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A Markov Chain Approach On Pattern of Rainfall Distribution

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NIGERIA



UYO Versus EKET

Uyo is the state capital of Akwa Ibom
 Population of about 882, 779 people

 Eket is the second largest city in Akwa Ibom, with a total population of 564, 489

(2006 Census)

Plots of RAINFALL statistics: UYO & EKET



EKET

The driest month is January, with July having the highest precipitation records It is highly difficult, if not impossible, to forecast what the weather will be like at a certain time in a very precise place

Brief Introduction

 Rainfall exhibits Strong variability in time and space across the globe.

 Its stochastic modelling is necessary for the prevention of natural disaster

Demand for rainfall to life has made its study a major focus to many researchers Mark (2005)
 Umoh, Akpan and Jacob (2013)

Motivation: Benefits of Rainfall

 Examining rainfall's Distribution and Pattern of extreme high and low Precipitation is good:

- For the Agriculture and the Economy of many African countries including Nigeria
- For determining best adapted Plant Species and the Optimum time for Planting
- For future planning
- For general Growth of a Nation

Motivation: Negative Impacts

The Hydrological extremes driven by rainfall include:

Floods- farm and communities



Motivation: Negative Impacts

Oroughts and Landslides



2012 Nigeria's floods Scenario

• Debris and Mud-flows



<u>https://www.youtube.com/watch?v=OU</u> <u>YY7cN7n8</u>

Model: Established Literature

- Many Probability Models developed in studies such as:
- Manning (1950);
- Feyerherm and Bark (1967);
- Kulandaivelu 1984;
- Phien and Ajirajah 1984;
- Topalogu 2002

Literature (Markov Models)....

- Aneja and Srivastava (1986, 1999) apply two-state (with two parameters) and three-state (with five independent parameters) models
- Purohit, Reddy, Bhaskar, and Chittora (2008) apply twostate Markov chain model to obtain probabilities of occurrence of dry and wet weeks
- Garg and Singh (2010) examine patterns of rainfall at Pantnagar for daily rainfall data of 42 years (1961-2002) using a three-state Markov chain model.

Methods & Design

Data: (15 YEARS)
UYO-Daily 1st Jan, 1995 to 31st Dec, 2009

Seasons: (1)Pre-Monsoon (Jan 1-March 31); (2)Monsoon (April 1 -September 30) ; (3)Post-Monsoon (Oct,1-Dec, 31)

• Regimes (States)-dry (d), wet (w) and rainy (r)

• Classifications: $\leq 2.50mm - dry$; $2.51mm \leq R < 5.0mm - wet$; $\geq 5.0mm - rainy$

Data & Methods

Data: (15 years)

EKET-Daily 1st Jan1993- 31st Dec.,2008
 Dec

Seasons: Monthly Grouped

• Regimes (States)-dry (d) and rainy (r)

• Classifications: $\leq 2.50mm - dry; \geq 2.51mm - rainy$

Methodology

• A Markov chain

a branch of stochastic process where the state space and index (time) are discrete A discrete-time process where future process behaviuor depends purely on the immediate past (Udom, 2010)

		Current			
		Dry (d)	Wet (w)	Rainy (r)	Total
Previous Day (i)	Dry (d)	n _{dd}	n_{dw}	n _{dr}	n _{d.}
	Wet (w)	n _{wd}	n _{ww}	n _{wr}	$n_{w.}$
	Rainy (r)	n _{rd}	n_{rw}	n_{rr}	$n_{r.}$

Methodology.....

• The Maximum Likelihood Estimators of $P_{ij}, i, j = \{d, w, r\}$ \hat{p}_{ij} are given by $\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=d}^{r} n_{ij}}$

		Current Day				
		Dry(d)	Wet(w)	Rainy(r)		
Dromious	Dry (d)	P _{dd}	P_{dw}	P _{dr}		
Pievious	Wet (w)	P_{wd}	P_{ww}	P_{wr}		
Day	Rainy (r)	P_{rd}	P_{rw}	P_{rr}		

Goodness-of-Fit Test

Wang and Maritz (1990) $WS = \frac{S_a + S_b - 1}{\sqrt{V(S_a + S_b - 1)}} \xrightarrow{P} N(0, 1)$

Methodology....

• Long Run Probabilities:

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = (p_1 \ p_2 \ p_3) \begin{pmatrix} P_{dd} & P_{dw} & P_{dr} \\ P_{wd} & P_{ww} & P_{wr} \\ P_{rd} & P_{rw} & P_{rr} \end{pmatrix}$$

Expected Length of Different Spells of Seasons and Weather Cycle (WC)

• The expected length of dry spell is given by

E(WC) = E(D) + E(W) + E(R)

E(D)

RESULTS: UYO

• Estimated Values of WS Test Statistic and the associated p-values

	Pre-Monsoon	Monsoon	Post-Monsoon
WS Statistic	2.26 (p=0.0119)	11.04	15.77
value		(p < 0.0001)	(p < 0.0001)

• Estimated Equilibrium State Probabilities, Expected Length of different Season's Spell, Weather Cycle and Total Number of days

Pexiod	Equilibrium state probability			Expected length of Season's Spell				
	Dry (π ₁)	$Wet(\pi_2)$	Rainy (π_3)	Dry Spell	Wet Spell	Rainy Spell	Weather Cycle	Total No. of Days
Pre-Monsoon	0.89	0.02	0.09	10.0	1.0	1.0	12.0	1363
Monscon	0.56	0.08	0.36	2.0	1.0	2.0	5.0	2744
Post-Monsoon	0.77	0.05	0.18	6.0	1.0	1.0	8.0	1380

RESULTS: EKET

• Probability of Rainfall Distribution per Month per season

Months	Initial Matrix (p)		Long Term Prob.			Prob. for either rain/dry on a day		
	P			p ⁿ		P(dry)	P(rains)	
January	$\begin{bmatrix} 0.71 & 0.2 \\ 0.61 & 0.3 \end{bmatrix}$	29] 39]	$P^{9}\begin{bmatrix}1\\0\end{bmatrix}$	1.000).999	$\begin{bmatrix} 0\\ 0.0001 \end{bmatrix}$	1.000	0	
February	$\begin{bmatrix} 0.91 & 0.0 \\ 0.82 & 0.1 \end{bmatrix}$)9] [8]	$P^{8}\begin{bmatrix}0\\0\end{bmatrix}$.9011 .9011	0.0989] 0.0989]	0.9011	0.0989	
March	$\begin{bmatrix} 0.741 & 0.2 \\ 0.63 & 0.3 \end{bmatrix}$	26] 37]	$P^5\begin{bmatrix}0\\0\end{bmatrix}$.7079 .7079	0.2921 0.2921]	0.7079	0.2921	
April	$\begin{bmatrix} 0.60 & 0.4 \\ 0.63 & 0.3 \end{bmatrix}$	10] 37]	$P^{6}\begin{bmatrix}0\\0\end{bmatrix}$.6117 .6117	0.3883] 0.3883]	0.6117	0.3883	
May	$\begin{bmatrix} 0.45 & 0.5 \\ 0.52 & 0.4 \end{bmatrix}$	55] [8]	$P^6\begin{bmatrix}0\\0\end{bmatrix}$.4860 .4860	$0.5140 \\ 0.5140 \end{bmatrix}$	0.4860	0.5140	
June	$\begin{bmatrix} 0.38 & 0.6 \\ 0.37 & 0.6 \end{bmatrix}$	52] 53]	$P^5\begin{bmatrix}0\\0\end{bmatrix}$.3737 .3737	0.6263 0.6263	0.3737	0.6263	
July	$\begin{bmatrix} 0.46 & 0.5 \\ 0.26 & 0.7 \end{bmatrix}$	54 74]	$P^{7}\begin{bmatrix}0\\0\end{bmatrix}$.3256 .3256	0.6750 0.6750	0.3256	0.6750	
August	$\begin{bmatrix} 0.43 & 0.5 \\ 0.26 & 0.7 \end{bmatrix}$	57] 74]	$P^{6}\begin{bmatrix}0\\0\end{bmatrix}$.3133 .3133	0.6867 0.6867	0.3133	0.6867	
September	$\begin{bmatrix} 1.00 & 0.0 \\ 0.30 & 0.7 \end{bmatrix}$	⁰⁰]	P ²⁸	$[1.000 \\ 1.000]$	0.00 0.00	1.000	0.00	
October	$\begin{bmatrix} 0.39 & 0.6 \\ 0.40 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 51\\ 50 \end{bmatrix}$	$P^5\begin{bmatrix}0\\0\end{bmatrix}$.6778 .6778	0.3222 0.3222	0.6778	0.3222	
November	$\begin{bmatrix} 0.71 & 0.2 \\ 0.61 & 0.3 \end{bmatrix}$	29] 39]	$P^5\begin{bmatrix}0\\0\end{bmatrix}$.6778 .6778	0.3222 0.3222]	0.6778	0.3222	
December	$\begin{bmatrix} 0.90 & 0.1 \\ 0.71 & 0.2 \end{bmatrix}$	[0] [9]	$P^7\begin{bmatrix}0\\0\end{bmatrix}$.8765 .8765	0.1235 0.1235]	0.8765	0.1235	

Conclusion

- The Data fit perfectly well into Markov's Assumptions
- The dry spell is highest in Pre-monsoon and Post-Monsoon compared to wet and rainy spells for Uyo community
- The highest probability of rainfall occur in August, followed by July; whereas the probability of perpetual dryness is a certainty for Eket community
- Findings Reflect near real picture of the rainfall situation in the two communities

Some Selected References

- Aneja D, Srivastava O (1986): A study technique for rainfall pattern." Aligarh Journal of Statistics, 6, 26-31
- Aneja D, Srivastava O (1999): Markov chain model for rainfall occurrence." Journal of Indian society of Agricultural Statistics, 52(2)
- **Garg V, Singh J (2010):** Markov chain Approach on the behaviour of Rainfall." International Journal of Agricultural and Statistical Sciences, 6(1)
- Mark S (2005). Atmospheric thermodynamics. University of Washington.
- Purohit R, Reddy G, Bhaskar S, Chittora A (2008): Markov Chain Model Probability of Dry, Wet weeks and Statistical Analysis of Weekly Rainfall for Agricultural Planning at Bangalore." Karnataka Journal of Agricultural Science, 2(1)
- Umoh A, Akpan A, Jacob B (2013): Rainfall And Relative Humidity Occurrence Patterns In Uyo Metropolis, Akwa Ibom State, South-South Nigeria." IOSR Journal of Engineering(IOSRJEN), 3(8)