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An application of Extreme Value Theory to Cryptocurrencies

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Abtract:
We study the tail behaviour of the returns of five major cryptocurrencies. By employing an extreme value analysis and estimating Value-at-Risk and Expected Shortfall as tail risk measures, we find that Bitcoin Cash is the riskiest, while Bitcoin and Litecoin are the least risky cryptocurrencies.

Keywords: Cryptocurrency, Bitcoin, Extreme value analysis, Value-at-Risk, Expected Shortfall

JEL classification: C46, F38, G01

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1. Introduction

The cryptocurrency market has recently grown immensely. Cryptocurrencies are a globally-spreading phenomenon which is frequently addressed by media as well as financial and governmental institutions (Glaser et al., 2014). Bitcoin is the largest cryptocurrency, representing 59% of the total estimated cryptocurrency capitalisation (Coinmarketcap.com accessed on Nov 1st, 2017). For this reason, a lot of academic research has been conducted on Bitcoin (see, e.g., Cheah and Fry, 2015; Dyhrberg, 2016; Urquhart, 2016, 2017; Bariviera, 2017; Katsiampa, 2017; Nadarajah and Chu, 2017). As of October 2017, there are more than 1000 cryptocurrencies, though. Bitcoin is followed in terms of total market capitalisation by Ethereum, Ripple, Bitcoin Cash and Litecoin, each one having a market capitalisation above 3 billion dollars. These five cryptocurrencies together represent 85% of the total cryptocurrency capitalisation at present (Coinmarketcap.com accessed on Nov 1st, 2017). However, despite the huge growth of the cryptocurrency market, research on cryptocurrencies other than Bitcoin is very limited. Among few authors who have studied additional cryptocurrencies are Osterrieder et al. (2017) and Chu et al. (2017).

It is now well-known that cryptocurrencies behave differently to traditional fiat currencies. In fact, cryptocurrency returns not only are more volatile and riskier than traditional currencies but also exhibit heavier tail behaviour (Osterrieder et al., 2017; Phillip et al., 2018). Cryptocurrencies could be therefore viewed as assets, and have a place in financial markets and portfolio management (Dyhrberg, 2016). Nevertheless, such assets display extreme price changes which violate the assumption of normality, and a major challenge of risk management is the appropriate selection of the distribution of asset returns (Longin, 2005). Consequently, examination of the tail behaviour of the returns of cryptocurrencies and of the underlying distribution is of high importance.

Although extreme value theory could be useful to better understand the characteristics of the distribution tails of asset returns (Longin, 2005), investigation of extreme value behaviour of cryptocurrencies is rather limited. To the best of the authors' knowledge only Osterrieder and Lorenz (2017) and Osterrieder et al. (2017) have applied extreme value theory to cryptocurrencies. However, the former studied the extreme value behaviour only of Bitcoin, while the latter considered several cryptocurrencies, but excluded Ethereum and Bitcoin Cash, which are among the largest cryptocurrencies, and used data only for the period between June 2014 and September 2016. Motivated by the emergence of cryptocurrencies as speculative assets and by the huge price fluctuations in the cryptocurrency market, in this paper we extend the study of Osterrieder et al. (2017) by using an updated dataset of major cryptocurrencies, including Ethereum and Bitcoin Cash. We also contribute to the literature by providing more accurate results based on an extreme value distribution, namely the generalized Pareto distribution (GPD). The GPD is the only nondegenerate distribution that approximates asymptotically the limiting distribution of exceedances (Balkema and De Haan, 1974; Pickands, 1975). We therefore consider only the relevant information of extremes providing more accurate risk estimates. Hence, by applying extreme value theory, the aim of this paper is to examine the tail behaviour of the major cryptocurrencies.

2. Data and methodology

In this study, we analyse the five largest cryptocurrencies, each one from the earliest date available to 23rd October 2017. More specifically, the dataset consists of the daily closing prices for the Bitcoin Coindesk Index (from 18th July 2010, 2655 observations), Ethereum (from 7th August 2015, 809 observations), for Ripple (from 4th August 2013, 1542 observations), Bitcoin Cash (from 23rd July 2017, 93 observations), and Litecoin (from 28th April 2013, 1640 observations). Although Bitcoin Cash was only recently launched (July 2017), some conclusions regarding its tail behaviour can still be drawn. The data are publicly available online at https://www.coindesk.com/price/ for Bitcoin and at https://coinmarketcap.com/coins/ for the remaining cryptocurrencies. We apply a data adjustment procedure similar to Longin and Pagliardi (2016) in order to obtain stationary time-series for the returns of the cryptocurrencies taking heteroskedasticity into consideration.

We start the risk analysis of the cryptocurrency returns by fitting a GPD to the marginal distribution of the returns of each cryptocurrency using the peaks-over-threshold method to extract extremes. We therefore estimate the two major tail risk measures of Value-at-Risk (VaR) and Expected Shortfall (ES) as extreme quantiles of the GPD. Then we proceed by applying a parametric bootstrap bias-correction approach to the two risk measures in order to reduce any uncertainty resulting from the estimation procedure of the asymptotic extreme value distribution and the threshold selection, as in Gkillas et al. (2016).

The distribution of univariate exceedances $(X - u)$ of a random variable X over a threshold, u, can be asymptotically approximated by the GPD, which is defined as

$$G_{\xi,\sigma}(x) = 1 - \zeta \left[1 + \frac{\xi x}{\sigma}\right]^{-1/\xi}, \; x > u, \tag{1}$$

where x represents the exceedance, $\sigma > 0$ represents the scale parameter, $\xi \in \mathbb{R}$ is the tail index or shape parameter, and ζ is the tail probability. In order to select the threshold, u, we apply a failure-to-reject method following Choulakian and Stephens (2012).

We then consider the risk measures of VaR and ES as functions of the parameters of the GPD. The VaR quantifies the maximum gain/loss occurring over a given time-period at a given percentile, p. We define the over one-day period VaR as

$$VaR_p^B = u + \frac{\sigma}{\xi}\left[\left(\frac{p}{\zeta}\right)^{-\xi} - 1\right], \tag{2}$$

where B represents the number of bootstrap iterations, and $\zeta \cong k/n$, where k is the number of exceedances over the threshold u and n is the sample size. On the other hand, the ES quantifies the expected size of the exceedance over the VaR. We define the ES as follows

$$ES_p^B = \frac{1}{1-\xi}\left[VaR_p^B + \sigma - u\xi\right], \tag{3}$$

where ES is a conditional mean, given that the VaR is exceeded.

3. Empirical results

Table 1 presents the asymptotic maximum likelihood estimates of the GPD and the estimates of the bootstrap bias-corrected risk measures of VaR and ES for 1000 bootstrap iterations for both the left and right distribution tail of each cryptocurrency. In panel A, the parameter estimates for the scale parameter, σ, the tail index, ξ, and the tail probability, ζ, are reported. In panel B, the bias-corrected risk measures of VaR and ES at the conventional 90^{th}, 95^{th} and 99^{th} percentiles (1^{th}, 5^{th} and 10^{th} for negative returns) are presented.

According to the results, the VaR measures for Bitcoin Cash are the highest of all at all the three percentiles for both negative and positive returns, being about twice as large as the ones for Bitcoin. This result indicates that Bitcoin Cash, which was only recently launched, has the highest potential loss, but also the highest potential gain. Bitcoin Cash also has the highest ES for both negative and positive returns at all these three percentiles.

On the other hand, the extreme returns of Litecoin in the left tail and of Bitcoin in the right tail are the lowest ones according to both VaR and ES. This result suggests that investment in Litecoin and Bitcoin can be viewed safer than in the other three cryptocurrencies considered in this study, and is somewhat consistent with the results of Osterrieder et al. (2017). However, all cryptocurrencies report higher values of risk than traditional currencies according to both risk measures. When the tails of return distributions are extremely heavy, diversification increases portfolio riskiness in terms of VaR, especially for a large class of dependent heavy tailed risks (Ibragimov and Prokhorov, 2016). According to our findings, this means that investors in cryptocurrencies are exposed to a high undifferentiated risk.

4. Conclusions

This paper employed extreme value theory to investigate the tail behaviour of the returns of the five largest cryptocurrencies. We found that Bitcoin Cash has the highest potential gain and loss and is thus the riskiest cryptocurrency, while Bitcoin and Litecoin were found to be the least risky, and hence position in those can be viewed safer than in the other cryptocurrencies considered in this study.

Examination of the tail behaviour of the returns of cryptocurrencies is of utmost importance for both investors and policy-makers. More specifically, our findings have important implications to investors, providing them with a better understanding of which investment choices in the cryptocurrency market are more susceptible to losses and gains as well as of potential bubbles due to exceedingly high returns. Our results are also useful to policymakers who could establish a policy intervention framework to protect investors from positions which have a significantly high financial risk but are not subject to any control and to limit the extent of a potential bubble, taking into consideration the high capitalisation as well as the significant amount of uninformed trades in cryptocurrencies.

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Table 1 Estimation of the univariate distribution of returns' exceedances.

Cryptocurrency	Bitcoin		Ethereum		Ripple		Bitcoin Cash		Litecoin	
Panel A: Univariate distribution estimates										
Parameter	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns
σ	0.0441	0.0458	0.0427	0.0497	0.0421	0.0512	0.0687	0.0530	0.0425	0.0502
	(0.0040)	(0.0044)	(0.0068)	(0.0109)	(0.0052)	(0.0062)	(0.1293)	(0.0226)	(0.0053)	(0.0059)
ξ	0.1766	0.1622	0.1491	0.1062	0.2716	0.3205	0.1293	0.5170	0.2403	0.2896
	(0.0746)	(0.0774)	(0.1279)	(0.0168)	(0.1004)	(0.1013)	(0.0272)	(0.0394)	(0.1005)	(0.0950)
ζ	0.1202	0.1100	0.1301	0.0607	0.1103	0.1305	0.1978	0.3076	0.1001	0.1105
	(0.0003)	(0.0030)	(0.0001)	(0.0007)	(0.0006)	(0.0007)	(0.0017)	(0.0023)	(0.0005)	(0.0006)
u	0.0443	0.0478	0.0540	0.1317	0.0489	0.0536	0.0654	0.0323	0.0485	0.0507
Panel B: Risk measures										
	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns
$VaR_{0.90}$	0.0525	0.0522	0.0654	0.1075	0.0531	0.0678	**0.1145**	**0.1132**	0.0485	0.0558
$VaR_{0.95}$	0.0861	0.0864	0.0979	0.1414	0.0861	0.1112	**0.1690**	**0.1924**	0.0806	0.0955
$VaR_{0.99}$	0.1820	0.1823	0.1875	0.2306	0.1915	0.2580	**0.3160**	**0.5334**	0.1794	0.2250
$ES_{0.90}$	0.1079	0.1078	0.1177	0.1603	0.1125	0.1500	**0.2008**	**0.3098**	0.1045	0.1286
$ES_{0.95}$	0.1487	0.1486	0.1558	0.1983	0.1578	0.2138	**0.2633**	**0.4736**	0.1468	0.1844
$ES_{0.99}$	0.2652	0.2630	0.2611	0.2980	0.3025	0.4299	**0.4322**	**0.6427**	0.2768	0.3667