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‘The unplanned impact of mathematics’ and its implications for research funding: a discussion-led educational activity

Peter Rowlett

Abstract

‘The unplanned impact of mathematics’ refers to mathematics which has an impact that was not planned by its originator, either as pure maths that finds an application or applied maths that finds an unexpected one. This aspect of mathematics has serious implications when increasingly researchers are asked to predict the impact of their research before it is funded and research quality is measured partly by its short term impact.

A session on this topic has been used in a UK undergraduate mathematics module that aims to consider topics in the history of mathematics and examine how maths interacts with wider society. First, this introduced the ‘unplanned impact’ concept through historical examples. Second, it provoked discussion of the concept through a fictionalised blog comments discussion thread giving different views on the development and utility of mathematics. Finally, a mock research funding activity encouraged a pragmatic view of how research funding is planned and funded.

The unplanned impact concept and the structure and content of the taught session are described.

1 Introduction

‘The unplanned impact of mathematics’ refers to mathematics that has an impact that was not planned by its originator. In mathematics, work can often find an unplanned impact many years after its introduction. This has implications particularly in relation to strategic allocation of research funding. The unplanned impact concept was used as the theme of a taught session in a module ‘Mathematics and Society’ at the University of Greenwich.

This paper introduces the unplanned impact concept and the teaching context, and describes the structure and content of the taught session.

2 The unplanned impact of mathematics

Time and again, pure mathematics displays an astonishing quality. A piece of mathematics originates with a mathematician who is, often, following his or her curiosity without a plan for meeting some identified need or application. Later, perhaps decades or centuries later, this mathematics fits perfectly into some need or application.

I became interested in this phenomenon as increasingly researchers are asked by funders to predict the impact of their research in advance, and research quality is measured partly by its short term impact. I spoke to research mathematicians on this issue and a pattern started to emerge: mathematicians tend to think of this aspect of mathematics as axiomatic, and generally come up with one of a small number of examples to justify this position: number theory in modern cryptography; work in logic leading to computing; and, imaginary numbers in fluid mechanics (particularly flight) and electricity. This left me a little concerned. If every mathematician you speak to comes up with one of the same few examples, is the story going to be sufficiently convincing?

A BSHM initiative collected several examples of ‘the unplanned impact of mathematics’: mathematics that later had some impact which had not been planned by its originator. This led to publication of seven vignettes in *Nature* (Rowlett *et al.*, 2011) and a series of five in *Mathematics Today* (Hoare, 2013a; Cambell-Kelly, 2013; Rowlett, 2013; Hoare, 2013b; Oxley, 2013). There are two types of unplanned impact demonstrated.

The first type of unplanned impact is those areas of pure mathematics which later found application. For example, Johannes Kepler suggested a way to pack spheres that is the most efficient in 1611. This conjecture was finally proved by Thomas Hales in 1998. Although the three-dimensional case took nearly four hundred years to prove, much work had been completed in the intervening time on higher-dimensional and other related cases. Harriss (in Rowlett *et al.*, 2011) describes how Gordon Lang in the 1960s and 1970s was designing systems for modems using this mathematics. Lang needed to send a signal over a noisy channel, such as a phone line, using a system of tones for signals. As the tone may become distorted during transmission, the original signal is reconstructed by mapping each sound received to the closest of the transmitted tones. Mathematically, these signals can be considered as spheres, to allow room around each tone for noise distortion. To maximise the information that can be sent, these 'spheres' must be packed as tightly as possible. Lang developed a modem with 8-dimensional signals using sphere packing.

The second type of unplanned impact is areas of applied mathematics that found application far beyond what the originator could have predicted. For example, George Green (1793–1841), a brilliant, almost entirely self-taught mathematician, made substantial contributions to 19th century mathematical physics by developing Green's theorem, Green's functions and the theory of potential and influencing Kelvin and others. Green's aim, stated in his original paper, was to "facilitate the application of analysis to one of the more interesting of the physical sciences" (Green, 1828). Rowlett (2013) suggests that the scale of application of Green's work must be beyond anything its author could have imagined, including 20th century work in microwave radar, particle accelerators and quantum electrodynamics.

3 'Mathematics and Society'

'Mathematics in Society' is a final year module in the BSc (Hons) Mathematics degree programme at the University of Greenwich. It aims to: consider selected topics in the history of mathematics and its uses; and, examine the role of the mathematical sciences in twenty-first century society. The learning outcomes are that students should be able to:

A critically discuss historical and contemporary examples of mathematical issues in society;

B make judgements, supported by evidence, on philosophical and ethical issues in the theory and application of mathematics.

Students discuss and analyse examples and issues during taught sessions. I was twice engaged as a guest lecturer for one taught session of three hours duration on the unplanned impact concept. This fits into the module because it relates to the professional practice of the mathematician, both by discussing the development of mathematical topics from initial research to applications, and as it relates to issues around contemporary research funding.

4 Unplanned impact taught session

The session consisted of both lecture and seminar elements, including transmission of information, whole- and small-group discussion and group presentations. The three hours, with appropriate breaks, were roughly partitioned into: the unplanned impact concept; discussion of the phenomenon; and, implications for research funding.

4.1 The unplanned impact concept

A short lecture introduced and defined the unplanned impact concept, giving examples of the two types — pure maths finding an application, and applied maths finding an unexpected one — much as set out in section 2 above.

Students were then separated into small groups and each group given a vignette from the unplanned impact publications. Groups were given twenty minutes to read and digest their story, then present it to the whole class. This has several advantages over a lecture that goes through each example in turn: it breaks the session up to maintain student interest; and, it encourages graduate skills development by asking students to digest and process new information quickly and give a presentation. I have observed that students like to plan a presentation very thoroughly and practice it several times before giving it for real, but in many contexts this may not be realistic. As a short-notice presentation on an unfamiliar topic, I believe this activity is providing an unusual, valuable experience.

The unplanned impact examples are short and generally state the area of original development followed by the unexpected application without technical detail. As such, I have found students understand these well and are surprised by them, without being lost through lack of prerequisite knowledge. Nevertheless, I found it useful to quickly summarise each vignette after all the student presentations have concluded. This served as an opportunity to refresh the unplanned impact concept ready for the discussion that followed, and allowed me to fill any gaps I felt had been missed.

4.2 Discussion of the phenomenon

There are issues with the unplanned impact concept as presented, principally that a small number of examples do not prove that this phenomenon exists in general. The second part of the session considered the question: 'is the phenomenon real?' Particularly in a module such as this, we should encourage students to question received wisdom and form their own views, in this case about the assertions I had made during the first hour.

This discussion was facilitated by a print-out of a fictionalised blog post and comment discussion thread. Students were asked to read the opinions expressed and discuss them in small groups for around twenty minutes, and then take part in whole-group discussion for around thirty minutes.

The fictionalised blog post used was an edited version of a real blog post by Castelvecchi (2013), using the following text:

> The current issue of *Nature* has a great feature about how mathematical inventions and discoveries often find unexpected applications, sometimes decades after their first appearance.
> Mathematicians usually pursue their theories out of curiosity, aesthetical interest, ambition, or that intangible quality of being mathematically deep (a notion that's worth a whole blog post of its own). They do so with a loftiness that frustrates just about everybody else, but especially scientists — and funding agencies. ...
> The list includes some predictable examples—though told with freshness—but also some that I had not heard of and that I found very intriguing. ...
> As much as I enjoyed the article, it must be said that picking some of the successful examples does not satisfactorily answer the broader question of whether the bulk of mathematical research is a 'waste of time,' in the sense that it will never find applications anywhere. It is a legitimate question, and one that I am not qualified to answer.

The handout then continued with a fictionalised comment thread under this blog post, actually compiled from multiple sources and from conversations I had had with people about unplanned impact. The intention was to present various views on Castelvecchi's 'legitimate question' and related issues. The advantage of a blog comment thread is that it is naturally a written discussion, rather than being a transcript of a spoken dialogue and, given the timescale, it is a more digestible form than, say, a longer article or essay. Comments can be 'nested', so that one is directly in reply to another, giving some structure to the discussion. In total, fifteen comments were included, some of which are highlighted below.

A piece by Harford in the Financial Times, partly in response to the *Nature* piece, provided a comment on the same lines as Castelvecchi. This asks whether such examples are 'typical of the unexpected bounties of pure mathematics — or an unrepresentative poster child?' (2011, 52).

A blog post by Freiberger (2011) announces the *Nature* paper, and several anonymous comments in response to that article provided useful text for the fictionalised thread. One comment below this suggests that the mathematics needed for the application could have been developed when the need arose, saying that 'presumably if mathematicians had concentrated instead more on things going on at the time, there would also have been faster progress then'. A reply to this opines that this approach will stunt progress overall, and points out that the development of a piece of mathematics is not a straightforward process and can not be planned.

A blog post by Kehoe (2011) directly addresses the question raised by Castelvecchi. Kehoe suggests the question is not legitimate, arguing that mathematical development arises from the human 'drive to *organize*' and explore 'organizational possibilities' through a process of 'fluid creativity (which is likely informed by things that are outside of our awareness)'. Then, Kehoe argues, we cannot direct this process nor evaluate whether an abstract mathematical concept might contain the key to solving some physical problem.

Hossenfelder (2011) uses the *Nature* piece to point out the difficulty of making 'predictions about future innovations, or the impact thereof', even in the relative short term. A statement that 'when it comes to predicting innovations, history shows such predictions are mostly entertaining speculations' was used on the handout to raise the flaw in assuming that future impact can be planned. A reply in the fictionalised thread agrees, saying that forcing researchers into an area of interest in order to manage discovery will preclude discoveries in unexpected areas.

Another comment below the piece by Freiberger questions the need to consider applications at all, pointing out that it is perfectly valid to 'study things for their own right, without caring that there is some spectacular real world use at the end'. In the fictionalised comment thread, I added a sentence pointing out that many aspects of art, literature and music are pursued for their intrinsic value.

Other short entries on the handout raised further issues: that the impact of a piece of work might be unexpected further development in mathematics rather than its applications; that mathematics may not be unusual among disciplines in demonstrating this phenomenon; that many applications of mathematics are so minor and commonplace, perhaps far in the past, that they are not easily discovered. Other comments agreed with those discussed above or reiterated the unplanned impact concept, including some repetition. Comments were not ordered particularly coherently, so students needed to draw on multiple parts when reading. The intention was to mimic somewhat a real blog discussion and to encourage critical analysis of what is written.

Such a discussion has no correct answers, and this is part of what makes it very suitable for such a module, but my experience is that mathematicians, including undergraduate mathematicians, will argue that the phenomenon exists and that pure mathematics should be valued without thought for applications. However, it does not seem likely that every apparently obscure pure mathematical topic is going to lead, eventually, to some unplanned impact, and this is the point to transition to thinking about research funding.

4.3 Implications for research funding

Students with whom I have had this discussion have generally been convinced that the phenomenon exists and that research in mathematics cannot always be directed or planned. Ultimately, this leads to a discussion of where to direct limited resources. For example, if students are drawn to the argument that art, literature and music are happily pursued without thought of applications, then it may be useful to point out that (some version of) these art forms can be funded by commercial enterprise, while people are not so likely to pay for the creations of pure mathematics.

The third part began with a short lecture on how research is funded, principally by the UK Government but also by private companies and charities. All these sources of funding have limited budgets and associated priorities. Particularly, government-funded research is naturally concerned to maximise the benefit (in some sense) of research to society.

The main exercise in this part of the session was to have students play the role of a funding agency. This had received three research proposals in pure mathematics, all judged worth funding, but could only award one grant. The task was to decide, individually or in small groups, which grant should be funded.

The following outline information was given to students about the proposals.

Proposal 1.
Applicant: The applicant is a new lecturer whose previous research experience comprises a PhD only.
Topic: Topological transformation groups. Students were told that this is a pure maths topic in dynamical systems, and that dynamical systems have wide applications.

Proposal 2.
Applicant: The applicant has thirty years of experience and has managed multiple large research grants. Students were told that this applicant had published many papers and established a new research area in pure mathematics during their career.
Topic: Number theory. Students were told this is a vast topic, some aspects of which are relevant to cryptography.

Proposal 3.
Applicant: The applicant has thirty years of experience and has managed multiple large research grants. Students were told that this applicant had published many papers, including one in pure maths which found unexpected applications in neural networks.
Topic: Category theory. Students were told that this is described as the most abstract area of mathematics and looks at properties of mathematical objects.

This is set up so that applicant 1 has the least experience, then the experience levels in proposals 2 and 3 are deliberately equivalent except that applicant 2 has a track record in pure maths, whereas applicant 3 has previously produced an unplanned impact. The intention is to make applicant 3 appear most likely to deliver (even though the nature of the *unplanned* impact makes this far-fetched), followed by applicants 2 and 1, with applicant 1's lack of a track record appearing as a risk. In topics, the situation is intended as reversed, with proposal 1 appearing to be the most likely to lead to applications, followed by proposals 2 and 3.

Students who believe that an unplanned impact could come from anywhere might suggest funding proposal 3, since the researcher is most likely to develop an unplanned impact and the topic is of little relevance to the decision; but as the impact is unplanned, can we really expect applicant 3 to repeat the feat? Pragmatically, though, it is relevant to consider that the topic of proposal 1 appears more likely to develop an application; but then are we guilty of attempting in vain to direct innovation?

Some students prefer to fund proposal 1 because applicant 1 'deserves the opportunity', while the others have established track records. This can lead to a discussion of early-career funding opportunities which are offered to develop researchers as well as research topics.

Of course, this is a task with no correct answer, in which justification of a position is of most interest and importance. However, when students have inevitably asked for a correct answer, I have tried to discuss the way that the people administering the funding might account for what happened if something goes wrong. If applicant 2 or 3 fails to deliver, the funder can hardly be blamed, since both have strong track records. Similarly, if proposal 1 or 2 fails to lead to an impact, this cannot be the fault of the funder because the topics are known to have application areas. Consequently, proposal 2 seems to be a happy medium, and I suggest this seems the most likely to get funded. My experience so far is that not many students have chosen proposal 2.

The session finishes with a short lecture outlining how research is funded in the UK and, in particular, the process of strategic planning undertaken by the Engineering and Physical Sciences Research Council called 'Shaping Capability' (EPSRC, 2013). This has as its goal 'to align [the EPSRC] portfolio to areas of UK strength and national importance'. Some response from mathematicians to this approach is presented, including that by Totaro (2012) and others. If there is time remaining at the end, a discussion of whether this is the right approach for EPSRC to take is potentially fruitful.

5 Discussion

Both times I have run this session, students have engaged positively with the concept and with the activities involved. Although there has been a little reluctance among some to give an impromptu presentation or get involved in a discussion, this reluctance has usually evaporated and most students engaged very positively with the activity, particularly when whole-group discussion has been rehearsed first in small-group discussion. The mixture of activities seems to keep students engaged over an extended time period, though the timing is very dependent on levels of student participation; low levels of discussion may mean the full three hours are not all used. The activity described could be modified for different timescales, or split over more than one session. It could be used in formal teaching in the way that is described here, or it could form the basis of an extra-curricular activity.

There is a danger that mathematics is presented as only valuable because of its utility, and this should be counterbalanced during discussion. Even so, this encourages a pragmatic view of how funding agencies must make decisions, which is useful for students looking to understand the wider societal context of research mathematics and the work of the professional mathematician. My experience suggests that students have very little idea how research is planned and funded, so the activity helps broaden their knowledge. It also exposes them to a little history, and to the idea that mathematics can find applications beyond those its originator intended. Finally, students take away some less common examples of the unplanned impact of mathematics.

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