How making for a purpose in mathematics can provoke mathematical meaning making

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HOW MAKING FOR A PURPOSE IN MATHEMATICS CAN PROVOKE MATHEMATICAL MEANING-MAKING[1]

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In this paper we are concerned to understand the connection that can occur for primary school children between relevant practical, "hands-on" engagement with the material world in partnership with others and the development of mathematical commitment, enthusiasm and understanding. We draw on our personal experiences of a European Union (EU) project, Mathematics in the Making (MiMa) *(http://www.mathematicsinthemaking.eu), (Project no. 539872-LLP-1-2013-1-IT-COMENIUS–CMP) funded through the Comenius Lifelong Learning Programme, which was based on a primary school mathematics intervention in five EU countries - England, Germany, Hungary, Italy and Portugal. We use the writings of David Jardine and Peter Applebaum to aid our thinking. We point to how absorbing, extended practical work recognises our humanity and can provide purposeful spaces for meaning-making in mathematics.*

INTRODUCTION

In this paper we begin by introducing a European Union (EU) project, *Mathematics in the Making (MiMa)* (http://www.mathematicsinthemaking.eu), (Project no. 539872-LLP-1-2013-1-IT-COMENIUS–CMP) funded through the Comenius Lifelong Learning Programme, which was based on a primary school mathematics intervention in five EU countries - England, Germany, Hungary, Italy and Portugal[2] We participated in this project in England as curriculum developers and as teacher educators and also contributed to the theoretical underpinnings of the project. The classroom activities which were generated as part of the project extended over time and involved practical "hands-on" work leading to mathematical artefacts of one kind or another which would be exhibited and have their meaning explained by the children in a public exhibition.

Our experience of the exhibition in England and the commitment, enthusiasm and mathematical meaning-making of the children surprised us. It led us to ask ourselves broadly philosophical questions about how using our hands in practical activities generates space to think, reflect and understand. Here, we draw on the writings of David Jardine and Peter Applebaum to aid our thinking, considering how absorbing, extended practical "work" (Arendt, 1958) provides respite from the fragmented and alienating piece-work of much contemporary mathematics in school in England and gives instead a space for meaning-making. We conclude that such experience also acknowledges and underscores our humanity.

[1] This paper draws on and extends Povey, Hilary, Adams, Gill, Jackson, Colin and Ughi, Emanuela (2016) The role of exhibitions by children in making mathematics. Unpublished paper presented at 13th International Congress on Mathematical Education, Hamburg, 24-31 July 2016.

[2] The partners were Università degli Studi di Perugia, P3 Poliedra Progetti in Partenariato, Eotvos Lorand University, Mathematikum, Universidade Nova de Lisboa and Sheffield Hallam University.

THE *MATHEMATICS IN THE MAKING (MiMa)* PROJECT

In this section, we give the background to the project and discuss its theoretical underpinnings. We then describe and illustrate the project itself from the perspective of England, including something about the classroom activities devised, the model of teacher professional development employed and the experience of the public exhibition in England with which the schools' engagement with the project culminated.

Background to the project

The project was the brain child of Emanuela Ughi, Professor of Mathematics at the University of Perugia. She had noticed over the years that many of her undergraduate mathematics students, despite being successful products of school mathematics, had very limited mental images of mathematical objects and found mathematical visualisation difficult. This impeded their understanding of the mathematical concepts they were studying. Emanuela and her students built mathematical models - for example, articulated systems of interconnected levers to explore surfaces or designing cylindrical mirror anamorphoses to explore the laws of reflection - and discussed them as part of the students' studies, deepening the students' grasp of key mathematical ideas. Emanuela realised that, had they had concrete, "hands-on", visceral, practical experience of mathematical objects earlier in their lives and the opportunity to make, handle and "play" with them, it would have enriched their previous learning.

The second key driver for the project was the recognition that for many young people mathematics is thought of as a "meaning-less" subject only accessible to the gifted, a subject in which it is not possible to make meaning but only to carry out routine and mechanical procedures (Organisation for Economic Co-operation and Development 2009). The combination of these two concerns produced the *MiMa* intervention project.

Theoretical underpinnings for *MiMa*

The recognition of the need for practical, hands-on experience in learning mathematics is not new. In the 19th Century, Johann Heinrich Pestalozzi (1801) argued that learning followed from *hand* to *heart* to *head*. Knowledge is gained by first providing motivating mathematical work for the hands. Such activity fosters joy and thus enters the heart of the child and from there a pathway opens up to the head where deep cognition can then take place. At this point, following Pestalozzi, we understood the significance of working with the hands to be about an active rather than a passive engagement with the learning process, with the child having direct experience of the world and using all the senses for learning. We also followed him in wanting to offer cross-curricular opportunities for learning.

We saw these ideas as being developed further by Jerome Bruner (1966). He argued that understanding develops from the *enactive* to the *iconic* and thence to the *symbolic* and that our approach to teaching should reflect this.

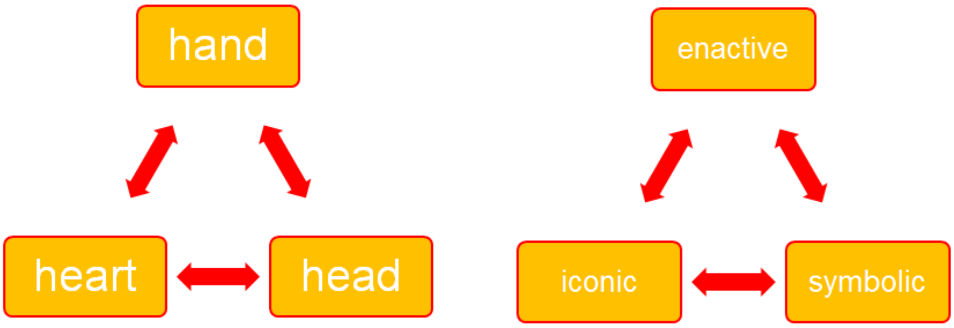

Figure 1: Two of the theoretical underpinnings for MiMa

In our view, the enactive, iconic and symbolic were not to be understood as disconnected nor progress through them to be thought of as rigidly linear. For example, learners return to the enactive at all levels of mathematical thinking to scrutinise and deepen existing knowledge and to resolve new uncertainties. Both the enactive and the iconic remain as metaphors in the mind to support symbolic thinking.

The project embraced the idea that learning should be understood as fundamentally social: Lev Vygotsky (1962; 1978) stressed the role of social interaction in the development of cognition, with children and their partners co-constructing knowledge within their zone of proximal development. Alongside this we saw effective learning as dialogic (Alexander, 2008) and that this is fostered through practical, hands-on activities: these enable learning to be *collective* with children engaged together in shared tasks; *reciprocal* with children listening to and experiencing each other's ideas; and *supportive* with a multiplicity of approaches and solutions being encouraged. Lastly, the project acknowledged that worthwhile learning is democratic (Dewey, 1949). We know that knowledge is shaped by the pedagogical context in which it is acquired (Povey & Burton, 2004). Practical, "hands-on" activities allow children to make mathematics whilst, together, they make things mathematical artefacts; and learning becomes open and consensual which accords with a democratic way of life.

The project

Working together and sharing ideas, the *MiMa* partners produced ten sets of activities (MiMa, 2015a, MiMa, 2015b) with teacher notes and explanatory introductory videos. The activities included, for example, making giant logic mazes; building a variety of polyhedra including a class beehive; creating a model of the solar system and taking "Neptune" two miles from the "sun" in the school playground; constructing frieze patterns in clay; and making and calibrating a variety of sundials. In England, participating teachers from five local primary schools experienced the activities themselves through two day-long professional development workshops in which they were able to work with concrete materials themselves making some of the artefacts and to discuss their mathematics before running "laboratories" based on the activities with their children in school.

Figure 2: Participants in the *MiMa* teacher professional development workshops in Sheffield, UK.

A distinctive feature of the *MiMa* project was that, throughout the laboratories, the teachers and children knew that they were preparing their objects and activities for public exhibition. Thus a fundamental aspect of the *MiMa* methodology was that children would display and explain their mathematics to others - their parents, other children and members of the general public. The physical models and activities produced in the project represented the concrete output of the children's mathematical thinking and acted as a supportive bridge to their reasoning and explanations. The reflection on their own learning and the mathematical strategies they had employed (which the process of explanation required) contributed to the metacognitive strength of the experience.

Figure 3: The MiMa exhibition in the Winter Gardens, Sheffield.

Visitors to the exhibition were encouraged to join in the children's mathematics and children and teachers were full of enthusiasm and praise for the project, planning to continue to use the activities and to involve neighbouring schools[3].

THEORETICAL UNDERPINNINGS REVISITED

We were at the exhibition ourselves and were very much surprised by what we saw and heard. We had not anticipated the commitment and the depth of understanding that the children displayed towards their mathematics. This led us to revisit our thinking about the theoretical underpinnings and to seek to think more deeply about the nature of the "hands-on" experiences of the children and their teachers. Our interest in exploring further was awoken:

> Something awakens our being in the middle (*inter*) of things (*esse*), and we find that there is a story already underway, one in which we are already moving and living. (Jardine, 2013, p. 1)

The first thing we noticed was that the sense of *time* experienced in the project: it made a place for extended, full-filled time very different from the clock time of contemporary schooling (Jardine, 2013). We have written about this elsewhere.[4]

Further, however, reading Jardine prompted us both to think again about the practical, "hands-on", even visceral and embodied *nature of the activity itself* which was exemplified by the *MiMa* tasks and also to reflect on the way in which this contributed to mathematical meaning making amongst the children. As well as providing tools to think with and metaphoric representations to revisit when developing mathematical understanding, the *MiMa* activities also provided the opportunity (sometimes, it has to be said, not taken up) to become absorbed and thoughtful, to work like an artisan creating a hand-crafted product, be this a series of scaled models of the planets or a gigantic carpet of tessellated shapes or, from a cardboard box, a decorated lion with head, mane and tail and visiting flea.

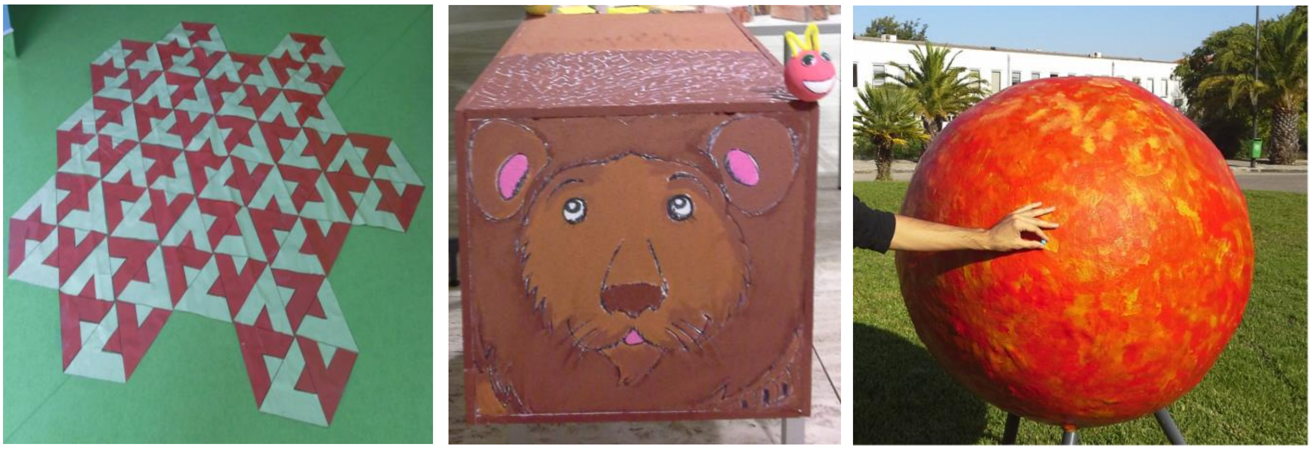

Figure 4: Children's work from Portugal.

[3] The teachers all noted how important the exhibition had been for the children's learning. The five themes which emerged during the initial analysis are reported in Povey, Hilary, Adams, Gill, Jackson, Colin and Ughi, Emanuela (2016) The role of exhibitions by children in making mathematics. Unpublished paper presented at 13th International Congress on Mathematical Education, Hamburg, 24-31 July 2016.

[4] We have explored this further in Povey, Hilary, Adams, Gill & Boylan, Mark (2016). Making time to make meaning: an embodied approach to learning. Paper presented as part of a symposium, British Education Research Association (BERA) Conference, Leeds, 13-15 September, 2016

We know from introspection that, as we cut or paste or colour or construct or mould or … the mind relaxes and expands, we become absorbed; and a space is created for thinking. (Hazlitt powerfully evokes a similar effect from a long country walk taken on one's own (2005).)

This is in sharp contrast to the fragmented mechanical tasks - the piece-work - of usual school mathematics (Boylan, 2004) which are reminiscent of the Taylor-esque world of factory production where 'no one of these isolated bits or pieces *requires* any prolonged attention' (Jardine, 2013, p. 9). Tasks are utterly fragmented in school mathematics for most English schoolchildren, with atomised "learning objectives" against which "progress" must be made in a single lesson (or, even worse, within twenty minutes, if the school inspectorate is around). The activity becomes meaning-less and alienating rather than allowing for 'the unity of thought and action, conception and execution, hand and mind' (Marx, 1975, p. 285) which occurs with meaning-full work. Giving attention and devotion to task at hand

> is very often not simply *unnecessary* but *impossible* because the school-matters at hand have been stripped of the very memorability and relatedness . . . that might require and sustain and reward such attention and devotion (Jardine, 2012, p. 175, initial emphasis).

Peter Applebaum reminds us of Hannah Arendt's distinction between "labor" "work" and "action" (Arendt, 1958) and suggests these as a way to think about teaching and learning mathematics:

> Mathematics educators and mathematics education researchers might use the distinctions across types of activity from Arendt to analyze the forms that are manifested in their practices. They would be interested in the relationships and potential for mathematics to be a catalyst of action rather than merely work, and hope to encourage forms of activity that avoid the alienating effects of what would be described as labour. (2016, p. 4).

For Arendt, inasmuch as we only "labor", without the possibility of experiencing "work" or "action", we are being denied the right to our humanity (1958, p. 84). We would describe usual school mathematics as "labor", defined by its remorseless repetition of isolated units of task which carry no meaning or purpose for the one "laboring". We speculate that, at times, the *MiMa* children and their teachers experienced instead a re-imagined school mathematics in which "work" - the interaction between the material world and human artisanship - produced whole and meaning-full artefacts. For the children these would be their mathematical objects and for the teachers, perhaps, lessons that fully made sense in human terms. It is possible - though, we think, at best, infrequently - that from time to time the "work" became "action", that is, purposeful, worthwhile activity with others with whom a common goal was shared and which motivated the endeavour. For Applebaum, "action" involves meeting as a group to work on the material world for some purpose, 'maybe even to specifically learn mathematics, or to use mathematics to learn something else' (2016, p. 3-4). He envisages classrooms where learners

> come together to create mathematics problems that they need to solve, rather than provide answers on tests that others score for no apparent purpose, disconnected from the classroom experience. (Applebaum, 2016, p. 4)

When we take hold of such practical "work" - or even, "action", - it in turn, reciprocall, takes hold of us:

> This possibility of, shall we say, “absorption” and being moved and addressed and, shall we say, summoned or beckoned by the work itself, is phenomenologically familiar. (Jardine, 2013, p. 23)

In the process of taking hold of and absorbing us, the task and our engagement with it have the capacity to open up possibilities for seeing aspects of the world and its mathematics differently - and thus to create a space for mathematical meaning-making.

CONCLUSION

We have described and discussed a primary mathematics intervention project which was based on practical, "hands-on", extended mathematical activities leading to a public exhibition of children's work at which they were called upon by the visitors to show and explain their work. Our experience of the exhibition and the children's engagement and their mathematical authority surprised us and prompted us to think more deeply about the connection between the nature of the activities with which the children were engaged and the mathematical meaning-making they showed. We have suggested that, sometimes, the *MiMa* activities and the preparation for the exhibition allowed children and their teachers to escape from "labor" and from piece-work and to "work" on tasks which commanded the children's attention and with which they were able to be absorbed.

The recognition that there is something to be understood about the relationship between the practical and embodied and the development of the intellect is not new - for example, Comenius, Locke, Rousseau and Franke (Olafsson & Thorsteinsson, 2011) all wrote about education and the role within it of artisanal craft. However, it is a link that has never occupied a central place in English schooling; and the understanding that there is such a link has been very largely lost generally in England school education and, even more so, in the teaching and learning of mathematics. In these neo-liberal times, this disconnection has been exacerbated by the performativity culture of schooling (Ball, 2003) which requires that teaching and learning becomes fragmented into measurable and auditable "bite size" pieces, thus precluding extended tasks which absorb us and for which we may not have a fixed and final end-product pre-determined.

Interrupting such ways of thinking is very far from easy and we are very far from wanting to suggest any sort of "magic bullet". Nevertheless our experience of children engaged in practical, "hands-on" mathematics in preparation for a public exhibition where their mathematical artefacts are viewed and mathematical explanations invited in genuine acts of communication - in other words, in response to enquiries from those who do not already know the answers - suggests that this is one way to provide some respite: being purposefully absorbed in meaning-full "work" acknowledges our humanity and allows us, sometimes, to make meaning in mathematics.

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