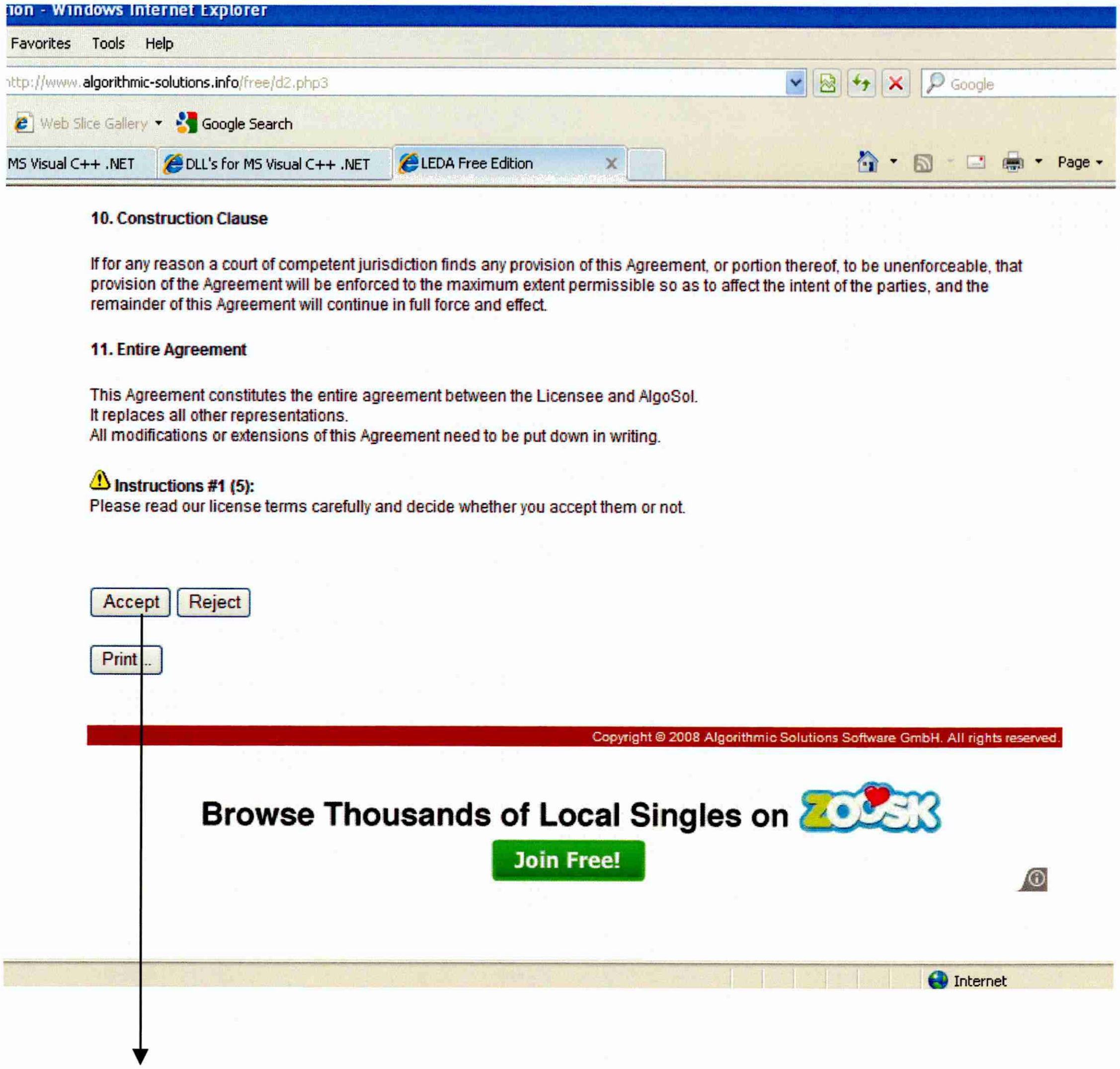
The numbers and instructions used in step 3 are an amalgam of those provided by AS and those provided by us.

2. Install LEDA

I installed the Free Edition of LEDA from

http://www.algorithmic-solutions.info/free/d1.php3





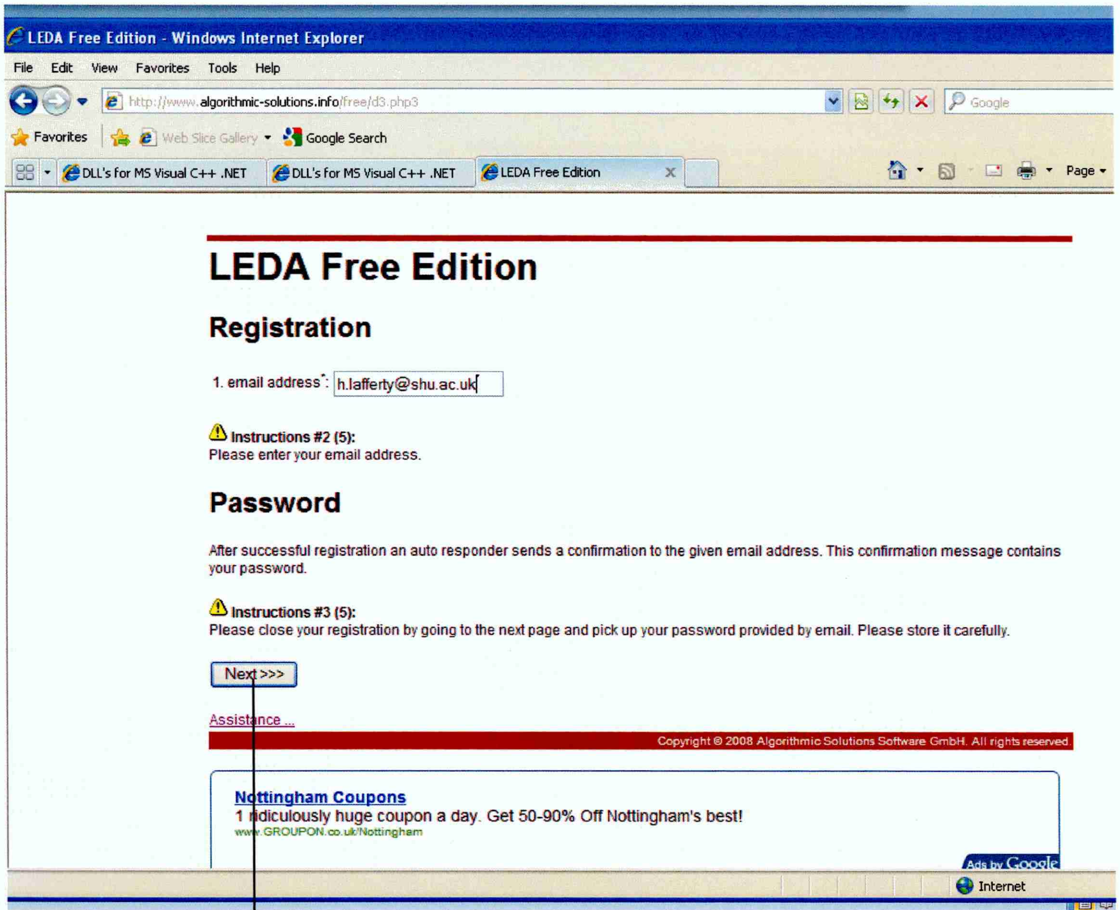
10. Construction Clause

If for any reason a court of competent jurisdiction finds any provision of this Agreement, or portion thereof, to be unenforceable, that provision of the Agreement will be enforced to the maximum extent permissible so as to affect the intent of the parties, and the remainder of this Agreement will continue in full force and effect.

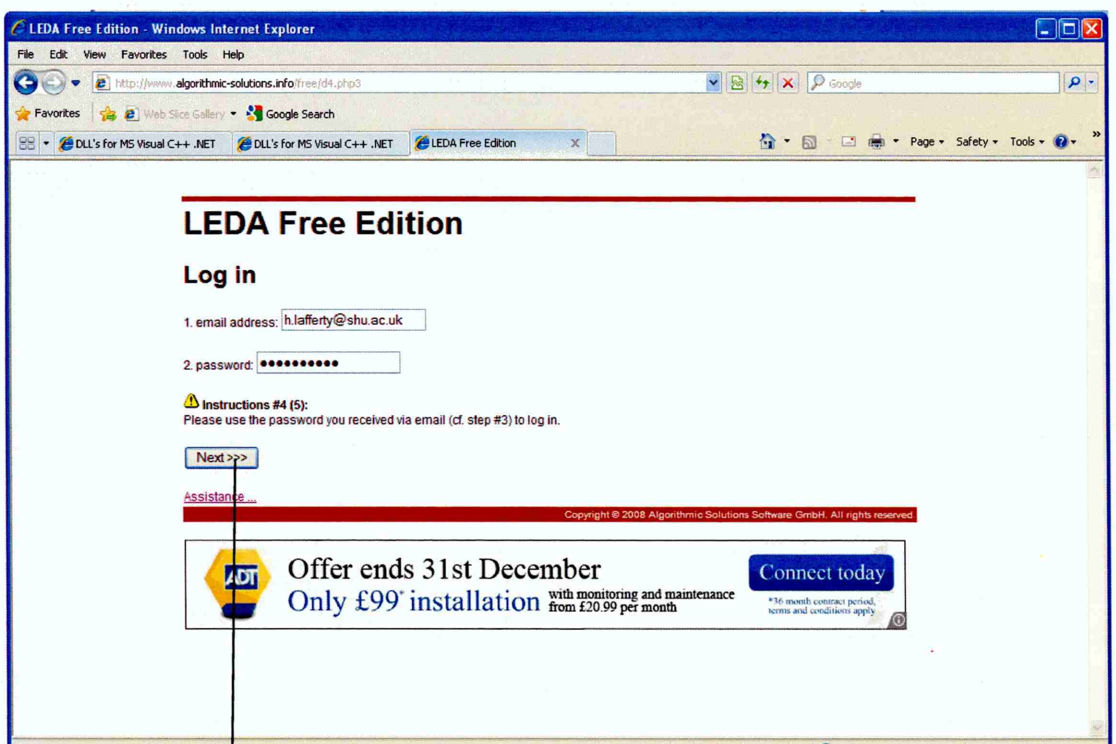
11. Entire Agreement

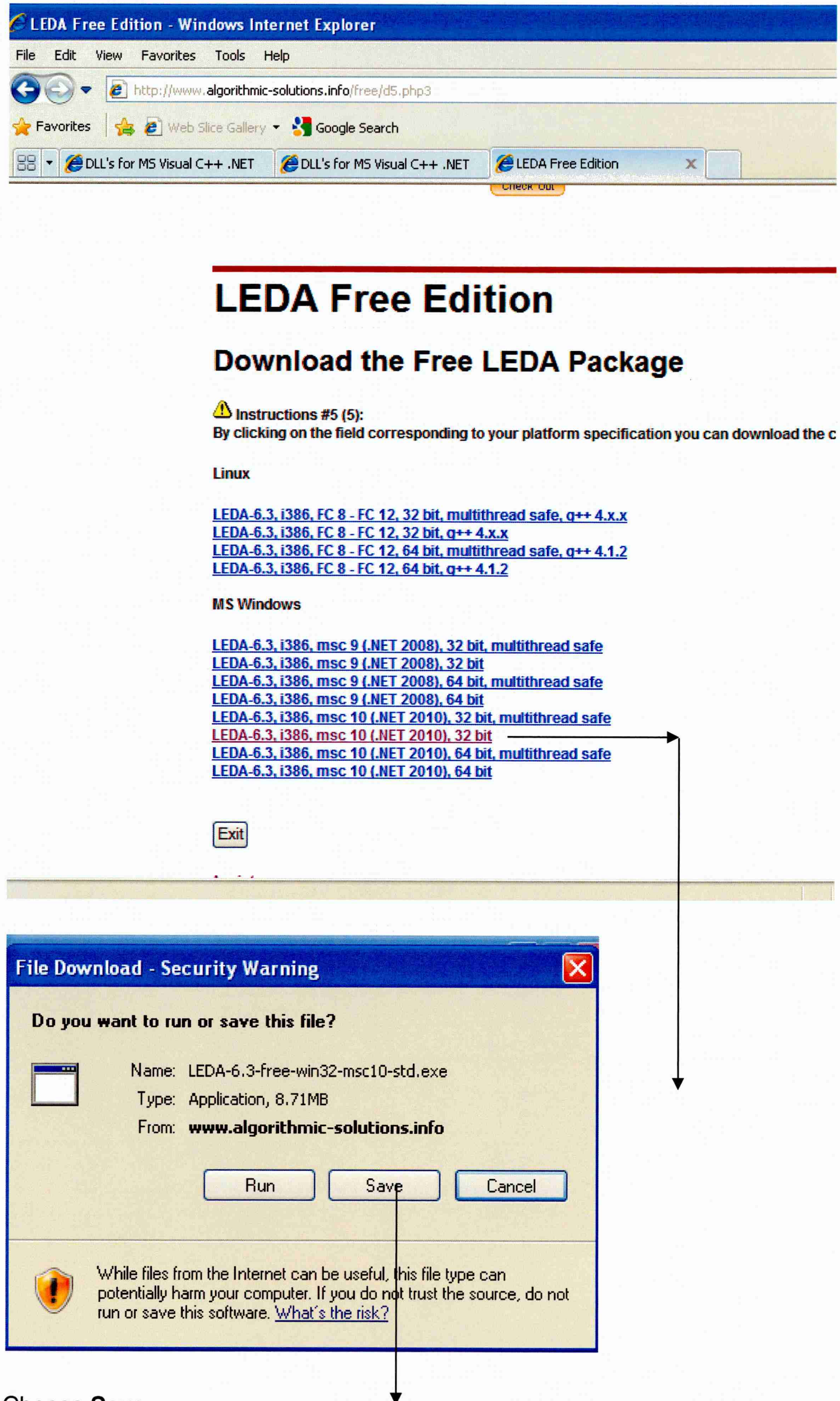
This Agreement constitutes the entire agreement between the Licensee and AlgoSol.
It replaces all other representations.
All modifications or extensions of this Agreement need to be put down in writing.

Instructions #1 (5):
Please read our license terms carefully and decide whether you accept them or not.



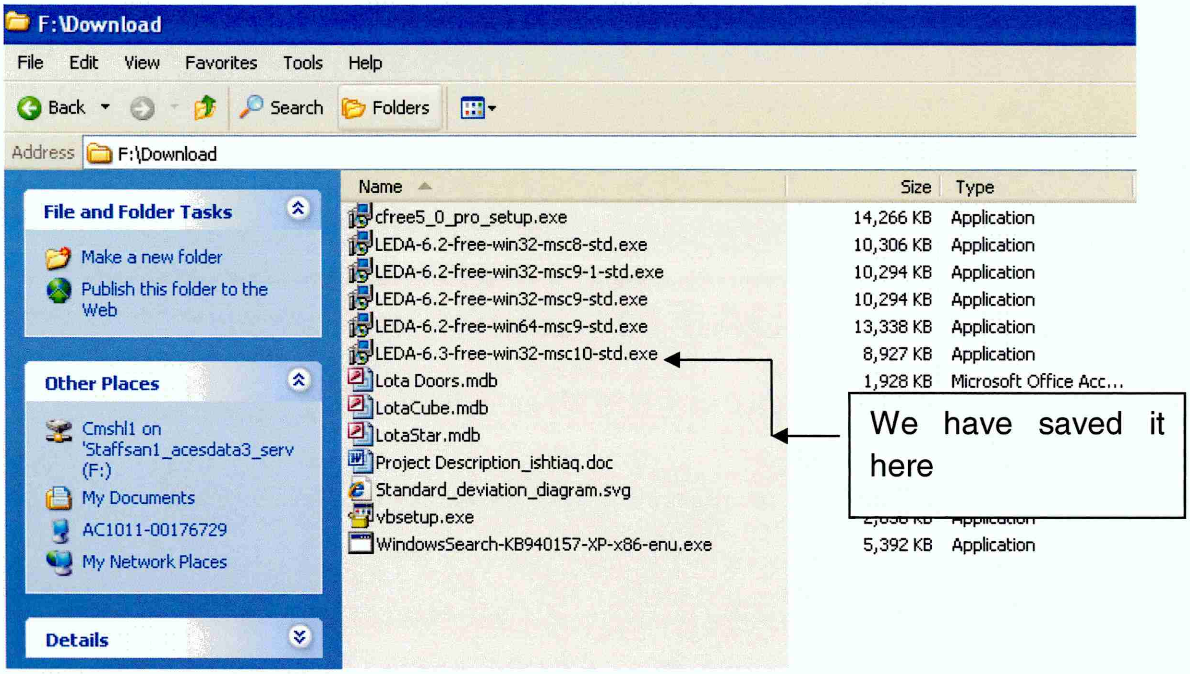
They will send you a password by e-mail (very quickly) and you copy this and paste it into the next screen



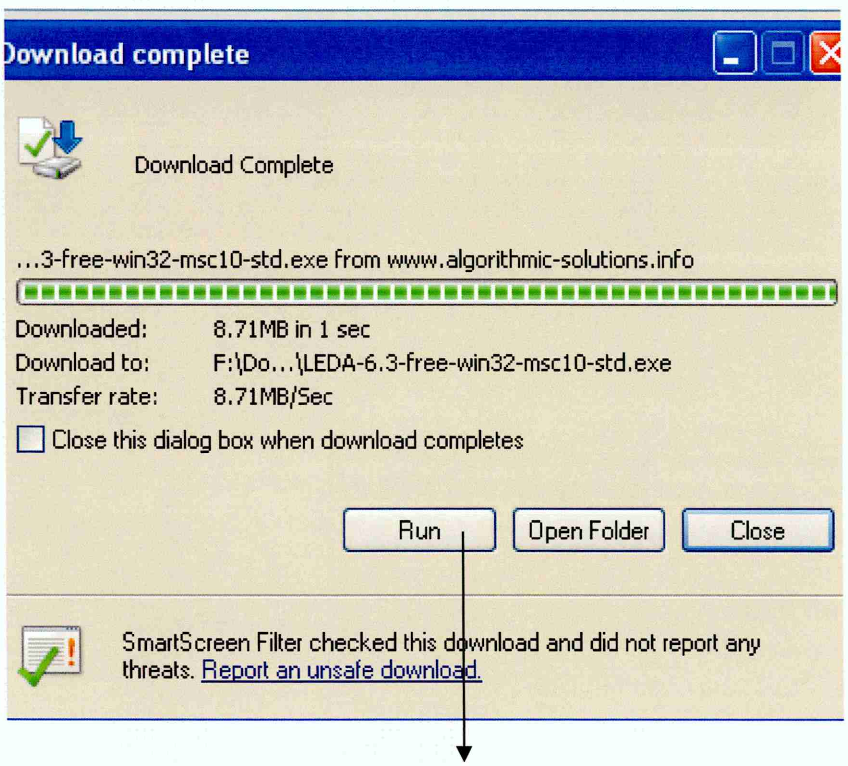

LEDA Free Edition - Windows Internet Explorer
File Edit View Favorites Tools Help
http://www.algorithmic-solutions.info/free/d5.php3
Favorites
Web Slice Gallery
Google Search
DLL's for MS Visual C++ .NET
DLL's for MS Visual C++ .NET
LEDA Free Edition
LEDA Free Edition
Download the Free LEDA Package
Instructions #5 (5):
By clicking on the field corresponding to your platform specification you can download the c
Linux
LEDA-6.3, i386, FC 8 - FC 12, 32 bit, multithread safe, g++ 4.x.x
LEDA-6.3, i386, FC 8 - FC 12, 32 bit, g++ 4.x.x
LEDA-6.3, i386, FC 8 - FC 12, 64 bit, multithread safe, g++ 4.1.2
LEDA-6.3, i386, FC 8 - FC 12, 64 bit, g++ 4.1.2
MS Windows
LEDA-6.3, i386, msc 9 (.NET 2008), 32 bit, multithread safe
LEDA-6.3, i386, msc 9 (.NET 2008), 32 bit
LEDA-6.3, i386, msc 9 (.NET 2008), 64 bit, multithread safe
LEDA-6.3, i386, msc 9 (.NET 2008), 64 bit
LEDA-6.3, i386, msc 10 (.NET 2010), 32 bit, multithread safe
LEDA-6.3, i386, msc 10 (.NET 2010), 32 bit
LEDA-6.3, i386, msc 10 (.NET 2010), 64 bit, multithread safe
LEDA-6.3, i386, msc 10 (.NET 2010), 64 bit
Exit
File Download - Security Warning
Do you want to run or save this file?
Name: LEDA-6.3-free-win32-msc10-std.exe
Type: Application, 8.71MB
From: www.algorithmic-solutions.info
Run
Save
Cancel
While files from the Internet can be useful, this file type can potentially harm your computer. If you do not trust the source, do not run or save this software. What's the risk?

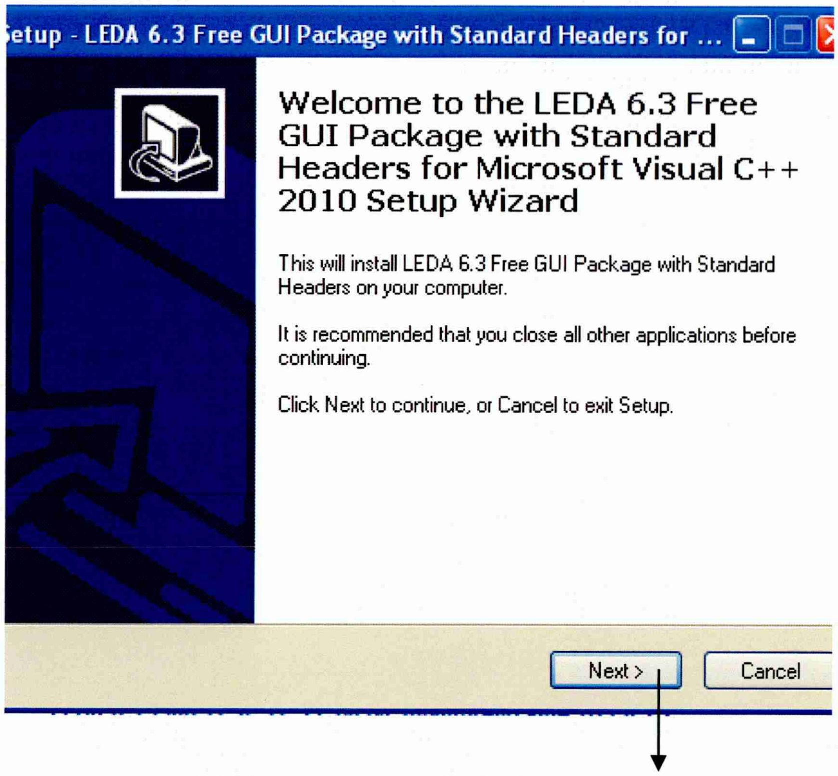
Choose **Save**

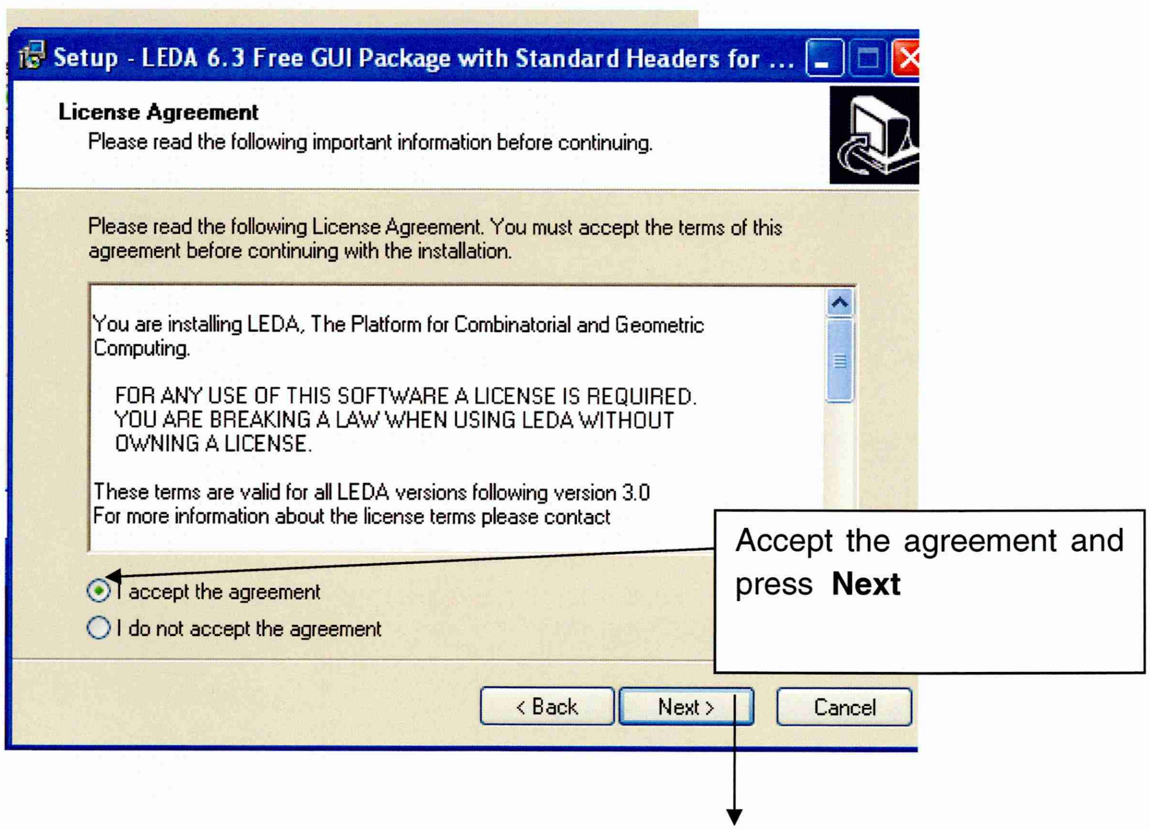
You will see that we have saved it on F: drive in folder Download

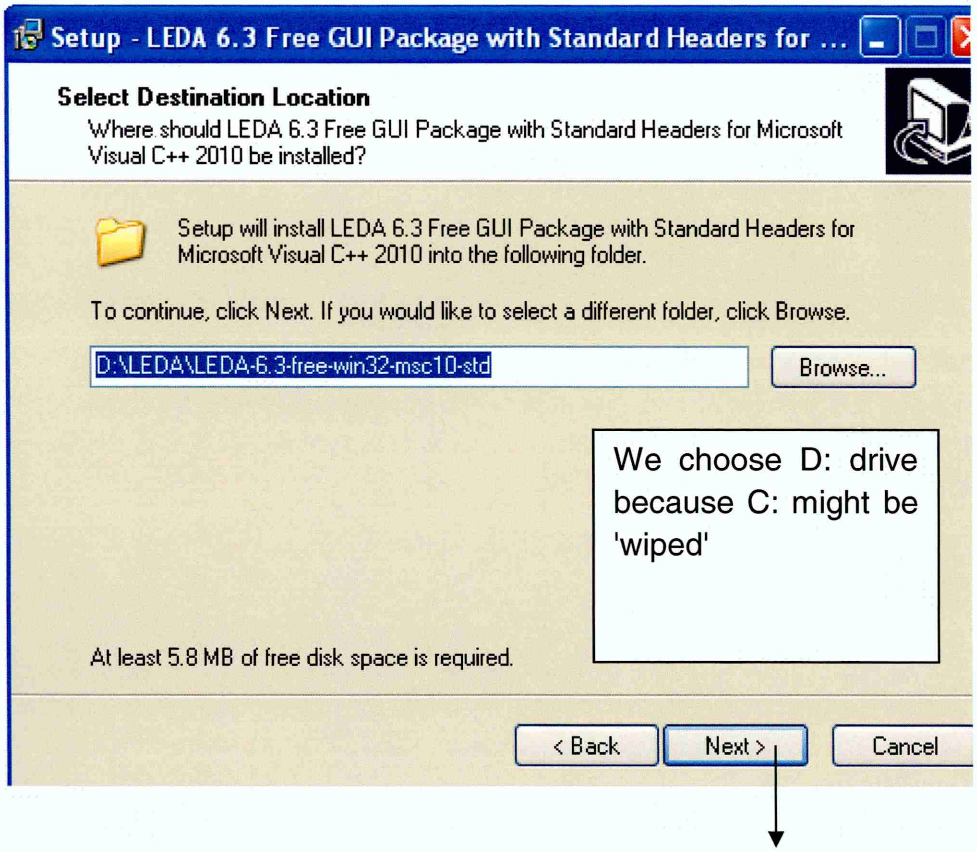


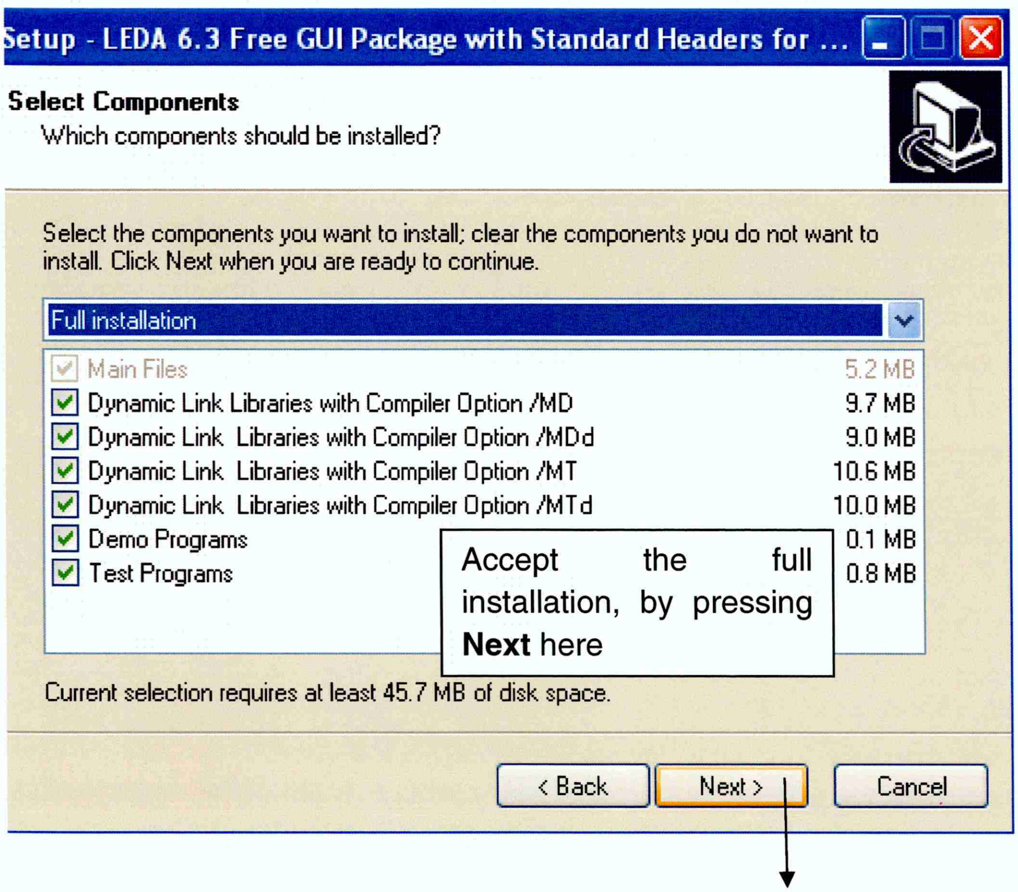
We then **Run** it

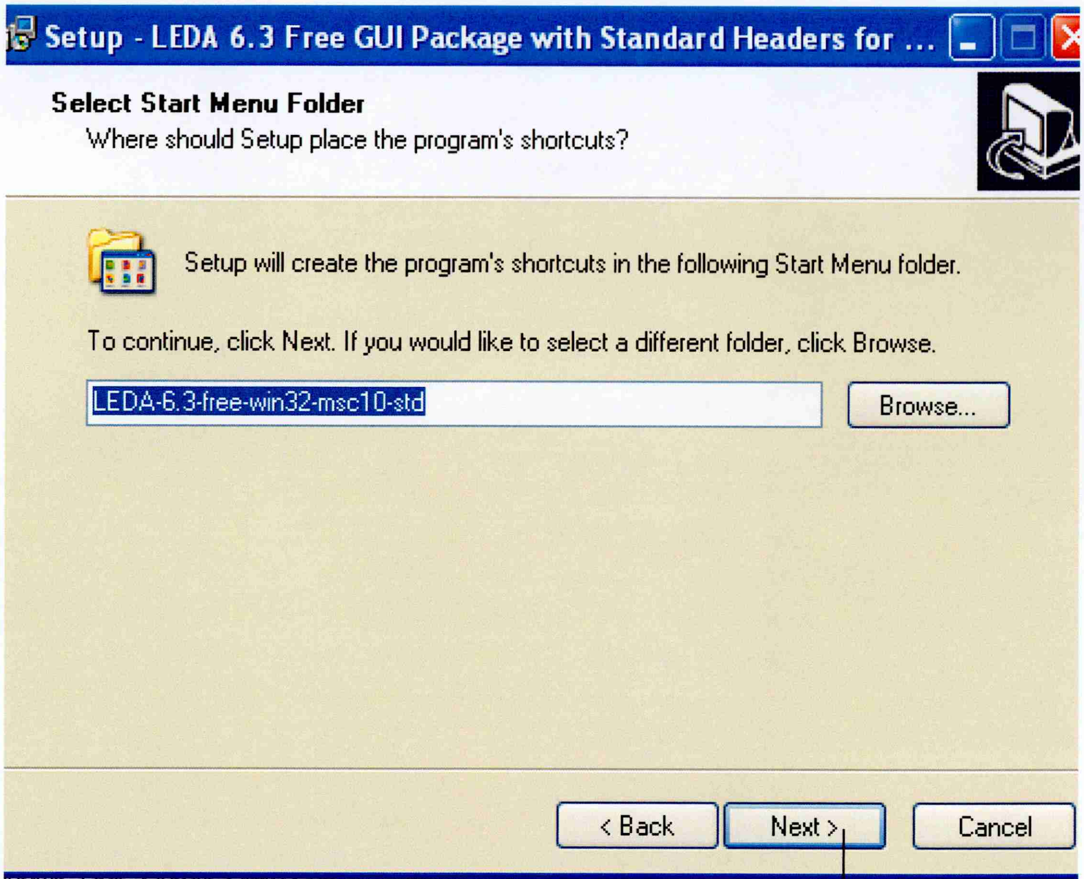


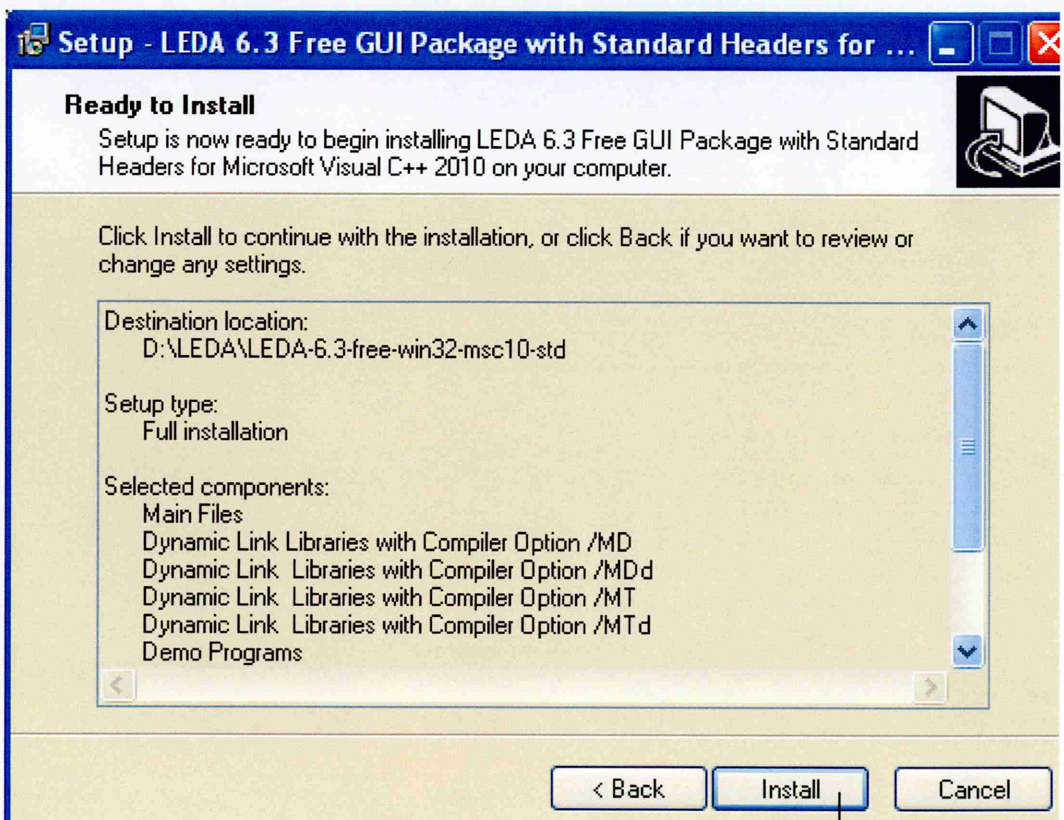

Setup - LEDA 6.3 Free GUI Package with Standard Headers for ...
Welcome to the LEDA 6.3 Free GUI Package with Standard Headers for Microsoft Visual C++ 2010 Setup Wizard
This will install LEDA 6.3 Free GUI Package with Standard Headers on your computer.
It is recommended that you close all other applications before continuing.
Click Next to continue, or Cancel to exit Setup.
Next >
Cancel


Setup - LEDA 6.3 Free GUI Package with Standard Headers for ...
License Agreement
Please read the following important information before continuing.
Please read the following License Agreement. You must accept the terms of this agreement before continuing with the installation.
You are installing LEDA, The Platform for Combinatorial and Geometric Computing.
FOR ANY USE OF THIS SOFTWARE A LICENSE IS REQUIRED. YOU ARE BREAKING A LAW WHEN USING LEDA WITHOUT OWNING A LICENSE.
These terms are valid for all LEDA versions following version 3.0
For more information about the license terms please contact
I accept the agreement
I do not accept the agreement
< Back
Next >
Cancel
Accept the agreement and press Next

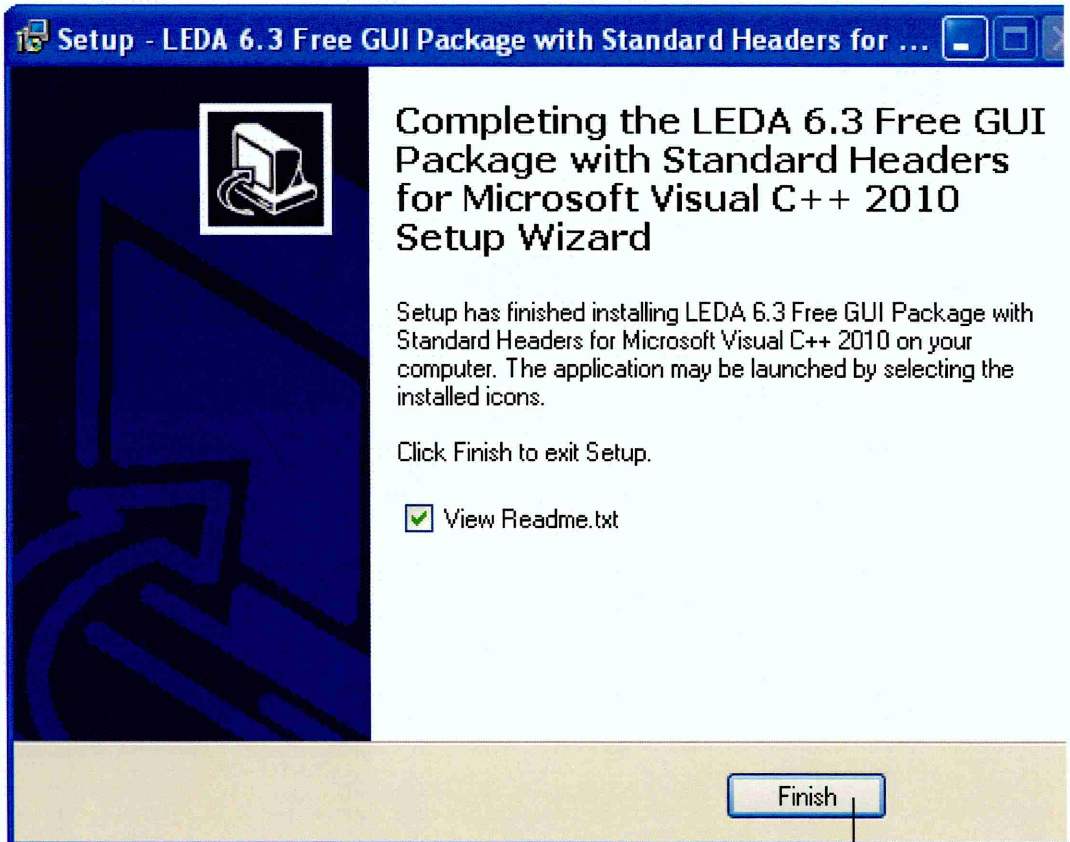

Setup - LEDA 6.3 Free GUI Package with Standard Headers for ...
Select Destination Location
Where should LEDA 6.3 Free GUI Package with Standard Headers for Microsoft Visual C++ 2010 be installed?
Setup will install LEDA 6.3 Free GUI Package with Standard Headers for Microsoft Visual C++ 2010 into the following folder.
To continue, click Next. If you would like to select a different folder, click Browse.
D:\LEDA\LEDA-6.3-free-win32-msc10-std
Browse...
We choose D: drive because C: might be 'wiped'
At least 5.8 MB of free disk space is required.
< Back
Next >
Cancel


Setup - LEDA 6.3 Free GUI Package with Standard Headers for ...
Select Components
Which components should be installed?
Select the components you want to install; clear the components you do not want to install. Click Next when you are ready to continue.
Full installation
Main Files 5.2 MB
Dynamic Link Libraries with Compiler Option /MD 9.7 MB
Dynamic Link Libraries with Compiler Option /MDd 9.0 MB
Dynamic Link Libraries with Compiler Option /MT 10.6 MB
Dynamic Link Libraries with Compiler Option /MTd 10.0 MB
Demo Programs 0.1 MB
Test Programs 0.8 MB
Accept the full installation, by pressing Next here
Current selection requires at least 45.7 MB of disk space.
< Back
Next >
Cancel

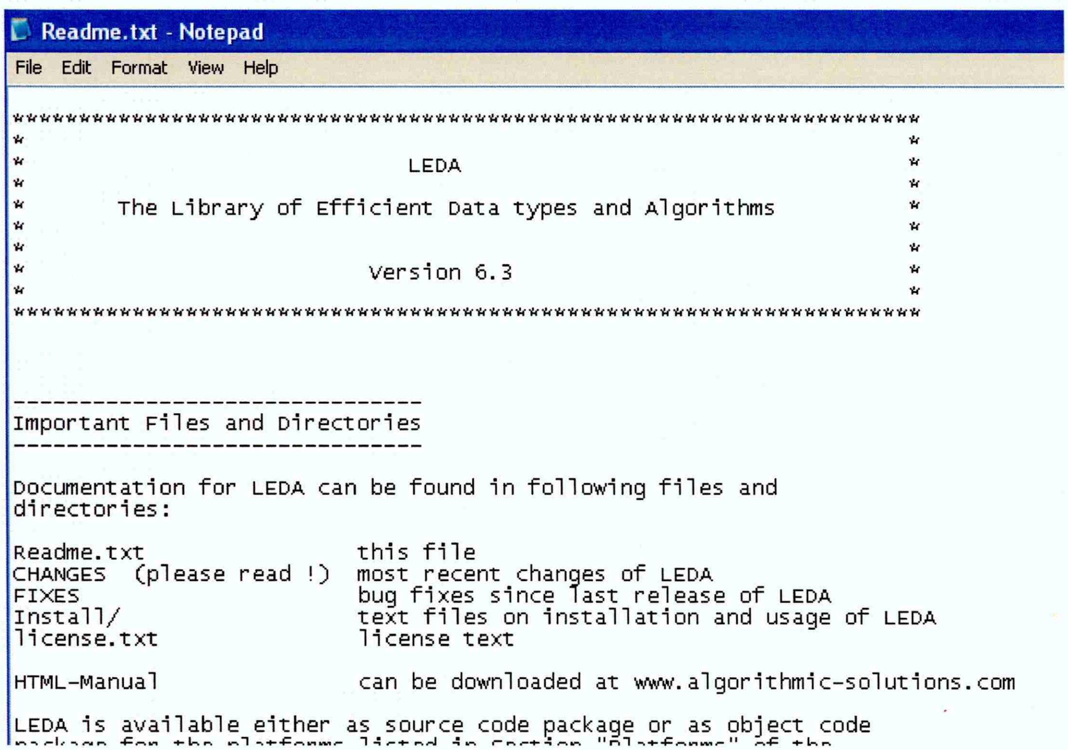




This takes about 20 seconds



The **Readme.txt** comes up in Notepad, a bit of which we show



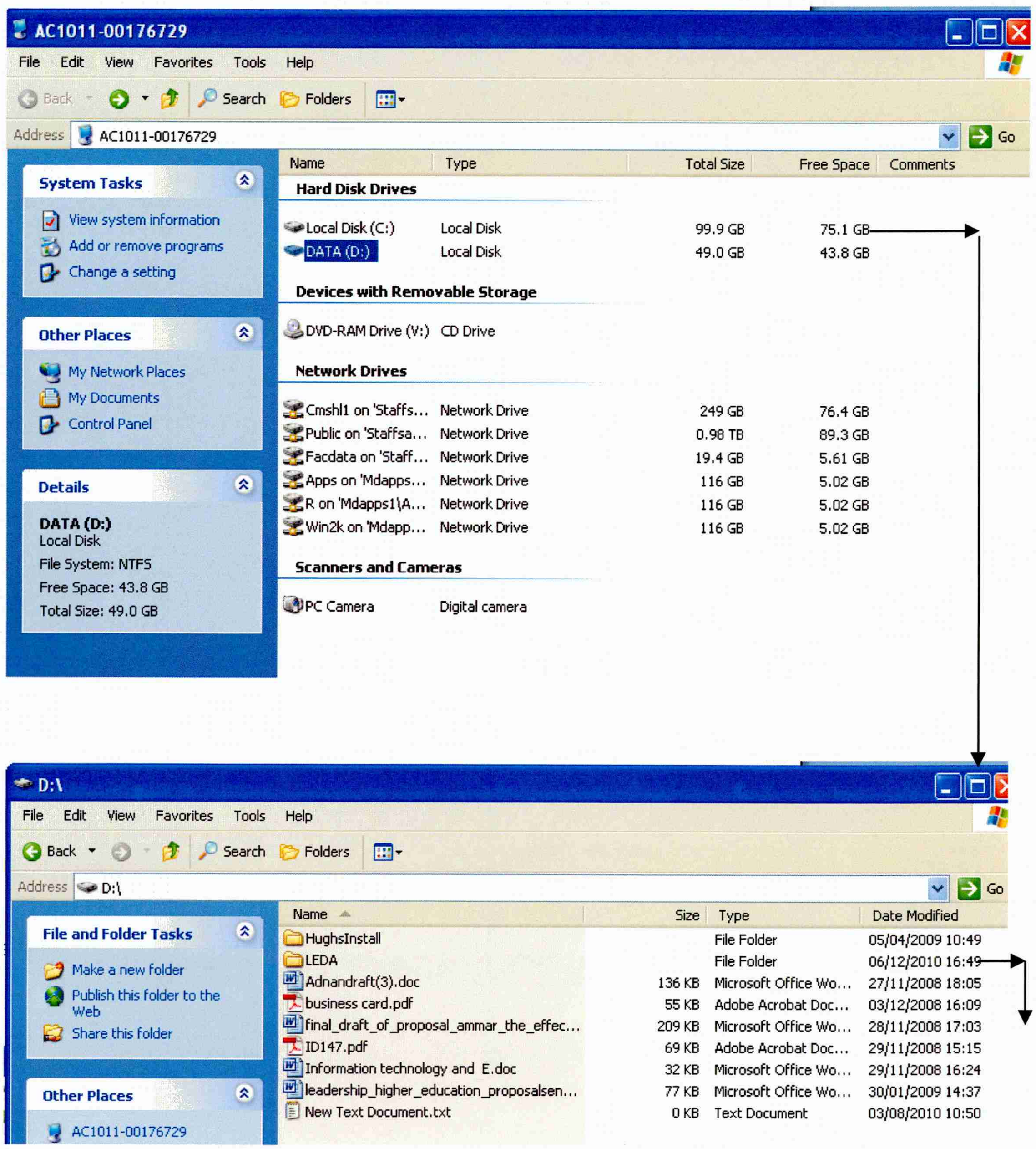

```
*********************************************************************
*                                                                   *
*                               LEDA                                *
*                                                                   *
*       The Library of Efficient Data types and Algorithms          *
*                                                                   *
*                                                                   *
*                            Version 6.3                            *
*                                                                   *
*********************************************************************


-------------------------------
Important Files and Directories
-------------------------------

Documentation for LEDA can be found in following files and
directories:

Readme.txt                this file
CHANGES  (please read !)  most recent changes of LEDA
FIXES                     bug fixes since last release of LEDA
Install/                  text files on installation and usage of LEDA
license.txt               license text

HTML-Manual               can be downloaded at www.algorithmic-solutions.com

LEDA is available either as source code package or as object code
```

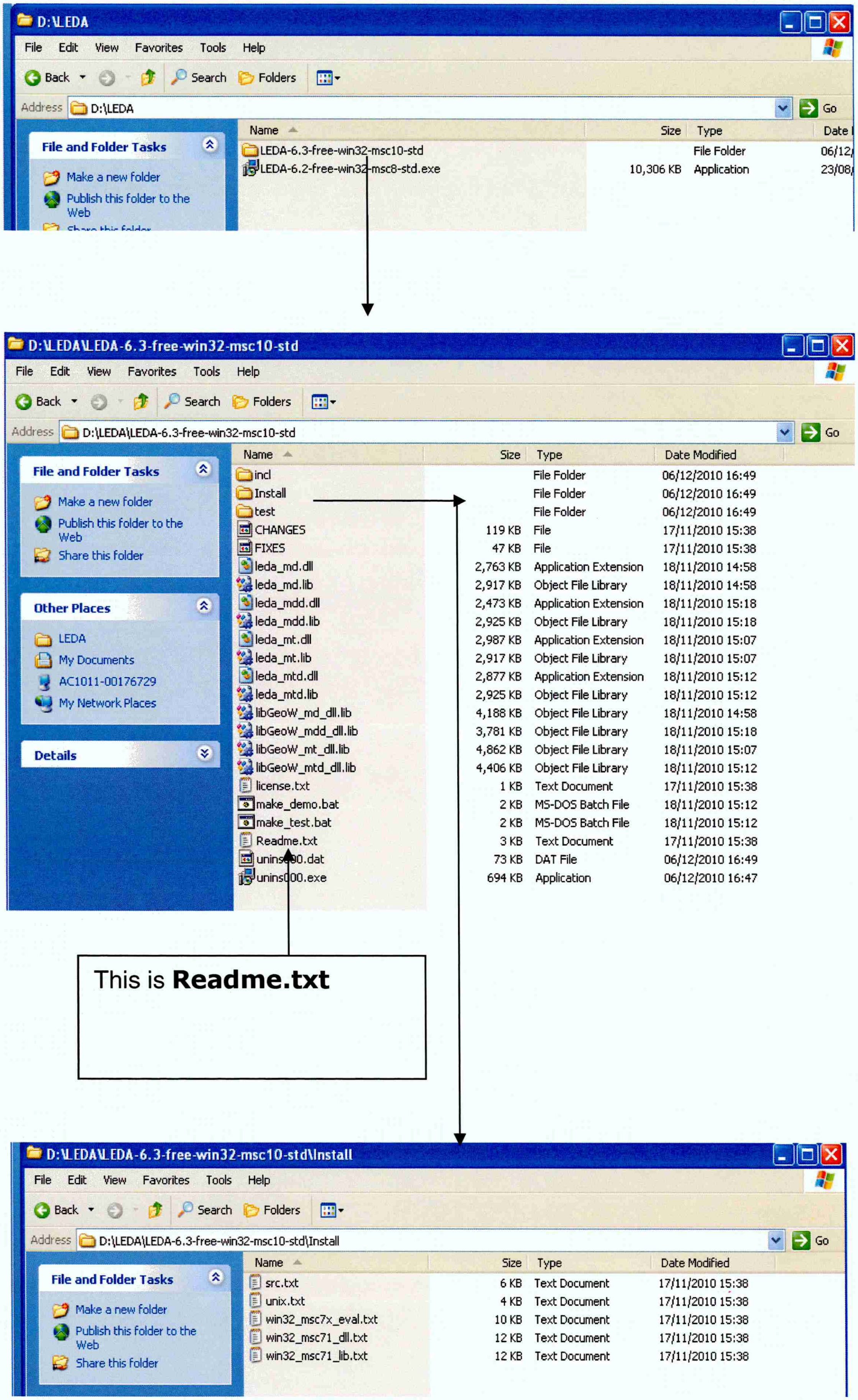
So, LEDA is now in

D:\LEDA\LEDA-6.3-free-win32-msc10-std

The LEDA libraries have been put on D: drive because if you put them on C: drive they will be 'wiped' every so often.

I think that you still have to configure VS every time C: drive is wiped, and we have to answer the question "What happens if D: drive is wiped or corrupted?" I think then you have to i) re-image C: and D: ii) download LEDA to D: iii) configure VS. Step ii could be shortened by saving LEDA to DVD or a stick.




D:\LEDA
File Edit View Favorites Tools Help
Back Search Folders
Address D:\LEDA
File and Folder Tasks
Make a new folder
Publish this folder to the Web
Name Size Type Date
LEDA-6.3-free-win32-msc10-std File Folder 06/12/
LEDA-6.2-free-win32-msc8-std.exe 10,306 KB Application 23/08/
D:\LEDA\LEDA-6.3-free-win32-msc10-std
Address D:\LEDA\LEDA-6.3-free-win32-msc10-std
File and Folder Tasks
Make a new folder
Publish this folder to the Web
Share this folder
Other Places
LEDA
My Documents
AC1011-00176729
My Network Places
Details
Name Size Type Date Modified
incl File Folder 06/12/2010 16:49
Install File Folder 06/12/2010 16:49
test File Folder 06/12/2010 16:49
CHANGES 119 KB File 17/11/2010 15:38
FIXES 47 KB File 17/11/2010 15:38
leda_md.dll 2,763 KB Application Extension 18/11/2010 14:58
leda_md.lib 2,917 KB Object File Library 18/11/2010 14:58
leda_mdd.dll 2,473 KB Application Extension 18/11/2010 15:18
leda_mdd.lib 2,925 KB Object File Library 18/11/2010 15:18
leda_mt.dll 2,987 KB Application Extension 18/11/2010 15:07
leda_mt.lib 2,917 KB Object File Library 18/11/2010 15:07
leda_mtd.dll 2,877 KB Application Extension 18/11/2010 15:12
leda_mtd.lib 2,925 KB Object File Library 18/11/2010 15:12
libGeoW_md_dll.lib 4,188 KB Object File Library 18/11/2010 14:58
libGeoW_mdd_dll.lib 3,781 KB Object File Library 18/11/2010 15:18
libGeoW_mt_dll.lib 4,862 KB Object File Library 18/11/2010 15:07
libGeoW_mtd_dll.lib 4,406 KB Object File Library 18/11/2010 15:12
license.txt 1 KB Text Document 17/11/2010 15:38
make_demo.bat 2 KB MS-DOS Batch File 18/11/2010 15:12
make_test.bat 2 KB MS-DOS Batch File 18/11/2010 15:12
Readme.txt 3 KB Text Document 17/11/2010 15:38
unins000.dat 73 KB DAT File 06/12/2010 16:49
unins000.exe 694 KB Application 06/12/2010 16:47
This is **Readme.txt**
D:\LEDA\LEDA-6.3-free-win32-msc10-std\Install
Address D:\LEDA\LEDA-6.3-free-win32-msc10-std\Install
File and Folder Tasks
Make a new folder
Publish this folder to the Web
Share this folder
Name Size Type Date Modified
src.txt 6 KB Text Document 17/11/2010 15:38
unix.txt 4 KB Text Document 17/11/2010 15:38
win32_msc7x_eval.txt 10 KB Text Document 17/11/2010 15:38
win32_msc71_dll.txt 12 KB Text Document 17/11/2010 15:38
win32_msc71_lib.txt 12 KB Text Document 17/11/2010 15:38

This is in line with

Files and Directories http://www.algorithmic-solutions.info/leda_manual/DLL_s_MS_Visual.html

To compile and link your programs with LEDA, the LEDA main directory should contain the following files and subdirectories:

Readme.txt	Readme File
Install \win32_msc70_dll.txt	txt-version of this section
incl \	the LEDA include directory
man_html \	HTML version of the LEDA user manual

and at least one of the following dll/library sets

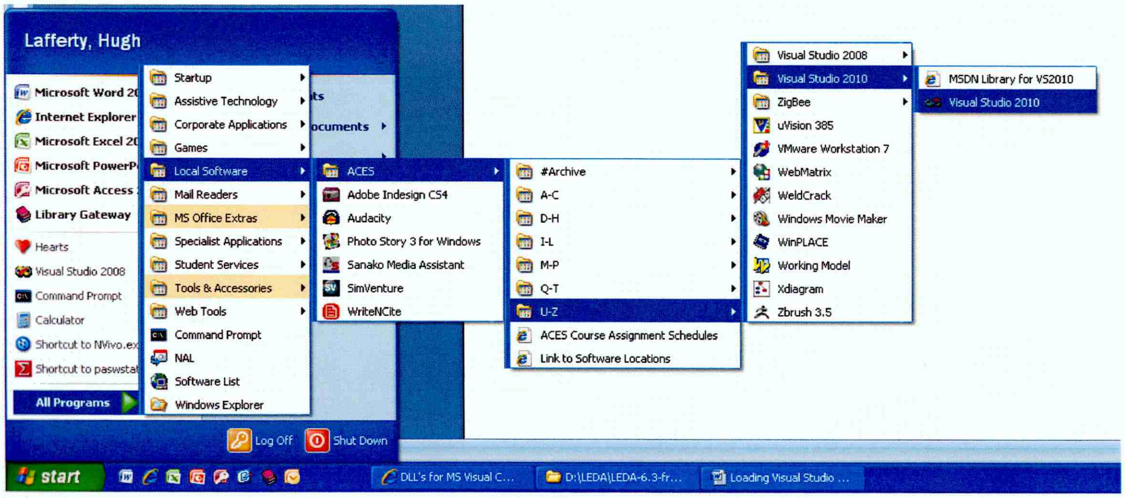
- `leda_ml.dll, leda_ml.lib, libGeoW_ml.lib`
- `leda_mld.dll, leda_mld.lib, libGeoW_mld.lib`
- `leda_md.dll, leda_md.lib, libGeoW_md.lib`
- `leda_mdd.dll, leda_mdd.lib, libGeoW_mdd.lib`
- `leda_mt.dll, leda_mt.lib, libGeoW_mt.lib`
- `leda_mtd.dll, leda_mtd.lib, libGeoW_mtd.lib`

Note: A DLL of GeoWin is currently not available. "

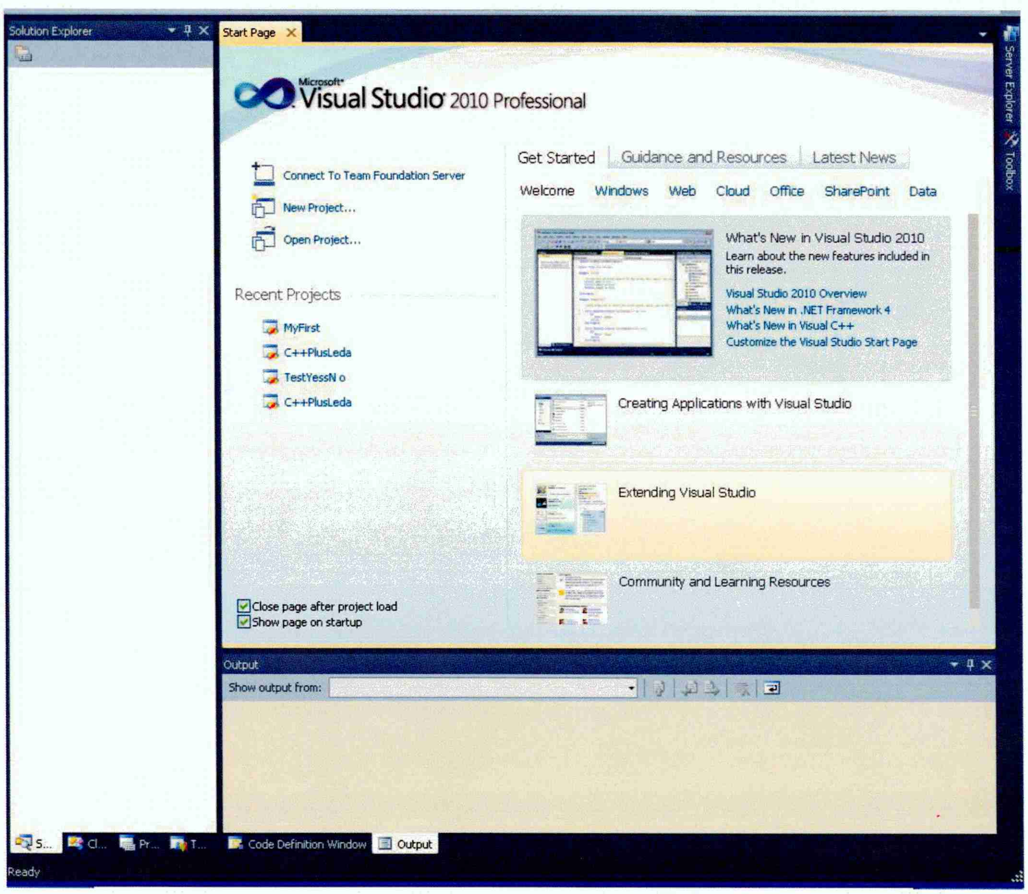
3. Configuring Visual Studio 10 (VS 10) to run with the LEDA routines

VS 10 is run from

Start → All programs→ Local Software →ACES→U-Z→Visual Studio 2010



and then you should see something like this



Compiling and Linking in Microsoft Visual C++ .NET

http://www.algorithmic-solutions.info/leda_manual/DLL_s_MS_Visual.html

You can choose between a manual and an automatic way to compile and link an application program using LEDA in Microsoft Visual C++ .NET.

Automatic Setting

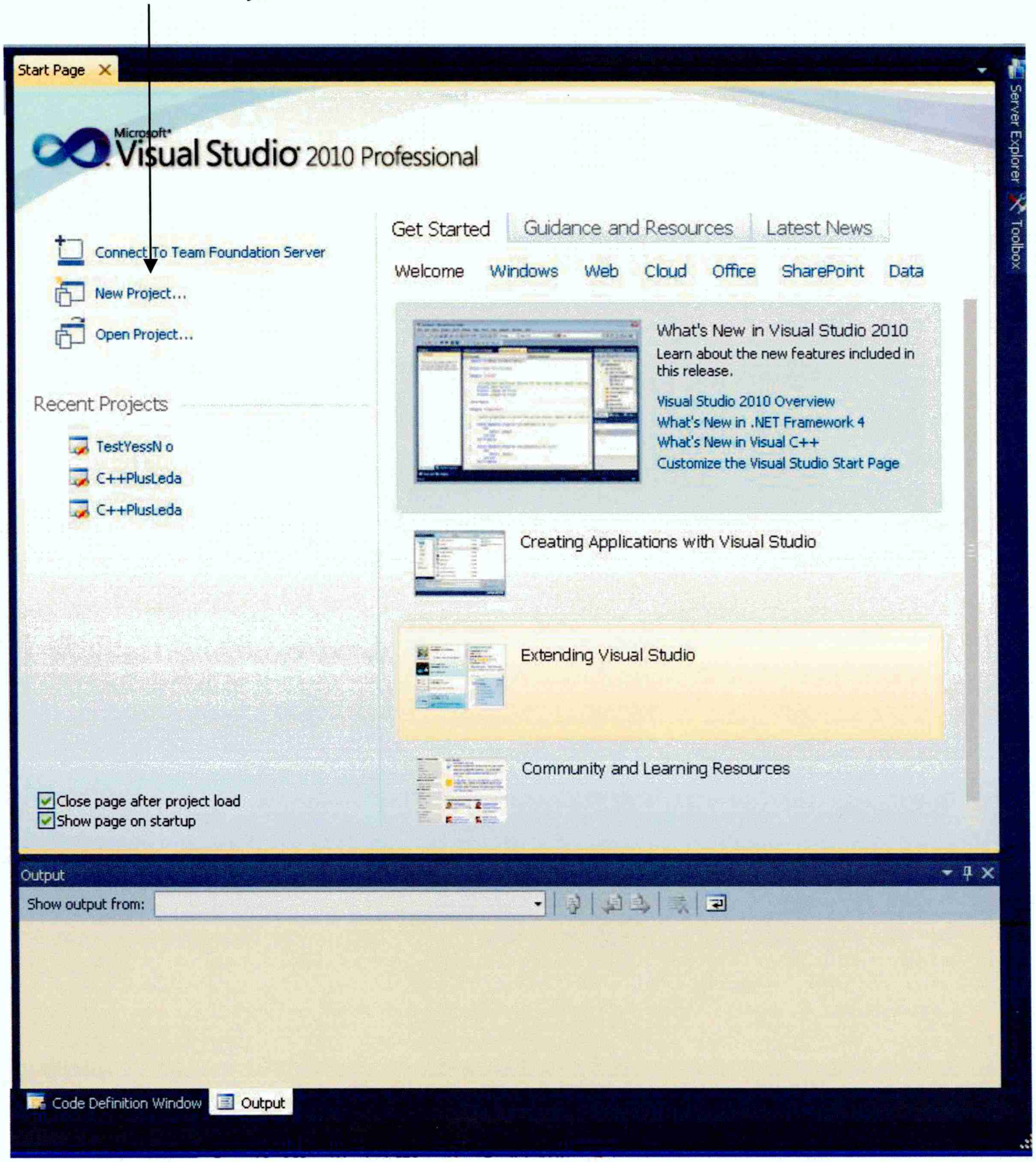
Include header file `LEDA/msc/autolink_dll.h` in one of your project's cpp-files. Depending on your build environment (Debug or Release build) the correct LEDA dll will be chosen automatically when you build your project.

> I do not understand this, as I do not know what a header file is, and so I am forced to use a Manual setting

Manual Setting

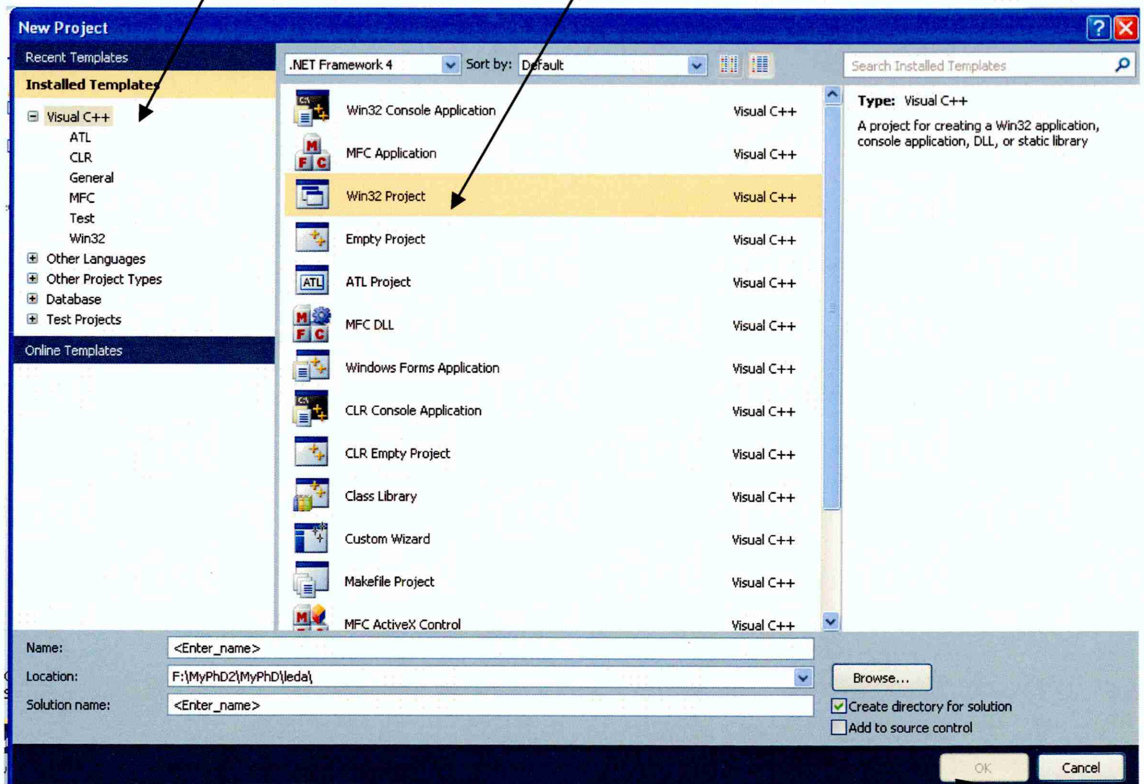
(1)

Click on "New->Project".

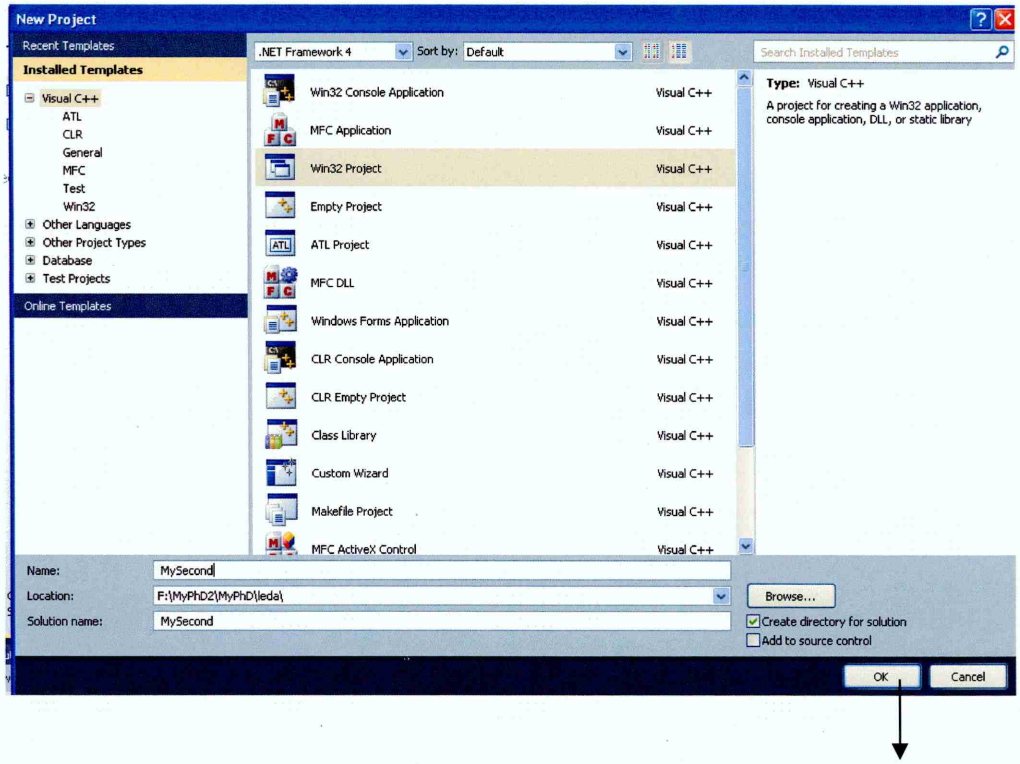


(2)

Choose "Visual C++ Projects" and "Win32 Project". Enter a project name, choose a directory for the project, and click "OK"



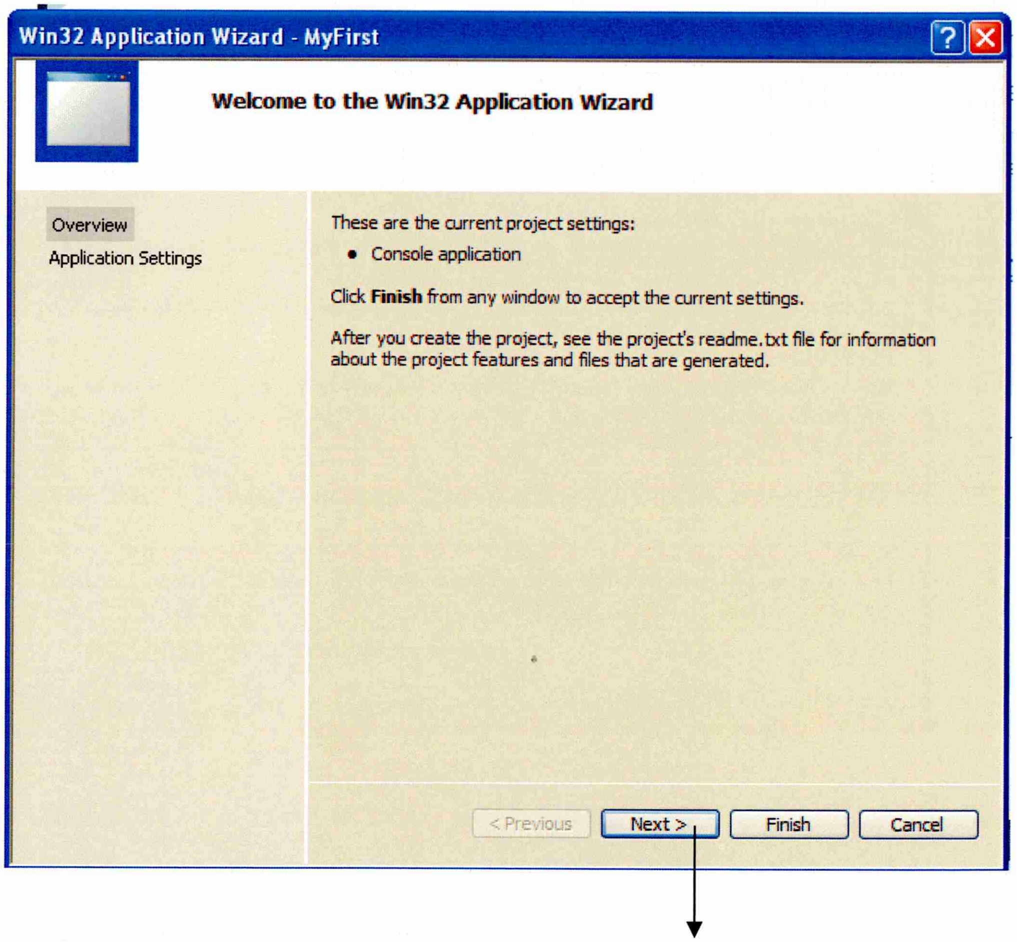
If you do not give the project a name then this is 'greyed' out



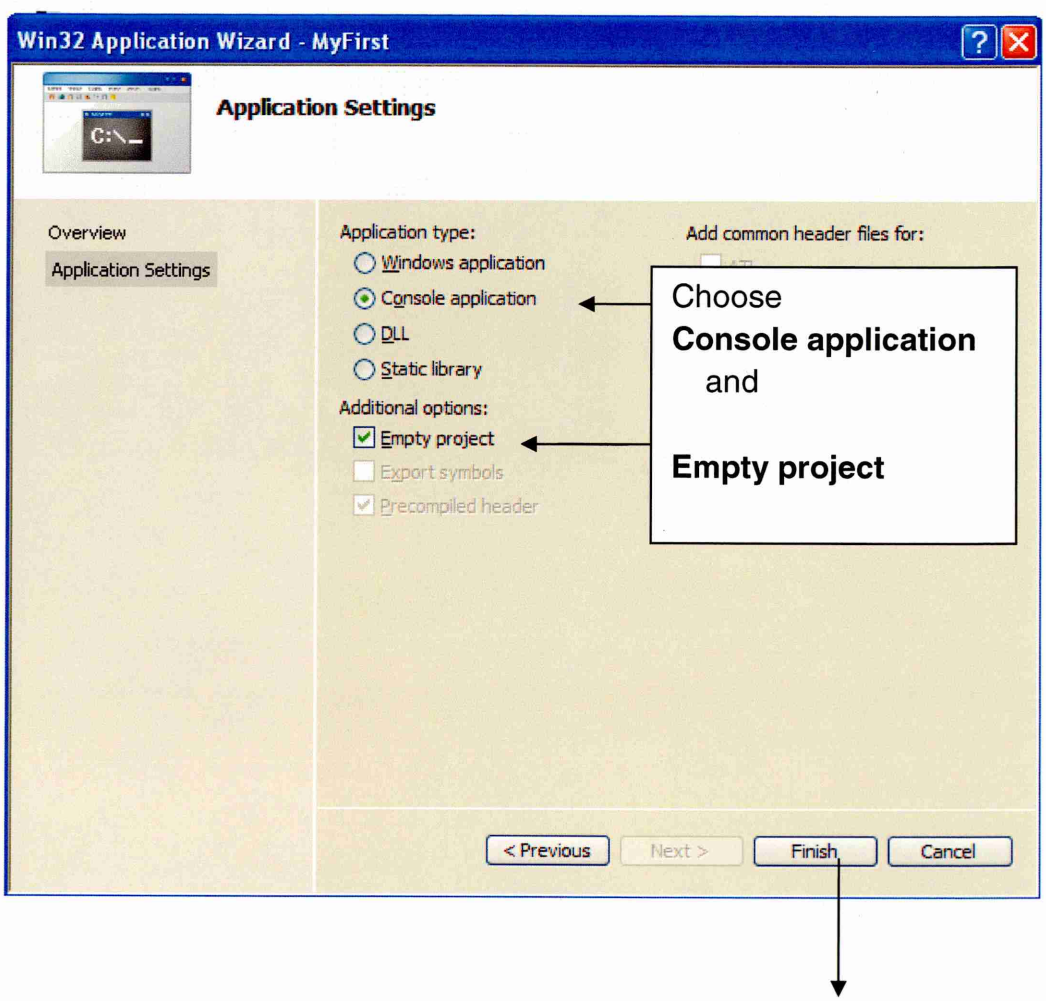
Name: =Project name
Location:= Where the project is to be put
Solution name:= is given the same value as **Name** by default

(3)

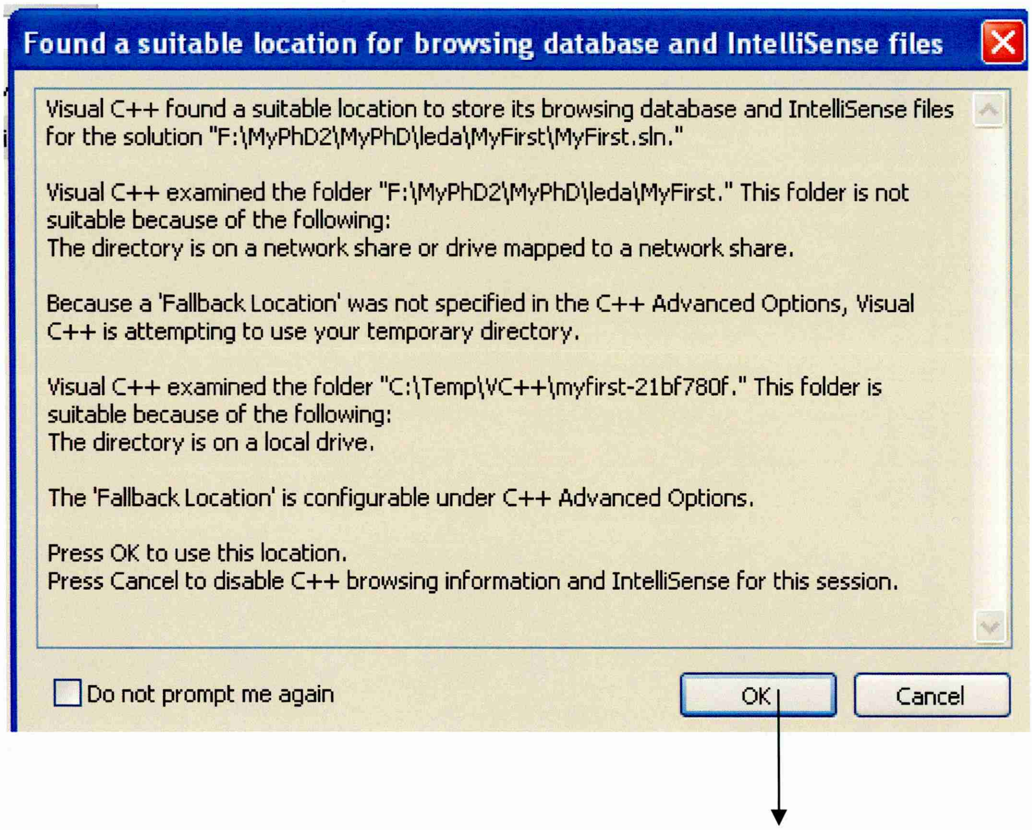
In the `Win32 Application Wizard` click on "Application Settings", choose "Console Application" and "Empty Project", and click "Finish"



Win32 Application Wizard - MyFirst
Welcome to the Win32 Application Wizard
Overview
Application Settings
These are the current project settings:
• Console application
Click **Finish** from any window to accept the current settings.
After you create the project, see the project's readme.txt file for information about the project features and files that are generated.
< Previous
Next >
Finish
Cancel

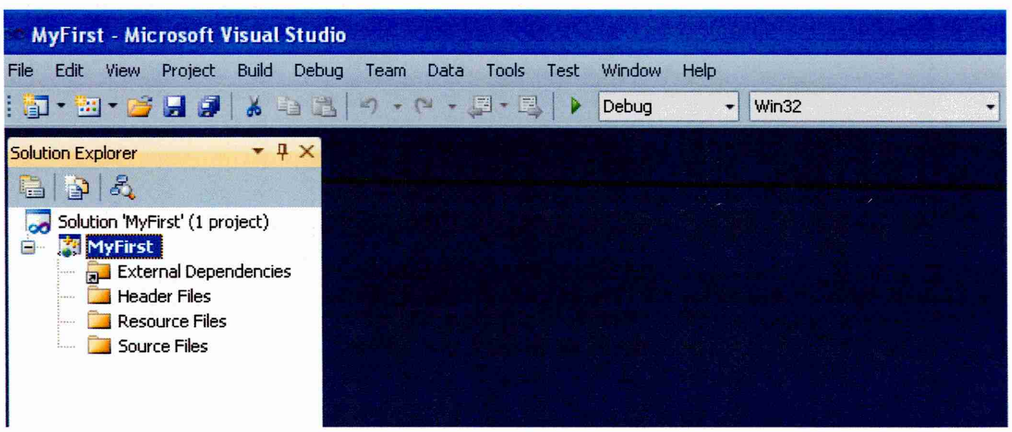


Win32 Application Wizard - MyFirst
Application Settings
C:_
Overview
Application Settings
Application type:
Windows application
Console application
DLL
Static library
Additional options:
Empty project
Export symbols
Precompiled header
Add common header files for:
Choose
Console application
and
Empty project
< Previous
Next >
Finish
Cancel



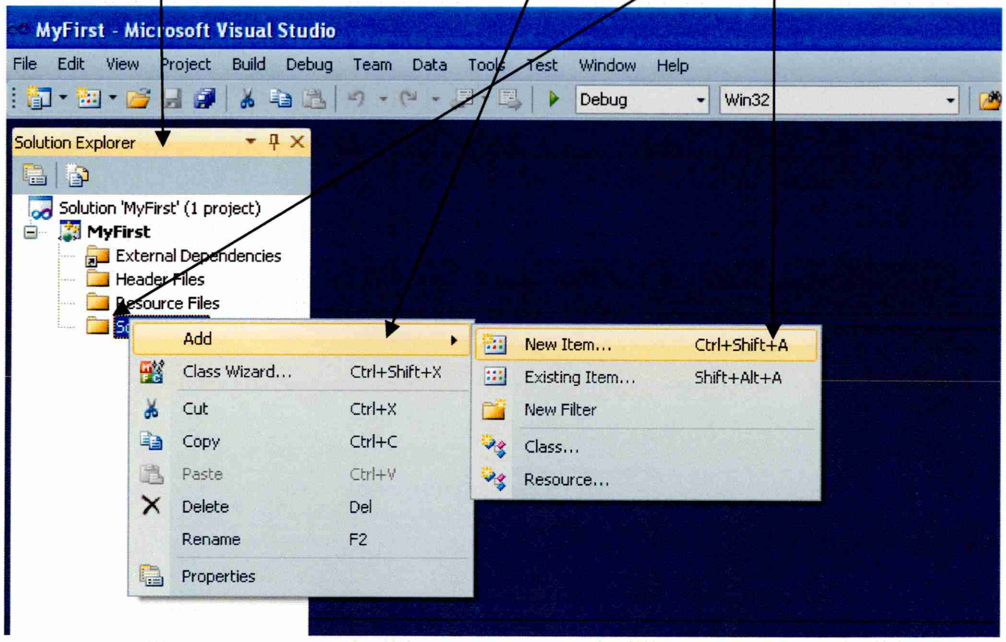
(4)

After clicking "**OK**" you have an empty project.



(5)

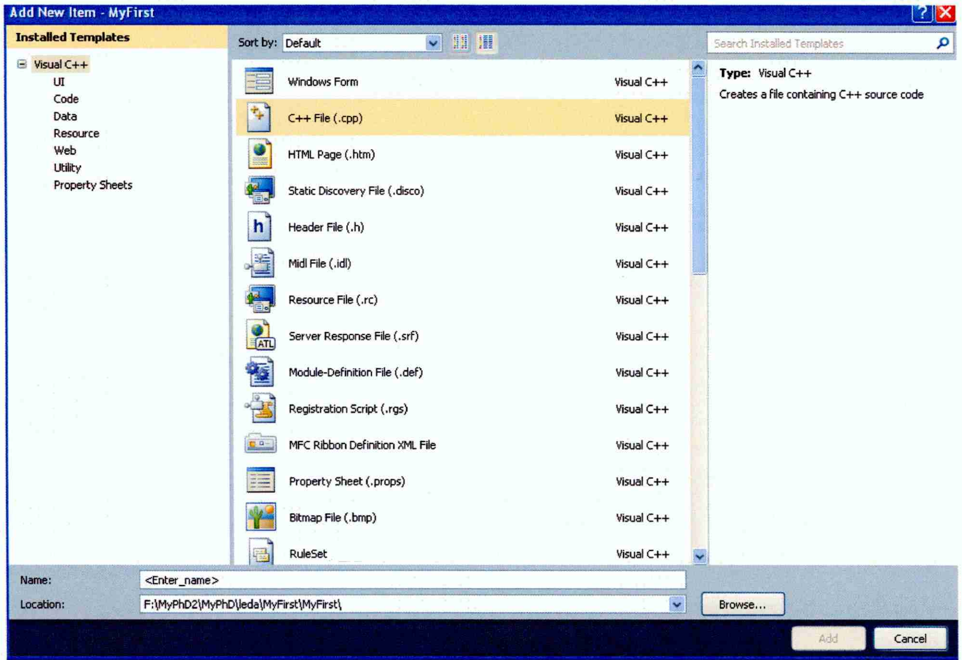
In the `Solution Explorer` of your project click on "Source Files" with the right mouse button, then click on "Add-> Add New Item" with the left mouse button.



You only have to add line numbers once per project.

(6)

Give your source file a name. and choose C++ File (.cpp)" in Installed Templates, and click "**Add**"

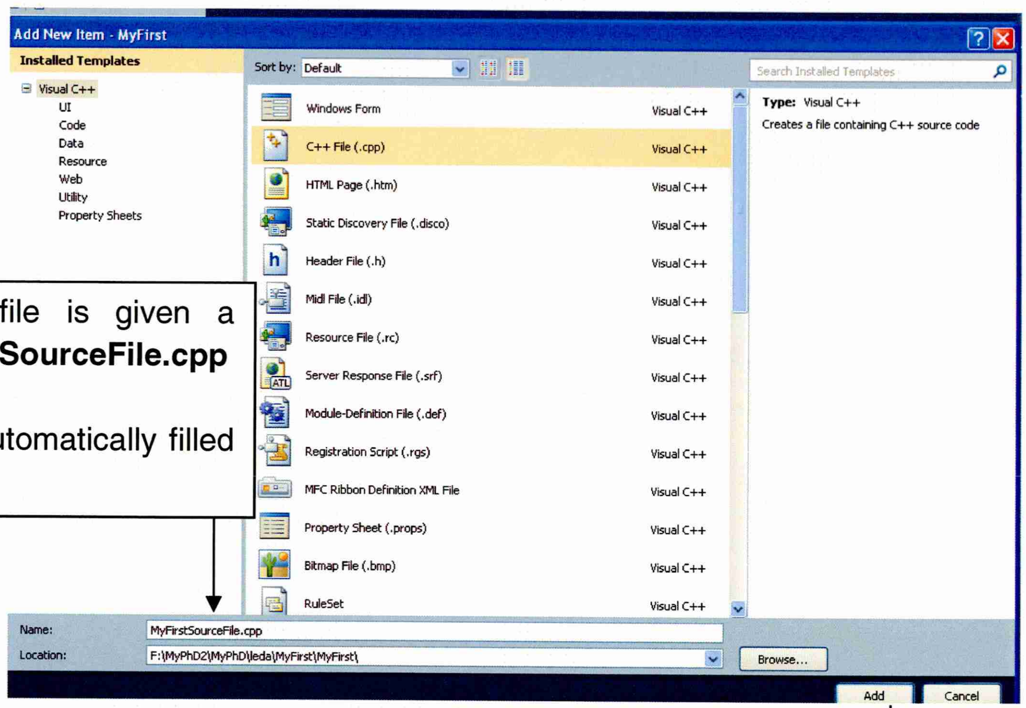


This is **Add,** but it is greyed out, because you must first give the source file a name

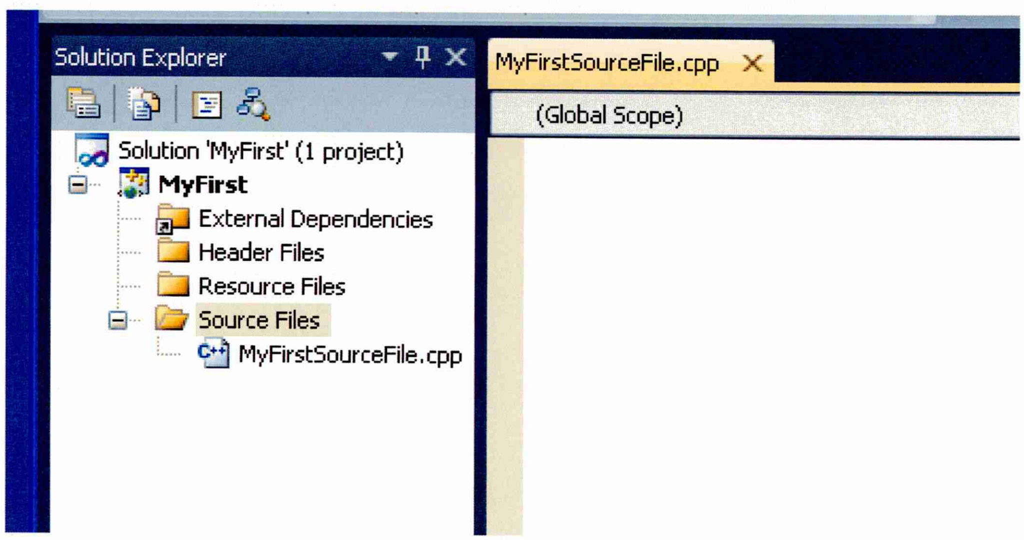
Going back and giving the source file a name, we have

The source file is given a name **MyFirstSourceFile.cpp**

The .cpp is automatically filled in



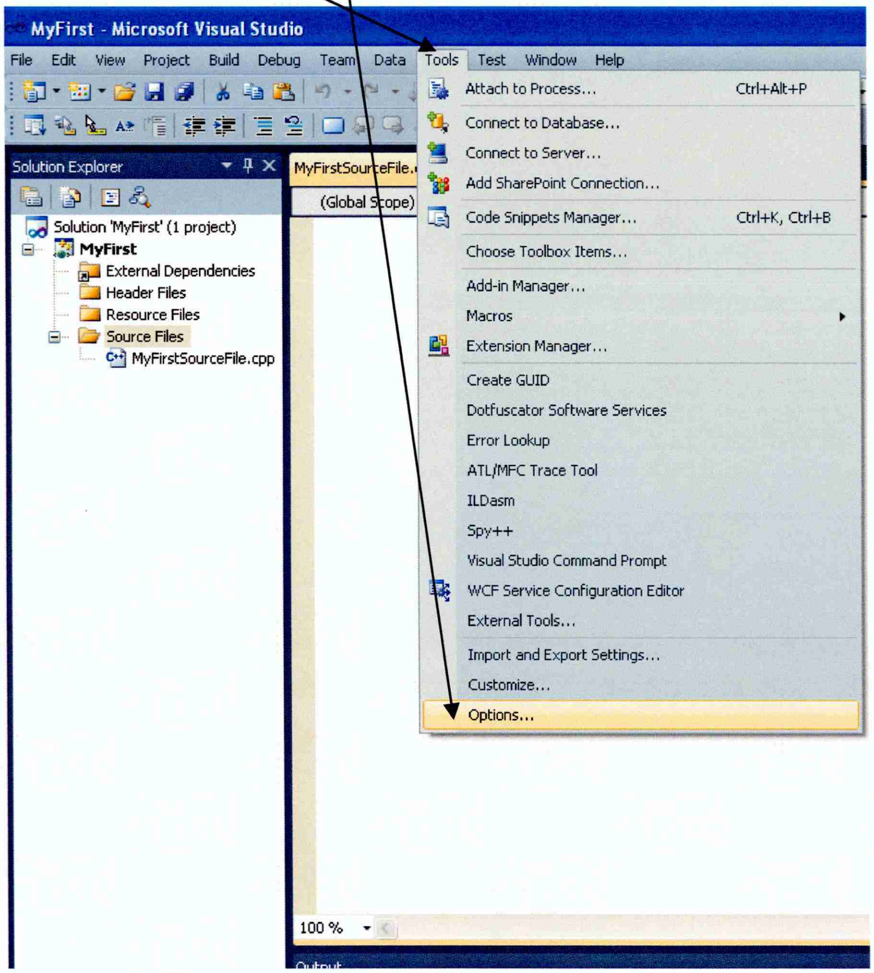
Add is now, *not* greyed out, and is clickable.

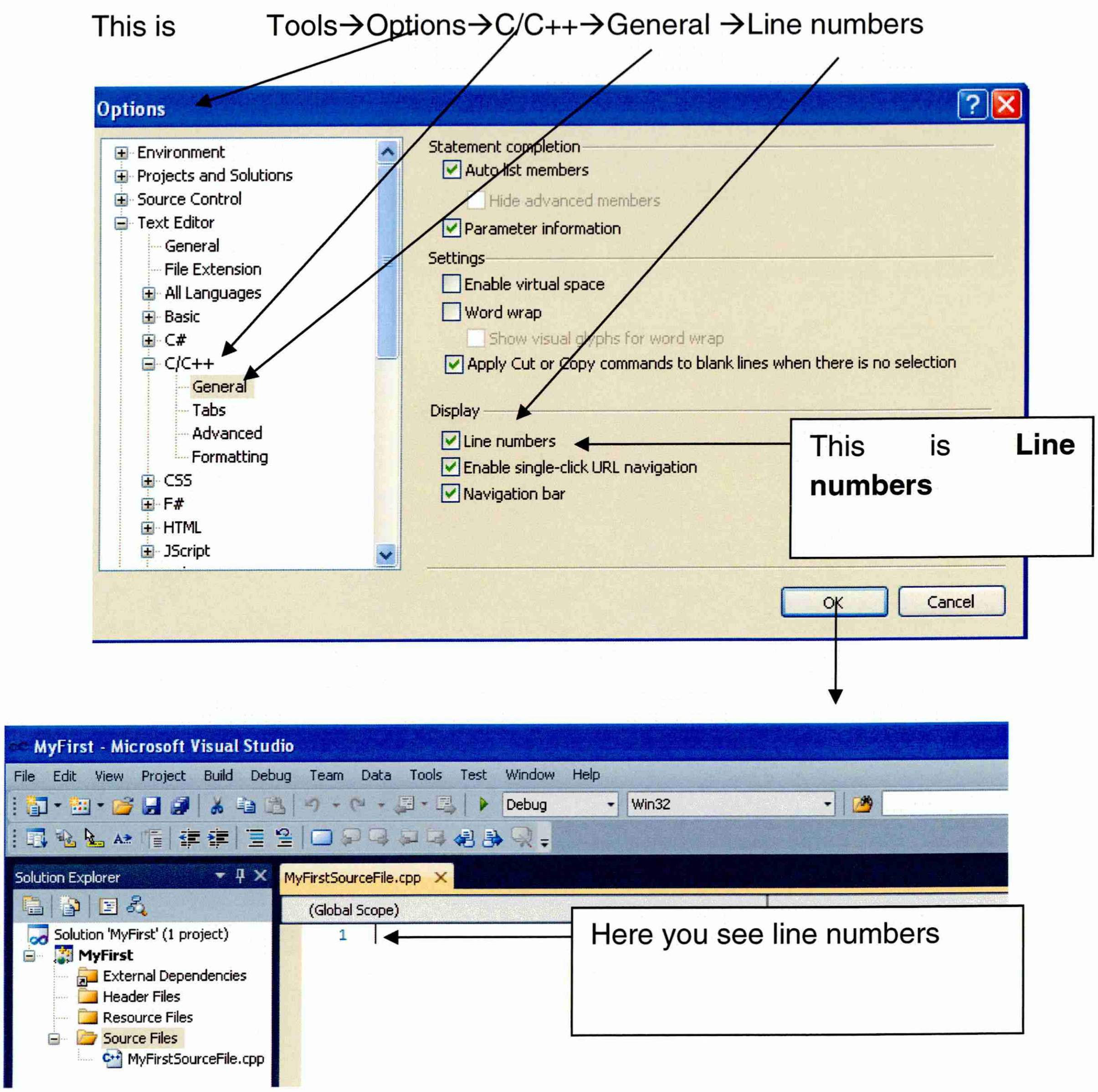


You will see that the source code now does not have line numbers, which makes de-bugging harder. So, **we add line numbers as follows**.(but you only have to do this once)

Tools→Options→C/C++→General →Line numbers

This is Tools→Options



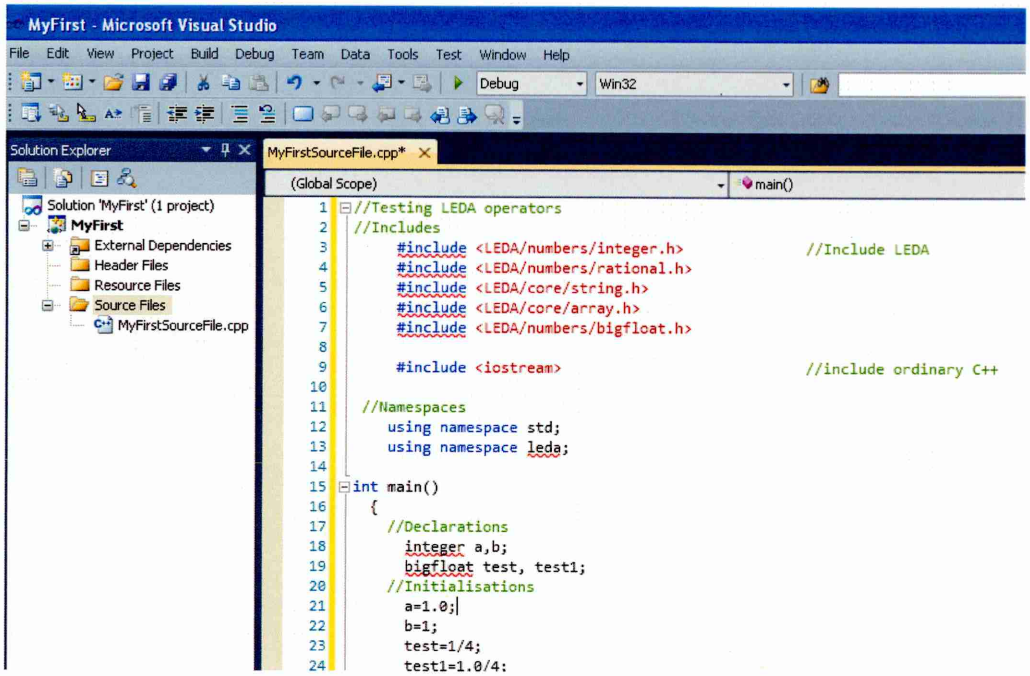

This is Tools→Options→C/C++→General →Line numbers
Options
Environment
Projects and Solutions
Source Control
Text Editor
General
File Extension
All Languages
Basic
C#
C/C++
General
Tabs
Advanced
Formatting
CSS
F#
HTML
JScript
Statement completion
Auto list members
Hide advanced members
Parameter information
Settings
Enable virtual space
Word wrap
Show visual glyphs for word wrap
Apply Cut or Copy commands to blank lines when there is no selection
Display
Line numbers
Enable single-click URL navigation
Navigation bar
This is **Line numbers**
OK
Cancel
MyFirst - Microsoft Visual Studio
File Edit View Project Build Debug Team Data Tools Test Window Help
Debug
Win32
Solution Explorer
MyFirstSourceFile.cpp
(Global Scope)
Solution 'MyFirst' (1 project)
MyFirst
External Dependencies
Header Files
Resource Files
Source Files
MyFirstSourceFile.cpp
1
Here you see line numbers

(7)

Enter your code.

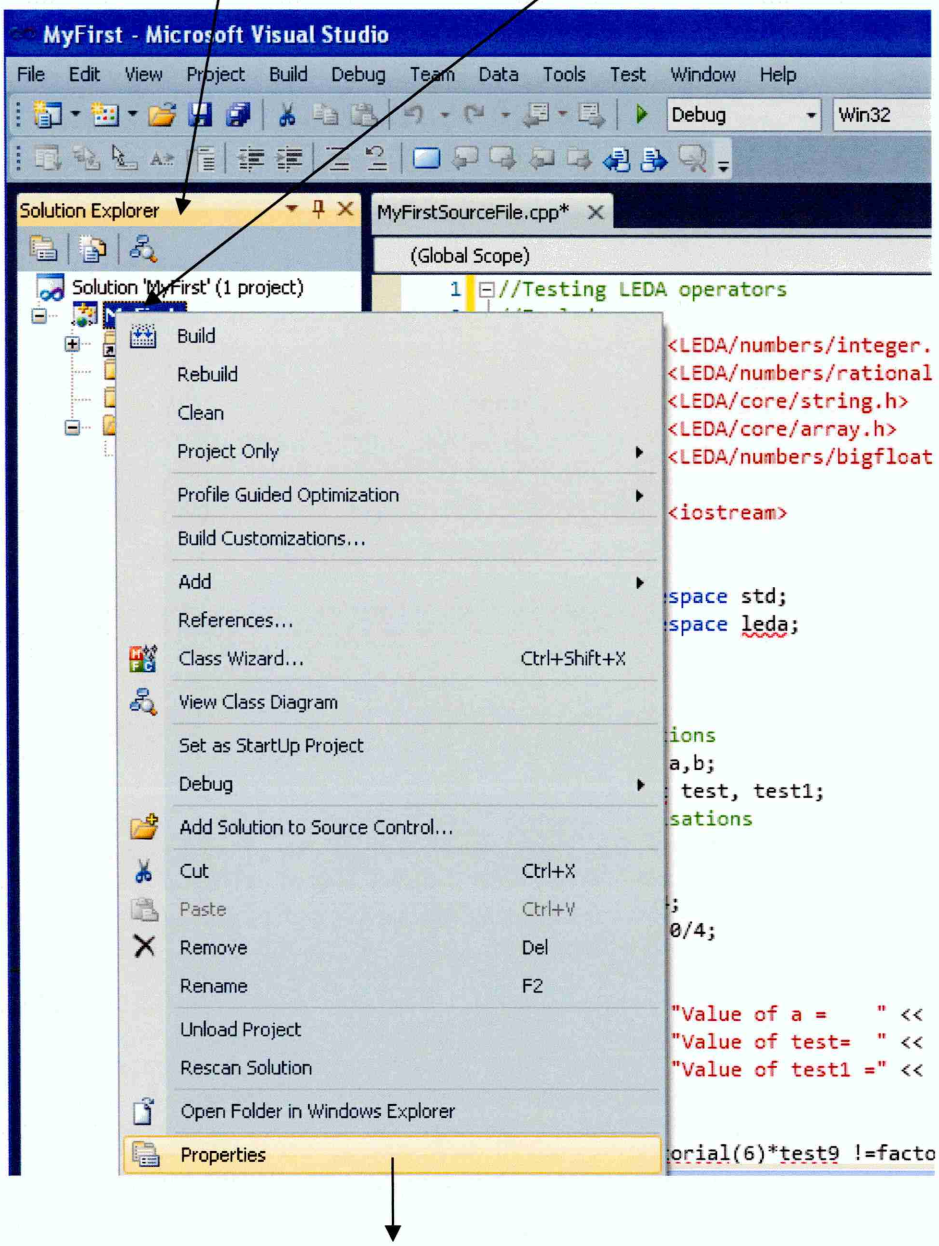
```
//Testing LEDA
//Includes
   #include <LEDA/numbers/integer.h>
   #include <LEDA/numbers/rational.h>
   #include <LEDA/core/string.h>
   #include <LEDA/core/array.h>
   #include <LEDA/numbers/bigfloat.h>

   #include <iostream>

//Namespaces
  using namespace std;
  using namespace leda;

  int main ()
  {
    //Declarations
         integer a,b;
         bigfloat test, test1;

    //Initialisation
         a=1.0;
         b=1;
         test=1/4;
         test1=1.0/4;

    //Output
         if (factorial(6) !=factorial(3))
         {
        cout << "Factorial(6)=" << factorial(6) << endl;
         }
         cout << "Value of test="  << test << endl;
         cout << "Value of test1=" << test1 << endl;
  }
```

and you will see



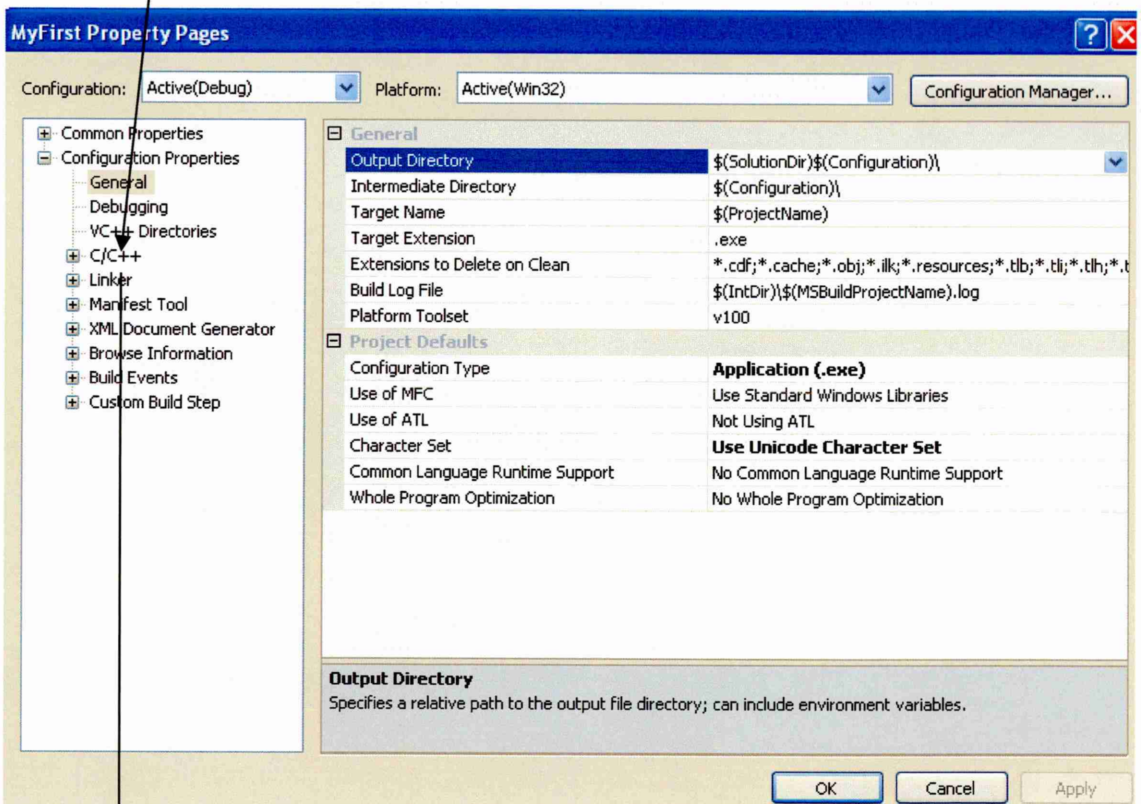
(8)

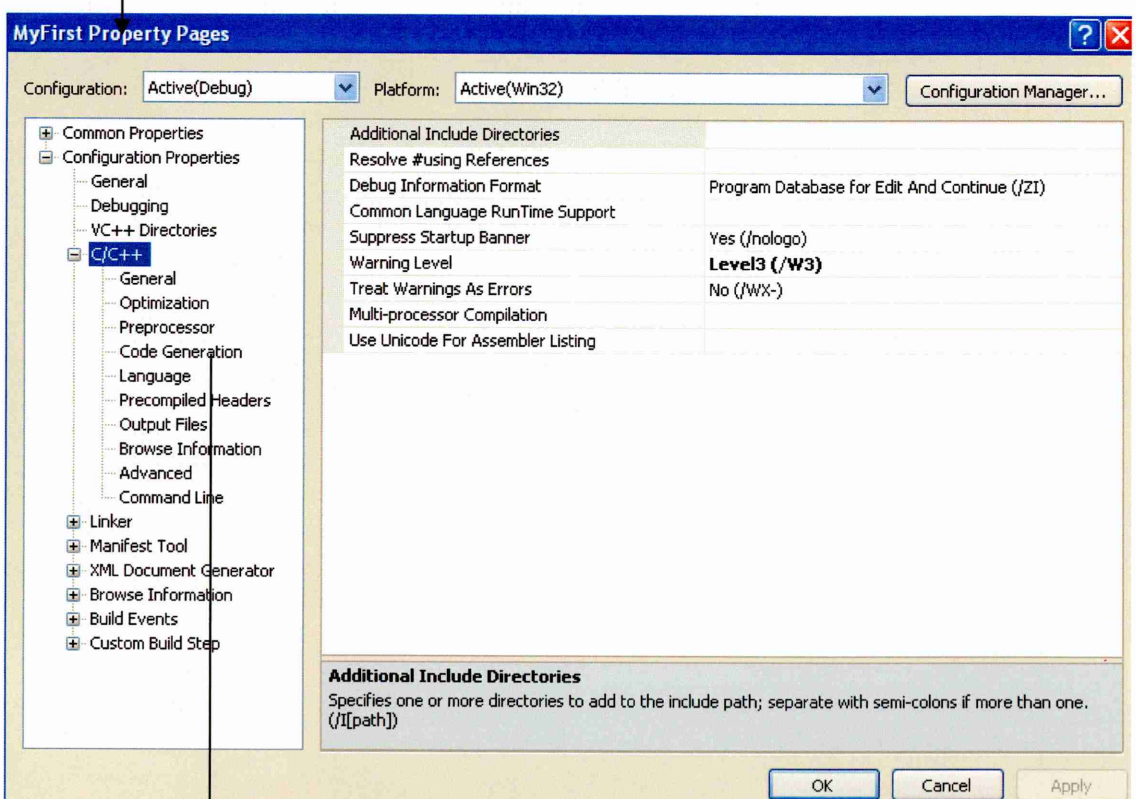
In the `Solution Explorer` right click on your project and left click on "Properties"

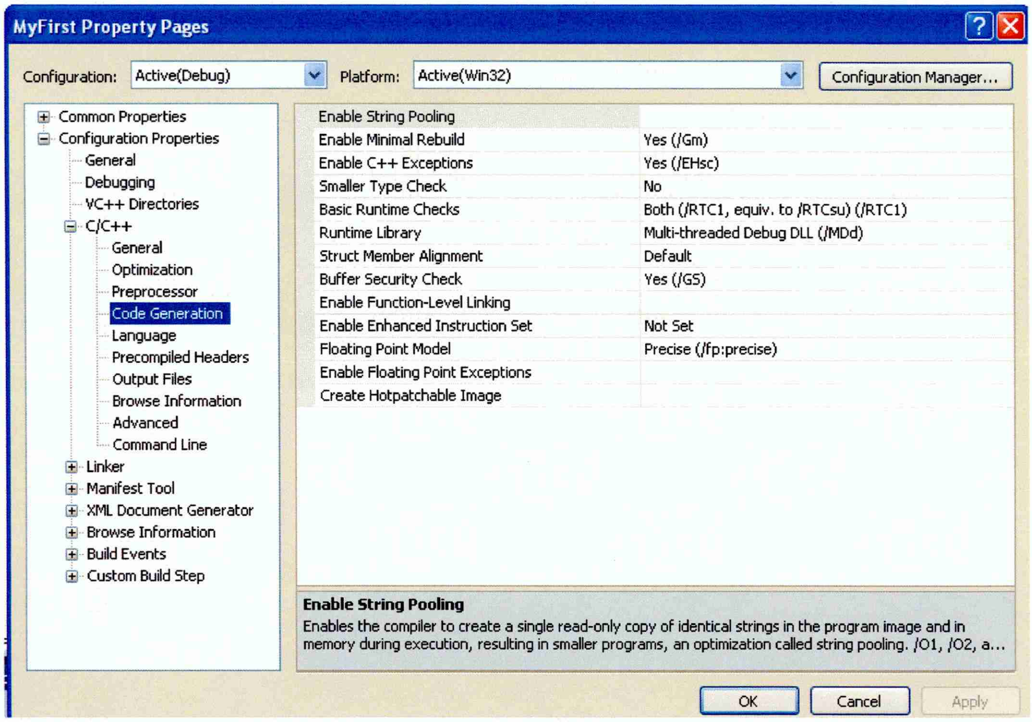


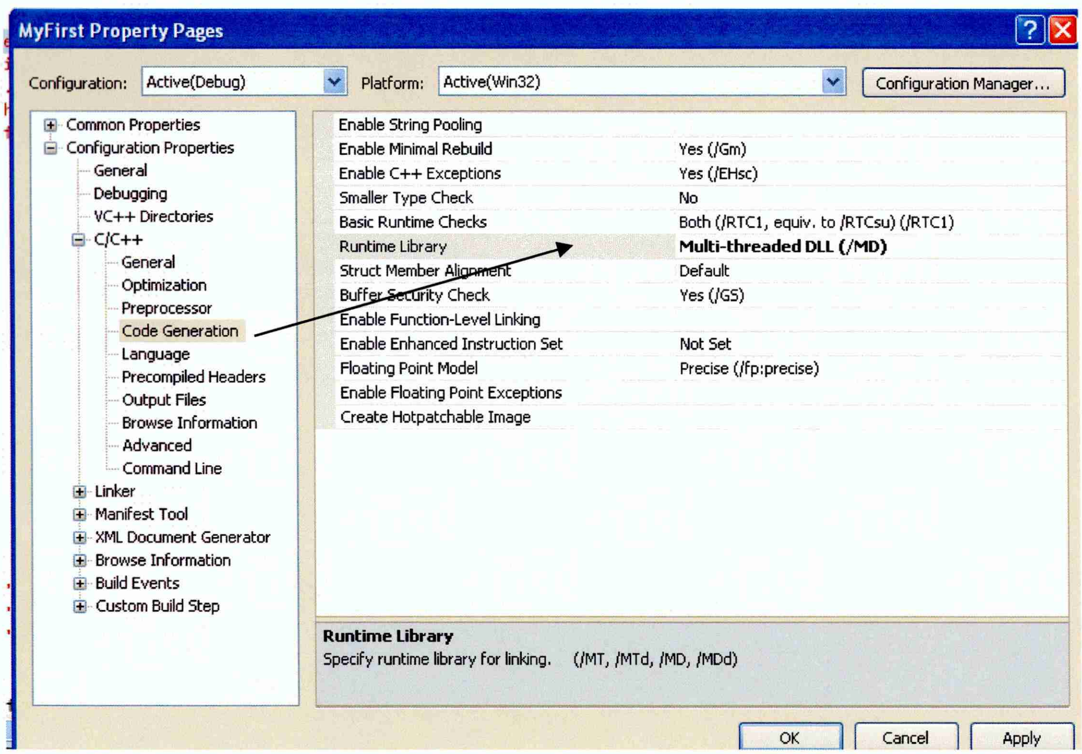
(9)

Click on "C/C++" and "Code Generation" and choose the "Run Time Library" (=compiler flag) you want to use.

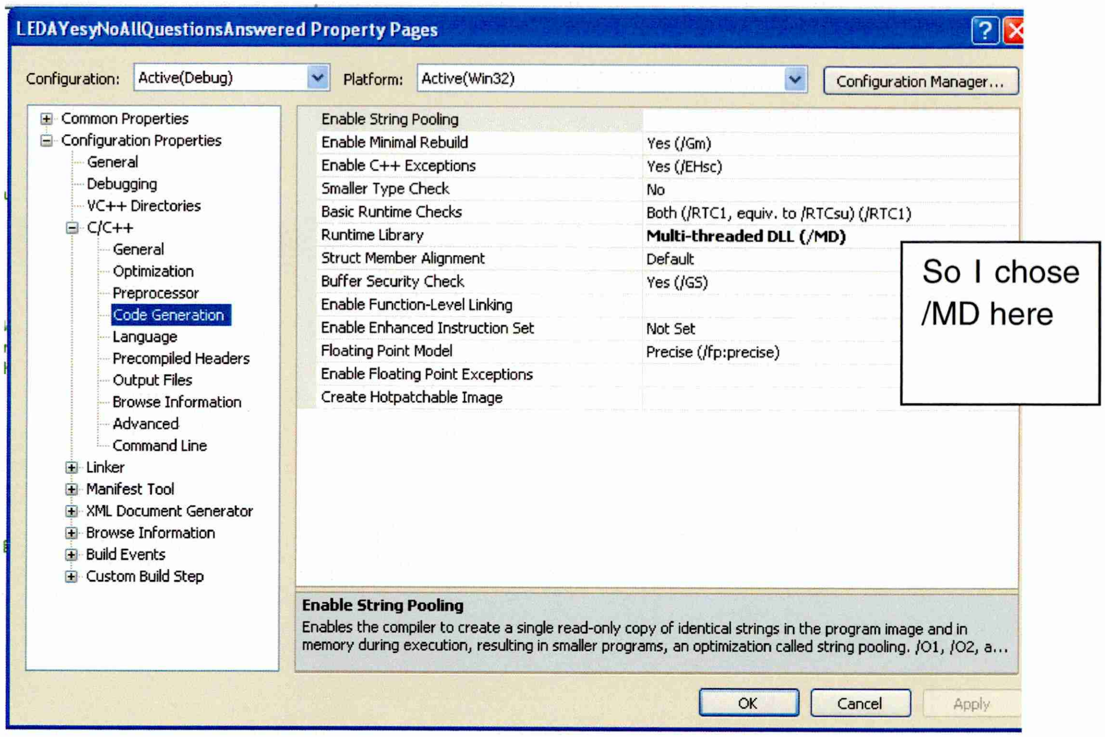






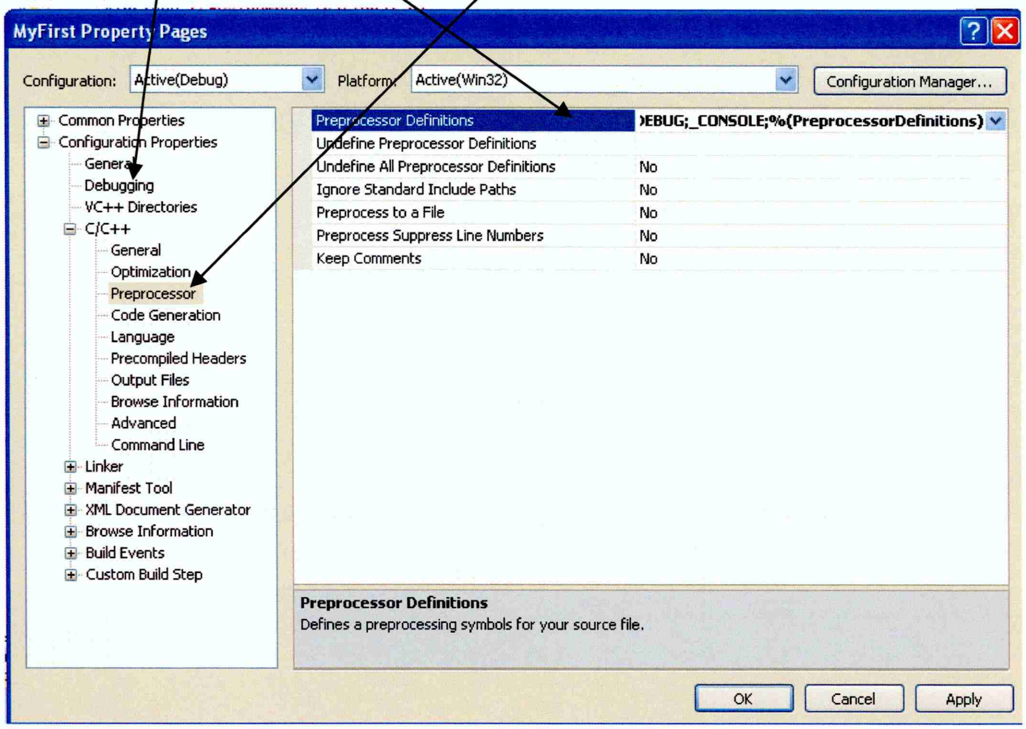


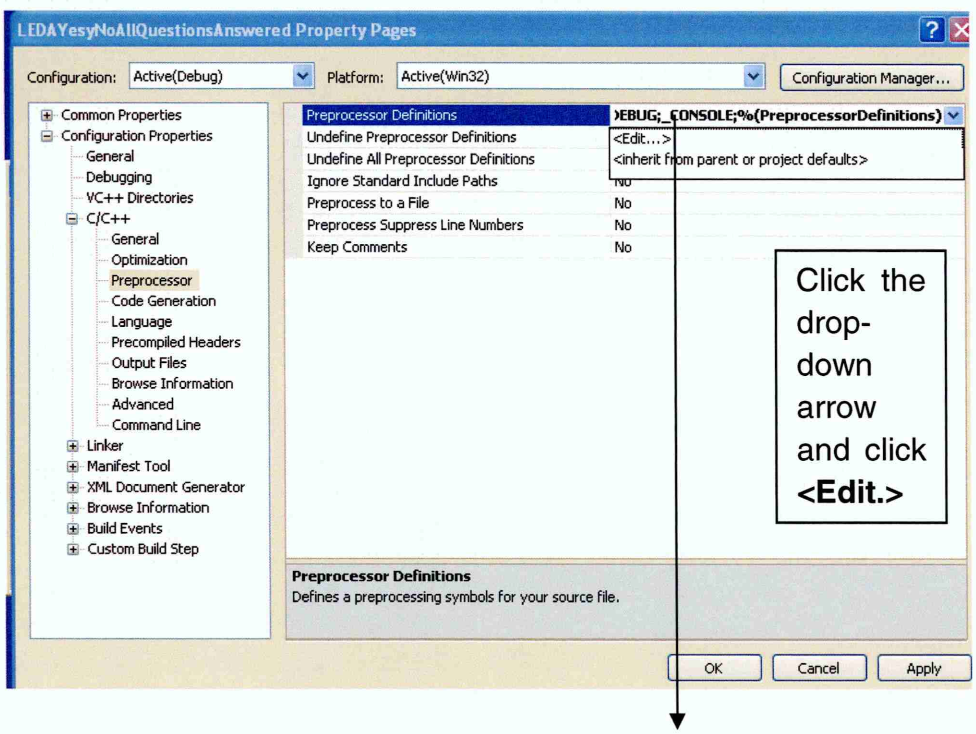
From the drop-down list choose **/MD**

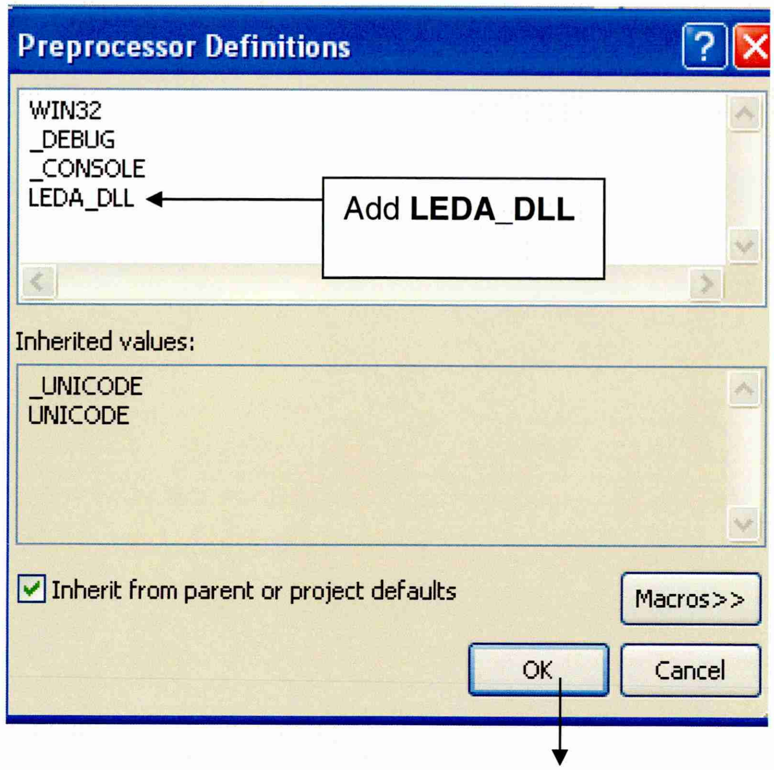

LEDAYesyNoAllQuestionsAnswered Property Pages
Configuration: Active(Debug)
Platform: Active(Win32)
Configuration Manager...
Common Properties
Configuration Properties
General
Debugging
VC++ Directories
C/C++
General
Optimization
Preprocessor
Code Generation
Language
Precompiled Headers
Output Files
Browse Information
Advanced
Command Line
Linker
Manifest Tool
XML Document Generator
Browse Information
Build Events
Custom Build Step
Enable String Pooling
Enable Minimal Rebuild
Yes (/Gm)
Enable C++ Exceptions
Yes (/EHsc)
Smaller Type Check
No
Basic Runtime Checks
Both (/RTC1, equiv. to /RTCsu) (/RTC1)
Runtime Library
Multi-threaded DLL (/MD)
Struct Member Alignment
Default
Buffer Security Check
Yes (/GS)
Enable Function-Level Linking
Enable Enhanced Instruction Set
Not Set
Floating Point Model
Precise (/fp:precise)
Enable Floating Point Exceptions
Create Hotpatchable Image
Enable String Pooling
Enables the compiler to create a single read-only copy of identical strings in the program image and in memory during execution, resulting in smaller programs, an optimization called string pooling. /O1, /O2, a...
OK
Cancel
Apply
So I chose /MD here

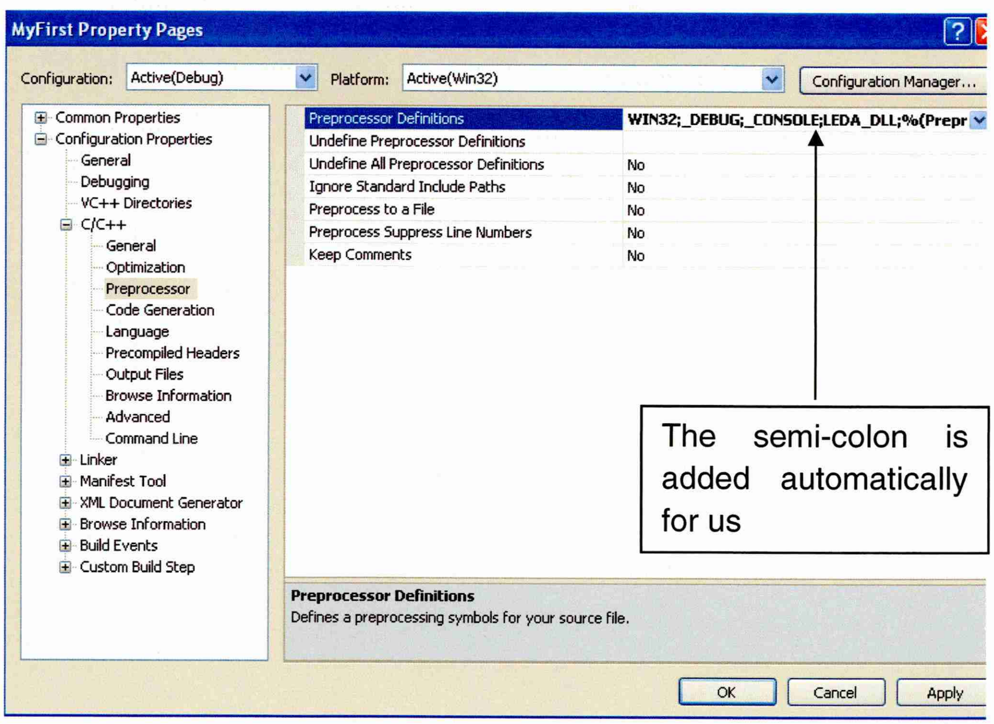
(10)

Click on "**C/C++**" and then "**Preprocessor**" add `";LEDA_DLL"` in "**Preprocessor Definitions**".



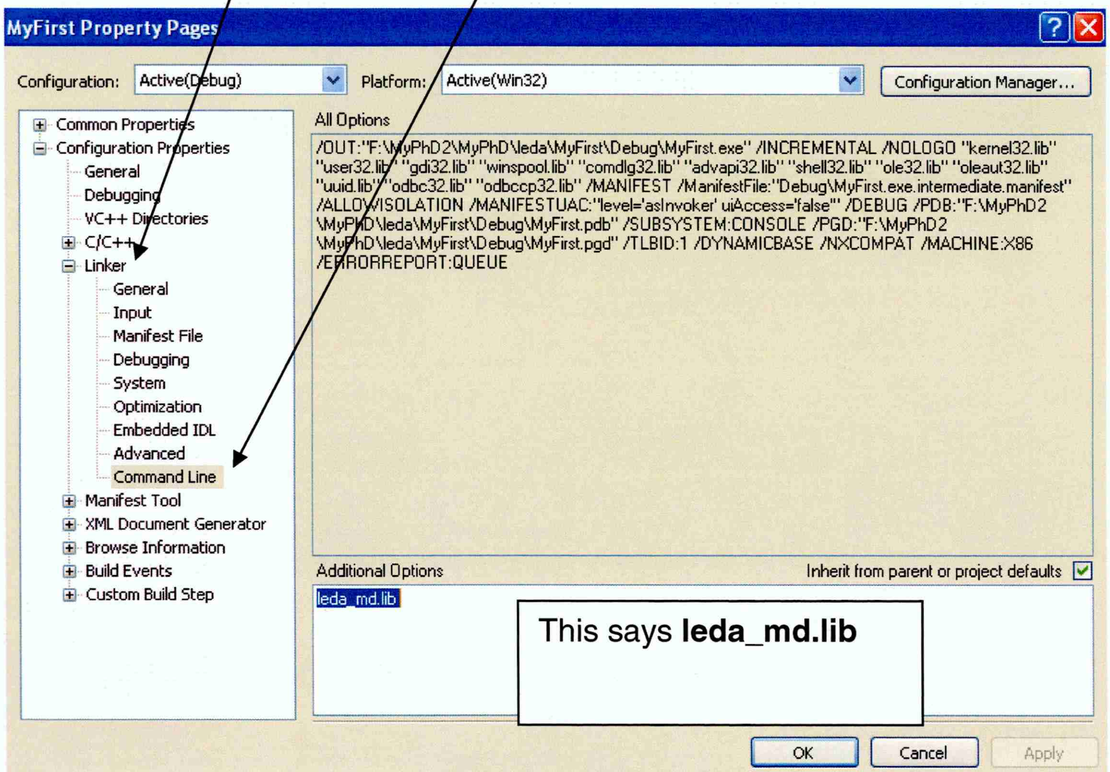



Preprocessor Definitions
WIN32
_DEBUG
_CONSOLE
LEDA_DLL
Add LEDA_DLL
Inherited values:
_UNICODE
UNICODE
Inherit from parent or project defaults
Macros>>
OK
Cancel


MyFirst Property Pages
Configuration: Active(Debug)
Platform: Active(Win32)
Configuration Manager...
Common Properties
Configuration Properties
General
Debugging
VC++ Directories
C/C++
General
Optimization
Preprocessor
Code Generation
Language
Precompiled Headers
Output Files
Browse Information
Advanced
Command Line
Linker
Manifest Tool
XML Document Generator
Browse Information
Build Events
Custom Build Step
Preprocessor Definitions
WIN32;_DEBUG;_CONSOLE;LEDA_DLL;%(Prepr
Undefine Preprocessor Definitions
Undefine All Preprocessor Definitions
No
Ignore Standard Include Paths
No
Preprocess to a File
No
Preprocess Suppress Line Numbers
No
Keep Comments
No
The semi-colon is added automatically for us
Preprocessor Definitions
Defines a preprocessing symbols for your source file.
OK
Cancel
Apply

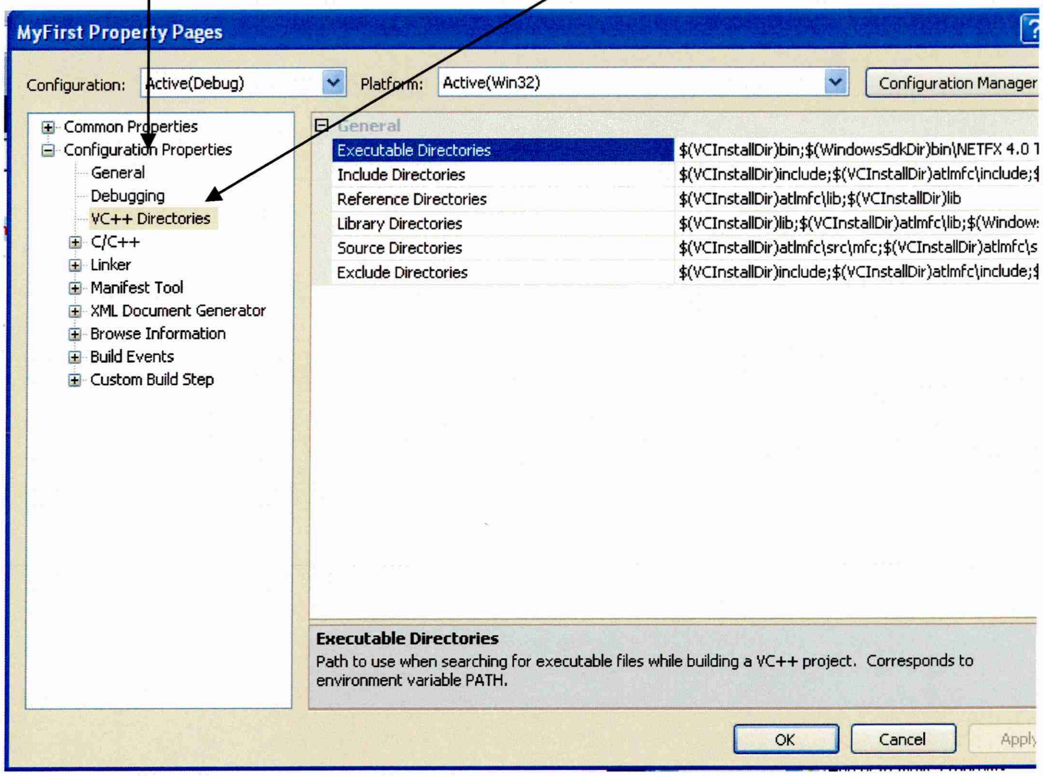
(11)

Click on "Linker" and "Command Line" and add the name of the LEDA libraries you want to use in "Additional Options" as follows.

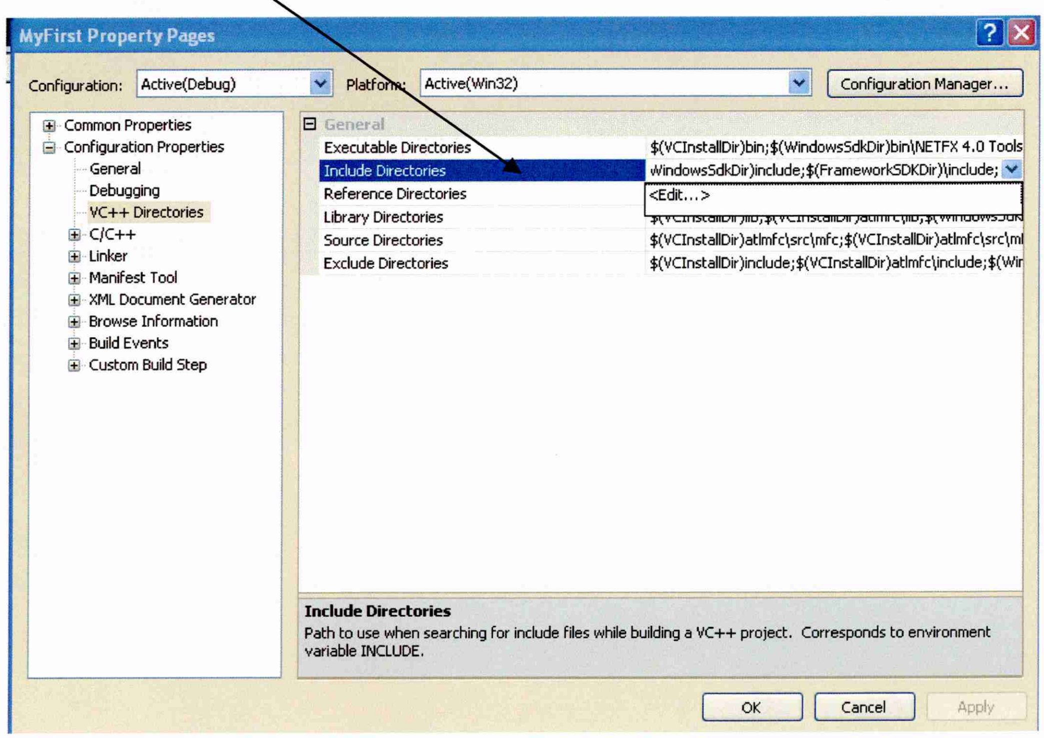


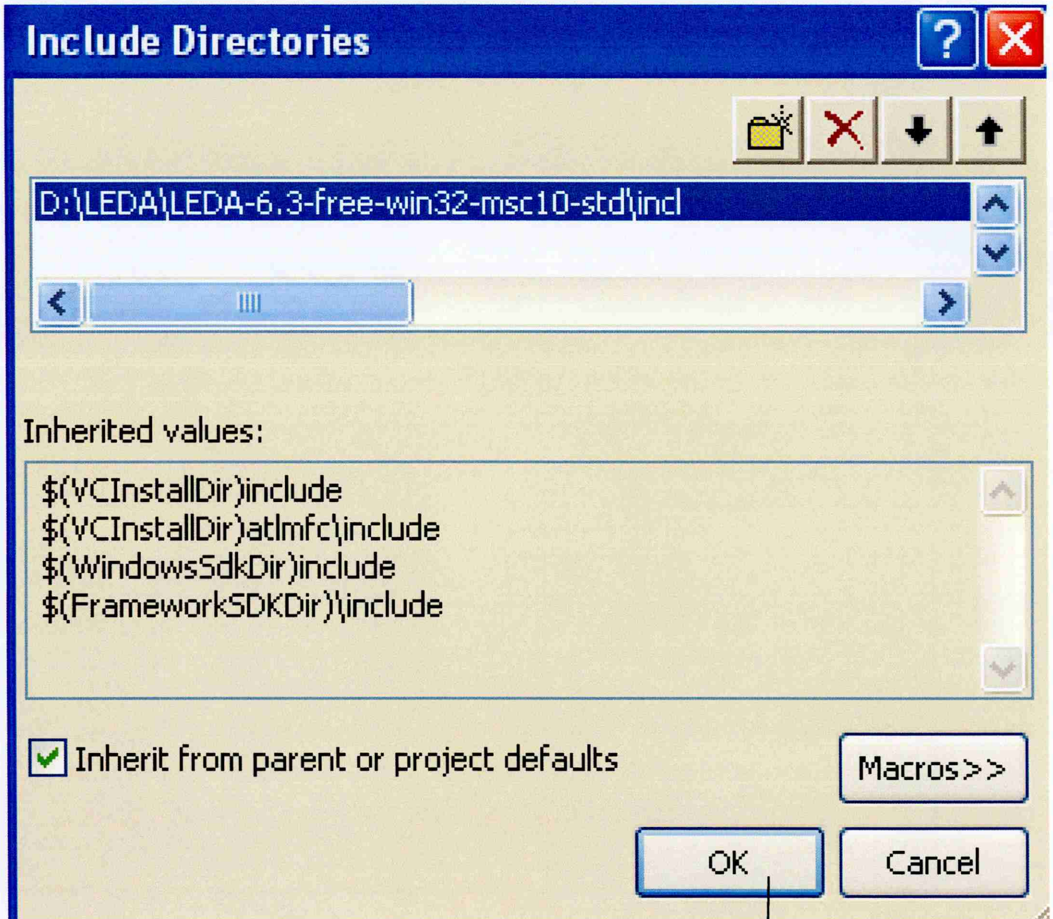
(12)

Click "**Configuration Properties**" → "**VC++ Directories**"

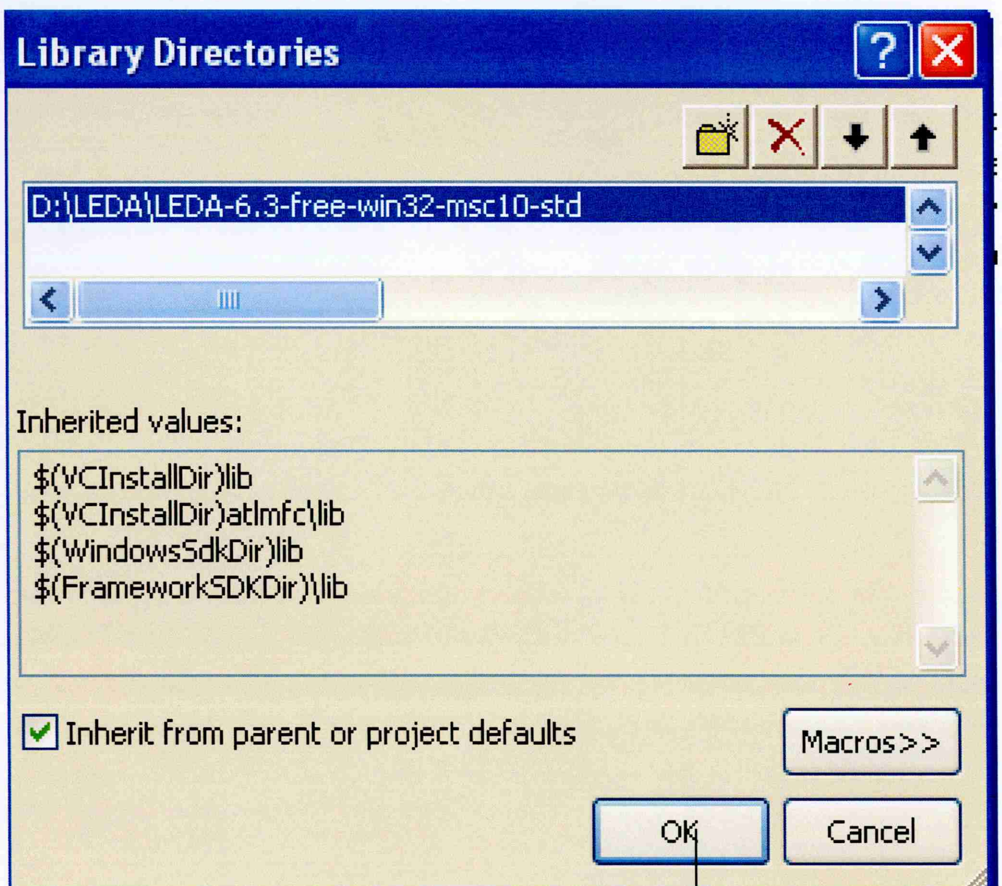


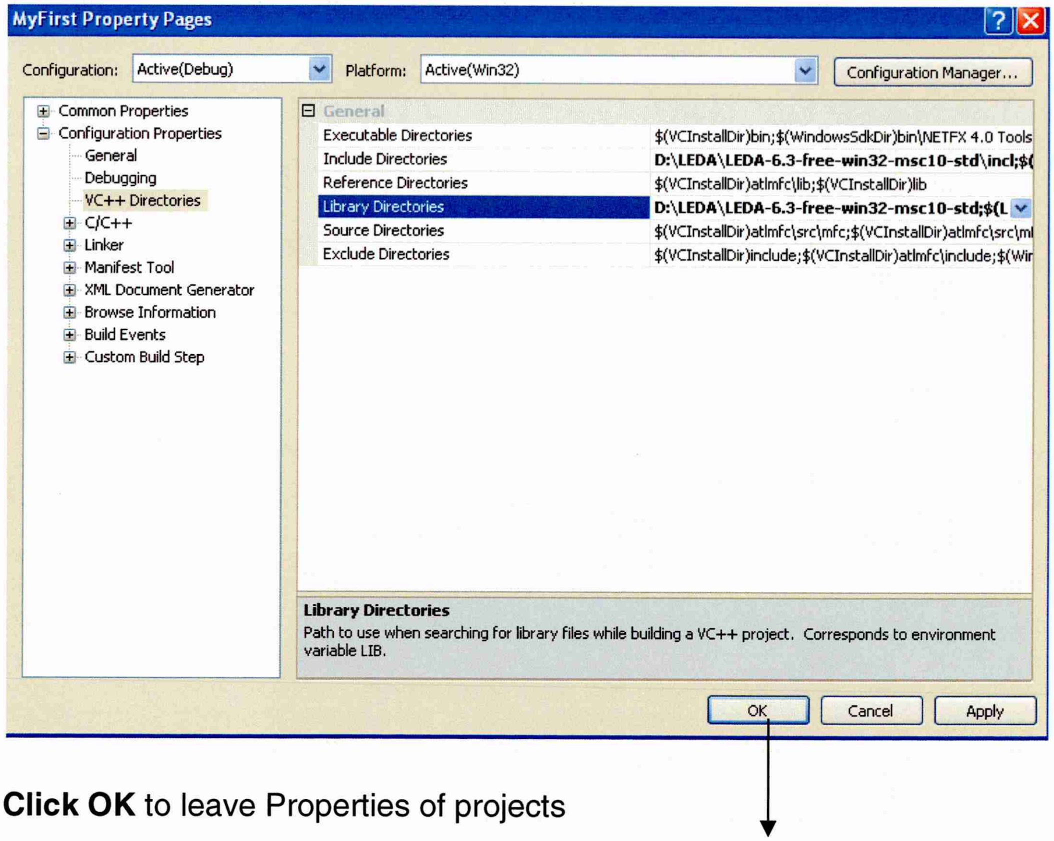
In **Include Directories** add where to look for #includes





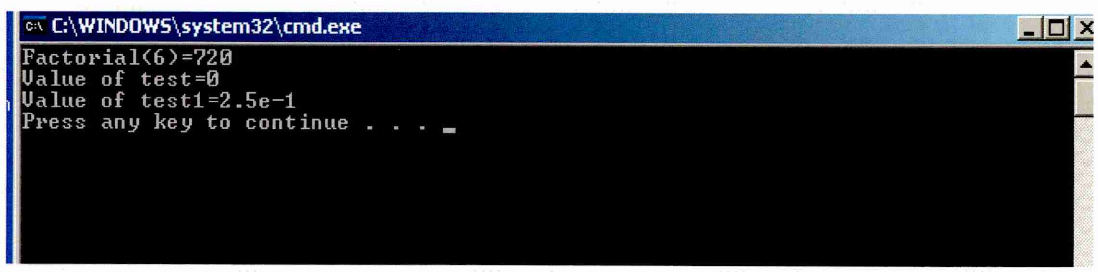
In **Library Directories** add where LEDA libraries are





(13) **Click OK** to leave Properties of projects

(14) To build and run press **Ctrl +F5** and you should see in the debug widow



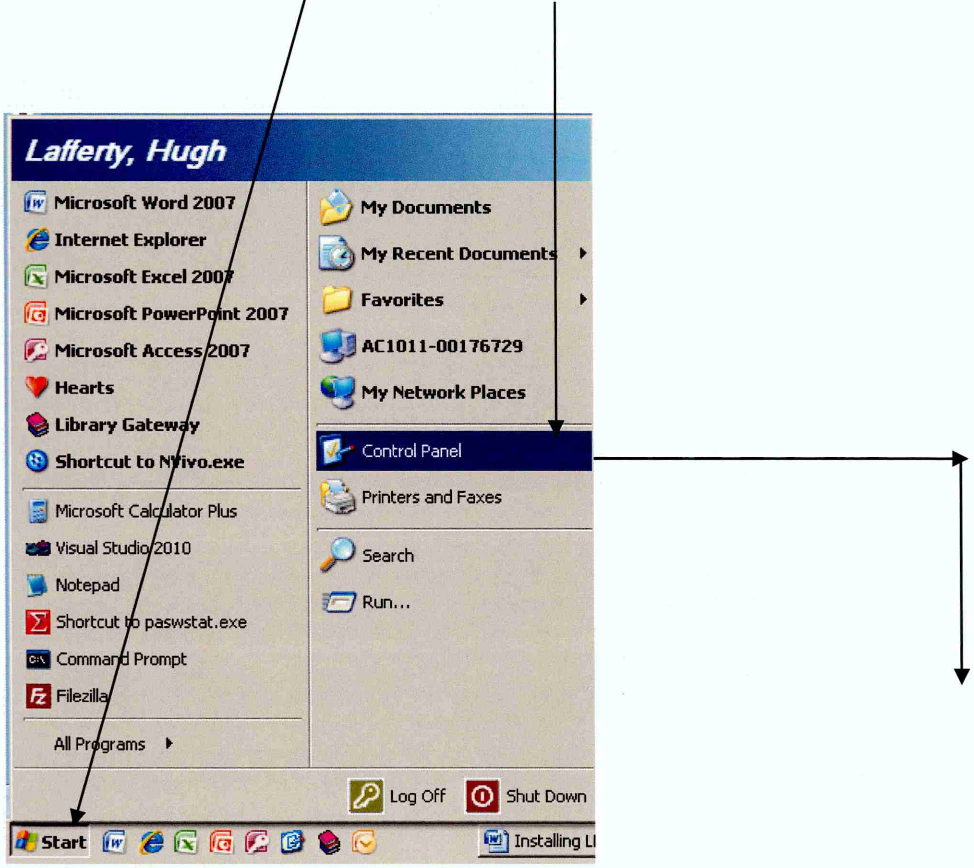
4. Setting up system variables.

(a) Setting the Environment Variables for Visual C++ .NET:

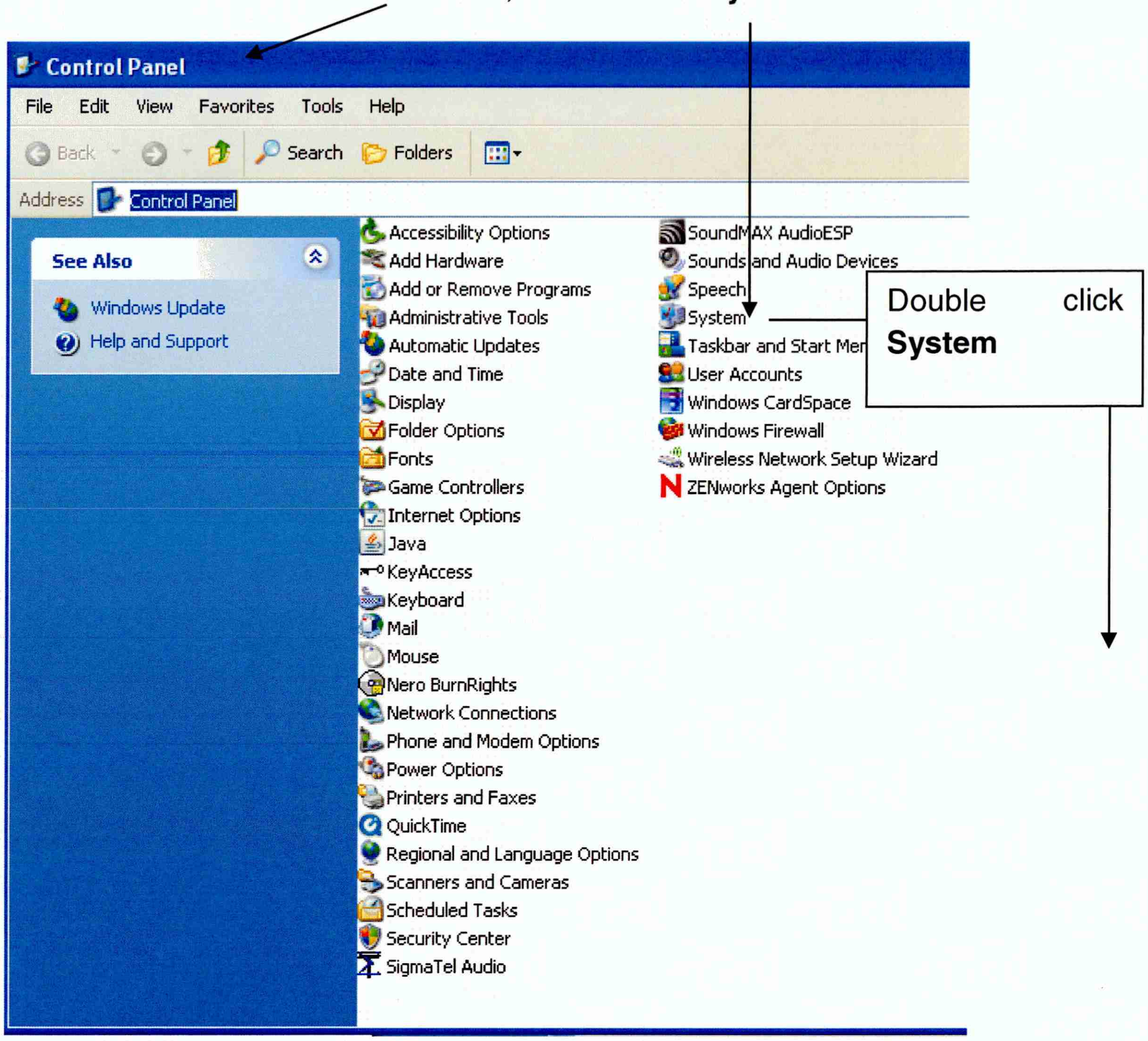
To compile programs together with LEDA, the environment variables PATH, LIB, and INCLUDE must contain the corresponding LEDA directories.

(b) Setting Environment Variables for LEDA:
(i) Windows NT/2000/XP:

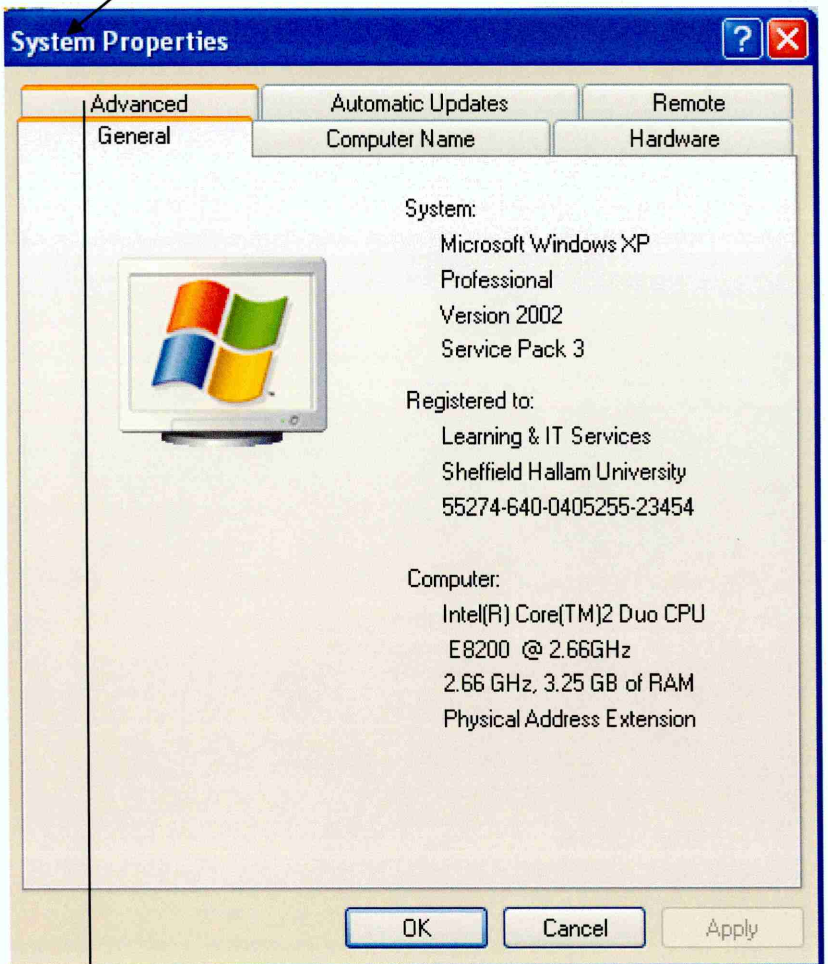
1. In the "**Start**" menu click "**Control Panel**".



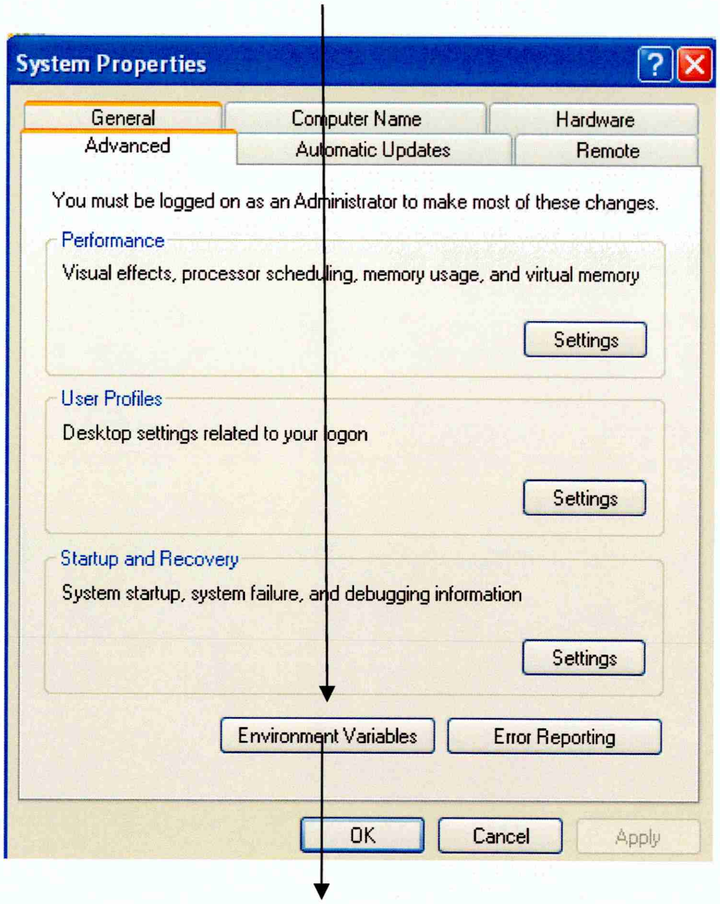
2. In the "**Control Panel**", double click "**System**".



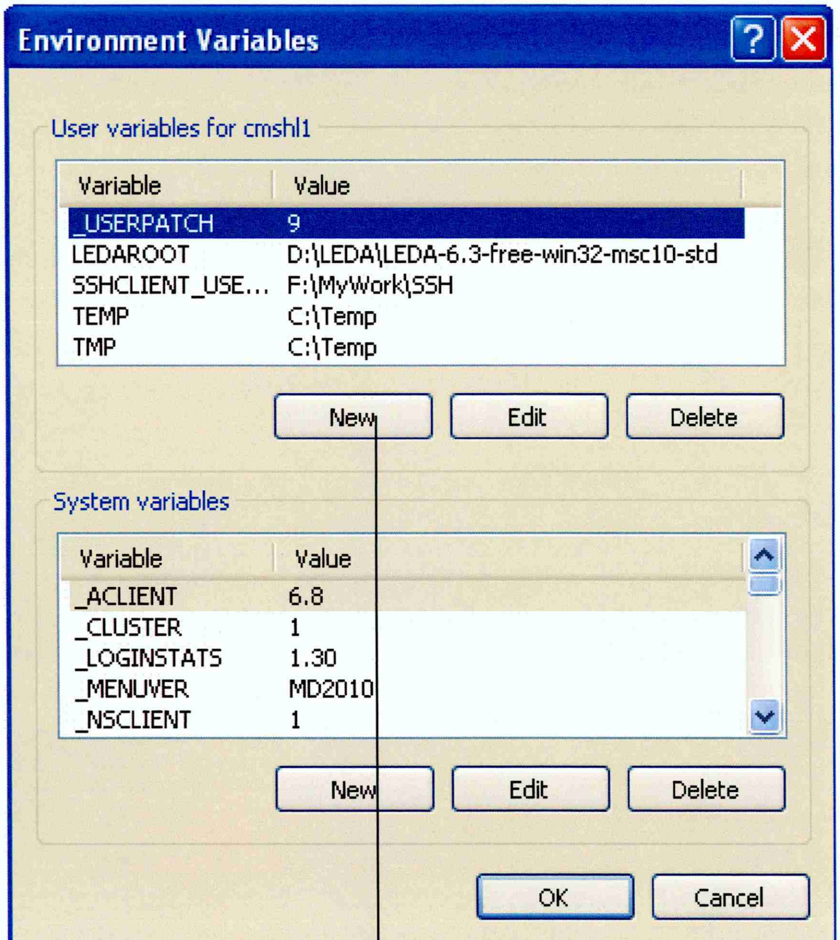
3. In the "**System Properties** "dialog box, click the "**Advanced**" tab.



4. Click on "**Environment Variables**"



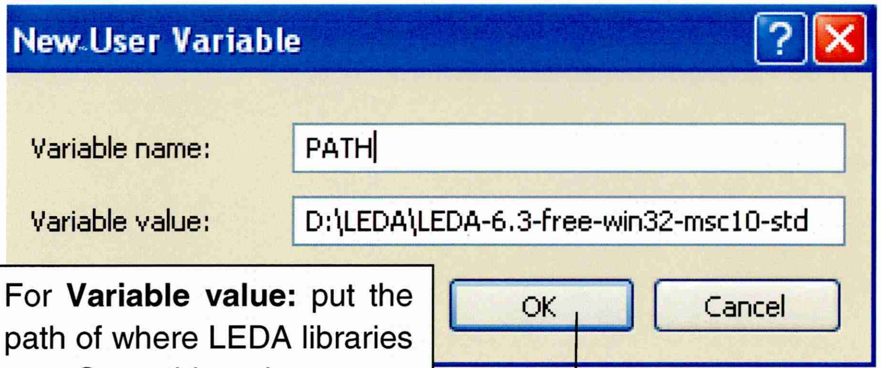
5. Create PATH, LIB and INCLUDE user variables



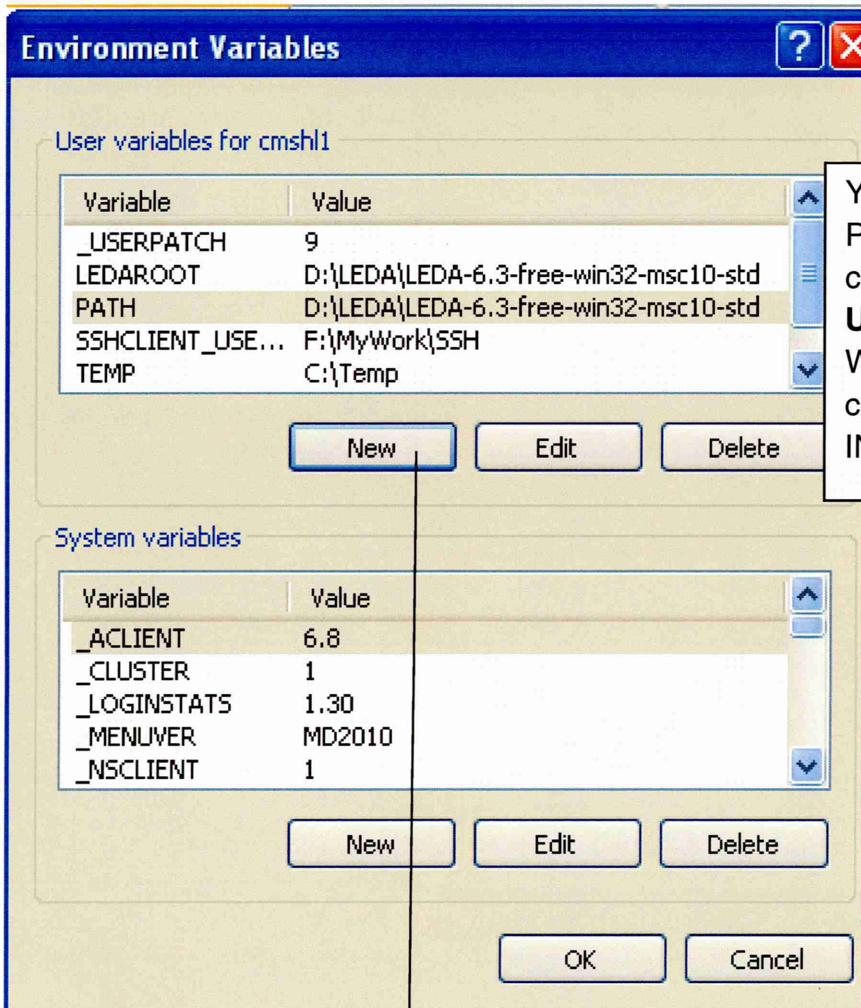
If a **user variable** PATH, LIB, or INCLUDE already exists, extend the current value as follows:

- extend PATH by <LEDA>
- extend INCLUDE by <LEDA> \incl
- extend LIB by <LEDA>

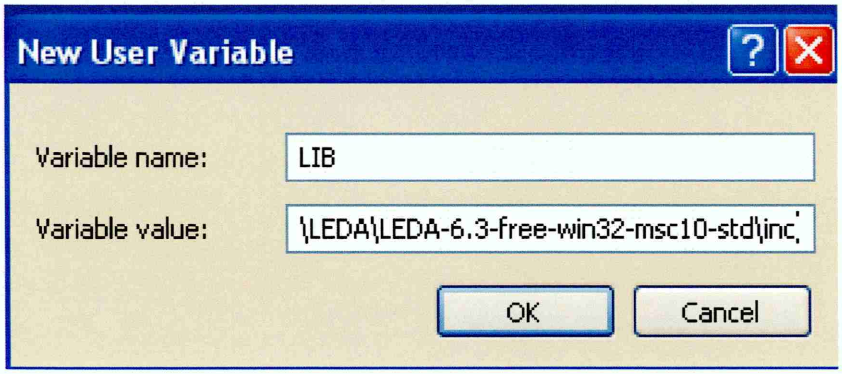
Otherwise add new a **user variable** PATH, INCLUDE, or LIB with value <LEDA>, respectively <LEDA> \incl.

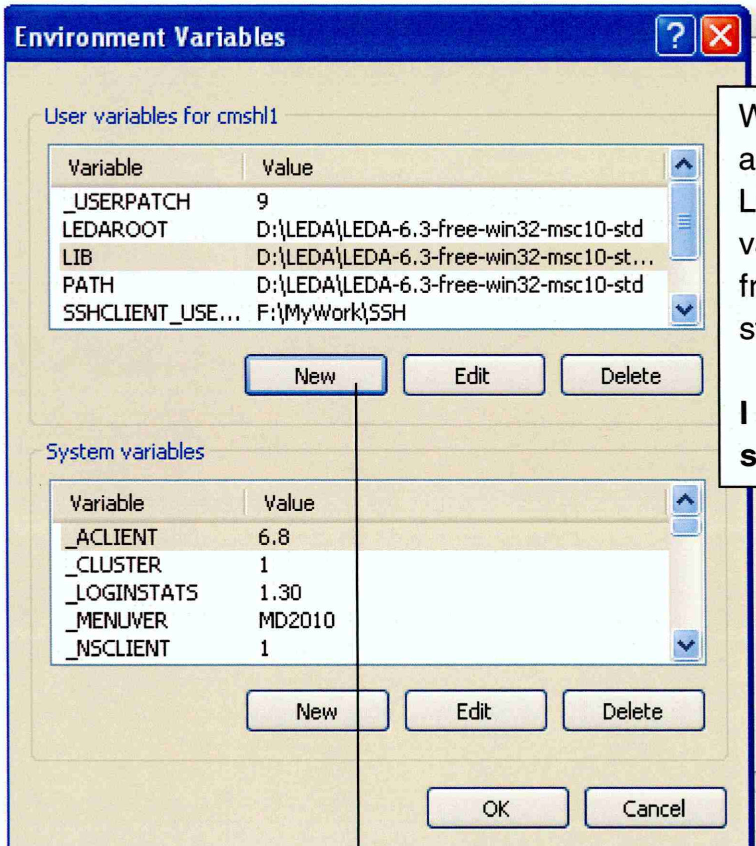


For **Variable value:** put the path of where LEDA libraries are. Copy this value as we are going to use it later.



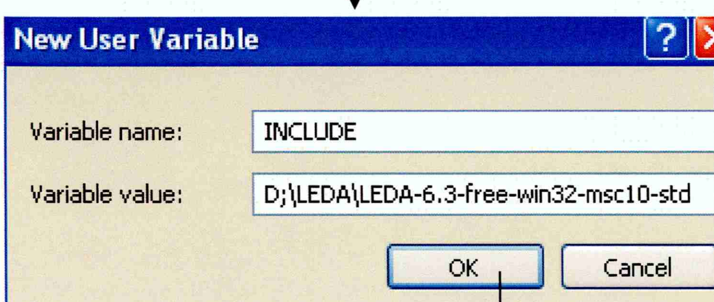
You will see that PATH has been created as a **User variable**. We now have to create LIB and INCLUDE



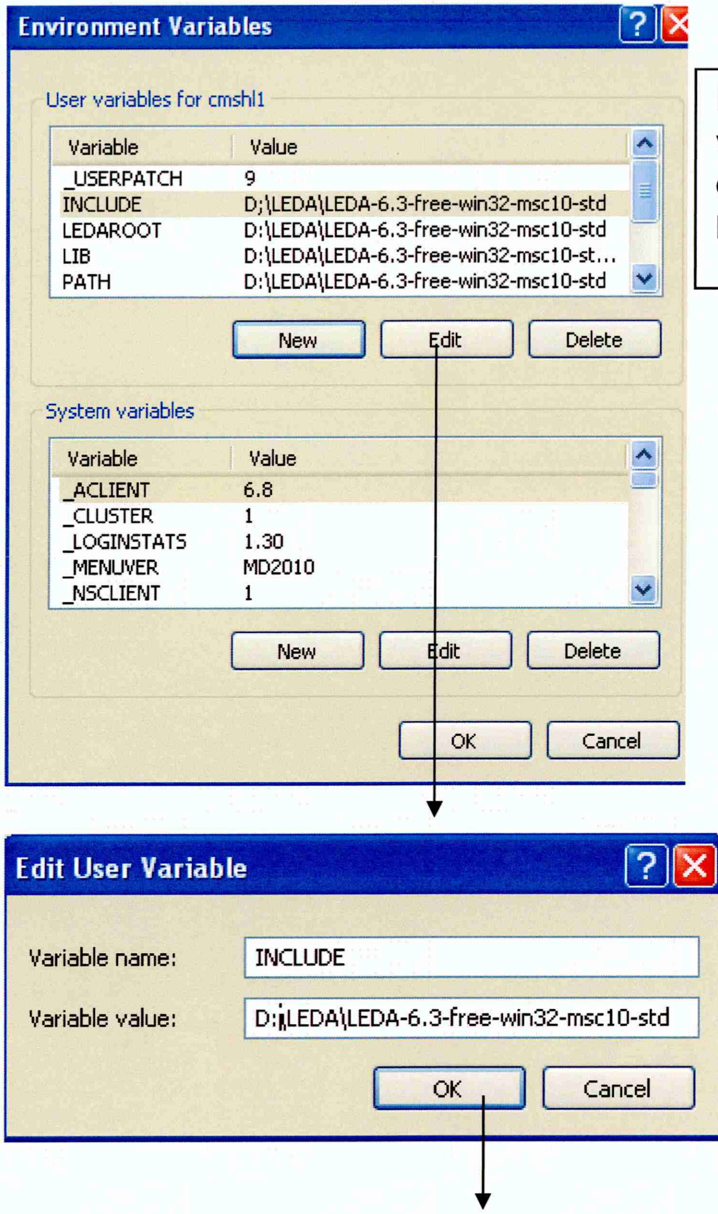


We have now created a User variable called LIB which has the value D:\LEDA\LEDA-free-win32-msc10-std**incl**

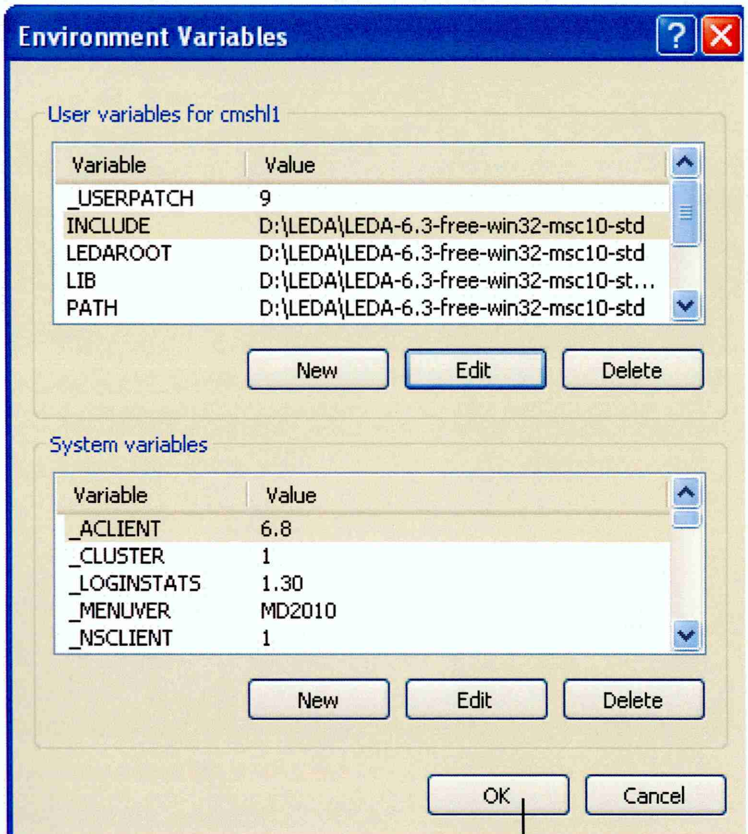
I do not know how to see all of it



You will notice a mistake here where we have written D; instead of D:. So, we have to **Edit** the **User variable** INCLUDE



Point, first, to the **User variable** that you want to change and then press **Edit**



You will see that INCLUDE is now changed

The only way to validate it is to see if this works, and it does.

Appendix 18. Difficulties we had.

18.1 Introduction

We describe the difficulties we encountered, which can be classed into:

- Difficulties with installing **V**isual **S**tudio (VS).
- Difficulties with the LEDA routines.
- Difficulties with making VS work with the LEDA routines.
- Difficulties with understanding how the programming language C works.
- Difficulties with understanding some Statistical concepts.
- Difficulties with SPSS.
- Difficulties with the limits of Excel™.
- Difficulties with the tools we wrote.

Some of the solutions involved more than 1 difficulty. For example, a new version of VS was loaded and that meant that a new version of the LEDA routines had to waited for (and the downloaded to a 'safe' place) and the programs that were written for the older versions of VS and LEDA had to be re-written to work with the newer versions.

18.2 Difficulties with installing Visual Studio (VS)

We wanted to use the latest version of VS and so the technicians were asked to install VS 8. They could not

> *"Still failed, I'm afraid, so I'll have to approach it from another angle…"*
>
> (Flowers, 2010)

Because I have administrator rights I could install VS 8, which I did onto **C:** drive and wrote programs in the programming language C that used the LEDA routines inside VS **8**.

Without me being told, a version of VS viz. VS **10**, was later installed and then the programs that had worked in VS 8 had to be re-written.

So, this was an installation problem.

18.3 Difficulties with the LEDA routines

We wanted to use the latest version of the LEDA routines, which, at the time was a version that worked inside VS **8.** We also did not want to pay anything for the privilege of using the LEDA routines that would run on our machine. The free version, which has no support, depended on what Operating System (OS) was used, a fact that we did not know. The LEDA free edition talked about i386, FC, Sparc Solaris and msc 8 and 9 (32 bit and 64 bit) and multithread safe. The newer version only talks about Linux OS and Windows OS (and not about Sparc Solaris OS). The technicians were asked whether we used 32 bits or 64, and after a long conversation it was thought that 32 bits was relevant (whether multithread was safe was unclear). When **A**lgorithmic **S**olutions (AS) of Germany were asked what version we should download we got a very 'frosty' reply,

> *"Are you writing a book or a blog about what you think are bad product descriptions or cryptic websites and you would like to engage us into some funny conversation that you can publish there?"*
> *(Support Desk 2010).*

But, they eventually replied that:

- **FC** stood for **F**edora **C**ore
- **i386** implies 32 bits
- **msc** stood for **M**icro**s**oft **C**orporation

and so, we downloaded **LEDA-6.6, i386, msc 8 (.NET 2005), 32 bit** to **C:\Algorithmic Solutions**.

The people around me complained that the system was running slowly and the technicians thought that was due to us using an old image. So, a new image was put onto **C:** that used VS 10 (despite the technicians saying they could not download VS 8), and that had 3 effects. Everything that was on **C:** was 'wiped'. A new version of the LEDA routines that worked in VS10 (when they had been written by AS) had to be downloaded into a 'safe' place, and it was decided that **D:** drive was 'safe' i.e. would not be 'wiped'. The programs that I had written in the programming language C for VS 8 had to be re-written to work in VS 10.

So, the problems break down into:

- *What* version of LEDA should be downloaded, despite problems with AS?
- *Where* should the LEDA routines be downloaded to?
- A new version of the LEDA routines, might cause the programs that use them to be re-written.

A good feature of the LEDA routines was that they included the function factorial(m)

18.4 Difficulties with making VS work with the LEDA routines

The difficulties of making VS work with the LEDA routines can be broken down into:

- configuring VS and telling VS where the LEDA routines are,
- telling the system what the PATH, INCLUDE and LIB environment variables are.

We attempted to do this in Appendix 17.

Configuring VS means telling it things like

1. the name of a project,
2. where is the solution,
3. what sort of application we are writing,
4. what sort of source file we are going to write (anything ending in .cpp is a C++ file),
5. what the source file is called,
6. how line numbers are included in the source file. This way we can understand, easily, the messages VS outputs. We only have to add line numbers *once* per project,
7. the source code,
8. what the compiler flag is,
9. where to find the LEDA dll,
10. where to find the LEDA library,
11. what directory to use for the #includes (this involves telling VS where the LEDA includes are),
12. what directory to use for the LEDA libraries.

This is a very error-prone process, because it is so long, and in order to reduce the number of errors, we wrote Appendix 17. But, there is still the difficulty of knowing whether to take out the _DEBUG statement and if there is an error we take it out

"The compiler defines _DEBUG when you specify the /MTd or /Mdd option. These options specify debug versions of the C runtime library." (Masur 2011)

Telling the system what 3 environment variables are is not a hard process and only has to be done *once*.

In order to build and run the source file we press Ctrl +F5.

So, the difficulties associated with VS are in configuring it to work with LEDA.

18.5 Difficulties with understanding how the programming language C works.

I had extensive knowledge of computer languages, but did not know C. The trouble with learning C which runs inside VS using the LEDA routines, was knowing whether an error:

a) was due to a lack of understanding how C works, or
b) due to a poor understanding of how VS works, or
c) due to a lack of understanding of how the LEDA routines work.

Words like namespace and define are new to me, although I am used to the concept of a constant (that define uses), and the concept of avoidance of name clashes (that namespace uses). include is not new to me but what is in what libraries sometimes baffles me, and so what to include is of concern. The idea of libraries is not new to me but getting C to work with the included libraries caused me great problems. So much so, that asking Algorithmic Solutions for help often implied that it was a C problem, not a LEDA problem

"The problems you have are not related to any problems with the LEDA package but to the correct use of the developer environment from Microsoft, the correct use of LEDA and correct C++ programming." (Support Desk Algorithmic Solutions 2010).

In the end they got completely fed up and refused any help at all

"We would like to kindly inform you, that the LEDA free edition comes without support." (Support Desk Algorithmic Solutions 2010)

18.6 C did not work as I expected it to.

The programming language C did not do what was expected of it in a number of ways

- in a statement like c=a/b; if a and b are both integers then this statement might give the wrong answer, especially if b is bigger than a, and thus c is given the value 0 (no matter what the type of c is). In the expression a/b if both sides of the operator / are integers then the operation yields an integer,
- this means that all divisions have to be looked at,
- in a loop like

```
for (i=1; i< k; i++)
        {

        }
```

  Is this repeat loop or a while loop? The basic question is "What is the minimum number of times that this loop can be executed?" The loop starts off with i=1 and then tests whether i is less than k, and then increments i. An even more basic question "Is the test done at the beginning of the loop or at the end?" So, if k is less than 1, how many times is this loop executed? Is it 0 or 1? I thought that this loop would be executed a minimum of 1 time. I was wrong.

  In Algol 60 a for-loop is executed at least once and in FORTRAN a do-loop is executed at least once.

- in an initialisation like maxForWorkingOutMedian=9999999999; (this has ten 9s). If maxForWorkingOutMedian is declared as of type integer (a LEDA type) then there is an error. A way round this is to initialise maxForWorkingOutMedian to nine 9s and then add to it e.g.

```
for (int i =1; i<5; i++)
  maxForWorkingOutMedian =10* maxForWorkingOutMedian +9;
```

- the main program is an integer function called **main** e.g.

```
int main()
{

}
```

- we can declare variables to be of type bool (i.e. boolean) but booleans are held as integers, with 0 representing false and 1 representing true.

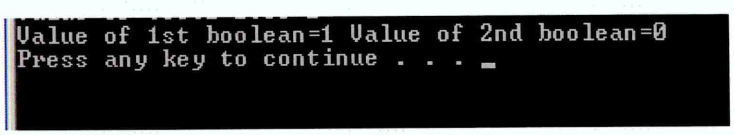
"When boolean values are evaluated, they actually don't evaluate to true or false. They evaluate to the numbers 0 (false) or 1 (true). Consequently, when we print their values with cout, it prints 0 for false, and 1 for true:"

(LearnCpp.com, 2012)

For example

```
int main ()
  {
   //Declarations

        bool firstBoolean,  secondBoolean;

   //Initialisation

        firstBoolean=true;
        secondBoolean=false;

   //Output
        cout <<  "Value of 1st boolean=" << firstBoolean;
        cout << " Value of 2nd boolean=" << secondBoolean << endl;
  }
```

and then in the debug window we should see



18.7 Difficulties of learning some statistical concepts

I had to learn many statistical concepts that include:

- What is a probability density function?
- What do *discrete* and *continuous* mean?
- What is a *distribution*?
- What does our data mean?
- What does *population* mean?
- What is a *census*?
- What is a *sample*?
- What does a sample have to be?
- What is *expectation*?
- What is *mean*?
- What is *mode*?
- What is *median*?
- What is *variance*?
- What is *standard deviation*?
- What is *skewness*?
- What is *kurtosis*?
- What does *symmetric* mean?
- What is the *moment* about the mean?
- What are *descriptive statistics*?
- What are *parametric* and *non-parametric* statistics?

18.8 What is a probability density function?

A probability density function of a *continuous* variable is the function that gives the probability of a value occurring.

18.9 What do discrete and continuous mean?

Variables come in two types discrete and continuous.

A discrete variable (like Mark) takes on a finite number of values.

A continuous variable takes on an infinite number of values.

18.10 What is a distribution?

A distribution has an x-value and a y-value.

In our case the x-value is a Mark, and the y-value is number of times that mark occurs.

18.11 What does our data mean?

The data, before a Pivot, might look like this

Mark	How many get that mark that way
0.111	278
0.222	3010
0.111	56

So, the data is *discrete*.

The first column is a mark and the 2nd column is the number obtaining that mark *that way*. So, 278 obtained the mark 0.111 *that way* and 56 also obtained the same mark in another way: explaining why a Pivot is necessary. How the mark was obtained is not recorded.

So, the 2nd column is the frequency of obtaining a mark, *that way*.

In order to explain the words *that way*, we will look at another example

0	265
1	304
0	27

and then after a Pivot, if we see

0	292
1	304

Then we can say there were 292 occurrences of the mark 0, and 304 occurrences of the mark 1. We can only say this after a Pivot is done.

So, the probability of the mark 0 occurring is 292/596=0.4899 and the probability of the mark 1 occurring is 304/596=0.5101. Their sum is 1.

The probability density function of the marks 0 and 1 occurring is (0.4899, 0.5101)

Excel™ can draw this distribution.

18.12 What does population mean?

A *population* is the whole set. In our case the *population* is all the marks that can be obtained in the quiz.

18.13 What is a census?

In our case a census looks at *all* the marks.

18.14 What is a sample?

A *sample* is a subset of the population. There are many types of sample e.g. random, systematic, convenience and snowball. For example, a systematic sample might be every 5^{th} one. In our case a *sample* is some of the marks that can be obtained in the quiz.

18.14.1 What does a sample have to be?

In order to generalise about the *sample*, it must **represent the population**. For example, if the population has a characteristic that the number of marks possible with a quiz consisting of m stems is 7 times the number of marks with a quiz consisting of (m - 1) stems, and a sample does not have this characteristic, then the sample does not represent the population. So, a sample of the marks of a quiz that is marked by CBM might not represent all the marks.

18.15 What is expectation?

The *expected* mark for a discrete distribution is defined as

$\sum$ Mark *(Probability of that mark occurring)

Generalising, where Mark=x

$E(x)= \sum x.\ p(x)$

18.16 What is mean?

The *mean* mark for a discrete distribution is defined as

$\sum$ (Mark)/(How many marks there are)

It can be shown that the mean mark and the expected mark are the same.

The mean for a sample is denoted by $\overline{x}$ and the mean for a census is denoted by μ.

18.17 What is mode?

The *mode* mark can be defined as the most frequently occurring mark, and in SPSS the mode is obtained under **Frequencies**.

But then we have to ask "What happens if there are 2 equal most frequently occurring marks?" In which case we have a bi-modal distribution. But if there is only one most frequently occurring mark we have a uni-modal distribution. In the case of a bi-modal distribution of marks obtained by a quiz then the mode mark is the mean of the two mode marks.

We then ask "What happens when there are more than 2 modes?" and then in our case, the mode is (the highest mode mark - the lowest mode mark)/(the number of modes). The tossing of an un-biased die is an example of a 6-mode distribution.

Finally we ask "What happens when mode does not exist?" as with the function tan(x). In that case, mode should not be calculated. In our case we mostly have uni and bi-modal distributions and very occasionally a tri-modal distribution, and so *tool2* always calculates mode. After a Pivot is done our data is

Mark **The number of times that mark occurs in the data**

So, the most frequently occurring mark is the *maximum* of the number of times that mark occurs in the data.

18.18 What is the median?

The *median* mark is the 'middle' mark. This has the difficulty of "what does **middle** mean?" It is best demonstrated by an example. Say, that our data, after a Pivot is

Mark	Number of times that mark occurs
0	5
1	4

Then there are 5 marks of 0 and 4 marks of 1

So, the marks are

0000 0 1111

and the 'middle' mark is 0.

If, however, the data is

Mark	Number of times that mark occurs
0	5
1	5

Then there are 5 marks of 0 and 5 marks of 1

So, the marks are

0000 01 1111

and the 'middle' mark is between the 0 and the 1, and so we take the mean and get 0.5 (a mark that is not possible, in this data).

So, the definition of the median mark is

If the number of data points (n) is **odd** then take the mark that corresponds to n/2 (in C), as long as n ≥ 2 (after a Pivot is done).

If the number of data points (n) is **even** then the median mark= (mean of the marks that correspond to most frequently occurring values) (after a Pivot is done).

(Here, n is NOT the number of stems, but the number of data points)

This means that the evenness or oddness of the number of data points has to be calculated. When Excel™ uses Scientific Notation the evenness or oddness cannot be calculated, because the number that get a mark is always even (e.g. 1.3 E^{27}) which is a multiple of 10 and thus even.

A natural question is "What are the advantages and disadvantages of mean, mode and median?" We can generalise this question a bit further by noting that mean, mode and median are all measures of the 'middle' of the data. There are many measures of the 'middle' of the data that include, arithmetic mean (the one we use), geometric mean, harmonic mean, weighted mean, truncated mean, midrange, midhinge, trimean, Winsorized mean, mode, median and geometric median. These are some of the measures of Central Tendency and so the question becomes "What are pros and cons of the measures of Central

Tendency?" The answer is left to the reader. But, one of the answers might be "Mean is distorted by high/low values". For example, the annual salaries of a sample of people might be £12,000, £26,000, £1,000,000. The mean salary of this sample is (12,000 + 26,000 +1,000,000)/3=£346,000 but in most of this sample people receive an annual salary much less than this. So, mean, here, is not a very good measure. Perhaps the median salary of £26,000 is a better measure. Perhaps, the *outlier* of £1,000,000 should be ignored, and that is where truncated mean or Winsorized mean come in. This opens a whole Pandora's box of what measures of Central Tendency are most appropriate to us and should outliers be ignored. We will not open this Pandora's box.

18.19 What is variance?

Variance measures the spread of a distribution about its mean $=\sum x\ p(x)$

18.20 What is standard deviation?

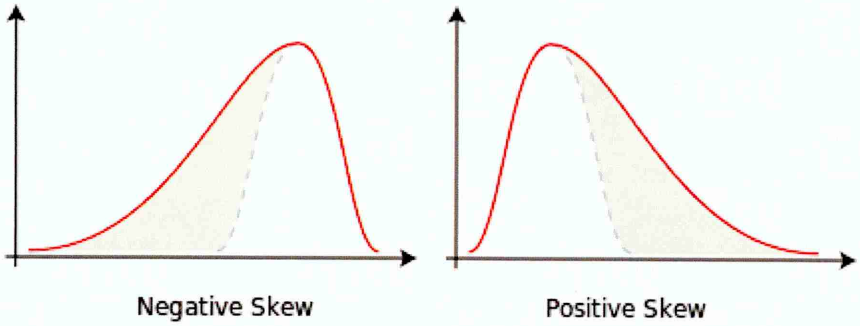
***S**tandard **D**eviation* (S.D.) of a distribution is the positive square root of the variance of the distribution.

The symbol for the S.D. of a census is $\boldsymbol{\sigma}$ and the symbol for the S.D. of a sample is **s**.

A natural question is "Why do you need both variance and S.D.?" and an answer is that you do *not* need both.

18.21 What is skewness?

Skewness measures whether a distribution has a long tail to the left or right.



18.22 What is kurtosis?

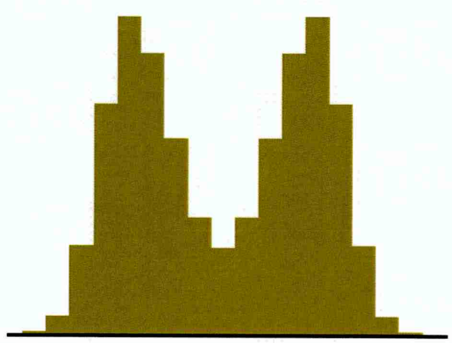
Kurtosis measures how 'peaky' the data is.

Distributions with a negative value of kurtosis are called *platykurtic*, and those with a positive value of kurtosis are called *leptokurtic.*

18.23 What does symmetric mean?

The distribution is not skewed (skewness=0) when a distribution is symmetric. This is a necessary condition but not sufficient.

> *"A situation in which the values of variables occur at regular frequencies, and the mean, median and mode occur at the same point" (Investopedia, 2011).*



This is an image of bi-modal, symmetric, discrete distribution.

18.24 What is moment about the mean?

The p^{th} moment about the mean for a census of a discrete distribution is

$$\sum (x_i - \mu)^p p(x)$$

Thus, *variance* is the 2^{nd} moment about the mean i.e. where p=2.
and *skewness* is the 3^{rd} (moment about the mean)/σ^3.
and *kurtosis* is the 4^{th} (moment about the mean)/σ^4.
and **excess** *kurtosis* is kurtosis -3.

18.25 What are descriptive statistics?

Descriptive statistics are measures used to describe a distribution, without drawing it. For example, we talk about *range* of the x-axis, and in our case it means what is the range of marks that we are considering e.g. from 0 to 100.

In this thesis we both draw a distribution and superimpose the descriptive statistics of the distribution by hand.

The descriptive statistics include, range, minimum x-value, maximum x-value, mean, mode, median, variance, standard deviation, skewness and kurtosis.

18.26 What are parametric and non-parametric statistics?

Parametric statistics assume that the data has parameters and non-parametric statistics do not make any assumptions about the data. Typically, parametric statistics assume that the data is Normal.

We do not assume that our data has any parameters and so we can be said to be using non-parametric statistics.

18.27 Difficulties of SPSS

SPSS treats the data as a *sample*, and not as a census, and hence uses **estimates** of its statistics which are inaccurate for small populations. For example,

App 18 Table 64. Samples and estimates.

	Estimates. **Formulae that SPSS uses.** **n=size of sample**	**Census** **Formulae** **N=size of population**
Variance	VarSam= $\frac{\sum (x_i - \bar{x})^2}{n-1}$	VarPop=∑X P(x) = $\frac{\sum (x_i - \mu)^2}{N}$
Standard Deviation	s= $\sqrt{VarSam}$	σ= $\sqrt{VarPop}$

where n is the size of the sample and N is the size of the population. So, the variance of the sample is divided by (n -1) and the variance of the population is divided by N. This leads to inaccuracies when n is 'small'. For example, the formulae 'settle down' when the number of stems is about 9 for Yes/No quizzes where all stems are answered (see Appendix 16).

We had to find out what formulae SPSS uses for its descriptive statistics and then see what the formulae should be used for a census. The following table shows the estimates that SPSS uses and the formulae that should be used for a census.

App 18 Table 19. Descriptive statistics, for estimates and measures.

	Estimates. **Formulae that SPSS uses.** **n=size of sample**	**Census** **Formulae** **N=size of population**
Mean	$\bar{x}=\frac{\sum x_i}{n}$	$\mu=\frac{\sum x_i}{N}=\sum x_i\, p(x_i)$
Variance	VarSam= $\frac{\sum (x_i-\bar{x})^2}{n-1}$	VarPop= $\frac{\sum (x_i-\mu)^2}{N}$
Standard Deviation	s= $\sqrt{VarSam}$	σ= $\sqrt{VarPop}$
Skewness	$\frac{n\sum_{i=1}^{n}(x_i-\bar{x})^3}{(n-1)(n-2)s^3}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^3}{N\sigma^3}$
Kurtosis	$\frac{n(n+1)\sum_{i=1}^{n}(x_i-\bar{x})^4}{(n-1)(n-2)(n-3)s^4}-\frac{3(n-1)^2}{(n-2)(n-3)}$	$\frac{\sum_{i=1}^{N}(x_i-\mu)^4}{N\sigma^4}-3$

Another difficulty we had was that we could not find out how SPSS calculates its descriptive statistics for a distribution (i.e. 2 columns). We could only find out how SPSS calculates its descriptive statistics for1 column. Our data consisted of 2 columns, a mark and the number that got that mark, and we ignored the mark, so that our data, finally, was in 1 column and we could compare estimate formulae with census formulae. We found that the results were different when n was 'small' and when SPSS could not handle very large numbers.

Another difficulty we had was that the textbooks appeared to be wrong about Kurtosis. Field (2005, p11) and an SPSS manual (Introduction to PASW, 2010, p14-11) say

> *" in a normal distribution the value of skew and kurtosis are 0..",*
> *"...normal curve (for which the kurtosis is zero)."*

But, the kurtosis for a Normal distribution is 3.

> *"The kurtosis for a standard normal distribution is* ***three.****"* (Engineering Statistics Handbook)

What Field and an SPSS manual mean is that for a normal distribution the ***excess*** kurtosis is 0, which they both continue to call kurtosis, and so will we.

18.27.1 The limits of SPSS

The data for SPSS is coming from the file **Output.txt** and when the numbers that it uses are 'too big' SPSS calculates the wrong answers for its descriptive statistics (see Appendix 16). The reason for this is that SPSS can only handle numbers in a file that occupy up to 40 characters (even if the variable is of type Scientific notation). That is why you see E39 such a lot where mean is concerned. The square of this is about E78, and so the variance is about E78 (see Appendix 16).

So, we cannot use SPSS because:

- it calculates the descriptive statistics wrongly, for a situation where a division by n results in an answer that is significantly different from the answer when a division is done by (n - 1). This is because SPSS treats the data as a sample, and thus uses estimates for its descriptive statistics.

- it calculates the descriptive statistics wrongly, for a situation where the data comes from a file and the numbers that the data represent occupy more than 40 characters.

- We do not know how to receive the descriptive statistics of a distribution.

- We do not know how to obtain the descriptive statistics of 2 columns (like the variance).
- The data that SPSS uses does not mean the same thing as we mean. For example, the data might be

Mark	
0	292
1	304
0	3

We mean that the mark 0 appears 295 times (after a Pivot), and if we see in SPSS, how many times the mark 0 appears SPSS says 2 (when we look at the column called **Mark**)

18.28 Difficulty with the limits of Excel™

The limits of Excel™ are easy to find as they are published (http://office.microsoft.com/en-001/Excel-help/excel-specifications-and-limits-HP010073849.aspx), and the ones that interest us include the number of rows that Excel™ could deal with is (about 1 million).

We never came across this restriction, but when we loosen the restriction on the types of stems in a quiz (e.g. have Yes/No stems and 4-option MCQ stems in the same quiz), we might.

- the maximum length of a cell (255 characters)

Excel™, like SPSS, uses Scientific Notation (when it has to), and so the evenness or oddness of a number cannot be calculated when Scientific Notation is used. Further, if we have a number that occupies more than 255 characters then a Pivot cannot be done. If we have a number that occupies, say, 300 characters, then the first 255 characters are put in cell A26 (say) and the characters of the number from 256 to 300 are put in cell A27. The number that corresponds to characters 1 to 255 in A26 is not the number that corresponds to the number in A26 and A27. If we ask for a Pivot to be done on the number in A26 we get the wrong answer.

- the maximum number that can be dealt with in a cell (about 1 E308).

18.28.1 Getting Excel ™ to import a tab-delimited text file.

We wrote Appendix 9 to deal with getting Excel ™ to *import* a tab-delimited text file.

18.28.2 Getting Excel™ to do a Pivot

The necessity for a Pivot is that the data before a Pivot is

Mark **Number getting that mark that way**

and the *same* mark might be obtained in 2, or more, ways.

After a Pivot, the data is now

Mark **Number getting that mark**

and so after the Pivot

- the marks increase, and there is only 1 mark 0 (say)
- the number getting that mark might involve numerous additions

Appendix 10 shows how Excel™ does a *Pivot.*

18.28.3 Getting Excel ™ to export a tab-delimited text file

We wrote Appendix 11 to deal with getting Excel™ to *export* a tab-delimited text file.

18.29 Difficulties with the tools we wrote

The tool, *tool1* calculated a mark and how many got that mark that way, and then outputted to a tab-delimited text file (see Appendices 12, 13 and 14) for Excel™ to do a Pivot on this data..

Excel™ imported this text file (see Appendix 9) did a Pivot (see Appendix 10) drew a distribution and then exported to another tab-delimited text file (see Appendix 11). The drawing of the distribution was copied to Word by hand.

The tool,*tool2* then imported this text file and calculates some descriptive and other statistics (see Appendices 15 and 16). These statistics were then superimposed on the distribution, by hand.

Both, a mark and how many get that mark that way are held in arrays, and there is a limit of about 86,000 on the index of an array in the programming language C. But arrays do not need to be used, if we are willing to calculate a mark again.

Thus there does not seem to be a limit on *tool1*.

The limits of Excel™ impose limits on *tool2*. Because Excel ™ can only handle about 1 million rows, if we want to use more than 1 million rows we have to use several intermediate files, between *tool1* and Excel™. This limitation was never encountered in our case, but it might be if the quiz can contain a mixture of stem types (e.g. Yes/No stems and 4-option MCQs). Excel™ cannot handle numbers that occupy more 255 characters, and in a few cases (e.g. a 20-option EMSQ quiz where 10 stems are correct and we use a quiz that has100 stems) we meet this restriction of Excel™. There is also a restriction that Excel ™ cannot handle numbers that are greater than about E308, and again this restriction was met for 'large' EMSQ quizzes. But, at least the mean mark of 'large' EMSQ quizzes could be calculated (which is about 0).

Thus, Excel™ has a few restrictions, which are rarely met, but they impinge upon *tool2*.

The tool, *tool2* comes after Excel™ has done its work and is therefore is subject to Excel's™™ restrictions. The tool, *tool2* calculates descriptive and other statistics of a distribution, and it does its calculations on any distribution, and is only limited by what it receives.

The tool, *tool2:*

a) tells us how many modes are,
b) works on census data and not samples,
c) does not use estimates for its descriptive statistics,
d) does more than SPSS e.g. tells us the proportion of passes,
e) works for 'small' samples,
f) uses the same formulae as SPSS until SPSS 'runs out of steam'.

So, we think that *tool2* is only restricted by the limitations of Excel ™.

Thus, the tools that we have built are only restricted by Excel ™.