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SHAHRASBI, Alireza, SHAMIZANJANI, Mehdi, ALAVIDOOST, M. H. and AKHGAR, Babak http://orcid.org/0000-0003-3684-6481

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An Aggregated Fuzzy Model for the Selection of a Managed Security Service Provider

Alireza Shahrasbi^a Department of Information Technology Management, University of Tehran, Tehran, Iran <u>A.shahrasbi@ut.ac.ir (Corresponding Author)</u>

Mehdi Shamizanjani^b Department of Information Technology Management, University of Tehran, Tehran, Iran <u>Mshami@ut.ac.ir</u>

> M. H. Alavidoost^c Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran <u>mostafa.alavidoost@aut.ac.ir</u>

Babak Akhgar^d Department of Informatics, Sheffield Hallam University, Sheffield, UK <u>b.akhgar@shu.ac.uk</u>

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ABSTRACT

In this study, by analyzing the related literature, the companies providing security services and, more importantly, the data provided by a group of experts, a novel set of 39 criteria is extracted which assists the Managed Security Service Provider (MSSP) selection process. The set is further categorized into eight general classes. The validity and weights of these criteria are measured by a group of experts in Iran. Due to the large number and often conflicting criteria, and the qualitative nature of the evaluations of the service providers, fuzzy multi-criteria decision-making methods (FMCDM) are adopted. In order to demonstrate the application of the proposed model, a numerical example is included, in which eight service providers are evaluated by four decision makers applying fuzzy TOPSIS, fuzzy VIKOR, fuzzy Group ELECTRE, and fuzzy SAW methods. Owing to the variations of the outputs of the applied MCDM methods, they are further analyzed by an aggregation method to propose a unique service provider. A comparison between the output of the aggregation method and the four applied Fuzzy MCDM methods is also made with the help of Euclidean, Hamming, Manhattan and Chebyshev distances. The comparison shows the minimum diversion between the outputs of the Fuzzy TOPSIS and the aggregation method, which indicates the appropriateness of the fuzzy TOPSIS method in this particular problem.

Key words: Managed security service provider selection; Information technology services outsourcing; Security outsourcing; Fuzzy multi-criteria decision making; Aggregation method; MCDM methods comparison

1- INTRODUCTION

Many researchers have focused on outsourcing for a long time. More or less, all agree on the definition of outsourcing; Outsourcing is the act of transferring one or more activities to external providers who will be responsible for managing and conducting that activity, on behalf of the outsourcer company [1].

Outsourcing of IT/IS¹ services is another topic which has also interested experts and researchers during the last years [2]. Willcocks, Lacity and Fitzgerald defined it as "handing over to third-party management, for required result, some or all of an organization's IT information systems and related services" [3]. In 1996, Grover and others presented it as "practice of turning over part or all of an organization's IS function to external service provider(s)." Three common components can be derived from all these definitions. Firstly, all or part of an organization's IS function will be delivered by an external provider; secondly, the external provider will be responsible for the outsourced activity; and finally, customer will transfer the responsibility of hiring employees and installing computer infrastructures to the external provider [4]. Researchers have mentioned the following points as the reasons for outsourcing of IS/IT activities: cost reduction [5], focus on company's core competencies, gaining flexibility in face of rapid business and environmental changes, access to efficient and effective resources [5], improved productivity, coping with continuous improvements, exploiting the supplier's deep expertise and knowledge, maximizing aggregate value for the firm, reducing risk, improving the quality of services, eliminating everyday problems and finally, focusing on IS strategic issues [6-8].

Gartner defines managed security services as "the remote management or monitoring of IT security functions delivered via remote security operations centers (SOCs), not through personnel on-site" [9, 10]. According to the Gartner research, conducted in 2013, the managed security service (MSS) market is a fast-growing one compared to other security market segments with a compound annual growth rate of 16.6%. They have also estimated that the MSS market will grow from US12\$ billion in 2013 to more than US22.5\$ billion in 2017 [10]. In addition, they have claimed that the highest future growth rate will be in the emerging Asia/Pacific region. The reasons for this rapid expansion in the MSS market are as follows: advancing threats across the globe, major data breaches, the expanding regulatory environment [10], reducing costs, and improving security capabilities [11]. According to the Forrester's research, a growth rate of 30% to 40% is estimated per year for the MSS market, and this prediction is based on three primary reasons: first, organizations can benefit from better resources, talent and lower price; second,

¹ Information Technology / Information System

the CIO's¹ need for a strategic and long term relationship with a security provider partner, and third, is the client's need for advanced technologies such as threat intelligence and correlation analysis, which requires experienced analysts [11].

In the past two decades, many researchers have used different decision-making methods for evaluation and selection of the right supplier, but most of which have been conducted in production sector with only a few works in the field of services [12]. Results provided by Yang, Wang, Liou, Oke, Jain, Gewald, Chen, Feng, Hsu [8, 12-17] are examples of such models in the field of service provider selection. Although there are many emerging outsourcing trends, which academics have contributed to practice, a few works have been done specifically on the MSSP selection and they are just limited to introducing some selection criteria. Therefore, the absence of a comprehensive model which covers a complete set of criteria as well as a decision model has motivated authors for investigating this research area. The selection of such a partner is crucial and necessitates the consideration of many criteria. Due to the weakness of human mind in processing conflicting criteria and linguistic evaluations of the service providers, fuzzy multi-criteria decision-making methods should be applied in such a selection [18]. The aim of this paper is to propose a comprehensive set of criteria as well as a fuzzy-based model for the selection of an MSSP. The model can be used by the industry practitioners to facilitate the selection process. The proposed decision model encompasses the application of four popular FMCDM methods included by an aggregation method. When selecting the FMCDM methods, two factors have been considered; first, being widely used in the literature and second, being compatible with the problem conditions. Decision-making process is always expected to provide a unique output, whereas MCDM methods usually yield different results. In order to unify the different outputs of the methods, the aggregation method has also been applied.

The remainder of this paper is organized as follows. In section 2, authors have precisely defined the problem, the necessity and the importance of such a selection and the research methodology. In section 3, the fuzzy set theory, the four applied FMCDM methods as well as the aggregation method and the mathematical distances, used for comparing the results, have been explained step by step. The derived criteria within their related categories have been elaborated in section 4, according to their relative importance. In Section 5, the results of the numerical example have been illustrated to better understand the proposed method. Finally, the conclusion and the future works have been presented in section 6.

2- PROBLEM DEFINITION AND THE RESEARCH METHODOLOGY

Data breach reports demonstrate that many companies in financial or retail sector may be the victims of attacks despite adopting security considerations [19]. The fact that every company can be considered as a target for an attack implies the necessity for adopting security implications. Hence, selecting the right service provider is a critical issue on which many researchers have focused in recent years. However, it is clear that when it comes to the security outsourcing, a complete set of criteria should be considered.

Apart from advantages of outsourcing [20], it has some disadvantages and risks [21], which professionals believe can be substantially reduced by considering a set of initial points. Probably the most important concern is MSSP's access to organization's mission-critical and sensitive information assets, which can be misused to ruin the company's reputation, and leave employees with a feeling of untrustworthiness [22, 23]. Ignoring the rational process of considering the service providers can lead to outsourcing failure. It is widely accepted that the high failure rate of IT/IS outsourcing projects, to a large extent, happens mostly as a result of wrong selection and incorrect decision-making. Many researchers have focused on IT/IS outsourcing decision-making models, and have proposed invaluable models for making this kind of intricate and critical decisions [4-7, 13, 24-27], which if not made cautiously, can cause a company to lose its competencies, reputation, and customer trust.

Authors have exhaustively reviewed the literature for a comprehensive model which can cover all aspects of the necessary evaluation for such a selection. Unfortunately, aside from some scattered efforts to define a complete set of criteria, there has been no complete and convergent set proposed by research groups and the companies working

¹ Chief Information Officer

in this field. This gap in academic literature has triggered the authors to look for a reliable comprehensive model for helping the decision makers.

In order to gather data for this research, content analysis methodology has been adopted. In the first step, the authors have reviewed articles regarding IT service outsourcing and IT service provider selection in general and not limited to security in order to find all researchers' proposed models and criteria. In the second step, the articles dealing with the reasons, advantages, and disadvantages of security outsourcing have been studied. Unfortunately, there has been only some scattered research performed by industry experts in the field of security service provider selection and very few has been done by academic researchers, although this is a field which demands a considerable amount of care and attention from academia. Based on this academic research gap, the authors have gathered related articles, along with performing reviews on products, services, and features of the leading MSSPs' services in North America (such as IBM, Dell Secure Works, Symantec, Verizon, TrustWave, CSC, and AT&T, which are ranked as the leaders by the Forrester research [11]). A set of criteria for the selection of the right MSSP has also been extracted. This set is first, validated by a group of experts. Then, it is further evaluated by another group of professionals in terms of their level of importance and effective weights in decision-making process. The detailed information of the experts and their affiliations has been demonstrated in Appendix 1.

Imprecision in MCDM models can be demonstrated using fuzzy set theory to define criteria weights and their levels of importance [28]. In this paper, authors have applied Fuzzy MCDM methods to solve the MSSP selection problem. Due to the variety of the MCDM models, choosing the most appropriate method is another MCDM problem by itself [29]. Actually, it is impossible to determine which MCDM model is the best amongst others; owing to the fact that they perform the selection operation by different algorithms [29]. However, virtually all MCDM methods, aside from their alternative selection algorithms, consist of common procedures of generating the alternatives, devising the related criteria, defining their weights and applying the ranking method [28].

In order to increase decision-making process reliability, some researchers apply different MCDM methods and aggregate their outputs [29]. Typical aggregation methods, which are further elaborated, includes average function, Borda and the Copeland.

Due to the criticality of the issue, in this paper, Fuzzy TOPSIS¹, Fuzzy VIKOR², Fuzzy Group ELECTRE³ and the Fuzzy SAW⁴ approaches have been applied. As expected, the methods have yielded different results [30]. Hence, an aggregation method, which is proposed by Jahan et al. [29], is applied to propose a unique reliable result. Furthermore, in order to compare the outputs, the Euclidean, Manhattan, Chebyshev and Hamming distances between the outputs of the mentioned FMCDM methods and the aggregation method are calculated. The overall procedure of this study is described through a series of steps depicted in Fig.1.

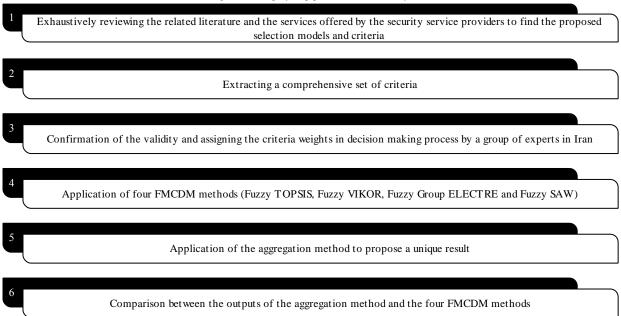
¹ Technique for Order Preference by Similarity to Ideal Solution

² Vlse Kriterijumska Optimizacija I Kompromisno Resenje

³ ELimination Et Choix Traduisant la REalité

⁴ Simple Additive Weighting

Fig. 1. The step by step procedure of the study



3-FUZZY SET THEORY AND THE FUZZY MULTI-CRITERIA DECISION-MAKING METHODS

When adopting different MCDM approaches [31-33] in the context of service provider selection [34], it is very hard for decision makers to determine the exact performance value of the alternatives in terms of each criterion. In addition, since the human perception is always vague and difficult to measure, the use of crisp data cannot actually present the real situation [18]. Statistical decision-making methods can only model some insufficient knowledge about the external environment [18]. Fuzzy set theory is the approach to help decision makers to deal with aforementioned vagueness, which plays a pivotal role in decision-making process to represent the decision makers' subjective means [35].

The fuzzy set theory handles the mentioned vagueness ambiguities by its membership degree which is calculated from the membership function [18, 36]. The membership degree can be a number between 0 and 1, which is different from the classical sets that are represented by either 0 or 1 [18]. There are different kinds of fuzzy numbers and membership functions. In this paper, triangular fuzzy numbers which is shown by the triplet (a_1,a_2,a_3) has been adopted [37]. After deriving a set of criteria in terms of the eligibility of an MSSP, the experts were asked to determine the weights of the criteria by a number from 1 to 10, which are also converted to fuzzy numbers [table 1]. While experts are capable of determining the weights of the criteria by a definite number, it will be very hard to exactly represent the status of a service provider by a crisp number for each criterion. Hence, the experts have applied linguistic evaluations of the alternatives, which are then converted to fuzzy numbers for the necessary calculations [table 2].

Score	Fuzzy Number	Score	Fuzzy Number
1	(1,1,1.5)	6	(5.5,6,6.5)
2	(1.5,2,2.5)	7	(6.5,7,7.5)
3	(2.5,3,3.5)	8	(7.5,8,8.5)
4	(3.5,4,4.5)	9	(8.5,9,9.5)
5	(4.5,5,5.5)	10	(9.5,10,10)

Table 1. The crisp and the related triangular fuzzy numbers used by the experts to determine the criteria weights [45]

Table 2. The linguistic variables and their related triangular fuzzy numbers for alternative evaluations [40]

Linguistic Variable	Fuzzy Number
Worst	(0.0, 0.0, 2.5)
Poor	(0.0, 2.5, 5.0)
Fair	(2.5, 5.0, 7.5)
Good	(5.0, 7.5, 10)
Best	(7.5, 10, 10)

As mentioned before, four popular types of MCDM methods, i.e. Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy Group ELECTRET, and Fuzzy SAW have been applied to ensure the selection of the most satisfactory alternative. A brief description of the applied MCDM methods is presented in the following section (for more information on the MCDM methods and their related concepts please refer to [38]).

3-1- Fuzzy TOPSIS

"TOPSIS was proposed by Hwang and Yoon (1981) to determine the best alternative based on the concepts of the compromise solution. The compromise solution can be regarded as choosing the solution with the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution" [38]. The steps of this method, regardless of whether it uses fuzzy numbers or the crisp ones, are as follows [39-41]: Suppose, there are "n" alternatives A={ $A_k \mid k=1,2, ..., n$ } and "m" criteria C = { $C_k \mid k=1,2, ..., m$ }. The X={ $X_{kj} \mid k=1,2, ..., n$; j= 1,2, ..., m} denotes the performance of the k_{th} alternative with regards to the j_{th} criterion and finally W={W_i | j=1,2, ..., m} is the weights of the criteria.

The Performance Matrix of MCDM methods					
Alternatives\ Criteria	C ₁	C ₂	Cm		
A ₁	X_{11}	X ₁₂	X _{1m}		
A ₂	X_{21}	X ₂₂	X_{2m}		
			••••		
A _n	X _{n1}	X _{n2}	X _{nm}		
W	W ₁	W_2	Wm		

3-1-1- Calculation of the normalized fuzzy ratings with the following equation:

$$\widetilde{r_{kj}}(x) = \widetilde{X}_{kj} \Big/ \sqrt{\sum_{k=1}^{n} \widetilde{X}_{kj}^{2}}$$

3-1-2- Calculation of the weighted normalized fuzzy ratings:

$$\widetilde{V_{k_l}}(x) = \widetilde{W_l} * \widetilde{r_{k_l}}(x)$$

3-1-3- In the next step, the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS) for each alternative are derived as follows:

$$\begin{split} \textbf{PIS} &= \widetilde{A^+} = \{ \ \widetilde{V_1^+}(x), \widetilde{V_2^+}(x), \dots, \widetilde{V_m^+}(x) \ \} = \{ (\max_k \widetilde{V_{kj}}(x) \mid j \in J_1) \ , \ (\min_k \widetilde{V_{kj}}(x) \mid j \in J_2) \mid \ k = l, 2, \dots, n \} \\ \textbf{NIS} &= \widetilde{A^-} = \{ \ \widetilde{V_1^-}(x), \widetilde{V_2^-}(x), \dots, \widetilde{V_m^-}(x) \ \} = \{ (\min_k \widetilde{V_{kj}}(x) \mid j \in J_1), (\max_k \widetilde{V_{kj}}(x) \mid j \in J_2) \mid \ k = l, 2, \dots, n \} \end{split}$$

Where J_1 and J_2 represent the set of benefit (larger is better) and cost (smaller is better) criteria respectively [38].

3-1-4- In this step the distance from the PIS $(\widetilde{S_k}^+)$ and the distance from the NIS $(\widetilde{S_k}^-)$ for each alternative are calculated by the following equations:

$$\begin{split} \widetilde{S_{k}^{+}} &= \sqrt{\sum_{j=1}^{m} [\ \widetilde{V_{kj}}(x) \ - \ \widetilde{V_{j}^{+}}(x) \]^{2}} \qquad k = 1, 2, ..., n \\ \widetilde{S_{k}^{-}} &= \sqrt{\sum_{j=1}^{m} [\ \widetilde{V_{kj}}(x) \ - \ \widetilde{V_{j}^{-}}(x) \]^{2}} \qquad k = 1, 2, ..., n \end{split}$$

The deterministic form of $\widetilde{S_k^+}$ and $\widetilde{S_k^-}$ are calculated by one of the related methods like CoA, which are described by $D(S_k^+)$ and $D(S_k^-)$. Finally, the order of the alternatives can be obtained by the comparison of the C_k^* in descending order, where: $C_k^* = D(S_k^-)/D(S_k^-) + D(S_k^+)$ k =1, 2, ..., n

The output of each step in this method is demonstrated in Appendix 4 through 6.

3-2- Fuzzy VIKOR

The VIKOR method was developed for multi-criteria optimization of complex systems [38] with noncommensurable and conflicting criteria [28, 42]. VIKOR provides the decision makers with a compromise solution, compromise ranking list and the weight stability intervals with the initial weights [27, 38, 43-45]. In this method, similar to fuzzy TOPSIS, there is a decision matrix consisting of 'J' alternatives (j = 1, 2, ..., J) in the vertical axis and 'n' criteria (i= 1,2,...,n) in its horizontal axis. The performance value of each alternative with respects to each criterion is demonstrated by $\tilde{f_{ij}} = (l_{ij}, m_{ij}, r_{ij})$, i = 1, 2, ..., n, j = 1, 2, ..., J. The set of the criteria is divided into two groups, the benefit (b) and the cost (c) ones.

3-2-1- As the first step for each criterion, the ideal $\tilde{f_i^*} = (l_i^*, m_i^*, r_i^*)$ and the nadir $\tilde{f_i^0} = (l_i^0, m_i^0, r_i^0)$ should be determined, i = 1, 2, ..., n:

For
$$i \in I^b$$
: $\widetilde{f}_i^* = \max_j \widetilde{f}_{ij}$, $\widetilde{f}_i^0 = \min_j \widetilde{f}_{ij}$
For $i \in I^c$: $\widetilde{f}_i^* = \min_j \widetilde{f}_{ij}$, $\widetilde{f}_i^0 = \max_j \widetilde{f}_{ij}$

3-2-2- In the second step, the normalized fuzzy difference $\widetilde{d_{ij}}$, J=1,2,...,j; i=1,2,...,n should be calculated:

For
$$i \in I^{b}$$
: $\widetilde{d_{ij}} = (\widetilde{f_{i}^{*}} \ominus \widetilde{f_{ij}})/(r_{i}^{*} - l_{i}^{0})$
For $i \in I^{c}$: $\widetilde{d_{ij}} = (\widetilde{f_{ij}} \ominus \widetilde{f_{i}^{*}})/(r_{i}^{0} - l_{i}^{*})$

3-3-3- In the third step, the fuzzy weighted sum $\tilde{S}_j = (S_j^l, S_j^m, S_j^r)$ and the fuzzy operator Max $\tilde{R}_j = (R_j^l, R_j^m, R_j^r)$ should be computed by the following relations:

$$\widetilde{S}_{j} = \sum_{i}^{m} \bigoplus (\widetilde{W}_{i} \otimes \widetilde{d}_{ij})$$
$$\widetilde{R}_{j} = MAX(\widetilde{W}_{i} \otimes \widetilde{d}_{ij})$$

Where \widetilde{W}_1 denotes the relative importance of the criteria [28].

3-2-4- In the fourth step, the $\widetilde{Q_j} = (Q_j^l, Q_j^m, Q_j^r)$, J= 1, 2, ..., j should be computed by the relation

 $\widetilde{Q}_j = v \big(\tilde{S}_j \ominus \tilde{S}^* \big) / \, (S^{0r} - \, S^{*l}) \oplus (1 - v) (\widetilde{R}_j \ominus \widetilde{R}^*) / \, (R^{0r} - \, R^{*l});$

Where $\widetilde{S^*} = \min_j \widetilde{S_j}$ · $S^{0r} = \max_j S_j^r$ · $\widetilde{R^*} = \min_j \widetilde{R_j}$, $R^{0r} = \max_j R_j^r$. The parameter 'v' is introduced as "Maximum Group Utility" and '1-v' as "The Weight of the Individual Regret". In this paper it has been assumed that 'v' is equal to 0.5.

3-2-5- This step is core ranking, where $\{A\}_{Q^m}$ is obtained by sorting the Q_j^m , J=1, 2, ..., in descending order.

3-2-6- The fuzzy values $\widetilde{S}_{J} \cdot \widetilde{R}_{J}$ and \widetilde{Q}_{J} are first defuzzified into crisp ones and then ranked in descending order, resulting in three ranking list $\{A\}_{R} \cdot \{A\}_{R}$ and $\{A\}_{Q}$.

Now it is the time to propose a compromise solution. The alternative $A^{(1)}$ that is best ranked by $\{A\}_Q$ is the compromise solution, when two conditions are satisfied:

- "Acceptable Advantage" in which $Adv \ge DQ$.

Where $Adv = [Q(A^{(2)}) - Q(A^{(1)})]/[Q(A^{(j)}) - Q(A^{(1)})]$. $A^{(2)}$ is the alternative with the second position in $\{A\}_Q$ and DQ = 1/(J-1).

- "Acceptable stability in decision making". The alternative A⁽¹⁾ must also be the first in the ranking list of R or/and S [28].

For a more detailed explanation of the fuzzy VIKOR method refer to [28]. The result of each step is demonstrated in Appendix 7 and 8.

3-3- Fuzzy Group ELECTRE

The ELECTRE method is another member of the MCDM methods used for ranking a set of alternatives, which is developed by Roy in 1973 [46]. After the development of its first version, named as ELECTRE I, it has been developed into other versions [30, 47, 48] among which ELECTRE II and III [49, 50] are the most applied versions by researchers [46]. The Fuzzy Group ELECTRE method is an extension of its first version. The following section provides a detailed explanation of these steps.

3-3-1- The first step is the construction of the fuzzy decision matrix. Like the past methods, there are 'm' alternatives $A = \{A_i | i = 1, 2, ..., m\}$, 'n' criteria $C = \{C_j | j = 1, 2, ..., n\}$, 'k' decision makers, $\widetilde{X}_{ijk} = (X_{ijk}^l, X_{ijk}^m, X_{ijk}^r)$ denoting the performance of the i_{th} alternative with regards to the j_{th}criterion in k_{th} decision maker's mind and finally $W = \{W_j | j = 1, 2, ..., n\}$ is the set of the weight of the criteria, where $\widetilde{W}_{jk} = (W_{jk}^l, W_{jk}^m, W_{jk}^r)$ denotes the weight of the j_{th} criterion in k_{th} decision maker's mind. For simplicity, the average function is used to get the consensus of the decision maker's opinions.

3-3-2- The second step is the normalization of the fuzzy decision matrix, which ensures that all the triangular fuzzy numbers belong to [0,1] and also have homogenous and comparable units [46].

$$\mathbf{R} = [\mathbf{r}_{ij}]_{m*n} \quad ;$$

The equation for the benefit criteria is:

$$\begin{split} \tilde{r}_{ij} &= \left(r_{ij}^l, r_{ij}^m, r_{ij}^r\right) = \left(\frac{x_{ij}^l}{c_j^*}, \frac{x_{ij}^m}{c_j^*}\right) , i = 1, 2, ..., m \text{ and } j \in B \qquad , \qquad C_j^* = \max\left(X_{ij}^r\right), \ j \in B \end{split}$$
For the cost criteria, the equation is as follows:

 $\tilde{r}_{ij} = \left(r_{ij}^l, r_{ij}^m, r_{ij}^r\right) = \left(\frac{a_j^-}{x_{ij}^r}, \frac{a_j^-}{x_{ij}^m}, \frac{a_j^-}{x_{ij}^l}\right) \qquad i = 1, 2, \dots, m \text{ and } j \in C \qquad , \qquad a_j^- = \text{ min}(X_{ij}^l)$

3-3-3- In the third step, the weighted normalized fuzzy decision matrix is obtained by:

 $\widetilde{V} = [\ \widetilde{V_{ij}}]_{m*n} \ ; \ \ \widetilde{V_{ij}} = \left(V_{ij}^l, V_{ij}^m, V_{ij}^r\right) = \ \widetilde{W}_j \ (*) \\ \widetilde{r}_{ij} = (\ w_j^l \ r_{ij}^l, w_j^m \ r_{ij}^m, w_j^r \ r_{ij}^r)$

3-3-4- The fourth step is the calculation of the distance between any two alternatives and the Concordance and Discordance sets [46]. Suppose, there are two alternatives A_p and A_q , the Concordance set is formed as $J^c = \{j | \widetilde{V_{pj}} \ge \widetilde{V_{qj}}\}$, in which $A_p \ S \ A_q$, meaning that " A_p is at least as good as A_q ". The Discordance set is also formed as $J^D = \{j | \widetilde{V_{pj}} \le \widetilde{V_{qj}}\}$ meaning exactly against the meaning of what is asserted in ' $A_p \ S \ A_q$ '. The hamming distance method, used here, is as follows:

 $\widetilde{V_{p_j}} \ge \widetilde{V_{q_j}} \Leftrightarrow d(\max(\widetilde{V_{p_j}}, \widetilde{V_{q_j}}), \widetilde{V_{q_j}}) \ge d(\max(\widetilde{V_{p_j}}, \widetilde{V_{q_j}}), \widetilde{V_{p_j}})$ and $\widetilde{V_{p_j}} \le \widetilde{V_{q_j}} \Leftrightarrow d(\max(\widetilde{V_{p_j}}, \widetilde{V_{q_j}}), \widetilde{V_{q_j}}) \le d(\max(\widetilde{V_{p_j}}, \widetilde{V_{q_j}}), \widetilde{V_{p_j}})$ 3-3-5- In the fifth step, the Concordance and the Discordance matrices are calculated:

$$\tilde{C} = \begin{bmatrix} - & \cdots & c_{1m} \\ \vdots & - & \vdots \\ c_{m1}^{-} & \cdots & - \end{bmatrix} \quad \text{where} \quad \widetilde{C_{pq}} = \left(\begin{array}{c} C_{pq}^{l}, C_{pq}^{m}, C_{pq}^{r} \right) = \sum_{j \in J^{c}} \widetilde{W_{j}} = \left(\sum_{j \in J^{c}} \widetilde{W_{j}}^{l}, \sum_{j \in J^{c}} \widetilde{W_{j}}^{m}, \sum_{j \in J^{c}} \widetilde{W_{j}}^{r} \right)$$

The concordance level is also obtained as:

 $\tilde{\tilde{C}} = (C^l, C^m, C^r) \quad C^l = \sum_{p=1}^m \sum_{q=1}^m \frac{C_{pq}^l}{m(m-1)} \quad , C^m = \sum_{p=1}^m \sum_{q=1}^m \frac{C_{pq}^m}{m(m-1)} \quad , C^r = \sum_{p=1}^m \sum_{q=1}^m \frac{C_{pq}^r}{m(m-1)}$

As mentioned before, in this step, the Discordance matrix is as follows:

$$\widetilde{D} = \begin{bmatrix} \dots & \dots & \dots & u_{1(m-1)} & u_{1m} \\ \dots & \dots & \dots & \dots & \dots \\ d_{p_1} & \dots & d_{p_q} & \dots & d_{p(m-1)} & d_{pm} \\ d_{m_1} & \dots & d_{m_q} & \dots & d_{m(m-1)} & - \end{bmatrix} \quad \text{and} \quad d_{pq} = \frac{\max_{j \in J^D} |\tilde{V_{p_j}} - \tilde{V_{q_j}}|}{\max_j |\tilde{V_{p_j}} - \tilde{V_{q_j}}|} = \frac{\max_{j \in J^D} |d(\max(\tilde{V_{p_j}}, \tilde{V_{q_j}}), \tilde{V_{q_j}})|}{\max_j |d(\max(\tilde{V_{p_j}}, \tilde{V_{q_j}}), \tilde{V_{q_j}})|}.$$

The discordance level is also calculated by the following equation: $\overline{D} = \sum_{p=1}^{m} \sum_{q=1}^{m} \frac{d_{pq}}{m(m-1)}$ 3-3-6- In this step the boolean matrices E and F are constructed as is defined here:

 $\text{If } \widetilde{\mathcal{C}_{pq}} \geq \tilde{\mathcal{C}} \ \leftrightarrow \ e_{pq} = 1 \ ; \text{If } \ \widetilde{\mathcal{C}_{pq}} < \ \tilde{\mathcal{C}} \ \leftrightarrow \ e_{pq} = 0$

$$\mathbf{E} = \begin{bmatrix} - & \dots & e_{1q} & e_{1m} \\ \dots & \dots & \dots & \dots \\ e_{p1} & \dots & e_{pq} & e_{pm} \\ e_{m1} & \dots & e_{mq} & - \end{bmatrix}$$

The formula for calculation of the F matrix is: If $d_{pq} < \overline{D} \leftrightarrow f_{pq} = 1$; If $d_{pq} \ge \overline{D} \leftrightarrow f_{pq} = 0$

$$\mathbf{F} = \begin{bmatrix} - & \dots & f_{1q} & f_{1m} \\ \dots & \dots & \dots & \dots \\ f_{p1} & \dots & f_{pq} & e_{pm} \\ f_{m1} & \dots & f_{mq} & - \end{bmatrix}$$

3-3-7- The seventh step is the construction of the general matrix, which is obtained by the peer to peer multiplication of the elements of the matrices E and F as G = E * F.

3-3-8- The last step is for developing the decision graph and ranking the alternatives (For a more detailed and comprehensive explanation of this method, please refer to [46]). The output of each step is depicted in Appendix 9 through 15.

3-4- Fuzzy SAW

The Simple Additive Weighting Method is probably the easiest, best known, and the most applicable MCDM method, which is first utilized in 1954 by Churchman and Ackoff. The method is explained in the following steps [38]:

3-4-1- Just as the last methods, the alternatives performance matrix should be calculated as the first step. Suppose, there are 'n' alternatives (i = 1, 2, ..., n), 'm' criteria (j = 1, 2, ..., m) and \widetilde{W}_j is defined as the importance weight of j_{th} criteria. Furthermore, $\widetilde{x_{ij}}$ is the preferred rating of the i_{th} alternative with respect to the j_{th} criterion.

3-4-2- In this step, the normalized preferred ratings $\tilde{r_{ij}}$ is calculated by:

 $\widetilde{r_{ij}}(x) = \frac{\widetilde{x_{ij}}}{\widetilde{x_{ij}^*}}$, where for the benefit criteria (the larger, the better) $\widetilde{x_{ij}^*} = \max_i \widetilde{x_{ij}}$.

For the cost criteria (the smaller, the better), the formula is: $\widetilde{r_{ij}}(x) = \frac{1/\widetilde{x_{ij}}}{1/\widetilde{x_{ij}}}$, where $\widetilde{x_{ij}} = \max_i \widetilde{x_{ij}}$.

3-4-3- After computing the \tilde{r}_{ij} , the utility of the i_{th} alternative is obtained by the equation $\tilde{u}_i(x) = \sum_{j=1}^n \tilde{w}_j \tilde{r}_{ij}(x)$. Where \tilde{u}_i is the utility of the i_{th} alternative, \tilde{w}_j is the weight of j_{th} criterion and \tilde{r}_{ij} is the normalized preferred rating of the i_{th} alternative with respect to the j_{th} criterion.

3-4-4- The last step is the computation of the A^* , where $A^* = {\tilde{u}_i(x)|\max_i \tilde{u}_i(x)|i = 1, 2, ..., n}$. The step by step output of this method is demonstrated in Appendix 16 through 18.

3-5- The Aggregation Method

Different MCDM methods often produce different outcomes for the selection or ranking a set of decision alternatives involving multiple attributes [51-53]. As Voogd has shown, for at least 40% of the times, each technique produces a different result from any other techniques [29]. The observed inconsistency in the outputs usually has a direct relation to the number of the alternatives [51, 54-56]. An empirical study performed by Yeh in 2002, with the help of sensitivity analysis, showed that even different data sets for a specific problem may result in the selection of different MCDM methods [51].

In recent years, considerable efforts have been made to the development of different MCDM methods, but the question of which method to use is still open since each method has its own algorithm and application [29, 51, 57, 58]. Because of the variety of different MCDM methods, the knowledge and experience of which method to use is an important issue [29], and no one can claim a single method to be the best amongst the others in different problems and even in different data sets [29, 51, 59]. However, there are some practical guidelines for the application of a suitable method in a situation. For example, the ELECTRE methods are more appropriate when the criteria outnumber the alternatives [29]. Some of the advantages of the ELECTRE method are the consideration of indifference and preference thresholds, when modeling the imperfect knowledge of data and also the ability to evaluate purely ordinal scales, without needing to convert the original scales [29, 60]. The TOPSIS method ranks the alternatives based on the shortest distance from the PIS and the longest distance from the NIS. Consequently, it is useful for risk-avoiding decision makers [29, 38, 57]. The VIKOR method proposes compromise solution with the maximum "group utility" of the "majority" and a minimum of "individual regret" of the "opponent" [38]. Therefore,

it can be used by decision makers who want to obtain maximum yield [29]. In spite of these guidelines, some researchers have also suggested a number of techniques to select a suitable MCDM method, which can be classified into three main categories [52]; the tree diagram [61], the criteria approach [62] and the expert systems [63]. Apart from the advantages of the mentioned approaches, they have some major disadvantages that reduce their applicability. Both an incomplete library of the MCDM methods, and the necessity of having some prerequisite knowledge about the MCDM methods, as well as being too simple are the most important critiques of the mentioned approaches [52]. Therefore, many researchers apply different MCDM methods for the sake of enhancing the accuracy of the decisions, especially when the performances of the alternatives are close together [29].

There are also some approaches for aggregating the rankings of different MCDM methods. Average function, Borda and the Copeland methods [29, 63] are the most common aggregation techniques [64]. In the average method, the decision makers usually apply different MCDM methods and calculate the average ranking of the alternatives as a basic aggregation strategy. However, there is no guarantee for obtaining the optimum result, when there are large differences between the rankings of the alternatives [29]. The Borda technique assigns more points to higher rankings and then adds up those points over all individual voters for every alternative. The alternative with the highest point in the voters' rankings is then selected [29]. Copeland's method is a single-winner strategy, in which the winner is identified by finding the candidate with the most pairwise victories minus the number of pairwise defeats [65]. The main weakness of the above methods is the probability of having a tied situation [29]. Hence, there is a need for a more logical procedure in order to enhance the reliability of the outputs of the MCDM methods. In this paper, authors have adopted the aggregation method, which is proposed by Jahan et al. [29] for a consensus ranking, which is a procedure for aggregating the different outcomes of the MCDM methods. The mentioned method has a number of steps, which are explained in the following section.

3-5-1- Suppose, there are 'm' alternatives. Thus, an (m×m) square matrix should be defined in which M_{ik} shows the number of times that the i_{th} alternative has obtained the j_{th} ranking as applied in different MCDM methods.

	Rank1	Rank2	Rank3	 Rank (m)
A_1				M_{1m}
<i>A</i> ₂	M ₂₁			M_{2m}
<i>A</i> ₃	M ₃₁	M ₃₂	M ₃₃	
A _m	M _{m1}	M_{m2}		 M _{mm}

3-5-2- In this step the $C_{ik} = M_{ik} + C_{i,k-1}$ is calculated, where i, k=1, 2, ... and $C_{i,0} = 0$.

	Rank1	Rank2	Rank3	 Rank (m)
<i>A</i> ₁				<i>C</i> _{1m}
A ₂				C_{2m}
<i>A</i> ₃	C ₃₁	<i>C</i> ₃₂	C ₃₃	
A _m	C_{m1}	C_{m2}		 C_{mm}

3-5-3- The final ranking of the alternatives is obtained by solving the following Linear Programming (L.P) problem by the regular simplex algorithm.

$$\max \sum_{i=1}^{m} \sum_{k=1}^{m} C_{i,k} * W_k * N_{i,k}$$
[1]

$$\sum_{k=1}^{m} N_{i,k} = 1 \quad i = 1, 2, ..., m$$
[2]

$$\sum_{i=1}^{m} N_{i,k} = 1 \quad k = 1, 2, ..., m$$
[3]

$$N_{i,k} = \begin{cases} 0 \\ 1 & \text{for all } i \text{ and } k \end{cases}$$

To inject the importance of high rankings to the model, the objective function is weighted by $W_k = \frac{m^2}{k}$, which gives more weight to higher rankings ('m' is the number of the alternatives). "N" is defined as an (m×m) permutation square matrix in which the N_{i,k}=1 if the i_{th} alternative is ranked as the k_{th} , and N_{i,k}=0 if otherwise [29].

The first relationship is the objective function. The second one implies that each alternative can only have one ranking, and finally the third one implies that each ranking can only be assigned to one alternative. The step by step output of the aggregation method is demonstrated in appendix 19 through 22.

3-6- Mathematical Distance

In order to compare and measure the closeness of rankings of the applied FMCDM methods to the aggregated ranking, the four following mathematical distances have been computed.

3-6-1- Euclidean Distance

The Euclidean distance function measures the 'as-the-crow-flies' distance. The formula for this distance between a point X (X1, X2, etc.) and a point Y (Y1, Y2, etc.) is: $d = \sqrt{\sum_{i=1}^{n} (X_i - Y_i)^2}$.

Deriving the Euclidean distance between two data points involves computing the square root of the sum of the squares of the differences between corresponding values [66].

3-6-2- Manhattan distance

The Manhattan distance function computes the distance that would be traveled to get from one data point to the other if a grid-like path is followed. The Manhattan distance between two items is the sum of the differences of their corresponding components. The formula for this distance between a point X= (X1, X2, etc.) and a point Y= (Y1, Y2, etc.) is: $d = \sum_{i=1}^{n} |X_i - Y_i|$

Where "n" is the number of variables, and X_i and Y_i are the values of the i_{th} variable, at points X and Y respectively [66].

3-6-3- Chebyshev distance

The Chebyshev distance is a <u>metric</u> induced by the <u>uniform norm</u>. It is an example of an <u>injective metric</u>. In a two-dimensional space, if the points P and Q have <u>Cartesian coordinates</u> (X_1, Y_1) and (X_2, Y_2) , their Chebyshev distance is [66]:

 $D_{chess} = \max(\mid \textbf{X}_2 \mid -\textbf{X}_1 \mid \text{ , } \mid \textbf{Y}_2 - \mid \textbf{Y}_1 \mid)$

3-6-4- Hamming Distance

By considering two objects of the same dimension, the Hamming distance can be defined as a metric that shows the distance between two objects by the number of mismatches among their corresponding variables. Although the Hamming distance is mainly used for string and bitwise analysis, it also has application in analysis of numerical variables. It should be noted that the basic Hamming distance is a metric. However, the version used in this study provides this opportunity to define a threshold. Variables that have an absolute difference smaller than the threshold are considered to be equal. By assigning values larger than 0 to this threshold, the triangle inequality could be violated for a number of calculated distances. The original Hamming distance is defined by considering the threshold equal to zero. In addition, thresholds below 0 are not defined.

$$d^{HAD}(i,j) = \sum_{k=0}^{n-1} [y_{i,k} \neq y_{j,k}]$$

In the equation, d^{HAD} is the Hamming distance between the objects *i* and *j*, and *k* is the index of the respective variable, reading *y* out of the total number of variables *n* [67].

4-RESEARCH FINDINGS

As mentioned before, the most important contribution of this paper is in the field of devising a novel set of criteria used in security service provider selection. This research has been done in three phases. First, exhaustively reviewing the literature and the services offered by the security service providers of North America; companies like AT&T, CSC, Verizon, Dell Secure Works, IBM, Hewlett Packard, Symantec, TrustWave and Wipro [11]. A set of criteria is extracted from this survey which is further validated by a group of experts. In the second phase, the weights of the derived criteria were assigned by a group of professionals using the questionnaires. They evaluated the importance degree of the criteria in decision-making process by choosing a number from 1 to 10. The assigned numbers are then converted to their related fuzzy numbers. Decision makers often use linguistic or qualitative variables when assessing the quality and the level of the services offered by a provider. Therefore, in the third phase, we decided to use fuzzy MCDM methods for selecting the best service provider. The four FMCDM methods which are included by an aggregation method virtually ensure the decision makers that the final selection is reliable.

In this section, the derived set of criteria will be explained according to the scores given by the decision makers. Authors have proposed 8 general categories of criteria which are further divided into 39 items which should be noticed by the decision makers. The great number of the criteria necessitates the use of the MCDM methods; otherwise it will be almost impossible to take all of them into consideration.

4-1- Staff: One of the most important factors differentiating MSSPs is their threat intelligence ability and discovering new trends. To achieve this capability, MSSPs require experienced analysts. In fact, the number of these analysts [11, 68], their related certifications [69], the positions they have held before in the client's working field (e.g. financial institutions) and finally the reputation of the MSSP principals [69] are the points that decision makers should take into account. Accordingly, the main criteria in this category are as follows:

4-1-1- The staffs' certificates

4-1-2- The MSSP's principals' experience and reputation in a particular field

4-1-3- The number of engineers and the SOC's analysts

4-2- Cost: One of the most important drivers of off-shoring services, especially in developed countries, is gaining cost-related advantages [26, 70]. But in developing countries such as India [16] and Iran, due to the low labor cost, off-shoring IT/IS services is done with the purpose of enhancing business value by improving operational efficiency, quality of service and access to new skills, resources and capabilities [16]. According to some studies conducted in the banking sector of India and Germany [16, 71], today businesses are more likely to look for value creation rather than reducing costs for outsourcing their services. Since outsourcing security is a crucial issue, enterprises usually look for MSSPs who can offer a better quality of service, with the purpose of enhancing business value, rather than a lower cost. Hence, it is assumed that the MSSP's service cost is important but not a high priority factor. Therefore, there would be three main criteria in this category:

4-2-1- The expected value of the service received in return for its cost

4-2-2- The transparency and well definition of the scope of the contract

4-2-3- The proposed cost of the MSSP's services aside from the quality – Which should be in the client's budget range

4-3- Market presence: Forrester has divided the MSSPs into three divisions on the basis of their services, the number of engineers and analysts, annual revenue and similar characteristics. The first category is the MSSPs who have proprietary technologies, whereas the MSSPs ranked in the second and third categories use licensed technologies which in some cases are extended [68]. According to this segmentation, the client should look for the MSSPs which best match their needs. Since not all clients need such a complete and expensive set of services offered by the leading MSSPs, the second and the third groups sometimes offer the same services as the first, but

with a lower cost [68]. The number of clients supported by the MSSP and the traffic volume which passes through its network are also important factors indicating the level of situational awareness, obtained by analyzing the traffic. The number and the location of the SOCs, reputation, experience in related fields, the amount of reliability and trust in vendor and the other MSSP's clients' opinions about the quality and continuity of the services are also noteworthy determining factors. Another important factor is the MSSP's ability to provide compliance with governmental and industry specific regulations and requirements such as Sarbanes Oxley, HIPPA, GLBA, SAS 70 [72, 73]. Therefore, there are seven main criteria in this category:

- 4-3-1- The reliability and the client's confidence in the MSSP
- 4-3-2- Customers' satisfaction
- 4-3-3- Experience and good reputation in the client's field
- 4-3-4- The MSSP's conformity to the government's and industry-specific regulations
- 4-3-5- The amount of the daily traffic passing through the provider's network
- 4-3-6- The MSSP's marketplace in its specific segment
- 4-3-7- The number and the location of the SOCs

4-4- Deliverable service package: Many think that hackers and malicious activities can be protected by implementing a firewall or some other solutions, but if these solutions are not continuously monitored and the required update patches are not installed, all these solutions and expenditures will be in vain and create a false sense of security. Therefore, it is of paramount importance to select an MSSP which not only provides a full package of services, but also can monitor the network traffic, the network and server sensors and firewall log files on a 24/7 basis. Generally speaking, apart from SLAs and the quality of services, the client should select an MSSP which in addition to satisfying its immediate needs, has the flexibility and capability to meet its future planned and even unplanned requirements [23]. Being vendor neutral or multi-vendor support of security devices is another important factor in preventing additional costs and putting constraints on clients [23]. Moreover, it is very important to consider MSSPs with capabilities and certifications necessary to manage and protect client's current equipment without the urge to reinvest in new infrastructure [25]. According to Gartner report, many of the SMBs¹ look for MSSPs offering consolidated architectures such as next generation firewalls or Unified Threat Management (UTM) due to the high network security expenditures [10]. Although there is some debate among client enterprises regarding the performance of intrusion prevention in a consolidated appliance, such as the UTM or the new firewalls, Gartner estimates a positive growth rate for them in the next years [10]. The other important factor to consider is whether an MSSP has the capability to offer cloud-based, premises-based or hybrid services [74]. Many clients still prefer keeping some of their security operations internal and only seek help and consultancy in some special fields, which is called co-sourcing. According to Gartner's findings, the target market for MSSPs is not only IT security departments but also all the C-level executives in client organizations who may become engaged with the MSSPs [10]. Ergo, another differentiating factor is having certified consultants available to guide the clients in some fields that need consultancy [73].

Most leading MSSPs provide the clients with a web-based console through which they can set the security configurations, track the security status, trends and receive the necessary reports when needed. This web-based portal is an important feature facilitating the interaction between the client and the provider [10, 11, 25, 73].

According to the Forrester's segmentation of the MSSP's market, clients should determine whether the technology used by that provider is proprietary, licensed or an extended form, and that what capabilities they have [68]. The other important issue is that some MSSPs offer OEM^2 services which clients should be aware of their existence and related SLAs³ [68, 69].

The use of employee-owned devices at work as well as remote workers is a rapidly growing trend, which if treated in an appropriate way, can cause agility, reduce hardware costs, increase operational efficiency, productivity as well as staff satisfaction and retention, but if not planned beforehand, it can endanger the whole company [75, 76]. The adoption of smart phones, tablets and lap-tops to access data and applications on corporate networks increases

¹ Small and Medium Business

² Original Equipment Manufacturer

³ Service Level Agreement

rapidly and so does the security risk [75]. Therefore, it is essential for companies of any size to adopt a holistic mobile security strategy, preventing the probable damages of this growing useful trend [73]. It is very important to select an MSSP, which can enhance device security requirements and expand the security policies from network to the mobile devices [73].

MSSP's capability to protect a company from DoS¹ and DDoS² attacks, the methods used, supporting and alerting the company representatives in such situations, and the average time they guarantee to recover everything back to a normal state, are other factors to consider. Another growing prominent trend is the concept of Big Data, Business Intelligence and trend analysis; which enable MSSPs to do forensic investigations and discover new threatening trends, attack patterns, malicious and out of compliance activities in order to take preventative actions rather than reacting to the attacks after they happen. Some clients have the required staff to perform these preventative measures, but others do not. Hence, it will also be important to determine whether the MSSP has the on-call staff in emergency situations for performing the recommended actions [77]. In this category, fourteen main criteria are proposed for the decision makers to consider:

4-4-1- Putting client's data within the boundaries of its own country

4-4-2- Providing BYOD³ services

4-4-3- The client's access to a web-based portal

4-4-4- Being vendor neutral

4-4-5- Having on-call staff for emergency situations

4-4-6- Conformance to standards like ITIL, ISO/IEC 27000 and etc. [10]

4-4-7- The way of monitoring the network and giving alarm in emergencies

4-4-8- Offering consolidated architectures such as the UTM and the next generation firewalls

4-4-9- Offering threat intelligence and forensic investigation services

4-4-10- Offering intrusion detection and prevention services

4-4-11- Having OEM services and their related SLAs

4-4-12- Their technology type such as proprietary, licensed or extended

4-4-13- Extended activity in multiple countries

4-4-14- Offering consultancy services

4-5- Support: Offering insurance coverage [69] and guarantee [25, 69] for the offered services and the way they alert clients and take immediate actions [68] are a number of important items to be considered in this category.

4-5-1- Offering guarantee

4-5-2- Offering insurance coverage for the provided services

4-5-3- Alerting clients about the issues raised and taking immediate actions

4-6-Environmental concerns: Physical and cultural proximity are the two factors that help to create a better relationship between clients and MSSPs [15, 23]. As mentioned before, nowadays organizations of all sizes look for MSSPs with which they can make a long-term relationship, that is referred to as strategic relationship. Therefore, physical and cultural proximity are two prominent factors, which help to establish a better contact and relation with the MSSP. Some researchers have found that offshore outsourcing poses additional challenges to clients, which have made security practitioners switch to near-shore alternatives [7]. Another element, which forces some clients to avoid being engaged with a number of MSSPs, is their government regulations and the MSSP's access to the client's mission-critical data. This is particularly true about MSSPs whose SOCs or Data Centers are not located in the client's home country. Accordingly, many large and even small organizations prefer domestic security service outsourcing [10].

When a company contracts with an MSSP, it puts its whole data to the exposure of the MSSP's access. Hence, the MSSP's reliability and trust play a key role in the selection process by decision makers. Reliability itself consists of some sub-criteria, such as political and societal stability. Any kind of instability in the country where the MSSP and

¹ Denial of Service

² Distributed Denial of Service

³Bring Your Own Device

its facilities are located, such as riot, revolution, strike and similar disorders can endanger client's data, the provider's reputation and its overall existence [15]. The three following main criteria are grouped in this category:

4-6-1- Societal and political stability in the MSSP's country

4-6-2- Physical proximity

4-6-3- Cultural proximity

4-7- Strategy: The nature of the client-provider relationship in Information Outsourcing of Services (IOS) can be divided into three forms. The first one is the tactical partnership, which is based on the Transaction Cost Economics (TCE) [24, 78]. This kind of relation is based on transactions aimed at cost reduction. It is short, task-based, and limited to a buyer-seller relation, which does not foster any kind of strategic relationship with providers. It is worth mentioning that value proposition for the client is the lowest. The second form is the strategic partnership, often referred to as the second generation of outsourcing [78]. In this case, a client seeks for a long term partnership, which enhances the value proposition with a number of best in class service providers instead of just reducing costs [24]. Since a client can benefit from the high cumulative experience and learning scope of the provider, this kind of partnership can be justified by the resource-based theory [79]. Therefore, organizations should look for the strategic partnership, especially when there are much competitive pressure and the necessity to focus on core competencies [80]. The mentioned situation is exactly what organizations face in today's competitive environment, and it is also the kind of partnership appropriate for security outsourcing [11, 23]. The last form of partnership is the transformational partnership, which is used when a client is in search of greater market share, gaining competitive advantage and is determined to redefine the existing business and its model [80].

Another important issue to consider is the security provider's strategy in research and development which demonstrate its ability to survive in this changing environment and adopting state-of-the-art technologies. The other considerable issue is client's and MSSP's strategy for future growth. There could be situations where a client needs to extend its work, but the security provider's strategy does not allow the extension. In addition, it will be too expensive for the client to change its MSS provider [11].

The following three main criteria are included in this category:

4-7-1- The possibility of making a long term relationship

4-7-2- The convergence of the future plans of clients and service providers

4-7-3- The convergence of the R&D strategies of clients and service providers

4-8- Financial status: Financial stability is one of the most important factors to consider when engaging with an MSSP. According to a white paper published by IBM in May 2011, managing security for a large number of customers requires significant capital and resources to operate a global network of SOCs, developing new technologies as well as attracting and training experts [25, 72]. Annual revenue and its growth rate are also two other important factors to be considered [81].

The three main criteria are as follows:

4-8-1- Financial stability

4-8-2- Annual revenue

4-8-3- Annual revenue growth rate

5- NUMERICAL EXAMPLE

After a comprehensive research about the selection of an IT service provider, especially in the field of security, a set of 39 criteria is extracted, which are further divided into 8 categories. The categories which have been elaborated in the previous section are sorted according to the experts' opinions, with regards to their level of importance in decision-making process. The large number of the criteria makes the decision-making problem an intricate process, which necessitates the use of the MCDM methods. Hence, in order to increase the accuracy of the result, the authors have applied four common FMCDM methods which not only are they consistent with the problem conditions but also they have had a wide application in supplier selection literature. Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy Group ELECTRE and Fuzzy SAW are the four applied methods, which are included by an aggregation method to propose a unique reliable result. The step-by-step implementations of all the mentioned FMCDM methods have been done in MATLAB. In order to show the applicability of the model, a numerical example is provided, in which the authors

have supposed 8 MSSPs with different levels of performance in each criterion, as well as 4 decision makers to evaluate the alternative's performance. As mentioned before, these evaluations have been done by linguistic variables, which are then converted to fuzzy numbers [table 2]. The performance value of each alternative, with regards to each criterion, is calculated by the average of the decision makers' opinions. The output of the four applied FMCDM methods, are as follows (The performance value of the alternatives and their related fuzzy numbers have been provided in appendix 2 and 3):

Ranking	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	Fuzzy Group ELECTRE
1	A1	A3	A6	A5=A6
2	A5	A6	A3	A1
3	A2	A1	A5	A4
4	A6	A5	A4	A3
5	A3	A2	A8	A2
6	A4	A4	A2	A8
7	A8	A8	A1	A7
8	A7	A7	A7	

Table 3. The outputs (rankings) of the four FMCDM methods

According to table 3, each method has yielded a different outcome. In spite of the similarities among the four methods, there is still the question of which method to rely on. Here, the mentioned aggregation method, elaborated in section 3-5, comes into use. The aggregated output is shown in table 4.

Final ranking by the aggregation method	Alternative
1	A6
2	A3
3	A1
4	A5
5	A2
6	A4
7	A8
8	А7

Table 4. The final ranking of the alternatives by the aggregation method

The aggregation method ranked the sixth alternative (A6) as the first, and the third one (A3) as the second and so forth. In order to compare the similarity between the outputs of the aggregation method and the four FMCDM methods and also their reliability, the Manhattan, Chebyshev, Euclidean and the Hamming Distances have been calculated. The method with the minimum distance can be considered as the most suitable MCDM method in this specific problem. The outputs of the mentioned distances are as follows:

Table 5. The Manhattan distance between the aggregated output and the outputs of the applied methods

Manhattan Distance	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	Fuzzy Group ELECTRE
A1	2	0	4	0
A2	2	0	1	1
A3	3	1	0	3
A4	0	0	2	2
A5	2	0	1	3
A6	3	1	0	0
A7	0	0	0	0
A8	0	0	2	0
	12.00	2.00	10.00	9.00

Chebyshev Distance	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	Fuzzy Group ELECTRE
A1	2	0	4	0
A2	2	0	1	1
A3	3	1	0	3
A4	0	0	2	2
A5	2	0	1	3
A6	3	1	0	0
A7	0	0	0	0
A8	0	0	2	0
	3.00	1.00	4.00	3.00

Table 6. The Chebyshev distance between the aggregated output and the outputs of the applied methods

Table 7. The Euclidean distance between the aggregated output and the outputs of the applied methods

Euclidean Distance	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	Fuzzy Group ELECTRE
A1	4	0	16	0
A2	4	0	1	1
A3	9	1	0	9
A4	0	0	4	4
A5	4	0	1	9
A6	9	1	0	0
A7	0	0	0	0
A8	0	0	4	0
	5.48	1.41	5.10	4.80

Table 8. The Hamming distance between the aggregated output and the outputs of the applied methods

Hamming Distance	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	Fuzzy Group ELECTRE
A1	1	0	1	0
A2	1	0	1	1
A3	1	1	0	1
A4	0	0	1	1
A5	1	0	1	1
A6	1	1	0	0
A7	0	0	0	0
A8	0	0	1	0
	5.00	2.00	5.00	4.00

As all of the four calculated distances show, in table 5 through 8, the Fuzzy TOPSIS method has the minimum distance from the aggregated outcome. Thus, it can be considered the most reliable Fuzzy MCDM method for this problem.

Table 9. The rankings of the FMCDM methods based on the calculated distance from the aggregated outcome

Rank per Distance	Manhattan Distance	Euclidean Distance	Chebyshev Distance	Hamming Distance
Fuzzy SAW	4	4	2	3
Fuzzy TOPSIS	1	1	1	1
Fuzzy VIKOR	3	3	4	3
Fuzzy Group ELECTRE	2	2	2	2

Based on table 9, all the calculated distances have ranked the Fuzzy TOPSIS as the first, the Fuzzy Group ELECTRE as the second, and almost the Fuzzy VIKOR as the third, and finally the Fuzzy SAW as the fourth method, with regards to the similarity to the aggregated outcome, and also the degree of reliability in such a situation.

6- CONCLUSION

There are many managerial and technical concerns regarding the outsourcing of security services, whether offshore or in-shore. Unfortunately, the literature in this field, which is usually proposed by the industry practitioners, is often limited to a number of technical criteria. In this paper, a comprehensive set of managerial and technical criteria is proposed by reviewing both the general IS/IT service outsourcing models, and the related security literature published in recent years. Authors have extracted 39 criteria in 8 general categories with the aid of the literature and the experts' opinions. Each criterion in this list has its own weight in decision-making process, which has been obtained by averaging the experts' scores about the level of importance of the criteria. The list of the criteria and their weights, sorted by their relative rankings, has been demonstrated in tables 10 and 11.

Category	Rank	Fuzzy Triangular Weight	The most important criterion
Staff	1	(7.50, 8.00, 8.43)	The staffs' certificates
Cost	2	(7.45, 7.95, 8.37)	The expected value in receiving the service in return for its cost
Market Presence	3	(7.23, 7.73, 8.17)	The reliability and the client's confidence in the MSSP
Deliverable Service Package	4	(7.09, 7.59, 8.02)	Putting the client's data in the boundaries of its home country
Support	5	(6.95, 7.45, 7.88)	Offering guarantee for the provided services
Environmental Concerns	6	(6.54, 7.03, 7.47)	Societal and political stability in the MSSP's country
Strategy	7	(5.98, 6.48, 6.97)	The possibility of making a long term relationship
Financial Status	8	(5.22, 5.71, 6.19)	Financial stability

ID	ID in Article	Criteria Description	Fuzzy Triangular Weight	Reference
1	4-2-1	The value expected in receiving the service in return for the service cost	(6.40, 6.90, 7.38)	[16, 68, 81]
2	4-4-1	Putting the client's data in the boundaries of its home country	(5.75, 6.25, 6.73)	[7]
3	4-3-1	The reliability and the client's confidence in the MSSP	(6.50, 7.00, 7.50)	[25, 69]
4	4-7-1	The possibility of making a long term relationship	(7.56, 8.15, 8.60)	[11, 23, 80]
5	4-5-1	Offering guarantee	(6.60, 7.10, 7.58)	[25, 69]
6	4-6-1	Societal and political stability in the MSSP's country	(4.73, 5.20, 5.70)	[15]
7	4-3-2	Customers' satisfaction	(6.70, 7.20, 7.68)	[68, 69, 81]
8	4-4-2	Providing BYOD services	(6.30, 6.80, 7.25)	[76]
9	4-1-1	The staffs' certificates	(7.20, 7.70, 8.15)	[23, 68, 69, 72]
10	4-4-3	The client's access to a web-based portal	(6.43, 6.90, 7.40)	[10, 11, 25, 73]
11	4-4-4	Being vendor neutral	(7.10, 7.60, 8.05)	[23, 25]
12	4-3-3	Experience and good reputation in the client's field	(6.25, 6.75, 7.25)	[25, 69]
13	4-2-2	The transparency and well definition of the scope of the contract	(8.25, 8.75, 9.13)	[25]
14	4-4-5	Having on-call staff for emergency situations	(7.75, 8.25, 8.68)	[77]
15	4-1-2	The MSSP's principals' experience and reputation in a particular field	(7.80, 8.30, 8.73)	[69]
16	4-4-6	Conformance to standards like ITIL, ISO/IEC 27000 and etc.	(7.80, 8.30, 8.70)	[10, 72]
17	4-3-4	The MSSP's conformity to the industry and government's specific regulations	(7.75, 8.25, 8.65)	[69, 72]
18	4-4-7	The way of monitoring the network and giving alarm in emergencies	(7.85, 8.35, 8.75)	[73]
19	4-1-3	The number of the engineers and the SOC's analysts	(7.50, 8.00, 8.40)	[11, 68]
20	4-4-8	Offering consolidated architectures such as the UTM and the next generation firewalls	(6.85, 7.35, 7.83)	[10, 68]
21	4-4-9	Offering threat intelligence and forensic investigation services	(7.35, 7.85, 8.30)	[10, 69]
22	4-6-2	Physical proximity	(6.90, 7.40, 7.88)	[7, 15]
23	4-8-1	Financial stability	(6.85, 7.35, 7.78)	[25, 69, 72]
24	4-4-10	Offering intrusion detection and prevention services	(6.95, 7.45, 7.93)	[10, 69, 73]
25	4-3-5	The amount of the daily traffic passing through the provider's network	(7.40, 7.90, 8.40)	[11]
26	4-5-2	Offering insurance coverage for the provided services	(8.00, 8.50, 8.83)	[69]
27	4-3-6	The MSSP's marketplace in its specific segment	(8.05, 8.55, 8.88)	[68, 81]

Table 10. The list of the derived categories and their level of importance according to the experts' opinions

28	4-4-11	Having OEM services and their related SLAs	(6.00, 6.50, 6.93)	[68, 69]
29	4-2-3	The proposed cost of the services apart from the quality of the services the MSSP offers – Which should be within the client's budget range	(7.70, 8.20, 8.60)	[68]
30	4-4-12	The kind of their technology such as proprietary, licensed or extended	(8.15, 8.65, 9.00)	[68]
31	4-3-7	The number and the location of the SOCs	(7.95, 8.45, 8.83)	[10, 11, 68]
32	4-5-3	Alerting the client about the raised issues and taking immediate actions	6.25, 6.75, 7.25)	[68]
33	4-4-13	Extended activity in multiple countries	(7.55, 8.05, 8.50)	[81]
34	4-4-14	Offering consultancy services	(7.40, 7.90, 8.33)	[73, 81]
35	4-7-2	The convergence of the future plans of the client and the service provider	(4.80, 5.30, 5.80)	[81]
36	4-7-3	The convergence of the R&D strategies of the client and the service provider	(5.50, 6.00, 6.50)	[69, 81]
37	4-8-2	Annual revenue	(4.80, 5.30, 5.80)	[11, 68]
38	4-6-3	Cultural proximity	(8.00, 8.50, 8.83)	[23]
39	4-8-3	Annual revenue growth rate	(4.00, 4.50, 5.00)	[25, 69, 72, 81

As most of the MSSPs' appraisals are vague and expressed by linguistic variables, the fuzzy approach has been adopted. Different MCDM methods often yield different results which are really confusing for the decision makers. In order to increase the accuracy of decision-making process, we have adopted Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy Group ELECTRE and Fuzzy SAW. These FMCDM methods are the ones which in addition to having consistency with the problem conditions, have a wide application and acceptance in supplier selection problems. As expected, the methods yielded different results. These variations happen as a result of the different algorithms of the FMCDM methods, and no one can claim that one method is always better than the others. Consequently, authors have used the aggregation method to obtain a unique reliable result. In order to have a comparison between the outcome of the aggregation and the FMCDM methods, the Euclidean, Hamming, Manhattan and Chebyshev distances have been calculated. The comparisons demonstrate the minimum divergence between the outputs of the aggregation and the fuzzy TOPSIS, which displays the suitability of this method for this particular problem.

The selection model provided by this paper can facilitate the selection of a managed security service provider by the industry decision makers. Adding more managerial and technical criteria to the proposed set of criteria to make it more exhaustive; determining the criteria weights by a larger group of experts; building a multi-criteria decision support system on the basis of the model, are some improvements that can be investigated in future works.

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APPENDIX 1

ID	Name	Family Name	Age	Education	Affiliation	Field	Experience (years)
1	Saeed	Rouhani	38	PhD	University of Tehran	Information Service Management	13 years – Industry consultant and university professor
2	Parviz	Agha Sadeghi	59	MSc.	Faratar az Danesh Institute	Data Specialist and Industry Consultant in IT Service Management	More than 30 years experience in various fields related to IT management and Operation
3	Mahmoud	Zibaie	50	MSc.	Central Bank of Iran	Systems Security and Development	23 years - Head of security and development division, Payment Systems Department
4	Isaac	Vaghefi	35	PhD	Binghamton University-State University of New York	Information Systems Management	About 10 years experience in academic and industry- related issues
5	Mehrdad	Gholamzadeh	48	BSc	Mellat Insurance Company / Samaneh Kish Company	IT Operation and Service Management	25 years – Head of IT department

The detailed information of the experts who have validated the set of criteria

The detailed information of the experts who have weighted and defined the level of importance of the criteria

ID	Name	Family Name	Age	Education	Affiliation	Experience
1	Nima	Rezaie	37	BSc	Mellat Insurance Company	11 years – Head of Network & Infrastructure department
2	Meysam	Mirzazadeh	35	MSc	Mellat Insurance Company	10 years – Vice president of IT department
3	Mehrdad	Gholamzadeh	48	BSc	Mellat Insurance Company / Samaneh Kish Company	25 years – Head of IT department
4	Saeed	Rouhani	38	PhD	University of Tehran	13 years – Industry consultant and university professor
5	Farzad	Shaygan	39	MSc	Mellat Insurance Company	Data center expert and industry consultant
6	Farshad	Shoushtari	46	MSc	Mellat Insurance Company	Senior programmer and head of technical division at IT department
7	Anahita	Pazhouheshfar	45	BSc	Mellat Insurance Company	Project manager and former head of IT department at ISIran company
8	Ali	Nemati	39	BSc	Mellat Insurance Company	Senior expert of data center
9	Ali	Kermanshah	58	PhD	Sharif University of Technology	University professor and Director general of New Technologies department at Central Bank of Iran
10	Babak	Sohrabi	49	PhD	University of Tehran	University professor and Electronic Training department manager of University of Tehran
11	Reza	Bayat	45	MSc	Central Bank of Iran	Hardware department manager
12	Mahmoud	Zibaie	49	MSc	Central Bank of Iran	Security & System Development manager
13	Davoud	Mohammad Beigi	48	MSc	Central Bank of Iran	Payment Systems department manager and former Software Development department manager
14	Nasser	Hakimi	50	BSc	Central Bank of Iran	Director general of IT department
15	Nasser	Shahrasbi	35	PhD	San Francisco State University College of	Assistant Professor

					Business	
16	Isaac	Vaghefi	35	PhD	Binghamton University- State University of New York	Assistant Professor of Information Systems
17	Soroosh	Nalchigar	34	PhD	Deloitte	Data Scientist
18	Mustafa	Purmehdi	35	PhD	HEC Montreal	Management Consultant and Researcher and Lecturer at HEC Montreal
19	Sina	Sabzevari	32	MSc	Samaneh Kish Company	IT Service management consultant
20	Hamidreza	Alaie	42	BSc	Remis Company	IT Service management consultant
21	Vahid	Esmaeeli	35	PhD	Assistant Professor at Wtiau	Management Consultant
22	Mahyar	Safaei	33	MSc	NooraNet and SooiaNet computer netwroking	Computer network instructor
23	Reza	Esfandiar	32	BSc	Informatics Services Corporation (ISC)	Computer and Network Security Engineer
24	Mostafa	Bahraini	32	BSc	Informatics Services Corporation (ISC)	Computer and Network Security Engineer
25	Saber	Jahanpour	36	MSc	Hanover Insurance Group	Certified Financial Risk Manager and Associate Data Scientist
26	Mohsen	Hoshyar	39	BSc	Fanava Company / Mellat Insurance Company	Senior Data Center Specialist and Consultant
27	Ali	Nemati	37	MSc	Mellat Insurance Company	Data Center Specialist
28	Ebrahim	Hadjarian	34	BSc	Respect General Trading	Data Center Consultant
29	Mona	Shahrasbi	32	MSc	Monash University / Informatics Services Corporation	Computer and Network Security Engineer
30	Ebrahim	Soleimani	36	MSc	Informatics Services Corporation	Computer and Network Security Engineer

APPENDIX 2

The scores of the eight alternatives with regards to each 39 criteria given by the four decision makers

ID	ID in Article	A1	A2	A3	A4	A5	A6	A7	A8
1	1-1	(B,G,G,B)	(F,G,F,F)	(F,P,P,W)	(F,G,G,F)	(B,B,G,G)	(B,G,G,G)	(F,P,P,F)	(G,G,G,G)
2	1-2	(G,G,F,G)	(P,P,F,W)	(P,P,W,G)	(P,P,W,P)	(F,F,P,P)	(G,G,G,F)	(P,F,P,G)	(F,G,F,G)
3	1-3	(F,F,P,F)	(W,P,G,P)	(G,F,G,W)	(F,P,F,F)	(F,G,F,G)	(F,G,B,F)	(G,G,F,F)	(F,F,P,G)
4	2-1	(B,G,G,B)	(B,F,G,G)	(F,G,G,F)	(B,F,G,G)	(G,G,F,G)	(G,B,B,F)	(F,P,P,P)	(G,F,F,B)
5	2-2	(B,F,G,G)	(G,B,G,G)	(G,G,F,F)	(G,F,F,G)	(F,P,P,F)	(F,F,F,F)	(P,P,W,P)	(G,G,G,G)
6	2-3	(G,F,F,F)	(B,G,B,B)	(G,G,G,F)	(W,P,F,P)	(F,W,F,F)	(G,G,G,P)	(F,F,F,F)	(G,G,F,G)
7	3-1	(G,F,G,F)	(B,G,G,B)	(F,F,F,P)	(F,G,F,G)	(G,F,G,B)	(B,G,B,F)	(P,F,P,F)	(B,G,G,F)
8	3-2	(G,G,F,F)	(G,B,G,B)	(G,F,G,F)	(G,F,G,B)	(G,B,G,G)	(F,F,P,F)	(F,F,G,G)	(F,F,P,G)
9	3-3	(F,F,G,G)	(G,F,F,G)	(G,G,B,G)	(F,P,F,P)	(P,F,F,G)	(G,B,F,P)	(P,P,F,P)	(F,G,G,F)
10	4-1	(G,G,F,B)	(F,F,B,G)	(G,F,F,G)	(W,F,P,W)	(W,P,W,W)	(F,F,P,F)	(W,W,P,G)	(G,F,P,G)
11	4-2	(B,B,G,B)	(P,F,W,P)	(G,G,F,F)	(W,F,W,P)	(F,F,P,F)	(G,B,B,P)	(P,P,F,P)	(G,G,G,G)
12	4-3	(W,W,W,P)	(P,W,P,P)	(F,P,F,F)	(F,G,G,F)	(G,G,B,G)	(P,P,P,F)	(B,B,B,P)	(F,F,G,G)
13	4-4	(G,B,G,F)	(B,F,G,B)	(P,W,P,P)	(P,W,W,P)	(F,G,G,G)	(F,P,G,F)	(P,P,F,G)	(F,F,P,W)
14	4-5	(G,F,G,B)	(B,G,F,B)	(B,G,B,B)	(G,B,F,B)	(B,F,B,G)	(G,F,G,P)	(P,P,W,P)	(P,P,F,P)
15	4-6	(B,F,B,G)	(G,F,G,B)	(W,F,F,P)	(F,G,G,F)	(B,G,B,B)	(G,B,B,F)	(W,W,F,F)	(P,P,F,F)
16	4-7	(F,G,F,P)	(G,F,G,F)	(F,G,B,G)	(G,B,G,G)	(F,G,G,F)	(G,B,B,F)	(W,W,P,F)	(G,F,F,F)
17	4-8	(G,B,F,B)	(F,G,G,F)	(F,G,F,G)	(G,B,G,G)	(F,G,G,B)	(G,F,F,P)	(W,F,P,F)	(F,F,P,F)
18	4-9	(F,P,F,P)	(B,G,B,B)	(B,G,G,G)	(P,P,P,P)	(G,G,G,G)	(G,F,G,F)	(P,W,P,F)	(W,F,F,P)
19	4-10	(G,G,G,B)	(G,B,B,B)	(G,F,G,F)	(F,W,F,G)	(G,F,G,F)	(B,G,B,F)	(P,W,P,F)	(G,G,F,F)
20	4-11	(G,G,G,G)	(G,F,F,F)	(B,F,G,B)	(F,F,G,P)	(F,P,G,G)	(F,B,G,P)	(P,P,P,G)	(F,P,F,F)
21	4-12	(W,F,W,W)	(F,G,F,G)	(B,B,B,B)	(B,B,B,B)	(B,B,G,B)	(B,G,F,F)	(F,F,P,G)	(F,P,F,G)
22	4-13	(B,G,B,B)	(G,F,G,B)	(B,F,G,B)	(B,F,B,B)	(B,F,B,G)	(F,B,B,F)	(W,P,W,P)	(P,F,G,G)
23	4-14	(G,F,G,F)	(P,W,P,W)	(B,G,B,B)	(G,P,F,G)	(P,P,W,P)	(F,G,F,G)	(P,P,W,G)	(F,P,F,G)
24	5-1	(G,P,F,G)	(G,F,F,G)	(F,P,P,F)	(F,W,F,F)	(F,P,W,P)	(P,W,P,P)	(P,P,F,G)	(G,G,G,F)
25	5-2	(G,G,P,G)	(B,G,F,G)	(B,G,B,B)	(F,P,F,F)	(G,G,F,F)	(W,F,P,F)	(G,F,F,F)	(G,F,F,F)
26	5-3	(F,P,F,F)	(G,G,F,F)	(F,B,G,G)	(F,W,P,W)	(G,F,G,G)	(G,B,G,P)	(F,P,P,F)	(F,F,G,F)
27	5-4	(W,F,P,P)	(P,W,P,P)	(B,G,F,G)	(F,W,W,P)	(B,F,B,G)	(G,G,F,G)	(F,F,P,F)	(P,W,P,P)
28	5-5	(F,G,F,G)	(F,F,P,F)	(F,B,B,B)	(F,G,P,F)	(F,G,G,G)	(F,P,F,F)	(F,G,G,G)	(F,F,P,F)
29	5-6	(G,F,B,B)	(G,F,G,G)	(F,G,F,F)	(G,P,G,F)	(G,F,G,F)	(F,F,F,F)	(P,P,F,F)	(F,G,G,F)
30	5-7	(G,F,F,G)	(F,G,B,B)	(F,P,F,P)	(G,F,B,G)	(F,G,G,F)	(G,G,G,P)	(F,G,G,F)	(F,G,G,F)
31	6-1	(W,P,W,W)	(W,W,W,W)	(P,F,P,P)	(F,G,G,F)	(P,W,W,P)	(F,G,G,F)	(F,G,G,F)	(F,P,F,P)
32	6-2	(G,F,G,B)	(P,W,P,W)	(F,P,G,F)	(F,G,G,F)	(G,F,G,G)	(B,B,B,P)	(P,W,F,F)	(F,F,F,G)
33	6-3	(G,B,B,G)	(G,B,G,G)	(B,G,F,G)	(F,P,F,P)	(G,G,G,F)	(G,G,F,F)	(F,P,F,F)	(P,W,W,P)
34	7-1	(B,G,B,G)	(G,G,F,G)	(P,P,F,F)	(F,F,F,G)	(W,W,W,W)	(F,P,P,G)	(F,P,P,W)	(G,G,F,F)
35	7-2	(F,F,P,F)	(G,F,F,G)	(B,B,G,G)	(F,G,G,F)	(G,G,F,F)	(B,F,G,F)	(F,G,F,G)	(F,W,F,F)
36	7-3	(G,G,B,G)	(P,G,F,F)	(F,G,F,P)	(F,G,F,G)	(F,P,F,P)	(B,G,F,G)	(F,P,F,G)	(F,P,F,F)
37	8-1	(P,W,P,W)	(F,P,P,F)	(P,W,F,P)	(G,P,G,B)	(P,G,G,F)	(G,F,G,P)	(P,F,P,F)	(F,W,P,W)
38	8-2	(W,F,P,P)	(P,F,P,F)	(G,F,G,B)	(P,W,P,P)	(F,G,G,B)	(G,F,G,F)	(F,P,G,F)	(F,W,W,P)
39	8-3	(B,B,B,B)	(B,B,B,B)	(B,B,B,B)	(B,B,B,B)	(B,B,B,B)	(F,F,G,P)	(W,F,F,P)	(B,B,B,G)

Fuzzy scores calculated for each alternative with regards to the criteria on the basis of the decision makers` I	linguistic scores

Criteria No.	Triangular Fuzzy Number	A1	A2	A3	A4	A5	A6	A7	A8
	L	6.25	3.125	0.7	3.75	6.25	5.625	1.3	5
1	М	8.75	5.625	125 0.7 3.75 6.25 5.625 1.3 125 5 8.75 8.125 3.75 125 5 8.75 10 10 6.25 125 5 8.75 10 10 6.25 125 5 8.75 10 10 6.25 7 1.325 0.1 1.3 4.375 1.925 55 5.625 4.375 6.25 9.375 6.875 15 5.025 4.375 6.25 6.875 8.75 5 3.75 5 4.375 5.625 0.7 5 6.25 7.5 6.875 8.125 3.125 5 3.75 3.75 1.3 2.5 0.1 125 6.25 7.5 8.125 3.125 5 6.25 3.75 1.3 2.5 0.1 125 125 6.25 0.7 1.9 3.75 5	7.5				
	R	10	8.125		8.75	10	10	6.25	10
	L	A1 A2 6.25 3.125 8.75 5.625 10 8.125 4.375 0.7 6.875 2.525 9.375 5 1.9 1.325 4.375 3.15 6.875 5.625 9.375 5 1.9 1.325 4.375 3.15 6.875 5.625 6.25 5 8.75 7.5 10 9.375 0 3.125 6.875 5.625 9.375 10 3.125 6.875 9.375 10 3.75 6.25 8.75 10 3.75 6.25 8.75 10 3.75 8.75 8.75 10 3.75 8.75 8.75 8.75 9.375 8.75 9.375 8.75 9.375	1.325	0.1	1.3	4.375	1.925	3.75	
2	М	6.875	2.525	3.15	1.9	3.75	6.875	1.3 3.75 6.25 1.925 4.375 6.875 3.75 6.25 8.75 0.7 3.125 5.625 0.1 1.9 4.375 2.5 7.5 1.3 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.7 1	6.25
	R				4.375	6.25			8.75
	L	1.9	1.325	3.15		3.75	4.375		2.525
3	М								5
-	R								7.5
	L								4.375
4	M								6.875
	R								8.75
	L								5
5	M							$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	7.5
U	R			0.7 3.75 6.25 5.625 1.3 2.525 6.25 8.75 8.125 3.75 7.5 5 8.75 10 10 6.25 11 1.325 0.1 1.3 4.375 6.25 4.375 6.25 5.625 4.375 6.25 6.875 6.25 6.875 6.25 5.025 4.375 6.25 6.875 6.25 0.7 4.3 5.025 4.375 6.25 0.7 4.3 6.25 0.7 4.3 6.25 7.5 6.875 8.125 3.125 6.8 8.75 9.375 9.375 9.375 5.625 8.8 8.75 9.375 5.625 1.9 7.5 6.875 8.75 9.375 6.25 5.625 1.3 2.55 4.375 6.25 7.5 8.125 3.75 7.5 9.375	10				
	L							$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	4.375
6	M							$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	6.875
0	R						5 5.625 1.3 5 8.125 3.75 0 10 6.25 3 4.375 1.925 5 6.875 4.375 5 9.375 6.875 5 9.375 6.25 5 4.375 3.75 5 6.875 6.25 5 8.75 8.75 75 5.625 0.7 75 9.375 5.625 3 2.5 0.1 5 5 1.9 5 7.5 4.375 9.375 5.625 1.3 5 8.75 7.5 5.625 1.3 5.625 5 8.75 7.5 5.625 1.3 5 8.75 7.5 5.625 1.3 5 8.75 5.625 6.75 8.75 5.625 <td></td> <td>9.375</td>		9.375
	L		-						5
7	M							1.3 3.75 6.25 1.925 4.375 6.875 3.75 6.25 8.75 0.7 3.125 5.625 0.1 1.9 4.375 2.5 5 7.5 1.3 3.75 6.25 3.125 5.625 7.5 1.3 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 5.625 1.325 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.625 5.6	7.5
,	R								9.375
	L		-						2.525
8	M							$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	5
0	R								7.5
	L		-			-			3.75
0	M								6.25
No.	R								8.75
	L							1.3 3.75 6.25 1.925 4.375 6.875 3.75 6.25 8.75 0.7 3.125 5.625 0.1 1.9 4.375 2.5 7.5 1.3 3.75 6.25 3.125 5.625 0.7 3.125 5.625 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 5.625 1.325 5.625 5.625 5.625 8.75 1.325 5.625 6.875 0.1 1.9 4.375 1.3 3.15 5.625 5 <td>3.15</td>	3.15
10	M							$\begin{array}{r} 6.25\\ 3.75\\ 6.25\\ 8.75\\ 0.7\\ 3.125\\ 5.625\\ 1.325\\ 2.55\\ 5\\ 0.7\\ 3.125\\ 5.625\\ 5\\ 5.625\\ 5.625\\ 5.65\\ 8.125\\ \end{array}$	5.625
10	R								8.125
	L R							$\begin{array}{c} 1.3\\ 3.75\\ 6.25\\ 1.925\\ 4.375\\ 6.875\\ 3.75\\ 6.25\\ 8.75\\ 6.25\\ 8.75\\ 0.7\\ 3.125\\ 5.625\\ 0.1\\ 1.9\\ 4.375\\ 2.5\\ 5\\ 7.5\\ 1.3\\ 3.75\\ 6.25\\ 5.625\\ 1.325\\ 5.625\\ 5.625\\ 5.625\\ 5.625\\ 5.65\\ 8.125\\ 5.5\\ 0.7\\ 1.925\\ 4.375\\ 1.3\\ 3.15\\ 5.625\\ 5\\ 0.7\\ 2.525\\ 5\\ 0.7\\ 2.525\\ 5\\ 0.7\\ 2.525\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 5\\ 0.7\\ 2.525\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5$	
11	L M							$\begin{array}{c} 1.3 \\ 3.75 \\ 6.25 \\ 1.925 \\ 4.375 \\ 6.875 \\ 3.75 \\ 6.25 \\ 8.75 \\ 0.7 \\ 3.125 \\ 5.625 \\ 0.1 \\ 1.9 \\ 4.375 \\ 2.5 \\ 5 \\ 7.5 \\ 1.3 \\ 3.75 \\ 6.25 \\ 5.625 \\ 1.325 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 1.325 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 0.7 \\ 1.925 \\ 4.375 \\ 1.3 \\ 3.15 \\ 5.625 \\ 0.7 \\ 2.525 \\ 5 \\ 0.7 \\ 2.525 \\ 5 \\ 1.325 \end{array}$	5 7.5
11									
	R L							$\begin{array}{c} 1.3 \\ 3.75 \\ 6.25 \\ 1.925 \\ 4.375 \\ 6.875 \\ 3.75 \\ 6.25 \\ 8.75 \\ 0.7 \\ 3.125 \\ 5.625 \\ 0.1 \\ 1.9 \\ 4.375 \\ 2.5 \\ 5 \\ 7.5 \\ 1.3 \\ 3.75 \\ 6.25 \\ 5.625 \\ 1.325 \\ 5.625 \\ 1.325 \\ 5.625 \\ 1.325 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 8.125 \\ 5.65 \\ 1.925 \\ 4.375 \\ 1.3 \\ 3.15 \\ 5.625 \\ 0.7 \\ 1.925 \\ 4.375 \\ 1.3 \\ 3.15 \\ 5.625 \\ 0.7 \\ 2.525 \\ 5 \\ 0.7 \\ 2.525 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ 5 \\ 1.325 \\ $	10
10								$\begin{array}{c} 1.3 \\ 3.75 \\ 6.25 \\ 1.925 \\ 4.375 \\ 6.875 \\ 3.75 \\ 6.25 \\ 8.75 \\ 0.7 \\ 3.125 \\ 5.625 \\ 0.1 \\ 1.9 \\ 4.375 \\ 2.5 \\ 5 \\ 7.5 \\ 1.3 \\ 3.75 \\ 6.25 \\ 5.625 \\ 1.325 \\ 5.625 \\ 5.625 \\ 1.325 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625 \\ 5.625 \\ 5.65 \\ 8.125 \\ 5.625$	3.75
12	M								6.25
	R								8.75
12	L							1.3 3.75 6.25 1.925 4.375 6.875 3.75 6.25 8.75 0.7 3.125 5.625 0.1 1.9 4.375 2.5 5.625 0.1 1.9 4.375 2.5 5 7.5 1.3 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 8.75 0.7 3.125 5.625 5.625 5.625 5.625 5.625 5.625 6.875 0.1 1.9 4.375 1.3 3.15 5.625 5 0.7 <td>1.3</td>	1.3
13	M							1.3 3.75 6.25 1.925 4.375 6.875 3.75 6.25 8.75 0.7 3.125 5.625 0.1 1.9 4.375 2.5 0.1 1.9 4.375 2.5 7.5 1.3 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 6.25 3.75 5.625 1.325 5.625 5.625 5.625 8.75 1.325 5.625 5.625 6.875 0.1 1.9 4.375 1.3 2.55	3.15
	R								5.625
1.4	L								0.7
14	M								3.125
	R								5.625
1.7	L								1.3
15	M								3.75
	R								6.25
	L								3.125
16	M								5.625
	R								8.125
	L								1.9
17	М								4.375
	R								6.875
	L								1.3
18	М								3.15
	R								5.625
	L				2.525				3.75
19	М	8.125	9.375	6.25	4.4	6.25	8.125	2.525	6.25
	R								8.75
20	L	5		5.625	2.525	3.15	3.775	1.325	1.9
20	М	7.5	5.625	8.125	5	5.625	6.25	3.75	4.375

	R	10	8.125	9.375	7.5	8.125	8.125	6.25	6.875
							4.375		2.525
21							6.875		5
21							8.75		7.5
						-	5		3.15
22							7.5		5.625
22							8.75		8.125
22							3.75	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	2.525
23							6.25		5
							8.75		7.5
24							0.1		4.375
24							1.9		6.875
							4.375		9.375
							1.3	1	3.125
25							3.15		5.625
							5.625		8.125
							4.4		3.125
26							6.875		5.625
							8.75		8.125
							4.375		0.1
27						8.125	6.875		1.9
		5		9.375	4.375	9.375	9.375		4.375
		3.75			2.525		1.9	4.375	1.9
28	М	6.25		8.75	5	6.875	4.375	6.875	4.375
	R	8.75		9.375	7.5	9.375	6.875	9.375	6.875
	L	5.625		3.125	3.15	3.75	2.5	1.3	3.75
29	М	8.125	6.875	5.625	5.625	6.25	5	3.75	6.25
	R	9.375	9.375	8.125	8.125	8.75	7.5	6.25	8.75
	L	3.75	5.625	1.3	5	3.75	3.775	2.525 5 7.5 0.1 1.3 3.75 1.325 3.15 5.625 1.925 4.375 6.875 3.125 5.625 1.3 3.75 6.25 1.9 4.375 6.875 4.375 6.875 4.375 6.875 1.3 3.75 6.25 1.3 3.75 6.25 3.75 6.25 3.75 6.25 8.75 1.3 3.15 5.625 1.3 3.15 5.625 1.3 3.15 5.625 5 7.5 1.3 3.75 6.25 5.75 <	3.75
30	М	6.25		3.75	7.5	6.25	6.25	6.25	6.25
		8.75		6.25		8.75	8.75	2.525 5 7.5 0.1 1.3 3.75 1.325 3.15 5.625 1.925 4.375 6.875 3.125 5.625 8.125 1.3 3.75 6.25 1.9 4.375 6.875 9.375 1.3 3.75 6.25 1.9 4.375 6.875 9.375 1.3 3.75 6.25 3.75 6.25 8.75 3.75 6.25 8.75 1.3 3.15 5.625 1.3 3.15 5.625 7.5 1.3 3.75 6.25 5 7.5 </td <td>8.75</td>	8.75
	L	0.1	0.1	0.7	3.75	0.1	3.75	$\begin{array}{r} 2.525\\ \hline 5\\ \hline 7.5\\ \hline 0.1\\ \hline 1.3\\ \hline 3.75\\ \hline 1.325\\ \hline 3.15\\ \hline 5.625\\ \hline 1.925\\ \hline 4.375\\ \hline 6.875\\ \hline 3.125\\ \hline 5.625\\ \hline 8.125\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 1.9\\ \hline 4.375\\ \hline 6.25\\ \hline 3.75\\ \hline 6.25\\ \hline 5.75\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.75\\ \hline 6.25\\ \hline 5\\ \hline 7.5\\ \hline 1.3\\ \hline 3.15\\ \hline \end{array}$	1.3
31	М	0.7	0.1	3.125	6.25	1.3	6.25		3.75
	R	3.125	2.5	5.625	8.75	3.75	8.75		6.25
		5		2.525			5.65		3.125
32	$\begin{tabular}{ c c c c c c c } \hline M & 6.25 & 7.5 & 9.375 \\ \hline R & 8.75 & 9.375 & 10 \\ \hline L & 1.9 & 3.75 & 5 \\ \hline M & 4.375 & 6.25 & 7.5 \\ \hline R & 6.875 & 8.75 & 9.375 \\ \hline L & 0.7 & 0.1 & 5 \\ \hline M & 2.525 & 1.9 & 7.5 \\ \hline R & 5 & 4.375 & 9.375 \\ \hline L & 3.75 & 1.9 & 6.25 \\ \hline M & 6.25 & 4.375 & 8.75 \\ \hline R & 8.75 & 6.875 & 9.375 \\ \hline L & 5.625 & 4.375 & 3.125 \\ \hline R & 8.125 & 6.875 & 9.375 \\ \hline L & 3.75 & 1.9 & 6.25 \\ \hline R & 8.125 & 6.875 & 9.375 \\ \hline L & 3.75 & 1.9 & 3.125 \\ \hline M & 6.25 & 8.125 & 3.75 \\ \hline R & 8.75 & 5.625 & 1.3 \\ \hline M & 6.25 & 8.125 & 3.75 \\ \hline R & 8.75 & 9.375 & 6.25 \\ \hline L & 0.1 & 0.1 & 0.7 \\ \hline M & 0.7 & 0.1 & 3.125 \\ \hline \end{tabular}$			8.125		5.625			
				7.5			8.75		8.125
							3.75		0.1
33		R 8.75 9.375 10 6.875 8.75 L 1.9 3.75 5 0.7 4.375 M 4.375 6.25 7.5 1.925 6.875 R 6.875 8.75 9.375 4.375 9.375 L 0.7 0.1 5 0.7 5.625 M 2.525 1.9 7.5 1.925 8.125 R 5 4.375 9.375 4.375 9.375 L 3.75 1.9 6.25 2.525 4.375 M 6.25 4.375 8.75 5 6.875 R 8.75 6.875 3.125 3.15 3.75 M 6.25 4.375 8.75 6.25 6.25 8.75 L 5.625 1.3 5 6.25 8.75 L 3.75 5.625 9.375 8.75 <td>6.25</td> <td></td> <td>1.3</td>	6.25		1.3				
							8.75		3.75
							1.925		3.75
34							4.375		6.25
							6.875		8.75
							4.375		1.9
35							6.875		3.775
20							8.75		6.25
							5		1.9
36							7.5		4.375
50							9.375		6.875
							3.15		0.875
37							5.625		1.925
51							8.125		4.375
							3.75		0.7
20									
38							6.25 8.75		1.925
							8.75		4.375
20							2.525		6.875
39							5		9.375
	ĸ	10	10	10	10	10	7.5	5.625	10

Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8
	L	0.254	0.127	0.028	0.152	0.254	0.228	0.053	0.203
1	М	0.457	0.294	0.132	0.327	0.457	0.424	0.196	0.392
	R	0.789	0.641	0.394	0.690	0.789	0.789	0.493	0.789
	L	0.215	0.034	0.065	0.005	0.064	0.215	0.095	0.184
2	М	0.503	0.185	0.231	0.139	0.274	0.503	0.320	0.457
	R	1.210	0.646	0.726	0.565	0.807	1.210	0.888	1.130
	L	0.088	0.061	0.146	0.088	0.173	0.202	0.173	0.117
3	М	0.292	0.211	0.336	0.292	0.418	0.460	0.418	0.334
	R	0.807	0.660	0.881	0.807	1.027	1.027	1.027	0.881
	L	0.248	0.198	0.149	0.198	0.173	0.223	0.028	0.173
4	М	0.438	0.376	0.313	0.376	0.344	0.407	0.157	0.344
	R	0.759	0.712	0.664	0.712	0.712	0.712	0.427	0.664
	L	0.212	0.239	0.159	0.159	0.055	0.106	0.004	0.212
5	М	0.434	0.470	0.361	0.361	0.217	0.289	0.110	0.434
	R	0.864	0.922		0.807		0.691	0.403	0.922
	L								0.189
6	М								0.398
									0.856
									0.204
7	М			0.229					0.392
									0.755
									0.101
8									0.264
0									0.617
									0.164
9									0.380
									0.890
									0.156
10									0.394
10									0.945
									0.229
11									0.229
11									0.430
10									0.179
12									0.401
									0.894
12		M 0.457 0.294 0.132 0.327 0.457 0.424 0.196 R 0.789 0.641 0.394 0.690 0.789 0.789 0.493 L 0.215 0.034 0.065 0.005 0.064 0.215 0.095 M 0.503 0.185 0.231 0.139 0.274 0.503 0.320 R 1.210 0.646 0.726 0.565 0.807 1.210 0.888 L 0.088 0.061 0.146 0.088 0.173 0.202 0.173 M 0.292 0.211 0.336 0.292 0.418 0.460 0.418 R 0.807 0.660 0.881 0.807 1.027 1.027 L 0.248 0.198 0.149 0.198 0.173 0.223 0.028 M 0.438 0.376 0.313 0.376 0.344 0.407 0.157 R 0.759 0.712	0.063						
13									0.208
									0.601
14									0.029
14									0.159
									0.422
1.5									0.056
15									0.202
									0.503
									0.131
16									0.311
									0.704
									0.080
17									0.244
		0.828	0.773		0.884		0.663	0.497	0.608
									0.058
18		0.222				0.444			0.186
	R	0.566	0.906	0.906	0.453	0.906	0.793	0.453	0.510
10	L		0.283	0.154	0.104	0.154	0.232	0.029	0.154
19				0.328	0.231			0.133	0.328

Normalized fuzzy ratings $\widetilde{\eta_{kj}}(x)$ in Fuzzy TOPSIS method

	R	0.792	0.792	0.693	0.544	0.693	0.742	0.396	0.693
	L	0.218	0.136	0.245	0.110	0.137	0.164	0.058	0.083
20	М	0.446	0.335	0.483	0.297	0.335	0.372	0.223	0.260
	R	0.989	0.803	0.927	0.741	0.803	0.803	0.618	0.680
	L	0.029	0.156	0.312	0.312	0.286	0.182	0.105	0.105
21	М	0.064	0.303	0.485	0.485	0.454	0.333	0.242	0.242
	R	0.261	0.609	0.696	0.696	0.696	0.609	0.522	0.522
	L	0.279	0.203	0.228	0.253	0.228	0.203	0.004	0.128
22	M	0.279	0.203	0.228	0.233	0.228	0.203	0.062	0.128
22									
	R	0.691	0.648	0.648	0.648	0.648	0.605	0.259	0.561
22	L	0.179	0.005	0.328	0.150	0.005	0.179	0.063	0.120
23	М	0.405	0.084	0.608	0.365	0.123	0.405	0.204	0.324
	R	0.905	0.388	1.034	0.840	0.453	0.905	0.582	0.776
	L	0.158	0.188	0.065	0.095	0.035	0.005	0.096	0.219
24	М	0.425	0.472	0.283	0.285	0.191	0.144	0.330	0.519
	R	1.120	1.206	0.862	0.862	0.689	0.603	0.948	1.292
	L	0.161	0.213	0.293	0.081	0.160	0.055	0.133	0.133
25	М	0.352	0.423	0.529	0.247	0.352	0.178	0.317	0.317
-	R	0.781	0.837	0.893	0.614	0.781	0.502	0.726	0.726
	L	0.085	0.168	0.224	0.031	0.196	0.197	0.058	0.140
26	M	0.272	0.389	0.467	0.120	0.428	0.428	0.233	0.350
20	R	0.272	0.907	0.407	0.120	0.428	0.428	0.233	0.842
	L R			0.972			0.907		
27		0.035	0.005		0.035	0.284		0.096	0.005
27	M	0.176	0.132	0.523	0.134	0.566	0.479	0.305	0.132
	R	0.558	0.488	1.046	0.488	1.046	1.046	0.767	0.488
	L	0.162	0.082	0.269	0.109	0.189	0.082	0.189	0.082
28	М	0.365	0.256	0.512	0.292	0.402	0.256	0.402	0.256
	R	0.840	0.660	0.900	0.720	0.900	0.660	0.900	0.660
	L	0.239	0.186	0.133	0.134	0.159	0.106	0.055	0.159
29	М	0.474	0.401	0.328	0.328	0.365	0.292	0.219	0.365
	R	0.908	0.908	0.787	0.787	0.848	0.727	0.606	0.848
	L	0.153	0.230	0.053	0.205	0.153	0.154	0.153	0.153
30	М	0.343	0.446	0.206	0.412	0.343	0.343	0.343	0.343
	R	0.771	0.826	0.551	0.826	0.771	0.771	0.771	0.771
	L	0.006	0.006	0.039	0.206	0.006	0.206	0.206	0.072
31	M	0.059	0.008	0.261	0.522	0.109	0.522	0.522	0.313
51	R	0.469	0.375	0.844	1.313	0.563	1.313	1.313	0.938
	L	0.409			1	1	0.254	1	
22			0.005	0.113	0.168	0.196		0.058	0.140
32	M	0.451	0.078	0.301	0.376	0.413	0.488	0.189	0.338
	R	0.902	0.361	0.722	0.842	0.902	0.842	0.541	0.782
	L	0.266	0.239	0.213	0.055	0.186	0.160	0.081	0.004
33	М	0.489	0.454	0.419	0.210	0.384	0.349	0.245	0.073
	R	0.863	0.863	0.809	0.540	0.809	0.755	0.593	0.324
	L	0.295	0.207	0.061	0.148	0.005	0.091	0.033	0.177
34	М	0.572	0.449	0.245	0.368	0.007	0.286	0.165	0.409
	R	1.066	1.000	0.667	0.867	0.267	0.733	0.533	0.933
	L	0.080	0.157	0.262	0.157	0.157	0.183	0.157	0.080
35	М	0.247	0.353	0.494	0.353	0.353	0.388	0.353	0.213
	R	0.623	0.793	0.907	0.793	0.793	0.793	0.793	0.567
	L	0.247	0.111	0.111	0.165	0.057	0.219	0.111	0.083
36	M	0.495	0.305	0.305	0.381	0.228	0.457	0.305	0.267
50	R	1.025	0.769	0.769	0.897	0.641	0.961	0.769	0.705
	L	0.005	0.070	0.038	0.237	0.041	0.901	0.709	0.038
27									
37	M	0.106	0.306	0.206	0.561	0.459	0.459	0.306	0.157
	R	0.568	0.947	0.757	1.326	1.231	1.231	0.947	0.663
	L	0.035	0.064	0.247	0.005	0.247	0.185	0.125	0.035
38	М	0.177	0.262	0.525	0.133	0.525	0.437	0.350	0.135
	R	0.585	0.731	1.096	0.512	1.096	1.023	0.877	0.512
	L	0.286	0.286	0.286	0.286	0.286	0.096	0.050	0.262
39	М	0.401	0.401	0.401	0.401	0.401	0.200	0.126	0.376
	R	0.545	0.545	0.545	0.545	0.545	0.409	0.307	0.545

Weighted normalized fuzzy ratings $(\widetilde{V_{kJ}}(x))$ and the Fuzzy PIS and NIS in Fuzzy TOPSIS method
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Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8	Fuzzy PIS	Fuzzy NIS
	L	0.008	0.004	0.001	0.005	0.008	0.007	0.002	0.006	0.007	0.001
1	М	0.014	0.009	0.004	0.010	0.014	0.013	0.006	0.012	0.013	0.004
	R	0.023	0.019	0.012	0.020	0.023	0.023	0.015	0.023	0.023	0.012
	L	0.004	0.001	0.001	0.000	0.001	0.004	0.002	0.003	0.004	0.000
2	М	0.009	0.003	0.004	0.003	0.005	0.009	0.006	0.008		0.003
	R	0.022	0.012	0.013	0.010	0.015	0.022	0.016	0.021		0.010
	L	0.002	0.001	0.003	0.002	0.004	0.004	0.004	0.002		0.001
3	M	0.006	0.004	0.007	0.006	0.009	0.010	0.009	0.007		0.004
	R	0.017	0.014	0.018	0.017	0.021	0.021	0.021	0.018		0.014
4	L M	0.006	0.005	0.004	0.005	0.004 0.009	0.006	0.001	0.004		0.001
4	R	0.011	0.010	0.008	0.010	0.009	0.010	0.004	0.009		0.004
	L	0.019	0.018	0.003	0.003	0.018	0.018	0.000	0.017		0.000
5	M	0.004	0.004	0.003	0.003	0.001	0.002	0.000	0.004		0.000
5	R	0.008	0.007	0.007	0.007	0.004	0.003	0.002	0.017		0.002
	L	0.002	0.005	0.003	0.001	0.001	0.003	0.002	0.003		0.000
6	M	0.005	0.009	0.006	0.002	0.003	0.006	0.005	0.006		0.002
	R	0.012	0.014	0.013	0.007	0.009	0.013	0.011	0.013	0.014	0.007
	L	0.004	0.007	0.002	0.004	0.005	0.006	0.001	0.005	0.007	0.001
7	М	0.009	0.012	0.006	0.009	0.011	0.011	0.005	0.011	0.012	0.005
	R	0.019	0.022	0.015	0.019	0.020	0.020	0.013	0.020	0.021	0.013
	L	0.004	0.007	0.004	0.006	0.007	0.002	0.004	0.003	0.007	0.002
8	М	0.010	0.013	0.010	0.011	0.012	0.007	0.010	0.008	0.013	0.007
	R	0.021	0.024	0.021	0.022	0.024	0.016	0.021	0.018	0.024	0.016
	L	0.005	0.005	0.007	0.002	0.003	0.005	0.001	0.005	0.007	0.001
9	М	0.011	0.011	0.014	0.006	0.008	0.011	0.005	0.011	0.014	0.005
	R	0.025	0.025	0.028	0.018	0.021	0.023	0.016	0.025	0.007 0.012 0.021 0.007 0.013 0.024 0.007 0.014 0.028 0.005 0.011 0.024 0.007 0.014 0.028 0.005 0.011 0.024 0.007 0.013 0.022 0.000 0.001 0.008 0.007 0.014 0.026 0.008 0.014 0.026 0.014 0.021	0.016
	L	0.005	0.005	0.004	0.001	0.000	0.002	0.001	0.003		0.000
10	М	0.011	0.010	0.010	0.003	0.001		0.004	0.009		0.001
	R	0.024	0.022	0.022	0.011	0.008		0.013	0.021		0.008
	L	0.007	0.001	0.004	0.001	0.002		0.001	0.005		0.001
11	M	0.013	0.004	0.009	0.003	0.006			0.011		0.003
	R	0.022	0.011	0.019	0.010	0.015			0.022		0.010
10	L M	0.000	0.000	0.002	0.004				0.004		0.006
12	R	0.001	0.003	0.007	0.010				0.010 0.021		0.012
		0.008	0.007	0.000	0.021				0.021		0.024
13	L M	0.008	0.007	0.000	0.000				0.002		0.000
15	R	0.013	0.014	0.003	0.002	0.012			0.000		0.002
	L	0.020	0.020	0.002	0.007	0.020		0.000	0.010		0.000
14	M	0.011	0.012	0.014	0.012	0.012	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0.003	0.005		0.003
	R	0.020	0.020	0.022	0.020	0.020		0.009	0.012		0.009
	L	0.007	0.006	0.002	0.005	0.009		0.002	0.002		0.002
15	М	0.013	0.012	0.005	0.010	0.015		0.004	0.006		0.004
	R	0.022	0.022	0.013	0.020	0.023		0.012	0.015	0.023	0.012
	L	0.003	0.005	0.006	0.007	0.005		0.001	0.004	0.007	0.001
16	М	0.008	0.010	0.012	0.013	0.010	0.013	0.003	0.009	0.013	0.003
	R	0.019	0.022	0.024	0.025	0.022	0.024	0.011	0.020	0.025	0.011
	L	0.006	0.004	0.004	0.006	0.005	0.003	0.001	0.002	0.006	0.001
17	М	0.012	0.009	0.009	0.012	0.011	0.007	0.005	0.006	0.012	0.004
	R	0.021	0.020	0.020	0.023	0.021	0.017	0.013	0.016	0.023	0.013
	L	0.002	0.008	0.007	0.000	0.006	0.005	0.001	0.002	0.008	0.000
18	М	0.006	0.015	0.013	0.004	0.012	0.010	0.004	0.005	0.015	0.004
	R	0.015	0.025	0.025	0.012	0.025	0.022	0.012	0.014	0.013 0.023 0.004 0.009 0.022 0.004 0.010 0.021 0.006 0.011 0.006 0.011 0.006 0.011 0.006 0.011 0.007 0.008 0.014 0.007 0.013 0.024 0.007 0.013 0.024 0.007 0.013 0.022 0.000 0.011 0.024 0.007 0.013 0.022 0.000 0.001 0.002 0.001 0.003 0.004 0.005 0.014 0.025 0.006 0.012 0.023 0.004 0.025 0.007 </td <td>0.012</td>	0.012
19	L	0.006	0.007	0.004	0.003	0.004	0.006	0.001	0.004		0.001
17	М	0.011	0.013	0.009	0.006	0.009	0.011	0.003	0.009	0.013	0.003

	R	0.020	0.020	0.018	0.014	0.018	0.019	0.010	0.018	0.020	0.010
	L	0.005	0.003	0.006	0.003	0.003	0.004	0.001	0.002	0.006	0.001
20	М	0.010	0.008	0.011	0.007	0.008	0.008	0.005	0.006	0.011	0.005
	R	0.022	0.018	0.021	0.017	0.018	0.018	0.014	0.015	0.021	0.014
	L	0.001	0.005	0.009	0.009	0.009	0.005	0.003	0.003	0.009	0.001
21	М	0.002	0.009	0.015	0.015	0.014	0.010	0.007	0.007	0.006 0.011 0.021	0.002
ľ	R	0.008	0.018	0.021	0.021	0.021	0.018	0.016	0.016		0.008
	L	0.008	0.006	0.006	0.007	0.006	0.006	0.000	0.004	0.008	0.000
22	М	0.012	0.010	0.011	0.012	0.011	0.010	0.002	0.008	0.012	0.002
	R	0.019	0.018	0.018	0.018	0.018	0.017	0.007	0.016	0.006 0.011 0.021 0.009 0.015 0.021 0.008 0.012 0.009 0.012 0.019 0.009 0.017 0.028 0.005 0.013 0.007 0.013 0.022 0.005 0.011 0.023 0.007 0.016 0.030 0.007 0.014 0.025 0.007 0.013 0.027 0.013 0.025 0.007 0.013 0.025 0.007 0.013 0.025 0.006 0.007 0.014 0.025 0.006 0.007 0.006 0.007 0.006 0.007 </td <td>0.007</td>	0.007
	L	0.005	0.000	0.009	0.004	0.000	0.005	0.002	0.003		0.000
23	M	0.003	0.002	0.017	0.010	0.003	0.011	0.002	0.009		0.002
25	R	0.025	0.002	0.017	0.010	0.003	0.011	0.000	0.00)		0.002
	L	0.023	0.001	0.028	0.023	0.012	0.023	0.010	0.021		0.000
24	M	0.004	0.003	0.002	0.002	0.001	0.000	0.002	0.003		0.000
24											
	R	0.027	0.029	0.021	0.021	0.017	0.015	0.023	0.031		0.015
	L	0.004	0.005	0.007	0.002	0.004	0.001	0.003	0.003		0.001
25	М	0.009	0.011	0.013	0.006	0.009	0.004	0.008	0.008		0.004
	R	0.020	0.021	0.022	0.015	0.020	0.013	0.018	0.018		0.013
	L	0.002	0.004	0.005	0.001	0.005	0.005	0.001	0.003	0.005	0.001
26	М	0.006	0.009	0.011	0.003	0.010	0.010	0.006	0.008		0.003
	R	0.017	0.021	0.023	0.011	0.023	0.021	0.015	0.020	0.023	0.011
	L	0.001	0.000	0.007	0.001	0.008	0.006	0.003	0.000	0.008	0.000
27	М	0.005	0.004	0.015	0.004	0.016	0.014	0.009	0.004	0.016	0.004
	R	0.016	0.014	0.030	0.014	0.030	0.030	0.022	0.014	0.030	0.014
	L	0.004	0.002	0.007	0.003	0.005	0.002	0.005	0.002	0.007	0.002
28	М	0.010	0.007	0.014	0.008	0.011	0.007	0.011	0.007		0.007
	R	0.023	0.018	0.025	0.020	0.025	0.018	0.025	0.018		0.018
	L	0.025	0.016	0.023	0.020	0.025	0.003	0.023	0.010		0.002
29	M	0.007	0.000	0.004	0.004	0.003	0.009	0.002	0.005		0.002
29	R	0.014	0.012	0.010	0.010	0.011	0.009	0.007	0.011		0.000
	L		0.027	0.023		0.023		0.018			
30		0.005			0.006		0.005		0.005		0.002
	M	0.010	0.013	0.006	0.012	0.010	0.010	0.010	0.010		0.006
	R	0.023	0.024	0.016	0.024	0.023	0.023	0.023	0.023		0.016
	L	0.000	0.000	0.001	0.005	0.000	0.005	0.005	0.002		0.000
31	М	0.001	0.000	0.006	0.013	0.003	0.013	0.013	0.008		0.000
	R	0.012	0.009	0.021	0.032	0.014	0.032	0.032	0.023		0.009
	L	0.007	0.000	0.003	0.005	0.006	0.008	0.002	0.004		0.000
32	М	0.013	0.002	0.009	0.011	0.012	0.014	0.006	0.010	0.014	0.002
	R	0.027	0.011	0.021	0.025	0.027	0.025	0.016	0.023	0.025	0.011
	L	0.006	0.006	0.005	0.001	0.004	0.004	0.002	0.000	0.006	0.000
33	М	0.011	0.011	0.010	0.005	0.009	0.008	0.006	0.002	0.011	0.002
ľ	R	0.020	0.020	0.019	0.013	0.019	0.018	0.014	0.008	0.020	0.008
	L	0.007	0.005	0.002	0.004	0.000	0.002	0.001	0.004	0.000	0.007
34	М	0.014	0.011	0.006	0.009	0.000	0.007	0.004	0.010		0.014
ľ	R	0.026	0.024	0.016	0.021	0.006	0.018	0.013	0.022		0.026
	L	0.002	0.005	0.008	0.005	0.005	0.006	0.005	0.002		0.002
35	M	0.002	0.003	0.000	0.003	0.003	0.012	0.003	0.002		0.002
55	R	0.008	0.011	0.013	0.011	0.011	0.012	0.011	0.007		0.000
	L	0.007	0.003	0.028	0.024	0.024	0.024	0.003	0.002		0.007
36	M	0.007	0.003	0.003	0.003	0.002	0.000	0.003	0.002		0.002
50			0.009			0.007		0.009	0.008		
	R	0.029		0.022	0.026		0.027				0.018
27	L	0.000	0.001	0.001	0.004	0.003	0.003	0.001	0.001		0.000
37	M	0.002	0.006	0.004	0.010	0.008	0.008	0.006	0.003		0.002
	R	0.010	0.017	0.014	0.024	0.022	0.022	0.017	0.012		0.010
	L	0.001	0.002	0.006	0.000	0.006	0.005	0.003	0.001		0.000
38	М	0.005	0.007	0.014	0.003	0.014	0.011	0.009	0.004	0.013	0.003
	R	0.015	0.019	0.028	0.013	0.028	0.026	0.023	0.013		0.013
	L	0.008	0.008	0.008	0.008	0.008	0.003	0.002	0.008	0.008	0.001
39	М	0.012	0.012	0.012	0.012	0.012	0.006	0.004	0.011		0.004
	R	0.016	0.016	0.016	0.016	0.016	0.012	0.009	0.016		0.009

Alternative	Fuzzy $\widetilde{S_k}^+$				Fuzzy $\widetilde{S_k}$		$D(\widetilde{S_k^+})$	$D(\widetilde{S_k})$	C_k^*
A1	(0.002	0.033	0.113)	(0.001	0.042	0.121)	0.049	0.055	0.527
A2	(0.000	0.035	0.118)	(0.000	0.039	0.118)	0.051	0.052	0.507
A3	(0.000	0.028	0.109)	(0.002	0.043	0.124)	0.046	0.056	0.553
A4	(0.000	0.038	0.122)	(0.002	0.035	0.111)	0.053	0.049	0.479
A5	(0.000	0.030	0.121)	(0.001	0.042	0.122)	0.050	0.055	0.521
A6	(0.000	0.026	0.119)	(0.000	0.042	0.124)	0.048	0.055	0.534
A7	(0.001	0.045	0.135)	(0.000	0.024	0.095)	0.060	0.040	0.397
A8	(0.000	0.038	0.129)	(0.000	0.030	0.110)	0.056	0.047	0.455

Calculated fuzzy $\widetilde{S_k}^+$ \cdot $\widetilde{S_k}^-$ and $\widetilde{C_k}^*$ in Fuzzy TOPSIS method

Criteria	Triangular Fuzzy No.	Fuzzy $\widetilde{f}_{\iota}^{*}$	Fuzzy $\widetilde{f}_{\iota}^{0}$	A1	A2	A3	A4	A5	A6	A7	A8
	L	6.250	0.700	-0.403	-0.202	0.134	-0.269	-0.403	-0.403	0.000	-0.403
1	М	8.750	2.525	0.000	0.336	0.669	0.269	0.000	0.067	0.538	0.134
	R	10.000	5.000	0.403	0.739	1.000	0.672	0.403	0.470	0.935	0.538
	L	4.375	0.100	-0.539	-0.067	-0.135	0.000	-0.202			-0.472
2	М	6.875	1.900	0.000	0.469	0.402	0.536	0.337	0.000	0.270	0.067
	R	9.375	4.375	0.539	0.935	0.868	1.000	0.871	0.539	0.000 0.538 0.935 -0.270 0.270 0.803 -0.589 0.084 0.673 0.067 0.605 1.000 0.126 0.629 1.000 -0.589 0.605 1.000 0.126 0.629 1.000 -0.677 0.470 0.806 0.000 0.575 1.000 0.538 1.000 0.538 1.000 0.534 0.672 1.000 0.534 0.672 1.000 0.255 0.750 0.874 -0.135 0.404 0.803 0.253 0.755 1.000 0.134 0.667 1.	0.606
	L	4.375	1.325	-0.337	-0.168	-0.421	-0.337	-0.589	-0.403 0.000 0.067 0.538 0.470 0.935 -0.539 -0.270 0.000 0.270 0.539 0.803 -0.539 0.803 -0.589 -0.589 0.000 0.084 0.589 0.673 -0.336 0.067 0.470 1.000 -0.189 0.126 0.316 0.629 0.758 1.000 -0.336 0.470 0.316 0.629 0.758 1.000 -0.202 -0.067 0.336 0.470 0.669 0.806 -0.359 0.000 0.072 0.575 0.503 1.000 -0.077 -0.309 0.540 0.309 1.000 0.772 -0.269 0.000 0.202 0.538 0.669 1.000 -0.202 0.134	-0.421	
2 3 4 5 6 7 8 9 10 11 12 13	М	6.875	3.150	0.337	0.502	0.249	0.337	0.084	0.000	0.084	0.253
	R	8.750	5.625	0.923	1.000	0.754	0.923	0.673	0.589	0.673	0.838
	L	6.250	0.700	-0.403	-0.336	-0.269	-0.336	-0.336		0.067	-0.269
4	М	8.750	3.125	0.000	0.134	0.269	0.134	0.202	0.067	0.605	0.202
	R	10.000	5.625	0.403	0.538	0.672	0.538	0.605	0.470	1.000	0.605
	L	5.625	0.100	-0.379	-0.442	-0.316	-0.316	-0.063	-0.189	0.126	-0.442
5	М	8.125	1.900	0.063	0.000	0.189	0.189	0.442	0.316	0.629	0.063
	R	10.000	4.375	0.505	0.442	0.631	0.631	0.879	0.758	1.000	0.505
	L	6.875	0.700	-0.134	-0.336	-0.269	0.202	0.067	-0.202	-0.067	-0.269
6	М	9.375	2.525	0.403	0.000	0.269	0.737	0.602		0.470	0.269
	R	10.000	5.000	0.739	0.336	0.605	1.000	0.871	0.669	0.806	0.605
	L	6.250	1.300	-0.287	-0.431	-0.072	-0.287	-0.359	-0.359	0.000	-0.359
7	М	8.750	3.750	0.287	0.000	0.503	0.287	0.144	0.072	0.575	0.144
	R	10.000	6.250	0.718	0.431	0.931	0.718	0.575	0.503	1.000	0.575
	L	6.250	1.900	-0.309	-0.463	-0.309	-0.386	-0.463	-0.077	-0.309	-0.154
8	М	8.750	4.375	0.309	0.000	0.309	0.154	0.077	0.540	0.309	0.463
1	R	10.000	6.875	0.772	0.463	0.772	0.617	0.540	1.000	0.000 0.538 0.935 -0.270 0.270 0.270 0.270 0.803 -0.589 0.084 0.673 0.067 0.605 1.000 0.126 0.629 1.000 -0.667 0.470 0.806 0.000 0.575 1.000 -0.309 0.309 0.772 0.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 0.134 0.672 1.000 0.255 0.750 0.874 -0.135 0.404 0.803 0.253 0.	0.923
	L	5.625	0.700	-0.336	-0.336	-0.470	-0.067	-0.202			-0.336
9	М	8.125	3.125	0.202	0.202	0.000	0.470	0.336		0.000 0.538 0.935 -0.270 0.270 0.270 0.803 -0.589 0.084 0.673 0.067 0.605 1.000 0.126 0.629 1.000 -0.575 1.000 0.575 1.000 0.575 1.000 0.575 1.000 0.575 1.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 1.000 0.538 0.134 0.868 0.134 0.867 1.000 0.253 0.755 1.000 0.216 0.752 1.000 0.13	0.202
l I	R	10.000	5.625	0.672	0.672	0.470	0.935	0.804			0.672
	L	5.000	0.100	-0.472	-0.404	-0.404	0.067	0.202			-0.337
10	М	7.500	0.700	0.000	0.067	0.135	0.601	0.733	0.337	0.534	0.202
	R	9.375	3.125	0.472	0.539	0.606	0.935	1.000			0.671
	L	6.875	0.700	-0.336	0.202	-0.202	0.269	0.000			-0.336
11	М	9.375	1.925	0.000	0.737	0.336	0.801	0.538	0.202	0.672	0.202
	R	10.000	4.375	0.336	1.000	0.672	1.000	0.871			0.538
	L	0.100	5.625	-0.306	-0.306	-0.124	0.063	0.253		0.255	0.063
12	М	0.700	8.125	0.000	0.121	0.371	0.561	0.750	0.245		0.561
	R	3.125	10.000	0.306	0.432	0.684	0.874	1.000			0.874
	L	5.625	0.100	-0.404	-0.404	0.135	0.202	-0.404			0.000
13	М	8.125	1.300	0.067	0.000	0.671	0.736	0.135			0.536
	R	9.375	3.750	0.472	0.404	1.000	1.000	0.539		0.803	0.871
	L	6.875	0.100	-0.253	-0.253	-0.316	-0.253	-0.253			0.126
14	M	9.375	1.900	0.189	0.126	0.000	0.126	0.126			0.631
	R	10.000	4.375	0.505	0.442	0.316	0.442	0.442			0.939
	L	6.875	1.300	-0.287	-0.287	0.144	-0.216	-0.359			0.072
15	M	9.375	2.550	0.144	0.216	0.716	0.359	0.000			0.647
	R	10.000	5.000	0.503	0.575	1.000	0.718	0.359			1.000
	L	5.625	0.700	-0.202	-0.336	-0.403	-0.470	-0.336			-0.269
16	M	8.125	1.925	0.336	0.202	0.067	0.000	0.202			0.269
-	R	10.000	4.375	0.804	0.672	0.538	0.470	0.672			0.739
	L	5.625	1.300	-0.431	-0.359	-0.359	-0.503	-0.431			-0.144
4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	M	8.125	3.150	0.000	0.216	0.216	0.000	0.072			0.431
	R	10.000	5.625	0.503	0.718	0.718	0.503	0.575			0.931
	L	6.875	0.100	0.063	-0.316	-0.316	0.189	-0.316			0.126
18	M	9.375	2.500	0.568	0.000	0.126	0.694	0.189			0.629
-	R	10.000	5.000	0.879	0.316	0.442	1.000	0.505			0.879
	L	6.875	0.700	-0.336	-0.336	-0.202	0.000	-0.202			-0.202
19							0.535		0.207	0.202	

Calculated \widetilde{f}_{i}^{0} , \widetilde{f}_{i}^{*} and \widetilde{d}_{ij} in Fuzzy VIKOR method

1	R	10.000	5.000	0.470	0.336	0.672	0.804	0.672	0.470	1.000	0.672
	L	5.625	1.325	-0.543	-0.311	-0.466	-0.233	-0.311	-0.311	-0.078	-0.155
20	М	8.125	3.750	0.078	0.311	0.000	0.388	0.311			0.466
-	R	9.375	6.250	0.543	0.776	0.466	0.851	0.773			0.929
	L	7.500	0.700	0.403	-0.134	-0.269	-0.269	-0.269			0.000
21	М	10.000	1.325	0.933	0.403	0.000	0.000	0.067			0.538
F	R	10.000	3.750	1.000	0.672	0.269	0.269	0.336	0.605	1.000 -0.078 0.543 1.000 0.000 0.538 0.804 0.316 0.816 1.000 0.126 0.629 0.876 -0.270 0.270 0.270 0.803 -0.144 0.431 0.790 -0.144 0.431 0.790 -0.144 0.432 0.931 -0.135 0.404 0.806 -0.418 0.251 0.669 -0.077 0.542 1.000 -0.578 0.669 -0.077 0.542 1.000 -0.578 0.003 0.575 0.861 -0.232 0.697 -0.578 0.003 0.575 0.861 -0.063 0.575 0.861 -0.245 0.495 -0.359 0.359 0.359 -0.214 0.359 0.359 -0.214 0.359 0.359 -0.214 0.359 0.221 0.359 0.359 -0.214 0.359 0.221 0.359 0.359 -0.214 0.361 0.359	0.804
22 23 24 25	L	6.875	0.100	-0.316	-0.253	-0.253	-0.253	-0.253		0.316	-0.126
22	М	9.375	1.300	0.000	0.189	0.126	0.063	0.126			0.379
F	R	10.000	3.750	0.316	0.505	0.442	0.379	0.442			0.692
	L	6.875	0.100	-0.189	0.316	-0.316	-0.126	0.253	1 -0.311 -0.078 1 0.233 0.543 3 0.696 1.000 9 -0.134 0.000 7 0.336 0.538 5 0.605 0.804 3 -0.189 0.316 5 0.189 0.816 2 0.505 1.000 3 -0.189 0.126 5 0.316 0.629 0 0.631 0.876 7 0.000 -0.270 0 0.536 0.270 5 1.000 0.803 6 0.144 -0.144 0 0.716 0.431 3 1.000 0.790 4 -0.432 -0.144 0 0.773 0.931 4 -0.404 -0.135 0 0.135 0.404 4 0.539 0.806	-0.063	
23	М	9.375	1.300	0.316	0.816	0.000	0.379	0.755			0.442
F	R	10.000	3.750	0.631	1.000	0.316	0.692	1.000	0.631	0.876	0.755
	L	4.375	0.100	-0.404	-0.472	-0.202	-0.202	-0.067			-0.539
24	М	6.875	1.900	0.135	0.067	0.337	0.334	0.469			0.000
F	R	9.375	4.375	0.671	0.606	0.871	0.806	0.935		0.803	0.539
	L	6.875	1.300	-0.216	-0.287	-0.359	0.000	-0.216	0.144	-0.144	-0.144
25	М	9.375	3.150	0.359	0.216	0.000	0.575	0.359	0.716	0.431	0.431
F	R	10.000	5.625	0.716	0.575	0.359	0.931	0.718			0.790
	L	5.000	0.700	-0.216	-0.432	-0.504	0.072	-0.504			-0.360
26	М	7.500	1.925	0.360	0.144	0.000	0.643	0.072			0.216
F	R	9.375	4.375	0.862	0.648	0.504	1.000	0.576			0.720
	L	5.625	0.100	0.067	0.135	-0.404	0.135	-0.404			0.135
27	М	8.125	1.900	0.604	0.671	0.067	0.668	0.000			0.671
F	R	9.375	4.375	0.935	1.000	0.472	0.935	0.404	0.539	0.806	1.000
	L	6.250	1.900	-0.334	-0.084	-0.418	-0.167	-0.418	-0.084	-0.418	-0.084
28	М	8.750	4.375	0.334	0.585	0.000	0.502	0.251	0.585	0.251	0.585
	R	9.375	6.875	0.753	1.000	0.418	0.916	0.669	1.000	0.669	1.000
	L	5.625	1.300	-0.464	-0.464	-0.310	-0.310	-0.387	-0.232	-0.077	-0.387
29	М	8.125	3.750	0.000	0.155	0.310	0.310	0.232	0.387	0.542	0.232
F	R	9.375	6.250	0.464	0.619	0.774	0.771	0.697	0.851	1.000	0.697
	L	5.625	1.300	-0.387	-0.464	-0.077	-0.464	-0.387	-0.387	-0.387	-0.387
30	М	8.125	3.750	0.232	0.000	0.542	0.077	0.232	0.232	0.232	0.232
F	R	9.375	6.250	0.697	0.464	1.000	0.542	0.697	0.693	0.697	0.697
	L	3.750	0.100	0.072	0.145	-0.217	-0.578	0.000	-0.578	-0.578	-0.289
31	М	6.250	0.100	0.642	0.711	0.361	0.000	0.572	0.000	0.000	0.289
E F	R	8.750	2.500	1.000	1.000	0.931	0.578	1.000	0.578	0.578	0.861
	L	5.650	0.100	-0.431	0.220	-0.214	-0.358	-0.431	-0.358	0.003	-0.286
32	М	8.125	1.300	0.072	0.789	0.361	0.217	0.145	0.000	0.575	0.289
E F	R	8.750	3.750	0.434	1.000	0.720	0.578	0.506	0.358	0.861	0.650
	L	6.250	0.100	-0.379	-0.379	-0.316	0.000	-0.316	-0.253	-0.063	0.253
33	М	8.750	1.300	0.000	0.063	0.126	0.505	0.189	0.253	0.442	0.753
	R	10.000	3.750	0.379	0.442	0.505	0.879	0.568	0.631	0.818	1.000
	L	0.100	6.250	0.379	0.189	-0.121	0.063	-0.242	-0.058	-0.182	0.126
34	М	0.100	8.750	0.874	0.684	0.369	0.558	0.000	0.432	0.245	0.621
	R	2.500	10.000	1.000	0.937	0.621	0.811	0.242	0.684	0.495	0.874
	L	6.250	1.900	-0.077	-0.309	-0.463	-0.309	-0.309	-0.309	-0.309	0.000
35	М	8.750	3.775	0.540	0.309	0.000	0.309	0.309	0.231	0.309	0.614
「	R	10.000	6.250	1.000	0.772	0.463	0.772	0.772	0.694	0.772	1.000
	L	5.625	1.300	-0.503	-0.216	-0.216	-0.359	-0.072		-0.216	-0.144
36	М	8.125	3.750	0.000	0.359	0.359	0.216	0.503	0.072	0.359	0.431
	R	10.000	6.250	0.503	0.859	0.859	0.718	1.000	0.575	0.859	0.931
	L	4.400	0.100	0.075	-0.214	-0.069	-0.503	-0.431	-0.431	-0.214	0.003
37	М	6.875	1.300	0.645	0.361	0.503	0.000	0.145	0.145	0.361	0.572
「	R	8.750	3.750	1.000	0.861	0.931	0.503	0.647	0.647	0.861	0.931
	L	5.000	0.100	0.000	-0.135	-0.472	0.067	-0.472	-0.404	-0.270	0.067
38	М	7.500	1.900	0.536	0.404	0.000	0.604	0.000	0.135	0.270	0.601
「	R	9.375	4.375	0.935	0.871	0.472	1.000	0.472	0.606	0.739	0.935
	L	7.500	1.300	-0.287	-0.287	-0.287	-0.287	-0.287	0.000	0.216	-0.287
39	М	10.000	3.150	0.000	0.000	0.000	0.000	0.000	0.575	0.787	0.072
	R	10.000	5.625	0.287	0.287	0.287	0.287	0.287	0.859		0.359

Alternative					$\widetilde{\mathbf{R}_{j}}$			$\widetilde{Q_{I}}$			
A1	-0.2505	0.2493	0.6403	0.0121	0.028	0.0303	-0.6745	0.195	0.8986		
A2	-0.2298	0.2727	0.6628	0.0086	0.0232	0.0295	-0.7298	0.117	0.8923		
A3	-0.2586	0.2358	0.6371	0.0041	0.0206	0.0295	-0.8268	0.051	0.881		
A4	-0.176	0.3448	0.7315	0.0063	0.0194	0.0272	-0.7492	0.0767	0.8805		
A5	-0.2627	0.2459	0.6435	0.0069	0.0207	0.0284	-0.7766	0.0571	0.8643		
A6	-0.2618	0.2489	0.657	0.0036	0.0179	0.0288	-0.8385	0.0058	0.8768		
A7	-0.0615	0.4941	0.8701	0.0088	0.0232	0.0296	-0.6524	0.2139	0.987		
A8	-0.1655	0.3823	0.7765	0.0059	0.0192	0.0303	-0.7525	0.0896	0.9587		

Calculated amounts of $\widetilde{S_j} \cdot \widetilde{R_j}$ $_{2} \widetilde{Q_j}$ for each alternative in Fuzzy VIKOR method

Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8
	L	0.625	0.313	0.07	0.375	0.625	0.563	0.13	0.5
1	М	0.875	0.563	0.253	0.625	0.875	0.813	0.375	0.75
	R	1	0.813	0.5	0.875	1	1	0.625	1
	L	0.467	0.075	0.141	0.011	0.139	0.467	0.205	0.4
2	М	0.733	0.269	0.336	0.203	0.4	0.733	0.467	0.667
	R	1	0.533	0.6	0.467	0.667	1	0.733	0.933
	L	0.217	0.151	0.36	0.217	0.429	0.5	0.429	0.289
3	М	0.5	0.36	0.574	0.5	0.714	0.786	0.714	0.571
	R	0.786	0.643	0.857	0.786	1	1	1	0.857
4	L	0.625	0.5	0.375	0.5	0.438	0.563	0.07	0.438
4	M R	0.875	0.75 0.938	0.625 0.875	0.75 0.938	0.688 0.938	0.813 0.938	0.313 0.563	0.688
		0.5		0.875	0.938		0.938	0.365	
5	L M	0.3	0.563 0.813	0.625	0.625	0.13 0.375	0.25	0.01	0.5
5	R	0.938	1	0.875	0.825	0.625	0.3	0.19	1
	L	0.313	0.688	0.438	0.875	0.023	0.75	0.438	0.438
6	M	0.563	0.088	0.438	0.07	0.378	0.625	0.23	0.438
0	R	0.813	1	0.938	0.233	0.625	0.875	0.75	0.088
	L	0.375	0.625	0.938	0.375	0.025	0.873	0.13	0.938
7	M	0.625	0.875	0.19	0.625	0.75	0.813	0.13	0.75
,	R	0.875	1	0.688	0.875	0.938	0.938	0.625	0.938
	L	0.375	0.625	0.375	0.5	0.563	0.19	0.375	0.253
8	M	0.625	0.875	0.625	0.75	0.813	0.438	0.625	0.255
Ū	R	0.875	1	0.875	0.938	1	0.688	0.875	0.75
	L	0.375	0.375	0.563	0.13	0.253	0.378	0.07	0.375
9	M	0.625	0.625	0.813	0.375	0.5	0.625	0.313	0.625
-	R	0.875	0.875	1	0.625	0.75	0.813	0.563	0.875
	L	0.533	0.467	0.4	0.075	0.011	0.203	0.141	0.336
10	М	0.8	0.733	0.667	0.205	0.075	0.467	0.272	0.6
	R	1	0.933	0.933	0.467	0.333	0.733	0.533	0.867
	L	0.688	0.07	0.375	0.07	0.19	0.503	0.07	0.5
11	М	0.938	0.253	0.625	0.193	0.438	0.75	0.313	0.75
	R	1	0.5	0.875	0.438	0.688	0.875	0.563	1
	L	0.032	0.023	0.015	0.011	0.01	0.018	0.011	0.011
12	М	0.143	0.053	0.023	0.016	0.012	0.032	0.012	0.016
	R	1	1	0.053	0.027	0.018	0.143	0.018	0.027
	L	0.533	0.6	0.011	0.011	0.467	0.269	0.205	0.139
13	М	0.8	0.867	0.203	0.139	0.733	0.533	0.467	0.336
	R	1	1	0.467	0.4	1	0.8	0.733	0.6
	L	0.5	0.563	0.688	0.563	0.563	0.315	0.01	0.07
14	М	0.75	0.813	0.938	0.813	0.813	0.563	0.19	0.313
	R	0.938	0.938	1	0.938	0.938	0.813	0.438	0.563
	L	0.563	0.5	0.13	0.375	0.688	0.563	0.13	0.13
15	M	0.813	0.75	0.315	0.625	0.938	0.813	0.255	0.375
	R	0.938	0.938	0.563	0.875	1	0.938	0.5	0.625
16	L	0.253	0.375	0.5	0.563	0.375	0.563	0.07	0.313
16	M	0.5	0.625	0.75	0.813	0.625	0.813	0.193	0.563
	R	0.75	0.875	0.938	1	0.875	0.938	0.438	0.813
17	L M	0.563 0.813	0.375 0.625	0.375 0.625	0.563 0.813	0.5 0.75	0.253 0.5	0.13 0.315	0.19
1/	R	0.938	0.825	0.825	0.815	0.75	0.5	0.563	0.438
	L	0.938	0.688	0.873	0.01	0.938	0.75	0.365	0.088
18	M	0.13	0.088	0.813	0.01	0.3	0.575	0.07	0.13
10	R	0.625	0.938	1	0.23	0.73	0.825	0.235	0.563
	L K	0.625	0.688	0.375	0.5	0.375	0.875	0.5	0.363
19	L M	0.565	0.088	0.375	0.255	0.375	0.565	0.07	0.375
17	R	1	1	0.875	0.44	0.875	0.813	0.235	0.823
	L K	0.5	0.313	0.873	0.088	0.875	0.938	0.133	0.873
20	M	0.75	0.563	0.813	0.233	0.563	0.378	0.135	0.19
411	141	0.75	0.505	0.015	0.5	0.505	0.025	0.375	0.450

Fuzzy normalized decision matrix ($\tilde{r}_{ij}(x)$) in Fuzzy Group ELECTRE method

	L	0.07	0.375	0.75	0.75	0.688	0.438	0.253	0.253
21	M	0.133	0.625	1	1	0.938	0.688	0.255	0.255
21	R	0.375	0.875	1	1	1	0.875	0.75	0.75
	L	0.688	0.5	0.563	0.625	0.563	0.5	0.01	0.315
22	M	0.938	0.75	0.813	0.875	0.813	0.75	0.13	0.513
22	R	1	0.938	0.938	0.938	0.938	0.875	0.375	0.813
	L	0.375	0.938	0.688	0.315	0.938	0.375	0.133	0.813
23	M	0.625	0.01	0.038	0.563	0.19	0.625	0.315	0.255
23	R	0.875	0.375	1	0.813	0.438	0.875	0.563	0.75
	L	0.336	0.375	0.139	0.203	0.075	0.011	0.205	0.467
24	M	0.6	0.667	0.135	0.403	0.269	0.203	0.467	0.733
24	R	0.867	0.933	0.667	0.667	0.533	0.467	0.733	1
	L	0.378	0.5	0.688	0.19	0.375	0.13	0.313	0.313
25	M	0.625	0.75	0.038	0.438	0.625	0.315	0.563	0.563
23	R	0.875	0.938	0.938	0.688	0.875	0.563	0.813	0.303
	L	0.873	0.938	0.533	0.088	0.467	0.363	0.139	0.333
26	M	0.203	0.4	0.335	0.205	0.733	0.469	0.139	0.555
20	R	0.467	0.007	1	0.203	1	0.733	0.4	0.867
	L K	0.733	0.933	0.533	0.467	0.6	0.933	0.007	0.867
27	M L	0.075	0.011	0.535	0.075		0.467	0.203	0.011
27	R	0.269	0.203	0.8		0.867	0.733	0.467	
	L K	0.535	0.467	0.667	0.467 0.269	0.467	0.203	0.733	0.467 0.203
28	M	0.4	0.203	0.007	0.269	0.467	0.203	0.467	0.203
20	R	0.007	0.467	0.955	0.335	0.755	0.467	0.755	0.467
	L	0.933	0.467	0.333	0.336	0.4	0.733	0.139	0.733
29	M			0.555			0.207		
29	R	0.867	0.733	0.867	0.6 0.867	0.667	0.335	0.4	0.667
	L	1 0.4		0.887		0.933	0.8	0.667	0.933
20	M		0.6	0.139	0.533		0.403	0.4	
30	R	0.667 0.933	0.867		0.8	0.667 0.933	0.007		0.667 0.933
	L	0.935	0.011	0.667 0.08	0.429	0.935	0.933	0.933 0.429	0.933
21	M		0.011	0.08	0.429		0.429	0.429	0.149
31	R	0.08 0.357	0.011	0.643	1	0.149 0.429	0.714	1	0.429
	L	0.537	0.286	0.043	0.4		0.603	0.139	0.714
32	M		0.139	0.209		0.467 0.733	0.867		0.555
52	R	0.8	0.139	0.335	0.667 0.933	0.755	0.867	0.336	0.867
		1		0.8		-	0.935		
22	L M	0.625	0.563	0.3	0.13	0.438		0.19	0.01
33		0.875	0.813	0.75	0.375	0.688	0.625	0.438	0.13
	R L	-	0.011	0.938	0.625 0.012	0.938 0.04	0.875 0.015	0.688	0.375
34	M L	0.01 0.011	0.011	0.016	0.012	0.04	0.015	0.02	0.011
54	R	0.011	0.013	0.027	0.018	1	0.023	0.04	0.016 0.027
	L K	0.016	0.023	0.625	0.032	0.375	0.052	0.143	0.027
35	M L	0.19	0.375	0.825	0.375	0.375	0.438	0.375	0.19
55	R		0.625			0.625			
	L K	0.688 0.563	0.875	1 0.253	0.875 0.375	0.875	0.875	0.875 0.253	0.625 0.19
36	M L	0.563	0.253	0.255	0.375	0.13	0.5	0.255	0.19
50	R	1	0.3	0.3	0.825	0.625	0.73	0.3	0.438
	L K	0.011	0.75	0.75	0.875	0.625	0.938	0.75	0.088
37	M L	0.011	0.149	0.08	0.503		0.36	0.149	0.08
57	R	0.149	0.429	0.289	0.786	0.643 0.929	0.643	0.429	0.22
	L K								
20		0.075	0.139	0.533	0.011	0.533	0.4	0.269	0.075
38	M D	0.269	0.4	0.8	0.203	0.8	0.667	0.533	0.205
	R	0.533	0.667	1	0.467	1	0.933	0.8	0.467
20	L	0.75	0.75	0.75	0.75	0.75	0.253	0.13	0.688
39	<u>M</u>	1	1	1	1	1	0.5	0.315	0.938
	R	1	1	1	1	1	0.75	0.563	1

L

М

R

L

Μ

R

19

20

0.0145

0.021 0.0258

0.0113

0.0169

0.0225

0.0178

0.0242 0.0258

0.007

0.0127

0.0183

Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8
	Ĺ	0.0184	0.0092	0.0021	0.011	0.0184	0.0166	0.0038	0.0147
1	М	0.0258	0.0166	0.0074	0.0184	0.0258	0.0239	0.011	0.0221
	R	0.0295	0.0239	0.0147	0.0258	0.0295	0.0295	0.0184	0.0295
	L	0.0086	0.0014	0.0026	0.0002	0.0025	0.0086	0.0038	0.0073
2	М	0.0135	0.0049	0.0062	0.0037	0.0073	0.0135	0.0086	0.0122
	R	0.0184	0.0098	0.011	0.0086	0.0122	0.0184	0.0135	0.0171
	L	0.0045	0.0031	0.0075	0.0045	0.0089	0.0104	0.0089	0.006
3	М	0.0104	0.0075	0.0119	0.0104	0.0149	0.0163	0.0149	0.0119
	R	0.0163	0.0134	0.0178	0.0163	0.0208	0.0208	0.0208	0.0178
	L	0.0159	0.0127	0.0096	0.0127	0.0111	0.0143	0.0018	0.0111
4	М	0.0223	0.0191	0.0159	0.0191	0.0175	0.0207	0.008	0.0175
	R	0.0255	0.0239	0.0223	0.0239	0.0239	0.0239	0.0143	0.0223
	L	0.0092	0.0103	0.0069	0.0069	0.0024	0.0046	0.0002	0.0092
5	М	0.0138	0.0149	0.0115	0.0115	0.0069	0.0092	0.0035	0.0138
	R	0.0172	0.0184	0.0161	0.0161	0.0115	0.0138	0.008	0.0184
	L	0.0049	0.0107	0.0068	0.0011	0.003	0.0059	0.0039	0.0068
6	М	0.0088	0.0146	0.0107	0.0039	0.0059	0.0097	0.0078	0.0107
	R	0.0127	0.0156	0.0146	0.0078	0.0097	0.0136	0.0117	0.0146
	L	0.01	0.0167	0.0051	0.01	0.0133	0.015	0.0035	0.0133
7	М	0.0167	0.0233	0.0117	0.0167	0.02	0.0217	0.01	0.02
	R	0.0233	0.0267	0.0183	0.0233	0.025	0.025	0.0167	0.025
	L	0.0108	0.018	0.0108	0.0144	0.0162	0.0055	0.0108	0.0073
8	М	0.018	0.0252	0.018	0.0216	0.0234	0.0126	0.018	0.0144
	R	0.0252	0.0288	0.0252	0.027	0.0288	0.0198	0.0252	0.0216
	L	0.0104	0.0104	0.0156	0.0036	0.007	0.0105	0.0019	0.0104
9	М	0.0173	0.0173	0.0225	0.0104	0.0139	0.0173	0.0087	0.0173
-	R	0.0243	0.0243	0.0277	0.0173	0.0208	0.0225	0.0156	0.0243
	L	0.0116	0.0101	0.0087	0.0016	0.0002	0.0044	0.0031	0.0073
10	М	0.0173	0.0159	0.0144	0.0044	0.0016	0.0101	0.0059	0.013
	R	0.0217	0.0202	0.0202	0.0101	0.0072	0.0159	0.0116	0.0188
	L	0.0162	0.0016	0.0088	0.0016	0.0045	0.0118	0.0016	0.0118
11	М	0.0221	0.006	0.0147	0.0045	0.0103	0.0177	0.0074	0.0177
	R	0.0236	0.0118	0.0206	0.0103	0.0162	0.0206	0.0133	0.0236
	L	0.0008	0.0005	0.0003	0.0003	0.0002	0.0004	0.0003	0.0003
12	М	0.0034	0.0013	0.0005	0.0004	0.0003	0.0008	0.0003	0.0004
	R	0.0239	0.0239	0.0013	0.0006	0.0004	0.0034	0.0004	0.0006
	L	0.014	0.0158	0.0003	0.0003	0.0123	0.0071	0.0054	0.0037
13	М	0.0211	0.0228	0.0053	0.0037	0.0193	0.014	0.0123	0.0088
	R	0.0263	0.0263	0.0123	0.0105	0.0263	0.0211	0.0193	0.0158
	L	0.0143	0.0161	0.0197	0.0161	0.0161	0.009	0.0003	0.002
14	М	0.0214	0.0232	0.0268	0.0232	0.0232	0.0161	0.0054	0.0089
	R	0.0268	0.0268	0.0286	0.0268	0.0268	0.0232	0.0125	0.0161
	L	0.0162	0.0144	0.0037	0.0108	0.0198	0.0162	0.0037	0.0037
15	М	0.0234	0.0216	0.0091	0.018	0.027	0.0234	0.0073	0.0108
<u> </u>	R	0.027	0.027	0.0162	0.0252	0.0288	0.027	0.0144	0.018
	L	0.0073	0.0109	0.0145	0.0163	0.0109	0.0163	0.002	0.009
16	М	0.0145	0.0181	0.0217	0.0235	0.0181	0.0235	0.0056	0.0163
	R	0.0217	0.0253	0.0271	0.0289	0.0253	0.0271	0.0127	0.0235
	L	0.0143	0.0096	0.0096	0.0143	0.0127	0.0064	0.0033	0.0048
17	М	0.0207	0.0159	0.0159	0.0207	0.0191	0.0127	0.008	0.0111
	R	0.0239	0.0223	0.0223	0.0255	0.0239	0.0191	0.0143	0.0175
	L	0.0035	0.0187	0.0153	0.0003	0.0136	0.0102	0.0019	0.0035
18	М	0.0102	0.0255	0.0221	0.0068	0.0204	0.017	0.0069	0.0086
	R	0.017	0.0272	0.0272	0.0136	0.0272	0.0238	0.0136	0.0153
	T	0.0145	0.0179	0.0007	0.0065	0.0007	0.0145	0.0019	0.0007

Weighted normalized fuzzy decision matrix ($\tilde{v}_{ij}(x)$) in Fuzzy Group ELECTRE method

0.0065

0.0114

0.0178

0.0057

0.0113

0.0169

0.0097

0.0161

0.0226

0.0071

0.0127

0.0183

0.0145

0.021

0.0242

0.0085

0.0141

0.0183

0.0018

0.0065 0.0129

0.003

0.0084

0.0141

0.0097

0.0161

0.0226

0.0043

0.0099

0.0155

0.0097

0.0161

0.0226

0.0127

0.0183

0.0211

	L	0.0021	0.0112	0.0225	0.0225	0.0206	0.0131	0.0076	0.0076
21	M	0.0021	0.0112	0.0225	0.0225	0.0281	0.0131	0.015	0.015
21	R	0.0112	0.0262	0.03	0.03	0.0201	0.0260	0.0225	0.0225
	L	0.0192	0.0139	0.0157	0.0174	0.0157	0.0202	0.0003	0.0088
22	M	0.0262	0.0209	0.0227	0.0244	0.0227	0.0209	0.0036	0.0157
	R	0.0279	0.0262	0.0262	0.0262	0.0262	0.0244	0.0105	0.0227
	L	0.0103	0.0003	0.0188	0.0086	0.0003	0.0103	0.0036	0.0069
23	M	0.0171	0.0036	0.0257	0.0000	0.0052	0.0103	0.0086	0.0137
23	R	0.024	0.0103	0.0274	0.0222	0.012	0.024	0.0154	0.0205
	L	0.0082	0.0097	0.0034	0.0049	0.0012	0.0003	0.005	0.0113
24	M	0.0146	0.0162	0.0097	0.0098	0.0065	0.0049	0.0113	0.0178
	R	0.021	0.0226	0.0162	0.0162	0.0129	0.0113	0.0178	0.0243
	L	0.0094	0.0125	0.0172	0.0047	0.0094	0.0032	0.0078	0.0078
25	M	0.0156	0.0187	0.0234	0.0109	0.0156	0.0079	0.014	0.014
	R	0.0218	0.0234	0.025	0.0172	0.0218	0.014	0.0203	0.0203
	L	0.0047	0.0094	0.0125	0.0017	0.0109	0.011	0.0032	0.0078
26	M	0.0109	0.0156	0.0187	0.0048	0.0172	0.0172	0.0094	0.014
	R	0.0172	0.0218	0.0234	0.0109	0.0234	0.0218	0.0156	0.0203
	L	0.0021	0.0003	0.0152	0.0021	0.0172	0.0133	0.0058	0.0003
27	M	0.0077	0.0058	0.0229	0.0059	0.0248	0.021	0.0133	0.0058
	R	0.0152	0.0133	0.0286	0.0133	0.0286	0.0286	0.021	0.0133
	L	0.011	0.0055	0.0183	0.0074	0.0128	0.0055	0.0128	0.0055
28	М	0.0183	0.0128	0.0256	0.0146	0.0201	0.0128	0.0201	0.0128
	R	0.0256	0.0201	0.0274	0.0219	0.0274	0.0201	0.0274	0.0201
	L	0.0178	0.0138	0.0099	0.01	0.0119	0.0079	0.0041	0.0119
29	М	0.0257	0.0217	0.0178	0.0178	0.0198	0.0158	0.0119	0.0198
	R	0.0296	0.0296	0.0257	0.0257	0.0277	0.0237	0.0198	0.0277
	L	0.0117	0.0176	0.0041	0.0156	0.0117	0.0118	0.0117	0.0117
30	М	0.0195	0.0254	0.0117	0.0234	0.0195	0.0195	0.0195	0.0195
	R	0.0273	0.0293	0.0195	0.0293	0.0273	0.0273	0.0273	0.0273
	L	0.0003	0.0003	0.002	0.0105	0.0003	0.0105	0.0105	0.0037
31	М	0.002	0.0003	0.0088	0.0176	0.0037	0.0176	0.0176	0.0105
	R	0.0088	0.007	0.0158	0.0246	0.0105	0.0246	0.0246	0.0176
	L	0.0157	0.0003	0.0079	0.0118	0.0137	0.0178	0.0041	0.0098
32	М	0.0236	0.0041	0.0157	0.0196	0.0216	0.0255	0.0099	0.0177
	R	0.0295	0.0118	0.0236	0.0275	0.0295	0.0275	0.0177	0.0255
	L	0.0146	0.0132	0.0117	0.003	0.0102	0.0088	0.0044	0.0002
33	М	0.0205	0.019	0.0175	0.0088	0.0161	0.0146	0.0102	0.003
	R	0.0234	0.0234	0.0219	0.0146	0.0219	0.0205	0.0161	0.0088
	L	0.0002	0.0003	0.0004	0.0003	0.001	0.0003	0.0005	0.0003
34	M	0.0003	0.0003	0.0006	0.0004	0.0239	0.0005	0.0009	0.0004
	R	0.0004	0.0005	0.0018	0.0008	0.0239	0.0012	0.0034	0.0006
25	L	0.0058	0.0114	0.019	0.0114	0.0114	0.0133	0.0114	0.0058
35	M	0.0133	0.019	0.0265	0.019	0.019	0.0208	0.019	0.0114
┝───┤	R	0.0208	0.0265	0.0303	0.0265	0.0265	0.0265	0.0265	0.019
20	L	0.016	0.0072	0.0072	0.0107	0.0037	0.0142	0.0072	0.0054
36	M	0.0231	0.0142	0.0142	0.0178	0.0107	0.0213	0.0142	0.0124
	R L	0.0284 0.0002	0.0213 0.0027	0.0213 0.0014	0.0249	0.0178 0.0065	0.0266 0.0065	0.0213	0.0195 0.0014
27	 M	0.0002	0.0027	0.0014	0.0091	0.0065	0.0065	0.0027	
37	R	0.0027	0.0077	0.0052	0.0142	0.0116	0.0116	0.0077	0.004
	L	0.0077	0.0129	0.0103	0.0003	0.0167	0.0107	0.0129	0.009
38	M	0.0019	0.0038	0.0137	0.00052	0.0137	0.0103	0.0009	0.0019
50	R	0.0069	0.0103	0.0205	0.0052	0.0205	0.0171	0.0137	0.0053
	L	0.0137	0.0171	0.0256	0.012	0.0256	0.0239	0.0205	0.012
39	M	0.0221	0.0221	0.0221	0.0221	0.0221	0.0074	0.0038	0.0203
37	R	0.0295	0.0295	0.0295	0.0295	0.0295	0.0221	0.0093	0.0276
	K	0.0295	0.0293	0.0295	0.0295	0.0295	0.0221	0.0100	0.0295

Concordance Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0.544	0.544	0.732	0.578	0.618	0.771	0.746
A2	0.513	0.000	0.506	0.630	0.545	0.512	0.750	0.778
A3	0.514	0.577	0.000	0.595	0.611	0.592	0.785	0.666
A4	0.345	0.484	0.483	0.000	0.442	0.396	0.645	0.591
A5	0.510	0.573	0.498	0.646	0.000	0.531	0.816	0.746
A6	0.457	0.515	0.408	0.629	0.487	0.000	0.871	0.756
A7	0.287	0.327	0.272	0.410	0.292	0.154	0.000	0.390
A8	0.312	0.306	0.376	0.433	0.365	0.271	0.694	0.000

Concordance Matrix $\tilde{C} = (C^l, C^m, C^r)$ in Fuzzy Group ELECTRE - C^l

Concordance Matrix $\tilde{C} = (C^l, C^m, C^r)$ in Fuzzy Group ELECTRE - C^m

Concordance Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0.544	0.544	0.732	0.578	0.618	0.771	0.746
A2	0.513	0.000	0.506	0.630	0.545	0.512	0.750	0.778
A3	0.514	0.577	0.000	0.595	0.611	0.592	0.785	0.666
A4	0.345	0.484	0.483	0.000	0.442	0.396	0.645	0.591
A5	0.510	0.573	0.498	0.646	0.000	0.531	0.816	0.746
A6	0.457	0.515	0.408	0.629	0.487	0.000	0.871	0.756
A7	0.287	0.327	0.272	0.410	0.292	0.154	0.000	0.390
A8	0.312	0.306	0.376	0.433	0.365	0.271	0.694	0.000

Concordance Matrix $\tilde{C} = (C^l, C^m, C^r)$ in Fuzzy Group ELECTRE - C^r

Concordance Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0.544	0.544	0.732	0.578	0.618	0.771	0.746
A2	0.513	0.000	0.506	0.630	0.545	0.512	0.750	0.778
A3	0.514	0.577	0.000	0.595	0.611	0.592	0.785	0.666
A4	0.345	0.484	0.483	0.000	0.442	0.396	0.645	0.591
A5	0.510	0.573	0.498	0.646	0.000	0.531	0.816	0.746
A6	0.457	0.515	0.408	0.629	0.487	0.000	0.871	0.756
A7	0.287	0.327	0.272	0.410	0.292	0.154	0.000	0.390
A8	0.312	0.306	0.376	0.433	0.365	0.271	0.694	0.000

Discordance Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0.957	1	0.978	0.974	1	0.836	0.85
A2	1	0	0.989	1	1	1	0.808	1
A3	0.919	1	0.000	1	1	1	0.929	1
A4	1	0.873	0.935	0	1	1	0.866	0.833
A5	1	0.867	0.845	0.903	0	0.9947	0.890	0.949
A6	0.842	0.871	0.908	0.864	1	0	0.808	0.849
A7	1	1	1.000	1	1	1	0	1
A8	1	0.938	0.963	1	1	1	0.7947	0

Discordance matrix "D" Fuzzy Group ELECTERE method

The Boolean matrix "E" in Fuzzy Group ELECTRE method

E Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	1	1	1	1	1	1	1
A2	0	0	0	1	1	0	1	1
A3	0	1	0	1	1	1	1	1
A4	0	0	0	0	0	0	1	1
A5	0	1	0	1	0	0	1	1
A6	0	0	0	1	0	0	1	1
A7	0	0	0	0	0	0	0	0
A8	0	0	0	0	0	0	1	0

Appendix 14

The Boolean matrix "F" in Fuzzy Group ELECTRE method

F Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0	0	0	0	0	1	1
A2	0	0	0	0	0	0	1	0
A3	1	0	0	0	0	0	1	0
A4	0	1	1	0	0	0	1	1
A5	0	1	1	1	0	0	1	0
A6	1	1	1	1	0	0	1	1
A7	0	0	0	0	0	0	0	0
A8	0	1	0	0	0	0	1	0

Appendix 15

The Boolean matrix "G" in Fuzzy Group ELECTRE method

G Matrix	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	0	0	0	0	0	1	1
A2	0	0	0	0	0	0	1	0
A3	0	0	0	0	0	0	1	0
A4	0	0	0	0	0	0	1	1
A5	0	1	0	1	0	0	1	0
A6	0	0	0	1	0	0	1	1
A7	0	0	0	0	0	0	0	0
A8	0	0	0	0	0	0	1	0

Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8
	Ĺ	0.625	0.313	0.070	0.375	0.625	0.563	0.130	0.500
1	М	1.000	0.643	0.289	0.714	1.000	0.929	0.429	0.857
	R	1.600	1.300	0.800	1.400	1.600	1.600	1.000	1.600
	L	0.467	0.075	0.141	0.011	0.139	0.467	0.205	0.400
	М	1.000	0.367	0.458	0.276	0.546	1.000	0.636	0.909
	R	2.143	1.143	1.286	1.000	1.429	2.143	1.571	2.000
	L	0.217	0.151	0.360	0.217	0.429	0.500	0.429	0.289
3	М	0.636	0.458	0.731	0.636	0.909	1.000	0.909	0.727
	R	1.571	1.286	1.714	1.571	2.000	2.000	2.000	1.714
	L	0.625	0.500	0.375	0.500	0.438	0.563	0.070	0.438
4	М	1.000	0.857	0.714	0.857	0.786	0.929	0.357	0.786
	R	1.600	1.500	1.400	1.500	1.500	1.500	0.900	1.400
	L	0.500	0.563	0.375	0.375	0.130			0.500
5	M	0.923	1.000	0.769	0.769	0.462			0.923
-	R	1.667	1.778	1.556	1.556	1.111			1.778
	L	0.313	0.688	0.438	0.070	0.190			0.438
6	M	0.600	1.000	0.733	0.269	0.403			0.733
0	R	1.182	1.455	1.364	0.727	0.909			1.364
	L	0.375	0.625	0.190	0.375	0.500			0.500
7	M	0.714	1.000	0.500	0.714	0.857			0.857
,	R	1.400	1.600	1.100	1.400	1.500			1.500
	L	0.375	0.625	0.375	0.500	0.563			0.253
8	M	0.714	1.000	0.714	0.857	0.929			0.233
0	R	1.400	1.600	1.400	1.500	1.600			1.200
	L	0.375	0.375	0.563	0.130	0.253			0.375
0	M	0.769	0.373	1.000	0.130	0.233			0.373
9	R	1.556	1.556	1.000	1.111	1.333			1.556
	K L	0.533							
10			0.467	0.400	0.075 0.257	0.011			0.336
10	M R	1.000	0.917	0.833		0.093			0.750
		1.875	1.750	1.750	0.875	0.625			1.625
11	L	0.688	0.070	0.375	0.070	0.190			0.500
11	M	1.000	0.269	0.667	0.205	0.467			0.800
	R	1.455	0.727	1.273	0.636	1.000			1.455
10	L	0.032	0.023	0.015	0.011	0.010			0.011
12	M	1.000	0.368	0.160	0.112	0.086			0.112
	R	31.250	31.250	1.645	0.833	0.556		0.563 0.070	0.833
2 3 4 5 6 7 8 9 10 11 12 13 14	L	0.533	0.600	0.011	0.011	0.467			0.139
	M	0.923	1.000	0.234	0.160	0.846			0.388
	R	1.667	1.667	0.778	0.667	1.667			1.000
	L	0.500	0.563	0.688	0.563	0.563	0.563 0.130 0.929 0.429 1.600 1.000 0.467 0.205 1.000 0.636 2.143 1.571 0.500 0.429 1.000 0.909 2.000 2.000 0.563 0.070 0.929 0.357 1.500 0.900 0.250 0.010 0.615 0.234 1.333 0.778 0.378 0.250 0.667 0.533 1.273 1.091 0.563 0.130 0.929 0.429 1.500 1.000 0.533 0.130 0.929 0.429 1.500 1.000 0.500 0.714 1.100 1.400 0.378 0.070 0.769 0.385 1.444 1.000 0.503 0.070 0.503 0.070 0.503 <td>0.070</td>	0.070	
14	М	0.800	0.867	1.000	0.867	0.867			0.333
	R	1.364	1.364	1.455	1.364	1.364		0.130 0.429 1.000 0.205 0.636 1.571 0.429 0.909 2.000 0.070 0.357 0.900 0.010 0.234 0.778 0.250 0.533 1.091 0.130 0.429 1.000 0.375 0.714 1.400 0.070 0.385 1.000 0.375 0.714 1.400 0.070 0.385 1.000 0.333 0.818 0.011 0.866 0.553 0.205 0.539 1.222 0.010 0.203 0.636 0.130 0.237 0.778 0.130 0.388 </td <td>0.818</td>	0.818
	L	0.563	0.500	0.130	0.375	0.688			0.130
15	М	0.867	0.800	0.336	0.667	1.000			0.400
	R	1.364	1.364	0.818	1.273	1.455	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.909
	L	0.253	0.375	0.500	0.563	0.375			0.313
16	М	0.615	0.769	0.923	1.000	0.769			0.692
	R	1.333	1.556	1.667	1.778	1.556	1.667		1.444
	L	0.563	0.375	0.375	0.563	0.500	0.253	0.130	0.190
17	М	1.000	0.769	0.769	1.000	0.923	0.615	0.388	0.539
	R	1.667	1.556	1.556	1.778	1.667	1.333	1.000	1.222
	L	0.130	0.688	0.563	0.010	0.500	0.375	0.070	0.130
18	М	0.400	1.000	0.867	0.267	0.800		0.269	0.336
	R	0.909	1.455	1.455	0.727	1.455			0.818
16	L	0.563	0.688	0.375	0.253	0.375			0.375
19	M	0.867	1.000	0.667	0.469	0.667			0.667

The normalized fuzzy decision matrix $(\widetilde{r_{ij}})$ in Fuzzy SAW method

	R	1.455	1.455	1.273	1.000	1.273	1.364	0.727	1.273
	L	0.533	0.333	0.600	0.269	0.336	0.403	0.141	0.203
20	М	0.923	0.692	1.000	0.615	0.692	0.769	0.462	0.539
	R	1.778	1.444	1.667	1.333	1.444	1.444	1.111	1.222
	L	0.070	0.375	0.750	0.750	0.688	0.438	0.253	0.253
21	М	0.133	0.625	1.000	1.000	0.938	0.688	0.500	0.500
	R	0.500	1.167	1.333	1.333	1.333	1.167	1.000	1.000
	L	0.688	0.500	0.563	0.625	0.563	0.500	0.010	0.315
22	М	L 0.533 M 0.923 R 1.778 L 0.070 M 0.133 R 0.500	0.800	0.867	0.933	0.867	0.800	0.139	0.600
			1.364	1.364	1.364	1.364	1.273		1.182
			0.010	0.688	0.315	0.010	0.375		0.253
23			0.139	1.000	0.600	0.203	0.667		0.533
-			0.546	1.455	1.182	0.636	1.273		1.091
			0.400	0.139	0.203	0.075	0.011		0.467
24			0.909	0.546	0.549	0.367	0.276		1.000
24			2.000	1.429	1.429	1.143	1.000		2.143
			0.500	0.688	0.190	0.375	0.130		0.313
25			0.800	1.000	0.467	0.667	0.336		0.600
23			1.364	1.455	1.000	1.273	0.336		1.182
26			0.400	0.533	0.075	0.467	0.469		0.333
26			0.833	1.000	0.257	0.917	0.917		0.750
			1.750	1.875	0.875	1.875	1.750		1.625
27			0.011	0.533	0.075	0.600	0.467		0.011
27			0.234	0.923	0.237	1.000	0.846		0.234
			0.778	1.667	0.778	1.667	1.667		0.778
• •			0.203	0.667	0.269	0.467	0.203		0.203
28			0.500	1.000	0.571	0.786	0.500	1	0.500
			1.100	1.500	1.200	1.500	1.100		1.100
			0.467	0.333	0.336	0.400	0.267		0.400
29			0.846	0.692	0.692	0.769	0.615	0.546 0.133 0.336 0.818 0.205 0.636 1.571 0.313 0.600 1.182 0.139 0.500 1.250 0.203 0.539 1.222 0.467 0.786 1.500 0.139 0.462 1.111 0.400 0.769 1.556 0.429 1.000 2.333 0.149 0.388 0.996 0.190 0.500 1.100 0.020 0.040 3.571 0.375 0.714 1.400 0.253 0.615 1.333	0.769
			1.667	1.444	1.444	1.556	1.333	1.111	1.556
	L	0.400	0.600	0.139	0.533	0.400	0.403	0.400	0.400
30			1.000	0.462	0.923	0.769	0.769		0.769
		1.556	1.667	1.111	1.667	1.556	1.556	1.556	1.556
	L	0.011	0.011	0.080	0.429	0.011	0.429	0.429	0.149
31	М	0.112	0.016	0.500	1.000	0.208	1.000	1.000	0.600
	R	0.833	0.667	1.500	2.333	1.000	2.333	2.333	1.667
	L	0.571	0.011	0.289	0.429	0.500	0.646	0.149	0.357
32	М	0.923	0.160	0.615	0.769	0.846	1.000	0.388	0.692
	R	1.659	0.664	1.327	1.549	1.659	1.549	0.996	1.438
	L	0.625	0.563	0.500	0.130	0.438	0.375	0.190	0.010
33	М	1.000	0.929	0.857	0.429	0.786	0.714	0.500	0.149
	R	1.600	1.600	1.500	1.000	1.500	1.400	1.100	0.600
	L	0.010	0.011	0.016	0.012	0.040	0.015	0.020	0.011
34			0.015	0.027	0.018	1.000	0.023		0.016
	R	0.400	0.571	1.923	0.800	25.000	1.299	0.141 0.462 1.111 0.253 0.500 1.000 0.133 0.546 0.133 0.336 0.818 0.205 0.636 1.571 0.313 0.600 1.182 0.139 0.500 1.250 0.203 0.539 1.222 0.467 0.786 1.500 0.139 0.462 1.111 0.400 0.769 1.556 0.429 1.000 2.333 0.149 0.388 0.996 0.190 0.500 1.100 0.202 0.4040 3.571 0.375 0.714 1.400 0.253 0.615<	0.667
			0.375	0.625	0.375	0.375	0.438		0.190
35			0.714	1.000	0.714	0.714	0.786	î.	0.431
			1.400	1.600	1.400	1.400	1.400		1.000
			0.253	0.253	0.375	0.130	0.500		0.190
36			0.615	0.615	0.769	0.462	0.923		0.539
20			1.333	1.333	1.556	1.111	1.667		1.222
			0.149	0.080	0.503	0.360	0.360		0.080
37			0.546	0.367	1.000	0.818	0.818		0.080
51	R	0.139	1.421	1.136	1.989	1.847	1.847		0.280
	L	0.032	0.139	0.533	0.011	0.533	0.400		0.994
29	M								
38		0.337	0.500	1.000	0.253	1.000	0.833		0.257
	R	1.000	1.250	1.875	0.875	1.875	1.750		0.875
20	L	0.750	0.750	0.750	0.750	0.750	0.253		0.688
39	M	1.000	1.000	1.000	1.000	1.000	0.500		0.938
	R	1.333	1.333	1.333	1.333	1.333	1.000	0.750	1.333

The calculated (\tilde{u}_{ij}) in Fuzzy SAW method

Criteria	Triangular Fuzzy No.	A1	A2	A3	A4	A5	A6	A7	A8
	L	0.018	0.009	0.002	0.011	0.018	0.017	0.004	0.015
1	М	0.030	0.019	0.009	0.021	0.030	0.027	0.013	0.025
	R	0.047	0.038	0.024	0.041	0.047	0.047	0.030	0.047
	L	0.009	0.001	0.003	0.000	0.003	0.009	0.004	0.007
2	М	0.018	0.007	0.008	0.005	0.010	0.010	0.012	0.017
	R	0.039	0.021	0.024	0.018	0.026	0.039	0.029	0.037
	L	0.005	0.003	0.008	0.005	0.009	0.010	0.009	0.006
3	М	0.013	0.010	0.015	0.013	0.019	0.021	0.019	0.015
	R	0.033	0.027	0.036	0.033	0.042	0.042	0.042	0.036
	L	0.016	0.013	0.010	0.013	0.011		0.002	0.011
4	М	0.026	0.022	0.018	0.022	0.020	0.024	0.009	0.020
	R	0.041	0.038	0.036	0.038	0.038	0.038	0.023	0.036
	L	0.009	0.010	0.007	0.007	0.002	0.005	0.000	0.009
5	М	0.017	0.018	0.014	0.014	0.009	0.011	0.004	0.017
	R	0.031	0.033	0.029	0.029	0.020	0.025	0.014	0.033
	L	0.005	0.011	0.007	0.001	0.003	0.006	0.004	0.007
6	М	0.009	0.016	0.011	0.004	0.006	0.010	0.008	0.011
	R	0.018	0.023	0.021	0.011	0.014	0.020	0.017	0.021
	L	0.010	0.017	0.005	0.010	0.013	0.015	0.004	0.013
7	М	0.019	0.027	0.013	0.019	0.023	0.025	0.011	0.023
	R	0.037	0.043	0.029	0.037	0.040	0.040	0.027	0.040
	L	0.011	0.018	0.011	0.014	0.016	0.006	0.011	0.007
8	М	0.021	0.029	0.021	0.025	0.027	0.014	0.021	0.016
	R	0.040	0.046	0.040	0.043	0.046	0.032	0.040	0.035
	L	0.010	0.010	0.016	0.004	0.007	0.011	0.002	0.010
9	М	0.021	0.021	0.028	0.013	0.017	0.021	0.011	0.021
	R	0.043	0.043	0.049	0.031	0.037	0.040		0.043
	L	0.012	0.010	0.009	0.002	0.000	0.004	0.028 0.003	0.007
10	М	0.022	0.020	0.018	0.006	0.002	0.013	0.007	0.016
	R	0.041	0.038	0.038	0.019	0.014	0.030	0.022	0.035
	L	0.016	0.002	0.009	0.002	0.005	0.012	0.002	0.012
11	М	0.024	0.006	0.016	0.005	0.011	0.019	0.008	0.019
	R	0.034	0.017	0.030	0.015	0.024	0.030	0.019	0.034
	L	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
12	М	0.024	0.009	0.004	0.003	0.002	0.005	0.002	0.003
9	R	0.747	0.747	0.039	0.020	0.013	0.107	0.013	0.020
	L	0.014	0.016	0.000	0.000	0.012	0.007	0.005	0.004
13	М	0.024	0.026	0.006	0.004	0.022	0.016	0.014	0.010
	R	0.044	0.044	0.021	0.018	0.044	0.035	0.032	0.026
	L	0.014	0.016	0.020	0.016	0.016	0.009	0.000	0.002
14	М	0.023	0.025	0.029	0.025	0.025	0.047 0.030 0.009 0.004 0.018 0.012 0.039 0.029 0.010 0.009 0.021 0.019 0.042 0.042 0.014 0.002 0.021 0.019 0.042 0.042 0.014 0.002 0.024 0.009 0.038 0.023 0.005 0.000 0.011 0.004 0.025 0.014 0.006 0.004 0.010 0.008 0.020 0.017 0.015 0.004 0.025 0.011 0.040 0.027 0.006 0.011 0.015 0.004 0.021 0.011 0.040 0.027 0.006 0.011 0.014 0.021 0.015 0.004 0.021 0.011 0.022 0.011 0.030 <td>0.006</td> <td>0.010</td>	0.006	0.010
	R	0.039	0.039	0.042	0.039	0.039		0.013 0.030 0.004 0.012 0.029 0.009 0.019 0.042 0.002 0.009 0.023 0.000 0.004 0.005 0.006 0.007 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.0011 0.027 0.011 0.021 0.002 0.0011 0.022 0.002 0.003 0.002 0.003 0.004 0.005 0.014 0.002 0.003 0.004 0.005 0.014 0.002 0.003 0.004 0.002 0.003	0.023
	L	0.016	0.014	0.004	0.011	0.020		0.004 0.013 0.030 0.004 0.012 0.029 0.009 0.019 0.042 0.009 0.002 0.009 0.023 0.000 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.0011 0.027 0.011 0.021 0.002 0.003 0.0013 0.002 0.0013 0.002 0.0013 0.002 0.0014 0.002 0.0013 0.002 0.0014 0.002	0.004
15	М	0.025	0.023	0.010	0.019	0.029	0.025		0.012
	R	0.039	0.039	0.024	0.037	0.042		0.013 0.030 0.004 0.012 0.029 0.009 0.019 0.042 0.002 0.009 0.023 0.000 0.004 0.005 0.007 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.004 0.011 0.027 0.011 0.021 0.040 0.002 0.011 0.022 0.003 0.001 0.002 0.013 0.002 0.013 0.002 0.014 0.002 0.001 0.002 0.001 0.002 0.003 0.004 0.002 </td <td>0.026</td>	0.026
	L	0.007	0.011	0.015	0.016	0.011	0.016	0.002	0.009
16	M	0.018	0.022	0.027	0.029	0.022			0.020
-	R	0.039	0.045	0.048	0.051	0.045			0.042
	L	0.014	0.010	0.010	0.014	0.013			0.005
17	M	0.026	0.020	0.020	0.026	0.024			0.014
- /	R	0.020	0.040	0.040	0.045	0.043			0.031
	L	0.004	0.019	0.015	0.045	0.043			0.004
18	M	0.004	0.017	0.013	0.000	0.022			0.004
10	R	0.025	0.027	0.024	0.007	0.022			0.009
	L	0.025	0.040	0.040	0.020	0.040			0.022
19	M	0.022	0.018	0.010	0.007	0.010			0.010

	R	0.038	0.038	0.033	0.026	0.033	0.035	0.019	0.033
	L	0.012	0.008	0.014	0.006	0.008	0.009	0.003	0.005
20	М	0.021	0.016	0.023	0.014	0.016	0.017	0.010	0.012
	R	0.040	0.033	0.038	0.030	0.033	0.033	0.025	0.028
	L	0.002	0.011	0.023	0.023	0.021	0.013	0.008	0.008
21	М	0.004	0.019	0.030	0.030	0.028	0.021	0.015	0.015
	R	0.015	0.035	0.040	0.040	0.040	0.035		0.030
	L	0.019	0.014	0.016	0.017	0.016	0.014		0.009
22	M	0.028	0.022	0.024	0.026	0.024	0.022		0.017
22	R	0.041	0.038	0.038	0.038	0.038	0.036		0.033
	L	0.010	0.000	0.019	0.009	0.000	0.010		0.007
23	M	0.018	0.004	0.027	0.016	0.006	0.018		0.015
23	R	0.035	0.015	0.040	0.032	0.017	0.035		0.030
	L	0.003	0.010	0.003	0.005	0.002	0.000		0.030
24	M	0.020	0.022	0.003	0.003	0.002	0.007		0.011
24	R	0.020	0.022	0.015	0.015	0.009	0.007		0.024
	L	0.043	0.049	0.033	0.005	0.028	0.024		0.032
25									
25	M	0.017	0.020	0.025	0.012	0.017	0.008		0.015
	R	0.032	0.034	0.036	0.025	0.032	0.020		0.030
26	L	0.005	0.009	0.013	0.002	0.011	0.011		0.008
26	M	0.014	0.020	0.023	0.006	0.021	0.021		0.018
	R	0.032	0.041	0.044	0.021	0.044	0.041		0.038
25	L	0.002	0.000	0.015	0.002	0.017	0.013		0.000
27	М	0.009	0.007	0.026	0.007	0.029	0.024		0.007
	R	0.025	0.022	0.048	0.022	0.048	0.048		0.022
	L	0.011	0.006	0.018	0.007	0.013	0.006		0.006
28	М	0.020	0.014	0.027	0.016	0.022	0.014		0.014
	R	0.038	0.030	0.041	0.033	0.041	0.030		0.030
	L	0.018	0.014	0.010	0.010	0.012	0.008	0.003 0.010 0.025	0.012
29	М	0.030	0.025	0.021	0.021	0.023	0.018		0.023
	R	0.049	0.049	0.043	0.043	0.046	0.040		0.046
	L	0.012	0.018	0.004	0.016	0.012	0.012	0.012	0.012
30	М	0.023	0.029	0.014	0.027	0.023	0.023	0.023	0.023
	R	0.046	0.049	0.033	0.049	0.046	0.046	0.046	0.046
	L	0.000	0.000	0.002	0.011	0.000	0.011	0.011	0.004
31	М	0.003	0.000	0.012	0.025	0.005	0.025	0.025	0.015
	R	0.021	0.016	0.037	0.057	0.025	0.057	0.057	0.041
	L	0.017	0.000	0.009	0.013	0.015	0.019	0.004	0.011
32	М	0.027	0.005	0.018	0.023	0.025	0.030	0.011	0.020
	R	0.049	0.020	0.039	0.046	0.049	0.046	0.029	0.042
	L	0.015	0.013	0.012	0.003	0.010	0.009	0.004	0.000
33	М	0.023	0.022	0.020	0.010	0.018	0.017	0.012	0.004
	R	0.037	0.037	0.035	0.023	0.035	0.033		0.014
	L	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000
34	M	0.000	0.000	0.001	0.000	0.024	0.001		0.000
	R	0.010	0.014	0.046	0.019	0.598	0.031	0.010 0.025 0.008 0.015 0.030 0.000 0.004 0.015 0.004 0.015 0.004 0.005 0.015 0.005 0.015 0.005 0.015 0.038 0.003 0.015 0.030 0.012 0.029 0.006 0.015 0.035 0.013 0.022 0.041 0.033 0.012 0.023 0.041 0.023 0.046 0.011 0.025 0.057 0.004 0.011 0.026 0.001 0.026 0.001 0.026 0.001 0.026 0.007 0.018 </td <td>0.016</td>	0.016
	L	0.006	0.011	0.019	0.011	0.011	0.013		0.006
35	M	0.000	0.022	0.030	0.022	0.022	0.024		0.000
55	R	0.013	0.043	0.049	0.043	0.043	0.043		0.030
	L	0.035	0.007	0.007	0.043	0.004	0.043		0.005
36	M	0.010	0.018	0.018	0.011	0.013	0.014		0.005
55	R	0.023	0.038	0.038	0.022	0.032	0.020	0.000 0.004 0.015 0.004 0.009 0.022 0.005 0.015 0.038 0.008 0.015 0.030 0.015 0.030 0.012 0.029 0.006 0.015 0.035 0.013 0.022 0.041 0.004 0.012 0.023 0.041 0.025 0.057 0.004 0.011 0.025 0.007 0.004 0.012 0.026 0.001 0.025 0.011 0.026 0.001 0.026 0.001 0.026 0.003 0.003 0.003 0.003 0.003	0.035
	L	0.001	0.003	0.038	0.044	0.032	0.047		0.001
37	M	0.000	0.003	0.001	0.009	0.007	0.007		0.001
51	R	0.003							
			0.026	0.021	0.036	0.033	0.033		0.018
20	L	0.002	0.004	0.014	0.000	0.014	0.010		0.002
38	M	0.009	0.013	0.026	0.007	0.026	0.021		0.007
	R	0.026	0.032	0.048	0.022	0.048	0.045		0.022
	L	0.022	0.022	0.022	0.022	0.022	0.007		0.020
39	М	0.030	0.030	0.030	0.030	0.030	0.015		0.028
	R	0.039	0.039	0.039	0.039	0.039	0.030	0.022	0.039

Alternative		$\widetilde{u}_i(x)$		$\mathbf{D}(\widetilde{\boldsymbol{u}}_i(\boldsymbol{x}))$	Final Ranking
A1	(0.39	0.73	2.1)	0.988	A1
A2	(0.37	0.69	2.06)	0.950	A5
A3	(0.39	0.72	1.43)	0.815	A2
A4	(0.31	0.61	1.27)	0.702	A6
A5	(0.39	0.73	1.96)	0.950	A3
A6	(0.38	0.72	1.5)	0.829	A4
A7	(0.18	0.46	1.17)	0.568	A8
A8	(0.28	0.58	1.26)	0.676	A7

Total fuzzy and deterministic utility of each alternative and their final ranking in Fuzzy SAW method

The $M_{\mathrm{i}\mathrm{k}}$ matrix in the aggregation method

M _{ik}	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8
A1	1	0	2	0	0	0	1	0
A2	0	0	1	0	1	1	1	0
A3	1	1	0	0	2	0	0	0
A4	0	0	0	2	0	2	0	0
A5	1	1	1	1	0	0	0	0
A6	2	1	0	1	0	0	0	0
A7	0	0	0	0	0	0	0	4
A8	0	0	0	0	1	0	3	0

Appendix 20

The $C_{i\mathbf{k}}$ matrix in the aggregation method

C _{ik}	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8
A1	1	1	3	3	3	3	4	4
A2	0	0	1	1	2	3	4	4
A3	1	2	2	2	4	4	4	4
A4	0	0	0	2	2	4	4	4
A5	1	2	3	4	4	4	4	4
A6	2	3	3	4	4	4	4	4
A7	0	0	0	0	0	0	0	4
A8	0	0	0	0	1	1	4	4

Lingo codes for solving the aggregation method

```
Sets:
      Alternative/1..8/;
                                 !i;
      Rank/1..8/:W;
                                 !k;
      Link(Alternative,Rank):N,C;
endsets
Data:
      W=64.0000
                    32.0000
                                21.3333
                                           16.0000
                                                      12.8000
                                                                  10.6667
                                                                               9.1429
      8.0000;
      C=
                           3
                  1
                                     3
                                               3
                                                         3
                                                                   4
                                                                             4
        1
                                               2
4
        0
                  0
                           1
                                     1
                                                         3
                                                                   4
                                                                             4
                                     2
                                                         4
                                                                             4
                  2
                           2
                                                                   4
        1
                                     2
4
                                               2
4
                                                                            4
4
                  0
                           0
                                                                   4
4
        0
                                                         4
        1
                  2
                           3
                                                         4
                  3
                           3
                                               4
                                                                   4
        2
                                     4
                                                         4
                                                                             4
                  0
                           0
                                     0
                                               0
                                                         0
                                                                   0
        0
                                                                             4
                                     0
        0
                  0
                           0
                                               1
                                                         1
                                                                   4
                                                                             4
EndData
!Objective;
Max = @sum(Link(i,k):C(i,k)*W(k)*N(i,k));
!Each alternative has only one ranking;
@for(Alternative(i): @sum(Rank(k):N(i,k))=1);
!Each ranking is assigned to only one alternative;
@for(Rank(k): @sum(Alternative(i):N(i,k))=1);
End
```

Appendix 22

Final ranking matrix in the Aggregation method

	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7	Rank 8
A1	0	0	1	0	0	0	0	0
A2	0	0	0	0	1	0	0	0
A3	0	1	0	0	0	0	0	0
A4	0	0	0	0	0	1	0	0
A5	0	0	0	1	0	0	0	0
A6	1	0	0	0	0	0	0	0
A7	0	0	0	0	0	0	0	1
A8	0	0	0	0	0	0	1	0