Recent progress in magnesium-based thermoelectric materials

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Recent progress in magnesium-based thermoelectric materials

Rafael Santos, a Sima Aminorroaya Yamini b, c and Shi Xue Dou a

Magnesium-based thermoelectric materials (MgX, X = Si, Ge, Sn) are considered one of the most attractive groups for large-scale application, due to their materials’ high availability, low cost, low mass density, and reasonably high efficiency. In this work, we present an overview of the recent developments relating to magnesium-based thermoelectric materials and review the current approaches towards high thermoelectric efficiency.

1. Introduction

The combined waste energy in electricity generation and transportation, only in the USA, represents 80.0 percent of the total losses as these industries are the most energy inefficient. Furthermore, the majority of the rejected energy is in the form of waste heat which has prompted electricity generation, transportation and other heavy industries such as metal processing to consider waste heat recovery technologies such as thermoelectric generation. Solid-state thermoelectric devices directly convert heat to electricity. Although traditional dynamic heat recovery technologies are more efficient, particularly in large-scale applications, thermoelectric technology shows promising advantages due to its having no moving parts, virtually eliminating the need for maintenance, as well as long life and reliability, silent operation, and high scalability.

The applicability of thermoelectric devices is highly dependent on the materials used in their fabrication. Higher efficiency devices based on bismuth or tellurium are usually not economically feasible, some industries are eliminating the use of toxic elements such as lead, and automotive applications have weight related restrictions, requiring low density materials. Magnesium-based thermoelectric materials possess the right characteristics to address these challenges while exhibiting high conversion efficiencies in the medium-to-high temperature range (250 to 650 °C), making them the target of great research interest.

Here, we summarise recent developments in magnesium-based thermoelectric materials, review the strategies used to obtain high efficiencies, and describe their associated mechanisms, while comparing the different synthesis techniques employed.

1.1 Thermoelectric efficiency

The main focus in thermoelectrics research has been driven by the pursuit of higher thermoelectric conversion efficiencies, defined by the dimensionless thermoelectric material figure-of-merit ZT.

\[ ZT = \frac{S^2 \sigma T}{\kappa_{tot}} \]  

where S is the Seebeck coefficient, \( \sigma \) the electrical conductivity, \( \kappa_{tot} \) the total thermal conductivity and \( T \) the absolute temperature. The \( ZT \) is dependent on the doping level of the material, making optimization of the charge carrier concentration essential for obtaining a fair comparison between different thermoelectric materials. Furthermore, the thermoelectric figure-of-merit \( ZT \) is a function of \( S, \sigma \) and \( \kappa \), which share dependencies on many fundamental thermoelectric properties, such as the charge carrier concentration. Balancing the effects of these overlapping dependencies creates significant challenges when selecting the correct strategy towards higher \( ZT \).

The thermoelectric quality factor, \( B \), summarizes the most important characteristics that a thermoelectric material should exhibit in order to achieve the maximum \( ZT \):

\[ B = \frac{2k_B^2h}{3\pi^2} \frac{N_vC_l}{m_1^*\Xi^2\kappa_{lat}} \]  

where \( k_B \) is the Boltzmann constant, \( h \) the reduced Planck constant, \( N_v \) the valley degeneracy, \( C_l \) the average longitudinal elastic modulus, \( m_1^* \) the inertial effective mass, \( \Xi \) the deformation potential coefficient, \( \kappa_{lat} \) the lattice component of the thermal conductivity, and \( T \) the absolute temperature. The \( B \) factor itself also has some limitations, as it operates on the assumption that changing the charge carrier concentration merely affects the Fermi level, without any significant impact on scattering or on the electronic structure. The inertial effective mass \( m_1^* \) is defined differently from the effective mass \( m^* \), which is used for semiconductors whose band...
Structures can be described by a single parabolic band model, where the conduction band minimum and the valence band maximum are positioned at the centre of the Brillouin zone (crystal momentum \( k = 0 \)). Many thermoelectric materials have several conduction band minima and valence band maxima at different positions in the Brillouin zone, creating degenerated valleys \( (N_V) \). In this case, the total density of states (DOS) effective mass is defined as \( m^* \), where:

\[
m^*_d = N^*_V / 3 m^*_b
\]

and \( m^*_b \) is the DOS effective mass of a single valley. \( m^*_d \) can be seen as the weighted average effective mass of valleys in semiconductors with anisotropic band structures, and is used to calculate the electrical conductivity. Generally, an increase in \( m^*_d \) results in an increase in \( m^*_b \). Defining the effective masses in this way is essential to understanding the thermoelectric quality factor, as increases in \( m^*_d \) usually lead to improvements in the Seebeck coefficient. According to Equation 3, this can be achieved either through a higher \( N_V \) or a large \( m^*_b \). The latter also means an increased \( m^*_d \), however, which is detrimental to \( B \) (Equation 2), while a higher \( N_V \) leads to a higher thermoelectric quality factor. This is also verified if the thermoelectric figure-of-merit is considered, as a higher \( m^*_d \) leads to a lower electrical conductivity and \( zT \). Moreover, the thermoelectric quality factor indicates that a low lattice thermal conductivity \( \kappa_{lat} \) should result in higher thermoelectric performance. This conclusion is commonly pursued in thermoelectric materials research, and specifically in magnesium-based thermoelectric materials, by promoting phonon-scattering through point defects in alloys, secondary phases, or pores, which generate additional boundaries, further contributing to the scattering of phonons and consequently to the reduction of the lattice thermal conductivity. The strategies currently used to depress the lattice thermal conductivity of magnesium-based thermoelectric materials are discussed in more detail later in this review.

The present market and safety regulations have steered the research and development of thermoelectric materials towards lead-free, environmentally friendly, and cheap materials made from abundant elements. Magnesium-based alloys are viewed as potential replacements for the heavily studied PbTe and others, due to their non-toxicity, high power factor (PF), and low mass density. Thermoelectric materials based on Bi₂Te₃, PbTe, CoSb₃, and GeTe compounds remain the most interesting materials for potential applications due to their high thermoelectric performances (Table 1). Figure 1 compares the \( zT \) values of the current best performing medium-to-high temperature thermoelectric materials, highlighting the place of magnesium-based thermoelectric materials.

### 2. Magnesium-based thermoelectric materials

\( \text{Mg}_xX (X = \text{Si, Ge, Sn}) \) materials are intermetallic alloys that have been studied for decades as potential high \( zT \) thermoelectric materials. The application of these materials in thermoelectric generators for the automotive industry, for example, requires other characteristics beyond thermoelectric performance such as plastic deformation and high fracture toughness in order to survive mechanical stress due to vibrations and temperature cycling. The automotive industry, in particular, requires these generators to be as light as possible, and low mass density materials such as magnesium-based thermoelectrics, with density of ~2 g/cm³, have the advantage over the more common PbTe, CoSb₃, or Bi₂Te₃ with densities between 6.5 and 8.5 g/cm³. The binary compounds of the Mgₓ(Si, Ge, Sn) system – Mg₅Si, Mg₅Ge, and Mg₅Sn – are known to have high thermal conductivity, making this class of compounds one of main approaches towards high \( zT \).

![Figure 1. State-of-art \( zT \) for medium-to-high temperature thermoelectric materials.](image)

**Table 1. Thermoelectric properties of state-of-art thermoelectric materials.**

<table>
<thead>
<tr>
<th>Material</th>
<th>( zT )</th>
<th>( S ) μV/K</th>
<th>( \sigma ) (Ωm)⁻¹</th>
<th>( \kappa_{tot} ) W/mK</th>
<th>( T ) K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Mg}<em>0.18(\text{Si}<em>0.3 \text{Ge}</em>{0.03} \text{Sn}</em>{0.65})<em>0.98 \text{Sb}</em>{0.02} )</td>
<td>1.45</td>
<td>-215</td>
<td>9.5×10⁻⁴</td>
<td>2.35</td>
<td>1.15</td>
</tr>
<tr>
<td>( \text{Bi}<em>2 \text{Sb}</em>{1.3} \text{Te}_3 )</td>
<td>1.86</td>
<td>242</td>
<td>6.5×10⁻⁴</td>
<td>0.65</td>
<td>0.32</td>
</tr>
<tr>
<td>( \text{PbTe}<em>{0.3} \text{PbS}</em>{0.7} )</td>
<td>2.30</td>
<td>240</td>
<td>3.5×10⁻⁴</td>
<td>0.82</td>
<td>0.35</td>
</tr>
<tr>
<td>( \text{CoSb}<em>{1.1} \text{Te}</em>{0.5} )</td>
<td>1.15</td>
<td>-200</td>
<td>1.3×10⁻⁵</td>
<td>3.50</td>
<td>1.50</td>
</tr>
<tr>
<td>( \text{Ge}<em>{0.8} \text{Sb}</em>{0.2} \text{Te} )</td>
<td>2.20</td>
<td>195</td>
<td>1.0×10⁻⁵</td>
<td>2.20</td>
<td>1.20</td>
</tr>
<tr>
<td>( \text{Si}<em>{0.8} \text{Ge}</em>{0.2} )</td>
<td>1.50</td>
<td>-290</td>
<td>3.5×10⁻⁴</td>
<td>2.30</td>
<td>1.38</td>
</tr>
</tbody>
</table>
All MgX (X = Si, Ge, Sn) compounds adopt the face-centered cubic (FCC) crystal structure, and their structural symmetry corresponds to the Fm-3m space group (No.225)\textsuperscript{48}. The unit cell is composed of 12 atoms, with the X\textsuperscript{2-} ions occupying the four face-centered cubic positions and the Mg\textsuperscript{2+} ions the eight centered tetrahedral sites (Figure 2a). These compounds have very similar band structures, with the valence band maximum (VBM) at the Γ point in the Brillouin zone (Figure 2b) and the conduction band minimum (CBM) at X, resulting in an indirect energy gap (Eg)\textsuperscript{47}. The CBM is characterized as having a split band, where the light (C\textsubscript{l}) and heavy (C\textsubscript{h}) bands are separated by an energy offset (ΔE)\textsuperscript{42}. This split conduction band is the focus of many studies due to its considerable influence on the electrical properties of these compounds\textsuperscript{28,41,49}.

The highest reported zT values for binary, ternary, and quaternary n- and p-type Mg-based thermoelectric alloys are shown in Figure 3. Most studies have explored n-type materials, resulting in a maximum zT of 1.45 at 775 K (502 °C), while for p-type Mg-based thermoelectrics, the maximum zT achieved is 0.5 at 700 K (427 °C). Despite the popularity of these materials, particularly for future applications in thermoelectric devices, there is limited information available on their thermal stability and mechanical properties. Table 3 summarizes the relevant mechanical properties of magnesium-based thermoelectric materials.

### Table 2. Summary of electronic and physical properties of Mg-based binary alloys\textsuperscript{31,42}.

<table>
<thead>
<tr>
<th>Composition</th>
<th>a (Å)</th>
<th>Density (g/cm\textsuperscript{3})</th>
<th>E\textsubscript{g} at 0 K (eV)</th>
<th>dE\textsubscript{g}/dT at 0 K (eV/K)</th>
<th>a\textsubscript{L} (W/mK)</th>
<th>Melting point (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MgSi</td>
<td>6.338</td>
<td>1.88</td>
<td>0.77</td>
<td>-6×10\textsuperscript{-4}</td>
<td>0.4</td>
<td>1375</td>
</tr>
<tr>
<td>MgGe</td>
<td>6.385</td>
<td>3.08</td>
<td>0.74</td>
<td>-8×10\textsuperscript{-4}</td>
<td>0.58</td>
<td>1388</td>
</tr>
<tr>
<td>MgSn</td>
<td>6.765</td>
<td>3.59</td>
<td>0.35</td>
<td>-3.2×10\textsuperscript{-4}</td>
<td>0.16</td>
<td>1051</td>
</tr>
</tbody>
</table>

3. Strategies to improve the zT of Mg-based thermoelectric materials

#### 3.1 Alloying

Early research on magnesium-based thermoelectric materials was dedicated to the study of the binary phases – MgSi, MgGe and MgSn – and their properties. The main leaps in the performance of these materials were obtained, however,

![Figure 2. (a) Crystal structure and (b) simplified band structure of MgX (X = Si, Ge, Sn) alloys. C\textsubscript{L}, C\textsubscript{h} and V represent the heavy conduction band, light conduction band, and valence band, respectively.\textsuperscript{41-47}](image1)

![Figure 3. Overview of the highest reported zT values for Mg-based thermoelectric materials. (a) n-type: Mg\textsubscript{2}Si\textsubscript{0.75}Ge\textsubscript{0.25}, Mg\textsubscript{2}Ge, Mg\textsubscript{2}Sn, Mg\textsubscript{2}Ge\textsubscript{0.5}Sn\textsubscript{0.5}, Mg\textsubscript{2}Ge\textsubscript{0.25}Sn\textsubscript{0.75}, Mg\textsubscript{2}Ge\textsubscript{0.375}Sn\textsubscript{0.625}, Mg\textsubscript{2}Ge\textsubscript{0.5}Sn\textsubscript{0.5}, Mg\textsubscript{2}Ge\textsubscript{0.75}Sn, Mg\textsubscript{2}Ge\textsubscript{0.9}Sn, Mg\textsubscript{2}Ge, Mg\textsubscript{2}Sn. (b) p-type: Mg\textsubscript{2}Si\textsubscript{1-x}Sn\textsubscript{x}, Mg\textsubscript{2}Si, Mg\textsubscript{2}Ge, Mg\textsubscript{2}Sn, Mg\textsubscript{2}Ge\textsubscript{0.5}Sn.\textsuperscript{51-55}](image2)
by studying the atomic substitution of one element by one or more elements, which is known as alloying. The main purpose of alloying in thermoelectrics research is the creation of point defects through solid solution alloying, which consists of point substitutions using isoelectronic elements. The crystal structure is maintained, while the increased atomic mass contrast increases phonon scattering\(^9\),\(^{20}\) leading to lower lattice thermal conductivity. At higher concentrations of the substituting element, alloying can have effects beyond creating point defects. The presence of atoms with different characteristics leads to the scattering of the crystal structure, which can have a significant influence on the electronic characteristics due to increased electron scattering and consequently, lower charge carrier mobility. These changes in the crystal structure, even if minimal, can have a substantial impact on the electronic band structure — with positive or negative consequences for the thermoelectric efficiency due to changes to the electronic band gap and the relative positions of multiple bands\(^{57}\). The latter effect has been the main motivation behind the study of Mg-based alloys, by exploiting the convergence of the two lowest conduction bands, leading to a greater power factor, or by increasing the band gap, which suppresses the detrimental impact of the bipolar effect\(^{20},^{57}\).

### 3.2 Solid solutions

The solid solutions of Mg\(_X\) (\(X = \text{Si, Ge, Sn}\)) pseudo-binary compounds — Mg\(_2\)Si\(_{1-x}\)Sn\(_x\), Mg\(_2\)Si\(_{1-x}\)Ge\(_x\), and Mg\(_2\)Ge\(_{1-x}\)Sn\(_x\) — have been the subject of great interest due to the significant effect alloying has on their lattice thermal conductivity. Mass-difference scattering is the main mechanism responsible for the decreased lattice thermal conductivity when alloying, and therefore, it is no surprise that the Mg\(_2\)Si\(_{1-x}\)Sn\(_x\) system exhibits the lowest \(\kappa_{\text{lat}}\)\(^{58},^{59}\) (Figure 4), as the atomic masses of Si, Ge, and Sn atoms are 28.1, 72.6, and 118.71 amu, respectively. For this reason, the Mg\(_2\)Si-Mg\(_2\)Sn pseudo-binaries are considered the most interesting compounds for thermoelectric applications in the Mg\(_X\) system\(^{50}\).

Alloying Mg\(_2\)Si\(_{0.6}\)Sn\(_{0.4}\) with small amounts of Ge was demonstrated\(^{60}\) to have a significant impact on the thermal conductivity of the matrix. The compound without Ge already exhibited nanosized phase-separated structures, and the addition of Ge led to the formation of more Ge-containing secondary phases, products of the reactions of Ge with Mg, Si, and Sn. These features were also refined with increased added Ge, which, along with the increased atomic mass difference due to Si substitution by Ge, led to lower lattice thermal conductivity at low Ge concentrations. The solid solutions of these compounds have naturally formed nanoscale features that contribute to the further scattering of phonons\(^{60}\) (Figure 5b). Despite reports on the formation of complete solid solutions of compounds such as Mg\(_2\)Si\(_{0.5}\)Sn\(_{0.5}\)\(^{61}\), many studies report the observation of Si- and Sn-rich phases\(^{60},^{62},^{63}\). The morphology and size of these secondary phases depend mostly on the synthesis and processing techniques, which can be tuned to induce the formation of phonon-scattering features such as nanoprecipitates located within the matrix and/or at the grain boundaries\(^{62},^{64}\), or distortions in the lattice\(^{65}\). Sb-doped Mg\(_2\)Si\(_{0.5}\)Sn\(_{0.5}\) prepared using spark plasma sintering (SPS) without the application of mechanical pressure resulted in samples with 37% porosity that exhibit a lattice thermal conductivity as low as 0.486 W/mK at 573 K\(^{25}\) (Figure 5g), achieving a \(zT\) of 1.63 at 615 K.

### 3.3 Nanostructuring

While alloying is an effective approach towards reducing the lattice thermal conductivity, the main mechanism through which it operates is the atomic mass contrast. The need to further reduce the lattice thermal conductivity led to the development of a collection of structural modifications at the nanoscale level commonly known as nanostructuring. This approach includes reduction of the grain size\(^{58},^{65}\) and nanoprecipitates\(^{23},^{24}\) (Figure 5a) and nano-inclusions\(^{63}\) (Figure 5c), with the intent of generating features with similar dimensions to the wavelength of phonons, thus scattering them. Obtaining nanosized crystalline matrices requires two main approaches: reduction of the grain size of the starting materials and control of the grain growth during synthesis/processing\(^{57}\).

#### 3.3.1 Grain size reduction

Theoretical studies on Mg\(_2\)Si suggest that a 40% reduction in the lattice thermal conductivity can be obtained with a grain size of 20 nm due to the formation of phonon-scattering grain boundaries, although these would also scatter electrons and significantly decrease the electron mobility by 20\%\(^{66}\). Powder processing techniques such as grinding and ball milling are commonly used techniques to process Mg-based thermoelectrics. Nonetheless, these present challenges, as

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**Figure 4.** Lattice thermal conductivity as a function of composition of magnesium-based ternary alloys. Adapted from\(^3^4\).
Magnesium particles tend to agglomerate instead of fracturing due to their high ductility\textsuperscript{53, 68}, conveying the need to react the materials first, forming brittle Mg\textsubscript{3}X (X = Si, Ge, Sn) compounds that can be easily milled to fine powders\textsuperscript{69}. The detrimental scattering of electrons by grain boundaries is not reported as a significant effect in Mg\textsubscript{3}Si\textsubscript{1-x}Sn\textsubscript{x}, solid solutions, and grain size reduction is seen as a pathway towards improving \( zT \)\textsuperscript{62, 69} (Figure 5e and f).

### 3.3.2 Addition of nanoparticles

The presence of nanosized features such as nanoprecipitates can be obtained through in-situ formation\textsuperscript{62, 67} (Figure 5d) or by the introduction of nanoparticles at some point during the synthesis. The addition of silicon carbide (SiC) nanoparticles to Mg\textsubscript{2}Si\textsubscript{0.676}Ge\textsubscript{0.324} has been recently reported\textsuperscript{75} (Figure 5a), leading to increased \( zT \). These nanoparticles have a significant impact on the thermal conductivity, despite the high thermal conductivity of SiC, lowering it to 1.82 W/mK at 773 K from 3.1 W/mK for Bi-doped Mg\textsubscript{2}Si, resulting in a small increase in \( zT \) despite the detection of a decrease in the electrical conductivity due to a detrimental effect on the electron mobility. The effect of TiO\textsubscript{2} nanoparticles in undoped Mg\textsubscript{2}Si was also reported to increase the \( zT \), although no significant effect on the thermal conductivity was observed\textsuperscript{70}. The authors reported a tenfold increase in the charge carrier concentration due to electrons donated by titanium atoms via a reaction of TiO\textsubscript{2} during sintering.

### 3.4 Band engineering

#### 3.4.1 Band convergence

Several works were recently dedicated to the study of the band structure of Mg\textsubscript{2}X (X = Si, Ge, Sn) compounds\textsuperscript{27, 42, 49, 71-75}, focusing mostly on alloys with potential convergence of the two lowest conduction bands. The most interesting compositions are around Mg\textsubscript{2}Si\textsubscript{1-x}Sn\textsubscript{x}, for \( x = 0.60 - 0.70 \) \textsuperscript{42, 49, 71, 76}, where the conduction band convergence of the bottom conduction bands (\( C_x \) and \( C_1 \)) is reported to occur (Figure 6), along with increased band degeneracy (\( N_x \)). The origins of the increased \( N_x \) are suggested to be owing to two main factors\textsuperscript{27}: (i) at \( x \sim 0.75 \), the Sn 4d states contribute by hybridizing with the Mg 3s states in the conduction band; (ii) the alloy-induced strain in the crystal lattice significantly affects the positions of the \( C_x \) and \( C_1 \) bands, due to the consequent variation in the lattice parameters.

![Figure 6. Relative position (in energy) of the two lowest conduction bands and the highest valence band as function of the Sn content in Mg\textsubscript{2}Si\textsubscript{1-x}Sn\textsubscript{x}\textsuperscript{49}.](image)
conduction bands is smaller than $2k_B T$, $N_v$ increases, leading to a higher carrier effective mass (Equation 3), which is reported to enhance the Seebeck coefficient \(^7\) (Equation 4) without decreasing the charge carrier mobility ($\mu$) and, consequently, the electrical conductivity ($\sigma$, $\sigma = ne\mu$) \(^42, 49\).

$$S = \frac{2k_B T}{3e\hbar^2} \left( \frac{\pi}{3\pi} \right)^2 m^*(1 + r)$$ (4)

where $k_B$ is the Boltzmann constant, $e$ the electron charge, $\hbar$ is the reduced Planck constant, $m^*$ the density of states effective mass, and $r$ the scattering parameter. Figure 6 shows the positions of the light and heavy conduction bands in relation to each other as a function of Sn content in Mg$_2$Si$_y$Sn$_{1-y}$. The composition at which the convergence of the conduction bands occurs corresponds to the Mg-based thermoelectrics reported to have the highest $zT$ – for $x = 0.65 - 0.70$ such as Mg$_2$Si$_{0.35}$Ge$_{0.05}$Sn$_{0.65}$ \(^32\) and Mg$_2$Si$_2$Sn$_{0.7}$ \(^78\) (Figure 3).

### 3.4.2 Doping

The charge carrier concentration governs the electronic characteristics of a thermoelectric material by having a direct influence on the electrical conductivity and the Seebeck coefficient through the positioning of the Fermi level in the band gap. Beyond that, the number of available carriers and the ratio between the electrons and holes play essential roles in these parameters, especially at higher temperature due to the bipolar effect \(^79\). Therefore, the primary step towards a higher $zT$ is through optimization of the charge carrier concentration, obtained through the doping of the matrix with a donor or acceptor element.

Numerous theoretical and experimental studies have reported on potential elemental dopants for $n$- and $p$-type Mg$_2$X ($X = Si$, Ge, Sn) thermoelectrics. The most commonly used $n$-type dopants are Bi \(^81, 84-86\) and Sb \(^42, 52, 62, 84, 87, 88\). Both contribute one electron per substitution \(^84, 87\) and are expected to occupy the $X$ site in Mg$_2$X when $X = Si$ \(^84\), Ge \(^85\), or Sn \(^86\). As shown in Figure 3, $p$-type doping of these materials has not been as successful, although the number of dedicated studies on the synthesis of these materials is still limited. The most successful method for improving the $zT$ in magnesium-based thermoelectric materials has been inducing the convergence of the two conduction bands closest to the band gap, which has a profound effect on the $n$-type properties of these materials. The lack of a similar structure for the valence band limits the pathways available for achieving high performance. Theoretical calculations have shown that the optimal charge carrier concentrations for $p$-type and $n$-type Mg$_2$Si to achieve the same $zT$ at 850 K are 5.3 x 10$^{19}$ cm$^{-3}$ and 3.7 x 10$^{20}$ cm$^{-3}$, respectively \(^89\). This further constrains the avenues through which improvement is possible, as higher charge carrier concentrations require larger additions of $p$-type dopants, which need to be considered with respect to limits of solubility. Some of these dopants have already been identified, however, with Ag \(^51, 54\), Li \(^43\), and Ga \(^55\) being the most commonly reported hole donors, which are expected to occupy the Mg site in the crystal structure \(^84, 86\). Table 4 lists elemental dopants used in magnesium-based thermoelectric materials.

#### Resonant states

Generally, the charge carrier (or carriers) donated by the dopant element occupies energy levels within the band gap of the host phase, defining the Fermi level ($E_F$). In the case of $n$-type dopants as an example, a loosely bound electron is available to populate the conduction band, when excited with the amount of energy required to bridge the small gap. Some dopants, however, create resonant levels which contain charge carriers that fall inside the band, with the same energy as already occupied states. These interact, resonate, and create new energy states that will then resonate with other already present energy states, repeating the process several times. These interactions lead to a DOS distortion of the band (Figure 7), increasing the carrier effective mass without changing the carrier concentration, resulting in an increased Seebeck coefficient (Equation 4) and power factor (i.e. without a significant impact on $\sigma$) \(^2, 80\).

Despite the impact that resonant states can have on the efficiency of thermoelectric materials, there are limited reports on doping Mg-based thermoelectrics with impurities that are expected to induce the formation of resonant states within the electronic bands. An exception to this is a work on first-principles calculations of the electronic band structure of Mg$_2$Si and possible dopants, which suggests that Ni could potentially form resonant states within the conduction band, and Zn and Cd within the valence band \(^91\).

<table>
<thead>
<tr>
<th>Composition</th>
<th>Dopant</th>
<th>Type</th>
<th>Max n/p $10^{30}$ cm$^{-3}$</th>
<th>$zT$</th>
<th>$T$ K</th>
<th>$S$ $\mu$V/K</th>
<th>$\sigma$ 10$^5$ $\Omega$ m</th>
<th>$\kappa_{tot}$ W/mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg$_2$Si</td>
<td>Bi</td>
<td>n</td>
<td>1.1</td>
<td>0.86</td>
<td>862</td>
<td>-250</td>
<td>5.6</td>
<td>3.5</td>
</tr>
<tr>
<td>Mg$_2$Si</td>
<td>Sb</td>
<td>n</td>
<td>2.4</td>
<td>0.62</td>
<td>823</td>
<td>-180</td>
<td>6.0</td>
<td>2.7</td>
</tr>
<tr>
<td>Mg$_2$Ge</td>
<td>Sb, Mg</td>
<td>n</td>
<td>4.6</td>
<td>0.20</td>
<td>740</td>
<td>-128</td>
<td>11.9</td>
<td>6.9</td>
</tr>
<tr>
<td>Mg$_2$Si$<em>2$Sn$</em>{0.5}$</td>
<td>Bi</td>
<td>n</td>
<td>n.a.</td>
<td>0.78</td>
<td>800</td>
<td>-252</td>
<td>2.3</td>
<td>1.5</td>
</tr>
<tr>
<td>Mg$_2$Si$<em>2$Sn$</em>{0.6}$</td>
<td>Sb, Mg</td>
<td>n</td>
<td>8.0</td>
<td>0.85</td>
<td>700</td>
<td>-217</td>
<td>6.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Mg$<em>2$Si$<em>2$Sn$</em>{0.7}$Ge$</em>{0.05}$</td>
<td>Bi</td>
<td>n</td>
<td>2.2</td>
<td>1.40</td>
<td>800</td>
<td>-219</td>
<td>7.3</td>
<td>2.0</td>
</tr>
<tr>
<td>Mg$_2$Si$<em>2$Sn$</em>{0.7}$</td>
<td>Li</td>
<td>p</td>
<td>4.9</td>
<td>0.50</td>
<td>750</td>
<td>161</td>
<td>6.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Mg$_2$Si$<em>2$Ge$</em>{0.4}$</td>
<td>Ga</td>
<td>p</td>
<td>0.3</td>
<td>0.36</td>
<td>625</td>
<td>350</td>
<td>8.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 4. List of dopants used successfully in magnesium-based thermoelectric materials and respective thermoelectric characteristics, with some literature reports on the doping influence of non-stoichiometric interstitial/magnesium.
properties of these materials. In general, excess
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rotating chilled copper roller, quickly solidifying in the form of thin ribbons. Independently of the reaction method used, the fabrication of these materials is normally complemented by a densification step such as hot pressing \(^{24, 99, 104}\) or spark plasma sintering\(^{24, 108, 112}\). The latter greatly reduces the duration of this step by promoting the sintering of particles through passage of an electrical current when pressure is applied, allowing for certain nanostructures to be maintained. Hot pressing, on the other hand, relies on the heating of powders through conduction of heat for extended periods of time at high temperature, usually inducing grain growth and homogenization of phases and structures within the material.

5. Solid solutions and phase diagrams

Despite the considerable volume of work on Mg-based thermoelectric materials, little is known about their stability at high operating temperatures. Most authors limit their research to supposedly single-phase, precipitate-free compositions as they are expected to exhibit higher performance and thermodynamic stability. Furthermore, there is no consensus on which compositions are single-phase, and most studies have focused on the significant enhancement of various properties through alloying\(^{69, 60}\). Early work on the Mg-Si-Mg\(_2\)Sn system focused on the optimization of the carrier concentration of \(x \sim 0.4\) and \(x \sim 0.6\) Mg\(_2\)Si\(_{1-x}\)Sn\(_x\) compounds, considering these as the limits of the miscibility gap for these alloys\(^{42}\). Subsequent reports on these compounds use this study as their main reference when discussing the solid solution limit of Mg\(_2\)Si\(_{1-x}\)Sn\(_x\). Study of the miscibility or solid solution gap of Mg\(_x\)Sn (\(x = Si, Ge, Sn\)) compounds has received relatively little attention, considering its importance for the stability of these materials when they are subject to high operating temperature and thermal cycling. There are, however, experimental and theoretical studies addressing this issue, along with reports on the solid solution stability range\(^{61, 115-116}\). For instance, ab-initio calculations were used to theoretically investigate the phase stability of Mg\(_2\)Si\(_{1-x}\)Sn\(_x\) and compare them with other theoretical and experimental thermodynamic reports\(^{115}\) on the same compound (Figure 8). These suggest that the miscibility gap is wider than what has been previously accepted, and the author concludes that reports on high \(zT\) thermoelectrics are likely not single-phase, questioning the stability of their thermoelectric properties in high temperature applications over an extended time. The discrepancy between reports on single-phase compounds and ab-initio phase stability calculations\(^{115}\) has been reinforced in recent works on Mg\(_2\)Si-Mg\(_2\)Sn thermoelectric materials, where electron microscopy techniques were used to observe more than one phase in these compounds, which could not be completely resolved by X-ray analysis\(^{60, 104}\).

The variation in the reported miscibility gap of Mg\(_2\)Si\(_{1-x}\)Sn\(_x\) is considered to be a result of the kinetics of the formation of Sn- or Si-rich phases\(^{116}\), suggesting a relationship with the synthesis method. Phase diagrams represent the existence of limits and transformations of compounds in thermodynamic equilibrium, and cannot be accurately used to predict the outcomes of most commonly used fabrication techniques. A systematic study of the structure of Mg\(_2\)Si\(_{1-x}\)Sn\(_x\) for \(0.1 \leq x \leq 0.8\), which was synthesized using a B\(_2\)O\(_3\) flux method\(^{111}\), reported significantly different miscibility gaps from those in the literature and compiled a list correlating the different results with the synthesis method used (Table 5). The authors point out the wide variability of reported miscibility gaps and suggest dependence on the final Mg stoichiometry obtained with each synthesis method.

6. Applications and devices

The functional unit of a thermoelectric device, generator, or cooling/heating system, is the thermoelectric couple or pair. A thermoelectric pair consists of \(n\)-type and \(p\)-type semiconducting materials connected electrically in series through metallic contacts and thermally in parallel, using electrically insulating but thermally conductive ceramic plates, in such a way that a temperature differential is allowed

Table 5. Reported miscibility gaps and corresponding synthesis methods for Mg\(_2\)Si\(_{1-x}\)Sn\(_x\)\(^{111}\).

<table>
<thead>
<tr>
<th>Synthesis method</th>
<th>Miscibility gap, (x)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux method</td>
<td>0.2-0.45</td>
<td>(^{113})</td>
</tr>
<tr>
<td>Melting</td>
<td>0.4-0.6</td>
<td>(^{117, 118})</td>
</tr>
<tr>
<td>Liquid-solid reaction and hot pressing</td>
<td>No gap</td>
<td>(^{119})</td>
</tr>
<tr>
<td>Mechanical alloying and hot uniaxial pressing</td>
<td>0.09-0.72</td>
<td>(^{114})</td>
</tr>
<tr>
<td>Synthesis in sealed corundum crucible</td>
<td>0.08-0.65</td>
<td>(^{120})</td>
</tr>
<tr>
<td>Calculated</td>
<td>0.08-0.66</td>
<td>(^{116})</td>
</tr>
</tbody>
</table>

Figure 8. Calculated phase diagram and comparable data from the literature on the Mg\(_2\)Si-Mg\(_2\)Sn quasi-binary phase diagram\(^{115}\).
through the length of the pair (Figure 9)\textsuperscript{15}. Connecting several of these pairs creates a thermoelectric module or Peltier device. When under operation, these devices are subjected to very large mechanical stress, even in the absence of vibration, due to thermal cycling and high temperature gradients. The effects of high temperature on the properties of these materials are also of great concern, particularly due to the fact that two distinct semiconducting materials are being used, raising the issue of compatibility with each other and with the electrical contacts due to diffusion of elements and chemical reactions at interfaces, often demanding the use of intermediate layers acting as diffusion barriers. Maintaining electrical contact between the thermoelectric legs and the electrical contacts presents another challenge, as simple solutions such as solders are not feasible at high temperatures, often requiring the direct bonding of the electrodes and the semiconductor.\textsuperscript{15, 121} The operational conditions for medium-to-high temperature thermoelectrics impose strict constraints on the selection of compatible materials for the assembly of a device. The simplest method when approaching these compatibility challenges is to use the same base thermoelectric material for both $n$- and $p$-type legs, reducing thermal expansion coefficient discrepancies between the two and possibly decreasing the complexity of the choice of electrode material. It is uncommon, however, for thermoelectric materials to exhibit high $zT$ for both $n$- and $p$-type materials: as shown in Figure 3. For example, $n$-type Mg$_2$Si$_{0.5}$Sn$_{0.45}$Ge$_{0.05}$Sn$_{0.65}$, and $p$-type Mg$_2$Ge$_{0.05}$Sn$_{0.95}$ only reach a maximum $zT$ at 0.5 at around the same temperature. Despite the low efficiency of $p$-type Mg-based thermoelectric materials, some preliminary results on device design and construction of devices based on these materials has already been achieved\textsuperscript{15, 56, 58, 121-124}. The low performance of the $p$-type Mg-based thermoelectric materials creates serious limitations on the construction of devices, and most reports on the use of these materials in modules are on $n$-type legs\textsuperscript{15, 56, 121}. Modules pairing $n$-type Mg$_2$Si$_{0.5}$Sn$_{0.45}$Ge$_{0.05}$Sn$_{0.65}$ and $p$-type Mn$_{0.1}$Si$_{0.2}$Ge$_{0.7}$ were reported to reach a maximum power output of 3.24 W at 735 °C, achieving an estimated efficiency of 5.3%\textsuperscript{122}. The authors report significant oxidation and decomposition of the hot side of the $n$-type leg, however, and they suggest that oxidation-preventing coatings could resolve this issue and that the development of effective diffusion layers and better bonding methods is needed.

Despite the significant challenges ahead for research on Mg-based thermoelectric devices, interest in these materials is not expected to fade. The highest $zT$ composition of this family of materials, Mg$_2$Si$_{0.5}$Sn$_{0.65}$, is reported to have a material cost of $4$/$kg, significantly cheaper than the currently commercialized materials such as Bi$_2$Te$_3$, PbTe, and SiGe, with material costs of $110$/$kg, $81$/$kg$, and $371$/$kg$, respectively\textsuperscript{125}. Table 6 compares state-of-art thermoelectric devices containing magnesium-based materials and other thermoelectric materials. There are at least two companies utilizing magnesium-based thermoelectric materials in commercially available products: Alphabet Energy, which produces modules and generators using an $n$-type Mg$_2$Si-Mg$_2$Sn compound paired with a $p$-type Cu-Sb-S compound, known as tetrahedrite, both thermoelectric systems that are recognized for their relatively good performance at medium-to-high temperatures and low price\textsuperscript{126}; and Rommy Scientific, which commercializes materials and modules fabricated using both $n$- and $p$-type Mg$_2$Si-Mg$_2$Sn\textsuperscript{127}.

### 6.1 Electrode bonding materials

Reports on thermoelectric generators using magnesium-based materials are still limited to Mg$_2$Si as the $n$-type leg, and therefore, development of bonding techniques and electrode materials are also in their preliminary stages. Bi-doped Mg$_2$Si has been used as the hot-side $n$-type thermoelectric material in a segmented leg\textsuperscript{122}, where Bi$_2$Te$_3$ is first bonded to Mg$_2$Si by evaporating 50 nm of Ti and 1 μm of Ag on both surfaces before using Ag solder. Ti is expected to increase the adhesion between the semiconductors and Ag acts as “glue” due its low wetting angle. Then, Cu electrode was brazed to the Mg$_2$Si using an Ag-Cu-Sn-Zn alloy. The cold side was bonded to the Cu electrode using a common Pb solder, because no significant reaction is expected to occur at low temperature. The total resistance of the electrodes, segmented legs, and respective bonding interfaces is reported to represent less than 2% (50 μΩcm$^2$) of the internal resistance of the materials used. Mg$_2$Si is normally bonded to Ni\textsuperscript{16, 123, 133} because at its working temperature, these materials are not expected to react\textsuperscript{16, 133}. Ni has been directly bonded by SPS to Sb-doped Mg$_2$Si powder between two layers of Ni powder\textsuperscript{133}, with its thermoelectric

<table>
<thead>
<tr>
<th>Materials</th>
<th>$T_\text{w}/T_\text{e}$ ($^\circ$C)</th>
<th>$\eta$ (%)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg$<em>2$Si$</em>{0.5}$Sn$<em>{0.4}$Ge$</em>{0.05}$/MnSi$<em>{1.7}$Ge$</em>{0.01}$</td>
<td>735/50</td>
<td>5.3</td>
<td>122</td>
</tr>
<tr>
<td>Bi$_2$Te$_3$ + CeFe$<em>3$Ge$<em>2$Mn$</em>{0.1}$Si$</em>{1.2}$/Bi$_2$Te$<em>2$Se$</em>{2.5}$ + Yb$_2$Co$<em>2$Sb$</em>{12}$</td>
<td>576/35</td>
<td>12.0</td>
<td>128</td>
</tr>
<tr>
<td>Bi$_2$Te$_3$ + PbTe/Bi$_2$Te$_3$ + PbTe-MgTe</td>
<td>600/30</td>
<td>11.0</td>
<td>129</td>
</tr>
<tr>
<td>Unilieg p-MgAg$<em>{0.965}$Ni$</em>{0.035}$Sb$_{0.99}$</td>
<td>250/25</td>
<td>8.5</td>
<td>130</td>
</tr>
<tr>
<td>Yb$_{0.3}$Co$<em>2$Sb$</em>{12}$/NdFe$_3$Co$<em>2$Sb$</em>{12}$</td>
<td>550/70</td>
<td>9.1</td>
<td>131</td>
</tr>
</tbody>
</table>
legs representing 12% (29.9 mOcm\(^2\)) of the total resistance. SPS was also used for bonding bulk Cu and Ni foil to Mg\(_2\)Si during the reaction of Mg and Si powders, forming a Cu/Ni/Mg\(_2\)Si/Ni/Cu structure.\(^{123}\) In this study, the authors performed a more in-depth analysis of the Ni/Mg\(_2\)Si interfaces and reported the formation of Ni-Si phases (Figure 10), obtaining a contact resistance of 1.44 mOcm\(^2\). The bonding of Ni foil to pre-reacted Mg\(_2\)Si powder by SPS\(^{16}\) led to an interface consisting of ternary phase Mg-Si-Ni and Si-Ni, resulting in high adhesion and very low contact resistance of 4.4 µOcm\(^2\). The synthesis of Mg\(_2\)Si and bonding with Ni in a single step consistently result in lower contact resistance due to the perfectly intimate contact between the two materials through the formation of intermediate phases. This intimate contact is not possible when Ni is bonded to bulk (already formed) Mg\(_2\)Si, resulting in contact resistance around an order of magnitude higher.\(^{124,\ 134}\) Ti, TiS\(_3\), and TiN were studied as candidate materials for diffusion barrier layers between Ni contacts and thermoelectric Mg\(_2\)Si, as these react at high temperature and form resistive phases.\(^{26}\) Deposited TiN has shown particularly promising results by preventing significant diffusion of Mg while maintaining low contact resistance after annealing. Bonding of Mo electrodes was also reported on the hot side of high \(zT\) Mg\(_2\)(Si\(_{0.5}\)Ge\(_{0.5}\))Te, Mg\(_2\)Si\(_2\)Sn\(_{1.0}\)Ge\(_{0.0}\)Ge\(_{0.0}\), and Mg\(_2\)Si\(_{0.5}\)Sn\(_{0.5}\)Ge\(_{0.0}\)Ge\(_{0.0}\) after deposition of thin layers of Ni, Pb, and Cr to improve contact and adhesion.\(^{122}\) At this point in the development of magnesium-based thermoelectric devices, research efforts are still focused on the development and optimization of bonding techniques and materials, although the long-term stability and impact of the materials used for the junctions on the thermoelectric and mechanical properties of the device are critical to the technology and are yet to be explored.

7. Summary and outlook

In this work, we have reviewed the general position occupied by magnesium-based thermoelectric materials in terms of performance, challenges, and advantages, in relation to other thermoelectric materials. This group of thermoelectric materials has been garnering considerable interest due to the high \(zT\) values obtained in its \(n\)-type materials at medium-to-high temperatures, while it is considered to be low-cost with high availability, non-toxicity, and very low density. Mg\(_2\)X (X = Si, Ge, Sn) binary and ternary alloys provide a wide range of compositions to explore, with many performance-enhancing pathways available, such as band engineering, by inducing band convergence\(^{41,\ 49}\), and/or resonant states;\(^{81}\) complete dissolution of elements, providing stable solid solutions;\(^{56,\ 59}\), or phonon-scattering features in intermetallic compositions;\(^{54,\ 60,\ 62}\), as well as the possibility of building thermoelectric modules using both \(n\)- and \(p\)-type magnesium-based materials, greatly reducing compatibility issues.\(^{15}\) Recent studies have been focused on understanding the mechanisms responsible for the current performance of magnesium-based materials, although an application-driven research approach may be necessary to solve challenges such as doping optimization – particularly for \(p\)-type compounds, long term and high temperature compositional and structural stability, the development of synthesis processes that provide precise control of the final stoichiometry, thermomechanical studies on the reliability of these systems under adverse conditions while in operation, and the bonding of compatible electrical contacts. Despite being mostly seen as potential alternatives to the higher efficiency materials, such as PbTe\(^{29}\), magnesium-based thermoelectric compounds exhibit exciting techno-economic characteristics that are relevant when considering viable thermoelectric materials for large-scale applications, particularly the aforementioned low mass density and low cost of the raw materials. Therefore, the development of high efficiency \(p\)-type magnesium-based compounds could be the breakthrough required for worldwide application of these materials. The particular challenges related to \(p\)-type optimization center mostly around the low charge carrier density achieved in these materials, aggravated by the limited number of compatible dopants, comprising Ag, Li, and Ga, with Li being the most promising dopant. A double-doping approach with combinations of these three elements might be the key to achieving high charge carrier concentrations in \(p\)-type magnesium-based thermoelectric materials. Most synthesis methods further contribute to a low charge carrier density, as excess magnesium in the final compounds occupies interstitial positions in the lattice and acts as an electron donor. Beyond their overall negative impact on the power factor, low doping densities generally lead to the manifestation of the bipolar effect at lower temperatures, which effectively decreases their maximum operating temperature. Reports on the variable solubility of dopants in the Mg-based binary phases depending on the fabrication method open the door to exploring new ways of increasing hole concentrations. The inconsistency concerning the solubility limits of dopants and solid solution ranges in these materials usher in exciting opportunities to explore. Concepts such as selective doping of secondary phases within a matrix, known as modulated doping – in the form of phonon disruptive structures such as nanodots, grain-surrounding nanolayers, or compositionally modulated structures;\(^{136,\ 137}\), with dopant elements having temperature dependent solubility, are techniques yet to be studied for this class of thermoelectric materials. Non-equilibrium techniques are often required in the development of these structures, but their benefits are undermined by low long-term thermal stability.

As the synthesis of magnesium-based thermoelectric materials is still under development, work on their application in thermoelectric devices is still in its infancy. Less attention has been devoted to the long-term stability of these compounds at high temperature, in terms of either thermoelectric or mechanical properties. Such studies are essential to ensure their future applicability. Consequently, reports on the bonding of electrical contacts are also still lacking, as this work offers significant engineering challenges by requiring high adhesion between the materials while maintaining their
mechanical and thermoelectric characteristics at high temperature. Despite the great number of challenges ahead for developing magnesium-based thermoelectric devices, their promising potential advantages make them very interesting, particularly from an commercial point-of-view, and they are sure to arouse further interest as their development approaches real-world device suitability.

Conflicts of interest
There are no conflicts to declare.

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Notes and references


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